CHAPTER I

INTRODUCTION

By the concept of innovation waves, first introduced by economist and historian Carlota Perez, the "Sixth Innovation Wave" refers to the current phase of technological development, building on previous waves of innovation (Perez, 1983). The sixth innovation wave represents a new era of technological and social development, with significant opportunities and challenges for individuals, organizations, and society as a whole. The sixth wave is characterized by the convergence of several technologies, including artificial intelligence, big data, the internet of things, robotics, blockchain, and quantum computing (Bangkok Post, 2020). Many of these technologies are associated with mathematical and statistical frameworks. Specifically, mathematics and statistics are critical for:

- Data analysis: These technologies generate vast amounts of data that can be analyzed to extract insights and make predictions. Statistics provides the tools and techniques for analyzing and interpreting this data.
- Machine learning: Machine learning is a subfield of artificial intelligence that involves using statistical algorithms to identify patterns and make predictions. Statistics provides the mathematical framework for these algorithms.
- Experimental design: In order to make accurate predictions and draw conclusions from data, experiments must be designed in a statistically sound manner. This involves determining sample sizes, controlling for confounding variables, and randomizing treatment assignments.
- Quality control: Many of these technologies are used in manufacturing and other industries where quality control is essential. Statistical process control methods are used to monitor and maintain the quality of products and processes.

- Risk management: Many of these technologies introduce new risks and challenges, such as cybersecurity threats, ethical concerns, and potential job displacement. Statistics provides the tools for assessing and managing these risks.

Probability distributions play a crucial role in statistical inference and modeling, particularly in describing the uncertainty associated with random variables. Classical probability distributions are suited for variables assumed to be linear and can take on values along an unbounded straight line. However, many variables in real life are constrained to a circular range and have a periodic nature. For example, angles and times of day are circular variables that can take on values between 0 and 2π or 0 and 24 hours, respectively. While classical probability distributions such as the normal distribution and the exponential distribution are well suited for linear variables, they do not account for the circular nature of these circular variables. In probability theory, a wrapped distribution is a circular version of a classical probability distribution that accounts for the circular nature of the variable being measured (Mardia, 1972; Mardia and Jupp, 2001).

However, traditional statistical methods are often limited by restrictive assumptions and the necessity of large sample sizes to achieve reliable results, which can be challenging when working with wrapped distributions. These limitations hinder their effective application in real-world data analysis. In contrast, Bayesian statistics provides a more flexible and effective approach, particularly in situations where data is scarce or conventional assumptions may not be entirely applicable. Bayesian statistics provides a powerful framework for estimating parameters within wrapped distributions, offering refinement beyond traditional approaches. Unlike conventional methods, which may rely on restrictive assumptions or require substantial sample sizes to achieve accuracy, the Bayesian approach incorporates prior knowledge into the estimation process, enabling more nuanced and robust parameter estimation even with limited data (Ravindran and Ghosh, 2004).

The thesis focuses on analyzing wrapped distributions. Wrapped distributions belong to the subclass of circular distributions, which describe the distribution of circular variables. The use of probability distributions is central to Bayesian analysis as it provides a way to represent the uncertainty associated with the parameters of interest. Probability distributions are used to model the prior information or beliefs about the parameters, as well as the likelihood of the observed data. The choice of prior distribution is crucial in Bayesian analysis as it impacts posterior distribution and resulting inference. The Bayesian analysis of wrapped distributions will be investigated, particularly the conjugate prior distributions associated with them.

1.1 Research Objective

The aim of this thesis is to advance the study of wrapped distributions by exploring an extended Bayesian analysis within this context. The focus is placed on illustrating examples of posterior wrapped distributions, with particular emphasis on their probability density functions (PDFs), cumulative distribution functions (CDFs), and graphical representations.

1.2 Scope and Limitations

This thesis primarily investigates continuous wrapped distributions within the framework of circular data, where the period is defined as 2π. The study aims to enhance the understanding of these distributions through an extended Bayesian analysis to derive posterior wrapped distributions. Key statistical properties, including the mean, variance, skewness, kurtosis, and moments, are examined, with particular emphasis on their PDFs, CDFs, and graphical representations.

In addressing the theoretical development of wrapped distributions, the study primarily focuses on their structural properties and estimation methods rather than empirical applications. Expressions for fundamental statistical measures are systematically derived, highlighting the role of Bayesian inference in improving parameter estimation. By emphasizing theoretical formulations, this work establishes a foundation for further exploration of wrapped distributions. The scope is explicitly confined to circular data with a fixed period of 2π, with non-circular and higher-dimensional extensions beyond the present analysis.

1.3 Research Procedure

The research work will proceed as follows:

1. Study Wrapped Distributions, with an emphasis on continuous forms within the context of circular data with period 2π.
2. Study Bayesian Statistics Theory with a focus on its application to wrapped distributions.
3. Analyze the wrapped form of probability distributions, particularly considering their probability density functions (PDFs), cumulative distribution functions (CDFs), and graphical representations.
4. Apply Bayesian Statistics Theory to derive key statistical measures, including mean, variance, skewness, kurtosis, and higher moments of wrapped distributions.
5. Investigate theoretical applications of the developed theory, focusing on examples that illustrate the methods and concepts.

1.4 Expected Result

The expected result of this research is the development of a deeper understanding of wrapped distributions, particularly through the application of Bayesian Statistics Theory. This includes a comprehensive analysis of continuous wrapped distributions within the context of circular data with period 2π, the derivation of important statistical measures such as mean, variance, skewness, kurtosis, and higher moments, and the clear representation of their probability density functions (PDFs), cumulative distribution functions (CDFs), and graphical depictions. Additionally, the research is expected to provide insights into the role of Bayesian methods in parameter estimation for wrapped distributions, focusing on theoretical development and statistical inference. The study will culminate in illustrative examples that demonstrate the methods and applications of the developed theory, contributing to a more thorough theoretical foundation for wrapped distributions and their potential applications in various contexts.