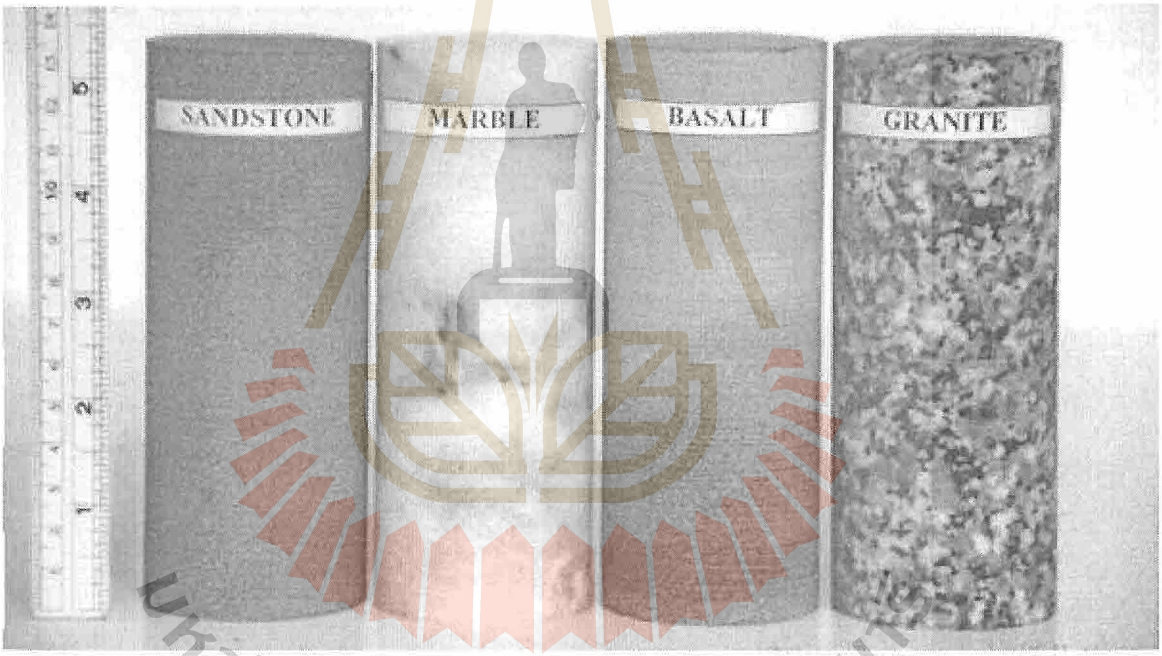
เอกสารประกอบการสอน

434370 ROCK MECHANICS



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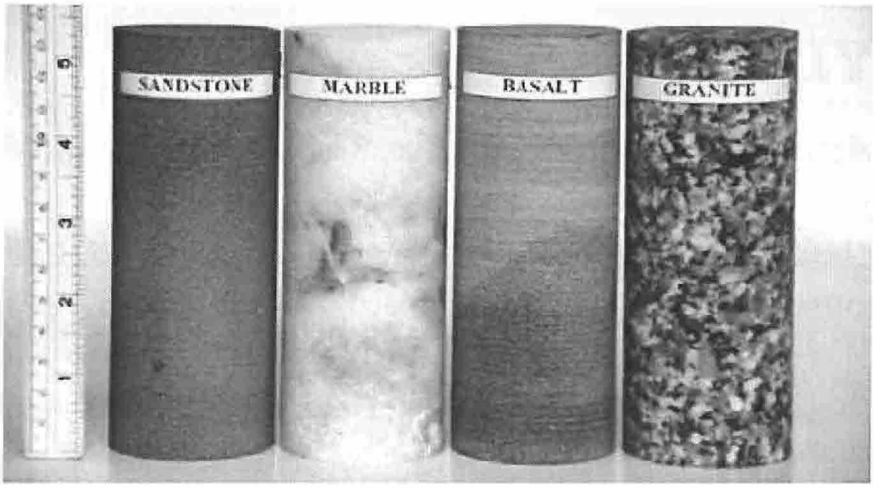

434370 Rock Mechanics
5 credits

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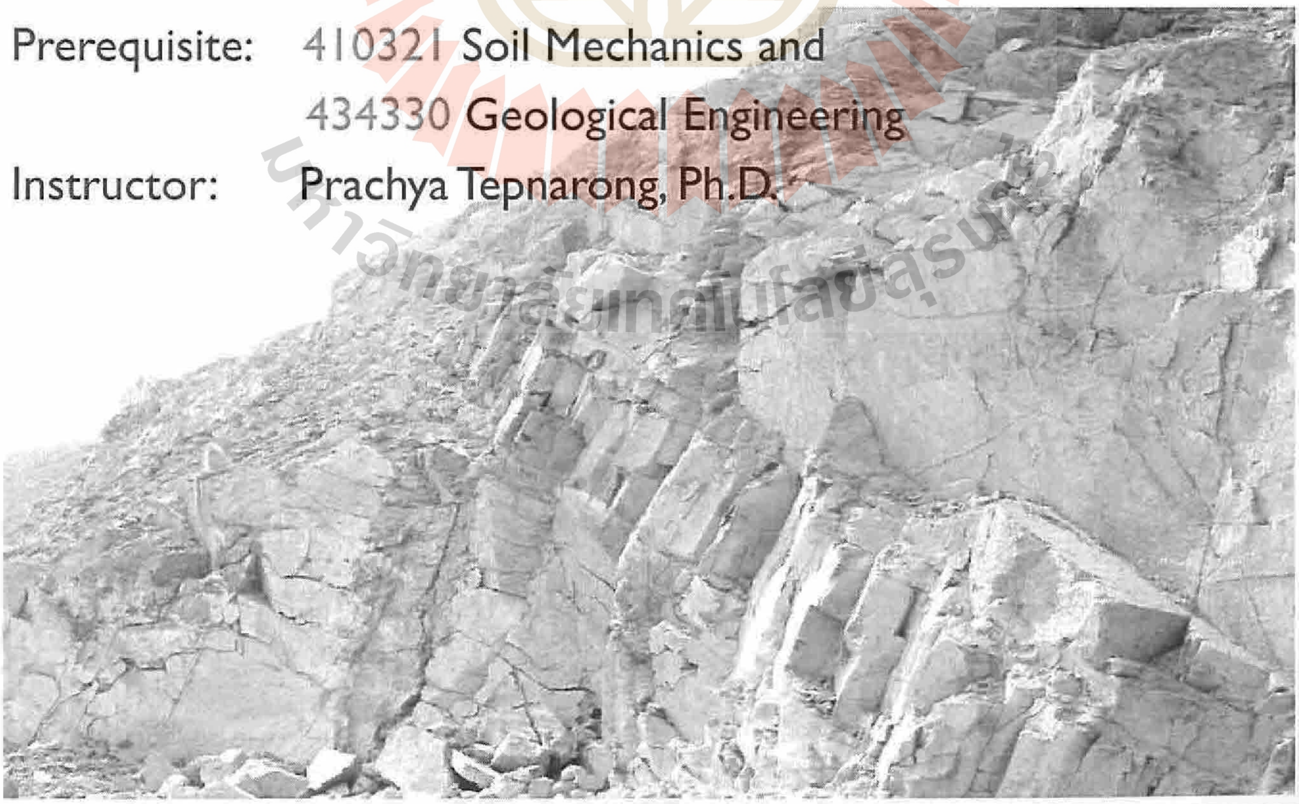
434370 Rock Mechanics

Prerequisite: 410321 Soil Mechanics and
434330 Geological Engineering

Instructor: Prachya Tepnarong, Ph.D.



SYLLABUS:

Topic	1:	Introduction to Rock Mechanics
Topic	2:	Stress Analysis
Topic	3:	Strain Analysis
Topic	4:	Mechanical Rock Properties and Behavior
Topic	5:	Linear Elasticity
Topic	6:	Laboratory Rock Mechanics Testing
	MIDTERM EXAM	
Topic	7:	Rock Mass Properties
Topic	8:	In-situ stress states in rock mass
Topic	9:	Field instrumentation and measurements
Topic	10:	Underground Opening
Topic	11:	Rock Slope Engineering
	FINAL EXAM	

Scoring

- Homework 20%
- Quiz 10%
- Lab Report 20%
- Mid-term Exam 25%
- Final Exam 25%

References:

- Goodman, R.E., 1980, *Introduction to rock mechanics*, John Wiley & Sons, New York, 478 p.
- Brown, E.T., (ed.) 1980, *Rock characterization testing and monitoring: ISRM suggested methods*, the Commission on Rock Testing Methods, International Society for Rock Mechanics, Pergamon Press, New York, 211 p.
- กิตติเทพ เฟื่องขจร, 2546, กลศาสตร์หินพื้นฐาน, สำนักวิชาวิศวกรรมศาสตร์ มหาวิทยาลัยเทคโนโลยีสุรนารี
- ปรัชญา เทพณรงค์, 2551, คู่มือปฏิบัติการกลศาสตร์หิน ROCK MECHANICS LABORATORY, สำนักวิชาวิศวกรรมศาสตร์ มหาวิทยาลัยเทคโนโลยีสุรนารี

Introduction to Rock Mechanics

1. Definition of Rock Mechanics
2. Objective of Rock Mechanics
3. Rock Mechanics Problems
4. Theoretical Basis of Rock Mechanics
5. Difference Between the Engineering Discipline of Strength of Materials and Rock Mechanics

Definition of Rock Mechanics

"**Rock Mechanics** is the theoretical and applied science of the mechanical behavior of rock; it is that branch of mechanics concerned with the response of rock to the force fields of its physical environment."

Defied by : Committee on Rock Mechanics of the Geological Society of America (Judd,1964)

Definition of Rock Mechanics

"*กลศาสตร์หิน*" เป็นการศึกษาเกี่ยวกับพฤติกรรมและคุณสมบัติของหินเชิงกลทั้งทางด้านทฤษฎี ปฏิบัติ และการนำไปประยุกต์ใช้

จาก : กลศาสตร์หินพื้นฐาน (รศ.ดร.กิตติเทพ เฟื่องขจร, 2546)

เนื้อหาที่เกี่ยวข้อง

- การศึกษาการตอบสนองของหินต่อแรงที่เข้ามากระทำภายใต้สภาวะแวดล้อมต่าง ๆ การวิเคราะห์แรงที่กระทำกับหิน
- การวิเคราะห์ผลกระทบภายในเนื้อหินในรูปของความเค้น ความเครียด และพลังงานที่เก็บอยู่
- การวิเคราะห์ผลที่ได้จากแรงที่มากระทำ ซึ่งอาจจะอยู่ในรูปของการเปลี่ยนรูป (deformation) การแตก/วิบัติ (Failure) และการเคลื่อนไหลของหิน (Creep)

สรุปได้ว่า กลศาสตร์หินคือแขนงหนึ่งของวิชากลศาสตร์วัสดุที่ได้ถูกพัฒนาขึ้นเพื่อให้เหมาะสมกับคุณสมบัติทางกายภาพของหิน

Background

การศึกษาหินในเชิงกลศาสตร์

อาศัยทฤษฎีและหลักการที่ได้พัฒนามาจากกลศาสตร์ดิน และกลศาสตร์วัสดุ

ข้อแตกต่าง

กลศาสตร์ดินและวัสดุอื่น ๆ วัสดุมีความต่อเนื่อง (Continuous medium)

กลศาสตร์หิน หินจะมีความไม่ต่อเนื่อง (Discontinuity)

โดยสรุป ทฤษฎี กฎ และหลักการที่นำมาใช้ในกลศาสตร์หินก็จะซ้ำซ้อนกับของกลศาสตร์ดินพอสมควร

Introduction to Rock Mechanics

- หินส่วนใหญ่ในธรรมชาติจะมีความไม่ต่อเนื่อง (Discontinuity) และมีความไม่เป็นเนื้อเดียวกัน (Non-homogeneous)
- ในการที่จะพิจารณาว่าหินนั้นมีความเป็นเนื้อเดียวกันและปราศจากรอยแตก หรือมีความไม่ต่อเนื่องจะขึ้นกับองค์ประกอบหลายอย่าง เช่น ขนาดของหินที่กำลังพิจารณา และจุดประสงค์ของการวิเคราะห์หรือการประยุกต์ใช้
- ในบางครั้งจะต้องสมมติให้หินนั้นมีความต่อเนื่อง เพื่อที่จะนำทฤษฎีทางด้านกลศาสตร์เข้ามาอธิบายถึงพฤติกรรมของโครงสร้างในหินนั้น
- แต่ในบางครั้งก็สามารถวิเคราะห์พฤติกรรมของหินที่ไม่มีความต่อเนื่องทางด้านกลศาสตร์ได้เช่นกัน

Introduction to Rock Mechanics

- ความไม่ต่อเนื่องหรือความไม่เป็นเนื้อเดียวกันของหินในธรรมชาติเกิดขึ้นจาก
  - รอยแตก (fracture)
  - รอยแยก (joint)
  - รอยเลื่อน (fault)
  - รอยชั้นหิน (Bedding)
  - รอยพับ (Folding)
  - โพรง (Cavern)
  - สิ่งเจือปน (inclusion)
  - และอื่น

Introduction to Rock Mechanics

- ความไม่ต่อเนื่องของหินหรือมวลหินนี้อาจจะมีความเป็นระเบียบอยู่บ้าง เช่น รอยแยกของมวลหินส่วนใหญ่จะมีลักษณะเป็นแนวเส้นที่ขนานกันในมวลหินนั้น รอยชั้นหินของมวลหินก็จะมีลักษณะที่ขนานกัน
- ความมีระเบียบอยู่บ้างเช่นนี้จึงสามารถนำทฤษฎีหรือกฎเกณฑ์ทางคณิตศาสตร์เข้ามาช่วยอธิบายพฤติกรรมทางด้านกลศาสตร์ของมวลหินนั้น และในการพัฒนาสูตรต่าง ๆ ที่เกี่ยวข้องอาจจะอาศัยประสบการณ์หรือผลที่ได้จากห้องทดลองเข้ามาช่วย

Introduction to Rock Mechanics

ROCK ENGINEERING

"*วิศวกรรมหิน*" จะหมายถึง วิชาที่กล่าวถึงการนำเอาความรู้ทางด้านกลศาสตร์หินมาประยุกต์ใช้ในงานวิศวกรรม เช่น การออกแบบอุโมงค์ในชั้นหิน การออกแบบฐานรากของเขื่อนใหญ่ อาคาร และสะพานบนชั้นหิน การออกแบบความลาดเอียงของชั้นหิน การขุดเจาะเพื่อผลิตน้ำบาดาล และปิโตรเลียม การทำเหมือง และการนำหินมาใช้เป็นวัสดุก่อสร้าง เป็นต้น

Introduction to Rock Mechanics

พื้นฐานความรู้ที่จำเป็นก่อนจะศึกษาวิชากลศาสตร์หิน

- ทางด้านธรณีวิทยา และทางด้านวิศวกรรม อาทิ ธรณีวิทยาโครงสร้าง อุทกธรณีวิทยา ธรณีวิทยาภาคสนาม ธรณีฟิสิกส์ แร่วิทยา กลศาสตร์วัสดุ กลศาสตร์ดิน เป็นต้น
- ทฤษฎีทางด้านแคลคูลัสชั้นสูงก็เป็นสิ่งจำเป็นที่จะทำให้เข้าใจที่มาของหลักสูตรต่าง ๆ ของกลศาสตร์หิน

Objective of Rock Mechanics

จุดประสงค์หลักของวิชากลศาสตร์หิน คือ

1) เพื่อศึกษาทฤษฎีทางด้านกลศาสตร์ที่สามารถนำมาอธิบายพฤติกรรมทางด้านกลศาสตร์ของหินและมวลหิน
2) เพื่อศึกษาคุณสมบัติทางด้านกลศาสตร์ของหินภายใต้สภาวะแวดล้อมต่าง ๆ ของแรงกด แรงดึง อุณหภูมิ และความดันน้ำ เป็นต้น
3) เพื่อนำความรู้มาประยุกต์ใช้ในกิจกรรมที่เกี่ยวข้องกับวิศวกรรมหิน
4) เพื่อทำการทดสอบในห้องปฏิบัติการ และตรวจวัดในภาคสนามเพื่อคาดคะเนพฤติกรรมและคุณสมบัติของมวลหินที่มีผลกระทบต่อโครงสร้างทางด้านวิศวกรรม

Objective of Rock Mechanics

ถ้าจะจำแนกจุดประสงค์เฉพาะของวิชากลศาสตร์หิน

- เพื่อสำรวจคุณสมบัติทางด้านวิศวกรรมของมวลหิน
- เพื่อการจำแนกหินและมวลหินในเชิงวิศวกรรม
- เพื่อจะพัฒนาเครื่องมือในการทดสอบคุณสมบัติของหิน
- เพื่อพัฒนาวิธีการทดสอบความแข็งของหินที่ได้มาตรฐาน
- เพื่อศึกษาพฤติกรรมของหินภายใต้อุณหภูมิสูง
- เพื่อศึกษาการเคลื่อนตัวของคลื่นในมวลหิน
- เพื่อเข้าใจถึงกลไกการแตกของหิน

Objective of Rock Mechanics

ถ้าจะจำแนกจุดประสงค์เฉพาะของวิชากลศาสตร์หิน (ต่อ)

- เพื่อคาดคะเนการเปลี่ยนรูปของอุโมงค์ในชั้นหินตามกาลเวลา
- เพื่อพัฒนาวิธีการตรวจวัดในภาคสนาม
- เพื่อเข้าใจถึงกลไกการเคลื่อนตัวของรอยเลื่อนในหิน
- เพื่อเข้าใจถึงการเลื่อนตัวของเปลือกโลก
- เพื่อเข้าใจการกระจายของความเค้นในชั้นหิน
- เพื่อประเมินความสามารถในการรับน้ำหนักของมวลหิน
- และอื่น ๆ

Rock Mechanics Problems

ปัญหาทางด้านกลศาสตร์หิน

ปัญหาทางด้านกลศาสตร์หินในเชิงวิศวกรรมจะมีอยู่หลายประเด็น เช่น การก่อสร้างฐานรากบนหิน การก่อสร้างอุโมงค์ในชั้นหิน การทำเหมือง การนำหินมาใช้เป็นสิ่งก่อสร้างเป็นต้น

Rock Mechanics Problems

ตัวอย่างของปัญหาทางด้านกลศาสตร์หิน

- มวลหินสามารถรับน้ำหนักได้มากเพียงใด ?
- หินจะประพฤติตัวอย่างไรภายใต้แรงกดที่ผันแปรขึ้นลงไปตามกาลเวลา ?
- ความต้านแรงเฉือนสูงสุดของหินมีค่าเท่าใด ?
- สัมประสิทธิ์ของความยืดหยุ่นของหินมีค่าเท่าใด ?
- รอยแตกและรอยเลื่อนในหินจะนำมาพิจารณาในการออกแบบได้อย่างไร ?
- ผลกระทบของความไม่ต่อเนื่องต่อความแข็งของหินมีมากน้อยเพียงใด ?
- กลไกการแตกของหินเป็นอย่างไร ?
- กฎอะไรที่สามารถอธิบายการไหลของหินได้ ?
- ผลที่ได้จากการทดลองในห้องปฏิบัติการจะแทนคุณสมบัติของหินในภาคสนามได้หรือไม่ ?

Rock Mechanics Problems

ตัวอย่างของปัญหาทางด้านกลศาสตร์หิน (ต่อ)

- ความเค้นที่เกิดจากน้ำหนักของชั้นหินภายใต้ความลึกมีค่าเท่าใด ?
- เราจะคาดคะเนแผ่นดินไหวได้อย่างไร ?
- แผ่นดินไหวมีผลกระทบต่อฐานรากของสิ่งก่อสร้างในหินอย่างไร ?
- การทดสอบในห้องปฏิบัติการวิธีใดที่จะให้ผลลัพธ์ใกล้เคียงกับพฤติกรรมของหินในภาคสนาม ?
- หินในอุโมงค์ควรจะมีการค้ำยันอย่างไร ?
- น้ำไหลอย่างไรในมวลหินที่มีความไม่ต่อเนื่องสูง ?
- หัวเจาะหินควรจะถูกออกแบบอย่างไรเพื่อให้มีประสิทธิภาพการเจาะสูงสุด ?
- หินมีการเปลี่ยนรูปหรือการไหลอย่างไรที่ความลึกมาก ๆ ?
- และปัญหาอื่น ๆ

Theoretical Basis of Rock Mechanics

ทฤษฎีพื้นฐานของกลศาสตร์หิน

- ทฤษฎี กฎเกณฑ์ และแนวคิดบางส่วนที่นำมาประยุกต์ใช้ในกลศาสตร์หินส่วนหนึ่งได้มาจากทฤษฎีของกลศาสตร์ดิน ทฤษฎีเหล่านี้บางส่วนได้พัฒนามาจากทฤษฎีทางด้านวิชาฟิสิกส์
- ทฤษฎีและกฎเกณฑ์อีกส่วนหนึ่งจะถูกพัฒนาขึ้นสำหรับกลศาสตร์หินโดยเฉพาะ ซึ่งได้พัฒนามาจากผลการทดสอบในห้องปฏิบัติการ และผลการตรวจวัดในภาคสนาม ดังนั้น กฎเกณฑ์บางอย่างที่ใช้ในกลศาสตร์หินอาจจะไม่มีทฤษฎีทางด้านฟิสิกส์เข้ามายืนยัน

Theoretical Basis of Rock Mechanics

ทฤษฎีพื้นฐานของกลศาสตร์หิน (ต่อ)

- ทฤษฎีและกฎเกณฑ์ต่าง ๆ ไม่ว่าจะถูกพัฒนาขึ้นมาด้วยวิธีใด สิ่งสำคัญซึ่งเป็นเอกลักษณ์อันหนึ่งของวิชากลศาสตร์หินและวิศวกรรมหิน คือ ทฤษฎีและกฎเกณฑ์เหล่านี้จะมีข้อจำกัดสูง และจะไม่สอดคล้องอย่างสมบูรณ์กับสภาพจริงของมวลหิน

Theoretical Basis of Rock Mechanics

ทฤษฎีพื้นฐานของกลศาสตร์หิน (ต่อ)

- สรุป การนำมาประยุกต์ใช้จะต้องอาศัยและชี้แจงข้อสมมติฐานที่เหมาะสม ซึ่งส่วนหนึ่งก็จะได้มาจากประสบการณ์ของผู้ปฏิบัติ
- นักกลศาสตร์หิน (Rock Mechanist) จะต้องใช้วิจารณญาณที่ดีเพื่อเพิ่มประสิทธิภาพและประสิทธิผลของงานนั้น ๆ
- นักกลศาสตร์หินควรจะตระหนักด้วยว่าประสบการณ์อย่างเดียวนั้นจะไม่เพียงพอ ควรจะต้องมีความรู้ที่ลึกซึ้งทางด้านทฤษฎี เพื่อใช้เป็นหลักการและแนวทางทางด้านวิชาการในการแก้ปัญหาที่ดำเนินการอยู่

Difference Between the Engineering Discipline of Strength of Materials and Rock Mechanics

ความแตกต่างระหว่างกลศาสตร์หินและกลศาสตร์แขนงอื่น

- กลศาสตร์หินจะแตกต่างกับกลศาสตร์วัสดุและกลศาสตร์ดิน ทั้งในเชิงทฤษฎีและปฏิบัติ ข้อแตกต่างนี้เกิดขึ้นเนื่องจากคุณสมบัติตามธรรมชาติของหิน กายภาพและการเกิดของหินจะแตกต่างกับของวัสดุทางวิศวกรรมอื่น ๆ (เช่น เสาคอนกรีต ท่อเหล็ก แท่งเหล็ก เป็นต้น) ข้อแตกต่างนี้สามารถจำแนกได้เป็น 6 ประการ คือ

Difference Between the Engineering Discipline of Strength of Materials and Rock Mechanics

ความแตกต่างระหว่างกลศาสตร์หินและกลศาสตร์แขนงอื่น

1. ความเป็นเนื้อเดียวกัน (Homogeneity)
2. คุณสมบัติที่เหมือนกันทุกทิศทาง (Isotropy)
3. ความต่อเนื่อง (Continuity)
4. ความสม่ำเสมอ (Uniformity)
5. ความสามารถในการคาดคะเน (Predictability)
6. ความเค้นแรกเริ่ม (Initial stress)

Homogeneity

- <u>ความเป็นเนื้อเดียวกัน</u> (Homogeneity) วัสดุทางวิศวกรรมจะถูกสร้างขึ้น เพื่อให้มีความเป็นเนื้อเดียวกันสูง หินหรือมวลหินถูกธรรมชาติสร้างขึ้น และจะมีการผันแปรของคุณสมบัติจากจุดหนึ่งไปอีกจุดหนึ่ง ความไม่เป็นเนื้อเดียวกันของหินนี้ (Inhomogeneity) จะปรากฏให้เห็นทั้งทางด้านกายภาพ กลศาสตร์ เคมี และอื่น ๆ ดังนั้น จึงเป็นการยากในการวิเคราะห์และออกแบบสิ่งก่อสร้างทางด้านวิศวกรรมธรณีที่มีผลกระทบจากคุณสมบัติของหินเข้ามาเกี่ยวข้อง

Isotropy

- <u>คุณสมบัติที่เหมือนกันทุกทิศทาง</u> (Isotropy) วัสดุทางวิศวกรรมส่วนใหญ่จะถูกสร้างขึ้นให้มีคุณสมบัติเหมือนกันในทุกทิศทางเพื่อให้ง่ายต่อการวิเคราะห์ ออกแบบ และก่อสร้าง มวลหินจะมีรอยของชั้นหินหรือรอยแตกอื่น ๆ เข้ามาแทรก ซึ่งทำให้คุณสมบัติทางด้านกลศาสตร์ในทิศทางที่ตั้งฉากกับรอยแตกหรือรอยชั้นหินจะต่างกับคุณสมบัติที่อยู่ในทิศทางที่ขนานกับรอยแตกหรือรอยชั้นหินนั้น (Anisotropy) ดังนั้น การศึกษาคุณสมบัติทางด้านกลศาสตร์ที่ต่างกันในแต่ละทิศทางของมวลหินจึงเป็นสิ่งจำเป็น

Continuity

- <u>ความต่อเนื่อง</u> (Continuity) วัสดุทางวิศวกรรมจะมีความต่อเนื่องสูง ส่วนใหญ่จะถูกสร้างขึ้นไม่ให้มีรอยแตกหรือรอยร้าว ในขณะเดียวกันความไม่ต่อเนื่องที่เกิดจากรอยแตกหรือรอยร้าวในหินหรือมวลหินเป็นสิ่งที่พบอยู่เสมอ (Discontinuity) ดังนั้น การวิเคราะห์หรือการออกแบบที่เกี่ยวข้องกับกลศาสตร์หินจึงจำเป็นต้องนำความไม่ต่อเนื่องของเนื้อหินนี้เข้ามาพิจารณา

Uniformity

- <u>ความสม่ำเสมอ</u> (Uniformity) คุณสมบัติทางด้านกลศาสตร์ของวัสดุทางวิศวกรรมที่ถูกประดิษฐ์หรือสร้างขึ้นจากวิธีเดียวกันจะมีคุณสมบัติเหมือนกัน มีความสม่ำเสมอของคุณสมบัติทางด้านกลศาสตร์ ซึ่งทำให้ง่ายต่อการวิเคราะห์และออกแบบ และง่ายต่อการนำไปใช้ประโยชน์
- แต่ไม่มีหินที่ไหนในโลกที่จะมีคุณสมบัติทางด้านกลศาสตร์เหมือนกันอย่างสมบูรณ์ อย่างมากที่สุดก็อาจจะมีความคล้ายคลึงกัน หรือมีค่าความแข็งหรือความอ่อนตัวที่ใกล้เคียงกัน เป็นต้น ดังนั้นในแต่ละพื้นที่ของมวลหินจึงจำเป็นต้องมีการสำรวจ วิเคราะห์ และออกแบบโดยเฉพาะ (Site-specific design) การนำเอาคุณสมบัติของมวลหินจากพื้นที่หนึ่งไปประยุกต์ใช้ในอีกพื้นที่หนึ่งอาจจะเป็นการไม่เหมาะสม

Predictability

- <u>ความสามารถในการคาดคะเน</u> (Predictability) โดยทั่วไปวิศวกรสามารถคาดคะเนคุณสมบัติทางด้านกลศาสตร์ของวัสดุทางวิศวกรรมได้ดี เนื่องจากรู้องค์ประกอบและขบวนการการผลิตวัสดุนั้น ๆ ยกตัวอย่างเช่น แท่งเหล็กที่ผลิตในประเทศสหรัฐอเมริกาก็จะมีคุณสมบัติทางด้านกลศาสตร์เหมือนกับแท่งเหล็กที่ผลิตในประเทศไทย ถ้าวัตถุดิบที่ใช้และขบวนการผลิตในสองประเทศเหมือนกัน แต่วิศวกรไม่อาจคาดคะเนคุณสมบัติทางด้านกลศาสตร์ของมวลหินได้ดีเท่า เช่น หินใกล้ผิวโลกที่ถูกสำรวจและรู้คุณสมบัติอาจมีคุณสมบัติต่างกับหินที่อยู่ลึกลงไปอย่างสิ้นเชิง ถึงแม้หินนี้จะเป็นหินชนิดเดียวกัน ทั้งนี้เนื่องจากขบวนการการเกิดของหินตามธรรมชาติเป็นขบวนการที่ซับซ้อนและไม่มีระเบียบ

Initial stress

- <u>ความเค้นแรกเริ่ม</u> (Initial stress) วัสดุทางวิศวกรรมที่ได้ถูกหล่อ หลอม หรือผสมขึ้น เมื่อแรกเริ่มจะอยู่ภายใต้สภาวะที่ไม่มีความเค้นเข้ามาเกี่ยวข้อง คือ ส่วนใหญ่จะมิได้ถูกประดิษฐ์ขึ้นภายใต้แรงกดหรือแรงดึงสูง
- ในทางตรงกันข้ามหินเกิดขึ้นภายใต้สภาวะของแรงกดและอุณหภูมิสูงมาก คือเกิดขึ้นที่ความลึกหลายสิบกิโลเมตรใต้ผิวโลก เมื่อแรกเริ่มนั้นหินจะอยู่ภายใต้ความเค้นของแรงกดที่เกิดจากน้ำหนักกดทับในความลึกนั้น ๆ ต่อมาเมื่อมีการยกตัว หรือสึกกร่อนของผิวโลกข้างบน หินเหล่านี้ก็จะปรากฏให้เห็นใกล้ผิวโลก ในสภาวะนี้ความเค้นที่อยู่ในเนื้อหินก็จะถูกระบายออกไป ซึ่งในบางครั้งอาจเห็นได้ในรูปของแนวการแตกของหินนั้น ในเชิงกลศาสตร์การวิเคราะห์ความเค้นในหินจึงแตกต่างกับการวิเคราะห์ความเค้นในวัสดุทางวิศวกรรมอื่น ๆ

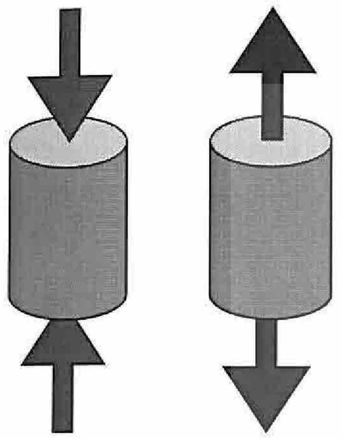
Summary

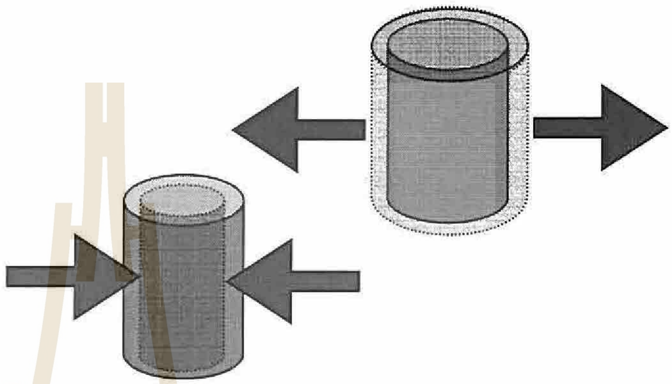
Engineering Material	Rock
Homogeneity	Inhomogeneity
Isotropy	Anisotropy
Continuity	Discontinuity
Uniformity	Non-uniformity
Predictability	Non-predictability
Non-initial stress	Initial stress

Topic 1: Stress Analysis (AN OVERVIEW)

For Rock Mechanics (= General Mechanics)

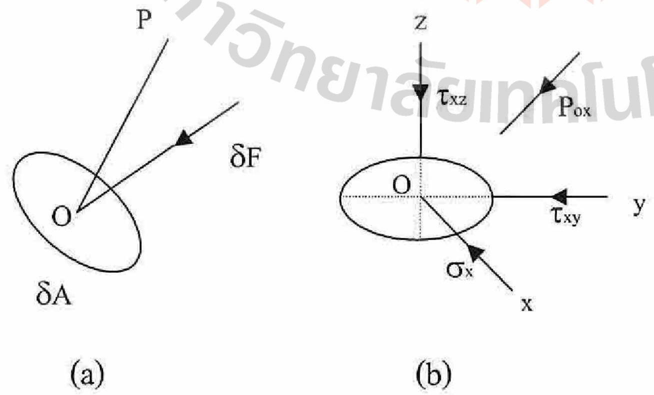
FORCE:	**DEFORMATION:**
Compression Force (+)	Contraction (+)
Tension Force (-)	Extension (-)





Stress at Point

$$P_{OP} = \lim_{\delta A \to O} \frac{\delta F}{\delta A}$$

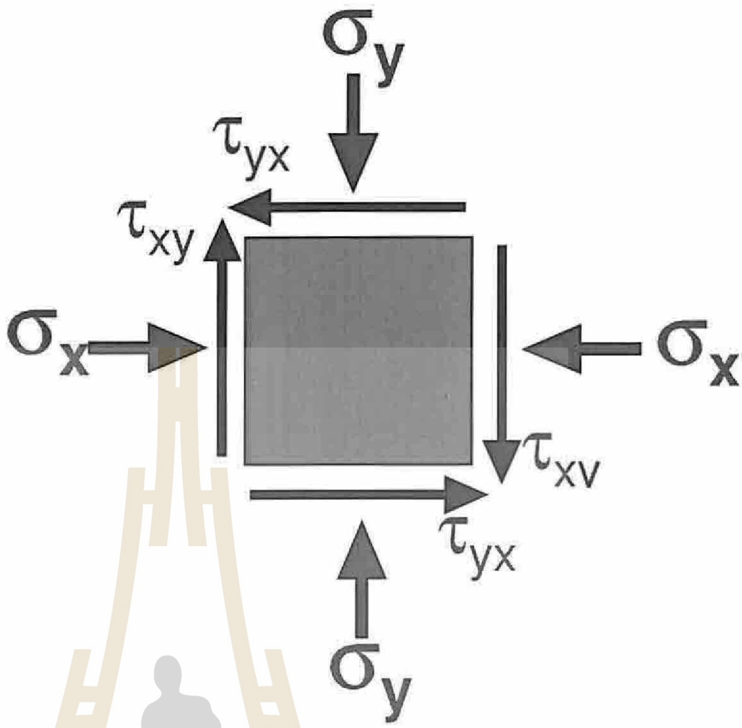


$$\begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix}$$

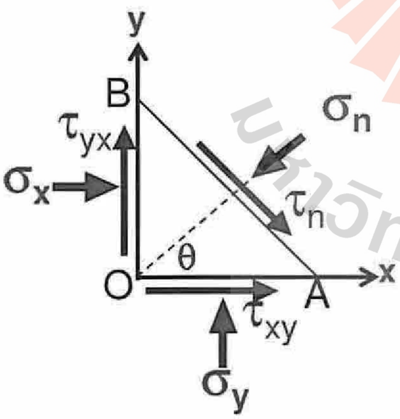
Rigid Body Analysis

in Equilibrium

- Translation
- Rotation



Stress Transformation

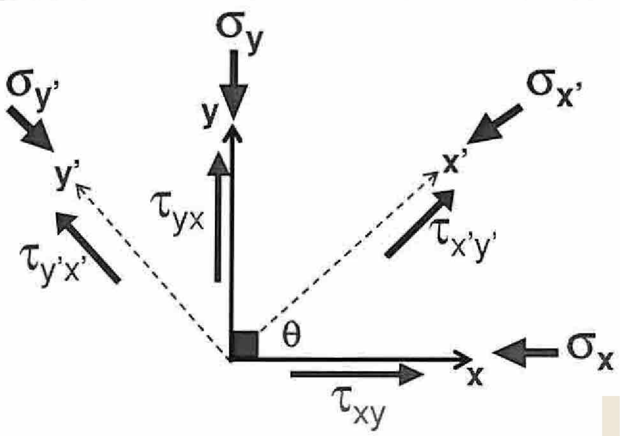


$$\sigma_n = \sigma_x \cos^2\theta + 2\tau_{yx} \sin\theta \cos\theta + \sigma_y \sin^2\theta$$

$$\tau_n = (\sigma_y - \sigma_x) \sin\theta \cos\theta + \tau_{xy} (\cos^2\theta - \sin^2\theta)$$

$$= \tfrac{1}{2} (\sigma_y - \sigma_x) \sin 2\theta + \tau_{xy} \cos 2\theta$$

Stress Transformation



$$\sigma_{x'} = \sigma_x \cos^2\theta + 2\tau_{xy} \sin\theta \cos\theta + \sigma_y \sin^2\theta$$
$$\sigma_{y'} = \sigma_x \sin^2\theta - 2\tau_{xy} \sin\theta \cos\theta + \sigma_y \cos^2\theta)$$
$$\tau_{x'y'} = \tfrac{1}{2}(\sigma_y - \sigma_x) \sin 2\theta + \tau_{xy} \cos 2\theta$$

Compatibility ($\Sigma\sigma$ = constant at every position)

$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$$

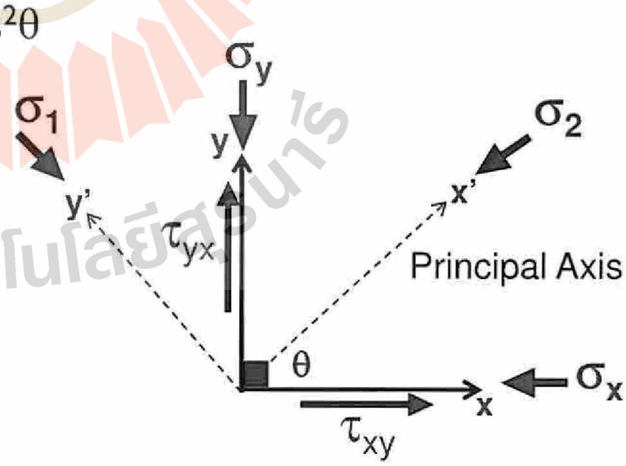
Principal Stress

from

$$\tau_{x'y'} = \tfrac{1}{2}(\sigma_y - \sigma_x) \sin^2\theta + \tau_{xy} \cos^2\theta$$

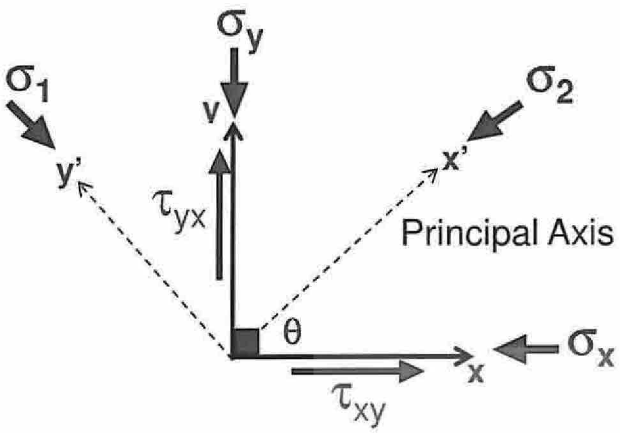
when shear stress, $\tau = 0$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$



Major Principal Stress (σ_1)
Minor Principal Stress (σ_2 for 2D, σ_3 for 3D)

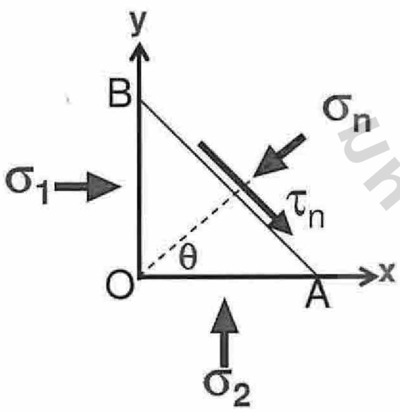
Principal Stress



$$\sigma_1 = \tfrac{1}{2}(\sigma_x + \sigma_y) + [\tau_{xy}^2 + \tfrac{1}{4}(\sigma_x - \sigma_y)^2]^{1/2}$$

$$\sigma_2 = \tfrac{1}{2}(\sigma_x + \sigma_y) - [\tau_{xy}^2 + \tfrac{1}{4}(\sigma_x - \sigma_y)^2]^{1/2}$$

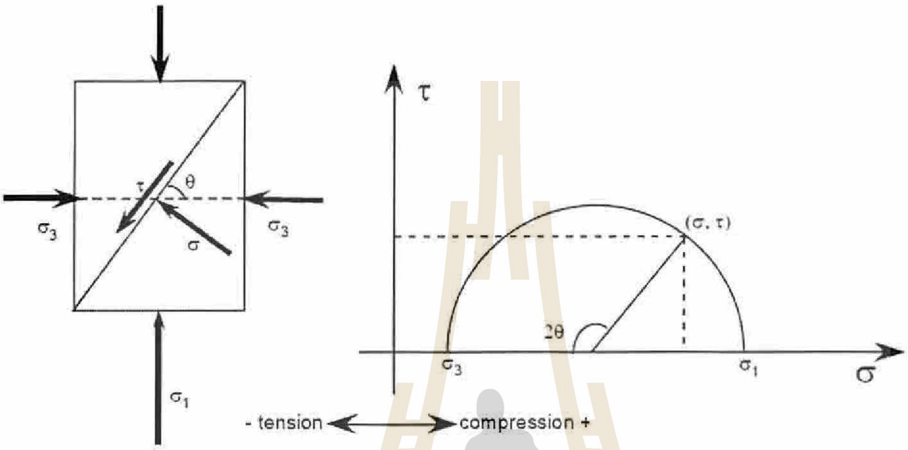
Principal Stress



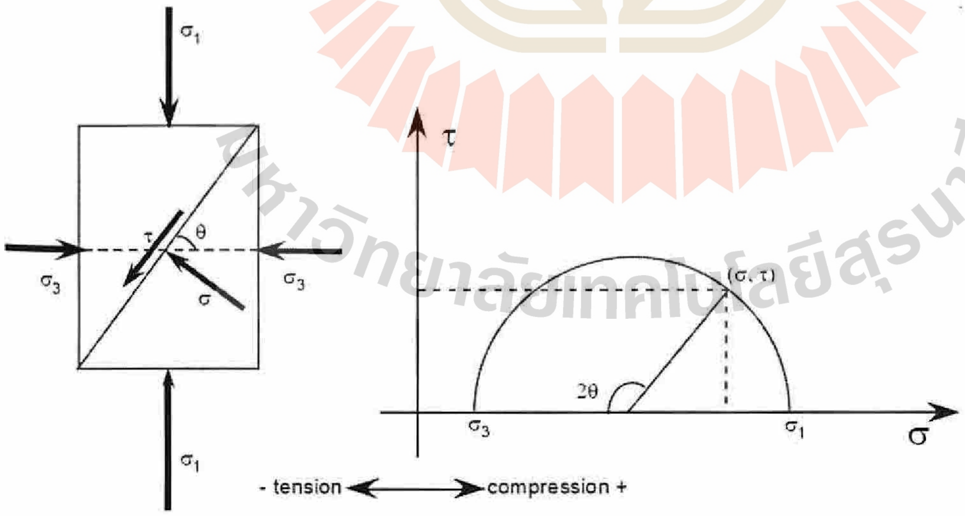
$$\sigma_n = \sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta$$
$$= \tfrac{1}{2}(\sigma_1 + \sigma_2) + \tfrac{1}{2}(\sigma_1 - \sigma_2)\cos 2\theta$$
$$\tau_n = -\tfrac{1}{2}(\sigma_1 - \sigma_2)\sin 2\theta$$

Mohr's Circle

Mathematically, it can be shown that the normal stress σ and the shear stress τ on any plane that has an angle of θ from the minimum principle stress σ_3 direction related to the maximum and minimum stress in the following equations. These relationships can also be expressed graphically by the Mohr's Circle:



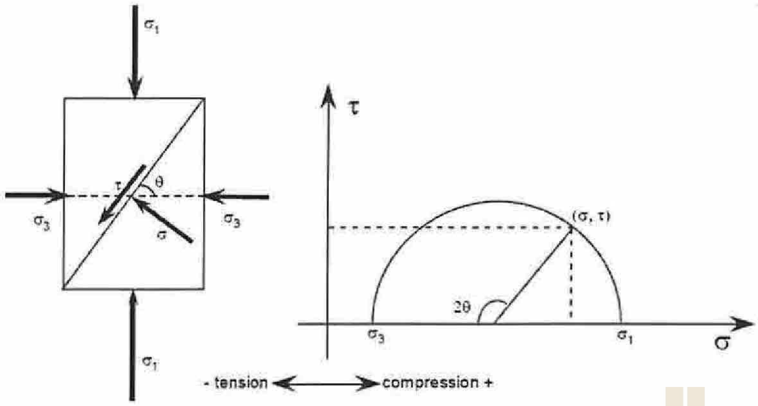
Mohr's Circle



$$\sigma = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos 2\theta$$

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta$$

Mohr's Circle

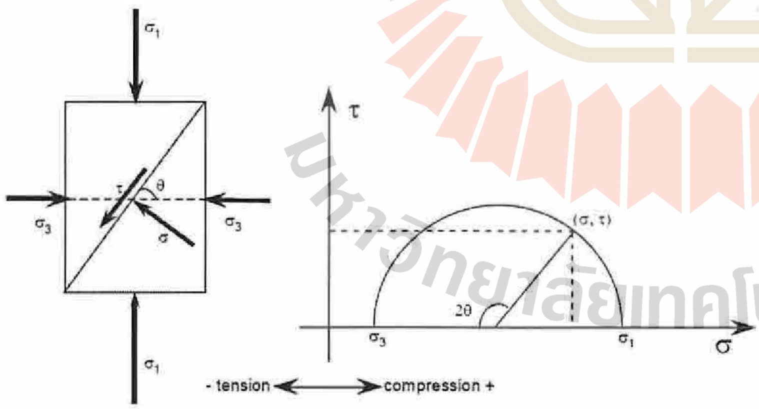


When θ=0°, (the plane is parallel to σ_3) we have

$$\sigma = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3) = \sigma_1$$

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta = 0$$

Mohr's Circle

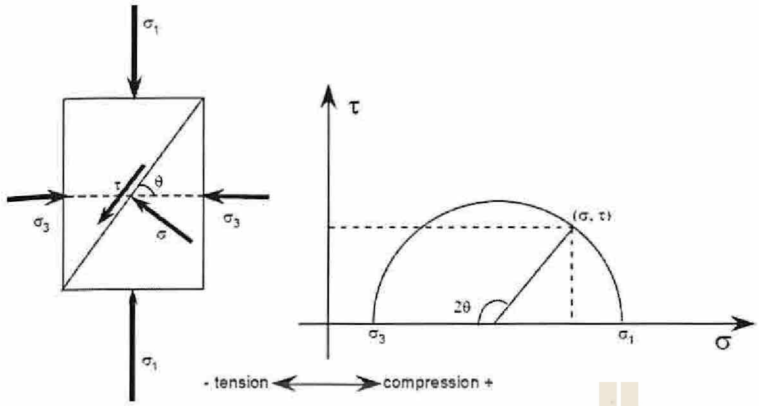


When θ=90° (the plane is parallel to σ_1), we have

$$\sigma = \frac{1}{2}(\sigma_1 + \sigma_3) - \frac{1}{2}(\sigma_1 - \sigma_3) = \sigma_3$$

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_3)\sin 2\theta = 0$$

Mohr's Circle

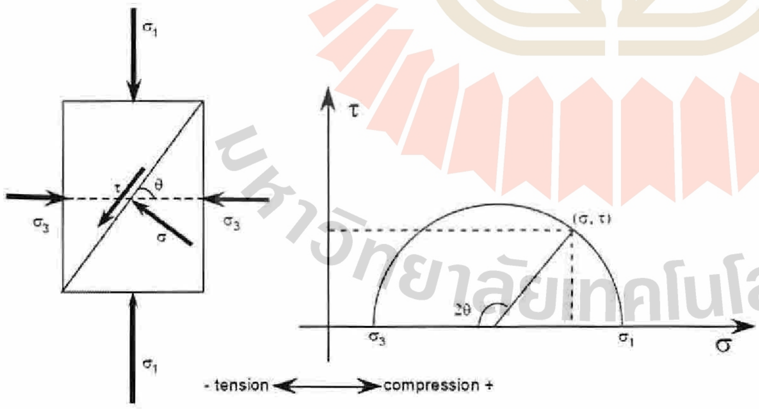


When θ=45°, we have

$$\sigma = \frac{1}{2}(\sigma_1 + \sigma_3) = \frac{1}{2}(\sigma_1 + \sigma_3) = \sigma_{ave}$$

$$\tau = \frac{1}{2}(\sigma_1 - \sigma_3)\sin(\frac{\pi}{2}) = \frac{1}{2}(\sigma_1 - \sigma_3) = \tau_{max}$$

Mohr's Circle



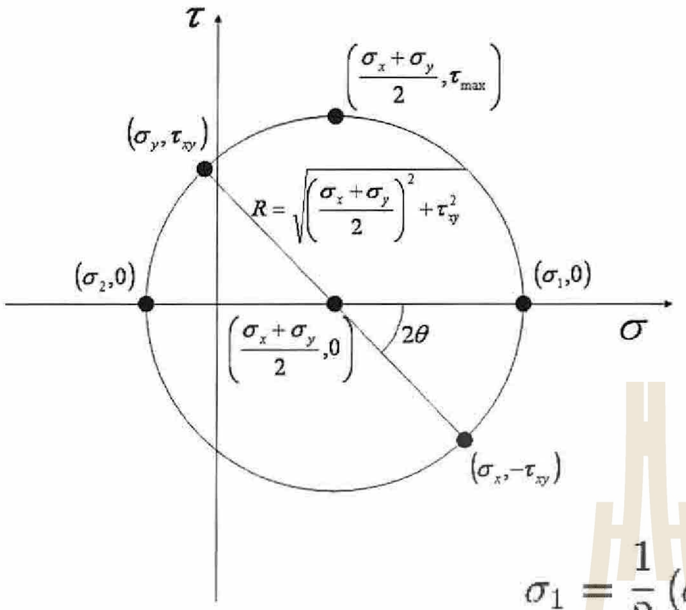
center $(\sigma_0, 0)$ with:

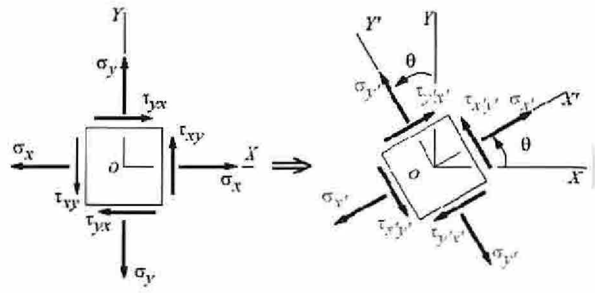
$$\sigma_0 = \frac{1}{2}(\sigma_1 + \sigma_3)$$

the radius r

$$r = \frac{1}{2}(\sigma_1 - \sigma_3)$$

Mohr's Circle

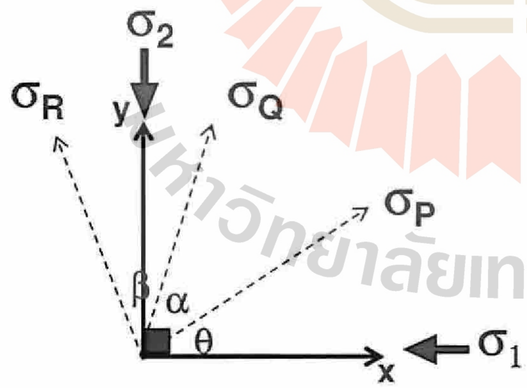




Stresses at given coordinate system Stresses transformed to another coordina

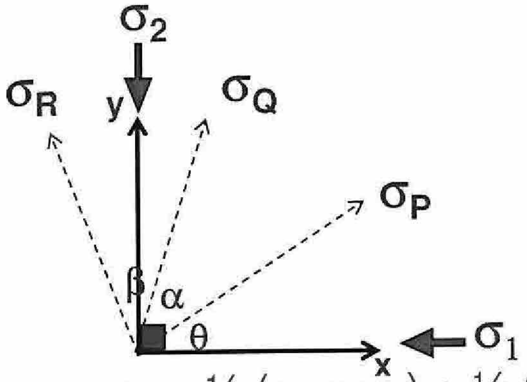
$$\sigma_1 = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$
$$\sigma_2 = \frac{1}{2}(\sigma_x + \sigma_y) - \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

In-situ Stress Measurements



$$\sigma_P = \tfrac{1}{2}(\sigma_1 + \sigma_2) + \tfrac{1}{2}(\sigma_1 - \sigma_2)\cos 2\theta$$
$$\sigma_Q = \tfrac{1}{2}(\sigma_1 + \sigma_2) + \tfrac{1}{2}(\sigma_1 - \sigma_2)\cos 2(\theta + \alpha)$$
$$\sigma_R = \tfrac{1}{2}(\sigma_1 + \sigma_2) + \tfrac{1}{2}(\sigma_1 - \sigma_2)\cos 2(\theta + \alpha + \beta)$$

In-situ Stress Measurements

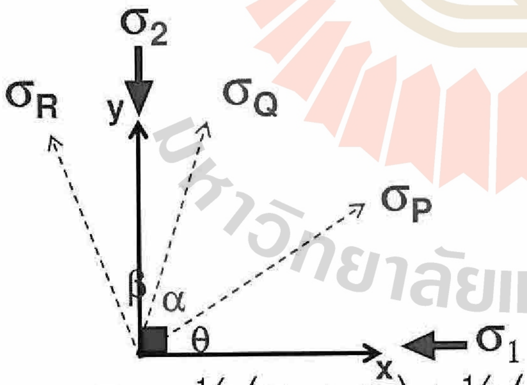


$$\sigma_P = \tfrac{1}{2}(\sigma_1 + \sigma_2) + \tfrac{1}{2}(\sigma_1 - \sigma_2)\cos 2\theta$$
$$\sigma_Q = \tfrac{1}{2}(\sigma_1 + \sigma_2) + \tfrac{1}{2}(\sigma_1 - \sigma_2)\cos 2(\theta + \alpha)$$
$$\sigma_R = \tfrac{1}{2}(\sigma_1 + \sigma_2) + \tfrac{1}{2}(\sigma_1 - \sigma_2)\cos 2(\theta + \alpha + \beta)$$

in case $\alpha = \beta = 45°$

$$\sigma_1 + \sigma_2 = \sigma_P + \sigma_R$$
$$\sigma_1 - \sigma_2 = \{(\sigma_P - 2\sigma_Q + \sigma_R)^2 + (\sigma_P - \sigma_R)^2\}^{1/2}$$
$$\tan 2\theta = (\sigma_P - 2\sigma_Q + \sigma_R)/(\sigma_P - \sigma_R)$$

In-situ Stress Measurements



$$\sigma_P = \tfrac{1}{2}(\sigma_1 + \sigma_2) + \tfrac{1}{2}(\sigma_1 - \sigma_2)\cos 2\theta$$
$$\sigma_Q = \tfrac{1}{2}(\sigma_1 + \sigma_2) + \tfrac{1}{2}(\sigma_1 - \sigma_2)\cos 2(\theta + \alpha)$$
$$\sigma_R = \tfrac{1}{2}(\sigma_1 + \sigma_2) + \tfrac{1}{2}(\sigma_1 - \sigma_2)\cos 2(\theta + \alpha + \beta)$$

in case $\alpha = \beta = 60°$

$$\sigma_1 + \sigma_2 = (2/3)(\sigma_P + \sigma_Q + \sigma_R)$$
$$(\sigma_1 - \sigma_2)^2 = (4/3)(\sigma_Q - \sigma_R)^2 + (4/9)(2\sigma_P - \sigma_R - \sigma_Q)^2$$
$$\tan 2\theta = 3^{1/2}(\sigma_Q - \sigma_R)/(\sigma_Q + \sigma_R - 2\sigma_P)$$

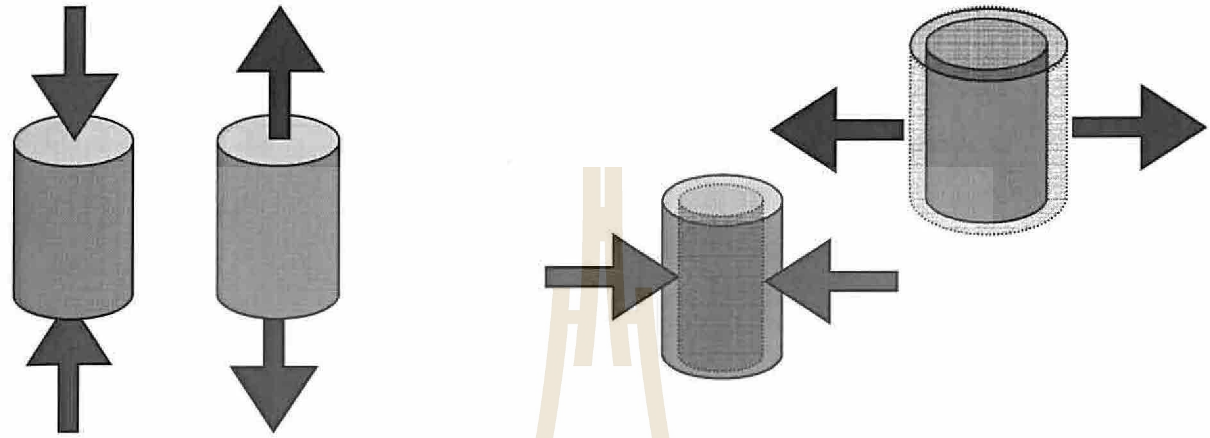
Greek Letter

Greek Letter		Name	Equivalent	Sound When Spoken
Α	α	Alpha	A	al-fah
Β	β	Beta	B	bay-tah
Γ	γ	Gamma	G	gam-ah
Δ	δ	Delta	D	del-tah
Ε	ε	Epsilon	E	ep-si-lon
Ζ	ζ	Zeta	Z	zay-tah
Η	η	Eta	E	ay-tay
Θ	θ	Theta	Th	thay-tah
Ι	ι	Iota	I	eye-o-tah
Κ	κ	Kappa	K	cap-ah
Λ	λ	Lambda	L	lamb-dah
Μ	μ	Mu	M	mew
Ν	ν	Nu	N	new
Ξ	ξ	Xi	X	zzEye
Ο	ο	Omicron	O	om-ah-cron
Π	π	Pi	P	pie
Ρ	ρ	Rho	R	row
Σ	σ	Sigma	S	sig-ma
Τ	τ	Tau	T	tawh
Υ	υ	Upsilon	U	oop-si-lon
Φ	φ	Phi	Ph	figh or fie
Χ	χ	Chi	Ch	kigh
Ψ	ψ	Psi	Ps	sigh
Ω	ω	Omega	O	o-may-gah

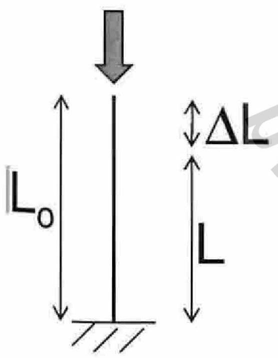
Topic 3: Strain Analysis (AN OVERVIEW)

For Rock Mechanics (≠ General Mechanics)

FORCE:	**DEFORMATION:**
Compression Force (+)	Contraction (+)
Tension Force (-)	Extension (-)



Strain of a Line Element



$(\Delta L = L_0 - L)$

Definition

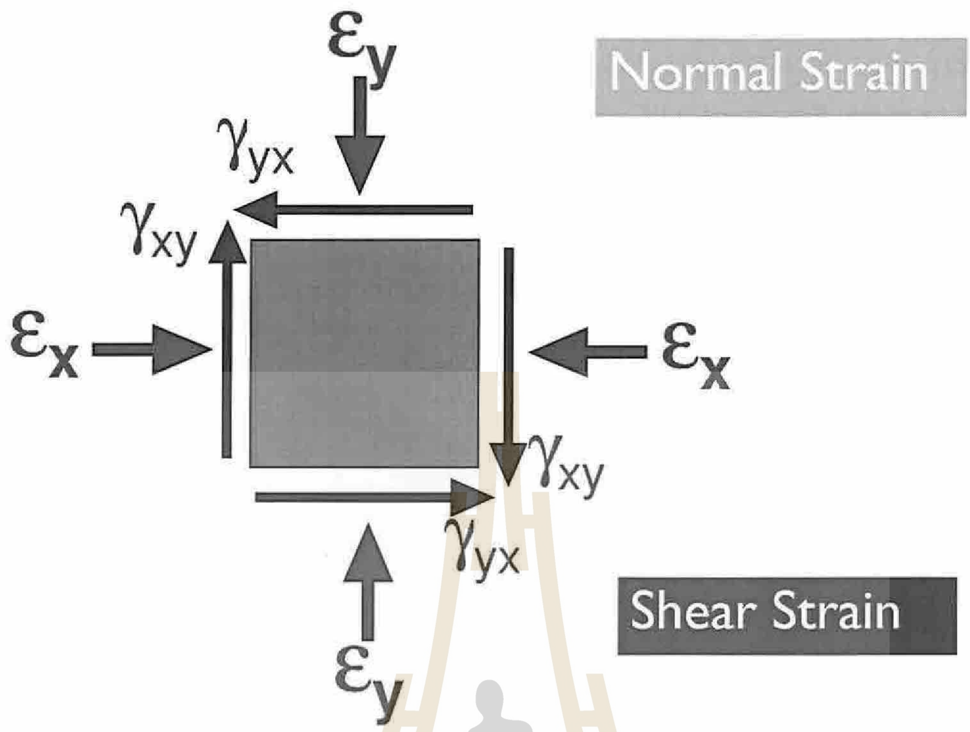
Engineering Strain : $\varepsilon_{eng} = \Delta L / L_0$

True Strain : $\varepsilon_{true} = \ln(L / L_0)$

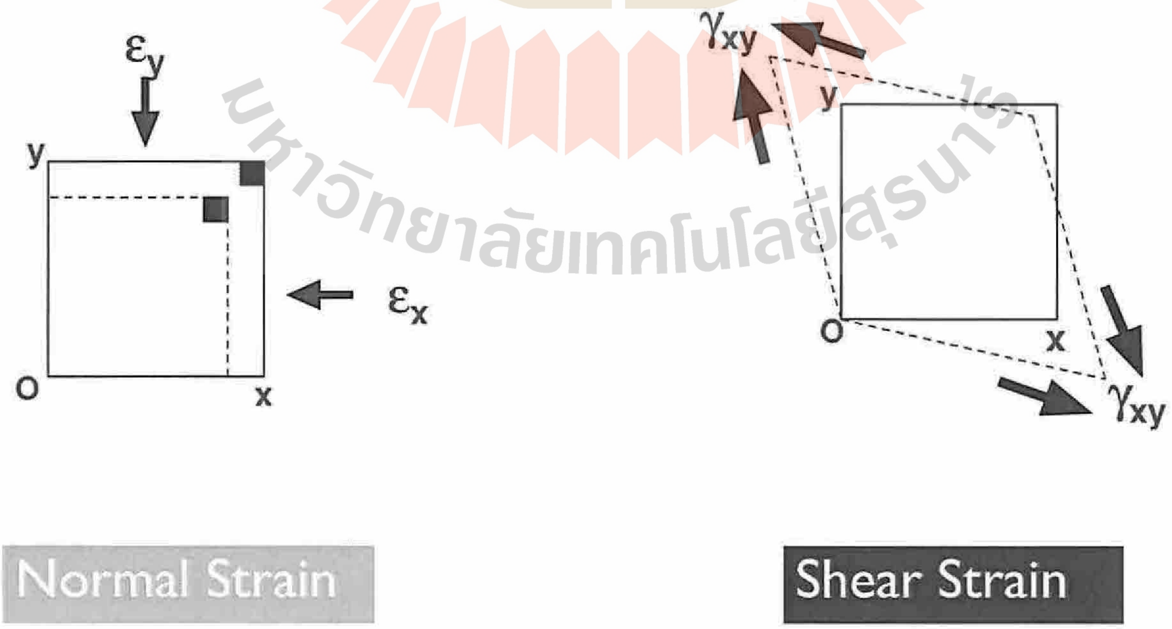
Relationship

$$\boxed{\varepsilon_{eng} = \ln(1 + \varepsilon_{true})}$$

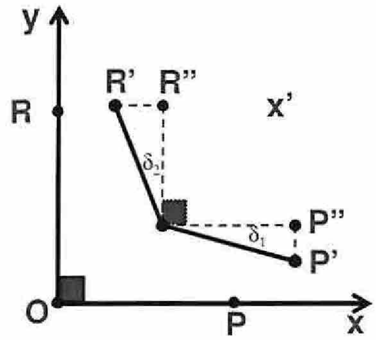
Strain Components in 2-D



Strain Components in 2-D



Strain in 2-D



Consider : ΔOPR -> ΔO'P'R'

Normal Strain

$$\varepsilon_x = (O'P''-OP)/OP = (du/dx)$$
$$\varepsilon_y = (O'R''-OR)/OR = (dv/dx)$$

u = displacement in x-direction

v = displacement in y-direction

Shear Strain

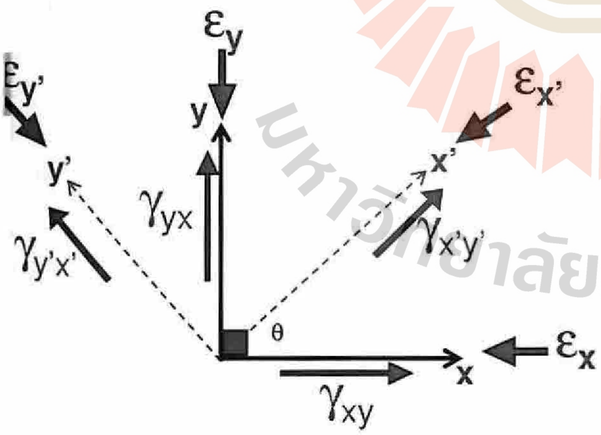
$$\boxed{\Gamma_{xy} = \tfrac{1}{2}\gamma_{xy}}$$

$$\Gamma_{xy} = \Gamma_{yx} = \frac{1}{2}\gamma_{xy} = \frac{1}{2}\gamma_{yx} = \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)$$

$$\gamma_{xy} = \delta_1 + \delta_2 = (du/dy + dv/dx)$$

$$\delta_1 = (du/dy)$$
$$\delta_2 = (dv/dx)$$

Strain Transformation



Set of strain equations as same as stress equation

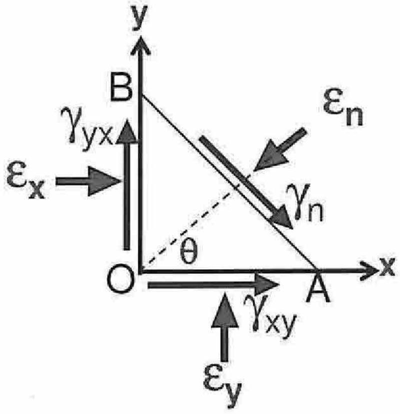
Replace symbols:

σ by ε

τ by Γ or ½ γ

$$\varepsilon_{x'} = \varepsilon_x\cos^2\theta + 2\Gamma_{xy}\sin\theta\cos\theta + \varepsilon_y\sin^2\theta$$
$$\varepsilon_{y'} = \varepsilon_x\sin^2\theta - 2\Gamma_{xy}\sin\theta\cos\theta + \varepsilon_y\cos^2\theta$$
$$\Gamma_{x'y'} = \tfrac{1}{2}(\varepsilon_y - \varepsilon_x)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

Strain Transformation



$$\varepsilon_n = \varepsilon_x \cos^2\theta + 2\Gamma_{xy} \sin\theta \cos\theta + \varepsilon_y \sin^2\theta$$

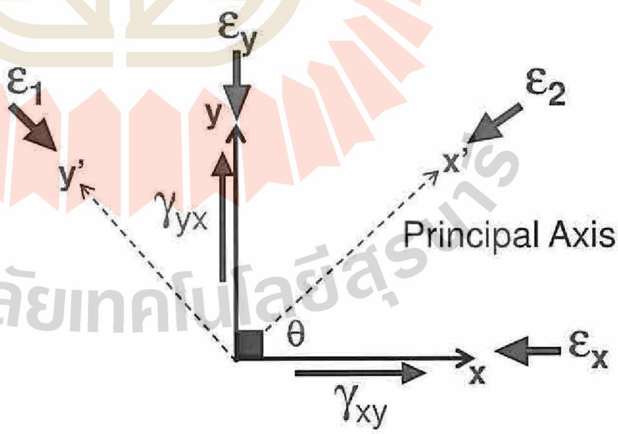
$$\Gamma_n = (\varepsilon_y - \varepsilon_x) \sin\theta \cos\theta + \Gamma_{xy} (\cos^2\theta - \sin^2\theta)$$

$$= \tfrac{1}{2} (\varepsilon_y - \varepsilon_x) \sin {}^{2}\theta + \Gamma_{xy} \cos 2\theta$$

Principal Strain

when shear strain, $\Gamma_{xy} = 0$

$$\tan 2\theta = \frac{2\Gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$



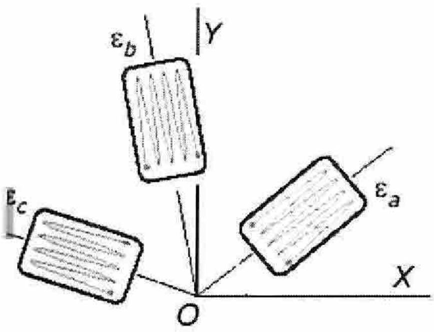
$$\varepsilon_1 = \tfrac{1}{2} (\varepsilon_x + \varepsilon_y) + [\Gamma_{xy}^2 + \tfrac{1}{4}(\varepsilon_x - \varepsilon_y)^2]^{1/2}$$

$$\varepsilon_2 = \tfrac{1}{2} (\varepsilon_x + \varepsilon_y) - [\Gamma_{xy}^2 + \tfrac{1}{4}(\varepsilon_x - \varepsilon_y)^2]^{1/2}$$

Volumetric strain, Δ (change in volume / original volume)

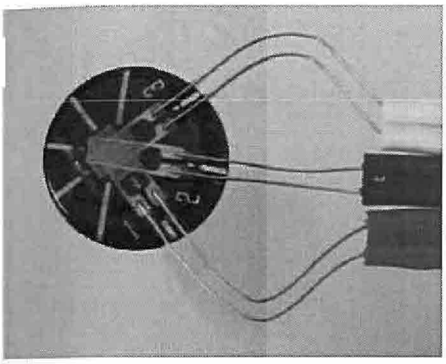
$$\Delta = \varepsilon_1 + \varepsilon_2 = \varepsilon_x + \varepsilon_y$$

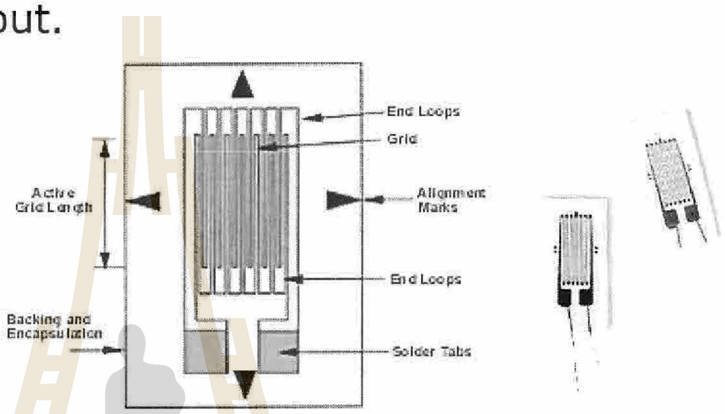
Strain Gage Rosettes



Strain Rosette for Strain Measurement

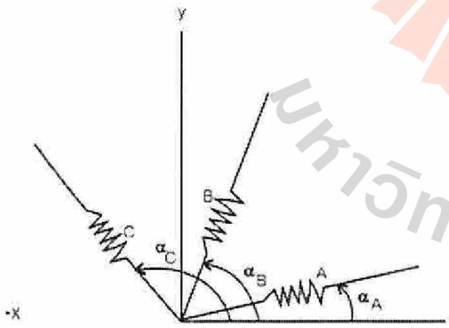
A wire strain gage can effectively measure strain in only one direction. To determine the three independent components of plane strain, three linearly independent strain measures are needed, i.e., three strain gages positioned in a rosette-like layout.





Strain Gage Rosettes

General Form



Consider a strain rosette attached on the surface with an angle a from the *x*-axis. The rosette itself contains three strain gages with the internal angles, as illustrated on the right. Suppose that the strain measured from these three strain gages are ε_A, ε_B, and ε_C, respectively.

Applying the coordinate transformation equation to each of the three strain gages results in the following system of equations

$$\begin{Bmatrix} \varepsilon_A \\ \varepsilon_B \\ \varepsilon_C \end{Bmatrix} = \begin{pmatrix} \cos^2\alpha_A & \sin^2\alpha_A & \frac{1}{2}\sin 2\alpha_A \\ \cos^2\alpha_B & \sin^2\alpha_B & \frac{1}{2}\sin 2\alpha_B \\ \cos^2\alpha_C & \sin^2\alpha_C & \frac{1}{2}\sin 2\alpha_C \end{pmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

Strain Gage Rosettes

General Form

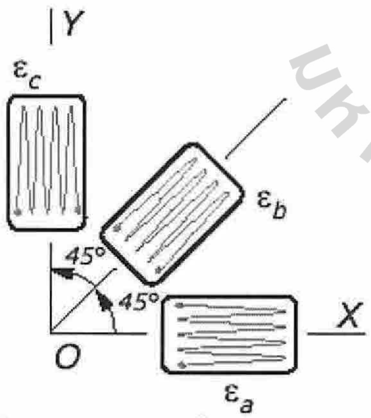
$$\begin{Bmatrix} \varepsilon_A \\ \varepsilon_B \\ \varepsilon_C \end{Bmatrix} = \begin{pmatrix} \cos^2\alpha_A & \sin^2\alpha_A & \frac{1}{2}\sin 2\alpha_A \\ \cos^2\alpha_B & \sin^2\alpha_B & \frac{1}{2}\sin 2\alpha_B \\ \cos^2\alpha_C & \sin^2\alpha_C & \frac{1}{2}\sin 2\alpha_C \end{pmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\varepsilon_A = \cos^2\alpha_A\,\varepsilon_x + \sin^2\alpha_A\,\varepsilon_y + \tfrac{1}{2}\sin 2\alpha_A$$
$$\varepsilon_B = \cos^2\alpha_B\,\varepsilon_x + \sin^2\alpha_B\,\varepsilon_y + \tfrac{1}{2}\sin 2\alpha_B$$
$$\varepsilon_C = \cos^2\alpha_C\,\varepsilon_x + \sin^2\alpha_C\,\varepsilon_y + \tfrac{1}{2}\sin 2\alpha_C$$

Strain Gage Rosettes

Special Cases of Strain Rosette Layouts

Case 1: 45° strain rosette aligned with the *x-y* axes, i.e., $\alpha_A = 0^0$, $\alpha_B = 45^0$ and $\alpha_C = 90^0$

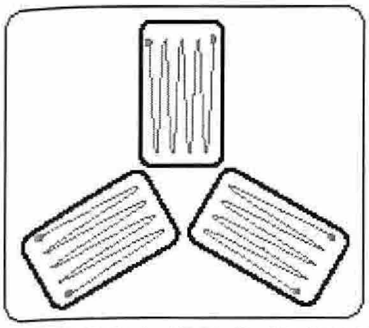


Rectangular Rosette

$$\begin{Bmatrix} \varepsilon_A \\ \varepsilon_B \\ \varepsilon_C \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0.5 & 0.5 & 0.5 \\ 0 & 1 & 0 \end{pmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -1 \end{bmatrix} \begin{Bmatrix} \varepsilon_A \\ \varepsilon_B \\ \varepsilon_C \end{Bmatrix}$$

Strain Gage Rosettes



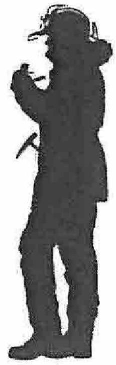
Planar 0°-60°-120° Delta Rosette

Delta Rosette

Case 2: 60º strain rosette, the middle of which is aligned with the *y*-axis, i.e., $\alpha_A = 0^0$, $\alpha_B = 60^0$ and $\alpha_C = 120^0$.

$$\begin{Bmatrix} \varepsilon_A \\ \varepsilon_B \\ \varepsilon_C \end{Bmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0.25 & 0.75 & 0.433 \\ 0.25 & 0.75 & -0.433 \end{pmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/3 & 2/3 & 2/3 \\ 0 & 1.1547 & -1.1547 \end{bmatrix} \begin{Bmatrix} \varepsilon_A \\ \varepsilon_B \\ \varepsilon_C \end{Bmatrix}$$



Topic #4 Friction of rock joints

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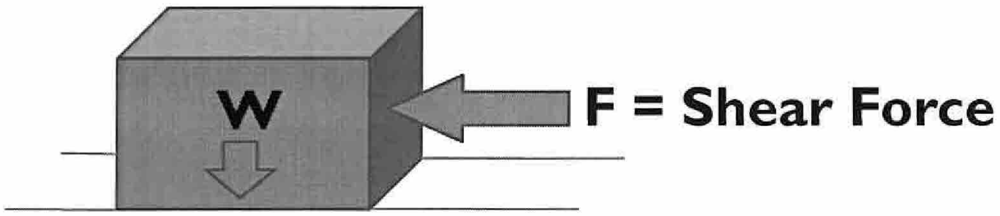
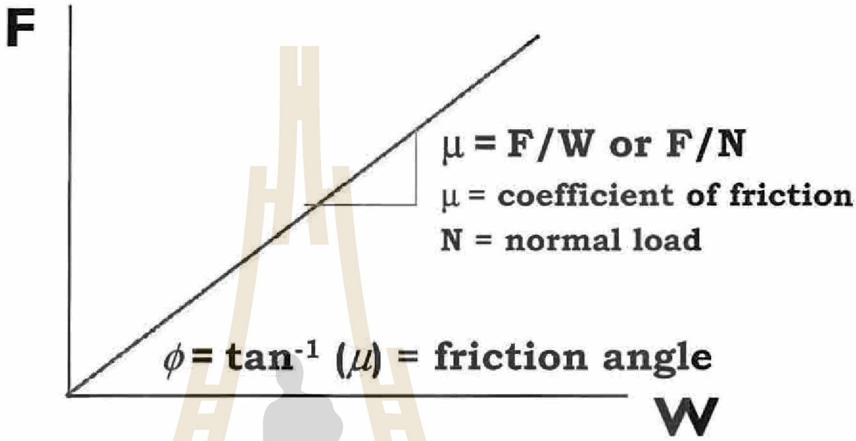
Friction in rock joints

- Microscopic Scale
  - in which friction is postulated between opposing surface of crack
- Large Scale
  - In which it occurs between individual grains of pieces of aggregate
- Geologic Scale
  - In friction on joint or fault surface in which the areas in question

Amonton's law (Boden & tobor, 1950)



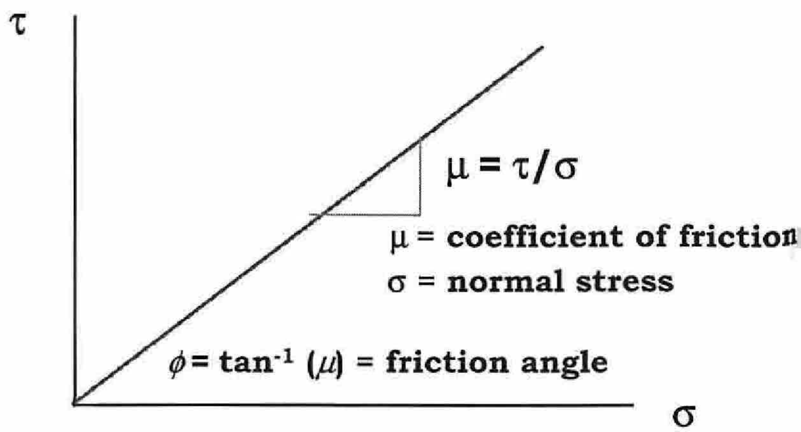


Amonton's law (Boden & tobor, 1950)

$F = \mu W$ or μN Divided by A (area) into both side

$F/A = \mu N/A$

$\tau = \mu\sigma = \sigma \tan \phi$

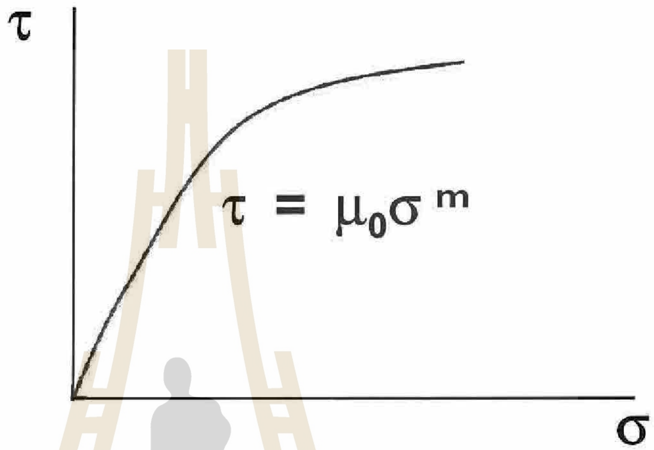


Power law (murrell, 1965)

$F = \mu_0 N^m$ ⟸ Divided by A (area) into both side

$F/A = \mu_0 N^m/A$ m = constant (2/3 - 1)

$$\boxed{\tau = \mu_0 \sigma^m}$$

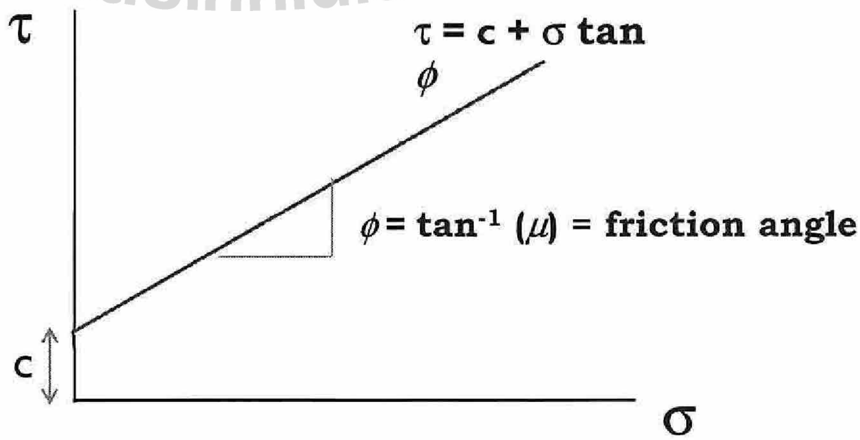


Coulomb's law (jaeger, 1959)

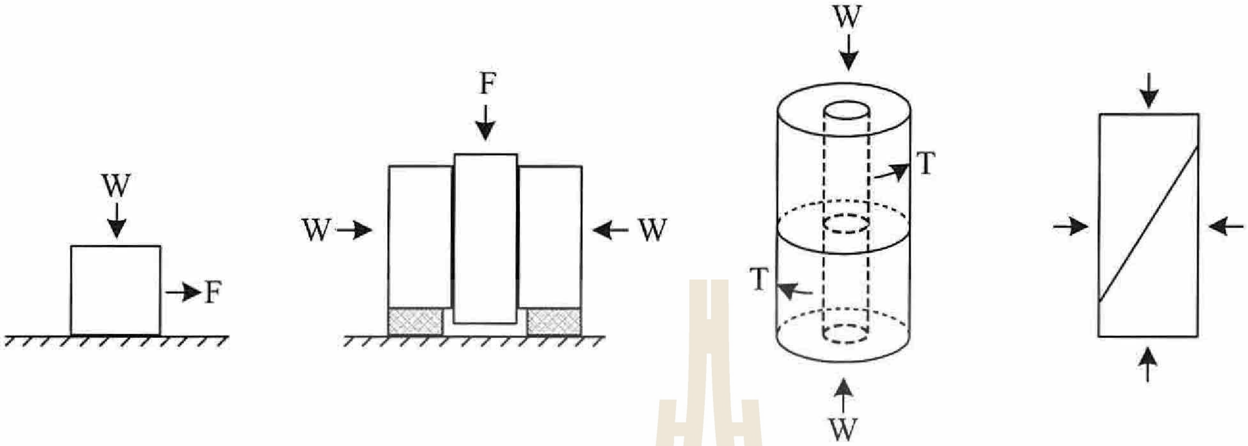
Linear Law

$$\boxed{\tau = c + \mu\sigma}$$

ϕ = friction angle
c = cohesion



Laboratory Testing for Friction of Rock Surface

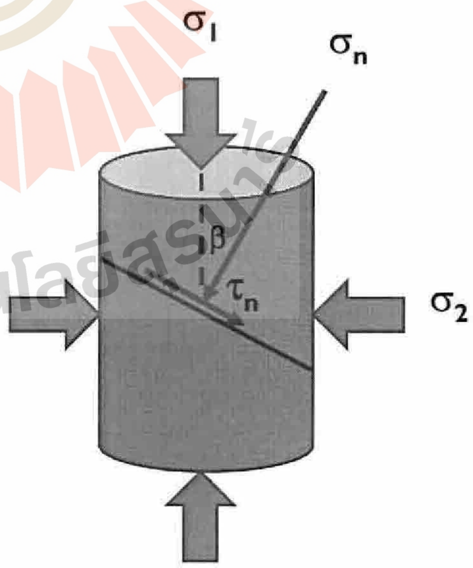


(a) (b) (c) (d)

Coulomb Criterion (jaeger, 1959)

$$|\tau| = c + \mu\sigma$$

τ= shear stress
σ = normal stress
ϕ= friction angle
c = cohesion



$$\sigma = \tfrac{1}{2}(\sigma_1 + \sigma_2) + \tfrac{1}{2}(\sigma_1 - \sigma_2)\cos 2\beta$$
$$\tau = -\tfrac{1}{2}(\sigma_1 - \sigma_2)\sin 2\beta$$

or

$$\sigma = \sigma_m + \tau_m \cos 2\beta$$
$$\tau = -\tau_m \sin 2\beta$$

$$\sigma_m = \tfrac{1}{2}(\sigma_1 + \sigma_2)$$
$$\tau_m = \tfrac{1}{2}(\sigma_1 - \sigma_2)$$

$$\tau_m \{\sin 2\beta - \tan\phi \cos 2\beta\} = c + \sigma_m \tan\phi$$
$$\tau_m = (\sigma_m + c \cot\phi)\tan\delta$$
$$\tan\delta = \sin\phi \operatorname{cosec}(2\beta - \phi)$$

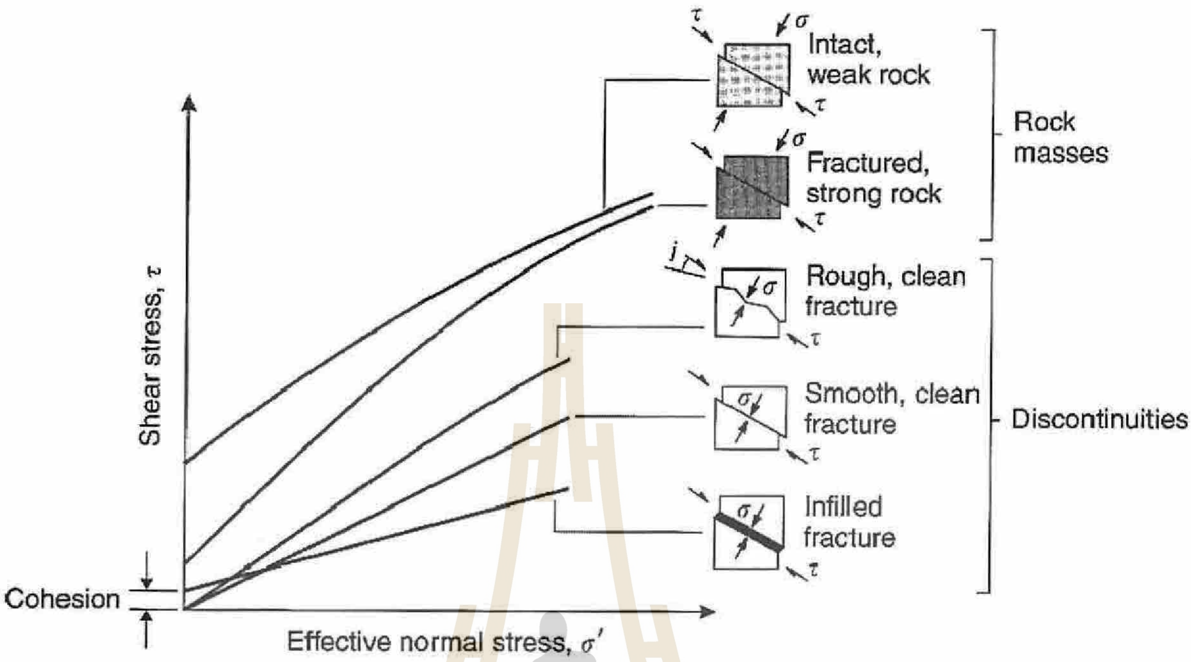
Example

Rock type	μ ($\tan\phi$)	c (kPa)
Granite	0.64	310
Gabbro	0.66	380
Trachyte	0.68	410
Sandstone	0.51	275
Marble	0.75	1,100

Joint shear strength criterion

1. **Coulomb Criterion** (Shear Strength of Planar Discontinuities)
2. **Patton Criterion** (Shear strength on an inclined plane)
3. **Ladanyi and Archambault Criterion** (Surface Roughness)
4. **Barton Criterion** (Surface Roughness)

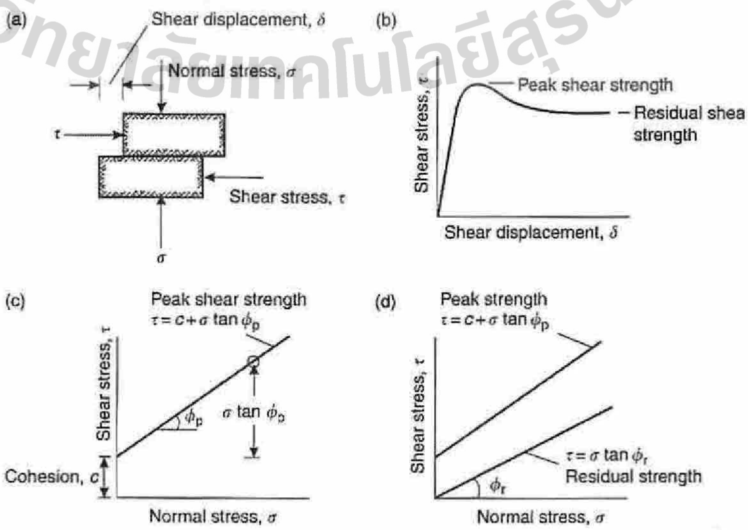
Shear stress vs. normal stress



Coulomb Criterion

Shear Strength of Planar Discontinuities

- Peak Shear Strength
- Residual Shear Strength



Coulomb Criterion

- **Peak Shear Strength**

$$\tau = c_p + \sigma \tan \phi_p$$

$$\tau = c_p + (\sigma\text{-}u) \tan \phi_p \qquad \textbf{(Effective Stress Law)}$$

- **Residual Shear Strength**

$$\tau = \sigma \tan \phi_r$$

$$\tau = (\sigma\text{-}u) \tan \phi_r \qquad \textbf{(Effective Stress Law)}$$

where u is the water pressure within the discontinuity

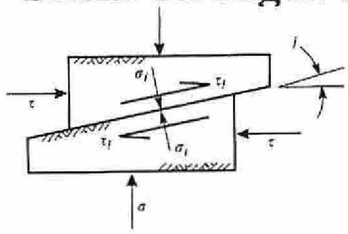
Coulomb Criterion

Table 4.1 Typical ranges of friction angles for a variety of rock types

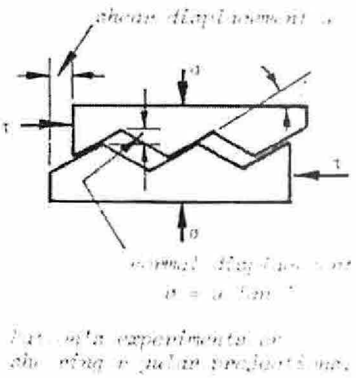
Rock class	*Friction angle range*	*Typical rock types*
Low friction	20–27°	Schists (high mica content), shale, marl
Medium friction	27–34°	Sandstone, siltstone, chalk, gneiss, slate
High friction	34–40°	Basalt, granite, limestone, conglomerate

Patton Criterion

- **Shear strength on an inclined plane**



i = asperities



$$\tau_i = \tau \, \mathrm{Cos}^2 i - \sigma \sin i \, \mathrm{Cos}\, i \qquad (6.3)$$

$$\sigma_i = \sigma \, \mathrm{Cos}^2 i - \sigma \sin i \, \mathrm{Cos}\, i \qquad (6.4)$$

If it is assumed that the discontinuity surface has zero cohesive strength and that its shear strength is given by

$$\tau_i = \sigma_i \, \mathrm{Tan}\, \phi \qquad (6.5)$$

sub equation 6.3 & 6.4 into equation 6.5

$$\tau = \sigma \, \mathrm{Tan}\, (\phi + i) \qquad (6.6)$$

Patton Criterion

- **Shear strength on an inclined plane**

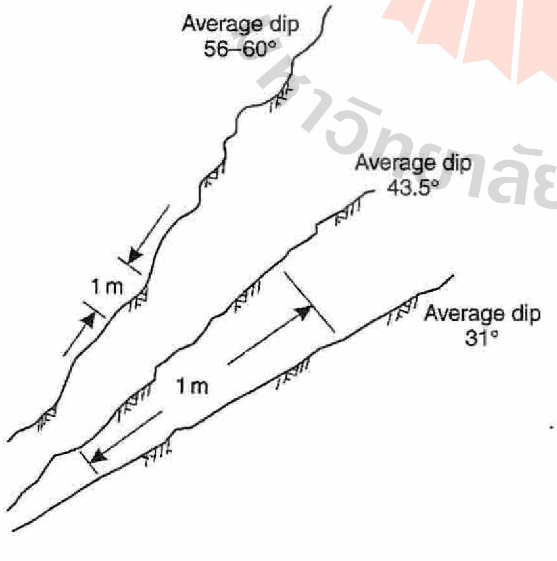


Figure 4.10 Patton's observations of bedding plane traces in unstable limestone slopes (Patton, 1966).

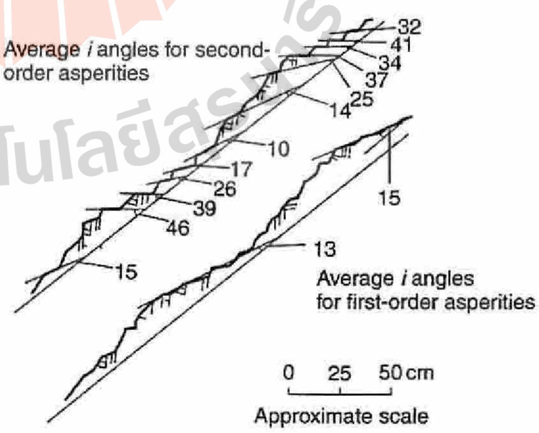


Figure 4.11 Measurement of roughness angles *i* for first- and second-order asperities on rough rock surfaces (Patton, 1966).

Ladanyi & Archambault Criterion

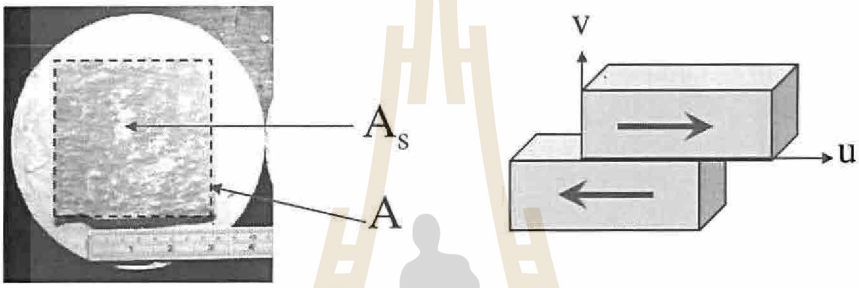
Surface Roughness

$$\tau = \frac{\sigma(1-a_s)(v+\tan\phi)+a_s.\tau_r}{1-(1-a_s)v\tan\phi}$$

where a_s = proportion of the discontinuity surface which is sheared through projections of intact rock material = A_s/A

v = dilation rate dv/du at peak shear strength

τ_r = shear strength of the intact rock material



Ladanyi & Archambault Criterion

Shear strength proposal by Fairhurst (1964):

$$\tau_r = \sigma_J \frac{\sqrt{1+n}-1}{n}\left(1+n\frac{\sigma}{\sigma_J}\right)^{\frac{1}{2}}$$

where σ_J = uniaxial compressive strength of the rock material (σ_C)

n = ratio of uniaxial compressive to uniaxial tensile strength (σ_C/σ_T)

Hoek (1968) has suggested that, for most hard rocks, n is approximately equal to 10

$$v = \left(1-\frac{\sigma}{\sigma_J}\right)^K \tan i \qquad \text{and} \qquad a_s = 1-\left(1-\frac{\sigma}{\sigma_J}\right)^L$$

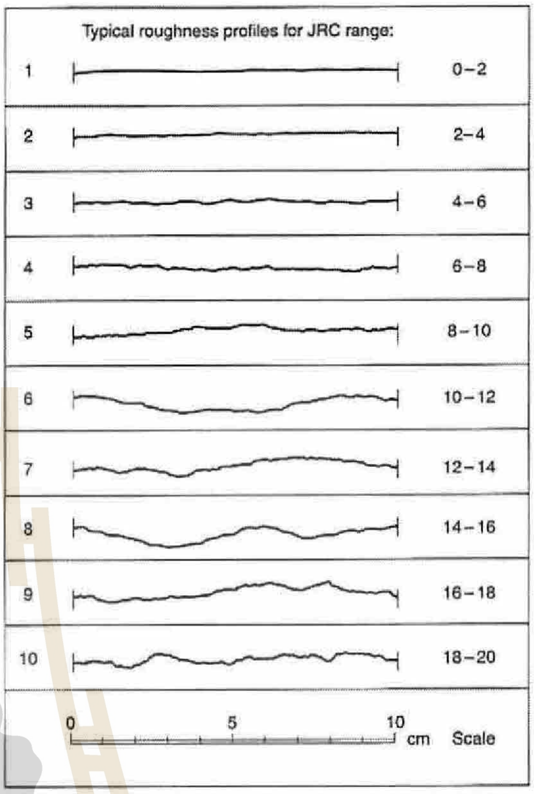
where, for rough rock surfaces, K = 4 and L = 1.5.

Barton Criterion

Predicting the shear strength of rough joints was proposed by Barton (1973)

$$\tau = \sigma \tan\left(\phi_b + JRC.Log_{10}\frac{\sigma_J}{\sigma}\right)$$

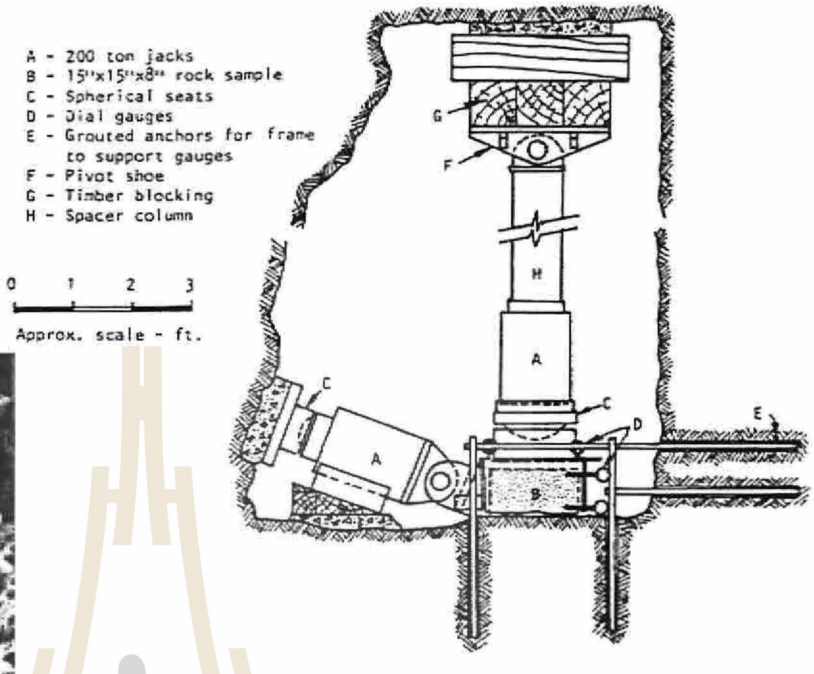
where JRC = Joint Roughness Coefficient



Shear strength testing

- Field (In-situ) Testing
- Laboratory Testing

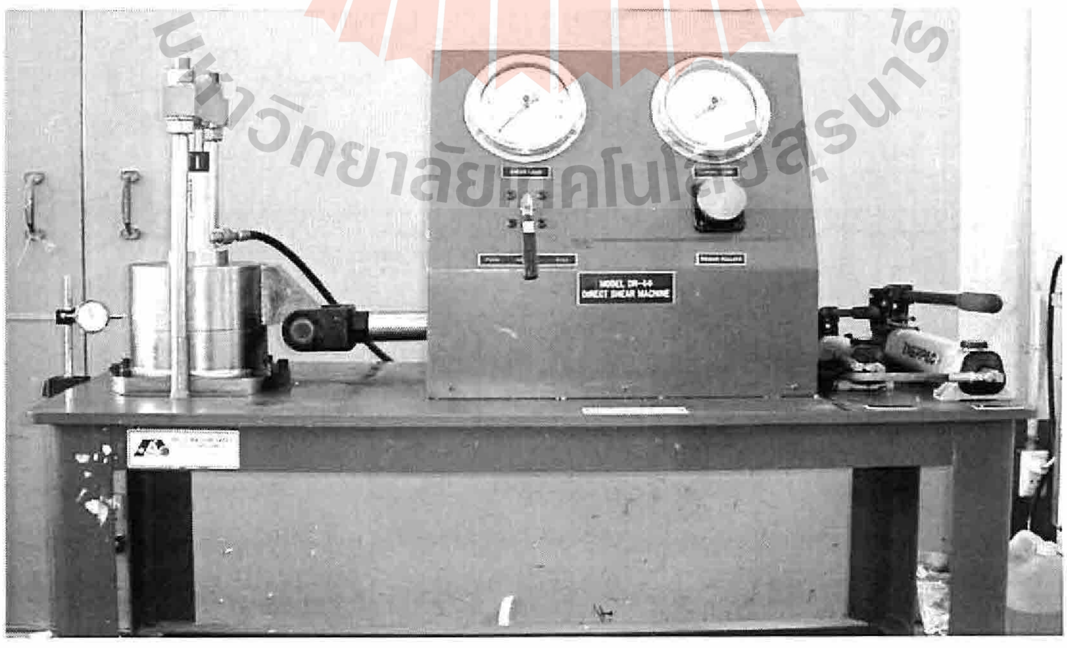
Field (In-situ) testing



In situ Shear Test

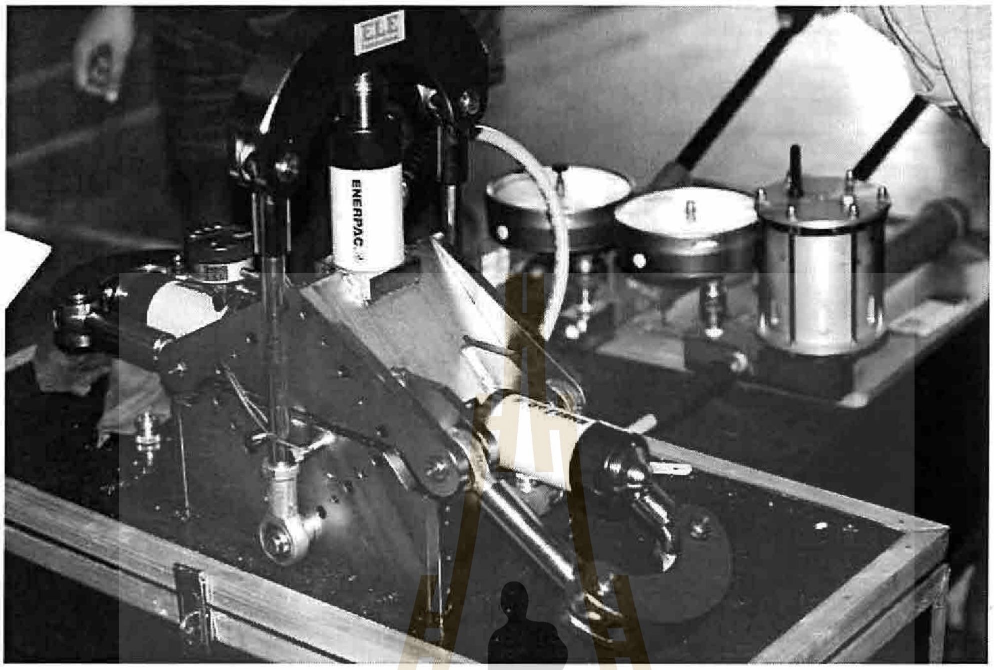
Laboratory testing

Large Scale Laboratory Tests

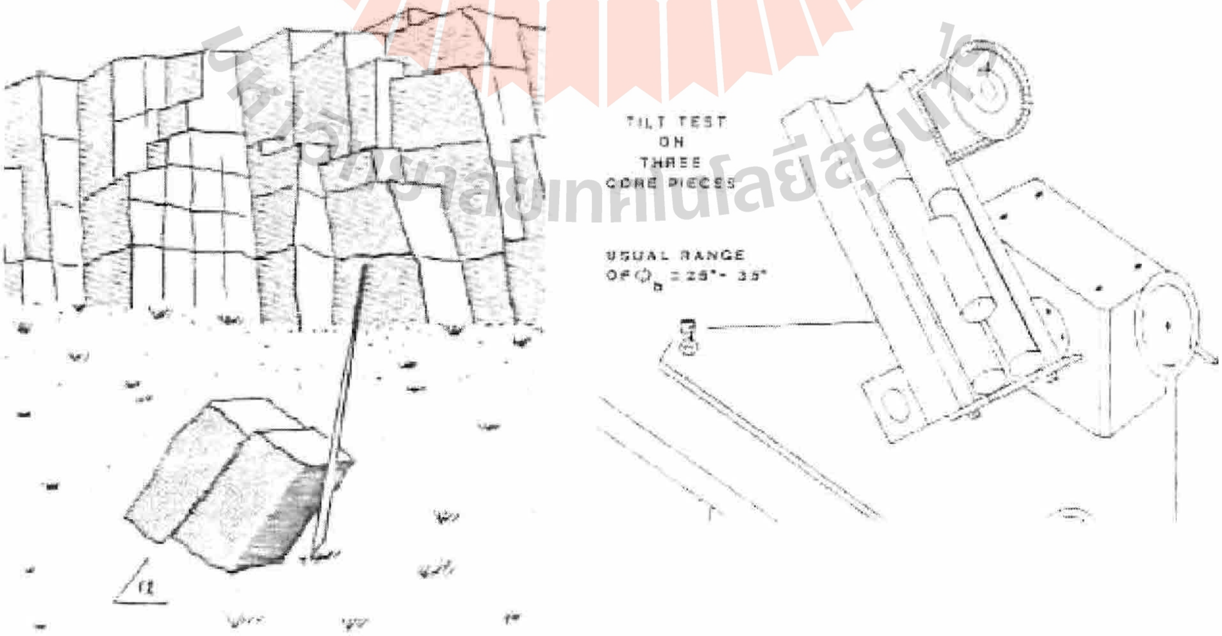


Laboratory testing

Portable Direct Shear Machine



Tilt Test



Topic 5
Mechanical Rock Properties and Behavior
คุณสมบัติและพฤติกรรมเชิงกลศาสตร์ของหิน

Prachya Tepnarong, Ph.D.
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วัตถุประสงค์

- เพื่อเข้าใจในคุณสมบัติและพฤติกรรมด้านกลศาสตร์ของหินซึ่งเป็นสิ่งสำคัญที่จะเชื่อมต่อระหว่างทฤษฎีและการนำไปประยุกต์ใช้ทางปฏิบัติ

คุณสมบัติของหินด้านกลศาสตร์ (Mechanical Properties)

- ความยืดหยุ่น (Elastic)
- ความเปราะ (Brittle)
- ความเหนียว (Ductile)
- ความเป็นพลาสติก (Plastic)
- การแตก (Failure)
- ความแข็ง/ทนทาน (Hardness/Durability)

(ถ้าพิจารณาโดยรวมแล้วคุณสมบัติและพฤติกรรมของหินจะแยกออกจากกันไม่ได้)

คุณสมบัติของหินด้านกายภาพ (Physical Properties)

- Mineral compositions
- Specific gravity / Unit weight
- Porosity / Void Ratio
- Moisture content
- Degree of saturation
- Chemical effect
- Thermal properties
- Electrical properties

ปัจจัยควบคุมคุณสมบัติของหิน

Control

- Nature of rock itself
- Stratigraphy (in-situ)
- Rock defect (discontinuities)
- Test Methodology

คุณสมบัติของหิน

- หินจะมีการตอบสนองต่อแรงที่มากระทำอย่างไร จะต้องอาศัยการตรวจสอบความสัมพันธ์ระหว่างความเค้นและความเครียดของหินนั้น ๆ เป็นหลัก
- ความสัมพันธ์เบื้องต้นนี้จะได้มาจากการทดสอบในห้องปฏิบัติการในรูปแบบต่าง ๆ (Uniaxial test, Triaxial test, Polyaxial test)

พฤติกรรมทางด้านกลศาสตร์ของหิน

สามารถแยกอธิบายเป็นสองลักษณะคือ

1. การเปลี่ยนรูปร่างหรือการวิรูปภายใต้แรงที่มากระทำ (Deformation)
2. การแตกหรือการวิบัติ (Failure) ภายใต้แรงที่มากระทำที่เกินกว่าหินนั้น ๆ จะรับได้ (Applied stress > Strength)

การเปลี่ยนรูปร่าง / การวิรูป (Rock Deformation)

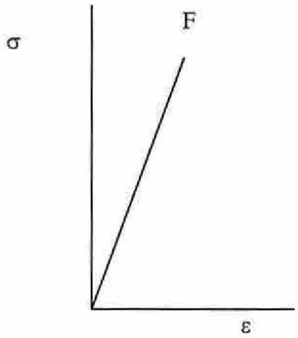
พฤติกรรมยืดหยุ่นอย่างสมบูรณ์ (Perfectly Elastic)

ความสัมพันธ์ระหว่างความเค้นและความเครียดจะต้องมีผลลัพธ์ที่แน่นอนและมีค่าเดียว (Unique relation)

$\sigma = f(\varepsilon)$

ความยืดหยุ่นเชิงเส้นตรง

ขั้นพื้นฐานความสัมพันธ์จถูกะสมมติให้เป็นเส้นตรง
คือ ความเค้นที่เพิ่มขึ้นจะเป็นสัดส่วนโดยตรงกับความเครียดที่เพิ่มขึ้น
จนในที่สุดหินจะแตกไปเมื่อได้รับแรงกดภายใต้ความเค้นที่ถึงจุดความเค้นแตกของหิ



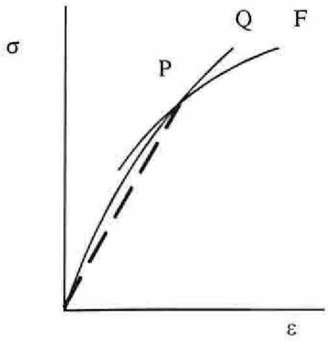
(a)

$$\boxed{\sigma = E\varepsilon}$$

โดยที่ E จะเรียกว่าเป็นค่า Young's modulus หรือสัมประสิทธิ์ของความยืดหยุ่น
สมการนี้เป็นไปตามทฤษฎีของความยืดหยุ่นเชิงเส้นตรง (Linearly elastic theory)

การเปลี่ยนรูปร่าง / การวิรูป (Rock Deformation)

ความยืดหยุ่นเชิงแบบเส้นโค้ง

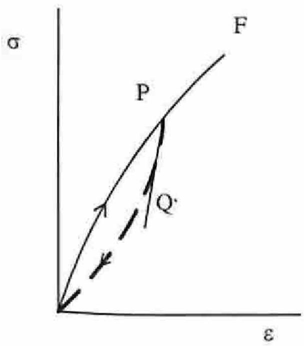


(b)

หินที่มีคุณสมบัติความยืดหยุ่นอย่างสมบูรณ์ก็ไม่จำเป็นที่ความเค้น
และความเครียดจะมีความสัมพันธ์เป็นเส้นตรง
โดยยังสามารถหาค่า E ของหินได้
คือ PQ จะเป็นค่า Tangent modulus
และเส้น OP จะเป็นค่า Secant modulus

การเปลี่ยนรูปร่าง / การวิรูป (Rock Deformation)

Hysteresis ของการกดและการคลายตัวออกของความเค้น



(c)

โดยสรุปแล้วหินที่มีคุณสมบัติยืดหยุ่นอย่างสมบูรณ์
จะต้องมีค่าความเครียดเป็นศูนย์เมื่อค่าความเค้นถูกลดลงมาเป็นศูนย์
การกลับคืนมาที่ศูนย์ของค่าความเค้นและความเครียดนี้
อาจจะมีลักษณะต่างกับการเพิ่มของค่าความเค้นและ
ความเครียดในขณะที่หินนั้นอยู่ภายใต้ความเค้น ซึ่งเรียกว่า **Hysteresis**
เกิดจากทางเดินของความเค้นที่ได้จากการกดและจากการคลายออกที่ต่างกัน

พฤติกรรมทางด้านกลศาสตร์ของหิน

สามารถแยกอธิบายเป็นสองลักษณะคือ

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$\sigma = f(\varepsilon)$

ความยืดหยุ่นเชิงเส้นตรง

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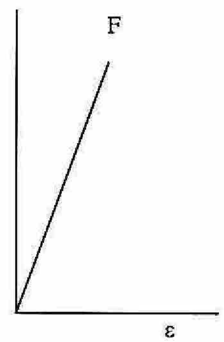

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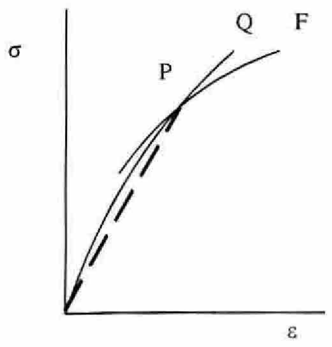
การเปลี่ยนรูปร่าง / การวิรูป (Rock Deformation)

ความยืดหยุ่นเชิงแบบเส้นโค้ง



(b)

หินที่มีคุณสมบัติความยืดหยุ่นอย่างสมบูรณ์ก็ไม่จำเป็นที่ความเค้น
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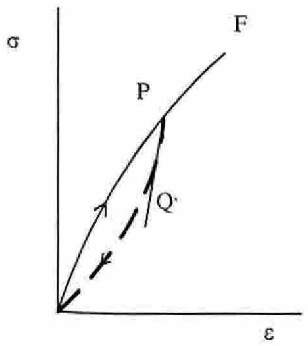

(c)

โดยสรุปแล้วหินที่มีคุณสมบัติยืดหยุ่นอย่างสมบูรณ์
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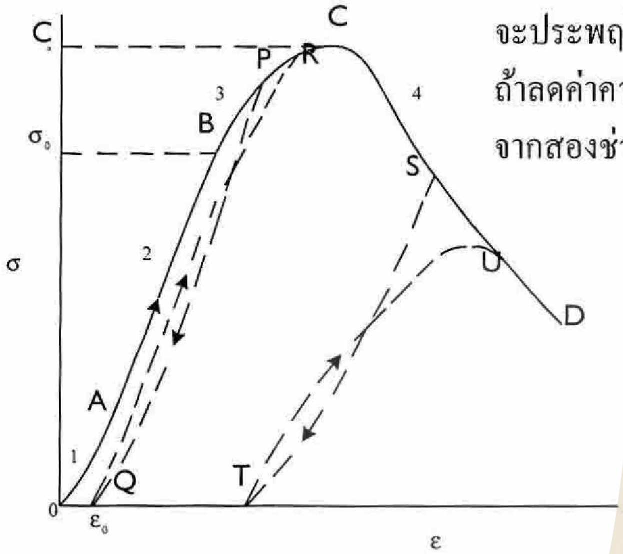
ความสัมพันธ์ระหว่างความเค้นและความเครียดที่สมบูรณ์ของหิน



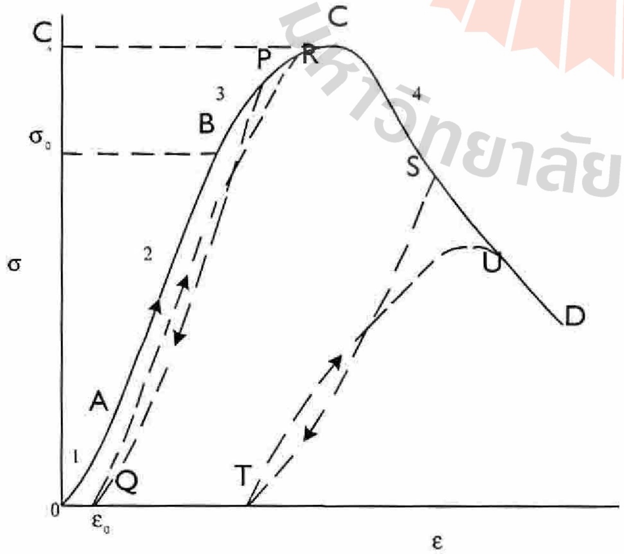
ช่วงที่ 1 (OA) + ช่วงที่ 2 (AB)

จะประพฤติตัวอย่างยืดหยุ่นเกือบสมบูรณ์

ถ้าลดค่าความเค้นลงมาที่ศูนย์ ความเครียดจะลดมาเป็นศูนย์เช่นกัน

จากสองช่วงนี้จะไม่มีการเปลี่ยนรูปอย่างถาวรของหินนั้น

ความสัมพันธ์ระหว่างความเค้นและความเครียดที่สมบูรณ์ของหิน



ช่วงที่ 3 (BC)

ส่วนใหญ่จะเริ่มจากเมื่อความเค้นมีค่าประมาณ 2 ใน 3 ของความเค้นสูงสุด

เส้นความสัมพันธ์จะค่อย ๆ ลดความชันลง

และในที่สุดจะถึงจุดสูงสุด

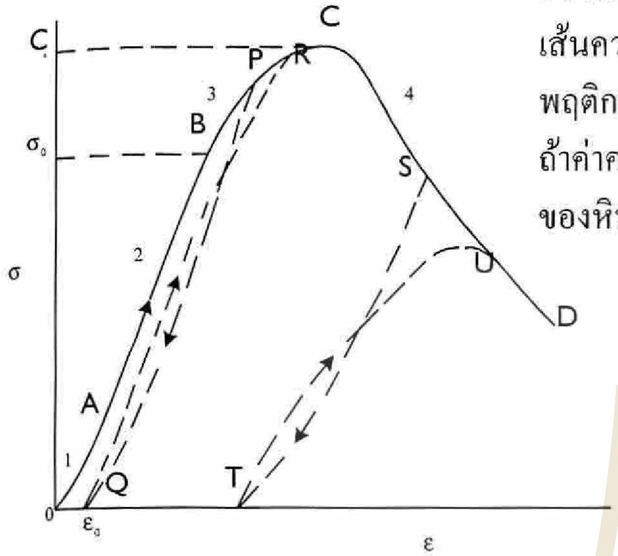
โดยที่เส้นความสัมพันธ์จะอยู่ในแนวนอนระยะหนึ่ง

ในช่วงนี้ถ้าความเค้นถูกลดลงมาเป็นศูนย์

ความเครียดจะไม่กลับไปที่ศูนย์

จะมีการเปลี่ยนรูปอย่างถาวรของหิน

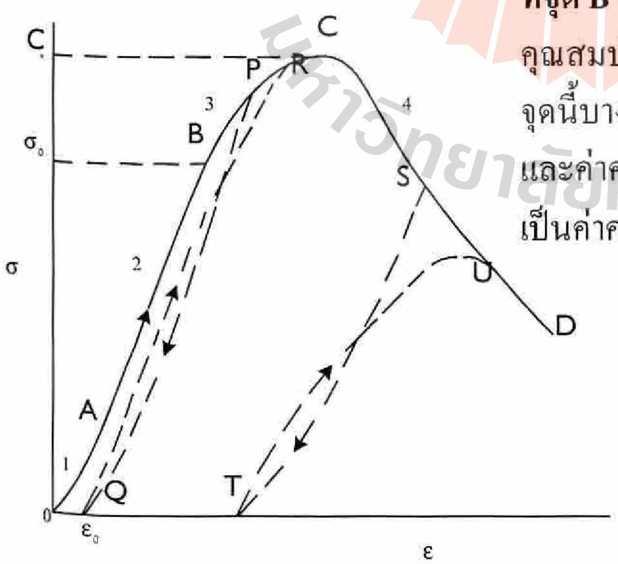
ความสัมพันธ์ระหว่างความเค้นและความเครียดที่สมบูรณ์ของหิน



ช่วงที่ 4 (CD)

เส้นความสัมพันธ์ระหว่างความเค้นและความเครียดจะลดลง พฤติกรรมเช่นนี้จะเรียกว่าความเปราะของหิน (Brittle behavior) ถ้าค่าความเค้นถูกลดมาที่ศูนย์ จากช่วงนี้จะมีการเปลี่ยนรูปของหินอย่างถาวรมากขึ้น

ความสัมพันธ์ระหว่างความเค้นและความเครียดที่สมบูรณ์ของหิน



ที่จุด B จะเป็นจุดที่แบ่งระหว่างหินที่มีคุณสมบัติอย่างยืดหยุ่น (Elastic) กับคุณสมบัติเหนียว (Ductile) จุดนี้บางทีเรียกว่าจุดอ่อนตัว (Yield point) และค่าความเค้นที่สัมพันธ์กับจุดนี้บางครั้งจะเรียกว่าเป็นค่าความเค้นอ่อนตัว (Yield stress) และใช้ตัวย่อว่า σ_0

การแตกของหิน (Failure)

ปัจจัยที่ทำให้หินแตกในหลายรูปแบบ

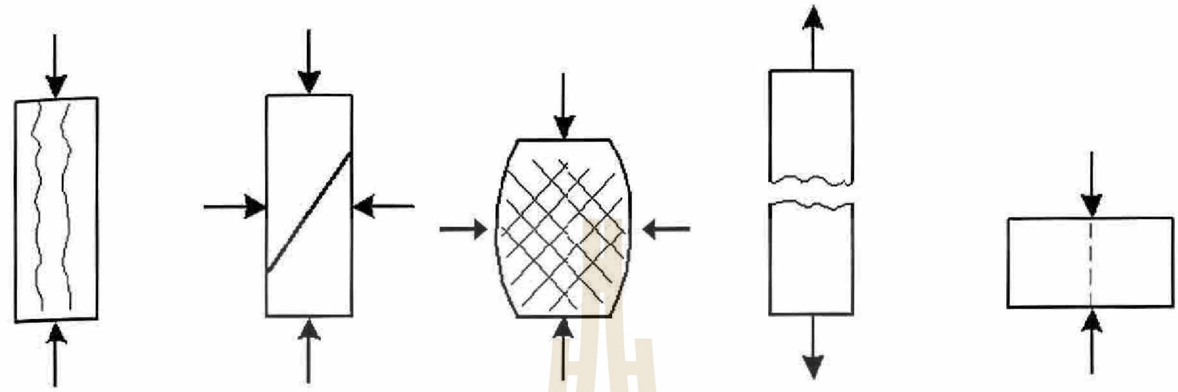
1. **ลักษณะของแรงที่มากระทำ (Load Characteristic)**
   - ขนาด (magnitude)
   - ทิศทาง (direction)
2. **คุณสมบัติของหิน (Rock Properties)**
   - ความหนาแน่น (density)
   - ปริมาณน้ำ/ความชื้น (water content)
   - ปริมาณและการกระจายตัวของรูพรุน (pore space)
   - รอยแตกเล็ก ๆ (micro-crack)
   - แรงยึดติดระหว่างผลึกแร่ (bonding b/w mineral crystal)

การแตกของหิน (Failure)

- เพื่อให้ง่ายต่อการศึกษาคุณลักษณะของรอยแตก หินรูปทรงกระบอกจะถูกใช้เป็นแบบจำลองเพื่อเทียบกับทิศทางของแรงที่มากระทำ ในการทดสอบคุณสมบัติทางด้านกลศาสตร์ในห้องปฏิบัติการ

Types of Fracture

ลักษณะของการแตก



(a) Longitudinal Spitting

(b) Shear Fracture

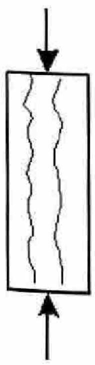
(c) Multiple Shear Fracture

(d) Extensional Fracture

(e) Extensional Fracture by Line Load

Types of Fracture

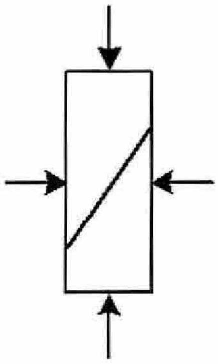
Longitudinal Spitting



- การแตกแบบแยกออกจากกันตามแนวยาวที่ขนานกับแรงกด
- มักพบในการทดสอบ UCS ที่มีขนาด L/D มากๆ (>3)

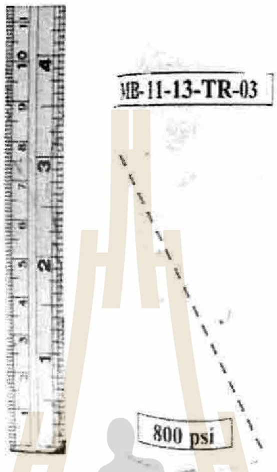
Types of Fracture

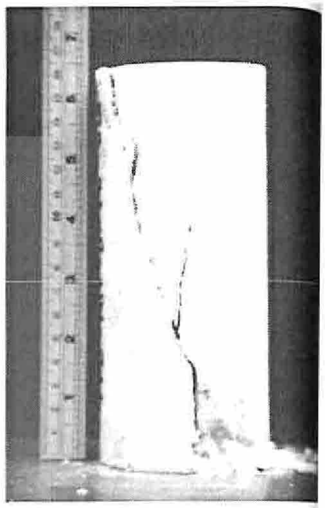
Shear Fracture



การแตกในแนวเฉือน

- จะเห็นได้บ่อยครั้งในการทดสอบความต้าน UCS หรือ Triaxial ที่ p ต่ำๆ
- ผิวของรอยแตกแบบเฉือนนี้จะมีการขูดและบดของหินที่อยู่ระหว่างรอยแ และจะพบเศษหินเล็ก ๆ หรือผงของหินในรอยแตกนั้น

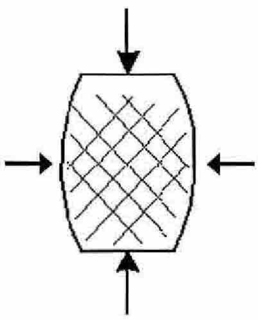




Single Shear Failure

Types of Fracture

Multiple Shear Fracture

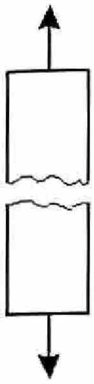


รอยแตกในแนวเฉือนแบบเป็นชุด

- พบได้จากการทดสอบหินภายใต้แรงกดในสามแกน (Triaxial compression test
- ขบวนการที่ทำให้เกิดรอยแตกเช่นนี้ก็จะเหมือนกับการแตกในแนวเฉือน แต่เนื่องจากในการทดสอบแรงกด ในสามแกนจะมีแรงกดเข้ามากระทำด้านข้าง ของหินตัวอย่างตลอดเวลา ซึ่งทำให้หินตัวอย่างนั้น มีการแตกร้าวอย่างเป็นชุดที่มีแนวขนานกัน

Types of Fracture

Extensional Fracture

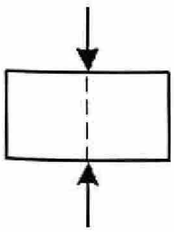


หินอยู่ภายใต้แรงดึง

- หินจะขาดออกจากกันทำให้เกิดรอยแตกในตำแหน่งที่มีแรงยึดติดน้อยที่สุด
- ผิวของรอยแตกจะสะอาดและไม่มีผงหรือเศษหินที่เกิดจากการขูดหรือไถของหิน

Types of Fracture

Extensional Fracture by Line Load



การแตกแบบดึงที่เกิดขึ้นจากการกด

- ลักษณะเช่นนี้ทิศทางของแรงกดจะขนานกับทิศทางของรอยแตก
- ซึ่งถ้าพิจารณาอย่างลึกซึ้งแล้วกลไกที่ทำให้เกิดการแตกเช่นนี้ก็จะคล้ายกับกลไกที่ทำให้เกิดการแตกแบบแยกออกจากกันตามแนวยาว

พฤติกรรมของหินที่ขึ้นกับเวลา

- การเปลี่ยนรูปของหินที่มีความอ่อนตัวมากหลายชนิดจะขึ้นกับระยะเวลาที่อยู่ภายใต้แรงที่มากระทำ (Time-dependent deformation)
- พฤติกรรมของการเปลี่ยนรูปเช่นนี้บางครั้งเรียกว่า "การเคลื่อนไหล" (Creep deformation)
- หินเหล่านี้เช่น เกลือหิน (Rock salt) ถ่านหิน (Coal) หินดินดาน (Shale) หินดิน (Claystone) หินโคลน (Mudstone) และหินทรายแป้ง (Siltstone) เป็นต้น

Deformation and Creep Mechanisms

1) Dislocation Glide (Cleavage sliding)

การเลื่อนของรอยแตกและรอยร้าวในผลึกของแร่

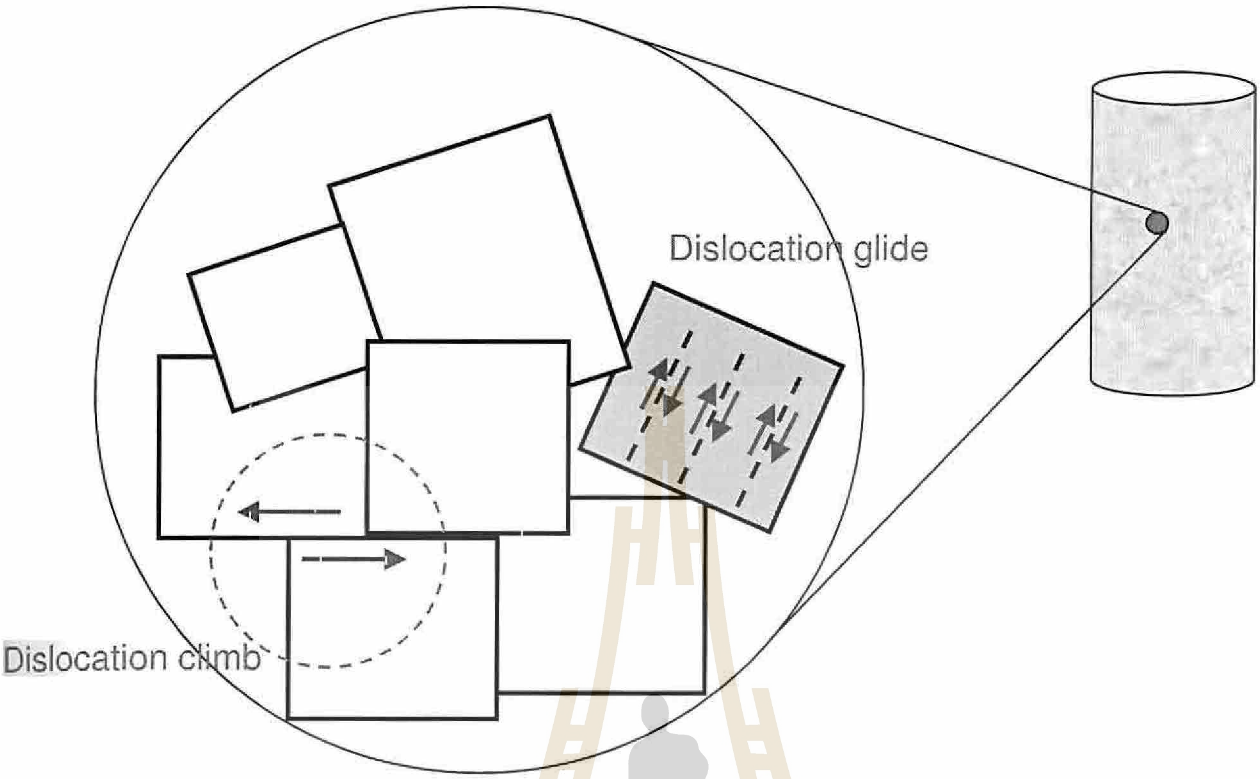
2) Dislocation Climb (Inter-granular sliding)

การเลื่อนของรอยต่อระหว่างผลึกของแร่

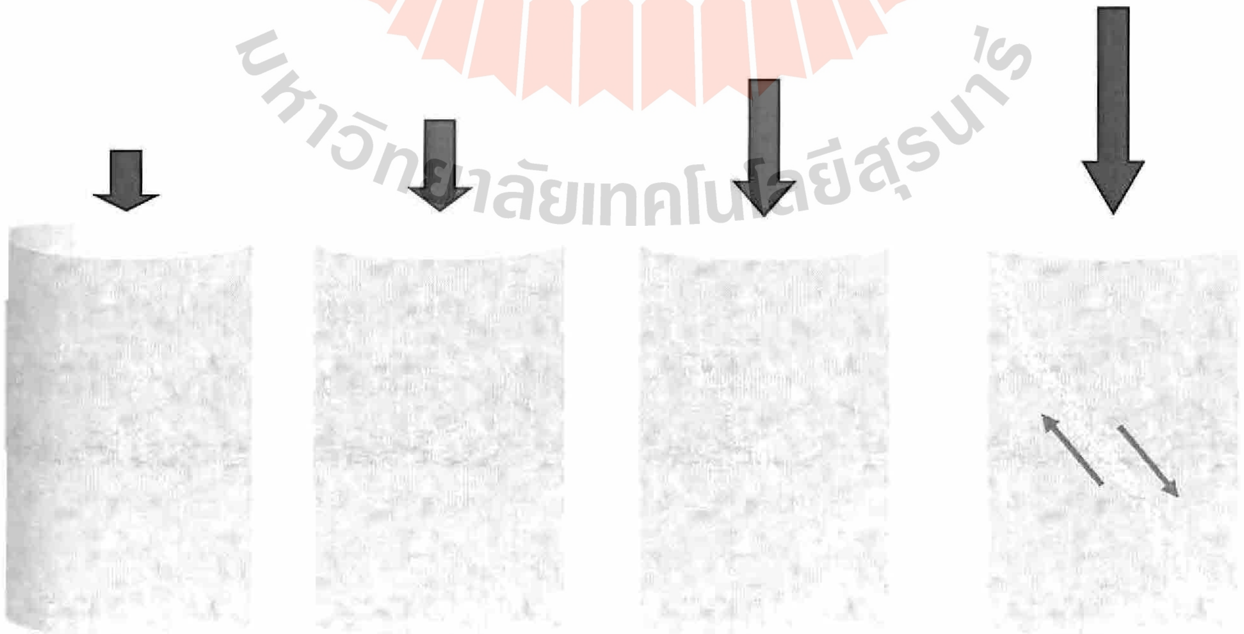
3) Micro-crack initiation and propagation

การเกิดขึ้นอย่างต่อเนื่องของรอยแตกเล็ก ๆ ในเนื้อหิน

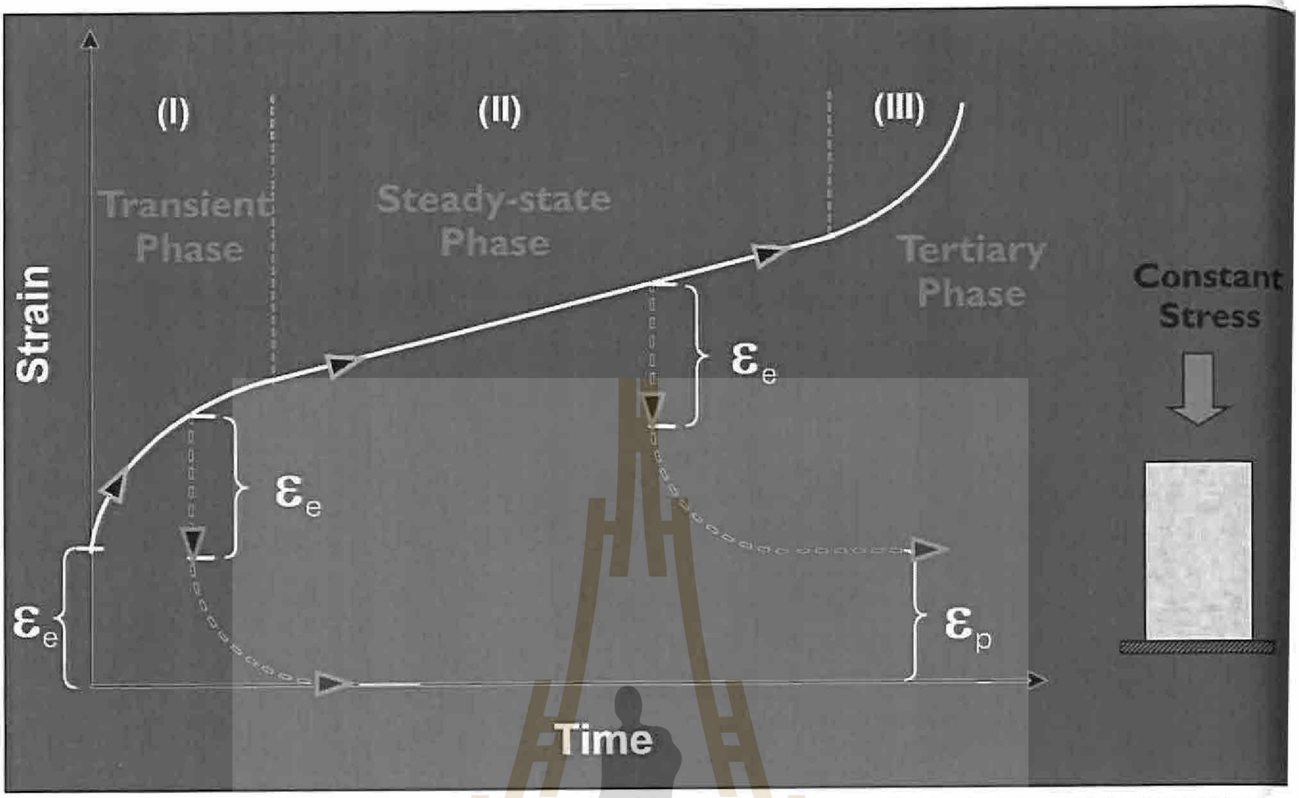
Dislocation Climb and Glide



Micro-Crack Initiation and Propagation



Time Dependency



Failure Strength criteria

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Failure Strength criterion

- Coulomb Criterion *
- Griffith Criterion
- Empirical Criterion
- Bieniawski Criterion
- Hoek and Brown Criterion*
- Yudhbir Criterion
- Kim and Lade Criterion
- Johnston Criterion

Coulomb Criterion (Jaeger and Cook, 1979)

- Linear Law

$$|\tau| = c + \mu\sigma$$

$$|\tau| = c + \sigma \tan \phi \qquad (\mu = \tan \phi)$$

where :

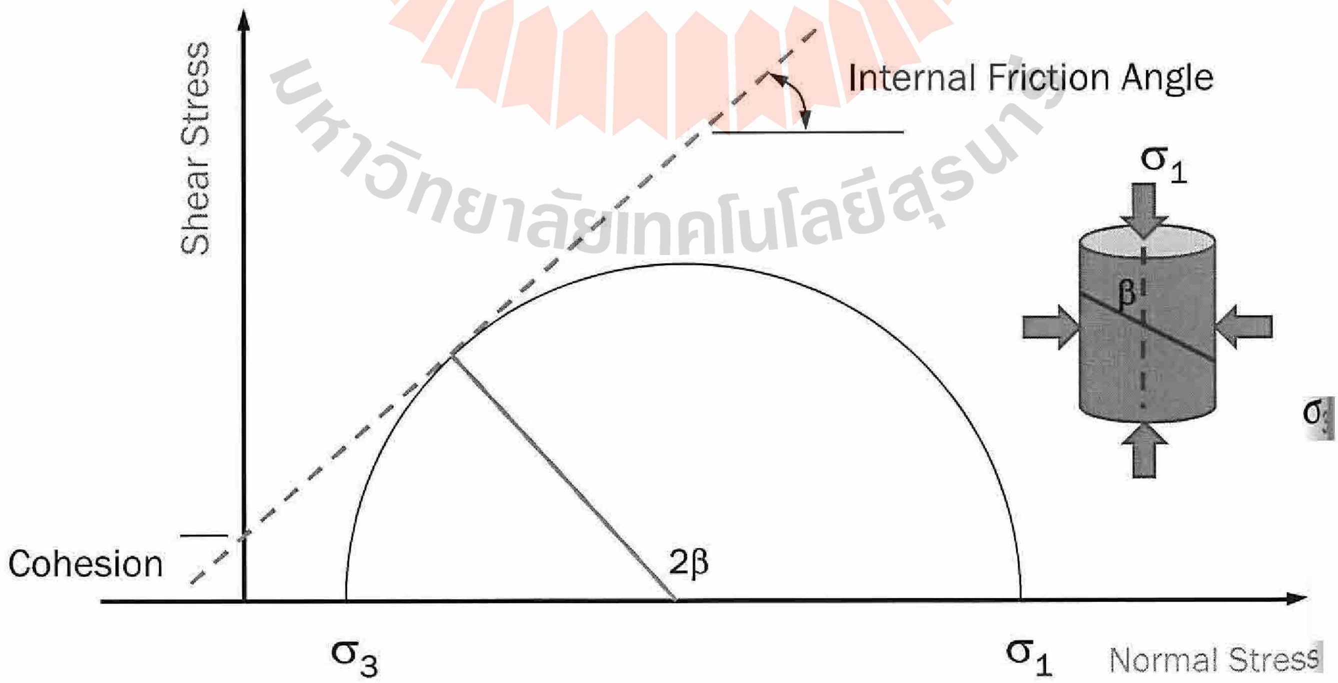
μ = coefficient of internal friction

c = cohesion

ϕ = angle of internal friction

Coulomb Criterion (Jaeger and Cook, 1979)

Mohr's Coulomb

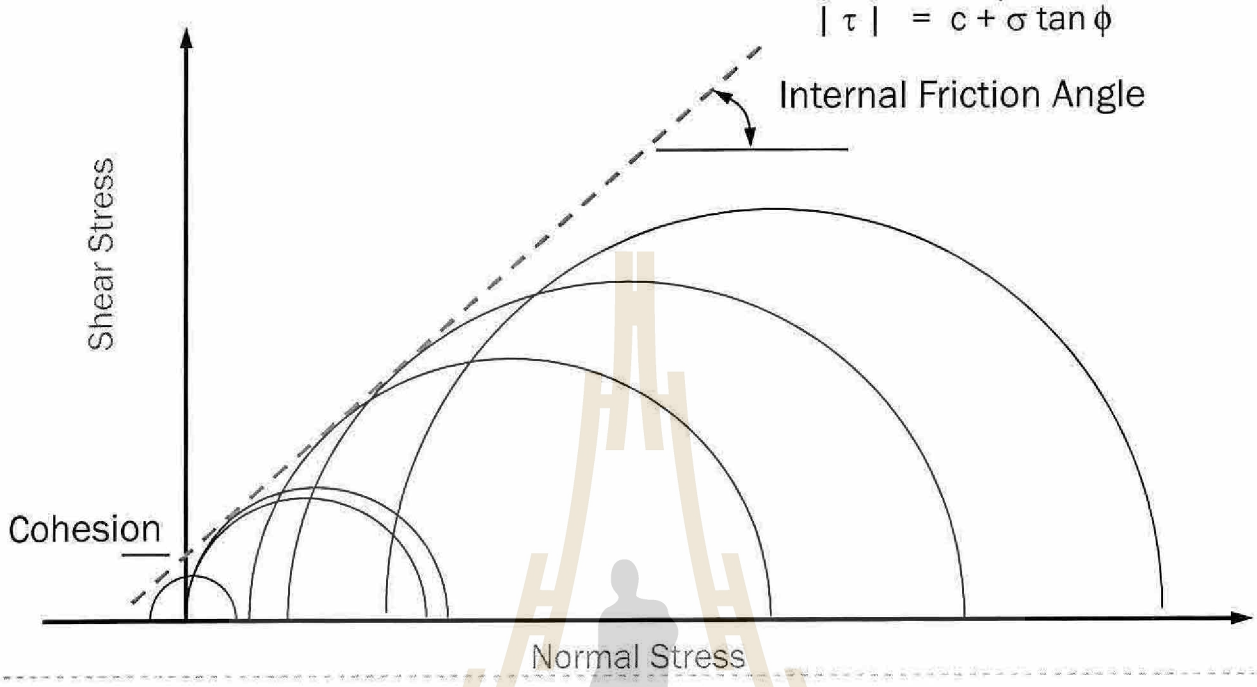


Coulomb Criterion (Jaeger and Cook, 1979)

Mohr's Coulomb

$|\tau| = c + \mu\sigma$

$|\tau| = c + \sigma \tan\phi$



Coulomb Criterion (Jaeger and Cook, 1979)

Failure Criteria: σ_1 vs. σ_3 Plot

The Strength of Rock Materials in Triaxial Compression

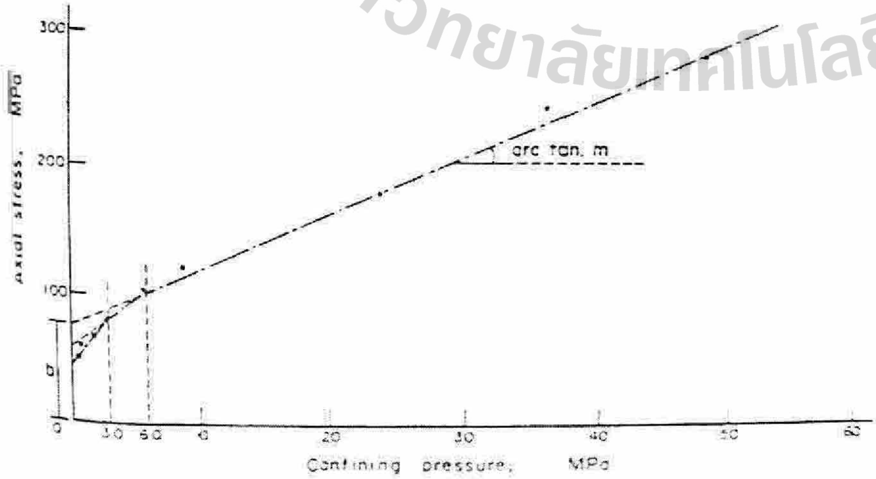


Fig. 2. Strength envelope.

Internal Friction Angle

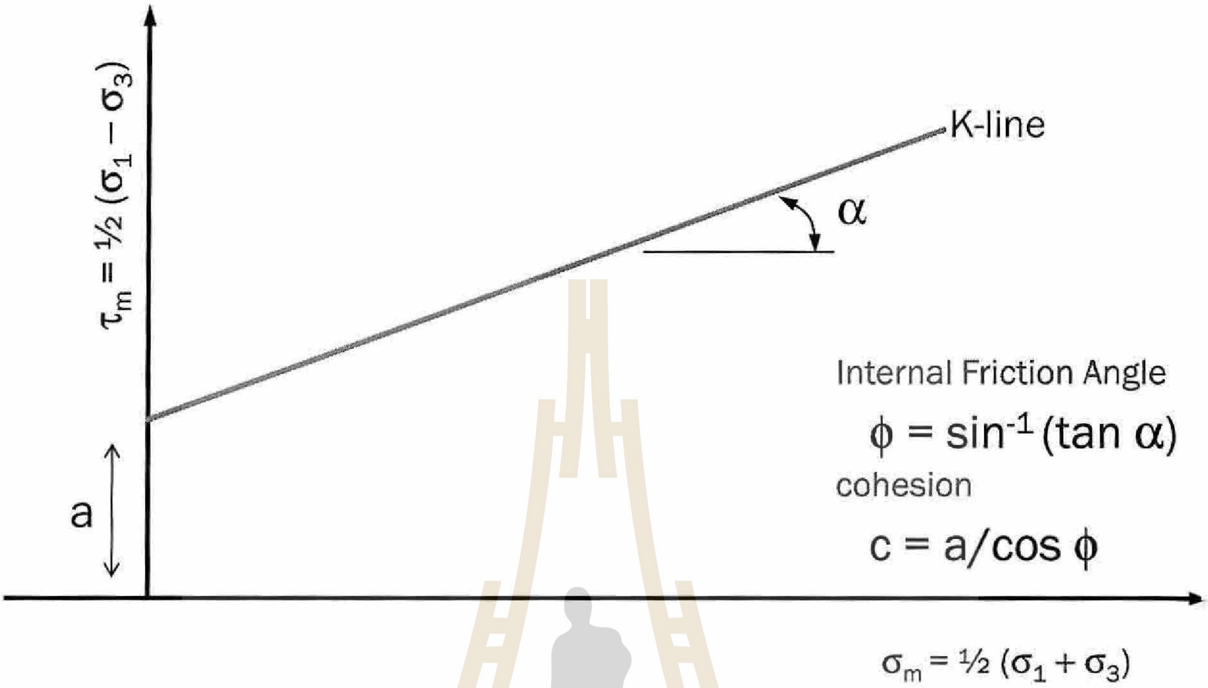
$$\phi = \arc\sin \frac{m-1}{m+1};$$

cohesion

$$C = b\,\frac{1-\sin\phi}{2\cos\phi}.$$

Coulomb Criterion (Jaeger and Cook, 1979)

Failure Criteria: τ_m vs. σ_m Plot



Griffith Criterion

กฎของ Griffith จะมีพื้นฐานมาจาก การแพร่กระจายของรอยแตกเล็ก ๆ (Micro-cracks)ในเนื้อหินที่มีความเปราะสูง ซึ่งในบางครั้งอาจจะไม่เหมาะสมที่จะนำมาอธิบายการแตกของหินขนาดใหญ่ กฎของ Griffith สามารถเขียนให้อยู่ในรูปของความเค้นหลักได้ดังนี้

$$(\sigma_1 - \sigma_3)^2 = 8T(\sigma_1 + \sigma_3) \qquad \text{ถ้า } \sigma_1 + 3\sigma_3 \geq 0$$
$$T = \sigma_3 \qquad \text{ถ้า } \sigma_1 + 3\sigma_3 < 0$$

T เป็นค่าแรงต้านความเค้นสูงสุดในแนวดึง กฎนี้ต่อมาได้ถูกพัฒนาโดย Murrell (1966) โดยสมมติให้ค่าแรงต้านความเค้นสูงสุดในแนวกดมีค่าเป็น 8 เท่าของค่าแรงต้านความเค้นสูงสุดในแนวดึง ดังนั้น

$$(\sigma_1 - \sigma_3)^2 - 8T(\sigma_1 + \sigma_3) = 16T^2$$

Empirical Criterion

มีกฎหลายชุดที่ได้ถูกพัฒนาขึ้นจากผลของการทดสอบในห้องปฏิบัติการ กฎเหล่านี้ไม่มีทฤษฎีทางฟิสิกส์มายืนยัน แต่ได้ถูกสร้างขึ้นในรูปแบบต่าง ๆ กัน โดยมุ่งไปที่การทำให้สูตรทางคณิตศาสตร์มีความสอดคล้องกับผลที่ได้จากห้องปฏิบัติการ (Empirical relationship) กฎเหล่านี้เช่น

$$\sigma_1 = A + B\sigma_3$$

$$\sigma_1 = A + B\sigma_3^{\,C}$$

$$\sigma_1 = A \log (B + \sigma_3)$$

$$\sigma_1 - \sigma_3 = \frac{A(\sigma_1 + \sigma_3) + B}{\sigma_1 + \sigma_3 + C}$$

$$\sigma_1 - \sigma_3 = A + BC^{\sigma_3}$$

$$\sigma_1 - \sigma_3 = A + B(\sigma_1 + \sigma_3)^C$$

$$\sigma_1 - \sigma_3 = A(\sigma_1 + \sigma_3)^B$$

A, B และ C เป็นค่าคงที่ ซึ่งขึ้นกับคุณสมบัติของหิน

Bieniawski's Criterion

Bieniawski (1974) ได้พัฒนากฎอันหนึ่งจากผลของการทดสอบในห้องปฏิบัติการ คือ

$$\frac{\sigma_1 - \sigma_3}{2\sigma_c} = 0.1 + B\left(\frac{\sigma_1 + \sigma_3}{2\sigma_c}\right)^a$$

โดยที่ a จะมีค่าอยู่ระหว่าง 0.85-0.93 และค่า B จะอยู่ระหว่าง 0.7-0.8

Hoek and Brown Criterion

Hoek and Brown (1980) ได้พยายามขยายผลของการแตกของหินที่ได้จากห้องปฏิบัติการ เพื่อคาดคะเนไปถึงการแตกของหินขนาดใหญ่ถึงระดับของมวลหิน กฎอันหนึ่งที่ถูกเสนอขึ้นมาคือ

$$\frac{\sigma_1}{\sigma_c} = \frac{\sigma_3}{\sigma_c} + \sqrt{m\frac{\sigma_3}{\sigma_c} + s}$$

โดยที่ m และ s เป็นค่าคงที่

Yudhbir Criterion

Yudhbir et al. (1983) ได้ทำการทดสอบหินปูน หินทราย และหินอัคนีหลายชนิด โดยทดสอบการกดในสามแกน และสรุปผลออกมาว่า กฎของ Hoek and Brown จะใช้ได้ดีต่อเมื่อหินมีความเปราะสูง และจะใช้ไม่ได้ดีถ้าหินมีความเหนียว (Ductile) Yudhbir และคณะจึงเสนอกฎใหม่คือ

$$\frac{\sigma_1}{\sigma_3} = A + B\left(\frac{\sigma_3}{\sigma_c}\right)^a$$

a มีค่าระหว่าง 0.65-0.75 ส่วน A และ B เป็นค่าคงที่ซึ่งขึ้นกับชนิดของหิน

Kim and Lade Criterion

Kim and Lade (1984) ได้เสนอกฎการแตกของหินในเชิงสามมิติ คือ

$$\left(\frac{I_1^3}{I_3} - 27\right)\left(\frac{I_1}{P_a}\right)^m = n_1$$

ค่า n_1 และ m เป็นค่าคงที่ขึ้นกับคุณสมบัติของหิน

ค่า $I_1 = \sigma_x + \sigma_y + \sigma_z$ ค่า $I_3 = \sigma_x \sigma_y \sigma_z$

ค่า P_a คือความกดของอากาศในขณะที่ทดสอบ

Johnston Criterion

Johnston (1985) เสนอกฎการแตกของหินที่มีความอ่อนเหมือนดินไปจนถึงหินที่มีความแข็งสูง

$$\frac{\sigma_1}{\sigma_c} = \left[\left(\frac{m}{B}\right)\left(\frac{\sigma_3}{\sigma_c}\right) + s\right]^B$$

สำหรับหินที่ไม่มีรอยแตก s จะมีค่าเท่ากับ 1 ค่า B จะผันแปรจาก 1 (สำหรับดิน) ไปจนถึง 0.5 (สำหรับหินแข็ง) ค่า m จะผันแปรจาก 2 (สำหรับดิน) ไปจนถึง 7-21 (สำหรับหินแข็ง)

Topic #7 Linear Elasticity

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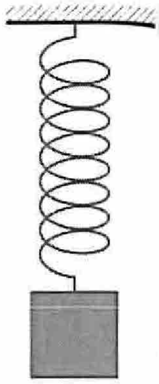
Linear Elasticity

- **Linear elasticity** is the mathematical study of how solid objects deform and become internally stressed due to prescribed loading conditions.
- Linear elasticity relies upon the continuum hypothesis and is applicable at macroscopic (and sometimes microscopic) length scales.
- Linear elasticity is a simplification of the more general nonlinear theory of elasticity and is a branch of continuum mechanics.
- The fundamental "linearizing" assumptions of linear elasticity are: infinitesimal strains or "small" deformations (or strains) and linear relationships between the components of stress and strain.
- In addition linear elasticity is only valid for stress states that do not produce yielding.
- These assumptions are reasonable for many engineering materials and engineering design scenarios.

Linear Elasticity

- **Hooke's law** of elasticity is an approximation that states that the extension of a spring is in direct proportion with the load added to it as long as this load does not exceed the elastic limit. Materials for which Hooke's law is a useful approximation are known as linear-elastic or "Hookean" materials.



$$\mathbf{F} = -k\mathbf{x}$$

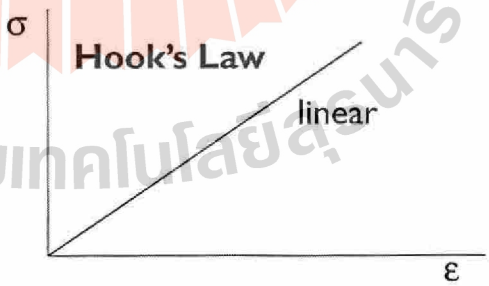
where:

x	is the distance that the spring has been stretched or compressed away from the equilibrium position, which is the position where the spring would naturally come to rest (meters),
F	is the restoring force exerted by the material (Newtons), and
k	is the force constant (or spring constant). The constant has units of force per unit length.

Hooke's law

- From stress – strain relation

$$\sigma = f\{\varepsilon\}$$



Assumption: Isotropic Rock

For elastic materials, Hooke's law represents the material behavior and relates the unknown stresses and strains. The general equation for Hooke's law is:

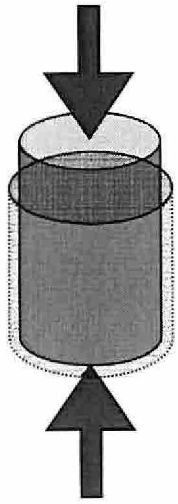
$$\sigma = E\varepsilon$$

Elastic Parameters

1. Elastic Modulus (Young's Modulus)
   - $E = \sigma_{axial} / \varepsilon_{axial}$

$\varepsilon_{axial} = \Delta L/L$

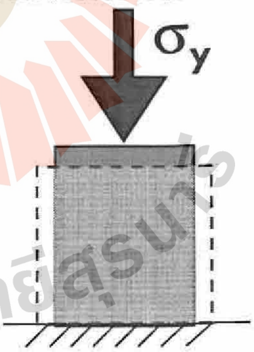
$\varepsilon_{lateral} = \Delta D/D$



2. Poisson's Ratio
   - $\nu = - \varepsilon_{lateral} / \varepsilon_{axial}$
   - $\nu = - \varepsilon_{lateral} \bullet (E/ \sigma_{axial})$ [sub by : $\varepsilon_{axial} = \sigma_{axial} /E$]
   - $\varepsilon_{lateral} = - (\nu / E) \bullet \sigma_{axial}$

Elasticity in 2D

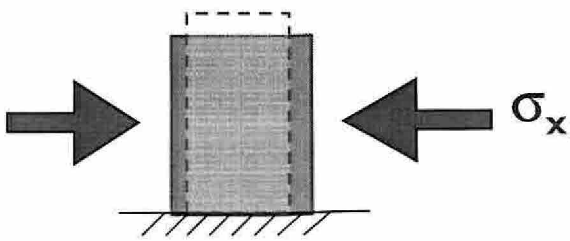
- Consider in σ_y



$\varepsilon_y = \sigma_y /E$

$\varepsilon_x = -\nu\sigma_y /E$

- Consider in σ_x



$\varepsilon_x = \sigma_x /E$

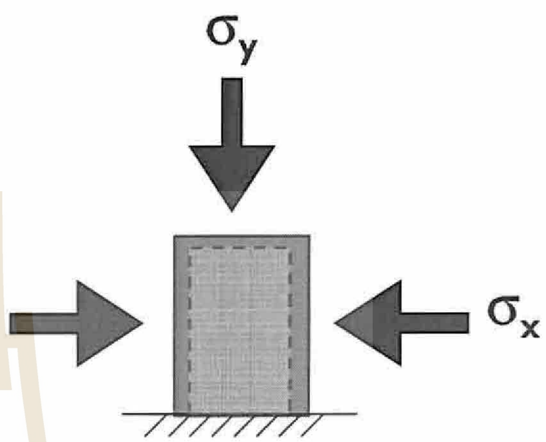
$\varepsilon_y = -\nu\sigma_x /E$

Elasticity in 2D

- Consider both in σ_x andσ_y

$$\varepsilon_x = (\sigma_x /E) - (\nu\sigma_y /E)$$
$$= (1/E)(\sigma_x - \nu\sigma_y)$$

$$\varepsilon_y = (\sigma_y /E) - (\nu\sigma_x /E)$$
$$= (1/E)(\sigma_y - \nu\sigma_x)$$

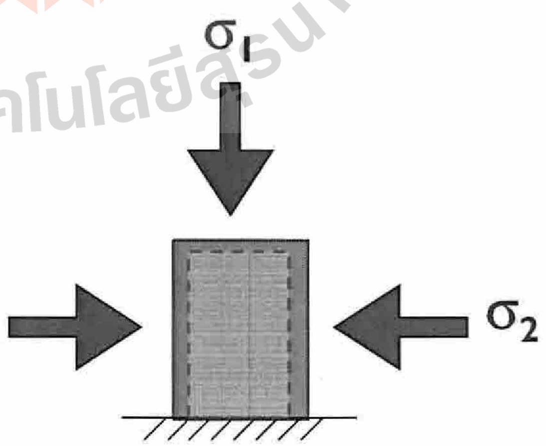


Principal Strain in 2D

- Consider in Principal Axis

$$\varepsilon_1 = (\sigma_1 /E) - (\nu\sigma_2 /E)$$
$$= (1/E)(\sigma_1 - \nu\sigma_2)$$

$$\varepsilon_2 = (\sigma_2 /E) - (\nu\sigma_1 /E)$$
$$= (1/E)(\sigma_2 - \nu\sigma_1)$$



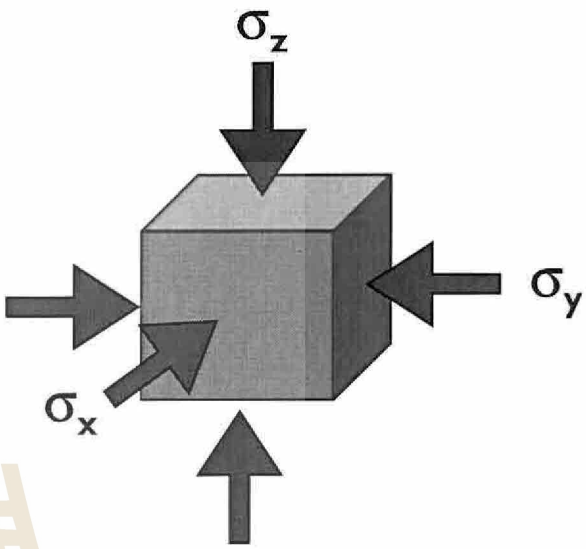
Elasticity in 3D

- Consider in σ_x, σ_y and σ_z

$$\varepsilon_x = (\sigma_x/E) - (\nu\sigma_y/E) - (\nu\sigma_z/E)$$
$$= (1/E)(\sigma_x - \nu\sigma_y - \nu\sigma_z)$$

$$\varepsilon_y = (\sigma_y/E) - (\nu\sigma_x/E) - (\nu\sigma_z/E)$$
$$= (1/E)(\sigma_y - \nu\sigma_x - \nu\sigma_z)$$

$$\varepsilon_z = (\sigma_z/E) - (\nu\sigma_x/E) - (\nu\sigma_y/E)$$
$$= (1/E)(\sigma_z - \nu\sigma_x - \nu\sigma_y)$$



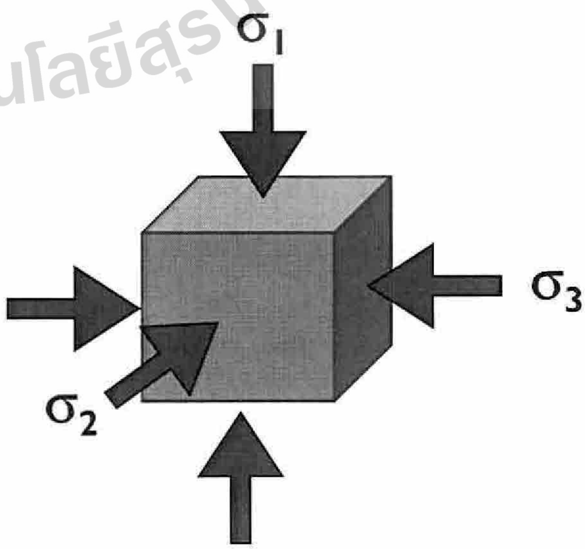
Principal Strain in 3D

- Consider in σ_1, σ_2 and σ_3

$$\varepsilon_1 = (\sigma_1/E) - (\nu\sigma_2/E) - (\nu\sigma_3/E)$$
$$= (1/E)(\sigma_1 - \nu\sigma_2 - \nu\sigma_3)$$

$$\varepsilon_2 = (\sigma_2/E) - (\nu\sigma_1/E) - (\nu\sigma_3/E)$$
$$= (1/E)(\sigma_2 - \nu\sigma_1 - \nu\sigma_3)$$

$$\varepsilon_3 = (\sigma_3/E) - (\nu\sigma_1/E) - (\nu\sigma_2/E)$$
$$= (1/E)(\sigma_3 - \nu\sigma_1 - \nu\sigma_2)$$



Shear Modulus

- **Shear modulus** or **modulus of rigidity**, denoted by G, or sometimes S, is defined as the ratio of shear stress to the shear strain.

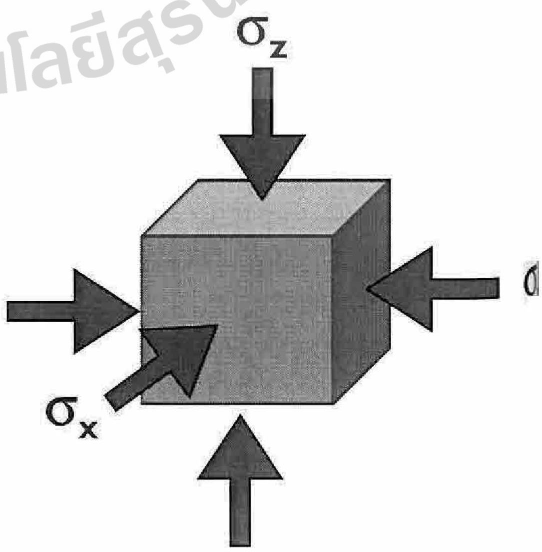
$$G = \tau_{xy} / \gamma_{xy} = \tau_{yz} / \gamma_{yz} = \tau_{zx} / \gamma_{zx}$$

$$\gamma_{xy} = \tau_{xy} / G \qquad \text{(sub } G = E/2(1+\nu)\text{)}$$
$$= \tau_{xy}\,(2(1+\nu))/E$$
$$\gamma_{yz} = \tau_{yx}\,(2(1+\nu))/E$$
$$\gamma_{zx} = \tau_{zx}\,(2(1+\nu))/E$$

Case of principal plans
$\tau_{xy} = \tau_{yz} = \tau_{zx} = 0$
$\gamma_{xy} = \gamma_{yz} = \gamma_{zx} = 0$

Elasticity in 3D

$$\varepsilon_x = (\sigma_x /E) - (\nu\sigma_y /E) - (\nu\sigma_z /E)$$
$$\varepsilon_y = (\sigma_y /E) - (\nu\sigma_x /E) - (\nu\sigma_z /E)$$
$$\varepsilon_z = (\sigma_z /E) - (\nu\sigma_x /E) - (\nu\sigma_y /E)$$
$$\gamma_{xy} = \tau_{xy}\,(2(1+\nu))/E$$
$$\gamma_{yz} = \tau_{yx}\,(2(1+\nu))/E$$
$$\gamma_{zx} = \tau_{zx}\,(2(1+\nu))/E$$



Matrix Form

$[\varepsilon_{ij}] = [S][\sigma_{ij}]$

[S] = Stiffness Matrix

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{v}{E} & -\frac{v}{E} & 0 & 0 & 0 \\ -\frac{v}{E} & \frac{1}{E} & -\frac{v}{E} & 0 & 0 & 0 \\ -\frac{v}{E} & -\frac{v}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{2(1+v)}{E} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{2(1+v)}{E} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{2(1+v)}{E} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

Matrix Form

$[\sigma_{ij}] = [S]^{-1}[\varepsilon_{ij}]$

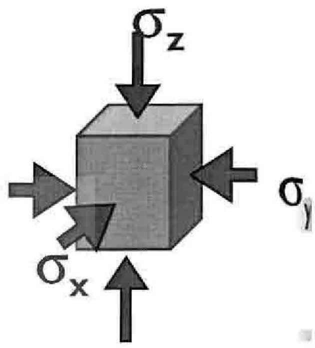
$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = \begin{pmatrix} \lambda+2G & \lambda & \lambda & 0 & 0 & 0 \\ \lambda & \lambda+2G & \lambda & 0 & 0 & 0 \\ \lambda & \lambda & \lambda+2G & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & G & 0 \\ 0 & 0 & 0 & 0 & 0 & G \end{pmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}$$

λ = *Lamé's parameter*

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

Anisotropic Rock

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{pmatrix} \frac{1}{E_x} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{xz}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{yx}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{zx}}{E_x} & -\frac{\nu_{zy}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{xy}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{zx}} \end{pmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$



where E_x, E_y and E_z are elastic modulus along x, y and z axis, G_x, G_y and G_z are shear modulus along x, y and z axis, and ν_{xy}, ν_{xz} and ν_{yz} are Poisson's ratio corresponding with x - y, x - z and y - z directions, respectively.

Principal Strain in 3D

- Consider in σ_1, σ_2 and σ_3

$$\varepsilon_1 = (\sigma_1/E) - (\nu\sigma_2/E) - (\nu\sigma_3/E)$$
$$\varepsilon_2 = (\sigma_2/E) - (\nu\sigma_1/E) - (\nu\sigma_3/E)$$
$$\varepsilon_3 = (\sigma_3/E) - (\nu\sigma_1/E) - (\nu\sigma_2/E)$$

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix}$$

Principal Strain in 3D

- Consider in σ_1, σ_2 and σ_3

$$\sigma_1 = (\lambda + 2G)\varepsilon_1 + \lambda\varepsilon_2 + \lambda\varepsilon_3$$
$$\sigma_2 = \lambda\varepsilon_1 + (\lambda + 2G)\varepsilon_2 + \lambda\varepsilon_3$$
$$\sigma_3 = \lambda\varepsilon_1 + \lambda\varepsilon_2 + (\lambda + 2G)\varepsilon_3$$

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix} = \begin{pmatrix} \lambda + 2G & \lambda & \lambda \\ \lambda & \lambda + 2G & \lambda \\ \lambda & \lambda & \lambda + 2G \end{pmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{Bmatrix}$$

Principal Strain in 3D

- Isotropic rock

$$\sigma_1 = (\lambda + 2G)\varepsilon_1 + \lambda\varepsilon_2 + \lambda\varepsilon_3$$
$$\sigma_2 = \lambda\varepsilon_1 + (\lambda + 2G)\varepsilon_2 + \lambda\varepsilon_3$$
$$\sigma_3 = \lambda\varepsilon_1 + \lambda\varepsilon_2 + (\lambda + 2G)\varepsilon_3$$

- The volumetric strain, $\Delta = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$

$$\sigma_1 = \lambda\Delta + 2G\varepsilon_1$$
$$\sigma_2 = \lambda\Delta + 2G\varepsilon_2$$
$$\sigma_3 = \lambda\Delta + 2G\varepsilon_3$$

Bulk Modulus

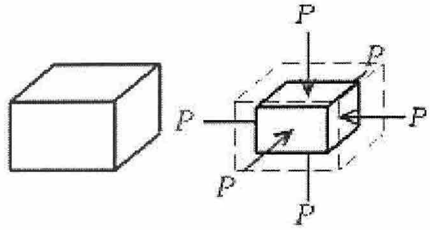
- The **bulk modulus** (*K*) of a substance measures the substance's resistance to uniform compression. It is defined as the pressure increase needed to cause a given relative decrease in volume.

$$K = \frac{p}{\Delta}$$ (Incompressibility)

p = Hydrostatic pressure,

Δ = Volumetric strain,

$\sigma_1 = \sigma_2 = \sigma_3 = p$



Compressibility = 1/K

Conversion Formulas of Elastic Parameters

- Young's modulus (*E*)
- Poisson's ratio (ν)
- Lamé's parameter (λ)
- Shear modulus (*G*)
- Bulk modulus (*K*)

	(λ, G)	(E, G)	(K, λ)	(K, G)	(λ, ν)	(G, ν)	(E, ν)	(K, ν)	(K, E)
$K =$	$\lambda + \frac{2G}{3}$	$\frac{EG}{3(3G-E)}$			$\lambda\frac{1+\nu}{3\nu}$	$\frac{2G(1+\nu)}{3(1-2\nu)}$	$\frac{E}{3(1-2\nu)}$		
$E =$	$G\frac{3\lambda+2G}{\lambda+G}$		$9K\frac{K-\lambda}{3K-\lambda}$	$\frac{9KG}{3K+G}$	$\frac{\lambda(1+\nu)(1-2\nu)}{\nu}$	$2G(1+\nu)$		$3K(1-2\nu)$	
$\lambda =$		$G\frac{E-2G}{3G-E}$		$K - \frac{2G}{3}$		$\frac{2G\nu}{1-2\nu}$	$\frac{E\nu}{(1+\nu)(1-2\nu)}$	$\frac{3K\nu}{1+\nu}$	$\frac{3K(3K-E)}{9K-E}$
$G =$			$3\frac{K-\lambda}{2}$		$\lambda\frac{1-2\nu}{2\nu}$		$\frac{E}{2(1+\nu)}$	$3K\frac{1-2\nu}{2(1+\nu)}$	$\frac{3KE}{9K-E}$
$\nu =$	$\frac{\lambda}{2(\lambda+G)}$	$\frac{E}{2G} - 1$	$\frac{\lambda}{3K-\lambda}$	$\frac{3K-2G}{2(3K+G)}$					$\frac{3K-E}{6K}$

Special cases

- **Poisson's Relation**

  Given $\lambda = G \rightarrow K = 5G/3, \quad E = 5G/2, \quad \nu = 1/4$

  ($\nu = 0.25$ mostly of rock)

- **Incompressible Material**

  $\lambda = K = \infty \rightarrow \nu = 1/2, \quad G = E/3$

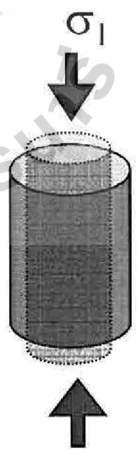
- **Compressible Fluid**

  $G \rightarrow 0, \quad E \rightarrow 0, \quad \nu \rightarrow 1/2, \quad \lambda \rightarrow K \rightarrow G/(1 - 2\nu)$

Special cases

- **Uniaxial Stress**

  given $\rightarrow \sigma_1 \neq 0, \ \sigma_2 = \sigma_3 = 0$



from
$$\begin{aligned}\sigma_1 &= (\lambda + 2G)\varepsilon_1 + \lambda\varepsilon_2 + \lambda\varepsilon_3\\ \sigma_2 &= \lambda\varepsilon_1 + (\lambda + 2G)\varepsilon_2 + \lambda\varepsilon_3\\ \sigma_3 &= \lambda\varepsilon_1 + \lambda\varepsilon_2 + (\lambda + 2G)\varepsilon_3\end{aligned}$$

become
$$\begin{aligned}\sigma_1 &= (\lambda + 2G)\varepsilon_1 + \lambda\varepsilon_2 + \lambda\varepsilon_3\\ 0 &= \lambda\varepsilon_1 + (\lambda + 2G)\varepsilon_2 + \lambda\varepsilon_3\\ 0 &= \lambda\varepsilon_1 + \lambda\varepsilon_2 + (\lambda + 2G)\varepsilon_3\end{aligned}$$

so; $\varepsilon_2 = \varepsilon_3 = -\dfrac{\lambda}{2(\lambda + G)}\varepsilon_1$ and $E = \dfrac{\sigma_1}{\varepsilon_1} = \dfrac{G(3\lambda + 2G)}{\lambda + G}$ and $\Delta = (1 - 2\nu)\sigma_1/E$

Special cases

Uniaxial Strain

given → $\varepsilon_1 \neq 0, \varepsilon_2 = \varepsilon_3 = 0$

from
$$\sigma_1 = (\lambda + 2G)\varepsilon_1 + \lambda\varepsilon_2 + \lambda\varepsilon_3$$
$$\sigma_2 = \lambda\varepsilon_1 + (\lambda + 2G)\varepsilon_2 + \lambda\varepsilon_3$$
$$\sigma_3 = \lambda\varepsilon_1 + \lambda\varepsilon_2 + (\lambda + 2G)\varepsilon_3$$

become
$$\sigma_1 = (\lambda + 2G)\varepsilon_1$$
$$\sigma_2 = \lambda\varepsilon_1$$
$$\sigma_3 = \lambda\varepsilon_1$$

so; $\sigma_1 = (\lambda + 2G)\varepsilon_1$ and $\sigma_2 = \sigma_3 = \lambda\varepsilon_1 = [\nu/(1 - \nu)]\,\sigma_1$

Special cases

Biaxial Stress or Plane Stress

given → $\sigma_1 \neq 0, \sigma_2 \neq 0, \sigma_3 = 0$

from
$$\varepsilon_1 = (1/E)(\sigma_1 - \nu\sigma_2 - \nu\sigma_3) \qquad \sigma_1 = (\lambda + 2G)\varepsilon_1 + \lambda\varepsilon_2 + \lambda\varepsilon_3$$
$$\varepsilon_2 = (1/E)(\sigma_2 - \nu\sigma_1 - \nu\sigma_3) \qquad \sigma_2 = \lambda\varepsilon_1 + (\lambda + 2G)\varepsilon_2 + \lambda\varepsilon_3$$
$$\varepsilon_3 = (1/E)(\sigma_3 - \nu\sigma_1 - \nu\sigma_2) \qquad \sigma_3 = \lambda\varepsilon_1 + \lambda\varepsilon_2 + (\lambda + 2G)\varepsilon_3$$

become
$$E\varepsilon_1 = \sigma_1 - \nu\sigma_2,$$
$$E\varepsilon_2 = \sigma_2 - \nu\sigma_1,$$
$$E\varepsilon_3 = -\nu(\sigma_1 + \sigma_2)$$

so;
$$(\lambda + 2G)\sigma_1 = 4G(\lambda + G)\varepsilon_1 + 2\lambda G\varepsilon_2$$
$$(\lambda + 2G)\sigma_2 = 2\lambda G\varepsilon_1 + 4G(\lambda + G)\varepsilon_2$$

and $\Delta = (\sigma_1 + \sigma_2)(1 - 2\nu)/E$ (case of pure shear, $\sigma_1 + \sigma_2 = 0, \varepsilon_3 = 0$)

Special cases

Biaxial Strain or Plane Strain

given → $\varepsilon_1 \neq 0, \varepsilon_2 \neq 0, \varepsilon_3 = 0$

from
$$\varepsilon_1 = (1/E)(\sigma_1 - \nu\sigma_2 - \nu\sigma_3) \qquad \sigma_1 = (\lambda + 2G)\varepsilon_1 + \lambda\varepsilon_2 + \lambda\varepsilon_3$$
$$\varepsilon_2 = (1/E)(\sigma_2 - \nu\sigma_1 - \nu\sigma_3) \qquad \sigma_2 = \lambda\varepsilon_1 + (\lambda + 2G)\varepsilon_2 + \lambda\varepsilon_3$$
$$\varepsilon_3 = (1/E)(\sigma_3 - \nu\sigma_1 - \nu\sigma_2) \qquad \sigma_3 = \lambda\varepsilon_1 + \lambda\varepsilon_2 + (\lambda + 2G)\varepsilon_3$$

become
$$\sigma_1 = (\lambda + 2G)\varepsilon_1 + \lambda\varepsilon_2$$
$$\sigma_2 = (\lambda + 2G)\varepsilon_2 + \lambda\varepsilon_1$$
$$\sigma_3 = \lambda(\varepsilon_1 + \varepsilon_2) = [\lambda/2(\lambda + G)](\sigma_1 + \sigma_2) = \nu(\sigma_1 + \sigma_2)$$

so;
$$E\varepsilon_1 = (1 - \nu^2)\sigma_1 - \nu(1 + \nu)\sigma_2 \qquad \text{or} \qquad E\varepsilon_x = (1 - \nu^2)\sigma_x - \nu(1 + \nu)\sigma_y$$
$$E\varepsilon_2 = (1 - \nu^2)\sigma_2 - \nu(1 + \nu)\sigma_1 \qquad\qquad E\varepsilon_y = (1 - \nu^2)\sigma_y - \nu(1 + \nu)\sigma_x$$

$$E\gamma_{xy} = 2E\Gamma_{xy} = 2(1 + \nu)\tau_{xy}$$

Special cases

plane strain ←→ plane stress

In case of principal axis

$$8G\varepsilon_1 = (x + 1)\sigma_1 + (x - 3)\sigma_2$$
$$8G\varepsilon_2 = (x - 3)\sigma_1 + (x + 1)\sigma_2$$

In case of any axis

$$8G\varepsilon_x = (x + 1)\sigma_x + (x - 3)\sigma_y$$
$$8G\varepsilon_y = (x - 3)\sigma_x + (x + 1)\sigma_y$$
$$\tau_{xy} = G\gamma_{xy} = 2G\Gamma_{xy}$$

where $x = 3 - 4\nu$ for plane strain
$x = (3 - \nu)/(1 + \nu)$ for plane stress

Special cases

plane strain ←→ plane stress

In case of any axis

$$8G\varepsilon_x = (x+1)\sigma_x + (x-3)\sigma_y$$
$$8G\varepsilon_y = (x-3)\sigma_x + (x+1)\sigma_y$$
$$\tau_{xy} = G\gamma_{xy} = 2G\Gamma_{xy}$$

where $x = 3 - 4\nu$ for plane strain

$x = (3-\nu)/(1+\nu)$ for plane stress

Thick wall cylinder

Assumption : Hydrostatic Pressure

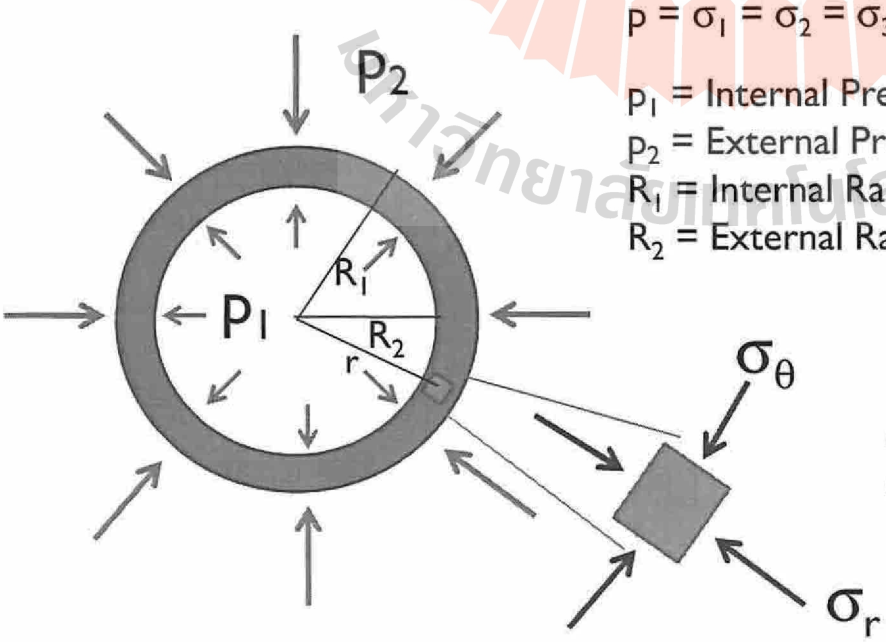
$p = \sigma_1 = \sigma_2 = \sigma_3$

p_1 = Internal Pressure

p_2 = External Pressure

R_1 = Internal Radius

R_2 = External Radius

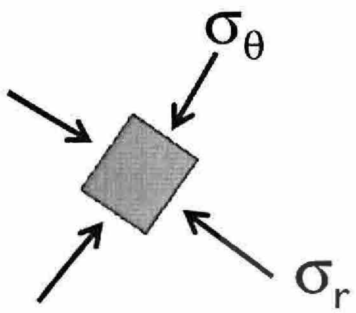


σ_r = Radial Stress

σ_θ = Tangential Stress

Thick wall cylinder

General Equation :



$$\sigma_r = \frac{p_2R_2^2 - p_1R_1^2}{(R_2^2 - R_1^2)} - \frac{(p_2 - p_1)R_1^2R_2^2}{r^2(R_2^2 - R_1^2)}$$

$$\sigma_\theta = \frac{p_2R_2^2 - p_1R_1^2}{R_2^2 - R_1^2} + \frac{(p_2 - p_1)R_1^2R_2^2}{r^2(R_2^2 - R_1^2)}$$

$$u = \frac{(p_2R_2^2 - p_1R_1^2)r}{2(\lambda + G)(R_2^2 - R_1^2)} + \frac{(p_2 - p_1)R_1^2R_2^2}{2G(R_2^2 - R_1^2)r}$$

u = Radial Displacement

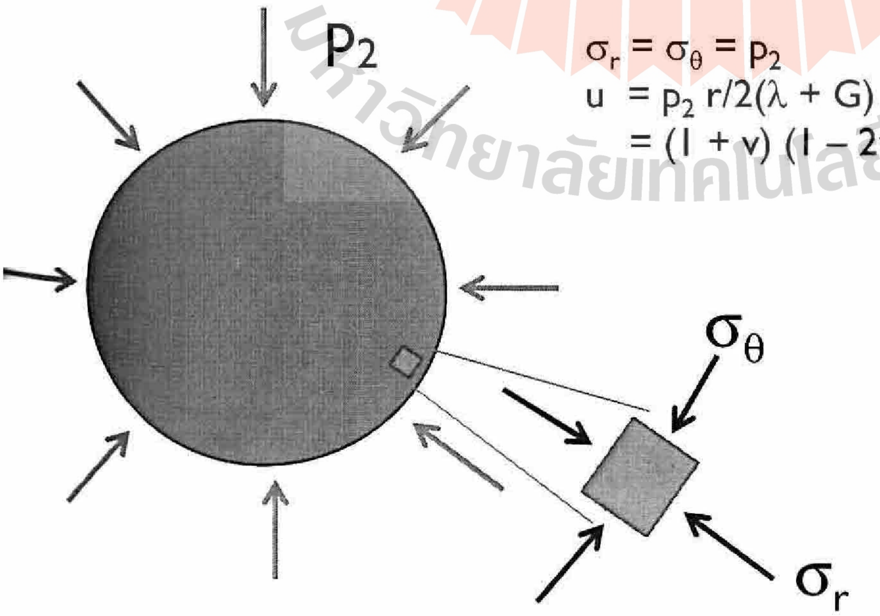
Thick wall cylinder

In case: $R_1 = 0$,

$\sigma_r = \sigma_\theta = p_2$

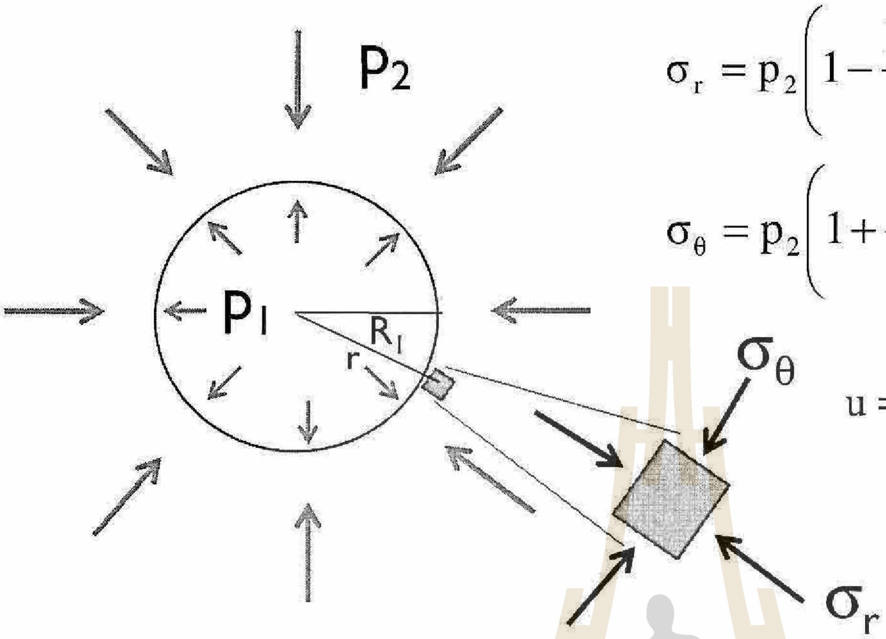
$u = p_2\, r/2(\lambda + G)$

$\quad = (1 + \nu)(1 - 2\nu) r\, p_2/E$



Thick wall cylinder

In case : $R_2 \to \infty$

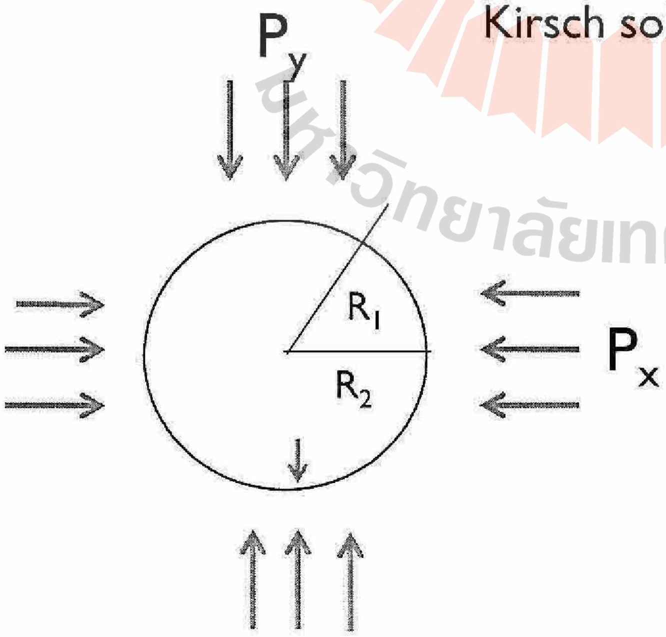


$$\sigma_r = p_2\left(1 - \frac{R_1^{\,2}}{r^2}\right) + \frac{p_1 R_1^{\,2}}{r^2}$$

$$\sigma_\theta = p_2\left(1 + \frac{R_1^{\,2}}{r^2}\right) - \frac{p_1 R_1^{\,2}}{r^2}$$

$$u = \frac{p_2 r}{2(\lambda + G)} + \frac{(p_2 - p_1)R_1^{\,2}}{2Gr}$$

Circular hole in infinite plate

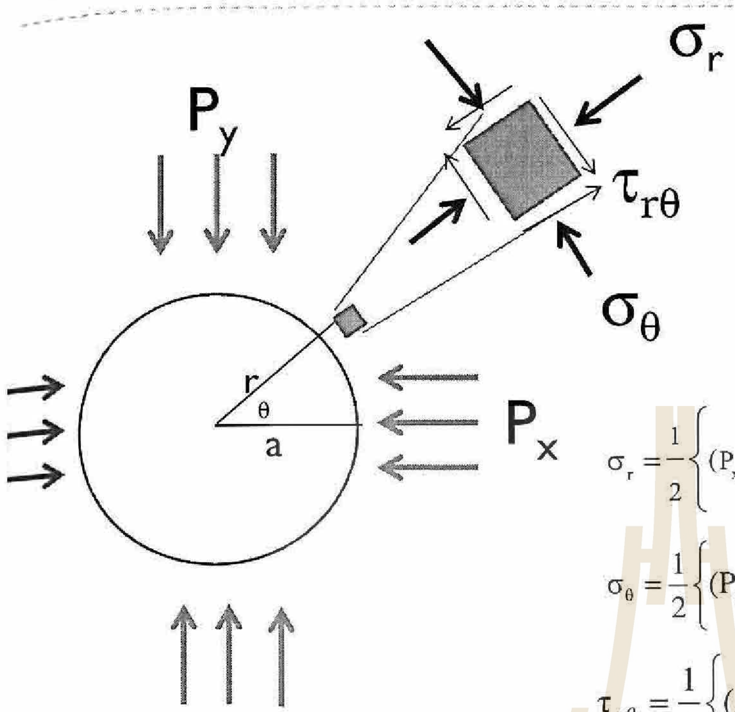


Kirsch solution (Brady and Brown, 1985)

Assumption : Perfectly Linear Elastic

P_x, P_y = Stress Field
a = Internal Radius

Circular hole in infinite plate

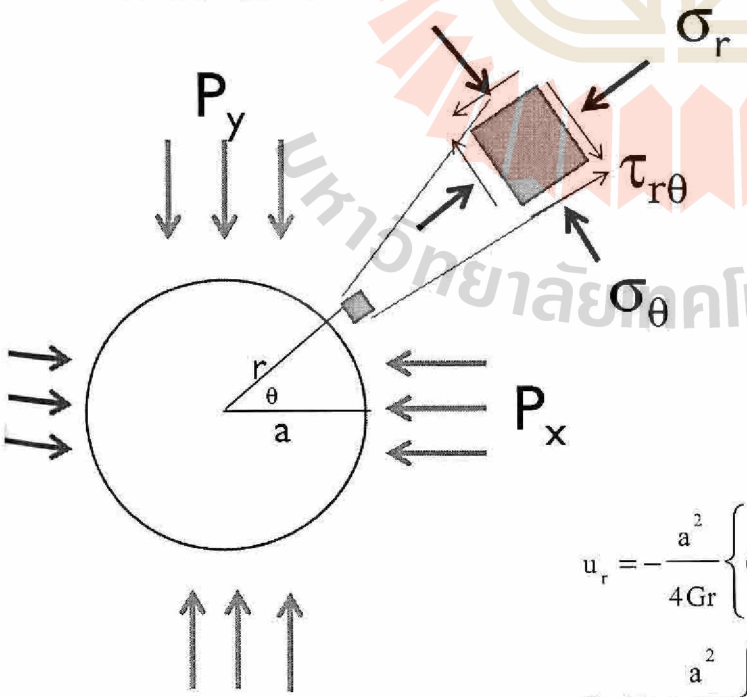


$$\sigma_r = \frac{1}{2}\left\{(P_x + P_y)\left(1 - \frac{a^2}{r^2}\right) + (P_x - P_y)\left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$\sigma_\theta = \frac{1}{2}\left\{(P_x + P_y)\left(1 + \frac{a^2}{r^2}\right) - (P_x - P_y)\left(1 + \frac{3a^4}{r^4}\right)\cos 2\theta\right\}$$

$$\tau_{r\theta} = \frac{1}{2}\left\{(P_y - P_x)\left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right)\sin 2\theta\right\}$$

Circular hole in infinite plate

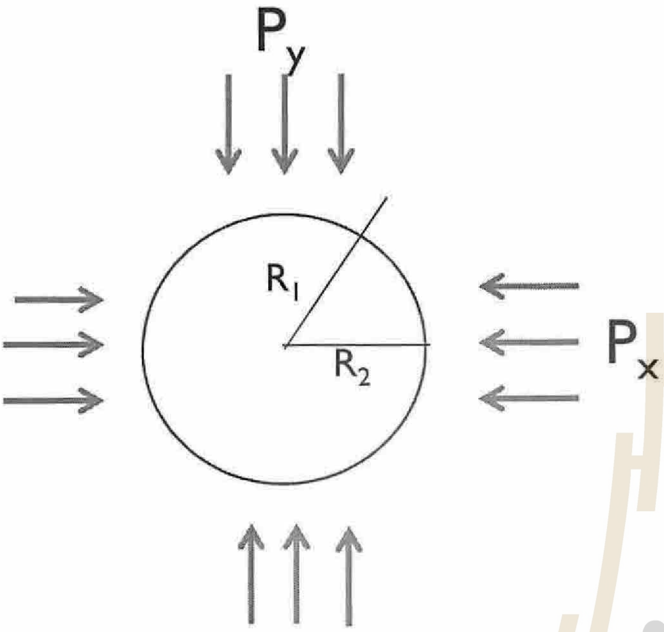


u_r = Radial Displacement
u_θ = Tangential Displacement

$$u_r = -\frac{a^2}{4Gr}\left\{(P_y + P_x) - (P_y - P_x)\left(2(1 - 2\nu) + \frac{a^2}{r^2}\right)\cos 2\theta\right\}$$

$$u_\theta = -\frac{a^2}{4Gr}\left\{(P_y - P_x)\left(2(1 - 2\nu) + \frac{a^2}{r^2}\right)\sin 2\theta\right\}$$

Circular hole in infinite plate



In case : Lithostatic Stress Field
(Uniform Stress Field)

$P_x = P_y$ =P= Stress Field

$$\sigma_r = P\left(1 - \frac{a^2}{r^2}\right)$$

$$\sigma_\theta = P\left(1 + \frac{a^2}{r^2}\right)$$

$$\tau_{r\theta} = 0$$

Topic 8 : ROCK MECHANICS LABORATORY

Out Line

Lab. 1: Rock Specimen Collection and Preparation
Lab. 2: Uniaxial Compressive Strength Testing
Lab. 3: Triaxial Compressive Strength Testing
Lab. 4: Brazilian Tensile Strength Testing
Lab. 5: Direct Shear Strength Testing
Lab. 6: Point Load Index Strength Testing
Lab. 7: Slake Durability Testing
Lab. 8: Dynamic Velocity Testing

Standard Testing

ASTM
- American Standards for Testing and Materials

ISRM
- International Society for Rock Mechanics

BS
- British Standards

Standard Codes (ASTM)

Lab. 1: Rock Specimen Collection and Preparation	**ASTM D4543**
Lab. 2: Uniaxial Compressive Strength Testing	**ASTM D7012**
Lab. 3: Triaxial Compressive Strength Testing	**ASTM D7012**
Lab. 4: Brazilian Tensile Strength Testing	**ASTM D3967**
Lab. 5: Direct Shear Strength Testing	**ASTM D5607**
Lab. 6: Point Load Index Strength Testing	**ASTM D5731**
Lab. 7: Slake Durability Testing	**ASTM D4644**
Lab. 8: Dynamic Velocity Testing	**ASTM D2845**

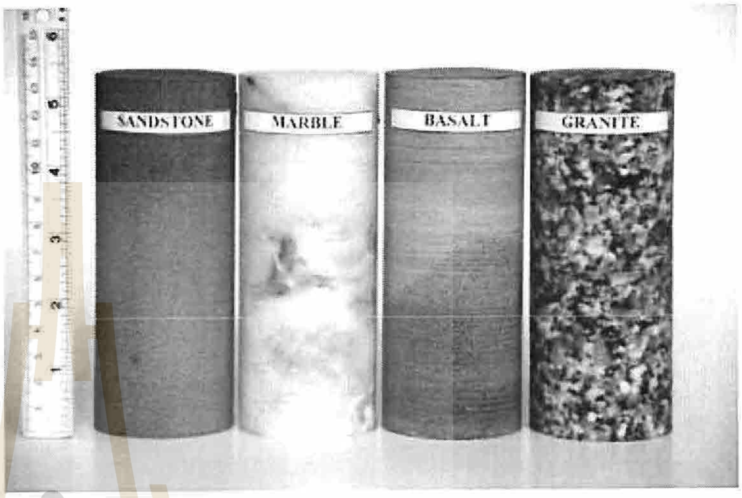
LABORATORY # 1

Rock Specimen Collection and Preparation

ASTM D4543-07 and ISRM Suggested Methods

Lab 1: Rock Specimen Collection and Preparation

Rock Samples



Rock Specimens

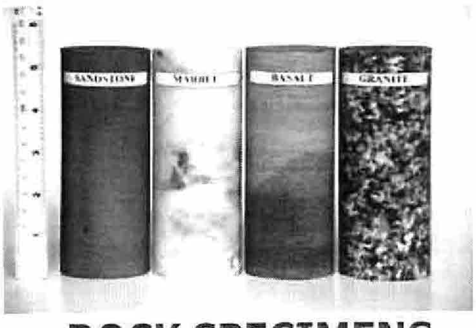
Lab 1: Rock Specimen Collection and Preparation

1) Collecting

2) Core drilling

3) Cutting

4) Grinding

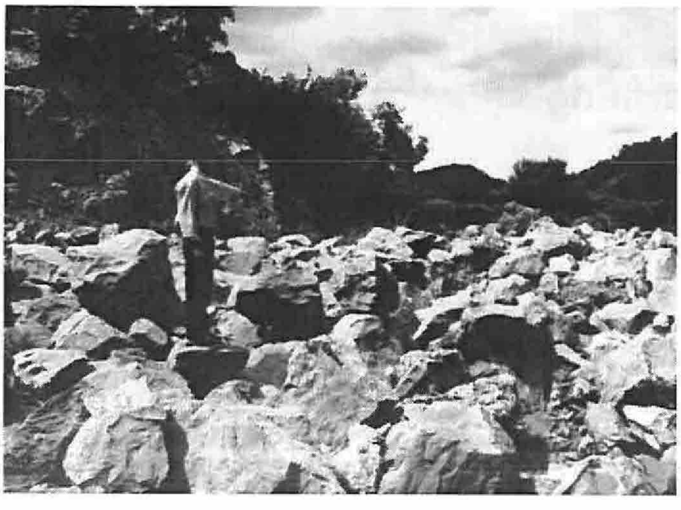


ROCK SPECIMENS

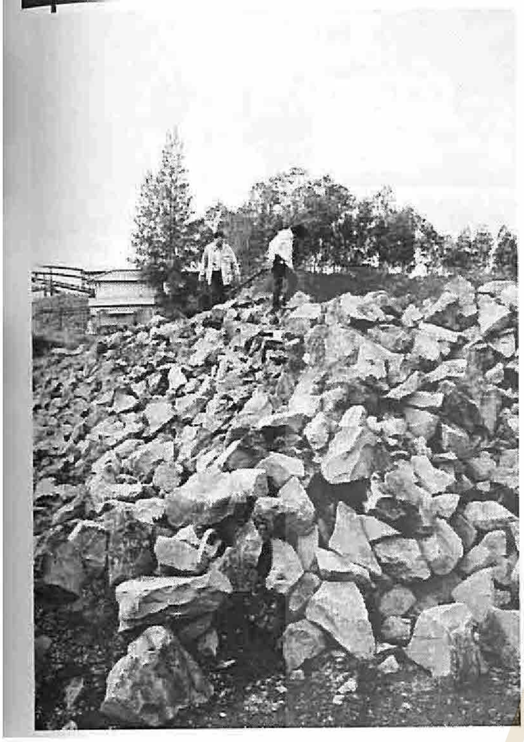
1) Sample Collection

1) Sample Collection

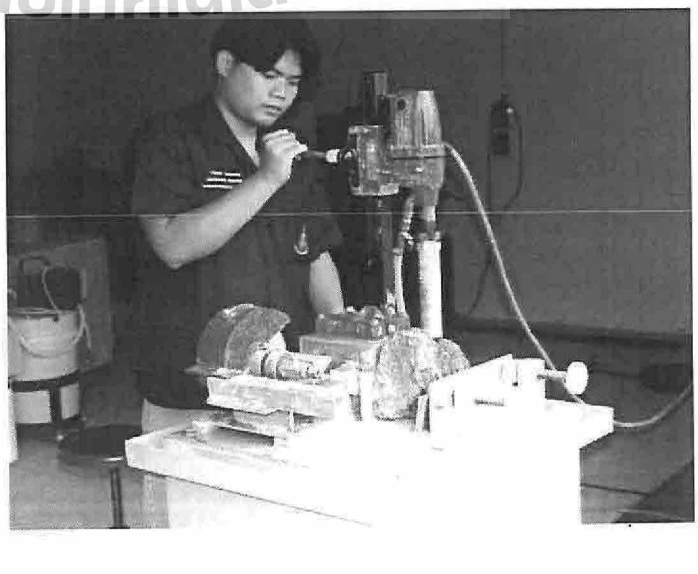
The Selection Criteria:
- should be homogeneous as much as possible
- should be convenient and repeatable



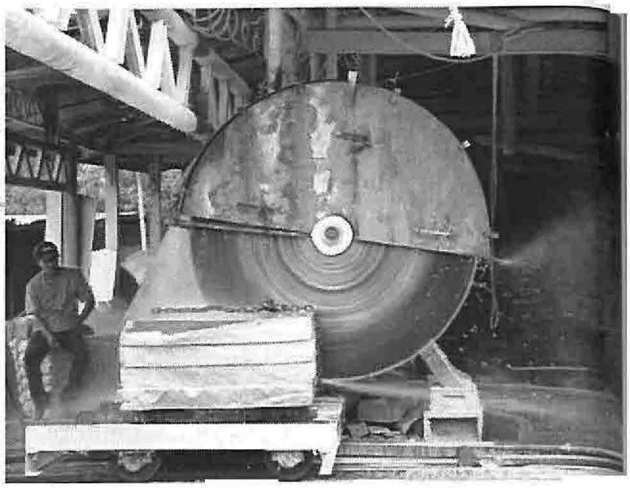
1) Sample Collection

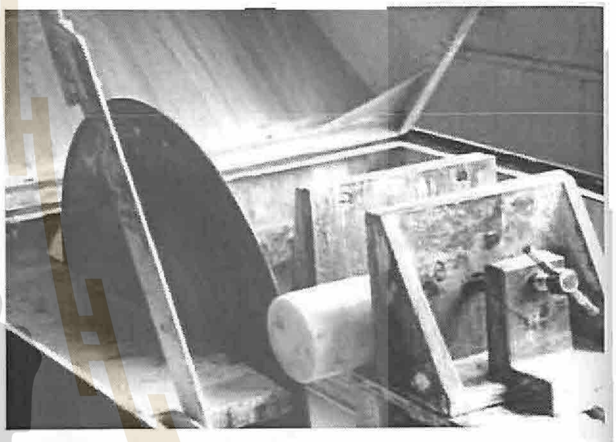


2) Core Drilling

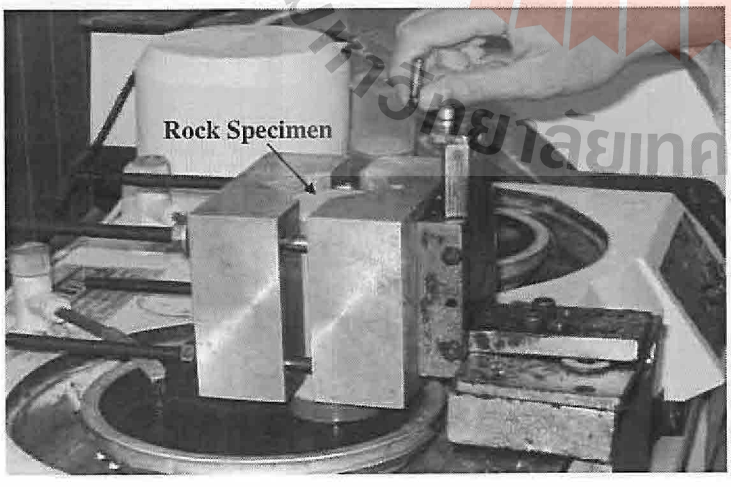


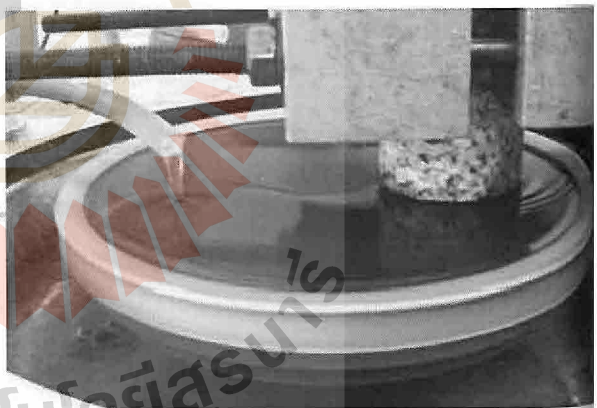
3) Cutting

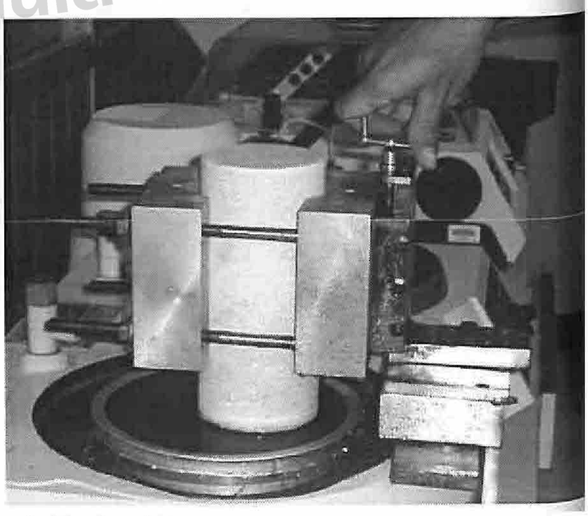




4) Grinding







Lab 1: Rock Specimen Preparation

Designation: D 4543 – 07

INTERNATIONAL

**Standard Practices for
Preparing Rock Core as Cylindrical Test Specimens and Verifying Conformance to Dimensional and Shape Tolerances[1]**

This standard is issued under the fixed designation D 4543; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

Objectives:

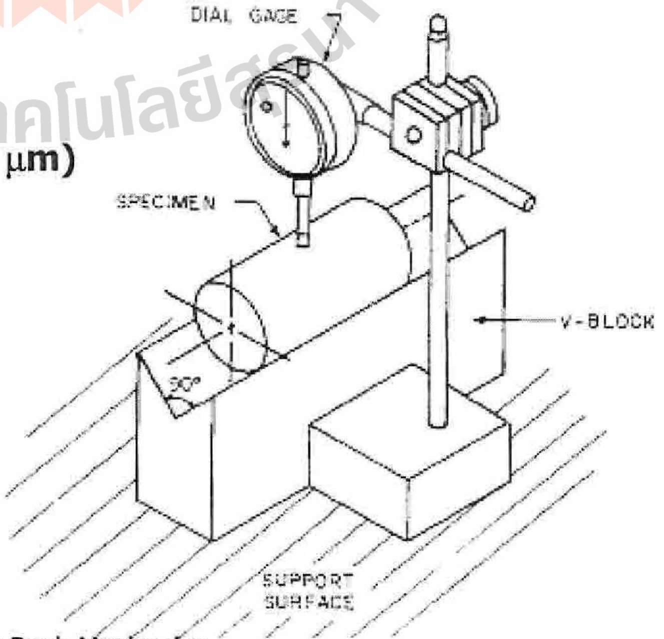
1) specimen preparing rock core
2) verifying conformance to dimensional and shape tolerances

Lab 1: Rock Specimen Preparation

Apparatus

1) Flat Surface
2) V-block
} **flat and smooth within 0.005 in. (13 μm)**

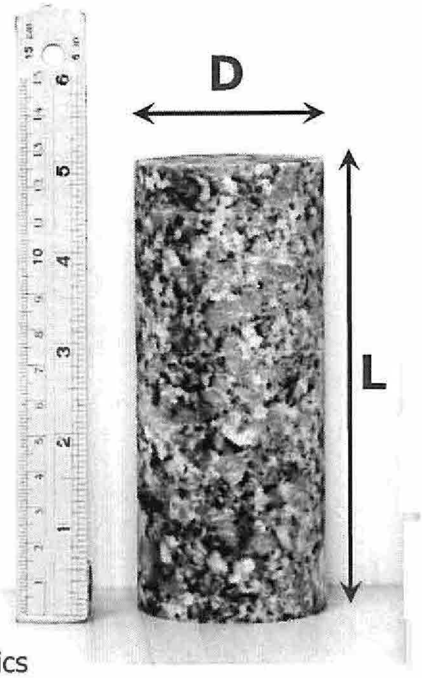
3) Dial gage
sensitivity < 0.001 in. (25 μm)



Lab 1: Rock Specimen Preparation

Specimens

L/D = 2.0-2.5
D > 1 $^{7}/_{8}$ in (47 mm)



Deviation Determination

1) Straightness / Side Smoothness
2) End Flatness & Parallelism
3) Perpendicularity

Lab 1: Rock Specimen Preparation

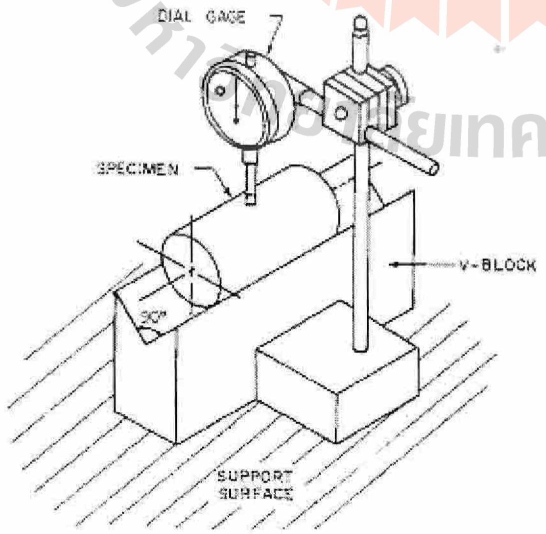


FIG. 1 Assembly for Determining the Straightness of Elements on the Cylindrical Surface (7.1.2, Procedure B)

Procedure B

Procedure A

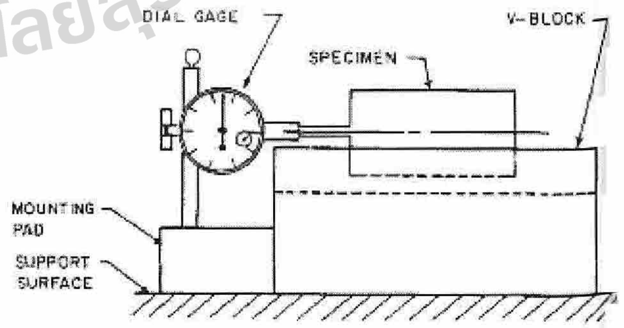
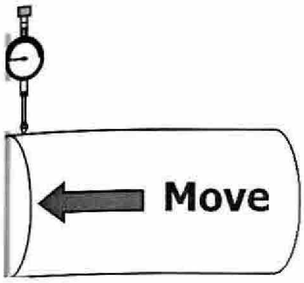


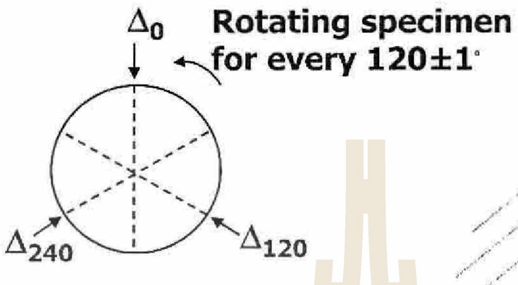
FIG. 2 Assembly for Determining the Flatness and Perpendicularity of End Surfaces to the Specimen Axis (7.2.1, Procedure A)

ASTM D4543-0

Lab 1: Rock Specimen Preparation

Straightness / Side Smoothness





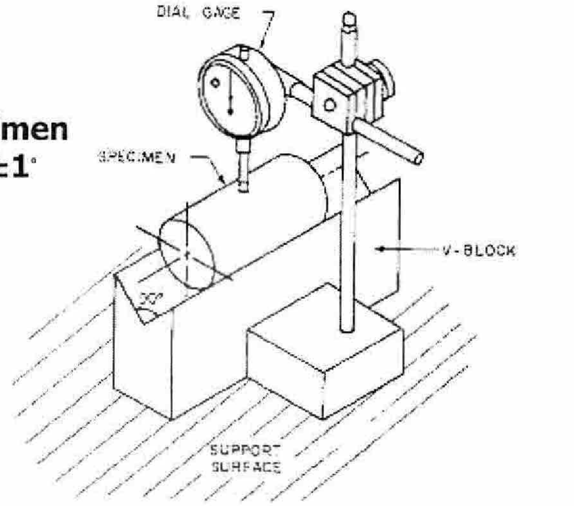


FIG. 1 Assembly for Determining the Straightness of Elements on the Cylindrical Surface (7.1.2, Procedure B)

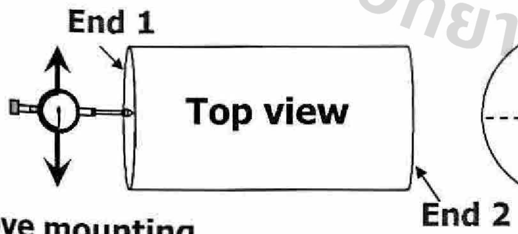
he difference (Δ)
= max reading – min reading

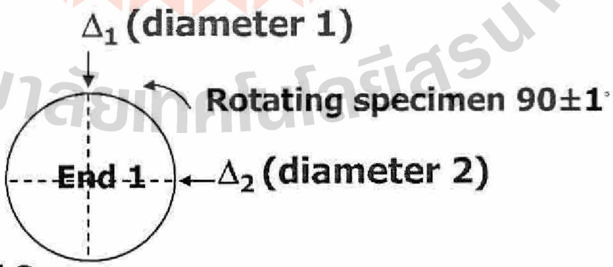
laximum value of differences (Δ) < 0.020 in (0.50 mm)

Lab 1: Rock Specimen Preparation

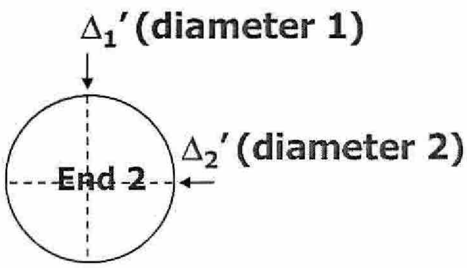
End Flatness & Parallelism

The difference (Δ)
Δ = max reading – min reading





ove mounting
ad horizontally,
eading every 1/8 in (3 mm)



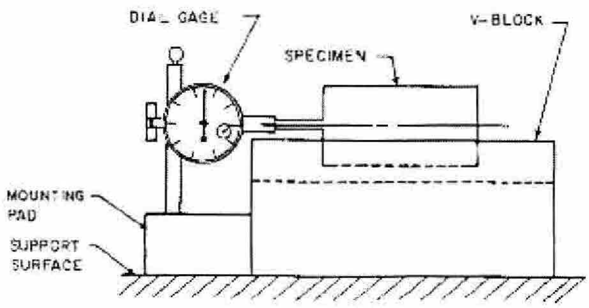
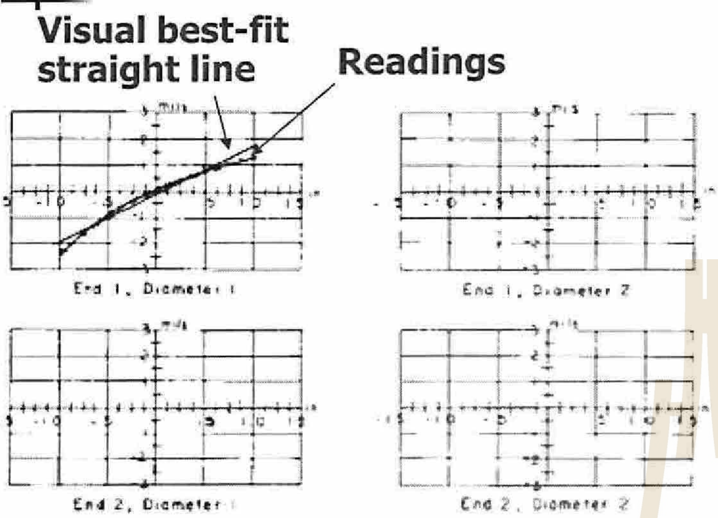


FIG. 2 Assembly for Determining the Flatness and Perpendicularity of End Surfaces to the Specimen Axis (7.2.1, Procedure A)

Lab 1: Rock Specimen Collection and Preparation



Difference between maximum and minimum readings for Diameter 1, End 1 = Δ_1 = _______
Difference for Diameter 2, End 1 = Δ_2 = _______
Difference for Diameter 1, End 2 = Δ_1' = _______
Difference for Diameter 2, End 2 = Δ_2' = _______
Use the largest of the four Δ, Δ_{max} = _______
Perpendicularity tolerance is met when $\frac{\Delta_{max}}{diam} \leq 0.0043$

FIG. 3 Suggested Format for Presenting Tolerance-Check Data

The Flatness tolerance

Difference b/w each smooth curve and visual best-fit straight line < 0.001 in. (25 μm

The Parallelism tolerance

Maximum angular difference b/w opposing best-fit straight line on each specimen end < 0.25 °
(for spherical seated test machine)
< 0.13 °
(for fixed end test machine)

Lab 1: Rock Specimen Collection and Preparation

3) Perpendicularity

$$\boxed{\frac{\Delta_i}{d} \text{ and } \frac{\Delta_i'}{d} \leq \frac{1}{230} = 0.0043}$$

$\tan^{-1}(0.0043) = 0.25°$

where:
i = 1 or 2,
d = diameter, and
Δ_i = diameter difference.

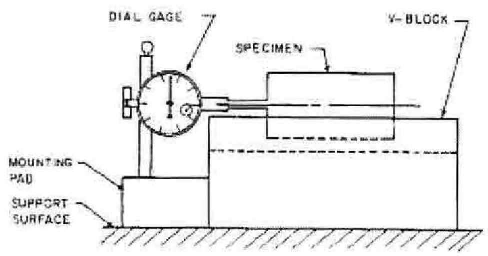
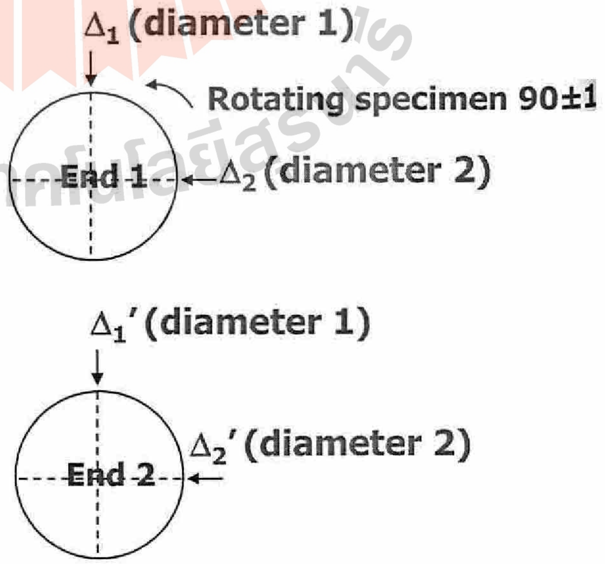


FIG. 2 Assembly for Determining the Flatness and Perpendicularity of End Surfaces to the Specimen Axis (7.2.1. Procedure A)



The difference (Δ)
Δ = max reading – min reading

Rock Specimen designation coding system

MB – 10 – 01 – UCS – 01

Rock Type.

- BA – Burirum Basalt
- GST – Phu Kradung Sandstone
- GR – Tak Granite
- MB – Saraburi Marble
  - – IR (Irregular Shape)
  - – SQ (Square Disk Shape)
- YST – Phu Phan Sandstone
- BD – Maha Sarakham Salt

Block No.

Core No.

Test Type

- BZ – Brazilian Tensile Strength Test
- CPL – Conventional Point Load Test
- MPL – Modified Point Load Test
- TR – Triaxial Compressive Strength Test
- UCS – Uniaxial Compressive Strength Test

Specimen No.

LABORATORY # 2

Uniaxial Compressive Strength Testing

ASTM D7012-07 and ISRM Suggested Methods

Lab 2: Uniaxial Compressive Strength Testing

Designation: D 7012 – 07

INTERNATIONAL

Standard Test Method for
Compressive Strength and Elastic Moduli of Intact Rock Core Specimens under Varying States of Stress and Temperatures[1]

This standard is issued under the fixed designation D 7012; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

Objectives: to determine

1) **Uniaxial Compressive Strength**
2) **Elastic Modulus (Young's Modulus)**
3) **Poisson's Ratio**

Lab 2: Uniaxial Compressive Strength Testing

ASTM D7012-07

1.1.1 This standard replaces and combines the following Standard Test Methods for: D 2664 Triaxial Compressive Strength of Undrained Rock Core Specimens Without Pore Pressure Measurements; D 5407 Elastic Moduli of Undrained Rock Core Specimens in Triaxial Compression Without Pore Pressure Measurements; $\boxed{\text{D 2938}}$ Unconfined Compressive Strength of Intact Rock Core Specimens; and $\boxed{\text{D 3148}}$ Elastic Moduli of Intact Rock Core Specimens in Uniaxial Compression.

Lab 2: Uniaxial Compressive Strength Testing

Apparatus

1) Loading Device
2) Platens
3) Strain/Deformation Measuring Devices
   (Axial & Lateral Strain)
   - resolution of at least 25 x 10^{-6} strain
   - accuracy within 2%

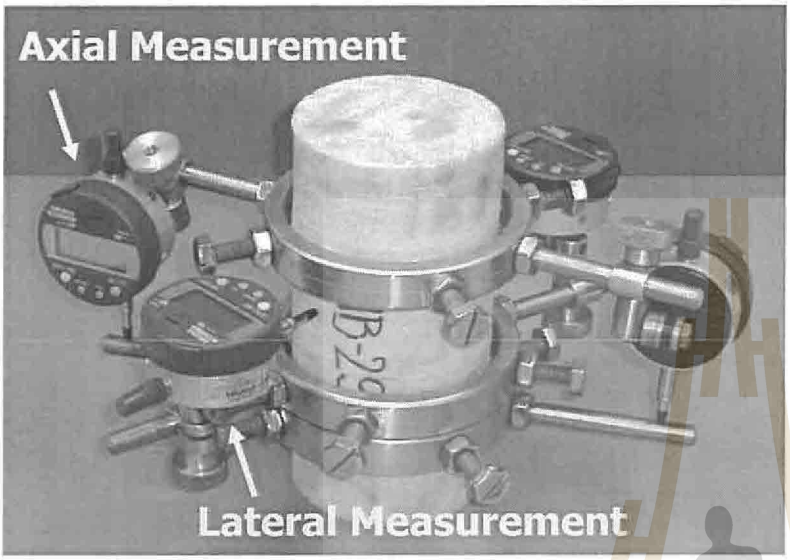
Specimens

L/D = 2.0-2.5 (ISRM Suggested, 2.5-3.0)
D > 1 $^{7}/_{8}$ in (47 mm) (ISRM Suggested, > 54 mm)

Lab 2: Uniaxial Compressive Strength Testing

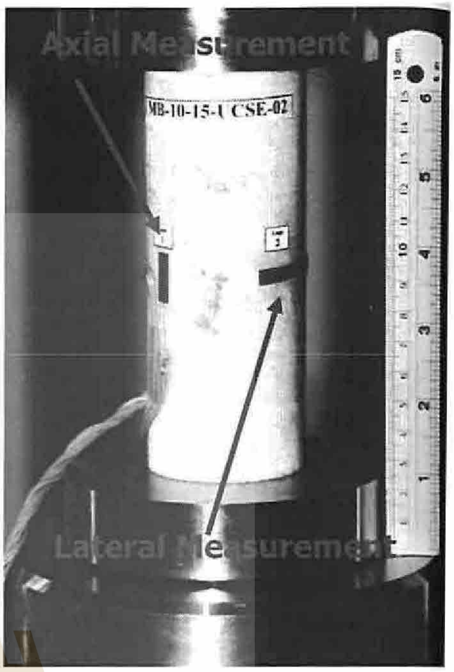
Loading Machine
- capacity > 2-4 times of failure load
- failure load within 25-75% of machine capacity
- constant loading rate of 0.5-1.0 MPa/s or failure occur within 5-10 min.

Lab 2: Uniaxial Compressive Strength Testing



Dial gages

Strain gages



Lab 2: Uniaxial Compressive Strength Testing

Uniaxial Compressive Strength

$$\boxed{\sigma_c = P_f / A}$$

P_f = Failure Load
$A = \pi D^2/4$

Axial Strain

$$\boxed{\epsilon_a = \frac{\Delta L}{L}}$$

where:
L = original undeformed axial gage length, and
ΔL = change in measured axial length (negative for decrease in length).

Lateral Strain

$$\boxed{\epsilon_l = \frac{\Delta D}{D}}$$

where:
D = original undeformed diameter, and
ΔD = change in diameter (positive for increase in diameter).

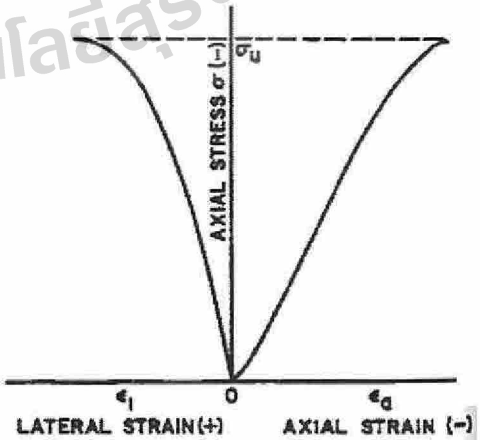
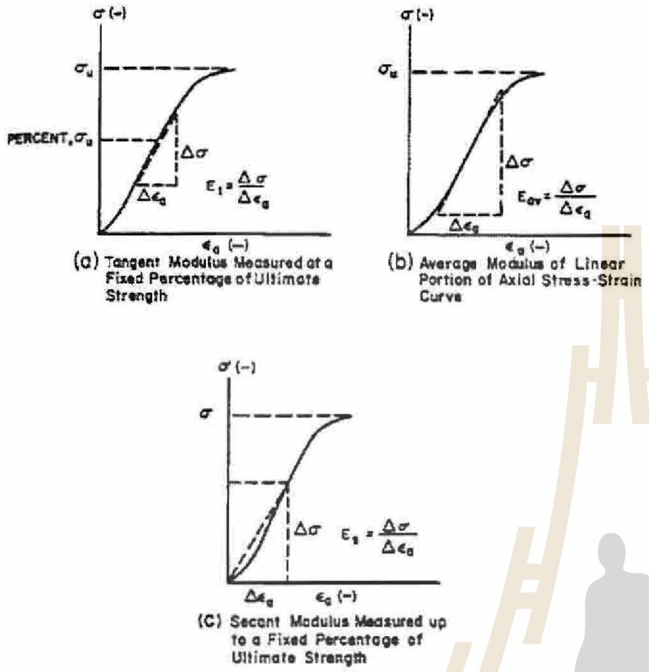


FIG. 1 Format for Graphical Presentation of Data

Lab 2: Uniaxial Compressive Strength Testing

Elastic Modulus Calculation



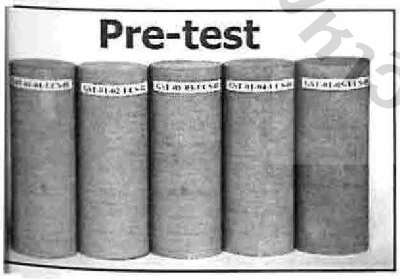
. 2 Methods for Calculating Young's Modulus from Axial Stress-Axial Strain Curve

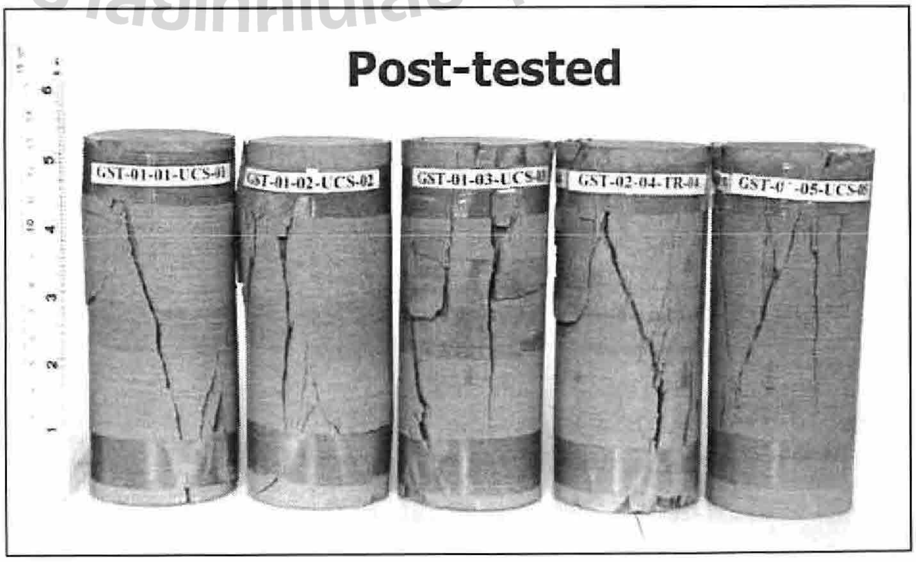
Poisson's Ration Calculation

$$\upsilon = -\frac{\text{slope of axial curve}}{\text{slope of lateral curve}} = -\frac{E}{\text{slope of lateral curve}}$$

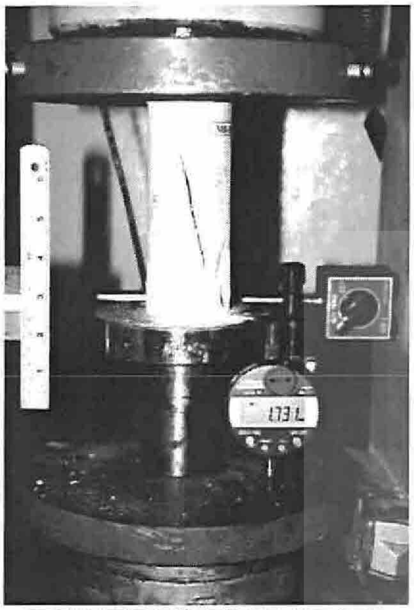
or $\nu = -\varepsilon_l / \varepsilon_a$ that measure point at 50% of compressive strength

Lab 2: Uniaxial Compressive Strength Testing





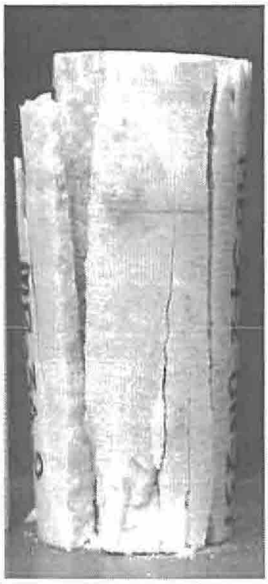
Lab 2: Uniaxial Compressive Strength Testing



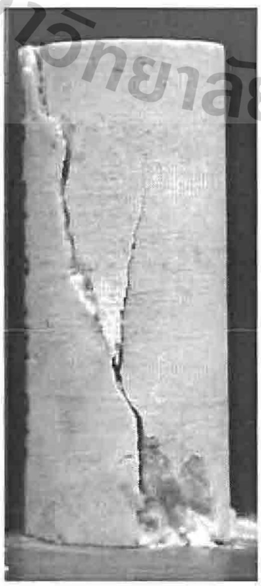
D = 54 mm (NX-size)
L/D = 2.5

Lab 2: Uniaxial Compressive Strength Testing

Modes of Failure

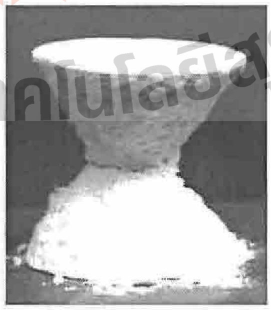


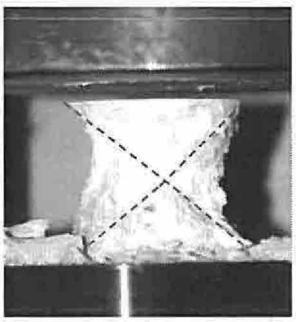
Extension Failure



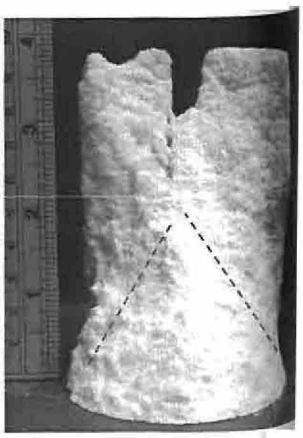
Shear Failure

Compressive Shear Failure





Conical Shape



LABORATORY # 3

Triaxial Compressive Strength Testing
ASTM D7012-07 and ISRM Suggested Methods

Lab 3: Triaxial Compressive Strength Testing

Designation: D 7012 – 07

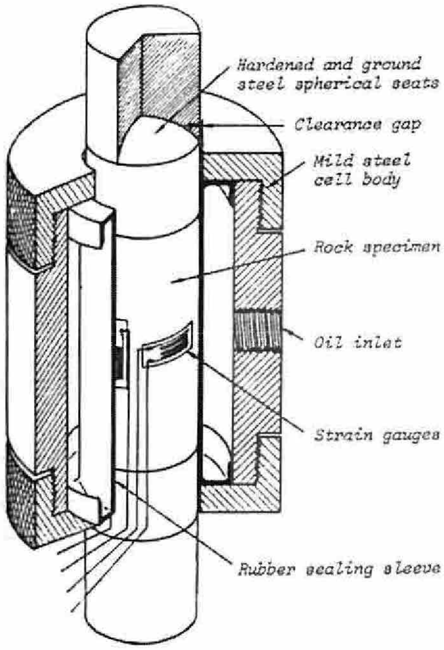
Standard Test Method for
Compressive Strength and Elastic Moduli of Intact Rock Core Specimens under Varying States of Stress and Temperatures[1]

Objectives: to determine

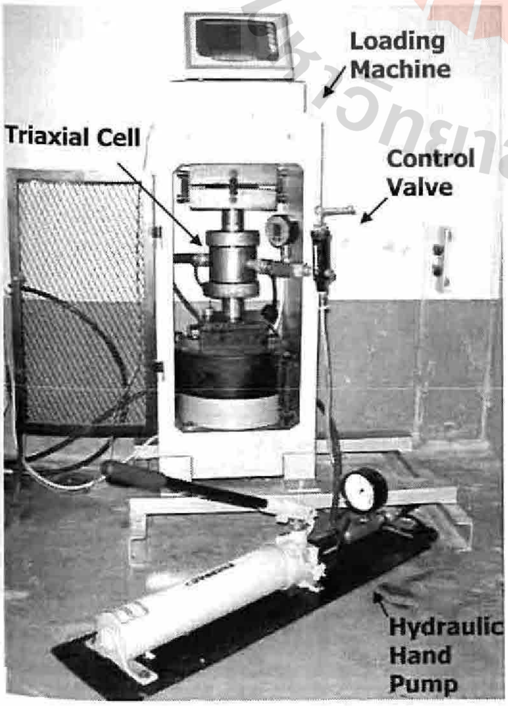
1) Triaxial Compressive Strength
2) Elastic Modulus (Young's Modulus)
3) Poisson's Ratio
4) Triaxial Strength Criterion

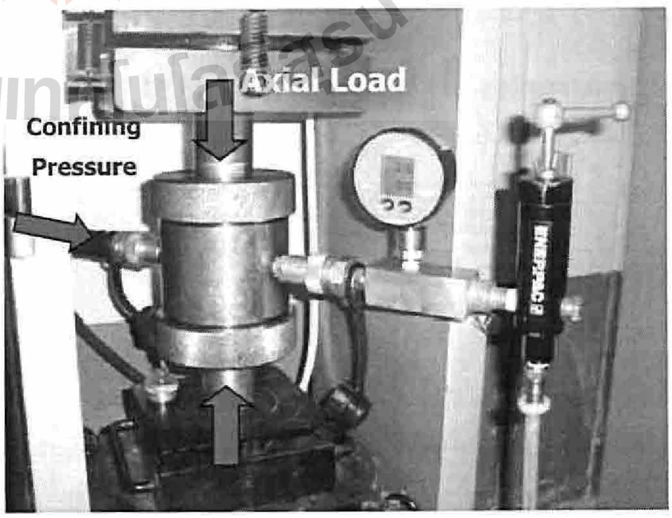
Lab 3: Triaxial Compressive Strength Testing

Triaxial Cell (Hoek Cell)



Lab 3: Triaxial Compressive Strength Testing





Lab 3: Triaxial Compressive Strength Testing

ASTM D7012-07

1.1.1 This standard replaces and combines the following Standard Test Methods for: D 2664 Triaxial Compressive Strength of Undrained Rock Core Specimens Without Pore Pressure Measurements; D 5407 Elastic Moduli of Undrained Rock Core Specimens in Triaxial Compression Without Pore Pressure Measurements; D 2938 Unconfined Compressive Strength of Intact Rock Core Specimens; and D 3148 Elastic Moduli of Intact Rock Core Specimens in Uniaxial Compression.

Lab 3: Triaxial Compressive Strength Testing

Apparatus

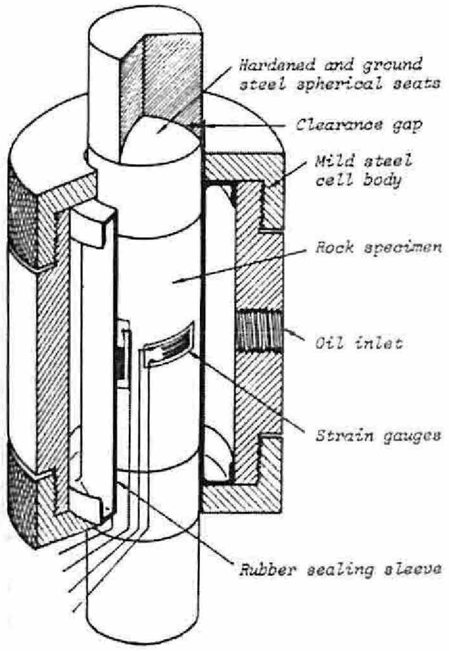
1) Loading Device & Platens
2) Triaxial Cell (Hoek Cell)
3) Hydraulic Pump & Control Valve
4) Strain/Deformation Measuring Devices
   (Axial & Lateral Strain)
   - resolution of at least 25 x 10^{-6} strain
   - accuracy within 2%

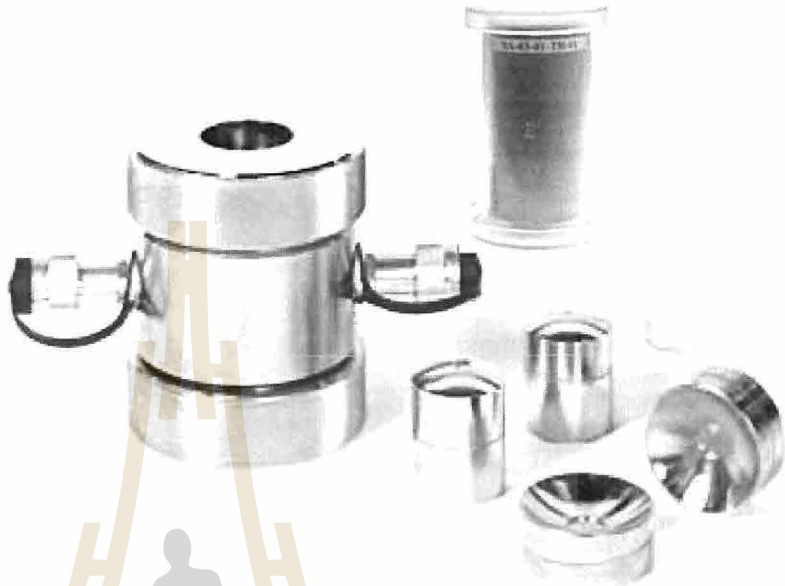
Specimens

L/D = 2.0-2.5 (ISRM Suggested, 2.5-3.0)
D > 1 $^7/_8$ in (47 mm) (ISRM Suggested, > 54 mm)

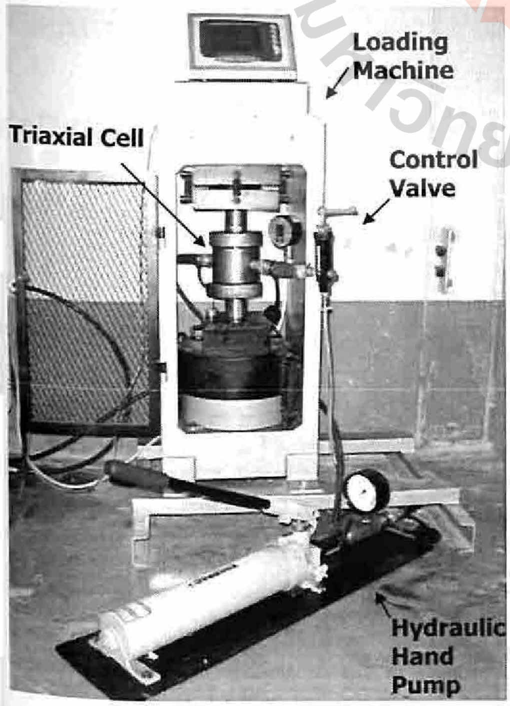
Lab 3: Triaxial Compressive Strength Testing

Triaxial Cell (Hoek Cell)





Lab 3: Triaxial Compressive Strength Testing



Lab 3: Triaxial Compressive Strength Testing

Triaxial Compressive Strength

$$\sigma = (\sigma_1 - \sigma_3)$$

where:
σ = differential failure stress,
σ_1 = total failure stress, and
σ_3 = confining stress.

Axial Strain

$$\epsilon_a = \frac{\Delta L}{L}$$

where:
L = original undeformed axial gage length, and
ΔL = change in measured axial length (negative for decrease in length).

Lateral Strain

$$\epsilon_1 = \frac{\Delta D}{D}$$

where:
D = original undeformed diameter, and
ΔD = change in diameter (positive for increase in diameter).

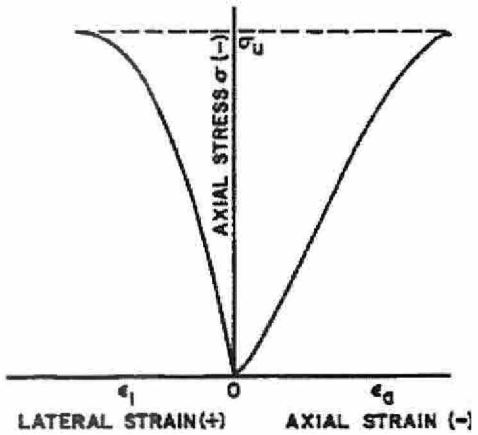


FIG. 1 Format for Graphical Presentation of Da

Lab 3: Triaxial Compressive Strength Testing

Elastic Modulus Calculation

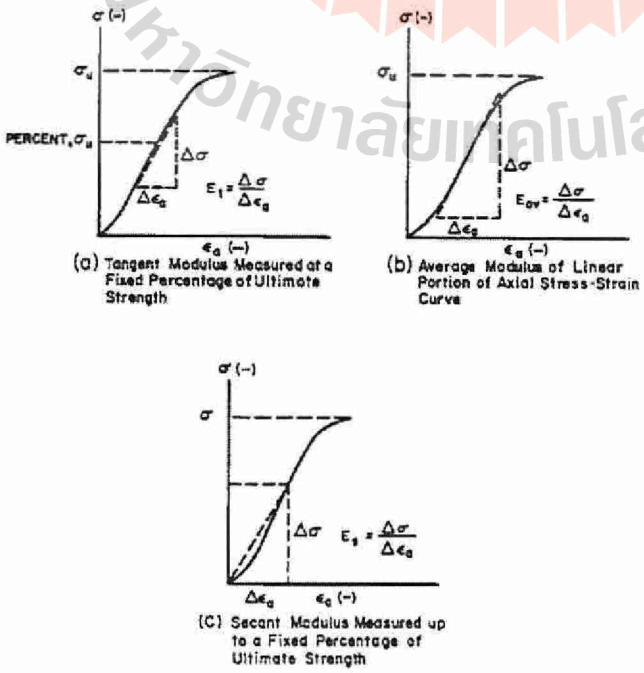


FIG. 2 Methods for Calculating Young's Modulus from Axial Stress-Axial Strain Curve

Poisson's Ration Calculation

$$\upsilon = -\frac{\text{slope of axial curve}}{\text{slope of lateral curve}} = -\frac{E}{\text{slope of lateral curve}}$$

or $\nu = -\varepsilon_l / \varepsilon_a$
that measure point at 50% of compressive strength

Lab 3: Triaxial Compressive Strength Testing

Failure Criteria: Mohr Stress Plot

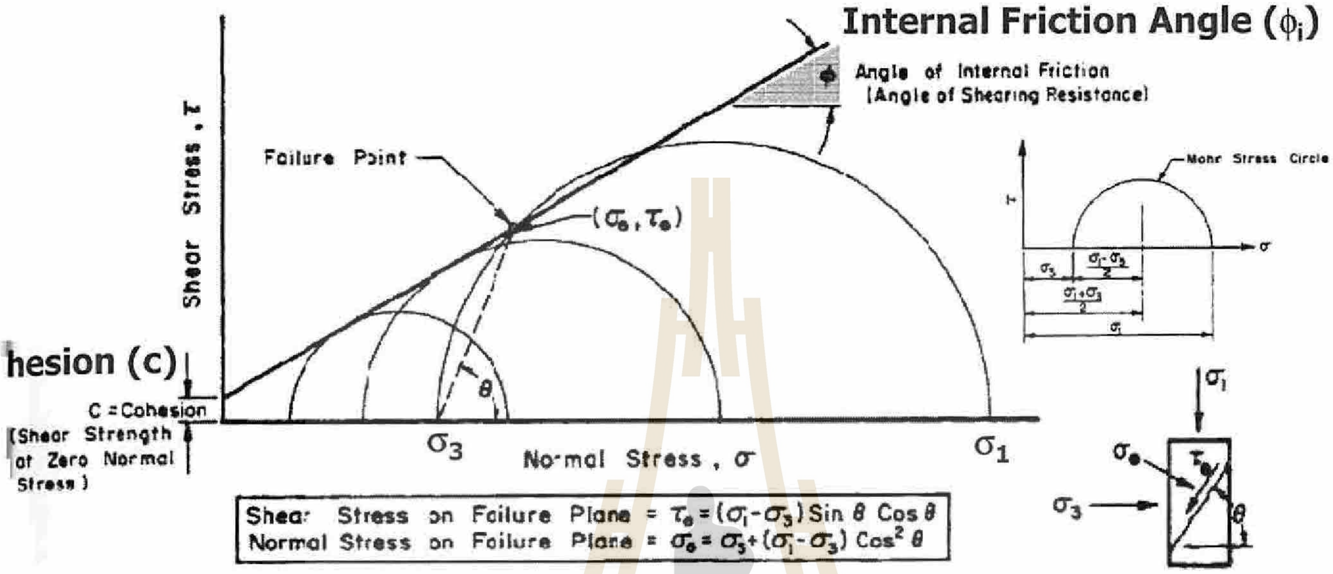


FIG. 3 Typical Mohr Stress Circles

Lab 3: Triaxial Compressive Strength Testing

Failure Criteria: σ_1 vs. σ_3 Plot

The Strength of Rock Materials in Triaxial Compression

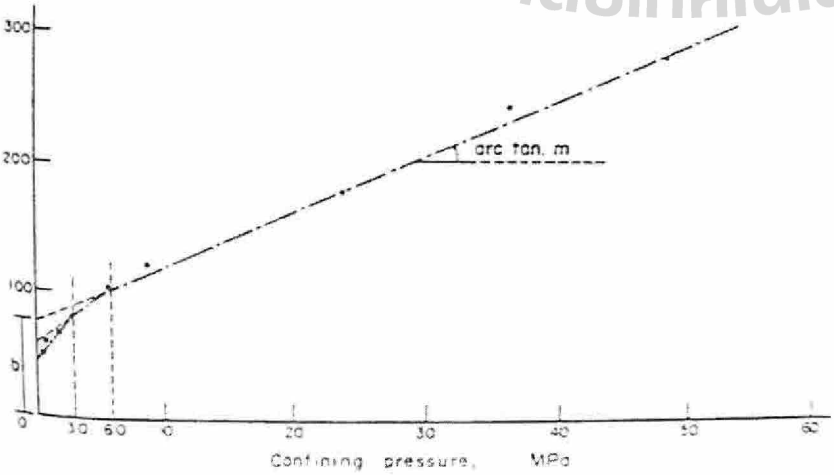


Fig. 2. Strength envelope

ISRM Suggested

Internal Friction Angle

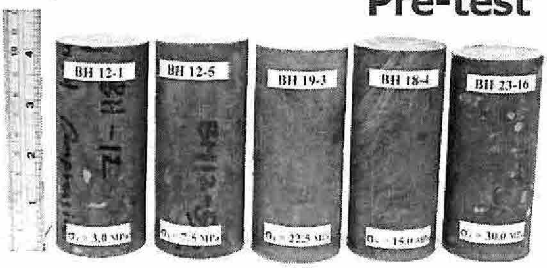
$$\phi = \arcsin \frac{m-1}{m+1};$$

cohesion

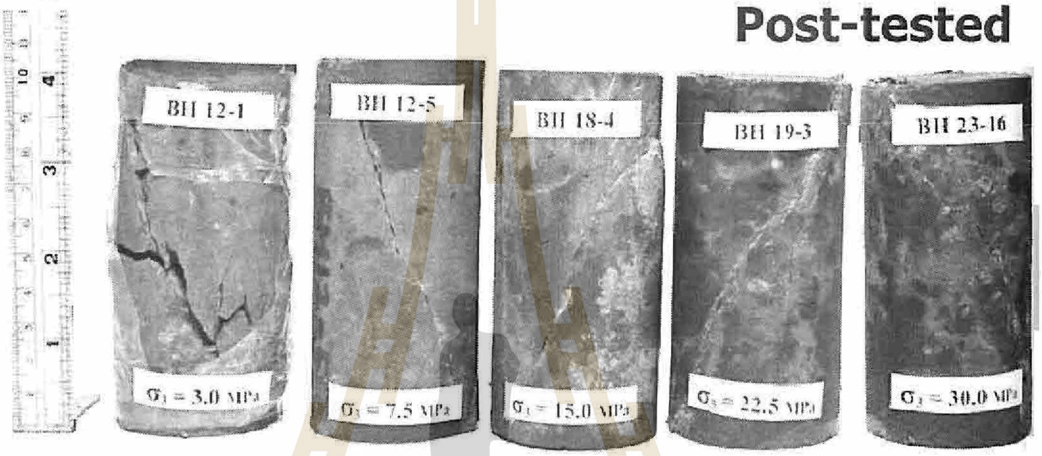
$$C = b\,\frac{1-\sin\phi}{2\cos\phi}.$$

Lab 3: Triaxial Compressive Strength Testing

Pre-test

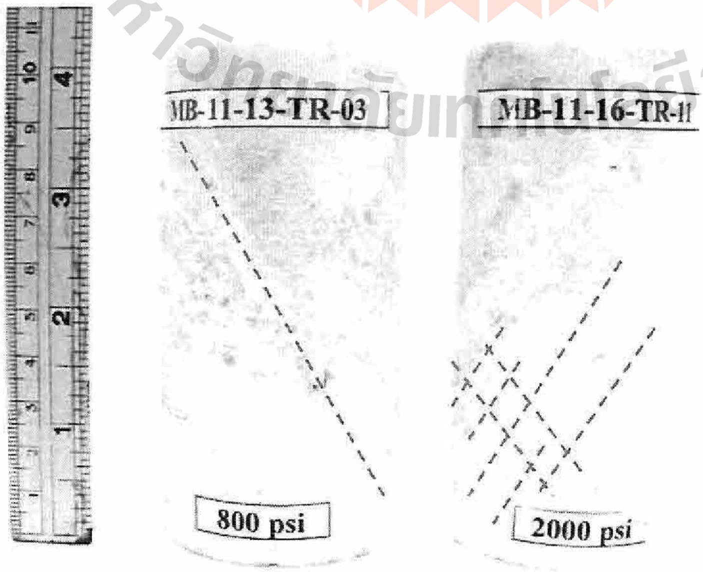


Post-tested



Lab 3: Triaxial Compressive Strength Testing

Post-tested specimens of the triaxial tests



Single Shear Failure

Multiple Shear Failure

LABORATORY # 4

Brazilian Tensile Strength Testing
ASTM D3967-05 and ISRM Suggested Methods

Lab 4: Brazilian Tensile Strength Testing

Designation: D 3967 – 05

Standard Test Method for
Splitting Tensile Strength of Intact Rock Core Specimens[1]

This standard is issued under the fixed designation D 3967; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

Objectives: to determine tensile strength (indirect) of intact rock

Lab 4: Brazilian Tensile Strength Testing

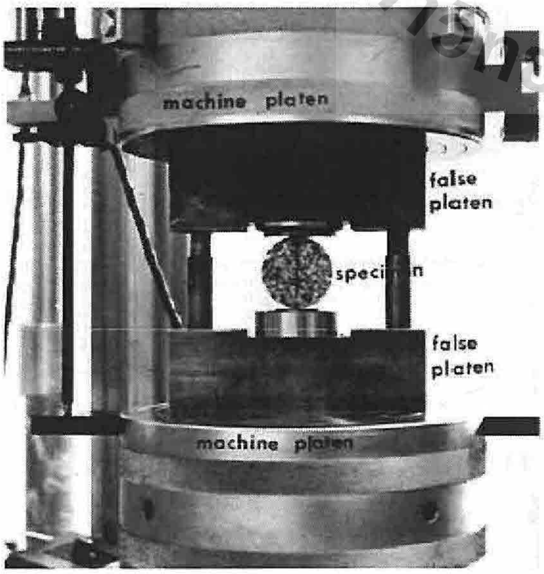
Apparatus

1) Loading Device & Platens
2) Flat Bearing Blocks or Curved Bearing Blocks
3) Venire Calliper 0.01 in

Specimens

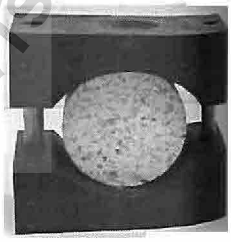
L/D = 0.2-0.75 (ISRM Suggested, 0.5)
D > 1 $^{7}/_{8}$ in (50 mm) (ISRM Suggested, > 54 mm)
at least 10 specimens

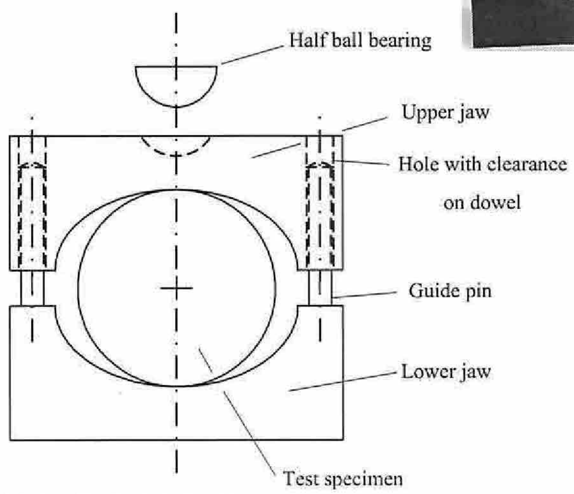
Lab 4: Brazilian Tensile Strength Testing



Flat Bearing Blocks

Curved Bearing Blocks





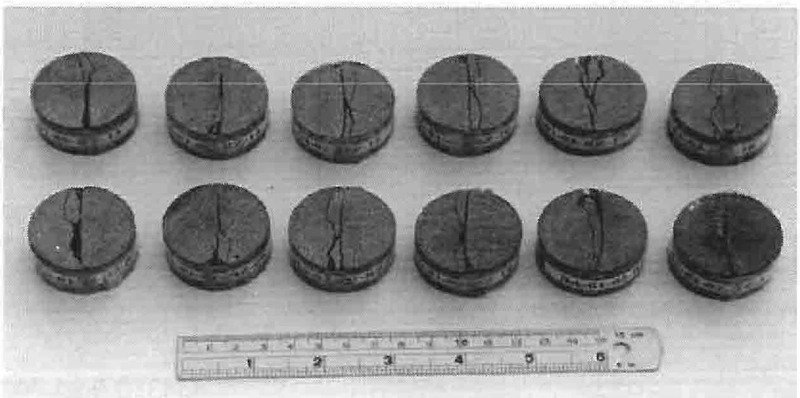
Lab 4: Brazilian Tensile Strength Testing

Pre-test

Post-tested

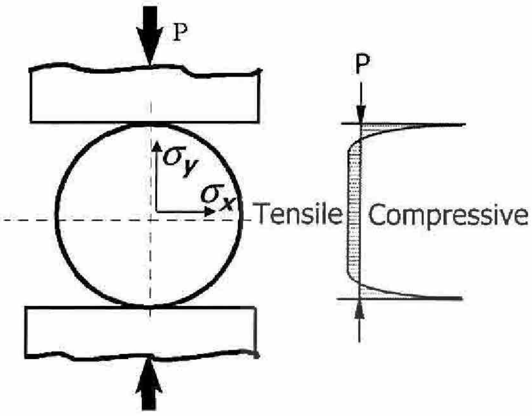
Lab 4: Brazilian Tensile Strength Testing

Pre-test



Post-tested

Lab 4: Brazilian Tensile Strength Testing



$\sigma_y = 3\sigma_x$

Compression

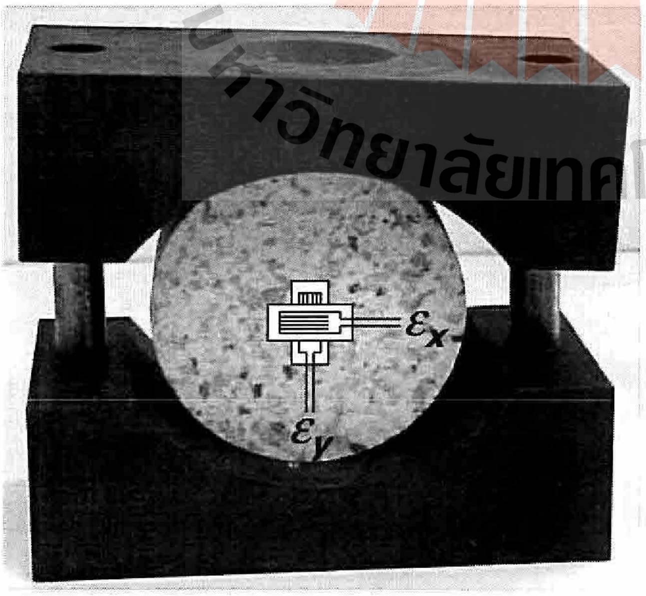
$$\boxed{\sigma_y = 6P/\pi LD}$$

Tension

$$\boxed{\sigma_x \text{ or } \sigma_t \text{ or } \sigma_B = 2P/\pi LD}$$

σ_t = splitting tensile strength, MPa (psi),
P = maximum applied load indicated by the testing machine, N (or lbf),
L = thickness of the specimen, mm (or in.), and
D = diameter of the specimen, mm (or in.).

Lab 4: Brazilian Tensile Strength Testing



Jaeger and Cook, 1979

$$E = 8\Delta P/\pi R(3\Delta\varepsilon_y+\Delta\varepsilon_x)$$

$$\nu = -(3\Delta\varepsilon_x+\Delta\varepsilon_y)/(3\Delta\varepsilon_y+\Delta\varepsilon_x)$$

where:
ΔP = change in stress
$\Delta\varepsilon_x$ = change in lateral strain
$\Delta\varepsilon_y$ = change in axial strain

LABORATORY # 5

Direct Shear Strength Testing
ASTM D5607-02 and ISRM Suggested Methods

434370 Rock Mechanics

Lab 5: Direct Shear Strength Testing

ASTM INTERNATIONAL

Designation: D 5607 – 02 (Reapproved 2006)

Standard Test Method for Performing Laboratory Direct Shear Strength Tests of Rock Specimens Under Constant Normal Force[1]

This standard is issued under the fixed designation D 5607; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

Objectives: to determine shear strength of intact rock

Lab 5: Direct Shear Strength Testing

Apparatus

1) Direct Shear Machine
2) Pressure Maintain Device
3) Displacement Measurement Device

Specimens

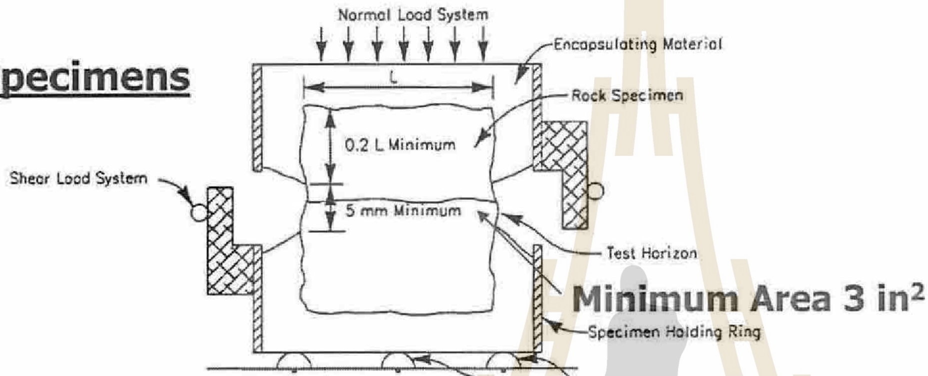
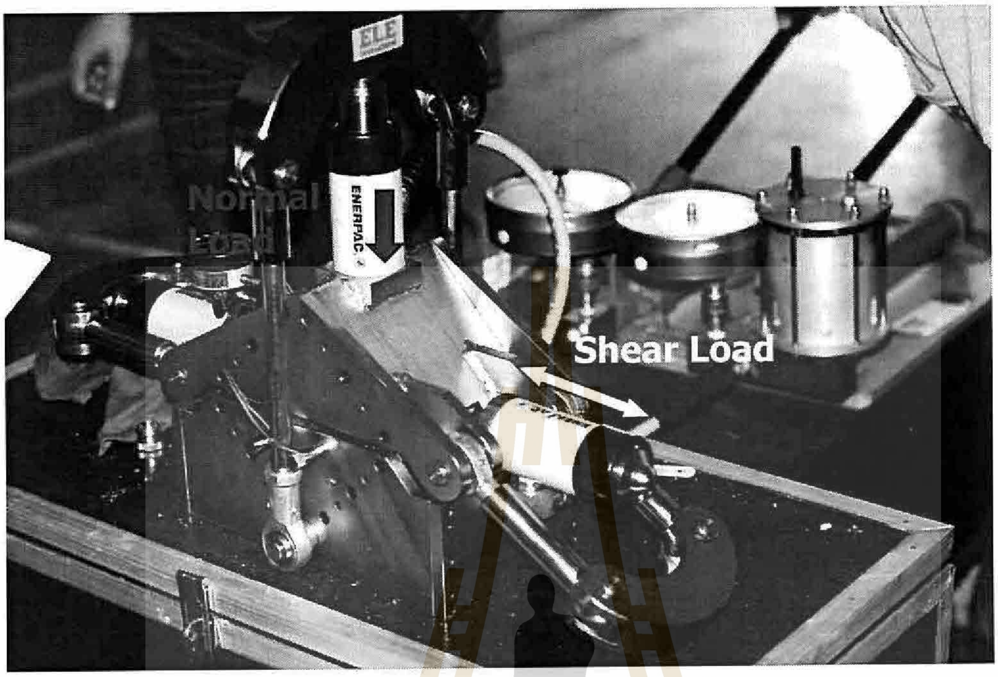


FIG. 2 Schematic Test Setup—Direct Shear Box with Encapsulated Specimen

Lab 5: Direct Shear Strength Testing

Laboratory Direct Shear Machine

Lab 5: Direct Shear Strength Testing



Portable Direct Shear Machine

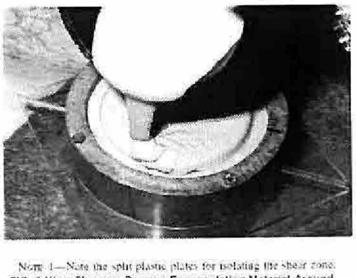
Lab 5: Direct Shear Strength Testing

FIG. 5 Specimen Supported in Place By Modeling Clay Pins Which Are Removed After Encapsulating Material Cures and the Resulting Holes Filled With Encapsulating Material

NOTE 1—In both Fig. 5 and Fig. 6 the shear box is cylindrical. Square boxes work just as well.

FIG. 6 Lower Half of a Specimen Encapsulated in Holding Ring

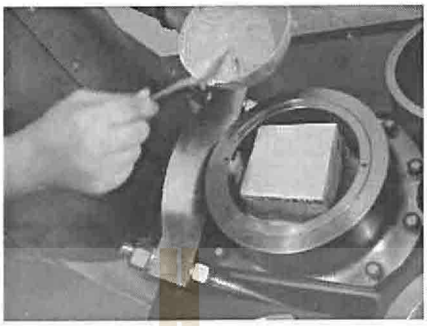
FIG. 7 Removing Spacer Plates After Encapsulating Material Has Cured

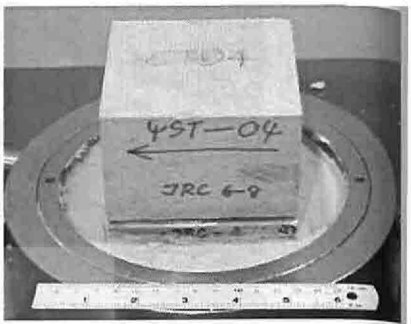


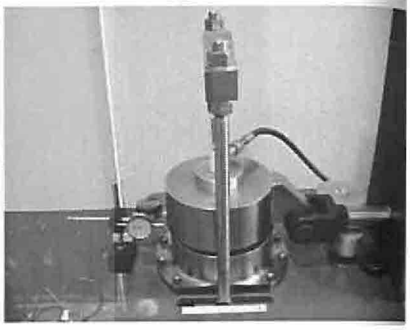
NOTE 1—Note the split plastic plates for isolating the shear zone.

FIG. 3 View Showing Pouring Encapsulating Material Around Upper Half of Specimen

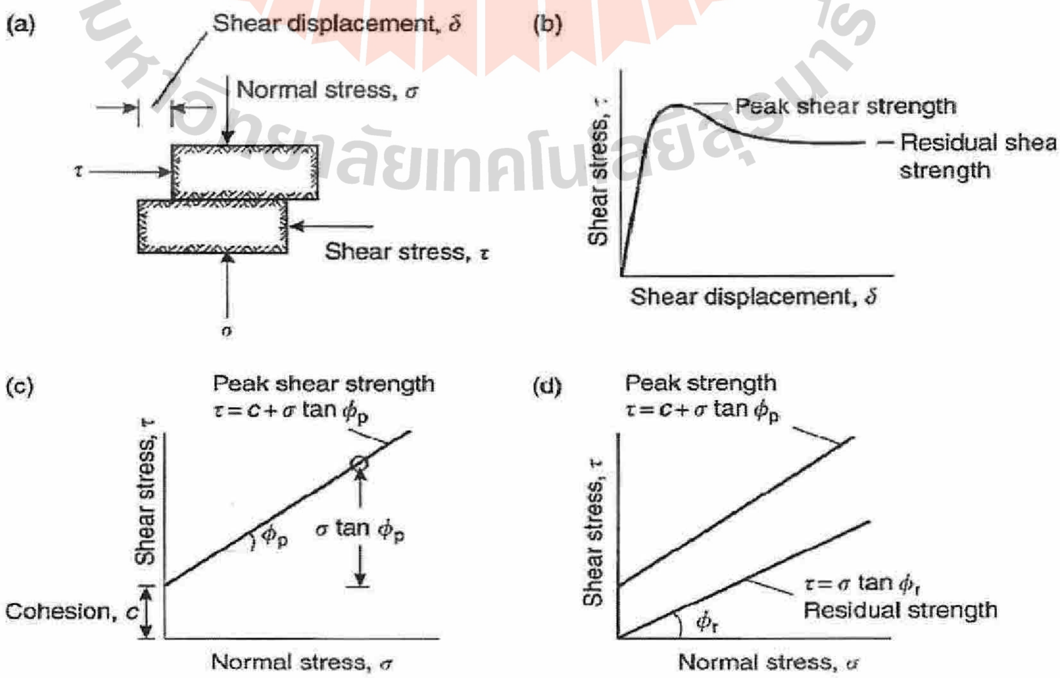
Lab 5: Direct Shear Strength Testing







Lab 5: Direct Shear Strength Testing



Lab 5: Direct Shear Strength Testing

Calculate the following engineering stresses:

$$\text{Apparent normal stress } \sigma = \frac{P_n}{A}$$

$$\text{Apparent shear stress } \tau = \frac{P_s}{A}$$

where:

P_n = normal load,
P_s = shear load, and
A = nominal initial cross-sectional area

For Core Specimens

the area is determined by:

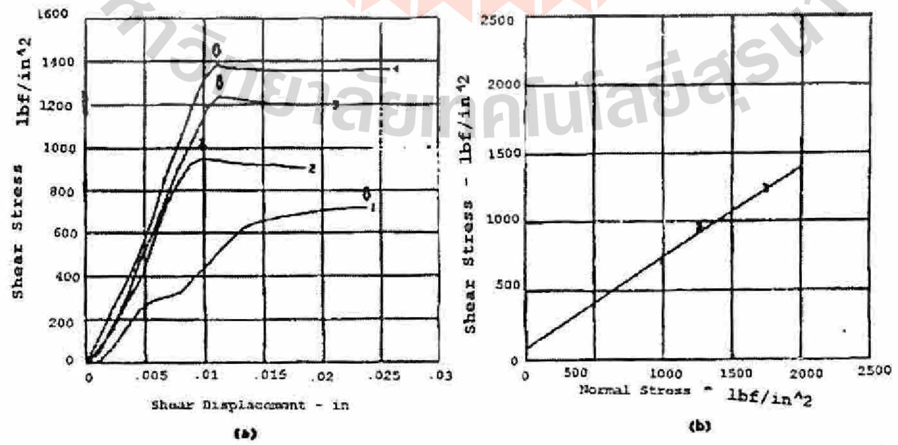
$$\boxed{A = \frac{\pi D^2}{4\cos\Theta}}$$

where:

D = core diameter, and
Θ = angle of tip.

Lab 5: Direct Shear Strength Testing

DIRECT SHEAR TEST



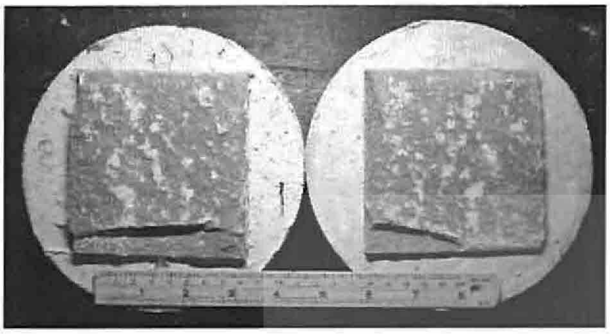
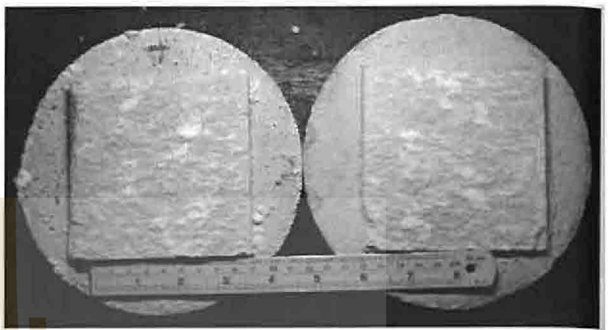
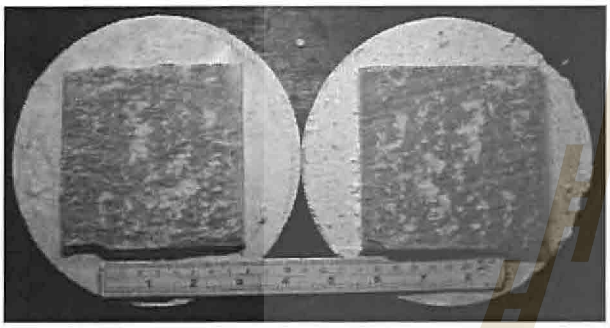
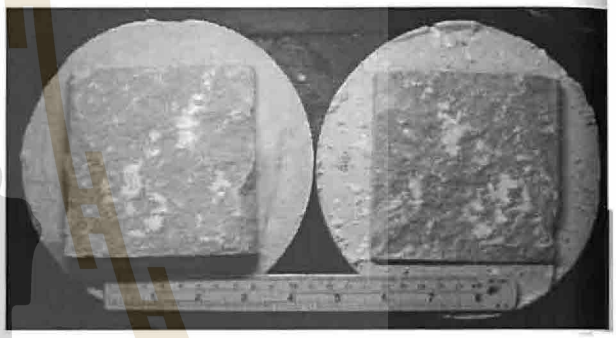
Project:
Feature:
Type:
Spec. no.:
Index no.:
Tested By:
Date Tested:
Area:

NORMAL STRESS lbf/in^2 | SHEAR STRESS lbf/in 2 | DISP in. | CYCLE NO. | SLIDING FRICTION RESULTS S= + (N) COHESION= lbf/in 2 PHI= deg COR COEF=

FIG. 8 Typical Presentation Sliding Friction Test Results: (a) Shear Stress and Shear Displacement and (b) Shear Strength and Normal Stress

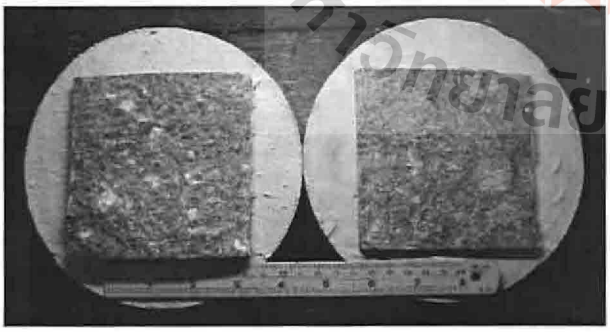
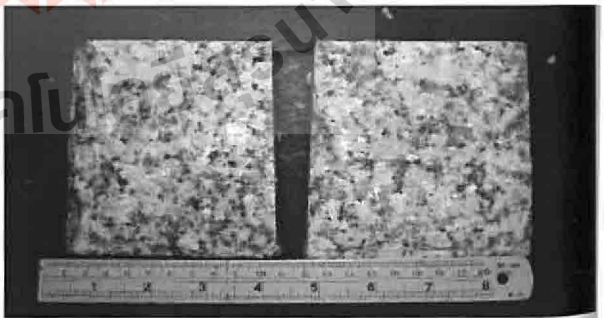
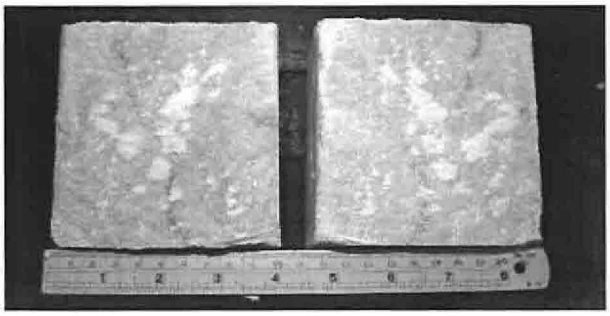
Lab 5: Direct Shear Strength Testing

Post-Tested

Lab 5: Direct Shear Strength Testing

Post-Tested


LABORATORY # 6

Point Load Index Strength Testing
ASTM D5731-07 and ISRM Suggested Methods

Lab 6: Point Load Index Strength Testing

Designation: D 5731 – 07

Standard Test Method for
Determination of the Point Load Strength Index of Rock and Application to Rock Strength Classifications[1]

This standard is issued under the fixed designation D 5731; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

Objective: to determine strength index of intact rock

Lab 6: Point Load Index Strength Testing

Apparatus

1) Point Load Testing Machine
2) Venire Calliper 0.01 in

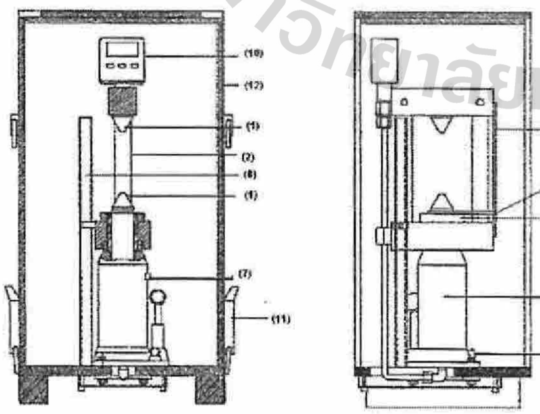
Specimens

- core/block
  at least 10 specimens

- lump (Irregular shaped specimens)
  at least 20 specimens

Lab 6: Point Load Index Strength Testing

Point Load Tester



NOTE—Load frame general information (figure 1)Load is applied to the specimens through two standard hardened points (1)Two column fixed crosshead frame (2)Scale (3)Scale pointer (4)Attached by a bolt (5) to the hydraulic pump body (6)Oil filler cap (7)The hydraulic piston assembly incorporated the oil reservoir, a single acting pump, pressure relief valve (9), and a handle (8)Pump handle (8)Pressure release valve (9)Case latched for top cover (10)Digital pressure readout (11)Point load tester top cover(12)

FIG. 1 Example of a Light-Weight Point Load Test Apparatus

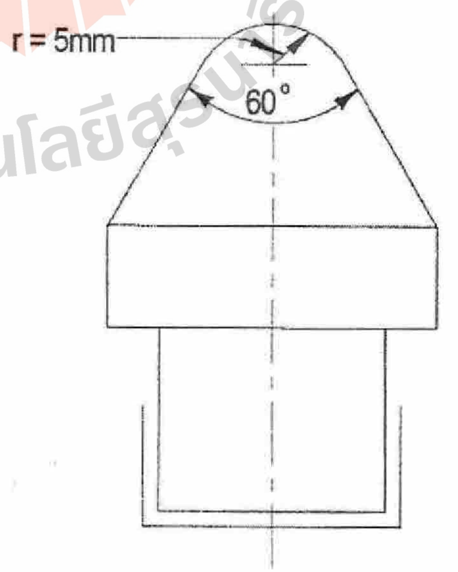
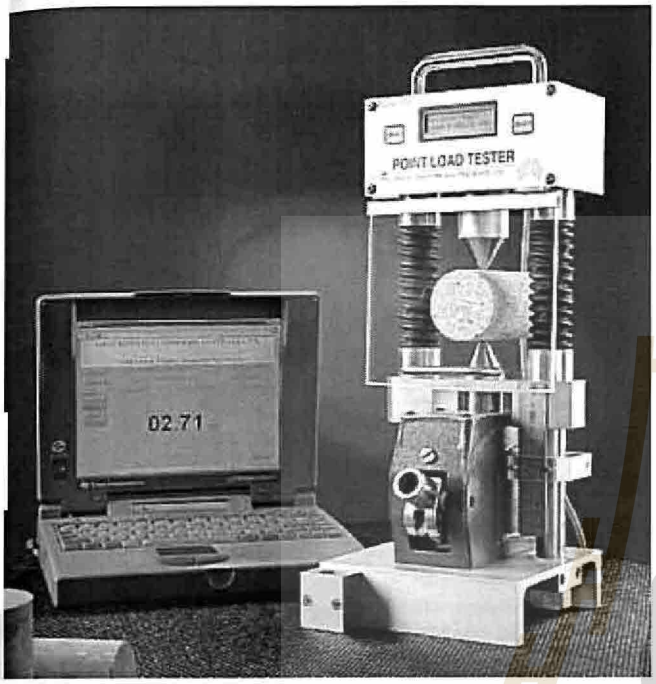


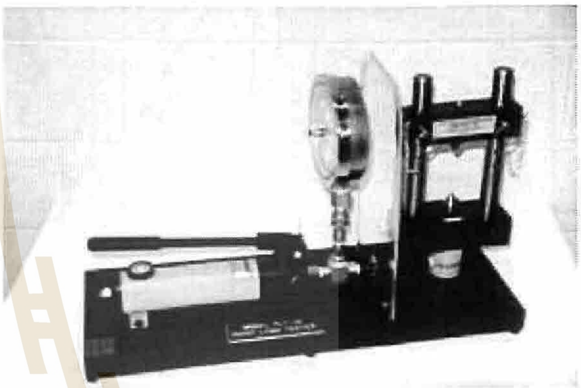
FIG. 2 Truncated, Conical Platen Dimensions for Point Load Apparatus

Point Load Platen

Lab 6: Point Load Index Strength Testing

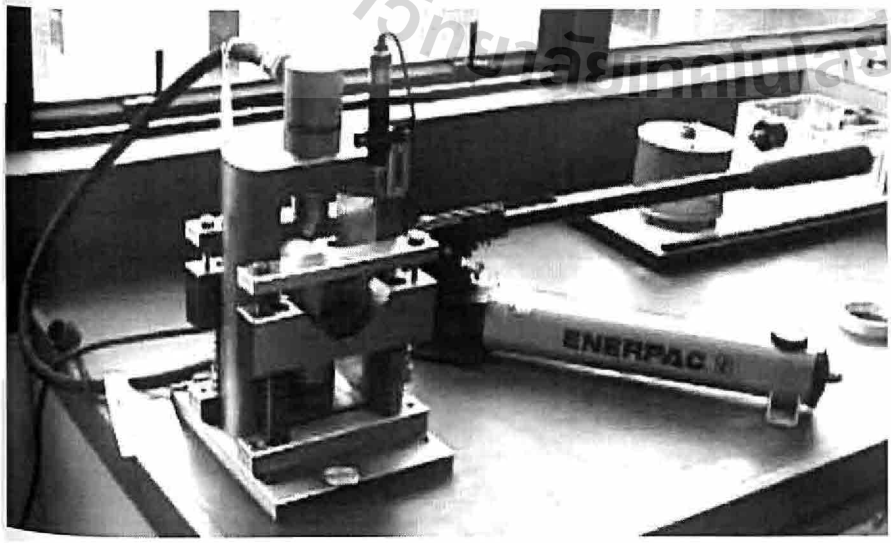
Point Load Tester





Lab 6: Point Load Index Strength Testing

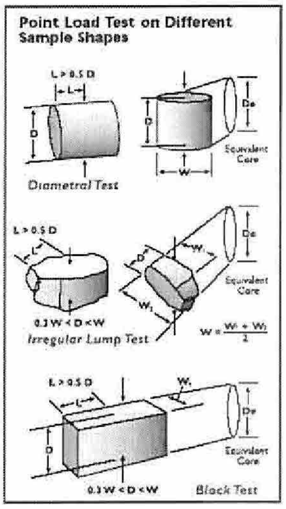
Point Load Tester

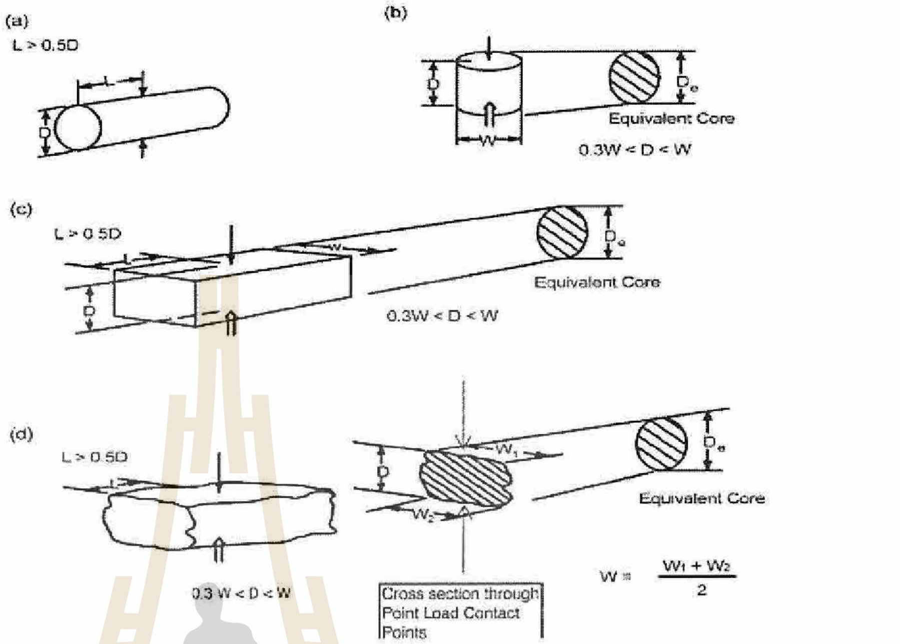


Lab 6: Point Load Index Strength Testing

Rock Specimens

$(D_e = 50 \text{ mm})$





NOTE 1—Legend: L = distance between contact points and nearest free face, and D_e = equivalent core diameter (see 10.1).

FIG. 3 Load Configurations and Specimen Shape Requirement for (a) the Diametral Test, (b) the Axial Test, (c) the Block Test, and the Irregular Lump Test[9]

Lab 6: Point Load Index Strength Testing

10.1 *Uncorrected Point Load Strength Index*—The uncorrected point load strength, I_s, is calculated as:

$$I_s = P/D_e^2, \text{ MPa} \qquad (1)$$

where:

P = failure load, N,

D_e = equivalent core diameter (see Fig. 3), mm, and is given by:

D_e^2 = D^2 for diametral core tests without penetration, mm^2, or

D_e^2 = $4A/\pi$ for axial, block, and lump tests, mm^2;

where:

A = WD = minimum cross-sectional area of a plane through the platen contact points (see Fig. 3).

Lab 6: Point Load Index Strength Testing

Size Corrected Point Load Index:

$$I_{s(50)} = F \times I_s$$

"Size Correction Factor F"

$$F = (D_e/50)^{0.45}$$

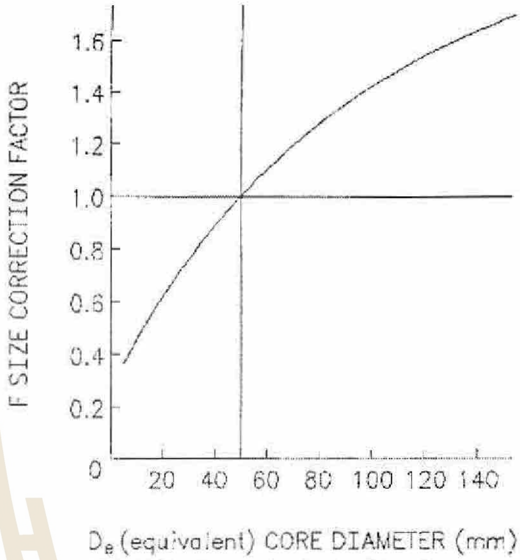


FIG. 8 Size Correction Factor Chart [8]

Lab 6: Point Load Index Strength Testing

Estimation of Uniaxial Compressive Strength

$$\sigma_c = K^* I_{s(D)}$$

where:

σ_c = uniaxial compressive strength, MPa

K = index to strength conversion factor that depends on site-specific correlation between σ_c and $I_{s(D)}$, MPa and

$I_{s(D)}$ = corrected point load strength index at a given (D).

Lab 6: Point Load Index Strength Testing

TABLE 1 Generalized Index to Strength Conversion Factor (K) for[A]

Core Size, mm	Value of "C" (Generalized)
21.5 (Ex Core)	18
30	19
42 (Bx Core)	21
50	23
54 (Nx Core)	24
60	24.5

[A]Bieniawski, Z.T. The Point-Load Test in Geotechnical Practice, Engineering Geology (9) 1-11.

Lab 6: Point Load Index Strength Testing

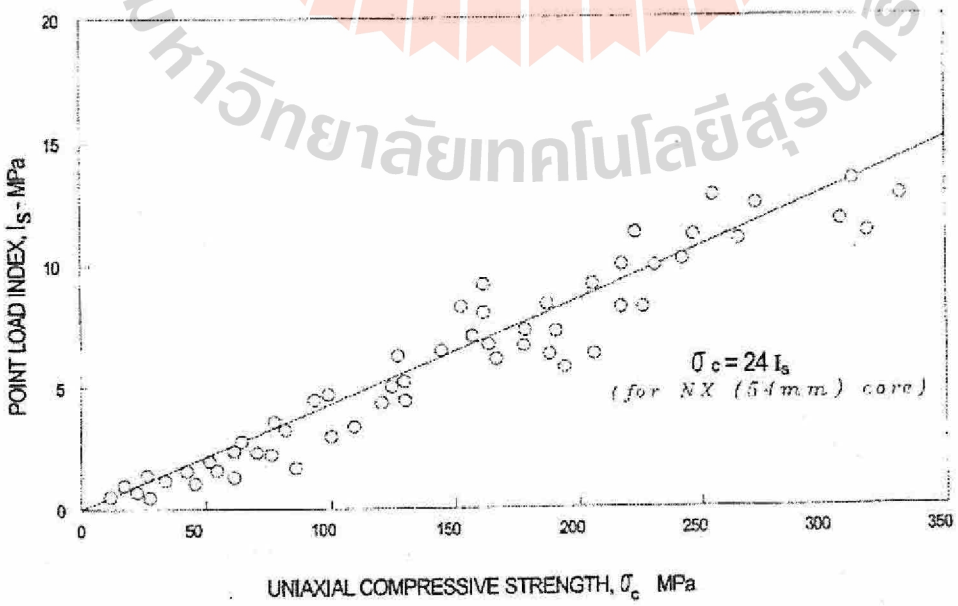
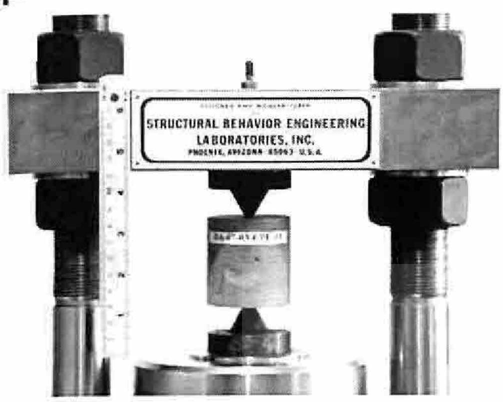


FIG. 9 Relationship Between Point Load Strength Index and Uniaxial Compressive Strength from 125 Tests On Sandstone, Quartzite, Marikana, Norite, and Belfast Norite[9]

Lab 6: Point Load Index Strength Testing



Axial Tests
D = 54 mm (NX-size)
L/D = 1.0

Saraburi Marble

Phu Kradung Sandstone

Burirum Basalt

Tak Granite

Post-tested specimen →

Lab 6: Point Load Index Strength Testing

On-site Testing

LABORATORY # 7

Slake Durability Testing
ASTM D4644-04 and ISRM Suggested Methods

Lab 7: Slake Durability Testing

INTERNATIONAL

Designation: D 4644 – 04

Standard Test Method for
Slake Durability of Shales and Similar Weak Rocks[1]

This standard is issued under the fixed designation D 4644; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

Objective:

to determine slake durability index of a shale or other similar rock

Lab 7: Slake Durability Testing

Apparatus

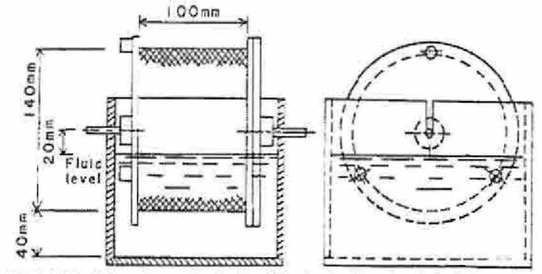


FIG. 1 Critical Dimensions of Slake Durability Device Showing Critical Dimensions

1) **Slake Durability Device**
   Drum - 200 mm square mesh (No.10)
   - diameter 140 mm (5.5 in)
   - length 100 mm (3.9 in)
2) **Drying Oven**
3) **Balance (sensitive to 1g & 2000g capacity)**
4) **Hammer & brush**

Lab 7: Slake Durability Testing

Specimens

- **roughly fragments weighing 40 – 60 g. (by breaking with a hammer)**
- **10 specimens (total weight 450g – 550g)**

Lab 7: Slake Durability Testing

Slake Durability Device

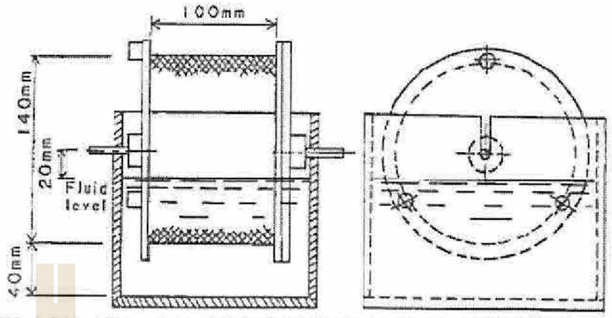
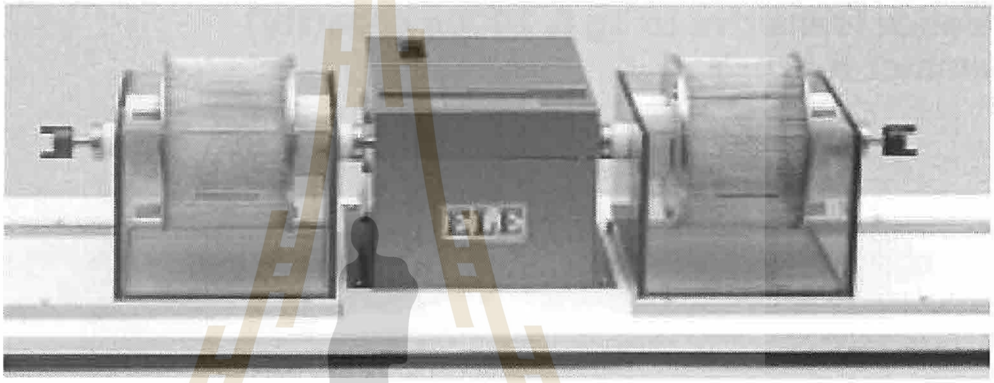
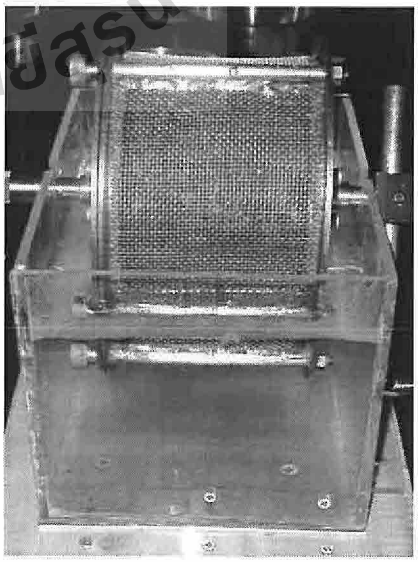


FIG. 1 Critical Dimensions of Slake Durability Device Showing Critical Dimensions

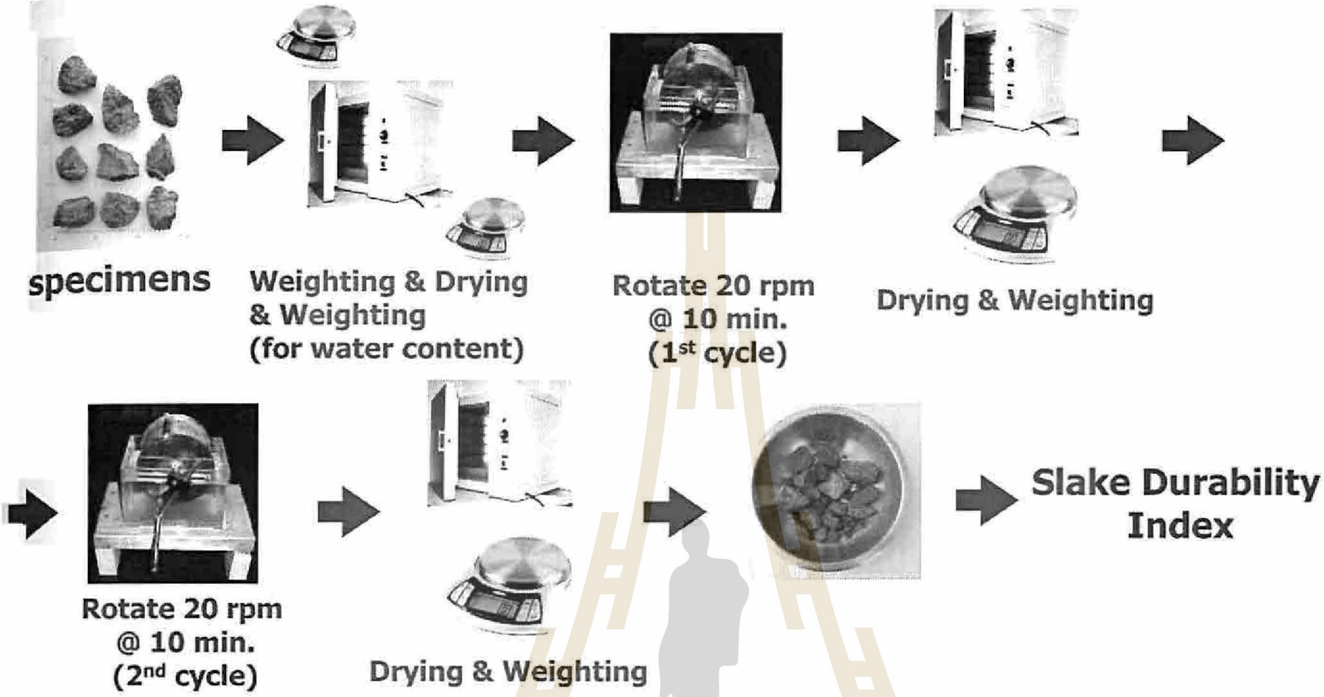


Lab 7: Slake Durability Testing



Lab 7: Slake Durability Testing

Procedure



Lab 7: Slake Durability Testing

Calculate the slake durability index (second cycle).

$$\boxed{I_d(2) = [(W_F - C)/(B - C)] \times 100}$$

where:

$I_d(2)$ = slake durability index (second cycle),
B = mass of drum plus oven-dried specimen before the first cycle, g,
W_F = mass of drum plus oven-dried specimen retained after the second cycle, g, and
C = mass of drum, g.

LABORATORY # 8

Dynamic Velocity Testing

ASTM D2845-05 and ISRM Suggested Methods

Lab 8: Dynamic Velocity Testing

Designation: D 2845 – 05

Standard Test Method for Laboratory Determination of Pulse Velocities and Ultrasonic Elastic Constants of Rock[1]

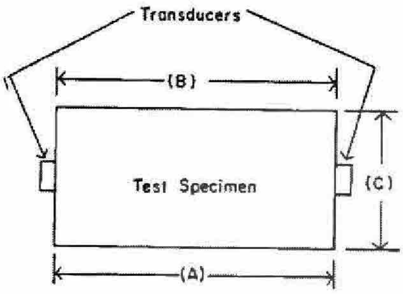
This standard is issued under the fixed designation D 2845; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

Objective:

to determine dynamic elastic modulus and Poisson's ratio

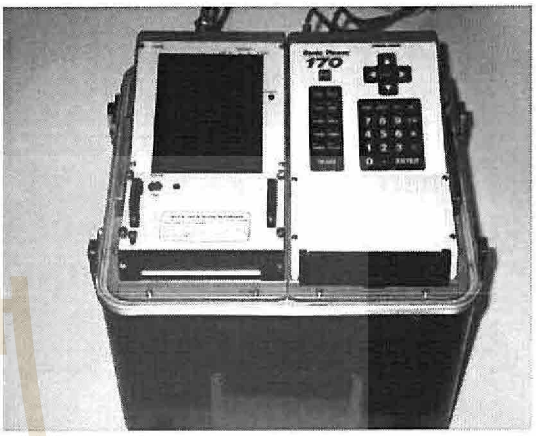
Lab 8: Dynamic Velocity Testing

Apparatus



NOTE 1—(A) must be within 0.1 mm of (B) for each 20 mm of width (C).

FIG. 3 Specification for Parallelism



Lab 8: Dynamic Velocity Testing

Calculation

9.1 Calculate the propagation velocities of the compression and shear waves, V_p and V_s respectively, as follows:

$$V_p = L_p/T_p$$

$$V_s = L_s/T_s$$

where:

V = pulse-propagation velocity, in./s (or m/s),

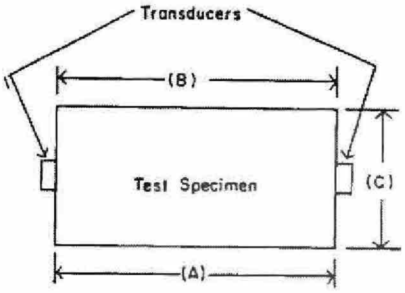
L = pulse-travel distance, in. (or m),

T = effective pulse-travel time (measured time minus zero time correction), s,

and subscripts $_p$ and $_s$ denote the compression wave and shear wave, respectively.

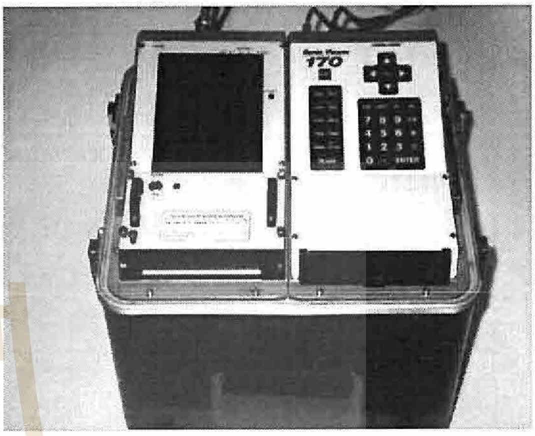
Lab 8: Dynamic Velocity Testing

Apparatus



NOTE 1—(A) must be within 0.1 mm of (B) for each 20 mm of width (C).

FIG. 3 Specification for Parallelism



Lab 8: Dynamic Velocity Testing

Calculation

9.1 Calculate the propagation velocities of the compression and shear waves, V_p and V_s respectively, as follows:

$$V_p = L_p/T_p$$
$$V_s = L_s/T_s$$

where:

V = pulse-propagation velocity, in./s (or m/s),

L = pulse-travel distance, in. (or m),

T = effective pulse-travel time (measured time minus zero time correction), s,

and subscripts $_p$ and $_s$ denote the compression wave and shear wave, respectively.

Lab 8: Dynamic Velocity Testing

Calculation

calculate the ultrasonic elastic constants

$$E = [pV_s^2(3V_p^2 - 4V_s^2)]/(V_p^2 - V_s^2)$$

where:
E = Young's modulus of elasticity, psi (or Pa), and
ρ = density, lb/in.^{3} (or kg/m^3);

$$G = \rho V_s^2$$

where:
G = modulus of rigidity or shear modulus, psi (or Pa);

Lab 8: Dynamic Velocity Testing

Calculation

$$\mu = (V_p^2 - 2V_s^2)/[2(V_p^2 - V_s^2)]$$

where:
μ = Poisson's ratio;

$$\lambda = p\,(V_p^2 - 2V_s^2)$$

where:
λ = Lamé's constant, psi (or Pa); and

$$K = \rho(3V_p^2 - 4V_s^2)/3$$

where:
K = bulk modulus, psi (or Pa).

Topic 9 Rock Mass

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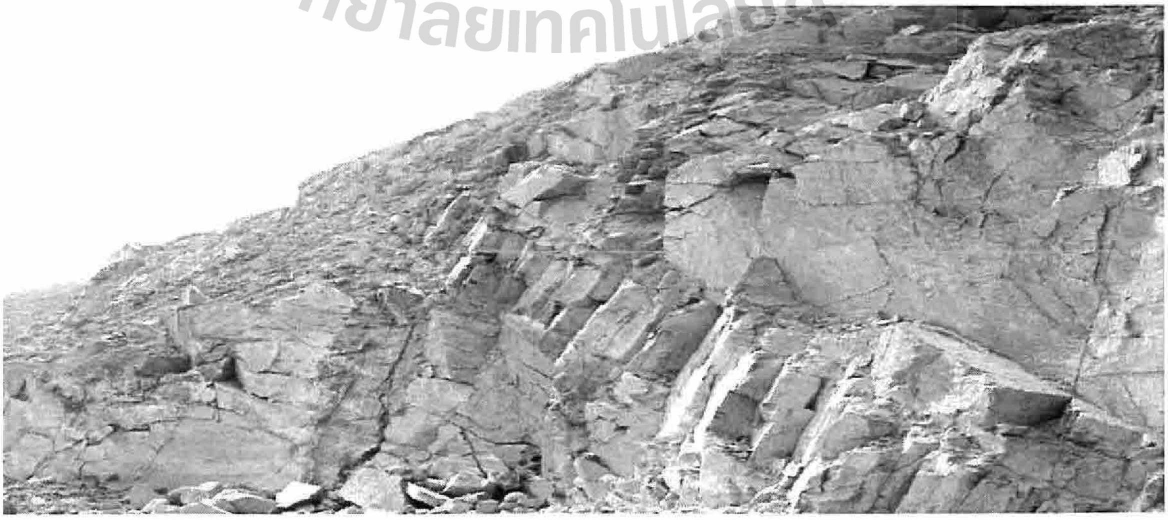
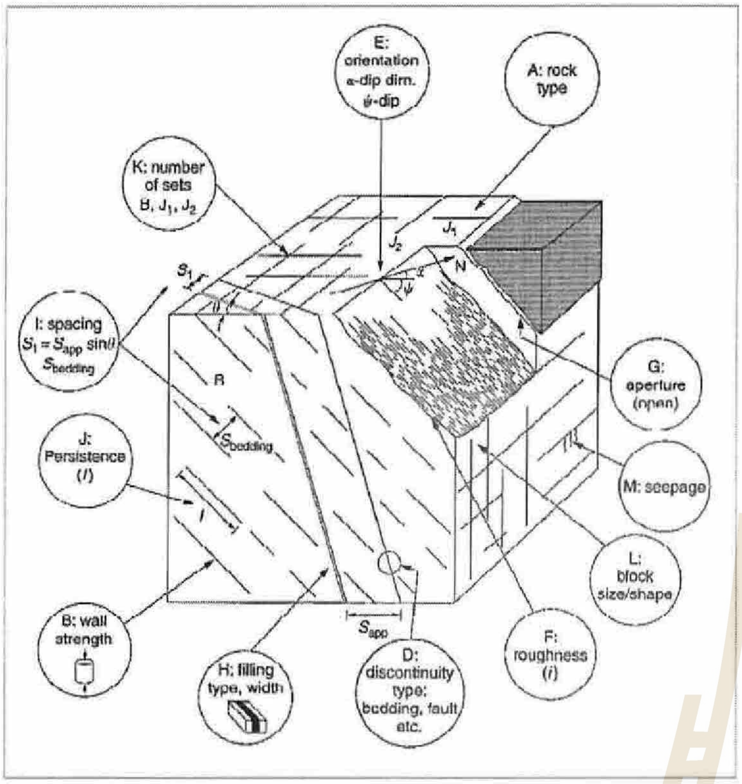
Outline

- Rock Mass Characterization
- Rock Mass Classifications
- Strength of Rock Mass
- In-situ Stresses



Quantitative Description of Discontinuities in Rock Masses (ISRM)



A - Rock type
B - Rock strength
C - Weathering
D - Discontinuity description
E - Discontinuity orientation
F - Roughness
G - Aperture
H - Infilling type and width
I - Spacing
J - Persistence
K - Number of sets
L - Block size and shape
M - Seepage

A-Rock type

Three primary characteristics of rock

1. **Color**, as well as whether light or dark minerals predominate
2. **Texture** or fabric ranging from crystalline, granular or glassy
3. **Grain size** that can range from clay particles to gravel

A-Rock type

Table II.1 Rock type classification

Genetic Group			Detrital Sedimentary			Pyroclastic	Chemical Organic	Metamorphic		Igneous			
Usual Structure			BEDDED			BEDDED		FOLIATED	MASSIVE	MASSIVE			
COMPOSITION										Light coloured minerals are quartz, feldspar, mica and feldspar-like minerals			Dark minerals
	Grain size (mm)		Grains of rock, quartz, feldspar and minerals	At least 50% of grains are of carbonate		At least 50% of grains are of fine-grained volcanic rock		Quartz, feldspars, micas, acicular dark minerals		Acid rocks	Intermediate rocks	Basic rocks	Ultra-basic rocks
Very coarse grained	60	RUDACEOUS	Grains are of rock fragments Rounded grains: CONGLOMERATE	LIMESTONE (undifferentiated)	CALCIRUDITE	Rounded grains AGGLOMERATE	SALINE ROCKS Halite Anhydrite Gypsum	MIGMATITE	HORNFELS	PEGMATITE			PYROXENITE and PERIDOTITE
Coarse grained	2		Angular grains: BRECCIA			Angular grains VOLCANIC BRECCIA		GNEISS Alternate layers of granular and flakey minerals	MARBLE GRANULITE	GRANITE	DIORITE	GABBRO	
Medium grained		ARENACEOUS	SANDSTONE: Grains are mainly mineral fragments QUARTZ SANDSTONE: 95% quartz, voids empty or cemented ARKOSE: 75% quartz, up to 25% feldspar: voids empty or cemented ARGILLACEOUS SANDSTONE: 75% quartz, 15% + fine detrital material		CALCARENITE	TUFF	CHERT	SCHIST PHYLLITE SLATE	QUARTZITE AMPHIBOLITE	MICRO-GRANITE	MICRO-DIORITE	DOLERITE	
	0.06						FLINT	MYLONITE					SERPENTINE
Fine grained	0.002	ARGILLACEOUS or LUTACEOUS	MUDSTONE SHALE: fissile mudstone SILTSTONE 50% fine-grained particles CLAYSTONE 50% very fine-grained particles CALCAREOUS MUDSTONE		CALCISILTITE	Fine-grained TUFF	COAL OTHERS			RHYOLITE	ANDESITE	BASALT	
Very fine grained					CALCILUTITE	Very fine-grained TUFF							
GLASSY										OBSIDIAN and PITCHSTONE		TACHYLYTE	

Note: Numbers can be used to identify rock types on data sheets (see Appendix III)
Reference. Geological Society Engineering Group Working Party (1977)

A-Rock type

Table II.2 Grain size scale

Description	*Grain size*
Boulders	200–600 mm (7.9–23.6 in)
Cobbles	60–200 mm (2.4–7.9 in)
Coarse gravel	20–60 mm (0.8–0.24 in)
Medium gravel	6–20 mm (0.2–0.8 in)
Fine gravel	2–6 mm (0.1–0.2 in)
Coarse sand	0.6–2 mm (0.02–0.1 in)
Medium sand	0.2–0.6 mm (0.008–0.02 in)
Fine sand	0.06–0.2 mm (0.002–0.008 in)
Silt, clay	<0.06 mm (<0.002 in)

B-Rock Strength

Table II.3 Classification of rock material strengths

Grade	Description	Field identification	Approximate compressive (MPa)	Range of strength (psi)
R6	Extremely strong rock	Specimen can only be chipped with geological hammer.	>250	>36,000
R5	Very strong rock	Specimen requires many blows of geological hammer to fracture it.	100–250	15,000–36,000
R4	Strong rock	Specimen requires more than one blow with a geological hammer to fracture it.	50–100	7000–15,000
R3	Medium weak rock	Cannot be scraped or peeled with a pocket knife; specimen can be fractured with single firm blow of geological hammer.	25–50	3500–7000
R2	Weak rock	Can be peeled with a pocket knife; shallow indentations made by firm blow with point of geological hammer.	5–25	725–3500
R1	Very weak rock	Crumbles under firm blows with point of geological hammer; can be peeled by a pocket knife.	1–5	150–725
R0	Extremely weak rock	Indented by thumbnail.	0.25–1	35–150
S6	Hard clay	Indented with difficulty by thumbnail.	>0.5	>70
S5	Very stiff clay	Readily indented by thumbnail.	0.25–0.5	35–70
S4	Stiff clay	Readily indented by thumb but penetrated only with great difficulty.	0.1–0.25	15–35
S3	Firm clay	Can be penetrated several inches by thumb with moderate effort.	0.05–0.1	7–15
S2	Soft clay	Easily penetrated several inches by thumb.	0.025–0.05	4–7
S1	Very soft clay	Easily penetrated several inches by fist.	<0.025	<4

C-Weathering

Table II.4 Weathering and alteration grades

Grade	Term	Description
I	Fresh	No visible sign of rock material weathering; perhaps slight discoloration on major discontinuity surfaces.
II	Slightly weathered	Discoloration indicates weathering of rock material and discontinuity surfaces. All the rock material may be discolored by weathering and may be somewhat weaker externally than in its fresh condition.
III	Moderately weathered	Less than half of the rock material is decomposed and/or disintegrated to a soil. Fresh or discolored rock is present either as a continuous framework or as corestones.
IV	Highly weathered	More than half of the rock material is decomposed and/or disintegrated to a soil. Fresh or discolored rock is present either as a discontinuous framework or as corestones.
V	Completely weathered	All rock material is decomposed and/or disintegrated to soil. The original mass structure is still largely intact.
VI	Residual soil	All rock material is converted to soil. The mass structure and material fabric are destroyed. There is a large change in volume, but the soil has not been significantly transported.

D-Discontinuity description

Type of Discontinuity

Fault – discontinuity along which there has been and observable amount of displacement

Bedding – surface parallel to the surface of deposition

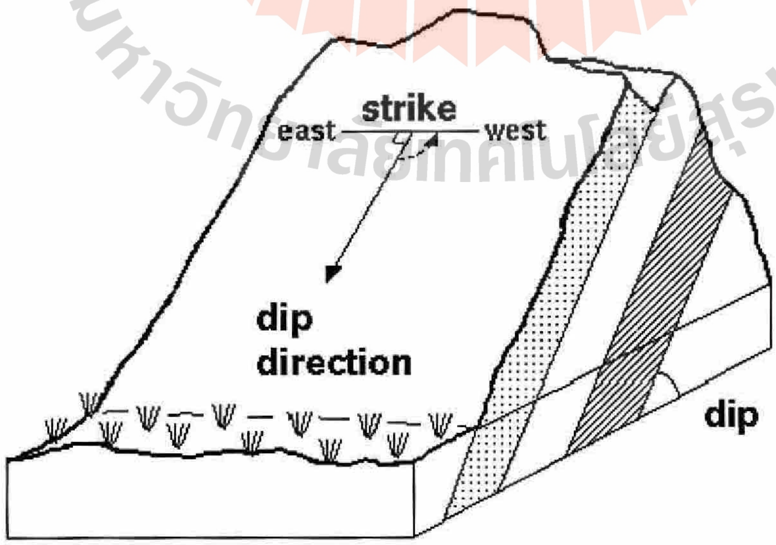
Foliation – parallel orientation of platy minerals, or mineral banding in metamorphic rocks

Joint – discontinuity in which there has been no observable relative moment

Cleavage – parallel discontinuities formed incompetent layers in a series of beds of varying degrees of competency

Schistosity – foliation in schist or other coarse grained crystalline rock

E-Discontinuity orientation



F-Roughness

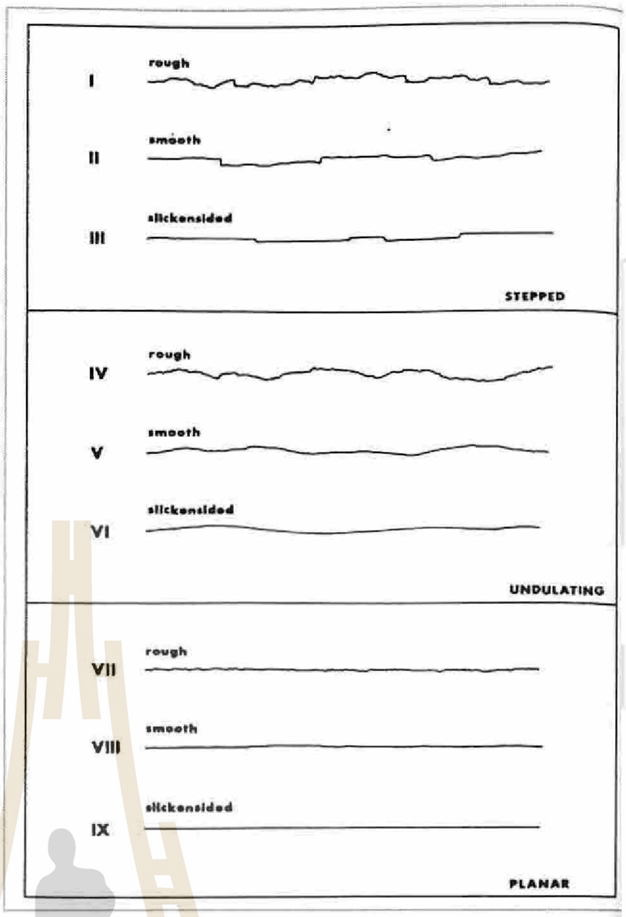


Table II.5 Descriptive terms for roughness

I	Rough, stepped
II	Smooth, stepped
III	Slickensided, stepped
IV	Rough, undulating
V	Smooth, undulating
VI	Slickensided, undulating
VII	Rough, planar
VIII	Smooth, planar
IX	Slickensided, planar

F-Roughness

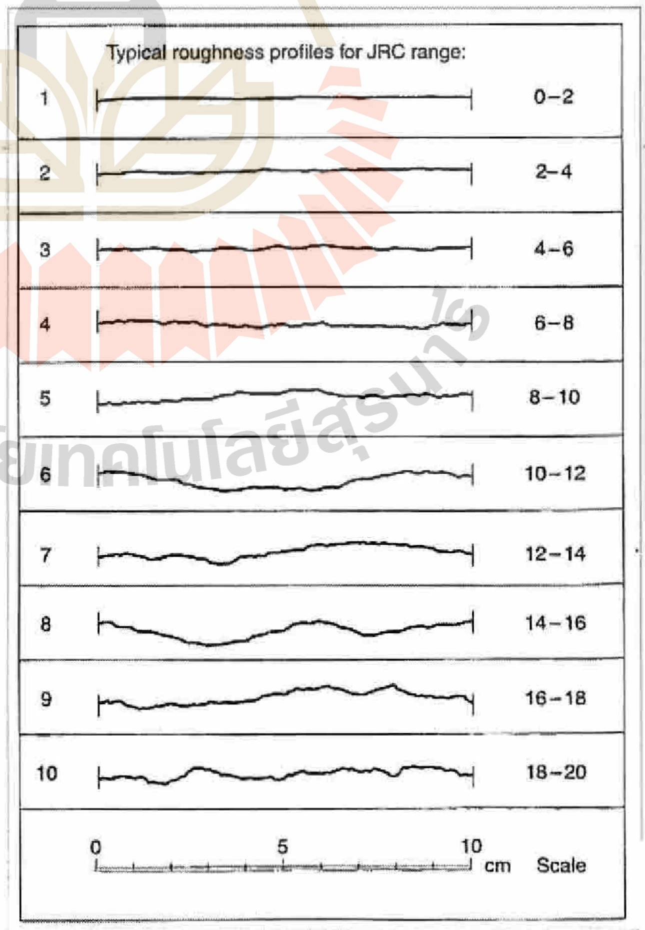
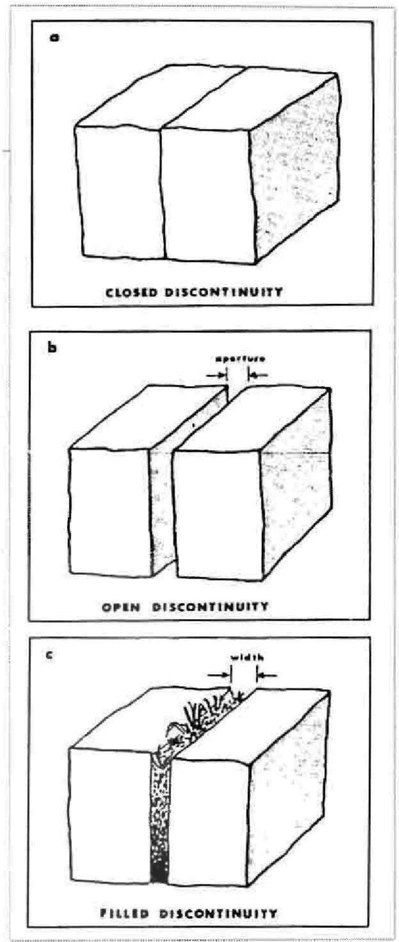


Figure II.3 Roughness profiles and corresponding range of JRC (joint roughness coefficient) values (ISRM, 1981a).

G-Aperture

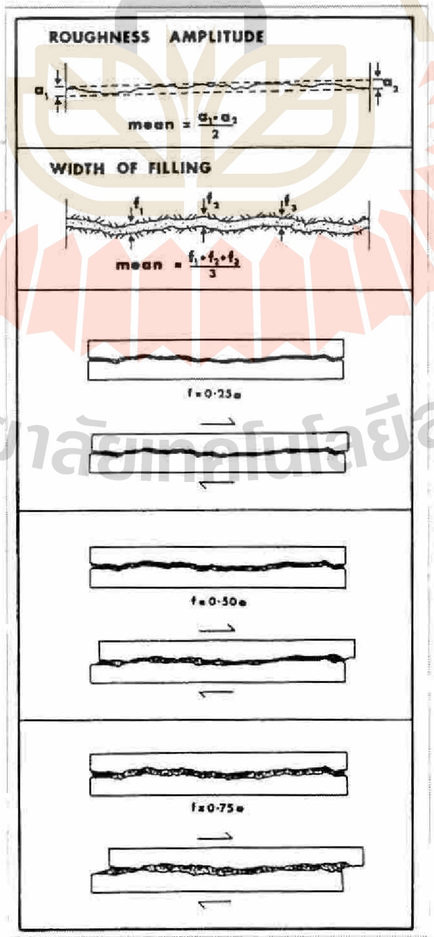
Table II.6 Aperture dimensions

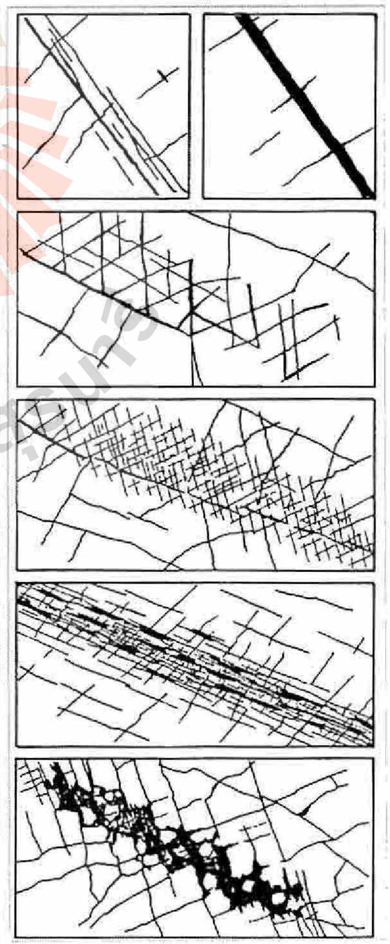
Aperture	*Description*	
<0.1 mm	Very tight	
0.1–0.25 mm	Tight	"Closed" features
0.25–0.5 mm	Partly open	
0.5–2.5 mm	Open	
2.5–10 mm	Moderately wide	"Gapped" features
>10 mm	Wide	
1–10 cm	Very wide	
10–100 cm	Extremely wide	"Open" features
>1 m	Cavernous	



H-Infilling type and width

- •Width
- •Weathering Grade
- •Mineralogy
- •Particle Size
- •Filling Strength
- •Previous Displacement
- •Water Content and Permeability





I-Spacing

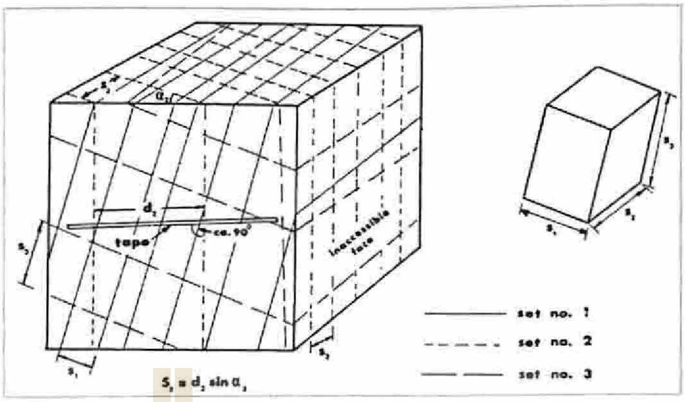
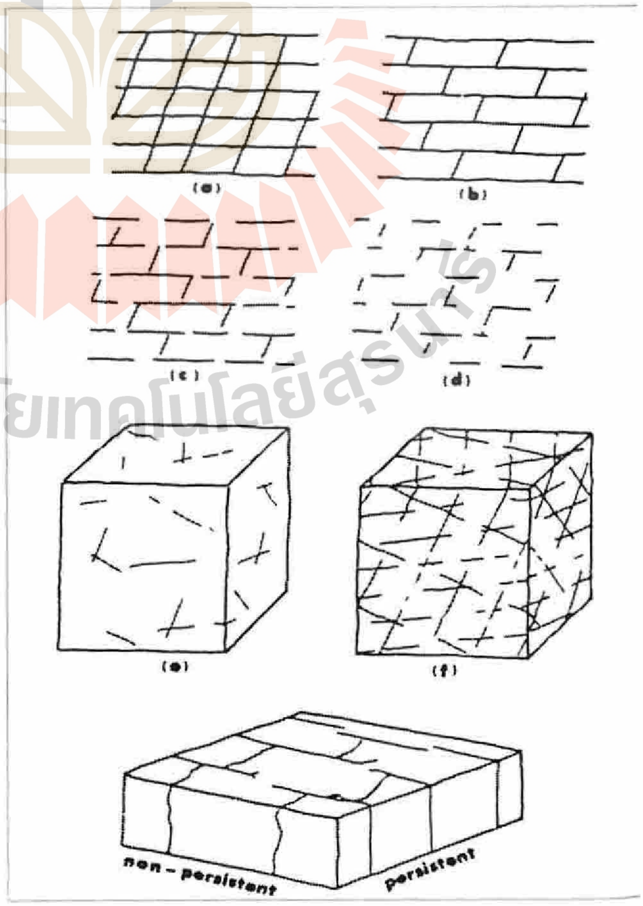


Table II.7 Spacing dimensions	
Description	*Spacing (mm)*
Extremely close spacing	<20
Very close spacing	20–60
Close spacing	60–200
Moderate spacing	200–600
Wide spacing	600–2000
Very wide spacing	2000–6000
Extremely wide spacing	>6000

J-Persistence

Table II.8 Persistence dimensions

Very low persistence	<1 m
Low persistence	1–3 m
Medium persistence	3–10 m
High persistence	10–20 m
Very high persistence	>20 m



K-Number of sets

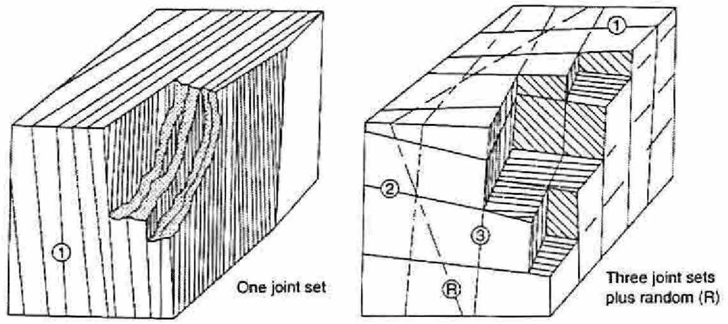


Figure II.4 Examples illustrating the effect of the number of joint sets on the mechanical behavior and appearance of rock masses (ISRM, 1981a).

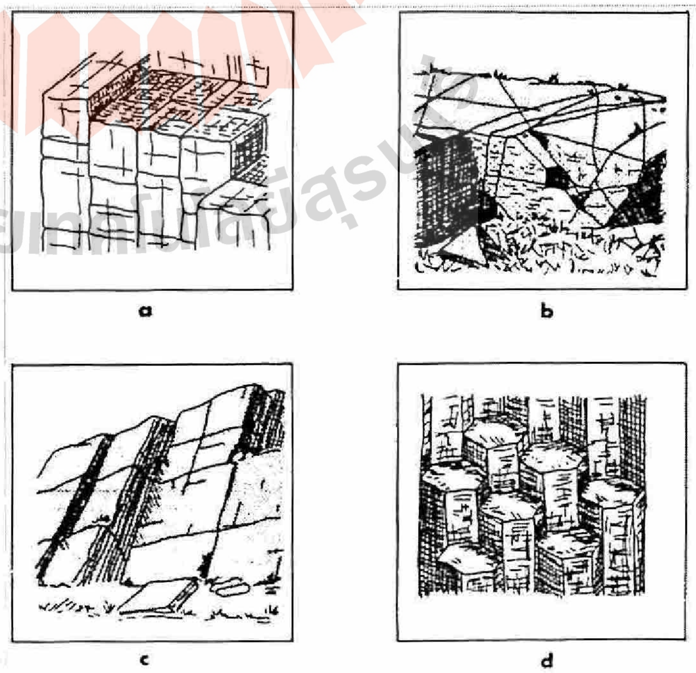
I	massive, occasional random joints
II	one joint set
III	one joint set plus random
IV	two joint sets
V	two joint sets plus random
VI	three joint sets
VII	three joint sets plus random
VIII	four or more joint sets
IX	crushed rock, earth-like

L-Block size and shape

Table II.9 Block dimensions

Description	J_v *(joints/m^3)*
Very large blocks	<1.0
Large blocks	1–3
Medium-sized blocks	3–10
Small blocks	10–30
Very small blocks	>30

(i)	*massive*	= few joints or very wide spacing
(ii)	*blocky*	= approximately equidimensional
(iii)	*tabular*	= one dimension considerably smaller than the other two
(iv)	*columnar*	= one dimension considerably larger than the other two
(v)	*irregular*	= wide variations of block size and shape
(vi)	*crushed*	= heavily jointed to "sugar cube"



M- Seepage

Table II.10 Seepage quantities in unfilled discontinuities

Seepage rating	*Description*
I	The discontinuity is very tight and dry, water flow along it does not appear possible.
II	The discontinuity is dry with no evidence of water flow.
III	The discontinuity flow is dry but shows evidence of water flow, that is, rust staining.
IV	The discontinuity is damp but no free water is present.
V	The discontinuity shows seepage, occasional drops of water, but no continuous flow.
VI	The discontinuity shows a continuous flow of water—estimate l/ min and describe pressure, that is, low, medium, high.

M- Seepage

Table II.11 Seepage quantities in filled discontinuities

Seepage rating	*Description*
I	The filling materials are heavily consolidated and dry, significant flow appears unlikely due to very low permeability.
II	The filling materials are damp, but no free water is present.
III	The filling materials are wet, occasional drops of water.
IV	The filling materials show signs of outwash, continuous flow of water—estimate l/ min.
V	The filling materials are washed out locally, considerable water flow along out-wash channels—estimate l/ min and describe pressure that is low, medium, high.
VI	The filling materials are washed out completely, very high water pressures experienced, especially on first exposure—estimate l/ min and describe pressure.

Rock Mass Classifications

Classification system	Form and type*	Main applications	Reference
Terzaghi rock load classification system	Descriptive and behaviouristic form Functional type.	Design of steel support in tunnels	Terzaghi, 1946
Lauffer's stand-up time classification	Descriptive form General type	Tunnelling design	Laufer H., 1958
New Australian tunneling method (NATM)	Descriptive and behaviouristic form Tunneling concept	Excavation and design in incompetent (overstressed) ground	Rabcewicz, Müller and Pacher, 1958–1964
Rock classification for rock mechanical purposes	Descriptive form General type	Input in rock mechanics	Patching and Coates, 1968
Unified classification of soils and rocks	Descriptive form General type	Based on particles and blocks for communication	Deer et al., 1969
Rock quality designation (RQD)	Numerical form General type	Based on core logging; used in other classification systems	Deer et al., 1967
Size-strength classification	Numerical form Functional type	Based on rock strength and block diameter, used mainly in mining	Franklin, 1975
Rock structure rating classification (RSR)	Numerical form Functional type	Design of (steel) support in tunnels	Wickham et al., 1972
Rock mass rating classification (RMR)	Numerical form Functional type	Design of tunnels, mines, and foundations	Bieniawski, 1973
Q-classification system	Numerical form Functional type	Design of support in underground excavation	Barton et al., 1974
Typological classification	Descriptive form General type	Use in communication	Maluta and Holzer, 1978
Unified rock classification system	Descriptive form General type	Use in communication	Williamson, 1980
Basic geotechnical classification (BGD)	Descriptive form General type	General applications	ISRM, 1981
Geological strength index (GSI)	Numerical form Functional type	Design of support in underground excavation	Hoek, 1994
Rock mass index system (RMi)	Numerical form Functional type	General characterization, design of support, TMB progress	Palmström, 1995

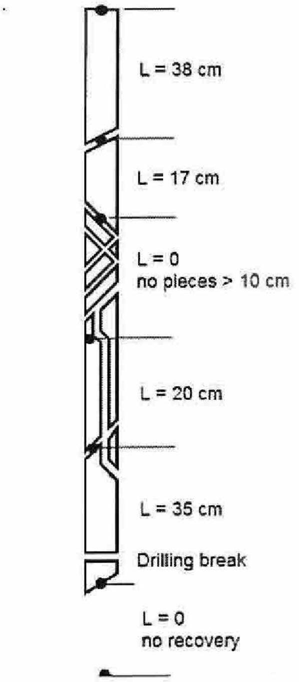
Rock Mass Classifications

Deer's Rock Quality Destination (RQD)

- Deere (1964) proposed a quantitative index of rock mass quality based upon core recovery by diamond drilling.
- RQD has come to be very widely used and has been shown to be particularly useful in classifying rock masses for the selection of tunnel support systems.
- RQD is defined as the percentage of intact core pieces longer than 100 mm (4 inches) in the total length of core.

Rock Mass Classifications

Deer's Rock Quality Destination (RQD)



Total length of core run = 200 cms

$$RQD = \frac{\Sigma \text{ Length of core pieces} > 10 \text{ cm length}}{\text{Total length of core run}} \times 100$$

$$RQD = \frac{38 + 17 + 20 +}{200} \times 100 = 55\ \%$$

Rock Mass Classifications

RQD Estimation from outcrop

- Palmström (1982) suggested that, when no core is available but discontinuity traces are visible in surface exposures or exploration adits, the *RQD may be estimated from the* number of discontinuities per unit volume. The suggested relationship for clay-free rock masses is:

$$RQD = 115 - 3.3\, J_v \qquad (J_v < 4.5)$$

$$RQD = 100 \exp(-0.1/S)\,(1 + 0.1/S)$$

- where J_v *is the sum of the number of joints per unit length for all joint (discontinuity)* sets known as the volumetric joint count and S is average spacing of joint.

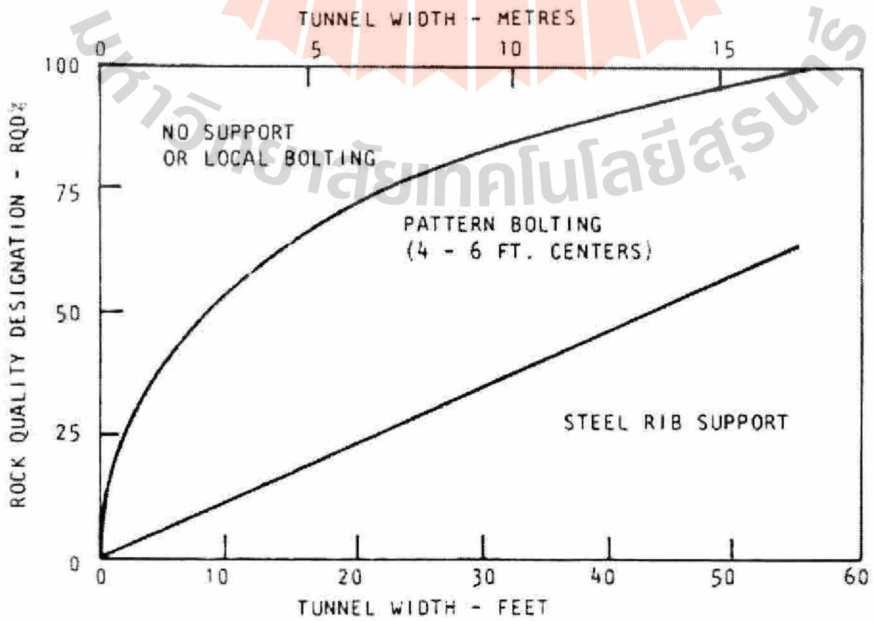
Rock Mass Classifications

Deer's Rock Quality Destination (RQD)

RQD	Rock Quality
< 25%	Very poor
25 –50 %	poor
50 – 75%	Fair
75 – 90%	Good
90 – 100%	Very good

Rock Mass Classifications

Deer's Rock Quality Destination (RQD)



Merritt, 1972

Rock Mass Classifications

Geomechanics Classification (RMR)

- Bieniawski (1976) published the details of a rock mass classification called the Geomechanics Classification or the Rock Mass Rating *(RMR) system.*
- The following six parameters are used to classify a rock mass using the *RMR system:*
  1. Uniaxial compressive strength of rock material.
  2. Rock Quality Designation *(RQD).*
  3. Spacing of discontinuities.
  4. Condition of discontinuities.
  5. Groundwater conditions.
  6. Orientation of discontinuities.

Geomechanics Classification (RMR)

A. CLASSIFICATION PARAMETERS AND THEIR RATINGS

	Parameter		Range of values						
1	Strength of intact rock material	Point-load strength index	>10 MPa	4 - 10 MPa	2 - 4 MPa	1 - 2 MPa	For this low range - uniaxial compressive test is preferred		
		Uniaxial comp strength	>250 MPa	100 - 250 MPa	50 - 100 MPa	25 - 50 MPa	5 - 25 MPa	1 - 5 MPa	< 1 MPa
	Rating		15	12	7	4	2	1	0
2	Drill core Quality *RQD*		90% - 100%	75% - 90%	50% - 75%	25% - 50%	< 25%		
	Rating		20	17	13	8	3		
3	Spacing of discontinuities		> 2 m	0.6 - 2 m	200 - 600 mm	60 - 200 mm	< 60 mm		
	Rating		20	15	10	8	5		
4	Condition of discontinuities (See E)		Very rough surfaces Not continuous No separation Unweathered wall rock	Slightly rough surfaces Separation < 1 mm Slightly weathered walls	Slightly rough surfaces Separation < 1 mm Highly weathered walls	Slickensided surfaces or Gouge < 5 mm thick or Separation 1-5 mm Continuous	Soft gouge >5 mm thick or Separation > 5 mm Continuous		
	Rating		30	25	20	10	0		
5	Groundwater	Inflow per 10 m tunnel length (l/m)	None	< 10	10 - 25	25 - 125	> 125		
		(Joint water press)/ (Major principal σ)	0	< 0.1	0.1 - 0.2	0.2 - 0.5	> 0.5		
		General conditions	Completely dry	Damp	Wet	Dripping	Flowing		
	Rating		15	10	7	4	0		

(After Bieniawski 1989)

Geomechanics Classification (RMR)

B. RATING ADJUSTMENT FOR DISCONTINUITY ORIENTATIONS (See F)						
Strike and dip orientations		Very favourable	Favourable	Fair	Unfavourable	Very Unfavourable
Ratings	Tunnels & mines	0	-2	-5	-10	-12
	Foundations	0	-2	-7	-15	-25
	Slopes	0	-5	-25	-50	

C. ROCK MASS CLASSES DETERMINED FROM TOTAL RATINGS					
Rating	100 ← 81	80 ← 61	60 ← 41	40 ← 21	< 21
Class number	I	II	III	IV	V
Description	Very good rock	Good rock	Fair rock	Poor rock	Very poor rock

D. MEANING OF ROCK CLASSES					
Class number	I	II	III	IV	V
Average stand-up time	20 yrs for 15 m span	1 year for 10 m span	1 week for 5 m span	10 hrs for 2.5 m span	30 min for 1 m span
Cohesion of rock mass (kPa)	> 400	300 - 400	200 - 300	100 - 200	< 100
Friction angle of rock mass (deg)	> 45	35 - 45	25 - 35	15 - 25	< 15

Geomechanics Classification (RMR)

E. GUIDELINES FOR CLASSIFICATION OF DISCONTINUITY conditions					
Discontinuity length (persistence)	< 1 m	1 - 3 m	3 - 10 m	10 - 20 m	> 20 m
Rating	6	4	2	1	0
Separation (aperture)	None	< 0.1 mm	0.1 - 1.0 mm	1 - 5 mm	> 5 mm
Rating	6	5	4	1	0
Roughness	Very rough	Rough	Slightly rough	Smooth	Slickensided
Rating	6	5	3	1	0
Infilling (gouge)	None	Hard filling < 5 mm	Hard filling > 5 mm	Soft filling < 5 mm	Soft filling > 5 mm
Rating	6	4	2	2	0
Weathering	Unweathered	Slightly weathered	Moderately weathered	Highly weathered	Decomposed
Ratings	6	5	3	1	0

F. EFFECT OF DISCONTINUITY STRIKE AND DIP ORIENTATION IN TUNNELLING**			
Strike perpendicular to tunnel axis		Strike parallel to tunnel axis	
Drive with dip - Dip 45 - 90°	Drive with dip - Dip 20 - 45°	Dip 45 - 90°	Dip 20 - 45°
Very favourable	Favourable	Very unfavourable	Fair
Drive against dip - Dip 45-90°	Drive against dip - Dip 20-45°	Dip 0-20 - Irrespective of strike°	
Fair	Unfavourable	Fair	

Geomechanics Classification (RMR)

- The *RMR value for the example under consideration is determined as follows:*

Table	*Item*	*Value*	*Rating*
A.1	Point load index	8 MPa	12
A.2	*RQD*	70%	13
A.3	Spacing of discontinuities	300 mm	10
E.4	Condition of discontinuities	Note 1	22
A.5	Groundwater	Wet	7
B	Adjustment for joint orientation	Note 2	-5
		Total	59

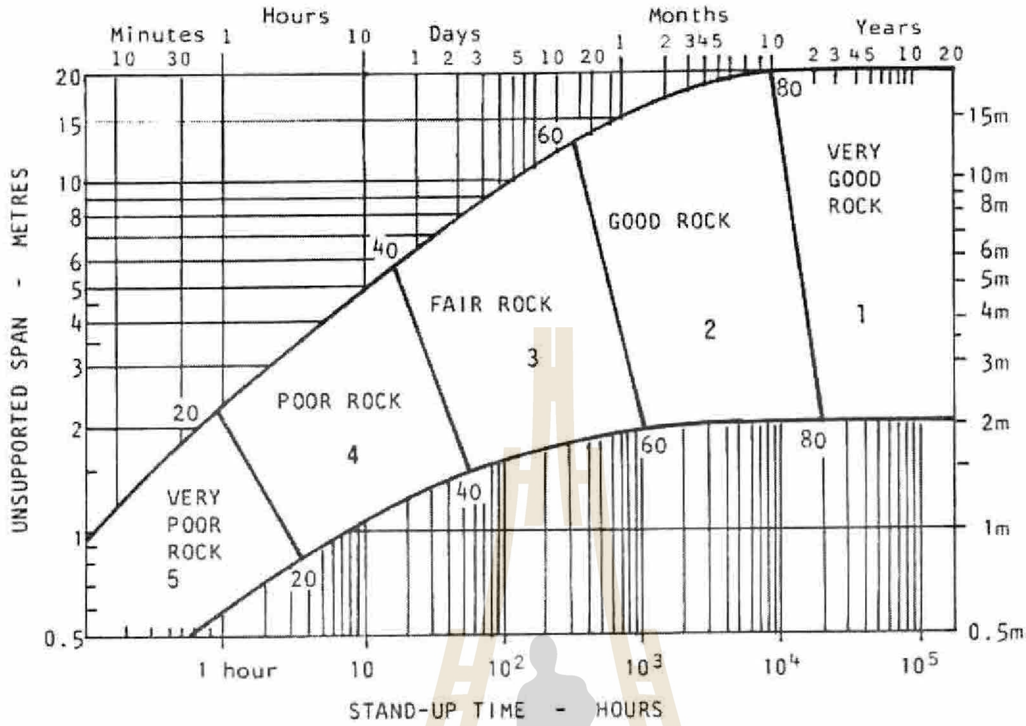
Geomechanics Classification (RMR)

- Guidelines for excavation and support of 10 m span rock tunnels in accordance with the *RMR system (After Bieniawski 1989).*

Rock mass class	Excavation	Rock bolts (20 mm diameter, fully grouted)	Shotcrete	Steel sets
I - Very good rock *RMR*: 81-100	Full face, 3 m advance.	Generally no support required except spot bolting.		
II - Good rock *RMR*: 61-80	Full face , 1-1.5 m advance. Complete support 20 m from face.	Locally, bolts in crown 3 m long, spaced 2.5 m with occasional wire mesh.	50 mm in crown where required.	None.
III - Fair rock *RMR*: 41-60	Top heading and bench 1.5-3 m advance in top heading. Commence support after each blast. Complete support 10 m from face.	Systematic bolts 4 m long, spaced 1.5 - 2 m in crown and walls with wire mesh in crown.	50-100 mm in crown and 30 mm in sides.	None.
IV - Poor rock *RMR*: 21-40	Top heading and bench 1.0-1.5 m advance in top heading. Install support concurrently with excavation, 10 m from face.	Systematic bolts 4-5 m long, spaced 1-1.5 m in crown and walls with wire mesh.	100-150 mm in crown and 100 mm in sides.	Light to medium ribs spaced 1.5 m where required.
V – Very poor rock *RMR*: < 20	Multiple drifts 0.5-1.5 m advance in top heading. Install support concurrently with excavation. Shotcrete as soon as possible after blasting.	Systematic bolts 5-6 m long, spaced 1-1.5 m in crown and walls with wire mesh. Bolt invert.	150-200 mm in crown, 150 mm in sides, and 50 mm on face.	Medium to heavy ribs spaced 0.75 m with steel lagging and forepoling if required. Close invert.

Geomechanics Classification (RMR)

- **By Bieniawski (1976)**



Rock Mass Classifications

Rock Tunnelling Quality Index, Q (NGI)

- On the basis of an evaluation of a large number of case histories of underground excavations, Barton et al (1974) of the Norwegian Geotechnical Institute proposed a Tunnelling Quality Index (Q) *for the determination of rock mass characteristics and* tunnel support requirements.
- The numerical value of the index *Q varies on a logarithmic* scale from 0.001 to a maximum of 1,000 and is defined by:

$$Q = \frac{RQD}{J_n} \times \frac{J_r}{J_a} \times \frac{J_w}{SRF}$$

Rock Tunnelling Quality Index, Q (NGI)

$$Q = \frac{RQD}{J_n} \times \frac{J_r}{J_a} \times \frac{J_w}{SRF}$$

where	RQD	*is the Rock Quality Designation*
	J_n	*is the joint set number*
	J_r	*is the joint roughness number*
	J_a	*is the joint alteration number*
	J_w	*is the joint water reduction factor*
	SRF	*is the stress reduction factor*

Rock Tunnelling Quality Index, Q (NGI)

- It appears that the rock tunnelling quality Q *can now be considered to be a function of* only three parameters which are crude measures of:
  1. Block size (RQD/J_n)
  2. Inter-block shear strength (J_r/J_a)
  3. Active stress (J_w/SRF)

Rock Tunnelling Quality Index, Q (NGI)

DESCRIPTION	VALUE	NOTES
1. ROCK QUALITY DESIGNATION	***RQD***	
A. Very poor	0 - 25	1. Where *RQD* is reported or measured as ≤ 10 (including 0),
B. Poor	25 - 50	a nominal value of 10 is used to evaluate Q.
C. Fair	50 - 75	
D. Good	75 - 90	2. *RQD* intervals of 5, i.e. 100, 95, 90 etc. are sufficiently
E. Excellent	90 - 100	accurate.
2. JOINT SET NUMBER	J_n	
A. Massive, no or few joints	0.5 - 1.0	
B. One joint set	2	
C. One joint set plus random	3	
D. Two joint sets	4	
E. Two joint sets plus random	6	
F. Three joint sets	9	1. For intersections use $(3.0 \times J_n)$
G. Three joint sets plus random	12	
H. Four or more joint sets, random, heavily jointed, 'sugar cube', etc.	15	2. For portals use $(2.0 \times J_n)$
J. Crushed rock, earthlike	20	

Rock Tunnelling Quality Index, Q (NGI)

DESCRIPTION	VALUE	NOTES
3. JOINT ROUGHNESS NUMBER	J_r	
a. Rock wall contact		
b. Rock wall contact before 10 cm shear		
A. Discontinuous joints	4	
B. Rough and irregular, undulating	3	
C. Smooth undulating	2	
D. Slickensided undulating	1.5	1. Add 1.0 if the mean spacing of the relevant joint set is
E. Rough or irregular, planar	1.5	greater than 3 m.
F. Smooth, planar	1.0	
G. Slickensided, planar	0.5	2. J_r = 0.5 can be used for planar, slickensided joints having
c. No rock wall contact when sheared		lineations, provided that the lineations are oriented for
H. Zones containing clay minerals thick enough to prevent rock wall contact	1.0 (nominal)	minimum strength.
J. Sandy, gravely or crushed zone thick enough to prevent rock wall contact	1.0 (nominal)	

Rock Tunnelling Quality Index, Q (NGI)

4. JOINT ALTERATION NUMBER	J_a	ϕr degrees (approx.)	
a. Rock wall contact			
A. Tightly healed, hard, non-softening, impermeable filling	0.75		1. Values of ϕr, the residual friction angle, are intended as an approximate guide to the mineralogical properties of the alteration products, if present.
B. Unaltered joint walls, surface staining only	1.0	25 - 35	
C. Slightly altered joint walls, non-softening mineral coatings, sandy particles, clay-free disintegrated rock, etc.	2.0	25 - 30	
D. Silty-, or sandy-clay coatings, small clay-fraction (non-softening)	3.0	20 - 25	
E. Softening or low-friction clay mineral coatings, i.e. kaolinite, mica. Also chlorite, talc, gypsum and graphite etc., and small quantities of swelling clays. (Discontinuous coatings, 1 - 2 mm or less)	4.0	8 - 16	

Rock Tunnelling Quality Index, Q (NGI)

4, JOINT ALTERATION NUMBER	J_a	ϕr degrees (approx.)
b. Rock wall contact before 10 cm shear		
F. Sandy particles, clay-free, disintegrating rock etc.	4.0	25 - 30
G. Strongly over-consolidated, non-softening clay mineral fillings (continuous < 5 mm thick)	6.0	16 - 24
H. Medium or low over-consolidation, softening clay mineral fillings (continuous < 5 mm thick)	8.0	12 - 16
J. Swelling clay fillings, i.e. montmorillonite, (continuous < 5 mm thick). Values of J_a depend on percent of swelling clay-size particles, and access to water.	8.0 - 12.0	6 - 12
c. No rock wall contact when sheared		
K. Zones or bands of disintegrated or crushed	6.0	
L. rock and clay (see G, H and J for clay	8.0	
M. conditions)	8.0 - 12.0	6 - 24
N. Zones or bands of silty- or sandy-clay, small clay fraction, non-softening	5.0	
O. Thick continuous zones or bands of clay	10.0 - 13.0	
P. & R. (see G.H and J for clay conditions)	6.0 - 24.0	

Rock Tunnelling Quality Index, Q (NGI)

5. JOINT WATER REDUCTION	J_w	approx. water pressure (kgf/cm^2)	
A. Dry excavation or minor inflow i.e. < 5 l/m locally	1.0	< 1.0	
B. Medium inflow or pressure, occasional outwash of joint fillings	0.66	1.0 - 2.5	
C. Large inflow or high pressure in competent rock with unfilled joints	0.5	2.5 - 10.0	1. Factors C to F are crude estimates; increase J_w if drainage installed.
D. Large inflow or high pressure	0.33	2.5 - 10.0	
E. Exceptionally high inflow or pressure at blasting, decaying with time	0.2 - 0.1	> 10	2. Special problems caused by ice formation are not considered.
F. Exceptionally high inflow or pressure	0.1 - 0.05	> 10	

Rock Tunnelling Quality Index, Q (NGI)

6. STRESS REDUCTION FACTOR	*SRF*	
a. Weakness zones intersecting excavation, which may cause loosening of rock mass when tunnel is excavated		
A. Multiple occurrences of weakness zones containing clay or chemically disintegrated rock, very loose surrounding rock any depth)	10.0	1. Reduce these values of *SRF* by 25 - 50% but only if the relevant shear zones influence do not intersect the excavation
B. Single weakness zones containing clay, or chemically dis-tegrated rock (excavation depth < 50 m)	5.0	
C. Single weakness zones containing clay, or chemically dis-tegrated rock (excavation depth > 50 m)	2.5	
D. Multiple shear zones in competent rock (clay free), loose surrounding rock (any depth)	7.5	
E. Single shear zone in competent rock (clay free). (depth of excavation < 50 m)	5.0	
F. Single shear zone in competent rock (clay free). (depth of excavation > 50 m)	2.5	
G. Loose open joints, heavily jointed or 'sugar cube', (any depth)	5.0	

Rock Tunnelling Quality Index, Q (NGI)

DESCRIPTION	VALUE			NOTES
6. STRESS REDUCTION FACTOR			*SRF*	
b. Competent rock, rock stress problems				
	σ_c/σ_1	σ_t/σ_1		2. For strongly anisotropic virgin stress field (if measured): when $5 \le \sigma_1/\sigma_3 \le 10$, reduce σ_c to $0.8\sigma_c$ and σ_t to $0.8\sigma_t$. When $\sigma_1/\sigma_3 > 10$, reduce σ_c and σ_t to $0.6\sigma_c$ and $0.6\sigma_t$, where σ_c = unconfined compressive strength, and σ_t = tensile strength (point load) and σ_1 and σ_3 are the major and minor principal stresses.
H. Low stress, near surface	> 200	> 13	2.5	
J. Medium stress	200 - 10	13 - 0.66	1.0	
K. High stress, very tight structure (usually favourable to stability, may be unfavourable to wall stability)	10 - 5	0.66 - 0.33	0.5 - 2	
L. Mild rockburst (massive rock)	5 - 2.5	0.33 - 0.16	5 - 10	
M. Heavy rockburst (massive rock)	< 2.5	< 0.16	10 - 20	3. Few case records available where depth of crown below surface is less than span width. Suggest *SRF* increase from 2.5 to 5 for such cases (see H).
c. Squeezing rock, plastic flow of incompetent rock under influence of high rock pressure				
N. Mild squeezing rock pressure			5 - 10	
O. Heavy squeezing rock pressure			10 - 20	
d. Swelling rock, chemical swelling activity depending on presence of water				
P. Mild swelling rock pressure			5 - 10	
R. Heavy swelling rock pressure			10 - 15	

Rock Tunnelling Quality Index, Q (NGI)

- Barton et al (1974) defined an additional parameter which they called the Equivalent Dimension, De, of the excavation.
- This dimension is obtained by dividing the span, diameter or wall height of the excavation by a quantity called the Excavation Support Ratio, ESR. Hence:

$$D_e = \frac{\text{Excavation span, diameter or height (m)}}{\text{Excavation Support Ratio } ESR}$$

Rock Tunnelling Quality Index, Q (NGI)

DESCRIPTION	VALUE			NOTES
6. STRESS REDUCTION FACTOR			*SRF*	
b. Competent rock, rock stress problems				
	σ_c/σ_1	$\sigma_t\sigma_1$		2. For strongly anisotropic virgin stress field (if measured): when $5 \leq \sigma_1/\sigma_3 \leq 10$, reduce σ_c to $0.8\sigma_c$ and σ_t to $0.8\sigma_t$. When $\sigma_1/\sigma_3 > 10$, reduce σ_c and σ_t to $0.6\sigma_c$ and $0.6\sigma_t$, where σ_c = unconfined compressive strength, and σ_t = tensile strength (point load) and σ_1 and σ_3 are the major and minor principal stresses.
H. Low stress, near surface	> 200	> 13	2.5	
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L. Mild rockburst (massive rock)	5 - 2.5	0.33 - 0.16	5 - 10	
M. Heavy rockburst (massive rock)	< 2.5	< 0.16	10 - 20	3. Few case records available where depth of crown below surface is less than span width. Suggest *SRF* increase from 2.5 to 5 for such cases (see H).
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Rock Tunnelling Quality Index, Q (NGI)

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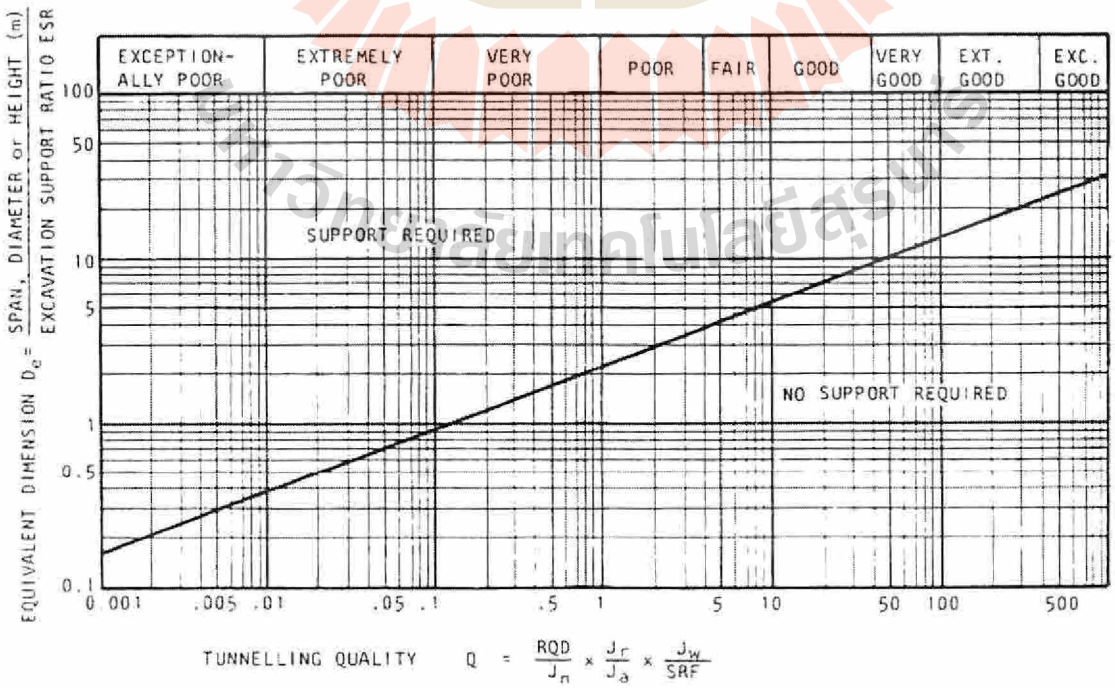
$$D_e = \frac{\text{Excavation span, diameter or height (m)}}{\text{Excavation Support Ratio } ESR}$$

Rock Tunnelling Quality Index, Q (NGI)

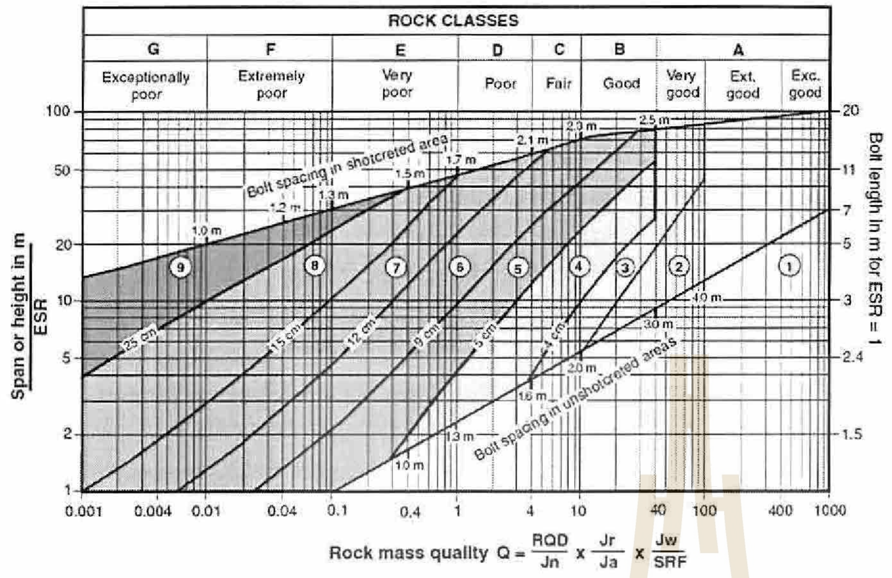
- The value of *ESR is related to the intended use of the excavation and to the degree of* security which is demanded of the support system installed to maintain the stability of the excavation.
- Barton et al (1974) suggest the following values:

Excavation category		*ESR*
A	Temporary mine openings.	3-5
B	Permanent mine openings, water tunnels for hydro power (excluding high pressure penstocks), pilot tunnels, drifts and headings for large excavations.	1.6
C	Storage rooms, water treatment plants, minor road and railway tunnels, surge chambers, access tunnels.	1.3
D	Power stations, major road and railway tunnels, civil defence chambers, portal intersections.	1.0
E	Underground nuclear power stations, railway stations, sports and public facilities, factories.	0.8

Rock Tunnelling Quality Index, Q (NGI)



Estimated support categories



Estimated support categories based on the tunnelling quality index Q *(After* Grimstad and Barton, 1993, reproduced from Palmstrom and Broch, 2006).

REINFORCEMENT CATEGORIES:

1) Unsupported
2) Spot bolting
3) Systematic bolting
4) Systematic bolting, (and unreinforced shotcrete, 4 - 10 cm)
5) Fibre reinforced shotcrete and bolting, 5 - 9 cm
6) Fibre reinforced shotcrete and bolting, 9 - 12 cm
7) Fibre reinforced shotcrete and bolting, 12 - 15 cm
8) Fibre reinforced shotcrete, > 15 cm, reinforced ribs of shotcrete and bolting
9) Cast concrete lining

Example

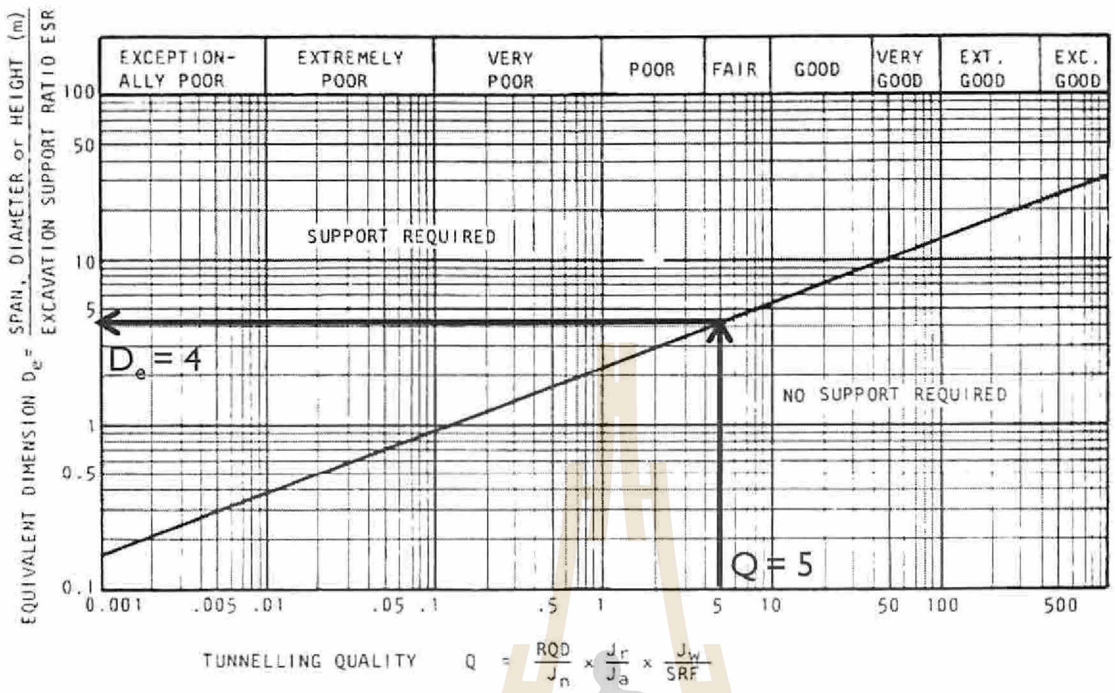
Item	Description	Value
1. Rock Quality	**Good**	**RQD = 80%**
2. Joint sets	**Two sets**	**Jn = 4**
3. Joint roughness	**Rough**	**Jr = 3**
4. Joint alteration	**Clay gouge**	**Ja = 4**
5. Joint water	**Large inflow**	**Jw = 0.33**
6. Stress reduction	**Medium stress**	**SRF = 1.0**

$$Q = \frac{80}{4} \times \frac{3}{4} \times \frac{0.33}{1} = 5$$

From the Figure 3.7, the maximum equivalent dimension D_e = 4 meters.

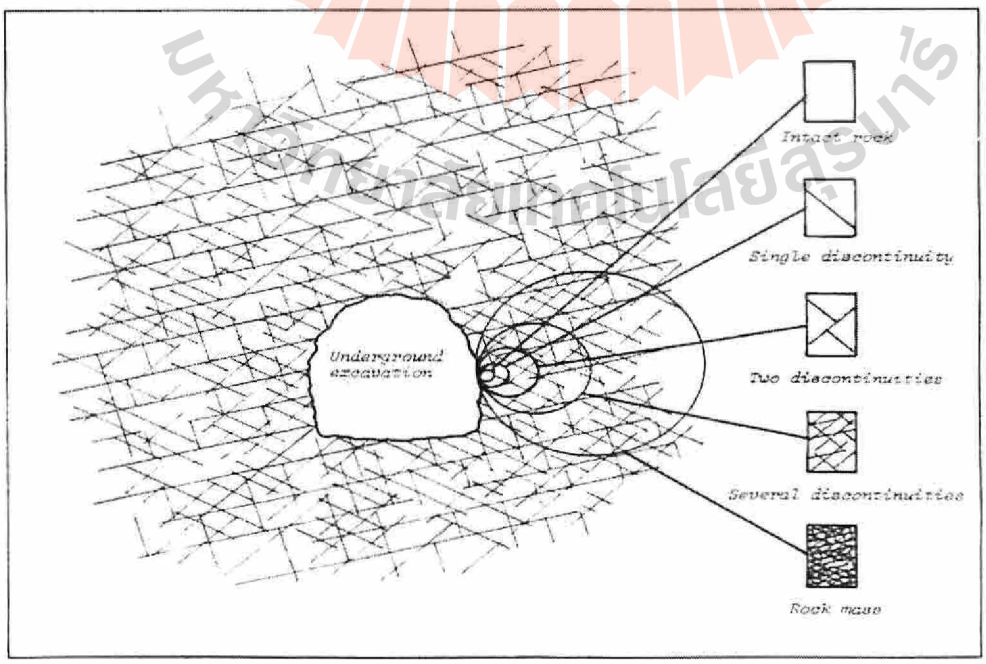
A permanent underground mine opening has an excavation support ratio ESR of 1.6 and, hence the maximum unsupported span which can be considered for this crusher station is ESR × De = 1.6 × 4 = 6.4 meters.

Rock Tunnelling Quality Index, Q (NGI)

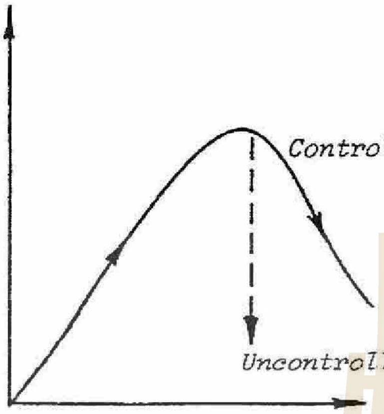


Strength of Rock and Rock Mass

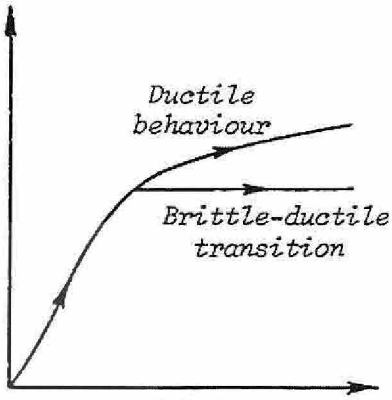
The transition from intact rock material to a heavily jointed rock mass



Brittle and Ductile Behavior

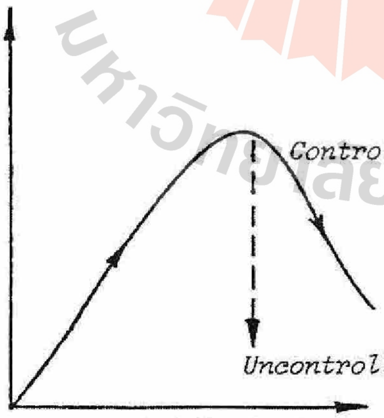


Stress-strain curves for brittle fracture in uniaxial compression



Stress-strain curves for ductile behaviour in compression

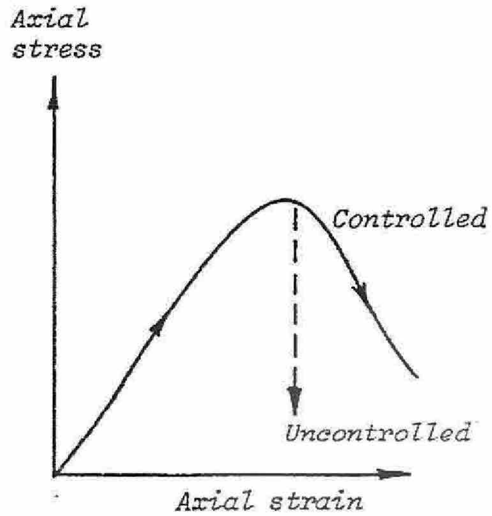
Brittle and Ductile Behavior



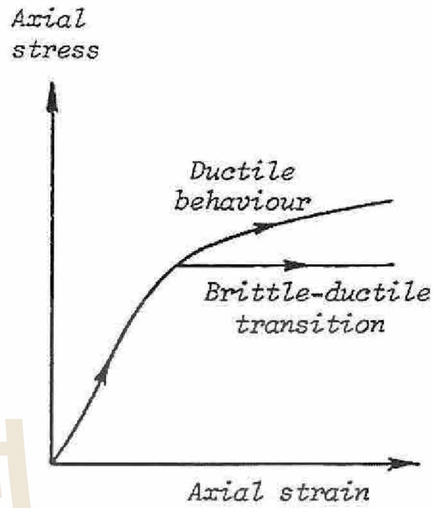
Stress-strain curves for brittle fracture in uniaxial compression

Brittle failure occurs when the ability of the rock to resist load decreases with increasing deformation. Brittle failure is often associated with little or no permanent deformation before failure and, depending upon the test conditions, may occur suddenly and catastrophically. Rock bursts in deep hard rock mines provide graphic illustrations of the phenomenon of explosive brittle fracture.

Brittle and Ductile Behavior

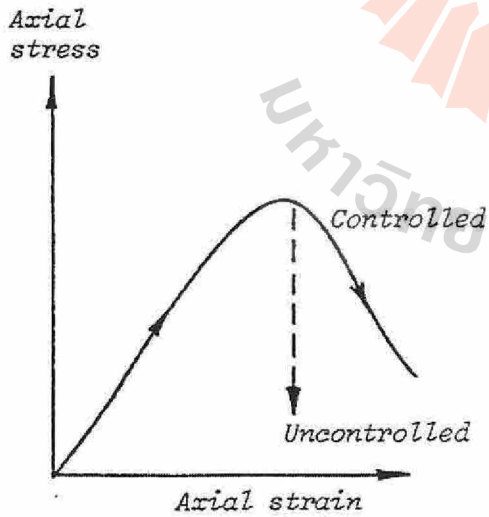


Stress-strain curves for brittle fracture in uniaxial compression



Stress-strain curves for ductile behaviour in compression

Brittle and Ductile Behavior



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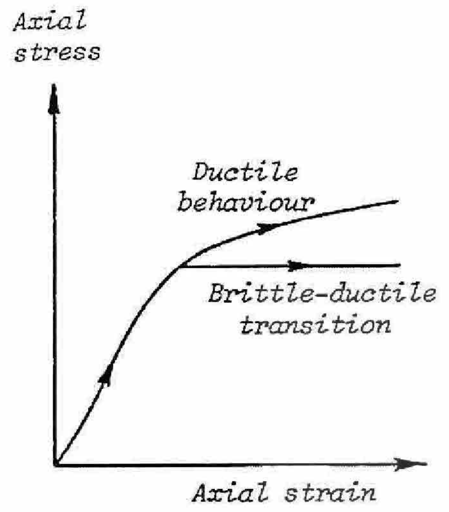
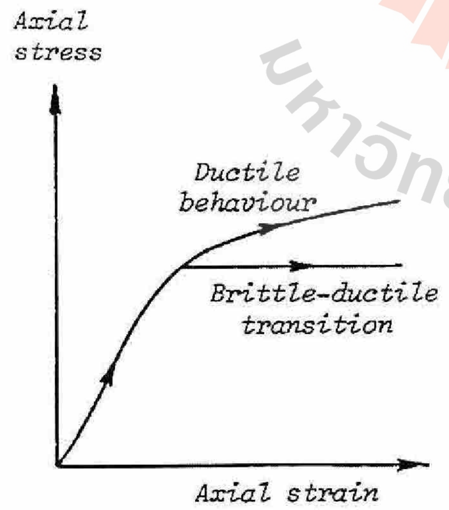
Brittle and Ductile Behavior



Stress-strain curves for ductile behaviour in compression

A material is said to be ***ductile*** when it can sustain permanent deformation without losing its ability to resist load. Most rocks will behave in a brittle rather than a ductile manner at the confining pressures and temperatures encountered in civil and mining engineering applications. *Ductility increases with increased confining pressure and temperature, but can also occur in weathered rocks,* heavily jointed rock masses and some weak rocks such as evaporites under normal engineering conditions.

Brittle and Ductile Behavior

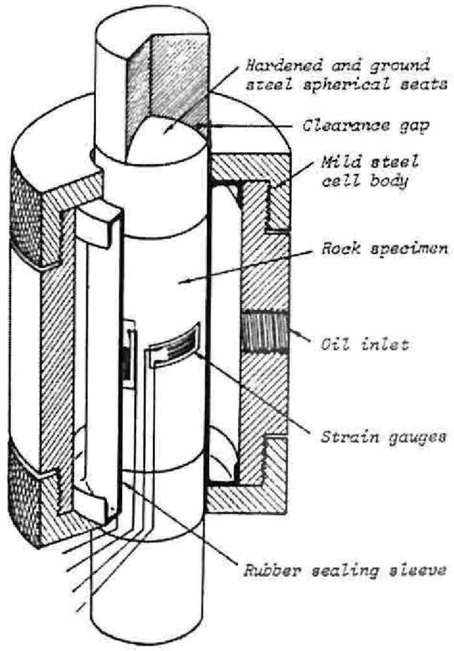


Stress-strain curves for ductile behaviour in compression

As the confining pressure is increased it will reach the ***brittle-ductile transition*** value at which there is a transition from typically brittle to fully ductile behaviour. The brittle-ductile transition pressure as the confining pressure is which the stress required to form a failure plane in a rock specimen is equal to the stress required to cause sliding on that plane.

Laboratory Testing of Intact Rock Specimens

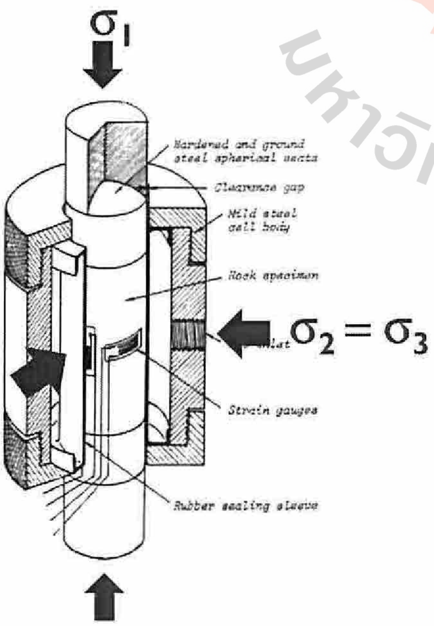
- Uniaxial and triaxial compression tests



Design by Hoek and Flankin (1964)

Laboratory Testing of Intact Rock Specimens

- Uniaxial and triaxial compression tests



Uniaxial compression

$\sigma_2 = \sigma_3 = 0$

$\sigma_1 = \sigma_c$ (uniaxial compressive strength)

Triaxial compression

$\sigma_2 = \sigma_3 = p$ (confining pressure)

An Empirical Failure Criterion of Rock

- Hoek and Brown (1980) have drawn on their experience in both theoretical and experimental aspects of rock behaviour to develop, by a process of trial and error, the following empirical relationship between the principal stresses associated with the failure of rock :

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_3\sigma_3 + s\sigma_c^2}$$ (The original Hoek and Brown Failure Criterion)

where

m = constant depending on the characteristics of the rock mass,
s = constant depending on the characteristics of the rock mass,
σ_c = uniaxial compressive strength of the intact rock material,
σ_1 = major principal stress at failure, and
σ_3 = minor principal stress at failure.

Hoek and Brown Failure Criterion

- The uniaxial compressive strength of the specimen is given by substitution $\sigma_3 = 0$

$$\sigma_{c,\,\mathrm{rockmass}} = \sigma_c \sqrt{s}$$ (Compressive Strength of Rock Mass)

- For intact rock, $\sigma_{c,\,\mathrm{rockmass}} = \sigma_c$ and s = 1.
- For previously broken rock, s < 1 and the strength at zero confining pressure, where σ_c is the uniaxial compressive strength of the pieces of *intact* rock material making up the specimen.

Hoek and Brown Failure Criterion

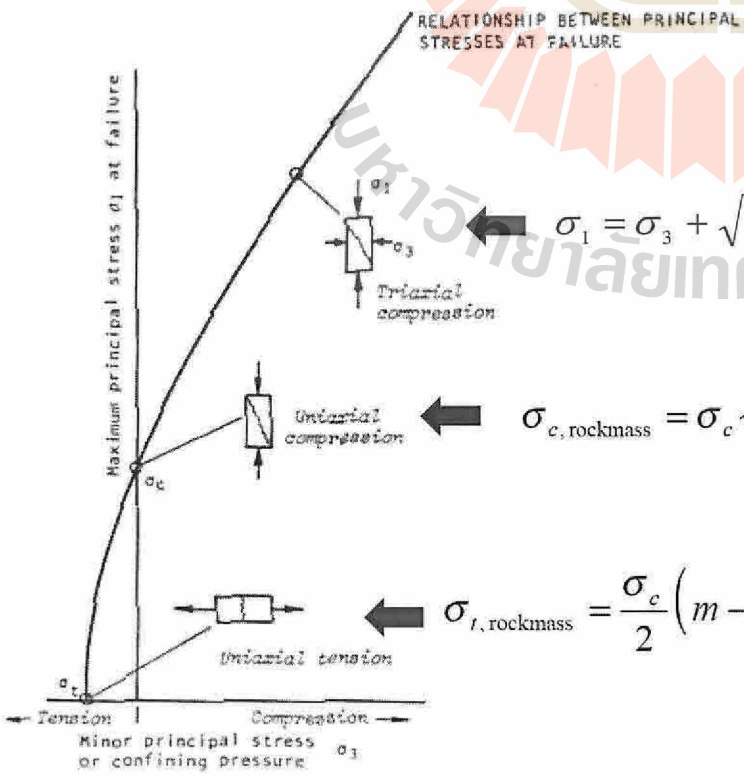
- The uniaxial tensile strength of the specimen is given by substitution of $\sigma_1 = 0$ in equation $\sigma_1 = \sigma_3 + \sqrt{m\sigma_3\sigma_3 + s\sigma_c^2}$ and by solving the resulting quadratic equation for σ_3

$$\sigma_{t,\,\mathrm{rockmass}} = \frac{\sigma_c}{2}\left(m - \sqrt{m^2 + 4s}\right)$$ (Tensile Strength of Rock Mass)

a **quadratic equation** is a polynomial equation of the second degree.

$$ax^2 + bx + c = 0, \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Hoek and Brown Failure Criterion



$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_3\sigma_3 + s\sigma_c^2}$$

$$\sigma_{c,\,\mathrm{rockmass}} = \sigma_c\sqrt{s}$$

$$\sigma_{t,\,\mathrm{rockmass}} = \frac{\sigma_c}{2}\left(m - \sqrt{m^2 + 4s}\right)$$

Use rock mass classification for strength prediction

Table 1 : Approximate relationship between rock mass quality and material constants

Disturbed rock mass *m* and *s* values — *undisturbed rock mass m and s values*

EMPIRICAL FAILURE CRITERION $\sigma_1' = \sigma_3' + \sqrt{m\sigma_c\sigma_3' + s\sigma_c^2}$ σ_1' = major principal effective stress σ_3' = minor principal effective stress σ_c = uniaxial compressive strength of intact rock, and *m* and *s* are empirical constants.		CARBONATE ROCKS WITH WELL DEVELOPED CRYSTAL CLEAVAGE *dolomite, limestone and marble*	LITHIFIED ARGILLACEOUS ROCKS *mudstone, siltstone, shale and slate (normal to cleavage)*	ARENACEOUS ROCKS WITH STRONG CRYSTALS AND POORLY DEVELOPED CRYSTAL CLEAVAGE *sandstone and quartzite*	FINE GRAINED POLYMINERALLIC IGNEOUS CRYSTALLINE ROCKS *andesite, dolerite, diabase and rhyolite*	COARSE GRAINED POLYMINERALLIC IGNEOUS & METAMORPHIC CRYSTALLINE ROCKS - *amphibolite, gabbro gneiss, granite, norite, quartz-diorite*
INTACT ROCK SAMPLES						
Laboratory size specimens free	m	7.00	10.00	15.00	17.00	25.00
from discontinuities	s	1.00	1.00	1.00	1.00	1.00
CSIR rating: RMR = 100	*m*	*7.00*	*10.00*	*15.00*	*17.00*	*25.00*
NGI rating: Q = 500	*s*	*1.00*	*1.00*	*1.00*	*1.00*	*1.00*
VERY GOOD QUALITY ROCK MASS						
Tightly interlocking undisturbed rock	m	2.40	3.43	5.14	5.82	8.56
with unweathered joints at 1 to 3m.	s	0.082	0.082	0.082	0.082	0.082
CSIR rating: RMR = 85	*m*	*4.10*	*5.85*	*8.78*	*9.95*	*14.63*
NGI rating: Q = 100	*s*	*0.189*	*0.189*	*0.189*	*0.189*	*0.189*
GOOD QUALITY ROCK MASS						
Fresh to slightly weathered rock, slightly	m	0.575	0.821	1.231	1.395	2.052
disturbed with joints at 1 to 3m.	s	0.00293	0.00293	0.00293	0.00293	0.00293
CSIR rating: RMR = 65	*m*	*2.006*	*2.865*	*4.298*	*4.871*	*7.163*
NGI rating: Q = 10	*s*	*0.0205*	*0.0205*	*0.0205*	*0.0205*	*0.0205*

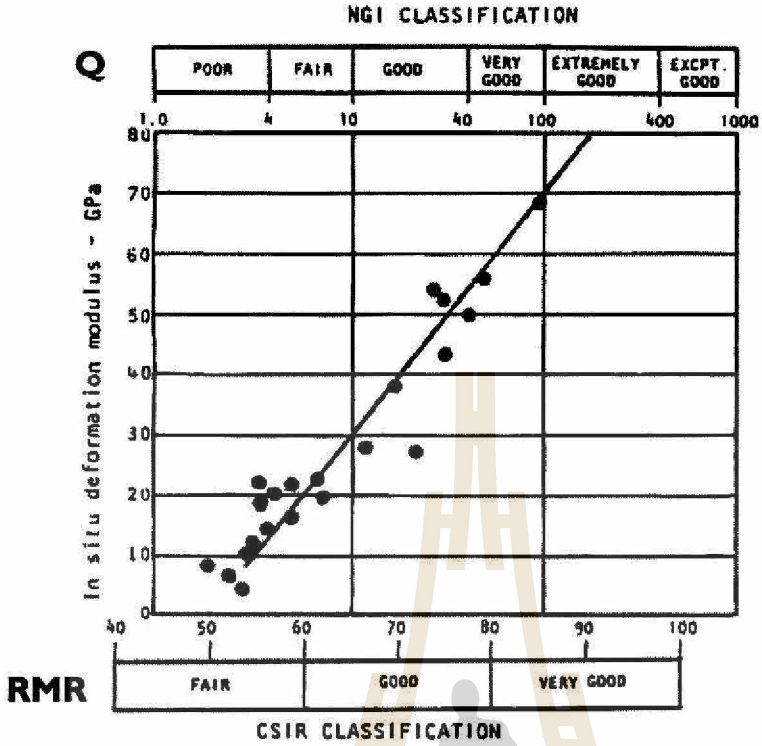
Use rock mass classification for strength prediction

Table 1 : Approximate relationship between rock mass quality and material constants

Disturbed rock mass *m* and *s* values — *undisturbed rock mass m and s values*

EMPIRICAL FAILURE CRITERION $\sigma_1' = \sigma_3' + \sqrt{m\sigma_c\sigma_3' + s\sigma_c^2}$ σ_1' = major principal effective stress σ_3' = minor principal effective stress σ_c = uniaxial compressive strength of intact rock, and *m* and *s* are empirical constants.		CARBONATE ROCKS WITH WELL DEVELOPED CRYSTAL CLEAVAGE *dolomite, limestone and marble*	LITHIFIED ARGILLACEOUS ROCKS *mudstone, siltstone, shale and slate (normal to cleavage)*	ARENACEOUS ROCKS WITH STRONG CRYSTALS AND POORLY DEVELOPED CRYSTAL CLEAVAGE *sandstone and quartzite*	FINE GRAINED POLYMINERALLIC IGNEOUS CRYSTALLINE ROCKS *andesite, dolerite, diabase and rhyolite*	COARSE GRAINED POLYMINERALLIC IGNEOUS & METAMORPHIC CRYSTALLINE ROCKS - *amphibolite, gabbro gneiss, granite, norite, quartz-diorite*
FAIR QUALITY ROCK MASS						
Several sets of moderately weathered	m	0.128	0.183	0.275	0.311	0.458
joints spaced at 0.3 to 1m.	s	0.00009	0.00009	0.00009	0.00009	0.00009
CSIR rating: RMR = 44	*m*	*0.947*	*1.353*	*2.030*	*2.301*	*3.383*
NGI rating: Q = 1	*s*	*0.00198*	*0.00198*	*0.00198*	*0.00198*	*0.00198*
POOR QUALITY ROCK MASS						
Numerous weathered joints at 30-500mm,	m	0.029	0.041	0.061	0.069	0.102
some gouge. Clean compacted waste rock	s	0.000003	0.000003	0.000003	0.000003	0.000003
CSIR rating: RMR = 23	*m*	*0.447*	*0.639*	*0.959*	*1.087*	*1.598*
NGI rating: Q = 0.1	*s*	*0.00019*	*0.00019*	*0.00019*	*0.00019*	*0.00019*
VERY POOR QUALITY ROCK MASS						
Numerous heavily weathered joints spaced	m	0.007	0.010	0.015	0.017	0.025
<50mm with gouge. Waste rock with fines.	s	0.0000001	0.0000001	0.0000001	0.0000001	0.0000001
CSIR rating: RMR = 3	*m*	*0.219*	*0.313*	*0.469*	*0.532*	*0.782*
NGI rating: Q = 0.01	*s*	*0.00002*	*0.00002*	*0.00002*	*0.00002*	*0.00002*

Deformability of Rock Mass



Deformability of Rock Mass

RMR > 55 (Bieniawski, 1978)

E ≅ 2 RMR – 100 (GPa)

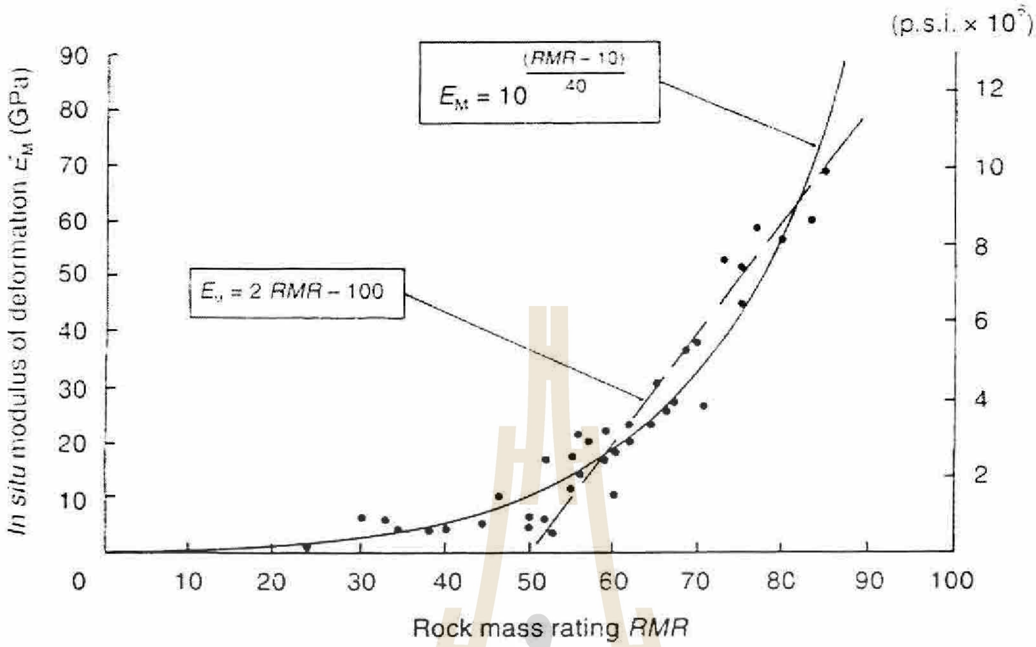
10 < RMR < 50 (Sarafim and Pereira, 1983)

E ≅ $10^{(RMR-10)/40}$ (GPa)

Hoek and Brown (1980)

E ≅ 17.5 ln (Q) – 10.17 (GPa)

Deformability of Rock Mass



Deformability of Rock Mass

The rock mass modulus of deformation is given by:

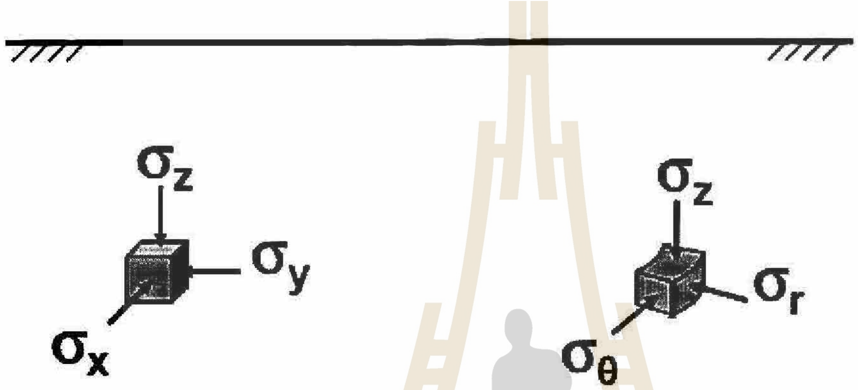
$$E_m(GPa) = \left(1 - \frac{D}{2}\right)\sqrt{\frac{\sigma_{ci}}{100}} \cdot 10^{((GSI-10)/40)} \qquad \text{for } \sigma_{ci} \leq 100 \text{ MPa}$$

$$E_m(GPa) = \left(1 - \frac{D}{2}\right) \cdot 10^{((GSI-10)/40)} \qquad \text{for } \sigma_{ci} > 100 \text{ MPa}$$

(Hoek et al, 2002)

In situ stresses field

- Stress field before excavation is represented by 3 principal stresses:
  1. **Vertical stress** is generally equal to the overburden stress
  2. **Horizontal stresses** are influenced by tectonic stress (in rock) and earth pressure coefficient (in soil)
- If the excavation is below water table, it is necessary to take into consideration **water pressure** (effective stress law)



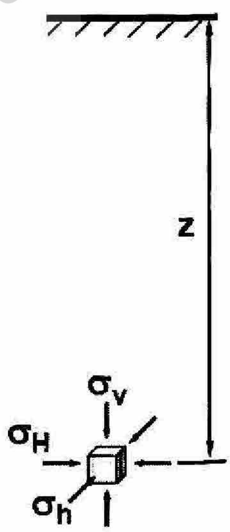
Vertical stress

- Depth below the surface, z = *1,000 m*
- Unit weight of the rock, γ = 0.027 MN/m^3
- The weight of the vertical column of rock? →*27 MPa*

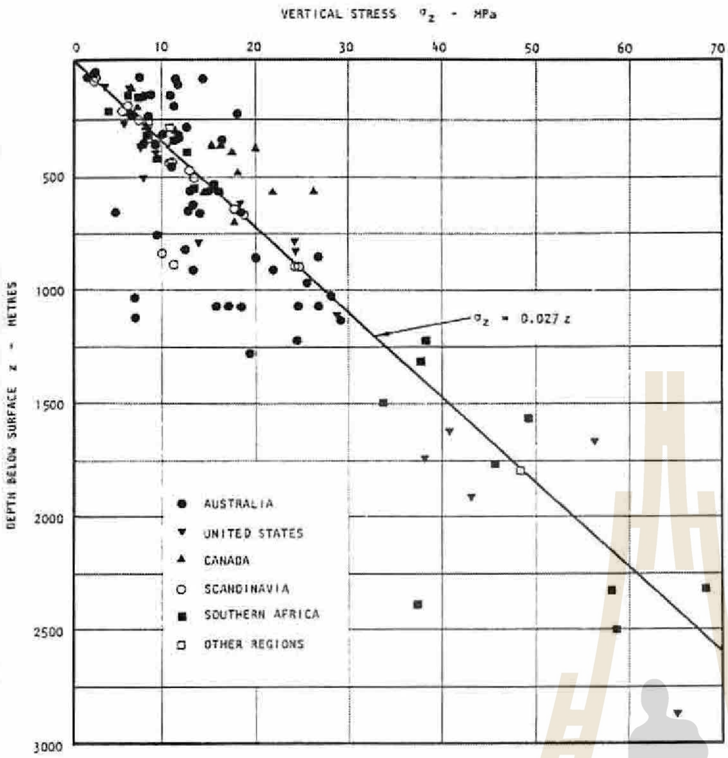
Vertical stress on the element

$$\boxed{\sigma_v = \gamma z}$$

where	σ_v	is the vertical stress
	γ	is the unit weight of the overlying rock and
	Z	is the depth below surface



Vertical stress



Vertical stress measurements from mining and civil engineering projects around the world. (After Brown and Hoek 1978).

$$\sigma_v \approx 0.027 \text{ MPa/m}$$
$$\approx 1 \text{ psi/ft}$$

As a rule of thumb, taking the average density of rock into account, 40 m of overlying rock induces 1 MPa stress.

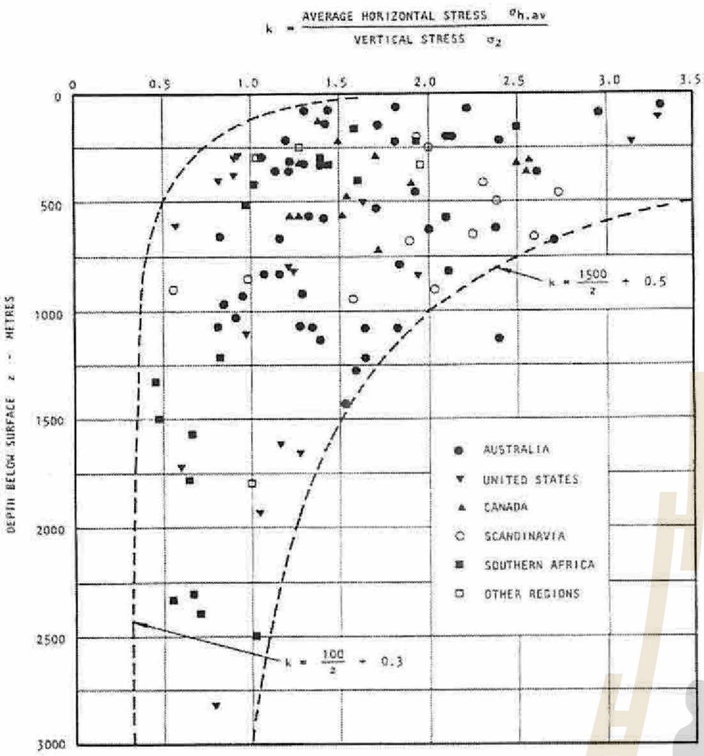
Horizontal stress

- Much more difficult to estimate than the vertical stresses
- Ratio of the average horizontal stress to the vertical stress, k
- k increases when shallow depth decreases

Horizontal stress on the element

$$\boxed{\sigma_h = k\sigma_v = k\gamma z}$$

Horizontal stress



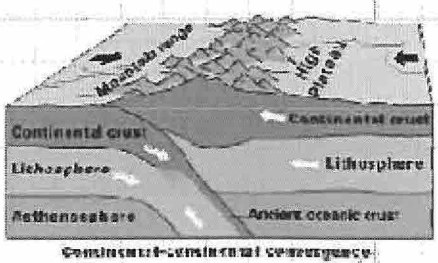
(100/z)+0.3 < k < (1500/z)+0.5

Reason for High Horizontal Stress

High horizontal stresses are caused by factors relating to erosion, tectonics, rock anisotropy, local effects near discontinuities, and scale effects:

Erosion - if horizontal stresses become 'locked in', then the erosion/removal of overburden (i.e. decrease in σ_V) will result in an increase in *K* ratio (σ_H/σ_V).

Tectonics - different forms of tectonic activity (e.g. subduction zones), can produce high horizontal stresses.



Horizontal stress

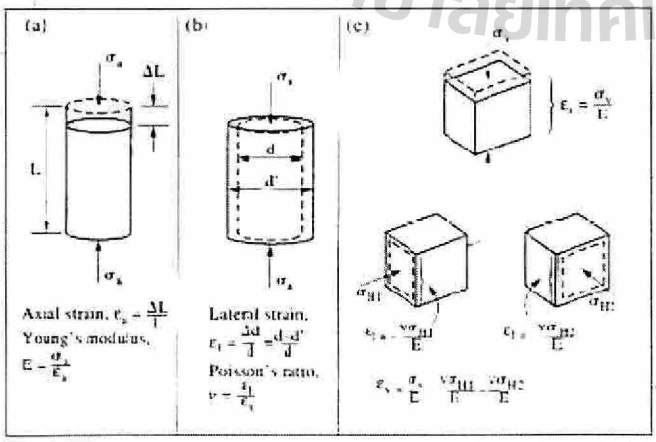
- Terzaghi and Richart (1952) suggested that, for a gravitationally loaded rock mass in which no lateral strain was permitted during formation of the overlying strata, the value of k is independent of depth and is given by

$$\boxed{k = \nu/(1-\nu)}$$

where ν is the Poisson's ratio of the rock mass.

Horizontal stress

The horizontal stress can be estimated using of elastic theory. If we consider the strain along any axis of a small cube at depth, then the total strain can be found from the strain due to the axial stress, subtracting the strain components due to the two perpendicular stresses.



Hudson & Harrison (1997)

For example:

$$\varepsilon_V = \frac{\sigma_V}{E} - \frac{\nu\sigma_{H1}}{E} - \frac{\nu\sigma_{H2}}{E}$$

$$\varepsilon_{H1} = \frac{\sigma_{H1}}{E} - \frac{\nu\sigma_{H2}}{E} - \frac{\nu\sigma_V}{E}$$

Horizontal stress

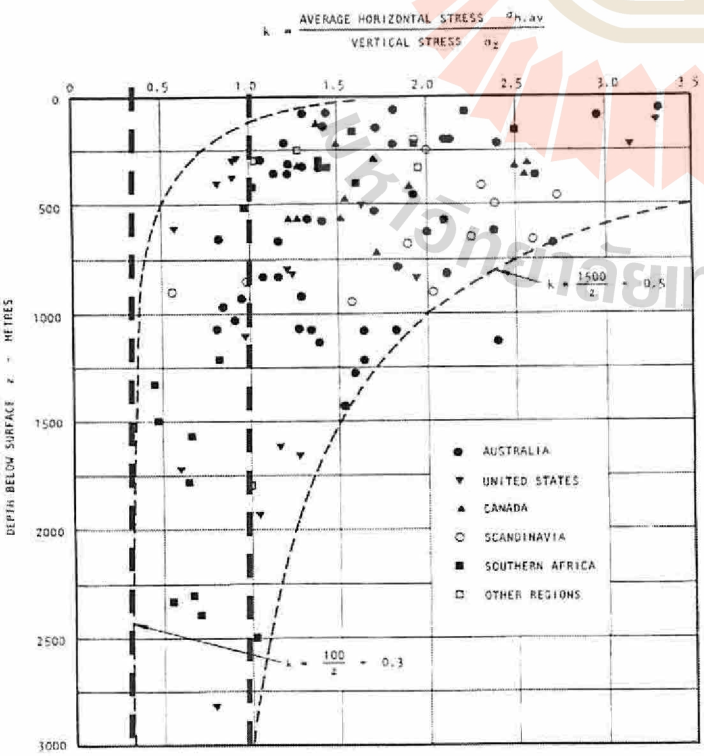
To provide an initial estimate of the horizontal stress, two assumptions are made:

- the two horizontal stresses are equal;
- there is no horizontal strain, i.e. both ε_{H1} and ε_{H2} are zero (e.g. because it is restrained by adjacent elements of rock).

Then we can take ε_{H1} as zero: $$0 = \frac{\sigma_{H1}}{E} - \frac{\nu\sigma_{H2}}{E} - \frac{\nu\sigma_V}{E}$$

And, because $\sigma_{H1} = \sigma_{H2}$: $$\sigma_H = \frac{\nu}{1-\nu}\sigma_V$$

Horizontal stress



Thus the ratio between the horizontal and vertical stress (referred to as $K = \sigma_H/\sigma_V$) is a function of the Poisson's ratio:

$$\frac{\sigma_H}{\sigma_V} = \frac{\nu}{1-\nu}$$

For a typical Poisson's ratio (ν) of 0.25, the resulting K ratio is 0.33. For a theoretical maximum of ν = 0.5, the maximum K ratio predicted is 1.0.

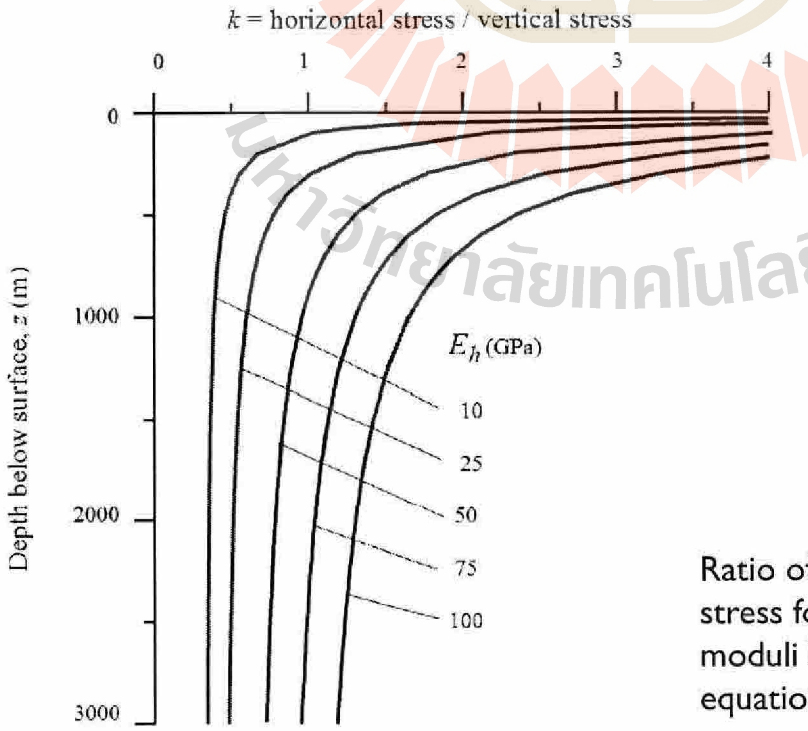
Horizontal stress

- Sheorey (1994) developed an elasto-static thermal stress model of the earth. This model considers curvature of the crust and variation of elastic constants, density and thermal expansion coefficients through the crust and mantle.
- He provide a simplified equation can be used for estimating the horizontal to vertical stress ratio *k*.

$$k = 0.25 + 7E_h\left(0.001 + \frac{1}{z}\right)$$

where z (m) is the depth below surface and E_h (GPa) is the average deformation modulus of the upper part of the earth's crust measured in a horizontal direction.

Horizontal stress

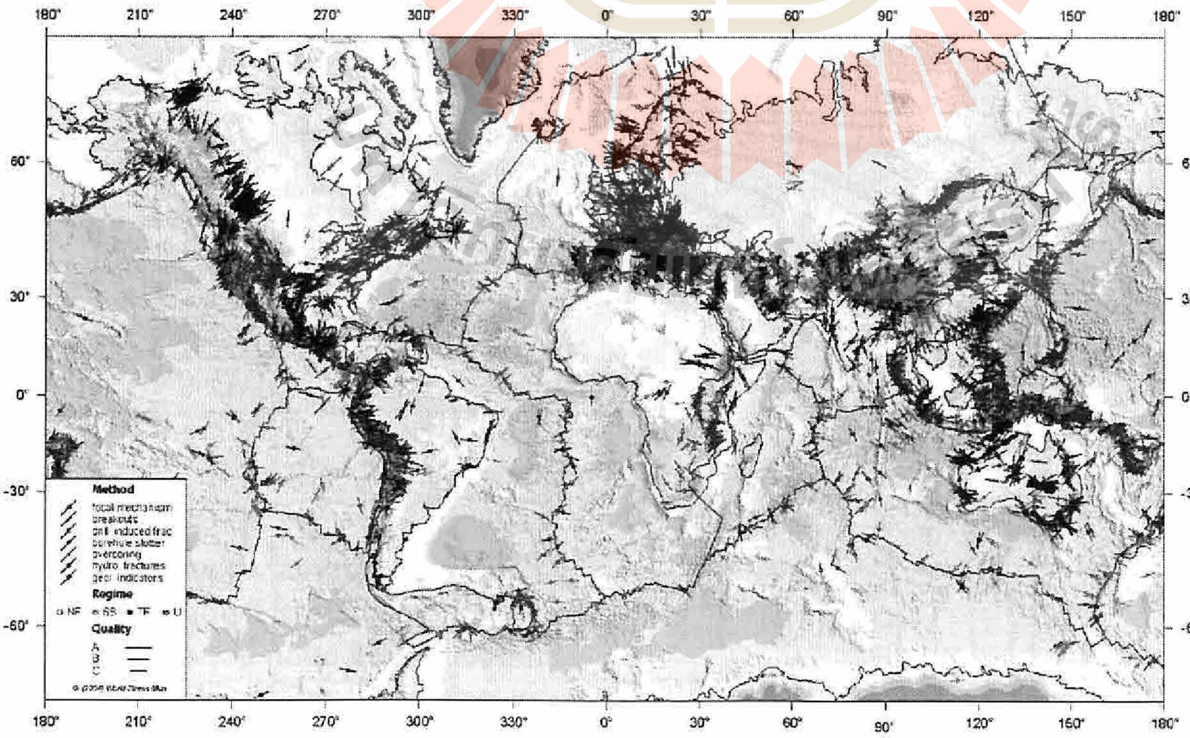


Ratio of horizontal to vertical stress for different deformation moduli based upon Sheorey's equation. (After Sheorey 1994).

The World stress map

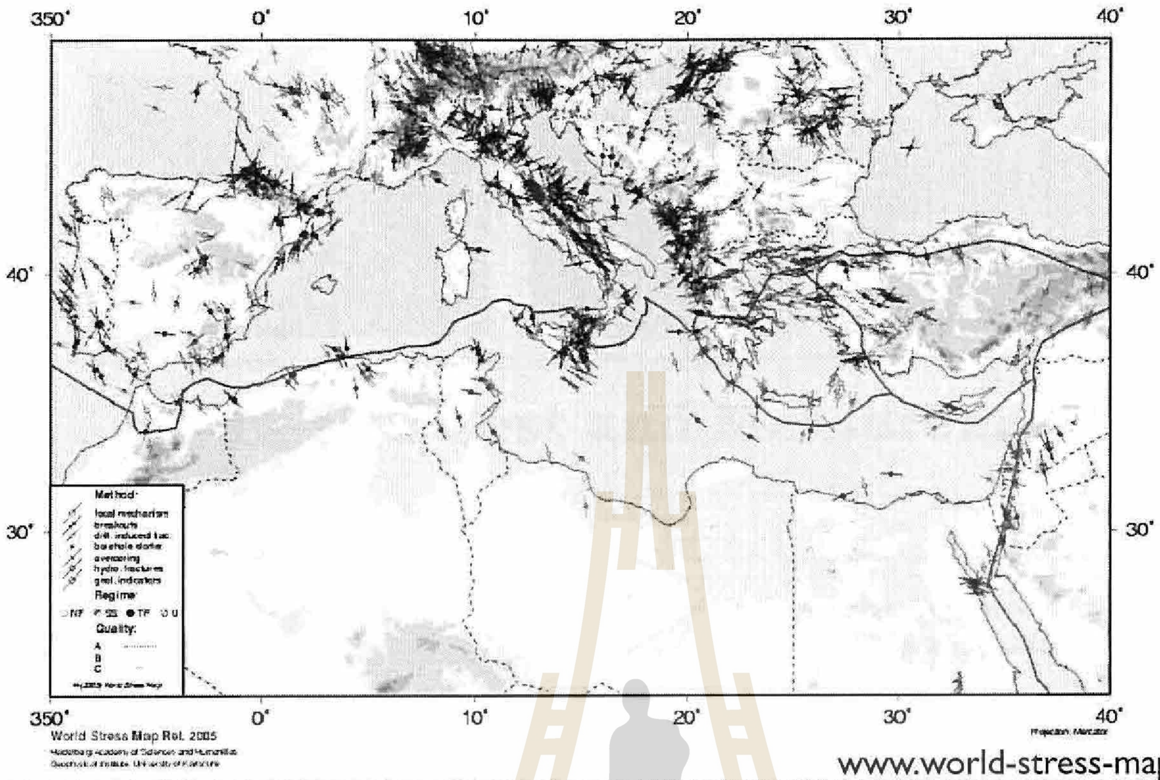
- The World Stress Map project, completed in July 1992, involved over 30 scientists from 18 countries and was carried out under the auspices of the International Lithosphere Project (Zoback, 1992).
- The aim of the project was to compile a global database of contemporary tectonic stress data.
- The World Stress Map (WSM) is now maintained and it has been extended by the Geophysical Institute of Karlsruhe University as a research project of the Heidelberg Academy of Sciences and Humanities.
- The WSM is an open-access database that can be accessed at www.world-stressmap.org (Reinecker et al, 2005)

World stress map giving orientations of the maximum horizontal compressive stress



www.world-stress-map.org

Stress map of the Mediterranean giving orientations of the maximum horizontal compressive stress



www.world-stress-map.org



In-situ Test and Measurements

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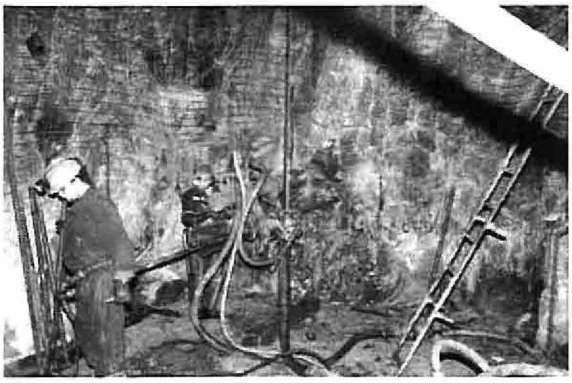
In-situ Test and Measurements

Objectives:

1. To determine in-situ stress (σ_v and σ_h)
2. To determine rock mass properties

Measurements from:

- Borehole / Drill hole
- Outcrop
- Tunnel wall / Pillar




In-situ Test and Measurements

In-situ stress Measurement Methods :

1. Hydraulic Fracturing *
2. Flat Jack *
3. Overcoring *
4. Doorstopper
5. Undercoring

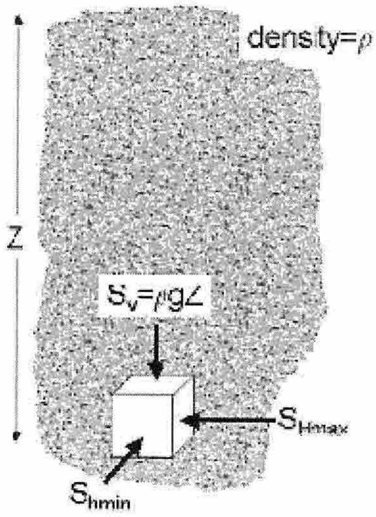
Elastic Modulus Measurement Methods :

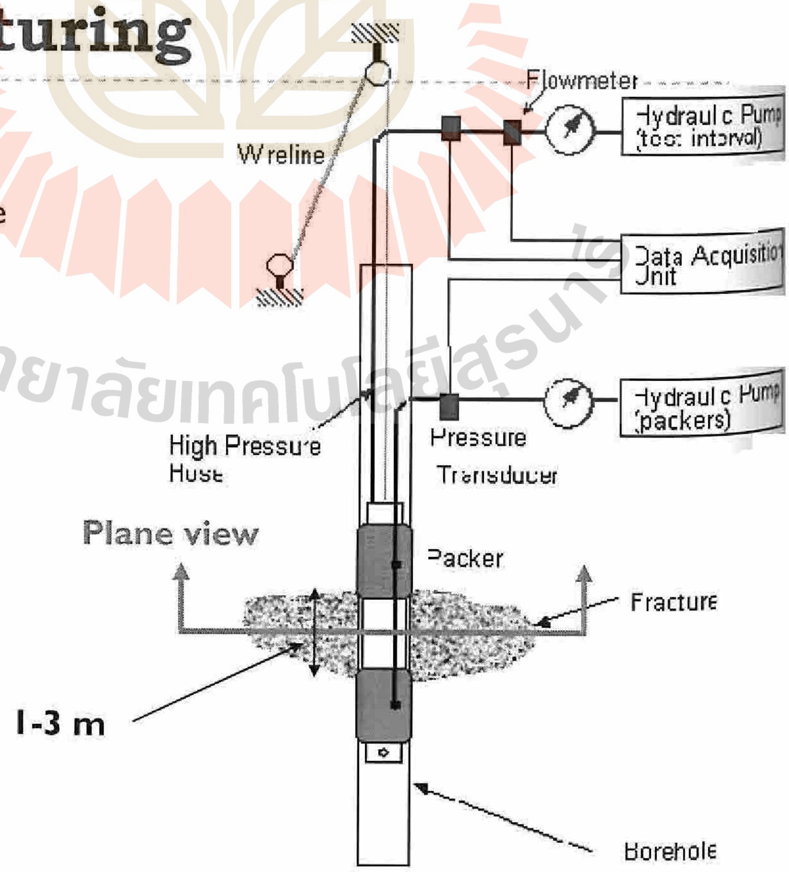
1. Plate Bearing Test
2. Dilatometer Test
3. Flat Jack Test

In-situ Direct Shear Test:

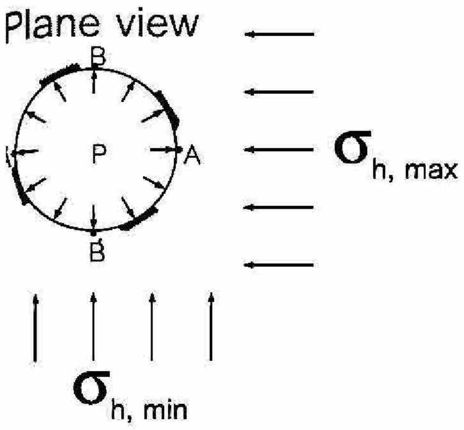
Hydraulic Fracturing

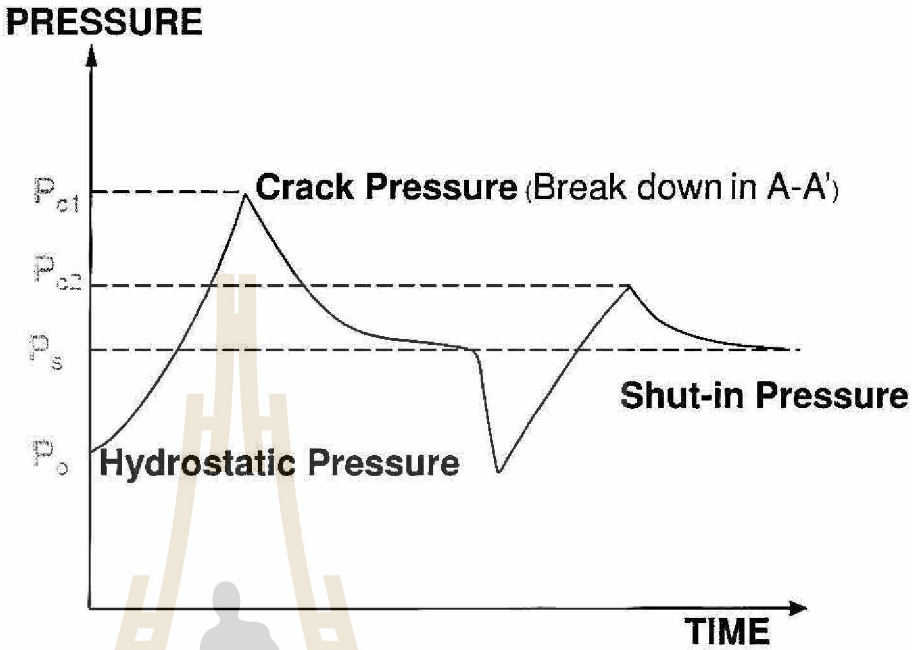
Measurement in Borehole/Drill hole





Hydraulic Fracturing Method

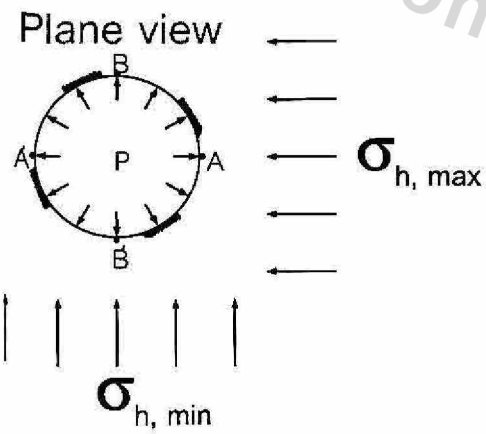




Hydraulic Fracturing Method

Assumptions:

- homogenous / continuous
- linear elastic
- isotropic



Applied from Kirsch Solution

Point A & A' (Radial Crack Occurred)

$P_x = \sigma_{h,max}$, $P_y = \sigma_{h,min}$, r=a, θ=0

and P=0 (internal pressure)

$$\sigma_\theta = \frac{1}{2}\left\{(P_x + P_y)\left(1+\cancel{\frac{a^2}{r^2}}\right)^{2} - (P_x - P_y)\left(1+\cancel{\frac{3a^4}{r^4}}\right)^{4}\cancel{\cos 2\theta}\right\}$$

$$\boxed{\sigma_\theta = 3\sigma_{h,min} - \sigma_{h,max}}$$

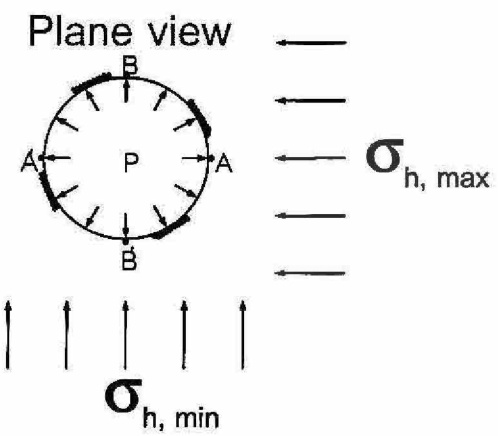
Hydraulic Fracturing Method

Applied from Kirsch Solution

Point A & A' (Radial Crack Occurred)

$P_x = \sigma_{h,max}$, $P_y = \sigma_{h,min}$, $r=a$, $\theta=0$

and $P=P_{c1}$ (internal pressure)



Tangential stress at Point A or A'

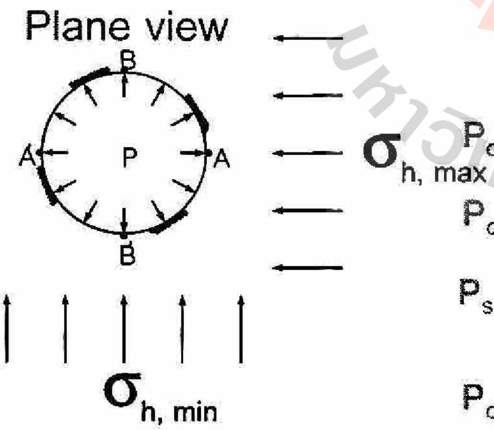
$$\sigma_\theta = 3\sigma_{h,\,min} - \sigma_{h,\,max} - P_{c1}$$

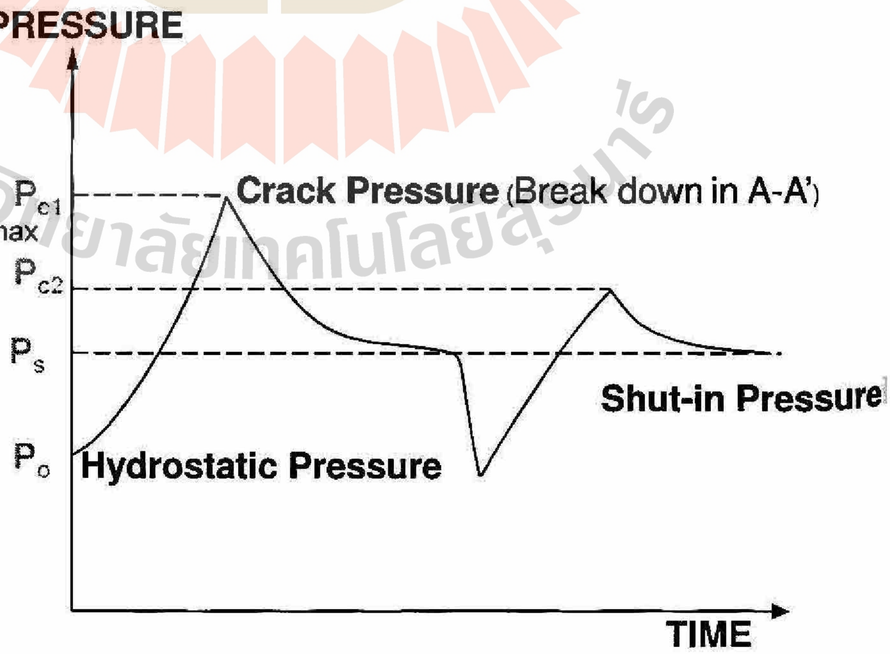
For Radial Crack Occurred ($\sigma_\theta = -T_o$)

T_0 = tensile strength of rock around boreholle

$$\boxed{3\sigma_{h,\,min} - \sigma_{h,\,max} - P_{c1} = -T_0}$$

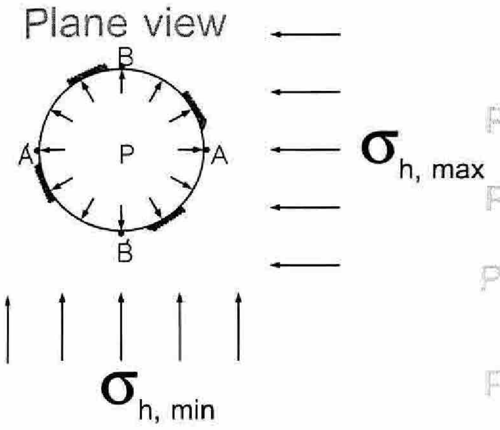
Hydraulic Fracturing Method

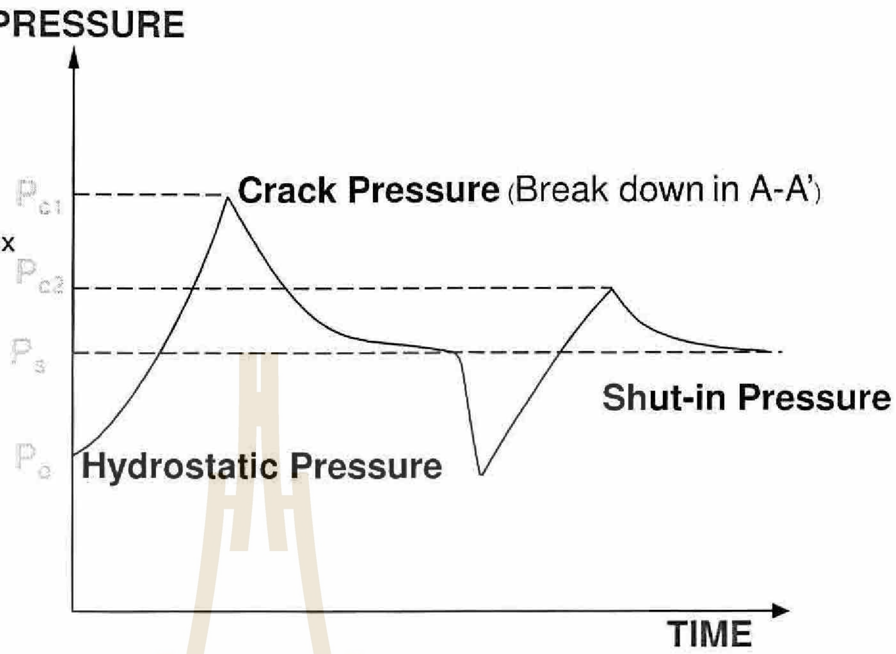




$$\boxed{\sigma_{h,\,min} = P_s}$$

Hydraulic Fracturing Method





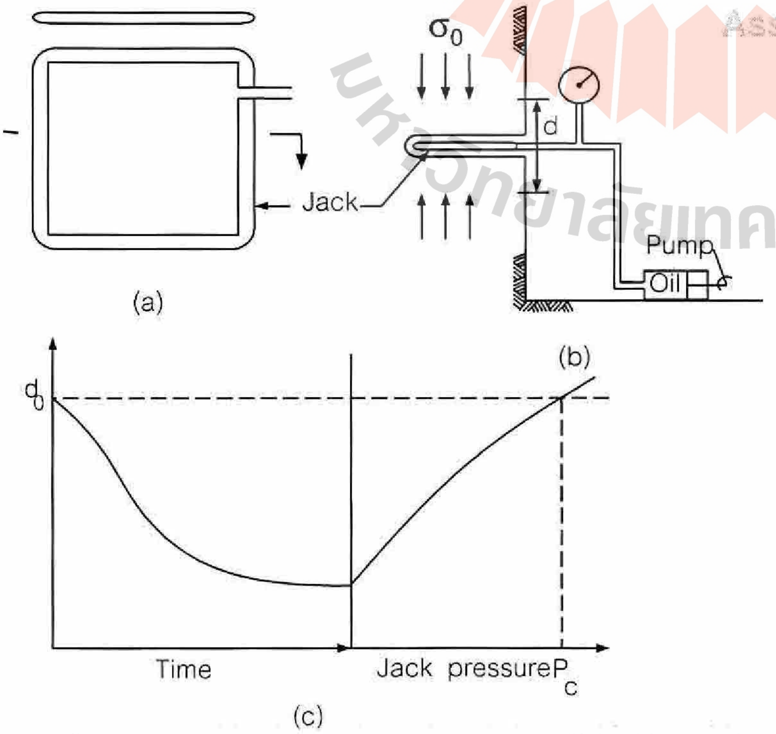
$$T_0 = P_{c1} - P_{c2}$$

Flat Jack Method

Measurement in Tunnel wall / Pillar

Assumption = Perfectly Elastic

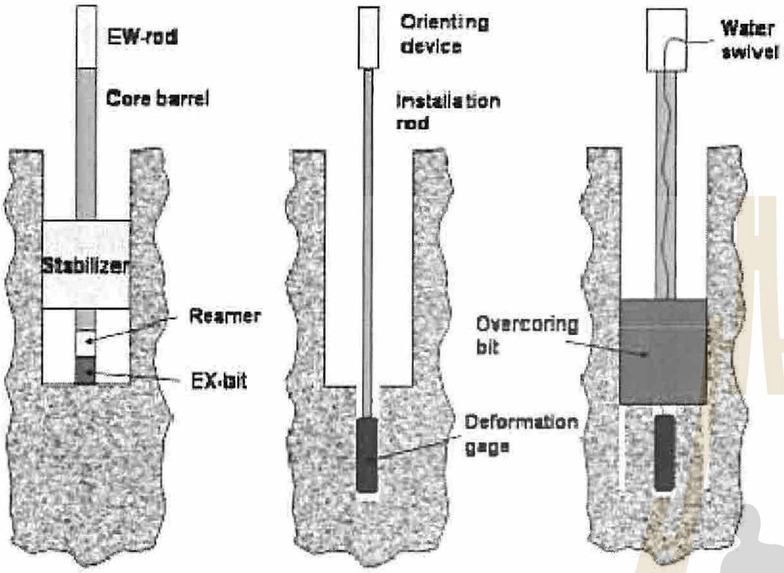
$$P_c = \sigma_0$$



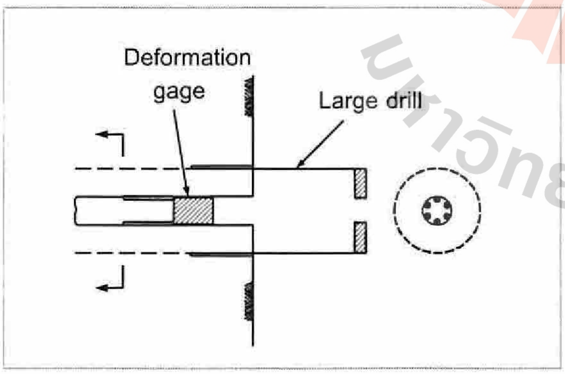
Overcoring Method

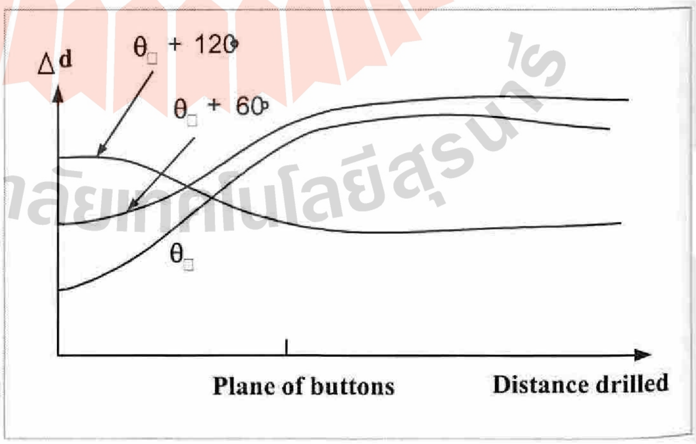
Measurement in Borehole/Drill hole

Tunnel wall / Pillar



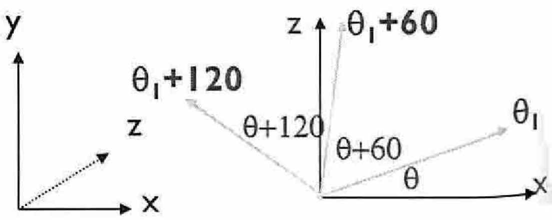
Overcoring Method





Change in diameter

$$\Delta d(\theta) = \sigma_x f_1 + \sigma_y f_2 + \sigma_z f_3 + \tau_{xz} f_4$$



Overcoring Method

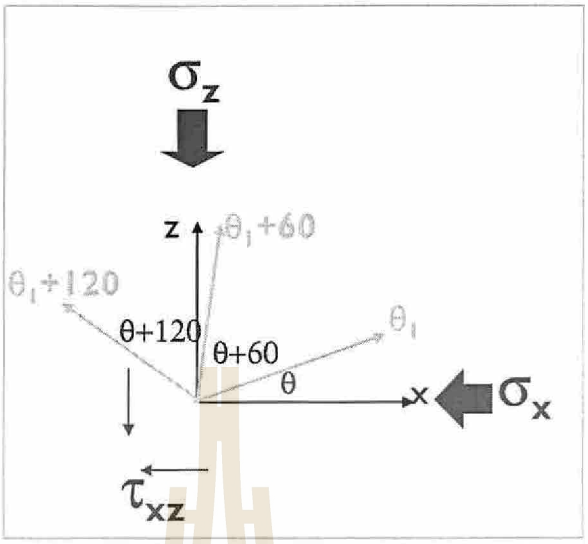
where

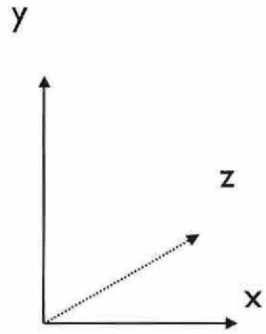
$$f_1 = d(1+2\cdot\cos 2\theta)\frac{1-\nu^2}{E} + \frac{d\nu^2}{E}$$

$$f_2 = -\frac{d\nu}{E}$$

$$f_3 = d(1-2\cdot\cos 2\theta)\frac{1-\nu^2}{E} + \frac{d\nu^2}{E}$$

$$f_4 = d(4\cdot\sin 2\theta)\frac{1-\nu^2}{E}$$





Overcoring Method

Equation

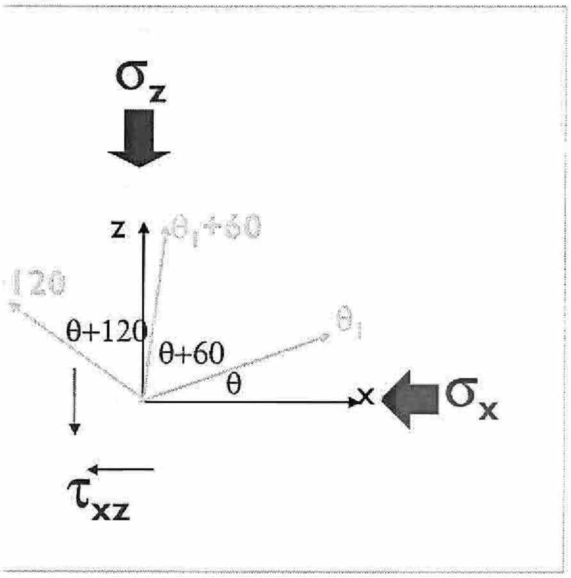
$$\begin{Bmatrix} \Delta d(\theta_1) & - f_2\,\sigma_y \\ \Delta d(\theta_1+60) & - f_2\,\sigma_y \\ \Delta d(\theta_1+120) & - f_2\,\sigma_y \end{Bmatrix} = \begin{bmatrix} f_{11} & f_{13} & f_{14} \\ f_{21} & f_{23} & f_{24} \\ f_{31} & f_{33} & f_{34} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{Bmatrix}$$

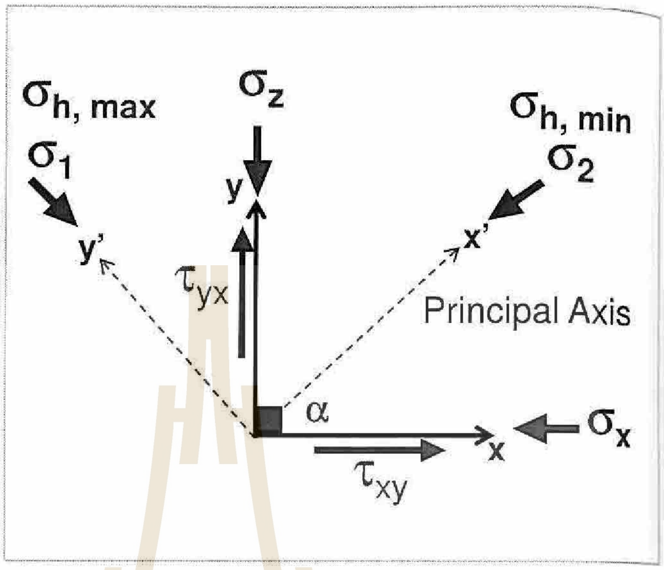
Δd = Change in diameter

f_{mn} ← n = 1,2,4 indicate the value of f_1, f_3 and f_4

m = 1,2,3 indicate the position of θ_1 , θ_1+60 and θ_1+120

vercoring Method

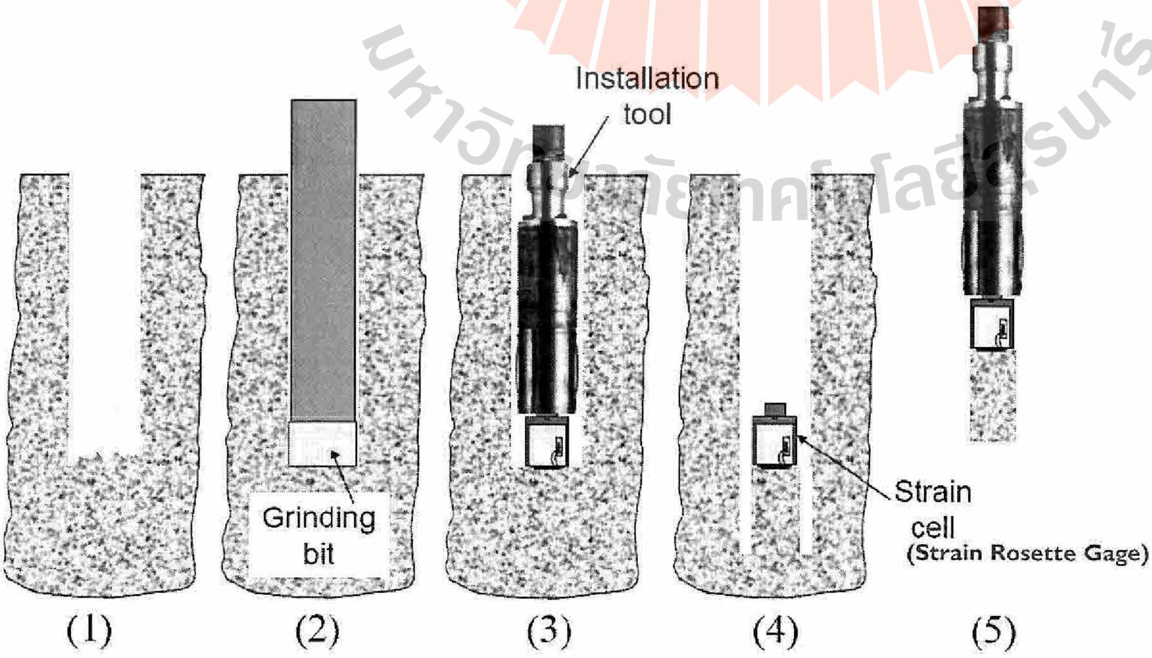




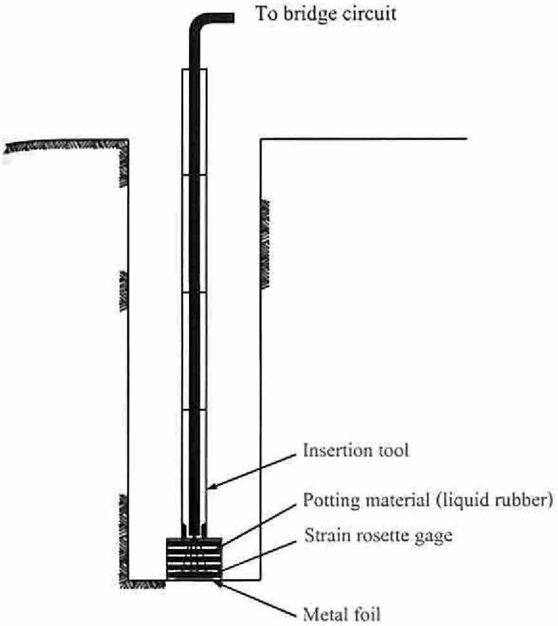
Horizontal Plane

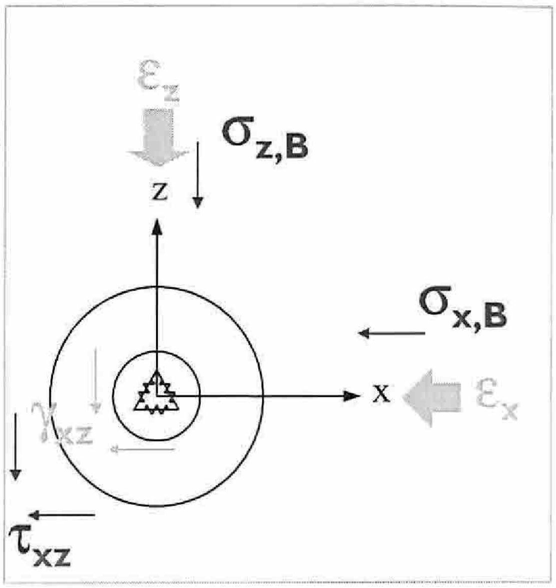
oorstopper

surement in Borehole/Drill hole
Tunnel wall / Pillar



Doorstopper



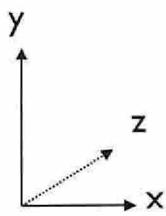


$$\begin{Bmatrix} \Delta\sigma_{x,B} \\ \Delta\sigma_{z,B} \\ \Delta\tau_{xz,B} \end{Bmatrix} = \frac{E}{1-\nu^2}\begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix}\begin{Bmatrix} \varepsilon_x \\ \varepsilon_z \\ \gamma_{xz} \end{Bmatrix}$$

Doorstopper

Calculate the stresses and shear stress

$$\begin{Bmatrix} \Delta\sigma_{x,B} \\ \Delta\sigma_{z,B} \\ \Delta\tau_{xz,B} \end{Bmatrix} = -\begin{pmatrix} a & c & b & 0 \\ b & c & a & 0 \\ 0 & 0 & 0 & d \end{pmatrix}\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xz} \end{Bmatrix}$$



$a = 1.30$

$b = (0.085 + 0.15\,\nu - \nu^2)$

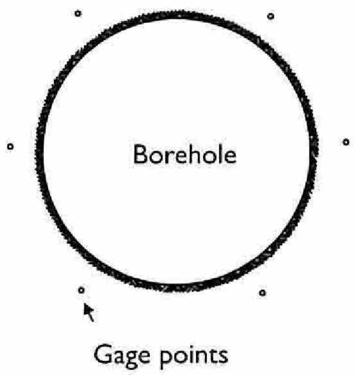
$c = (0.473 + 0.91\nu)$

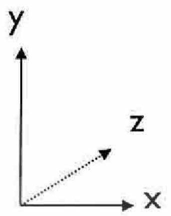
$d = (1.423 - 0.027\nu)$

(Goodman, 1989)

Undercoring

Measurement in Borehole/Drill hole
Tunnel wall / Pillar





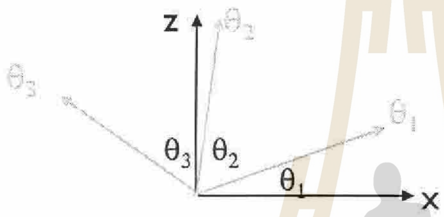
$$u_r = \sigma_x \cdot f_1 + \sigma_z \cdot f_2 + \tau_{xz} \cdot f_3$$

$$f_1 = \frac{1}{2E} \cdot \frac{a^2}{r}[(1+\nu) + H \cdot \cos 2\theta]$$

$$f_2 = \frac{1}{2E} \cdot \frac{a^2}{r}(1+\nu) - H \cdot \cos 2\theta$$

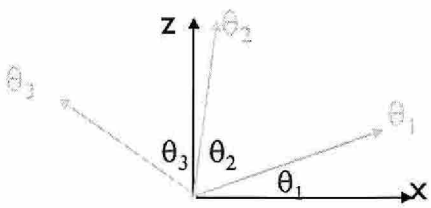
$$f_3 = \frac{1}{E} \cdot \frac{a^2}{r} H \cdot \sin 2\theta$$

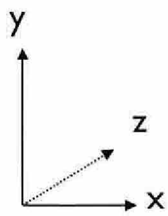
$$H = 4 - (1+\nu)a^2/r^2$$



Undercoring

$$\begin{Bmatrix} u_{r,1} \\ u_{r,2} \\ u_{r,3} \end{Bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_z \\ \tau_{xz} \end{Bmatrix}$$





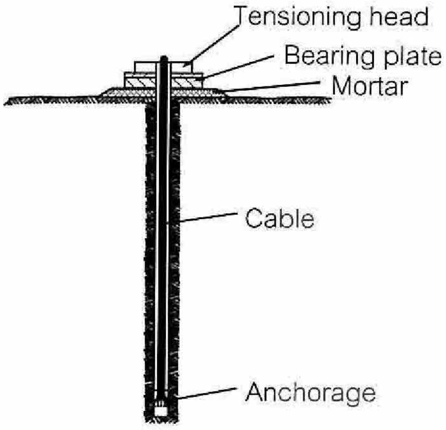
f_{mn} ← n = 1,2,3 indicate the value of f_1, f_2 and f_3

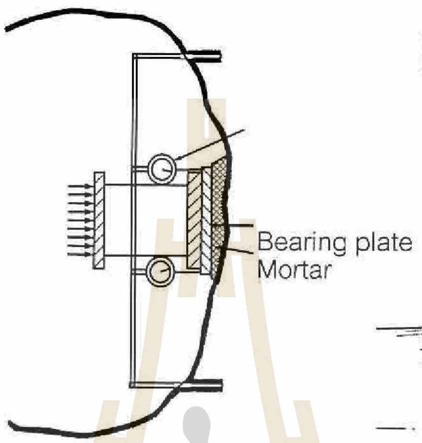
m = 1,2,3 indicate the position of θ_1, θ_2 and θ_3

Plate Bearing Test

Measurement in Outcrop/Tunnel wall

Rigid Plate Bearing Test (ASTM D3494)





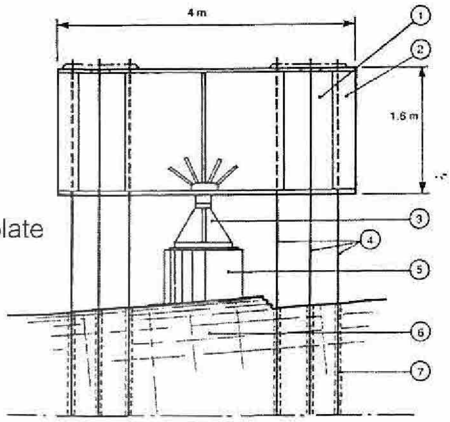


Plate Load Test

- Rigid Plate Bearing Test ASTM D3494
- Flexible Plate Bearing Test ASTM D3495

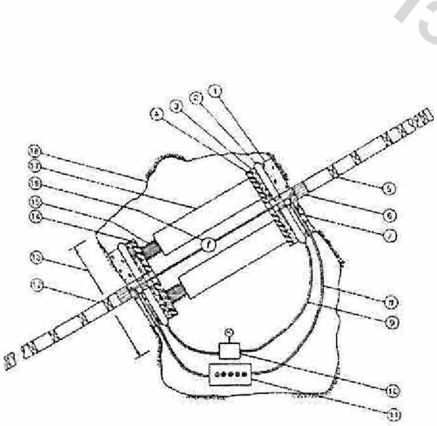


Figure 4.17 Typical set up for a uniaxial jacking test in which the load is applied through hydraulic flatjacks (Misterek *et al.*, 1974, © ASTM, reprinted with permission).
1. Concrete pad. 2. 1 m diameter flatjack. 3. Particle board pad. 4. Top plate. 5. MPBX anchors – 5 or more/hole. 6. MPBX sensor head. 7. Rubber sleeve over lead wires. 8. Transducer lead wire. 9. Hydraulic hoses. 10. Hydraulic pump, 70 MPa. 11. Data-acquisition system MPa. 12. NX drill hole, depth = 6 flatjack diameters. 13. Prepared diameter, 1.5 to 2 × flat jack diameter. 14. Base plate. 15. Screws for set up and removal. 16. Tunnel diameter gauge. 17. 254 mm diameter aluminum columns. 18. Tunnel surface.

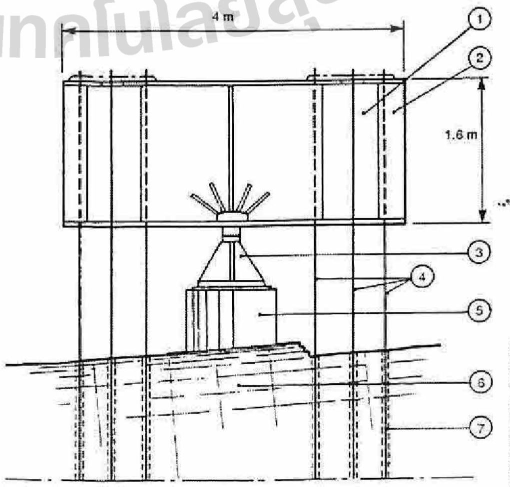


Figure 4.18 Typical arrangement of plate load test at ground surface (Pusch, 1992).
1. Hydraulic jacks. 2. Steel beam reaction head. 3. Steel lid. 4. Tie rods. 5. Concrete foundation. 6. Schistose gneiss. 7. 100 mm dia. anchor holes.

Rigid Plate Bearing Test

Designation: D 4394 – 04

Standard Test Method for
Determining the In Situ Modulus of Deformation of Rock Mass Using the Rigid Plate Loading Method[1]

Rigid Plate Bearing Test

D 4394 – 04

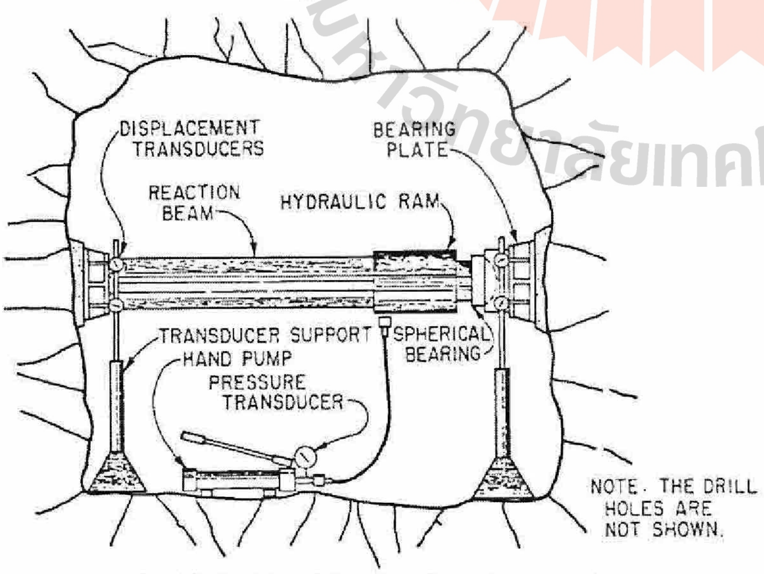


FIG. 4 Typical Rigid Plate Bearing Test Setup Schematic

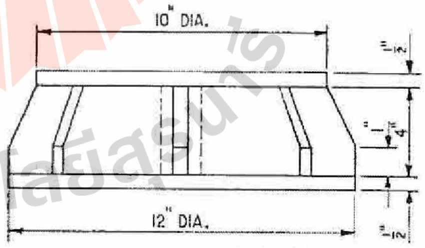
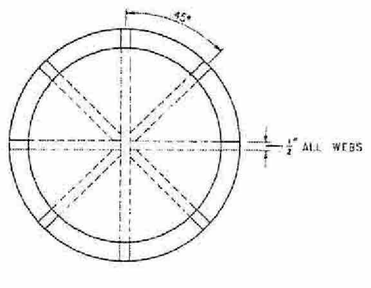


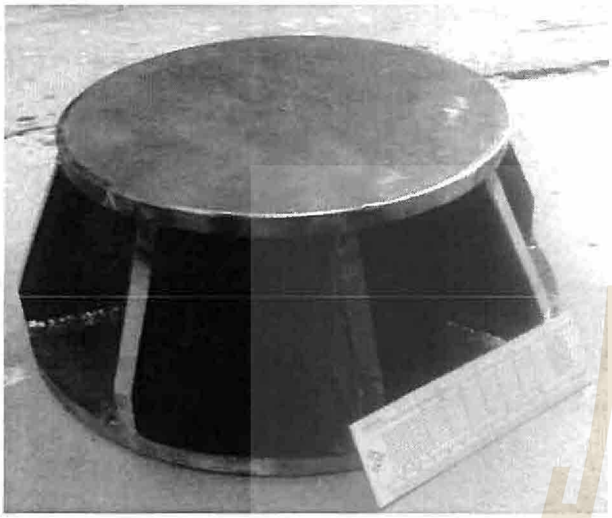
FIG. 3 Rigid Bearing Plate for 12 in. Diameter Test



NOTE: ALL JOINTS FULLY WELDED

Rigid Plate Bearing Test

Nam Ngum 3 Project



Rigid Plate Bearing Test

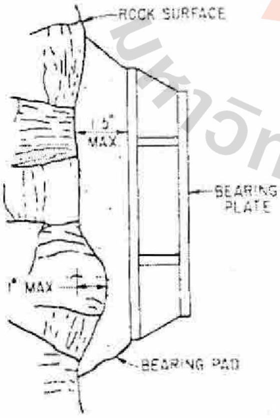


FIG. 5 Allowable Dimensions for Rock Surface and Bearing Pad

$$E = \frac{(1-\mu^2)\cdot P}{2W_a \cdot R}$$

where:

μ = Poisson's ratio of the rock,
P = total load on the rigid plate, lbf (kN),
W_a = average deflection of the rigid plate, in. (mm), and
R = radius of the rigid plate, in. (mm).

Rigid Plate Bearing Test

Designation: D 4394 – 04

Standard Test Method for Determining the In Situ Modulus of Deformation of Rock Mass Using the Rigid Plate Loading Method[1]

Rigid Plate Bearing Test

D 4394 – 04

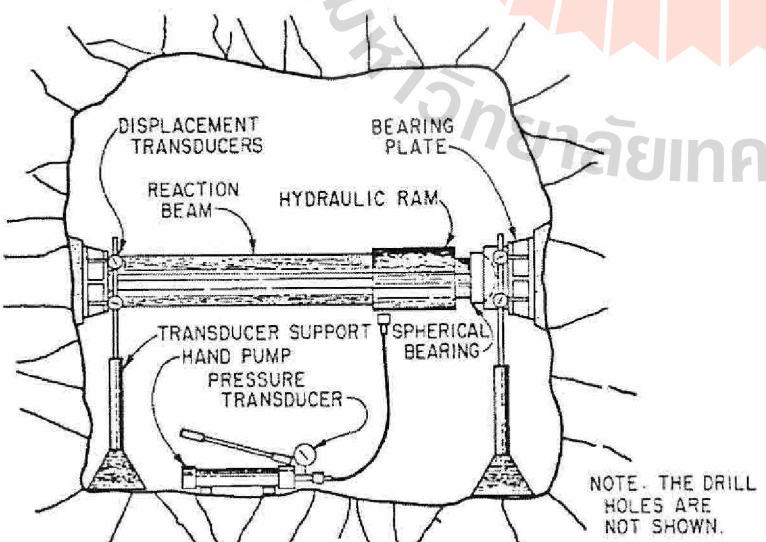


FIG. 4 Typical Rigid Plate Bearing Test Setup Schematic

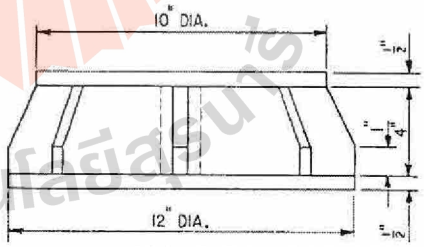
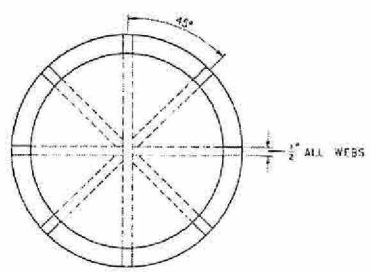


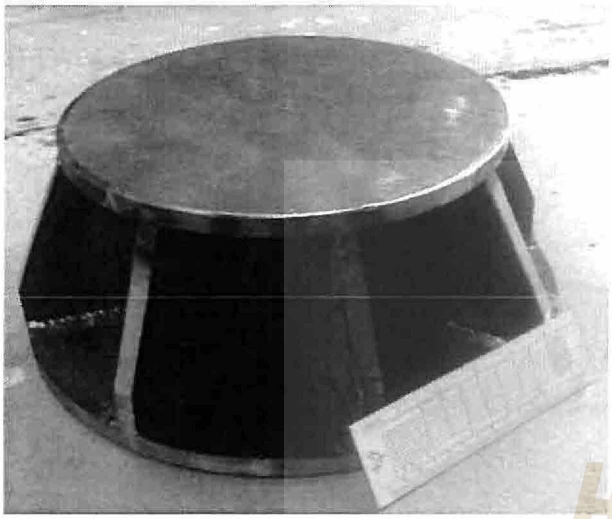
FIG. 3 Rigid Bearing Plate for 12 in. Diameter Test



NOTE: ALL JOINTS FULLY WELDED

Rigid Plate Bearing Test

Nam Ngum 3 Project



Rigid Plate Bearing Test

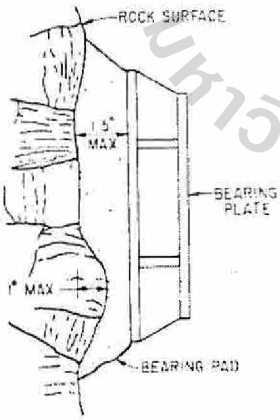


FIG. 5 Allowable Dimensions for Rock Surface and Bearing Pad

$$E = \frac{(1-\mu^2)\cdot P}{2W_a \cdot R}$$

where:

μ = Poisson's ratio of the rock.
P = total load on the rigid plate, lbf (kN),
W_a = average deflection of the rigid plate, in. (mm), and
R = radius of the rigid plate, in. (mm).

Flexible Plate Bearing Test

Designation: D 4395 – 04

Standard Test Method for
Determining the In Situ Modulus of Deformation of Rock Mass Using the Flexible Plate Loading Method[1]

Flexible Plate Bearing Test

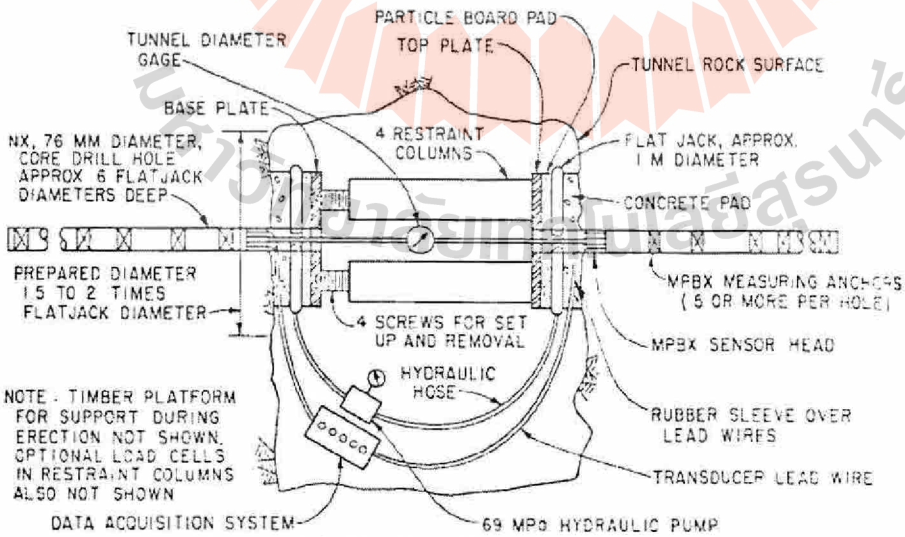


FIG. 3 Typical Flexible Plate Bearing Test Setup Schematic

Flexible Plate Bearing Test

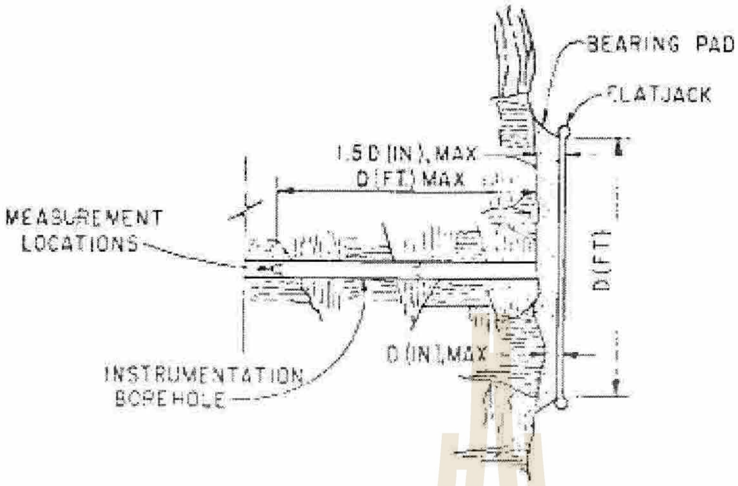


FIG. 4 Allowable Dimensions for Rock Surface and Bearing Pad, Flexible Plate Loading Test

Flexible Plate Bearing Test

E—Calculate the modulus, *E*, from the deflection at the center of a circularly loaded area at the rock surface as follows:

$$E = \frac{2(1-\gamma^2)QR}{W_c}$$

where:

γ = Poisson's ratio of the rock,
Q = pressure on loaded area, lbf/in^2 (MPa),
R = radius of loaded area, in. (mm), and
W_c = deflection at center of loaded area, in. (mm).

Flexible Plate Bearing Test

Calculate the modulus, E from the deflection at the edge of a circularly loaded area at the rock surface as follows:

$$E = \frac{4(1-\gamma^2)QR}{\pi W_e}$$

where:

γ = Poisson's ratio of the rock,
Q = pressure on loaded area, lbf/in^2 (MPa),
R = radius of loaded area, in. (mm), and
W_e = deflection at the edge of the loaded area, in. (mm).

Flexible Plate Bearing Test

Calculate the modulus, E, from the deflection at a point within the rock mass beneath the center of a circularly loaded area as follows:

$$E = \frac{2Q(1-\gamma^2)}{W_z}((R^2+Z^2)^{1/2}-Z) - \frac{QZ(1+\gamma)}{W_z}(Z(R^2+Z^2)^{-1/2}-1)$$

where:

Z = depth beneath center of loaded area, in. (mm), and
W_z = deflection at depth z, in. (mm).

Flexible Plate Bearing Test

Calculate the modulus, E, from the deflection at the center of an annularly loaded area at the rock surface as follows:

$$E = \frac{2Q(1-\gamma^2)(R_2 - R_1)}{W_c}$$

where:

R_2 = outside radius of annulus, in. (mm), and
R_1 = inside radius of annulus, in. (mm).

Calculate the modulus, E, from the deflection at the edge of an annularly loaded area at the rock surface as follows:

$$E = \frac{4Q(1-\gamma^2)(R_2 - R_1)}{\pi W_e}$$

Flexible Plate Bearing Test

Calculate the modulus, E, from the deflection at a point within the rock mass beneath the center of an annularly loaded area as follows:

$$E = \frac{2Q(1-\gamma^2)}{W_z}[(R_2^{\,2} + Z^2)^{1/2} - (R_1^{\,2} + Z^2)^{1/2}] + \frac{Z^2 \cdot Q(1+\gamma)}{W_z}[(R_1^{\,2} + Z^2)^{-1/2} - (R_2^{\,2} + Z^2)^{-1/2}]$$

The deflection, W_z, along the center line beneath the loaded area may be expressed in a general form (inches or millimetres) from equations Eq 3 or Eq 6 as follows:

$$W_z = \frac{Q}{E} \cdot K_z$$

Flexible Plate Bearing Test

From this, it follows that the modulus, E, may be calculated from the relative deflection between two positions below the center of the loaded area as follows:

$$\boxed{E = Q\frac{K_{z_1} - K_{z_2}}{w_{z_1} - w_{z_2}}}$$

where:

K_{z_1}, K_{z_2} = geometric coefficients for depths z_1 and z_2, respectively, and

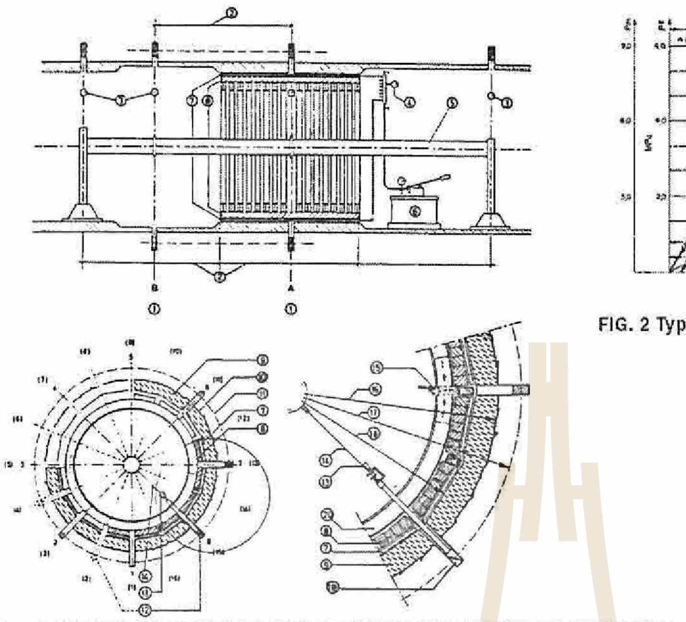
w_{z_1}, w_{z_2} = deflection at depths z_1 and z_2, respectively.

Radial Jacking Test

Designation: D 4506 – 02 (Reapproved 2006)

Standard Test Method for
Determining the In Situ Modulus of Deformation of Rock Mass Using a Radial Jacking Test[1]

Radial Jacking Test



1. Measuring profile. 2. Distance equal to the length of active loading. 3. Control extensometer. 4. Pressure gage. 5. Reference beam. 6. Hydraulic pump. 7. Flat jack. 8. Hardwood lagging. 9. Shotcrete. 10. Excavation diameter. 11. Measuring diameter. 12. Extensometer drillholes. 13. Dial gage extensometer. 14. Steel rod. 15. Expansion wedges. 16. Excavation radius. 18. Inscribed Circle. 19. Rockbolt anchor. 20. Steel ring.

FIG. 1 Radial Jacking Test

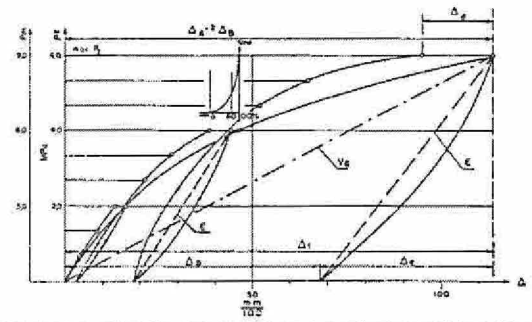


FIG. 2 Typical Graph of Applied Pressure Versus Displacement

Radial Jacking Test

$$p_1 = \frac{\Sigma b}{2 \cdot \pi \cdot r_1} p_m$$

where:

p_1 = distributed pressure on the lining at r_1, psi (MPa),
r_1 = radius, ft (m),
p_m = pressure in the flat jacks, psi (MPa), and
b = flat jack width (see Fig. 3), ft (m).

$$P_2 = \frac{r_1}{r_2} \cdot P_1 = \frac{\Sigma b}{2 \cdot \pi \cdot r_2} \cdot P_m$$

$$P_m \Sigma b = P_1 \cdot 2 \cdot r_1 \cdot \pi$$

$$P_1 = \frac{P_m \Sigma b}{2 \cdot \pi \cdot r_1}$$

$$P_2 = P_1 \frac{r_1}{r_2}$$

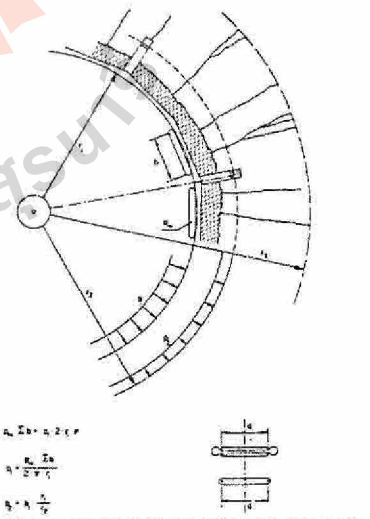


FIG. 3 Scheme of Loading Showing Symbols Used in the Calculations

Radial Jacking Test

$$\Delta_t = \Delta_p + \Delta_e$$

$$E = \frac{p_2 \cdot r_2}{\Delta_e} \cdot \frac{(1+\nu)}{\nu}$$

$$D = \frac{p_2 \cdot r_2}{\Delta_t} \cdot \frac{(1+\nu)}{\nu}$$

where:
p_2 = maximum test pressure, and
ν = estimated value for Poisson's Ratio.

$$E = \frac{p_2 \cdot r_2}{\Delta_e} \cdot \left(\frac{\nu+1}{\nu} + ln\frac{r_3}{r_2}\right)$$

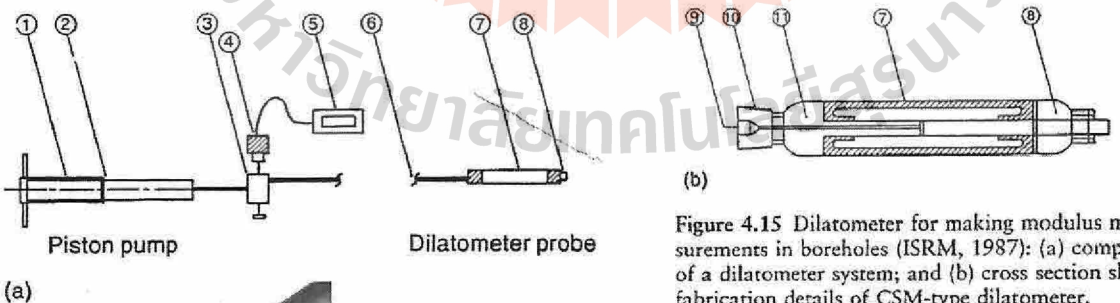
$$D = \frac{p_2 \cdot r_2}{\Delta_t} \cdot \left(\frac{\nu+1}{\nu} + ln\frac{r_3}{r_2}\right)$$

where:
r_3 = radius to the limit of the assumed fissured and loosened zone, ft (m), and
ln = natural logarithm.

Dilatometer Test

Measurement in Borehole/Drill hole

- Advantage : can be made remote from surface as part of the exploration program.



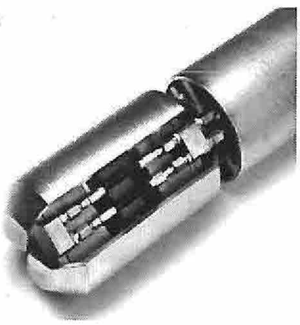


Figure 4.15 Dilatometer for making modulus measurements in boreholes (ISRM, 1987): (a) components of a dilatometer system; and (b) cross section showing fabrication details of CSM-type dilatometer.
1. Piston actuator.
2. Vernier.
3. Valve.
4. Pressure transducer.
5. Pressure readout.
6. High-pressure stainless-steel tubing.
7. Polyurethane rubber membrane.
8. Removable end cap.
9. High-pressure connection.
10. Pipe thread for insertion tool.
11. Fluid passage.

Dilatometer Test

$$G_d = k_R \frac{\pi L d^2}{\rho}$$

and

$$E_d = 2(1 + \nu_R) G_d$$

⇒ Shear Modulus

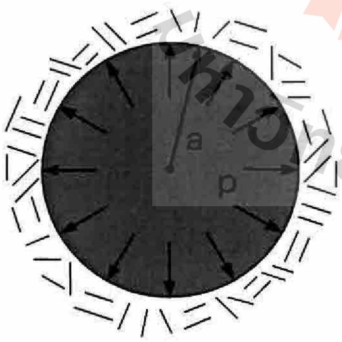
⇒ Modulus of Elasticity

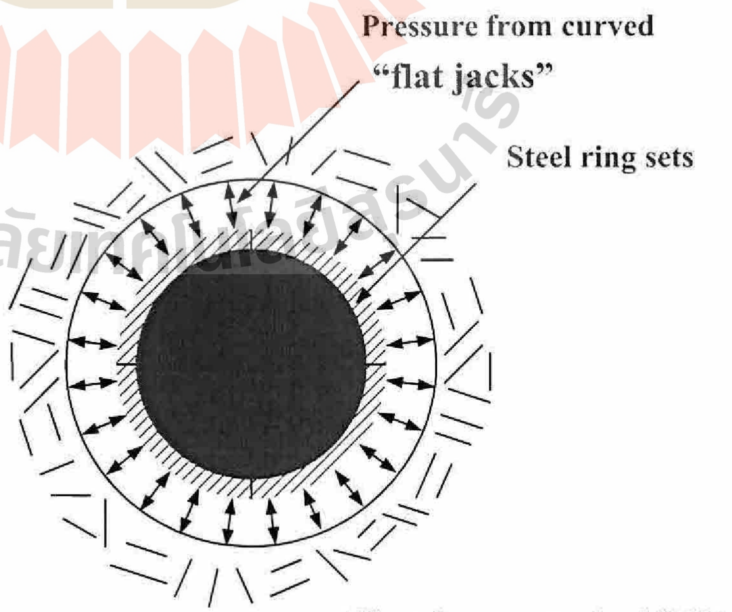
Where L = length of test section (cell membrane)
d = diameter of drill hole test section
ν_R = Poisson's ratio of rock
ρ = pump constant
(fluid volume displaced per turn of pump wheel)

$$k_R = \frac{k_s k_T}{(k_s - k_T)} \text{ (MPa/turn)}$$

⇒ Stiffness of rock

Dilatometer Test

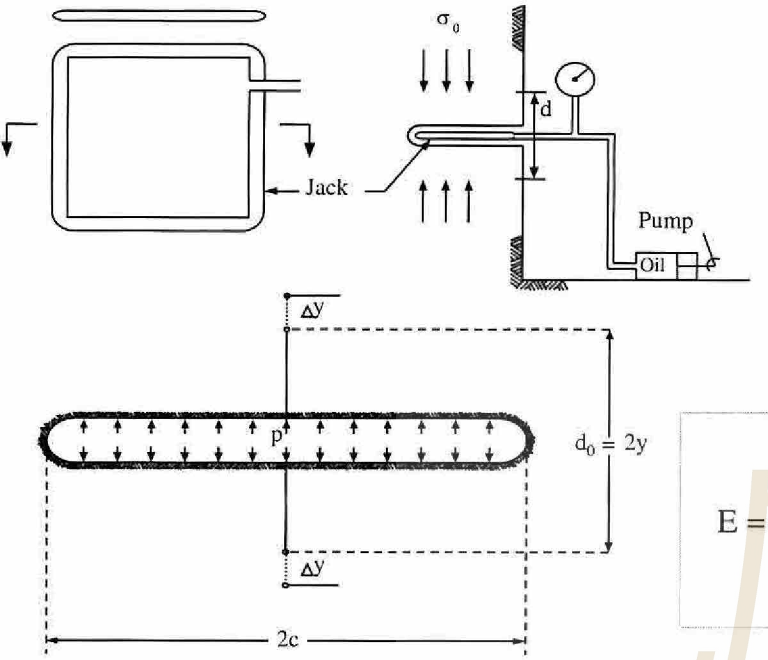




$$E = (1 + \nu)\Delta p \frac{a}{\Delta u}$$

(Goodman et al., 1972)

Flat Jack Test



$$E = \frac{p(2c)}{2\Delta y}\left[(1-\nu)\left(\sqrt{1+\frac{y^2}{c^2}}-\frac{y}{c}\right)+\frac{(1+\nu)}{\sqrt{1+\frac{y^2}{c^2}}}\right]$$

(Goodman, 1989)

In-situ Direct Shear Tests

Designation: D 4554 – 02 (Reapproved 2006)

Standard Test Method for
In Situ Determination of Direct Shear Strength of Rock Discontinuities[1]

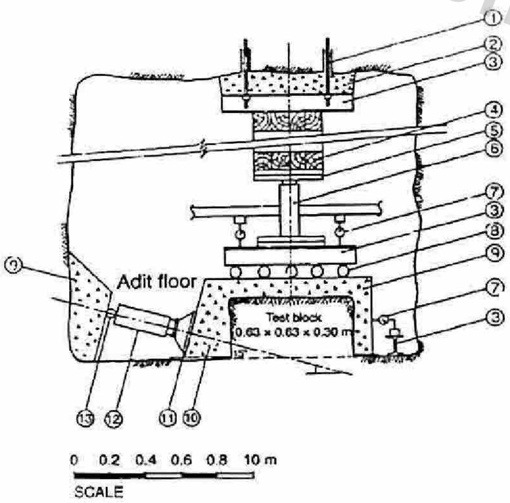


Figure 4.24 Typical set up for an *in situ* direct-shear test in an adit (Saint Simon *et al.*, 1979).
1. Rock anchor. 2. Hand-placed concrete. 3. WF beam. 4. Hardwood. 5. Steel plates. 6. 30 ton jack. 7. Dial gauge. 8. Steel rollers. 9. Reinforced concrete. 10. Bearing plate. 11. Styrofoam. 12. 50 ton jack. 13. Steel ball.

Direct Shear Tests

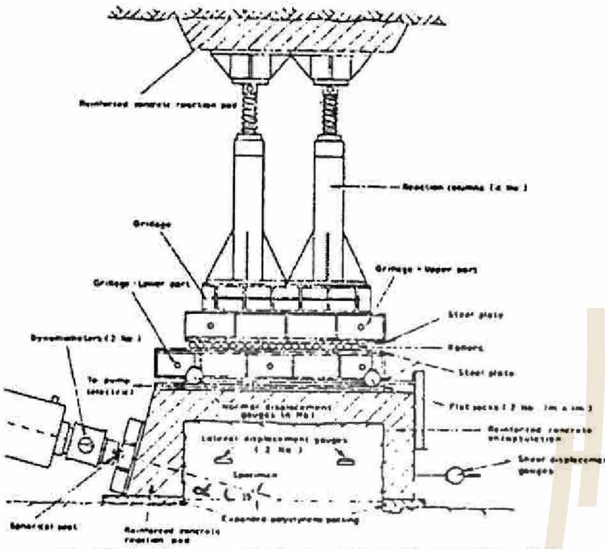
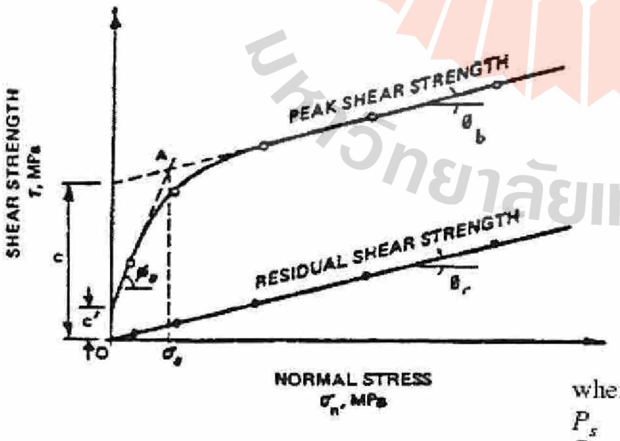


FIG. 3 Typical Arrangement of Equipment for In Situ Direct Shear Test

In situ Shear Test

Direct Shear Tests

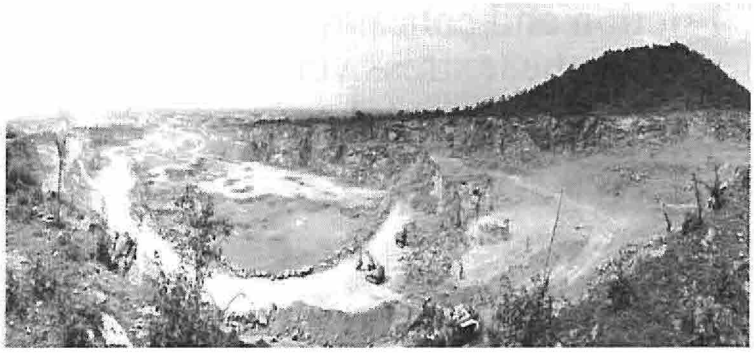


$$\text{Shear stress, } \tau = \frac{P_s}{A} = \frac{P_{sa}(\cos\alpha)}{A}$$

$$\text{Normal stress, } \sigma_n = \frac{P_n}{A} = \frac{P_{na} + P_{sa}(\sin\alpha)}{A}$$

where:

P_s = total shear force, MPa.
P_n = total normal force, MPa.
P_{sa} = applied shear force, MPa
P_{na} = applied normal force, MPa.
α = inclination of the applied shear force to the shear plane; if $\alpha = 0$, $\cos\alpha = 1$, and $\sin\alpha = 0$, and
A = area of shear surface overlap (corrected to account for shear displacement), mm.



Rock Slope Engineering

Prachya Tepnarong, Ph.D.
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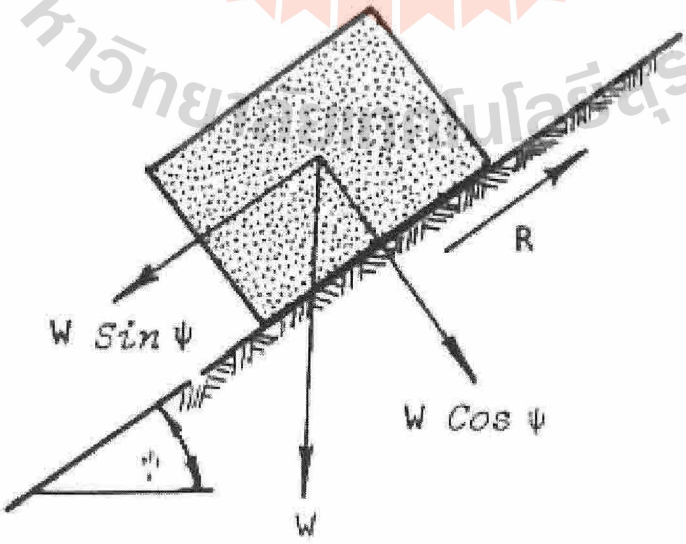
Basic Mechanics of Slope Failure



Friction, Cohesion and Unit Weight

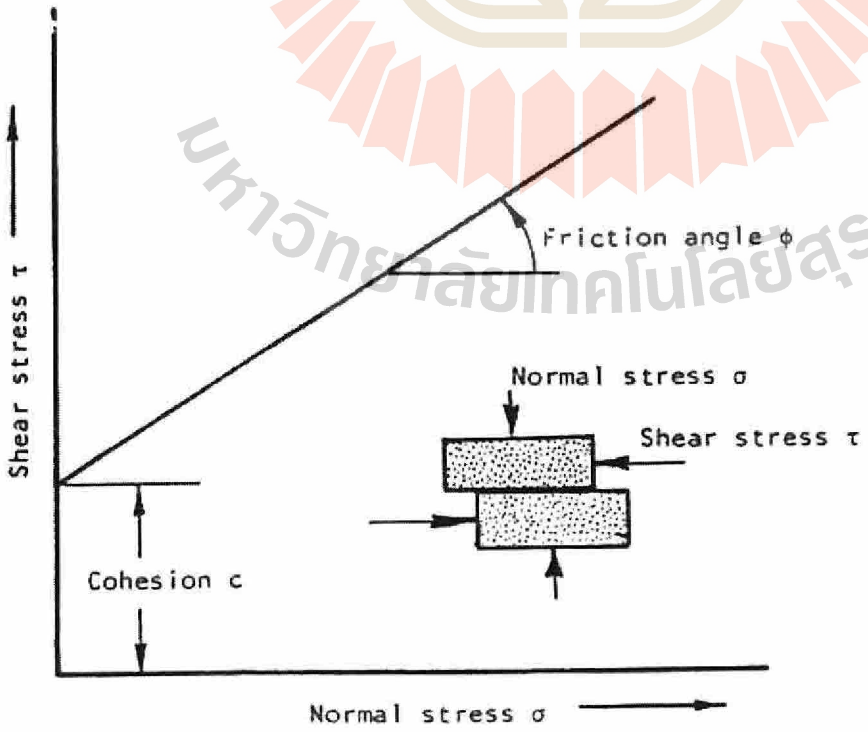
- Friction and cohesion are best defined in term of the plot of shear stress versus normal stress
- The relationship between shear and normal stresses for a typical rock surface or for a soil sample can be expressed as:

$$\tau = c + \sigma \tan \phi$$

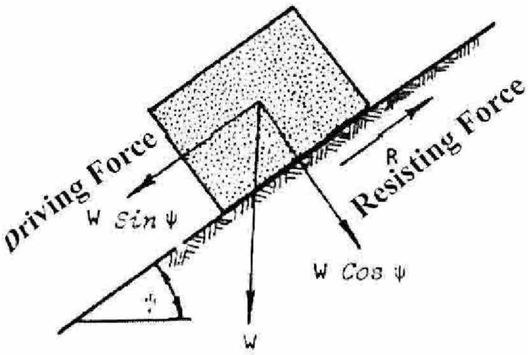
where τ = shear stress
σ = normal stress
c = cohesion
ϕ = basic friction angle

} from direct shear test

Shear stress-normal stress relationship



Sliding due to gravitational Loading



- Coulomb Criterion:

$$\tau = c + \sigma \tan \phi \qquad (1)$$

- The normal stress σ which acts across the potential sliding surface is given by

$$\sigma = (W \cos \psi) / A \qquad (2)$$

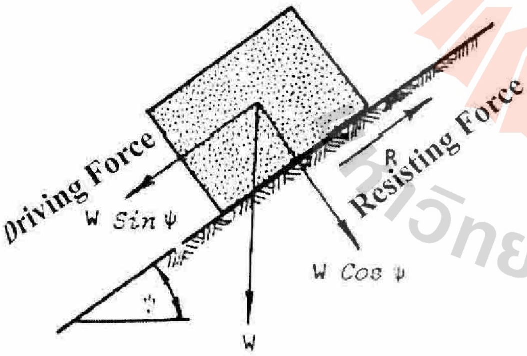
where A is the base area of the block

- Sub (2) into (1); and Shear Force, $\mathbf{R = \tau A}$

$$\tau = c + \frac{W \cos \psi}{A} . \tan \phi$$

$$R = cA + W \cos \psi . \tan \phi \qquad (3)$$

Sliding due to gravitational Loading



- Condition of Limiting Equilibrium

Driving Force = Resisting Force

$$W \sin \psi = cA + W \cos \psi . \tan \phi \qquad (4)$$

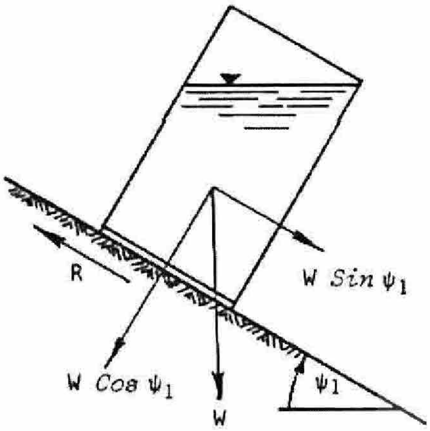
- If the cohesion c = 0, the condition of limiting equilibrium defined by equation (4) simplifies to

$$\cancel{W} \sin \psi = \cancelto{0}{cA} + \cancel{W} \cos \psi . \tan \phi$$

$$\sin \psi = \cos \psi . \tan \phi$$

$$\psi = \phi \qquad (5)$$

Influence of Water Pressure on Shear Strength



- The influence of water pressure upon the shear strength of two surfaces in contact can most effectively be demonstrated by the beer can experiment.
- An opened beer can filled with water rests on an inclined piece of wood as shown in sketch.
- For simplicity the cohesion between the beer can base and the rood is assumed to be zero. According to equation (5) the can with its contents of water will slide down the plank when $\psi_1 = \phi$.
- The base of the can is now punctured sot that water can enter the gap between the base and the plank, giving rise to a water pressure u or to an uplift force

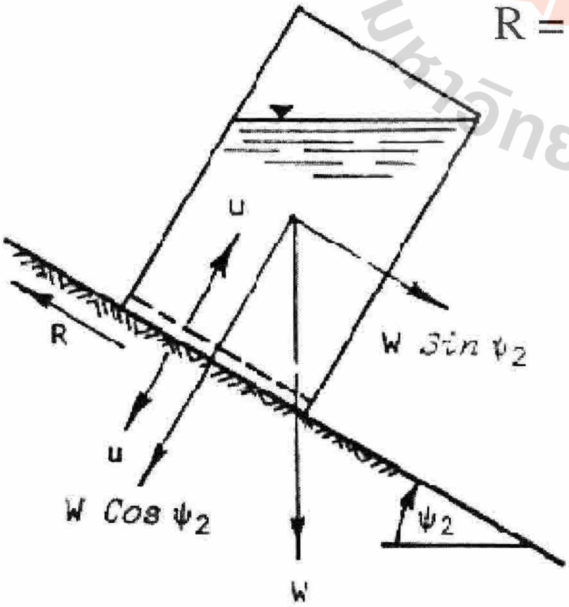
$$\mathbf{U = uA}$$

where A is the base area of the can.

Influence of Water Pressure on Shear Strength

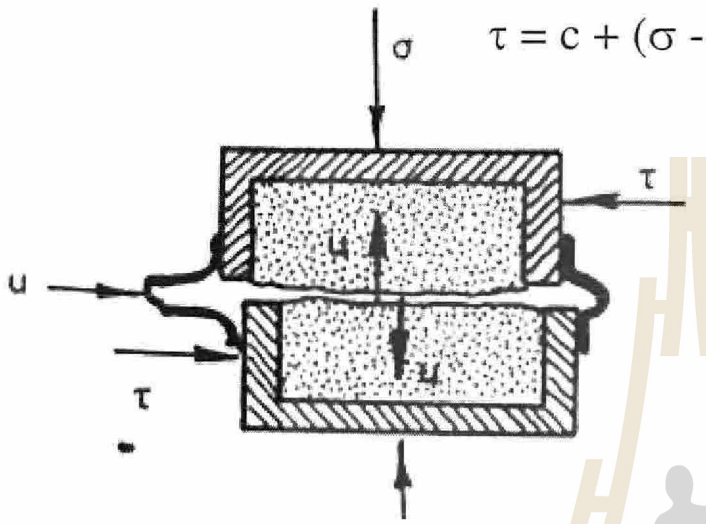
- The normal force W cos ψ_2 is now reduced by this uplift force U and the resistance to sliding is now

$$R = (W \cos \psi_2 - U) \tan \phi \qquad (6)$$



Effective Stress Law

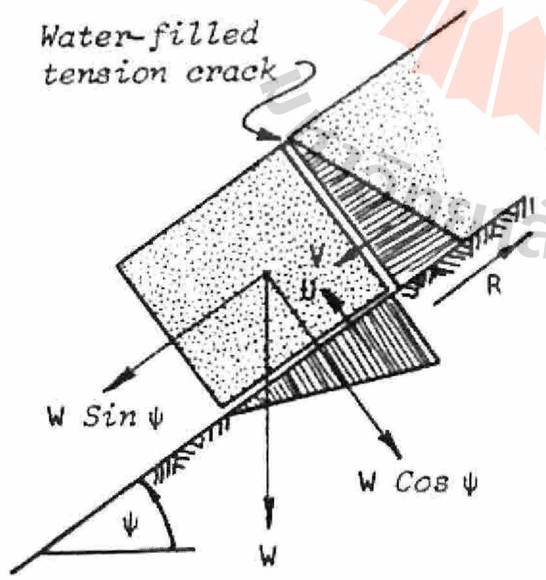
- The normal stress σ acting across the failure surface is reduced to the effective stress (σ - u) by the water pressure u. The relationship between shear strength and normal strength defined by equation (1) now becomes



$$\tau = c + (\sigma - u) \tan \phi \qquad (10)$$

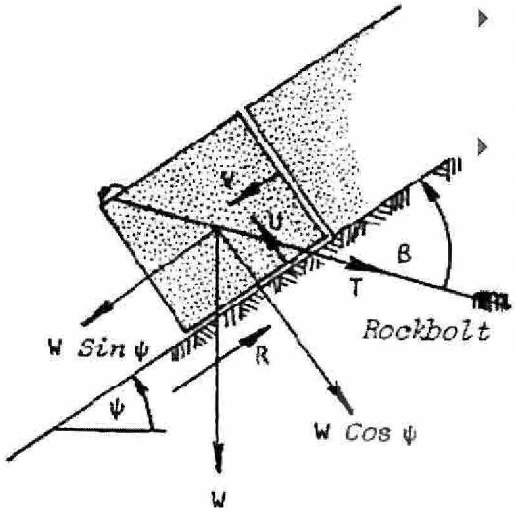
The Effect of Water Pressure in a tension Crack

- The condition of limiting equilibrium for this case of a block acted upon by water forces **V** and **U** in addition to its own weight W is defined by
- From this equation it will be seen that the disturbing force tending to induce sliding down the plane is increased and the frictional force resisting sliding is decreased and hence, both **V** and **U** result in decreases in stability.



$$W \sin \psi + V = cA + (W \cos \psi - U) \tan \phi \qquad (11)$$

Reinforcement to Prevent Sliding



- Consider the block resting on the inclined plane and acted upon by the uplift force **U** and the force **V** due to water pressure in the tension crack.
- A rockbolt, tensioned to a load T is installed at an angle β to the plane. The resolved component of the bolt tension T acting parallel to the plane is **T Cos** β while the component acting across the surface upon which the block rests is T sin β. The condition of limiting equilibrium for the case is defined by

$$W \sin \psi + V - T \cos \beta = cA + (W \cos \psi - U + T \sin \beta) \tan \phi \qquad (12)$$

Factor of Safety of Slope

- In order to compare the stability of slopes under conditions other than those of limiting equilibrium, some form of index is required and the most commonly used index is the *factor of Safety (F.S or F)*

$$F.S. = \frac{\text{Resisting Force}}{\text{Driving Force}}$$

- Considering the case of the block acted upon by water forces and stabilised by a tensioned rockbolt the factor of safety is given by

$$F.S. = \frac{cA + (W\cos\psi - U + T\sin\beta)\tan\phi}{W\sin\psi + V - T\cos\beta}$$

Factor of Safety of Slope

- The bolt tension required to provide a specified factor of safety of F is a minimum when the angle β satisfies the equation

$$\tan \beta = \tan \phi / F.S. \qquad (14)$$

- This result is obtained by differentiating equation (13) with respect to β, and setting

$$\frac{dT}{d\beta} = 0 \text{ and } \frac{dF}{d\beta} = 0.$$

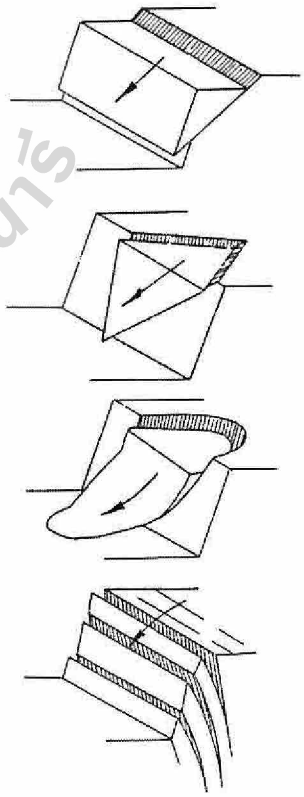
Minimum F.S.

Mining Slope (Shot Life Slope)	**F.S. = 1.1-1.3**
Civil (Long Term Slope)	**F.S. = 1.5**
Natural Slope	**F.S. = 1.1-1.3**

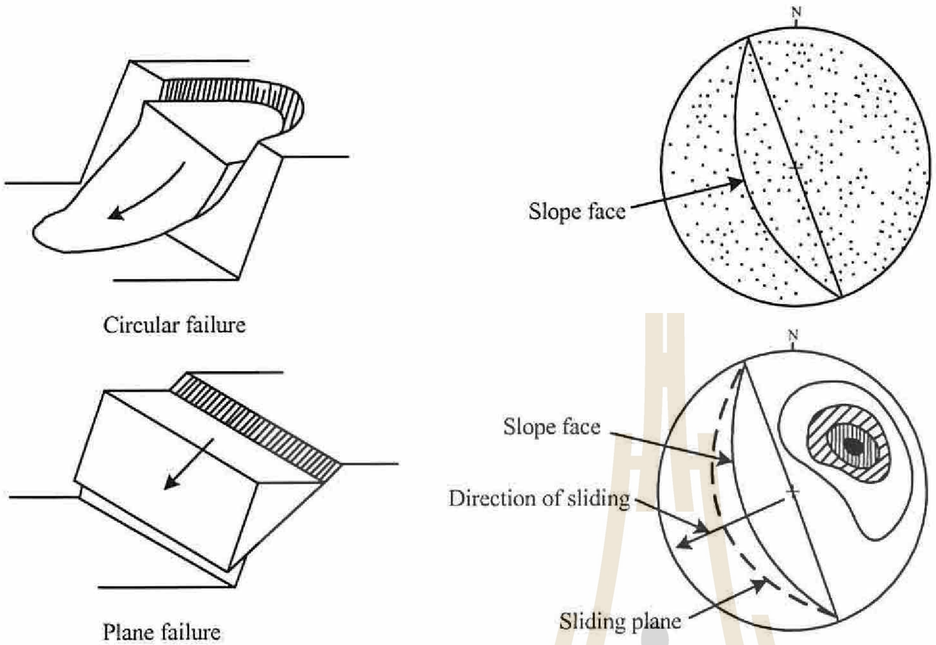
Type of Slope Failure

Failure Modes:

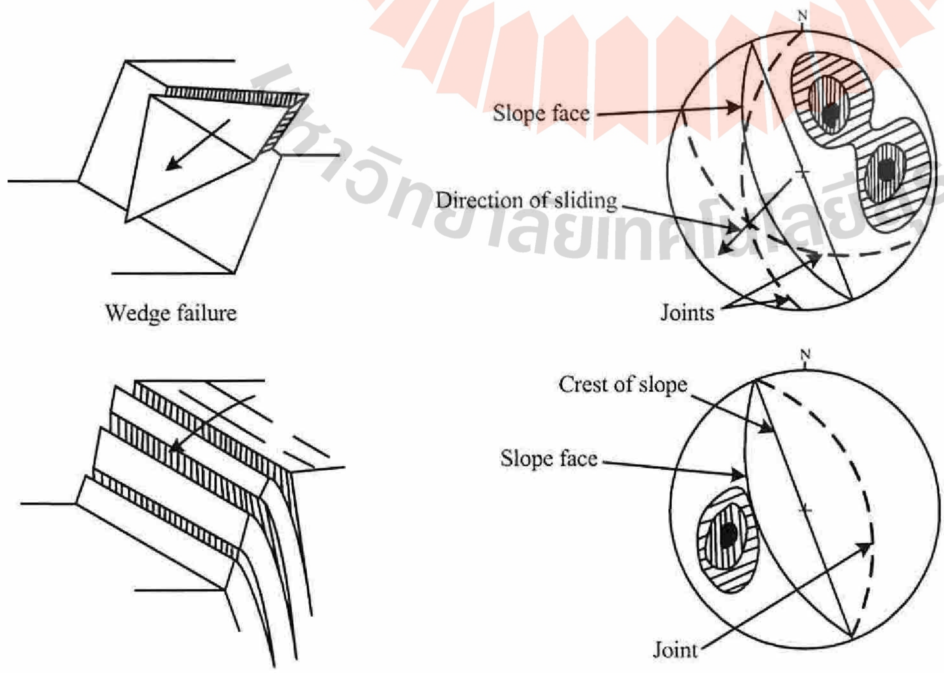
1. Plane Failure
2. Wedge Failure
3. Circular Failure
4. Toppling Failure



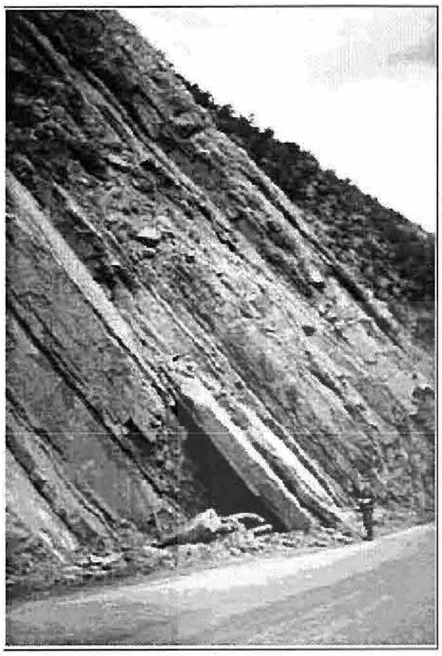
Type of Slope Failure vs. Stereonet Plot



Type of Slope Failure vs. Stereonet Plot

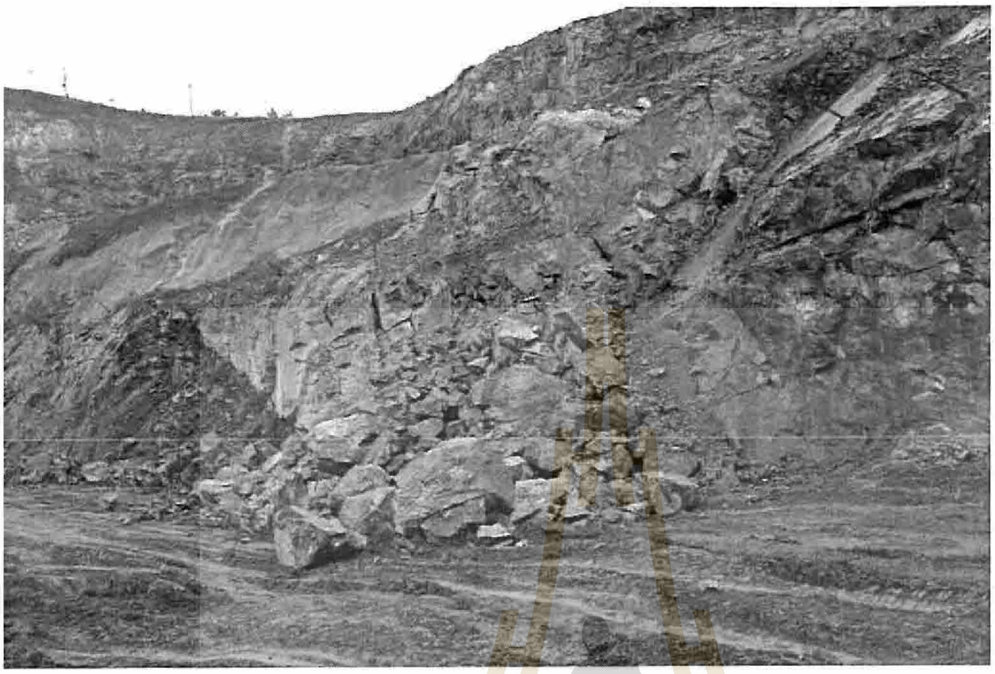


Plane Failure



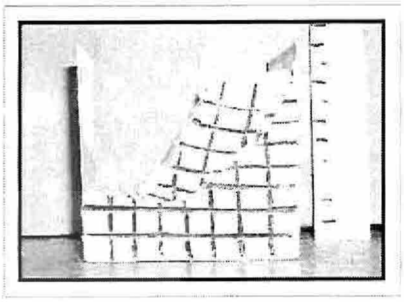
Wedge Failure

Wedge Failure



Wedge Failure

Circular Failure

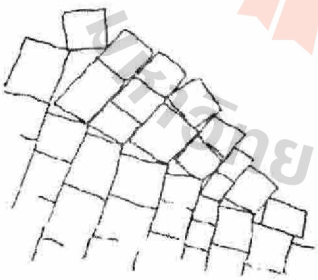


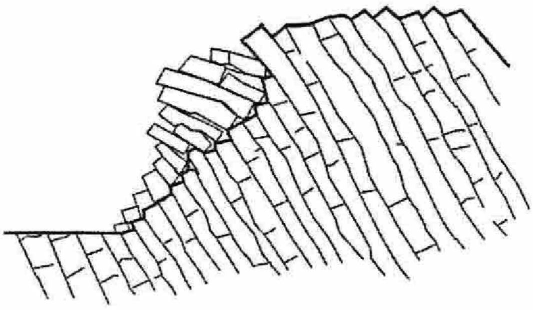
Circular Failure

Circular Failure

Toppling Failure

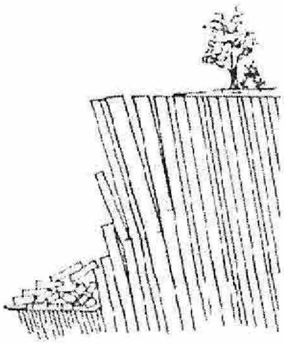
1. Block Toppling

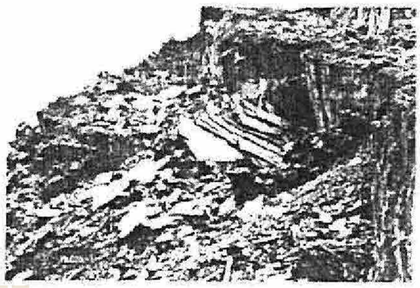


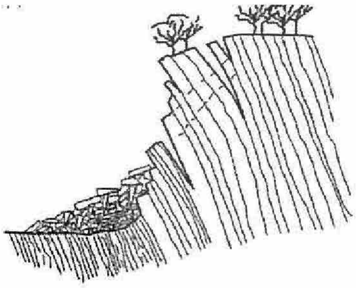


Toppling Failure

2. Flexural Toppling

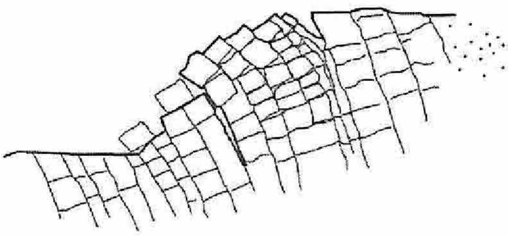






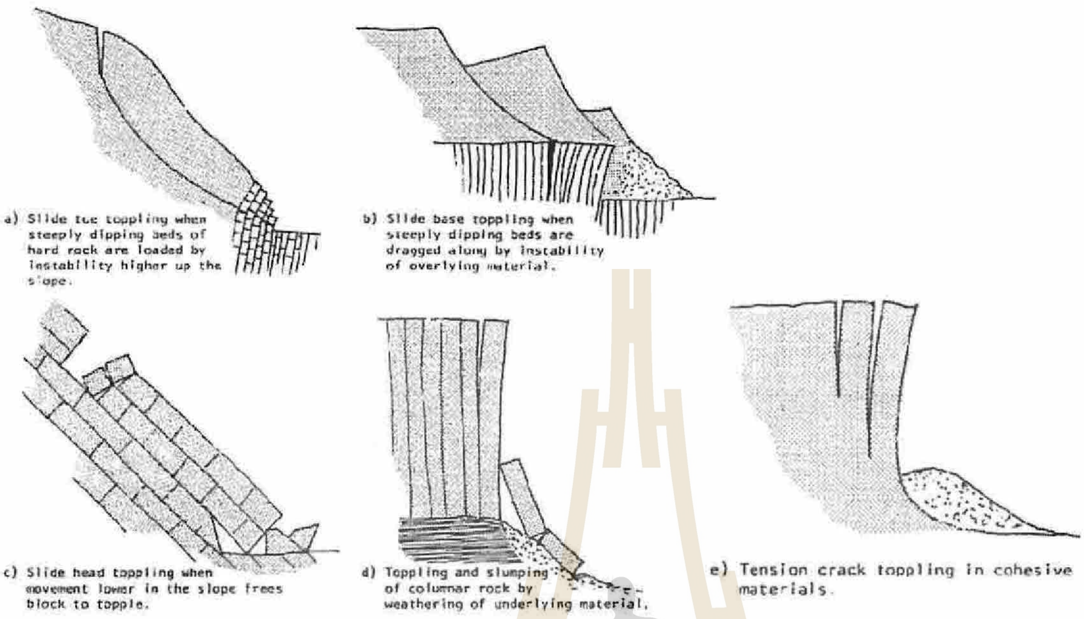
Toppling Failure

3. Block-Flexural Toppling

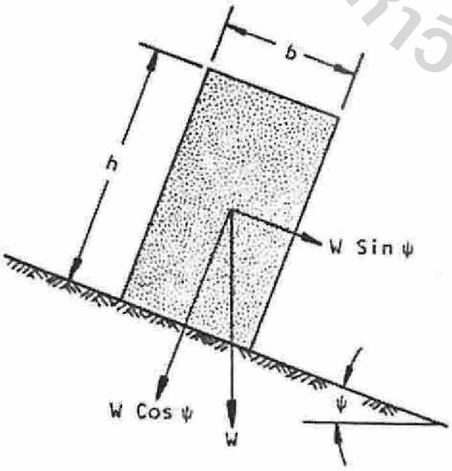


Toppling Failure

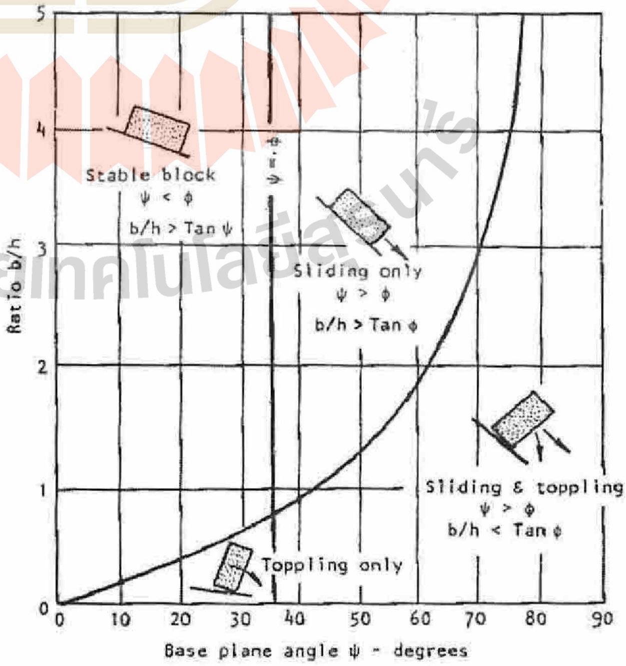
4. Secondary Toppling



Conditions for sliding and toppling



Geometry of block on inclined plane.



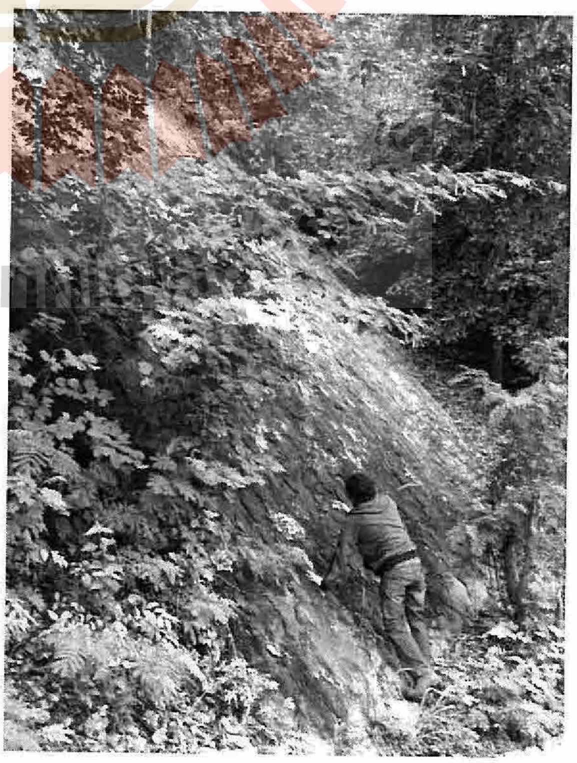
Conditions for sliding and toppling of a block on an inclined plane.

Plane Failure

Plane Failure

Plane Failure

Plane Failure



Plane Failure

General Condition for Plane Failure

- Rare
- Strike of sliding plane // strike of slope face (± 20 degrees)
- Daylight ($\psi_f > \psi_p$)
- Overcome friction angle ($\psi_p > \phi$)
- Upper end of sliding surface intersects upper slope / tension crack
- Release surface

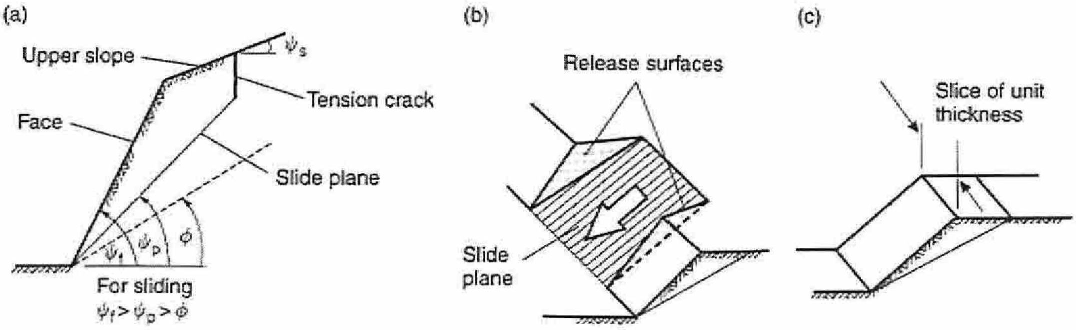


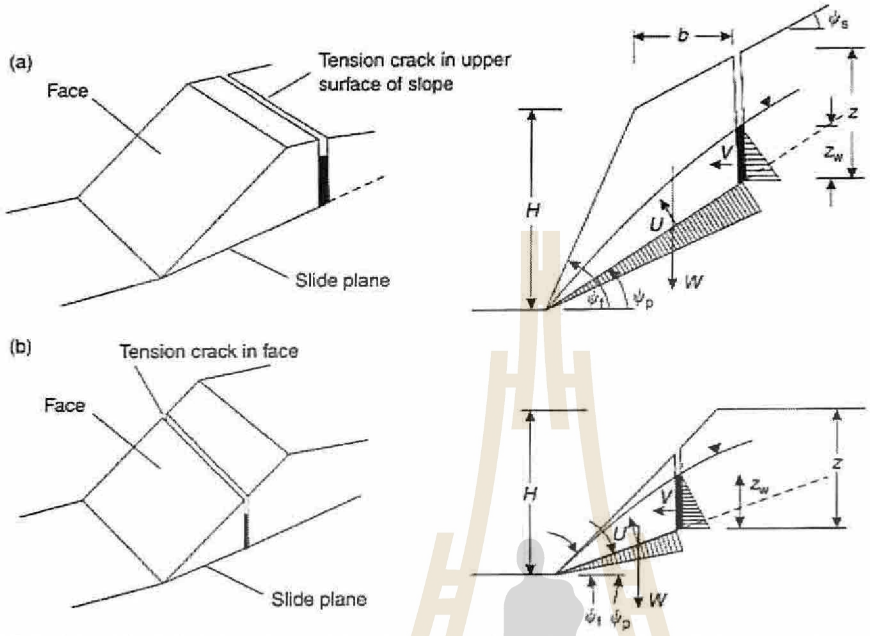
Figure 6.2 Geometry of slope exhibiting plane failure: (a) cross-section showing planes forming a plane failure; (b) release surfaces at ends of plane failure; (c) unit thickness slide used in stability analysis.

Plane Failure Analysis

The geometry of the slope is defined two cases:

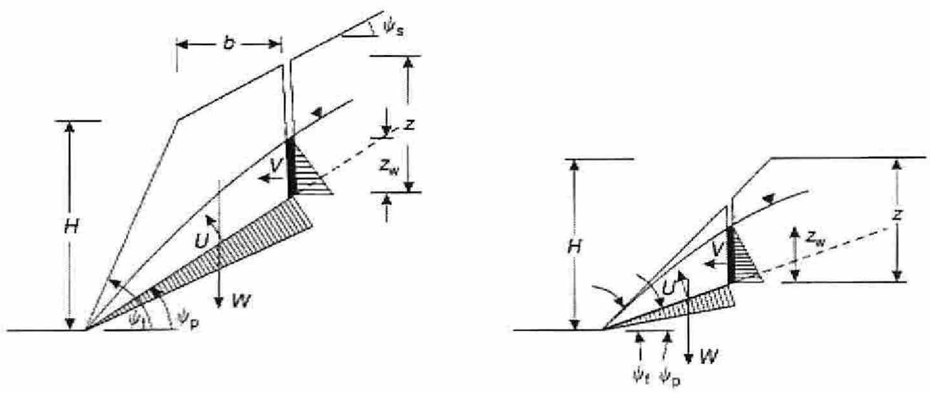
(a) A slope having a tension crack in its upper surface

(b) A slope with a tension crack in its face.

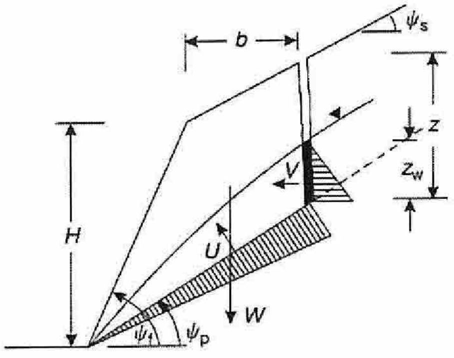


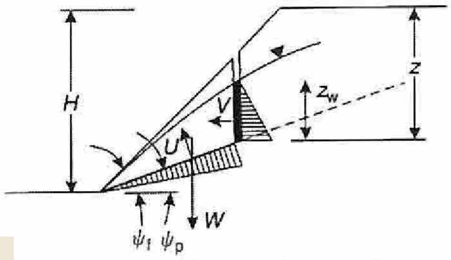
Assumptions Required for Analysis

- Both sliding surface and tension crack strike parallel to the slope surface.
- The tension crack is vertical and is filed with water to a depth z_w.
- Water in sliding surface and tension crack subjected to atmospheric pressure.
- All forces act through the centroid of the sliding mass.
- Using Coulomb criterion, $\tau = c + \sigma \tan \phi$
- Release surfaces is no resistance to sliding.



Symbols





A	=	area of sliding block
U	=	uplift force
V	=	water pressure in tension crack
H	=	slope height
b	=	horizontal distance b/w slope crest & tension crack
W	=	weight of sliding block
ψ_f	=	dip angle of slope face
ψ_p	=	dip angle of failure plane
ψ_s	=	dip angle of upper slope face
γ_w	=	unit weight of water
γ_r	=	unit weight of rock
z	=	depth of tension crack
z_w	=	vertical depth of filled water

F.S. Calculations

$$F.S. = \frac{\text{Resisting Force}}{\text{Driving Force}}$$

$$F.S. = \frac{cA + (W.\cos\psi_p - U - V.\sin\psi_p)\tan\phi}{W.\sin\psi_p + V.\cos\psi_p}$$

where

$$A = (H + b.\tan\psi_s - z).\ \mathrm{cosec}\ \psi_p$$
$$U = \tfrac{1}{2}\gamma_w . z_w (H + b.\tan\psi_s - z) .\ \mathrm{cosec}\ \psi_p$$
$$V = \tfrac{1}{2}\gamma_w . z^2_w$$

F.S. Calculations

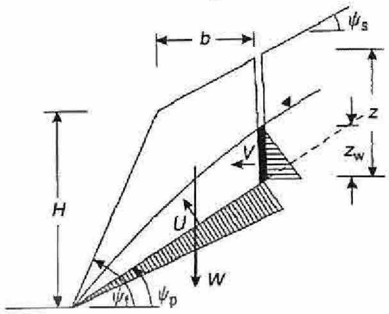
For the tension crack in the upper slope surface

$$W = \gamma_r[(1-\cot\psi_f \tan\psi_p)(bH + ½ H^2 \cot\psi_f) + ½ b^2(\tan\psi_s - \tan\psi_p)]$$

(for ψ_s= dip angle of upper slope face)

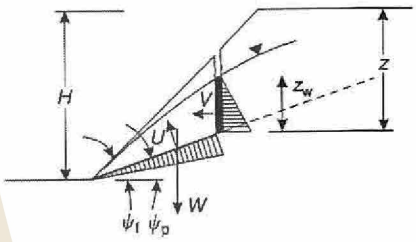
$$W = ½ \gamma_r H^2 [(1 - (z/H)^2) \cot\psi_p - \text{Cot}\,\psi_f]$$

(for ψ_s= 0, upper slope face is horizontal)



For the tension crack in the slope face

$$W = ½ \gamma_r H_2 [(1 - z/H)^2 \cot\psi_p (\cot\psi_p . \tan\psi_f - 1)]$$



Analysis of Failure on a Rough Plane

For dry slope, U=V=0

$$F.S. = \frac{\tau A}{W \sin\psi_p}$$

$$F.S. = \frac{\sigma \tan(\phi + JRC \log_{10}(\sigma_j/\sigma))A}{W \sin\psi_p}$$

Sub $\sigma = \frac{W \cos\psi_p}{A}$ in Equation

$$F.S. = \frac{\tan(\phi + JRC \log_{10}(\sigma_j/\sigma))}{\tan\psi_p}$$ Barton Criterion

$$F.S. = \frac{\tan(\phi + i)}{\tan\psi_p}$$ Patton Criterion

Reinforcement of a Slope

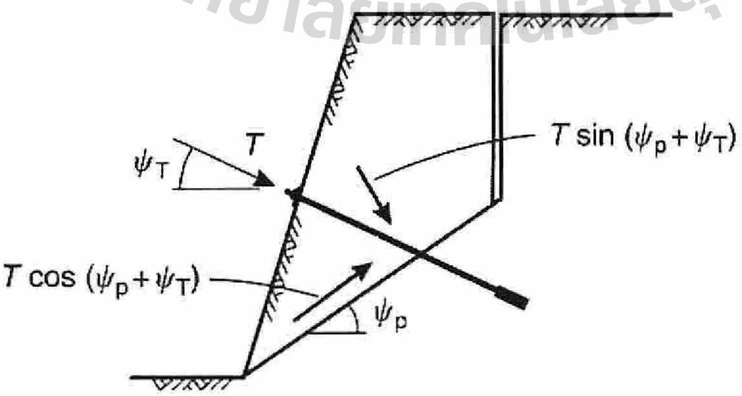
-Reinforcement with Tensioned Anchors

-Reinforcement with Fully Grouted Untensioned Dowels

-Reinforcement with Buttresses

Reinforcement with Tensioned Anchors

$$F.S. = \frac{cA + (W.\cos\psi_p - U - V.\sin\psi_p + T.\cos(\psi_T + \psi_p))\tan\phi}{W.\sin\psi_p + V.\cos\psi_p - T.\sin(\psi_T + \psi_p)}$$



Topic 12
Introduction to Underground Excavation in Rock

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Planning Considerations

- **Underground Mining** (temporary opening)
  - old mine (small scale / 10 years)
  - modern mine (large scale / 50-70 years)
- **Modern Mining**
  - semi-permanent opening
- **Civil Engineering Excavations**
  - tunnel
  - subway
  - underground storage

Type of Underground Excavation

by degree of stability [mechanical & security (human)]

<u>Type A</u> **Temporary mine openings.**

<u>Type B</u> **Vertical shafts.**

<u>Type C</u> **Permanent mine opening, water tunnels for hydroelectric projects (excluding high pressure penstocks), pilot tunnels, drifts and headings for large excavations.**

<u>Type D</u> **Storage rooms, water treatment plants, minor road and railway tunnels, surge chambers & access tunnels in hydro-electric projects.**

<u>Type E</u> **Underground power station caverns, major road and railway tunnels, civil defense chambers, tunnel portals and intersections.**

<u>Type F</u> **Underground nuclear power stations, railway stations, sports and public facilities, underground factories.**

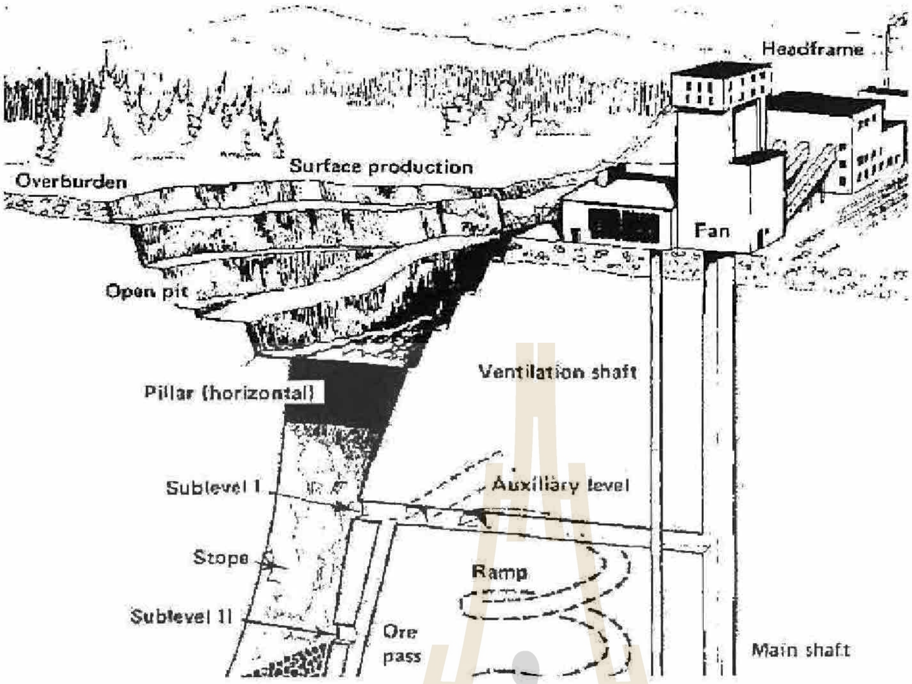
Support Costs
Factor of Safety

Sources of Instability

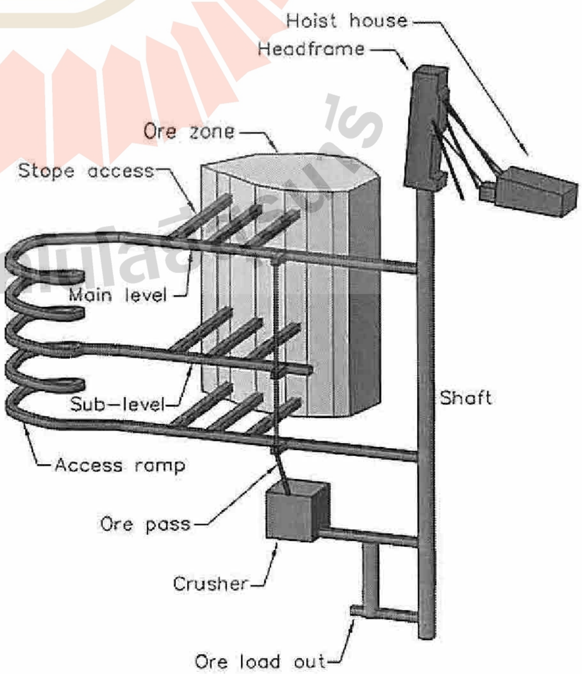
Hoek and Brown (1980) and Hoek et al. (1995)

- Structure-controlled instability
- Stress-controlled instability
- Weathering and erosion
- Groundwater pressure and flow

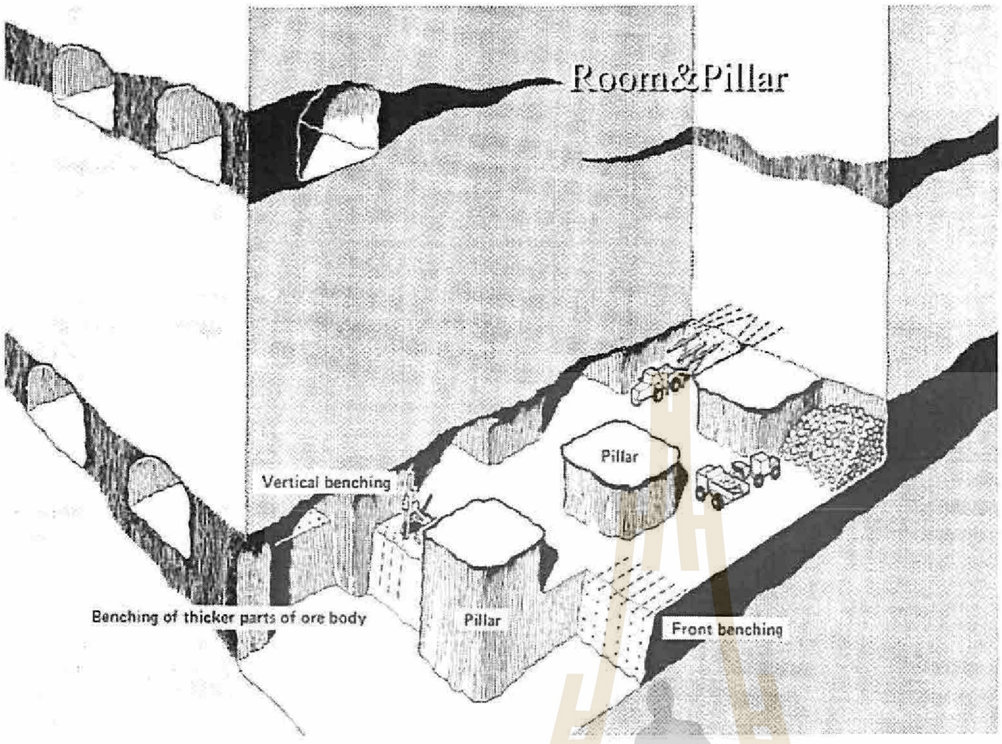
Introduction to U.G. mining



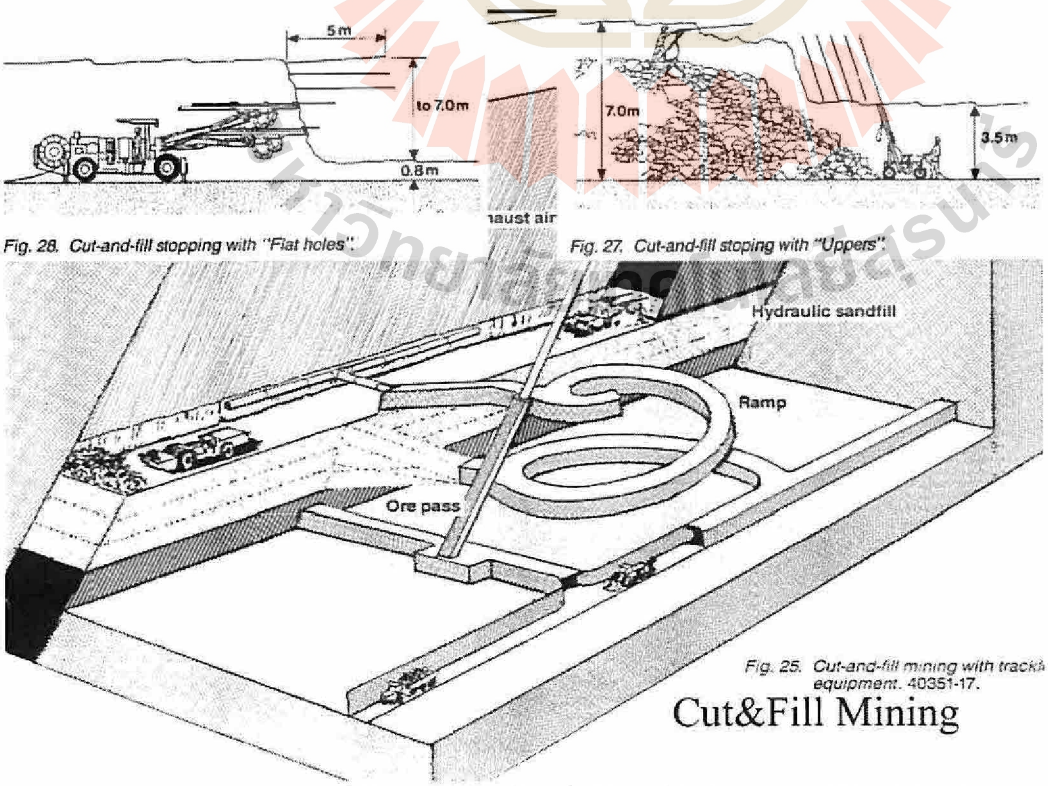
Introduction to U.G. mining



Introduction to U.G. mining

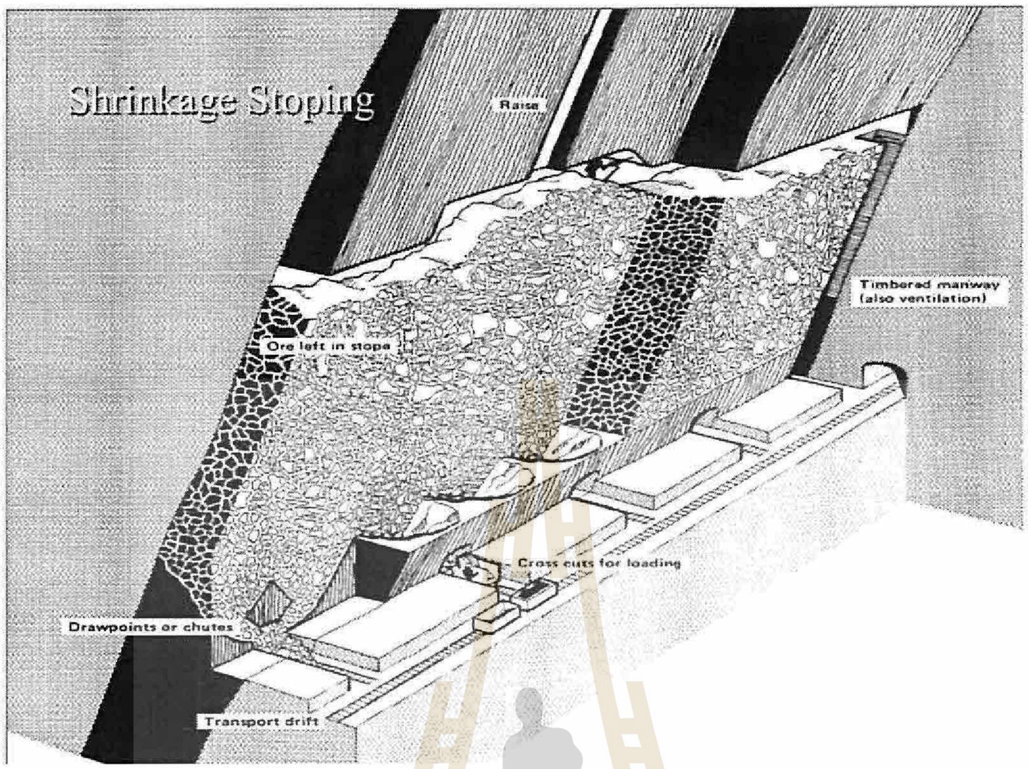


Introduction to U.G. mining



Cut&Fill Mining

Introduction to U.G. mining



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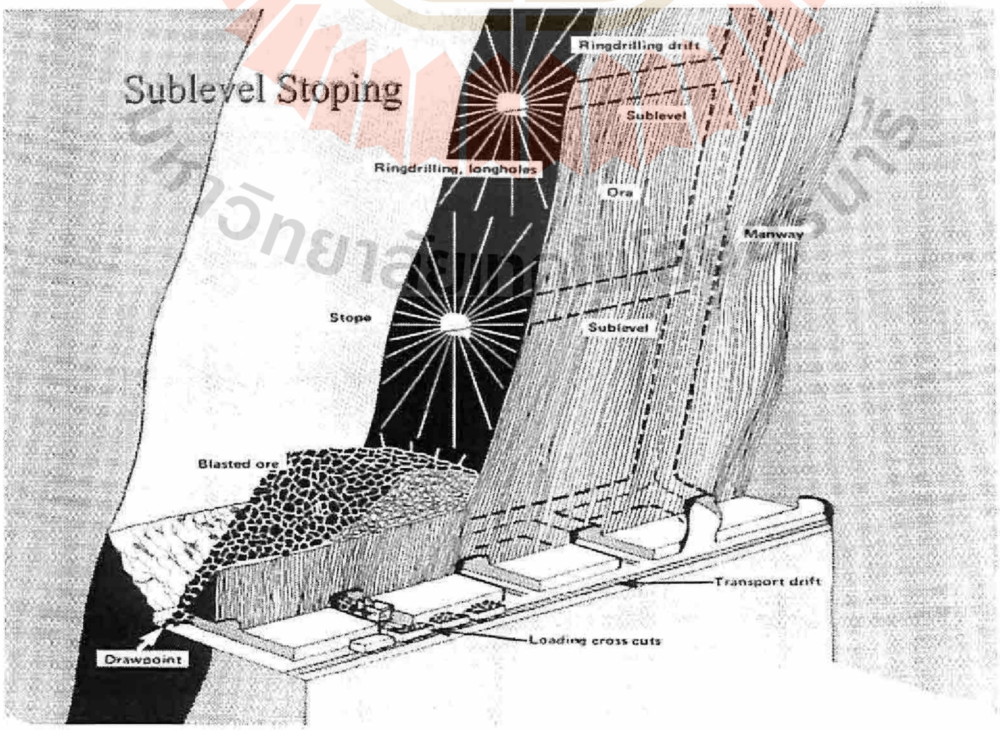
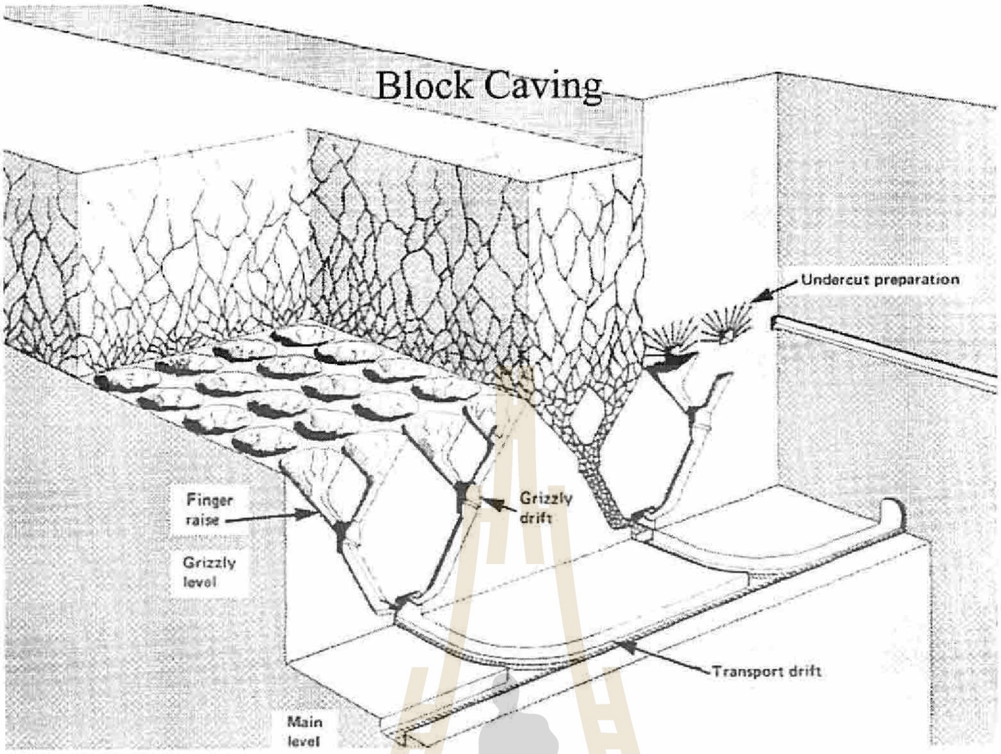
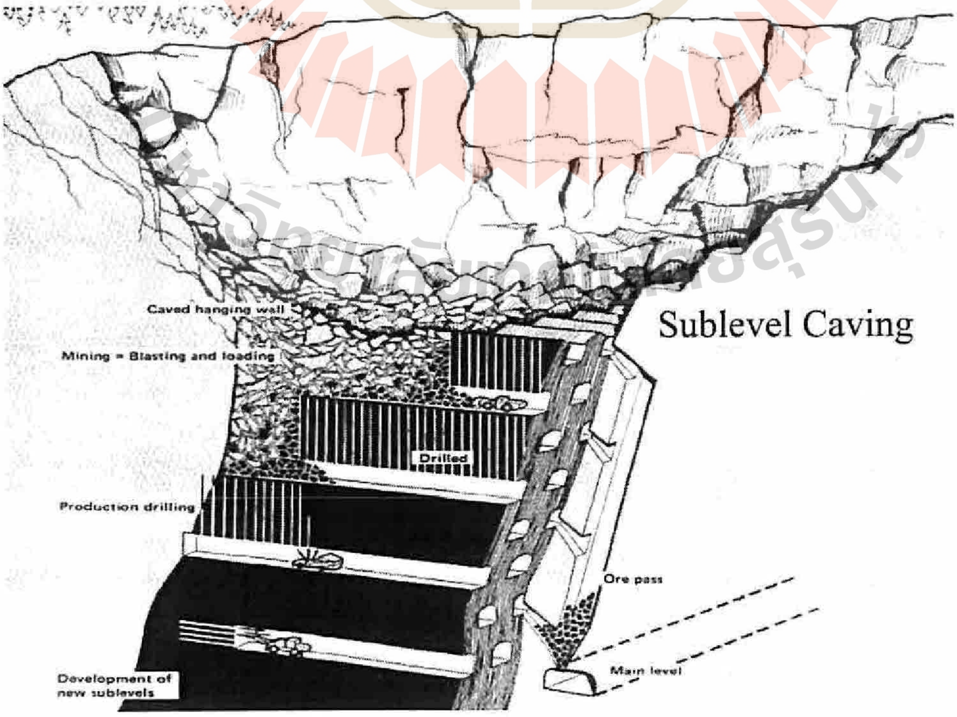


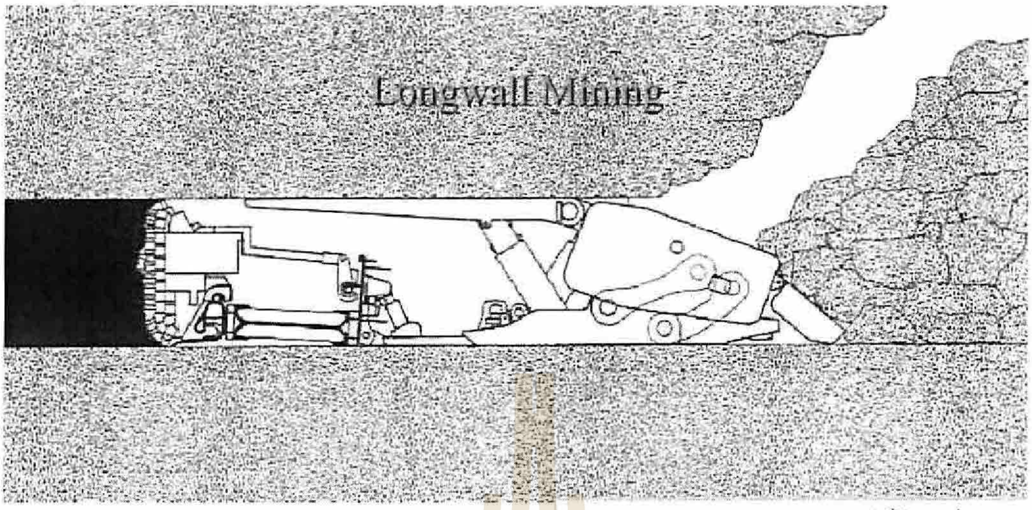
Introduction to U.G. mining



Introduction to U.G. mining



Introduction to U.G. mining



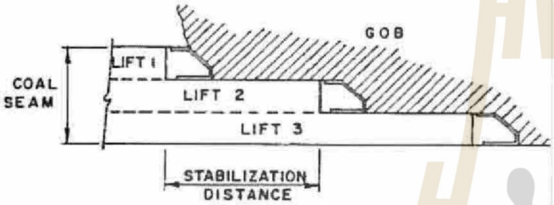


Fig. 17. Thick seam simultaneous lifts.

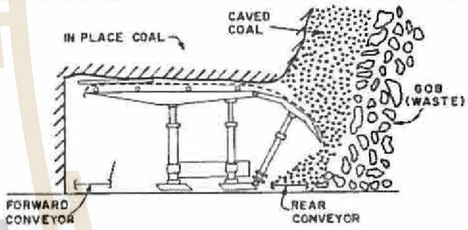
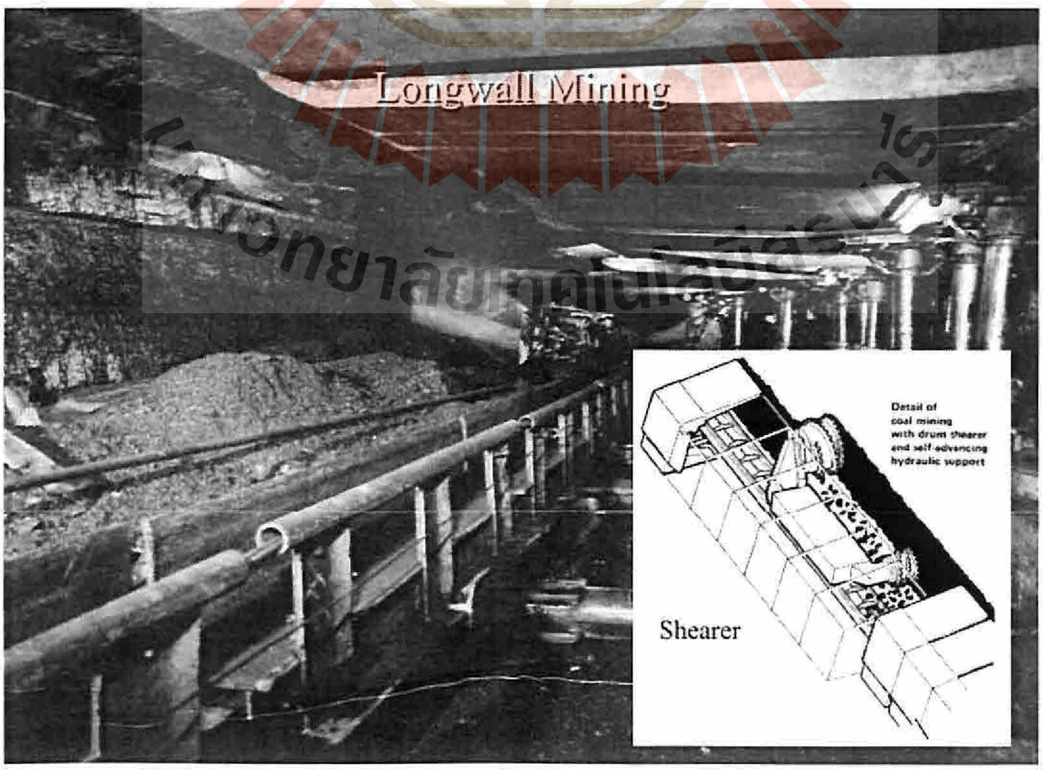


Fig. 19. Typical longwall caving face arrangement.

Introduction to U.G. mining



Introduction to Types of Tunnels

Tunnels Classified by Usages

- Road tunnel, rail tunnel,
- rapid transit tunnel,
- water tunnel, sewage
- tunnel, hydroelectric
- tunnel, service and
- utilities tunnel, and
- others

Types of Tunnels

Tunnels Classified by Ground

- Rock Tunnel
- Soil Tunnel
- Mixed Ground Tunnel

Types of Tunnels

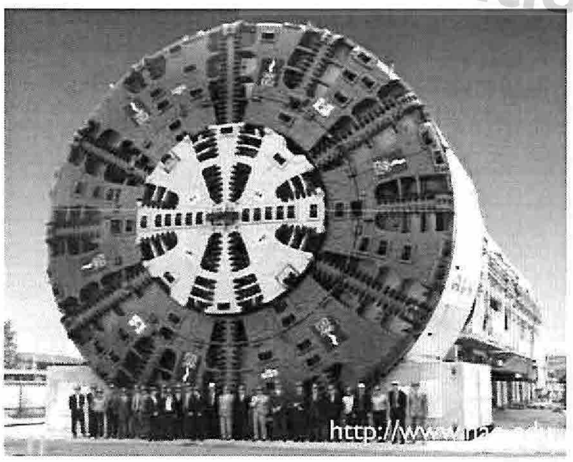
Tunnels Classified by Construction Methods

- Rock and Soil TBM Bored Tunnels
- Conventional Mined and Drill-and-Blast
- Pipe Jacking
- Cut-and-Cover Tunnels
- Immersed Tube and Floating Tube

Types of Tunnels

TBM Bored Tunnel

- Tunnel is excavated by tunnel boring machines (TBM).
- Construction is done underground.



Types of Tunnels

Conventional Mined Tunnel

- Tunnel is excavated by conventional mining methods (e.g., by excavator, drill-and-blast), and supported.
- Construction is done underground.

Opening in Competent Rock

Assumptions:

- Homogenous
- isotropic
- continuous
- linearly elastic material

Kirsch's solution

Opening in Competent Rock

Kirsch's solution

$$\sigma_r = \frac{P_x + P_y}{2}\left(1 - \frac{a^2}{r^2}\right) + \frac{P_x - P_y}{2}\left(1 - \frac{4a^2}{r^2} + \frac{3a^4}{r^4}\right)\cdot \cos 2\theta$$

$$\sigma_\theta = \frac{P_x + P_y}{2}\left(1 + \frac{a^2}{r^2}\right) - \frac{P_x - P_y}{2}\left(1 + \frac{3a^4}{r^4}\right)\cdot \cos 2\theta$$

$$\tau_{r\theta} = -\frac{P_x + P_y}{2}\left(1 + \frac{2a^2}{r^2} - \frac{3a^4}{r^4}\right)\cdot \sin 2\theta$$

Opening in Competent Rock

Kirsch's solution

$$u_r = \frac{P_x - P_y}{4G}\cdot\frac{a^2}{r} + \frac{P_x - P_y}{4G}\cdot\frac{a^2}{r}\left(4(1-\nu) - \frac{a^2}{r^2}\right)\cdot \cos 2\theta$$

$$v_\theta = -\frac{P_x - P_y}{4G}\cdot\frac{a^2}{r}\left(2(1-2\nu) + \frac{a^2}{r^2}\right)\cdot \sin 2\theta$$

โดยที่ u_r และ v_θ คือค่าการเคลื่อนตัวของมวลหินที่จุด (r, θ) ในแนวรัศมีและในแนวสัมผัสตามลำดับ

Underground Mine

- Room-and-Pillar Mine
- Room-and Rib Mine

Tributary Area Theory

ทฤษฎีการแบ่งพื้นที่รับน้ำหนักของเสาค้ำยันแต่ละอัน

โดย Obert and Duvall (1967)

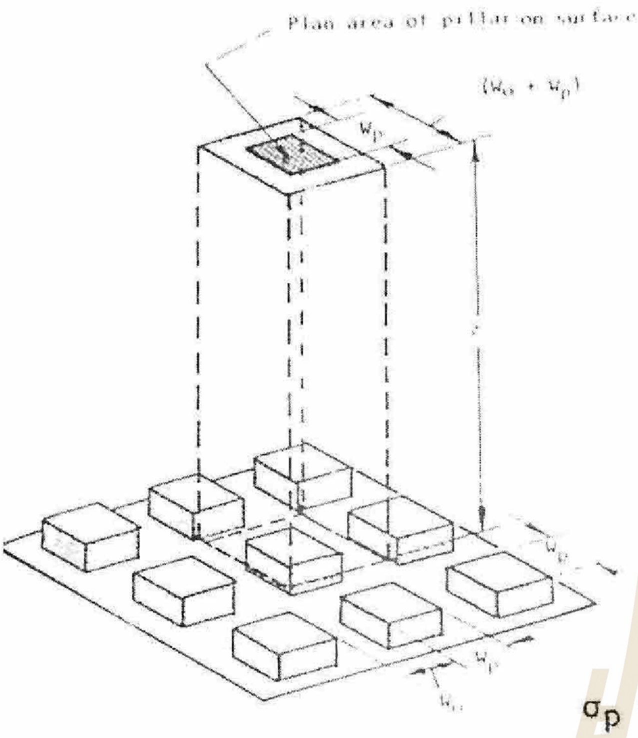
$$\boxed{\sigma_p = (A_t/A_p) \cdot \sigma_v}$$

โดย σ_p คือความเค้นกดเฉลี่ยในแนวดิ่งของเสาค้ำยัน (Pillar)

σ_v คือความเค้นในมวลหิน (ก่อนเจาะอุโมงค์) ที่ระดับหลังคาของอุโมงค์ A_t คือพื้นที่หน้าตัดของมวลหินที่รับน้ำหนักโดยเสาค้ำยัน

A_p คือพื้นที่หน้าตัดของเสาค้ำยัน

Average Pillar Stresses



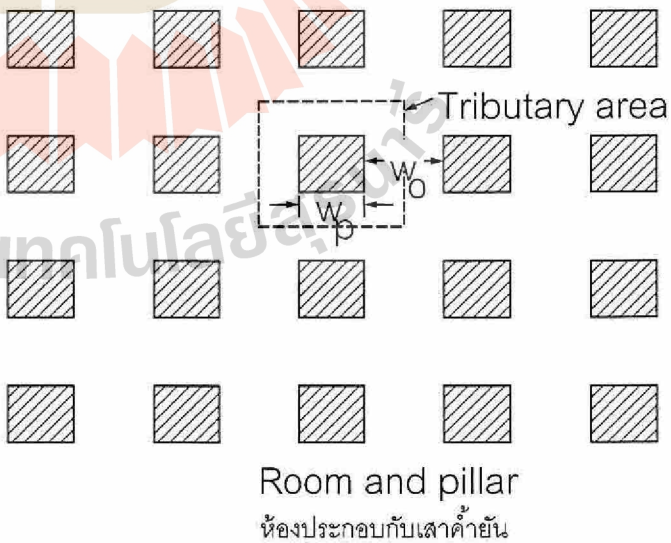
Tributary Area Theory
Obert and Duvall (1967)

$$\sigma_p = (A_t/A_p) \cdot P_z$$

σ_p = Average Pillar Stress
P_z = Vertical Stress at Roof level (= γz)
A_t = Rock Column Area
A_p = Cross Section Area of Pillar

$$\sigma_p = P_z(1 + W_o/W_p)^2 = \gamma z(1 + W_o/W_p)^2$$

Room-and –Pillar Mine



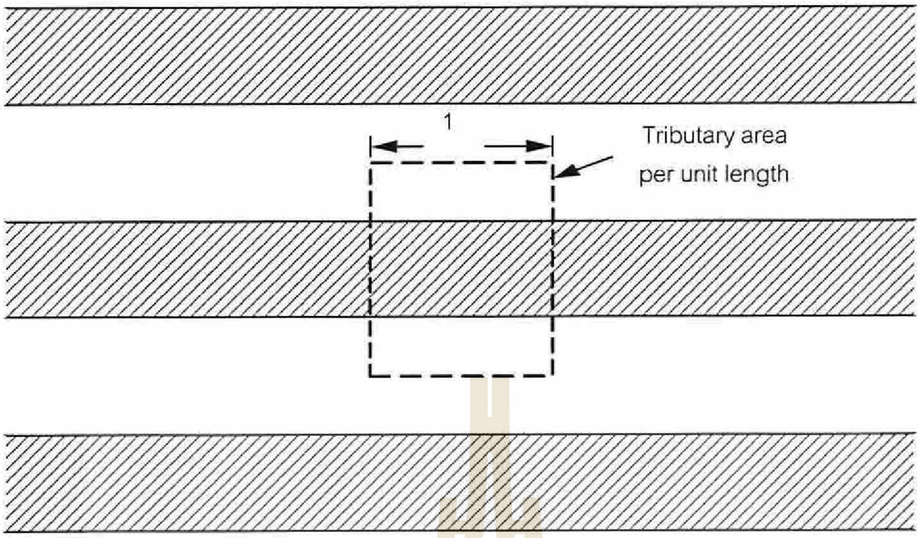
Square Pillar

$$A_t = (w_o + w_p)^2$$
$$A_p = (w_p)^2$$

Rectangular Pillar

$$A_t = [(w_o + w_{p1}) + (w_o + w_{p2})]^2$$
$$A_p = (w_{p1} \times w_{p2})$$

Room-and –Rib Mine



Room and rib

ชุดของอุโมงค์ที่ขนานกันโดยมีความกว้างของเสาค้ำยันเท่ากัน

Room-and –Pillar Mine

Factor of Safety

$$F.S. = \text{Strength of Pillar} / \text{Stress in Pillar}$$
$$= \sigma_c / \sigma_p$$

Extraction Ratio (r)

$$r = \text{Extraction Ore} / \text{Total Ore} \ (\times 100\%)$$
$$= (A_t - A_p) / (A_t)$$

$$\sigma_p = 1 / (1-r) \cdot \sigma_v$$