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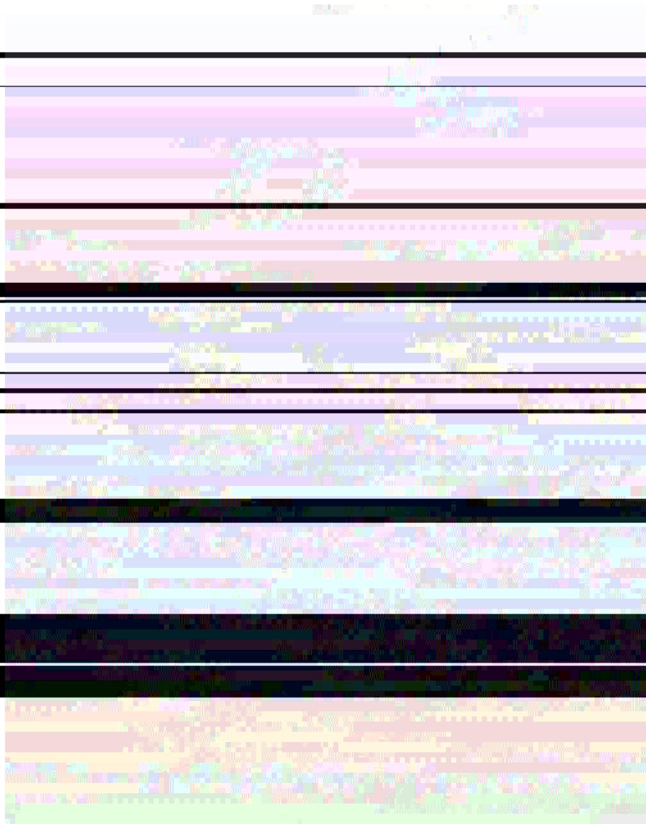

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Introduction

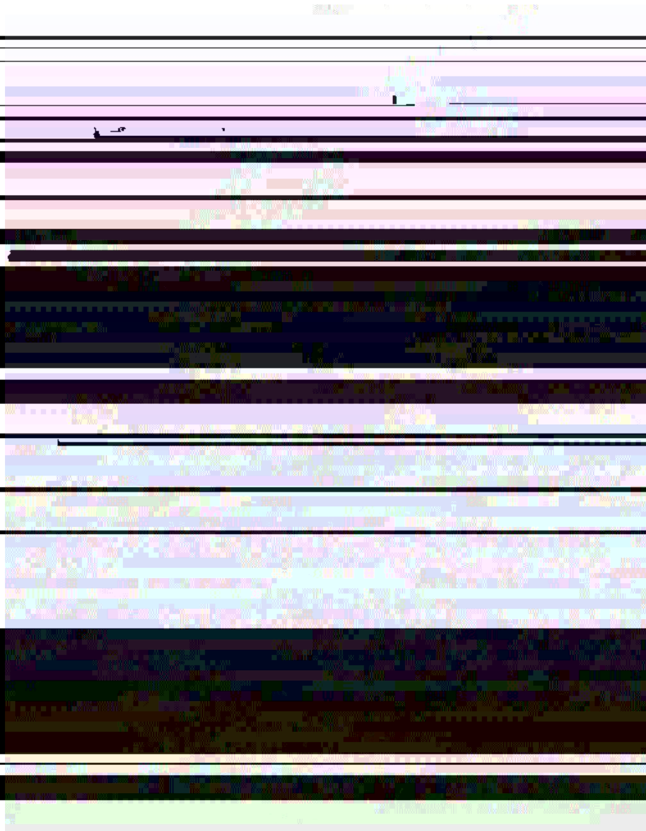
[illegible] and Background of the Research

One of the main problems of fluid dynamics is the motion of fluid in [illegible] incompressible fluid flow through a bounded domain with the inflow and outflow parts of boundaries have not yet been considered in detail. The goal of this thesis is to study the numerical methods for approximating solutions of such

flowing-through problems.

tions

This section has an introductory nature, wherein we discuss the funda-



$$\mathbf{u} = (u_1, u_2, 0), \quad u_i = u_i(x_1, x_2)$$

only the third component of the vorticity $\omega_3 = \omega$ is present and the right-hand

... (1.3.7) vanishes. Therefore, the Euler equations for the vorticity in two

Figure 1.2: Normal vector on surface

when the wall of the region moves with a given law. The Euler equations become books in fluid mechanics such as G.K. Batchelor (1970), J. Serrin (1959)

Through Problems

We will present here the three kinds of [illegible]

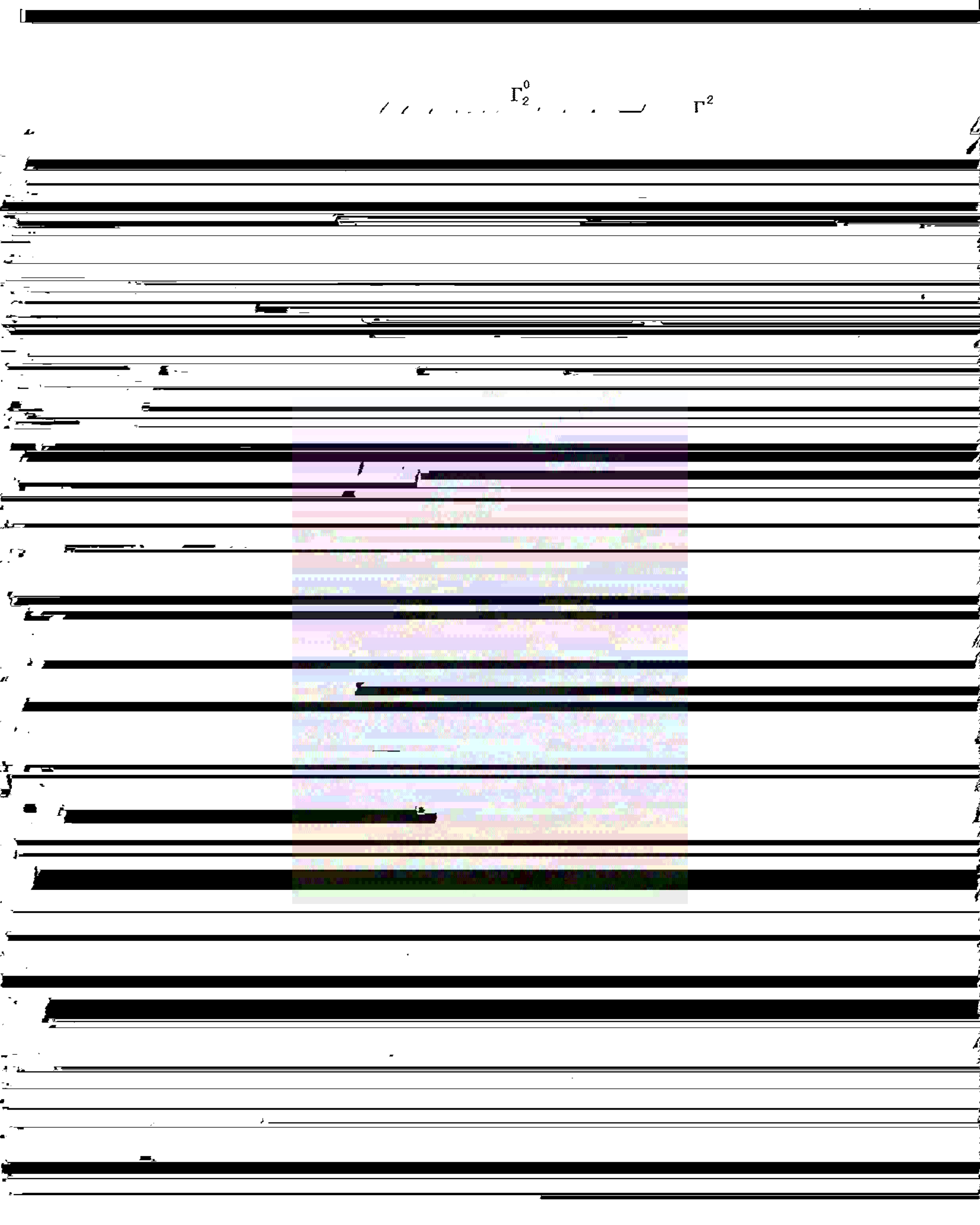
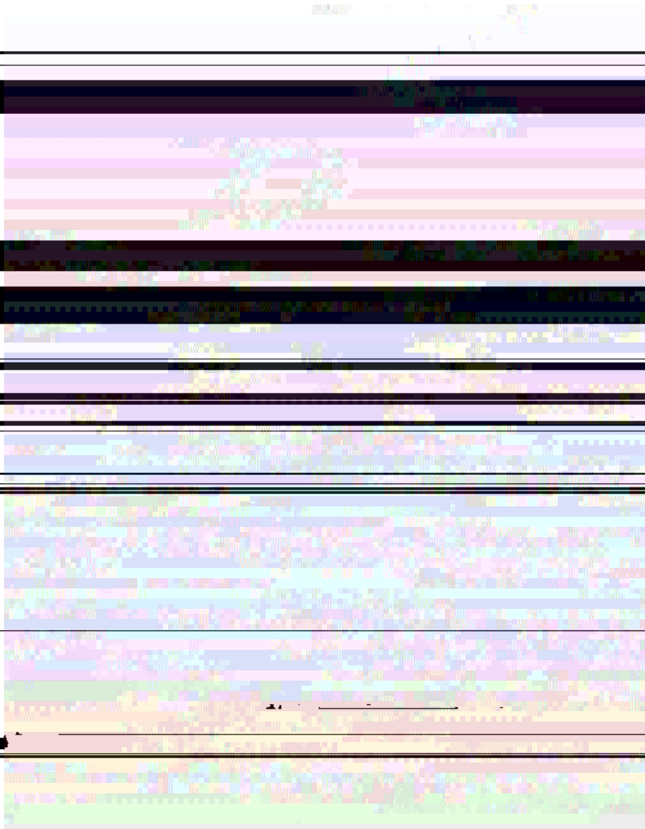


Figure 1.4: Sketch of a domain.



situation is not only very mathematically interesting but also corresponding to

[redacted]

eration of numerical oscillations have been proposed (and successfully employed)

[redacted]

component of the velocity vector and the tangent components of the vorticity

where $u(x, y)$ and $v(x, y)$ are components of the velocity vector in the x and y di-

we set the density equal to one ($\rho = 1$).

$$
\begin{aligned}
q &= q_1(x, y), \\
w &= w_1(x, y).
\end{aligned}
\qquad (2.2.5)
$$

Outflow part CD: $(x, y) \in \Gamma^2$

[illegible]

Find the solution of equations (2.2.1)–(2.2.3) with the following boundary condi-

[illegible]

tions:

Impermeable boundaries AD and BC: $(x, y) \in \Gamma' \cup \Gamma''$

[illegible]

$$\left(\frac{\partial u}{\partial x}\right)\left(\frac{\partial v}{\partial x}\right) + u\frac{\partial^2 v}{\partial x^2} + \left(\frac{\partial v}{\partial x}\right)\left(\frac{\partial v}{\partial y}\right) + v\frac{\partial^2 v}{\partial x \partial y} = -\frac{\partial^2 P}{\partial y \partial x}.$$

We have

[illegible]

after simplification. we obtain

$$x = x(\varphi, \psi), \qquad y = y(\varphi, \psi).$$

$$dx = \frac{\partial x}{\partial \varphi} d\varphi + \frac{\partial x}{\partial \psi} d\psi.$$

$$dy = \frac{\partial y}{\partial \varphi} d\varphi + \frac{\partial y}{\partial \psi} d\psi$$

or in a matrix form

$$\begin{bmatrix} dx \\ dy \end{bmatrix} = \begin{bmatrix} \dfrac{\partial x}{\partial \varphi} & \dfrac{\partial x}{\partial \psi} \\ \dfrac{\partial y}{\partial \varphi} & \dfrac{\partial y}{\partial \psi} \end{bmatrix} \begin{bmatrix} d\varphi \\ d\psi \end{bmatrix}.$$

Solving this matrix equation, for the right-hand column matrix, we have

$$\left| \frac{\partial \varphi}{} \quad \frac{\partial \varphi}{} \right|$$

Since [illegible] $I \neq 0$ then [illegible] and [illegible] are not equal to zero. Finally, we can

$$\frac{1}{\rho}\frac{\partial w}{\partial \varphi} + c w^2 \frac{\partial q}{\partial \psi} = 0.$$

The second step: We have to use the condition

$$\partial^2\varphi \quad = \quad \partial^2\varphi \qquad (2.2.26)$$

and $\psi|_{AB}$ and $\psi|_{BC}$ are constants. The function $\psi(x, y)$ is determined up to an arbitrary constant, and without loss of generality, we can choose

$$\psi|_{AD} = \psi(A) = \text{const.}$$

into equation (2.2.31) yields

$$(y_\varphi - y'_{AB}(x)x_\varphi)\,d\varphi + (y_\psi - y'_{AB}(x)x_\psi)\,d\psi = 0. \qquad (2.2.32)$$

In (φ, ψ)-plane, equation (2.2.32) is ODE for unknown function $\varphi_{A'B'}(\psi)$

Let us consider boundary CD. In the [illegible]

[illegible]

$$[illegible] \in \mathbb{R}^2$$

[illegible]

$P_2(x, y)$, $(x, y) \in \Gamma_2$. Here, the situation is slightly more complicated. Let us

assume that equation of boundary $C'D'$ is given by the formula

and the equation of boundary CD is given by the formula

By analogy with two

In order to summarize, we write the formulation of problem 3′ and problem 2′ in a compact form in terms of new unknown functions and new independent variables (φ, ω).

Problem 3′:

$$\varphi_{A'B'}(0) = 0, \tag{2.2.45}$$

$$w = w_0(\varphi, \psi), \quad \varphi = \varphi_{A'B'}(\psi)$$

dq

[redacted]

$$\begin{aligned}
\varphi_{A'B'}(0) &= 0 \\
q &= q_1(\varphi,\psi), \qquad \varphi = \varphi_{A'B'}(\psi) \\
w &= w_1(\varphi,\psi), \qquad \varphi = \varphi_{A'B'}(\psi)
\end{aligned} \tag{2.2.49}$$

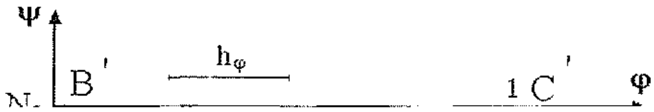
$B'C'$: $\varphi(\cdot, 0) = \varphi_0, \quad \varphi_0 \neq 0$

[redacted]

$$\frac{dq}{dl} = k_{BC'}(l), \tag{2.2.50}$$

[redacted]

To [illegible] the solution of equations (3.3.43)–(3.3.44) [illegible] the following [illegible]



ψ
B'
h_φ
1 C'
φ
N

approximate the integral $\int^{w_{i+2}} (q_c) dw$

$$i = N_1 - 2, \ldots, 1.$$

$$\left(\frac{1}{\ } \frac{q_{i_0,j+1} - q_{i_0,j-1}}{\ } \quad j \neq 1 \text{ and } j \neq N \right.$$

When the values $w^{(n)}$ and $\phi^{(n)}$ are found for all grid points, we can solve or after integration

$$\tilde{z}^{(n)} \quad \tilde{z}^{(n)} \quad \int^{t_j}$$

where $z(n)$, $z(n)$ denote the values on n-th iteration

$n = 1$

where ... $\frac{1}{\ }$ (...) ... $\frac{1}{\ }$ (...) and $\tilde{a}$ ... is the ...

intermediate values of the unknown vector ...

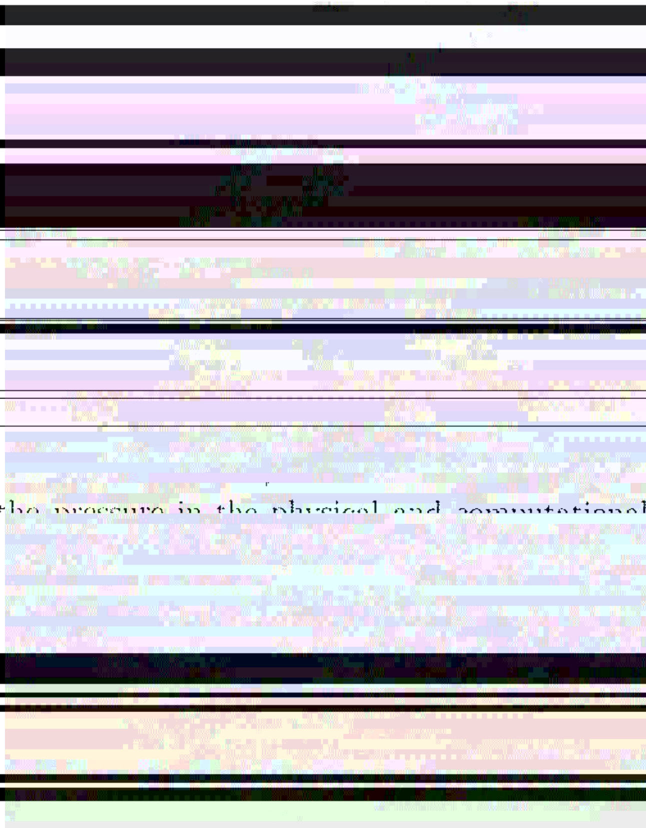
The sweep method for the solution of (2.3.6) is given by the following formulas

$$\tilde{q}_{i,j} = \alpha_{i+1}\tilde{q}_{i+1,j} + \beta_{i+1}$$

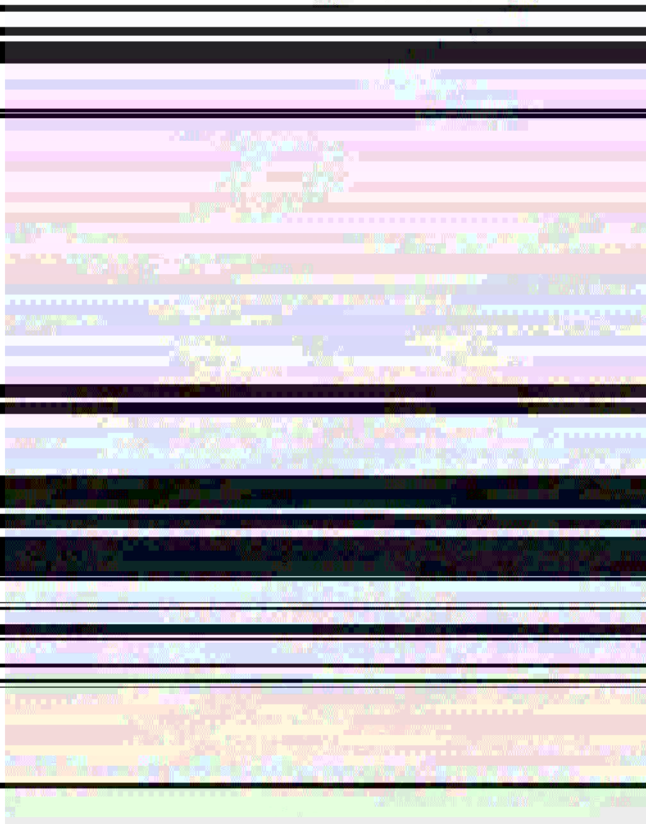
$$i = 1, \ldots, N_1 - 1.$$

$$\tilde{q}_{1,j} = \tilde{q}_{AD}(j).$$

In the SOR scheme, the solution of (2.3.6) is combined with desirable. When the iterative process is completed, we have to transform back



bution of the pressure in the physical and computational domain. To find the



$$\left(\frac{\partial w}{\partial \varphi}\right)_{i,j-1} = \begin{cases} \dfrac{w_{i,j} - w_{i,j-2}}{2h_2} & : \quad j \neq N_2, \\ -3w_{i,1} + 4w_{i,2} - w_{i,3} & : \quad j = 2. \end{cases}$$

2.4 Results and Di...