การเกิดอนุภาคฟีในอันตรกิริยาเฮดรอนิกและการละเมิดกฎของโอเส็ดไอ

นายสรไกร ศรีศุภผล

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรดุษฎีบัณฑิต
สาขาวิชาฟิสิกส์
มหาวิทยาลัยเทคโนโลยีสุรนารี
ปีการศึกษา 2554

ϕ PRODUCTION IN HADRONIC INTERACTIONS AND THE VIOLATION OF THE OZI RULE

Sorakrai Srisuphaphon

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Physics

Suranaree University of Technology

Academic Year 2011

ϕ PRODUCTION IN HADRONIC INTERACTIONS AND THE VIOLATION OF THE OZI RULE

Suranaree University of Technology has approved this thesis submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy.

Thesis Examining Committee

____

(Asst. Prof. Dr. Chinorat Kobdaj)

Chairperson

____

(Prof. Dr. Yupeng Yan)

Member (Thesis Advisor)

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วิทยานิพนธ์นี้ได้ทำการพิจารณาองค์ประกอบของควาร์กเอสภายในนิวคลีออนที่มีการละเมิดกฎของโอเส็ดไอในปฏิกิริยาแบบทำลายระหว่างนิวคลีออนกับปฏินิวคลีออน ซึ่งพบว่าการมีองค์ประกอบของคู่ควาร์กเอสกับปฏิควาร์กเอสในฟังก์ชันคลื่นของนิวคลีออนสามารถก่อให้เกิดอนุภาคเมซอนฟีได้โดยไม่มีการละเมิดกฎของโอเส็ดไอ งานวิจัยนี้ได้ทำการศึกษาแบบจำลองของฟังก์ชันคลื่นที่มีควาร์กเอสเป็นองค์ประกอบ 3 แบบจำลองดังนี้ แบบที่หนึ่งเป็นแบบจำลองของฟังก์ชันคลื่นที่ประกอบด้วยกลุ่มก้อนของควาร์กสองกลุ่มได้แก่ กลุ่มของควาร์กยูสองตัวและควาร์กดีหนึ่งตัว กับ กลุ่มก้อนควาร์กเอสกับปฏิควาร์กเอส ส่วนแบบจำลองที่สองอยู่บนพื้นฐานของแบบจำลองไคเรลควาร์กซึ่งฟังก์ชันคลื่นมีลักษณะเป็นกลุ่มก้อนของอนุภาคเคออนกับไฮเปอร์ออน และ แบบจำลองสุดท้ายเป็นแบบจำลองในรูปของเพนตาควาร์ก ในกรณีที่ไม่พิจารณาแบบจำลองของควาร์กแบบสัมพัทธภาพ การคำนวณพบว่าค่าโมเมนต์แม่เหล็ก และสปินของควาร์กเอสมีผลต่ออนุภาคโปรตรอน โดยในแบบจำลองที่หนึ่งและสองนั้นให้ค่าโมเมนต์แม่เหล็กและสปินเป็นลบ สอดคล้องกับผลการทดลองที่ได้มีการรายงานออกมาเร็วๆ นี้ ในขณะที่แบบจำลองที่สามสามารถพบค่าโมเมนต์แม่เหล็ก และสปินเป็นลบได้เฉพาะในกรณีที่โครงร่างของควาร์กสี่ตัวเป็น $[31]_{FS}\,[211]_F\,[22]_S$ และ $[31]_{FS}[31]_F[22]_S$

นอกจากนั้นแล้ว วิทยานิพนธ์นี้ยังได้แสดงผลการคำนวณค่าสัดส่วนการเกิดปฏิกิริยาระหว่างโปรตอนกับปฏิอนุภาคโปรตอนนิ่ง ที่ถูกทำลายไปเป็นอนุภาคฟีกับอนุภาคเมซอน ชนิด ไพ อีตา โรห์ และโอเมกา โดยใช้แบบจำลองทริปเปตพีศูนย์ร่วมกับแผนผังแบบเส้นของควาร์กยังผล ผลลัพธ์ที่ได้พบว่าฟังก์ชันคลื่นตามแบบจำลองที่หนึ่งและสามนั้น สัดส่วนการเกิดปฏิกิริยาของอนุภาคฟีกับอนุภาคเมซอนจากสถานะคลื่นเอสในอะตอมของโปรตอนกับปฏิอนุภาคโปรตอนขึ้นกับชนิดของปฏิกิริยาอย่างมาก ซึ่งผลที่ได้สอดคล้องกับการทดลองเป็นอย่างดี

	ลายมือชื่อนักศึกษา_______________
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	ลายมือชื่ออาจารย์ที่ปรึกษาร่วม_______________

SORAKRAI SRISUPHAPHON : ϕ PRODUCTION IN HADRONIC INTERACTIONS AND THE VIOLATION OF THE OZI RULE. THESIS ADVISOR : PROF. YUPENG YAN, Ph.D. 91 PP.

THE VIOLATION OF THE OZI RULE/STRANGENESS OF THE NUCLEON

Apparent channel-dependent violations of the OZI rule in nucleon-antinucleon annihilation reactions are discussed in the presence of an intrinsic strangeness component in the nucleon. Admixture of $s\bar{s}$ quark pairs in the nucleon wave function enables the direct coupling to the ϕ - meson in the annihilation channel without violating the OZI rule. Three forms are considered in this work for the strangeness content of the proton wave function, namely, the uud cluster with a $s\bar{s}$ sea quark component, kaon-hyperon clusters based on a simple chiral quark model, and the pentaquark picture $uuds\bar{s}$. Nonrelativistic quark model calculations reveal that the strangeness magnetic moment μ_s and the strangeness contribution to the proton spin σ_s from the first two models are consistent with recent experimental data where μ_s and σ_s are negative. For the third model, the $uuds$ subsystem with the configurations $[31]_{FS}[211]_F[22]_S$ and $[31]_{FS}[31]_F[22]_S$ leads to negative values of μ_s and σ_s.

With effective quark line diagrams incorporating the $^3P_0$ model we give estimates for the branching ratios of the annihilation reactions at rest $p\bar{p} \rightarrow \phi X$ ($X = \pi^0, \eta, \rho^0, \omega$). Results for the branching ratios of ϕX production from atomic $p\bar{p}$ s-wave states are for the first and third model found to be strongly channel dependent, in good agreement with measured rates.

School of Physics

Academic Year 2011

Student's Signature ____________________

Advisor's Signature ____________________

Co-Advisor's Signature ____________________

ACKNOWLEDGEMENTS

It is my great pleasure to thank people who have made this thesis possible. I would like to express my deep and sincere gratitude to my supervisor, Prof. Dr. Yupeng Yan, for his valuable advices, comments and suggestions which have provided a good basis for my thesis. I am deeply grateful to my supervisor, Prof. Dr. Thomas Gutsche at Institut für Theoretische Physik, Universität Tübingen, Germany, for his detailed and constructive comments, and for his important support throughout this work as well as his important guidance and untiring help during my stay in Germany. I wish to express my warm and sincere thanks to all the thesis examining committee members for their efforts throughout the entire process. I am thankful to all professors at the School of Physics, Suranaree University of Technology (SUT) for their kindness and providing me with invaluable knowledge. My gratitude is extended to all members in the Laboratory of Nuclear and Particle Physics, ThEP and all graduate students and staff members at School of Physics, SUT, for their friendly support, encouragement, and extensive discussions of my work and other interesting explorations. I am grateful to the German Academic Exchange Service (DAAD), Germany, for supporting this research project and thankful to Burapha University for financial support. Last, but certainly not least, it is my pleasure to thank my beloved family for their support, believe, and encouragement.

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LIST OF ABBREVIATIONS

QCD	Quantum Chromodynamics
CQM	Constituent quark Models
EMC	The European Muon Collaboration
CCFR	Columbia-Chicago-Fermilab-Rochester
HAPPEX	Hall a Proton Parity Experiment
GB	Goldstone Boson
ChQM	Chiral Quark Model

CHAPTER I

INTRODUCTION

The fundamental constituents of matter are quarks and leptons interacting through gauge bosons which is the basis for the understanding of matter at the level of 10^{-18} m (Halzen and Martin, 1984). The different kinds of quarks are distinguished by a quantity usually called flavor. There are six flavors of quarks (q), named up (u), down (d), strange (s), charm (c), top (t), and bottom (b). However, in experiments quarks have not been observed as isolated objects, but are only seen as clusters of quarks called hadrons. All hadrons are made up of quarks and antiquarks, with baryons of half integer spin being bound states of three quarks, whereas the mesons with even numbered spins are quark-antiquark configurations. The quarks in the hadrons are bound by the strong interaction mediated through gluons.

Quantum Chromodynamics (QCD) is the fundamental theory of the strong interaction. QCD has registered remarkable success in the limit $Q \to \infty$ or at very short distances where the effective QCD coupling, $\alpha_s(Q^2)$ (Q^2 is the scale of the four-momentum transfer squared) is very small and a perturbative calculation is amenable. On the other hand, in the low energy limit where QCD cannot be treated perturbatively, the vast amount of low energy data are usually explained through constituent quark models (CQM) which are based on simplifying assumptions that all hadrons are made up of quarks and antiquarks (Thomas and Weise, 2001). In the CQM, the proton is made of two u quarks and a d quark while the neutron consists of two d quarks and a u quark. A hyperon is any baryon containing one or more strange quarks (s) while a meson containing one s or $\bar{s}$ quark and a non-strange, light quark/antiquark is called kaon. The CQM has been very successful in explaining many experimental data such as hadron spectroscopy data (Hendry and Lichtenberg, 1978), the neutron charge radius (Gupta and Kaur, 1983), baryon magnetic moments, etc.

However, experimental results for the value of the pion-nucleon sigma term, the strange magnetic moment μ_s, the strangeness contribution to the nucleon form factor (von Harrach, 2005) as well as the apparent violations in nucleon-antinucleon annihilation reactions involving ϕ mesons (Amsler, 1992) indicate that the proton might contain a substantial strange quark-antiquark ($s\bar{s}$) component. The strangeness sigma term appears to lie somewhere in the range of $2-7\%$ of the nucleon mass (Young, 2010). The substantial Okubo-Zwieg-Iizuka (OZI) rule violations in the $N\bar{N}$ annihilation reactions involving the ϕ meson may suggest the presence of an intrinsic $s\bar{s}$ in the nucleon wave function (Ellis et al., 1995), for instance, the presence of a $q^3s\bar{s}(\bar{q}^3s\bar{s})$ piece in the $N(\bar{N})$ wave function. With such an assumption, the ϕ meson could be produced in $N\bar{N}$ annihilation reactions via a shake-out or rearrangement of the strange quarks already stored in the nucleon without the violation of the OZI rule. There are other explanations of the OZI rule violation without introducing a strange component in the nucleon such as the resonance interpretation, instanton induced interactions (Kochelev, 1996), and rescattering (Locher and Lu, 1995).

The European Muon Collaboration (EMC) spin experiment (Ashman et al., 1988) on deep inelastic scattering of longitudinally polarised muons by longitudinally polarised protons revealed the first time that the polarization of the strange quark sea may contribute to the proton spin σ_s a significant negative value. This experimental result was confirmed by the subsequent deep inelastic double polarization experiments. Then, in Ref. (Ellis and Karliner, 1995) all the available data were analyzed in a systematic way and a value of $\sigma_s = -0.10 \pm 0.03$ was found. Among a large number of theoretical works, Cheng and Li apply the chiral quark model (ChQM) to explain the spin and flavor structure of the proton (Cheng and Li, 1995). With the fluctuation of the proton into a kaon and a hyperon, they can explain the negative polarization of the strange quark sea and get other theoretical results consistent with the DIS experimental data.

However, the configuration of strange quarks in the nucleon is still an open question. The strangeness magnetic moment μ_s can be extrapolated from the strange magnetic form factor $G_M^s(Q^2)$ at the momentum transfer $Q^2 = 0$ measured in parity violation experiments of electron scattering from a nucleon (Diehl et al., 2008). Most experimental measurements suggest a positive value for μ_s, in contrast to the recent experiment data (Baunack et al., 2009) and most theoretical calculations which have obtained negative values for this observable (Beck and McKeown, 2001; Lyubovitskij et al., 2002). A recent work (An et al., 2006) has proposed a different form for the strangeness content of the proton which has the strange quark piece in terms of pentaquark configurations instead of the 5-quark component which consists of a uud cluster and a $s\bar{s}$ pair proposed for solving the puzzle of violation the OZI rule. Different pentaquark configurations that may be contained in the proton may yield both positive and negative values for the strangeness spin and magnetic moment of the proton.

The experimental results on μ_s, which is extracted from experimental data on $G_M^s(Q^2)$, are rather uncertain due to the large uncertainties in $G_M^s(Q^2)$ and the extrapolation approach. So it is believed that the proton-antiproton reactions involving ϕ production may be another platform to be applied to tackle the possible configuration of strange quarks in the proton. In the present work we consider the strange content in the proton wave function in three models, namely, the uud cluster with a $s\bar{s}$ sea quark component, kaon-hyperon clusters based on the chiral quark model, and the pentaquark picture $uuds\bar{s}$. The theoretical σ_s, μ_s and branching ratios of the reactions $p\bar{p} \to \phi X$ $(X = \pi^0, \eta, \rho^0, \omega)$ will be compared to experimental data. We resort to the $^3P_0$ quark model (Le Yaouanc et al., 1988) and the nearest threshold dominance model (Vandermeulen, 1988) to obtain quantitative predictions for the branching ratios of the annihilation reactions from atomic $p\bar{p}$ states with the relative orbital angular momentum $L = 0$ (Gutsche et al., 1997).

This thesis is organized as follows. In Chapter II we show some detailed description of the violation of the OZI rule and experimental data that suggested the presence of strange in the nucleon. detailed description of the proton wave functions for each models as mentions and their corresponding strangeness spin and magnetic moment are presented in Chapter III. In Chapter IV we briefly review the description of the reaction $p\bar{p} \to \phi X$ with a strange component in the proton wave functions by using $^3P_0$ model, the possible quark line diagrams and the model predictions of branching ratios in comparison with experimental data are contained in this section. Finally, the conclusions are given in Chapter V.

CHAPTER II

STRANGENESS IN THE NUCLEON

The constituent quark model has been very successful in explaining not only the nucleon magnetic moment but also the magnetic moments and masses of other baryons. However, the model is still incomplete to explain some experimental findings which indicate a possible, sizeable occurrence of strange quarks in the nucleon. In this chapter we discuss experimental evidences and theoretical results that support the knowledge of the presence of the strange quark in the proton.

2.1 The OZI Rule

The quark model has not only been applied to predict the ratio of the proton to neutron magnetic moments but it was also applied to predict the decay and collision of hadrons by picturing/describing these processes in terms of interactions of their constituent quarks. Accordingly, the quark line diagram was employed to describe the processes explicitly in term of quarks. The decays and collisions of hadrons with the strong interaction, represented by the quark line diagram, can be classified in two main categories (the OZI rule forbidden and the OZI allowed transitions) according to the quark line topology. Hence, if the transition from initial to final state hadrons can only be separated by cutting a quark line the diagram is classified to be allowed by the OZI rule. Otherwise, if the quark line diagram can be divided without cutting a quark line the process is classified to be OZI forbidden(Le Yaouanc et al., 1988). An explanation of the OZI rule can be seen from the fact that the coupling constant in QCD decreases with increasing energy (or momentum transfer). For the OZI suppressed channels the gluons must have high Q^2 (at least as much as the rest mass energies of the quarks into which they decay) and so the coupling constant will appear small to these gluons. Consider for example the decay of the ϕ and ω mesons. Among the nine lightest vector mesons, the ϕ and ω mesons are linear combinations of the singlet and the isospin-zero octet state, that is

$$\phi = \omega_8 \cos\theta - \omega_1 \sin\theta \tag{2.1}$$

$$\omega = \omega_8 \sin\theta + \omega_1 \cos\theta \tag{2.2}$$

where ω_8 and ω_1 are the isospin-zero octet state and the singlet state, respectively. They contain the quark constituents as follows:

$$\omega_8 = (u\overline{u} + d\overline{d} - 2s\overline{s})/\sqrt{6} \tag{2.3}$$

$$\omega_1 = (u\overline{u} + d\overline{d} + s\overline{s})/\sqrt{3} \tag{2.4}$$

The so-called ideal mixing angle is the one which leads to a pure $s\overline{s}$ configuration of the ϕ meson. From Eqs. (2.3) and (2.4), one finds the ideal mixing angle

$$\theta_0 \;=\; \tan^{-1}(1/\sqrt{2}) \;=\; 35.16^o \tag{2.5}$$

With the ideal mixing angle the ϕ and ω meson states may be rewritten as follows:

$$\phi = (s\bar{s})\cos(\theta_0 - \theta) + \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\sin(\theta_0 - \theta), \tag{2.6}$$

$$\omega = (s\bar{s})\sin(\theta_0 - \theta) + \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})\cos(\theta_0 - \theta). \tag{2.7}$$

The physical value of the mixing angle θ may be estimated from the quadratic Gell-Mann-Okubo mass formula (Okubo, 1977)

$$4m_{K^*}^2 - m_\rho^2 = 3\,(\,m_\phi^2\cos^2\theta + m_\omega^2\sin^2\theta). \tag{2.8}$$

Inserting into Eq. (2.8) the latest mass values of the Particle Data Group (Eidelman et al., 2004) for the neutral mesons, $m_{K^*} = 1680$ MeV, $m_\rho = 783$ MeV, $m_\phi = 1020$ MeV, $m_\omega = 783$ MeV, we get

$$\theta = 38.5^o \tag{2.9}$$

The mixing angle θ is not far from the ideal mixing angle, therefore, the ϕ consists predominantly of $s\bar{s}$ quarks while the ω consists predominantly of $u\bar{u}$ and $d\bar{d}$ quarks. These two mesons have the same quantum numbers $J^P = 1^-$ but ϕ has a larger mass than the ω and one would naively also expect a larger decay width. But the experimental finding shows that the decay width of the ω into 3π is about fifteen times of the ϕ into the same decay channel. The mechanism in these decays can be explained by the OZI rule in terms of the quark line diagrams of Figure 2.1 (Hendry and Lichtenberg, 1978).

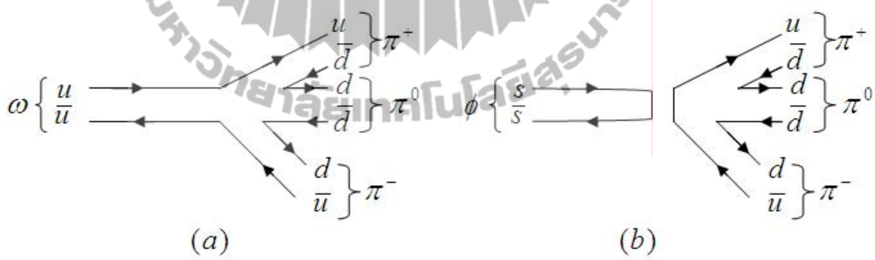

Figure 2.1 Quark line diagrams demonstrate (a) the allowed decay of the ω into three pions, (b) the OZI rule-forbidden decay of the ϕ into three pions.

According to the constituent quark picture of ω and ϕ with the nearly ideal mixing angle, each quark line in Figure 2.1(a) is a part of two hadrons but the s quark line of Figure 2.1(b) is by far dominant for the ϕ. This is an illustration of the OZI rule which states that any process which incorporates a disconnected quark line diagram is forbidden.

A disconnected diagram can be defined as one in which one or more hadrons can be isolated by a line which does not cut any quark lines. Therefore, the OZI rule inhibits the decay $\phi \to 3\pi$ while allowing the decays $\omega \to 3\pi$.

2.2 Apparent Violation of the OZI Rule

The OZI rule predictions have been tested many times in different reactions but there is a long-standing problem of the apparent violation of the OZI rule, such as in $N\bar{N}$ reactions involving the ϕ meson in the final state, $N\bar{N} \to \phi X$. For the collision involved in this work let us consider the creation of a $q\bar{q}$ pair in the reaction of hadron annihilation

$$A + B \to C + q\bar{q}, \qquad \text{for } q = u, d, s, \tag{2.10}$$

where A, B and C are hadrons. If the hadrons A, B have no strange quark component then the OZI rule demands that

$$\varepsilon = \frac{T(A + B \to X + s\bar{s})}{T(A + B \to X + u\bar{u}) + T(A + B \to X + d\bar{d})} = 0 \tag{2.11}$$

where $T(A + B \to X + q\bar{q})$ is the amplitude of the corresponding process and X stand for a non-strange meson. It means that if the ϕ meson is a pure $s\bar{s}$ state, it could not have been produced in the interaction of hadrons composed from u and d quarks only. The ratio of cross sections for production of the ϕ and ω mesons in case of $A, B = N, \bar{N}$ is derived in terms of the parameter ε as:

$$R = \frac{\sigma(N\bar{N} \to X\phi)}{\sigma(N\bar{N} \to X\omega)} = \beta^2 \cdot f \tag{2.12}$$

with

$$\beta = \frac{\varepsilon + \tan(\theta - \theta_0)}{1 - \varepsilon \tan(\theta - \theta_0)}, \tag{2.13}$$

where f is the kinetic phase space factor depending on the masses of final state mesons. If it is true that the N and $\bar{N}$ have no strange quark components, then the ϕ meson can only be produced by coupling to its non-strange components in $N\bar{N}$ annihilation reactions. For instance, creation of the ϕ and ω together with a π in the $p\bar{p}$ annihilation should be described by the quark line diagrams in Figure 2.2 (Nomokonov and Sapozhnikov, 2003). The quark lines of a $s\bar{s}$ pair in the final state are not connected with the quark lines of the initial state (Figure 2.2(a)), this reaction should be suppressed in comparison with the production of the ω meson, Figure 2.2(b). The theoretically expected ratio of branching ratios is

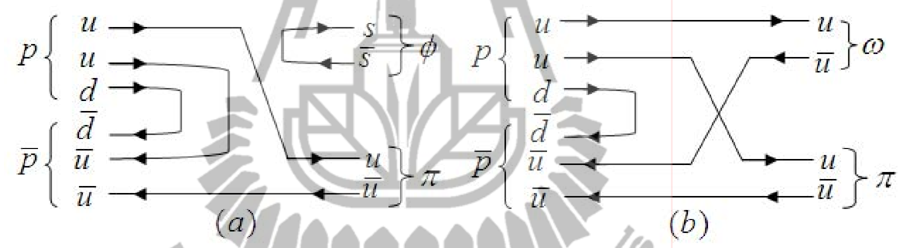

Figure 2.2 Quark line diagrams for the ϕ (a) and ω (b)- meson production in $p\bar{p}$ annihilation.

$$\frac{\sigma(N\bar{N} \to X\phi)}{\sigma(N\bar{N} \to X\omega)} = \beta^2 \cdot f \approx 0.001 - 0.003. \qquad (2.14)$$

However, as shown in Table 2.1 experimental data by the ASTERIX Collaboration at LEAR for the branching ratios of $N\bar{N}$ annihilations at rest show that the $\phi X/\omega X$ ratios are generally larger than the naive estimate of Eq. (2.14). Especially for the $^{33}S_1$ initial state, the $BR(\pi\phi)/BR(\pi\omega)$ of order 1/10 for the $N\bar{N}$ annihilation from the $^{33}S_1$ initial state indicates a considerable OZI violation. The violation of the OZI rule was observed not only in the proton-antiproton annihilation, but also in the reactions with protons $pd \to 3He\phi$, $pp \to pp\phi$ and pions $\phi p \to \phi\pi p$. The discrepancy with the OZI rule in these reactions was found to

Table 2.1 Ratios for ϕ, ω production from S and P waves of the $N\overline{N}$ atoms.

Transition	Ratios $\phi X/\omega X$
${}^{33}S_1 \rightarrow \pi^-\phi,\ \pi^-\omega$	0.077
${}^{13}S_1 \rightarrow \eta\phi,\ \eta\omega$	0.003
${}^{31}S_0 \rightarrow \rho^0\phi,\ \rho^0\omega$	0.018
${}^{11}S_0 \rightarrow \omega\phi,\ \omega\omega$	0.019
${}^{33}P_J \rightarrow \rho^0\phi,\ \rho^0\omega$	0.006

be larger by a factor of 10 – 100. Large OZI rule violations in these reactions may signal some interesting new physics. Naively, one would expect that the presence of intrinsic strangeness in the nucleon leads to an overall enhancement of ϕ production in $N\bar{N}$ annihilation, which is contrary to experimental data.

2.3 Strangeness of the Nucleon

The violation of the OZI rule in $N\bar{N}$ interactions could be evaded if there is a sizable $s\bar{s}$ component in the nucleon wave function. Then new classes of connected quark line diagrams could be drawn for the production of the ϕ meson and other $s\bar{s}$ mesons. Not only the apparent violation of the OZI rule but also several different experimental indications, including theoretical considerations which were summarized in (von Harrach, 2005; Diehl et al., 2008; Nomokonov and Sapozhnikov, 2003; Beck and McKeown, 2001; Ellis, 2005), suggest that the nucleon wave function contains a $s\bar{s}$ component. For example, CCFR measures the fraction of the nucleon momentum carried by strange quarks: $P_s = 4\%$ at momentum transfer $Q^2 = 20$ Gev^2. One other prominent example is the strange scalar density which is parameterized with the ratio

$$y = \frac{2\langle p|s\bar{s}|p\rangle}{\langle p|u\bar{u} + d\bar{d}|p\rangle} \tag{2.15}$$

which should be vanishing or very small for a nonstrange hadron such as the proton $|p\rangle$. This ratio is related to the pion-nucleon sigma term σ which can be extracted from the pion-nucleon scattering data. Analysis of the σ term indicates that the $s\bar{s}$ pairs may contribute ~15% to the mass of the nucleon (Beck and McKeown, 2001). Different groups have evaluated the sigma term making use of different data sets and values for coupling constants. From the values of σ the strangeness content is derived as $y = 0.1 - 0.6$ (von Harrach, 2005). Not only mass and momentum fraction but also strange quark content has a contribution to the spin and magnetic moment of the nucleon. The EMC and successor experiments with polarized lepton beams and nucleon targets, and experiments on elastic neutrino scattering gave indications that the $s\bar{s}$ pair in the nucleon is polarized negatively with respect to the direction of the nucleon spin: $\Delta s = -0.10 \pm 0.02$ and -0.15 ± 0.07, respectively. The intrinsic nucleon strangeness spins from those experimental data are consistent with theoretical considerations. Such as, the analysis of the baryon magnetic moment: $\Delta s = -0.19 \pm 0.05$, the QCD lattice calculation: $\Delta s = -0.12 \pm 0.07$ and the SU(3) flavor chiral quark model: $\Delta s = -(0.11 - 0.22)$ also indicated the negative polarization of strange quarks in the proton (Nomokonov and Sapozhnikov, 2003). Other examples are the strange form factors which can be extracted from parity violation in elastic electron scattering on a nucleon (Diehl et al., 2008). Experiments typically measure a linear combination of the electric and magnetic form factors (G_E^s and G_M^s) at a low value of the momentum transfer. For example, the recent HAPPEX data, which have significantly smaller errors than the other measurements, correspond at $Q^2 = 0.109$ GeV^2 to the linear combination $G_E^2 + 0.09G_M^2 = 0.007 \pm 0.011 \pm 0.006$. At a momentum transfer of $Q^2 = 0.077$ GeV^2 it corresponds to the electric form factor $G_E^s = 0.002 \pm 0.014 \pm 0.007$. Furthermore, the strangeness magnetic moment of the nucleon can be obtained from the magnetic form factors with the relation

$$\mu_s \equiv G_M(Q^2 = 0). \tag{2.16}$$

A variety of theoretical methods has been employed in efforts to compute the form factor G_M including μ_s. A reasonably complete compilation of theoretical results for μ_s are listed in Ref.(Beck and McKeown, 2001). From most of the results one should expect $\mu_s < 0$ but with few exceptions, generally μ_s is in the range of -0.8→0.0 nuclear magnetons (μ_N). A derivation making use of additional experimental input (Leinweber et al., 2005) has obtained a strange magnetic moment of $\mu_s = -0.051 \pm 0.021\mu_N$. Essentially, the same value $\mu_s = -0.048 \pm 0.012\mu_N$ has been obtained earlier in a chiral quark model (Lyubovitskij et al., 2002). The experiments on parity violation in electron-proton scattering suggest a positive value of μ_s (Spayde et al., 2004; Aniol et al., 2004; Maas et al., 2005; Armstrong et al., 2005). This is in contrast to a recent one (Baunack et al., 2009) and the most of the theoretical calculations discussed above. However, the μ_s from the experimental data are rather uncertain due to the large uncertainties in the measurement of the electromagnetic form factors and the extrapolation to $G_M^s(Q^2 = 0)$. For instance, an analysis from the SAMPLE collaboration indicates $G_M^s(Q^2 = 0.1) = 0.14 \pm 0.29 \pm 0.31$ (the third error is related to uncertainties in the electromagnetic form factor). This value is corrected to obtain a result for the strange magnetic moment: $\mu_s = 0.01 \pm 0.29 \pm 0.31 \pm 0.07$ where the last uncertainty accounts for the additional uncertainty associated with the theoretical extrapolation to $Q^2 = 0$ (Beck and McKeown, 2001). From above discussion, the apparent violation of the OZI rule and the other studies suggest that there could be a sizeable strange quark component present in the nucleon. Therefore, the nucleon wave function may be improved by taking the admixture of strange quarks into account. In the next chapter we consider some possible models and give explicit forms of the proton wave function containing strange quarks/antiquarks.

CHAPTER III

THE PROTON WAVE FUNCTIONS

The inclusion of strange quarks in the form of a $s\bar{s}$ pair may not change the proton quantum numbers. The constituent quark wave function of the proton in the presence of the strange quark pair $s\bar{s}$ may be constructed by including a $qqqs\bar{s}$ part in addition to the naive uud quark model component. In general, the proton wave function may be written in the form

$$|p\rangle = A|uud\rangle + B|uuds\bar{s}\rangle \tag{3.1}$$

where A and B are the amplitudes for the 3-quark and 5-quark components in the proton, respectively. Recently, there is some evidence that the strange quark contribution to the strangeness sigma term appears to lie somewhere in the range of $2-7\%$ of the nucleon mass (Young, 2010). Therefore, the strangeness admixture can be treated as a small perturbation in the proton wave function, that is, $B^2 << A^2$. However, the 5-quark component is the main contribution to the ϕ production in $p\bar{p}$ annihilation. In this work we restrict ourselves to three models of the proton wave function, where strange sea quarks are included.

3.1 Meson and Baryon Wave Functions

According to the intrinsic properties of quarks, in the quark model hadron wave functions include a spatial, a spin, a flavor and a color part. Generally the wave functions can be written as

$$\psi_{hadron} = \varphi_{spatial}\varphi_{spin}\varphi_{flavor}\varphi_{color} \tag{3.2}$$

The spatial part of a hadron wave function cannot be written down explicitly, as its form depends on the unknown details of quark dynamics under the influence of the strong interaction. However, the interaction between quarks can be assumed to be basically confining, the lowest states will have a symmetric spatial wave function. A reasonable choice for the potential, associated with the strong interaction between quarks, is the harmonic oscillator potential (Hendry and Lichtenberg, 1978). Not only because of its exact solvability and simplicity but also because the energies of mesons are roughly equally spaced in energy (like for a harmonic oscillator potential for instance) (Close, 1979) this first choice for the leading part of the potential seems reasonable.

For the spin part, due to quarks are spin $\frac{1}{2}$ fermions, the total spin S and the third component S_z of the quark systems can be obtained by the addition of angular momentum. In case of mesons, the $q\bar{q}$ state is coupled to give a total spin of $S = 0$ and $S = 1$, which corresponds to the spin singlet and triplet states, respectively. The composition of theses states

$$|S, S_z\rangle = |(\frac{1}{2} \otimes \frac{1}{2})_{S,S_z}\rangle, \tag{3.3}$$

in terms of the standard Clebsch-Gordan coefficients is

$$|0, 0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow), \tag{3.4}$$

$$|1, 0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \tag{3.5}$$

$$|1, 1\rangle = \uparrow\uparrow, \tag{3.6}$$

$$|1, -1\rangle = \downarrow\downarrow, \tag{3.7}$$

where $\uparrow$ and $\downarrow$ denote the single quark spin function with the third component $\frac{1}{2}$ and $-\frac{1}{2}$, respectively. For baryons the spin states of the qqq system are formed by combining 3 $S = \frac{1}{2}$ objects:

$$|S, S_z\rangle = |((\frac{1}{2} \otimes \frac{1}{2})_{S_{12}} \otimes \frac{1}{2})_{S,S_z}\rangle. \tag{3.8}$$

This scheme results in $2^3 = 8$ independent states as follows:

$$|\frac{3}{2}, \frac{3}{2}\rangle = \uparrow\uparrow\uparrow \tag{3.9}$$

$$|\frac{3}{2}, -\frac{3}{2}\rangle = \downarrow\downarrow\downarrow \tag{3.10}$$

$$|\frac{3}{2}, \frac{1}{2}\rangle = \frac{1}{\sqrt{3}}(\uparrow\uparrow\downarrow + \uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow) \tag{3.11}$$

$$|\frac{3}{2}, -\frac{1}{2}\rangle = \frac{1}{\sqrt{3}}(\uparrow\downarrow\downarrow + \downarrow\uparrow\downarrow + \downarrow\downarrow\uparrow) \tag{3.12}$$

$$|\frac{1}{2}, \frac{1}{2}\rangle_+ = \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \tag{3.13}$$

$$|\frac{1}{2}, \frac{1}{2}\rangle_- = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \tag{3.14}$$

$$|\frac{1}{2}, -\frac{1}{2}\rangle_+ = -\frac{1}{\sqrt{6}}(2\downarrow\downarrow\uparrow - \downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow) \tag{3.15}$$

$$|\frac{1}{2}, -\frac{1}{2}\rangle_- = -\frac{1}{\sqrt{2}}(\downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow) \tag{3.16}$$

The subscript $(+)$ denotes a state that is symmetric and $(-)$ denotes a state that is antisymmetric under the exchange of the spin of the first two quarks, these states are known as mixed symmetric and mixed antisymmetric states, respectively.

According to the QCD quark-quark-gluon interaction that the strength is flavor-independent the mass differences of the u, d, s quarks are not very large, therefore, in limit $m_u = m_d = m_s = m$ the Hamiltonian is then invariant under SU(3) flavor transformations of the u, d, s quarks. Quarks are then assigned to the 3 representation of SU(3) (antiquarks to the conjugate $\bar{3}$ representation). In this representation the generators are the 3×3 Gell-Mann SU(3) matrices acting on the 3-dimensional flavor space. It follows that the physical states, mesons and baryons, will be collected into approximately degenerate multiplets corresponding to irreducible representations of SU(3). In the case of mesons, the combination of $q\bar{q}$ from u, d and s quarks and antiquarks generates nonets of mesons. In the framework of the flavor SU(3) symmetry, the SU(3) nonets can be reduced to an octet and a singlet state as in group theory the product $3 \otimes \bar{3} = 8 \oplus 1$ reduces to the irreducible representations 8 and 1. The meson nonets are classified according to total angular momentum $J = L + S$ (L is the relative orbital angular momentum) and parity $P = (-1)^{L+1}$. The nine lightest mesons with $J^P = 0^-$ and $J^P = 1^-$ are known as pseudoscalar and vector mesons, respectively, which has been observed and their corresponding wave functions are listed in Table 3.1 (Yan, 2006).

The baryons are constructed by forming qqq states. The direct product of three flavor triplets leads to a symmetric decuplet (10_S), one octet of mixed symmetric (8_{M_S}), one octet of mixed antisymmetric (8_{M_A}) and a singlet of anti-symmetric (1_S) states: $3 \otimes 3 \otimes 3 = 10_S \oplus 8_{M_S} \oplus 8_{M_A} \oplus 1_A$. The explicit form of the octet flavor wave functions, involved in the study of this thesis, is listed in the Table 3.2 (Lichtenberg, 1978).

There is indirect evidence that quarks have, in addition to flavor, another internal degree of freedom called color. The reason for introducing color has to do with quark statistics. Quarks are supposed to be particles of spin 1/2. The wave function of a collection of identical particles of half-integral spin must be antisymmetric under the interchange of any two of them. But the wave function of quarks inside a baryon (including just the usual quantum numbers) appears to be symmetric under this interchange. A way out of the difficulty is to assume that quarks carry a color degree of freedom and that the wave function is antisymmetric in the color variable. A baryon contains three quarks, so it is natural to let the color degree of freedom take on three values. In other words, a quark of a given flavor comes in three colors, say red (R), green (G) and blue (B). A baryon then

Table 3.1 Flavor wave functions of the pseudoscalar ($J^P = 0^-$) and vector meson ($J^P = 1^-$) nonets.

Flavor	Pseudoscalar	Vector
$u\bar{d}$	π^+	ρ^+
$d\bar{u}$	π^-	ρ^-
$\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$	π^0	ρ^0
$\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$	η_1	ω_1
$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$	η_8	ω_8
$u\bar{s}$	K^+	K^{*+}
$d\bar{s}$	K^0	K^{*0}
$-s\bar{u}$	K^-	K^{*-}
$s\bar{d}$	$\bar{K}^0$	$\bar{K}^{*0}$

Table 3.2 Baryon octet wave functions constructed from three quarks.

Baryon	Octet 8_{M_S}	Octet 8_{M_A}
p	$\frac{1}{\sqrt{6}}(2uud - udu - duu)$	$\frac{1}{\sqrt{2}}(udu - duu)$
n	$\frac{1}{\sqrt{6}}(udd + dud - 2ddu)$	$\frac{1}{\sqrt{2}}(udd - dud)$
Σ^0	$\frac{1}{\sqrt{12}}(2uds + 2dus - usd - dsu - sud - sdu)$	$\frac{1}{2}(usd + dsu - sud - sdu)$
Σ^+	$\frac{1}{\sqrt{6}}(2uus - usu - suu)$	$\frac{1}{\sqrt{2}}(usu - suu)$
Λ^0	$\frac{1}{2}(usd + sud - dsu - sdu)$	$\frac{1}{\sqrt{12}}(2uds - 2dus + sdu - dsu + usd - sud)$

is made up of three quarks, each one with a different color in an antisymmetric combination. Such a combination is a color singlet and is said to be colorless. The color wave function can be written down explicitly using the same method as constructing the flavor part. Then the baryon color wave function is

$$B_{color} = \frac{1}{\sqrt{6}}(\mathrm{RGB} - \mathrm{RBG} - \mathrm{GRB} + \mathrm{GBR} - \mathrm{BGR} + \mathrm{BRG}). \tag{3.17}$$

Likewise, the meson color wave function is

$$M_{color} = \frac{1}{\sqrt{3}}(\mathrm{R\bar{R}} + \mathrm{G\bar{G}} + \mathrm{B\bar{B}}). \tag{3.18}$$

The complete wave functions we consider in this thesis are the ones of the ground state octet of baryons. For an ordinary attractive interaction between quarks, the bound state of lowest energy has a configuration in which the quarks are all in relative S states ($L = 0$). It follows that the wave function of a baryon is symmetric under the interchange of the spatial coordinates of any two quarks. Then, with orbital angular momentum $L = 0$, the baryons of lowest mass should have total angular momentum J equal to the total spin (of either 1/2 or 3/2). Since the color part is assumed to be a singlet, the remaining part of the baryon wave function (flavor and spin) is therefore totally symmetric under any interchange of quarks. The symmetrized wave function in flavor and spin is expressed as

$$\psi_{flavor}\psi_{spin} = \frac{1}{\sqrt{2}}(\phi_{M_S}\chi_{M_S} + \phi_{M_A}\chi_{M_A}), \tag{3.19}$$

where ϕ and χ denotes the respective flavor and spin wave functions with the types of symmetry under quark exchange. For the proton, which is the baryon of lowest mass with $I_3 = 1/2$ and $J = 1/2$, its flavor and spin wave function in case of a spin-up proton ($J_z = +1/2$) is then given by

$$\phi_{M_S} = \frac{1}{\sqrt{6}}(2uud - udu - duu), \quad \phi_{M_A} = \frac{1}{\sqrt{2}}(udu - duu), \tag{3.20}$$

$$\chi_{M_S} = \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow), \quad \chi_{M_A} = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow). \tag{3.21}$$

3.2 Spin Polarization and Magnetic Moment of the Nucleon in the Quark Model

From the flavor and spin parts as discussed in the previous section, the explicit form in the quark spin flavor and spin structure of a spin-up proton is given by

$$|p\uparrow\rangle = \frac{1}{\sqrt{6}}(2u\uparrow u\uparrow d\downarrow - u\uparrow u\downarrow d\uparrow - u\downarrow u\uparrow d\uparrow). \tag{3.22}$$

From this explicit wave function we can count the number (or probabilities) of quark flavors with spin parallel (q_+) or antiparallel (q_-) to the proton spin:

$$u_+ = \frac{5}{3}, \quad u_- = \frac{1}{3}, \quad d_+ = \frac{1}{3}, \quad d_- = \frac{2}{3} \tag{3.23}$$

summing up to two u and one d quark. With the spin polarizations defined as:

$$\Delta q = q_+ - q_- \tag{3.24}$$

we also obtain the contribution by each of the quark flavors to the proton spin:

$$\Delta u = \frac{4}{3} \quad \Delta d = -\frac{1}{3} \quad \Delta s = 0, \quad \text{and} \quad \Delta\Sigma = 1, \tag{3.25}$$

where $\Delta\Sigma = \Delta u + \Delta d + \Delta s$ is the sum of quark polarizations. These spin polarizations can be used to calculate the nucleon magnetic moment, an observable successfully described in the quark model. With the constituent quarks the magnetic moment for a nucleon is defined as

$$\mu(N) = \Delta u^N \mu_u + \Delta d^N \mu_d + \Delta s^N \mu_s \tag{3.26}$$

Thus, the magnetic moments of the proton and neutron are given by

$$\mu(p) = \frac{4}{3}\mu_u - \frac{1}{3}\mu_d, \tag{3.27}$$

$$\mu(n) = \frac{4}{3}\mu_d - \frac{1}{3}\mu_u. \tag{3.28}$$

The magnetic moment of a point-like spin-$\frac{1}{2}$ particle of charge e is $e/2m$. Thus a quark assumed to be point particle of charge $Q_i e$ and mass m_i has the magnetic moment

$$\mu_i = Q_i \left(\frac{e}{2m_i}\right). \tag{3.29}$$

In the limit that $m_u = m_d$, then $\mu_u = -2\mu_d$, so the quark model prediction is $\mu_d/\mu_u = -0.677$ which is very well in agreement with the experimental result of $\mu_d/\mu_u = -0.68497945 \pm 0.00000058$ (Halzen and Martin, 1984).

3.3 The Proton Wave Function with $s\bar{s}$ Sea-Quark Components

For the 5-quark component, we first consider the idea that strange quarks are in the form of a $s\bar{s}$ sea-quark component in the proton state. This idea was proposed for describing the production of ϕ mesons in nucleon-antinucleon annihilation reactions (Ellis et al., 1995). Consequently, the production of a ϕ meson can be interpreted in terms of the shake-out or the rearrangement of an intrinsic $s\bar{s}$ component of the nucleon wave function without the violation of the OZI rule.

The corresponding 5-quark component for the proton can be written in Fock space as (Henley et al., 1992)

$$|uuds\bar{s}\rangle = a_0|(uud)_{1/2}(s\bar{s})_0\rangle_{1/2} + a_1|(uud)_{1/2}(s\bar{s})_1\rangle_{1/2}\,. \quad (3.30)$$

The subscripts denote the spin coupling of the quark clusters, a_0 and a_1 represent the amplitudes for the spin 0 and spin 1 components of the admixed $s\bar{s}$ pairs. There are a number of possible quantum numbers of the $s\bar{s}$ cluster in the nucleon which can be considered (Ellis et al., 2000). The assumption that the $s\bar{s}$ has the quantum numbers of a ϕ (the second term) may seem attractive a priori. One would expect in addition that ϕ production is due to a shake-out of this cluster in the transition process.

3.4 The Proton Wave Function from the Chiral Quark Model

The chiral quark model (ChQM) was essentially developed to refine the understanding in the successes of the CQM. The idea of the ChQM is based on the picture that a quark inside a nucleon emits a quark-antiquark pairs as a Goldstone boson (GB), for example

$$q_{\pm} \rightarrow GB + q'_{\pm} \rightarrow (q\bar{q}') + q'_{\pm}. \quad (3.31)$$

This process could be described in the effective Lagrangian valid below the chiral symmetry breaking scale Λ_{ChQM}, which involves quarks, gluons and Goldstone bosons. The first few terms in this Lagrangian are (Shu et al., 2007):

$$\mathcal{L}_{ChQM} = \bar{\psi}(iD_\mu + V_\mu)\gamma^\mu\psi + ig_A\bar{\psi}A_\mu\gamma^\mu\gamma^5\psi - m\bar{\psi}\psi + \frac{1}{4}f_\pi^2 \mathrm{Tr}\partial^\mu\Sigma^\dagger\partial_\mu\Sigma + ... \quad (3.32)$$

where $D_\mu = \partial_\mu + igG_\mu$ is the gauge-covariant derivative of QCD, G_μ the gluon field, and g the strong coupling constant. The dimensionless axial-vector coupling

$g_A = 0.7524$ is determined from the axial charge of the nucleon. The symbol m represents the constituent quark masses due to the spontaneous breaking of chiral symmetry breaking. The pseudoscalar decay constant is $f_\pi \approx 93$ MeV. The Σ field, vector V_μ and axial-vector currents A_μ are given in terms of the Goldstone boson fields Φ_M as

$$\Phi_M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & \frac{-2\eta}{\sqrt{6}} \end{pmatrix}, \tag{3.33}$$

$$\Sigma = \exp(i\frac{\sqrt{2}\Phi}{f_\pi}), \tag{3.34}$$

$$\begin{pmatrix} V_\mu \\ A_\mu \end{pmatrix} = \frac{1}{2}(\xi^\dagger \partial_\mu \xi \pm \xi \partial_\mu \xi^\dagger), \tag{3.35}$$

$$\xi = \exp(i\frac{\Phi_M}{\sqrt{2}f_\pi}). \tag{3.36}$$

An expansion of the currents in powers of Φ/f_π yields the effective interaction between GB and q

$$\mathcal{L}_I = -\frac{g_A}{\sqrt{2}f_\pi}\bar{\psi}\partial_\mu \Phi_M \gamma^\mu \gamma_5 \psi. \tag{3.37}$$

This allows the fluctuation of a quark into a recoil quark plus a Goldstone boson. The basic interaction causes a modification of the spin content because a quark can change its helicity by emitting a spin zero meson. It causes a modification of the flavor content because the GB fluctuation is flavor dependent. The spin flip process makes it possible to understand the spin content of the nucleon, which was not possible in the conventional constituent quark model. Nevertheless, in the absence of interactions, the proton is made up of two u quarks and one d quark. For example, let us consider the fluctuation of u quark with this interaction. Suppressing all the space-time structure and only displaying the flavor content, the basic GB-quark interaction vertices are given by (Cheng and Li, 1995)

$$\mathcal{L}_I = g_8 \left[\bar{d}\pi^- + \bar{s}K^- + \bar{u}(\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}}) \right] u + ... \,. \quad (3.38)$$

Thus after one emission process the u quark wave function has the components

$$\Psi(u) \sim \left[d\pi^+ + sK^+ + u(\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}}) \right]. \quad (3.39)$$

which can be entirely expressed in terms of the quark content by using $\pi^+ = u\bar{d}$, and $K^+ = u\bar{s}$, etc. Hence, the parameter $|g_8|^2$ denotes the probability for the transition $u \to d + \pi^+$. In the quark model, a hadron is built up from constituent quarks. In accordance with the ChQM, a constituent quark should be treated as a composite particle including such components q' and mesons. With the admixture of mesons to the nucleon wave function, one finds that only one third of the nucleon spin is carried by the quarks. Moreover, for the other spin-flavor observables, such as magnetic moments, sea quark distributions and the Gottfried sum rule, the agreement with experimental data is also improved using this model (Gupta and Kaur, 1983).

To obtain the proton wave function we consider the SU(3) invariant meson-baryon interaction Lagrangian (Stoks, 1998)

$$\mathcal{L}_I = -g_8\sqrt{2}\left(\alpha[\bar{\Phi}_B\Phi_B\Phi_M]_F + (1-\alpha)[\bar{\Phi}_B\Phi_B\Phi_M]_D\right) - g_1\frac{1}{\sqrt{3}}[\bar{\Phi}_B\Phi_B\Phi_M]_S \quad (3.40)$$

where g_8 and g_1 are coupling constants, and α is known as the $F/(F+D)$ ratio with $F \simeq 0.51$, $D \simeq 0.76$ (Thomas and Weise, 2001). The square parentheses denote the SU(3) invariant combinations:

$$[\bar{\Phi}_B\Phi_B\Phi_M]_F = \mathrm{Tr}(\bar{\Phi}_B\Phi_M\Phi_B) - \mathrm{Tr}(\bar{\Phi}_B\Phi_B\Phi_M) \quad (3.41)$$

$$[\bar{\Phi}_B\Phi_B\Phi_M]_D = \mathrm{Tr}(\bar{\Phi}_B\Phi_M\Phi_B) + \mathrm{Tr}(\bar{\Phi}_B\Phi_B\Phi_M) - \frac{2}{3}\mathrm{Tr}(\bar{\Phi}_B\Phi_B)\mathrm{Tr}(\Phi_M) \quad (3.42)$$

$$[\bar{\Phi}_B\Phi_B\Phi_M]_S = \mathrm{Tr}(\bar{\Phi}_B\Phi_B)\mathrm{Tr}(\Phi_M). \quad (3.43)$$

where Φ_B is the baryon matrix

$$\Phi_B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}. \quad (3.44)$$

Hence, the part of the interaction Lagrangian which allows the fluctuation of the proton into kaons and hyperon is

$$\mathcal{L}_I = -g_1\bar{p}\eta_1 p + g_8[\bar{p}\pi^0 + \frac{1-4\alpha}{\sqrt{3}}\bar{p}\eta_8 + \frac{1+2\alpha}{\sqrt{3}}\bar{\Lambda}K^- + (2\alpha-1)\Sigma^0 K^- \\ -\sqrt{2}\bar{n}\pi^- + \sqrt{2}(2\alpha-1)\Sigma^- K^0]p + ... \quad (3.45)$$

The final state resulting from pseudoscalar meson emission by the proton is

$$\Psi(p) \sim -g_1|p\eta_1\rangle + g_8[\frac{1-4\alpha}{\sqrt{3}}|p\eta_8\rangle + |p\pi^0\rangle + \frac{1+2\alpha}{\sqrt{3}}|\Lambda K^+\rangle \\ +(2\alpha-1)|\Sigma^0 K^+\rangle - \sqrt{2}|n\pi^+\rangle + \sqrt{2}(2\alpha-1)|\Sigma^+ K^0\rangle]. \quad (3.46)$$

In the absence of fluctuations the proton is made up of the conventional two u quarks and one d quark. Thus $\Psi(p)$ may be interpreted as the 5-quark component of the proton wave function which is given by

$$|uuds\bar{s}\rangle^{ChQM} = G_1|p\eta_1\rangle + G_2|p\eta_8\rangle + G_3|\Sigma^0 K^+\rangle + G_4|\Sigma^+ K^0\rangle + G_5|\Lambda^0 K^+\rangle, \quad (3.47)$$

where the G are the coefficients corresponding to each factor of $\Psi(p)$. Each component in above equation can be represented in terms of a quark cluster as

$$|p\eta_{1,8}\rangle = |(uud)_{1/2}(s\bar{s})_0\rangle_{1/2},\ |\Sigma^0 K^+\rangle = |(uds)_{1/2}(u\bar{s})_0\rangle_{1/2}, \\ |\Sigma^+ K^0\rangle = |(uus)_{1/2}(d\bar{s})_0\rangle_{1/2}, |\Lambda^0 K^+\rangle = |(usd)_{1/2}(u\bar{s})_0\rangle_{1/2}. \quad (3.48)$$

3.5 Proton Wave Function with Explicit Configurations of the *uuds* Subsystem

There is another form for the 5-quark component proposed by An, Zou and Riska (An et al., 2006). Instead of a meson coupling to a baryon cluster, they considered the 5-quark component in the configuration of a $q^4\bar{q}$ pentaquark. This idea was introduced for considering the possible values of the strangeness magnetic moment and spin of the proton. The flavor wave functions for the $uuds\bar{s}$ components are usually constructed by coupling $uuds$ flavor wave functions to the $\bar{s}$ flavor wave function. In the language of group theory, there are four possible flavor symmetry patterns for the $uuds$ system: $[4]_F, [31]_F, [22]_F$ and $[211]_F$ characterized by the S_4 Young tableaux. Combination of these with the antiquark with flavor symmetry $[1]_F^*$ leads to the following pentaquark multiplet representations of SU(3):

$$[4]_F \otimes [1]_F^* = 10 \oplus 35, \tag{3.49}$$

$$[31]_F \otimes [1]_F^* = 8 \oplus 10 \oplus 27, \tag{3.50}$$

$$[22]_F \otimes [1]_F^* = 8 \oplus \bar{10}, \tag{3.51}$$

$$[211]_F \otimes [1]_F^* = 1 \oplus 8. \tag{3.52}$$

However, the possible flavor symmetry patterns for the $uuds$ in the proton are limited to $[31]_F, [22]_F$ and $[211]_F$ because the proton belongs to the octet representation. The corresponding independent flavor wave functions for these flavor symmetry representations are as following:

$$\begin{aligned}\chi_{[31]_{F_\lambda}} = \frac{1}{\sqrt{18}}(2|uusd\rangle + 2|suud\rangle + 2|usud\rangle - |sudu\rangle \\ -|usdu\rangle - |dusu\rangle - |udsu\rangle \\ -|dsuu\rangle - |sduu\rangle),\end{aligned} \tag{3.53}$$

$$\begin{aligned}\chi_{[31]_{F_\rho}} = \frac{1}{12}(6|uuds\rangle - 3|duus\rangle - 3|udus\rangle - 4|dsuu\rangle \\ -4|sduu\rangle + 5|sudu\rangle + 5|usdu\rangle + 2|uusd\rangle \\ -|suud\rangle - |dusu\rangle - |usud\rangle - |udsu\rangle),\end{aligned} \tag{3.54}$$

$$\chi_{[31]_{F_\eta}} = \frac{1}{\sqrt{48}}(-3|duus\rangle + 3|udus\rangle - 3|dusu\rangle + 3|udsu\rangle$$
$$-2|dsuu\rangle + 2|sduu\rangle - |sudu\rangle + |usdu\rangle$$
$$-|suud\rangle + |usud\rangle), \qquad (3.55)$$

$$\chi_{[22]_{F_\lambda}} = \frac{1}{\sqrt{24}}(2|uuds\rangle + 2|uusd\rangle + 2|dsuu\rangle + 2|sduu\rangle$$
$$-|duus\rangle - |udus\rangle - |sudu\rangle - |usdu\rangle$$
$$-|suud\rangle - |dusu\rangle - |usud\rangle - |udsu\rangle), \qquad (3.56)$$

$$\chi_{[22]_{F_\rho}} = \frac{1}{\sqrt{8}}(|udus\rangle + |sudu\rangle + |dusu\rangle + |usud\rangle$$
$$-|duus\rangle - |usdu\rangle - |suud\rangle - |udsu\rangle), \qquad (3.57)$$

$$\chi_{[211]_{F_\lambda}} = \frac{1}{4}(2|uuds\rangle - 2|uusd\rangle - |duus\rangle - |udus\rangle$$
$$-|sudu\rangle - |usdu\rangle + |suud\rangle + |dusu\rangle$$
$$+|usud\rangle + |udsu\rangle), \qquad (3.58)$$

$$\chi_{[211]_{F_\rho}} = \frac{1}{\sqrt{48}}(3|udus\rangle - 3|duus\rangle + 3|suud\rangle - 3|usud\rangle$$
$$+2|dsuu\rangle - 2|sduu\rangle - |sudu\rangle + |usdu\rangle$$
$$+|dusu\rangle - |udsu\rangle), \qquad (3.59)$$

$$\chi_{[211]_{F_\eta}} = \frac{1}{\sqrt{6}}(|sudu\rangle + |udsu\rangle + |dsuu\rangle$$
$$-|usdu\rangle - |dusu\rangle - |sduu\rangle). \qquad (3.60)$$

The flavor wave functions for the proton with zero strangeness is given by

$$\chi_F(uuds\bar{s}) = \bar{s}\chi_{F_j}(uuds), \qquad (3.61)$$

where $j = \lambda, \rho$ and η.

For the color wave functions, according to group theory, the color part of the antiquark in pentaquark states is a [11] antitriplet thus the color symmetry of all the *uuds* configurations is limited to a [211] triplet in order to form a pentaquark color singlet. That is, the color part of the pentaquark wave function must be a [222] singlet. There are three independent color wave functions of the q^4 for the [211] triplet

$$\begin{aligned}\chi_{[211]_{C_\lambda}}(RGB) = \frac{1}{\sqrt{16}}(2|RRGB\rangle - 2|RRBG\rangle - |GRRB\rangle \\ -|RGRB\rangle - |BRGR\rangle - |RBGR\rangle \\ +|BRRG\rangle + |GRBR\rangle + |RBRG\rangle \\ +|RGBR\rangle),\end{aligned} \tag{3.62}$$

$$\begin{aligned}\chi_{[211]_{C_\rho}}(RGB) = \frac{1}{\sqrt{48}}(3|RGRB\rangle - 3|GRRB\rangle + 3|BRRG\rangle \\ -3|RBRG\rangle + 2|GBRR\rangle - 2|BGRR\rangle \\ -|BRGR\rangle + |RBGR\rangle + |GRBR\rangle \\ -|RGBR\rangle),\end{aligned} \tag{3.63}$$

$$\begin{aligned}\chi_{[211]_{C_\eta}}(RGB) = \frac{1}{\sqrt{6}}(|BRGR\rangle + |RGBR\rangle + |GBRR\rangle \\ -|RBGR\rangle - |GRBR\rangle - |BGRR\rangle).\end{aligned} \tag{3.64}$$

Thus, the corresponding color singlet wave function χ_C of the pentaquark at color symmetry pattern $j = \lambda, \rho, \eta$ is given by

$$\chi_{[222]_{C_j}} = \frac{1}{\sqrt{3}}(\bar{R}\chi_{[211]_{C_j}}(RGB) + \bar{G}\chi_{[211]_{C_j}}(GBR) + \bar{B}\chi_{[211]_{C_j}}(BRG)). \tag{3.65}$$

Since the pentaquark state should be antisymmetric under any permutation of the four quark configuration with the spatial wave function being symmetric, hence the spin-flavor part of *uuds* must be a [31] state in order to form the antisymmetric wave function of the pentaquark. The corresponding q^4 wave function may take the form

$$\chi(uuds) = \frac{1}{\sqrt{3}}(\chi_{C_\lambda}\chi_{SF_\rho} - \chi_{C_\rho}\chi_{SF_\lambda} + \chi_{C_\eta}\chi_{SF_\eta}), \tag{3.66}$$

here χ_{SF_j} denote a spin-flavor triplet of the four quark configuration. For example, the flavor symmetry representations [31] and [211] combine with the spin symmetry states $[22]_S$ to form the mixed symmetry state $[31]_{FS}$ given by

$$|[31]_{FS_\eta}[31]_F[22]_S\rangle = \frac{1}{\sqrt{2}}(\chi_{[31]_{F_\lambda}}\chi_{[22]_{S_\lambda}} + \chi_{[31]_{F_\rho}}\chi_{[22]_{S_\rho}}), \tag{3.67}$$

$$|[31]_{FS_\lambda}[31]_F[22]_S\rangle = \frac{1}{2}(\sqrt{2}\chi_{[31]_{F_\lambda}}\chi_{[22]_{S_\lambda}} + \chi_{[31]_{F_\rho}}\chi_{[22]_{S_\lambda}} - \chi_{[31]_{F_\eta}}\chi_{[22]_{S_\rho}}), \tag{3.68}$$

$$|[31]_{FS_\rho}[31]_F[22]_S\rangle = \frac{1}{2}(\sqrt{2}\chi_{[31]_{F_\lambda}}\chi_{[22]_{S_\rho}} - \chi_{[31]_{F_\rho}}\chi_{[22]_{S_\rho}} - \chi_{[31]_{F_\eta}}\chi_{[22]_{S_\lambda}}), \tag{3.69}$$

$$|[31]_{FS_\eta}[211]_F[22]_S\rangle = \frac{1}{\sqrt{2}}(\chi_{[211]_{F_\lambda}}\chi_{[22]_{S_\lambda}} + \chi_{[211]_{F_\rho}}\chi_{[22]_{S_\rho}}), \tag{3.70}$$

$$|[31]_{FS_\lambda}[211]_F[22]_S\rangle = \frac{1}{2}(-\sqrt{2}\chi_{[211]_{F_\eta}}\chi_{[22]_{S_\rho}} + \chi_{[211]_{F_\rho}}\chi_{[22]_{S_\rho}} - \chi_{[211]_{F_\lambda}}\chi_{[22]_{S_\lambda}}), \tag{3.71}$$

$$|[31]_{FS_\rho}[211]_F[22]_S\rangle = \frac{1}{2}(\chi_{[211]_{F_\lambda}}\chi_{[22]_{S_\rho}} + \chi_{[211]_{F_\rho}}\chi_{[22]_{S_\lambda}} + \sqrt{2}\chi_{[211]_{F_\eta}}\chi_{[22]_{S_\lambda}}). \tag{3.72}$$

The spin wave functions with the [22] symmetry are given by

$$\begin{aligned}\chi_{[22]_{S_\lambda}} = & \frac{1}{\sqrt{3}}|\uparrow\uparrow\downarrow\downarrow\rangle - \frac{1}{2\sqrt{3}}|\uparrow\downarrow\uparrow\downarrow\rangle - \frac{1}{2\sqrt{3}}|\downarrow\uparrow\uparrow\downarrow\rangle \\ & -\frac{1}{2\sqrt{3}}|\uparrow\downarrow\downarrow\uparrow\rangle - \frac{1}{2\sqrt{3}}|\downarrow\uparrow\downarrow\uparrow\rangle + \frac{1}{\sqrt{3}}|\downarrow\downarrow\uparrow\uparrow\rangle,\end{aligned} \tag{3.73}$$

$$\chi_{[22]_{S_\rho}} = \frac{1}{2}\left(|\uparrow\downarrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle\right). \tag{3.74}$$

Finally, to obtain the pentquark wave function we have to couple the spin state of the anti-strange quark to the *uuds* spin. Hence, in case of the [22] spin symmetry corresponding to spin zero, we have

$$\chi(uuds\bar{s}) = \frac{1}{\sqrt{3}}\big(\chi_{222_{C_\lambda}}\chi_{22_{SF_\rho}} - \chi_{222_{C_\rho}}\chi_{22_{SF_\lambda}} + \chi_{222_{C_\eta}}\chi_{22_{SF_\eta}}\big)\bar{s}|\uparrow(\downarrow)\rangle. \tag{3.75}$$

The spin projection state of the anti-strange quark is denoted by $|\uparrow(\downarrow)\rangle$. The construction of the spin, color and flavor wave functions of the four quark configuration of the pentaquark states are shown in Appendix D. The detailed description of the spin-flavor of the proton wave function with the 5-quark component for these three models, and the corresponding strangeness magnetic moment and spin of the proton are given in the next section.

3.6 Strangeness Spin and Magnetic Moment of the Proton

In the nonrelativistic quark model the strangeness magnetic moment $\vec{\mu}_s$ and the strangeness contribution to the proton spin $\vec{\sigma}_s$ operator are defined as

$$\vec{\mu}_s = \frac{e}{2m_s}\sum_i \widehat{S}_i(\widehat{\ell}_s + \widehat{\sigma}_s) \tag{3.76}$$

$$\vec{\sigma}_s = \widehat{\sigma}_s + \widehat{\sigma}_{\bar{s}} \tag{3.77}$$

where $\widehat{\ell}_s$ and $\widehat{\sigma}_s$ are the angular momentum and spin projection operators, respectively. The operator $\widehat{S}_i$ is the strangeness counting operator with eigenvalue $+1$ for s and -1 for an $\bar{s}$ quark and m_s is the constituent mass of the strange quark.

For the first proton wave function, in which the $s\bar{s}$ sea-quark components are in the S and P-states, the spin-flavor wave function can be constructed by coupling the $|s\bar{s}\rangle_{j_s=0,1}$ wave function with the $|uud\rangle_{1/2}$ wave function. However, since the admixed meson carries negative intrinsic parity an orbital p-wave ($\ell = 1$) has to be introduced into the proton quark cluster wave function. The spin-flavor wave function for the proton state with spin $+1/2$ can then be written in the general form:

$$|p;\frac{1}{2},m_p=\frac{1}{2}\rangle = A|(uud)\rangle_{\frac{1}{2},m_p=\frac{1}{2}} + B\sum_{j_s,j_i=0,1}\alpha_{j_sj_i}|[(s\bar{s})_{j_s}\otimes\ell=1]_{j_i}\otimes(uud)_{\frac{1}{2}}\rangle_{\frac{1}{2},m_{ps\bar{s}}=\frac{1}{2}} \tag{3.78}$$

with the normalization $\sum_{j_s,j_i=0,1} |\alpha_{j_sj_i}|^2 = 1$. To evaluate the strangeness spin contribution we decouple the 5-quark component to be

$$|uuds\bar{s}\rangle^{s\bar{s}} = \sum_{j_s,j_i}\sum_{m_i,m_{uud}}\sum_{j_{s_z},\mu}\alpha_{j_sj_i}\langle j_i, j_s, m_i, m_{uud}|\frac{1}{2},\frac{1}{2}\rangle \langle j_s, 1, j_{s_z}, \mu|j_i, m_i\rangle|s\bar{s}\rangle_{j,j_{s_z}}|1,\mu\rangle|uud\rangle_{\frac{1}{2},m_{uud}} \quad (3.79)$$

where $\langle\ |\ \rangle$ are Clebsch-Gordon Coefficients. The spin-flavor decoupled states in above equation are given by

$$|s\bar{s}\rangle_{j,j_{s_z}} = s\bar{s}|(\frac{1}{2}\otimes\frac{1}{2})_{j,j_{s_z}}\rangle = \begin{cases} s\uparrow\bar{s}\uparrow, & j=1, j_{s_z}=1\ ; \\ s\downarrow\bar{s}\downarrow, & j=1, j_{s_z}=-1\ ; \\ \frac{1}{\sqrt{2}}(s\uparrow\bar{s}\downarrow + s\downarrow\bar{s}\uparrow), & j=1, j_{s_z}=0\ ; \\ \frac{1}{\sqrt{2}}(s\uparrow\bar{s}\downarrow - s\downarrow\bar{s}\uparrow), & j=0, j_{s_z}=0, \end{cases} \quad (3.80)$$

$$|uud\rangle_{\frac{1}{2},m_{uud}=\pm\frac{1}{2}} = \frac{1}{\sqrt{2}}\left(\phi_{M_S}|\frac{1}{2},\pm\frac{1}{2}\rangle_+ + \phi_{M_A}|\frac{1}{2},\pm\frac{1}{2}\rangle_-\right), \quad (3.81)$$

the symmetry flavor $\phi_{M_{S,A}}$ and spin states $|\frac{1}{2},\pm\frac{1}{2}\rangle_\pm$ are given by Eq. (3.20) and Eq. (3.21) respectively, $|1,\mu\rangle$ denotes the relative angular momentum between the quark clusters.

With the above three equations the probabilities of finding a strange quark pair ($s\bar{s}$) with spin parallel (↑) and opposite (↓) to the proton spin are given by

$$P_{s\uparrow\bar{s}\uparrow} = |\langle s\uparrow\bar{s}\uparrow|p;\frac{1}{2},m_p=\frac{1}{2}\rangle|^2 = \left(\frac{\alpha_{1,0}^2}{3} - \frac{1}{3}\sqrt{2}\alpha_{1,1}\alpha_{1,0} + \frac{\alpha_{1,1}^2}{2}\right)B^2, \quad (3.82)$$

$$P_{s\downarrow\bar{s}\downarrow} = |\langle s\downarrow\bar{s}\downarrow|p;\frac{1}{2},m_p=\frac{1}{2}\rangle|^2 = \left(\frac{\alpha_{1,0}^2}{3} + \frac{1}{3}\sqrt{2}\alpha_{1,1}\alpha_{1,0} + \frac{\alpha_{1,1}^2}{6}\right)B^2, \quad (3.83)$$

$$P_{s\uparrow\bar{s}\downarrow} = |\langle s\uparrow\bar{s}\downarrow|p;\frac{1}{2},m_p=\frac{1}{2}\rangle|^2 = \left(\frac{\alpha_{0,1}^2}{2} + \frac{1}{3}\alpha_{1,0}\alpha_{0,1} - \frac{1}{3}\sqrt{2}\alpha_{1,1}\alpha_{0,1} + \frac{\alpha_{1,0}^2}{6} + \frac{\alpha_{1,1}^2}{6}\right)B^2, \quad (3.84)$$

$$P_{s\downarrow\bar{s}\uparrow} = |\langle s\downarrow \bar{s}\uparrow |p; \frac{1}{2}, m_p = \frac{1}{2}\rangle|^2$$
$$= \left(\frac{\alpha_{0,1}^2}{2} - \frac{1}{3}\alpha_{1,0}\alpha_{0,1} + \frac{1}{3}\sqrt{2}\alpha_{1,1}\alpha_{0,1} + \frac{\alpha_{1,0}^2}{6} + \frac{\alpha_{1,1}^2}{6}\right) B^2. \tag{3.85}$$

We then can calculate the strangeness contribution to the proton spin:

$$\sigma_s = \Delta_s + \Delta_{\bar{s}}$$
$$= \sum(P_{s\uparrow} - P_{s\downarrow}) + \sum(P_{\bar{s}\uparrow} - P_{\bar{s}\downarrow})$$
$$= 2P_{s\uparrow\bar{s}\uparrow} - 2P_{s\downarrow\bar{s}\downarrow}$$
$$= -1.22|\alpha|^2|B|^2. \tag{3.86}$$

Here, we have fixed the parameters as $\alpha_{1,0} = \alpha_{1,1} \equiv \alpha$. Using the operator from Eq. (3.76) we can drive the strangeness magnetic moment

$$\mu_s = B^2\langle uuds\bar{s}|\hat{\mu}_s|uuds\bar{s}\rangle$$
$$= \frac{eB^2}{2m_s}(\langle\ell_s\rangle - \langle\ell_{\bar{s}}\rangle + \langle\sigma_s\rangle - \langle\sigma_{\bar{s}}\rangle)$$
$$= \frac{eB^2}{2m_s}(2P_{s\uparrow\bar{s}\downarrow} - 2P_{s\uparrow\bar{s}\downarrow})$$
$$= -0.55\alpha\alpha_{0,1}\frac{eB^2}{2m_s}, \tag{3.87}$$

where $\langle\ell_s\rangle - \langle\ell_{\bar{s}}\rangle = 0$ because both of the strange and antistrange quarks are in same quark cluster.

Similarly, for the proton wave function from the chiral quark model, since the $\bar{s}$ quark in η and kaons carries negative parity it is necessary that η and kaon be in relative p-wave about the uud and the hyperon cluster. The spin-flavor wave function of the 5-quark component with spin $+1/2$ can be expressed as

$$|uuds\bar{s}\rangle^{\chi}_{\frac{1}{2},m_\chi=\frac{1}{2}} = \sum_{i=1,5} G_i|[(q\bar{q})^i_{j_s=0} \otimes \ell = 1]_{j_i,m_i} \otimes (qqq)^i_{\frac{1}{2},m}\rangle_{\frac{1}{2},m_\chi=\frac{1}{2}}, \tag{3.88}$$

here $j_s = 0$ because all of the $(q\bar{q})^i$ quark clusters are pseudoscalar mesons. The 5-quark component can be decoupled to be

$$|uuds\bar{s}\rangle^{\chi}_{\frac{1}{2},m_\chi=\frac{1}{2}} = \sum_{i=1,5}\sum_{\mu,m} G_i\langle 1, \frac{1}{2}, \mu, m|\frac{1}{2}, \frac{1}{2}\rangle|q\bar{q}\rangle^i_{0,0}|1,\mu\rangle|qqq\rangle^i_{\frac{1}{2},m} \tag{3.89}$$

where

$$G_i|q\bar{q}\rangle^i_{0,0} = |0,0\rangle^{spin} G_i|q\bar{q}\rangle^i|qqq\rangle^i$$
$$= \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)\begin{cases} -g_1|\eta_1\rangle|p(uud)\rangle_{\frac{1}{2},m}, & i=1; \\ -0.266g_8|\eta_8\rangle|p(uud)\rangle_{\frac{1}{2},m}, & i=2; \\ g_8|K^+\rangle|\Lambda^0(usd)\rangle_{\frac{1}{2},m}, & i=3; \\ -0.27g_8|K^+\rangle|\Sigma^0(uds)\rangle_{\frac{1}{2},m}, & i=4; \\ -0.381g_8|K^0\rangle|\Sigma^+(uud)\rangle_{\frac{1}{2},m}, & i=5. \end{cases} \tag{3.90}$$

The baryon clusters are

$$|qqq\rangle^i_{\frac{1}{2},m_\chi=\pm\frac{1}{2}} = \frac{1}{\sqrt{2}}\left(\phi^i_{M_S}|\frac{1}{2},\pm\frac{1}{2}\rangle_+ + \phi^i_{M_A}|\frac{1}{2},\pm\frac{1}{2}\rangle_-\right), \tag{3.91}$$

where the baryon octet wave functions are given by the explicit form as listed in Table 2.3. The probability of finding strange quarks ($s\bar{s}$) with spin parallel ($\uparrow$) and opposite ($\downarrow$) to the proton spin are as follows

$$P_{s\uparrow\bar{s}\uparrow} = |\langle s\uparrow\bar{s}\uparrow|p;\frac{1}{2},m_p=\frac{1}{2}\rangle|^2 = 0.277g_8^2B^2, \tag{3.92}$$

$$P_{s\downarrow\bar{s}\downarrow} = |\langle s\downarrow\bar{s}\downarrow|p;\frac{1}{2},m_p=\frac{1}{2}\rangle|^2 = 0.382g_8^2B^2, \tag{3.93}$$

$$\begin{aligned} P_{s\uparrow\bar{s}\downarrow} &= |\langle s\uparrow\bar{s}\downarrow|p;\frac{1}{2},m_p=\frac{1}{2}\rangle|^2 \\ &= \frac{1}{6}B^2g_1^2 + 0.251B^2g_8^2 - 0.125B^2g_1g_8, \end{aligned} \tag{3.94}$$

$$\begin{aligned} P_{s\downarrow\bar{s}\uparrow} &= |\langle s\downarrow\bar{s}\uparrow|p;\frac{1}{2},m_p=\frac{1}{2}\rangle|^2 \\ &= \frac{1}{6}B^2g_1^2 + 0.405B^2g_8^2 - 0.125B^2g_1g_8. \end{aligned} \tag{3.95}$$

The strangeness contribution to the proton spin for this case is found to be

$$\sigma_s = \Delta_s + \Delta_{\bar{s}} = -0.31g_8^2|B|^2. \tag{3.96}$$

By using $\hat{S}_i\hat{\ell}_s\bar{s}|1,\mu\rangle = -\mu\bar{s}|1,\mu\rangle$ one obtains

$$\langle\hat{S}_i\ell_{\bar{s}}\rangle = -0.80g_8^2B^2. \tag{3.97}$$

The expectation value for μ_s for this proton wave function is then

$$\mu_s = -1.1\frac{eg_8^2|B|^2}{2m_s}\,. \tag{3.98}$$

The negative value of the strangeness spin contribution of these two proton wave functions is consistent with deep inelastic scattering with the polarization of the strange quark helicity opposite to the proton spin.

For the configuration with a *uuds* subsystem the 5-quark states are constructed by coupling the *uuds* flavor wave function with the $\bar{s}$ flavor wave function. According to the requirement of positive parity for the proton wave function, if the *uuds* system is in ground state then $\bar{s}$ has to be in the P state. The spin-flavor wave function of the 5-quark component of the proton state with spin +1/2 for this configuration is given by

$$|uuds\bar{s}\rangle_{\frac{1}{2},m_{uuds}=\frac{1}{2}} = |[(\bar{s})_{\frac{1}{2},m_{\bar{s}}}\otimes(\ell=1)]\otimes(uuds)_s\rangle_{\frac{1}{2},m_{uuds}=\frac{1}{2}} \tag{3.99}$$

In configurations where the *uuds* quarks are in their ground state with spin $S=0$, the spin-flavor wave functions may be written in the general form

$$\begin{aligned}|uuds\bar{s}\rangle_{\frac{1}{2},m_{uuds}} = &\sum_{m_{\bar{s}}}A_{m_{\bar{s}}}|\frac{1}{2},m_{\bar{s}}\rangle|1,\mu\rangle\bar{s}\\ &\frac{1}{\sqrt{3}}\big(\chi_{222_{C_\lambda}}\chi_{22_{SF_\rho}} - \chi_{222_{C_\rho}}\chi_{22_{SF_\lambda}} + \chi_{222_{C_\eta}}\chi_{22_{SF_\eta}}\big)\end{aligned} \tag{3.100}$$

where

$$A_{m_{\bar{s}}} = \sum_\mu\langle\frac{1}{2},1,m_{\bar{s}},\mu|\frac{1}{2},\frac{1}{2}\rangle. \tag{3.101}$$

For this configuration, the spin symmetry of the *uuds* cluster are described by $[22]_S$ corresponding to spin 0. Their spin-flavor part can be coupled to be a mixed symmetric state by using the mixed symmetry $[31]_{FS}$. Namely, $[31]_{FS}[211]_F[22]_S$ and $[31]_{FS}[31]_F[22]_S$ but it gives no contribution because the total spin of the *uuds* cluster is zero. Thus, only the $\bar{s}$ quark contributes to μ_s and σ_s, that is (An et al., 2006)

$$\sigma_s = B^2 \left(\frac{1}{3} - \frac{2}{3}\right) = -\frac{1}{3}B^2, \tag{3.102}$$

and

$$\mu_s = -\frac{1}{3}\left(\frac{eB^2}{2m_s}\right). \tag{3.103}$$

There are other configurations that were considered for the corresponding strangeness spin and magnetic moment contribution to the proton. For example, $[31]_{FS}[211]_F[31]_S$ and $[31]_{FS}[22]_F[31]_S$ also give a μ_s which is negative, but σ_s is positive in contradiction with experiment. Therefore, in the consideration of the branching ratios from the proton wave function with a *uuds* subsystem, we consider only the configurations $[31]_{FS}[211]_F[22]_S$ and $[31]_{FS}[31]_F[22]_S$ that also yield negative values both for the strangeness spin and magnetic moment as the other two models.

CHAPTER IV

PRODUCTION OF THE ϕ MESON IN PROTON-ANTIPROTON ANNIHILATION

In this chapter we apply three types of proton wave functions, as given in Chapter III, to study the proton-antiproton annihilation reactions $p\bar{p} \rightarrow \phi\omega$, $\phi\pi^0$, $\phi\rho^0$ and $\phi\eta$. The strong interaction mechanism of $p\bar{p}$ annihilation into two or more mesons is still not understood yet in detail. Since this process occurs in the nonperturbative regime of QCD, phenomenological models have to be applied. The $^3P_0$ quark model is one of the most attractive among a number of quark models dealing with the strong interaction, because of its simplicity and successes in studying baryon and meson decays. In this chapter we begin with the formulation of the $^3P_0$ model. Then we describe in the $^3P_0$ model the reactions $p\bar{p} \rightarrow \phi X$ with the strange components included explicitly in the proton wave functions as discussed in the previous section. In the second subsection we work out the relevant transition amplitudes. The model predictions for branching ratios in comparison with experimental data are given in the last subsection.

4.1 The $^3P_0$ Model

In the $^3P_0$ model three possible quark line diagrams as shown in Figure. 4.1. contribute to the process of $p\bar{p}$ annihilation into ϕX with the 5-quark component in the proton wave functions. Two $q\bar{q}$ quark pairs are destroyed with vacuum quantum numbers $J^{PC}(I^G) = 0^{++}(0^+)$ or in the $^3P_0$ state. The $q\bar{q}$ $^3P_0$ interaction vertices are defined according to (Dover et al., 1992)

$$V^{ij} = \sum_{\mu} \sigma^{ij}_{-\mu} Y_{1\mu}(\vec{q}_i - \vec{q}_j)\delta^{(3)}(\vec{q}_i + \vec{q}_j)(-1)^{1+\mu} 1^{ij}_F 1^{ij}_C \ , \tag{4.1}$$

where $Y_{1\mu}(\vec{q}) = |\vec{q}|\mathcal{Y}_{1\mu}(\widehat{q})$ represents the spherical harmonic in momentum space and $1^{ij}_{F(C)}$ is a unit matrix in flavor (color) space. The $\sigma^{ij}_{-\mu}$ arises from the $^3P_0$ vertex indicating the coupling of the annihilated quark-antiquark pairs to spin 1. The transition amplitude for the reaction $p\bar{p}$ can be written in general as

$$T = \langle \phi X | \mathcal{O} | p\bar{p} \rangle \tag{4.2}$$

where $\mathcal{O}$ is the transition operator that involves the $^3P_0$ vertex. With the proton wave function that consists of the 3- and 5-quark components, the transition amplitude corresponding to the ϕ production is then given by

$$\begin{aligned} T &= AB\langle \phi X | \mathcal{O} | uuds\bar{s} \otimes \bar{u}\bar{u}\bar{d} \rangle \\ &\quad + AB\langle \phi X | \mathcal{O} | uud \otimes \bar{u}\bar{u}\bar{d}\bar{s}s \rangle \end{aligned} \tag{4.3}$$

Since the 5-quark component can be treated as a small perturbative admixture in the proton ($B^2 << A^2$), the amplitude corresponding to $B^2\langle \phi X | \mathcal{O} | (uuds\bar{s}) \otimes (\bar{u}\bar{u}\bar{d}\bar{s}s) \rangle$ can be ignored. In above equation the two terms are symmetric to each other, therefore we need to calculate only the first term and multiply the result by the factor 2. In momentum space representation the transition amplitude for a quark diagram topology A_I is given by

$$\begin{aligned} T_{A_I} = \int d^3q_1..d^3q_8 d^3q_{1'}..d^3q_{4'} \langle \phi X | \vec{q}_{1'}..\vec{q}_{4'} \rangle \\ \langle \vec{q}_{1'}..\vec{q}_{4'} | \mathcal{O}_{A_I} | \vec{q}_1..\vec{q}_8 \rangle \langle \vec{q}_1..\vec{q}_8 | (uuds\bar{s}) \otimes (\bar{u}\bar{u}\bar{d}) \rangle \end{aligned} \tag{4.4}$$

where the operators $\mathcal{O}_{A_I}$ for each diagram are given by

$$\mathcal{O}_{A_1} = \lambda_{A_1} \delta^{(3)}(\vec{q}_1 - \vec{q}_{1'})\delta^{(3)}(\vec{q}_2 - \vec{q}_{2'})\delta^{(3)}(\vec{q}_3 - \vec{q}_{3'})\delta^{(3)}(\vec{q}_8 - \vec{q}_{4'}) V^{56} V^{47} \ , \tag{4.5}$$

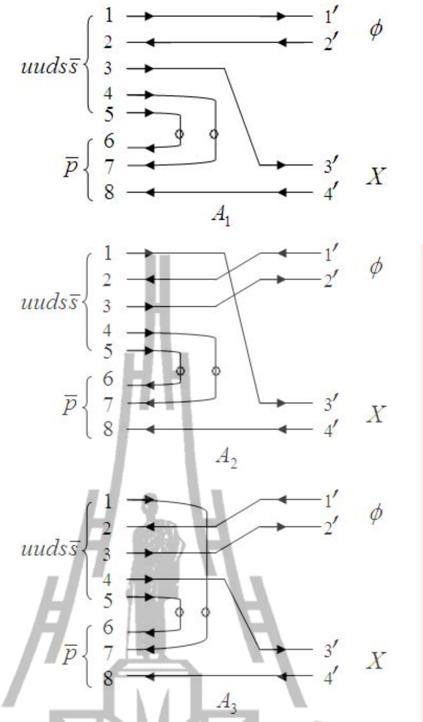

Figure 4.1 Quark line diagrams corresponding to the production of two meson final states in $p\bar{p}$ annihilation. The dots refer to the effective operator $\mathcal{O}_{A_I}$ for $q\bar{q}$ annihilation. The first diagram correspond to the effective quark line diagram for the shake-out of the intrinsic $s\bar{s}$ component of the proton wave function (Ellis et al., 1995; Gutsche et al., 1997).

$$\mathcal{O}_{A_2} = \lambda_{A_2}\delta^{(3)}(\vec{q}_2 - \vec{q}_{1'})\delta^{(3)}(\vec{q}_3 - \vec{q}_{2'})\delta^{(3)}(\vec{q}_1 - \vec{q}_{3'})\delta^{(3)}(\vec{q}_8 - \vec{q}_{4'})V^{56}V^{47} \ , \qquad (4.6)$$

$$\mathcal{O}_{A_3} = \lambda_{A_3}\delta^{(3)}(\vec{q}_2 - \vec{q}_{1'})\delta^{(3)}(\vec{q}_3 - \vec{q}_{2'})\delta^{(3)}(\vec{q}_4 - \vec{q}_{3'})\delta^{(3)}(\vec{q}_8 - \vec{q}_{4'})V^{56}V^{17} \ . \qquad (4.7)$$

The δ-functions represent the noninteracting and continuous quark-antiquark lines in the diagram. The constant λ_I is a free dimensionless pair annihilation parameter that describes the effective strength of the transition topology and is fitted to the experimental data. Here the factor $2AB$ can be included into the pair annihilation parameter λ_I, nevertheless, this factor will be fixed by experimental data in evaluating the branching ratios.

4.2 The Transition Amplitude of $p\bar{p} \to \phi X$ Reaction

According to Eq. (4.4) the full wave functions of the involved hadrons, consisting of spatial, flavor, spin and color parts, have to be used for calculating the transition amplitude. In this work the internal spatial wave functions are taken in the harmonic oscillator approximation. For the mesons $M(\phi$ and $X)$ the wave function can be expressed in terms of the quark momenta as

$$\langle M|\vec{q}_{i'}\vec{q}_{j'}\rangle \equiv \varphi_M(\vec{q}_{i'},\vec{q}_{j'})\chi_M = N_M \text{exp}\left\{-\frac{R_M^2}{8}(\vec{q}_{i'}-\vec{q}_{j'})^2\right\}\chi_M, \qquad (4.8)$$

with $N_M = (R_M^2/\pi)^{3/4}$ and R_M is the meson radial parameter. The spin-color-flavor wave function is denoted by χ. The baryon wave functions are given by

$$\langle B|\vec{q}_i\vec{q}_j\vec{q}_k\rangle \equiv \varphi_B\chi_B = N_B \text{exp}\left\{-\frac{R_B^2}{2}[(\frac{\vec{q}_j-\vec{q}_k}{\sqrt{2}})^2+(\frac{\vec{q}_j+\vec{q}_k-2\vec{q}_i}{\sqrt{6}})^2]\right\}\chi_B, \qquad (4.9)$$

where $N_B = (3R_B^2/\pi)^{3/2}$ and R_B is the baryon radial parameter. In case of the $s\bar{s}$ sea model and the ChQM, where a meson is coupled with a baryon wave function, the full 5-quark component wave function is given by

$$\begin{aligned}
\langle\vec{q}_1...\vec{q}_5|uuds\bar{s}\rangle &= \varphi_{uuds\bar{s}}(\vec{q}_1,..,\vec{q}_5)\chi_{uuds\bar{s}} \\
&= N_{uuds\bar{s}}\text{exp}\left\{-\frac{R_B^2}{2}[(\frac{\vec{q}_4-\vec{q}_5}{\sqrt{2}})^2+(\frac{\vec{q}_4+\vec{q}_5-2\vec{q}_3}{\sqrt{6}})^2]\right\} \\
&\quad \text{exp}\left\{-\frac{R^2}{8}(\vec{q}_3+\vec{q}_4+\vec{q}_5-\vec{q}_1-\vec{q}_2)^2\right\} \\
&\quad Y_{1\mu}(\vec{q}_3+\vec{q}_4+\vec{q}_5-\vec{q}_1-\vec{q}_2) \\
&\quad \text{exp}\left\{-\frac{R_M^2}{8}(\vec{q}_1-\vec{q}_2)^2\right\} \\
&\quad (\chi_B\otimes\chi_M). \qquad (4.10)
\end{aligned}$$

The radial parameter R and the spherical harmonic $Y_{1\mu}$ represented the internal relative P-wave between the 3-quark and the 2-quark cluster, respectively. For the pentaquark, a symmetric function of the coordinates of the $uuds\bar{s}$, where the strange antiquark has orbital angular momentum $(1, \mu)$, may be written as

$$\varphi_{uuds\bar{s}}(\vec{q}_1, .., \vec{q}_5) = N_{uuds\bar{s}} \exp\{-\frac{R_B^2}{2}[\frac{1}{2}(\vec{q}_2 - \vec{q}_3)^2 + \frac{1}{6}(\vec{q}_2 + \vec{q}_3 - 2\vec{q}_4)^2 + \frac{1}{12}(\vec{q}_2 + \vec{q}_3 + \vec{q}_4 - 3\vec{q}_5)^2]\} Y_{1\mu}(\frac{\vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5 - 4\vec{q}_1}{\sqrt{20}}) \exp\left\{-\frac{R_B^2}{2}(\frac{\vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5 - 4\vec{q}_1}{\sqrt{20}})^2\right\}, \quad (4.11)$$

where the first exponential term in above equation corresponds to the spatial part of the the 4-quark cluster. By choosing a plane wave basis for the relative motion of the proton and antiproton, the initial state wave functions in the center of momentum system ($\vec{k} = \vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5$) are obtained as:

$$\langle \vec{q}_1 ... \vec{q}_8 | (uuds\bar{s}) \otimes (\bar{u}\bar{u}\bar{d}) \rangle = \varphi_{uuds\bar{s},\bar{p}} [\chi_{uuds\bar{s}} \otimes \chi_{\bar{p}}]_{S,S_z} \quad (4.12)$$

with

$$\varphi_{uuds\bar{s},\bar{p}} = \varphi_{uuds\bar{s}} \varphi_{\bar{p}} \delta^{(3)}(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5 - \vec{k}) \delta^{(3)}(\vec{q}_6 + \vec{q}_7 + \vec{q}_8 + \vec{k}) \ . \quad (4.13)$$

The spins of the p and $\bar{p}$ are coupled to total spin S with projections S_z. Similarly, the final state ϕX wave functions in the center of momentum system are given by ($\vec{q} = \vec{q}_{1'} + \vec{q}_{2'}$):

$$\langle \phi X | \vec{q}_{1'} ... \vec{q}_{4'} \rangle = \varphi_{\phi, X} [\chi_\phi \otimes \chi_X]_{j_i, m_\epsilon} \quad (4.14)$$

with

$$\varphi_{\phi,X} = \varphi_\phi \varphi_X \delta^{(3)}(\vec{q} - \vec{q}_{1'} - \vec{q}_{2'}) \delta^{(3)}(\vec{q} + \vec{q}_{3'} + \vec{q}_{4'}) \ . \quad (4.15)$$

The spins of the two meson states are coupled to j_i with projection m_ϵ. In a low-momentum approximation the transition amplitude T_{fi} for the S to P transition from the initial state i to final state f with a quark line diagram A_I is evaluated as

$$T_{fi}(\vec{q}, \vec{k}) = \lambda_{A_I} F_{L=0,\ell_f=1} q \exp\left\{-Q_q^2 q^2 - Q_k^2 k^2\right\} \langle f | O_{A_I} | i \rangle \quad (4.16)$$

The index i represents the initial state ${}^{2I+1,2S+1}L_J$ (L is the orbital angular momentum, S is the spin, J is the total angular momentum and I is the isospin). The final state f is represented by the set of quantum numbers $f = \{\ell_f j J'\}$ (ℓ_f is the relative orbital angular momentum). The coefficients $F_{0,1}$, Q_q^2 and Q_k^2 are geometrical constants depending on the radial parameters. The matrix element $\langle f|O_{A_I}|i\rangle$ is the spin-flavor weight for a quark line diagram A_I. The analytic evaluation of the expression of Eq. (4.16) and the numerical evaluations of $\langle f|O_{A_I}|i\rangle$ are summarized in Appendix A, B and C. Since the proton and the neutron give the same the spin-flavor weight, the ϕ production from the nucleon-antinucleon annihilation at rest can be described by the transition amplitude Eq. (4.16) multiplied with factor $\sqrt{2}$. Since we consider the $p\bar{p}$ system in annihilation at rest the proton-antiproton wave function is strongly correlated. Thus the initial state interaction for the atomic state of the $p\bar{p}$ system has to be included into the transition amplitude (Kercek et al., 1999), which results in

$$T_{f,LSJ}(\vec{q}) = \int d^3k \; T_{fi}(\vec{q},\vec{k})\phi^I_{LSJ}(\vec{k}), \tag{4.17}$$

where $\phi^I_{LSJ}(\vec{k})$ is the protonium wave function in the momentum space for fixed isospin I.

4.3 The Branching Ratios

In particle physics, it well known that the likelihood of interaction between particles can be expressed by using the concept of a cross section. For the case of two particles the cross section is a measure of the interaction event between the two particles when they are colliding with each other. Additionally, in high energy physics experiments, not only the cross section but also the branching ratio is measured to be compared with theoretical predictions. The branching ratio (branching fraction) is the fraction of events for the decay in a certain way of a chosen particle measured; accordingly the sum of branching ratios for a particle is one. The transition (the rate of number of events) depends on the corresponding decay width. Hence, instead of the transitional fraction, the branching ratio can be defined as the partial decay width divided by the total width

$$\Gamma_{p\bar{p}\to\phi X} = \frac{1}{2E}\int \frac{d^3p_\phi}{2E_\phi}\frac{d^3p_X}{2E_X}\delta^{(3)}(\vec{p}_\phi + \vec{p}_X)\delta(E - E_\phi - E_X)|T_{f,LSJ}(\vec{q})|^2 \ , \quad (4.18)$$

where E is the total energy ($E = 1.876$ GeV) and $E_{\phi,X} = \sqrt{m^2_{\phi,X} + \vec{p}^2_{\phi,X}}$ is the energy of the outgoing mesons ϕ and X with mass $m_{\phi,X}$ and momentum $\vec{p}_{\phi,X}$. With the explicit form of the transition amplitude given by Eqs (4.16) and (4.17), the partial decay width for the S to P transition ($L = 0$, $\ell_f = 1$) is written as

$$\Gamma_{p\bar{p}\to\phi X} = \lambda^2_{A_I} f(\phi, X)\langle f|O_{A_I}|i\rangle^2\gamma(I, J), \quad (4.19)$$

with

$$\gamma(I, J) = |F_{0,1}\int d^3k\ \phi^I_{LSJ}(\vec{k})\exp\left\{-Q^2_k k^2\right\}|^2 \ . \quad (4.20)$$

The kinematical phase-space factor is defined by

$$f(\phi, X) = \frac{q^3}{8E}\ \exp\left\{-2Q^2_q q^2\right\}. \quad (4.21)$$

The spin-flavor weights of the different transitions including the three different versions of the proton wave functions discussed previously were calculated and are shown in the Appendix A, B and C. Due to the projection of the initial values of J the statistical weight 1/4 and 3/4 has to be added for $J = 0$ and $J = 1$, the branching ratio of S-wave $p\bar{p}$ annihilation to the final state ϕX is then given by

$$BR(\phi, X) = \frac{(2J + 1)\Gamma_{p\bar{p}\to\phi X}}{4\Gamma_{tot}(J)}, \quad (4.22)$$

where $\Gamma_{tot}(J)$ is the total annihilation width of the $p\bar{p}$ atomic state with fixed principal quantum number (Dover et al., 1991). However, in order to evaluate the branching ratio, the model dependence in Eq. (4.19) might be reduced by choosing a simplified phenomenological approach that has been applied in studies for two-meson branching ratios in nucleon-antinucleon (Kercek et al., 1999) and radiative protonium annihilation (Gutsche et al., 1999). To state it explicitly, instead of the phase space factor of Eq. (4.21) that depends on the relative momentum and the masses of the ϕX system, a kinematical phase-space factor of following form was used

$$f(\phi, X) = q \cdot \exp\{-1.2\ GeV^{-1}(s - s_{\phi X})^{1/2}\} \tag{4.23}$$

where $s_{\phi X} = (m_\phi + m_X)^{1/2}$ and $\sqrt{s} = (m_\phi^2 + q^2)^{1/2} + (m_X^2 + q^2)^{1/2}$. Last form was obtained from the fit of the momentum dependence of the cross section of various annihilation channels (Vandermeulen, 1988). In addition, the functions $\gamma(I, J)$, depending on the initial-state interaction, are related to the probability for a protonium state to have isospin I and spin J with the normalization condition $\gamma(0, J) + \gamma(1, J) = 1$. The probability for a protonium state $\gamma(I, J)$ and the total decay width $\Gamma_{tot}(J)$, which are obtained from an optical potential calculation (Carbonell et al., 1989) and which is listed in (Dover et al., 1991), were adopted in this work. The results for the branching ratios of Eq. (4.22) which are compared with the experimental data are demonstrated in Table 4.1. The calculation of branching ratios from a proton wave function with $s\bar{s}$ quark sea by using this approach has been discussed in Ref(Gutsche et al., 1997). In case of $s\bar{s}$ and $uuds$ the annihilation process can be described by the quark line diagram A_1 but the effective strength parameter λ_{A_1} corresponding to each transition is not known priori. Therefore, in the model predictions one entry (as indicated by $\star$) is normalized to the experimental number. In case of thhe ChQM, according to their $5q$-components, we need to use all three quark line diagrams to describe the annihilation process. However, the process via the diagram A_1 with the component

Table 4.1 Branching ratios $BR(\times 10^4)$ for the transition $p\bar{p} \to \phi X$ ($X = \pi^0, \eta, \rho^0, \omega$) in $p\bar{p}$ annihilation at rest. The results indicated by $\star$ are normalized to the experimental values.

Transition	BR^{exp}	$BR^{s\bar{s}}$	BR^{ChQM}	$BR^{[31][31][22]}$	$BR^{[31][211][22]}$
$^{11}S_0 \to \omega\phi$	6.3 ± 2.3	6.3 $\star$	6.3 $\star$	6.3 $\star$	6.3 $\star$
$^{33}S_1 \to \pi^0\phi$	5.5 ± 0.7	5.4	1.6	5.4	5.4
$^{31}S_0 \to \rho^0\phi$	3.4 ± 1.0	3.8	0.87	3.8	3.8
$^{13}S_1 \to \eta\phi$	0.9 ± 0.3	1.4−1.8	0.20−0.27	1.4−1.8	1.4−1.8

$|p\eta\rangle$ has no contribution to the transition because the vector meson ϕ cannot be produced or shaken out from the pseudoscalar η cluster. Therefore, the annihilation process can be described by the quark line diagrams A_2 and A_3. Nevertheless, according to the same annihilation of two quark pairs in these two diagrams, the two unknown strength parameter λ_{A_2} and λ_{A_3} can be taken to be equal. A model prediction is also normalized to an experimental number. For final states with $X = \eta$ the physical η meson is produced by its nonstrange component η_{ud} with $\eta = \eta_{ud}(\sqrt{1/3}cos\theta - \sqrt{2/3}sin\theta)$ where we consider a variation of the pseudoscalar mixing angle θ from $\theta = -10.7^o$ to $\theta = -20^o$. As shown in Table 4.1, the model predictions are in good agreement with the experimental data. In particular, excellent agreement is found in the case of the $s\bar{s}$ sea-quark (Gutsche et al., 1997) and the *uuds* subsystem. For both of these two cases, the annihilation processes $p\bar{p} \to \phi X$ are described with the quark line diagram A_1.

CHAPTER V

CONCLUSIONS

Three models have been studied for the proton involving intrinsic strangeness in the form of a 5-quark component $qqqs\bar{s}$ in the wave function. In particular, the proton wave function is made up of a uud configuration and a uud cluster with a $s\bar{s}$ sea-quark component, kaon-hyperon clusters based on the simple chiral quark model, or a pentaquark component $uuds\bar{s}$. We have calculated the strangeness magnetic moment μ_s and spin σ_s for the first and second models are able to generate negative values in line with recent experimental indication. Similarly, for the third model we pick these $uuds$ cluster configurations $[31]_{FS}[211]_F[22]_S$ and $[31]_{FS}[31]_F[22]_S$, where negative values for μ_s and σ_s result (An et al., 2006). We further applied quark line diagrams supplemented by the ${}^3P_0$ vertex to study the annihilation reactions $p\bar{p} \to \phi X$ ($X = \pi^0, \eta, \rho^0, \omega$) with the three types of proton wave functions. Excellent agreement of the model predictions in the first and third models with the experimental data are found for the branching ratios of the reactions of the $L = 0$ atomic $p\bar{p}$ state to ϕX ($X = \pi^0, \eta, \rho^0, \omega$). In this work we have supposed that the five quark component in the proton wave function can be treated as a small admixture. However, we have adjusted the parameter B with other model parameters together to experimental data. Following the work here we may study other reactions to nail down the size parameters and effective coupling constants first, and then estimate the parameter B for each model. In the third model we have picked two $uuds$ cluster configurations $[31]_{FS}[211]_F[22]_S$ and $[31]_{FS}[31]_F[22]_S$ among a large number of configurations. Here, it might be interesting to check even other wave functions for the ϕX annihilation process.

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APPENDICES

APPENDIX A

TRANSITION AMPLITUDE OF THE PROTON WAVE FUNCTION WITH $S\bar{S}$ SEA QUARKS

To describe the annihilation process of $p\bar{p} \to \phi X$ where $X = \pi^0, \eta, \rho^0, \omega$ with the proton wave function with $s\bar{s}$ sea quarks we consider the shake-out of the intrinsic $s\bar{s}$ component of the proton wave function as indicated in the diagram A_1. With the operator $\mathcal{O}_{A_1}$ and the full account of the spin-flavor-color-orbital structure of the initial and final state, the transition amplitude can be written as

$$T_{if}^{s\bar{s}} = \lambda_{A_1} \langle f | \sum_{\nu,\lambda} (-1)^{\nu+\lambda} \sigma_{-\nu}^{56} \sigma_{-\lambda}^{47} 1_F^{56} 1_F^{47} 1_C^{56} 1_C^{47} I_{spatial}^{s\bar{s}} | i \rangle \tag{A.1}$$

where

$$|i\rangle = |\{\chi_{\frac{1}{2},m_{ps\bar{s}}}(uuds\bar{s}) \otimes \chi_{\frac{1}{2},m_{\bar{p}}}(\bar{u}\bar{u}\bar{d})\}_{S,S_z} \otimes (L,M)\rangle_{J,J_z}, \tag{A.2}$$

$$|f\rangle = |\{\chi_{1,m_\alpha}(\phi) \otimes \chi_{j_m,m_{3',4'}}(X)\}_{j,m_\epsilon} \otimes (\ell_f, m_f)\rangle_{J',J'_z}, \tag{A.3}$$

and the total angular momentum of the initial state $|i\rangle$ (the final state $|f\rangle$) are coupled to J with projection J_z (J' with projection J'_z). Subscripts refer to the corresponding orbital structure. The spin-flavor-color content of the clusters is denoted by $\chi (= \chi_\sigma \otimes \chi_F \otimes \chi_C)$. The 5-quark component $\chi_{\frac{1}{2},m_{ps\bar{s}}}(uuds\bar{s})$ is defined as

$$\chi_{\frac{1}{2},m_{ps\bar{s}}}(uuds\bar{s}) = |\{\chi_{j_s,m_s}(s\bar{s}) \otimes (\ell = 1, \mu)\}_{j_i,m_i} \otimes \chi_{\frac{1}{2},m_p}(uud)\rangle_{\frac{1}{2},m_{ps\bar{s}}}. \tag{A.4}$$

The spatial wave amplitude $I^{s\bar{s}}_{spatial}$ is explicitly given by

$$I^{s\bar{s}}_{spatial} = \int d^3q_1...d^3q_8 d^3q_{1'}...d^3q_{4'} \varphi_{\phi,X} \mathcal{O}^{spatial}_{A_1} \varphi_{uuds\bar{s},\bar{p}} \qquad \text{(A.5)}$$

where

$$\mathcal{O}^{spatial}_{A_1} = Y_{1\lambda}(\vec{q}_4 - \vec{q}_7)\delta^{(3)}(\vec{q}_4 + \vec{q}_7)Y_{1\nu}(\vec{q}_5 - \vec{q}_6)\delta^{(3)}(\vec{q}_5 + \vec{q}_6)$$
$$\delta^{(3)}(\vec{q}_1 - \vec{q}_{1'})\delta^{(3)}(\vec{q}_2 - \vec{q}_{2'})\delta^{(3)}(\vec{q}_3 - \vec{q}_{3'})\delta^{(3)}(\vec{q}_8 - \vec{q}_{4'}). \qquad \text{(A.6)}$$

By choosing a plane wave basis and the harmonic oscillator approximation for the relative motions and the internal spatial wave functions, respectively, the spatial wave functions in $I^{s\bar{s}}_{spatial}$ are given by

$$\varphi_{uuds\bar{s},\bar{p}} = \varphi_{uuds\bar{s}}\varphi_{\bar{p}}\delta^{(3)}(\vec{q}_1 + \vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5 - \vec{k})\delta^{(3)}(\vec{q}_6 + \vec{q}_7 + \vec{q}_8 + \vec{k}), \qquad \text{(A.7)}$$

$$\varphi_{\phi,X} = \varphi_\phi\varphi_X\delta^{(3)}(\vec{q} - \vec{q}_{1'} - \vec{q}_{2'})\delta^{(3)}(\vec{q} + \vec{q}_{3'} + \vec{q}_{4'}), \qquad \text{(A.8)}$$

$$\varphi_{uuds\bar{s}} = N_{uuds\bar{s}}\exp\left\{-\frac{R_B^2}{2}[(\frac{\vec{q}_4 - \vec{q}_5}{\sqrt{2}})^2 + (\frac{\vec{q}_4 + \vec{q}_5 - 2\vec{q}_3}{\sqrt{6}})^2]\right\}$$
$$\exp\left\{-\frac{R^2}{8}(\vec{q}_3 + \vec{q}_4 + \vec{q}_5 - \vec{q}_1 - \vec{q}_2)^2\right\}$$
$$Y_{1\mu}(\vec{q}_3 + \vec{q}_4 + \vec{q}_5 - \vec{q}_1 - \vec{q}_2)$$
$$\exp\left\{-\frac{R_M^2}{8}(\vec{q}_1 - \vec{q}_2)^2\right\}, \qquad \text{(A.9)}$$

$$\varphi_{\bar{p}} = N_{\bar{p}}\exp\left\{-\frac{R_B^2}{2}[(\frac{\vec{q}_6 - \vec{q}_7}{\sqrt{2}})^2 + (\frac{\vec{q}_6 + \vec{q}_7 - 2\vec{q}_8}{\sqrt{6}})^2]\right\}, \qquad \text{(A.10)}$$

$$\varphi_\phi = N_\phi\exp\left\{-\frac{R_M^2}{8}(\vec{q}_{1'} - \vec{q}_{2'})^2\right\}, \qquad \text{(A.11)}$$

$$\varphi_X = N_X\exp\left\{-\frac{R_M^2}{8}(\vec{q}_{3'} - \vec{q}_{4'})^2\right\}. \qquad \text{(A.12)}$$

Integrating $d\vec{q}_{1'}..d\vec{q}_{4'}d\vec{q}_6 d\vec{q}_7$ with the delta functions $\delta^{(3)}(\vec{q}_3-\vec{q}_{3'})\delta^{(3)}(\vec{q}_8-\vec{q}_{4'})\delta^{(3)}(\vec{q}_4+\vec{q}_7)\delta^{(3)}(\vec{q}_5+\vec{q}_6)$, $I^{s\bar{s}}_{spatial}$ can be reduced to

$$\begin{aligned}I^{s\bar{s}}_{spatial} = \int d^3q_1...d^3q_5 d^3q_8 N\exp\Big(\frac{1}{24}(-3R^2(-\vec{q}_1-\vec{q}_2+\vec{q}_3+\vec{q}_4+\vec{q}_5)^2 \\ -2(3(\vec{q}_4-\vec{q}_5)^2+(-2\vec{q}_3+\vec{q}_4+\vec{q}_5)^2)R_B^2-2(3(\vec{q}_6-\vec{q}_7)^2 \\ +(\vec{q}_6+\vec{q}_7-2\vec{q}_8)^2)R_B^2-6(\vec{q}_1-\vec{q}_2)^2R_M^2-3(\vec{q}_3-\vec{q}_8)^2R_M^2)\Big) \\ \delta(-\vec{k}+\vec{q}_1+\vec{q}_2+\vec{q}_3+\vec{q}_4+\vec{q}_5)\delta(\vec{q}_5+\vec{q}_6)\delta(\vec{q}_4+\vec{q}_7) \\ \delta(\vec{q}+\vec{q}_3+\vec{q}_8)\delta(\vec{k}+\vec{q}_6+\vec{q}_7+\vec{q}_8)\delta(\vec{q}-\vec{q}_1-\vec{q}_2) \\ Y_{1,\lambda}(\vec{q}_5-\vec{q}_6)Y_{1,\mu}(-\vec{q}_1-\vec{q}_2+\vec{q}_3+\vec{q}_4+\vec{q}_5)Y_{1,\nu}(\vec{q}_4-\vec{q}_7)\end{aligned} \quad (A.13)$$

where $N = N_\phi N_X N_{uuds\bar{s}} N_{\bar{p}}$. The integrals $d\vec{q}_1 d\vec{q}_5 d\vec{q}_8$ can be done with the delta functions $\delta(\vec{q}-\vec{q}_1-\vec{q}_2)\delta(-\vec{k}+\vec{q}_1+\vec{q}_2+\vec{q}_3+\vec{q}_4+\vec{q}_5)\delta(\vec{q}+\vec{q}_3+\vec{q}_8)$, so $I^{s\bar{s}}_{spatial}$ becomes

$$\begin{aligned}I^{s\bar{s}}_{spatial} = \int d^3q_1...d^3q_5 d^3q_8 N\delta^{(3)}(\vec{0})\exp\Big(\frac{1}{24}(-8(2\vec{k}^2-5\vec{q}\vec{k}+4\vec{q}^2+6\vec{q}_3^2 \\ +6\vec{q}_4(-\vec{k}+\vec{q}+\vec{q}_4)+\vec{q}_3(-6\vec{k}+9\vec{q}+6\vec{q}_4))R_B^2 \\ -3((\vec{k}-2\vec{q})^2R^2+(3\vec{q}^2-8\vec{q}_2\vec{q}+8\vec{q}_2^2+4\vec{q}_3(\vec{q} \\ +\vec{q}_3))R_M^2))\Big)Y_{1,\mu}(\vec{k}-2\vec{q})Y_{1,\nu}(2\vec{q}_4) \\ Y_{1,\lambda}(2(\vec{k}-\vec{q}-\vec{q}_3-\vec{q}_4))\end{aligned} \quad (A.14)$$

Applying the transformation $\vec{q}_2 = \vec{x}_2+\alpha_2\vec{k}+\beta_2\vec{q}$, $\vec{q}_3 = \vec{x}_3+\alpha_3\vec{k}+\beta_3\vec{q}$ and $\vec{q}_4 = \vec{x}_4+\alpha_4\vec{k}+\beta_4\vec{q}$, the cross terms $\vec{x}_i\cdot\vec{k}$ and $\vec{x}_i\cdot\vec{q}$ in the exponent can be eliminated by choosing

$$\begin{aligned}\beta_2 = \frac{1}{2},\ \alpha_2 = 0 \\ \beta_3 = -\frac{R_M^2+4R_B^2}{2\left(R_M^2+3R_B^2\right)},\ \alpha_3 = \frac{R_B^2}{R_M^2+3R_B^2} \\ \beta_4 = -\frac{R_M^2+2R_B^2}{4\left(R_M^2+3R_B^2\right)},\ \alpha_4 = \frac{1}{2}-\frac{R_B^2}{2\left(R_M^2+3R_B^2\right)}.\end{aligned} \quad (A.15)$$

The integral is reduced to

$$
\begin{aligned}
I^{s\bar{s}}_{spatial} = \int d^3x_2 d^3x_3 d^3x_4 N\delta^{(3)}(\vec{0})\exp(-\frac{1}{24(R_M^2+3R_B^2)}(12\vec{x}_3^2R_M^4 \\
+4k^2R_B^2R_M^2 + 5q^2R_B^2R_M^2 - 4kqR_B^2R_M^2 + 3k^2R^2R_M^2 \\
+12q^2R^2R_M^2 - 12kqR^2R_M^2 + 12R_{s\bar{s}}^2\vec{x}_2^2R_M^2 \\
+12R_\phi^2\vec{x}_2^2R_M^2 + 84R_B^2\vec{x}_3^2R_M^2 + 48R_B^2\vec{x}_4^2R_M^2 \\
+48R_B^2\vec{x}_3\cdot\vec{x}_4R_M^2 + 12q^2R_B^4 + 9k^2R_B^2R^2 \\
+36q^2R_B^2R^2 - 36kqR_B^2R^2 + 36R_B^2R_{s\bar{s}}^2\vec{x}_2^2 \\
+36R_B^2R_\phi^2\vec{x}_2^2 + 144R_B^4\vec{x}_3^2 + 144R_B^4\vec{x}_4^2 + 144R_B^4\vec{x}_3\cdot\vec{x}_4)) \\
Y_{1,\lambda}(2(-\alpha_3\vec{k} - \alpha_4\vec{k} + \vec{k} - \vec{q} - \vec{x}_3 - \vec{x}_4 - \vec{q}\beta_3 - \vec{q}\beta_4)) \\
Y_{1,\mu}(\vec{k} - 2\vec{q})Y_{1,\nu}(2(\vec{x}_4 + \vec{k}\alpha_4 + \vec{q}\beta_4)). \qquad \text{(A.16)}
\end{aligned}
$$

To eliminate the cross terms $\vec{x}_3\cdot\vec{x}_4$ we apply the transformations $\vec{x}_2 = \vec{p}_2$, $\vec{x}_3 = \vec{p}_3$ and $\vec{x}_4 = b\vec{p}_3 + \vec{p}_4$ with $b = -1/2$, the spatial wave amplitude is reduced to

$$
\begin{aligned}
I^{s\bar{s}}_{spatial} = \int d^3p_2 d^3p_3 d^3p_4 N\delta^{(3)}(\vec{0})\exp(-Q_k^2\vec{k}^2 - Q_q^2\vec{q}^2 - Q_{kq}^2\vec{k}\cdot\vec{q} \\
-Q_{p_2}^2\vec{p}_2^2 - Q_{p_3}^2\vec{p}_3^2 - Q_{p_4}^2\vec{p}_4^2) \\
Y_{1,\lambda}(2(-\alpha_3\vec{k} - \alpha_4\vec{k} + \vec{k} - \vec{q} - \frac{\vec{p}_3}{2} - \vec{p}_4 - \vec{q}\beta_3 - \vec{q}\beta_4)) \\
Y_{1,\mu}(\vec{k} - 2\vec{q})Y_{1,\nu}(2(-\frac{\vec{p}_3}{2} + \vec{p}_4 + \vec{k}\alpha_4 + \vec{q}\beta_4)) \qquad \text{(A.17)}
\end{aligned}
$$

where

$$
\begin{aligned}
Q_k^2 &= \frac{4R_M^2R_B^2 + 9R^2R_B^2 + 3R_M^2R^2}{24\left(R_M^2 + 3R_B^2\right)}, \\
Q_q^2 &= \frac{12R_B^4 + 5R_M^2R_B^2 + 36R^2R_B^2 + 12R_M^2R^2}{24\left(R_M^2 + 3R_B^2\right)}, \\
Q_{kq}^2 &= -\frac{R_M^2R_B^2 + 9R^2R_B^2 + 3R_M^2R^2}{6\left(R_M^2 + 3R_B^2\right)}, \\
Q_{p_2}^2 &= R_M^2, \\
Q_{p_3}^2 &= \frac{1}{2}\left(R_M^2 + 3R_B^2\right), \\
Q_{p_4}^2 &= 2R_B^2. \qquad \text{(A.18)}
\end{aligned}
$$

To described the annihilation process at rest, the partial wave amplitude can be obtained by projecting the transition amplitude onto the partial wave corresponding to S to P transition. So the spatial partial wave amplitude for the $L = 0$ to $\ell_f = 1$ transition is given by

$$I_{spatial,L=0,\ell_f=1} = \int d\Omega_k d\Omega_q \mathcal{Y}^*_{0,0}(\hat{k})\mathcal{Y}^*_{1,m_f}(\hat{q}) I_{spatial}. \tag{A.19}$$

In order to integrate $d\Omega_k$, $d\Omega_q$, d^3p_2, d^3p_3 and d^3p_4 we apply the spherical harmonic identity $Y_{1,m}(\vec{a}+\vec{b}) = Y_{1,m}(\vec{a}) + Y_{1,m}(\vec{b})$ and $Y_{1,m}(\vec{a}) = a\mathcal{Y}_{1,m}(\hat{a})$ with the Gaussian integral

$$\int d^3\vec{p}_i \exp(-Q^2_{p_i}\vec{p}_i^2) = \left(\frac{\pi}{Q^2_{p_i}}\right)^{3/2}, \tag{A.20}$$

$$\int d^3\vec{p}_i \vec{p}_i \exp(-Q^2_{p_i}\vec{p}_i^2) = \int d^3\vec{p}_i \vec{p}_i^3 \exp(-Q^2_{p_i}\vec{p}_i^2) = 0, \tag{A.21}$$

$$\int d^3\vec{p}_i \vec{p}_i^2 \exp(-Q^2_{p_i}\vec{p}_i^2)\mathcal{Y}_{1,a}(\hat{p}_i)\mathcal{Y}_{1,b}(\hat{p}_i) = \frac{3(-1)^a\sqrt{\pi}\delta_{a,-b}}{8\left(Q^2_{p_i}\right)^{5/2}}. \tag{A.22}$$

In a low-momentum approximation $\exp\{Q^2_{kq}\vec{k}\cdot\vec{q}\} \approx j_0(Q^2_{kq}\vec{k}\cdot\vec{q}) \approx 1$ the integrals can be done analytically. The spatial wave amplitude corresponding to the leading order in the external momenta q is given by

$$I^{s\bar{s}}_{spatial,0,1} = qF^{s\bar{s}}_{0,1} f^{s\bar{s}}_{0,1}(\nu, \lambda, \mu, m_f) \exp\left\{-Q^2_q q^2 - Q^2_k k^2\right\}. \tag{A.23}$$

The geometrical constant $F^{s\bar{s}}_{0,1}$ and the spin-angular momentum function $f^{s\bar{s}}_{0,1}(\nu, \lambda, \mu, m_f)$ are given by

$$F^{s\bar{s}}_{0,1} = 2N\pi^2\left(\frac{1}{Q^2_{p_2}}\right)^{3/2}\left(\frac{3\sqrt{\pi}}{\left(Q^2_{p_4}\right)^{5/2}} - \frac{3\sqrt{\pi}}{4\left(Q^2_{p_3}\right)^{5/2}}\right)\delta^{(3)}(\vec{0}),$$
$$f^{s\bar{s}}_{0,1}(\nu, \lambda, \mu, m_f) = (-1)^\nu \delta_{\nu,-\lambda}\delta_{\mu,m_f}. \tag{A.24}$$

With the spatial wave amplitude $I^{s\bar{s}}_{spatial}$ the transition amplitude $T^{s\bar{s}}_{if}$ has the form as Eq. (4.16) with the spin-flavor weight:

$$\langle f|O_{A_1}|i\rangle = \langle f|\sum_{\nu,\lambda}(-1)^{\nu+\lambda}\sigma^{56}_{-\nu}\sigma^{47}_{-\lambda}1^{56}_F 1^{47}_F 1^{56}_C 1^{47}_C (-1)^\nu \delta_{\nu,-\lambda}\delta_{\mu,m_f}|i\rangle. \tag{A.25}$$

To evaluate the spin-color-flavor weight $\langle f|O_{A_1}|i\rangle$, for the S to P transition, firstly we decouple the coupled spins and angular momenta form the spin of quark clusters:

$$\langle f|O_{A_1}|i\rangle = \sum_{m_\epsilon,m_f}\sum_{m_\alpha,m_{3,8}}\sum_{m_{ps\bar{s}},m_{\bar{p}}}\sum_{m_i,m_p}\sum_{m_s,\mu}\sum_{\nu,\lambda}\langle j,1,m_\epsilon,m_f|J',J_z'\rangle$$
$$\langle 1,j_m,m_\alpha,m_{3,8}|j,m_\epsilon\rangle\langle\frac{1}{2},\frac{1}{2},m_{ps\bar{s}},m_{\bar{p}}|J,J_z\rangle$$
$$\langle j_i,\frac{1}{2},m_i,m_p|\frac{1}{2},m_{ps\bar{s}}\rangle\langle j_s,1,m_s,\mu|j_i,m_i\rangle$$
$$(-1)^{\nu+\lambda}(-1)^{\nu}\delta_{\nu,-\lambda}\delta_{\mu,m_f}\langle SCF\rangle^{s\bar{s}} \qquad (A.26)$$

with the matrix element

$$\langle SCF\rangle^{s\bar{s}} = \langle\chi_{1,m_\alpha}(\phi)\chi_{j_m,m_{3',4'}}(X)|$$
$$\sigma^{56}_{-\nu}\sigma^{47}_{-\lambda}1^{56}_F 1^{47}_F 1^{56}_C 1^{47}_C|\chi_{j_s,m_s}(s\bar{s})\chi_{\frac{1}{2},m_p}(uud)\chi_{\frac{1}{2},m_{\bar{p}}}(\bar{u}\bar{u}\bar{d})\rangle. \qquad (A.27)$$

The spin-color-flavor wave function (χ) takes the form

$$\chi = (\chi_\sigma\otimes\chi_F)\chi_C. \qquad (A.28)$$

The spin-flavor part $\chi_\sigma\chi_F$ wave functions for the mesons and baryons, that will be used in calculating the matrix element $\langle SCF\rangle^{s\bar{s}}$, are given by

$$\chi_{1,m_\alpha}(\phi) = \chi_F(\phi)\otimes\chi_\sigma(1,m_\alpha)\simeq|s\bar{s}\rangle|1,m_\alpha\rangle, \qquad (A.29)$$

$$\chi_{j_m,m_{3',4'}}(X) = |(\omega,\pi^0,\rho^0,\eta)\rangle|j_m,m_{3',4'}\rangle, \qquad (A.30)$$

$$\chi_{j_s,m_s}(s\bar{s}) = |s\bar{s}\rangle|j_s,m_s\rangle, \qquad (A.31)$$

$$\chi_{\frac{1}{2},m_p}(uud) = \chi_F(uud)\otimes\chi_\sigma(1,m_p)$$
$$= \frac{1}{\sqrt{2}}\{|uud\rangle_+|\frac{1}{2},m_p\rangle_+ + |uud\rangle_-|\frac{1}{2},m_p\rangle_-\}, \qquad (A.32)$$

where

$$|\omega\rangle = \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})|1,m_{3',4'}\rangle, \qquad (A.33)$$

$$|\pi^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})|0,0\rangle, \tag{A.34}$$

$$|\rho^0\rangle = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})|1, m_{3',4'}\rangle, \tag{A.35}$$

$$|\eta\rangle = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})|0,0\rangle, \tag{A.36}$$

$$|uud\rangle_+ = \frac{1}{\sqrt{6}}(2uud - udu - duu) \tag{A.37}$$

$$|uud\rangle_- = \frac{1}{\sqrt{2}}(udu - duu) \tag{A.38}$$

$$|\frac{1}{2}, m_p = \frac{1}{2}\rangle_+ = \frac{1}{\sqrt{6}}(2\uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \tag{A.39}$$

$$|\frac{1}{2}, m_p = \frac{1}{2}\rangle_- = \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \tag{A.40}$$

$$|\frac{1}{2}, m_p = -\frac{1}{2}\rangle_+ = -\frac{1}{\sqrt{6}}(2\downarrow\downarrow\uparrow - \downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow) \tag{A.41}$$

$$|\frac{1}{2}, m_p = -\frac{1}{2}\rangle_- = -\frac{1}{\sqrt{2}}(\downarrow\uparrow\downarrow - \uparrow\downarrow\downarrow) \tag{A.42}$$

$$|j_{s(m)} = 0, m_{s(3'4')} = 0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow) \tag{A.43}$$

$$|j_{s(m)} = 1, m_{s(3'4')} = 0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow) \tag{A.44}$$

$$|j_{s(m)} = 1, m_{s(3'4')} = 1\rangle = \uparrow\uparrow \tag{A.45}$$

$$|j_{s(m)} = 1, m_{s(3'4')} = -1\rangle = \downarrow\downarrow \tag{A.46}$$

The subscript $(+)$ denotes a state that is symmetric and $(-)$ denotes a state that is antisymmetric under exchange of the spin or isospin of the first two quarks. The quark indices $3'4'$ become 38 corresponding to the quark line in diagram A_1.

For the color state wave function of meson and baryon that have to be singlet, the corresponding color state wave function for each $2q$ and $3q$ is given by

$$\chi_C(q\bar{q}) = \frac{1}{\sqrt{3}}(\mathrm{R\bar{R} + G\bar{G} + B\bar{B}}), \tag{A.47}$$

$$\chi_C(qqq) = \frac{1}{\sqrt{6}}(\mathrm{RGB - RBG - GRB + GBR - BGR + BRG}). \tag{A.48}$$

According to the $^3P_0$ model the matrix element $\langle SCF\rangle^{s\bar{s}}$ can be evaluated by using the two-body matrix elements for spin, flavor and color which are given by

$$\langle 0|\sigma_{\upsilon}^{ij}|\chi_{m_{ij}}^{J_{ij}}(ij)\rangle = \delta_{J_{ij},1}\delta_{m_{ij},-\upsilon}(-1)^{\upsilon}\sqrt{2}, \tag{A.49}$$

$$\langle 0|1_F^{ij}|\chi_{t_{ij}}^{T_{ij}}(ij)\rangle = \delta_{T_{ij},0}\delta_{t_{ij},0}\sqrt{2}, \tag{A.50}$$

and

$$\langle 0|1_C^{ij}|q_{\alpha}^{i}\bar{q}_{\beta}^{j}\rangle = \delta_{\alpha\beta}, \tag{A.51}$$

where α and β are the color indices.

With the quark labeling as defined in the spatial part wave functions, we have the matrix elements $\langle SCF\rangle^{s\bar{s}}$ for each proton and antiproton spin projection (m_p and $m_{\bar{p}}$):

$$\begin{aligned}\langle SCF\rangle^{s\bar{s}}_{m_p=\frac{1}{2},m_{\bar{p}}=\frac{1}{2}} = -\frac{1}{9\sqrt{6}}(-1)^{\lambda+\nu}\delta_{1,j_m}\delta_{1,j_s}\delta_{m_s,m_\alpha}(&3\delta_{0,T}\delta_{0,\nu}\delta_{0,m_{38}}\delta_{1,\lambda}\\ &-\delta_{0,\nu}\delta_{0,m_{38}}\delta_{1,T}\delta_{1,\lambda}-6\delta_{-1,m_{38}}\delta_{0,T}\delta_{1,\nu}\delta_{1,\lambda}\\ &+2\delta_{-1,m_{38}}\delta_{1,T}\delta_{1,\nu}\delta_{1,\lambda}-6\delta_{-1,\nu}\delta_{0,T}\delta_{1,m_{38}}\delta_{1,\lambda}\\ &-4\delta_{-1,\nu}\delta_{1,T}\delta_{1,m_{38}}\delta_{1,\lambda}+3\delta_{0,T}\delta_{0,\lambda}\delta_{0,m_{38}}\delta_{1,\nu}\\ &-\delta_{0,\lambda}\delta_{0,m_{38}}\delta_{1,T}\delta_{1,\nu}+3\delta_{0,T}\delta_{0,\lambda}\delta_{0,\nu}\delta_{1,m_{38}}\\ &+5\delta_{0,\lambda}\delta_{0,\nu}\delta_{1,T}\delta_{1,m_{38}}-6\delta_{-1,\lambda}\delta_{0,T}\delta_{1,\nu}\delta_{1,m_{38}}\\ &-4\delta_{-1,\lambda}\delta_{1,T}\delta_{1,\nu}\delta_{1,m_{38}}),\end{aligned} \tag{A.52}$$

$$\langle SCF\rangle^{s\bar{s}}_{m_p=\frac{1}{2},m_{\bar{p}}=-\frac{1}{2}} = -\frac{1}{18\sqrt{3}}(-1)^{\lambda+\nu}\delta_{1,j_s}\delta_{m_s,m_\alpha}(3\delta_{0,T}\delta_{0,\lambda}\delta_{0,\nu}\delta_{0,j_m}\delta_{0,m_{38}}$$
$$+5\delta_{0,\lambda}\delta_{0,\nu}\delta_{0,j_m}\delta_{1,T}\delta_{0,m_{38}} - 3\delta_{-1,\nu}\delta_{0,T}\delta_{0,j_m}\delta_{1,\lambda}\delta_{0,m_{38}}$$
$$-5\delta_{-1,\nu}\delta_{0,j_m}\delta_{1,T}\delta_{1,\lambda}\delta_{0,m_{38}} - 3\delta_{-1,\lambda}\delta_{0,T}\delta_{0,j_m}\delta_{1,\nu}\delta_{0,m_{38}}$$
$$-5\delta_{-1,\lambda}\delta_{0,j_m}\delta_{1,T}\delta_{1,\nu}\delta_{0,m_{38}} + 9\delta_{0,T}\delta_{0,\lambda}\delta_{0,\nu}\delta_{1,j_m}\delta_{0,m_{38}}$$
$$+3\delta_{0,\lambda}\delta_{0,\nu}\delta_{1,T}\delta_{1,j_m}\delta_{0,m_{38}} - 3\delta_{-1,\nu}\delta_{0,T}\delta_{1,\lambda}\delta_{1,j_m}\delta_{0,m_{38}}$$
$$-5\delta_{-1,\nu}\delta_{1,T}\delta_{1,\lambda}\delta_{1,j_m}\delta_{0,m_{38}} - 3\delta_{-1,\lambda}\delta_{0,T}\delta_{1,\nu}\delta_{1,j_m}\delta_{0,m_{38}}$$
$$-5\delta_{-1,\lambda}\delta_{1,T}\delta_{1,\nu}\delta_{1,j_m}\delta_{0,m_{38}} - 3\delta_{-1,m_{38}}\delta_{0,T}\delta_{0,\nu}\delta_{1,\lambda}\delta_{1,j_m}$$
$$+\delta_{-1,m_{38}}\delta_{0,\nu}\delta_{1,T}\delta_{1,\lambda}\delta_{1,j_m} - 3\delta_{-1,m_{38}}\delta_{0,T}\delta_{0,\lambda}\delta_{1,\nu}\delta_{1,j_m}$$
$$+\delta_{-1,m_{38}}\delta_{0,\lambda}\delta_{1,T}\delta_{1,\nu}\delta_{1,j_m} - 3\delta_{-1,\nu}\delta_{0,T}\delta_{0,\lambda}\delta_{1,j_m}\delta_{1,m_{38}}$$
$$-3\delta_{-1,\lambda}\delta_{0,T}\delta_{0,\nu}\delta_{1,j_m}\delta_{1,m_{38}} + \delta_{-1,\nu}\delta_{0,\lambda}\delta_{1,T}\delta_{1,j_m}\delta_{1,m_{38}}$$
$$+\delta_{-1,\lambda}\delta_{0,\nu}\delta_{1,T}\delta_{1,j_m}\delta_{1,m_{38}}), \quad (A.53)$$

$$\langle SCF\rangle^{s\bar{s}}_{m_p=-\frac{1}{2},m_{\bar{p}}=\frac{1}{2}} = \frac{1}{18\sqrt{3}}(-1)^{\lambda+\nu}\delta_{1,j_s}\delta_{m_s,m_\alpha}(3\delta_{0,T}\delta_{0,\lambda}\delta_{0,\nu}\delta_{0,j_m}\delta_{0,m_{38}}$$
$$+5\delta_{0,\lambda}\delta_{0,\nu}\delta_{0,j_m}\delta_{1,T}\delta_{0,m_{38}} - 3\delta_{-1,\nu}\delta_{0,T}\delta_{0,j_m}\delta_{1,\lambda}\delta_{0,m_{38}}$$
$$-5\delta_{-1,\nu}\delta_{0,j_m}\delta_{1,T}\delta_{1,\lambda}\delta_{0,m_{38}} - 3\delta_{-1,\lambda}\delta_{0,T}\delta_{0,j_m}\delta_{1,\nu}\delta_{0,m_{38}}$$
$$-5\delta_{-1,\lambda}\delta_{0,j_m}\delta_{1,T}\delta_{1,\nu}\delta_{0,m_{38}} - 9\delta_{0,T}\delta_{0,\lambda}\delta_{0,\nu}\delta_{1,j_m}\delta_{0,m_{38}}$$
$$-3\delta_{0,\lambda}\delta_{0,\nu}\delta_{1,T}\delta_{1,j_m}\delta_{0,m_{38}} + 3\delta_{-1,\nu}\delta_{0,T}\delta_{1,\lambda}\delta_{1,j_m}\delta_{0,m_{38}}$$
$$+5\delta_{-1,\nu}\delta_{1,T}\delta_{1,\lambda}\delta_{1,j_m}\delta_{0,m_{38}} + 3\delta_{-1,\lambda}\delta_{0,T}\delta_{1,\nu}\delta_{1,j_m}\delta_{0,m_{38}}$$
$$+5\delta_{-1,\lambda}\delta_{1,T}\delta_{1,\nu}\delta_{1,j_m}\delta_{0,m_{38}} + 3\delta_{-1,m_{38}}\delta_{0,T}\delta_{0,\nu}\delta_{1,\lambda}\delta_{1,j_m}$$
$$-\delta_{-1,m_{38}}\delta_{0,\nu}\delta_{1,T}\delta_{1,\lambda}\delta_{1,j_m} + 3\delta_{-1,m_{38}}\delta_{0,T}\delta_{0,\lambda}\delta_{1,\nu}\delta_{1,j_m}$$
$$-\delta_{-1,m_{38}}\delta_{0,\lambda}\delta_{1,T}\delta_{1,\nu}\delta_{1,j_m} + 3\delta_{-1,\nu}\delta_{0,T}\delta_{0,\lambda}\delta_{1,j_m}\delta_{1,m_{38}}$$
$$+3\delta_{-1,\lambda}\delta_{0,T}\delta_{0,\nu}\delta_{1,j_m}\delta_{1,m_{38}} - \delta_{-1,\nu}\delta_{0,\lambda}\delta_{1,T}\delta_{1,j_m}\delta_{1,m_{38}}$$
$$-\delta_{-1,\lambda}\delta_{0,\nu}\delta_{1,T}\delta_{1,j_m}\delta_{1,m_{38}}), \quad (A.54)$$

and

$$\langle SCF\rangle^{s\bar{s}}_{m_p=-\frac{1}{2},m_{\bar{p}}=-\frac{1}{2}} = -\frac{1}{9\sqrt{6}}(-1)^{\lambda+\nu}\delta_{1,j_m}\delta_{1,j_s}\delta_{m_s,m_\alpha}(3\delta_{-1,m_{38}}\delta_{0,T}\delta_{0,\lambda}\delta_{0,\nu}$$
$$+3\delta_{-1,\lambda}\delta_{0,T}\delta_{0,m_{38}}\delta_{0,\nu} + 5\delta_{-1,m_{38}}\delta_{0,\lambda}\delta_{1,T}\delta_{0,\nu}$$
$$-\delta_{-1,\lambda}\delta_{0,m_{38}}\delta_{1,T}\delta_{0,\nu} + 3\delta_{-1,\nu}\delta_{0,T}\delta_{0,\lambda}\delta_{0,m_{38}}$$
$$-\delta_{-1,\nu}\delta_{0,\lambda}\delta_{0,m_{38}}\delta_{1,T} - 6\delta_{-1,\nu}\delta_{-1,m_{38}}\delta_{0,T}\delta_{1,\lambda}$$
$$-4\delta_{-1,\nu}\delta_{-1,m_{38}}\delta_{1,T}\delta_{1,\lambda} - 6\delta_{-1,\lambda}\delta_{-1,m_{38}}\delta_{0,T}\delta_{1,\nu}$$
$$-4\delta_{-1,\lambda}\delta_{-1,m_{38}}\delta_{1,T}\delta_{1,\nu} - 6\delta_{-1,\lambda}\delta_{-1,\nu}\delta_{0,T}\delta_{1,m_{38}}$$
$$+2\delta_{-1,\lambda}\delta_{-1,\nu}\delta_{1,T}\delta_{1,m_{38}}). \quad (A.55)$$

With the corresponding spin and isospin quantum number of each meson X

$$|\omega\rangle \equiv |j_m = 1, T_{38} = 0\rangle,\ |\pi^0\rangle \equiv |j_m = 0, T_{38} = 1\rangle,$$
$$|\rho^0\rangle \equiv |j_m = 1, T_{38} = 1\rangle,\ |\eta\rangle \equiv |j_m = 0, T_{38} = 0\rangle \quad (A.56)$$

the spin-flavor weight $\langle f|O_{A_1}|i\rangle$ of the different transitions are calculated as listed in Table A.1.

Table A.1 Spin-flavor matrix elements $\langle f|O_{A_1}|i\rangle$ for the transition $p\bar{p}(L=0) \rightarrow \phi X(\ell_f = 1)$ of the proton wave function with a $s\bar{s}$ sea quark component. Here, η_{ud} refers to the nonstrange flavor combination $\eta_{ud} = (u\bar{u} + d\bar{d})/\sqrt{2}$.

Transition	$\langle f\|O_{A_1}\|i\rangle$
$^{11}S_0 \rightarrow \omega\phi$	$\frac{5}{3\sqrt{3}}$
$^{33}S_1 \rightarrow \pi^0\phi$	$\frac{5}{9}$
$^{31}S_0 \rightarrow \rho^0\phi$	$\frac{13}{9\sqrt{3}}$
$^{13}S_1 \rightarrow \eta\phi$	$\frac{1}{3}$

APPENDIX B

TRANSITION AMPLITUDE OF THE PROTON WAVE FUNCTION FROM THE CHIRAL QUARK MODEL

In case of the ChQM, where the annihilation process can be described by the quark line diagrams A_2 and A_3, we obtain the transition amplitude as

$$T_{if}^{ChQM} = T_{if}^{ChQM}(\mathcal{O}_{A_2}) + T_{if}^{ChQM}(\mathcal{O}_{A_3}). \tag{B.1}$$

The corresponding transition amplitude for the two quarks line diagrams are given by

$$T_{if}^{ChQM}(\mathcal{O}_{A_2}) = \lambda_{A_2}\langle f|\sum_{\nu,\lambda}(-1)^{\nu+\lambda}\sigma_{-\nu}^{56}\sigma_{-\lambda}^{47}1_F^{56}1_F^{47}1_C^{56}1_C^{47}I_{spatial,A_2}^{ChQM}|i\rangle \tag{B.2}$$

and

$$T_{if}^{ChQM}(\mathcal{O}_{A_3}) = \lambda_{A_3}\langle f|\sum_{\nu,\lambda}(-1)^{\nu+\lambda}\sigma_{-\nu}^{56}\sigma_{-\lambda}^{17}1_F^{56}1_F^{17}1_C^{56}1_C^{17}I_{spatial,A_3}^{ChQM}|i\rangle. \tag{B.3}$$

The initial state $|i\rangle$ and final state $|f\rangle$ have the form as defined by Eq. (A.2) and Eq. (A.3), but the 5-quark component in this case is given by

$$\chi_{\frac{1}{2},m_{KY}}(uuds\bar{s}) = g_8\sum_{i=1}^{3} b_i|\{\chi_{j_s,m_s}^i(q\bar{s})\otimes(\ell=1,\mu)\}_{j_i,m_i}\otimes\chi_{\frac{1}{2},m_Y}^i(qqs)\rangle_{\frac{1}{2},m_{KY}}, \tag{B.4}$$

where $i = 1, 2, 3$ represent the kaon-hyperon cluster $K^+\Sigma^0$, $K^0\Sigma^+$ and $K^+\Lambda^0$ respectively. The coefficients corresponding to each component are represented by b_i:

$$b_1 = 2\alpha - 1,\ b_2 = \frac{2\alpha+1}{\sqrt{3}},\ b_3 = \sqrt{2}(2\alpha - 1). \tag{B.5}$$

We first calculate the spatial partial wave amplitude $I^{ChQM}_{spatial,A_2}$

$$I^{ChQM}_{spatial,A_2} = \int d^3\vec{q}_1...d^3\vec{q}_8 d^3\vec{q}_{1'}...d^3\vec{q}_{4'}\ \varphi_{\phi X}\mathcal{O}^{spatial}_{A_2}\varphi^{ChQM}_{uuds\bar{s},\bar{p}} \tag{B.6}$$

where

$$\begin{aligned}\mathcal{O}^{spatial}_{A_2} &= Y_{1\lambda}(\vec{q}_4-\vec{q}_7)\delta^{(3)}(\vec{q}_4+\vec{q}_7)Y_{1\nu}(\vec{q}_5-\vec{q}_6)\delta^{(3)}(\vec{q}_5+\vec{q}_6)\\ &\quad \delta^{(3)}(\vec{q}_2-\vec{q}_{1'})\delta^{(3)}(\vec{q}_3-\vec{q}_{2'})\delta^{(3)}(\vec{q}_1-\vec{q}_{3'})\delta^{(3)}(\vec{q}_8-\vec{q}_{4'}).\end{aligned} \tag{B.7}$$

Substituting the spatial wave functions as defined in Eq. (A.7)– Eq. (A.12) and integrating with the delta functions defined in above equation, $I^{ChQM}_{spatial,A_2}$ becomes

$$\begin{aligned}I^{ChQM}_{spatial,A_2} = \int d^3q_1d^3q_3d^3q_4N\delta^{(3)}(\vec{0})\exp\Big(\frac{1}{24}\big(&-3R^2(\vec{k}-2\vec{q}-2\vec{q}_1+2\vec{q}_3)^2\\ &-2((\vec{k}-3\vec{q}-3\vec{q}_1)^2+3(-\vec{k}+\vec{q}+\vec{q}_1+2\vec{q}_4)^2)R_B^2\\ &-2((-\vec{k}+\vec{q}+\vec{q}_1+2\vec{q}_3)^2+3(-\vec{k}+\vec{q}+\vec{q}_1\\ &+2\vec{q}_4)^2)R_B^2-3(\vec{q}+2\vec{q}_1)^2R_M^2-3(\vec{q}\\ &-2\vec{q}_3)^2R_M^2-3(-\vec{q}+\vec{q}_1+\vec{q}_3)^2R_M^2\big)\Big)\\ &Y_{1,\lambda}(2\vec{q}_4)Y_{1,\mu}(\vec{k}-2\vec{q}-2\vec{q}_1+2\vec{q}_3)\\ &Y_{1,\nu}(2(\vec{k}-\vec{q}-\vec{q}_1-\vec{q}_4)).\end{aligned} \tag{B.8}$$

Applying the transformations $\vec{q}_1 = \vec{x}_1+\alpha_1\vec{k}+\beta_1\vec{q}$, $\vec{q}_3 = \vec{x}_3+\alpha_3\vec{k}+\beta_3\vec{q}$, $\vec{q}_4 = \vec{x}_4+\alpha_4\vec{k}+\beta_4\vec{q}$, $\vec{x}_1 = \vec{p}_1$, $\vec{x}_3 = \vec{p}_3+a_3\vec{p}_1$ and $\vec{x}_4 = \vec{p}_4+a_4\vec{p}_1$, the spatial wave amplitude can be reduced to

$$\begin{aligned}I^{ChQM}_{spatial,A_2} = \int d^3q_1d^3q_3d^3q_4N\delta^{(3)}(\vec{0})\ &\exp(-Q_k^2\vec{k}^2-Q_q^2\vec{q}^2-Q_{kq}^2\vec{k}\cdot\vec{q}\\ &-Q_{p_1}^2\vec{p}_1^2-Q_{p_3}^2\vec{p}_3^2-Q_{p_4}^2\vec{p}_4^2)Y_{1,\lambda}(2(a_4\vec{p}_1+\vec{p}_4+\vec{k}\alpha_4+\vec{q}\beta_4))\\ Y_{1,\mu}(\vec{k}-2\vec{q}-2(\vec{p}_1&+\vec{k}\alpha_1+\vec{q}\beta_1)+2(a_3\vec{p}_1+\vec{p}_3+\vec{k}\alpha_3+\vec{q}\beta_3))\\ Y_{1,\nu}(2(-\alpha_1\vec{k}-\alpha_4\vec{k}&+\vec{k}-\vec{q}-a_4\vec{p}_1-\vec{p}_1-\vec{p}_4-\vec{q}\beta_1-\vec{q}\beta_4)),\end{aligned} \tag{B.9}$$

where the constants depend on the size parameters as given by

$$
\begin{aligned}
Q_k^2 = & \frac{1}{24}(8(4\alpha_1^2 + (\alpha_3 + 6\alpha_4 - 5)\alpha_1 + (\alpha_3 - 1)\alpha_3 + 6(\alpha_4 - 1)\alpha_4 + 2)R_B^2 \\
& + 3(4\alpha_3 R^2 + R^2 + (4R^2 + 5R_M^2)\alpha_1^2 + (4R^2 + 5R_M^2)\alpha_3^2 \\
& + 2\alpha_1((R_M^2 - 4R^2)\alpha_3 - 2R^2))), \\
Q_q^2 = & \frac{1}{24}(8(4\beta_1^2 + (\beta_3 + 6\beta_4 + 8)\beta_1 + \beta_3^2 + \beta_3 + 6\beta_4(\beta_4 + 1) + 4)R_B^2 \\
& + 3((5\beta_1^2 + 2(\beta_3 + 1)\beta_1 + \beta_3(5\beta_3 - 6) + 3)R_M^2 \\
& + 4R^2(\beta_1 - \beta_3 + 1)^2)), \\
Q_{kq}^2 = & \frac{1}{12}(6(2\alpha_1 - 2\alpha_3 - \beta_1 - 1)R^2 + 3(2(2(\alpha_1 - \alpha_3)(\beta_1 - \beta_3) \\
& + \beta_3)R^2 + R_M^2(\alpha_1(5\beta_1 + \beta_3 + 1) + \alpha_3(\beta_1 + 5\beta_3 - 3))) \\
& + 4R_B^2(-5\beta_1 - \beta_3 + \alpha_3(\beta_1 + 2\beta_3 + 1) - 6\beta_4 \\
& + 6\alpha_4(\beta_1 + 2\beta_4 + 1) + \alpha_1(8\beta_1 + \beta_3 + 6\beta_4 + 8) - 5)), \\
Q_{p_1}^2 = & \frac{1}{24}((12R^2 + 8R_B^2 + 15R_M^2)a_3^2 + (8R_B^2 + 6(R_M^2 - 4R^2))a_3 \\
& + 16(3a_4(a_4 + 1) + 2)R_B^2 + 3(4R^2 + 5R_M^2)), \\
Q_{p_3}^2 = & \frac{R^2}{2} + \frac{R_B^2}{3} + \frac{5R_M^2}{8}, \; Q_{p_4}^2 = R_N^2 + R_B^2,
\end{aligned}
\tag{B.10}
$$

$$
\begin{aligned}
\alpha_4 &= \left(2R_B^2 + 3R_M^2\right)\left(9R^2 + 4R_B^2 + 6R_M^2\right)/\Delta, \\
\alpha_3 &= 2\left(4R_B^4 + 3R_M^2 R_B^2 - 9R^2 R_M^2\right)/\Delta, \\
\alpha_1 &= 2\left(4R_B^4 + 18R^2 R_B^2 + 9R_M^2 R_B^2 + 9R^2 R_M^2\right)/\Delta, \\
\beta_4 &= -4R_M^2\left(6R^2 + R_B^2 + 3R_M^2\right)/\Delta, \\
\beta_1 &= -2\left(12R_B^4 + 36R^2 R_B^2 + 29R_M^2 R_B^2 + 6R_M^4 + 12R^2 R_M^2\right)/\Delta, \\
\beta_3 &= 8R_M^2\left(6R^2 + 4R_B^2 + 3R_M^2\right)/\Delta, \\
a_3 &= \frac{12R^2 - 4R_B^2 - 3R_M^2}{12R^2 + 8R_B^2 + 15R_M^2}, \; a_4 = -\frac{1}{2}, \\
\Delta &= 24R_B^4 + 72R^2 R_B^2 + 66R_M^2 R_B^2 + 36R_M^4 + 72R^2 R_M^2.
\end{aligned}
\tag{B.11}
$$

Similarly, for the S to P transition in the low-momentum approximation, the spatial partial wave amplitude corresponding to leading order in the external momenta q takes the form

$$I^{ChQM}_{spatial,0,1,A_2} = qF^{A_2}_{0,1} f^{A_2}_{0,1}(\nu, \lambda, \mu, m_f) \exp\left\{-Q^2_q q^2 - Q^2_k k^2\right\}, \tag{B.12}$$

where the geometrical constant and the spin-angular momentum function are given by

$$F^{A_2}_{0,1} = -6g_8 N\pi^4 (a_3 - 1) a_4 \left(\frac{1}{Q^2_{3_3}}\right)^{3/2} \sqrt{\frac{1}{Q^{10}_{p_1}}} \left(\frac{1}{Q^2_{p_4}}\right)^{3/2} (\beta_1 + \beta_4 + 1)\, \delta(\vec{0}),$$
$$f^{A_2}_{0,1}(\nu, \lambda, \mu, m_f) = (-1)^\mu \delta_{\lambda, m_f} \delta_{\mu,-\nu} + (-1)^\lambda \delta_{\lambda,-\nu} \delta_{\mu, m_f} + (-1)^\mu \delta_{\mu,-\lambda} \delta_{\nu, m_f}. \tag{B.13}$$

Substituting the obtained spatial wave amplitude (Eq. (B.12)) into the transition amplitude Eq. (B.2), we obtain

$$T^{ChQM}_{if}(\mathcal{O}_{A_2}) = \lambda_{A_2} q F^{A_2}_{0,1} \exp\left\{-Q^2_q q^2 - Q^2_k k^2\right\} \langle f|O_{A_2}|i\rangle \tag{B.14}$$

where

$$\langle f|O_{A_2}|i\rangle = \langle f| \sum_{\nu,\lambda} (-1)^{\nu+\lambda} \sigma^{56}_{-\nu} \sigma^{47}_{-\lambda} 1^{56}_F 1^{47}_F 1^{56}_C 1^{47}_C f^{A_2}_{0,1} |i\rangle. \tag{B.15}$$

The spin-orbital coupling in $\langle f|O_{A_2}|i\rangle$ can be decoupled to be

$$\begin{aligned}\langle f|O_{A_2}|i\rangle = \sum_{m_\epsilon, m_f} \sum_{m_\alpha, m_{1,8}} \sum_{m_\epsilon, m_f} \sum_{m_{KY}, m_{\bar{p}}} \sum_{m_i, m_Y} \sum_{m_s, \mu} \sum_{\nu,\lambda} (-1)^{\nu+\lambda} \\ \langle j, 1, m_\epsilon, m_f | J', J'_z\rangle \langle 1, j_m, m_\alpha, m_{1,8} | j, m_\epsilon\rangle \\ \langle \frac{1}{2}, \frac{1}{2}, m_{KY}, m_{\bar{p}} | J, J_z\rangle \langle j_i, \frac{1}{2}, m_i, m_Y | \frac{1}{2}, m_{KY}\rangle \\ \langle j_s, 1, m_s, \mu | j_i, m_i\rangle f^{A_2}_{0,1} \langle SCF\rangle^{ChQM}_{A_2}\end{aligned} \tag{B.16}$$

where $\langle SCF\rangle^{ChQM}_{A_2}$ is given by

$$\begin{aligned}\langle SCF\rangle^{ChQM}_{A_2} = \sum_{i=1}^{3} b_i \langle \chi_{1,m_\alpha}(\phi) \chi_{j_m, m_{1,8}}(X)| \\ \sigma^{56}_{-\nu} \sigma^{47}_{-\lambda} 1^{56}_F 1^{47}_F 1^{56}_C 1^{47}_C | \chi^i_{j_s, m_s}(q\bar{s}) \chi^i_{\frac{1}{2}, m_Y}(qqs) \chi_{\frac{1}{2}, m_{\bar{p}}}(\bar{u}\bar{u}\bar{d})\rangle.\end{aligned} \tag{B.17}$$

The spin-color-flavor wave functions in above equation (χ) also takes the form as Eq. (A.28) with the spin-flavor wave function of the kaons $\chi^i_{j_s,m_s}(q\bar{s})$ and the hyperons $\chi^i_{\frac{1}{2},m_Y}(qqs)$ are given by

$$\chi^i_{0,0}(q\bar{s}) = \chi_F^{i=1,3(2)}(q\bar{s}) \otimes \chi_\sigma(0,0) = |K^{+(0)}\rangle \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow), \tag{B.18}$$

$$\chi^i_{\frac{1}{2},m_Y}(qqs) = \frac{1}{\sqrt{2}}\{|\chi^i(qqs)\rangle_+|\frac{1}{2},m_Y\rangle_+ + |\chi^i(qqs)\rangle_-|\frac{1}{2},m_Y\rangle_-\}, \tag{B.19}$$

where

$$\chi_F^{i=1,3}(q\bar{s}) = |K^+\rangle = u\bar{s}, \tag{B.20}$$

$$\chi_F^{i=2}(q\bar{s}) = |K^0\rangle = d\bar{s}, \tag{B.21}$$

$$|\chi^{i=1}(qqs)\rangle_+ = |\Sigma^0\rangle_+ = \frac{1}{\sqrt{12}}(2uds + 2dus - usd - dsu - sud - sdu), \tag{B.22}$$

$$|\chi^{i=1}(qqs)\rangle_- = |\Sigma^0\rangle_- = \frac{1}{2}(usd + dsu - sud - sdu), \tag{B.23}$$

$$|\chi^{i=2}(qqs)\rangle_+ = |\Sigma^+\rangle_+ = \frac{1}{\sqrt{6}}(2uus - usu - suu), \tag{B.24}$$

$$|\chi^{i=2}(qqs)\rangle_- = |\Sigma^+\rangle_- = \frac{1}{\sqrt{2}}(usu - suu), \tag{B.25}$$

$$|\chi^{i=3}(qqs)\rangle_+ = |\Lambda^0\rangle_+ = \frac{1}{2}(usd + sud - dsu - sdu), \tag{B.26}$$

$$|\chi^{i=3}(qqs)\rangle_- = |\Lambda^0\rangle_- = \frac{1}{\sqrt{12}}(2uds - 2dus + sdu - dsu + usd - sud), \tag{B.27}$$

and the spin state wave functions $|\frac{1}{2},m_Y\rangle_\pm$ are given by Eq. (A.39)-Eq. (A.42).

By using the color wave function (Eq. (A.47) and Eq. (A.48)) and the two-body matrix elements (Eq. (A.49), Eq. (A.50) and Eq. (A.51)) with the quark labeling as defined in the spatial part wave functions, the spin-color-flavor weights $\langle f|O_{A_2}|i\rangle$ are calculated as listed in Table B.1.

For the transition amplitude of the quark line diagram A_3 given by Eq. (B.3), the corresponding spatial wave amplitude is given by

$$I^{CQM}_{spatial,A_3} = \int d^3\vec{q}_1...d^3\vec{q}_8 d^3\vec{q}_{1'}...d^3\vec{q}_{4'}\ \varphi_{\phi X}\mathcal{O}^{spatial}_{A_3}\varphi^{CQM}_{uuds\bar{s},\bar{p}} \tag{B.28}$$

Table B.1 The spin-color-flavor weights $\langle f|O_{A_2}|i\rangle$ corresponding to the transition $p\bar{p}(L=0) \to \phi X(\ell_f = 1)$ with the $5q$ component from the chiral quark model.

Transition	$\langle f\|O_{A_2}\|i\rangle$
$^{11}S_0 \to \omega\phi$	$\frac{5\left(-\sqrt{2}b_1+2b_2+\sqrt{6}b_3\right)}{54\sqrt{6}}$
$^{33}S_1 \to \pi^0\phi$	$\frac{5}{486}\left(b_1+\sqrt{2}b_2+3\sqrt{3}b_3\right)$
$^{31}S_0 \to \rho^0\phi$	$\frac{5\left(-\sqrt{2}b_1-2b_2+\sqrt{6}b_3\right)}{54\sqrt{6}}$
$^{13}S_0 \to \eta\phi$	$\frac{5}{486}\left(b_1-\sqrt{2}b_2+3\sqrt{3}b_3\right)$

where

$$\mathcal{O}_{A_3}^{spatial} = Y_{1\lambda}(\vec{q}_1 - \vec{q}_7)\delta^{(3)}(\vec{q}_1 + \vec{q}_7)Y_{1\nu}(\vec{q}_5 - \vec{q}_6)\delta^{(3)}(\vec{q}_5 + \vec{q}_6)$$
$$\delta^{(3)}(\vec{q}_2 - \vec{q}_{1'})\delta^{(3)}(\vec{q}_3 - \vec{q}_{2'})\delta^{(3)}(\vec{q}_4 - \vec{q}_{3'})\delta^{(3)}(\vec{q}_8 - \vec{q}_{4'}). \quad \text{(B.29)}$$

After integrating with the delta functions that are defined in $\mathcal{O}_{A_3}^{spatial}$, we have

$$I_{spatial,A_3}^{ChQM} = \int d^3q_1 d^3q_3 d^3q_4 N\delta^{(3)}(\vec{0})\exp\Big(\frac{1}{24}(-3R^2(\vec{k} - 2\vec{q} - 2\vec{q}_1 + 2\vec{q}_3)^2$$
$$-2((\vec{k} - 3\vec{q} - 3\vec{q}_4)^2 + 3(-\vec{k} + \vec{q} + 2\vec{q}_1 + \vec{q}_4)^2)R_B^2 - 2((-\vec{k} + \vec{q}$$
$$+\vec{q}_1 + 2\vec{q}_3)^2 + 3(-\vec{k} + \vec{q} + \vec{q}_1 + 2\vec{q}_4)^2)R_B^2 - 3(\vec{q} - 2\vec{q}_3)^2 R_M^2$$
$$-3(-\vec{q} + \vec{q}_1 + \vec{q}_3)^2 R_M^2 - 3(\vec{q} + 2\vec{q}_4)^2 R_M^2)\Big)$$
$$Y_{1,\lambda}(2\vec{q}_1)Y_{1,\mu}(\vec{k} - 2\vec{q} - 2\vec{q}_1 + 2\vec{q}_3)$$
$$Y_{1,\nu}(2(\vec{k} - \vec{q} - \vec{q}_1 - \vec{q}_4)) \quad \text{(B.30)}$$

By applying the transformations $\vec{q}_1 = \vec{x}_1 + \alpha_1\vec{k} + \beta_1\vec{q}$, $\vec{q}_3 = \vec{x}_3 + \alpha_3\vec{k} + \beta_3\vec{q}$, $\vec{q}_4 = \vec{x}_4 + \alpha_4\vec{k} + \beta_4\vec{q}$, $\vec{x}_1 = \vec{p}_1$, $\vec{x}_3 = \vec{p}_3 + a_3\vec{p}_1$ and $\vec{x}_4 = \vec{p}_4 + a_4\vec{p}_1$, the spatial wave amplitude is reduced to

$$I_{spatial,A_2}^{ChQM} = \int d^3q_1 d^3q_3 d^3q_4 N\delta^{(3)}(\vec{0})\exp\Big(-Q_k^2\vec{k}^2 - Q_q^2\vec{q}^2 - Q_{kq}^2\vec{k}\cdot\vec{q}$$
$$-Q_{p_1}^2\vec{p}_1^2 - Q_{p_3}^2\vec{p}_3^2 - Q_{p_4}^2\vec{p}_4^2\Big)Y_{1,\lambda}(2(a_4\vec{p}_1 + \vec{p}_4 + \vec{k}\alpha_4 + \vec{q}\beta_4))$$
$$Y_{1,\lambda}(2(\vec{p}_1 + \vec{k}\alpha_1 + \vec{q}\beta_1))Y_{1,\mu}(\vec{k} - 2\vec{q} - 2(\vec{p}_1 + \vec{k}\alpha_1 + \vec{q}\beta_1)$$
$$+2(a_3\vec{p}_1 + \vec{p}_3 + \vec{k}\alpha_3 + \vec{q}\beta_3))Y_{1,\nu}(-2(\alpha_1\vec{k} + \alpha_4\vec{k}$$
$$-\vec{k} + \vec{q} + (a_4 + 1)\vec{p}_1 + \vec{p}_4 + \vec{q}\beta_1 + \vec{q}\beta_4)). \quad \text{(B.31)}$$

The constants appearing in above equation are given by

$$
\begin{aligned}
Q_k^2 =& \frac{1}{24}(8(4\alpha_1^2 + (\alpha_3 + 6\alpha_4 - 5)\alpha_1 + (\alpha_3 - 1)\alpha_3 + 6(\alpha_4 - 1)\alpha_4 \\
&+ 2)R_B^2 + 3((\alpha_1^2 + 2\alpha_3\alpha_1 + 5\alpha_3^2 + 4\alpha_4^2)R_M^2 \\
&+ R^2(-2\alpha_1 + 2\alpha_3 + 1)^2)), \\
Q_q^2 =& \frac{1}{24}(8(4\beta_1^2 + (\beta_3 + 6\beta_4 + 5)\beta_1 + \beta_3^2 + 6\beta_4^2 + \beta_3 + 9\beta_4 \\
&+ 4)R_B^2 + 3((\beta_1^2 + 2(\beta_3 - 1)\beta_1 + \beta_3(5\beta_3 - 6) + 4\beta_4 \\
&(\beta_4 + 1) + 3)R_M^2 + 4R^2(\beta_1 - \beta_3 + 1)^2)), \\
Q_{kq}^2 =& \frac{1}{12}(4(-5\beta_1 - \beta_3 + \alpha_3(\beta_1 + 2\beta_3 + 1) - 6\beta_4 + 3\alpha_4(2\beta_1 \\
&+ 4\beta_4 + 3) + \alpha_1(8\beta_1 + \beta_3 + 6\beta_4 + 5) - 5)R_B^2 \\
&+ 3(2(2\alpha_1 - 2\alpha_3 - 1)(\beta_1 - \beta_3 + 1)R^2 \\
&+ R_M^2(\alpha_1(\beta_1 + \beta_3 - 1) + \alpha_3(\beta_1 + 5\beta_3 \\
&- 3) + 2\alpha_4(2\beta_4 + 1)))), \\
Q_{p_1}^2 =& \frac{1}{24}(8(a_3^2 + a_3 + 6a_4(a_4 + 1) + 4)R_B^2 + 3(4(a_3 \\
&- 1)^2R^2 + (4a_4^2 + a_3(5a_3 + 2) + 1)R_M^2)), \\
Q_{p_3}^2 =& \frac{R_B^2}{3} + \frac{R^2}{2} + \frac{5R_M^2}{8},\ Q_{p_4}^2 = \frac{1}{2}\left(4R_B^2 + R_M^2\right), \qquad \text{(B.32)}
\end{aligned}
$$

$$
\begin{aligned}
\alpha_4 &= \frac{1}{\Delta}(2R_B^2(8R_B^4 + 2(9R^2 + 8R_M^2)R_B^2 + 3(R_M^4 + 5R^2R_M^2))), \\
\alpha_3 &= \frac{1}{\Delta}((2R_B^2 + R_M^2)(8R_B^4 - (4R_B^2 + 3R^2)R_M^2)), \\
\alpha_1 &= \frac{1}{\Delta}(16R_B^6 + 24(3R^2 + 2R_M^2)R_B^4 + 6R_M^2 \\
&\quad (11R^2 + 4R_M^2)R_B^2 + 9R^2R_M^4), \\
\beta_4 &= -\frac{1}{2} - \frac{1}{\Delta}(2R_B^2(6R_B^4 + 3R_M^4 + 2(5R_B^2 + 3R^2)R_M^2)), \\
\beta_3 &= \frac{1}{2} + \frac{1}{\Delta}(-48R_B^6 - 14R_M^2R_B^4 + 4R_M^2(6R^2 + R_M^2)R_B^2 + 6R^2R_M^4), \\
\beta_1 &= -\frac{1}{2\Delta}((2R_B^2 + R_M^2)(-3R_M^4 + 2(7R_B^2 + 6R^2)R_M^2 + 72R_B^2R^2)), \\
a_3 &= \frac{-4R_B^2 + 12R^2 - 3R_M^2}{8R_B^2 + 12R^2 + 15R_M^2},\ a_4 = -\frac{2R_B^2}{4R_B^2 + R_M^2}, \\
\Delta &= 3R_M^6 + 4(13R_B^2 + 6R^2)R_M^4 + 24R_B^2(5R_B^2 \\
&\quad + 6R^2)R_M^2 + 48(R_B^6 + 3R^2R_B^4).
\end{aligned} \tag{B.33}
$$

The spatial wave amplitude for S to P, in the low-momentum approximation, of this diagram is given by

$$I^{ChQM}_{spatial,0,1,A_3} = qF^{A_3}_{0,1}f^{A_3}_{0,1}(\nu,\lambda,\mu,m_f)\exp\left\{-Q_q^2q^2 - Q_k^2k^2\right\}. \tag{B.34}$$

The corresponding geometrical constant are coupled to the spin-angular momentum:

$$F^{A_3}_{0,1}f^{A_3}_{0,1}(\nu,\lambda,\mu,m_f) = \Omega_{1,A_3}f_{1,A_3} + \Omega_{2,A_3}f_{2,A_3} + \Omega_{3,A_3}f_{3,A_3}, \tag{B.35}$$

where

$$\Omega_{1,A_3} = 6g_8N\pi^4\left(a_4+1\right)\sqrt{\frac{1}{Q_{p_1}^{10}}}\left(\frac{1}{Q_{p_3}^2}\right)^{3/2}\left(\frac{1}{Q_{p_4}^2}\right)^{3/2}\left(\beta_1-\beta_3+1\right)\delta(\vec{0}), \tag{B.36}$$

$$f_{1,A_3} = (-1)^\lambda\delta_{\lambda,-\nu}\delta_{\mu,m_f}, \tag{B.37}$$

$$\Omega_{2,A_3} = -6g_8N\pi^4\left(a_3-1\right)\left(a_4+1\right)\sqrt{\frac{1}{Q_{p_1}^{10}}}\left(\frac{1}{Q_{p_3}^2}\right)^{3/2}\left(\frac{1}{Q_{p_4}^2}\right)^{3/2}\delta(\vec{0}) \tag{B.38}$$

$$f_{2,A_3} = (-1)^{\mu}\delta_{\lambda,m_f}\delta_{\mu,-\nu}, \tag{B.39}$$

$$\Omega_{3,A_3} = -6g_8N\pi^4\,(a_3-1)\sqrt{\frac{1}{Q_{p_1}^{10}}}\left(\frac{1}{Q_{p_3}^2}\right)^{3/2}\left(\frac{1}{Q_{p_4}^2}\right)^{3/2}(\beta_1+\beta_4+1)\,\delta(\vec{0}), \tag{B.40}$$

and

$$f_{3,A_3} = (-1)^{\mu}\delta_{\mu,-\lambda}\delta_{\nu,m_f}. \tag{B.41}$$

Using the partial wave amplitude Eq. (B.34) the transition amplitude $T_{if}^{ChQM}(\mathcal{O}_{A_3})$ can be written as

$$T_{if}^{ChQM}(\mathcal{O}_{A_3}) = \sum_{r=1}^{3}\lambda_{A_3}q\Omega_{r,A_3}\exp\left\{-Q_q^2q^2-Q_k^2k^2\right\}\langle f|O_{A_3}|i\rangle_r, \tag{B.42}$$

with the spin-color-flavor weight

$$\langle f|O_{A_3}|i\rangle_r = \langle f|\sum_{\nu,\lambda}(-1)^{\nu+\lambda}\sigma_{-\nu}^{56}\sigma_{-\lambda}^{17}1_F^{56}1_F^{17}1_C^{56}1_C^{17}f_{r,A_3}|i\rangle. \tag{B.43}$$

As the procedure we have done in the first two diagrams, $\langle f|O_{A_3}|i\rangle$ can be decoupled to be

$$\begin{aligned}\langle f|O_{A_3}|i\rangle_r = &\sum_{m_\epsilon,m_f}\sum_{m_\alpha,m_{4,8}}\sum_{m_\epsilon,m_f}\sum_{m_{KY},m_{\bar p}}\sum_{m_i,m_Y}\sum_{m_s,\mu}\sum_{\nu,\lambda}(-1)^{\nu+\lambda}\\ &\langle j,1,m_\epsilon,m_f|J',J_z'\rangle\langle 1,j_m,m_\alpha,m_{4,8}|j,m_\epsilon\rangle\\ &\langle\frac{1}{2},\frac{1}{2},m_{KY},m_{\bar p}|J,J_z\rangle\langle j_i,\frac{1}{2},m_i,m_Y|\frac{1}{2},m_{KY}\rangle\\ &\langle j_s,1,m_s,\mu|j_i,m_i\rangle f_{r,A_3}\langle SCF\rangle_{A_3}^{ChQM}\end{aligned} \tag{B.44}$$

where $\langle SCF\rangle_{A_3}^{ChQM}$ is given by

$$\begin{aligned}\langle SCF\rangle_{A_2}^{ChQM} = &\sum_{i=1}^{3}b_i\langle\chi_{1,m_\alpha}(\phi)\chi_{j_m,m_{4,8}}(X)|\\ &\sigma_{-\nu}^{56}\sigma_{-\lambda}^{17}1_F^{56}1_F^{17}1_C^{56}1_C^{17}|\chi_{j_s,m_s}^{i}(q\bar s)\chi_{\frac{1}{2},m_Y}^{i}(qqs)\chi_{\frac{1}{2},m_{\bar p}}(\bar u\bar u\bar d)\rangle.\end{aligned} \tag{B.45}$$

Similarly, with the quark labeling as used in the spatial wave functions, we have $\langle f|O_{A_3}|i\rangle_r$ as shown in Table B.2.

Table B.2 The spin-color-flavor weight $\langle f|O_{A_3}|i\rangle_r$ corresponding to the transition $p\bar{p}(L=0) \rightarrow \phi X(\ell_f=1)$ with the $5q$ component from the chiral quark model.

Transition	$\langle f\|O_{A_3}\|i\rangle_1$	$\langle f\|O_{A_3}\|i\rangle_2$	$\langle f\|O_{A_3}\|i\rangle_3$
${}^{11}S_0 \rightarrow \omega\phi$	$-\frac{3\sqrt{2}b_1-6b_2+3\sqrt{6}b_3}{162\sqrt{6}}$	$-\frac{7\sqrt{2}b_1-14b_2-3\sqrt{6}b_3}{162\sqrt{6}}$	0
${}^{33}S_1 \rightarrow \pi^0\phi$	$-\frac{3\sqrt{6}b_1-4\sqrt{3}b_2+9\sqrt{2}b_3}{486\sqrt{6}}$	$-\frac{3\sqrt{6}b_1+4\sqrt{3}b_2+9\sqrt{2}b_3}{486\sqrt{6}}$	$-\frac{9\sqrt{6}b_1-3\sqrt{2}b_3}{486\sqrt{6}}$
${}^{31}S_0 \rightarrow \rho^0\phi$	$-\frac{2\sqrt{6}b_3-6b_2}{162\sqrt{6}}$	$-\frac{-6\sqrt{2}b_1-14b_2+2\sqrt{6}b_3}{162\sqrt{6}}$	$-\frac{\sqrt{6}b_3-9\sqrt{2}b_1}{162\sqrt{6}}$
${}^{13}S_1 \rightarrow \eta_{ud}\phi$	$-\frac{\sqrt{2}b_1-2b_2}{243\sqrt{2}}$	$-\frac{\sqrt{2}b_1-2b_2}{243\sqrt{2}}$	$\frac{\sqrt{2}\left(\sqrt{2}b_1-2b_2\right)}{243}$

In order to combine the two transition amplitudes, we choose the radial parameters for the baryons and mesons: $R_B = 3.1\ GeV^{-1}$, $R_M = 4.1\ GeV^{-1}$ (Gutsche et al., 1997) and the size parameter between the quark clusters as $R = 4.1\ GeV^{-1}$. According to the vertex in the two quark line diagram, the effective strength can take the same value, that is $\lambda_{A_2} = \lambda_{A_3} = \lambda_{ChQM}$. We have the total transition amplitude Eq. (B.1) as

$$T_{if}^{ChQM} = \lambda_{ChQM} F_{0,1}^{ChQM} q \exp\left\{-Z_q^2 q^2 - Z_k^2 k^2\right\} \langle f|O_{ChQM}|i\rangle, \tag{B.46}$$

where $F_{0,1}^{ChQM} = \Omega^{A_2} = 4.9 \times 10^{-4}\ GeV^{-11}$, $Z_q \simeq 2.3\ GeV^{-1}$ and $Z_k \simeq 1.3\ GeV^{-1}$. The total spin-color-flavor weight is given by

$$\langle f|O_{ChQM}|i\rangle \simeq \langle f|O_{A_2}|i\rangle + 2(\langle f|O_{A_3}|i\rangle_1 - \langle f|O_{A_3}|i\rangle_2 + \langle f|O_{A_3}|i\rangle_3), \tag{B.47}$$

as listed in Table B.3.

Table B.3 The total spin-color-flavor weight $\langle f|O_{ChQM}|i\rangle$ corresponding to the transition $p\bar{p}(L=0) \to \phi X(\ell_f = 1)$ with the $5q$ component from the chiral quark model.

Transition	$\langle f\|O_{ChQM}\|i\rangle \times 10^{-2}$
$^{11}S_0 \to \omega\phi$	-9.7
$^{33}S_1 \to \pi^0\phi$	3.1
$^{31}S_0 \to \rho^0\phi$	4.0
$^{13}S_0 \to \eta\phi$	1.3

APPENDIX C

TRANSITION AMPLITUDE OF THE PROTON WAVE FUNCTION WITH PENTAQAURK CONFIGURATION

Finally the proton wave function with $5q$ component in form of pentaquark, the ϕ production can only be described by the quark line diagram A_1. Therefore, the transition amplitude has the form as Eq. (A.1) but the 5-quark component $|uuds\bar{s}\rangle$ is defined as

$$\chi_{\frac{1}{2},m_{ps\bar{s}}}(uuds\bar{s}) = |\{\chi_{1/2,m_{\bar{s}}}(\bar{s}) \otimes (\ell = 1, \mu)\}_{j_i,m_i} \otimes \chi_{s,s_z}(uuds)\rangle_{\frac{1}{2},m_{ps\bar{s}}}. \qquad \text{(C.1)}$$

According to this configuration, which is constructed by coupling an $\bar{s}$ quark momentum $\vec{q}_1$ to the 4-quark $uuds$ momentum $\vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5$ with orbital angular momentum $(1, \mu)$, the 5-quark component spatial wave function is given by

$$\begin{aligned} \varphi_{uuds\bar{s}}(\vec{q}_1, .., \vec{q}_5) = N_{uuds\bar{s}} \text{exp}\{-\frac{R_B^2}{2}[\frac{1}{2}(\vec{q}_2 - \vec{q}_3)^2 + \frac{1}{6}(\vec{q}_2 + \vec{q}_3 - 2\vec{q}_4)^2 \\ + \frac{1}{12}(\vec{q}_2 + \vec{q}_3 + \vec{q}_4 - 3\vec{q}_5)^2]\} Y_{1\mu}(\frac{\vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5 - 4\vec{q}_1}{\sqrt{20}}) \\ \exp\left\{-\frac{R_B^2}{2}(\frac{\vec{q}_2 + \vec{q}_3 + \vec{q}_4 + \vec{q}_5 - 4\vec{q}_1}{\sqrt{20}})^2\right\}. \qquad \text{(C.2)} \end{aligned}$$

With the spatial operator $\mathcal{O}_{A_1}^{spatail}$ and the spatial wave function of the antiproton and the mesons as defined in Appendix A, and integrating with the delta functions as done in the case of $s\bar{s}$-sea quark , $I^{uuds}_{spatial}$ can be reduced to

$$\begin{aligned} I^{uuds}_{spatial} = \int d^3q_2 d^3q_3 d^3q_4 N\delta^{(3)}(\vec{0})\exp\Big((-\frac{11\vec{k}^2}{15} + 2\vec{q}\vec{k} - 2\vec{q}^2 - \vec{q}_2^2 - 2\vec{q}_3^2 \\ +\vec{q}\vec{q}_2 - 2\vec{q}_4(-\vec{k} + \vec{q} + \vec{q}_4) - \vec{q}_3(-2\vec{k} + 3\vec{q} + 2\vec{q}_4))R_B^2 \\ -\frac{1}{4}(\vec{q}^2 - 2\vec{q}_2\vec{q} + 2\vec{q}_2^2 + 2\vec{q}_3(\vec{q} + \vec{q}_3))R_M^2\Big) \\ Y_{1,\lambda}(2(\vec{k} - \vec{q} - \vec{q}_3 - \vec{q}_4))Y_{1,\nu}(2\vec{q}_4) \\ Y_{1,\mu}(\frac{\vec{k} - \vec{q} - 4(\vec{q} - \vec{q}_2) + \vec{q}_2}{2\sqrt{5}}) \end{aligned} \tag{C.3}$$

where $N = N_\phi N_X N_{uuds\bar{s}} N_{\bar{p}}$. Applying the transformations $\vec{q}_2 = \vec{x}_2 + \alpha_2\vec{k} + \beta_2\vec{q}$, $\vec{q}_3 = \vec{x}_3 + \alpha_3\vec{k} + \beta_3\vec{q}$, $\vec{q}_4 = \vec{x}_4 + \alpha_4\vec{k} + \beta_4\vec{q}$, $\vec{x}_2 = \vec{p}_2$, $\vec{x}_3 = \vec{p}_3$ and $\vec{x}_4 = b\vec{p}_3 + \vec{p}_4$ with $b = -1/2$, the spatial wave amplitude can be reduced to

$$\begin{aligned} I^{uuds}_{spatial} = \int d^3p_2 d^3p_3 d^3p_4 N\delta^{(3)}(\vec{0})\exp(-Q_k^2\vec{k}^2 - Q_q^2\vec{q}^2 - Q_{kq}^2\vec{k}\cdot\vec{q} \\ -Q_{p_2}^2\vec{p}_2^2 - Q_{p_3}^2\vec{p}_3^2 - Q_{p_4}^2\vec{p}_4^2) \\ Y_{1,\mu}(\frac{\alpha_2\vec{k} + \vec{k} - \vec{q} + \vec{p}_2 + \vec{q}\beta_2 - 4(-\beta_2\vec{q} + \vec{q} - \vec{p}_2 - \vec{k}\alpha_2)}{2\sqrt{5}}) \\ Y_{1,\lambda}(2(-\alpha_3\vec{k} - \alpha_4\vec{k} + \vec{k} - \vec{q} - \frac{\vec{p}_3}{2} - \vec{p}_4 - \vec{q}\beta_3 - \vec{q}\beta_4)) \\ Y_{1,\nu}(2(-\frac{\vec{p}_3}{2} + \vec{p}_4 + \vec{k}\alpha_4 + \vec{q}\beta_4)). \end{aligned} \tag{C.4}$$

The constants depending on the size parameters are given by

$$\begin{aligned} Q_k^2 &= \frac{7R_B^2}{30} - \frac{R_B^4}{2\,(3R_B^2 + R_M^2)},\ Q_q^2 = \frac{1}{8}R_B^2\left(5 - \frac{R_B^2}{3R_B^2 + R_M^2}\right), \\ Q_{kq}^2 &= \frac{1}{2}R_B^2\left(\frac{R_B^2}{3R_B^2 + R_M^2} - 1\right),\ Q_{p_2}^2 = R_B^2 + \frac{R_M^2}{2}, \\ Q_{p_3}^2 &= \frac{1}{2}\left(3R_B^2 + R_M^2\right),\ Q_{p_4}^2 = 2R_B^2, \\ \beta_2 &= \frac{1}{2},\ \alpha_2 = 0,\ \beta_3 = -\frac{4R_B^2 + R_M^2}{2\,(3R_B^2 + R_M^2)}, \\ \alpha_3 &= \frac{R_B^2}{3R_B^2 + R_M^2},\ \beta_4 = -\frac{2R_B^2 + R_M^2}{4\,(3R_B^2 + R_M^2)}, \\ \alpha_4 &= \frac{1}{2} - \frac{R_B^2}{2\,(3R_B^2 + R_M^2)}. \end{aligned} \tag{C.5}$$

As in previous cases, the spatial partial wave amplitude for the S to P transition in the low-momentum approximation is given by

$$I^{uuds}_{spatial,0,1} = qF_{0,1}f_{0,1}(\nu,\lambda,\mu,m_f)\exp\left\{-Q_q^2q^2 - Q_k^2k^2\right\}, \tag{C.6}$$

with the geometrical constant and the spin-angular momentum function are given by

$$F_{0,1} = \frac{3}{8}\sqrt{5}N\pi^4\left(\frac{1}{Q^2_{p_2}}\right)^{3/2}\left(\frac{\left(\frac{1}{Q^2_{p_4}}\right)^{3/2}}{\left(Q^2_{p_3}\right)^{5/2}} - \frac{4\left(\frac{1}{Q^2_{p_3}}\right)^{3/2}}{\left(Q^2_{p_4}\right)^{5/2}}\right)(\beta_2-1)\,\delta^{(3)}(\vec{0}),$$
$$f_{0,1}(\nu,\lambda,\mu,m_f) = (-1)^{\nu}\delta_{\nu,-\lambda}\delta_{\mu,m_f}. \tag{C.7}$$

Decoupling the spin-color-flavor weight $\langle f|O_{A_1}|i\rangle^{uuds}$:

$$\begin{aligned}\langle f|O_{A_1}|i\rangle^{uuds} = \sum_{m_\epsilon,m_f}\sum_{m_\alpha,m_{3,8}}\sum_{m_{ps\bar{s}},m_{\bar{p}}}\sum_{m_i,s_z}\sum_{m_{\bar{s}},\mu}\sum_{\nu,\lambda}&\langle j,1,m_\epsilon,m_f|J',J'_z\rangle\\ &\langle J_s,j_m,m_\alpha,m_{3,8}|j,m_\epsilon\rangle\langle\frac{1}{2},\frac{1}{2},m_{ps\bar{s}},m_{\bar{p}}|J,J_z\rangle\\ &\langle j_i,s,m_i,s_z|\frac{1}{2},m_{ps\bar{s}}\rangle\langle\frac{1}{2},1,m_{\bar{s}},\mu|j_i,m_i\rangle\\ &(-1)^{\nu+\lambda}(-1)^{\nu}\delta_{\nu,-\lambda}\delta_{\mu,m_f}\langle SCF\rangle^{uuds},\end{aligned} \tag{C.8}$$

so the corresponding matrix element is given by

$$\begin{aligned}\langle SCF\rangle^{uuds} = &\langle\chi_{1,m_\alpha}(\phi)\chi_{j_m,m_{3,8}}(X)|\\ &\sigma^{56}_{-\nu}\sigma^{47}_{-\lambda}1^{56}_F1^{47}_F1^{56}_C1^{47}_C|\chi_{\frac{1}{2},m_{\bar{s}}}(\bar{s})\chi_{s,s_z}(uuds)\chi_{\frac{1}{2},m_{\bar{p}}}(\bar{u}\bar{u}\bar{d})\rangle.\end{aligned} \tag{C.9}$$

The wave function $(\chi_{\frac{1}{2},m_{\bar{s}}}(\bar{s})\chi_{s,s_z}(uuds))$ can be substituted by the spin-color-flavor wave function which is given by Eq. (3.75). By using the explicit form of the spin-flavor-color configurations $|[31]_{FS}[211]_F[22]_S\rangle$ and $|[31]_{FS}[22]_F[31]_S\rangle$ as given by Eq. (3.67)– Eq. (3.70) with the two-body matrix elements and the quark labeling as done in case of $s\bar{s}$, the corresponding spin-flavor weights can be obtained as shown in Table C.1.

Table C.1 Spin-flavor matrix elements $\langle f|O_{A_I}|i\rangle$ for the transition $p\bar{p}(L=0) \rightarrow \phi X(\ell_f = 1)$ which are described by quark line diagram A_I with the 5-quark component of a pentaquark.

Transition	$[31][31][22]_{A_1}$	$[31][211][22]_{A_1}$
$^{11}S_0 \rightarrow \omega\phi$	$\frac{5}{36\sqrt{6}}$	$\frac{5}{36\sqrt{6}}$
$^{33}S_1 \rightarrow \pi^0\phi$	$\frac{5}{108\sqrt{2}}$	$\frac{5}{108\sqrt{2}}$
$^{31}S_0 \rightarrow \rho^0\phi$	$\frac{13}{108\sqrt{6}}$	$\frac{13}{108\sqrt{6}}$
$^{13}S_1 \rightarrow \eta_{ud}\phi$	$\frac{1}{36\sqrt{2}}$	$\frac{1}{36\sqrt{2}}$

APPENDIX D

THE CONSTRUCTION OF THE PENTAQUARK WAVE FUNCTIONS

The spin, color and flavor wave functions of the q^4 configuration of the pentaquark can be worked out in the the Yamanochi technique (Yan, 2006). Here we present some detail description to illustrate how to construct the presentation matrices of the irreducible representations of S_4. For instance, there are three Young tableaux for [3, 1] with three corresponding basis functions:

$$
\begin{aligned}
\phi_{\eta}^{[3,1]} &= \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array} = |[3,1](2111)\rangle, \\
\phi_{\lambda}^{[3,1]} &= \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1}\end{array} = |[3,1](1211)\rangle, \\
\phi_{\rho}^{[3,1]} &= \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array} = |[3,1](1121)\rangle,
\end{aligned} \tag{D.1}
$$

where $\phi_r^{[\bar{\lambda}]} = |[\bar{\lambda}](r_n, r_{n-1}, .., r_2, r_1)\rangle$ represents the Yamanochi basis or standard basis where r_i stands for the row from which a box is removed. Thus, there are three projection operators for the irreducible representations [3, 1]:

$$
\begin{aligned}
W_{(2111)}^{[3,1]} &= \frac{3}{4!}\sum_i \langle[3,1](2111)|P_i|[3,1](2111)\rangle P_i, \\
W_{(1211)}^{[3,1]} &= \frac{3}{4!}\sum_i \langle[3,1](1211)|P_i|[3,1](1211)\rangle P_i, \\
W_{(1121)}^{[3,1]} &= \frac{3}{4!}\sum_i \langle[3,1](1121)|P_i|[3,1](1121)\rangle P_i,
\end{aligned} \tag{D.2}
$$

where P_i stand for all the permutations of S_4 and the factor 3 is the dimension of the representation [3, 1]. In order to evaluate the representation matrices for the permutation P_i, in case of permutation between object $n-1$ and n , we apply the operation of the element $(n-1,n)$ on the standard basis which satisfies the following:

$$(n-1,n)|[\bar{\lambda}](r,r,r_{n-2},..,r_2,1)\rangle = |[\bar{\lambda}](r,r,r_{n-2},..,r_2,1)\rangle,$$
$$(n-1,n)|[\bar{\lambda}](r,r-1,r_{n-2},..,r_2,1)\rangle = -|[\bar{\lambda}](r,r-1,r_{n-2},..,r_2,1)\rangle, \qquad \text{(D.3)}$$

when $|[\bar{\lambda}](r,r-1,r_{n-2},..,r_2,1)\rangle$ dose not exist, and

$$(n-1,n)|[\bar{\lambda}](r,s,r_{n-2},..,r_2,r_1)\rangle = \sigma_{rs}|[\bar{\lambda}](r,s,r_{n-2},..,r_2,r_1)\rangle + \sqrt{1-\sigma_{rs}^2}|[\bar{\lambda}](s,r,r_{n-2},..,r_2,r_1)\rangle, \qquad \text{(D.4)}$$

where

$$\sigma_{rs} = \frac{1}{(\lambda_r - r) - (\lambda_s - s)}, \qquad \text{(D.5)}$$

for $|[\bar{\lambda}](r,s,r_{n-2},..,r_2,r_1)\rangle$ and $|[\bar{\lambda}](s,r,r_{n-2},..,r_2,r_1)\rangle$ all exist and $r \neq s$. While the matrices of the elements (i,n) can be obtained by using the relation

$$(i,n) = (n-1,n)(i,n-1)(n-1,n). \qquad \text{(D.6)}$$

For example, the matrix of the element $(3,4)$ is:

$$D^{[3,1]}(34) = \begin{pmatrix} \langle\phi_\eta^{[3,1]}|(34)|\phi_\eta^{[3,1]}\rangle & \langle\phi_\lambda^{[3,1]}|(34)|\phi_\eta^{[3,1]}\rangle & \langle\phi_\rho^{[3,1]}|(34)|\phi_\eta^{[3,1]}\rangle \\ \langle\phi_\eta^{[3,1]}|(34)|\phi_\lambda^{[3,1]}\rangle & \langle\phi_\lambda^{[3,1]}|(34)|\phi_\lambda^{[3,1]}\rangle & \langle\phi_\rho^{[3,1]}|(34)|\phi_\lambda^{[3,1]}\rangle \\ \langle\phi_\eta^{[3,1]}|(34)|\phi_\rho^{[3,1]}\rangle & \langle\phi_\lambda^{[3,1]}|(34)|\phi_\rho^{[3,1]}\rangle & \langle\phi_\rho^{[3,1]}|(34)|\phi_\rho^{[3,1]}\rangle \end{pmatrix}$$
$$= \begin{pmatrix} -1/3 & 2\sqrt{2}/3 & 0 \\ 2\sqrt{2}/3 & 1/3 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \qquad \text{(D.7)}$$

A cycle permutation can be resolved into a product of transpositions, where we have

$$D(ijk) = D(ik)D(ij),$$
$$D(ijkl) = D(il)D(ik)D(ij). \qquad \text{(D.8)}$$

By applying Eq. (D.6) and Eq. (D.8) with the matrix of the element $(n-1, n)$, all of the permutation matrices $D^{[3,1]}$ can be obtained such as

$$D(123) = D(13)D(12) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{3} & -\frac{\sqrt{2}}{3} & \sqrt{\frac{2}{3}} \\ -\frac{\sqrt{2}}{3} & \frac{5}{6} & \frac{1}{2\sqrt{3}} \\ \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}} & \frac{1}{2} \end{pmatrix}, \tag{D.9}$$

$$D(1234) = D(14)D(13)D(12)$$

$$= \begin{pmatrix} -\frac{1}{3} & -\frac{\sqrt{2}}{3} & -\sqrt{\frac{2}{3}} \\ -\frac{\sqrt{2}}{3} & \frac{5}{6} & -\frac{1}{2\sqrt{3}} \\ -\sqrt{\frac{2}{3}} & -\frac{1}{2\sqrt{3}} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{1}{3} & \frac{2\sqrt{2}}{3} & 0 \\ -\frac{\sqrt{2}}{3} & -\frac{1}{6} & \frac{\sqrt{3}}{2} \\ -\sqrt{\frac{2}{3}} & \frac{1}{4\sqrt{3}} - \frac{\sqrt{3}}{4} & -\frac{1}{2} \end{pmatrix}. \tag{D.10}$$

Substituting $\langle \phi_i^{[3,1]} | (34) | \phi_i^{[3,1]} \rangle$ for all permutations into Eq. (D.2) we obtain the projection operator

$$\begin{aligned} W_{(2111)}^{[3,1]} = \frac{1}{8}(&P_1 - \frac{1}{3}P_{1,4}P_{2,3} - \frac{1}{3}P_{1,3}P_{2,4} - \frac{1}{3}P_{1,2}P_{3,4} + P_{1,2} + P_{1,3} \\ &- \frac{P_{1,4}}{3} + P_{2,3} - \frac{P_{2,4}}{3} - \frac{P_{3,4}}{3} + P_{1,2,3} - \frac{1}{3}P_{1,2,4} + P_{1,3,2} \\ &- \frac{1}{3}P_{1,3,4} - \frac{1}{3}P_{1,4,2} - \frac{1}{3}P_{1,4,3} - \frac{1}{3}P_{2,3,4} - \frac{1}{3}P_{2,4,3} \\ &- \frac{1}{3}P_{1,2,3,4} - \frac{1}{3}P_{1,2,4,3} - \frac{1}{3}P_{1,3,2,4} - \frac{1}{3}P_{1,3,4,2} \\ &- \frac{1}{3}P_{1,4,2,3} - \frac{1}{3}P_{1,4,3,2}), \end{aligned} \tag{D.11}$$

$$
\begin{aligned}
W^{[3,1]}_{(1211)} = \frac{1}{8}(P_1 &- \frac{2}{3}P_{1,4}P_{2,3} - \frac{1}{6}P_{1,3}P_{2,4} + \frac{1}{3}P_{1,2}P_{3,4} + P_{1,2} - \frac{P_{1,3}}{2} \\
&+ \frac{5P_{1,4}}{6} - \frac{P_{2,3}}{2} + \frac{5P_{2,4}}{6} + \frac{P_{3,4}}{3} - \frac{1}{2}P_{1,2,3} + \frac{5}{6}P_{1,2,4} \\
&- \frac{1}{2}P_{1,3,2} - \frac{1}{6}P_{1,3,4} + \frac{5}{6}P_{1,4,2} - \frac{1}{6}P_{1,4,3} - \frac{2}{3}P_{2,3,4} \\
&- \frac{2}{3}P_{2,4,3} - \frac{1}{6}P_{1,2,3,4} - \frac{1}{6}P_{1,2,4,3} - \frac{2}{3}P_{1,3,2,4} \\
&- \frac{1}{6}P_{1,3,4,2} - \frac{2}{3}P_{1,4,2,3} - \frac{1}{6}P_{1,4,3,2}), \qquad \text{(D.12)}
\end{aligned}
$$

and

$$
\begin{aligned}
W^{[3,1]}_{(1121)} = \frac{1}{8}(P_1 &+ \frac{1}{2}P_{1,3}P_{2,4} - P_{1,2}P_{3,4} - P_{1,2} + \frac{P_{1,3}}{2} + \frac{P_{1,4}}{2} + \frac{P_{2,3}}{2} \\
&+ \frac{P_{2,4}}{2} + P_{3,4} - \frac{1}{2}P_{1,2,3} - \frac{1}{2}P_{1,2,4} - \frac{1}{2}P_{1,3,2} + \frac{1}{2}P_{1,3,4} \\
&- \frac{1}{2}P_{1,4,2} + \frac{1}{2}P_{1,4,3} - \frac{1}{2}P_{1,2,3,4} - \frac{1}{2}P_{1,2,4,3} \\
&- \frac{1}{2}P_{1,3,4,2} - \frac{1}{2}P_{1,4,3,2}), \qquad \text{(D.13)}
\end{aligned}
$$

where P denotes the permutation on the objects that are labeled by the subscript. Similarly, for the irreducible representation [2, 2] and [2, 1, 1] which correspond to the Young tableaux

$$
\begin{aligned}
\phi^{[2,2]}_{\lambda} &= \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array} = |[2,2](2211)\rangle, \\
\phi^{[2,2]}_{\rho} &= \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline\end{array} = |[2,2](2121)\rangle, \qquad \text{(D.14)}
\end{aligned}
$$

$$
\begin{aligned}
\phi^{[2,1,1]}_{\eta} &= \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 \\ \cline{1-1} 3 \\ \cline{1-1}\end{array} = |[2,1,1](2111)\rangle, \\
\phi^{[2,1,1]}_{\lambda} &= \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array} = |[2,1,1](1211)\rangle, \\
\phi^{[2,1,1]}_{\rho} &= \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array} = |[2,1,1](1121)\rangle, \qquad \text{(D.15)}
\end{aligned}
$$

we have

$$W^{[2,2]}_{(2211)} = \frac{1}{24}(2P_1 + 2P_{1,4}P_{2,3} + 2P_{1,3}P_{2,4} + 2P_{1,2}P_{3,4} + 2P_{1,2} - P_{1,3} - P_{1,4} - P_{2,3} - P_{2,4} + 2P_{3,4} - P_{1,2,3} - P_{1,2,4} - P_{1,3,2} - P_{1,3,4} - P_{1,4,2} - P_{1,4,3} - P_{2,3,4} - P_{2,4,3} - P_{1,2,3,4} - P_{1,2,4,3} + 2P_{1,3,2,4} - P_{1,3,4,2} + 2P_{1,4,2,3} - P_{1,4,3,2}), \quad (D.16)$$

$$W^{[2,2]}_{(2121)} = \frac{1}{24}(2P_1 + 2P_{1,4}P_{2,3} + 2P_{1,3}P_{2,4} + 2P_{1,2}P_{3,4} - 2P_{1,2} + P_{1,3} + P_{1,4} + P_{2,3} + P_{2,4} - 2P_{3,4} - P_{1,2,3} - P_{1,2,4} - P_{1,3,2} - P_{1,3,4} - P_{1,4,2} - P_{1,4,3} - P_{2,3,4} - P_{2,4,3} + P_{1,2,3,4} + P_{1,2,4,3} - 2P_{1,3,2,4} + P_{1,3,4,2} - 2P_{1,4,2,3} + P_{1,4,3,2}), \quad (D.17)$$

$$W^{[2,1,1]}_{(2111)} = \frac{1}{8}(P_1 - \frac{1}{3}P_{1,4}P_{2,3} - \frac{1}{3}P_{1,3}P_{2,4} - \frac{1}{3}P_{1,2}P_{3,4} - P_{1,2} - P_{1,3} + \frac{P_{1,4}}{3} - P_{2,3} + \frac{P_{2,4}}{3} + \frac{P_{3,4}}{3} + P_{1,2,3} - \frac{1}{3}P_{1,2,4} + P_{1,3,2} - \frac{1}{3}P_{1,3,4} - \frac{1}{3}P_{1,4,2} - \frac{1}{3}P_{1,4,3} - \frac{1}{3}P_{2,3,4} - \frac{1}{3}P_{2,4,3} + \frac{1}{3}P_{1,2,3,4} + \frac{1}{3}P_{1,2,4,3} + \frac{1}{3}P_{1,3,2,4} + \frac{1}{3}P_{1,3,4,2} + \frac{1}{3}P_{1,4,2,3} + \frac{1}{3}P_{1,4,3,2}), \quad (D.18)$$

$$W^{[2,1,1]}_{(1211)} = \frac{1}{8}(P_1 + \frac{1}{2}P_{1,3}P_{2,4} - P_{1,2}P_{3,4} + P_{1,2} - \frac{P_{1,3}}{2} - \frac{P_{1,4}}{2} - \frac{P_{2,3}}{2} - \frac{P_{2,4}}{2} - P_{3,4} - \frac{1}{2}P_{1,2,3} - \frac{1}{2}P_{1,2,4} - \frac{1}{2}P_{1,3,2} + \frac{1}{2}P_{1,3,4} - \frac{1}{2}P_{1,4,2} + \frac{1}{2}P_{1,4,3} + \frac{1}{2}P_{1,2,3,4} + \frac{1}{2}P_{1,2,4,3} + \frac{1}{2}P_{1,3,4,2} + \frac{1}{2}P_{1,4,3,2}), \quad (D.19)$$

$$W^{[2,1,1]}_{(1121)} = \frac{1}{8}(P_1 - \frac{2}{3}P_{1,4}P_{2,3} - \frac{1}{6}P_{1,3}P_{2,4} + \frac{1}{3}P_{1,2}P_{3,4} - P_{1,2} + \frac{P_{1,3}}{2} - \frac{5P_{1,4}}{6} + \frac{P_{2,3}}{2} - \frac{5P_{2,4}}{6} - \frac{P_{3,4}}{3} - \frac{1}{2}P_{1,2,3} + \frac{5}{6}P_{1,2,4} - \frac{1}{2}P_{1,3,2} - \frac{1}{6}P_{1,3,4} + \frac{5}{6}P_{1,4,2} - \frac{1}{6}P_{1,4,3} + \frac{1}{6}P_{1,2,4,3} + \frac{2}{3}P_{1,3,2,4} + \frac{1}{6}P_{1,3,4,2} - \frac{2}{3}P_{2,3,4} - \frac{2}{3}P_{2,4,3} + \frac{1}{6}P_{1,2,3,4} + \frac{2}{3}P_{1,4,2,3} + \frac{1}{6}P_{1,4,3,2}). \quad \text{(D.20)}$$

By acting the obtained projection operators onto the four quark state $uuds$, the flavor wave functions are obtained as given by Eq. (3.53)– Eq. (3.60). The two corresponding spin wave functions $\chi_{[22]_{S_\lambda}}$ and $\chi_{[22]_{S_\rho}}$ can be constructed by the substitutions $u \leftrightarrow \uparrow$ and $d, s \leftrightarrow \downarrow$ in the flavor wave function, $\chi_{[22]_{F_\lambda}}$ and $\chi_{[22]_{F_\rho}}$, with an additional $1/\sqrt{2}$ in the normalization factor. In analogy, the color symmetry [211] is constructed by replacing $u \leftrightarrow R$, $d \leftrightarrow G$ and $s \leftrightarrow B$ in the flavor wave function $\chi_{[211]_F}$.

Finally, in order to complete the q^4 wave function we have to couple the [211] color with [31] the spin-flavor wave function to be [31] spin-color-flavor wave function for the $uuds$ configuration. We write down all possible multiplications of $\chi_{[211]_{C_i}}$ with $\chi_{[31]_{FS_j}}$ in form of the linear combination

$$\chi_{[31]_{CSF}} = \sum_{i,j=\lambda,\rho,\eta} a_{i,j}\chi_{[211]_{C_i}}\chi_{[31]_{FS_j}}, \quad \text{(D.21)}$$

where $a_{i,j}$ is the coefficient for making the combination correspond to the [31] symmetry of the spin-color-flavor wave function. There are only three components that give the four quark configuration which is antisymmetric,

$$\chi_{[31]_{CSF}} = a_{\lambda,\rho}\chi_{[211]_{C_\lambda}}\chi_{[31]_{FS_\rho}} + a_{\rho,\lambda}\chi_{[211]_{C_\rho}}\chi_{[31]_{FS_\lambda}} + a_{\eta,\eta}\chi_{[211]_{C_\eta}}\chi_{[31]_{FS_\eta}}. \quad \text{(D.22)}$$

Transforming $\chi_{[31]_{CSF}}$ with the corresponding permutation $D(34)$, we have

$$\begin{aligned}(34)\chi_{[31]_{CSF}} = &-a_{\lambda,\rho}\chi_{[211]_{C_\lambda}}\chi_{[31]_{FS_\rho}} \\ &+a_{\rho,\lambda}\big(-\frac{1}{3}\chi_{[211]_{C_\rho}}+\frac{2\sqrt{2}}{3}\chi_{[211]_{C_\eta}}\big)\big(\frac{1}{3}\chi_{[31]_{FS_\lambda}}+\frac{2\sqrt{2}}{3}\chi_{[31]_{FS_\eta}}\big) \\ &+a_{\eta,\eta}\big(\frac{2\sqrt{2}}{3}\chi_{[211]_{C_\rho}}+\frac{1}{3}\chi_{[211]_{C_\eta}}\big)\big(\frac{2\sqrt{2}}{3}\chi_{[31]_{FS_\lambda}}-\frac{1}{3}\chi_{[31]_{FS_\eta}}\big). \quad \text{(D.23)}\end{aligned}$$

The antisymmetric wave function require that

$$(34)\chi_{[31]_{CSF}} = -\chi_{[31]_{CSF}}, \quad \text{(D.24)}$$

this give the condition $a_{\rho,\lambda} = -a_{\eta,\eta}$ while $a_{\lambda,\rho}$ is an arbitrary number. By choosing $a_{\lambda,\rho} = 1$ and normalizing $\chi_{[31]_{CSF}}$ with the obtained condition we get

$$\chi_{[31]_{CSF}} = \frac{1}{\sqrt{3}}\big(\chi_{[211]_{C_\lambda}}\chi_{[31]_{FS_\rho}} - \chi_{[211]_{C_\rho}}\chi_{[31]_{FS_\lambda}} + \chi_{[211]_{C_\eta}}\chi_{[31]_{FS_\eta}}\big), \quad \text{(D.25)}$$

as given by Eq. (3.66). For the spin-flavor $\chi_{[31]_{FS}}$ such as $|[31]_{FS_\eta}[31]_F[22]_S\rangle$ it corresponds to the Young tableau

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array} \quad \text{(D.26)}$$

There are only two products of the spin $\chi_{[22]_S}$ and flavor $\chi_{[31]_F}$ wave functions, that are consistent with the characteristic permutation of $\chi_{[31]_{FS}}$

$$\begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array} \otimes \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline\end{array}, \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array} \otimes \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline\end{array} \quad \text{(D.27)}$$

Hence, the spin-flavor $|[31]_{FS_\eta}[31]_F[22]_S\rangle$ can be assumed as

$$|[31]_{FS_\eta}[31]_F[22]_S\rangle = a_{\rho,\rho}\chi_{[31]_{F_\rho}}\chi_{[22]_{S_\rho}} + a_{\lambda,\lambda}\chi_{[31]_{F_\lambda}}\chi_{[22]_{S_\lambda}}. \quad \text{(D.28)}$$

The coefficients $a_{\rho,\rho}$ and $a_{\lambda,\lambda}$ can be evaluated by acting the matrix $D^{[3,1]}(123)$ (Eq. (D.9)) and

$$D^{[22]}(123) = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \quad \text{(D.29)}$$

onto the corresponding wave function of Eq. (D.28)

$$D^{[31]}(123)|[31]_{FS_\eta}[31]_F[22]_S\rangle = a_{\rho,\rho}D^{[31]}(123)\chi_{[31]_{F_\rho}}D^{[22]}(123)\chi_{[22]_{S_\rho}} + a_{\lambda,\lambda}D^{[31]}(123)\chi_{[31]_{F_\lambda}}D^{[22]}(123)\chi_{[22]_{S_\lambda}}. \quad \text{(D.30)}$$

We obtain

$$|[31]_{FS_\eta}[31]_F[22]_S\rangle = a_{\rho,\rho}(-\frac{1}{2}\chi_{[31]_{F_\rho}} - \frac{\sqrt{3}}{2}\chi_{[31]_{F_\lambda}})(-\frac{1}{2}\chi_{[22]_{S_\rho}} - \frac{\sqrt{3}}{2}\chi_{[22]_{S_\lambda}}) + a_{\lambda,\lambda}(-\frac{1}{2}\chi_{[31]_{F_\lambda}} + \frac{\sqrt{3}}{2}\chi_{[31]_{F_\rho}})(\frac{\sqrt{3}}{2}\chi_{[22]_{S_\rho}} - \frac{1}{2}\chi_{[22]_{S_\lambda}}). \quad \text{(D.31)}$$

Taking Eq. (D.28) equal to Eq. (D.31) and normalizing gives $a_{\rho,\rho} = a_{\lambda,\lambda} = 1/\sqrt{2}$ so we have the explicit form of the $|[31]_{FS_\eta}[31]_F[22]_S\rangle$ as given by Eq. (3.67).

CURRICULUM VITAE

NAME Mr. Sorakrai Srisuphaphon

DATE OF BIRTH September 13, 1976

PLACE OF BIRTH Bangkok, Thailand

PERSONAL ADDRESS 67/108, Pracha Chuen 33 Bang Sue,
Bangkok 10800, Thailand
Tel. 66-2587-3861, Mobile. 66-83-9090427

E-MAIL Edsorakrai@gmail.com

EDUCATIONS

Master of Science (1998 – 2002)

Department of Physics, Chulalongkorn University, Thailand.

Supervisors: Professor Virulh Sa-yakanit, F.D.

Bachelor of Science (1994 – 1998)

Department of Physics, Srinakharinwiron University, Thailand.

WORKING

Assistant Researcher (1998)

Ultrasonic Sound Laboratory, Department of Physics, Srinakharinwiron University, Thailand.

Lecturer (2002 – present)

Department of Physics Burapha University, Thailand.