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BARYON CHIRAL PERTURBATION THEORY WITH VIRTUAL PHOTONS AND LEPTONS

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BARYON CHIRAL PERTURBATION THEORY

WITH VIRTUAL PHOTONS AND LEPTONS

Suranaree University of Technology has approved this thesis submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy.

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วิทยานิพนธ์นี้เกี่ยวข้องกับการสร้างลากรานเจียนทั่วไปของระบบไพออนและนิวคลีออนในทฤษฎีการรบกวนแบบไครัลสำหรับแบริออนเชิงสัมพัทธภาพโดยมีการรวมโฟตอนเสมือนและเลปตอนเข้าไปด้วยโดยพิจารณาจนถึงอันดับที่สี่ ในการศึกษาครั้งนี้การรวมเลปตอนเบาเข้าไปเป็นองศาเสรีแบบชัดแจ้งทำได้โดยการเสนอส่วนประกอบใหม่ซึ่งเป็นตัวแทนของเลปตอนเหล่านั้น จากนั้นนำผลของลากรานเจียนที่ได้มาประมาณค่าตรวจแก้ของการแผ่รังสี ที่มีต่อแอมพลิจูดของปฏิกิริยาการสลายของนิวตรอนแบบเบตา ซึ่งเกิดจากเทอมใหม่นั้น ผลของแอมพลิจูดที่ได้จะนำมาซึ่งเวกเตอร์และเวกเตอร์แกนของกระแสอย่างอ่อนของนิวคลีออน และเมื่อกระจายแล้วจะได้ค่าของแฟกเตอร์รูปแบบต่างๆ ที่อยู่ในเทอมของค่าคงที่พลังงานต่ำที่เกิดจากเทอมใหม่นั้นที่มีต่อค่าการตรวจแก้ของการแผ่รังสี ในที่นี้เราจะไม่พิจารณาแผนภาพแบบห่วงทั้งหมด เราทำการคำนวณแบบห่วงสำหรับการรีนอมัลไลซ์ฟังก์ชันคลื่นเท่านั้น

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CHIRAL PERTURBATION THEORY/ChPT

This thesis is concerned with the construction of the general pion-nucleon Lagrangian with the inclusion of both virtual photons and leptons for relativistic baryon chiral perturbation theory to fourth order. We include the light leptons as explicit dynamical degrees of freedom by introducing new building blocks which represent these leptons. The result is used to evaluate the contributions of these new terms to the radiative corrections to the amplitude for neutron beta decay. Then we get the full vector and axial vector weak nucleon currents which, allows us to extract explicit expressions for the contributions of the Low Energy Constants from these new terms to the radiative corrections to the various form factors. We do not consider all of the loop diagrams. We do the loop calculations only for the wavefunction renormalizations.

School of Physics	Student's Signature ________________
Academic Year 2009	Advisor's Signature ________________

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CHAPTER I

INTRODUCTION

In the Standard Model (SM) hadrons are comprised of quarks which are held together by strong forces. Quantum Chromodynamics (QCD) is used to formulate in terms of quarks and leptons degrees of freedom and its fundamental coupling constant is the strong coupling constant α_S. At high energies, observables can be expanded in terms of α_S. But at low energies, the strong coupling constants become large so that the perturbative methods are not applicable. This leads to use of effective field theory (EFT) method which is called Chiral Perturbation Theory (ChPT). ChPT is just the EFT of the SM at low energies. It was initiated by S. Weinberg in 1979 (Weinberg, 1979) and then developed by J. Gasser and H. Leutwyler in 1984 and 1985 (Gasser and Leutwyler, 1984; Gasser and Leutwyler, 1985) and described in terms of the degrees of freedom relevant to low-energy strong processes. Physical quantities are calculated as expansions in terms of small momenta. The ChPT Lagrangian contains an infinite number of terms and Feynman diagrams contributing to any physical process can be derived.

ChPT has proven to be a useful method for studying low energy processes for example $\pi\pi$ scattering which is a fundamental process for QCD at low energies (Bijnens et al., 1996; Bijnens et al., 1997). However, the extension to processes which involve a nucleon (Gasser et al., 1988) caused some problems. In the mesonic sector the Feynman diagrams which are relevant for calculation are chosen by a scheme called Weinberg's power counting (Weinberg, 1979). It gives a chiral order D to each diagram. But in the case with nucleon there are terms which do not obey the power counting. These problems can be solved by the application of a renormalization scheme, which regenerates the power counting. The different methods were subsequently developed for the description of the baryonic sector. Heavy baryon chiral perturbation theory (HBChPT) received most success (Bernard et al., 1992; Ecker and Mojžiš, 1996). HBChPT is constructed similarly as heavy-quark effective theory. The nucleon field is divided into two components which are heavy and light where the heavy components are integrated out. Subsequently the nonlocal contributions created by integrating out, the heavy components are expanded in local interaction terms suppressed with powers of the nucleon mass. Later the new manifestly Lorentz-invariant formulations of baryonic ChPT have been introduced (Ellis and Tang, 1998; Becher and Leutwyler, 1999; Gegelia and Japaridze, 1999; Goity et al., 2001; Schindler et al., 2004). These formulations were used to restore the power counting in the baryonic sector.

The complete chiral Lagrangian in baryonic case based on the relativistic ChPT has been constructed up to fourth order and applied to describe the dynamics of πN scattering (Fettes et al., 1998; Fettes and Meißner, 2000; Fuchs et al., 2003). In the framework of ChPT the presence of the electromagnetism was originally proposed by Urech (Urech, 1995). In principle, it is straightforward to establish the theoretical framework for the description of EM effects. First, the photon field was included as an additional dynamical degree of freedom and then the most general Lagrangian of the desired order was constructed. The divergences of the generating function to one loop were calculated and the structure of the local action that incorporates the counterterms which cancel the divergences was determined. This method was used to calculate the electromagnetic corrections for the low energy $\pi\pi$ scattering (Ecker et al., 1989; Meißner et al., 1997; Knecht and

Urech, 1998). About a decade ago, the general Lagrangian with virtual photons for baryonic case was constructed by Müller and Meißner with the same procedure (Müller and Meißner, 1999). They applied this method to calculate the nucleon self energy, nucleon mass and the scalar form factor of the nucleon based on the heavy baryon chiral perturbation theory.

The inclusion of virtual photons and leptons in the chiral Lagrangian was introduced for mesonic sector by Knecht et al. (2000). The full treatment of isospin breaking effects in semileptonic weak interaction was allowed. They enlarge the ChPT Lagrangian with virtual photons (Urech, 1995) by including the light leptons as dynamical degree of freedom and determine the additional one-loop divergences generated by the presence of virtual leptons and give the full list of associated counterterms. This method was applied to the pion and kaon decays to calculate their decay rates.

There has been no development of the chiral Lagrangian with both virtual photons and leptons for baryonic sector. Therefore, in this thesis we construct the general pion-nucleon Lagrangian in which both virtual photons and leptons are included. Then, we use this Lagrangian to calculate the tree level contributions of the new terms involving photons to the weak form factors. This is an important first step in the calculation of radiative corrections to weak processes, such as beta decay or muon capture, in the framework of ChPT. We then consider as a specific example radiative corrections to neutron beta decay.

In Chapter II, a review of ChPT with the construction of the Lagrangian both in mesonic and baryonic sectors will be presented. The inclusion of virtual photons and leptons to the mesonic Lagrangian and virtual photons in the pion-nucleon Lagrangian up to fourth order will be shown in Chapter III. In Chapter IV, we construct the new Lagrangian in the pion-nucleon sector, including both virtual photons and leptons. Renormalization scheme and the calculations of the wavefunction renormalizations of pion, nucleons and leptons are in Chapter V. In Chapter VI we apply our new Lagrangian to the neutron beta decay to evaluate the weak form factors to the nucleon current. A summary and conclusion can be found in Chapter VII, while the appendices contain theoretical details.

CHAPTER II

CHIRAL PERTURBATION THEORY

Chiral perturbation theory is based on an effective Lagrangian which can be used to describe strong interactions at low energies. In the effective Lagrangian, the quark and gluon fields are replaced by meson and baryon fields and the quark interactions are replaced by a series of effective vertices. Since the effective vertices are reformulated from QCD, they must possess the same symmetry properties, which are chiral, Lorentz, parity, charge conjugation and time reversal symmetries.

In this chapter, the definition of chiral symmetry will be explained and the building blocks which are used to construct the effective Lagrangian for both pion and pion-nucleon systems will be introduced.

2.1 Chiral symmetry in QCD

The form of the QCD Lagrangian is

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle + \bar{q}(i\gamma^{\mu}D_{\mu} - M)q. \tag{2.1}$$

The matrix $G_{\mu\nu}$ is the gluon field strength tensor, the vector $\bar{q}$ and q are the quark fields, D_μ is the gauge covariant derivative, and M is quark mass matrix.

The quark field is decomposed into the sum of the left and right handed helicity components,

$$q = \frac{1}{2}(1-\gamma_5)q + \frac{1}{2}(1+\gamma_5)q = q_L + q_R. \tag{2.2}$$

By using this, the QCD Lagrangian is then written as

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}\langle G_{\mu\nu}G^{\mu\nu}\rangle + \bar{q}_R(i\gamma^\mu D_\mu - M)q_R + \bar{q}_L(i\gamma^\mu D_\mu - M)q_L$$
$$-\bar{q}_R M q_L - \bar{q}_L M q_R. \tag{2.3}$$

We notice that if $M = 0$, q_L and q_R do not interact with each other. In this case they are each invariant under its own transformations, and there is a new symmetry group $SU(N)_R \times SU(N)_L$ which is referred to as chiral symmetry group.

One assumes that the vacuum state does not obey chiral symmetry even be invariant under $SU(N)$. From this result $SU(N)_R \times SU(N)_L$ is spontaneously broken down to $SU(N)$. Goldstone's theorem predicts the occurrence of $N^2 - 1$ massless bosons which are called pseudoscalar Goldstone bosons.

The mass of the three lightest quarks, the up, down, and strange, are small compared to typical energy scales of QCD. This means that the chiral symmetry is valid for just the light quarks:

$$q = \begin{bmatrix} u \\ d \\ s \end{bmatrix}. \tag{2.4}$$

The spontaneous breaking of this chiral symmetry form the pseudoscalar octet.

In this work, the strange quark mass is considered to be large at very low energies so only the up and down quarks are involved

$$q = \begin{bmatrix} u \\ d \end{bmatrix}, \tag{2.5}$$

which is invariant under the chiral symmetry group $SU(2)_R \times SU(2)_L$. The spontaneous breaking of this symmetry predicts the occurrence of three massless bosons, which are the pions.

2.2 Pion chiral perturbation theory

2.2.1 Definitions

The pion fields obey an $SU(2)$ symmetry which is isospin symmetry. This symmetry was created to describe the symmetry of the nucleons where the masses of proton and neutron are almost identical. Then the nucleons can be treated as a single particle with isospin state either up or down.That means each state represents either proton or neutron.

This spin symmetry is used for the three pion states, with the isospin quantum number $I_3 = 0, \pm 1$ instead of $I_3 = \pm 1/2$. But this representation is not used in chiral perturbation theory. One can write a new representation for the $SU(2)$ symmetry by using a three dimensional basis composed of the Pauli matrices.Then he pion wavefunction is written as the triplet

$$\phi = \sum_{i=1}^{3} \tau_i \pi_i, \tag{2.6}$$

where the τ_i's are the Pauli matrices:

$$\tau_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \tau_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \qquad \tau_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{2.7}$$

The π_i's are fundamental pion fields which are convenient notations; however we have to transform to the physical pion fields that are eigenstates of electric charge and defined as follow:

$$\begin{aligned} \pi^+ &= \frac{\pi_1 - i\pi_2}{\sqrt{2}}, \\ \pi^- &= \frac{\pi_1 + i\pi_2}{\sqrt{2}}, \\ \pi^0 &= \pi_3, \end{aligned} \tag{2.8}$$

which gives the general pion wavefunction as

$$\phi = \begin{bmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{bmatrix}. \tag{2.9}$$

In chiral perturbation theory, the nonlinear function of the meson field ϕ is used in the chiral Lagrangian (Weinberg, 1979). The exponential representation is the most common choice

$$U(x) = exp[\frac{i\phi(x)}{F_0}], \tag{2.10}$$

where F_0 is a constant with the proper dimensions. The matrix U is the building block from which an effective Lagrangian is constructed and must be invariant under chiral symmetry,

$$U(x) \longrightarrow g_R(x)U(x)g_L^\dagger(x) \quad g_R(x), g_L(x) \in SU(2). \tag{2.11}$$

The matrix U has always appeared in the effective Lagrangian for the processes which do not have nucleons. For the processes which involve nucleons, one defines (Ecker et al., 1989)

$$U = u^2, \tag{2.12}$$

which in the exponential parameterization gives

$$u(\phi(x)) = \exp[\frac{i\phi(x)}{2F_0}], \tag{2.13}$$

where the constant F_0 can be identified with the pion decay constant. The matrix u can be written in two components u_L and u_R with $u^2 = u_R u_L^\dagger = u_L^\dagger u_R$ and each component transforms chirally as

$$\begin{aligned} u_L &\rightarrow g_L u_L h^{-1}(g_L, g_R, \phi), \\ u_R &\rightarrow g_R u_R h^{-1}(g_L, g_R, \phi), \\ g_R, g_L &\in G = SU(2)_L \times SU(2)_R, \end{aligned} \tag{2.14}$$

where the compensator $h(g_L, g_R, \phi)$ is a nonlinear function of the pion field ϕ and the chiral symmetry group G.

One defines the covariant derivative as (Fearing and Scherer, 1996)

$$\begin{array}{lcl} A \xrightarrow{G} h(g_L, g_R, \phi)A & : & D_\mu A = \partial_\mu A + \Gamma_\mu A, \\ B \xrightarrow{G} h(g_L, g_R, \phi)Bh^{-1}(g_L, g_R, \phi) & : & D_\mu B = \partial_\mu B + [\Gamma_\mu, B], \end{array} \tag{2.15}$$

where

$$\Gamma_\mu = \frac{1}{2}\left[u_R^\dagger(\partial_\mu - ir_\mu)u_R + u_L^\dagger(\partial_\mu - il_\mu)u_L\right], \tag{2.16}$$

with $r_\mu = v_\mu + a_\mu$ and $l_\mu = v_\mu - a_\mu$. v_μ and a_μ are the external vector and axial-vector fields. Note that the definition of the covariant derivative depends on the transformation property of the object it acts on and the covariant derivative transforms in the same way as that object. The Γ_μ is the so-called chiral connection. It transforms under local transformation as

$$\Gamma_\mu \rightarrow h\Gamma_\mu h^{-1} + h\partial_\mu h^{-1}. \tag{2.17}$$

The connection Γ_μ contains one derivative. Another object with one derivative is called the axial-vector object and defined as

$$u_\mu = i\left[u_R^\dagger(\partial_\mu - ir_\mu)u_R - u_L^\dagger(\partial_\mu - il_\mu)u_L\right], \tag{2.18}$$

which transforms homogeneously, $u_\mu \rightarrow hu_\mu h^{-1}$. Field strength tensors are defined by

$$F_{\mu\nu}^{\pm} = u_R^\dagger F_{\mu\nu}^R u_R \pm u_L^\dagger F_{\mu\nu}^L u_L, \tag{2.19}$$

where

$$F_{\mu\nu}^R = \partial_\mu r_\nu - \partial_\nu r_\mu - i[r_\mu, r_\nu], \tag{2.20}$$

$$F_{\mu\nu}^L = \partial_\mu l_\nu - \partial_\nu l_\mu - i[l_\mu, l_\nu]. \tag{2.21}$$

Under a chiral transformation, these two tensors transform as

$$F^R_{\mu\nu} \to g_R F^R_{\mu\nu} g^\dagger_R, \tag{2.22}$$

$$F^L_{\mu\nu} \to g_L F^L_{\mu\nu} g^\dagger_L. \tag{2.23}$$

Therefore, they make the transformation of $F^\pm_{\mu\nu}$ as

$$F^\pm_{\mu\nu} \to h F^\pm_{\mu\nu} h^{-1}. \tag{2.24}$$

The last component corresponds to the explicit symmetry breaking created by the non-zero quark mass. It is introduced as

$$\chi_\pm = u^\dagger_R \chi u_L \pm u^\dagger_L \chi^\dagger u_R, \tag{2.25}$$

with

$$\chi = 2B_0(s + ip), \tag{2.26}$$

where s and p are scalar and pseudoscalar densities and B_0 is related to the quark condensate in the chiral limit. It is used to set up a general Lagrangian which has symmetry breaking. It is then assumed that for the real world, the scalar density is the quark mass matrix

$$M = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix}, \tag{2.27}$$

and the pseudoscalar is zero. One assumes that χ transforms as

$$\chi \to g_R \chi g^\dagger_L, \tag{2.28}$$

under chiral symmetry, so it makes

$$\chi_\pm \to h \chi h^{-1}. \tag{2.29}$$

In order to construct the Lagrangian we have to know the order of each building blocks. One assign the following chiral dimensions to the building blocks:

$$\Psi, \partial_\mu \Psi = \mathcal{O}(1), \tag{2.30}$$

$$u_\mu = \mathcal{O}(p), \tag{2.31}$$

$$\chi_\pm, F^\pm_{\mu\nu} = \mathcal{O}(p^2). \tag{2.32}$$

Here, p denotes a small momentum or meson mass with respect to the typical hadronic scale of about 1 GeV. The covariant derivative of each building block has the same order as the building block it acts on.

2.2.2 Chiral invariants

In the Lagrangian, all terms must be invariant under chiral transformation. This requires that the trace of matrices has to be taken. The lowest order contribution to the Lagrangian is the second order $\mathcal{O}(p^2)$. There are three terms which obey Lorentz and chiral symmetry

$$\begin{aligned} &\langle u_\mu u^\mu \rangle, \\ &\langle \chi_+ \rangle, \\ &\langle \chi_- \rangle, \end{aligned} \tag{2.33}$$

where $\langle \cdots \rangle$ represents the trace in flavor space. This method is used to derive the fourth order effective Lagrangian as well. Each term is comprised of four axial-vector objects, two axial-vector objects and one $\chi_\pm$, two axial-vector objects and one $F^\pm_{\mu\nu}$, one $\chi_\pm$ and one $F^\pm_{\mu\nu}$, two $\chi_\pm$ or two $F^\pm_{\mu\nu}$. There are too many chiral invariant terms, so they will not be written explicitly.

2.2.3 Parity

The parity transformation acts on the fields as follow:

$$\begin{aligned}
u_R(t,\mathbf{x}) &\rightarrow u_R^\dagger(t,-\mathbf{x}) = u_L(t,-\mathbf{x}) \\
u_L(t,\mathbf{x}) &\rightarrow u_L^\dagger(t,-\mathbf{x}) = u_R(t,-\mathbf{x}) \\
\partial_\mu u_R(t,\mathbf{x}) &\rightarrow \partial^\mu u_R^\dagger(t,-\mathbf{x}) = \partial^\mu u_L(t,-\mathbf{x}) \\
\partial_\mu u_L(t,\mathbf{x}) &\rightarrow \partial^\mu u_L^\dagger(t,-\mathbf{x}) = \partial^\mu u_R(t,-\mathbf{x}) \\
v_\mu(t,\mathbf{x}) &\rightarrow v^\mu(t,-\mathbf{x}) \\
a_\mu(t,\mathbf{x}) &\rightarrow -a^\mu(t,-\mathbf{x}) \\
s(t,\mathbf{x}) &\rightarrow s(t,-\mathbf{x}) \\
p(t,\mathbf{x}) &\rightarrow -p(t,-\mathbf{x}).
\end{aligned} \tag{2.34}$$

They lead to the transformation of the building blocks as

$$\begin{aligned}
u_\mu &\rightarrow -u^\mu \\
\chi_+ &\rightarrow \chi_+ \\
\chi_- &\rightarrow -\chi_- \\
F_{\mu\nu}^+ &\rightarrow F^{\mu\nu+} \\
F_{\mu\nu}^- &\rightarrow -F^{\mu\nu-}.
\end{aligned} \tag{2.35}$$

By using these transformation properties. The three chiral invariant terms are reduced to

$$\begin{aligned}
&\langle u_\mu u^\mu \rangle, \\
&\langle \chi_+ \rangle.
\end{aligned} \tag{2.36}$$

2.2.4 Charge conjugation

The last symmetry of the QCD Lagrangian which is considered in the chiral Lagrangian is charge conjugation. The transformation properties of the fields under this symmetry are

$$
\begin{aligned}
u_R &\rightarrow u_R^T \\
u_L &\rightarrow u_L^T \\
\partial_\mu u_R &\rightarrow \partial_\mu u_R^T \\
\partial_\mu u_L &\rightarrow \partial_\mu u_L^T \\
v_\mu &\rightarrow -v_\mu^T \\
a_\mu &\rightarrow a_\mu^T \\
s &\rightarrow s^T \\
p &\rightarrow p^T.
\end{aligned}
\tag{2.37}
$$

From the properties above, one gets

$$
\begin{aligned}
u_\mu &\rightarrow u_\mu^T \\
\chi_+ &\rightarrow \chi_+^T \\
\chi_- &\rightarrow \chi_-^T \\
F_{\mu\nu}^+ &\rightarrow -F_{\mu\nu}^+ \\
F_{\mu\nu}^- &\rightarrow F_{\mu\nu}^-.
\end{aligned}
\tag{2.38}
$$

Using the property of trace

$$\langle A^T \rangle = \langle A \rangle, \tag{2.39}$$

and one has for a pair of matrices,

$$\langle A^T B^T \rangle = \langle (BA)^T \rangle = \langle BA \rangle = \langle AB \rangle. \tag{2.40}$$

As a result, there are still two possible terms which are invariant under all QCD symmetries. They are

$$
\begin{aligned}
&\langle u_\mu u^\mu \rangle, \\
&\langle \chi_+ \rangle.
\end{aligned}
\tag{2.41}
$$

2.2.5 The Lagrangian

At the second order, there are two independent terms left so there should be two constants in the Lagrangian which are taken to be the pion decay constant F_0 and the parameter related to the strength of the quark-antiquark condensate B_0. The resulting Lagrangian is

$$\mathcal{L}_\pi^{(2)} = \frac{F_0^2}{4}\langle u_\mu u^\mu \rangle + \frac{F_0^2}{4}\langle \chi_+ \rangle. \tag{2.42}$$

At fourth order, there are twelve terms which satisfy all QCD symmetries. The constants which are usually called the low energy constants are defined as ℓ_i. Then, the fourth order Lagrangian is (Gasser and Leutwyler, 1985)

$$\begin{aligned} \mathcal{L}_\pi^{(4)} &= \ell_1 \langle u_\mu u^\mu \rangle^2 + \ell_2 \langle u_\mu u^\nu \rangle \langle u^\mu u_\nu \rangle + \ell_3 \langle u_\mu u^\mu u_\nu u^\nu \rangle \\ &\quad + \ell_4 \langle u_\mu u^\mu \rangle \langle \chi_+ \rangle + \ell_5 \langle u_\mu u^\mu \chi_+ \rangle + \ell_6 \langle \chi_+ \rangle^2 \\ &\quad + \ell_7 \langle \chi_- \rangle^2 + \frac{1}{4}(2\ell_8 + \ell_{12}) \langle \chi_+^2 \rangle + \frac{1}{4}(2\ell_8 - \ell_{12}) \langle \chi_-^2 \rangle \\ &\quad - i\ell_9 \langle F^{\mu\nu +} u_\mu u_\nu \rangle + \frac{1}{4}(\ell_{10} + 2\ell_{11}) \langle F_{\mu\nu}^+ F^{\mu\nu +} \rangle \\ &\quad - \frac{1}{4}(\ell_{10} - 2\ell_{11}) \langle F_{\mu\nu}^- F^{\mu\nu -} \rangle. \end{aligned} \tag{2.43}$$

2.3 Baryon chiral perturbation theory

2.3.1 Definitions

The inclusion of baryons in the effective Lagrangian was first systematized by (Gasser et al., 1988). The construction of the Lagrangian for baryon is more difficult because the nucleon mass does not vanish in the chiral limit. In the derivation of the Lagrangian for pion it was assumed that the energy and momentum of the fields were very much less than 1 GeV which is called chiral scale, however the nucleon mass is close to the 1 GeV scale, which implies that an expansion in terms of the nucleon energy will not converge. In this section the pion-nucleon Lagrangian will be derived and the convergence will be studied in the next section.

The nucleon fields will be defined in the spinor

$$\Psi = \begin{pmatrix} p \\ n \end{pmatrix}, \tag{2.44}$$

where p and n are proton and neutron wavefunction respectively. When we consider massless quarks in QCD the axial symmetry $U(1)$ must be included. Under this symmetry the quark fields are invariant and transform as (Donoghue et al., 1995)

$$q \rightarrow e^{-i\theta\gamma^5} q. \tag{2.45}$$

The axial symmetry should be satisfied in the chiral Lagrangian. In the previous section, the terms in the pion Lagrangian are unchanged under axial transformation so it was ignored. The symmetry group which is used in the pion-nucleon Lagrangian is $SU(2)_R \times SU(2)_L \times U(1)_A$ and the transformation of the nucleon field is

$$\Psi \rightarrow h(g_L, g_R, \phi)\Psi, \qquad \overline{\Psi} \rightarrow \overline{\Psi} h^{-1}(g_L, g_R, \phi). \tag{2.46}$$

From Equation (2.15), the nucleon covariant derivative is defined as

$$D_\mu \Psi = \partial_\mu \Psi + \Gamma_\mu \Psi, \tag{2.47}$$

and transforms as

$$D_\mu \Psi \rightarrow h(g_L, g_R, \phi) D_\mu \Psi. \tag{2.48}$$

For the construction of invariant terms and phenomenological applications, one treats isosinglet and isotriplet components of the external fields separately and defines the traceless operator as

$$\tilde{X} = X - \frac{1}{2}\langle X \rangle, \tag{2.49}$$

Table 2.1 Chiral dimension and the transformation properties of the building blocks and the covariant derivative of the nucleon field

	u_μ	χ_+	χ_-	$F^+_{\mu\nu}$	$F^-_{\mu\nu}$	D_μ
chiral dimension	1	2	2	2	2	1
parity	$-$	$+$	$-$	$+$	$-$	$+$
charge conjugation	$+$	$+$	$+$	$-$	$+$	$+$
hermitian conjugation	$+$	$+$	$-$	$+$	$+$	$+$

where $\langle \ldots \rangle$ represents the trace. To construct a hermitian Lagrangian which is chiral, parity and charge conjugation invariant, we need to know the transformation properties of the fields under all transformations. Under parity the building block transforms to $\pm$ itself with changing Lorentz indices from lower to upper. The building block transforms to $\pm$ its transposed under charge conjugation and to $\pm$ itself under hermitian conjugation where the signs are given in Table 2.1 (Fettes et al., 2000).

The pion-nucleon Lagrangian also includes the Clifford algebra elements which are γ_5, γ_μ and $\gamma_\mu\gamma_5$ and $\sigma_{\mu\nu}$, the metric $g_{\mu\nu}$ and the Levi-Civita tensor $\varepsilon_{\mu\nu\alpha\beta}$ and the covariant derivative of the nucleon field to contract Lorentz indices. Each matrix transforms to $\pm$ itself with changing Lorentz indices from lower to upper under parity. Under charge conjugation the matrix transforms to $\pm$ its transposed under charge conjugation and transforms to $\pm\gamma^0$(itself) γ^0 under hermitian conjugation where the signs are given in Table 2.2 (Fettes et al., 2000).

Table 2.2 Transformation properties and chiral dimension of the Clifford algebra elements, the metric, the Levi-Civita tensors with the covariant derivative of the nucleon field.

	γ_5	γ_μ	$\gamma_\mu\gamma_5$	$\sigma_{\mu\nu}$	$g_{\mu\nu}$	$\varepsilon_{\mu\nu\alpha\beta}$	$D_\mu\Psi$
chiral dimension	1	0	0	0	0	0	0
parity	$-$	$+$	$-$	$+$	$+$	$-$	$+$
charge conjugation	$+$	$-$	$+$	$-$	$+$	$+$	$-$
hermitian conjugation	$-$	$+$	$+$	$+$	$+$	$+$	$-$

2.3.2 Chiral invariants

In this section, we will construct all possible chiral and Lorentz invariant terms by combining all building blocks. Any invariant term in the pion-nucleon Lagrangian is of the form

$$\overline{\Psi}A^{\mu\nu\cdots}\Theta_{\mu\nu\ldots}\Psi + \text{h.c.}, \tag{2.50}$$

where $A^{\mu\nu\cdots}$ is a product of pion and/or external fields and the covariant derivative thereof. $\Theta_{\mu\nu\ldots}$ is a product of an element of Clifford algebra $\Gamma_{\mu\nu}$ and n covariant derivatives acting on the nucleon field $D^n_{\alpha\beta\ldots}$.

One expects that the first term in the pion-nucleon Lagrangian will be the Lagrangian for a free Dirac field

$$\mathcal{L}_{\text{free}} = \overline{\Psi}(i\gamma_\mu D^\mu - m_N)\Psi, \tag{2.51}$$

where m_N is the nucleon mass and this Lagrangian is counted as $\mathcal{O}(p)$. All terms which are chiral and Lorentz invariant are

$$\overline{\Psi}(i\gamma_\mu D^\mu - m_N)\Psi, \tag{2.52}$$

$$\overline{\Psi}\gamma^\mu u_\mu \Psi, \tag{2.53}$$

$$\overline{\Psi}\gamma^\mu \gamma_5 u_\mu \Psi, \tag{2.54}$$

$$\overline{\Psi}\gamma_5 \Psi, \tag{2.55}$$

$$\overline{\Psi}u_\mu D^\mu \Psi + \text{h.c.} \tag{2.56}$$

At second order, the chiral invariant terms are

$$\overline{\Psi}u_\mu u^\mu \Psi, \tag{2.57}$$

$$\overline{\Psi}\sigma^{\mu\nu} u_\mu u_\nu \Psi, \tag{2.58}$$

$$\overline{\Psi}D^\mu u_\mu \Psi, \tag{2.59}$$

$$\overline{\Psi}\gamma^5 u_\mu D^\mu \Psi + \text{h.c.}, \tag{2.60}$$

$$\overline{\Psi}\sigma^{\mu\nu} D_\mu u_\nu \Psi, \tag{2.61}$$

$$\overline{\Psi}u_\mu u_\nu D^\mu D^\nu \Psi + \text{h.c.}, \tag{2.62}$$

$$\overline{\Psi}u_\mu u_\nu D^\nu D^\mu \Psi + \text{h.c.}, \tag{2.63}$$

$$\overline{\Psi}\langle u_\mu u_\nu \rangle D^\mu D^\nu \Psi + \text{h.c.}, \tag{2.64}$$

$$\overline{\Psi}\langle u_\mu u_\nu \rangle D^\mu D^\nu \Psi + \text{h.c.}, \tag{2.65}$$

$$\overline{\Psi}\varepsilon^{\mu\nu\alpha\beta} u_\mu u_\nu D_\alpha D_\beta \Psi + \text{h.c.}, \tag{2.66}$$

$$\overline{\Psi}\chi_+ \Psi, \tag{2.67}$$

$$\overline{\Psi}\langle \chi_+ \rangle \Psi, \tag{2.68}$$

$$\overline{\Psi}\chi_- \Psi, \tag{2.69}$$

$$\overline{\Psi}\langle \chi_- \rangle \Psi, \tag{2.70}$$

$$\overline{\Psi}\sigma^{\mu\nu} F^+_{\mu\nu} \Psi, \tag{2.71}$$

$$\overline{\Psi}\sigma^{\mu\nu} \langle F^+_{\mu\nu} \rangle \Psi, \tag{2.72}$$

$$\overline{\Psi} F^{+}_{\mu\nu} D^{\mu} D^{\nu} \Psi + \text{h.c.}, \tag{2.73}$$

$$\overline{\Psi} F^{+}_{\mu\nu} D^{\nu} D^{\mu} \Psi + \text{h.c.}, \tag{2.74}$$

$$\overline{\Psi} \varepsilon^{\mu\nu\alpha\beta} F^{+}_{\mu\nu} D_{\alpha} D_{\beta} \Psi + \text{h.c.}, \tag{2.75}$$

$$\overline{\Psi} \langle F^{+}_{\mu\nu} \rangle D^{\mu} D^{\nu} \Psi + \text{h.c.}, \tag{2.76}$$

$$\overline{\Psi} \langle F^{+}_{\mu\nu} \rangle D^{\nu} D^{\mu} \Psi + \text{h.c.}, \tag{2.77}$$

$$\overline{\Psi} \varepsilon^{\mu\nu\alpha\beta} \langle F^{+}_{\mu\nu} \rangle D_{\alpha} D_{\beta} \Psi + \text{h.c.}, \tag{2.78}$$

$$\overline{\Psi} \sigma^{\mu\nu} F^{-}_{\mu\nu} \Psi, \tag{2.79}$$

$$\overline{\Psi} \sigma^{\mu\nu} \langle F^{-}_{\mu\nu} \rangle \Psi, \tag{2.80}$$

$$\overline{\Psi} F^{-}_{\mu\nu} D^{\mu} D^{\nu} \Psi + \text{h.c.}, \tag{2.81}$$

$$\overline{\Psi} F^{-}_{\mu\nu} D^{\nu} D^{\mu} \Psi + \text{h.c.}, \tag{2.82}$$

$$\overline{\Psi} \varepsilon^{\mu\nu\alpha\beta} F^{-}_{\mu\nu} D_{\alpha} D_{\beta} \Psi + \text{h.c.}, \tag{2.83}$$

$$\overline{\Psi} \langle F^{-}_{\mu\nu} \rangle D^{\mu} D^{\nu} \Psi + \text{h.c.}, \tag{2.84}$$

$$\overline{\Psi} \langle F^{-}_{\mu\nu} \rangle D^{\nu} D^{\mu} \Psi + \text{h.c.}, \tag{2.85}$$

$$\overline{\Psi} \varepsilon^{\mu\nu\alpha\beta} \langle F^{-}_{\mu\nu} \rangle D_{\alpha} D_{\beta} \Psi + \text{h.c.} \tag{2.86}$$

The terms which involve derivatives of the nucleon field are not hermitian therefore these terms must be the sum of the term and its hermitian conjugate.

2.3.3 Parity

The parity transformation of each factor are given in Table 2.1 and Table 2.2. The lowest order parity invariant terms are

$$\overline{\Psi}(i\gamma_{\mu} D^{\mu} - m_N)\Psi, \tag{2.87}$$

$$\overline{\Psi} \gamma^{\mu} \gamma_5 u_{\mu} \Psi. \tag{2.88}$$

The second order $\mathcal{O}(p^2)$ terms are

$$\overline{\Psi}u^{\mu}u_{\mu}\Psi, \tag{2.89}$$

$$\overline{\Psi}\langle u^{\mu}u_{\mu}\rangle\Psi, \tag{2.90}$$

$$\overline{\Psi}\sigma^{\mu\nu}u_{\mu}u_{\nu}\Psi, \tag{2.91}$$

$$\overline{\Psi}\sigma^{\mu\nu}\langle u_{\mu}u_{\nu}\rangle\Psi, \tag{2.92}$$

$$\overline{\Psi}\gamma^{5}u_{\mu}D^{\mu}\Psi + \text{h.c.}, \tag{2.93}$$

$$\overline{\Psi}\gamma^{5}\langle u_{\mu}\rangle D^{\mu}\Psi + \text{h.c.}, \tag{2.94}$$

$$\overline{\Psi}u_{\mu}u_{\nu}D^{\mu}D^{\nu}\Psi + \text{h.c.}, \tag{2.95}$$

$$\overline{\Psi}\rangle u_{\mu}u_{\nu}\rangle D^{\mu}D^{\nu}\Psi + \text{h.c.}, \tag{2.96}$$

$$\overline{\Psi}u_{\mu}u_{\nu}D^{\nu}D^{\mu}\Psi + \text{h.c.}, \tag{2.97}$$

$$\overline{\Psi}\langle u_{\mu}u_{\nu}\rangle D^{\nu}D^{\mu}\Psi + \text{h.c.}, \tag{2.98}$$

$$\overline{\Psi}\chi_{+}\Psi, \tag{2.99}$$

$$\overline{\Psi}\langle\chi_{+}\rangle\Psi, \tag{2.100}$$

$$\overline{\Psi}\sigma^{\mu\nu}F^{+}_{\mu\nu}\Psi, \tag{2.101}$$

$$\overline{\Psi}\sigma^{\mu\nu}\langle F^{+}_{\mu\nu}\rangle\Psi, \tag{2.102}$$

$$\overline{\Psi}F^{+}_{\mu\nu}D^{\mu}D^{\nu}\Psi + \text{h.c.}, \tag{2.103}$$

$$\overline{\Psi}F^{+}_{\mu\nu}D^{\nu}D^{\mu}\Psi + \text{h.c.}, \tag{2.104}$$

$$\overline{\Psi}\langle F^{+}_{\mu\nu}\rangle D^{\mu}D^{\nu}\Psi + \text{h.c.}, \tag{2.105}$$

$$\overline{\Psi}\langle F^{+}_{\mu\nu}\rangle D^{\nu}D^{\mu}\Psi + \text{h.c.}, \tag{2.106}$$

$$\overline{\Psi}\varepsilon^{\mu\nu\alpha\beta}F^{-}_{\mu\nu}D_{\alpha}D_{\beta}\Psi + \text{h.c.}, \tag{2.107}$$

$$\overline{\Psi}\varepsilon^{\mu\nu\alpha\beta}\langle F^{-}_{\mu\nu}\rangle D_{\alpha}D_{\beta}\Psi + \text{h.c.} \tag{2.108}$$

2.3.4 Charge conjugation

The first order parity invariant terms which are also invariant under charge conjugation are

$$\overline{\Psi}(i\gamma_\mu D^\mu - m_N)\Psi, \tag{2.109}$$

$$\overline{\Psi}\gamma^\mu\gamma^5 u_\mu \Psi, \tag{2.110}$$

and for the second order Lagrangian there are

$$\overline{\Psi}\langle u^\mu u_\mu\rangle\Psi, \tag{2.111}$$

$$\overline{\Psi}\sigma^{\mu\nu}\left[u_\mu, u_\nu\right]\Psi, \tag{2.112}$$

$$\overline{\Psi}\langle u_\mu u_\nu\rangle D^\mu D^\nu\Psi + \text{h.c.}, \tag{2.113}$$

$$\overline{\Psi}\chi_+\Psi, \tag{2.114}$$

$$\overline{\Psi}\langle\chi_+\rangle\Psi, \tag{2.115}$$

$$\overline{\Psi}\sigma^{\mu\nu}F^+_{\mu\nu}\Psi, \tag{2.116}$$

$$\overline{\Psi}\sigma^{\mu\nu}\langle F^+_{\mu\nu}\rangle\Psi, \tag{2.117}$$

$$\overline{\Psi}\varepsilon^{\mu\nu\alpha\beta}F^-_{\mu\nu}D_\alpha D_\beta\Psi + \text{h.c.}, \tag{2.118}$$

$$\overline{\Psi}\varepsilon^{\mu\nu\alpha\beta}\langle F^-_{\mu\nu}\rangle D_\alpha D_\beta\Psi + \text{h.c.} \tag{2.119}$$

2.3.5 The pion-nucleon Lagrangian

The list of invariant terms generated above still contains linearly dependent term which can be reduced by using various identities. First of all, the property which is frequently used in the construction of the Lagrangian is provided by the Cayley-Hamilton theorem. For 2×2 matrices A and B, the anti-commutator of these two matrices can be written in terms of their traces as

$$\{A, B\} = A\langle B\rangle + \langle A\rangle B + \langle AB\rangle - \langle A\rangle\langle B\rangle. \tag{2.120}$$

Another identity is the curvature relation

$$[D_\mu, D_\nu] = \frac{1}{4}[u_\mu, u_\nu] - \frac{i}{2}F^+_{\mu\nu}. \tag{2.121}$$

We notice that the terms on the right hand side are both second order, so the term in Equation (2.118) can be rewritten as

$$\overline{\Psi}\varepsilon^{\mu\nu\alpha\beta}F^-_{\mu\nu}(D_\beta D_\alpha + \mathcal{O}(p^2))\Psi \approx \overline{\Psi}\varepsilon^{\mu\nu\alpha\beta}F^-_{\mu\nu}F^+_{\beta\alpha}\Psi. \tag{2.122}$$

The Lorentz indices can be interchanged, which gives

$$\overline{\Psi}\varepsilon^{\mu\nu\beta\alpha}F^-_{\mu\nu}D_\alpha D_\beta\Psi = -\overline{\Psi}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}D_\alpha D_\beta\Psi. \tag{2.123}$$

The result is that the term in Equation (2.118) is not actually a second order term. The same proof can be use to eliminate Equation (2.119).

Another set of identities is based on the equation of motion (EOM) deduced from the lowest order pion Lagrangian

$$[D_\mu, u^\mu] = \frac{i}{2}\widetilde{\chi}_-, \tag{2.124}$$

and pi-nucleon Lagrangian

$$\left(i\gamma_\mu D^\mu - m_N + \frac{1}{2}g_A\gamma_\mu\gamma_5 u^\mu\right)\Psi = 0, \tag{2.125}$$

$$\left(i\gamma_\mu \overleftarrow{D}^\mu + m_N - \frac{1}{2}g_A\gamma_\mu\gamma_5 u^\mu\right)\Psi = 0 \tag{2.126}$$

where m_N and g_A are the nucleon mass and the axial-vector coupling constant in the $SU(3)$ chiral limit. The EOM is used to simplify things, e.g. it allows us to disregard $\gamma_\mu D^\mu$ in many terms.

For the construction of higher order Lagrangian we need more relations to reduce to the minimal set of terms. Some of these relations have already appeared in (Fettes et al., 1998) as following:

$$\overline{\Psi}A^\mu iD_\mu\Psi + \text{h.c.} \doteq 2m_N\overline{\Psi}\gamma_\mu A^\mu\Psi, \tag{2.127}$$

$$\overline{\Psi}A^{\mu\nu}D_\nu D_\mu\Psi + \text{h.c.} \doteq -m_N(\overline{\Psi}\gamma_\mu A^{\mu\nu}iD_\nu\Psi + \text{h.c.}), \tag{2.128}$$

$$\overline{\Psi}A^{\mu\nu\alpha}iD_\alpha D_\nu D_\mu\Psi+\text{h.c.} \doteq m_N(\overline{\Psi}\gamma_\mu A^{\mu\nu\alpha}D_\alpha D_\nu\Psi+\text{h.c.}), \quad (2.129)$$

$$\overline{\Psi}\gamma_5\gamma_\nu A^{\mu\nu}iD_\mu\Psi+\text{h.c.} \doteq 2im_N\overline{\Psi}\gamma_5\sigma_{\mu\nu}A^{\mu\nu}\Psi +(\overline{\Psi}\gamma_5\gamma_\mu A^{\mu\nu}iD_\nu\Psi+\text{h.c.}), \quad (2.130)$$

$$\overline{\Psi}\gamma_5\gamma_\mu A^{\mu\nu\alpha}D_\alpha D_\mu\Psi+\text{h.c.} \doteq m_N(\overline{\Psi}\gamma_5\sigma_{\mu\nu}A^{\mu\nu\alpha}D_\alpha\Psi+\text{h.c.}) +(\overline{\Psi}\gamma_5\gamma_\mu A^{\mu\nu\alpha}D_\alpha D_\nu\Psi+\text{h.c.}), \quad (2.131)$$

$$\overline{\Psi}\gamma_5\gamma_\nu A^{\mu\nu\alpha\beta}iD_\beta D_\alpha D_\mu\Psi+\text{h.c.} \doteq m_N(i\overline{\Psi}\gamma_5\sigma_{\mu\nu}A^{\mu\nu\alpha\beta}D_\beta D_\alpha\Psi+\text{h.c.}) +(\overline{\Psi}\gamma_5\gamma_\mu A^{\mu\nu\alpha\beta}iD_\beta D_\alpha D_\nu\Psi+\text{h.c.}), \quad (2.132)$$

$$\overline{\Psi}\sigma_{\alpha\beta}A^{\alpha\beta\mu}iD_\mu\Psi+\text{h.c.} \doteq -2m_N\overline{\Psi}\varepsilon_{\alpha\beta\mu\nu}\gamma_5\gamma^\nu A^{\alpha\beta\mu}\Psi -(\overline{\Psi}\sigma_{\beta\mu}A^{\alpha\beta\mu}iD_\alpha\Psi+\text{h.c.}) +(\overline{\Psi}\sigma_{\alpha\mu}A^{\alpha\beta\mu}iD_\beta\Psi+\text{h.c.}), \quad (2.133)$$

$$\overline{\Psi}\sigma_{\alpha\beta}A^{\alpha\beta\nu\mu}D_\nu D_\mu\Psi+\text{h.c.} \doteq m_N(i\overline{\Psi}\varepsilon_{\alpha\beta\mu\nu}\gamma_5\gamma^\nu A^{\alpha\beta\mu\nu}D_\nu\Psi+\text{h.c.}) -(\overline{\Psi}\sigma_{\beta\mu}A^{\alpha\beta\nu\mu}D_\nu D_\alpha\Psi+\text{h.c.}) +(\overline{\Psi}\sigma_{\alpha\mu}A^{\alpha\beta\nu\mu}D_\nu D_\beta\Psi+\text{h.c.}), \quad (2.134)$$

$$\overline{\Psi}\gamma_5\sigma_{\alpha\beta}A^{\alpha\beta\mu}D_\mu\Psi+\text{h.c.} \doteq -(\overline{\Psi}\gamma_5\sigma_{\beta\mu}A^{\alpha\beta\mu}D_\alpha+\text{h.c.}) +(\overline{\Psi}\gamma_5\sigma_{\alpha\mu}A^{\alpha\beta\mu}D_\beta\Psi+\text{h.c.}), \quad (2.135)$$

$$i\overline{\Psi}\gamma_5\sigma_{\alpha\beta}A^{\alpha\nu\mu}D_\nu D_\mu\Psi+\text{h.c.} \doteq -(i\overline{\Psi}\gamma_5\sigma_{\beta\mu}A^{\alpha\beta\nu\mu}D_\nu D_\alpha\Psi+\text{h.c.}) +(i\overline{\Psi}\gamma_5\sigma_{\alpha\mu}A^{\alpha\beta\nu\mu}D_\nu D_\beta\Psi+\text{h.c.}), \quad (2.136)$$

$$\overline{\Psi}\gamma_\mu[iD^\mu,A]\Psi \doteq \frac{g_A}{2}\overline{\Psi}\gamma^\mu\gamma_5[A,u_\mu]\Psi, \quad (2.137)$$

$$\overline{\Psi}\gamma_5\gamma_\mu[iD^\mu,A]\Psi \doteq -2m_N\overline{\Psi}\gamma_5A\Psi-\frac{g_A}{2}\overline{\Psi}\gamma^\mu[A,u_\mu]\Psi, \quad (2.138)$$

$$\overline{\Psi}\gamma_5\gamma_\nu[D^\mu,A^\nu]iD_\mu\Psi+\text{h.c.} \doteq 0 \quad (2.139)$$

$$\overline{\Psi}\varepsilon_{\alpha\beta\mu\nu}\gamma^\nu[D^\lambda,A^{\alpha\beta\mu}]iD_\lambda\Psi+\text{h.c.} \doteq 0, \quad (2.140)$$

$$\overline{\Psi}[D^\mu,A^\nu]D_\nu D_\mu\Psi+\text{h.c.} \doteq 0, \quad (2.141)$$

$$\overline{\Psi}\sigma_{\alpha\beta}[D^\mu,A^{\alpha\beta\nu}]D_\nu D_\mu\Psi+\text{h.c.} \doteq 0, \quad (2.142)$$

where the symbol $\doteq$ means equal up to terms of higher order.

Another set of identities is provided by the connection between curvature relation in Equation (2.121) and the Bianchi identity for covariant derivatives

$$[D_\alpha, [D_\mu, D_\nu]] + \text{cyclic} = 0, \tag{2.143}$$

where "cyclic" refers to cyclic permutations It leads

$$[D_\alpha, F^+_{\mu\nu}] + \text{cyclic} = \frac{i}{2}[u_\alpha, F^-_{\mu\nu}] + \text{cyclic}, \tag{2.144}$$

by using the Leibniz rule and the relation that

$$[D_\mu, u_\nu] - [D_\nu, u_\mu] = F^-_{\mu\nu}. \tag{2.145}$$

The effective πN Lagrangian is given by the combination of terms with increasing chiral dimension,

$$\mathcal{L}^{\text{eff}}_{\pi N} = \mathcal{L}^{(1)}_{\pi N} + \mathcal{L}^{(2)}_{\pi N} + \mathcal{L}^{(3)}_{\pi N} + \mathcal{L}^{(4)}_{\pi N} + \dots. \tag{2.146}$$

At lowest order, the effective πN Lagrangian is given by

$$\mathcal{L}^{(1)}_{\pi N} = \overline{\Psi}\left(i\gamma_\mu D^\mu - m_N + \frac{g_A}{2}\gamma_\mu\gamma_5 u^\mu\right)\Psi. \tag{2.147}$$

At second order, there are seven independent terms with their low energy constants (LECs) (Gasser et al., 1988),

$$\begin{aligned}\mathcal{L}^{(2)}_{\pi N} \;=\; & \overline{\Psi}\Big\{c_1\langle\chi_+\rangle - \frac{c_2}{8m_N^2}\left(\langle u_\mu u_\nu\rangle\{D^\mu, D^\nu\} + \text{h.c.}\right) + \frac{c_3}{2}\langle u^2\rangle \\ & + \frac{ic_4}{4}\sigma^{\mu\nu}[u_\mu, u_\nu] + c_5\widetilde{\chi}_+ + \frac{c_6}{8m_N}\sigma^{\mu\nu}F^+_{\mu\nu} + \frac{c_7}{8m_n}\sigma^{\mu\nu}\langle F^+_{\mu\nu}\rangle\Big\}\Psi.\end{aligned} \tag{2.148}$$

The third order pion-nucleon Lagrangian has 23 independent terms and 118 terms for the fourth order. We will write these two Lagrangian in the form (Fettes et al.,

Table 2.3 The terms of dimension three for the relativistic Lagrangian.

i	$\mathcal{O}^{(3)}$
1	$-\frac{1}{2m_N}[u_\mu,[D_\nu,u^\mu]]D^\nu+\text{h.c.}$
2	$-\frac{1}{2m_N}[u_\mu,[D^\mu,u_\nu]]D^\nu+\text{h.c.}$
3	$\frac{1}{12m_N^3}[u_\mu,[D_\nu,u_\alpha]](D^\mu D^\nu D^\alpha+\text{sym.})+\text{h.c.}$
4	$-\frac{1}{2m_N}\varepsilon^{\mu\nu\alpha\beta}\langle u_\mu u_\nu u_\alpha\rangle D_\beta+\text{h.c.}$
5	$\frac{1}{2m_N}i[\chi_-,u_\mu]D^\mu+\text{h.c.}$
6	$\frac{1}{2m_N}i[D^\mu,\widetilde{F}^+_{\mu\nu}]D^\nu+\text{h.c.}$
7	$\frac{1}{2m_N}i[D^\mu,\langle F^+_{\mu\nu}\rangle]D^\nu+\text{h.c.}$
8	$\frac{1}{2m_N}i\varepsilon^{\mu\nu\alpha\beta}\langle\widetilde{F}^+_{\mu\nu}u_\alpha\rangle D_\beta+\text{h.c.}$
9	$\frac{1}{2m_N}i\varepsilon^{\mu\nu\alpha\beta}\langle F^+_{\mu\nu}\rangle u_\alpha D_\beta+\text{h.c.}$
10	$\frac{1}{2}\gamma^\mu\gamma_5\langle u^2\rangle u_\mu$
11	$\frac{1}{2}\gamma_\mu\gamma_5\langle u_\mu u_\nu\rangle u^\nu$
12	$-\frac{1}{8m_N^2}\gamma^\mu\gamma_5\langle u_\alpha u_\nu\rangle u_\mu\,\{D^\alpha,D^\nu\}+\text{h.c.}$
13	$-\frac{1}{8m_N^2}\gamma^\mu\gamma_5\langle u_\mu u_\nu\rangle u_\alpha\,\{D^\alpha,D^\nu\}+\text{h.c.}$
14	$\frac{1}{4m_N}i\sigma^{\mu\nu}\langle[D_\alpha,u_\mu]u_\nu\rangle D^\alpha+\text{h.c.}$
15	$\frac{1}{4m_N}i\sigma^{\mu\nu}\langle u_\mu[D_\nu,u_\alpha]\rangle D^\alpha+\text{h.c.}$

2000)

$$\mathcal{L}^{(3)}_{\pi N} = \sum_{i=1}^{23} d_i\overline{\Psi}\mathcal{O}^{(3)}_i\Psi, \tag{2.149}$$

$$\mathcal{L}^{(4)}_{\pi N} = \sum_{i=1}^{118} e_i\overline{\Psi}\mathcal{O}^{(4)}_i\Psi, \tag{2.150}$$

and the monomials $\mathcal{O}^{(3)}$ and $\mathcal{O}^{(4)}$ are in Table 2.3 and Table 2.4, respectively.

Table 2.3 (Continued.)

i	$\mathcal{O}^{(3)}$
16	$\frac{1}{2}\gamma^\mu\gamma_5\langle\chi_+\rangle u_\mu$
17	$\frac{1}{2}\gamma^\mu\gamma_5\langle\chi_+ u_\mu\rangle$
18	$\frac{1}{2}i\gamma^\mu\gamma_5[D_\mu, \chi_-]$
19	$\frac{1}{2}i\gamma^\mu\gamma_5[D_\mu, \langle\chi_-\rangle]$
20	$-\frac{1}{8m_N^2}i\gamma^\mu\gamma_5[\widetilde{F}^+_{\mu\nu}, u_\alpha]\left\{D^\alpha, D^\nu\right\} + \text{h.c.}$
21	$\frac{1}{2}i\gamma^\mu\gamma_5[\widetilde{F}^+_{\mu\nu}, u^\nu]$
22	$\frac{1}{2}\gamma^\mu\gamma_5[D^\nu, F^-_{\mu\nu}]$
23	$\frac{1}{2}\gamma_\mu\gamma_5\varepsilon^{\mu\nu\alpha\beta}\langle u_\nu F^-_{\alpha\beta}\rangle$

Table 2.4 The independent terms for fourth order relativistic pion-nucleon Lagrangian.

i	$\mathcal{O}^{(4)}$
1	$\langle u\cdot u\rangle\langle u\cdot u\rangle$
2	$\langle u_\mu u_\nu\rangle\langle u^\mu u^\nu\rangle$
3	$\frac{1}{4m_N^2}\langle u\cdot u\rangle\langle u_\mu u_\nu\rangle\left\{D^\mu, D^\nu\right\} + \text{h.c.}$
4	$-\frac{1}{4m_N^2}\langle u_\alpha u_\mu\rangle\langle u^\alpha u_\nu\rangle\left\{D^\mu, D^\nu\right\} + \text{h.c.}$
5	$\frac{1}{48m_N^4}\langle u_\alpha u_\mu\rangle\langle u_\nu u_\beta\rangle(D^\alpha D^\mu D^\nu D^\beta + \text{sym.}) + \text{h.c.}$
6	$\frac{i}{2}\sigma^{\mu\nu}[u_\mu, u_\nu]\langle u\cdot u\rangle$
7	$-\frac{i}{8m_N^2}\sigma^{\alpha\mu}[u_\alpha, u_\mu]\langle u_\nu u_\beta\rangle\left\{D^\nu, D^\beta\right\} + \text{h.c.}$
8	$\frac{i}{2}\sigma^{\mu\nu}\langle[u_\mu, u_\nu]u_\alpha\rangle u^\alpha$
9	$-\frac{i}{8m_N^2}\sigma^{\alpha\mu}\langle[u_\alpha, u_\mu]u_\nu\rangle u_\beta\left\{D^\nu, D^\beta\right\} + \text{h.c.}$
10	$-\frac{1}{4m_N^2}\varepsilon^{\mu\nu\beta}_{\tau}\langle h_{\alpha\mu}u_\nu\rangle u_\beta\left\{D^\alpha, D^\tau\right\} + \text{h.c.}$
11	$\frac{1}{4m_N}\gamma^\mu\gamma_5\langle h_{\alpha\mu}[u^\alpha, u_\nu]\rangle D^\nu + \text{h.c.}$

Table 2.4 (Continued.)

i	$\mathcal{O}^{(4)}$
12	$\frac{1}{4m_N}\gamma^\nu\gamma_5\langle h_{\alpha\mu}[u^\alpha, u_\nu]\rangle D^\mu + \text{h.c.}$
13	$-\frac{1}{24m_N^3}\gamma^\beta\gamma_5\langle h_{\alpha\mu}[u_\nu, u_\beta]\rangle(D^\alpha D^\mu D^\nu + \text{sym.}) + \text{h.c.}$
14	$\langle h_{\mu\nu}h^{\mu\nu}\rangle$
15	$-\frac{1}{4m_N^2}\langle h_{\alpha\mu}h^\alpha_\mu\rangle\{D^\mu, D^\nu\} + \text{h.c.}$
16	$\frac{1}{48m_N^4}\langle h_{\alpha\mu}h^{\nu\beta}\rangle(D^\alpha D^\mu D^\nu D^\beta + \text{syms.}) + \text{h.c.}$
17	$\frac{i}{2}\sigma^{\mu\nu}[h_{\alpha\mu}, h^\alpha_\nu]$
18	$-\frac{i}{8m_N^2}\sigma^{\mu\nu}[h_{\alpha\mu}, h_{\nu\beta}]\left\{D^\alpha, D^\beta\right\} + \text{h.c.}$
19	$\langle\chi_+\rangle\langle u\cdot u\rangle$
20	$-\frac{1}{4m_N^2}\langle\chi_+\rangle\langle u_\mu u_\nu\rangle\{D^\mu, D^\nu\}$
21	$\frac{i}{2}\sigma^{\mu\nu}\langle\chi_+\rangle[u_\mu, u_\nu]$
22	$[D_\mu, [D^\mu, \langle\chi_+\rangle]]$
23	$\widetilde{\chi}_+\langle u\cdot u\rangle$
24	$-\frac{1}{4m_N^2}\widetilde{\chi}_+\langle u_\mu u_\nu\rangle\{D^\mu, D^\nu\}$
25	$u_\mu\langle\widetilde{\chi}_+ u^\mu\rangle$
26	$-\frac{1}{4m_N^2}u_\mu\langle\widetilde{\chi}_+ u_\nu\rangle\{D^\mu, D^\nu\} + \text{h.c.}$
27	$\frac{i}{2}\sigma^{\mu\nu}\langle\widetilde{\chi}_+[u_\mu, u_\nu]\rangle$
28	$\frac{1}{4m_N}\gamma^\mu\gamma_5[\widetilde{\chi}_+, h_{\mu\nu}]D^\nu + \text{h.c.}$
29	$\frac{1}{4m_N}\gamma^\mu\gamma_5[[D_\mu, \widetilde{\chi}_+], u_\nu]D^\nu + \text{h.c.}$
30	$[D_\mu, [D^\mu, \widetilde{\chi}_+]]$
31	$-\frac{i}{4m_N}\gamma^\mu\gamma_5\langle\chi_-\rangle[u_\mu, u_\nu]D^\nu + \text{h.c.}$
32	$-\frac{i}{4m_N^2}\langle\chi_-\rangle h_{\mu\nu}\{D^\mu, D^\nu\} + \text{h.c.}$
33	$iu_\mu[D^\mu, \langle\chi_-\rangle]$
34	$-\frac{i}{4m_N}\gamma^\mu\gamma_5\langle\widetilde{\chi}_-[u_\mu, u_\nu]\rangle D^\nu + \text{h.c.}$
35	$-\frac{i}{4m_N^2}\langle\widetilde{\chi}_- h_{\mu\nu}\rangle\{D^\mu, D^\nu\} + \text{h.c.}$

Table 2.4 (Continued.)

i	$\mathcal{O}^{(4)}$
36	$i\langle u_\mu[D^\mu, \widetilde{\chi}_-]\rangle$
37	$-\frac{1}{2}\sigma^{\mu\nu}[u_\mu, [D_\nu, \widetilde{\chi}_-]]$
38	$\langle\chi_+\rangle\langle\chi_+\rangle$
39	$\widetilde{\chi}_+\langle\chi_+\rangle$
40	$\langle\widetilde{\chi}_+\widetilde{\chi}_+\rangle$
41	$\widetilde{\chi}_-\langle\chi_-\rangle$
42	$i\langle F^+_{\mu\nu}\rangle[u^\mu, u^\nu]$
43	$-\frac{i}{4m_N^2}\langle F^+_{\alpha\mu}\rangle[u^\alpha, u_\nu]\left\{D^\mu, D^\nu\right\} + \text{h.c.}$
44	$-\frac{1}{2}\sigma^{\mu\nu}\langle F^+_{\mu\nu}\rangle\langle u\cdot u\rangle$
45	$-\frac{1}{2}\sigma^{\mu\nu}\langle F^+_{\alpha\mu}\rangle\langle u^\alpha u_\nu\rangle$
46	$\frac{1}{8m_N^2}\sigma^{\alpha\mu}\langle F^+_{\alpha\mu}\rangle\langle u_\nu u_\beta\left\{D^\nu, D^\beta\right\} + \text{h.c.}$
47	$-\frac{1}{8m_N^2}\sigma^{\mu\nu}\langle F^+_{\alpha\mu}\rangle\langle u_\nu u_\beta\rangle\left\{D^\alpha, D^\beta\right\} + \text{h.c.}$
48	$-\frac{i}{4m_N}\gamma^\mu\gamma_5\langle F^+_{\alpha\mu}\rangle h^\alpha_\nu D^\nu + \text{h.c.}$
49	$-\frac{i}{4m_N}\gamma^\nu\gamma_5\langle F^+_{\alpha\mu}\rangle h^\alpha_\nu D^\mu + \text{h.c.}$
50	$-\frac{i}{24m_N^3}\gamma^\alpha\gamma_5\langle F^+_{\alpha\mu}\rangle h_{\nu\beta}(D^\mu D^\nu D^\beta + \text{sym.}) + \text{h.c.}$
52	$-\frac{i}{4m_N}\gamma^\mu\gamma_5 u_\mu[D^\alpha, \langle F^+_{\alpha\nu}\rangle]D^\nu + \text{h.c.}$
53	$-\frac{i}{4m_N}\gamma^\nu\gamma_5 u_\mu[D^\alpha, \langle F^+_{\alpha\nu}\rangle]D^\mu + \text{h.c.}$
54	$-\frac{1}{2}\sigma^{\mu\nu}[D^\alpha, [D_\alpha, \langle F^+_{\mu\nu}\rangle]]$
55	$i\langle\widetilde{F}^+_{\mu\nu}[u^\mu, u^\nu]\rangle$
56	$-\frac{i}{4m_N^2}\langle\widetilde{F}^+_{\alpha\mu}[u^\alpha, u_\nu]\rangle\left\{D^\mu, D^\nu\right\} + \text{h.c.}$
57	$-\frac{1}{2}\sigma^{\mu\nu}\widetilde{F}^+_{\mu\nu}\langle u\cdot u\rangle$
58	$-\frac{1}{2}\sigma^{\mu\nu}\widetilde{F}^+_{\alpha\mu}\langle u^\alpha u_\nu\rangle$
59	$\frac{1}{8m_N^2}\sigma^{\mu\nu}\widetilde{F}^+_{\mu\nu}\langle u_\alpha u_\beta\rangle\left\{D^\alpha, D^\beta\right\} + \text{h.c.}$
60	$\frac{1}{8m_N^2}\sigma^{\mu\nu}\widetilde{F}^+_{\alpha\mu}\langle u_\nu u_\beta\rangle\left\{D^\alpha, D^\beta\right\} + \text{h.c.}$

Table 2.4 (Continued.)

i	$\mathcal{O}^{(4)}$
61	$-\frac{1}{2}\sigma^{\mu\nu}u^{\alpha}\langle\widetilde{F}^{+}_{\mu\nu}u_{\alpha}\rangle$
62	$-\frac{1}{2}\sigma^{\mu\nu}u^{\alpha}\langle\widetilde{F}^{+}_{\alpha\mu}u_{\nu}\rangle$
63	$-\frac{1}{2}\sigma^{\mu\nu}u_{\mu}\langle\widetilde{F}^{+}_{\alpha\nu}u^{\alpha}\rangle$
64	$-\frac{1}{8m_N^2}\sigma^{\mu\nu}u_{\alpha}\langle\widetilde{F}^{+}_{\mu\nu}u_{\beta}\rangle\left\{D^{\alpha},D^{\beta}\right\}+\text{h.c.}$
65	$\frac{1}{8m_N^2}\sigma^{\mu\nu}u_{\beta}\langle\widetilde{F}^{+}_{\mu\alpha}u_{\nu}\rangle\left\{D^{\alpha},D^{\beta}\right\}+\text{h.c.}$
66	$\frac{1}{8m_N^2}\sigma^{\mu\nu}u_{\nu}\langle\widetilde{F}^{+}_{\mu\alpha}u_{\beta}\rangle\left\{D^{\alpha},D^{\beta}\right\}+\text{h.c.}$
67	$-\frac{i}{4m_N}\gamma^{\mu}\gamma_5\langle\widetilde{F}^{+}_{\alpha\mu}h^{\alpha}_{\nu}\rangle D^{\nu}+\text{h.c.}$
68	$-\frac{i}{4m_N}\gamma^{\nu}\gamma_5\langle\widetilde{F}^{+}_{\alpha\mu}h^{\alpha}_{\nu}\rangle D^{\nu}+\text{h.c.}$
69	$\frac{i}{24m_N^3}\gamma^{\alpha}\gamma_5\langle\widetilde{F}^{+}_{\alpha\mu}h_{\nu\beta}\rangle(D^{\mu}D^{\nu}D^{\beta}+\text{sym.})+\text{h.c.}$
70	$-\frac{i}{4m_N^2}\varepsilon^{\alpha\mu\nu}_{\tau}[\widetilde{F}^{+}_{\alpha\mu},h_{\nu\beta}]\left\{D^{\beta},D^{\tau}\right\}+\text{h.c.}$
71	$-\frac{i}{4m_N}\gamma^{\mu}\gamma_5\langle u^{\alpha}[D_{\alpha},\widetilde{F}^{+}_{\mu\nu}]\rangle D^{\nu}+\text{h.c.}$
72	$-\frac{i}{4m_N}\gamma^{\mu}\gamma_5\langle u^{\mu}[D^{\alpha},\widetilde{F}^{+}_{\alpha\nu}]\rangle D^{\nu}+\text{h.c.}$
73	$-\frac{i}{4m_N}\gamma^{\nu}\gamma_5\langle u^{\alpha}[D_{\alpha},\widetilde{F}^{+}_{\mu\nu}]\rangle D^{\mu}+\text{h.c.}$
74	$-\frac{1}{2}\sigma^{\mu\nu}[D^{\alpha},[D_{\alpha},\widetilde{F}^{+}_{\mu\nu}]]$
75	$\varepsilon^{\alpha\mu\nu\beta}u_{\alpha}\langle F^{-}_{\mu\nu}u_{\beta}\rangle$
76	$-\frac{1}{4m_N^2}\varepsilon^{\mu\nu\beta}_{\tau}u_{\alpha}\langle F^{-}_{\mu\nu}u_{\beta}\rangle\left\{D^{\alpha},D^{\tau}\right\}+\text{h.c.}$
77	$-\frac{1}{4m_N^2}\varepsilon^{\alpha\mu\nu}_{\tau}u_{\alpha}\langle F^{-}_{\mu\nu}u_{\beta}\rangle\left\{D^{\beta},D^{\tau}\right\}+\text{h.c.}$
78	$-\frac{1}{4m_N^2}\varepsilon^{\alpha\mu\nu}_{\tau}F^{-}_{\alpha\mu}\langle u_{\nu}u_{\beta}\rangle\left\{D^{\beta},D^{\tau}\right\}+\text{h.c.}$
79	$\frac{1}{4m_N}\gamma^{\mu}\gamma_5\langle F^{-}_{\alpha\mu}[u^{\alpha},u_{\nu}]\rangle D^{\nu}+\text{h.c.}$
80	$\frac{1}{4m_N}\gamma^{\nu}\gamma_5\langle F^{-}_{\alpha\mu}[u^{\alpha},u_{\nu}]\rangle D^{\mu}+\text{h.c.}$
81	$-\frac{1}{4m_N^2}\langle F^{-}_{\alpha_\mu}h^{\alpha}_{\nu}\rangle\left\{D^{\mu},D^{\nu}\right\}+\text{h.c.}$
82	$\frac{i}{2}\sigma^{\mu\nu}[F^{-}_{\alpha\mu},h^{\alpha}_{\nu}]$
83	$-\frac{i}{8m_N^2}\sigma^{\mu\nu}[F^{-}_{\alpha\mu},h_{\nu\beta}]\left\{D^{\alpha},D^{\beta}\right\}+\text{h.c.}$
84	$-\frac{i}{8m_N^2}\sigma^{\alpha\mu}[F^{-}_{\alpha\mu},h_{\nu\beta}]\left\{D^{\nu},D^{\beta}\right\}+\text{h.c.}$

Table 2.4 (Continued.)

i	$\mathcal{O}^{(4)}$
85	$\langle u^\mu [D^\nu, F^-_{\mu\nu}]\rangle$
86	$-\frac{1}{m_N^2}\langle u_\mu [D^\alpha, F^-_{\alpha\nu}]\rangle \{D^\mu, D^\nu\} + \text{h.c.}$
87	$\frac{i}{2}\sigma^{\mu\nu}[u^\alpha, [D_\alpha, F^-_{\mu\nu}]]$
88	$\frac{i}{2}\sigma^{\mu\nu}[u_\mu, [D^\alpha, F^-_{\alpha\nu}]]$
89	$\langle F^+_{\mu nu}\rangle\langle F^{\mu\nu+}\rangle$
90	$-\frac{1}{4m_N^2}\langle F^+_{\alpha\mu}\rangle\langle F^{\alpha+}_{\nu}\rangle \{D^\mu, D^\nu\} + \text{h.c.}$
91	$\widetilde{F}^+_{\mu\nu}\langle F^{\mu\nu+}\rangle$
92	$-\frac{1}{4m_N^2}\widetilde{F}^+_{\alpha\mu}\langle F^{\alpha+}_{\nu}\rangle \{D^\mu, D^\nu\} + \text{h.c.}$
93	$\langle \widetilde{F}^+_{\mu\nu}\widetilde{F}^{\mu\nu+}\rangle$
94	$-\frac{1}{4m_N^2}\langle \widetilde{F}^+_{\alpha\mu}\widetilde{F}^{\alpha+}_{\nu}\rangle \{D^\mu, D^\nu\} + \text{h.c.}$
95	$\frac{i}{2}\sigma^{\mu\nu}[\widetilde{F}^+_{\alpha\mu}, \widetilde{F}^{\alpha+}_{\nu}]$
96	$-\frac{i}{8m_N^2}\sigma^{\mu\nu}[\widetilde{F}^+_{\alpha\mu}, \widetilde{F}^+_{\beta\nu}]\left\{D^\alpha, D^\beta\right\} + \text{h.c.}$
97	$\frac{i}{2}\sigma^{\mu\nu}[F^-_{\alpha\mu}, F^{\alpha-}_{\nu}]$
98	$-\frac{i}{8m_N^2}\sigma^{\mu\nu}[F^-_{\alpha\mu}, F^-_{\beta\nu}]\left\{D^\alpha, D^\beta\right\} + \text{h.c.}$
99	$-\frac{i}{4m_N}\gamma^\mu\gamma_5 F^-_{\alpha\mu}\langle F^{\alpha+}_{\nu}\rangle D^\nu + \text{h.c.}$
100	$-\frac{i}{4m_N}\gamma^\nu\gamma_5 F^-_{\alpha\mu}\langle F^{\alpha+}_{\nu}\rangle D^\mu + \text{h.c.}$
101	$-\frac{i}{4m_N}\gamma^\mu\gamma_5 \langle F^-_{\alpha\mu}F^{\alpha+}_{\nu}\rangle D^\nu + \text{h.c.}$
102	$-\frac{i}{4m_N}\gamma^\nu\gamma_5 \langle F^-_{\alpha\mu}F^{\alpha+}_{\nu}\rangle D^\mu + \text{h.c.}$
103	$i\varepsilon^{\alpha\mu\nu\beta}[F^-_{\alpha\mu}, \widetilde{F}^+_{\nu\beta}]$
104	$-\frac{i}{2m_N^2}\varepsilon^{\alpha\mu\nu}_{\tau}[F^-_{\alpha\mu}, \widetilde{F}^+_{\nu\beta}]\left\{D^\beta, D^\tau\right\} + \text{h.c.}$
105	$-\frac{1}{2}\sigma^{\mu\nu}\langle F^+_{\mu\nu}\rangle\langle \chi_+\rangle$
106	$-\frac{1}{2}\sigma^{\mu\nu}\widetilde{F}^+_{\mu\nu}\langle \chi_+\rangle$
107	$-\frac{1}{2}\sigma^{\mu\nu}\langle F^+_{\mu\nu}\rangle\widetilde{\chi}_+$
108	$-\frac{1}{2}\sigma^{\mu\nu}\widetilde{F}^+_{\mu\nu}\widetilde{\chi}_+$

Table 2.4 (Continued.)

i	$\mathcal{O}^{(4)}$
109	$\frac{1}{4m_N}\gamma^\mu\gamma_5[F^-_{\mu\nu},\widetilde{\chi}_+]D^\nu + \text{h.c.}$
110	$\frac{1}{4m_N}\gamma^\mu\gamma_5\langle F^+_{\mu\nu}\rangle\langle\chi_-\rangle D^\nu + \text{h.c.}$
111	$\frac{1}{4m_N}\gamma^\mu\gamma_5\widetilde{F}^+_{\mu\nu}\langle\chi_-\rangle D^\nu + \text{h.c.}$
112	$\frac{1}{4m_N}\gamma^\mu\gamma_5\langle F^+_{\mu\nu}\rangle\widetilde{\chi}_- D^\nu + \text{h.c.}$
113	$\frac{1}{4m_N}\gamma^\mu\gamma_5\langle\widetilde{F}^+_{\mu\nu}\widetilde{\chi}_-\rangle D^\nu + \text{h.c.}$
114	$-\frac{1}{2}\sigma^{\mu\nu}[F^-_{\mu\nu},\widetilde{\chi}_-]$
115	$\frac{1}{4}\langle\chi_+^2 - \chi_-^2\rangle$
116	$-\frac{1}{4}(\langle\chi_-^2\rangle - \langle\chi_-\rangle^2 + \langle\chi_+^2\rangle - \langle\chi_+\rangle^2)$
117	$-\frac{1}{8m_N^2}\langle F^-_{\alpha\mu}F^{\alpha-}_{\nu} + F^+_{\alpha\mu}F^{\alpha+}_{\nu}\rangle\,\{D^\mu, D^\nu\} + \text{h.c.}$
118	$\frac{1}{2}\langle F^-_{\mu\nu}F^{\mu\nu-} + F^+_{\mu\nu}F^{\mu\nu+}\rangle$

2.4 Power counting

In this section, we will set up a scheme to organize the infinite number of terms contributing to the most general effective Lagrangian which can be ordered according to the number of derivatives acting on pion fields and powers of pion masses,

$$\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \ldots \tag{2.151}$$

The mesonic Lagrangian contains terms of even power, while in the baryonic case all orders appear. The order of a Feynman diagram will be defined corresponding to the order of each term in the Lagrangian. For the mesonic sector this is achieved by Weinberg's power counting. Any Feynman diagram contributing to a physical matrix element $\mathcal{M}$ is a function of the quark masses an the momenta of pions

$$\mathcal{M} = \mathcal{D}_1(m_q, p_i) + \mathcal{D}_2(m_q, p_i) + \ldots. \tag{2.152}$$

Consider the behavior of a physical matrix element $\mathcal{M}(m_q, p_i)$ under linear rescaling of the external pion momenta, $p_i \mapsto tp_i$, and quadratic rescaling of quark masses, $m_q \mapsto t^2 m_q$,

$$\mathcal{M}(m_q, p_i) \mapsto \mathcal{M}(t^2 m_q, tp_i) = t^D \mathcal{M}(m_q, p_i). \tag{2.153}$$

Here, D is the chiral dimension and is given by

$$D = 2 + \sum_{n=0}^{\infty} 2(n-1)N_{2n} + 2N_L \tag{2.154}$$

where N_{2n} is the number of vertices form $\mathcal{L}_{2n}$ and N_L is the number of loop integrations. For small values of t diagrams with an increasing order D are suppressed and those with smaller D dominate. If the order D is fixed only a limited number of diagrams from the most general Lagrangian contribute. Using the relation

$$N_V = N_I - N_L + 1, \tag{2.155}$$

where N_V is the total number of vertices and N_I stands for the number of internal pion lines, one obtains

$$D = 4N_L - 2N_I + \sum_{n=0}^{\infty} 2nN_{2n}. \tag{2.156}$$

Therefore, one assigns the following chiral order to individual parts of Feynman diagrams:

1. Loop integration in 4 dimensions counts as chiral order 4,

2. a pion propagator counts as chiral order -2,

3. a vertex from $\mathcal{L}_{2n}$ counts as chiral order $2n$.

Next we will extend to the baryonic sector. The power counting in the baryonic sector was first stated by Gasser, Sainio, and Svarc (Gasser et al., 1988). The generalization of the power counting from the mesonic sector is realized by assigning the following chiral orders to the individual component of diagram:

1. The nucleon propagator counts as chiral order -1,

2. vertices from the Lagrangian $\mathcal{L}_{\pi N}^{(n)}$ count as chiral order n,

3. the mesonic power counting is still the same.

Finally loop corrections are arised at some stage in a perturbative calculation which have to be treated carefully. In the mesonic sector diagrams are evaluated by using dimensional regularization and the so-called modified minimal substraction scheme of ChPT ($\widetilde{MS}$). When ChPT was extend to include processes with one nucleon, one saw a breakdown of the power counting. The breakdown consist of terms with smaller chiral dimension of the nucleon. Let us establish the chiral orders of the diagrams in Figure 2.1 using the above power counting.

1. The diagram of the left in Figure 2.1 has chiral order

$$D = n + 2 \cdot 1 - 1 - 2 = n - 1 \to 3 \tag{2.157}$$

2. The right diagram in Figure 2.1 has chiral order

$$D = n + 1 \cdot 2 - 2 = n \to 4 \tag{2.158}$$

where n is the space-time dimension. However the lowest-order term has chiral order

$$D = 2, \tag{2.159}$$

We will see explicitly that the calculation of the diagrams in dimensional regularization combined with the $\widetilde{MS}$-scheme shoes that in Figure 2.1, the left diagram contains terms violating power-counting, but the right diagram satisfies power counting. One has to remember that the power counting was obtained by rescaling the momenta and quark masses of a physical element and attending to the behavior of diagrams contributing to this matrix element case

Figure 2.1 Self-energy diagrams

by case. Therefore, power counting should be applied to renormalized diagrams only, it was realized in (Gasser et al., 1988) that the validity of the power counting scheme is related to the choice of the renormalization scheme.

CHAPTER III

INCLUSION OF PHOTONS AND LEPTONS

In the previous chapter we have presented the construction of the most general effective Lagrangian only for strong interactions. In this chapter we will construct the complete effective chiral Lagrangian including photons and leptons for the mesonic and photons for the baryonic sectors. The next chapter will be devoted to the extension of this procedure to obtain the full Lagrangian in the baryonic sector including both virtual photons and leptons. Since the details of the construction are similar to the previous chapter, we will concentrate on the parts due to the inclusion of the photons and leptons.

3.1 Chiral perturbation theory with virtual photons

From the reason that Chiral perturbation theory is a nonrenormalizable theory, loops generate divergences. One needs a set of counterterms for each order and thus an infinite number to renormalize to all orders. In ChPT, one considers only order by order, and renormalizes only be order. The divergences can be absorbed by introducing counterterms. The associated coupling constants of these counterterm absorb the divergences that are produced by loop-graphs with a virtual photon or a vertex from the Lagrangian of $\mathcal{O}(e^2)$.

The counterterms of the effective Lagrangian for electromagnetic interactions were introduced firstly by G. Ecker et al. (Ecker et al., 1989) but they considered only up to second order for mesonic sector. Then, the counterterms for fourth order effective Lagrangian in mesonic (Urech, 1995) and baryonic sector

(Müller and Meißner, 1999) were constructed later.

To introduce photons in the effective field theory, one firstly has to set up the power counting scheme for the electric charge e. From the observation that $e^2/4\pi \simeq m_\pi^2/(4\pi F_\pi)^2 \sim 1/100$, one counts the electric charge as a small momentum (Müller and Meißner, 1999),

$$e = \mathcal{O}(p). \tag{3.1}$$

Since the electric charge is always quadratic there are only terms of order e^2 at second order, e^2p at third order and e^2p^2 or e^4 at fourth order .

3.1.1 Definitions for meson case

The effective Lagrangian in mesonic sector with the inclusion of virtual photons had been constructed (Urech, 1995) up to fourth order. The building blocks which corresponded to the electromagnetic effects are defined via the spurions Q_L and Q_R with a definite transformation property under chiral $SU(3)_L \times SU(3)_R$,

$$Q_I \longrightarrow g_I Q_I g_I^\dagger, \quad g_I \in SU(3), \quad I = L, R. \tag{3.2}$$

In our case we consider chiral $SU(2)$ symmetry and follow the procedure and notation of Knecht et al. (Knecht et al., 2000) in which the photon field A_μ is introduced in

$$u_\mu = i[u_R^\dagger(\partial_\mu - ir_\mu)u_R - u_L^\dagger(\partial_\mu - il_\mu)u_L], \tag{3.3}$$

by adding the term which corresponds to the electromagnetic field to the usual external vector field v_μ. The result gives

$$r_\mu = v_\mu + a_\mu - eQ_R^{\mathrm{em}} A_\mu, \tag{3.4}$$

$$l_\mu = v_\mu - a_\mu - eQ_L^{\mathrm{em}} A_\mu. \tag{3.5}$$

In this work we consider $SU(2)$ symmetry. The quark charge matrix for $SU(2)$ symmetry is

$$Q^{\rm em}_{L,R} = \left(\begin{array}{cc} 2/3 & 0 \\ 0 & -1/3 \end{array} \right), \tag{3.6}$$

and one introduces spurion fields $Q^{\rm em}_{L,R}$ with the transformation properties

$$Q^{\rm em}_{L} \xrightarrow{G} g_L Q^{\rm em}_{L} g_L^\dagger, \qquad Q^{\rm em}_{R} \xrightarrow{G} g_R Q^{\rm em}_{R} g_R^\dagger, \tag{3.7}$$

under chiral $SU(2)_L \times SU(2)_R$. In accord with the definitions of the building blocks defined in the previous chapter, one also defines

$$\mathcal{Q}^{\rm em}_{L} \equiv u_L^\dagger Q^{\rm em}_{L} u_L, \tag{3.8}$$

$$\mathcal{Q}^{\rm em}_{R} \equiv u_R^\dagger Q^{\rm em}_{L} u_R, \tag{3.9}$$

which transform under chiral group as,

$$\mathcal{Q}^{\rm em}_{L} \xrightarrow{G} h(g,\phi)\mathcal{Q}^{\rm em}_{L} h(g,\phi)^{-1}, \tag{3.10}$$

$$\mathcal{Q}^{\rm em}_{R} \xrightarrow{G} h(g,\phi)\mathcal{Q}^{\rm em}_{R} h(g,\phi)^{-1}. \tag{3.11}$$

Furthermore, under parity (P) and charge conjugation (C) transformations, one finds

$$\mathcal{Q}^{\rm em}_{R} \xrightarrow{P} \mathcal{Q}^{\rm em}_{L}, \qquad \mathcal{Q}^{\rm em}_{L} \xrightarrow{C} \mathcal{Q}^{\rm em}_{R}, \tag{3.12}$$

$$\mathcal{Q}^{\rm em}_{R} \xrightarrow{P} \mathcal{Q}^{{\rm em}T}_{L}, \qquad \mathcal{Q}^{\rm em}_{L} \xrightarrow{C} \mathcal{Q}^{{\rm em}T}_{R}. \tag{3.13}$$

One also defines the derivative of $\mathcal{Q}^{\rm em}_{R}$ and $\mathcal{Q}^{\rm em}_{L}$ as (Knecht et al., 2000)

$$\widehat{\nabla}_\mu \mathcal{Q}^{\rm em}_{L} = \nabla_\mu \mathcal{Q}^{\rm em}_{L} + \frac{i}{2}[u_\mu, \mathcal{Q}^{\rm em}_{L}] = u_L^\dagger (D_\mu Q^{\rm em}_{L}) u_L, \tag{3.14}$$

$$\widehat{\nabla}_\mu \mathcal{Q}^{\rm em}_{R} = \nabla_\mu \mathcal{Q}^{\rm em}_{R} - \frac{i}{2}[u_\mu, \mathcal{Q}^{\rm em}_{R}] = u_R^\dagger (D_\mu Q^{\rm em}_{R}) u_R, \tag{3.15}$$

where

$$D_\mu Q^{\rm em}_{L} = \partial_\mu Q^{\rm em}_{L} - i[l_\mu, Q^{\rm em}_{L}], \tag{3.16}$$

$$D_\mu Q^{\rm em}_{R} = \partial_\mu Q^{\rm em}_{R} - i[r_\mu, Q^{\rm em}_{R}], \tag{3.17}$$

which transform in the same way as $\mathcal{Q}_L^{\text{em}}$ and $\mathcal{Q}_R^{\text{em}}$ and the definitions of $\nabla_\mu \mathcal{Q}_L^{\text{em}}$ and $\nabla_\mu \mathcal{Q}_R^{\text{em}}$ are

$$\nabla_\mu \mathcal{Q}_L^{\text{em}} = \partial_\mu \mathcal{Q}_L^{\text{em}} + [\Gamma_\mu, \mathcal{Q}_L^{\text{em}}], \tag{3.18}$$

$$\nabla_\mu \mathcal{Q}_R^{\text{em}} = \partial_\mu \mathcal{Q}_R^{\text{em}} + [\Gamma_\mu, \mathcal{Q}_R^{\text{em}}]. \tag{3.19}$$

3.1.2 The EM lagrangian in meson case

With these building blocks the lowest order effective pion Lagrangian with virtual photons takes the form

$$\mathcal{L}_{\pi\pi,\text{em}}^{(2)} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\lambda(\partial_\mu A^\mu)^2 + \frac{F_0^2}{4}\langle u_\mu u^\mu\rangle + \frac{F_0^2}{4}\langle \chi_+\rangle + e^2 F_0^4 Z\langle \mathcal{Q}_L^{\text{em}}\mathcal{Q}_R^{\text{em}}\rangle, \tag{3.20}$$

where $F_{\mu\nu}$ is the field strength tensor of the photon field A_μ, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. λ is the gauge fixing parameter and will be kept at $\lambda = 1$ (we are using the Feynman gauge). The coupling constant Z can be determined from the difference of the charged pion and the neutral pion masses.

For the fourth order, there are two minimal set of terms. One has order e^2p^2 and another one has order e^4. For the first set Urech's Lagrangian can be rewritten in Knecht's notation as (Knecht et al., 2000)

$$\begin{aligned}
\mathcal{L}_{\pi\pi,\text{em}}^{(4),e^2p^2} = \; & e^2 F_0^2 \Big\{ \frac{1}{2}k_1\langle(\mathcal{Q}_L^{\text{em}})^2 + (\mathcal{Q}_R^{\text{em}})^2\rangle\langle u_\mu u^\mu\rangle + k_2\langle \mathcal{Q}_L^{\text{em}}\mathcal{Q}_R^{\text{em}}\rangle\langle u_\mu u^\mu\rangle \\
& -k_3[\langle \mathcal{Q}_L^{\text{em}} u_\mu\rangle\langle \mathcal{Q}_L^{\text{em}} u^\mu\rangle + \langle \mathcal{Q}_R^{\text{em}} u_\mu\rangle\langle \mathcal{Q}_R^{\text{em}} u^\mu\rangle] \\
& +k_4\langle \mathcal{Q}_L^{\text{em}} u_\mu\rangle\langle \mathcal{Q}_R^{\text{em}} u^\mu\rangle + k_5\langle[(\mathcal{Q}_L^{\text{em}})^2 + (\mathcal{Q}_R^{\text{em}})^2]u_\mu u^\mu\rangle \\
& +k_6\langle(\mathcal{Q}_L^{\text{em}}\mathcal{Q}_R^{\text{em}} + \mathcal{Q}_R^{\text{em}}\mathcal{Q}_L^{\text{em}})u_\mu u^\mu\rangle \\
& +\frac{1}{2}k_7\langle(\mathcal{Q}_L^{\text{em}})^2 + (\mathcal{Q}_R^{\text{em}})^2\rangle\langle\chi_+\rangle \\
& +k_8\langle \mathcal{Q}_L^{\text{em}}\mathcal{Q}_R^{\text{em}}\rangle\langle\chi_+\rangle + k_9\langle[(\mathcal{Q}_L^{\text{em}})^2 + (\mathcal{Q}_R^{\text{em}})^2]\chi_+\rangle \\
& +k_{10}\langle(\mathcal{Q}_L^{\text{em}}\mathcal{Q}_R^{\text{em}} + \mathcal{Q}_R^{\text{em}}\mathcal{Q}_L^{\text{em}})\chi_+\rangle \\
& -k_{11}\langle(\mathcal{Q}_L^{\text{em}}\mathcal{Q}_R^{\text{em}} - \mathcal{Q}_R^{\text{em}}\mathcal{Q}_L^{\text{em}})\chi_-\rangle
\end{aligned}$$

$$
\begin{aligned}
& -ik_{12}\langle\Big[(\widehat{\nabla}_\mu \mathcal{Q}_L^{\rm em})\mathcal{Q}_L^{\rm em} - \mathcal{Q}_L^{\rm em}\widehat{\nabla}_\mu \mathcal{Q}_L^{\rm em} \\
& - (\widehat{\nabla}_\mu \mathcal{Q}_R^{\rm em})\mathcal{Q}_R^{\rm em} + \mathcal{Q}_R^{\rm em}\widehat{\nabla}_\mu \mathcal{Q}_R^{\rm em}\Big] u^\mu\rangle \\
& +k_{13}\langle(\widehat{\nabla}_\mu \mathcal{Q}_L^{\rm em})(\widehat{\nabla}^\mu \mathcal{Q}_R^{\rm em})\rangle \\
& +k_{14}\langle(\widehat{\nabla}_\mu \mathcal{Q}_L^{\rm em})(\widehat{\nabla}^\mu \mathcal{Q}_L^{\rm em}) + (\widehat{\nabla}_\mu \mathcal{Q}_R^{\rm em})(\widehat{\nabla}^\mu \mathcal{Q}_R^{\rm em})\rangle\Big\}, \qquad (3.21)
\end{aligned}
$$

The latter set which has order e^4 and which we will need, was not considered by Knecht et al. Therefore, we have rewritten at order e^4 based on the Knecht's notation as

$$
\begin{aligned}
\mathcal{L}_{\pi\pi,\rm em}^{(4),e^4} &= e^2 F_0^4 \Big\{k_{15}\langle \mathcal{Q}_R^{\rm em}\mathcal{Q}_L^{\rm em}\rangle^2 + \frac{k_{16}}{2}\left(\langle \mathcal{Q}_R^{\rm em}\mathcal{Q}_L^{\rm em}\rangle\langle(\mathcal{Q}_R^{\rm em})^2 + (\mathcal{Q}_L^{\rm em})^2\right) \\
& + \frac{k_{17}}{4}\langle(\mathcal{Q}_R^{\rm em})^2 + (\mathcal{Q}_L^{\rm em})^2\rangle^2\Big\}. \qquad (3.22)
\end{aligned}
$$

3.1.3 Definitions for baryon case

Electromagnetic corrections to pion-nucleon systems were first emphasized by Weinberg (Weinberg, 1977). He pointed out that reactions involving nucleon and neutral pions might lead to violations of isospin symmetry and argued that the mass difference of the up and down quarks can produce a 30% effect in the difference of the $\pi^0 p$ and $\pi^0 n$ S-wave scattering length. This calculation was extended to the so-called pion-nucleon σ-term by Meißner and Steininger (Meißner and Steininger, 1998). The effective chiral pion-nucleon Lagrangian with the inclusion of virtual photons to one loop was constructed up to third order in that paper and extended to fourth order by Müller and Meißner (Müller and Meißner, 1999).

To introduce virtual photons in the effective pion-nucleon field theory one defines (Meißner and Steininger, 1998)

$$
Q_\pm = \frac{1}{2}\left(uQu^\dagger \pm u^\dagger Q u\right), \qquad (3.23)
$$

which can be rewritten in Knecht's notation as

$$Q_{\pm} = \frac{1}{2}\left(\mathcal{Q}_L^{\text{em}} \pm \mathcal{Q}_R^{\text{em}}\right), \tag{3.24}$$

where the definitions of $\mathcal{Q}_L^{\text{em}}$ and $\mathcal{Q}_R^{\text{em}}$ are in Equation (3.8) and Equation (3.9). It is natural here to use the nucleon charge matrix

$$Q_{L,R}^{\text{em}} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{3.25}$$

We can also define the covariant derivative of $Q_{\pm}$ via

$$[\nabla_\mu, Q_{\pm}] = \frac{i}{2}[u_\mu, Q_{\mp}] + c_\mu^{\pm}, \tag{3.26}$$

where

$$c_\mu^{\pm} = \frac{1}{2}\left\{u_L(\partial_\mu Q_L^{\text{em}} - i[l_\mu, Q_L^{\text{em}}])u_L^\dagger \pm u_R^\dagger(\partial_\mu Q_R^{\text{em}} - i[r_\mu, Q_R^{\text{em}}]u_R)\right\}. \tag{3.27}$$

Under parity and charge conjugation $Q_{\pm}$ transform as

$$Q_{\pm} \xrightarrow{P} \pm Q_{\pm}, \quad Q_{\pm} \xrightarrow{C} \pm {Q_{\pm}}^T. \tag{3.28}$$

3.1.4 The EM lagrangian in baryon case

At first order, from substituting Equation (3.4) and Equation (3.5) to Equation (3.3) one finds

$$\mathcal{L}_{\pi N,\text{em}}^{(1)} = \overline{\Psi}\left(i\gamma_\mu \widetilde{D}^\mu - m_N + \frac{1}{2}g_A\gamma^\mu\gamma_5\widetilde{u}_\mu\right)\Psi, \tag{3.29}$$

with

$$\widetilde{D}_\mu = D_\mu - iQ_+A_\mu, \tag{3.30}$$

$$\widetilde{u}_\mu = u_\mu - 2Q_-A_\mu. \tag{3.31}$$

Table 3.1 Monomials $\mathcal{O}_i$ of third order for the relativistic EM lagrangian.

i	$\mathcal{O}^{(3)}_{i,\text{em}}$	i	$\mathcal{O}^{(3)}_{i,\text{em}}$
1	$\gamma^\mu\gamma_5\langle\widetilde{Q}_+u_\mu\rangle\widetilde{Q}_+$	7	$\gamma^\mu\langle\widetilde{Q}_-u_\mu\rangle\widetilde{Q}_+$
2	$\gamma^\mu\gamma_5\langle\widetilde{Q}_-u_\mu\rangle\widetilde{Q}_-$	8	$\gamma^\mu\langle\widetilde{Q}_-u_\mu\rangle\langle Q_+\rangle$
3	$\gamma^\mu\gamma_5\langle\widetilde{Q}_+u_\mu\rangle\langle Q_+\rangle$	9	$\gamma^\mu[\widetilde{Q}_+,[i\nabla_\mu,\widetilde{Q}_+]]$
4	$\gamma^\mu\gamma_5\langle\widetilde{Q}^2_+ + \widetilde{Q}^2_-\rangle$	10	$\gamma^\mu[\widetilde{Q}_-,[i\nabla_\mu,\widetilde{Q}_-]]$
5	$\gamma^\mu\gamma_5\langle\widetilde{Q}^2_+ - \widetilde{Q}^2_-\rangle$	11	$\gamma^\mu\gamma_5[\widetilde{Q}_+,[i\nabla_\mu,\widetilde{Q}_-]]$
6	$\gamma^\mu\langle\widetilde{Q}_+u_\mu\rangle\widetilde{Q}_-$	12	$\gamma^\mu\gamma_5[\widetilde{Q}_-,[i\nabla_\mu,\widetilde{Q}_+]]$

At second order, local contact terms with their low-energy constants (LECs) f_i appear. The EM Lagrangian is given in terms of squares of $Q_\pm$ as expressed before (Müller and Meißner, 1999),

$$\mathcal{L}^{(2)}_{\pi N,\text{em}} = \sum_{i=1}^{3} e^2F_0^2 f_i\overline{\Psi}\mathcal{O}^{(2)}_i\Psi, \tag{3.32}$$

with

$$\mathcal{O}^{(2)}_1 = \langle\widetilde{Q}^2_+ - \widetilde{Q}^2_-\rangle, \quad \mathcal{O}^{(2)}_2 = \langle Q_+\rangle\widetilde{Q}_+, \quad \mathcal{O}^{(2)}_3 = \langle\widetilde{Q}^2_+ + \widetilde{Q}^2_-\rangle, \tag{3.33}$$

where $\widetilde{Q}_\pm$ represents the traceless part of $Q_\pm$. The EM Lagrangian to third order has been constructed with LECs g_i (Müller and Meißner, 1999)

$$\mathcal{L}^{(3)}_{\pi N,\text{em}} = \sum_{i=1}^{12} e^2F^2 g_i\overline{\Psi}\mathcal{O}^{(3)}_i\Psi, \tag{3.34}$$

with the $\mathcal{O}^{(3)}_i$ are in Table 3.1.

Next, we will consider the terms of fourth order. The complete fourth order EM pion-nucleon Lagrangian with corresponding LECs is written as (Müller and Meißner, 1999)

$$\mathcal{L}^{(4)}_{\pi N,\text{em}} = \sum_{i=1}^{5} e^4F_0^4 h_i\overline{\Psi}\mathcal{O}^{(e^4)}_i\Psi + \sum_{i=6}^{90} e^2F_0^2 h_i\overline{\Psi}\mathcal{O}^{(e^2p^2)}_i\Psi. \tag{3.35}$$

Table 3.2 The monomials for $\mathcal{O}(e^4)$.

i	$\mathcal{O}(e^4)$
1	$\langle \widetilde{Q}_+^2 + \widetilde{Q}_-^2 \rangle^2$
2	$\langle \widetilde{Q}_+^2 - \widetilde{Q}_-^2 \rangle^2$
3	$\langle \widetilde{Q}_+^2 + \widetilde{Q}_-^2 \rangle \langle \widetilde{Q}_+^2 - \widetilde{Q}_-^2 \rangle$
4	$\langle \widetilde{Q}_+^2 + \widetilde{Q}_-^2 \rangle \langle Q_+ \rangle \widetilde{Q}_+$
5	$\langle \widetilde{Q}_+^2 - \widetilde{Q}_-^2 \rangle \langle Q_+ \rangle \widetilde{Q}_+$

Table 3.3 The monomials for $\mathcal{O}(e^2p^2)$.

i	$\mathcal{O}(e^2p^2)$	i	$\mathcal{O}(e^2p^2)$
6	$\langle \widetilde{Q}_+^2 + \widetilde{Q}_-^2 \rangle \langle u^2 \rangle$	12	$\langle \widetilde{Q}_+^2 + \widetilde{Q}_-^2 \rangle \langle u_\mu u_\nu \rangle D^\mu D^\nu + \text{h.c.}$
7	$\langle \widetilde{Q}_+^2 - \widetilde{Q}_-^2 \rangle \langle u^2 \rangle$	13	$\langle \widetilde{Q}_+^2 - \widetilde{Q}_-^2 \rangle \langle u_\mu u_\nu \rangle D^\mu D^\nu + \text{h.c.}$
8	$\langle \widetilde{Q}_+ u_\mu \rangle \langle \widetilde{Q}_+ u^\mu \rangle$	14	$\langle \widetilde{Q}_+ u_\mu \rangle \langle \widetilde{Q}_+ u_\nu \rangle D^\mu D^\nu + \text{h.c.}$
9	$\langle \widetilde{Q}_- u_\mu \rangle \langle \widetilde{Q}_- u^\mu \rangle$	15	$\langle \widetilde{Q}_- u_\mu \rangle \langle \widetilde{Q}_- u_\nu \rangle D^\mu D^\nu + \text{h.c.}$
10	$\langle Q_+ \rangle \langle u^2 \rangle \widetilde{Q}_+$	16	$\langle Q_+ \rangle \langle u_\mu u_\nu \rangle \widetilde{Q}_+ D^\mu D^\nu + \text{h.c.}$
11	$\langle Q_+ \rangle \langle \widetilde{Q}_+ u_\mu \rangle u^\mu$	17	$\langle Q_+ \rangle \langle \widetilde{Q}_+ u_\mu \rangle u_\nu D^\mu D^\nu + \text{h.c.}$

Note that the first five terms are all $\mathcal{O}(e^4)$ and given in Table 3.2. The other terms are $\mathcal{O}(e^2p^2)$ and given in Table 3.3.

Table 3.3 (Continued.)

i	$\mathcal{O}(e^2p^2)$	i	$\mathcal{O}(e^2p^2)$
18	$i\sigma^{\mu\nu}\langle\widetilde{Q}_+[u_\mu,u_\nu]\rangle\widetilde{Q}_+$	42	$\sigma^{\mu\nu}\langle\widetilde{Q}_+^2-\widetilde{Q}_-^2\rangle\langle F^+_{\mu\nu}\rangle$
19	$i\sigma^{\mu\nu}\langle\widetilde{Q}_-[u_\mu,u_\nu]\rangle\widetilde{Q}_-$	43	$\sigma^{\mu\nu}\langle Q_+\rangle\widetilde{Q}_+\langle F^+_{\mu\nu}\rangle$
20	$i\sigma^{\mu\nu}\langle\widetilde{Q}_+^2+\widetilde{Q}_-^2\rangle[u_\mu,u_\nu]$	44	$\sigma^{\mu\nu}\gamma_5\langle[\widetilde{Q}_+,\widetilde{Q}_-]\widetilde{F}^+_{\mu\nu}\rangle$
21	$i\sigma^{\mu\nu}\langle\widetilde{Q}_+^2-\widetilde{Q}_-^2\rangle[u_\mu,u_\nu]$	45	$\sigma^{\mu\nu}\gamma_5[\widetilde{Q}_+,\widetilde{Q}_-]\langle F^+_{\mu\nu}\rangle$
22	$i\sigma^{\mu\nu}\langle\widetilde{Q}_+u_\mu\rangle[\widetilde{Q}_+,u_\nu]$	46	$\sigma^{\mu\nu}\gamma_5\langle Q_+\rangle[\widetilde{Q}_-,\widetilde{F}^+_{\mu\nu}]$
23	$i\sigma^{\mu\nu}\langle\widetilde{Q}_-u_\mu\rangle[\widetilde{Q}_-,u_\nu]$	47	$\sigma^{\mu\nu}\langle Q_+\rangle\langle\widetilde{Q}_-\widetilde{F}^-_{\mu\nu}\rangle$
24	$i\sigma^{\mu\nu}\langle Q_+\rangle\langle\widetilde{Q}_+[u_\mu,u_\nu]\rangle$	48	$\sigma^{\mu\nu}\widetilde{Q}_+\langle\widetilde{Q}_-\widetilde{F}^-_{\mu\nu}\rangle$
25	$\gamma^\mu\gamma_5\langle\widetilde{Q}_+u_\mu\rangle\langle\widetilde{Q}_-u_\nu\rangle iD^\nu+\text{h.c.}$	49	$\sigma^{\mu\nu}\widetilde{Q}_-\langle\widetilde{Q}_+\widetilde{F}^-_{\mu\nu}\rangle$
26	$\gamma^\nu\gamma_5\langle\widetilde{Q}_+u_\mu\rangle\langle\widetilde{Q}_-u_\nu\rangle iD^\mu+\text{h.c.}$	50	$\sigma^{\mu\nu}\gamma_5\langle Q_+\rangle[\widetilde{Q}_+,\widetilde{F}^-_{\mu\nu}]$
27	$\gamma^\mu\gamma_5\langle Q_+\rangle\langle\widetilde{Q}_-u_\mu\rangle u_\nu iD^\nu+\text{h.c.}$	51	$\langle[i\nabla_\mu,\widetilde{Q}_+][\widetilde{Q}_-,u^\mu]\rangle$
28	$\gamma^\nu\gamma_5\langle Q_+\rangle\langle\widetilde{Q}_-u_\mu\rangle u_\nu iD^\mu+\text{h.c.}$	52	$\langle[i\nabla_\mu,\widetilde{Q}_-][\widetilde{Q}_+,u^\mu]\rangle$
29	$\langle\widetilde{Q}_+^2+\widetilde{Q}_-^2\rangle\widetilde{\chi}_+$	53	$\langle[i\nabla_\mu,\widetilde{Q}_+][\widetilde{Q}_-,u_\nu]D^\mu D^\nu+\text{h.c.}$
30	$\langle\widetilde{Q}_+^2-\widetilde{Q}_-^2\rangle\widetilde{\chi}_+$	54	$\langle[i\nabla_\mu,\widetilde{Q}_-][\widetilde{Q}_+,u_\nu]D^\mu D^\nu+\text{h.c.}$
31	$\langle Q_+\rangle\langle\widetilde{Q}_+\widetilde{\chi}_+\rangle$	55	$\langle[i\nabla_\mu,u_\nu][\widetilde{Q}_+,\widetilde{Q}_-]D^\mu D^\nu+\text{h.c.}$
32	$\langle\widetilde{Q}_+^2+\widetilde{Q}_-^2\rangle\langle\chi_+\rangle$	56	$\langle\widetilde{Q}_-[\nabla^\mu,[\nabla_\mu,\widetilde{Q}_-]]\rangle$
33	$\langle\widetilde{Q}_+^2-\widetilde{Q}_-^2\rangle\langle\chi_+\rangle$	57	$\langle\widetilde{Q}_-[\nabla_\mu,[\nabla_\nu,\widetilde{Q}_-]]\rangle D^\mu D^\nu+\text{h.c.}$
34	$\langle Q_+\rangle\widetilde{Q}_+\langle\chi_+\rangle$	58	$\langle[\nabla_\mu,\widetilde{Q}_-][\nabla^\mu,\widetilde{Q}_-]\rangle$
35	$\langle Q_+\rangle[i\widetilde{Q}_-,\widetilde{\chi}_-]$	59	$\langle[\nabla_\mu,\widetilde{Q}_-][\nabla_\nu,\widetilde{Q}_-]\rangle D^\mu D^\nu+\text{h.c.}$
36	$\langle[\widetilde{Q}_+,\widetilde{Q}_-]\widetilde{\chi}_-\rangle$	60	$\langle Q_+\rangle[[i\nabla_\mu,\widetilde{Q}_-],u^\mu]$
37	$[\widetilde{Q}_+,\widetilde{Q}_-]\langle\chi_-\rangle$	61	$\langle Q_+\rangle[[i\nabla_\mu,\widetilde{Q}_-,u_\nu]]D^\mu D^\nu+\text{h.c.}$
38	$\sigma^{\mu\nu}\langle\widetilde{Q}_+^2+\widetilde{Q}_-^2\rangle\widetilde{F}^+_{\mu\nu}$	62	$\langle Q_+\rangle[[i\nabla_\mu,u_\nu],\widetilde{Q}_-]D^\mu D^\nu+\text{h.c.}$
39	$\sigma^{\mu\nu}\langle\widetilde{Q}_+^2-\widetilde{Q}_-^2\rangle\widetilde{F}^+_{\mu\nu}$	63	$\langle[\nabla^\mu,\widetilde{Q}_+][\nabla_\mu,\widetilde{Q}_+]\rangle$
40	$\sigma^{\mu\nu}\langle Q_+\rangle\langle\widetilde{Q}_+\widetilde{F}^+_{\mu\nu}\rangle$	64	$\langle[\nabla_\mu,\widetilde{Q}_+][\nabla_\nu,\widetilde{Q}_+]\rangle D^\mu D^\nu+\text{h.c.}$
41	$\sigma^{\mu\nu}\langle\widetilde{Q}_+^2+\widetilde{Q}_-^2\rangle\langle F^+_{\mu\nu}\rangle$	65	$\langle\widetilde{Q}_+[\nabla^\mu,[\nabla_\mu,\widetilde{Q}_+]]\rangle$

Table 3.3 (Continued.)

i	$\mathcal{O}(e^2p^2)$	i	$\mathcal{O}(e^2p^2)$
66	$\langle\widetilde{Q}_+[\nabla_\mu,[\nabla_\nu,\widetilde{Q}_+]]\rangle D^\mu D^\nu+\text{h.c.}$	79	$i\sigma^{\mu\nu}[\widetilde{Q}_-,[\nabla_\mu,[\nabla_\nu,\widetilde{Q}_-]]]$
67	$[\nabla^\mu,[\nabla_\mu,\widetilde{Q}_+]]\langle Q_+\rangle$	80	$\gamma^\mu\gamma_5\langle Q_+\rangle[[\nabla_\mu,\widetilde{Q}],u_\nu]D^\nu+\text{h.c.}$
68	$[\nabla_\mu,[\nabla_\nu,\widetilde{Q}_+]]\langle Q_+\rangle D^\mu D^\nu+\text{h.c.}$	81	$\gamma^\nu\gamma_5\langle Q_+\rangle[[\nabla_\mu,\widetilde{Q}],u_\nu]D^\mu+\text{h.c.}$
69	$\sigma^{\mu\nu}[\nabla_\mu,\widetilde{Q}_+]\langle\widetilde{Q}_-u_\nu\rangle$	82	$\gamma^\mu\gamma_5\langle[\nabla_\mu,\widetilde{Q}_+][\widetilde{Q}_+,u_\nu]\rangle D^\nu+\text{h.c.}$
70	$\sigma^{\mu\nu}\widetilde{Q}_+\langle[\nabla_\mu,\widetilde{Q}_-]u_\nu\rangle$	83	$\gamma^\nu\gamma_5\langle[\nabla_\mu,\widetilde{Q}_+][\widetilde{Q}_+,u_\nu]\rangle D^\mu+\text{h.c.}$
71	$\sigma^{\mu\nu}\langle Q_+\rangle\langle[\nabla_\mu,\widetilde{Q}_-]u_\mu\rangle$	84	$\gamma^\mu\gamma_5\langle[\nabla_\mu,\widetilde{Q}_-][\widetilde{Q}_-,u_\nu]\rangle D^\nu+\text{h.c.}$
72	$\sigma^{\mu\nu}[\nabla_\mu,\widetilde{Q}_-]\langle\widetilde{Q}_+u_\nu$	85	$\gamma^\nu\gamma_5\langle[\nabla_\mu,\widetilde{Q}_-][\widetilde{Q}_-,u_\nu]\rangle D^\mu+\text{h.c.}$
73	$\sigma^{\mu\nu}\widetilde{Q}_-\langle[\nabla_\mu,\widetilde{Q}_+]u_\nu\rangle$	86	$\gamma^\nu\gamma_5\langle[\nabla_\mu,\widetilde{Q}_+][\nabla_\nu,\widetilde{Q}_-]\rangle iD^\mu+\text{h.c.}$
74	$\sigma^{\mu\nu}u_\mu\langle[\nabla_\nu,\widetilde{Q}_+]\widetilde{Q}_-\rangle$	87	$\gamma^\mu\gamma_5\langle[\nabla_\mu,\widetilde{Q}_+][\nabla_\nu,\widetilde{Q}_-]\rangle iD^\nu+\text{h.c.}$
75	$\sigma^{\mu\nu}u_\mu\langle[\nabla_\nu,\widetilde{Q}_-]\widetilde{Q}_+\rangle$	88	$i\sigma^{\mu\nu}\gamma_5\langle[\nabla_\mu,[\nabla_\nu,\widetilde{Q}_+]]\widetilde{Q}_-\rangle$
76	$i\sigma^{\mu\nu}[[\nabla_\mu,\widetilde{Q}_+],[\nabla_\nu,\widetilde{Q}_+]]$	89	$i\sigma^{\mu\nu}\gamma_5\langle[\nabla_\mu,[\nabla_\nu,\widetilde{Q}_-]]\widetilde{Q}_+\rangle$
77	$i\sigma^{\mu\nu}[[\nabla_\mu,\widetilde{Q}_-],[\nabla_\nu,\widetilde{Q}_-]]$	90	$i\sigma^{\mu\nu}\gamma_5\langle Q_+\rangle[\nabla_\mu,[\nabla_\nu,\widetilde{Q}_-]]$
78	$i\sigma^{\mu\nu}[\widetilde{Q}_+,[\nabla_\mu,[\nabla_\nu,\widetilde{Q}_+]]]$		

3.2 Chiral perturbation theory with photons and leptons

3.2.1 Definitions in meson case

The theoretical results in mesonic sector mentioned in previous section have been used to calculate the electromagnetic corrections to the elastic $\pi\pi$ scattering amplitude, in particular to its S-wave threshold parameters. These electromagnetic effects are found to be comparable size to the $\mathcal{O}(p^6)$ strong interaction contributions (Meißner et al., 1997; Knecht and Urech, 1998).

The further extension of ChPT is the analysis of the electromagnetic corrections in semileptonic reactions. For the complete treatment of the electromagnetic and weak interactions within the framework of the ChPT, the photons and light leptons have to be included as explicit dynamical degrees of freedom in a suitable effective Lagrangian.

The photon field A_μ and the leptons $\ell, \nu_\ell (\ell = e, \mu)$ are also introduced in Equation (3.3) with (Knecht et al., 2000)

$$l_\mu = v_\mu - a_\mu - eQ_L^{\rm em}A_\mu + \sum_\ell \left(\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell Q_L^{\rm wk} + \overline{\nu_\ell}\gamma_\mu(1-\gamma_5)\ell Q_L^{\rm wk\dagger}\right), \tag{3.36}$$

$$r_\mu = v_\mu + a_\mu - eQ_R^{\rm em}A_\mu. \tag{3.37}$$

The matrix $Q_L^{\rm wk}$ is the new building block which corresponds to the weak field and transforms as

$$Q_L^{\rm wk} \xrightarrow{G} g_L Q_L^{\rm wk} g_L^\dagger, \tag{3.38}$$

under chiral symmetry. The weak spurion in $SU(2)$ symmetry is taken at

$$Q_L^{\rm wk} = -2\sqrt{2}G_F \begin{pmatrix} 0 & V_{ud} \\ 0 & 0 \end{pmatrix}, \tag{3.39}$$

where G_F is the Fermi coupling constant and V_{ud} is Kobayashi-Maskawa matrix element. To work with the usual generalizations one defines

$$\mathcal{Q}_L^{\rm wk} = u_L^\dagger Q_L^{\rm wk} u_L, \tag{3.40}$$

which transforms as

$$\mathcal{Q}_L^{\rm wk} \xrightarrow{G} h(g,\phi)\mathcal{Q}_L^{\rm wk} h^{-1}(g,\phi). \tag{3.41}$$

In the leptonic case we have to consider the CP transformation of the building blocks, we find

$$\mathcal{Q}_L^{\rm wk} \xrightarrow{CP} (-\mathcal{Q}_L^{\rm wk\dagger})^T, \tag{3.42}$$

where $(\cdots)^T$ is the transpose of the matrix in $(\cdots)$.

3.2.2 The leptonic lagrangian in meson case

The lowest order effective Lagrangian takes the form

$$\begin{aligned}\mathcal{L}^{(2)}_{\pi\pi,\mathrm{wk}} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{F_0^2}{4}\langle u_\mu u^\mu\rangle + \frac{F_0^2}{4}\langle\chi_+\rangle + e^2F_0^4 Z\langle\mathcal{Q}_L^{\mathrm{em}}\mathcal{Q}_R^{\mathrm{em}}\rangle \\ &\quad + \sum_\ell \left[\bar{\ell}(i\gamma^\mu\partial_\mu + e\gamma^\mu A_\mu - m_\ell)\ell + \overline{\nu_\ell} i\gamma^\mu(1-\gamma_5)\partial_\mu\nu_\ell\right]. \qquad (3.43)\end{aligned}$$

For the fourth order Lagrangian with LECs x_i is given by (Knecht et al., 2000)

$$\begin{aligned}\mathcal{L}^{(4)}_{\pi\pi,\mathrm{wk}} &= e^2\sum_\ell \Big\{F^2\big[x_1\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\langle u^\mu\{\mathcal{Q}_R^{\mathrm{em}},\mathcal{Q}_L^{\mathrm{wk}}\}\rangle \\ &\qquad + x_2\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\langle u^\mu[\mathcal{Q}_R^{\mathrm{em}},\mathcal{Q}_L^{\mathrm{wk}}]\rangle \\ &\qquad + x_3 m_\ell\bar{\ell}(1-\gamma_5)\nu_\ell\langle\mathcal{Q}_L^{\mathrm{wk}}\mathcal{Q}_R^{\mathrm{em}}\rangle \\ &\qquad + ix_4\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\langle\mathcal{Q}_L^{\mathrm{wk}}\widehat{\nabla}^\mu\mathcal{Q}_L^{\mathrm{em}}\rangle \\ &\qquad + ix_5\bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell\langle\mathcal{Q}_L^{\mathrm{wk}}\widehat{\nabla}^\mu\mathcal{Q}_R^{\mathrm{em}}\rangle + \mathrm{h.c.}\big] \\ &\qquad + x_6\bar{\ell}(i\gamma^\mu\partial_\mu + e\gamma^\mu A_\mu)\ell \\ &\qquad + x_7 m_\ell\bar{\ell}\ell\Big\}. \qquad (3.44)\end{aligned}$$

In $\mathcal{L}_{\pi\pi,\mathrm{wk}}$ we consider only quadratic terms in the weak fields and linear in G_F. The coupling constants $x_1,\ldots,x_5$ are real in the limit of CP invariance and the reality of x_6 and x_7 is a consequence of the hermiticity of the associated action.

One has used

$$\nabla_\mu u^\mu = \frac{i}{2}\left(\chi_- - \frac{1}{2}\langle\chi_-\rangle\right) + 2ie^2F_0^2 Z[\mathcal{Q}_R^{\mathrm{em}},\mathcal{Q}_L^{\mathrm{em}}], \qquad (3.45)$$

which is the mesonic equation of motion for $SU(2)$ symmetry and the following relations

$$\mathcal{Q}_L^{\mathrm{em}}\mathcal{Q}_L^{\mathrm{wk}} = \frac{2}{3}\mathcal{Q}_L^{\mathrm{wk}}, \qquad \mathcal{Q}_L^{\mathrm{wk}}\mathcal{Q}_L^{\mathrm{em}} = -\frac{1}{3}\mathcal{Q}_L^{\mathrm{wk}}, \qquad \langle\mathcal{Q}_L^{\mathrm{wk}}\rangle = 0, \qquad (3.46)$$

to get a minimal set of terms in Equation (3.44). This equation was applied to perform a complete one-loop analysis of semileptonic pion and kaon decays including the electromagnetic contributions of $\mathcal{O}(e^2p^2)$. The theoretical results for the decay rates of $\pi \rightarrow \ell\nu_\ell$ and $K \rightarrow \ell\nu_\ell$ were illustrated (Knecht et al., 2000).

CHAPTER IV

THE LEPTONIC LAGRANGIAN FOR BARYONIC SECTOR

The inclusion of virtual photons and leptons has been worked out completely only for meson case by Knecht et al. (2000) which the details have been shown in previous Chapter.

In this thesis we will extend the work of Knecht et al. to calculate the electromagnetic corrections to the neutron beta decays which involve both the weak leptonic and nucleonic currents. Thus we will have photons loops connecting to both leptons and nucleons. Therefore, we have to include both virtual photons and leptons in the effective Lagrangian as dynamical degrees of freedom as in the meson case, but we also have nucleons which bring in the Dirac structure. Thus this is much more complicated than it is in the purely mesonic sector thus we will devote this chapter to the consideration of both virtual photons and leptons in baryonic sector.

4.1 Weak current building block

In this section, we will be concerned with the construction of the effective Lagrangian for baryon ChPT involving virtual photons and leptons. Since the details of the construction with virtual photons involved has been demonstrated in Section 3.1.3 and 3.1.4, we will concentrate on discussing the new aspects due to the inclusion of the leptons.

To introduce leptons in the effective pion-nucleon Lagrangian we have to define the building block which represents weak current. This building block is $\mathcal{Q}_L^{\rm wk}$ which has been defined in Equation (3.40). In the real world there is only left-handed weak current so the subscript L will be neglected.

We recall the definition and the transformation properties of the weak current building block to make it easier to follow the argument. One defines the weak current building block as

$$\mathcal{Q}^{\rm wk} = u_L^\dagger Q_L^{\rm wk} u_L \tag{4.1}$$

where

$$Q_L^{\rm wk} = -2\sqrt{2}G_F \begin{pmatrix} 0 & V_{ud} \\ 0 & 0 \end{pmatrix}, \tag{4.2}$$

and it transforms as

$$\mathcal{Q}^{\rm wk} \xrightarrow{G} h(g,\phi)\mathcal{Q}^{\rm wk}h^{-1}(g,\phi), \qquad \mathcal{Q}^{\rm wk} \xrightarrow{CP} (-\mathcal{Q}^{\rm wk\dagger})^T \tag{4.3}$$

under the chiral group and charge conjugation with parity invariance (CP), respectively. And we define

$$j_\mu^{\rm wk} = \sum_\ell \bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell, \qquad \ell = e, \mu \tag{4.4}$$

where $(1-\gamma_5)$ reflects the parity violation.

4.2 The construction of leptonic lagrangian

In this section, we give a detailed exposition of how to get the effective chiral leptonic lagrangian for baryon case. The way to form the invariant monomials is to combine the building block which have been introduced formerly into invariant terms in the form of Equation (2.50). For third order leptonic lagrangian we have to consider terms of the form

$$e^2 j_\mu^{\rm wk}\overline{\Psi}Q_\pm\mathcal{Q}^{\rm wk}\Theta^\mu\Psi, \tag{4.5}$$

with the possibilities of Θ^μ are γ^μ, $\gamma^\mu\gamma_5$, D^μ and $\sigma^{\mu\nu}D_\nu$. The multiplication of $Q_\pm$ and $\mathcal{Q}^{\rm wk}$ can be rewritten in terms of commutators, anticommutators, single and multiple traces, which are

$$[Q_\pm, \mathcal{Q}^{\rm wk}], \quad \{Q_\pm, \mathcal{Q}^{\rm wk}\}, \quad \langle Q_\pm\rangle \mathcal{Q}^{\rm wk}, \quad Q_\pm\langle \mathcal{Q}^{\rm wk}\rangle, \quad \langle Q_\pm \mathcal{Q}^{\rm wk}\rangle. \tag{4.6}$$

From the definitions of Q_- and $\mathcal{Q}^{\rm wk}$ in Equation (3.23) and Equation (4.1), they give $\langle Q_-\rangle = \langle \mathcal{Q}^{\rm wk}\rangle = 0$ and for the anticommutator terms we use Equation (2.120) which leads

$$\begin{aligned} \left\{Q_+, \mathcal{Q}^{\rm wk}\right\} &= Q_+\langle \mathcal{Q}^{\rm wk}\rangle + \langle Q_+\rangle \mathcal{Q}^{\rm wk} + \langle Q_+ \mathcal{Q}^{\rm wk}\rangle - \langle Q_+\rangle\langle \mathcal{Q}^{\rm wk}\rangle \\ &= \langle Q_+\rangle \mathcal{Q}^{\rm wk} + \langle Q_+ \mathcal{Q}^{\rm wk}\rangle \end{aligned} \tag{4.7}$$

$$\begin{aligned} \left\{Q_-, \mathcal{Q}^{\rm wk}\right\} &= Q_-\langle \mathcal{Q}^{\rm wk}\rangle + \langle Q_-\rangle \mathcal{Q}^{\rm wk} + \langle Q_- \mathcal{Q}^{\rm wk}\rangle - \langle Q_-\rangle\langle \mathcal{Q}^{\rm wk}\rangle \\ &= \langle Q_- \mathcal{Q}^{\rm wk}\rangle \end{aligned} \tag{4.8}$$

All possible chiral invariant terms are

$$e^2\overline{\Psi} j_\mu^{\rm wk} \left\{ \begin{array}{c} [Q_\pm, \mathcal{Q}^{\rm wk}] \\ \langle Q_+\rangle \mathcal{Q}^{\rm wk} \\ \langle Q_\pm \mathcal{Q}^{\rm wk}\rangle \end{array} \right\} \Theta^\mu \Psi + \text{h.c.} \tag{4.9}$$

Next, we apply the CP transformation to these terms. The transformation properties of the Clifford algebra elements have been shown in Table 2.2 and under CP transformation $Q_\pm$, $\mathcal{Q}^{\rm wk}$ and $j_\mu^{\rm wk}$transform as

$$Q_\pm \xrightarrow{CP} (Q_\pm)^T, \qquad \mathcal{Q}^{\rm wk} \xrightarrow{CP} -(\mathcal{Q}^{\rm wk\dagger})^T, \qquad j_\mu^{\rm wk} \xrightarrow{CP} j^{\mu,{\rm wk}\dagger} \tag{4.10}$$

So that the CP invariant terms are

$$e^2\overline{\Psi}\gamma^\mu j_\mu^{\rm wk}[Q_+, \mathcal{Q}^{\rm wk}]\Psi + \text{h.c.} \tag{4.11}$$

$$e^2\overline{\Psi}\gamma^\mu j_\mu^{\rm wk}[Q_-, \mathcal{Q}^{\rm wk}]\Psi + \text{h.c.} \tag{4.12}$$

$$e^2\overline{\Psi}\gamma^\mu j_\mu^{\rm wk}\langle Q_+\rangle \mathcal{Q}^{\rm wk}\Psi + \text{h.c.} \tag{4.13}$$

$$e^2\overline{\Psi}\gamma^\mu j_\mu^{\rm wk}\langle Q_+ \mathcal{Q}^{\rm wk}\rangle\Psi + \text{h.c.} \tag{4.14}$$

$$e^2\overline{\Psi}\gamma^\mu j_\mu^{\mathrm{wk}}\langle Q_-\mathcal{Q}^{\mathrm{wk}}\rangle\Psi + \mathrm{h.c.} \tag{4.15}$$

$$e^2\overline{\Psi}\gamma^\mu\gamma_5 j_\mu^{\mathrm{wk}}[Q_+, \mathcal{Q}^{\mathrm{wk}}]\Psi + \mathrm{h.c.} \tag{4.16}$$

$$e^2\overline{\Psi}\gamma^\mu\gamma_5 j_\mu^{\mathrm{wk}}[Q_-, \mathcal{Q}^{\mathrm{wk}}]\Psi + \mathrm{h.c.} \tag{4.17}$$

$$e^2\overline{\Psi}\gamma^\mu\gamma_5 j_\mu^{\mathrm{wk}}\langle Q_+\rangle\mathcal{Q}^{\mathrm{wk}}\Psi + \mathrm{h.c.} \tag{4.18}$$

$$e^2\overline{\Psi}\gamma^\mu\gamma_5 j_\mu^{\mathrm{wk}}\langle Q_+\mathcal{Q}^{\mathrm{wk}}\rangle\Psi + \mathrm{h.c.} \tag{4.19}$$

$$e^2\overline{\Psi}\gamma^\mu\gamma_5 j_\mu^{\mathrm{wk}}\langle Q_-\mathcal{Q}^{\mathrm{wk}}\rangle\Psi + \mathrm{h.c.} \tag{4.20}$$

$$e^2\overline{\Psi}j_\mu^{\mathrm{wk}}[Q_+, \mathcal{Q}^{\mathrm{wk}}]D^\mu\Psi + \mathrm{h.c.} \tag{4.21}$$

$$e^2\overline{\Psi}j_\mu^{\mathrm{wk}}[Q_-, \mathcal{Q}^{\mathrm{wk}}]D^\mu\Psi + \mathrm{h.c.} \tag{4.22}$$

$$e^2\overline{\Psi}j_\mu^{\mathrm{wk}}\langle Q_+\rangle\mathcal{Q}^{\mathrm{wk}}D^\mu\Psi + \mathrm{h.c.} \tag{4.23}$$

$$e^2\overline{\Psi}j_\mu^{\mathrm{wk}}\langle Q_+\mathcal{Q}^{\mathrm{wk}}\rangle D^\mu\Psi + \mathrm{h.c.} \tag{4.24}$$

$$e^2\overline{\Psi}j_\mu^{\mathrm{wk}}\langle Q_-\mathcal{Q}^{\mathrm{wk}}\rangle D^\mu\Psi + \mathrm{h.c.} \tag{4.25}$$

$$e^2\overline{\Psi}\sigma^{\mu\nu}j_\mu^{\mathrm{wk}}[Q_+, \mathcal{Q}^{\mathrm{wk}}]D_\nu\Psi + \mathrm{h.c.} \tag{4.26}$$

$$e^2\overline{\Psi}\sigma^{\mu\nu}j_\mu^{\mathrm{wk}}[Q_-, \mathcal{Q}^{\mathrm{wk}}]D_\nu\Psi + \mathrm{h.c.} \tag{4.27}$$

$$e^2\overline{\Psi}\sigma^{\mu\nu}j_\mu^{\mathrm{wk}}\langle Q_+\rangle\mathcal{Q}^{\mathrm{wk}}D_\nu\Psi + \mathrm{h.c.} \tag{4.28}$$

$$e^2\overline{\Psi}\sigma^{\mu\nu}j_\mu^{\mathrm{wk}}\langle Q_+\mathcal{Q}^{\mathrm{wk}}\rangle D_\nu\Psi + \mathrm{h.c.} \tag{4.29}$$

$$e^2\overline{\Psi}\sigma^{\mu\nu}j_\mu^{\mathrm{wk}}\langle Q_-\mathcal{Q}^{\mathrm{wk}}\rangle D_\nu\Psi + \mathrm{h.c.} \tag{4.30}$$

We can use the total derivative argument on Equations (4.21) - (4.25) to put D^μ on $Q_\pm$ or $\mathcal{Q}^{\mathrm{wk}}$, these equations become higher order. In Equations (4.26) - (4.30), we use the Dirac matrices relation,

$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]. \tag{4.31}$$

For example, let us consider Equation (4.26)

$$\begin{aligned} &e^2\overline{\Psi}\sigma^{\mu\nu}j_\mu^{\mathrm{wk}}[Q_+, \mathcal{Q}^{\mathrm{wk}}]D_\nu\Psi + \mathrm{h.c.} \\ &\qquad = 2m_N e^2\overline{\Psi}j_\mu^{\mathrm{wk}}\gamma^\mu[Q_+, \mathcal{Q}^{\mathrm{wk}}]\Psi + \text{higher order}, \end{aligned} \tag{4.32}$$

Table 4.1 The monomials $\mathcal{O}_{i,\text{wk}}(e^2p)$ of third order for the leptonic Lagrangian.

i	$\mathcal{O}_{\text{wk}}(e^2p)$
1	$\gamma^\mu j_\mu^{wk}\langle\widetilde{Q}_+\mathcal{Q}^{\text{wk}}\rangle$ + h.c.
2	$\gamma^\mu j_\mu^{wk}\mathcal{Q}^{\text{wk}}\langle Q_+\rangle$ + h.c.
3	$\gamma^\mu j_\mu^{wk}[\widetilde{Q}_+,\mathcal{Q}^{\text{wk}}]$ + h.c.
4	$\gamma^\mu j_\mu^{wk}\langle\widetilde{Q}_-\mathcal{Q}^{\text{wk}}\rangle$ + h.c.
5	$\gamma^\mu j_\mu^{wk}[\widetilde{Q}_-,\mathcal{Q}^{\text{wk}}]$ + h.c.
6	$\gamma^\mu\gamma_5 j_\mu^{wk}\langle\widetilde{Q}_+\mathcal{Q}^{\text{wk}}\rangle$ + h.c.
7	$\gamma^\mu\gamma_5 j_\mu^{wk}\mathcal{Q}^{\text{wk}}\langle Q_+\rangle$ + h.c.
8	$\gamma^\mu\gamma_5 j_\mu^{wk}[\widetilde{Q}_+,\mathcal{Q}^{\text{wk}}]$ + h.c.
9	$\gamma^\mu\gamma_5 j_\mu^{wk}\langle\widetilde{Q}_-\mathcal{Q}^{wk}\rangle$ + h.c.
10	$\gamma^\mu\gamma_5 j_\mu^{wk}[\widetilde{Q}_-,\mathcal{Q}^{\text{wk}}]$ + h.c.

which yields Equations (4.26) - (4.30) are not independent terms. To make our result consistent with the previous Lagrangian, we switch $Q_\pm$ to the traceless version $\widetilde{Q}_\pm$ where

$$\widetilde{Q}_\pm = Q_\pm - \frac{1}{2}\langle Q_\pm\rangle, \tag{4.33}$$

4.3 The leptonic lagrangian in baryon case

For the third order, the leptonic Lagrangian is written shortly as

$$\mathcal{L}^{(3)}_{\pi N,\text{wk}} = \sum_i^{10} e^2F_0^2 n_i\overline{\Psi}\mathcal{O}^{(e^2p)}_{i,\text{wk}}\Psi, \tag{4.34}$$

where $\mathcal{O}_{i,\text{wk}}$ denotes the minimal chiral CP invariance terms which are shown in Table 4.1.

For fourth order leptonic Lagrangian we construct only for the specific case which has no pion i.e. no u_μ. The leptonic Lagrangian can be written in the same

Table 4.2 The monomials $\mathcal{O}_{i,\mathrm{wk}}(e^2p^2)$ of fourth order for the leptonic Lagrangian.

i	$\mathcal{O}_{\mathrm{wk}}(e^2p^2)$	i	$\mathcal{O}_{\mathrm{wk}}(e^2p^2)$
1	$i(\nabla^\mu j_\mu^{\mathrm{wk}})\langle\widetilde{Q}_+\mathcal{Q}^{\mathrm{wk}}\rangle$ + h.c.	11	$\sigma^{\mu\nu}\gamma_5(\nabla_\nu j_\mu^{\mathrm{wk}})\langle\widetilde{Q}_+\mathcal{Q}^{\mathrm{wk}}\rangle$ + h.c.
2	$i(\nabla^\mu j_\mu^{\mathrm{wk}})\langle Q_+\rangle\mathcal{Q}^{\mathrm{wk}}$ + h.c.	12	$\sigma^{\mu\nu}\gamma_5(\nabla_\nu j_\mu^{\mathrm{wk}})\langle Q_+\rangle\mathcal{Q}^{\mathrm{wk}}$ + h.c.
3	$i(\nabla^\mu j_\mu^{\mathrm{wk}})\left[\widetilde{Q}_+,\mathcal{Q}^{\mathrm{wk}}\right]$ + h.c.	13	$\sigma^{\mu\nu}\gamma_5(\nabla_\nu j_\mu^{\mathrm{wk}})\left[\widetilde{Q}_+,\mathcal{Q}^{\mathrm{wk}}\right]$ + h.c.
4	$i(\nabla^\mu j_\mu^{\mathrm{wk}})\langle\widetilde{Q}_-\mathcal{Q}^{\mathrm{wk}}\rangle$ + h.c.	14	$\sigma^{\mu\nu}\gamma_5(\nabla_\nu j_\mu^{\mathrm{wk}})\langle\widetilde{Q}_-\mathcal{Q}^{\mathrm{wk}}\rangle$ + h.c.
5	$i(\nabla^\mu j_\mu^{\mathrm{wk}})\left[\widetilde{Q}_-,\mathcal{Q}^{\mathrm{wk}}\right]$ + h.c.	15	$\sigma^{\mu\nu}\gamma_5(\nabla_\nu j_\mu^{\mathrm{wk}})\left[\widetilde{Q}_-,\mathcal{Q}^{\mathrm{wk}}\right]$ + h.c.
6	$\sigma^{\mu\nu}(\nabla_\nu j_\mu^{\mathrm{wk}})\langle\widetilde{Q}_+\mathcal{Q}^{\mathrm{wk}}\rangle$ + h.c.	16	$i\gamma_5(\nabla^\mu j_\mu^{\mathrm{wk}})\langle\widetilde{Q}_+\mathcal{Q}^{\mathrm{wk}}\rangle$ + h.c.
7	$\sigma^{\mu\nu}(\nabla_\nu j_\mu^{\mathrm{wk}})\langle Q_+\rangle\mathcal{Q}^{\mathrm{wk}}$ + h.c.	17	$i\gamma_5(\nabla^\mu j_\mu^{\mathrm{wk}})\langle Q_+\rangle\mathcal{Q}^{\mathrm{wk}}$ + h.c.
8	$\sigma^{\mu\nu}(\nabla_\nu j_\mu^{\mathrm{wk}})\left[\widetilde{Q}_+,\mathcal{Q}^{\mathrm{wk}}\right]$ + h.c.	18	$i\gamma_5(\nabla^\mu j_\mu^{\mathrm{wk}})\left[\widetilde{Q}_+,\mathcal{Q}^{\mathrm{wk}}\right]$ + h.c.
9	$\sigma^{\mu\nu}(\nabla_\nu j_\mu^{\mathrm{wk}})\langle\widetilde{Q}_-\mathcal{Q}^{\mathrm{wk}}\rangle$ + h.c.	19	$i\gamma_5(\nabla^\mu j_\mu^{\mathrm{wk}})\langle\widetilde{Q}_-\mathcal{Q}^{\mathrm{wk}}\rangle$ + h.c.
10	$\sigma^{\mu\nu}(\nabla_\nu j_\mu^{\mathrm{wk}})\left[\widetilde{Q}_-,\mathcal{Q}^{\mathrm{wk}}\right]$ + h.c.	20	$i\gamma_5(\nabla^\mu j_\mu^{\mathrm{wk}})\left[\widetilde{Q}_-,\mathcal{Q}^{\mathrm{wk}}\right]$ + h.c.

way as

$$\mathcal{L}_{\pi N,\mathrm{wk}}^{(4)} = \sum_i^{20} e^2 F_0^2 s_i \overline{\Psi}\mathcal{O}_{i,\mathrm{wk}}^{(e^2p^2)}\Psi, \tag{4.35}$$

with the monomials $\mathcal{O}_{i,\mathrm{wk}}(e^2p^2)$ are shown in Table 4.2.

CHAPTER V

RENORMALIZATION

From the exploring work of Weinberg (1979), effective field theory has been developed to one of the most important tools for investigating strong interaction processes in the low-energy regime. It is based on a completely general lagrangian requires an infinite number of counterterms and is not renormalizable, infinities encountered in the calculation of loop diagrams need to be removed by a renormalization of the infinite number of free parameters of the lagrangian.

If the lagrangian is actually the most general one possible, up to a given order, then a renormalization can be set up for that order, and relations between all physical observables will be finite. Infinities arising in the calculation are all absorbed into the definitions of the free parameters of the contact terms in the lagrangian.

In this work, we do not do a full renormalization which gives lots of loop diagrams for the beta decay process and would be better after renormalization. The calculations in this chapter relate to the contributions of the electromagnetic LECs to the weak current. We will discuss the wavefunction renormalizations of all particles involved in the beta decay. Since these renormalizations appear in the calculations of the observables for beta decay and thus will contribute to the contributions of the electromagnetic LECs to the observables.

To determine the pion and nucleon wave function renormalizations Z_π and Z_N, we have to calculate the pion self energy and nucleon self energy. The calculation of self energy for pion and nucleon will be shown in later sections.

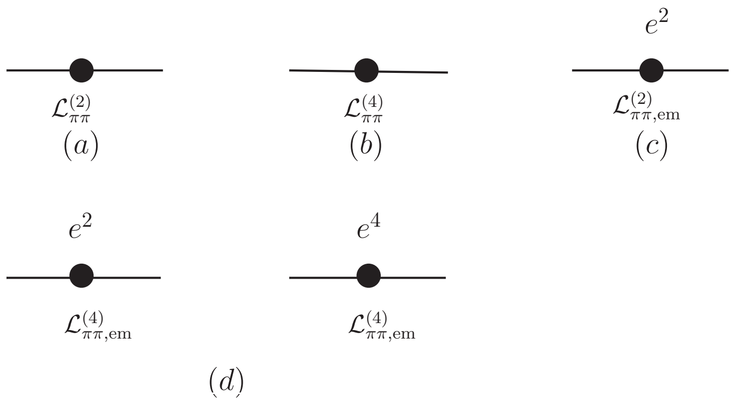

Figure 5.1 Tree-level diagrams contributing to the pion self-energy.

5.1 Pion self energy

According to Weinberg's power counting scheme, we have to include a tree-level contribution from $\mathcal{L}_{\pi\pi}^{(2)}$ and a tree-level contribution from $\mathcal{L}_{\pi\pi}^{(4)}$. The pion self energy Σ_π can therefore be written as

$$\Sigma_\pi = \Sigma_\pi^{(2)} + \Sigma_\pi^{(4)}, \tag{5.1}$$

where the superscript refer to the contributions from the Lagrangian $\mathcal{L}_{\pi\pi}^{(2)}$ and $\mathcal{L}_{\pi\pi}^{(4)}$. The tree-level diagrams contributing to the pion self-energy are shown in Figure 5.1, the results for the tree-level diagrams read

$$-i\Sigma_{(a)} = i\mathcal{L}_{\pi\pi}^{(2)} = \frac{i}{2}\left(p^2 - \mathring{m}_\pi^2\right)\pi^2, \tag{5.2}$$

$$-i\Sigma_{(b)} = i\mathcal{L}_{\pi\pi}^{(4)} = i\frac{\mathring{m}_\pi^2}{F_0^2}\left[l_4(p^2 - \mathring{m}_\pi^2) - l_3\mathring{m}_\pi^2\right]\pi^2, \tag{5.3}$$

$$-i\Sigma_{(c)} = i\mathcal{L}_{\pi\pi,em}^{(2)} = ie^2F_0^2Z\left[(\vec{\pi}\cdot\hat{z})^2 - \pi^2\right], \tag{5.4}$$

$$\begin{aligned}
-i\Sigma_{(d)} &= i\mathcal{L}^{(4)}_{\pi\pi,em}, \\
&= ie^2\left[\frac{10}{9}p^2(k_1+k_2+k_5+k_6)\pi^2 + p^2(-2k_3+k_4)(\vec{\pi}\cdot\hat{z})^2\right. \\
&\quad -\frac{1}{9}\overset{\circ}{m}{}^2_\pi(10k_7+46k_8+10k_9+46k_{10}+36k_{11})\pi^2 \\
&\quad \left.+4\overset{\circ}{m}{}^2_\pi(k_8+k_{10}+k_{11})(\vec{\pi}\cdot\hat{z})^2\right] \\
&\quad +ie^4F_0^2\left[-\frac{5}{9}(2k_{15}+k_{16})\pi^2+\frac{5}{9}(2k_{15}+k_{16})(\vec{\pi}\cdot\hat{z})^2\right]. \qquad (5.5)
\end{aligned}$$

5.2 Pion wave function renormalization constant

From H. Fearing et al. (Fearing et al., 1997), they defined $\Gamma_{\pi\pi}(q^2)$ as the Green function which is the sum of $i\mathcal{L}_{\pi\pi}$ for tree-level and one-loop calculations. The Green Function is related to the pion self energy $\Sigma_\pi(q^2)$ via

$$i\mathcal{L}_{\pi\pi}(p^2) = i\left(p^2 - \overset{\circ}{m}{}^2_\pi - \Sigma(p^2)\right), \qquad (5.6)$$

where $\overset{\circ}{m}{}^2_\pi$ is the square of pion mass in chiral limit.

$$\begin{aligned}
-\frac{1}{i\mathcal{L}_{\pi\pi}(p^2)} &= \frac{i}{p^2-\overset{\circ}{m}{}^2_\pi-\Sigma(p^2)}, \\
&= \frac{i}{p^2-\overset{\circ}{m}{}^2_\pi-\Sigma(m_\pi^2)-(p^2-m_\pi^2)\Sigma'(m_\pi^2)-\tilde{\Sigma}(p^2)}. \qquad (5.7)
\end{aligned}$$

The last equation is obtained from expanding the self energy about the point $p^2 = m_\pi^2$.

The pion mass is obtained from the condition that the propagator has a pole at the physical mass which means

$$i\mathcal{L}_{\pi\pi}(m_\pi^2) = m_\pi^2 - \overset{\circ}{m}{}^2_\pi - \Sigma(m_\pi^2) = 0, \qquad (5.8)$$

which gives

$$m_\pi^2 = \overset{\circ}{m}{}^2_\pi + \Sigma(m_\pi^2). \qquad (5.9)$$

So the full propagator can be written as

$$
\begin{aligned}
-\frac{1}{i\mathcal{L}_{\pi\pi}(p^2)} &= \frac{i}{(p^2-m_\pi^2)-(p^2-m_\pi^2)\Sigma'(m_\pi^2)-\tilde{\Sigma}(p^2)}, \\
&= \frac{i}{(p^2-m_\pi^2)[1-\Sigma'(m_\pi^2)]-\tilde{\Sigma}(p^2)}, \\
&= \frac{i}{[1-\Sigma'(m_\pi^2)]\left[p^2-m_\pi^2-\frac{\tilde{\Sigma}(p^2)}{1-\Sigma'(m_\pi^2)}\right]}. \qquad (5.10)
\end{aligned}
$$

One defines $-\frac{1}{i\mathcal{L}_{\pi\pi}(p^2)}=\frac{iZ_\pi}{p^2-m_\pi^2-Z_\pi\tilde{\Sigma}(p^2)}$. Then

$$
Z_\pi=\frac{1}{1-\Sigma'(m_\pi^2)}=\left[\frac{i(p^2-m_\pi^2)}{i\mathcal{L}_{\pi\pi}(p^2)}\right]_{p^2=m_\pi^2}. \qquad (5.11)
$$

In our case, we consider only tree-level calculations.

$$
\begin{aligned}
i\mathcal{L}_{\pi\pi}(p^2) &= \frac{i}{2}(p^2-\overset{\circ}{m}{}_\pi^2)\pi^2+i\frac{\overset{\circ}{m}{}_\pi^2}{F_0^2}\left[p^2l_4-\overset{\circ}{m}{}_\pi^2(l_3+l_4)\right]\pi^2+ie^2F_0^2Z[(\vec{\pi}\cdot\hat{z})^2-\pi^2] \\
&+ie^2\left[\frac{10}{9}p^2(k_1+k_2+k_5+k_6)\pi^2+p^2(-2k_3+k_4)(\vec{\pi}\cdot\hat{z})^2\right. \\
&\quad-\frac{\overset{\circ}{m}{}_\pi^2}{9}(10k_7+46k_8+10k_9+46k_{10}+36k_{11})\pi^2 \\
&\quad\left.+4\overset{\circ}{m}{}_\pi^2(k_8+k_{10}+k_{11})(\vec{\pi}\cdot\hat{z})^2\right] \\
&+ie^4F_0^2\left[\frac{5}{9}(2k_{15}+k_{16})((\vec{\pi}\cdot\hat{z})^2-\pi^2)\right]. \qquad (5.12)
\end{aligned}
$$

Then calculate the propagator for pion that correspond to the annihilation of $\pi_\pm$ to create $\pi_\mp(\pi^2={\pi^0}^2+2\pi^+\pi^-)$,

$$
\begin{aligned}
\Gamma_{\pi\pi}(p^2) &= i(p^2-\overset{\circ}{m}{}_\pi^2)+i\frac{\overset{\circ}{m}{}_\pi^2}{F_0^2}\left[2p^2l_4-2\overset{\circ}{m}{}_\pi^2(l_3+l_4)\right]-2ie^2F_0^2Z \\
&+\frac{20ie^2}{9}p^2(k_1+k_2+k_5+k_6) \\
&-\frac{2ie^2}{9}\overset{\circ}{m}{}_\pi^2(10k_7+46k_8+10k_9+46k_{10}+36k_{11}) \\
&-\frac{10ie^4F_0^2}{9}(2k_{15}+k_{16}). \qquad (5.13)
\end{aligned}
$$

Then the pion self energy can be obtained as

$$\begin{aligned}-\Sigma(p^2) &= \frac{\overset{\circ}{m}{}_\pi^2}{F_0^2}\left[2p^2 l_4 - 2\overset{\circ}{m}{}_\pi^2(l_3+l_4)\right] - 2e^2F_0^2 Z \\ &\quad + e^2\left[\frac{20}{9}p^2(k_1+k_2+k_5+k_6)\right] \\ &\quad - e^2\left[\frac{2}{9}\overset{\circ}{m}{}_\pi^2(10k_7+46k_8+10k_9+46k_{10}+36k_{11})\right] \\ &\quad - e^4F_0^2\frac{10}{9}(2k_{15}+k_{16}). \end{aligned} \tag{5.14}$$

To get the pion wavefunction renormalization constant we calculate the derivative of the pion self energy as

$$-\left.\frac{d\Sigma(p^2)}{d(p^2)}\right|_{p^2=m_\pi^2} = \frac{2m_\pi^2}{F_0^2}l_4 + \frac{20e^2}{9}(k_1+k_2+k_5+k_6). \tag{5.15}$$

From the relation that

$$Z_\pi = \frac{1}{1-\Sigma'(m_\pi^2)},$$

we get

$$\begin{aligned}Z_\pi &= \frac{1}{1+\frac{2m_\pi^2}{F_0^2}l_4+\frac{20e^2}{9}(k_1+k_2+k_5+k_6)} \\ &\sim 1-\left[\frac{2m_\pi^2}{F_0^2}l_4+\frac{20e^2}{9}(k_1+k_2+k_5+k_6)\right]. \end{aligned} \tag{5.16}$$

which is the charged pion wave function renormalization constant.

5.3 Nucleon self energy

A one-particle state in the spectrum of a Hamiltonian has the physical mass m_N if $P^2 = m_N^2$ for this state. The corresponding full propagator $\Gamma(P)$ has a simple pole at $P^2 = m_N^2$. The full propagator can be written in terms of free nucleon propagator as

$$\Gamma(P) = \frac{i}{\not{P} - \overset{\circ}{m}_N - \Sigma(P) + i0^+}, \tag{5.17}$$

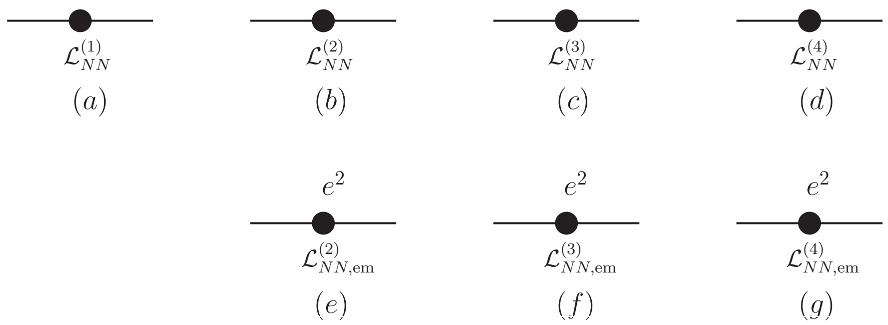

Figure 5.2 Tree-level diagrams contributing to the nucleon self-energy.

where $\overset{\circ}{m}_N$ is the bare nucleon mass and $-i\Sigma(P)$ represents the summation of one particle irreducible diagrams. The physical mass of the nucleon is given by the pole in the full propagator. The mass of the nucleon is given by $\not{P}_0 = m_N$ where $\not{P}_0$ satisfies

$$\not{P}_0 - \overset{\circ}{m}_N - \Sigma(P_0) = 0. \tag{5.18}$$

When the nucleon momentum is close to the pole, the nucleon propagator is of the form

$$\Gamma(P) = \frac{iZ(\not{P} + m_N)}{P^2 - m_N^2 + i0^+}, \tag{5.19}$$

where the renormalization constant is

$$Z^{-1} = 1 - \frac{\partial\Sigma(P_0)}{\partial\not{P}}. \tag{5.20}$$

The external nucleon fields must also be renormalized, which results in a factor of $\sqrt{Z}$ in the amplitude for each external fields.

The tree-level diagrams contributing to the nucleon self-energy from contact interaction are shown in Figure 5.2 and can be evaluated as

$$-i\Sigma_{(a)} = i(i\not{\partial} - \overset{\circ}{m}_N), \tag{5.21}$$

$$-i\Sigma_{(b)} = 4ic_1\overset{\circ}{m}{}_\pi^2, \tag{5.22}$$

$$-i\Sigma_{(c)} = 0, \tag{5.23}$$

$$-i\Sigma_{(d)} = i(16\overset{\circ}{m}{}_\pi^4 e_{38} + 2\overset{\circ}{m}{}_\pi^4 e_{115} + 2\overset{\circ}{m}{}_\pi^4 e_{116}), \tag{5.24}$$

$$-i\Sigma_{(e)} = i\frac{e^2F_0^2}{2}(f_1 + \vec{\tau}\cdot\hat{z}f_2 + f_3), \tag{5.25}$$

$$-i\Sigma_{(f)} = 0, \tag{5.26}$$

$$\begin{aligned} -i\Sigma_{(g)} &= i\frac{e^4F_0^4}{4}(h_1 + h_2 + h_3 + \vec{\tau}\cdot\hat{z}h_4 + \vec{\tau}\cdot\hat{z}h_5) \\ &\quad +2ie^2F_0^2\overset{\circ}{m}{}_\pi^2(h_{32} + h_{33} + \vec{\tau}\cdot\hat{z}h_{34}). \end{aligned} \tag{5.27}$$

Next, we will consider the loop diagrams which are shown in Figure 5.3. Only one-loop diagrams are calculated in this work. The detail of loop calculation is in Appendix B. The diagram 5.3(h) is the third order contribution to $\Sigma(P)$, the vertices are given by the first order term

$$\overline{\Psi}\frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu\Psi = \overline{\Psi}\frac{-g_A}{2F_0}\gamma^\mu\gamma_5\partial_\mu\pi^a\tau^a\Psi, \tag{5.28}$$

where a is the pion isospin index. The contribution to the matrix element from each pion-nucleon vertex is $\frac{ig_A}{2F_0}\gamma_5\not{\partial}\pi^a\tau^a$. The total matrix element is

$$\begin{aligned} -i\Sigma_{(h)} &= \mu^{4-d}\int\frac{d^d\ell}{(2\pi)^d}\overline{\Psi}\left(\frac{-ig_A}{2F_0}\gamma^\mu\gamma_5\partial_\mu\pi^a\tau^a\right)\Psi\overline{\Psi}\left(\frac{-ig_A}{2F_0}\gamma^\nu\gamma_5\partial_\nu\pi^b\tau^b\right)\Psi, \\ &= \frac{-ig_A^2}{4F_0^2}\mu^{4-d}\int\frac{d^d\ell}{(2\pi)^d}\frac{\gamma^\mu\gamma_5\tau^a[-(\ell+k)^a]\pi^a\gamma^\nu\gamma_5\tau^b[(\ell+k)^b]\pi^b}{(\not{\ell}+\not{k}+\not{p}) - \overset{\circ}{m}_N + i0^+}. \end{aligned} \tag{5.29}$$

We will always redefine the integration variable ℓ so to make the first pion momentum $k=0$,

$$\begin{aligned} -i\Sigma_{(h)} &= \frac{ig_A^2}{4F_0^2}\mu^{4-d}\int\frac{d^d\ell}{(2\pi)^4}\frac{\gamma^\mu\gamma_5\tau^a\ell^a\pi^a[(\not{\ell}+\not{p}) + \overset{\circ}{m}_N]\gamma^\nu\gamma_5\tau^b\ell^b\pi^b}{(\ell+p)^2 - \overset{\circ}{m}{}_N^2 + i0^+}, \\ &= \frac{ig_A^2}{4F_0^2}\mu^{4-d}\int\frac{d^d\ell}{(2\pi)^d}\frac{\not{\ell}\gamma_5[(\not{\ell}+\not{p}) + \overset{\circ}{m}_N]\ \not{\ell}\gamma_5 i\delta^{ab}\tau^a\tau^b}{[(\ell+p)^2 - \overset{\circ}{m}{}_N^2 + i0^+][\ell^2 - \overset{\circ}{m}{}_\pi^2 + i0^+]}. \end{aligned} \tag{5.30}$$

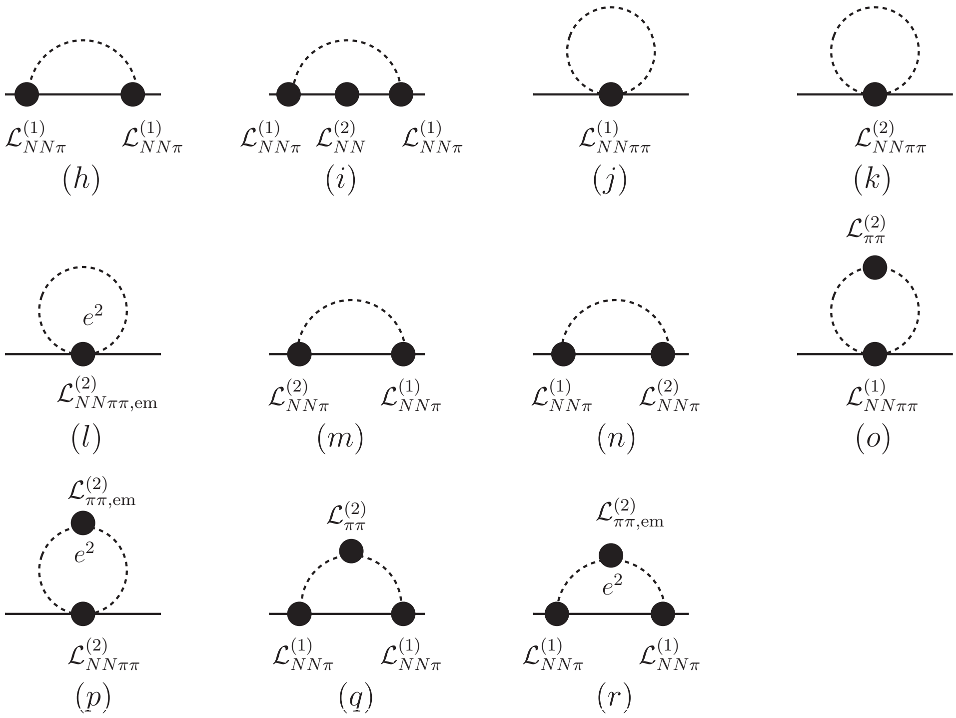

Figure 5.3 loop diagrams contributing to the nucleon self-energy.

The isospin indices are summed over including the neutral and charged pion loops, and the sum contributes a factor of 3 to the matrix element. The loop integral can be written as a sum of the integral in Appendix. Then the contribution to $\Sigma(p)$ from diagram (h) is

$$-i\Sigma_{(h)} = \frac{3ig_A^2}{4F_0^2} I_{\pi N}[p, \not\ell\gamma_5(\not\ell + \not p + \mathring{m})\ \not\ell\gamma_5]. \tag{5.31}$$

The contribution from the remaining diagrams in Figure 5.3 are

$$\begin{aligned} -i\Sigma_{(i)} &= \frac{-3ic_1\mathring{m}_\pi^2 g_A^2}{F_0^2}\left[i\mu^{4-d}\int\frac{d^d\ell}{(2\pi)^d}\frac{\not\ell\gamma_5[\not\ell + \not p + \mathring{m}_N]^2\ \not\ell\gamma_5}{[(\ell+p)^2 - \mathring{m}_N^2 + i0^+]^2[\ell^2 - \mathring{m}_\pi^2 + i0^+]}\right], \\ &= \frac{-3ic_1\mathring{m}_\pi^2 g_A^2}{F_0^2} I_{\pi NN}[p, p, \not\ell\gamma_5[\not\ell + \not p + \mathring{m}_N][\not\ell + \not p + \mathring{m}_N]\ \not\ell\gamma_5], \qquad (5.32) \\ -i\Sigma_{(j)} &= 0. \qquad (5.33) \end{aligned}$$

The contribution from diagram (k) and (l) are given by the three terms in the

Lagrangian which contain 2 pion fields.

$$-i\Sigma^{c_1}_{(k)} = \frac{-6ic_1\overset{\circ}{m}{}^2_\pi}{F_0^2}\left[i\mu^{4-d}\int\frac{d^d\ell}{(2\pi)^d}\frac{1}{[\ell^2-\overset{\circ}{m}{}^2_\pi+i\epsilon]}\right],$$
$$= \frac{-6ic_1\overset{\circ}{m}{}^2_\pi}{F_0^2}I_\pi[1], \tag{5.34}$$
$$-i\Sigma^{c_2}_{(k)} = \frac{3ic_2\overset{\circ}{m}{}^2_\pi p^2}{dF_0^2\overset{\circ}{m}{}^2_N}I_\pi[1], \tag{5.35}$$

$$-i\Sigma^{c_3}_{(k)} = \frac{3ic_3\overset{\circ}{m}{}^2_\pi}{F_0^2}I_\pi[1]. \tag{5.36}$$

Then,

$$-i\Sigma_{(k)} = \frac{3i\overset{\circ}{m}{}^2_\pi}{F_0^2}\left[-2c_1+\frac{c_2p^2}{d\overset{\circ}{m}{}^2_N}+c_3\right]I_\pi[1]. \tag{5.37}$$

For diagram (l),

$$-i\Sigma^{f_1}_{(l)} = \frac{-5if_1e^2}{2}I_\pi[1],$$
$$-i\Sigma^{f_2}_{(l)} = \frac{-3if_2e^2\vec{\tau}\cdot\hat{z}}{4}I_\pi[1],$$
$$-i\Sigma^{f_3}_{(l)} = \frac{-if_3e^2}{2}I_\pi[1].$$

So the final result for diagram (l) is

$$-i\Sigma_{(l)} = \frac{-ie^2}{2}\left[5f_1+3f_2\vec{\tau}\cdot\hat{z}+f_3\right]I_\pi[1]. \tag{5.38}$$

The results of the rest diagrams in Figure 5.3 are

$$-i\Sigma_{(m)} = 0, \tag{5.39}$$
$$-i\Sigma_{(n)} = 0, \tag{5.40}$$
$$-i\Sigma_{(o)} = 0, \tag{5.41}$$
$$-i\Sigma_{(p)} = 0, \tag{5.42}$$
$$-i\Sigma_{(q)} = \frac{-9ig_A^2}{8}I_{\pi N}[p,\gamma_5\not\ell(\not\ell+\not p+\overset{\circ}{m}_N)\gamma_5\not\ell], \tag{5.43}$$
$$-i\Sigma_{(r)} = \frac{-15ig_A^2e^2Z}{8}I_{\pi\pi N}[q,p,\gamma_5\not\ell(\not\ell+\not p+\overset{\circ}{m}_N)\gamma_5\not\ell]. \tag{5.44}$$

5.4 Nucleon renormalization constant

The wave function renormalization constant for nucleon, defined by

$$Z_N^{-1} = 1 - \frac{\partial \Sigma}{\partial \not{p}}|_{\not{p} = m_N} \tag{5.45}$$

is used to renormalize the nucleon propagator and the external nucleon fields. In our case, we consider only tree-level diagrams. Therefore, the nucleon wave function renormalization constant is

$$Z_N \sim 1. \tag{5.46}$$

CHAPTER VI

CALCULATION OF THE FORM FACTORS

We will apply our method to the neutron beta decay process, since this decay is a low energy process. In this work we will calculate the contributions of the LECs to the radiative corrections, but loop calculation will not be considered because they would be too complicated. Thus, our result will miss out on finite contributions from loops. In this chapter, we calculate unrenormalized LECs, but we know that they can be renormalized and that the expressions for the contributions from the LECs will look the same only with unrenormalized quantities replaced by renormalized ones.

6.1 Weak form factor of the nucleon current

We consider the neutron beta decay process,

$$n(p_i) \to p(p_f) + e^-(p_e) + \bar{\nu}_e(p_\nu), \tag{6.1}$$

where p_i, p_f, p_e and p_ν denote the four-momentum of the neutron, proton, electron and anti-neutrino, respectively. The S-matrix amplitude is indicated in a common (V-A) form (Bjorken and Drell, 1964) and given by

$$\mathcal{M} = -\frac{iG_F V_{ud}}{\sqrt{2}}\bar{u}(p_e)\gamma_\alpha(1-\gamma_5)u(p_\nu)\bar{u}(p_f)\tau_+[V^\alpha - A^\alpha]u(p_i), \tag{6.2}$$

with

$$V^\alpha = G_V(q^2)\gamma^\alpha + \frac{iG_M(q^2)}{2m_N}\sigma^{\alpha\beta}q_\beta + \frac{G_S(q^2)}{m_\mu}q^\alpha, \tag{6.3}$$

$$A^\alpha = G_A(q^2)\gamma^\alpha\gamma_5 + \frac{G_P(q^2)}{m_\mu}q^\alpha\gamma_5 + \frac{iG_T(q^2)}{2m_N}\sigma^{\alpha\beta}q_\beta\gamma_5, \tag{6.4}$$

where $q^\alpha = p_f^\alpha - p_i^\alpha$. Here G_F is the Fermi constant, V_{ud} is an element of the CKM matrix, m_μ is the physical muon mass, m_N is the average of the physical neutron and proton masses, $m_N = \frac{1}{2}(m_n + m_p)$, and τ_+ is the isospin raising operator, $\langle p|\tau_+|n\rangle = 1$. Here we also consider second class currents.

The fact that the nucleon has a complicated internal structure means a deviation from the V-A structure, i.e. $G_V/G_A \neq 1$ and induces non-zero scalar G_S, pseudo-scalar G_P, weak-magnetic G_M, and axial-tensor G_T, form factors. $G_A(0)$ is most exactly determined from neutron beta decay rate.

The contributions to the weak-nucleon-nucleon vertex involve coupling of the nucleons to an external vector field and to an external axial field for strong, em and weak interactions. The diagrams which contribute to these contributions are given in Figure 6.1.

The tree level contributions to the amplitude for strong terms correspond to diagram $(a) - (d)$ in Figure 6.1 and are given by (Ando and Fearing, 2007)

$$\begin{aligned}
\mathcal{M}_{(a)V} &= i\sqrt{Z_N}\overline{\Psi}_f(v_\mu^{(s)} + \vec{\tau}\cdot\tilde{v}_\mu)\gamma^\mu\Psi_i\sqrt{Z_N}, \\
&= i\sqrt{Z_N}\overline{\Psi}_f(v_\mu^{(s)} + \tau_+\tilde{v}_\mu^+ + \tau_-\tilde{v}_\mu^-)\gamma^\mu\Psi_i\sqrt{Z_N}, \qquad (6.5)\\
\mathcal{M}_{(a)A} &= ig_A\sqrt{Z_N}\overline{\Psi}_f\vec{\tau}\cdot\tilde{a}_\mu\gamma^\mu\gamma_5\Psi_i\sqrt{Z_N}, \\
&= ig_A\sqrt{Z_N}\overline{\Psi}_f(\tau_+\tilde{a}_\mu^+ + \tau_-\tilde{a}_\mu^-)\gamma^\mu\gamma_5\Psi_i\sqrt{Z_N}, \qquad (6.6)\\
\mathcal{M}_{(b)V} &= i\sqrt{Z_N}\overline{\Psi}_f\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}[(c_6 + 2c_7)v_\mu^{(s)} + c_6\vec{\tau}\cdot\tilde{v}_\mu]\Psi_i\sqrt{Z_N}, \\
&= i\sqrt{Z_N}\overline{\Psi}_f\frac{i\sigma^{\mu\nu}q_\nu}{2m_N}[(c_6 + 2c_7)v_\mu^{(s)} + c_6(\tau_+\tilde{v}_\mu^+ + \tau_-\tilde{v}_\mu^-)]\Psi_i\sqrt{Z_N}, \qquad (6.7)\\
\mathcal{M}_{(b)A} &= 0, \qquad (6.8)\\
\mathcal{M}_{(c)V} &= i\overline{\Psi}_f(q^\mu q^\nu - q^2 g^{\mu\nu})(p_i + p_f)_\nu(\frac{2d_7}{m_N}v_\mu^{(s)} + \frac{d_6}{m_N}\vec{\tau}\cdot\tilde{v}_\mu)\Psi_i, \qquad (6.9)
\end{aligned}$$

by using the Gordon decomposition

$$\bar{u}\gamma^\mu u = \bar{u}\left[\frac{(p_f + p_i)^\mu}{2m_N} + \frac{i\sigma^{\mu\nu}(p_f - p_i)_\nu}{2m_N}\right]u, \qquad (6.10)$$

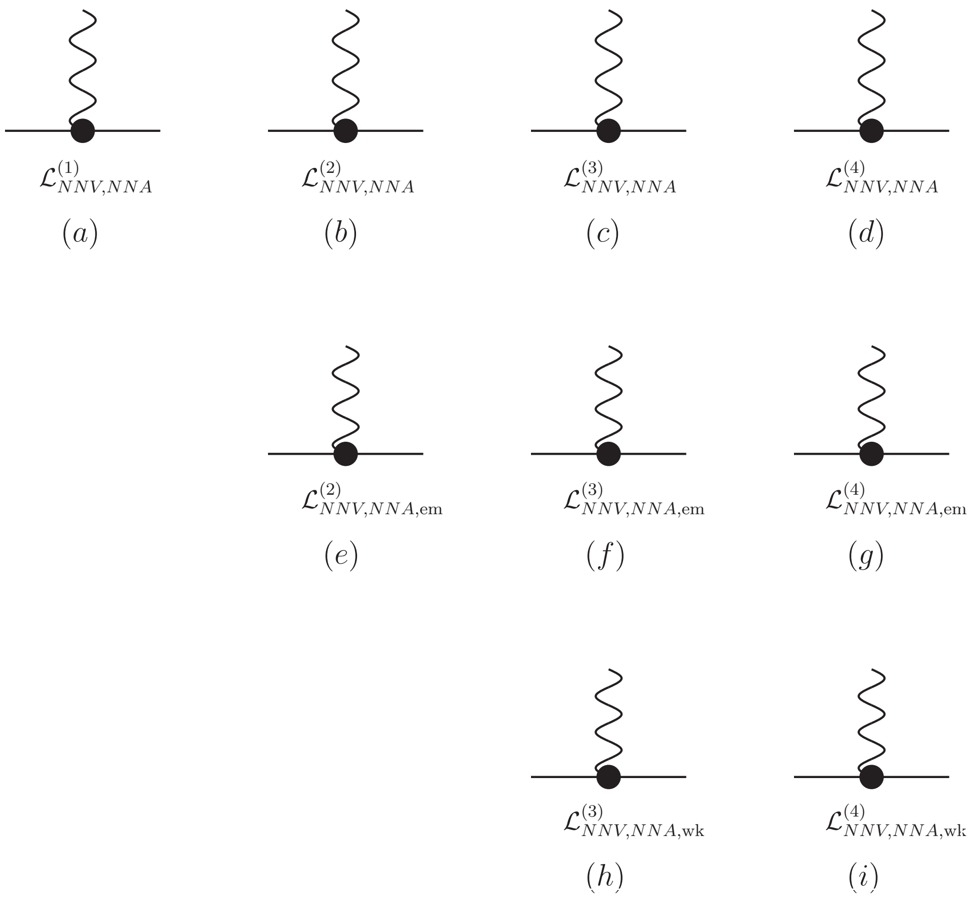

Figure 6.1 Diagrams which contribute to the coupling of nucleon to the external vector and axial vector currents. $\mathcal{L}^{(1,\ldots,4)}_{NNV,NNA}$ is the pion-nucleon Lagrangian from the strong part, $\mathcal{L}^{(2,\ldots,4)}_{NNV,NNA,\text{em}}$ is the Lagrangian from the EM part and $\mathcal{L}^{(3,4)}_{NNV,NNA,\text{wk}}$ is the Lagrangian from the weak part up to fourth order.

we can replace,

$$(p_f + p_i)_\nu \to 2m_N\gamma_\nu + i\sigma_{\alpha\nu}(p_f - p_i)^\alpha. \tag{6.11}$$

So we get

$$\mathcal{M}_{(c)V} = i\overline{\Psi}_f(q^\mu q^\nu - q^2 g^{\mu\nu})(2m_N\gamma_\nu + i\sigma_{\alpha\nu}(p_f - p_i)^\alpha)\left[\frac{2d_7}{m_N}v_\mu^{(s)} + \frac{d_6}{m_N}\vec{\tau}\cdot\tilde{v}_\mu\right]\Psi_i, \tag{6.12}$$

since the momentum transfer $q^\alpha = p_f^\alpha - p_i^\alpha$ therefore,

$$\begin{aligned}
\mathcal{M}_{(c)V} &= i\overline{\Psi}_f(q^\mu q^\nu - q^2 g^{\mu\nu})(2m_N\gamma_\nu + i\sigma_{\alpha\nu}q^\alpha)\left[\frac{2d_7}{m_N}v_\mu^{(s)} + \frac{d_6}{m_N}\vec{\tau}\cdot\tilde{v}_\mu\right]\Psi_i,\\
&= i\overline{\Psi}_f\left\{\left[-2m_N\gamma^\mu q^2 + i\sigma^{\mu\nu}q^2 q_\nu\right]\times\left[\frac{2d_7}{m_N}v_\mu^{(s)} + \frac{d_6}{m_N}(\tau_+\tilde{v}_\mu^+ + \tau_-\tilde{v}_\mu^-)\right]\right\}\Psi_i,
\end{aligned} \tag{6.13}$$

$$\begin{aligned}
\mathcal{M}_{(c)A} &= i\overline{\Psi}_f\gamma^\mu\gamma_5[4\overset{\circ}{m}{}_\pi^2 d_{16}\vec{\tau}\cdot\tilde{a}_\mu + d_{22}(q^2 g_{\mu\nu} - q_\mu q_\nu)\vec{\tau}\cdot\tilde{a}^\nu]\Psi_i,\\
&= i\overline{\Psi}_f\left[4\gamma^\mu\gamma_5\overset{\circ}{m}{}_\pi^2 d_{16} + \gamma^\mu\gamma_5 d_{22}q^2 - 2m_N\gamma_5 q^\mu d_{22}\right]\vec{\tau}\cdot\tilde{a}_\mu\Psi_i,\\
&= i\overline{\Psi}_f\left[4\gamma^\mu\gamma_5\overset{\circ}{m}{}_\pi^2 d_{16} + \gamma^\mu\gamma_5 d_{22}q^2 - 2m_N\gamma_5 q^\mu d_{22}\right](\tau_+\tilde{a}_\mu^+ + \tau_-\tilde{a}_\mu^-)\Psi_i, \quad (6.14)\\
\mathcal{M}_{(d)V} &= i\overline{\Psi}_f i\sigma^{\mu\nu}q_\nu[4(q^2 e_{54} - 4\overset{\circ}{m}{}_\pi^2 e_{105})v_\mu^{(s)} + 2(q^2 e_{74} - 4\overset{\circ}{m}{}_\pi^2 e_{106})\vec{\tau}\cdot\tilde{v}_\mu]\Psi_i,\\
&= i\overline{\Psi}_f i\sigma^{\mu\nu}q_\nu[4(q^2 e_{54} - 4\overset{\circ}{m}{}_\pi^2 e_{105})v_\mu^{(s)}\\
&\quad +2(q^2 e_{74} - 4\overset{\circ}{m}{}_\pi^2 e_{106})(\tau_+\tilde{v}_\mu^+ + \tau_-\tilde{v}_\mu^-)]\Psi_i, \quad (6.15)\\
\mathcal{M}_{(d)A} &= 0. \quad (6.16)
\end{aligned}$$

The subscript V or A refers to coupling to vector and axial current respectively and the number refers to the diagram number in Figure 6.1. The contributions from electromagnetic terms originating from the Lagrangian constructed in Ref. (Müller and Meißner, 1999) in diagram (e)-(g) are given by

$$\mathcal{M}_{(e)V} = 0, \tag{6.17}$$

$$\mathcal{M}_{(e)A} = 0, \tag{6.18}$$

$$\begin{aligned}
\mathcal{M}_{(f)V} &= ie^2F_0^2\overline{\Psi}_f\gamma^\mu g_9(-\vec{\tau}\cdot\tilde{v}_\mu+\vec{\tau}\cdot\hat{z}\,\tilde{v}_\mu\cdot\hat{z})\Psi_i, \\
&= ie^2F_0^2\overline{\Psi}_f\gamma^\mu g_9(-\tau_+\tilde{v}_\mu^+-\tau_-\tilde{v}_\mu^-+\tau_3\tilde{v}_\mu^3)\Psi_i, \qquad (6.19)\\
\mathcal{M}_{(f)A} &= ie^2F_0^2\overline{\Psi}_f\gamma^\mu\gamma_5[g_1\vec{\tau}\cdot\hat{z}\,\tilde{a}_\mu\cdot\hat{z}+2g_3\tilde{a}_\mu\cdot\hat{z}+(g_4+g_5)(\vec{\tau}\cdot\tilde{a}_\mu)]\Psi_i, \\
&= ie^2F_0^2\overline{\Psi}_f\gamma^\mu\gamma_5[g_1\tau_3\tilde{a}_\mu^0+2g_3\tilde{a}_\mu^0+(g_4+g_5)(\tau_+\tilde{a}_\mu^++\tau_-\tilde{a}_\mu^-)]\Psi_i, \\
&= ie^2F_0^2\overline{\Psi}_f\gamma^\mu\gamma_5[(g_4+g_5)(\tau_+\tilde{a}_\mu^++\tau_-\tilde{a}_\mu^+)]\Psi_i \qquad (6.20)\\
\mathcal{M}_{(g)V} &= -ie^2F_0^2\overline{\Psi}_f\{2(h_{38}+h_{39})i\sigma^{\mu\nu}q_\nu\vec{\tau}\cdot\tilde{v}_\mu+4h_{40}i\sigma^{\mu\nu}\tilde{v}_\mu\cdot\hat{z} \\
&\quad +4(h_{41}+h_{42})i\sigma^{\mu\nu}q_\nu v_\mu^{(s)}+4h_{43}i\sigma^{\mu\nu}q_\nu\vec{\tau}\cdot\hat{z}\,v_\mu^{(s)} \\
&\quad -h_{67}iq_\mu\vec{\tau}\cdot\tilde{v}^\mu\times\hat{z}-2h_{68}iq_\mu(p_i^\mu p_i^\nu+p_f^\mu p_f^\nu)\vec{\tau}\cdot\tilde{v}_\nu\times\hat{z} \\
&\quad +h_{78}i\sigma^{\mu\nu}q_\nu(-\vec{\tau}\cdot\tilde{v}_\mu+\vec{\tau}\cdot\hat{z}\,\tilde{v}_\mu\cdot\hat{z})\}\,\Psi_i, \\
&= ie^2F_0^2\overline{\Psi}_f\left\{2(h_{38}+h_{39})i\sigma^{\mu\nu}q_\nu(\tau_+\tilde{v}_\mu^++\tau_-\tilde{v}_\mu^-)+4h_{40}i\sigma^{\mu\nu}q_\nu\tilde{v}_\mu^3\right. \\
&\quad +4(h_{41}+h_{42})i\sigma^{\mu\nu}q_\nu v_\mu^{(s)}+4h_{43}i\sigma^{\mu\nu}q_\nu\tau_3 v_\mu^{(s)} \\
&\quad -h_{67}iq_\mu[i\sqrt{2}(\tau_+\tilde{v}_+^\mu-\tau_-\tilde{v}_-^\mu)]+h_{68}iq_\mu q^2[i\sqrt{2}(\tau_+\tilde{v}_+^\mu-\tau_-\tilde{v}_-^\mu)] \\
&\quad \left.+h_{78}i\sigma^{\mu\nu}q_\nu(-\tau_+\tilde{v}_\mu^+-\tau_-\tilde{v}_\mu^-+\tau_3\tilde{v}_\mu^3)\right\}\Psi_i \qquad (6.21)\\
\mathcal{M}_{(g)A} &= ie^2F_0^2\overline{\Psi}_f 4h_{50}\sigma^{\mu\nu}\gamma_5 q_\nu\vec{\tau}\cdot\tilde{a}_\mu\times\hat{z}\Psi_i \\
&= ie^2F_0^2\overline{\Psi}_f i\sigma^{\mu\nu}\gamma_5 q_\nu 4h_{50}\sqrt{2}(\tau_+\tilde{a}_\mu^+-\tau_-\tilde{a}_\mu^-)\Psi_i. \qquad (6.22)
\end{aligned}$$

The tree level contributions to the amplitude from our new terms correspond to diagram (h)-(i) in Figure 6.1 are

$$\begin{aligned}
\mathcal{M}_{(h)} &= i\mathcal{L}^{(3)}_{NNA,\mathrm{wk}}, \\
&= i\sqrt{2}G_F V_{ud}e^2F_0^2\overline{\Psi}_f\left\{-(n_2\gamma^\mu+n_7\gamma^\mu\gamma_5)[j_\mu^{\mathrm{wk}}\vec{\tau}\cdot(\hat{x}+i\hat{y})+j_\mu^{\mathrm{wk}\dagger}\vec{\tau}\cdot(\hat{x}-i\hat{y})]\right. \\
&\qquad \left.-(n_3\gamma^\mu+n_8\gamma^\mu\gamma_5)[j_\mu^{\mathrm{wk}}\vec{\tau}\cdot(\hat{x}+i\hat{y})+j_\mu^{\mathrm{wk}\dagger}\vec{\tau}\cdot(\hat{x}-i\hat{y})]\right\}\Psi_i,
\end{aligned}$$

(6.23)

$$
\begin{aligned}
\mathcal{M}_{(i)} &= i\mathcal{L}^{(4)}_{NNA,\mathrm{wk}}, \\
&= -i\sqrt{2}G_F V_{ud} e^2 F_0^2 \overline{\Psi}_f \\
&\Big\{ [(s_2+s_3)q^\mu - (s_{12}+s_{13}) i\sigma^{\mu\nu}\gamma_5 q_\nu] \left[j^{\mathrm{wk}}_\mu \vec{\tau}\cdot(\hat{x}+i\hat{y}) - j^{\mathrm{wk}\dagger}_\mu \vec{\tau}\cdot(\hat{x}-i\hat{y}) \right] \\
&- [(s_7+s_8) i\sigma^{\mu\nu} q_\nu + (s_{17}+s_{18})\gamma_5 q^\mu] \left[j^{\mathrm{wk}}_\mu \vec{\tau}\cdot(\hat{x}+i\hat{y}) + j^{\mathrm{wk}\dagger}_\mu \vec{\tau}\cdot(\hat{x}-i\hat{y}) \right] \Big\} \Psi_i.
\end{aligned}
\tag{6.24}
$$

In the weak case, we have to rewrite $j^{\mathrm{wk}}_\mu \vec{\tau}\cdot(\hat{x}+i\hat{y})$ in terms of a_μ and v_μ which is

$$
-\sqrt{2}G_F V_{ud} \left[j^{\mathrm{wk}}_\mu \vec{\tau}\cdot(\hat{x}+i\hat{y}) + \mathrm{h.c.} \right] \rightarrow 2\vec{\tau}\cdot\tilde{v}_\mu \text{ or } -2\vec{\tau}\cdot\tilde{a}_\mu, \tag{6.25}
$$

$$
-\sqrt{2}G_F V_{ud} \left[j^{\mathrm{wk}}_\mu \vec{\tau}\cdot(\hat{x}+i\hat{y}) - \mathrm{h.c.} \right] \rightarrow -\sqrt{2}i\vec{\tau}\cdot\tilde{v}_\mu\times\hat{z} \text{ or } \sqrt{2}i\vec{\tau}\cdot\tilde{a}_\mu\times\hat{z} \tag{6.26}
$$

This holds only for the v_+ and v_- components. By picking up either v_μ or a_μ appropriately. Then $\mathcal{M}_{(h)}$ and $\mathcal{M}_{(i)}$ are

$$
\begin{aligned}
\mathcal{M}_{(h)} &= ie^2F_0^2\overline{\Psi}_f \left\{ 2(n_2+n_3)\gamma^\mu\vec{\tau}\cdot\tilde{v}_\mu - 2(n_7+n_8)\gamma^\mu\gamma_5\vec{\tau}\cdot\tilde{a}_\mu \right\} \Psi_i, \\
&= ie^2F_0^2\overline{\Psi}_f \left\{ 2(n_2+n_3)\gamma^\mu(\tau_+\tilde{v}^+_\mu + \tau_-\tilde{v}^-_\mu) \right. \\
&\qquad \left. -2(n_7+n_8)\gamma^\mu\gamma_5(\tau_+\tilde{a}^+_\mu + \tau_-\tilde{a}^-_\mu) \right\} \Psi_i,
\end{aligned}
\tag{6.27}
$$

$$
\begin{aligned}
\mathcal{M}_{(i)} &= ie^2F_0^2\overline{\Psi}_f \Big\{ \left[-\sqrt{2}(s_2+s_3)q^\mu\vec{\tau}\cdot\tilde{v}_\mu\times\hat{z} - \sqrt{2}(s_{12}+s_{13})i\sigma^{\mu\nu}\gamma_5 q_\nu\vec{\tau}\cdot\tilde{a}_\mu\times\hat{z} \right] \\
&\qquad + \left[-2(s_7+s_8)i\sigma^{\mu\nu}q_\nu\vec{\tau}\cdot\tilde{v}_\mu - 2(s_{17}+s_{18})\gamma_5 q^\mu\vec{\tau}\cdot\tilde{a}_\mu \right] \Big\} \Psi_i, \\
&= ie^2F_0^2\overline{\Psi}_f \Big\{ \Big[-\sqrt{2}(s_2+s_3)q^\mu(\tau_+\tilde{v}^+_\mu - \tau_-\tilde{v}^-_\mu) \\
&\qquad -2(s_7+s_8)i\sigma^{\mu\nu}q_\nu(\tau_+\tilde{v}^+_\mu + \tau_-\tilde{v}^-_\mu) \\
&\qquad -\sqrt{2}(s_{12}+s_{13})i\sigma^{\mu\nu}\gamma_5 q_\nu(\tau_+\tilde{a}^+_\mu - \tau_-\tilde{a}^-_\mu) \\
&\qquad -2(s_{17}+s_{18})\gamma_5 q^\mu(\tau_+\tilde{a}^+_\mu + \tau_-\tilde{a}^-_\mu) \Big] \Big\} \Psi_i.
\end{aligned}
\tag{6.28}
$$

Another contribution which we have to calculate comes from the $NN\pi$ vertex and will contribute to the axial current. The diagrams are shown in Figure

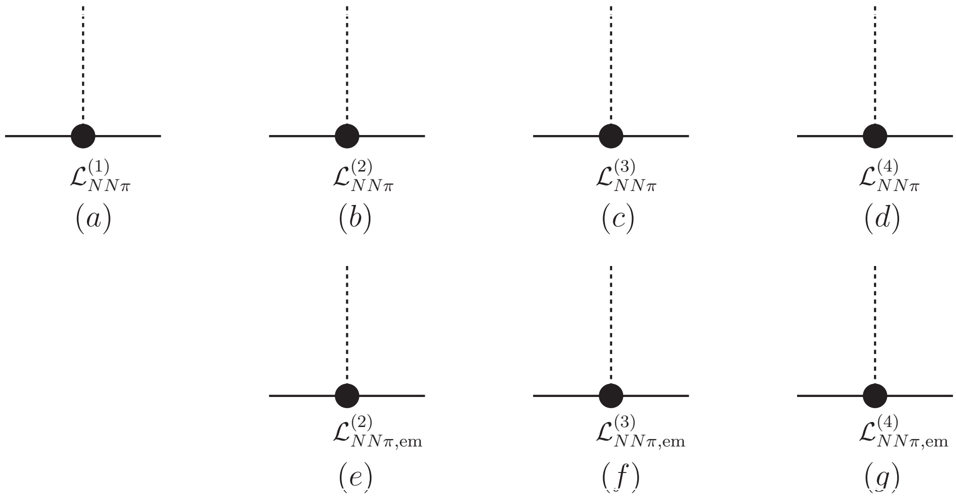

Figure 6.2 Tree level diagrams which contribute to the $NN\pi$ vertex.

6.2.

$$\mathcal{M}_{(a)\pi} = -\frac{g_A}{2F_0}\sqrt{Z_N}\overline{\Psi}_f\vec{\tau}\cdot\vec{\pi}\gamma_\mu\gamma_5 q^\mu\Psi_i\sqrt{Z_N}\sqrt{Z_\pi}, \tag{6.29}$$

$$\mathcal{M}_{(b)\pi} = 0, \tag{6.30}$$

$$\mathcal{M}_{(c)\pi} = \frac{\overset{\circ}{m}{}_\pi^2}{F_0}(d_{18}-2d_{16})\overline{\Psi}_f\gamma_\mu\gamma_5\vec{\tau}\cdot\vec{\pi}q^\mu\Psi_i, \tag{6.31}$$

$$\mathcal{M}_{(d)\pi} = 0. \tag{6.32}$$

The contributions from EM corrections are

$$\mathcal{M}_{(f)\pi} = 0, \tag{6.33}$$

$$\mathcal{M}_{(g)\pi} = -\frac{e^2F_0}{2}\overline{\Psi}_f\gamma^\mu\gamma_5 q_\mu\left\{(g_1+g_{11})\vec{\tau}\cdot\hat{z}\vec{\pi}\cdot\hat{z}+2g_3\vec{\pi}\cdot\hat{z}\right.$$
$$\left.+(g_4+g_5-g_{11})\vec{\tau}\cdot\vec{\pi}\right\}\Psi_i, \tag{6.34}$$

$$\mathcal{M}_{(h)\pi} = 0. \tag{6.35}$$

To get the contribution to the weak nucleon-nucleon current form πNN amplitude we combine the πNN amplitudes with the πA amplitudes to get the pion pole diagram. The amplitudes for πA vertex are

$$\mathcal{M}_{\pi A}^{(2)} = 2F_0 q_\mu \vec{\pi}\cdot\tilde{a}^\mu \sqrt{Z_\pi} \tag{6.36}$$

$$\mathcal{M}_{\pi A}^{(4)} = \frac{4}{F_0} m_\pi^2 \ell_4 q_\mu \vec{\pi}\cdot\tilde{a}^\mu \tag{6.37}$$

for the second and fourth order respectively. The contraction between $\mathcal{M}_{(a)\pi}$ with $\mathcal{M}_{\pi A}^{(2)}$ gives

$$\begin{aligned}
\mathcal{M}_{NNA}^{(1)} &= -\frac{g_A}{2F_0}\sqrt{Z_N}\overline{\Psi}_f \vec{\tau}\cdot\vec{\pi}\gamma^\nu\gamma_5 q_\nu (2F_0 q_\mu)\vec{\pi}\cdot\tilde{a}^\mu\sqrt{Z_\pi}\Psi_i \\
&= -2ig_A m_N Z_N Z_\pi \overline{\Psi}_f \frac{\gamma_5 q_\mu \vec{\tau}\cdot\tilde{a}^\mu}{q^2 - m_\pi^2}\Psi_i \\
&= -2ig_A m_N Z_N Z_\pi \overline{\Psi}_f \frac{\gamma_5 q_\mu (\tau_+ \tilde{a}^{+,\mu} + \tau_- \tilde{a}^{-,\mu})}{q^2 - m_\pi^2}\Psi_i
\end{aligned} \tag{6.38}$$

Contracting $\mathcal{M}_{(c)\pi}$ and $\mathcal{M}_{\pi A}^{(2)}$, we get

$$\begin{aligned}
\mathcal{M}_{NNA}^{(3)} &= \frac{m_\pi^2}{F_0}(d_{18} - 2d_{16})\overline{\Psi}_f \vec{\tau}\cdot\vec{\pi}\gamma_\nu\gamma_5 q_\nu (2F_0 q_m u)\vec{\pi}\cdot\tilde{a}^\mu\sqrt{Z_\pi}\Psi_i \\
&= 4im_\pi^2 m_N \overline{\Psi}_f \frac{\gamma_5 q_\mu (d_{18} - 2d_{16})\vec{\tau}\cdot\tilde{a^\mu}}{q^2 - m_\pi^2}\Psi_i \\
&= 4im_\pi^2 m_N \overline{\Psi}_f \frac{\gamma_5 q_\mu (d_{18} - 2d_{16})(\tau_+ \tilde{a}^{+,\mu} + \tau_- \tilde{a}^{-,\mu})}{q^2 - m_\pi^2}\Psi_i
\end{aligned} \tag{6.39}$$

The contraction between the EM correction of $NN\pi$ amplitude and the second order πA amplitude is calculated as

$$\begin{aligned}
\mathcal{M}_{NNA,\mathrm{em}}^{(3)} &= -e^2 F_0^2 \overline{\Psi}_f \{\gamma^\nu\gamma_5 q_\nu [(g_1 + g_{11})\vec{\pi}\cdot\hat{z}\vec{\tau}\cdot\hat{z} + 2g_3\vec{\pi}\cdot\hat{z} \\
&\quad + (g_4 + g_5 - g_{11})\vec{\tau}\cdot\vec{\pi}] q_\mu \vec{\pi}\cdot\tilde{a}^\mu\}\Psi_i \\
&= \frac{2ie^2 F_0^2 m_N}{q^2 - m_\pi^2}\overline{\Psi}_f \{\gamma_5 q_\mu [(g_1 + g_{11})\tilde{a}^\mu\cdot\hat{z}\vec{\tau}\cdot\hat{z} + g_3\tilde{a}^\mu\cdot\hat{z} \\
&\quad + (g_4 + g_5 - g_{11})\vec{\tau}\cdot\tilde{a}^\mu]\}\Psi_i \\
&= \frac{2ie^2 F_0^2 m_N}{q^2 - m_\pi^2}\overline{\Psi}_f \left\{\gamma_5 q_\mu \left[(g_4 + g_5 - g_{11})(\tau_+ \tilde{a}^{+,\mu} + \tau_- \tilde{a}^{-,\mu})\right]\right\}\Psi_i
\end{aligned} \tag{6.40}$$

Another third order NNA amplitude is from the contraction between the lowest order $NN\pi$ vertex and the fourth order πA vertex for strong, EM and weak parts. Firstly, we contract $NN\pi$ vertex with the fourth order strong πA vertex,

$$\begin{aligned}\mathcal{M}^{(3)}_{NNA,\text{strong}} &= -\frac{4ig_A m_\pi^2 m_N}{F_0^2}\overline{\Psi}_f \frac{\gamma_5 q_\mu \vec{\tau}\cdot\tilde{a}_\mu \ell_4}{q^2-m_\pi^2}\Psi_i \\ &= -\frac{4ig_A m_\pi^2 m_N}{F_0^2}\overline{\Psi}_f \frac{\gamma_5 q_\mu \ell_4(\tau_+\tilde{a}^{+,\mu}+\tau_-\tilde{a}^{-,\mu})}{q^2-m_\pi^2}\Psi_i \qquad (6.41)\end{aligned}$$

For the contraction between the $NN\pi$ vertex and the fourth order EM πA vertex, it yields the EM NNA amplitude as

$$\begin{aligned}\mathcal{M}^{(3)}_{NNA,\text{em}} &= -ig_A m_N e^2\overline{\Psi}_f\gamma_5 q_\mu\Bigg\{\left[\frac{40}{9}(k_1+k_2+k_5+k_6)+4k_{12}\right]\vec{\tau}\cdot\tilde{a}^\mu \\ &\quad +4(k_4-2k_3-k_{12})\vec{\tau}\cdot\hat{z}\tilde{a}^\mu\cdot\hat{z}\Bigg\}\Psi_i \\ &= -ig_A m_N e^2\overline{\Psi}_f\gamma_5 q_\mu\Bigg\{\frac{1}{q^2-m_\pi^2}\left[\frac{40}{9}(k_1+k_2+k_5+k_6)+4k_{12}\right] \\ &\quad \times(\tau_+\tilde{a}^{+,\mu}+\tau_-\tilde{a}^{-,\mu})\Bigg\}\Psi_i \qquad (6.42)\end{aligned}$$

The third order weak NNA amplitude is

$$\mathcal{M}^{(3)}_{NNA,\text{wk}} = -2ig_A m_N e^2\overline{\Psi}_f\left\{\gamma_5 q_\mu\frac{[-\frac{2}{3}x_1-2x_2+2x_3](\tau_+\tilde{a}^{+,\mu}-\tau_-\tilde{a}^{-,\mu})}{q^2-m_\pi^2}\right\}\Psi_i \qquad (6.43)$$

6.2 The calculations

The most general form for the vector and axial-vector currents evaluated for neutron beta decay are given by

$$\begin{aligned}\langle p(p_f)|V_\mu^+|n(p_i)\rangle &= \bar{u}(p_f)\left[G_V\gamma^\mu+\frac{iG_M}{2m_N}\sigma^{\mu\nu}q_\nu+\frac{G_S}{m_\mu}q^\mu\right]\tau_+u(p_i), \qquad (6.44) \\ \langle p(p_f)|A_\mu^+|n(p_i)\rangle &= \bar{u}(p_f)\left[G_A\gamma^\mu\gamma_5+\frac{G_P}{m_\mu}q^\mu\gamma_5+\frac{iG_T}{2m_N}\sigma^{\mu\nu}q_\nu\gamma_5\right]\tau_+u(p_i). \qquad (6.45)\end{aligned}$$

By adding up the vector contributions of Equations (6.5), (6.7), (6.13), (6.15), (6.17), (6.19) and (6.21), the vector current operator can be calculated as

$$\begin{aligned} V^\mu &= \gamma^\mu \left[1 + q^2(-2d_6) + e^2 F_0^2 \left[-g_9 + 2(n_2 + n_3)\right]\right. \\ &+ \frac{i\sigma^{\mu\nu} q_\nu}{2m_N} \left[c_6 - 16 e_{106} m_N \overset{\circ}{m}{}_\pi^2\right. \\ &+ 2e^2 F_0^2 (2h_{38} + 2h_{39} - h_{78} - 2(s_7 + s_8)) m_N \\ &\left. + q^2(2d_6 + 4e_{74} m_N)\right] \\ &+ \frac{q^\mu}{m_\mu} \left[e^2 F_0^2 m_\mu \left(-\sqrt{2}(s_2 + s_3) + \sqrt{2} h_{67}\right)\right]. \end{aligned} \tag{6.46}$$

The axial vector current can be received from Equations (6.6), (6.8), (6.14), (6.16), (6.18), (6.20), (6.22), (6.27), (6.28), (6.38), (6.39), (6.40), (6.41), (6.42) and (6.43) and written as

$$\begin{aligned} A^\mu &= \gamma^\mu \gamma_5 \left[g_A + 4\overset{\circ}{m}{}_\pi^2 d_{16} + q^2 d_{22} + e^2 F_0^2 \left(g_4 + g_5 - 2(n_7 + n_8)\right)\right] \\ &+ \frac{q^\mu \gamma_5}{m_\mu} \left\{ -\frac{-2m_N m_\mu}{q^2 - m_\pi^2} \left[Z_\pi g_A - m_\pi^2(2d_{18} - 4d_{16}) - e^2 F_0^2 (g_4 + g_5 - g_{11})\right.\right. \\ &\left. + e^2 g_A [\frac{20}{9}(k_1 + k_2 + k_5 + k_6) + 2k_{12}] - e^2 g_A (\frac{2}{3} x_1 + x_2 - x_3)\right] \\ &\left. - 2m_N m_\mu d_{22} - 2e^2 F_0^2 m_\mu (s_{17} + s_{18})\right\} \\ &+ \frac{i\sigma^{\mu\nu} q_\nu \gamma_5}{2m_N} e^2 F_0^2 \left[2m_N(-4\sqrt{2} h_{50} - \sqrt{2}(s_{12} + s_{13}))\right]. \end{aligned} \tag{6.47}$$

From above equations we get

$$G_V(q^2) = 1 + q^2(-2d_6) + e^2 F_0^2 \left[-g_9 + 2(n_2 + n_3)\right], \tag{6.48}$$

$$\begin{aligned} G_M(q^2) &= c_6 - 16 e_{106} m_N \overset{\circ}{m}{}_\pi^2 + 2e^2 F_0^2 \left(2h_{38} + 2h_{39} - h_{78} - 2(s_7 + s_8)\right) m_N \\ &+ q^2(2d_6 + 4e_{74} m_N), \end{aligned} \tag{6.49}$$

$$G_S(q^2) = e^2 F_0^2 m_\mu \left(2(s_2 + s_3) - \sqrt{2} h_{67}\right), \tag{6.50}$$

$$G_A(q^2) = g_A + 4\overset{\circ}{m}{}_\pi^2 d_{16} + q^2 d_{22} + e^2 F_0^2 \left(g_4 + g_5 - 2(n_7 + n_8)\right), \tag{6.51}$$

$$
\begin{aligned}
G_P(q^2) &= -\frac{2m_N m_\mu}{q^2-m_\pi^2}\Big\{Z_\pi g_A - m_\pi^2(2d_{18}-4d_{16}) - e^2F_0^2(g_4+g_5-g11) \\
&\quad + e^2 g_A\Big[\frac{20}{9}(k_1+k_2+k_5+k_6)+2k_{12}-\frac{2}{3}x_1-x_2+x_3\Big]\Big\} \\
&\quad -2m_N m_\mu d_{22} - 2e^2F_0^2 m_\mu(s_{17}+s_{18}), \qquad (6.52)\\
G_T(q^2) &= 2m_N e^2 F_0^2\left[4\sqrt{2}h_{50}-2(s_{12}+s_{13})\right]. \qquad (6.53)
\end{aligned}
$$

The pion-nucleon-nucleon coupling $G_{\pi NN}(q^2)$ is defined by the πNN amplitudes by

$$\mathcal{M}_{\pi NN}(q^2) = -G_{\pi NN}(q^2)\overline{\Psi}_f \vec{\tau}\cdot\vec{\pi}\gamma_5\Psi_i. \qquad (6.54)$$

The πNN amplitudes have been calculated in Ref. (Ando and Fearing, 2007).

$$\mathcal{M}_{\pi NN} = -\overline{\Psi}_f \vec{\tau}\cdot\vec{\pi}\gamma_\mu\gamma_5 q^\mu\left[\frac{g_A}{2F_0} - \frac{\overset{\circ}{m}{}_\pi^2}{F_0}(d_{18}-2d_{16})\right]\Psi_i. \qquad (6.55)$$

Substituting $\gamma_\mu q^\mu = 2m_N$. Then,

$$G_{\pi NN}(q^2) = \frac{m_N}{F_0}\left[g_A - \overset{\circ}{m}{}_\pi^2(2d_{18}-4d_{16})\right]. \qquad (6.56)$$

Therefore,

$$g_A - \overset{\circ}{m}{}_\pi^2(2d_{18}-4d_{16}) = \frac{G_{\pi NN}(q^2)F_0}{m_N}, \qquad (6.57)$$

and $G_P(q^2)$ can be rewritten as

$$
\begin{aligned}
G_P(q^2) &= -\frac{2m_N m_\mu}{q^2-m_\pi^2}\Big\{\frac{G_{\pi NN}(q^2)F_0}{m_N} - e^2F_0^2(g_4+g_5-g11) \\
&\quad + e^2 g_A\Big[\frac{20}{9}(k_1+k_2+k_5+k_6)+2k_{12}-\frac{2}{3}x_1-x_2+x_3\Big]\Big\} \\
&\quad -2m_N m_\mu d_{22} - 2e^2F_0^2 m_\mu(s_{17}+s_{18}), \qquad (6.58)
\end{aligned}
$$

6.3 Dimensional analysis of the LECs

Since there is a lack of experimental information for the electromagnetic and weak low-energy constants (LECs) f_i, g_i, h_i, n_i and s_i, we have to look for some method estimating them. One possibility is dimensional analysis (Müller and Meißner, 1999). It is a tool for estimating the dimensionless parameters appearing in a low energy effective theory. The application of dimension analysis is an estimation of the size of the LECs. The accepted estimation is $\alpha \approx 4\pi$.

From the effective Lagrangian, we can notice that each power whether of electromagnetic or weak charge matrices appearing in any monomial is accompanied by a factor of F_0 so that the corresponding LECs have the same mass dimension as their strong counterparts. Therefore, the f_i, g_i and $h_{1...5}$ scale as $[\text{mass}]^{-1}$, $[\text{mass}]^{-2}$ and $[\text{mass}]^{-3}$, respectively. Most of h_{6-90} have dimensionality $[\text{mass}]^{-3}$ except for the terms which have single and double of the covariant derivative acting on wavefunction have $[\text{mass}]^{-4}$ and $[\text{mass}]^{-5}$ respectively. Furthermore, the factors of F_0 are proportional to the natural low energy scale and defined through $\langle 0|A^a_\mu(0)|\pi^b(x)\rangle = e^{ipx}p_\mu\delta^{ab}F_0$ where $|\pi^a(p)\rangle$ is the exact one-pion eigenstate and $|0\rangle$ is the corresponding vacuum. The appearing of the pion decay constant in the chiral limit is necessary and sufficient conditio for spontaneous chiral symmetry breaking. The origin of electromagnetic and weak LECs is the integration of hard photon loops. Therefore, each power in e^2 is a power in the fine structure constant $\alpha = e^2/4\pi$. Thus the natural scale of chiral symmetry breaking is $\Lambda \sim m_N \sim 1\text{GeV}$, one can deduce the following estimates on the renormalized electromagnetic and weak LECs at the typical hadronic scale:

$$f_i = \frac{\tilde{f}_i}{4\pi}, \quad g^r_i = \frac{\tilde{g}_i}{4\pi}, \quad h^r_{1...5} = \frac{\tilde{h}_{1...5}}{(4\pi)^2}, \quad h^r_{6...90} = \frac{\tilde{h}_{6...90}}{4\pi} \tag{6.59}$$

$$n^r_i = \frac{\tilde{n}_i}{4\pi}, \qquad s^r_i = \frac{\tilde{s}_i}{4\pi}, \tag{6.60}$$

with the $\tilde{f}_i, \tilde{g}_i, \tilde{h}_i, \tilde{n}_i$ and $\tilde{s}_i$ are number of order one,

$$\tilde{f}_i \sim \tilde{g}_i \sim \tilde{h}_i \sim \tilde{n}_i \sim \tilde{s}_i = \mathcal{O}(1). \tag{6.61}$$

Because our formulas also contain c_i, d_i and e_i which are the strong LECs for the second, third and fourth order of the pion-nucleon Lagrangian. We will apply the dimensional analysis to get their dimensionality. The dimension of the strong LECs for second order is considered as $[\text{mass}]^{-1}$, for the third order scale as $[\text{mass}]^{-2}$ whereas the dimension for the LECs of the most terms in fourth order is $[\text{mass}]^{3}$. By the same reason as the consideration of the dimension for h_{6-90}, the dimension of LECs of the terms which have single derivative acting on wavefunction is $[\text{mass}]^{-4}$ and the dimensionality $[\text{mass}]^{-5}$ is for the LECs of the terms which have double derivative acting on wavefunction.

CHAPTER VII

CONCLUSIONS

We have constructed the most general effective pion-nucleon Lagrangian for the relativistic baryon chiral perturbation theory with the consideration of the electromagnetic corrections in weak processes. The electromagnetic reactions requires the inclusion of the virtual photons and the light leptons fields as explicitly dynamical degrees of freedom in the chiral Lagrangian. We applied our new terms of the pion-nucleon Lagrangian to the neutron beta decay and evaluate their weak form factors.

In order to obtain and simplify the radiative corrections to the weak form factors we define

$$\tilde{G} = G[1 + \frac{\alpha}{4\pi}(e_V - e_A)] \tag{7.1}$$

where e_V and e_A are the α-order corrections. The values of G's correspond to the physical value with short-range radiative corrections have been removed. From Equation (6.48) to Equation (6.53), we get the radiative corrections for all weak form factor as following:

$$G_V^r = \frac{4\pi}{\alpha} e^2 F_0^2 \big[-g_9 + 2(n_2 + n_3)\big] \tag{7.2}$$

$$G_M^r = \frac{4\pi}{\alpha} 2e^2 F_0^2 \big[2h_{38} + 2h_{39} - h_{78} - 2(s_7 + s_8)\big] \tag{7.3}$$

$$G_S^r = \frac{4\pi}{\alpha} m_\mu e^2 F_0^2 \big[-\sqrt{2}h_{67} + 2(s_2 + s_3)\big] \tag{7.4}$$

$$G_A^r = \frac{4\pi}{\alpha} e^2 F_0^2 \big[g_4 + g_5 - 2(n_7 + n_8)\big] \tag{7.5}$$

$$\begin{aligned} G_P^r = {} & \frac{2m_N m_\mu}{q^2 m_\pi^2} e^2 \big(\frac{4\pi}{\alpha}\big) \Big\{ \Big[F_0^2(g_4 + g_5 - g_{11}) - g_A\big(\frac{20}{9}(k_1 + k_2 + k_5 + k_6) \\ & + 2k_{12} - \frac{2}{3}x_1 - x_2 + x_3\big)\Big]\Big\} - \frac{4\pi}{\alpha} 2m_\mu e^2 F_0^2 (s_{17} + s_{18}) \end{aligned} \tag{7.6}$$

$$G_T^r = \frac{4\pi}{\alpha} 2m_N e^2 F_0^2 \big[4\sqrt{2}h_{50} - 2(s_{12} + s_{13})\big] \tag{7.7}$$

From the result of weak form factors the interesting things happened when we get non-zero G_S and G_T, which are corresponded to the second-class currents and do not figure in the standard model.

From Equation (6.50) and Equation (6.53) we may conclude that the induced scalar and induced tensor form factors come from the radiative corrections. To date, there is no unambiguous evidence for the presence of second class currents in nuclear beta decay, but it is nonetheless an interesting experimental challenge to place limits on the possible presence of these second-class currents.

The induced scalar in the vector current is second class is zero by the conserved vector current (CVC) hypothesis, we do not question that here. We look for second-class effects by a finite value for the induced-tensor G_T in the axial current. An early estimate based on the dynamics of relativistic current quarks (Halprin et al., 1976) gave:

$$G_T \approx (m_d - m_u)/2m_N\omega \tag{7.8}$$

where ω is a single-quark energy of about 400 MeV and m_N is the nucleon mass, so that we might then expect $G_T \approx 4 \times 10^{-6}$ MeV^{-1}. More reliable estimation of G_T derived from the application of QCD sum rules to m_u and m_d symmetry-breaking which yields (Shiomi, 1996) $G_T = (1.0 \pm 0.4) \times 10^{-5}$ MeV^{-1}.

If the LECs are in the natural order, the value of G_T is estimated to be of the order 10^{-7}. This leads to an estimation of G_T about 100 times smaller than that of Ref. (Shiomi, 1996).

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APPENDICES

APPENDIX A

INFRARED REGULARIZATION OF BECHER AND LEUTWYLER

The method of infrared regularization is based on dimensional regularization and the analytic properties of loop integrals. It is applicable to one-loop integral in the one-nucleon sector of ChPT. Consider the general integral

$$H_{\pi\cdots N\cdots}(q_1,\ldots,p_1,\ldots) = i\int \frac{d^D}{(2\pi)^D}\frac{1}{a_1\cdots a_m b_1\cdots b_l}, \tag{A.1}$$

where $a_i = (k+q_i)^2 - m_\pi^2 + i0^+$ and $b_j = (k+p_j)^2 - m_N^2 + i0^+$ represent pion and nucleon propagators, respectively, D is the number of space-time dimensions and the masses m_π and m_N refer to the lowest-order pion mass and the nucleon mass in the chiral limit. One combine the pion propagators by using

$$\frac{1}{a_1\cdots a_m} = \left(\frac{\partial}{\partial m_\pi^2}\right)^{(m-1)} \int_0^1 dx_1 \cdots \int_0^1 dx_{m-1}\frac{X}{A}, \tag{A.2}$$

with the numerator given by

$$X = \begin{cases} 1 & \text{for } m=2, \\ x_2(x_3)^2\cdots(x_{m-1})^{m-2} & \text{for } m>2, \end{cases} \tag{A.3}$$

and the recursion relation for the denominator

$$\begin{aligned} A &= A_m \\ A_1 &= a_1 \\ A_{p+1} &= x_p A_p + (1-x_p)a_{p+1} \quad (p=1,\ldots,m-1) \end{aligned} \tag{A.4}$$

The denominator A can be written as

$$A = (k+\bar{q})^2 - \bar{A} + i0^+, \tag{A.5}$$

where $\bar{q}$ is a linear combination of the external momenta q_i and $\bar{A}$ ia a constant.

We combine the nucleon propagators in the same way

$$\frac{1}{b_1 \cdots b_l} = \left(\frac{\partial}{\partial m_N^2}\right)^{(l-1)} \int_0^1 dy_1 \cdots \int_0^1 dy_{l-1} \frac{Y}{B}, \tag{A.6}$$

where the numerator reads

$$Y = \begin{cases} 1 & \text{for } l = 2, \\ y_2 (y_3)^2 \cdots (y_l - 1)^{l-2} & \text{for } l > 2, \end{cases} \tag{A.7}$$

and the recursion relation for the denominator B is given by

$$\begin{aligned} B &= B_l \\ B_1 &= b_1 \\ B_{p+1} &= y_p B_p + (1 - y_p) b_{p+1} \quad (p = 1, \ldots, l-1) \end{aligned} \tag{A.8}$$

The result for the denominator is

$$B = (k + \bar{p})^2 - \bar{B} + i0^+, \tag{A.9}$$

where $\bar{p}$ is a linear combination of the external momenta p_i. The two resulting denominators can be combined by using the identity

$$\frac{1}{AB} = \int_0^1 \frac{dz}{[(1-z)A + zB]^2}, \tag{A.10}$$

giving

$$\begin{aligned} H_{\pi \cdots N \cdots}(q_1, \ldots, p_1, \ldots) &= i \left(\frac{\partial}{\partial m_\pi^2}\right)^{(m-1)} \left(\frac{\partial}{\partial m_N^2}\right)^{(l-1)} \\ &\quad \int_0^1 dz \int_0^1 dx_i \int_0^1 dy_i XY \int \frac{d^D k}{(2\pi)^D} \frac{1}{[(1-z)A + zB]^2} \end{aligned} \tag{A.11}$$

where

$$\int_0^1 dx_i = \int_0^1 dx_1 \cdots \int_0^1 dx_{m-1}, \quad \int_0^1 dy_i = \int_0^1 dy_1 \cdots \int_0^1 dy_{l-1}. \tag{A.12}$$

one obtains

$$H_{\pi\cdots N\cdots}(q_1,\ldots,p_1,\ldots) = \frac{(-1)^{1-l-m}}{(4\pi)^{D/2}}\Gamma(l+m-\frac{D}{2}) \int_0^1 dz z^{l-1}(1-z)^{m-1}\int_0^1 dx_i \int_0^1 dy_i XY[f(z)]^{(D/2)-l-m}, \tag{A.13}$$

with

$$f(z) = \bar{p}^2 z^2 - \left(\bar{p}^2 - \bar{B}\right) z + \bar{A}(1-z) - \left(\bar{q}^2 - 2\bar{p}\cdot\bar{q}\right) z - i0^+. \tag{A.14}$$

The infrared regularization consists of the z integration as

$$\int_0^1 dz \ldots = \int_0^\infty dz \cdots - \int_1^\infty dz \cdots . \tag{A.15}$$

The first term on the right-handed of Equation. A.15 is called the infrared singular part I, and the second term is called the infrared regular part R,

$$H_{\pi\cdots N\cdots} = I_{\pi\cdots N\cdots} + R_{\pi\cdots N\cdots}, \tag{A.16}$$

it can be written shortly

$$H = I + R. \tag{A.17}$$

The infrared singular I satisfies power counting, while R contains terms that violate the power counting. In contrast to the infrared singular part the regular part allows for an expansion in a Taylor series in the external momenta and the quark masses. Therefore using an appropriate renormalization procedure one can compensate these terms in the redefinition of the coupling constants and fields of the most general Lagrangian. So the Green functions obtained from a one-loop diagram separated into an infrared singular and regular part separately satisfy the Ward identities of the theory. This guarantees that regular part can be combined in the coupling constants and fields of the most general Lagrangian. If one removes the regular part of the integral $I_{\pi N}$, the resulting expression satisfies power counting. Note that I and R contain additional divergences which are not shown in $I_{\pi N}$ therefore, these divergences have to be taken away.

APPENDIX B

LOOP INTEGRALS

We will define the general loop integral in d dimensions containing i pion propagators and j nucleon propagators and corresponding to the momenta as in Figure (B.1) as

$$\begin{aligned} &I_{\pi\pi\ldots\pi NN\ldots N}[k_1, k_2, \ldots, k_i, p_1, p_2, \ldots, p_j, A] \\ &\qquad = i\mu^{4-d} \int \frac{d^d\ell}{(2\pi)^d} \frac{A}{D_\pi(k_1)\ldots D_\pi(k_i) D_N(p_1)\ldots D_N(p_j)} \end{aligned} \tag{B.1}$$

Here μ is a scale factor and A is the numerator function, which may contain anything. $D_\pi(k) = (\ell + k)^2 - \mathring{m}_\pi^2 + i\epsilon$ and $D_N(p) = (\ell + p)^2 - m_N^2 + i\epsilon$ are the pion and nucleon propagator denominators , respectively. $\mathring{m}_\pi^2$ and m_N are the unrenormalized pion and nucleon masses appearing in the original Lagrangian. The number of subscripts π and N correspond to the number of pion and nucleon propagators, respectively.

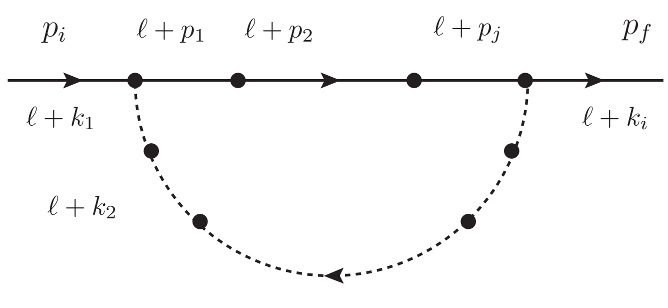

Figure B.1 The general loop integral.

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