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การกำทอนโรเปอร์ในแบบจำลองควาร์ก-กลูออน

(Roper Resonance in Quark-Gluon Configurations)

คณะผู้วิจัย

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ABSTRACT IN THAI

การกำทอนโรเปอร์ N*(1440) ซึ่งเป็นนิวคลีออนในสภาวะกระตุ้นที่ต่ำสุด ได้ถูกอธิบายมากขึ้นนับตั้งแต่มีการค้นพบเมื่อปี ค.ศ. 1964 โดยเมื่อพิจารณาตามแบบจำลองควาร์กสามอนุภาค การกำทอนโรเปอร์นับว่าเป็นการกระตุ้นในแนวรัศมีของนิวคลีออน เพราะการมีเลขควอนตัมที่เหมือนกับนิวคลีออน แต่จากการศึกษาในรายละเอียดพบว่าเป็นการยากมากที่จะให้ความหมายของการกำทอนเช่นนี้ว่าเป็นสภาวะที่ประกอบด้วยควาร์กสามอนุภาคเพราะมวลที่ต่ำและค่าคงตัวของการคู่ควบกับนิวคลีออนและเมซอนที่แปลก จากความล้มเหลวที่จะอธิบายการกำทอนโรเปอร์ด้วยแบบจำลองควาร์กสามอนุภาค จึงได้มีการเสนอแบบจำลองต่างๆอีกมากมาย แต่ยังไม่มีแบบจำลองใดที่ประสบความสำเร็จเป็นอย่างดี

งานวิจัยนี้ เป็นการศึกษาธรรมชาติของการกำทอนโรเปอร์ผ่านกระบวนการการสลายตัวควบคู่ไปกับข้อสันนิษฐานที่ว่า การกำทอนโรเปอร์เป็นสถานะที่ประกอบด้วยควาร์กสามอนุภาคและกลูออนในภาวะไฟฟ้าตามขวางอีกหนึ่งอนุภาค ในงานวิจัยนี้ได้มีการนำแบบจำลองควาร์กกลูออนแบบไม่สัมพัทธภาพมาใช้ โดยแบบจำลองดังกล่าวอธิบายจลนศาสตร์ของระบบควาร์ก แอนติควาร์กและกลูออนด้วยจุดยอดแบบ 3S1 ซึ่งคู่ของควาร์กแอนติควาร์กเกิดขึ้นจากกลูออนหรือสลายไปเป็นกลูออน ทั้งนี้การสร้างฟังก์ชันคลื่นของการกำทอนโรเปอร์ได้คำนึงถึงองศาเสรีของกลูออน ซึ่งมีอิทธิพลต่อฟิสิกส์ของควอนตัมโครโมไดนามิกส์แบบนอนเพอร์เทอร์เบทีฟ

จากการคำนวณอัตราส่วนของค่าการสลายตัวจากปฏิกิริยา $N^*(1440) \to N\rho,\ N\eta,\ N\sigma,\ \Delta\pi$ ต่อค่าการสลายตัวจากปฏิกิริยา $N^*(1440) \to N\pi$ โดยมีพารามิเตอร์อิสระหนึ่งตัวที่ใช้ระบุถึงองค์ประกอบของการกำทอนโรเปอร์ พบว่าค่าอัตราส่วนที่ได้จากการคำนวณสอดคล้องกับผลการทดลองเป็นอย่างดี ทำให้สามารถระบุได้ว่าการกำทอนโรเปอร์เป็นรูปแบบที่ประกอบด้วย $|^2N_g'\rangle - |^4N_g'\rangle$ ซึ่ง $|^2N_g\rangle$ หมายถึงระบบกลูออนกับควาร์กสามอนุภาคที่มีสปิน 1/2 และ $|^4N_g\rangle$ หมายถึงระบบกลูออนกับควาร์กสามอนุภาคที่มีสปิน 3/2

ABSTRACT IN ENGLISH

The Roper resonance $N^*(1440)$, the lowest nucleon excited state, has been subject to intense discussions since its discovery in 1964. In the three-quark picture the Roper resonance had been commonly assigned to a radial excitation of the nucleon since its quantum numbers are the same as the nucleon's. But detailed studies found that it is very difficult to interpret the resonance as a three quark state due to its low mass and strange coupling constants with nucleon and meson. Because of the failure of the three-quark picture, various other models have also been suggested, but none is very successful.

In this work we study the nature of the Roper resonance via its decay processes. We go along with the argument that the Roper resonance is a state of three quarks and one transverse-electric (TE) gluon. A nonrelativistic quark-gluon model is employed, where the dynamics of $\bar{q}qG$ is described in the effective $^3S_1$ vertex in which a quark-antiquark pair is created (destroyed) from (into) a gluon. The wave function of the Roper resonance has been constructed to properly establish the gluonic degree of freedom, which has been a fascinating challenge in nowadays non-perturbative QCD physics.

The ratios of the decay widths of the reactions $N^*(1440) \to N\rho$, $N\eta$, $N\sigma$, $\Delta\pi$ to the one of the reaction $N^*(1440) \to N\pi$, have been derived in the work with only one free parameter which tells how the Roper resonance is made up by the two components $|^2N_g\rangle$ (spin 1/2 three quark core plus a gluon) and $|^4N_g\rangle$ (spin 3/2 three quark core plus a gluon). The theoretical predictions are consistent with experimental data and suggest that the Roper resonance is likely to take the form $|^2N_g'\rangle - |^4N_g'\rangle$.

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CHAPTER 1

INTRODUCTION

All matter consists of atoms. Atoms in turn are built up by nuclei and electrons orbiting around. Nuclei are bound states of protons and neutrons by the strong interaction. Proton and neutrons possess a further substructure of even smaller constituents, the quarks. Quark has not been observed in experiments as isolated objects, but only as clusters such as mesons (quark-antiquark system) and baryons (three quark system). Quark model of hadrons (baryons and mesons) has made a considerable success and been widely accepted, but it is still a challenge to understand the natures of all the observed baryons and mesons in various quark models.

The study of baryon excitation states plays an important role in understanding of the nucleon internal structure, the quark model and hence the nature of the strong interaction. Information is usually extracted from the properties of nucleon excitation state N^*'s, such as their mass spectrum, various production and decay rate(Burkert, 1994). Since the late 1970's, very little has happened in experimental N^* baryon spectroscopy (Moorhouse and Roper, 1974). Considering its importance for the understanding of the baryon structure and for distinguishing various picture of the nonperturbative regime of quantum chromodynamics (QCD), a new generation of experiments on N^* physics with electromagnetic probes has recently been started at new facilities such as Continuous Electron Beam Accelerator Facility (CEBAF) at Jefferson Laboratory (JLAB), Electron Stretcher Accelerator (ELSA) at Bonn, Grenoble Anneau Accelerateur

Laser (GRAAL) at Grenoble.

The contribution of the lowest-lying baryon resonance, the $\Delta(1232)$, to a wide range of nuclear phenomena has been extensively studied (Ericson and Weise, 1988). This resonance ($J = 3/2, I = 3/2, P = +1$) is the dominant feature of the pion-nucleon scattering amplitude at low energy. As such, it strongly influences the creation, propagation and absorption of π in the nuclear medium and acts as an independent degree of freedom of nuclear dynamics at energy scales of the order of a few hundred MeV.

The next baryon resonance, the Roper resonance or $N^*(1440)$, has the same quantum numbers as the nucleon ($J = 1/2, I = 1/2, P = +1$) and is therefore regarded as its first intrinsic excitation, at an energy of about 500 MeV. The structure of this excitation appears rather complex and its properties could have profound consequences on the understanding of the baryon spectrum. The understanding of the Roper resonance has been a long-standing problem in N^* physics. Its very small branching ratios of electromagnetic decay modes, unusual couplings to the $N\pi$ and $N\sigma$ channels, and its low mass together make it is difficult to identify the resonance as a simple three-quark bound state.

The $N^*(1440)$ is a very wide resonance with a full width of (350 ± 100) MeV while the neighboring nucleon excitation states $N^*(1520)$ and $N^*(1535)$ are twice as narrow (Groom, 2000). It is found that $N^*(1440)$ couples strongly (60-70%) to the π-nucleon channel and significantly (5-10%) to the σ-nucleon (more properly $(\pi-\pi)^{I=0}_{S-wave}$-nucleon) channel (Groom, 2000). There are no data on its coupling to the vector meson-nucleon channels, except for an upper limit of 8% to the ρ-nucleon channel. The branching ratios to radiative final states (0.035-0.048% for $p\gamma$ and 0.009-0.032% for $n\gamma$) are unusually small (compared for example to the branching ratio of 0.52-0.60% for $\Delta \rightarrow N\gamma$). The general impression one gets from these data is that the transition of the nucleon to the $N^*(1440)$ (or vice-versa) is induced mainly by scalar fields (π and σ) and very little by vector fields.

We consider the coupling of the $N^*(1440)$ to the $N\pi$ channel. From the partial decay width of the $N^*(1440)$ into the $N\pi$ channel, $\Gamma_{N^*\to N\pi} = (228 \pm 82)$ MeV, one can deduce the values of the coupling constants $g_{\pi NN^*}$ and $f_{\pi NN^*}$ characterizing the strength of the πNN^* pseudoscalar coupling and pseudovector coupling, respectively. We find $g^2_{\pi NN^*}/4\pi = 3.4 \pm 1.2$ and $f^2_{\pi NN^*} = 0.011 \pm 0.004$.

The coupling constant $g_{\sigma NN^*}$ depends on the σ mass and on the width $\Gamma_{\sigma\to\pi\pi}(m^0_\sigma)$ of the σ-meson at the peak of the resonance. The σ-meson of relevance in the many-body problem is the effective degree of freedom accounting for the exchange of two uncorrelated as well as two resonating pions in the scalar-isoscalar channel. It is expected to have mass of the order of 500-550 MeV and to be a broad state. It can be shown that $g_{\sigma NN^*}$ depends weakly on the value of $\Gamma_{\sigma\to\pi\pi}(m^0_\sigma)$ but rather strongly on m^0_σ. The latter effect is a consequence of the coincidence between the σ mass and the difference between the mass of the $N^*(1440)$ and of the nucleon, which determines the phase space limit for the $N^*(1440) \to N\pi\pi$ decay. Fixing $\Gamma_{\sigma\to\pi\pi}(m^0_\sigma) = 250$ MeV, we obtain for example, $g^2_{\sigma NN^*}/4\pi = 0.34 \pm 0.21$ for $m^0_\sigma = 500$ MeV and $g^2_{\sigma NN^*}/4\pi = 0.56 \pm 0.35$ for $m^0_\sigma = 550$ MeV.

Comparing the πNN^* and σNN^* coupling constants to the corresponding values for the πNN and σNN vertices, we have

$$\frac{g_{\pi NN^*}}{g_{\pi NN}} \simeq \frac{1}{2} \quad \text{and} \quad \frac{g_{\sigma NN^*}}{g_{\sigma NN}} \simeq \frac{1}{4}.$$

This seems to depart somewhat from the scaling law

$$\frac{g_{\pi NN^*}}{g_{\pi NN}} = \frac{g_{\sigma NN^*}}{g_{\sigma NN}} = \frac{g_{\omega NN^*}}{g_{\omega NN}} = \frac{g_{\rho NN^*}}{g_{\rho NN}},$$

often used on the basis of constituent quark model arguments. There are however large uncertainties.

To have more constraints on the couplings discussed above, it is useful to make meson-exchange models of simple processes in which the Roper resonance is excited and compare to the coupling constants needed to understand the data on these processes to their values derived from the $N^*(1440)$ partial decay widths. One should keep in mind however the limits of such determinations: the exchanged mesons are effective degrees of freedom and meson-baryon vertices involve not only coupling constants but also form factors which may affect significantly the strength of the couplings.

In the simple three-quark picture of baryons the Roper resonance would be the first radial excitation state of the nucleon if one considers only its quantum numbers. But we will find that the resonance possesses a too low mass to be a radial excitation state.

In a simple model where quarks are confined in an oscillator potential whose slope is independent of the flavor quantum numbers, the Hamiltonian of a three-quark system may take the form

$$H = \frac{\vec{p}_1^2}{2m} + \frac{\vec{p}_2^2}{2m} + \frac{\vec{p}_3^2}{2m} + \frac{1}{2}\mu\omega(\vec{r}_2 - \vec{r}_3)^2 + \frac{1}{2}\mu\omega(\vec{r}_1 - \vec{r}_2)^2 + \frac{1}{2}\mu\omega(\vec{r}_3 - \vec{r}_1)^2. \quad (1.1)$$

Here we have supposed all quarks involved have the same mass. Introducing the Jacobi coordinates

$$\vec{\rho} = \frac{(\vec{r}_1 - \vec{r}_2)}{\sqrt{2}}, \quad \vec{\lambda} = \frac{(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)}{\sqrt{6}}, \quad \vec{R}_{cm} = \frac{(\vec{r}_1 + \vec{r}_2 + \vec{r}_3)}{3}, \quad (1.2)$$

and eliminating the center-of-mass motion, the Schödinger equation of the Hamiltonian in eq (1.1) gives the wave function of a three-quark system,

$$\Psi_{N^*(1440)} = \Psi^{\text{spatial}}(\vec{\rho},\vec{\lambda})\Psi^{\text{spin-flavor-color}} \tag{1.3}$$

with the spatial wave function Ψ^{spatial} taking the form

$$\Psi^{\text{spatial}} = \psi_{n_\rho l_\rho}(\vec{\rho})\psi_{n_\lambda l_\lambda}(\vec{\lambda}), \tag{1.4}$$

where

$$\psi_{n_\rho l_\rho}(\vec{\rho}) = R_{n_\rho l_\rho}(\rho)\mathcal{Y}_{n_\rho l_\rho}(\hat{\rho}), \tag{1.5}$$

$$\psi_{n_\lambda l_\lambda}(\vec{\lambda}) = R_{n_\lambda l_\lambda}(\lambda)\mathcal{Y}_{n_\lambda l_\lambda}(\hat{\lambda}). \tag{1.6}$$

The energy of a state is specified by the quantum number N

$$E_N = \left(N + \frac{3}{2}\right) \quad , \quad N = N_\rho + N_\lambda = (2n_\rho + l_\rho) + (2n_\lambda + l_\lambda), \tag{1.7}$$

and parity $P = (-1)^{l_\rho + l_\lambda}$. The $N = 1$ states have mixed symmetry:

$$\Psi_{11}(\rho) = \psi_{01}(\vec{\rho})\psi_{00}(\vec{\lambda}), \tag{1.8}$$

$$\Psi_{11}(\lambda) = \psi_{00}(\vec{\rho})\psi_{01}(\vec{\lambda}), \tag{1.9}$$

and parity $P = -1$. These states are not corresponding to the parity of the Roper resonance.

For the Roper resonance with positive parity, the only possibility is

$$n_\rho = 1 \;\; , \;\; n_\lambda = 0 \;\; , \;\; l_\rho = l_\lambda = 0 \qquad \text{or} \qquad n_\rho = 0 \;\; , \;\; n_\lambda = 1 \;\; , \;\; l_\rho = l_\lambda = 0.$$

In this case the Roper resonance has $N = 2$ band in a harmonic oscillator basis. The lightest of these states with a totally symmetrical spatial wave function is usually attributed to the Roper resonance. Its low mass has present some problems for simple three-quark picture as these models are not able to describe the right level ordering of positive and negative parity states (Groom, 2000; Høgaasen and Richard, 1983). Indeed, various quarks models (Liu and Wong, 1983; Isgur and Karl, 1978; Høgaasen and Richard, 1983) met difficulties to explain its mass and electromagnetic couplings.

The N^* is not visible as a well-defined peak in the total pion-nucleon cross section. It is established as a pion-nucleon resonance in the P_{11} channel only through detailed partial wave analyzes (Cutkosky et al., 1979; Cutkosky and Wang, 1990). In contrast to the negative parity baryon resonance observed in the 1500–1700 MeV range, which can be described by constituent quark models with harmonic confining potentials (Isgur and Karl, 1978). The Roper resonance has been considered a good candidate for a collective excitation and interpreted as a breathing mode of the nucleon in bag models (DeGrand and Rebbi, 1978). A recent coupled-channel calculation (Schütz et al., 1998), involving the πN, $\pi\Delta$ and σN channels, suggest that the $N^*(1440)$ could be explained as a dynamical effect, without an associated genuine three-quark state. It has therefore been suggested to be a gluonic excitation state of the nucleon, i.e., a "hybrid baryon".

The aim of the whole project is to investigate if the Roper resonance could be reasonably interpreted as a bound state of three-quark and one-gluon through studying all its decay modes such as to $N\pi$, $N\pi\pi$, $N\rho$, $\Delta\pi$ and $N\gamma$. The work services as a pioneer study to pave the way for the whole project. We will first construct the wave function of the Roper resonance to properly include the gluon freedom in the nonrelativistic regime, then evaluate the transition amplitude of the process $N^*(1440) \to N\pi$ in a very general method which could be used, without modification, to other decay channels. In the work we will mainly employ nonrelativistic quark-gluon models where the dynamics of the quark-gluon interaction is described by the effective vertex $^3S_1$ in which a quark-antiquark pair is created/destroyed from/into a gluon and the wave functions of the Roper resonance, nucleon and mesons are nonrelativistic.

The work is structured as follows: in Chapter 2 the wave functions of mesons, nucleons, and the Roper resonance are worked out in a quark-gluon model with aid of group theory. In Chapter 3 we introduce the $^3S_1$ model for the description of the decay process. The transition amplitude for the decay of the Roper resonance is evaluated in Chapter 4. Finally, Chapter 5 gives our results of the decay widths of the Roper resonance to various channels.

CHAPTER 2

WAVE FUNCTIONS

In this chapter we provide some details on how to construct the wave functions of mesons and baryons in the quark model with the aid of group theory. The wave function of the Roper resonance is properly constructed to include the gluon freedom in the nonrelativistic regime, which is a pioneer work as we know.

2.1 Flavor SU(3) Symmetry

The fundamental assumption of the quark model for hadrons is that mesons are quark-antiquark bound states and that baryons are three-quark states. The observed hadrons are eigenstates of the Hamilton operator for the strong interaction H_{st}. We begin by considering a world with only three quarks u, d and s and make the following assumptions:

(1) Flavor university of strong interaction, that is, the strong forces should act in the same way on quarks with different flavors.

(2)Equality of the masses of u, d and s quarks:

$$m_u = m_d = m_s. \tag{2.1}$$

The Hamilton operator of the strong interaction H_{st} is then invariant under SU(3) transformation of the quarks u, d and s. In the framework of the flavor SU(3) symmetry, u, d and s quarks form the fundamental representation of the group.

Quark states $|q\rangle$ are transformed according to

$$|q\rangle' = U|q\rangle, \tag{2.2}$$

with

$$U^\dagger U = UU^\dagger = 1, \tag{2.3}$$

$$\det U = 1. \tag{2.4}$$

The unitary, unimodular matrix U can be written in the form

$$U = \exp\left(-i\frac{1}{2}\lambda_i\theta_i\right), \tag{2.5}$$

where λ_i with $i = 1, \ldots, 8$ are linearly independent, hermitian and traceless 3×3 matrices. Conveniently, the matrices are chosen to be the Gell-Mann matrices (Close, 1981)

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \lambda_8 = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{2.6}$$

2.2 Spin-Flavor Wave Functions of Mesons

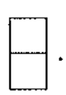
In the framework of the flavor SU(3) symmetry, u, d and s quarks and their anti-objects form nine lightest pseudoscalar and nine lightest vector mesons, see Table. 2.1. Those mesons are the ground states of the Hamiltonian of the quark-antiquark strong interaction.

Table 2.1 Meson nonet

	Charge	Strangeness	Examples	
$u\bar{d}$	$+1$	0	π^+	ρ^+
$d\bar{u}$	-1	0	π^-	ρ^-
$u\bar{u}$			π^0	ρ^0
$d\bar{d}$	0	0	η^0	ω^0
$s\bar{s}$			η'^0	ϕ^0
$u\bar{s}$	$+1$	$+1$	K^+	K^{*+}
$d\bar{s}$	0		K^0	K^{*0}
$\bar{u}s$	-1	-1	K^-	K^{*-}
$\bar{d}s$	0		$\bar{K}^0$	$\bar{K}^{*0}$

In the language of group theory, the fundamental representation of SU(3) is denoted by the Young tableaux

while the conjugation representation in which the antiquark states are transformed is depicted by



The Young tableaux for mesons are formed as follows:

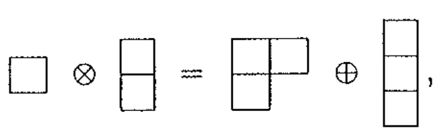


Table 2.2 Flavor wave functions of the pseudoscalar and vector meson nonets

Pseudoscalar	Vector	Flavor
π^+	ρ^+	$u\bar{d}$
π^-	ρ^-	$d\bar{u}$
π^0	ρ^0	$\frac{1}{\sqrt{2}}(d\bar{d} - u\bar{u})$
η_1	ω_1	$\frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$
η_8	ω_8	$\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$
K^+	K^{*+}	$u\bar{s}$
K^0	K^{*0}	$d\bar{s}$
K^-	K^{*-}	$-s\bar{u}$
$\bar{K}^0$	$\bar{K}^{*0}$	$s\bar{d}$

with the corresponding dimensions being:

$$\mathbf{3} \otimes \bar{\mathbf{3}} = \mathbf{8} \oplus \mathbf{1}. \tag{2.7}$$

The $q\bar{q}$ mesons build a nonet (one singlet and one octet) of the flavor SU(3) group.

Since each quark or antiquark can be in a spin-up or spin-down state, namely $S_z = \pm\frac{1}{2}$, the two states form a fundamental representation of the SU(2) group in spin space. The representations of mesons in spin space are

$$\mathbf{2} \otimes \bar{\mathbf{2}} = \mathbf{3} \oplus \mathbf{1}, \tag{2.8}$$

where the spin wave functions of mesons can take the well-known singlet (spin $S = 0$) or triplet (spin $S = 1$) form. The possible spin-flavor conjugation for mesons are

$$(\mathbf{1}, \mathbf{1}),\ \ (\mathbf{1}, \mathbf{3}),\ \ (\mathbf{8}, \mathbf{1}),\ \ (\mathbf{8}, \mathbf{3}). \tag{2.9}$$

The nine lightest pseudoscalar and the nine lightest vector mesons which have been observed and their corresponding flavor wave functions are listed in Table.2.2

2.3 Spin-Flavor Wave Functions of Baryons

Baryons are three-quark bound states of the strong interaction in the quark model. The total wave function of a baryon must be antisymmetric since it is a system of three identical fermions. All particles observed are color singlet, that is, the color part of the wave function of a hadron is antisymmetric. Therefore, the spatial-spin-flavor part of the wave function of a baryon must be symmetric. For the baryons in the ground state of the strong interaction Hamiltonian, the spatial wave functions are usually S-states, hence symmetric. The spin-flavor wave function of a baryon in the ground state is required to be symmetric so that its total wave function is antisymmetric.

Taking the SU(3) fundamental representation (uds) and combining it with the SU(2) ($\uparrow\downarrow$) one can form a six-dimensional fundamental representation of SU(6), $u\uparrow$, $d\uparrow$, $s\uparrow$, $u\downarrow$, $d\downarrow$, $s\downarrow$. Physically in the quark model the intrinsic SU(3) degrees of freedom will be multiplied by the SU(2) spin of the quarks. We will quote the following rules for combining states of different permutation symmetry and verify it by writing out the states explicitly. Denoting symmetric, mixed and antisymmetric states by S, M, A respectively, the symmetry properties that arise are shown in the matrix

	S	M	A
S	S	M	A
M	M	S, M, A	M

Recalling that in SU(3) we found $\mathbf{10}_S, \mathbf{8}_M, \mathbf{1}_A$, while in SU(2) $\mathbf{4}_S$ and $\mathbf{2}_M$ emerged, then the above rules imply, for instance, that the **10** with spin $\frac{3}{2}$ (**4** in SU(2)) will be totally symmetric; the **10** with spin $\frac{1}{2}$ (**2** in SU(2)) will be totally mixed and so forth.

To classify under SU(6) we collect together those states which are symmetric, then those which are mixed and finally those which are antisymmetric. These are listed below together with their subgroup dimensionalities. The total number of such states is given on the right.

$$S : (\mathbf{10}, \mathbf{4}) \quad + (\mathbf{8}, \mathbf{2}) \quad = \quad \mathbf{56} \tag{2.10}$$

$$M : (\mathbf{10}, \mathbf{2}) + (\mathbf{8}, \mathbf{4}) + (\mathbf{8}, \mathbf{2}) + (\mathbf{1}, \mathbf{2}) \quad = \quad \mathbf{70} \tag{2.11}$$

$$A : \quad (\mathbf{8}, \mathbf{2}) \quad + (\mathbf{1}, \mathbf{4}) \quad = \quad \mathbf{20} \tag{2.12}$$

We can immediately verify these results by using the Young tableaux as

$$\boxed{1} \otimes \boxed{2} \otimes \boxed{3} = \boxed{1}\boxed{2}\boxed{3} \oplus \begin{array}{ll} \boxed{1} & \boxed{2} \\ \boxed{3} & \end{array} \oplus \begin{array}{ll} \boxed{1} & \boxed{3} \\ \boxed{2} & \end{array} \oplus \begin{array}{l} \boxed{1} \\ \boxed{2} \\ \boxed{3} \end{array}$$

The corresponding dimensions are

$$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} \;=\; \mathbf{56} \oplus \mathbf{70} \oplus \mathbf{70} \oplus \mathbf{20}, \tag{2.13}$$

hence the $\mathbf{56}_S, \mathbf{70}_{M,S}, \mathbf{70}_{M,A}$, and $\mathbf{20}_A$ representation are seen. After this a mixed-symmetric wave function will be labelled by the superscript λ and ρ for mixed-antisymmetric wave function. In table.2.3, we list, for completeness, the spin-flavor wave function of various permutation symmetries.

2.4 The Spatial Wave Functions of Mesons and Baryons

The spatial wave functions of hadrons are very much model dependent since the interaction among quarks is still an open question. What has been commonly accepted is that the interaction must confine the quarks as clusters

Table 2.3 Spin-flavor wave functions of a baryon classified according to permutation symmetry

56 (S)	
(10,4) : $\phi^S\chi^S$	**(8,2)** : $(\phi^\rho\chi^\rho + \phi^\lambda\chi^\lambda)/\sqrt{2}$
20 (A)	
(1,4) : $\phi^A\chi^S$	**(8,2)** : $(\phi^\lambda\chi^\rho - \phi^\rho\chi^\lambda)/\sqrt{2}$
70 (ρ)	
(10,2) : $\phi^S\chi^\rho$	**(8,4)** : $\phi^\rho\chi^S$
(8,2) : $(\phi^\lambda\chi^\rho + \phi^\rho\chi^\lambda)/\sqrt{2}$	**(1,2)** : $\phi^A\chi^\rho$
70 (λ)	
(10,2) : ϕ^S	**(8,4)** : $\phi^\lambda\chi^S$
(8,2) : $(\phi^\rho\chi^\rho - \phi^\lambda\chi^\lambda)/\sqrt{2}$	**(1,2)** : $\phi^A\chi^\lambda$

since experiments have never observed any free quark. The most simplest but well accepted form of the interaction is the harmonic oscillator potential. In this work we will employ the harmonic oscillator approximation for the quark interaction in setting up the quark cluster wave function of the mesons and baryons. The wave functions in the approximation take analytical forms both in coordinate and momentum spaces.

With the oscillator potential

$$V(r) = \frac{1}{2}\mu\omega^2 r^2, \tag{2.14}$$

where μ is the reduced mass of the quark-antiquark pair and r is the relative coordinate, the momentum space wave function for s-wave and p-wave mesons take the form

$$\Phi_s(\vec{p}) = N_s \exp\left(-\frac{1}{2}b^2p^2\right)\frac{1}{\sqrt{4\pi}}, \tag{2.15}$$

$$\Phi_p(\vec{p}) = N_p(bp)\exp\left(-\frac{1}{2}b^2p^2\right)\mathcal{Y}_{1m}(\hat{p}), \tag{2.16}$$

where $\vec{p}$ is the relative momentum with

$$\vec{p} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2),$$

and

$$\begin{aligned} N_s &= \frac{2b^{3/2}}{\pi^{1/4}}, \\ N_p &= \frac{2(2/3)^{1/2}b^{3/2}}{\pi^{1/4}}, \end{aligned}$$

with $b^2 = \frac{1}{\mu\omega}$.

For the three-quark objects we assume, as for mesons, that the interaction between quarks are well represented by the harmonic oscillator potential. The Schrödinger equation for the three-quark system takes the form

$$\begin{aligned} E\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) &= \left(\frac{p_1^2}{2m} + \frac{p_2^2}{2m} + \frac{p_3^2}{2m}\right)\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) \\ &+ \left[\frac{1}{2}m\omega^2(\vec{r}_1 - \vec{r}_2)^2 + \frac{1}{2}m\omega^2(\vec{r}_2 - \vec{r}_3)^2 \right. \\ &\left. + \frac{1}{2}m\omega^2(\vec{r}_3 - \vec{r}_1)^2\right]\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3), \end{aligned} \quad (2.17)$$

where the three quarks have the same mass m for simplicity. We introduce the Jacobi coordinates

$$\vec{\xi}_1 = \frac{\vec{r}_2 - \vec{r}_1}{\sqrt{2}}, \quad (2.18)$$

$$\vec{\xi}_2 = \frac{\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3}{\sqrt{6}}, \quad (2.19)$$

$$\vec{R} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3}{3}. \quad (2.20)$$

In these new coordinates the Schrödinger equation is rewritten as

$$\begin{aligned} E\Psi(\vec{\xi}_1,\vec{\xi}_2,\vec{R}) &= \left[\frac{1}{2\mu_R}\nabla^2_R + \frac{1}{2\mu_1}\nabla^2_{\xi_1} + \frac{1}{2\mu_2}\nabla^2_{\xi_2}\right]\Psi(\vec{\xi}_1,\vec{\xi}_2,\vec{R}) \\ &+ \left[\frac{3}{2}m\omega^2\xi_1^2 + \frac{3}{2}m\omega^2\xi_2^2\right]\Psi(\vec{\xi}_1,\vec{\xi}_2,\vec{R}), \end{aligned} \tag{2.21}$$

where $\mu_R = 3m$ and $\mu_1 = \mu_2 = m$. The solution for the ground state in the center of mass system is

$$\Psi(\vec{\xi}_1,\vec{\xi}_2) = N_B \exp\left(-\frac{1}{2a^2}\xi_1^2\right)\exp\left(-\frac{1}{2a^2}\xi_2^2\right), \tag{2.22}$$

where $a^2 = 1/(3m\omega)$ and $N_B = 3^{3/4}/(\pi^{3/2}a^3)$. The solution in momentum space is obtained by Fourier transformation as follows

$$\Psi(\vec{p}_1,\vec{p}_2,\vec{p}_3) = N_{B_p}\exp\left[-\frac{1}{2}a^2\left(\frac{\vec{p}_1-\vec{p}_2}{\sqrt{2}}\right)^2\right]\exp\left[-\frac{1}{2}a^2\left(\frac{\vec{p}_1+\vec{p}_2-2\vec{p}_3}{\sqrt{6}}\right)^2\right] \tag{2.23}$$

where $N_{B_p} = (3^{3/4}a^3)/\pi^{3/2}$.

The root-mean-square radii for mesons and baryons might be defined in terms of the size parameters as follows (Yan, 1994):

For a s-wave meson

$$\begin{aligned} \langle r^2\rangle_s^{1/2} &= \frac{1}{2}\sqrt{\langle\Phi_s|r^2|\Phi_s\rangle} \\ &= \frac{1}{2}\sqrt{\frac{3}{2}}b \simeq 0.5 \text{ fm}. \end{aligned} \tag{2.24}$$

For a p-wave meson

$$\begin{aligned} \langle r^2\rangle_p^{1/2} &= \frac{1}{2}\sqrt{\langle\Phi_p|r^2|\Phi_p\rangle} \\ &= \frac{1}{2}\sqrt{\frac{5}{2}}b \simeq 0.64 \text{ fm}. \end{aligned} \tag{2.25}$$

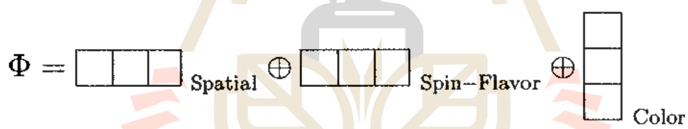
For the nucleon

$$
\begin{aligned}
\langle r^2 \rangle_N^{1/2} &= \frac{1}{2}\sqrt{\langle \Psi | \xi_1^2 | \Psi \rangle} \\
&= a \simeq 0.61 \ \text{fm}. \qquad (2.26)
\end{aligned}
$$

Here we have used $a = 3.1$ GeV^{-1} and $b = 4.1$ GeV^{-1} (Maruyama et al., 1987; Gutsche et al., 1989), which are determined by fitting to the nucleon and meson sizes and nucleon-nucleon, nucleon-antinucleon and pion-nucleon reactions.

2.5 The Nucleon Wave Function

In the quark model, a nucleon is composed of three quarks, with totally antisymmetric wave function and should take the form



The spin-flavor part takes the form:

$$
|N\rangle^{\text{Spin-Flavor}} = \frac{1}{\sqrt{2}} \sum_{J=0,1} \left| \left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)} \right)_J \otimes \frac{1}{2}^{(3)} \right\rangle^{\text{Spin}}_{\frac{1}{2}, S_z'} \left| \left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)} \right)_J \otimes \frac{1}{2}^{(3)} \right\rangle^{\text{Flavor}}_{\frac{1}{2}, T_z'} \qquad (2.27)
$$

The color singlet wave function of the nucleon is given by

$$
|N\rangle^{\text{color}} = \frac{1}{\sqrt{6}} \sum_{i,j,k} \epsilon_{ijk} |q_1\rangle_i |q_2\rangle_j |q_3\rangle_k, \qquad (2.28)
$$

where ϵ_{ijk} is the totally antisymmetric Levi-Civita tensor. For the harmonic oscillator interaction

$$V(r) = \frac{1}{2}m\omega^2 r^2, \tag{2.29}$$

the spatial wave function in momentum space takes the form

$$\begin{aligned}\Psi(\vec{p}_1,\vec{p}_2,\vec{p}_3) &= N_N \exp\left[-\frac{1}{2}a^2\left(\frac{\vec{p}_1-\vec{p}_2}{\sqrt{2}}\right)^2\right] \\ &\cdot \exp\left[-\frac{1}{2}a^2\left(\frac{\vec{p}_1+\vec{p}_2-2\vec{p}_3}{\sqrt{6}}\right)^2\right]\end{aligned} \tag{2.30}$$

where $N_N = (3^{\frac{3}{4}}a^3)/\pi^{\frac{3}{2}}$, with $a = 1/\sqrt{3m\omega}$.

Then putting all the parts together, one obtains the total wave function of nucleon as

$$\begin{aligned}\Psi_N &= \frac{N_N}{\sqrt{2}} \exp\left[-\frac{1}{2}a^2\left(\frac{\vec{p}_1-\vec{p}_2}{\sqrt{2}}\right)^2\right] \exp\left[-\frac{1}{2}a^2\left(\frac{\vec{p}_1+\vec{p}_2-2\vec{p}_3}{\sqrt{6}}\right)^2\right] \\ &\sum_{J=0,1}\left|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_J\otimes\frac{1}{2}^{(3)}\right\rangle^{\text{spin}}_{\frac{1}{2},S'_z}\left|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_J\otimes\frac{1}{2}^{(3)}\right\rangle^{\text{flavor}}_{\frac{1}{2},T'_z} \\ &\cdot\frac{1}{\sqrt{6}}\sum_{i,j,k}\epsilon_{ijk}|q_1\rangle_i|q_2\rangle_j|q_3\rangle_k.\end{aligned} \tag{2.31}$$

2.6 The π Meson Wave Function

In the quark-antiquark interaction of harmonic oscillator type, π-meson is s-wave meson and the wave function in momentum space is written as

$$\Psi^{\text{s-wave}}_{\text{meson}} = N^{\text{s-wave}}_{\text{meson}} \exp\left(-\frac{1}{2}b^2p^2\right)\frac{1}{\sqrt{4\pi}}, \tag{2.32}$$

where $\vec{p}$ is the relative momentum with $\vec{p} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$, and $N^{\text{s-wave}}_{\text{meson}} = 2b^{\frac{3}{2}}/\pi^{\frac{1}{4}}$ with $b^2 = 1/(\mu\omega)$.

The spin-flavor part of the π-meson can be written in form:

$$|\pi\rangle^{\text{spin-flavor}} = \left|\left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right)\right\rangle_{00}^{\text{spin}} \left|\left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right)\right\rangle_{1t_z}^{\text{flavor}} \quad (2.33)$$

The color singlet wave function of π-meson is

$$|\pi\rangle^{\text{color}} = \frac{1}{\sqrt{3}} \sum_{l=1}^{3} |\bar{q}_1\rangle_l |q_2\rangle_l. \quad (2.34)$$

The π-meson wave function is then

$$\begin{aligned}\Psi_\pi &= N_\pi \exp\left[-\frac{1}{8} b^2 (\vec{p}_1 - \vec{p}_2)^2\right] \left|\left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right)\right\rangle_{00}^{\text{spin}} \\ &\quad \left|\left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right)\right\rangle_{1t_z}^{\text{flavor}} \frac{1}{\sqrt{3}} \sum_{l=1}^{3} |\bar{q}_1\rangle_l |q_2\rangle_l, \end{aligned} \quad (2.35)$$

where $N_\pi = N_\pi^{\text{s-wave}}/\sqrt{4\pi} = (b/\pi)^{3/2}$

2.7 The Roper Resonance Wave Function

Hybrid resonances (q^3G and $q\bar{q}G$) have been studied mainly in bag model. A general conclusion of those studies is that the hybrid states should have their rooms in nature. It has also been concluded that the confined gluon might be both TE (transverse electric) and TM (transverse magnetic) modes, and a TE mode is the lowest eigenmode. Considering its low mass, we presume that the Roper resonance is composed of three valence quarks and a TE gluon, denoted by q^3G.

Three quarks states and hybrid states may have the same quantum numbers; a study of the spectroscopic assignments will therefore not be sufficient to discriminate between the q^3 and q^3G states. A hybrid state is excited in the spin-flavor space, and has an SU(6) spin-flavor wave function orthogonal to that of the nucleon, where as the spin-flavor wave function of a radial excited state is identical to that of the nucleon. The gluon is in the color octet representation of SU(3) (Perkins, 1986). In analogy with the (flavor) octet of mesons in Table.2.2 we can write the color-anticolor states of the 8 gluons as follows:

$$r\bar{b}, r\bar{g}, b\bar{g}, b\bar{r}, g\bar{r}, g\bar{b}, \frac{r\bar{r}-b\bar{b}}{\sqrt{2}}, \frac{r\bar{r}+b\bar{b}-2g\bar{g}}{\sqrt{6}} \tag{2.36}$$

With 3 colors and 3 anticolors, we expect $3^2 = 9$ combinations, but one of these is a color singlet and has to be excluded.

Let ϕ, χ and ψ denote flavor, spin, and color wave functions for three quarks and let superscripts S, ρ, λ and a denote the permutation symmetry (S/a is totally symmetric/antisymmetric under any exchange among the three quarks, and λ/ρ is symmetric/antisymmetric under the exchange of the first two quarks). The quantum number of the q^3G states are dictated mainly by the requirements that the three-quark state transform as a color octet. The totally antisymmetric q^3G states are explicitly (Li et al., 1992; Li, 1991)

$$\begin{aligned} |^2N_g\rangle &= \frac{1}{2}\left[\left(\phi^\rho\chi^\rho - \phi^\lambda\chi^\lambda\right)\psi^\rho - \left(\phi^\rho\chi^\lambda + \phi^\lambda\chi^\rho\right)\psi^\lambda\right]\otimes|G\rangle\,, && (2.37)\\ |^4N_g\rangle &= \frac{1}{\sqrt{2}}\left[\left(\phi^\lambda\psi^\rho - \phi^\rho\psi^\lambda\right)\chi^S\right]\otimes|G\rangle, && (2.38)\end{aligned}$$

where superscript 2 and 4 denote the total quark spins as $2S+1$. In the spin-flavor-color wave functions $|^2N_g\rangle$ and $|^4N_g\rangle$ above, the color components of the three-quark core take the form

$$\psi^\rho_\alpha = \frac{1}{2}\sum_{i,j,k,l} |q_3\rangle_i \lambda^\alpha_{ij}\cdot\epsilon_{jkl}|q_1\rangle_k|q_2\rangle_l, \tag{2.39}$$

$$\psi_\alpha^\lambda = \frac{1}{2}\sum_{i,j,k,l}(|q_1\rangle_i|q_2\rangle_j + |q_1\rangle_j|q_2\rangle_i)\lambda_{il}^\alpha \cdot \epsilon_{jkl}|q_3\rangle_k, \tag{2.40}$$

where λ^a are the Gell-Mann matrices. One may write $|^4N_g\rangle$ and $|^2N_g\rangle$ explicitly

$$\begin{aligned}|^4N_g\rangle \;=\; & \frac{1}{\sqrt{2}}\Bigg[\Bigg|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_1\otimes\frac{1}{2}^{(3)}\Bigg\rangle_{\frac{3}{2},m_{123}}\otimes \\ & \left(\mathcal{Y}_{1m_1}(\vec{p}_1+\vec{p}_2+\vec{p}_3-3R\,\vec{p}_4)\otimes \mathbf{e}_{1m_s}\right)_{1m_2'}\Bigg]^{\text{spin}}_{\frac{1}{2},S_z''} \\ & \Bigg\{\Bigg[\Bigg|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_1\otimes\frac{1}{2}^{(3)}\Bigg\rangle^{\text{flavor}}_{\frac{1}{2},T_z''}\frac{1}{\sqrt{8}}\sum_\alpha \psi_\alpha^\rho|g\rangle_\alpha^{\text{color}}\Bigg] \\ & -\Bigg[\Bigg|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_0\otimes\frac{1}{2}^{(3)}\Bigg\rangle^{\text{flavor}}_{\frac{1}{2},T_z''}\frac{1}{\sqrt{8}}\sum_\alpha \psi_\alpha^\lambda|g\rangle_\alpha^{\text{color}}\Bigg]\Bigg\},\end{aligned} \tag{2.41}$$

$$
\begin{aligned}
|^2N_g\rangle \;=\; & \frac{1}{2}\sum_{J_{12}}\Bigg\{\Bigg((-1)^{J_{12}}\Bigg[\Bigg|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_{J_{12}}\otimes\frac{1}{2}^{(3)}\Bigg\rangle_{\frac{1}{2},m_{123}}\otimes \\
& (\mathcal{Y}_{1m_1}(\vec{p}_1+\vec{p}_2+\vec{p}_3-3R\,\vec{p}_4)\otimes \mathrm{e}_{1m_s})_{1m_2'}\Bigg]^{\text{spin}}_{\frac{1}{2},S_z''} \\
& \Bigg|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_{J_{12}}\otimes\frac{1}{2}^{(3)}\Bigg\rangle^{\text{flavor}}_{\frac{1}{2},T_z''}\frac{1}{\sqrt{8}}\sum_{\alpha}\psi^{\rho}_{\alpha}|g\rangle^{\text{color}}_{\alpha}\Bigg) \\
& -\Bigg(\Bigg[\Bigg|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_{1-J_{12}}\otimes\frac{1}{2}^{(3)}\Bigg\rangle_{\frac{1}{2},m_{123}}\otimes \\
& (\mathcal{Y}_{1m_1}(\vec{p}_1+\vec{p}_2+\vec{p}_3-3R\,\vec{p}_4)\otimes \mathrm{e}_{1m_s})_{1m_2'}\Bigg]^{\text{spin}}_{\frac{1}{2},S_z''} \\
& \Bigg|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_{J_{12}}\otimes\frac{1}{2}^{(3)}\Bigg\rangle^{\text{flavor}}_{\frac{1}{2},T_z''}\frac{1}{\sqrt{8}}\sum_{\alpha}\psi^{\lambda}_{\alpha}|g\rangle^{\text{color}}_{\alpha}\Bigg)\Bigg\} \qquad (2.42)
\end{aligned}
$$

The total wave function of the Roper resonance is the linear combination of the $|^4N_g\rangle$ component and the $|^2N_g\rangle$ one, taking the form

$$
\Psi^{N^*(1440)} = \Psi^{\text{spatial}}_{N^*(1440)}\left[A|^2N_g\rangle + B|^4N_g\rangle\right], \qquad (2.43)
$$

with

$$
A^2 + B^2 = 1 \qquad (2.44)
$$

In the approximation of the harmonic oscillator interaction among quarks and gluon, the spatial part $\Psi^{\text{spatial}}_{N^*(1440)}$ may take the form

$$\begin{aligned}\Psi^{\text{spatial}}_{N^*(1440)} &= N_{N^*} \exp\left[-\frac{1}{2}a^2\left(\frac{\vec{p}_1-\vec{p}_2}{\sqrt{2}}\right)^2\right]\exp\left[-\frac{1}{2}a^2\left(\frac{\vec{p}_1+\vec{p}_2-2\vec{p}_3}{\sqrt{6}}\right)^2\right]\\ &\quad \exp\left[-\frac{1}{2}a^2d^2\left(\frac{\vec{p}_1+\vec{p}_2+\vec{p}_3-3R\,\vec{p}_4}{\sqrt{12}}\right)^2\right] \qquad (2.45)\end{aligned}$$

where N_{N^*} is the normalization factor, $d = 4/(1+3R)$ and R is the ratio of the quark mass to the gluon one, $R = m_q/m_g$.

Then, the total wave function of the Roper resonance can be written as

$$\begin{aligned}\Psi_{N^*(1440)} &= N_{N^*} \exp\left[-\frac{1}{2}a^2\left(\frac{\vec{p}_1-\vec{p}_2}{\sqrt{2}}\right)^2\right]\\ &\quad \exp\left[-\frac{1}{2}a^2\left(\frac{\vec{p}_1+\vec{p}_2-2\vec{p}_3}{\sqrt{6}}\right)^2\right]\\ &\quad \exp\left[-\frac{1}{2}a^2d^2\left(\frac{\vec{p}_1+\vec{p}_2+\vec{p}_3-3R\,\vec{p}_4}{\sqrt{12}}\right)^2\right]\\ &\quad \mathcal{Y}_{1m_1}(\vec{p}_1+\vec{p}_2+\vec{p}_3-3R\vec{p}_4)\left[A|^2N'_g\rangle + B|^4N'_g\rangle\right], \qquad (2.46)\end{aligned}$$

with

$$
\begin{aligned}
|^2N_g'\rangle &= \frac{1}{2}\sum_{m_{123},m_1} C\left(\frac{1}{2}\frac{1}{2}1; S_z'' m_{123}(S_z''-m_{123})\right) \\
&\quad C\left(111; (S_z''-m_{123})m_1(S_z''-m_{123}-m_1)\right) \\
&\quad \sum_{J_{12}}\Bigg\{\Bigg(\Bigg[\Bigg|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_{J_{12}}\otimes\frac{1}{2}^{(3)}\Bigg\rangle_{\frac{1}{2},m_{123}}\mathbf{e}_{1m_s}\Bigg]^{\text{spin}}_{\frac{1}{2},S_z''} \\
&\qquad \Bigg|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_{J_{12}}\otimes\frac{1}{2}^{(3)}\Bigg\rangle^{\text{flavor}}_{\frac{1}{2},T_z''}(-1)^{J_{12}}\frac{1}{\sqrt{8}}\sum_a\psi_a^\rho|g\rangle_a^{\text{color}}\Bigg) \\
&\quad -\Bigg(\Bigg[\Bigg|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_{1-J_{12}}\otimes\frac{1}{2}^{(3)}\Bigg\rangle_{\frac{1}{2},m_{123}}\mathbf{e}_{1m_s}\Bigg]^{\text{spin}}_{\frac{1}{2},S_z''} \\
&\qquad \Bigg|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_{J_{12}}\otimes\frac{1}{2}^{(3)}\Bigg\rangle^{\text{flavor}}_{\frac{1}{2},T_z''}\frac{1}{\sqrt{8}}\sum_\alpha\psi_\alpha^\lambda|g\rangle_\alpha^{\text{color}}\Bigg)\Bigg\}, \quad (2.47)
\end{aligned}
$$

and

$$
\begin{aligned}
|^4N_g'\rangle \;=\; & \frac{1}{\sqrt{2}} \sum_{m_{123},m_1} C\left(\frac{1}{2}\frac{3}{2}1; S_z'' m_{123}(S_z''-m_{123})\right) \\
& C\left(111;(S_z''-m_{123})m_1(S_z''-m_{123}-m_1)\right) \\
& \left(\left| \left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right)_1 \otimes \frac{1}{2}^{(3)} \right\rangle^{\text{spin}}_{\frac{3}{2},m_{123}} \mathbf{e}_{1m_s} \right) \\
& \left\{ \left| \left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right)_1 \otimes \frac{1}{2}^{(3)} \right\rangle^{\text{flavor}}_{\frac{1}{2},T_z''} \frac{1}{\sqrt{8}} \sum_{\alpha} \psi^{\rho}_{\alpha} |g\rangle^{\text{color}}_{\alpha} \right. \\
& \left. - \left| \left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right)_0 \otimes \frac{1}{2}^{(3)} \right\rangle^{\text{flavor}}_{\frac{1}{2},T_z''} \frac{1}{\sqrt{8}} \sum_{\alpha} \psi^{\lambda}_{\alpha} |g\rangle^{\text{color}}_{\alpha} \right\}, \qquad (2.48)
\end{aligned}
$$

where the first Clebsch-Gordon coefficients in both eq. (2.47) and (2.48) come from the coupling between the spin of the three-quark core and the total angular momentum of the TE gluon, and the second from the coupling between the spin and orbital angular momenta of the gluon.

CHAPTER 3

THE INTERACTION OPERATORS

In this chapter, we provide some detail on how to construct the interaction operator for the decay of the Roper resonance. First, we discuss the relevant $q\bar{q}g$ dynamics, as defined in the so-called $^3S_1$ model. Then we work out the interaction operator of our model in the nonrelativistic approximation.

The dynamics of a $q\bar{q}g$ vertex is effectively described by vector $^3S_1$ interaction. We start with the quark-antiquark-gluon vertex, which can be deduced from the interaction Lagrangian density

$$\mathcal{L}_{int} = \bar{\psi} g \gamma^\mu A_\mu^\alpha \frac{\lambda^\alpha}{2} \psi, \tag{3.1}$$

where g is the strong coupling constant, $\frac{\lambda^\alpha}{2}$ is the SU(3) generators, A_μ^α is the gluon fields ($\alpha = 1, \ldots, 8$) and ψ is the quark field. The gluon field A_μ^α might be rewritten as

$$A_\mu^\alpha = A^\alpha A_\mu,$$

with A^α responsible for only the color sector. For free quarks, ψ are just the four-momentum eigensolutions of Dirac's equation, taking the form

$$\psi = u(\vec{p})e^{ip\cdot x}, \tag{3.2}$$

where u is a four-component spinor independent of x.

For the transition, depicted in Fig. 3.1, of a quark with momentum $\vec{p}_i$ into an antiquark with momentum $\vec{p}_f$ and a gluon with Lorentz index μ and color label

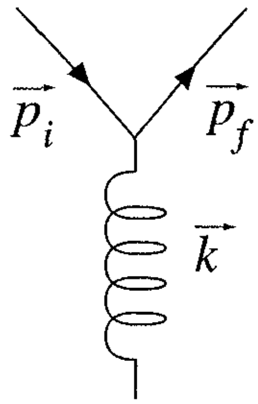


Figure 3.1 Diagram for the quark-antiquark-gluon vertex.

α $(= 1, \ldots, 8)$, the Lagrangian is simply

$$\mathcal{L}_{int} = \bar{v}(\vec{p}_f)e^{ip_f \cdot x} g\gamma^\mu A_\mu A^\alpha \frac{\lambda^\alpha}{2} u(\vec{p}_i)e^{ip_i \cdot x}, \tag{3.3}$$

where the Dirac spinors for the quark (u_i) and antiquark (v_f) are defined as

$$u_i(\vec{p}_i, \vec{\sigma}_i) = \sqrt{\frac{E_i + m}{2m}} \begin{pmatrix} \chi_i \\ \frac{\vec{\sigma}_i \cdot \vec{p}_i}{E_i + m}\chi_i \end{pmatrix}, \tag{3.4}$$

$$v_f(\vec{p}_f, \vec{\sigma}_f) = \sqrt{\frac{E_f + m}{2m}} \begin{pmatrix} \frac{\vec{\sigma}_f \cdot \vec{p}_f}{E_f + m}\bar{\chi}_f \\ \bar{\chi}_f \end{pmatrix}, \tag{3.5}$$

with

$$\bar{v}_f = v^\dagger \gamma^0,$$

where χ and $\bar{\chi}$ are 2-component spinors respectively for quark and antiquark, defined as:

$$\chi(\text{spin up}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \chi(\text{spin down}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$
$$\bar{\chi}(\text{spin up}) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \quad \bar{\chi}(\text{spin down}) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}. \tag{3.6}$$

The vector potential A_μ is given by a plane wave:

$$A_\mu(x,\vec{k}) = \varepsilon_\mu(\vec{k}) N_k \left(e^{-ik\cdot x} + e^{ik\cdot x}\right), \tag{3.7}$$

where $\varepsilon_\mu(\vec{k})$ is the polarization vector of gluon and N_k is the normalization constant. The general polarization vectors of gluon is given by

$$\varepsilon_\mu(\vec{k}) = \left(\frac{\vec{k}\cdot\vec{\varepsilon}}{m_g}, \vec{\varepsilon} + \frac{\vec{k}\cdot\vec{\varepsilon}}{m_g(k_0+m_g)}\vec{k}\right). \tag{3.8}$$

We choose only one direction of the gluon then eq.(3.7) is reduced to

$$A_\mu(x,\vec{k}) = \varepsilon_\mu(\vec{k}) N_k \left(e^{-ik\cdot x}\right). \tag{3.9}$$

Substituting eq.(3.9) in eq.(3.3) gives

$$\begin{aligned} \mathcal{L}_{int} &= \bar{v}(\vec{p}_f) e^{ip_f\cdot x} g\gamma^\mu \varepsilon_\mu(\vec{k}) A^\alpha \frac{\lambda^\alpha}{2} e^{ip_i\cdot x} e^{-ik\cdot x} u(\vec{p}_i) \\ &= \bar{v}(\vec{p}_f) g\gamma^\mu \varepsilon_\mu(\vec{k}) A^\alpha \frac{\lambda^\alpha}{2} e^{i(p_i+p_f-k)\cdot x} u(\vec{p}_i). \end{aligned} \tag{3.10}$$

The interaction Lagrangian in eq.(3.10) can be written out explicitly using the following representation of the Dirac 4×4 matrices

$$\gamma^0 = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} \mathbf{0} & \sigma^i \\ -\sigma^i & \mathbf{0} \end{pmatrix}, \tag{3.11}$$

where $\sigma^i (i = 1, 2, 3)$ is the indicated Pauli matrix, $\mathbf{1}$ denotes the 2×2 unit matrix, and $\mathbf{0}$ is the 2×2 matrix of zeroes.

Now

$$\gamma^\mu \varepsilon_\mu = \gamma^0 \varepsilon_0 - \vec{\gamma} \cdot \vec{\varepsilon}_\mu, \tag{3.12}$$

hence

$$\begin{aligned}
\mathcal{L}_{int} &= \bar{v}(\vec{p}_f) g \left(\gamma^0 \varepsilon_0 - \vec{\gamma} \cdot \vec{\varepsilon}_\mu\right) A^\alpha \frac{\lambda^\alpha}{2} e^{i(p_i + p_f - k)\cdot x} u(\vec{p}_i) \\
&= \bar{v}(\vec{p}_f) g \gamma^0 \left(\frac{\vec{k} \cdot \vec{\varepsilon}}{m_g}\right) A^\alpha \frac{\lambda^\alpha}{2} e^{i(p_i + p_f - k)\cdot x} u(\vec{p}_i) \\
&\quad - \bar{v}(\vec{p}_f) g \vec{\gamma} \cdot \left(\vec{\varepsilon} + \frac{\vec{k} \cdot \vec{\varepsilon}}{m_g(k_0 + m_g)} \vec{k}\right) A^\alpha \frac{\lambda^\alpha}{2} e^{i(p_i + p_f - k)\cdot x} u(\vec{p}_i) \\
&= \bar{v}(\vec{p}_f) g \gamma^0 \left(\frac{\vec{k} \cdot \vec{\varepsilon}}{m_g}\right) A^\alpha \frac{\lambda^\alpha}{2} e^{i(p_i + p_f - k)\cdot x} u(\vec{p}_i) \\
&\quad - \bar{v}(\vec{p}_f) g \vec{\gamma} \cdot \vec{\varepsilon} A^\alpha \frac{\lambda^\alpha}{2} e^{i(p_i + p_f - k)\cdot x} u(\vec{p}_i) \\
&\quad - \bar{v}(\vec{p}_f) g \left(\frac{\vec{k} \cdot \vec{\varepsilon}}{m_g(k_0 + m_g)}\right) (\vec{\gamma} \cdot \vec{k}) A^\alpha \frac{\lambda^\alpha}{2} e^{i(p_i + p_f - k)\cdot x} u(\vec{p}_i). \\
&= \sqrt{\frac{E_f + m}{2m}} \sqrt{\frac{E_i + m}{2m}} \chi_f{}^\dagger \left[\frac{\vec{\sigma}_f \cdot \vec{p}_f}{E_f + m} + \frac{\vec{\sigma}_i \cdot \vec{p}_i}{E_i + m} - \left(\frac{\vec{\sigma}_f \cdot \vec{p}_f}{E_f + m}\right) \left(\frac{\vec{\sigma}_i \cdot \vec{p}_i}{E_i + m}\right) \vec{\sigma} \cdot \vec{\varepsilon} \right. \\
&\quad - \vec{\sigma} \cdot \vec{\varepsilon} - \left(\frac{\vec{\sigma}_f \cdot \vec{p}_f}{E_f + m}\right) \left(\frac{\vec{\sigma}_i \cdot \vec{p}_i}{E_i + m}\right) \left(\frac{\vec{k} \cdot \vec{\varepsilon}}{m_g(k_0 + m_g)}\right) \vec{\sigma} \cdot \vec{k} \\
&\quad \left. - \vec{\sigma} \cdot \vec{k} \right] g A^\alpha \frac{\lambda^\alpha}{2} e^{i(p_i + p_f - k)\cdot x} \chi_i.
\end{aligned} \tag{3.13}$$

The interaction is given by

$$W_{fi} = \int \mathcal{L}_{int} d^4x = \bar{\chi}_f{}^{\dagger} V_{fi} \chi_i, \tag{3.14}$$

with

$$V_{fi} = \sqrt{\frac{E_f+m}{2m}}\sqrt{\frac{E_i+m}{2m}}\left\{\frac{\vec{\sigma}_f\cdot\vec{p}_f}{E_f+m} + \frac{\vec{\sigma}_i\cdot\vec{p}_i}{E_i+m} - \left(\frac{\vec{\sigma}_f\cdot\vec{p}_f}{E_f+m}\right)\left(\frac{\vec{\sigma}_i\cdot\vec{p}_i}{E_i+m}\right)\vec{\sigma}\cdot\vec{\varepsilon} - \vec{\sigma}\cdot\vec{\varepsilon} - \left(\frac{\vec{\sigma}_f\cdot\vec{p}_f}{E_f+m}\right)\left(\frac{\vec{\sigma}_i\cdot\vec{p}_i}{E_i+m}\right)\left(\frac{\vec{k}\cdot\vec{\varepsilon}}{m_g(k_0+m_g)}\right)\vec{\sigma}\cdot\vec{k} - \vec{\sigma}\cdot\vec{k}\right\} gA^{\alpha}\frac{\lambda^{\alpha}}{2}\delta(\vec{p}_i+\vec{p}_f-\vec{k}). \tag{3.15}$$

In the nonrelativistic quark model, the quark-antiquark-gluon transition operator corresponds to the nonrelativistic quark-antiquark-gluon interaction of lowest order QCD, where the created $q\bar{q}$ carries the quantum number $^{2S+1}L_J = {}^3S_1$. In the nonrelativistic approximation, namely $E \simeq m$, $k_0 \simeq m_g$, and $|\vec{p}_i| = |\vec{p}_f| = |\vec{k}| \simeq 0$, we have

$$W_{fi} = \bar{\chi}_f{}^{\dagger} V_{fi} \chi_i, \tag{3.16}$$

with

$$V_{fi} = -gA^{\alpha}\frac{\lambda^{\alpha}}{2}\vec{\sigma}\cdot\vec{\varepsilon}\delta(\vec{p}_i+\vec{p}_f-\vec{k}) = gA^{\alpha}\frac{\lambda^{\alpha}}{2}(-1)^{\mu+1}\sigma^{\mu}\varepsilon_{-\mu}\delta(\vec{p}_i+\vec{p}_f-\vec{k}). \tag{3.17}$$

Here we have used

$$\vec{A}\cdot\vec{B} = \sum_{\mu}(-1)^{\mu}A_{\mu}B_{-\mu}, \tag{3.18}$$

with

$$
\begin{aligned}
A_1 &= -\frac{1}{\sqrt{2}}\left(A_x + iA_y\right), \\
A_0 &= A_z, \\
A_{-1} &= \frac{1}{\sqrt{2}}\left(A_x - iA_y\right).
\end{aligned} \tag{3.19}
$$

It can be easily proven that

$$
\begin{aligned}
\langle \bar{b}|\sigma^{-1}|a\rangle &= -\sqrt{2}\delta_{a,\bar{b}}\delta_{a,\frac{1}{2}}, \\
\langle \bar{b}|\sigma^{0}|a\rangle &= \sqrt{2}\delta_{a,-\bar{b}}, \\
\langle \bar{b}|\sigma^{1}|a\rangle &= -\sqrt{2}\delta_{a,\bar{b}}\delta_{a,-\frac{1}{2}},
\end{aligned} \tag{3.20}
$$

where a and $\bar{b}$ are spin states of quark and antiquark respectively and σ^{μ} are defined as

$$
\begin{aligned}
\sigma^1 &= -\frac{1}{\sqrt{2}}(\sigma^x + i\,\sigma^y), \\
\sigma^0 &= \sigma^z, \\
\sigma^{-1} &= \frac{1}{\sqrt{2}}(\sigma^x - i\,\sigma^y).
\end{aligned} \tag{3.21}
$$

The operation of the $\vec{\sigma}$ could be understand as that it operates a quark state to an antiquark state, or that it projects a quark-antiquark pair onto a spin-1 state. We may write eq. (3.20) in the form

$$
\langle 0,0|\sigma^{\mu}_{ij}|[\bar{\chi}_i \otimes \chi_j]_{JM}\rangle = (-1)^M\sqrt{2}\delta_{J,1}\delta_{M,-\mu}. \tag{3.22}
$$

For the flavor a quark-antiquark pair which annihilates into a gluon must have zero isospin. So we may introduce an unit operator $\mathbf{1}^F_{ij}$ with the property

$$\langle 0,0|\hat{\mathbf{1}}^F_{ij}|T,T_z\rangle = \sqrt{2}\delta_{T,0}\delta_{T_z,0}. \tag{3.23}$$

The operation of the $\vec{\varepsilon}$ which operates on the unit vector of gluon can be in form

$$\langle 0,0|\varepsilon_{-\mu}|\mathbf{e}_{sm_s}\rangle = \delta_{-\mu,m_s}. \tag{3.24}$$

Finally one may write the $^3S_1$ operator in the form

$$V_{ij} = g\sum_{\mu}(-1)^{\mu}\sigma^{\mu}_{ij}\,\mathbf{1}^F_{ij}\varepsilon_{-\mu}\,A^{\alpha}\frac{\lambda^{\alpha}}{2}\delta(\vec{p}_i+\vec{p}_j-\vec{k}), \tag{3.25}$$

where g is the effective coupling strength, and the two body matrix elements given by

$$\langle 0,0|\sigma^{\mu}_{ij}|[\bar{\chi}_i\otimes\chi_j]_{JM}\rangle = (-1)^M\sqrt{2}\delta_{J,1}\delta_{M,-\mu}, \tag{3.26}$$

$$\langle 0,0|\hat{\mathbf{1}}^F|T,T_z\rangle = \sqrt{2}\delta_{T,0}\delta_{T_z,0}, \tag{3.27}$$

$$\langle 0,0|\varepsilon_{-\mu}|\mathbf{e}_{sm_s}\rangle = \delta_{-\mu,m_s} \tag{3.28}$$

CHAPTER 4

TRANSITION AMPLITUDE OF N*(1440) DECAY

The transition amplitude of the Roper resonance decay into nucleon and pion is evaluated in hybrid baryon picture. The method developed in the chapter is general, and could be applied to other decay channels without modification. The whole transition amplitude is derived by calculating the spatial part, spin-flavor part and color part separately.

The transition amplitude for the decay of the Roper resonance $N^*(1440)$ into nucleon and π-meson is defined as

$$T_{N^*(1440)\to N\pi} = \int \prod_{i=1}^{9} d\vec{p}_i \Psi^{\dagger}_{N\pi} \mathcal{O} \Psi_{N^*(1440)}, \tag{4.1}$$

where $\Psi_{N\pi}$ and $\Psi_{N^*(1440)}$ are the final and initial wave functions, respectively. The

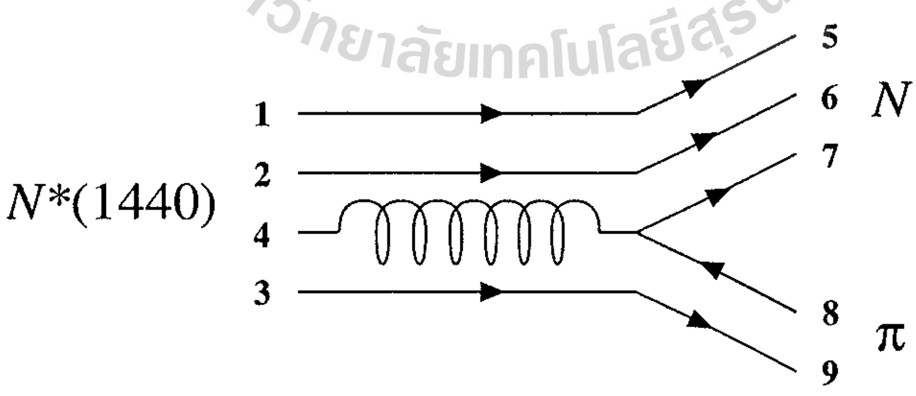


Figure 4.1 Feynman diagram for $N^*(1440) \to N\pi$

momenta $\vec{p}_i$ are labelled as in Fig. 4.1. The operator $\mathcal{O}$ is defined as

$$\mathcal{O} = \delta(\vec{p}_1 - \vec{p}_5)\delta(\vec{p}_2 - \vec{p}_6)\delta(\vec{p}_3 - \vec{p}_9)V_{78}^{\dagger}(^3S_1), \tag{4.2}$$

with the $^3S_1$ interaction vertex

$$V_{ij}(^3S_1) = g\sum_{\mu}(-1)^{\mu+1}\sigma_{ij}^{\mu}\mathbf{1}_{ij}^{F}\varepsilon_{-\mu}A^{\alpha}\frac{\lambda^{\alpha}}{2}\delta(\vec{p}_i + \vec{p}_j - \vec{k}). \tag{4.3}$$

In order to evaluate the transition amplitude for a certain final state, we expand the total transition amplitude in partial waves

$$T_{N^*(1440)\to N\pi} = \sum_{lm_l} T_{lm_l}\mathcal{Y}_{lm_l}^*(\hat{p}), \tag{4.4}$$

where $\mathcal{Y}_{lm_l}^*$ is the spherical harmonics with l and m_l denoting the total orbital angular momentum and its projection of the final $N\pi$ state.

The partial wave transition amplitude T_{lm_l} for the $N^*(1440)$ decay into nucleon and π-meson takes the form

$$\begin{aligned} T_{lm_l} = g\sum_{m_1,m_{123}} P_{lm_l}\Bigg[& \frac{A}{2}C\left(\frac{1}{2}\frac{1}{2}1; S_z'' m_{123}(S_z'' - m_{123})\right) \\ & C\left(111; (S_z'' - m_{123})m_1(S_z'' - m_{123} - m_1)\right)Q_1 \\ & + \frac{B}{\sqrt{2}}C\left(\frac{1}{2}\frac{3}{2}1; S_z'' m_{123}(S_z'' - m_{123})\right) \\ & C\left(111; (S_z'' - m_{123})m_1(S_z'' - m_{123} - m_1)\right)Q_2\Bigg], \end{aligned} \tag{4.5}$$

with

$$
\begin{aligned}
P_{lm_l} &= N_{N^*}N_N N_\pi \int d\hat{p}\prod_{i=1}^{9} d\vec{p}_i\,\mathcal{Y}_{lm_l}(\hat{p})\cdot\exp\left[-\frac{a^2}{2}\left(\frac{\vec{p}_5-\vec{p}_6}{\sqrt{2}}\right)^2\right] \\
&\cdot\exp\left[-\frac{a^2}{2}\left(\frac{\vec{p}_5+\vec{p}_6-2\vec{p}_7}{\sqrt{6}}\right)^2\right]\exp\left[-\frac{b^2}{8}\left(\vec{p}_8-\vec{p}_9\right)^2\right] \\
&\cdot\delta(\vec{p}_8+\vec{p}_9-\vec{p})\exp\left[-\frac{a^2}{2}\left(\frac{\vec{p}_1-\vec{p}_2}{\sqrt{2}}\right)^2\right]\exp\left[-\frac{a^2}{2}\left(\frac{\vec{p}_1+\vec{p}_2-2\vec{p}_3}{\sqrt{6}}\right)^2\right] \\
&\cdot\exp\left[-\frac{1}{2}a^2d^2\left(\frac{\vec{p}_1+\vec{p}_2+\vec{p}_3-3R\vec{p}_4}{\sqrt{12}}\right)^2\right]\delta\left(\vec{p}_1+\vec{p}_2+\vec{p}_3+\vec{p}_4\right)\delta\left(\vec{p}_1-\vec{p}_5\right) \\
&\cdot\delta\left(\vec{p}_2-\vec{p}_6\right)\delta\left(\vec{p}_3-\vec{p}_9\right)\delta\left(\vec{p}_4-\vec{p}_7-\vec{p}_8\right) \\
&\mathcal{Y}_{1m_1}(\vec{p}_1+\vec{p}_2+\vec{p}_3-3R\vec{p}_4), \qquad (4.6)
\end{aligned}
$$

$$
Q_1 = \left\langle \Psi_{N\pi}^{\text{spin-flavor-color}} \middle| \mathcal{O}_{\text{spin-flavor-color}} \middle| {}^2N_g'' \right\rangle, \qquad (4.7)
$$

and

$$
Q_2 = \left\langle \Psi_{N\pi}^{\text{spin-flavor-color}} \middle| \mathcal{O}_{\text{spin-flavor-color}} \middle| {}^4N_g'' \right\rangle, \qquad (4.8)
$$

where

$$
\begin{aligned}
|{}^2N_g''\rangle &= \sum_{J_{12}}\Bigg\{\Bigg(\Bigg[\left|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_{J_{12}}\otimes\frac{1}{2}^{(3)}\right\rangle_{\frac{1}{2},m_{123}}\mathbf{e}_{1m_s}\Bigg]^{\text{spin}}_{\frac{1}{2},S_z''} \\
&\qquad\left|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_{J_{12}}\otimes\frac{1}{2}^{(3)}\right\rangle^{\text{flavor}}_{\frac{1}{2},T_z''}(-1)^{J_{12}}\frac{1}{\sqrt{8}}\sum_\alpha\psi_\alpha^\rho|g\rangle_\alpha^{\text{color}}\Bigg) \\
&\quad-\Bigg(\Bigg[\left|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_{1-J_{12}}\otimes\frac{1}{2}^{(3)}\right\rangle_{\frac{1}{2},m_{123}}\mathbf{e}_{1m_s}\Bigg]^{\text{spin}}_{\frac{1}{2},S_z''} \\
&\qquad\left|\left(\frac{1}{2}^{(1)}\otimes\frac{1}{2}^{(2)}\right)_{J_{12}}\otimes\frac{1}{2}^{(3)}\right\rangle^{\text{flavor}}_{\frac{1}{2},T_z''}\frac{1}{\sqrt{8}}\sum_\alpha\psi_\alpha^\lambda|g\rangle_\alpha^{\text{color}}\Bigg)\Bigg\}, \qquad (4.9)
\end{aligned}
$$

and

$$|{}^4N_g''\rangle = \left[\left|\left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right)_1 \otimes \frac{1}{2}^{(3)}\right\rangle^{\text{spin}}_{\frac{3}{2},m_{123}} \mathbf{e}_{1m_s}\right]_{\frac{1}{2},S_z''}$$
$$\left\{\left|\left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right)_1 \otimes \frac{1}{2}^{(3)}\right\rangle^{\text{flavor}}_{\frac{1}{2},T_z''} \frac{1}{\sqrt{8}}\sum_\alpha \psi_\alpha^\rho |g\rangle_\alpha^{\text{color}}\right.$$
$$\left. - \left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right)_0 \otimes \frac{1}{2}^{(3)}\right\rangle^{\text{flavor}}_{\frac{1}{2},T_z''} \frac{1}{\sqrt{8}}\sum_\alpha \psi_\alpha^\lambda |g\rangle_\alpha^{\text{color}}\Bigg\} \tag{4.10}$$

After a tedious evaluation we derive the partial-wave transition amplitude for $N^*(1440)$ decay,

$$\begin{aligned} T_{lm_l} &= g\sum_{m_1,m_{123}} P_{lm_l}\left[\frac{A}{2}C\left(\frac{1}{2}\frac{1}{2}1; S_z''m_{123}(S_z''-m_{123})\right)\right. \\ &\quad C\left(111;(S_z''-m_{123})m_1(S_z''-m_{123}-m_1)\right)Q_1 \\ &\quad +\frac{B}{\sqrt{2}}C\left(\frac{1}{2}\frac{3}{2}1; S_z''m_{123}(S_z''-m_{123})\right) \\ &\quad \left. C\left(111;(S_z''-m_{123})m_1(S_z''-m_{123}-m_1)\right)Q_2\right] \\ &= g\delta_{S_z,S_z''}\delta_{T_z,T_z''}P_{1m_l}\left[\frac{A}{2}Q_1' + \frac{B}{\sqrt{2}}Q_2'\right], \end{aligned} \tag{4.11}$$

with

$$
\begin{aligned}
Q_1' \;=\; & -\frac{2\sqrt{2}}{3}\sum_{m_{123}}\Bigg[\left\langle\left(0\frac{1}{2}\right)_{\frac{1}{2}},\left(\frac{1}{2}\frac{1}{2}\right)_{0};TT_z\middle|\left(0\frac{1}{2}\right)_{\frac{1}{2}},\left(\frac{1}{2}\frac{1}{2}\right)_{1};TT_z\right\rangle \\
& \left\langle\left(0\frac{1}{2}\right)_{\frac{1}{2}},\left(\frac{1}{2}\frac{1}{2}\right)_{1};SS_z\middle|\left(0\frac{1}{2}\right)_{\frac{1}{2}},\left(\frac{1}{2}\frac{1}{2}\right)_{0};SS_z\right\rangle \\
& \left\langle\frac{1}{2}T_z'',00\middle|TT_z\right\rangle\left\langle\frac{1}{2}m_{123},1(S_z-m_{123})\middle|SS_z\right\rangle \\
& +\left\langle\left(1\frac{1}{2}\right)_{\frac{1}{2}},\left(\frac{1}{2}\frac{1}{2}\right)_{0};TT_z\middle|\left(1\frac{1}{2}\right)_{\frac{1}{2}},\left(\frac{1}{2}\frac{1}{2}\right)_{1};TT_z\right\rangle \\
& \left\langle\left(1\frac{1}{2}\right)_{\frac{1}{2}},\left(\frac{1}{2}\frac{1}{2}\right)_{1};SS_z\middle|\left(1\frac{1}{2}\right)_{\frac{1}{2}},\left(\frac{1}{2}\frac{1}{2}\right)_{0};SS_z\right\rangle \\
& \left\langle\frac{1}{2}T_z'',00\middle|TT_z\right\rangle\left\langle\frac{1}{2}m_{123},1(S_z-m_{123})\middle|SS_z\right\rangle\Bigg] \\
& C\left(\frac{1}{2}\frac{1}{2}1;S_z''m_{123}(S_z''-m_{123})\right) \\
& C\left(111;(S_z''-m_{123})(-m_l)(S_z''-m_{123}+m_l)\right), \qquad (4.12)
\end{aligned}
$$

$$
\begin{aligned}
Q_2' \;=\; & -\frac{2\sqrt{2}}{3}\sum_{m_{123}}\left\langle\left(1\frac{1}{2}\right)_{\frac{1}{2}},\left(\frac{1}{2}\frac{1}{2}\right)_{0};TT_z\middle|\left(1\frac{1}{2}\right)_{\frac{1}{2}},\left(\frac{1}{2}\frac{1}{2}\right)_{1};TT_z\right\rangle \\
& \left\langle\left(1\frac{1}{2}\right)_{\frac{3}{2}},\left(\frac{1}{2}\frac{1}{2}\right)_{1};SS_z\middle|\left(1\frac{1}{2}\right)_{\frac{1}{2}},\left(\frac{1}{2}\frac{1}{2}\right)_{0};SS_z\right\rangle \\
& \left\langle\frac{1}{2}T_z'',00\middle|TT_z\right\rangle\left\langle\frac{3}{2}m_{123},1(S_z-m_{123})\middle|SS_z\right\rangle \\
& C\left(\frac{1}{2}\frac{3}{2}1;S_z''m_{123}(S_z''-m_{123})\right) \\
& C\left(111;(S_z''-m_{123})(-m_l)(S_z''-m_{123}+m_l)\right), \qquad (4.13)
\end{aligned}
$$

$$
P_{1m_l} \;=\; (-1)^{m_l}\beta p\exp(-\alpha p^2) \qquad (4.14)
$$

where α and β are constants consist of the size parameters a and b, the normalization factors N_{N^*}, N_N and N_π, and the mass ratio $R = m_q/m_g$.

CHAPTER 5

N*(1440) DECAY WIDTHS

We are now ready to evaluate the partial decay widths of the various decay processes of $N^*(1440)$. With the the transition amplitude of the reaction is expressed in the partial wave expansion

$$T = \sum_{lm_l} T_{lm_l} Y^*_{lm_l}, \tag{5.1}$$

the partial wave amplitudes T_{lm_l} are derived in the previous section, for example, for the reaction $N^*(1440) \to N\pi$. The decay width of the process $N^*(1440) \to N\pi$ can be evaluated in the formula

$$\Gamma_{N^*(1440)\to N\pi} = 2\pi \frac{E_N E_\pi p}{M_{N^*}} \int d\Omega \, |T_{N^*(1440)\to N\pi}|^2, \tag{5.2}$$

where $T_{N^*(1440)\to N\pi}$ is the total transition amplitude and p the magnitude of the final momentum of π or N. Integrating over the solid angle Ω of the final particle N or π and averaging over the initial states, one derives the unpolarized decay width in the partial wave transition amplitudes

$$\begin{aligned}
\Gamma_{N^*(1440)\to N\pi} &= 2\pi \frac{E_N E_\pi p}{M_{N^*}} \frac{1}{2} \sum_{S_z} \sum_{lm_l} | T_{lm_l} |^2 \\
&= 2\pi \frac{E_N E_\pi p}{M_{N^*}} \frac{1}{2} \sum_{S_z} \sum_{m_l} | T_{1m_l} |^2 \\
&= 2\pi \frac{E_N E_\pi}{M_{N^*}} g^2 \beta^2 p^3 \exp\left(-2\alpha p^2\right) \frac{1}{2} \sum_{S_z} \sum_{m_l} \left| (-1)^{m_l} \left(\frac{A}{2} Q'_1 + \frac{B}{\sqrt{2}} Q'_2 \right) \right|^2,
\end{aligned} \tag{5.3}$$

where the factor $\frac{1}{2}$ comes from the average over the initial states. It is found that the Roper resonance decays into $N\pi$ through only the $l = 1$ channel in the hybrid picture.

5.1 Results

Shown in Table 5.1 are the model predictions for the ratios of the decay widths of the reactions $N^*(1440) \to N\rho$, $N\eta$, $N\sigma$, $\Delta\pi$ to the one of the reaction $N^*(1440) \to N\pi$. The experimental data ((Amsler, 2008)) are listed in the last row of the table. In our calculation we have employed $a = 3.1$ GeV^{-1}, $b = 4.1$ GeV^{-1} and $R \equiv m_q/m_g = 1$ and considered various combinations of the mixing parameters A and B. Numerical calculations show that the theoretical results are independent of the effective coupling constant g and insensitive to the parameter $R = m_q/m_g$ and the size parameters a and b. The theoretical predictions with $A = -B = 1/\sqrt{2}$ are consistent with experimental data.

Table 5.1 $N^*(1440)$ decay width ratios with various A and B combinations.

	$A = 0$, $B = 1$	$A = 1$, $B = 0$	$A = \frac{1}{\sqrt{2}}$, $B = \frac{1}{\sqrt{2}}$	$A = \frac{1}{\sqrt{2}}$, $B = -\frac{1}{\sqrt{2}}$	Data
$\Gamma_{N\rho}/\Gamma_{N\pi}$	0.027	0.018	0.016	0.021	< 0.15
$\Gamma_{N\eta}/\Gamma_{N\pi}$	0.784	0.196	0.577	0.124	–
$\Gamma_{\Delta\pi}/\Gamma_{N\pi}$	1.812	0.065	0.179	0.392	$0.27 - 0.55$
$\Gamma_{N\sigma}/\Gamma_{N\pi}$	0.508	0.115	0.396	0.057	$0.05 - 0.10$

5.2 Conclusions

In this work we have evaluated the transition amplitudes of the decay processes of the Roper resonance $N^*(1440)$ to $N\pi$, $N\rho$, $N\eta$, $N\sigma$ and $\Delta\pi$ in a nonrelativistic quark-gluon model. The Roper resonance is treated as a hybrid, that is, composed of three valence quarks and a gluon of the TE (transverse electric) mode (a TE mode is the lowest eigenmode). The wave function of the Roper resonance has been constructed to properly establish the gluonic degree of freedom, which has been a fascinating challenge in nowadays non-perturbative QCD physics.

The theoretical results, for the ratios of the decay widths of the reactions $N^*(1440) \to N\rho$, $N\eta$, $N\sigma$, $\Delta\pi$ to the one of the reaction $N^*(1440) \to N\pi$, have been derived with only one free parameter, A/B which tells how the Roper resonance is made up by the two components $|^2N_g\rangle$ and $|^4N_g\rangle$. The experimental data prefer a Roper resonance of the form $|^2N_g'\rangle - |^4N_g'\rangle$.

The hybrid picture of the Roper resonance in the work is in line with experimental data. Indeed, it has been indicated in the study of photoproduction of baryons ((Li, 1991)) that not only the Roper resonance but also other lower-lying baryons like N and Δ may have a component of gluon. To confirm or rule out the argument, a systematical study for the strong process of those baryons are essential. The presence of the gluonic degrees of freedom may solve the long-standing puzzle of the Roper resonance, and hopefully provide an explanation of the observation that the spin content of nucleon is not carried dominantly by valence quarks. It might be argued that nucleon may also include glunic degrees of freedom since q^3 and q^3G states could be strongly mixed in physical baryon resonances because of the quark-gluon coupling. The gluonic components of nucleon do not change the isospin and flavor structure, and therefore the ratio of the magnetic moments will be the same as in the conventional q^3 picture, namely, $\mu_p/\mu_n = -3/2$.

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APPENDICES

APPENDIX A

CONFERENCE ABSTRACT

The following abstract below was presented as oral presentation at the International Workshop on the Physics of Excited Nucleon (NSTAR 2009) which hosted by the Institute of High Energy Physics, Chinese Academy of Sciences (CAS) in Beijing, China on April 19-22, 2009.

Decay of Roper resonance in hybrid baryon model

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In this work we study the structure of the Roper resonance via its decay processes. We go along with the argument that the Roper resonance is a state of three quarks and one transverse-electric (TE) gluon. A nonrelativistic quark-gluon model is employed, where the dynamics of antiquark-quark-gluon is described in the effective 3S1 vertex in which a quark-antiquark pair is created (destroyed) from (into) a gluon. The wave function of the Roper resonance is properly constructed to take account into the gluon freedom in the nonrelativistic regime. The branching decay widths of Roper resonance to the N-pi, N-rho, N-eta, N-pi-pi and Delta-pi channels will be shown.

Keywords: Roper resonance, Hybrid baryon

APPENDIX B

PUBLICATION PAPER

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NUCLEAR PHYSICS A
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N*(1440) decays in a hybrid baryon model*

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In this work we study the nature of the Roper resonance via its decay processes. We go along with the argument that the Roper resonance is a state of three quarks and one transverse-electric (TE) gluon. A nonrelativistic quark-gluon model is employed, where the dynamics of $\bar{q}qG$ is described in the effective $^3S_1$ vertex in which a quark-antiquark pair is created (destroyed) from (into) a gluon. The wave function of the Roper resonance is properly constructed to take account into the gluon freedom in the nonrelativistic regime. The evaluated decay width ratios $\Gamma_{N^*(1440)\to N\rho}/\Gamma_{N^*(1440)\to N\pi}$ and $\Gamma_{N^*(1440)\to N\eta}/\Gamma_{N^*(1440)\to N\pi}$ are in good agreement with experimental data.

1. INTRODUCTION

The study of baryon excitation states plays an important role in understanding of the nucleon internal structures, the quark model and hence the nature of the strong interaction. Information is usually extracted from the properties of nucleon excitation state N^*'s such as their mass spectrum, various production and decay rate [1].

The understanding of the Roper resonance has been a long-standing problem in N^* physics. Its very small branching ratios of electromagnetic decay modes, unusual couplings to the $N\pi$ and $N\sigma$ channels and its low mass together make difficult to identify the resonance as a simple three-quark bound state. The Roper resonance has been considered a good candidate for a collective excitation and interpreted as a breathing mode of the nucleon in bag models [2]. A recent coupled-channel calculation [3] involving the $N\pi$, $\Delta\pi$ and $N\sigma$ channels, suggest that the $N^*(1440)$ could be explained as a dynamical effect, without an associated genuine three-quark state. It has therefore been suggested to be a gluonic excitation state of the nucleon, i.e., a "hybrid baryon".

The aim of this work is to investigate if the Roper resonance could be reasonably interpreted as a bound state of three-quark and one-gluon, hybrid baryon, through studying its decay modes such as to $N\pi$, $N\rho$ and $N\eta$. We will first construct the wave functions of the mesons, nucleon and the Roper resonance to properly include the gluon freedom in the nonrelativistic regime. Then, we introduce the $^3S_1$ interaction vertex for the description

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of the decay process and evaluate the transition amplitudes. The decay width ratios and conclusions are shown in the last section.

2. WAVE FUNCTIONS

The wave functions of mesons, nucleons and the Roper resonance are worked out in a quark-gluon model. The spatial wave function of hadrons are very much model dependent since the interaction among quarks is still an open question. The most simplest but well accepted form of the interaction is the harmonic oscillator potential. In this work we will employ the harmonic oscillator approximation for the quark interaction in setting up the quark cluster wave functions of the mesons and nucleons.

The nucleon wave function in momentum space is given by

$$\begin{aligned} |\Psi\rangle_N &= N_N\, e^{-\frac{1}{4}a^2(\vec{p}_1-\vec{p}_2)^2}\, e^{-\frac{1}{12}a^2(\vec{p}_1+\vec{p}_2-2\vec{p}_3)^2}\, \frac{1}{\sqrt{6}} \sum_{i,j,k} \epsilon_{ijk}\, |q_1\rangle_i\, |q_2\rangle_j\, |q_3\rangle_k \\ & \sum_{J_{12}=0,1} \left| \left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right)_{J_{12}} \otimes \frac{1}{2}^{(3)} \right\rangle^{\text{spin}}_{\frac{1}{2},S_Z} \left| \left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right)_{J_{12}} \otimes \frac{1}{2}^{(3)} \right\rangle^{\text{flavor}}_{\frac{1}{2},T_Z} \end{aligned} \qquad (1)$$

and s-wave meson wave functions in momentum space take the form

$$|\Psi\rangle^{\text{S-wave}}_{\text{meson}} = N_S\, e^{-\frac{1}{8}b^2(\vec{p}_1-\vec{p}_2)^2} \frac{1}{\sqrt{3}} \sum_{l=1}^{3} |\bar{q}_l\rangle\, |q_l\rangle \left| \left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right) \right\rangle_{S_i} \left| \left(\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right) \right\rangle_{T_i} \qquad (2)$$

where N_N and N_S are the normalization factors for the nucleon and meson, respectively. The size parameters $a = 3.1\,\text{GeV}^{-1}$ and $b = 4.1\,\text{GeV}^{-1}$, determined by fitting to the nucleon and meson sizes and nucleon-nucleon, nucleon-antinucleon and pion-nucleon reactions [4,5].

The wave function of the Roper resonance is properly constructed to include the gluon freedom in the nonrelativistic regime. The confined gluon might be both the TE (transverse electric) and TM (transverse magnetic) modes, with the TE mode the lowest eigenmode. Considering its low mass, we presume that the Roper resonance is composed of three valence quarks and a TE gluon, denoted by q^3G.

Let ϕ, χ and ψ denote flavor, spin and color wave functions for the three quarks and let superscripts S, ρ, λ and A denote the permutation symmetry. (S/A is totally symmetric/antisymmetric under any exchange among the three quarks, and λ/ρ is symmetric/antisymmetric under the exchange of the first two quarks). The quantum number of the q^3G states are dictated mainly by the requirements that the three-quark state transform as a color octet. The totally antisymmetric q^3G states are explicitly [6,7]

$$\left|^2N_g\right\rangle = \frac{1}{2}\left[(\phi^\rho\chi^\rho - \phi^\lambda\chi^\lambda)\,\psi^\rho - (\phi^\rho\chi^\lambda + \phi^\lambda\chi^\rho)\,\psi^\lambda\right] \otimes |G\rangle \qquad (3)$$

$$\left|^4N_g\right\rangle = \frac{1}{\sqrt{2}}\left[(\phi^\lambda\chi^\rho - \phi^\rho\chi^\lambda)\,\psi^S\right] \otimes |G\rangle \qquad (4)$$

where superscript 2 and 4 denote the total quark spins as $2S+1$. In the spin-flavor-color wave functions $|^2N_g\rangle$ and $|^4N_g\rangle$ above, the color components of the three-quark core take the form

$$\psi_\alpha^\rho = \frac{1}{2}\sum_{i,j,k,l} |q_3\rangle_i \lambda_{ij}^\alpha \cdot \epsilon_{jkl} |q_1\rangle_k |q_2\rangle_l , \quad (5)$$

$$\psi_\alpha^\lambda = \frac{1}{2}\sum_{i,j,k,l} \left(|q_1\rangle_i |q_2\rangle_j + |q_1\rangle_j |q_2\rangle_i\right) \lambda_{il}^\alpha \cdot \epsilon_{jkl} |q_3\rangle_k , \quad (6)$$

where λ^a are the Gell-Mann matrices. The total wave function of the Roper resonance is the linear combination of the $|^2N_g\rangle$ and $|^4N_g\rangle$ ones, taking the form

$$\Psi^{N^*(1440)} = \Psi_{N^*(1440)}^{\text{spatial}} \left[A\left|^2N_g\right\rangle + B\left|^4N_g\right\rangle\right] \quad (7)$$

with $A^2 + B^2 = 1$.

In the approximation of the harmonic oscillator interaction among quarks and gluon, the spatial part $\Psi_{N^*(1440)}^{\text{spatial}}$ may take the form

$$\Psi_{N^*(1440)}^{\text{spatial}} = N_{N^*}\, e^{-\frac{1}{4}a^2(\vec{p}_1-\vec{p}_2)^2}\, e^{-\frac{1}{12}a^2(\vec{p}_1+\vec{p}_2-2\vec{p}_3)^2}\, e^{-\frac{2}{3}\frac{a^2}{(3R+1)^2}(\vec{p}_1+\vec{p}_2+\vec{p}_3-3R\vec{p}_4)^2} , \quad (8)$$

where N_{N^*} is the normalization factor and R is the ratio of the quark mass to the gluon one, $R = m_q/m_g$. $\vec{p}_i$ (with $i = 1,2,3$) is the momentum of the ith quark in the Roper resonance while $\vec{p}_4$ is the momentum of the gluon.

3. THE INTERACTION OPERATOR

We start with the quark-antiquark-gluon vertex, which can be deduced from the interaction Lagrangian density

$$\mathcal{L}_{\text{int}} = \bar{\psi} g \gamma^\mu A_\mu^\alpha \frac{\lambda^\alpha}{2} \psi \quad (9)$$

where g is the strong coupling constant, $\lambda^\alpha/2$ are the SU(3) generators, A_μ^α are the gluon fields and ψ is the quark field. In the nonrelativistic quark model, the quark-antiquark-gluon transition operator corresponds to the nonrelativistic quark-antiquark-gluon interaction of lowest order QCD, where the created $q\bar{q}$ carries the quantum number ${}^{2S+1}L_J = {}^3S_1$. It can be easily shown that the interaction is given by $W_{ij} = \bar{\chi}_i^\dagger V_{ij} \chi_j$ with

$$V_{ij}({}^3S_1) = g\sum_\mu (-1)^\mu \sigma_{ij}^\mu \mathbf{1}_{ij}^F \varepsilon_{-\mu} A^\alpha \frac{\lambda^\alpha}{2} \delta(\vec{p}_i + \vec{p}_j - \vec{k}) \quad (10)$$

where $\varepsilon_\mu(\vec{k})$ is the polarization vector of the gluon. $\vec{p}_i$ and $\vec{p}_j$ are the momenta of quark and antiquark created from the gluon of momentum $\vec{k}$. The $\vec{\sigma}_{ij}$ in the vertex can be understood as an operator projecting a quark-antiquark pair onto a spin-1 state. Then the transition amplitude for the process $N^*1440 \to N\pi$ takes the form

$$T = \langle N\pi | V_{ij}^\dagger({}^3S_1) | N^*(1440)\rangle \quad (11)$$

With the wave functions of meson, nucleon and the Roper resonance and the interaction operator above, one is ready to evaluate the decay width of the reaction $N^*(1440) \to N\pi$.

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4. RESULTS AND CONCLUSIONS

There are in the model five free parameters, the effective coupling constant g, the size parameters a and b, the mass ratio $R = m_q/m_g$, and the parameter (A or B) describing the mixture of the states $|^2N_g\rangle$ and $|^4N_g\rangle$. The length parameters a and b may be determined by fitting to the nucleon and meson sizes and nucleon-nucleon, nucleon-antinucleon and pion-nucleon reactions. We take in the work $a = 3.1\,\mathrm{GeV}^{-1}$ and $b = 4.1\,\mathrm{GeV}^{-1}$ as determined in [4,5].

Table 1
The decay width ratio for $N^*(1440)$ decay in some channel for different A and B.

	$A=0$, $B=1$	$A=1$, $B=0$	$A=\frac{1}{\sqrt{2}}$, $B=\frac{1}{\sqrt{2}}$	$A=\frac{1}{\sqrt{2}}$, $B=-\frac{1}{\sqrt{2}}$	Data
$\Gamma_{N^*(1440)\to N\rho}/\Gamma_{N^*(1440)\to N\pi}$	0.010	0.003	0.008	0.002	$\frac{0.00\pm0.01}{0.65\pm0.10}$
$\Gamma_{N^*(1440)\to N\eta}/\Gamma_{N^*(1440)\to N\pi}$	0.165	0.041	0.122	0.026	$\frac{0.00\pm0.01}{0.65\pm0.10}$

The experimental data are given in [8].

The decay width ratios $\Gamma_{N^*(1440)\to N\rho}/\Gamma_{N^*(1440)\to N\pi}$ and $\Gamma_{N^*(1440)\to N\eta}/\Gamma_{N^*(1440)\to N\pi}$ are evaluated in the work. The effective coupling constant g has no effect on the ratios, but there are still two free parameters, that is, the mass ratio $R = m_q/m_g$ and the wave mixture parameter A or B. It is found that the results is independent of the mass ratio R. The dependence of the model results on the wave mixture parameter is shown in Table 1. Comparing our theoretical predictions for the decay width ratios with the experimental data in Table 1, one finds that the results are quite reasonable when $A = -B = 1/\sqrt{2}$ which are the same as the ones used in work [7]. One may conclude at this stage that the hybrid baryon model for the Roper resonance is promising.

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NRCT Fund (NRCT: National Reaearch Council of Thailand) (Oct. 2001 - Sep. 2003) Project: Heavy Ion Reactions at Ultra-Relativistic Energies

RGJ Grant (RGJ: Royal Golden-Jubilee Ph.D. Project of Thailand, for more information see http://rgj.trf.or.th/eng.htm) (Oct. 2001 - Sep. 2006) Project: Low-Energy Pion-Proton Processes in Chiral Quark Models

RGJ Grant (October 2001 - September 2006) Project: Two-Pion and Two-Kaon Bound States in Chiral Quark Models

RGJ Grant (October 1998 - September 2003) Project: Baryon Weak and Electromagnetic Decays in Chiral Quark Models

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International Workshop on the Physics of Excited Nucleon: NSTAR 2009 (April 19-22, 2009)

Talk: Electron-positron annihilation to nucleon-antinucleon pairs at low energies

The Fourth Asia-Pacific Conference on Few Body Problems in Physics (August 19-23, 2008)

Talk: Accurate evaluation of wave functions of pionium, kaonium and kaonic atom

The Third Asia-Pacific Conference on Few Body Problems in Physics (July 26-30, 2005)

Talk: Accurate Evaluation of Antiproton-Deuteron Atoms

CCAST World Laboratory Workshop (April 2-6, 2001)

China Center of the Advanced Science and Technology (CCAST), Beijing, P. R. China

Lectures: Proton-antiproton annihilation into two and three mesons (10 hours)

Nucleon-antinucleon atomic states (4 hours)

Quantum object is merely particle (4 hours)

(CCAST directed by T.D. Lee invites every year one outstanding young Chinese scholar working in each field abroad to give a series of lectures in Beijing)

Germany-East Asia Symposium of Nuclear and Particle Physics (May, 1998)

Talk: Proton-antiproton annihilation to two pions and two kaons

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