PIYANUCH SIRIWAT : GROUP CLASSIFICATION OF THE THREE-DIMENSIONAL EQUATIONS OF FLUIDS WITH INTERNALINERTIA THESIS ADVISOR : PROF. SERGEY MELESHKO, Ph.D. 85 PP.

ACKNOWLEDGEMENT

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scribing the behavior of a dispersive continuum are obtained as an Euler-Lagrange equation for the Lagrangian of the form

$$L = L(\rho, \rho_t, \quad \rho, u)$$

where t is time, L

$$\dot{u} + \psi \ \rho_x = 0.$$

Hence, in the case $W_{\dot{\rho}\dot{\rho}} = 0$ equations (1.1), (1.2), (1.3) are similar to the gas dynamics equations. This case was completely studied in (Chirkunov, 1989). the one-dimensional case of equations (1.1), (1.2), (1.3) was studied in (Hematulin,

CHAPTER II

FLUIDS WITH INTERNAL INERTIA

Notice that if W is a linear function with respect to $\dot{\rho}$, then these equations

The gas pressure

CHAPTER III

GROUP ANALYSIS METHOD

In this chapter, the group analysis method is discussed. An introduction to this method can be found in various textbooks (cf. Ovsiannikov 1978), (Olver,

where

Note that after substituting (3.

where

$$\zeta^{u_x}(x, u, u_x) = \frac{\partial h(x, u, u_x; a)}{\partial a}\bigg|_{a=0}$$

For constructing prolongations of an infinitesimal generator in case $n, m \geq 2$ one proceeds similarly.

Let $x = \{x_i\}$

where

ζ^u

Here n, m is the numbers of independent and dependent variables, respectively and r is the total rank of the matrix composed by the coe□cients of the generators X_i, $(i = 1, 2, ..., r)$.

Definition 5. A set M is said to be invariant with respect to the group G

The generator X

3.3 Classification of subalgebras

One of the main aims of group analysis is to construct exact solutions of differential equations. The set of all solutions can be divided into equivalence classes of solutions:

Definition 10.

Let *L*

where

The notion of partially invariant solutions generalizes the notion of an invariant solution, and extends the scope of applications of group analysis for constructing exact solutions of partial di□erential equations. The algorithm of finding invariant and partially invariant solutions consists of the following steps.

Let L^r be a Lie algebra with the basis $X_1, ..., X_r$. The universal invariant J consists of $s = m + n - r$ functionally independent invariants

$$J = \square\, J^1(x, u), J^2(x, u), ..., J\psi f\psi\psi\psi F$$

The number l satisfies the inequality $1 \leq l \leq q \leq m$. The representation of the $H($

CHAPTER IV

GROUP CLASSIFICATION OF THE THREE-DIMENSIONAL EQUATIONS

4.1 Introduction

where $u_4 = \rho$ and $x_4 = t$. An infinitesimal operator X^e of the equivalence Lie group is sought in the form (Meleshko, 2005),

$$X^e = \xi^i \partial_{x_i} + \zeta^{u_j} u \quad +$$

group

$$X_1^e = \partial_{x_1},\ X_2^e = \partial_{x_2},\ X_3^e = \partial_{x_3},$$

$$X_4^e = t\partial_{x_1} + \partial_{u_1},\ X_5^e = t\partial_{x_2} + \partial_{u_2},\ X' \ X t\partial \ e_{u_2}$$

x 210436111511558682552412243382161813225840311011

where *k*

with some function $B(\rho, \dot{\rho}) = 0$. Because $W_{\dot{\rho}\dot{\rho}} = 0$, one has that β

If $\phi_2 = C_2\rho^{-\mu} = 0$, the extension of the kernel is given by the generator

$$(1-\mu)X_1 + 2(X_{14} + \varphi X_7),$$

where $\varphi = (\mu + \lambda + p - 2)/p$. If $\phi_2 = 0$, the extension is given by the generators

$$pX_1 - 2X_7, \quad (p + \lambda - 1)X_1 + 2X_{14}.$$

If $k = -2$, then integrating (4.14), one obtains

$$W(\rho, \dot\rho) = -q_o\rho^\lambda \ln(\dot\rho) + \dot\rho\phi_1(\rho) + \phi_2(\rho), \quad (q_o = 0).$$

Substituting this into equations (4.2)-(4.4), we obtain

$$c_1 = c_{15}(\lambda - 1)/2,$$

and the condition

$$c_{15}(\rho\phi_2' - \phi_2(\lambda + 2)) + q_o\lambda(\lambda - 1)(c_{15} - c_7)\rho^{\lambda-2} = 0.$$

If $\lambda(\lambda - 1) = 0$ and ϕ_2 is arbitrary, then the extension is given only by the generator

$$X_7.$$

If $\lambda(\lambda - 1) = 0$ and $\phi_2 = C_2\rho^{\lambda+2}$, then the extension of the kernel consists of the generators

$$(\lambda - 1)X_1 + 2X_{14}, \quad X_7.$$

If $\lambda(\lambda - 1) = 0$ and $\phi_2 = C_2\rho^{\lambda+2} - \frac{q_o}{4}\lambda(\lambda - 1)\mu\rho^{\lambda-2}$, then the extension is

$$(\lambda - 1)X_1 + 2(X_{14} + (\mu + 1)X_7)$$

where $c_7 = (\mu + 1)c_{15}$.

If $k = -1$, then integrating (4.14), one obtains

$$W(\rho, \dot\rho) = -q_o\rho^\lambda\dot\rho \ln(\dot\rho) + \dot\rho\phi_1(\rho) + \phi_2(\rho),$$

If $\phi_2 = 0$, then $\phi_2 = C_2\rho^{-\mu}$, where $\mu = 2c_7/c_{15}$. The extension of the kernel consists of the generator

$$(1-\mu)X_1 + 2$$

The characteristic system of this equation is

The general solution of this equation is

In the second case, we assume that

$$(2\lambda + 3p - 4) - \mu(p - 2) = 0.$$

Equation (5.22) gives

$$k_4 = -(2p + 3((2\lambda + 3p - 4) - \mu(p - 2)))k_3.$$

The extension of the kernel becomes

$2(-\lambda+(p-1)\mu-2p+2)X_1+(2\lambda+(p+2)\mu$

Substituting (5.29) into equations (5.3)-(5.6), we have

$$k_4 = 5k_3$$

and the condition

$$\phi_2 \rho k_3 + \phi_2(k_1 + 2k_3).$$

If $\phi_2 = 0$, then the extension of the kernel is given by the generators

$$X_1, \quad X_3 + 5X_4.$$

If $\phi_2 = 0$, then $k_3 = 0$ ans $\phi_2 = C_2\rho^{-\mu}$, where $\mu = k$

ordinary differential equation. Here also all dependent variables can be defined through the function $h(r)$, but the equation for $h(r)$ is a fourth-order ordinary differential equation. In fact, since $H = 0$, from (5.30) one obtains that $U = 0$. Hence, $\alpha = \alpha_o$, where α_o is constant. From the first and third equations of (5.30), one finds

$$\rho = R_o$$

commutators:

	X_0	X_1	X_2	X_3
X_0	0	X_0	X_3	$2X_0$
X_1		0	X_2	0
X_2			0	$-2X_2$
X_3				0

Solving the Lie equations for the automorphisms, one obtains:

$$A_0 : \left\{ \bar{x}_0 = x_0 + a_0(x_1 + 2x_3) + a_0^2 x_2, \right.$$

5.2.5 One-dimensional subalgebras

One can decompose the Lie algebra L_4 as $L_4 = I \quad N$, where $I = L_3$ is an ideal and $N = \{X_1\}$ is a subalgebra of L_4. Classification of the subalgebra $N = \{X_1\}$ is simple: it consists of the subalgebras:

$$N_1 = \{0\}, \quad N_2 = \{X_1\}.$$

According to the algorithm (Ovsiannikov, 1993) for constructing an optimal system of one-dimensional subalgebras one has to consider two types of generators: (a) Xnr

Case (b)

Assuming that $x_0 = 0$, choosing $a_2 = -x_3/x_0$, one maps x_3 into zero. In this case $x_2(A_2) \rightarrow \tilde{x}_2 =$ (

To find invariants, one needs to solve the equation

$$XJ = 0,$$

CHAPTER VI

INVARIANT SOLUTIONS OF ONE OF MODELS

This chapter is focused on obtaix

If $\beta = 0$, then there is one more admitted generator,

$$Y_6 = t\partial_t - u\partial_u.$$

The six-dimensional Lie algebra with the generators $\{Y_1, Y_2, ..., Y_6\}$ is denoted by L_6.

The structural constants of the Lie algebra are defined by the table of commutators:

	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6
Y_1	0	0	Y_2	$-2Y_1$	$-Y_4$	Y_1
Y_2		0	0	$-Y_2$	Y_3	0
Y_3			0	Y_3	0	$-Y_3$
Y_4				0	$-2Y_5$	0
Y_5					0	$-Y_5$
Y_6						0

Solving the Lie equations (3.22) for the automorphisms, one obtains:

$$A_1 : \begin{cases} y_1' = y_1 + \tau_1(y_6 - 2y_4) + \tau_1^2 y_5, \\ y' \end{cases}$$

Case (b)

Assuming that $y_1 = 0$, choosing $\tau_5 = -y$

	Subalgebra		Subalgebra
1	$Y_5 \pm Y_1$		
2	$Y_1 + Y_3$	6	$\alpha Y_1 + Y_6 + Y_5$
3	Y_1	7	$Y_6 + \alpha Y_2$
4	Y_2	8	$2Y_6 + Y_4$
5	Y_3		

Here $\alpha = 0$ is an arbitrary constant.

Remark 1. Since the automorphism A_4 for $W = -a\rho^{-3}\dot{\rho}^2$ di□ers from the automorphism A_4 for the Green-Naghdi model, the subalgebras $Y_1 + \gamma Y_3$, $(\gamma = 0)$ considered in (Bagderina and Chupakhin, 2005) are equivalent here to $Y_1 + Y_3$.

Remark 2. Because of the automorphism A_4 the subalgebras $\{Y_5 + \beta Y_1\}$ are equivalent to one of the subalgebras: $\{Y_5 + Y_1\}$, $\{Y_5 - Y_1\}$ or $\{Y_5\}$. The subalgebra $\{Y_5 - Y_1\}$ is equivalent to $\{Y_4\}$. The subalgebra $\{Y_5\}$ is equivalent to $\{Y_1\}$. Notice also that the subalgebra $\{Y_6 + Y_5\}$ g $Y_6 + Y + Y$5401791791329751350[4172277(0221793t5128650[]1449

Let $\alpha = 1/4 + y^2$, $y = 0$. In this case, invariants of the Lie group are

$$U = s\left[\left((t+1/2)^2 + y^2\right)u - xt\right], \quad R = x\rho,$$

where

$$s = \left((t+1/2)^2 + y^2\right)^{-1/2} e^{\frac{1}{2y}\arctan\left(\frac{2t+1}{2y}\right)}.$$

The representation of an invariant solution is

$$s\left[\left((t+1/2)^2 + y^2\right)u - xt\right] = U(y), \quad \rho = x^{-1}R(y), \quad y = xs.$$

Substituting the representation of a solution into (4.1), one obtains two ordinary differential equations. The general solution of the first equation (conservation of mass) is

$$U = kyR^{-1}.$$

The second equation becomes a third-order ordinary e242third-ordernonathe1(f)]TJ0-25.28Tfunc1

One can easily see that these equations have the constant solution $f = 1$.

6.2.3 Invariant solutions of 2

In the case $\beta = 0$ this equation is reduced by the substitution $R = f(R)/y$

6.2.6 Invariant solutions of $Y_1 + Y_3$

Invariants of the generator

$$Y_1 + Y_3 = \partial_t + t\partial_x + \partial_u$$

are

Substitution into equations (6.2) gives

$$R = 0, \quad RU = 0.$$

6.2.8 Invariant solutions of Y_3

CHAPTER VII

CONCLUSIONS

original three-dimensional system of equations is reduced to a system with two independent variables. Group classification of the reduced system is obtained. All invariant solutions of the reduced system with the potential function $W q_0 \rho^{5} \rho^{2} +$

The last part of the thesis c

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