Electron-Positron Annihilation into Hadron-Antihadron Pairs

Y. Yan *, C. Kobdaj *, P. Suebka *, Y. M. Zheng *,
Amand Faessler †, Th. Gutsche †, V. E. Lyubovitskij †‡

* *School of Physics, Suranaree University of Technology, Nakhon Ratchasima 30000, Thailand*

† *Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

‡ *On leave of absence from Department of Physics, Tomsk State University, 634050 Tomsk, Russia*

Abstract

The reactions $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \to \overline{N}N$ with $N = p, n$ are studied in a non-perturbative quark model. The work suggests that the two-step process, in which the primary $\overline{q}q$ pair forms first a vector meson which in turn decays into a hadron pair, is dominant over the one-step process in which the primary $\overline{q}q$ pair is directly dressed by additional $\overline{q}q$ pairs to form a hadron pair. To reproduce the experimental data of the reaction $e^+e^- \to \overline{n}n$ and $\overline{p}p$ a D-wave ω-like vector meson with a mass of around 2 GeV has to be introduced.

I. INTRODUCTION

Experimental data on the reaction $e^+e^- \to \overline{n}n$ from the FENICE collaboration [1], earlier data on the reaction $e^+e^- \to \overline{p}p$ from the FENICE and DM2 collaborations [2] and also data collected at the LEAR antiproton ring at CERN on the time-reversed reaction $\overline{p}p \to e^+e^-$ [3] which are summarized in FIG.1 indicate a ratio $\sigma(e^+e^- \to \overline{n}n)/\sigma(e^+e^- \to \overline{p}p) > 1$ at energies around the $\overline{N}N$ threshold with $E_{c.m.} \sim 2$ GeV. Averaging over the available data on both the direct and time-reversed reactions, one finds [4]

$$\frac{\sigma(e^+e^- \to \overline{p}p)}{\sigma(e^+e^- \to \overline{n}n)} = 0.66^{+0.16}_{-0.11}. \tag{1}$$

In a naive perturbative description of e^+e^- annihilation into baryons the virtual time-like photon first decays into a $\overline{q}q$ pair, then the $\overline{q}q$ pair is dressed by two additional quark-antiquark pairs excited out of the vacuum to form a baryon pair. The dressing process does not distinguish between u and d quarks at high momentum transfers since in the description of perturbative QCD the gluon couplings are flavor blind. In the conventional perturbative picture the only difference between the proton and neutron production arises from the different electric charges of the primary $\overline{q}q$ pairs. One expects to get

$$\frac{\sigma(e^+e^- \to \overline{p}p)}{\sigma(e^+e^- \to \overline{n}n)} > 1 \tag{2}$$

at large momentum transfers where the u quark contribution dominates in the proton and the d quark in the neutron.

The reaction $e^+e^- \to \overline{N}N$ at energies around the $\overline{N}N$ threshold is highly nonperturbative, hence the problem must be tackled in a nonperturbative manner. In this work we model the reactions by the nonperturbative $^3P_0$ quark dynamics which describes quark-antiquark annihilation and creation. It was shown that the $^3P_0$ approach is phenomenologically successful in the description of hadronic couplings [5, 6, 7, 8, 9].

The reaction $e^+e^- \to \overline{N}N$ may arise from two different processes: (1) the primary $\overline{q}q$ pair is dressed directly by two additional quark-antiquark pairs created out of the vacuum to form a baryon pair; and (2) the primary $\overline{q}q$ pair forms a virtual vector meson first, then the virtual vector meson decays into a baryon pair. We expect that the second process is dominant over the first because of the considerable success of the vector dominance model. However, it is difficult to extract a solid conclusion by studying the reaction itself since

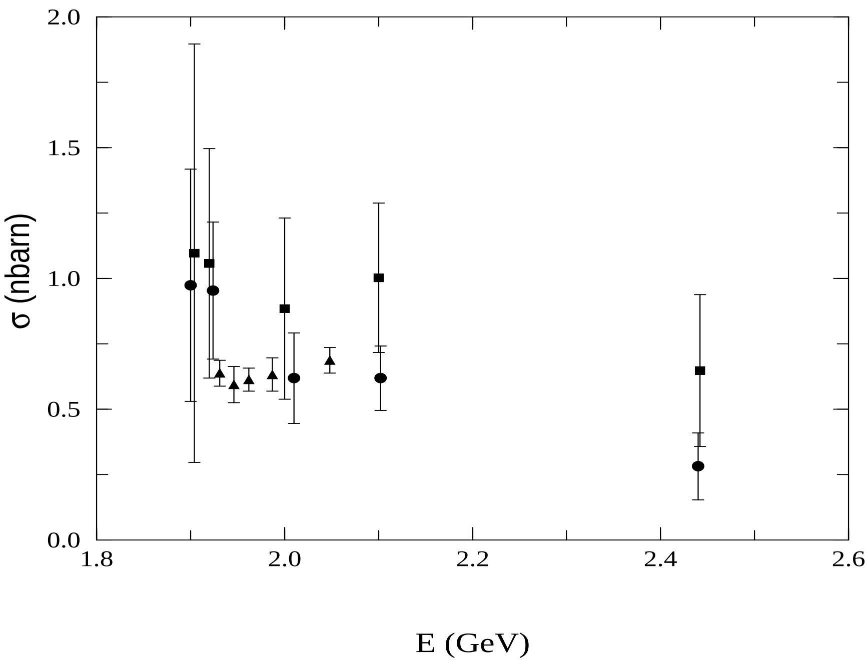

FIG. 1: Comparison of the cross sections for $e^+e^- \to \overline{p}p$ and $e^+e^- \to \overline{n}n$ at the $\overline{N}N$ threshold region. The solid circles [2] and triangles [3] are for the reaction $e^+e^- \to \overline{p}p$ while the squares [1] are for $e^+e^- \to \overline{n}n$.

there are only very limited experimental data available and the effective strength of the quark-antiquark vertex may vary largely from one process to another. We therefore study first a much simpler process, the reaction $e^+e^- \to \pi^+\pi^-$, where a large number of high quality data are available. The work is arranged as follows: In Sec. II we study the reaction $e^+e^- \to \pi^+\pi^-$ with the parameters determined in the reactions $\rho^0 \to e^+e^-$ and $\rho^0 \to \pi^+\pi^-$. The reaction $e^+e^- \to \overline{N}N$ is studied in Sec. III in the two-step process described above. We give our conclusions in Sec. IV. In Appendices A and B we discuss the calculations of the transition amplitudes $\rho^0 \to \pi^+\pi^-$ and $V \to \overline{N}N$ in the $^3P_0$ model.

II. REACTION $e^+e^- \to \pi^+\pi^-$

The reaction $e^+e^- \to \pi^+\pi^-$ may arise in the valence quark dominated picture from the following process: the e^+e^- pair annihilates into a virtual time-like photon, the virtual photon decays into a $\overline{q}q$ pair, and finally the $\overline{q}q$ pair is dressed by an additional quark-antiquark pair created out of the vacuum to form a meson pair, as shown in FIG.2a. The

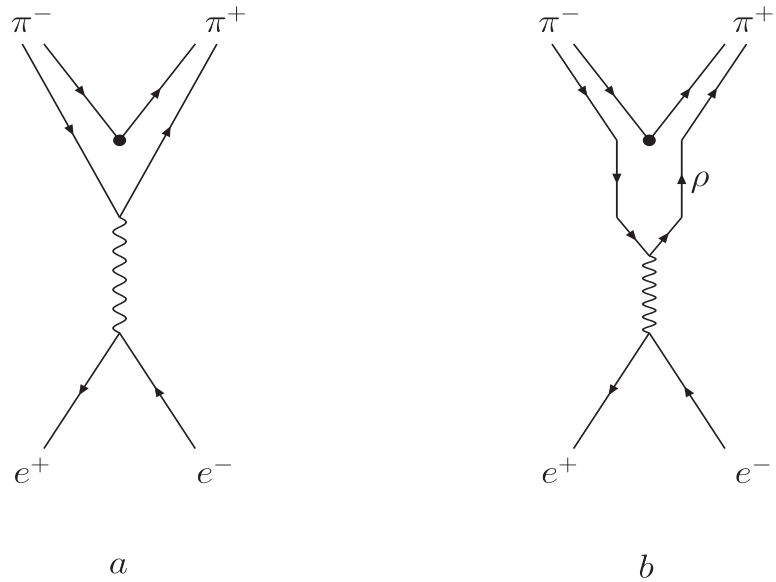

FIG. 2: The reaction $e^+e^- \to \pi^+\pi^-$ in the one-step process (a), and the two-step process (b).

transition amplitude is expressed formally as

$$T_1 = \langle \pi^+\pi^- | V_{\overline{q}q} | \overline{q}q \rangle \langle \overline{q}q | G | \overline{q}q \rangle \langle \overline{q}q | T | e^+e^- \rangle \tag{3}$$

where $\langle \overline{q}q | T | e^+e^- \rangle$ is simply the transition amplitude of e^+e^- to a primary quark pair, $\langle \overline{q}q | G | \overline{q}q \rangle$ is the Green function describing the propagation of the intermediate $\overline{q}q$ state and $\langle \pi^+\pi^- | V_{\overline{q}q} | \overline{q}q \rangle$ denotes the amplitude of the process of a $\overline{q}q$ pair to a $\pi^+\pi^-$ pair. $V_{\overline{q}q}$ is the effective vertex for creation and destruction of a quark-antiquark pair in quark models, which is identified in the context of the $^3P_0$ quark-antiquark dynamics. At an energy scale of about 1 GeV the intermediate quark-antiquark state can be assumed to be saturated by the $\rho^0(770)$ resonance, depicted in FIG.2b, as in the context of the vector dominance model. We refer to this process as the two-step reaction, whereas the former, more general one, is the one-step reaction. The corresponding transition amplitude then takes the form

$$T_2 = \langle \pi^+\pi^- | V_{\overline{q}q} | \rho \rangle \langle \rho | G | \rho \rangle \langle \rho | \overline{q}q \rangle \langle \overline{q}q | T | e^+e^- \rangle \tag{4}$$

where $\langle \rho | \overline{q}q \rangle$ is simply the wave function of the intermediate meson ρ, $\langle \rho | G | \rho \rangle$ the Green function describing the propagation of the intermediate meson, and $\langle \pi^+\pi^- | V_{\overline{q}q} | \rho \rangle$ the transition amplitude of ρ^0 annihilation into a $\pi^+\pi^-$ pair.

The size parameter of the ρ meson associated with its wave function may be determined by studying the reaction $\rho^0 \to e^+e^-$. The transition amplitude of a vector meson annihilation into an electron-positron pair takes the general form

$$T = \langle e^+e^-|T|q\overline{q}\rangle\langle q\overline{q}|V\rangle \tag{5}$$

where $|V\rangle$ is the vector meson state (see Appendix A), and $\langle e^+e^-|T|q\overline{q}\rangle$ the transition amplitude of a quark-antiquark pair to an electron-positron pair. The transition amplitude can be evaluated by a standard method as for example outlined in [10]. One has

$$\langle e^+e^-|T|q\overline{q}\rangle = -\frac{e_q\, e}{s}\,\overline{u}_e(p_{e^-}, m_{e^-})\gamma^\mu\, v_e(p_{e^+}, m_{e^+})\,\overline{v}_q(p_{\overline{q}}, m_{\overline{q}})\gamma_\mu\, u_q(p_q, m_q) \tag{6}$$

where $s = (p_q + p_{\overline{q}})^2$, e_q is the charge of quarks, and the Dirac spinors are normalized according to $\overline{u}u = \overline{v}v = 2m$. In the small quark momentum approximation, the decay width for the transition of a vector meson to an electron-positron pair can be easily evaluated. One has

$$\Gamma_{\rho^0\to e^+e^-} = \frac{16\pi\alpha^2 Q^2}{M_V^2}|\psi(0)|^2 \tag{7}$$

where Q^2 is the squared sum of the charges of the quarks in the meson, with $Q^2 = 1/2$ for ρ, 1/18 for ω and 1/9 for ϕ, and $\psi(0) = 1/(\pi b^2)^{3/4}$ is the coordinate space wave function of the vector meson at the origin. Using as an input $M_\rho = 0.7758$ GeV, $\alpha = 1/137$ and the experimental value of $\Gamma_{\rho^0\to e^+e^-} = 7.02 \pm 0.11$ KeV, we get $b = 3.847$ GeV^{-1} for the size parameter of the ρ meson with the spatial wave function set up in the harmonic oscillator approximation (see details in Appendix A). The size parameter b in Eq. (A2) might be slightly different from meson to meson.

We use the reaction $\rho^0 \to \pi^+\pi^-$ to determine the effective strength parameter λ in the quark-antiquark $^3P_0$ vertex

$$V_{ij} = \lambda\,\vec{\sigma}_{ij}\cdot(\vec{p}_i - \vec{p}_j)\,\hat{F}_{ij}\,\hat{C}_{ij}\,\delta(\vec{p}_i + \vec{p}_j) = \lambda\sum_\mu\sqrt{\frac{4\pi}{3}}(-1)^\mu\sigma^\mu_{ij}y_{1\mu}(\vec{p}_i - \vec{p}_j)\,\hat{F}_{ij}\,\hat{C}_{ij}\,\delta(\vec{p}_i + \vec{p}_j) \tag{8}$$

where $y_{1\mu}(\vec{q}) = |\,\vec{q}\,|\,Y_{1\mu}(\hat{q})$, $\vec{\sigma}_{ij} = (\vec{\sigma}_i + \vec{\sigma}_j)/2$, $\vec{p}_i$ and $\vec{p}_j$ are the momenta of quark and antiquark created out of the vacuum. $\hat{F}_{ij}$ and $\hat{C}_{ij}$ are the flavor and color operators projecting a quark-antiquark pair to the respective vacuum quantum numbers. The derivation and interpretation of the quark-antiquark $^3P_0$ dynamics may be found in literature [5, 6].

The decay width of the reaction $\rho^0 \to \pi^+\pi^-$ takes the form

$$\Gamma_{\rho^0\to\pi^+\pi^-} = \frac{\pi}{4}\,M_\rho^2\,\sqrt{1 - \frac{4M_\pi^2}{M_\rho^2}}\,|T_{\rho^0\to\pi^+\pi^-}|^2, \tag{9}$$

where $T_{\rho^0\to\pi^+\pi^-}$ is the corresponding transition amplitude defined in the center-of-mass system. Substituting $T_{\rho^0\to\pi^+\pi^-}$ as calculated in our approach (see Appendix A) we get

$$\Gamma_{\rho^0\to\pi^+\pi^-} = \lambda^2 \left(\frac{2}{3}\right)^7 \sqrt{\pi} M_\rho \,(bk)^3 \, e^{-\frac{1}{6} b^2 k^2} \qquad (10)$$

where M_ρ is the mass of ρ meson and $k = \sqrt{M_\rho^2/4 - M_\pi^2}$ the momentum of the outgoing pions. The result obtained here is consistent with the ones of Refs. [5, 11, 12], the different magnitude of the strength parameter λ just depends on the different normalization of the $^3P_0$ vertex. With the size parameter $b = 3.847$ GeV^{-1} determined from the reaction $\rho^0 \to e^+e^-$, the experimental value $\Gamma = 150$ MeV for the decay width of $\rho^0 \to \pi^+\pi^-$ requires the effective strength parameter λ to take the value $\lambda = 0.98$. In [11], the size parameter b is taken to be 2.5 GeV^{-1} and the effective strength is fitted to be 0.39, which according to our normalization corresponds to $\lambda = 0.96$.

Based on the evaluations for the reactions $\rho^0 \to \pi^+\pi^-$ and $\rho^0 \to e^+e^-$, it is straightforward to work out the transition amplitude of the two step diagram shown in FIG.2b in the reaction $e^+e^- \to \pi^+\pi^-$

$$T_{e^+e^-\to\pi^+\pi^-} = T_{\rho^0\to\pi^+\pi^-} \frac{1}{E - M_\rho} T_{e^+e^-\to\rho^0}. \qquad (11)$$

The transition amplitude for the process $\rho \to e^+e^-$ is

$$T_{\rho\to e^+e^-} = \langle e^+e^-|T|q\overline{q}\rangle\langle q\overline{q}|V\rangle = \int \frac{d\vec{p}_q \, d\vec{p}_{\overline{q}}}{(2\pi)^{3/2} 2E_q} \, \delta(\vec{p}_q + \vec{p}_{\overline{q}}) \, \psi_\rho(\vec{p}_q, \vec{p}_{\overline{q}}) \, T_{q\overline{q}\to e^+e^-}(\vec{p}_q, \vec{p}_{\overline{q}}) \qquad (12)$$

where ψ_ρ is the wave function of the ρ meson in momentum space, and $T_{q\overline{q}\to e^+e^-}$ is given in Eq. (6). The delta function $\delta(\vec{p}_q + \vec{p}_{\overline{q}})$ indicates that we work in the ρ meson rest frame.

Note that only the P-wave contributes to the process $e^+e^- \to \pi^+\pi^-$ since the spin of the intermediate ρ is 1. Furthermore, there is no free parameter since the size and the effective strength parameters have been determined by the processes $\rho^0 \to e^+e^-$ and $\rho^0 \to \pi^+\pi^-$, respectively.

In FIG.3 we give the prediction for the cross section of the reaction $e^+e^- \to \pi^+\pi^-$ in the model for the two step process. The result seems to indicate that the reaction $e^+e^- \to \pi^+\pi^-$ is completely dominated by the intermediate vector meson. One may therefore conclude that the one step process is completely saturated by the relevant resonances entering at this energy scale.

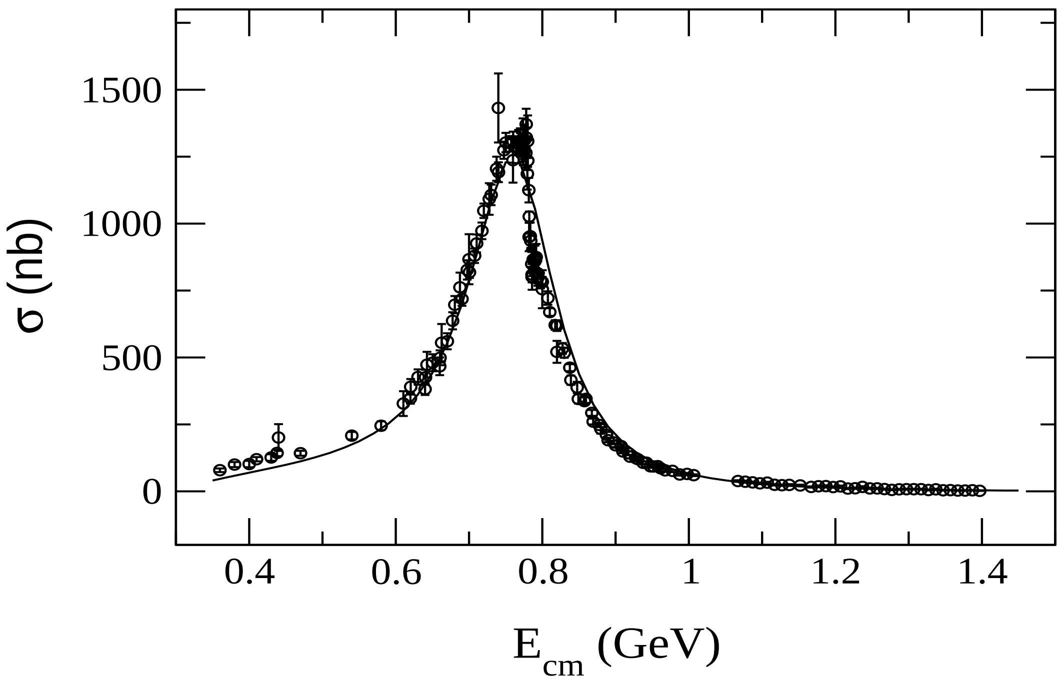

FIG. 3: Theoretical prediction (solid line) in the two-step model shown in FIG.2b for the cross section of the reaction $e^+e^- \to \pi^+\pi^-$ compared with experimental data taken from [13, 14]

III. REACTION $e^+e^- \to \overline{N}N$

In the following we assume that the reaction $e^+e^- \to \overline{N}N$ is described by a two-step process, just as for the reaction $e^+e^- \to \pi^+\pi^-$. This is again consistent with the vector dominance model.

Here we study the two-step process shown in FIG.4: the e^+e^- pair annihilates into a virtual time-like photon, the photon decays into a $\overline{q}q$ pair, the $\overline{q}q$ pair forms a virtual vector meson, finally the virtual vector meson is dressed by two additional quark-antiquark pairs created out of the vacuum to form a baryon pair. The meson $\rho(2150)$ with the quantum number $I^G(J^{PC}) = 1^+(1^{--})$ is a good candidate [15] for such an intermediate state. The transition amplitude in such a two step process takes the form

$$T_{e^+e^-\to\overline{N}N} = \langle\overline{N}N|V(^3P_0)|V\rangle\langle V|G|V\rangle\langle V|\overline{q}q\rangle\langle\overline{q}q|T|e^+e^-\rangle \tag{13}$$

Here $\langle V|\overline{q}q\rangle$ is simply the wave function of the intermediate vector meson with both isospin $I = 0$ and 1 (see Appendix B), $\langle V|G|V\rangle$ the Green's function describing the propagation of the intermediate vector meson, $\langle\overline{N}N|V(^3P_0)|V\rangle$ the transition amplitude of the intermediate

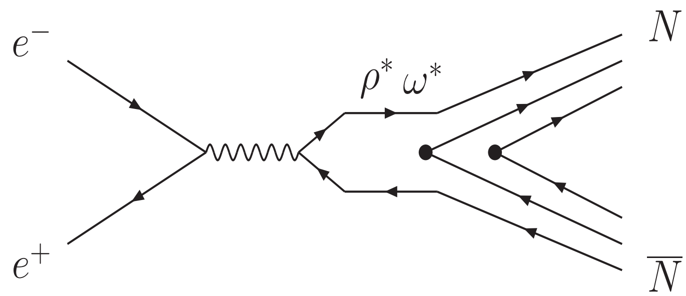

FIG. 4: Electron-positron annihilation into nucleon-antinucleon pairs in a two-step process via intermediate vector meson states.

meson annihilation into a nucleon-antinucleon pair, and $\langle \overline{q}q|T|e^+e^-\rangle$ the transition amplitude of an electron-positron pair to a quark-antiquark pair as given in Eq. (6). The transition amplitude $\langle V|\overline{q}q\rangle\langle\overline{q}q|T|e^+e^-\rangle$ for the process of the intermediate vector meson to an election-position pair is defined as in Eq. (5) but with different meson wave functions. The evaluation of the transition amplitude $\langle \overline{N}N|V(^3P_0)|V\rangle$ is worked out in Appendix B. The energy scale of the intermediate $q^2\bar{q}^2$ state in the $V \to N\bar{N}$ transition is simply set by associating in average an equal share of the total energy to each valence quark involved.

Considering that both isospin $I = 0$ and 1 vector mesons could be the intermediate states for the reaction $e^+e^- \to \overline{N}N$, we have the transition amplitudes

$$\begin{aligned} T_{e^+e^-\to\overline{p}p} &= \frac{1}{\sqrt{2}}[T(e^+e^- \to V(I=1) \to \overline{N}N) + T(e^+e^- \to V(I=0) \to \overline{N}N)] \\ T_{e^+e^-\to\overline{n}n} &= \frac{1}{\sqrt{2}}[T(e^+e^- \to V(I=1) \to \overline{N}N) - T(e^+e^- \to V(I=0) \to \overline{N}N)] \end{aligned} \tag{14}$$

for the reactions $e^+e^- \to \overline{p}p$ and $e^+e^- \to \overline{n}n$, respectively. It is clear that the cross sections of the reactions $e^+e^- \to \overline{p}p$ and $e^+e^- \to \overline{n}n$ would be the same if either a single isospin 0 or isospin 1 vector meson dominates the intermediate state at this energy scale. However, the experimental ratio of Eq. (1) indicates that at least two vector mesons with isospin 0 and 1 are involved as intermediate states. In addition to the confirmed vector meson $\rho(2150)$, there are clues [18, 19] for the existence of an ω-like meson lying in the energy region near the $\overline{N}N$ threshold. The vector meson with isospin 0 has mass and width of about 2150 MeV and 220 MeV, respectively. The contribution of the $\omega(2150)$ as well as the $\rho(2150)$ are included in our calculation. The mesons $\rho(2150)$ and $\omega(2150)$ are assumed to be superpositions of $3S$ and $2D$ states, considering that the lower lying states have been occupied (see Table I).

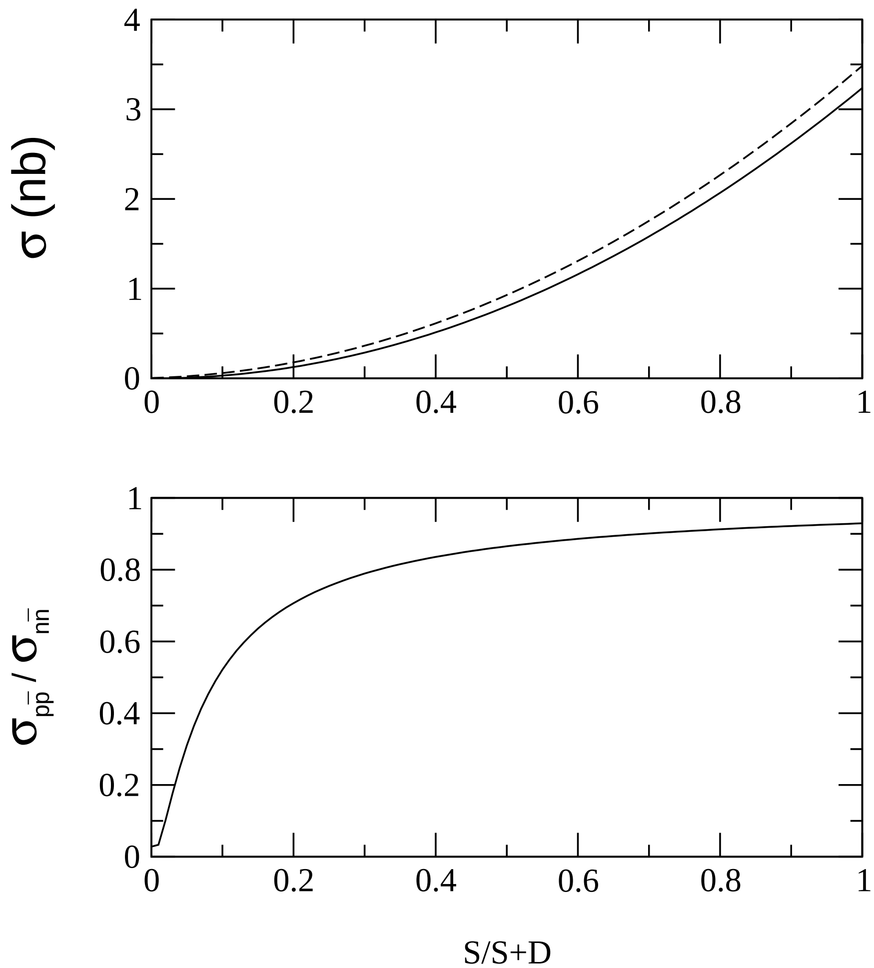

FIG. 5: Model predictions for the cross sections of the reactions $e^-e^+ \to \overline{n}n$ (dashed line in the upper figure) and $e^-e^+ \to \overline{p}p$ (solid line in the upper figure) and for the ratio $\sigma(e^+e^- \to \overline{p}p)/\sigma(e^+e^- \to \overline{n}n)$ (in the lower figure) versus the S-wave probability of the meson $\rho(2150)$.

TABLE I: ρ and ω mesons coming in pairs

1S	$\rho(770)$	$\omega(782)$
2S	$\rho(1450)$	$\omega(1420)$
1D	$\rho(1700)$	$\omega(1650)$
3S or 2D	$\rho(2150)$	$\omega(2150)$?

It is found in our study that the experimental data suggest a $\omega(2150)$ being in a D wave, and prefer the $\rho(2150)$ meson as a mixture of S and D waves. It may be interesting to mention that the work [12], which studied the decay of higher quarkonia in the $^3P_0$ quark model, reveals that the lower energy counterparts $\rho(1450)$ and $\rho(1700)$ of the meson $\rho(2150)$ are likely to be mixtures of $2S$ or $1D$ states.

Presented in FIG.5 are the predictions of the present model for the total cross sections of the reactions $e^+e^- \to \overline{p}p$ and $e^+e^- \to \overline{n}n$ at the $\overline{N}N$ threshold, with the $\omega(2150)$ in the $2D$ state and the $\rho(2150)$ varying from the $2D$ state to the $3S$ state. Note that, except the parameters describing the S and D wave admixture in the mesons $\rho(2150)$ and $\omega(2150)$, there are no more free parameters. For the size parameter we employ $b = 3.847$ GeV^{-1} as already fixed in the reaction $\rho^0 \to e^+e^-$, the $^3P_0$ strength with $\lambda = 0.98$ is fixed in the reaction $\rho^0 \to \pi^+\pi^-$. The size parameter of the nucleon with $a = 3.1$ GeV^{-1} is fixed from other considerations [6, 20], and the masses and widths of the mesons $\rho(2150)$ and $\omega(2150)$ are taken from [15, 18, 19]. The energy denominator of the intermediate $q^2\bar{q}^2$ state ΔE is roughly approximated as $\Delta E = E_{c.m.}/3$, assuming that the reaction energy $E_{c.m.}$ is shared by the six quarks equally [16, 17].

With $\omega(2150)$ in the $2D$ state and $\rho(2150)$ half in the $3S$ and half in the $2D$ state, we get

$$\sigma(e^+e^- \to \overline{p}p) \approx 0.65 \text{ nb}, \qquad \sigma(e^+e^- \to \overline{n}n) \approx 0.76 \text{ nb} \tag{15}$$

and hence

$$\frac{\sigma(e^+e^- \to \overline{p}p)}{\sigma(e^+e^- \to \overline{n}n)} \approx 0.85 \tag{16}$$

The model results for the cross sections of Eq. (15) are sensitive to the length parameters b and a and the effective parameter λ. However, the ratio $\sigma(e^+e^- \to \overline{p}p)/\sigma(e^+e^- \to \overline{n}n)$ is of course independent of the strength parameter and rather independent of the length parameters involved.

IV. CONCLUSIONS

The puzzling experimental result that $\sigma(e^+e^- \to \overline{p}p)/\sigma(e^+e^- \to \overline{n}n) < 1$ can be understood in the framework of a phenomenological nonrelativistic quark model. All parameters employed in the model, except the ones describing the mixture of the S and D waves for the intermediate vector mesons $\rho(2150)$ and $\omega(2150)$, are not free but determined by other reactions.

The experimental data suggest the existence of a D-wave ω meson with a mass of about 2100 MeV. The conclusion is quite general, independent of the special values of the size parameters a, b and the $^3P_0$ strength λ.

Acknowledgments

This work was supported in part by the National Research Council of Thailand (NRCT) under Grant No. 1.CH7/2545 and the National Natural Science Foundation of China under Grant Nos. 19975074 and 10275096, by the DFG under contracts FA67/25-3 and GRK683. This research is also part of the EU Integrated Infrastructure Initiative Hadronphysics project under contract number RII3-CT-2004-506078 and President grant of Russia "Scientific Schools" No. 1743.2003.

APPENDIX A: TRANSITION $\rho \to \pi^+\pi^-$ IN $^3P_0$ MODEL

We study the reaction $\rho \to \pi^+\pi^-$ shown in FIG.6 to determine the effective strength parameter λ in the quark-antiquark $^3P_0$ vertex of Eq. (8). The $\vec{\sigma}_{ij}$ in the vertex can be understood as a operater acting on a quark and antiquark state, or it projects a quark-antiquark pair onto a spin-1 state. It can be easily proven that

$$\langle 0,0|\sigma^{\mu}_{ij}|[\bar{\chi}_i \otimes \chi_j]_{JM}\rangle \;=\; (-1)^M \sqrt{2}\delta_{J,1}\delta_{M,-\mu}. \tag{A1}$$

Concerning SU(2) flavor a quark-antiquark pair which annihilates into the vacuum must have zero isospin. So the operator $\hat{F}_{ij}$ has the similar property $\langle 0,0|\hat{F}_{ij}|T,T_z\rangle = \sqrt{2}\delta_{T,0}\delta_{T_z,0}$. For the color part, one simplify has $\langle 0,0|\hat{C}_{ij}|q^i_\alpha \bar{q}^j_\beta\rangle = \delta_{\alpha\beta}$ where α and β are color indices. The transition amplitude for meson decay into two mesons in the $^3P_0$ model shown in FIG.6 is defined as $T = \langle\Psi_i|V^\dagger_{45}|\Psi_f\rangle$, where $|\Psi_i\rangle$ and $|\Psi_f\rangle$ are the initial and final states, respectively. For simplicity, we consider here only the S-wave mesons, that is, all the mesons involved have orbital angular momentum equal to 0. The initial state is simply the one meson wave function (WF) having the form

$$|\Psi_i\rangle = N\, e^{-\frac{1}{8}\,b^2(\vec{p}_1-\vec{p}_2)^2} \left[\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right]_{S_i,M_i} \left[\frac{1}{2}^{(1)} \otimes \frac{1}{2}^{(2)}\right]_{T,T_z} \tag{A2}$$

We have spin $S_i = 1$ and isospin $T_i = 1$ for the ρ meson, and the isospin projection $T_z = 0$ for ρ^0. Here we have employed the harmonic oscillator interaction between quark and antiquark. The final state $|\Psi_f\rangle$ is formed by coupling the WFs of the two final mesons. For two S-wave mesons we have

$$|\Psi_f\rangle \;=\; N_1 N_2\, e^{-\frac{1}{8}\,b^2(\vec{p}_3-\vec{p}_4)^2}\; e^{-\frac{1}{8}\,b^2(\vec{p}_5-\vec{p}_6)^2} \left[\left[\frac{1}{2}^{(3)} \otimes \frac{1}{2}^{(4)}\right]_{S_1} \otimes \left[\frac{1}{2}^{(5)} \otimes \frac{1}{2}^{(6)}\right]_{S_2}\right]_{S_f,M_f}$$

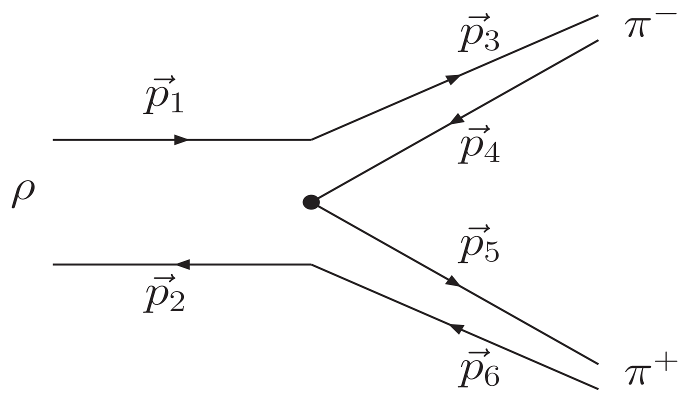

FIG. 6: $\rho \to \pi^+\pi^-$ in the $^3P_0$ nonrelativistic quark model.

$$\times \left[\left[\frac{1}{2}^{(3)} \otimes \frac{1}{2}^{(4)}\right]_{T_1} \otimes \left[\frac{1}{2}^{(5)} \otimes \frac{1}{2}^{(6)}\right]_{T_2}\right]_{T,T_z} \tag{A3}$$

The transition amplitude is derived as

$$T_{spatial} \;=\; \lambda \frac{2^4}{3^3\sqrt{3}\,\pi^{1/4}} b^{3/2} k\, e^{-\frac{1}{12}b^2k^2} \tag{A4}$$

Note that we have, for simplicity, set the ρ and π mesons to have the same size parameter b, that is $N = N_1 = N_2 = (b^2/\pi)^{3/4}$.

APPENDIX B: TRANSITION $V \to \overline{N}N$ IN $^3P_0$ MODEL

In the $^3P_0$ model the transition amplitude for a vector meson decaying into a $\overline{N}N$ pair might be written in the form

$$\begin{aligned} \langle \overline{N}N|V(^3P_0)|V\rangle \;&\equiv\; \langle \overline{N}N|V_{25}^{\dagger}\frac{1}{\Delta E}V_{36}^{\dagger}|V\rangle \\ &= \frac{4\pi}{3}\,\lambda^2\,\frac{1}{\Delta E}\sum_{S'_z} C(L\,J_z - S'_z, S'S'_z, 1J_z)\cdot T_{color}T_{sf}T_{spatial} \end{aligned} \tag{B1}$$

where the Clebsch-Gordon coefficient $C(L\,J_z - S'_z, S'S'_z, 1J_z)$ results from the spin-orbital coupling of the intermediate meson having the orbital angular momentum $L = 0, 2$ and spin S', and the factor $1/\Delta E$ accounts for the energy propagation between the two quark-antiquark vertices $V_{25}^{\dagger}$ and $V_{36}^{\dagger}$ which are defined as in Eq. (8). Here we have supposed that ΔE is constant for a given reaction energy as, for examplem, discussed in [16, 17]. Using the wave functions defined in the previous sections we get for the color part $T_{color} = \frac{1}{\sqrt{3}}$, for the spin-flavor part

$$
\begin{aligned}
T_{sf} &= \frac{1}{2}\left\langle J^{[ij]}\right|_{S'S'_z}(-1)^{\mu}\sigma^{25}_{-\mu}\hat{F}_{25}(-1)^{\nu}\sigma^{36}_{-\nu}\hat{F}_{36}\sum_{J_{23},J_{56}}\left|J^{[231;564]}\right\rangle^{\rm Spin}_{SS_z}\left|J^{[231;564]}\right\rangle^{\rm Flavor}_{TT_z} \qquad \text{(B2)}\\
J^{[ij]} &= \frac{1}{2}^{(7)}\otimes\frac{1}{2}^{(8)},\; J^{[ijk;lmn]} = \left[\left(\frac{1}{2}^{(i)}\otimes\frac{1}{2}^{(j)}\right)_{J_{ij}}\otimes\frac{1}{2}^{(k)}\right]_{1/2}\otimes\left[\left(\frac{1}{2}^{(l)}\otimes\frac{1}{2}^{(m)}\right)_{J_{lm}}\otimes\frac{1}{2}^{(n)}\right]_{1/2}
\end{aligned}
$$

and for the spatial part

$$
\begin{aligned}
T_{spatial} &= \int\prod d^3q_i\,\Psi^{\dagger}_{N\overline{N}}Y^*_{1\mu}(\vec{q}_{25})\delta^{(3)}(\vec{q}_{25})Y^*_{1\nu}(\vec{q}_{36})\delta^{(3)}(\vec{q}_{36}) \qquad \text{(B3)}\\
&\times\;\Psi_m(\vec{q}_{78})\delta^{(3)}(\vec{q}_{17})\delta^{(3)}(\vec{q}_{48})\delta^{(3)}(\vec{q}_{123}-\vec{k})\delta^{(3)}(\vec{q}_{456}+\vec{k})
\end{aligned}
$$

where $\vec{q}_{ij}=\vec{q}_i+\vec{q}_j$, $\vec{q}_{ijk}=\vec{q}_i+\vec{q}_j+\vec{q}_k$, $\Psi_{N\overline{N}}$ is the spatial wave function of the $N\overline{N}$ state

$$
\Psi_{N\overline{N}} = N_b^2\,e^{-\frac{1}{4}a^2[\vec{q}^{\,2}_{23}+\vec{q}^{\,2}_{56}]}\,e^{-\frac{1}{12}a^2[(\vec{q}_{12}-\vec{q}_{13})^2+(\vec{q}_{46}-\vec{q}_{45})^2]} \qquad \text{(B4)}
$$

and Ψ_m with $m=s,d$ the spatial wave function of the intermediate meson which are taken as $3S$ and $2D$ states

$$
\begin{aligned}
\Psi_s(\vec{p}) &= N_s\,e^{-\frac{1}{2}b^2p^2}\left(\frac{15}{4}-5b^2p^2+b^4p^2\right), \qquad \text{(B5)}\\
\Psi_d(\vec{p}) &= N_d\,e^{-\frac{1}{2}b^2p^2}(bp)^2\left(\frac{7}{2}-b^2p^2\right)Y_{2L_z}(\hat{p}).
\end{aligned}
$$

At the $\overline{N}N$ threshold, that is, $k\approx 0$, one may evaluate $T_{spatial}$ analytically. For the process where the vector meson is in a S-wave, we obtain

$$
\begin{aligned}
T_{spatial} &= 4\cdot(4\pi)\cdot 8\cdot\delta_{\mu,-\nu}(-1)^{\nu}\cdot N_sN_b^2\left\{f(2,\alpha)\left[\frac{15}{4}f(4,\beta)-20b^2f(4,\beta)+14b^4f(8,\beta)\right]\right.\\
&-\left.f(4,\alpha)\left[\frac{15}{4}f(2,\beta)-20b^2f(4,\beta)+14b^4f(6,\beta)\right]\right\} \qquad \text{(B6)}
\end{aligned}
$$

For the process where the vector meson is in a D-wave, we have

$$
T_{spatial} = 16\cdot(4\pi)\cdot 8\cdot b^2N_sN_b^2\cdot I\cdot f(2,\alpha)\left[\frac{7}{2}f(6,\beta)-4b^2f(8,\beta)\right] \qquad \text{(B7)}
$$

with

$$
I = \frac{3}{\sqrt{5}}\frac{1}{\sqrt{4\pi}}C(10,10,20)C(1\mu,1\nu,2\,\mu+\nu). \qquad \text{(B8)}
$$

In the above equations α and β are constants defined as $\alpha=2a^2$ and $\beta=2b^2+6a^2$. The function $f(n,u)$ is given as

$$
f(n,u) = \int_0^{\infty}dx\,x^ne^{-ux^2} = \frac{1}{2}u^{-\frac{n+1}{2}}\Gamma\left(\frac{n+1}{2}\right). \qquad \text{(B9)}
$$

[1] A. Antonelli *et al.*, Nucl. Phys. B **517**, 3 (1998).

[2] A. Antonelli *et al.*, Phys. Lett. B **334**, 431 (1994); D. Bisello *et al.* [DM2 Collaboration], Z. Phys. C **48**, 23 (1990).

[3] G. Bardin *et al.*, Nucl. Phys. B **411**, 3 (1994).

[4] J. R. Ellis and M. Karliner, New J. Phys. **4**, 18 (2002) [arXiv:hep-ph/0108259].

[5] A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D **8**, 2223 (1973); Phys. Rev. D **9**, 1415 (1974); Phys. Rev. D **11**, 1272 (1975).

[6] M. Maruyama, S. Furui and A. Faessler, Nucl. Phys. A **472**, 643 (1987); M. Maruyama, S. Furui, A. Faessler and R. Vinh Mau, Nucl. Phys. A **473**, 649 (1987); T. Gutsche, M. Maruyama and A. Faessler, Nucl. Phys. A **503**, 737 (1989).

[7] C. B. Dover, T. Gutsche, M. Maruyama and A. Faessler, Prog. Part. Nucl. Phys. **29**, 87 (1992).

[8] A. Muhm, T. Gutsche, R. Thierauf, Y. Yan and A. Faessler, Nucl. Phys. A **598**, 285 (1996).

[9] Y. Yan, R. Tegen, T. Gutsche and A. Faessler, Phys. Rev. C **56**, 1596 (1997).

[10] A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, *Hadron Transitions in the Quark Model* (Gordon and Breach, Amsterdam, 1988).

[11] R. Kokoski and N. Isgur, Phys. Rev. D **35**, 907 (1987).

[12] E. S. Ackleh, T. Barnes and E. S. Swanson, Phys. Rev. D **54**, 6811 (1996) [arXiv:hep-ph/9604355]; T. Barnes, F. E. Close, P. R. Page and E. S. Swanson, Phys. Rev. D **55**, 4157 (1997) [arXiv:hep-ph/9609339].

[13] L. M. Barkov *et al.*, Nucl. Phys. B **256**, 365 (1985).

[14] R. R. Akhmetshin *et al.* [CMD-2 Collaboration], Phys. Lett. B **527**, 161 (2002) [arXiv:hep-ex/0112031].

[15] S. Eidelman *et al.* [Particle Data Group Collaboration], Phys. Lett. B **592**, 1 (2004).

[16] T. Gutsche, R. D. Viollier and A. Faessler, Phys. Lett. B **331**, 8 (1994).

[17] Y. Yan, S. W. Huang and A. Faessler, Phys. Lett. B **354**, 24 (1995).

[18] A. V. Anisovich *et al.*, Phys. Lett. B **476**, 15 (2000).

[19] A. V. Anisovich *et al.*, Phys. Lett. B **507**, 23 (2001).

[20] N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1978); Phys. Rev. D **20**, 1191 (1979).