Nonlinear Dynamics **36:** 47–68, 2004.

A Particular Class of Partially Invariant Solutions of the Navier–Stokes Equations

SERGEY V. MELESHKO
School of Mathematics, Institute of Science, Suranaree University of Technology, Nakhon Ratchasima, 30000, Thailand (e-mail: sergey@math.sut.ac.th)

(Received: 27 October 2003; accepted: 17 November 2003)

Abstract. One class of partially invariant solutions of the Navier–Stokes equations is studied here. This class of solutions is constructed on the basis of the four-dimensional algebra L_4 with the generators

$$X_1 = \phi_1\partial_x + \phi_1'\partial_u - x\phi_1''\partial_p, \qquad X_2 = \phi_2\partial_x + \phi_2'\partial_u - x\phi_2''\partial_p,$$
$$Y_1 = \psi_1\partial_y + \psi_1'\partial_v - y\psi_1''\partial_p, \qquad Y_2 = \psi_2\partial_y + \psi_2'\partial_v - y\psi_2''\partial_p.$$

Systematic investigation of the case, where the Monge–Ampere equation (10) is hyperbolic ($Lf_z + k + l \geq 0$) is given. It is shown that this class of solutions is a particular case of the solutions with linear velocity profile with respect to one or two space variables.

Key words: Group classification, group stratification, invariant and partially invariant solutions, Navier–Stokes equations

1. Introduction

An unsteady motion of incompressible viscous fluid is governed by the Navier–Stokes equations

$$\mathbf{u}_t + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla p + \Delta\mathbf{u}, \qquad \nabla\cdot\mathbf{u} = 0, \tag{1}$$

where $\mathbf{u} = (u, v, w)$ is the velocity field, p is the fluid pressure, ∇ is the gradient operator in the three-dimensional space $\mathbf{x} = (x, y, z)$ and Δ is the Laplacian. The Navier–Stokes equations contain complete information about the structure of flows under usual temperature and pressure. Despite progress in numerical methods and techniques, there is considerable interest in finding exact solutions of the Navier–Stokes equations. Each exact solution has value, first, as the exact description of the real process in the framework of a given model; secondly, as a model to compare various numerical methods; and thirdly, as theoretical tool to improve the models used.

One method of constructing exact solutions is group analysis [1]. A historical review of the development of group analysis can be found in [2]. Many results obtained by group analysis are collected in [3]. The method is based on symmetries of given equations. Note that many of invariant solutions of the Navier–Stokes equations have been known for a long time: these solutions were obtained by assuming a form of the representation of the solution. Group analysis gives a method for obtaining a representation of a solution. The first group classification of the Navier–Stokes equations in the three-dimensional case was done in [4]. The first classification of the two-dimensional Navier–Stokes equations was studied in [5]. It was shown that the Lie algebra admitted by the Navier–Stokes equations is infinite-dimensional