

PREDICTABILITY OF BARTON'S JOINT SHEAR STRENGTH CRITERION USING FIELD-IDENTIFICATION PARAMETERS

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Abstract

A series of direct shear tests have been performed in an attempt at assessing the predictive capability of Barton's joint shear strength criterion derived from field-identified parameters. Ten rock types have been tested, including basalt, two marbles, three granites and four sandstones. Testing on saw-cut surface specimens determines the relationship between the basic friction angle (ϕ_b) and the rock compressive strength (UCS). Testing on specimens with tension-induced fractures yields joint shear strengths under different JRC's, for use in the verification. The results indicate that Barton's criterion using the field-identified parameters can satisfactorily predict the shear strengths of rough joints in marbles and sandstones from all source locations, and slightly over-predicts the shear strength in the basalt specimens. It cannot however describe the joint shear strengths for the granite specimens. This is probably because the saw-cut surfaces for coarse-grained and strong crystalline rocks are very smooth resulting in an unrealistically low ϕ_b . Barton's shear strength is more sensitive to ϕ_b than to UCS and JRC. For all sandstones the ϕ_b values are averaged as 33 ± 8 degrees, apparently depending on their cementing materials. The averaged ϕ_b for the tested marbles and for the limestone recorded elsewhere is 35 ± 3 degrees, and is independent of UCS. The ϕ_b values for other rock types apparently increase with UCS particularly for very strong rocks (R5 and R6). The factors governing ϕ_b for crystalline rocks are probably crystal sizes, mineral compositions, and the cutting process, and for clastic rocks are grain size and shape, and the strength of cementing materials.

Keywords: Rock joint, shear strength, friction, roughness

Introduction

Barton's joint shear strength criterion (Barton, 1972, 1973; Barton and Bandis, 1990) has long been widely used in practice for determining the strength of discontinuities in rock mass (Hoek and Bray, 1981; Grasselli and Egger, 2003). This empirical criterion has several advantages over other shear strength criteria (Patton, 1966; Ladanyi and Archambault, 1970), e.g., an ease

of application, capability of describing non-linear behavior of shear strength in respect to normal stress, and permitting the incorporation of the actual joint morphology into the calculation. Barton's criterion [$\tau = \sigma_n \tan (\phi_b + \text{JRC} \cdot \log (\sigma_j / \sigma_n))$] requires three parameters that depend on rock mechanical properties and fracture characteristics; i.e., joint roughness coefficient (JRC),

basic friction angle (ϕ_b), and joint wall strength (σ_j , normally assumed equal to the uniaxial compressive strength, UCS). JRC is normally obtained by visual comparison between joint morphology and the standard profiles, which can be easily performed on site. A profilometer or digital coordinate measuring machine may be used to determine the detailed profile of the fracture surface. The obtained data can be analyzed by several methods, e.g., Gaussian distribution, fractal method, Fourier transform method and spectral method. These measuring techniques and data analyses are time-consuming and may not be able to be performed on-site.

The basic friction angle of a rock joint is normally determined by laboratory testing, e.g., direct shear test and tilt test. The uniaxial compressive strength test or point load strength index test is usually performed to obtain the joint wall strength. Applications of Barton's criterion therefore rely on laboratory testing to determine ϕ_b and UCS. For the past decades extensive studies have been carried out on various aspects of Barton's criterion, e.g., scale effect by Barton and Bandis (1990), joint dilation by Indraratna and Haque (2000), and joint infill by Phien-wej *et al.* (1990). Rare efforts however have been made to apply a simpler, quicker or more economic approach to determine ϕ_b and UCS and incorporate them into the joint shear strength calculation without laboratory testing. This raises two key questions. Can ϕ_b be evaluated or inferred from the mineralogical or mechanical rock properties that can be visually examined in the field? And are the existing field methods for determining the UCS of intact rock adequate for use in the application of Barton's criterion?

The objective of this research is to assess the predictive capability of Barton's joint shear strength criterion derived from the field-identified parameters. Experimental efforts involve a series of direct shear testing on smooth (saw-cut) surfaces, and on rough (tension-induced) surfaces of rock specimens prepared from ten different source locations. The reliability of the field methods used to evaluate the basic friction angle, the strength of the intact rock, and the JRC are also evaluated. Correlation between the

basic friction angle and the rock mechanical properties has been made. For comparison purposes the joint wall strengths have been determined using both the standard laboratory test method and the suggested field-identification methods. Comparison of the predicted joint shear strengths with those obtained from actual testing will reveal the predictability of Barton's criterion derived from the field-identified parameters. All tested fractures or joints are clean, tight, and perfectly matched. The peak shear strength is of interest. The effects of dilation, joint aperture, filling materials, joint alteration, water pressure, and shearing rate (Indraratna and Haque, 2000) are excluded from this study. These factors are isolated primarily to reveal the effects of the rock conditions and basic friction angle on the Barton's shear strength equation.

Rock Samples

Rock samples used in this research have been collected from ten different source locations, which represent the most commonly encountered rocks in the construction and mining industries in Thailand. They can be categorized here into four groups: basalt, two marbles, three granites and four sandstones. The main selection criteria are the availability and the mechanical homogeneity of the specimens, while aiming at the mineralogical diversity among different rock types. Thin sections are prepared from each rock type for the petrographic analysis. Mineral compositions and grain (crystal) sizes are determined. Table 1 gives rock names, brief mineral compositions, the geologic formation or unit to which they belong, and the location from which they are obtained.

Uniaxial Compressive Strength

The uniaxial compressive strength (UCS) or the rock strength on the joint walls is required for applications of Barton's criterion. Two approaches have been used here: the standard laboratory test method designated by the American Society for Testing and Materials

(ASTM), and the field-identification method suggested by the International Society of Rock Mechanics (ISRM).

Sample preparation and test procedure for the laboratory determination of the UCS strictly follow the ASTM D4543 and ASTM D2938. Core specimens with a nominal diameter of 54 mm and length-to-diameter ratio of 2.5 are

prepared. Ten specimens have been tested for each rock type. Each specimen is axially loaded to failure at a constant rate of 1 MPa/s. The axial load and deformation are monitored. The calculated UCS's are summarized in Table 2 and the stress-strain curves are plotted in Figure 1. The tangent Young's modulus (E) is calculated from the stress-strain curves at 50% of the maximum stress level.

Table 1. Brief description of rock samples obtained from ten source locations

Rock Name	Mineral compositions	Rock unit / Location
Aphanitic Basalt	50% Pyroxene (0.5-1 mm) and 50% plagioclase (0.3-0.8 mm)	Burirum Basalt Unit / Burirum Province
Limestone Marble (from Saraburi)	100% Calcite (1-5 mm)	Saraburi Group / Saraburi Province
Limestone Marble (from Lopburi)	100% Calcite (1-2 mm)	Saraburi Group / Lopburi Province
Quartz Syenite	75% Orthoclase (0.3-2 cm), 10% quartz (2-5 mm), 10% plagioclase (1-3 mm), and 5% amphibole (1-2 mm)	"Unknown" / Vietnam
Plagiogranite	40% Plagioclase (0.5-1 mm), 30% quartz (2-5 mm), 5% orthoclase (3-5 mm), 3% amphibole (1-2 mm), and 2% biotite (1-2 mm)	Tak Batholith / Tak Province
Quartz Monzonite	70% Plagioclase (0.5-2 cm), 15% quartz (3-5 mm), 7% orthoclase (2-3 mm), 5% amphibole (1-2 mm), and 3% biotite (2-3 mm)	"Unknown" / China
Calcareous Lithic Sandstone	70% Lithic fragment (0.1-0.3 mm), 18% quartz (0.1-0.5 mm), 7% mica (0.1-0.5 mm), 3% feldspar (0.1-0.5 mm), and 2% other (0.1-0.8 mm)	Phu Kradung Formation / Nakhon Ratchasima Province
Quartz Sandstone	72% Quartz (0.2-0.8 mm), 20% feldspar (0.1-0.8 mm), 3% mica (0.1-0.3 mm), 3% rock fragment (0.5-2mm), and 2% other (0.5-1 mm)	Phu Phan Formation / Nakhon Ratchasima Province
White Quartz Sandstone	75% Quartz (0.1-0.5 mm), 15% feldspar (0.2-0.5 mm), 7% mica (0.1-0.5 mm), and 3% lithic fragment (0.1-1 mm)	Phra Wihan Formation / Nakhon Ratchasima Province
Arkosic Feldspathic Sandstone	70% Feldspar (0.1-0.5 mm), 18% quartz (0.1-0.5 mm), 7% mica (0.1-0.2 mm), 3% rock fragment (0.1-0.3 mm), and 2% other (0.1-0.3 mm)	Sao Khua Formation / Nakhon Ratchasima Province

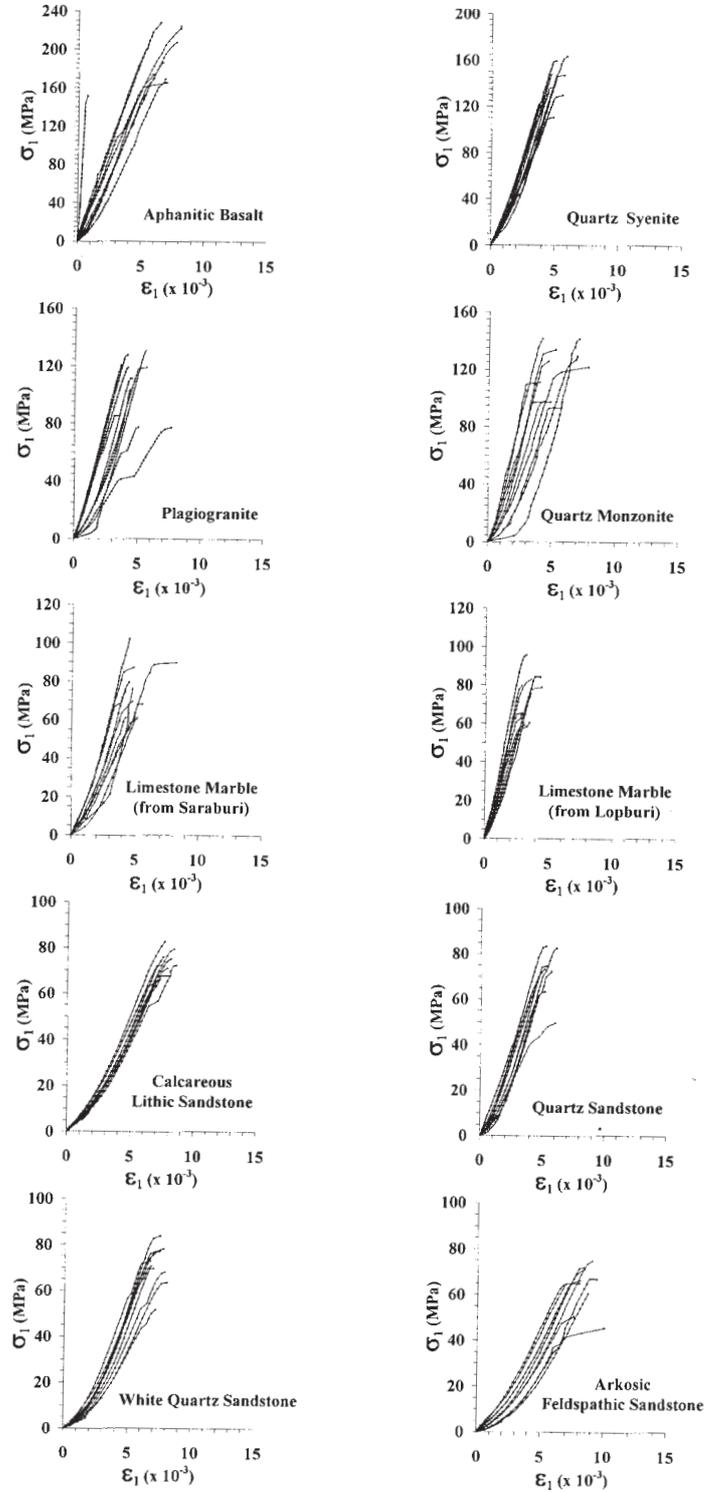


Figure 1. Results of uniaxial compressive strength testing from ten rock types



Figure 2. Arkosic feldspathic sandstone block specimens with saw-cut surface prepared for ϕ_b determination

Table 2. Some basic mechanical properties of rock specimens obtained from ten source locations

Rock Name	Density	Density (g/cc)	ϕ_b (degrees)	E (GPa)	UCS (MPa)	
					(ASTM Lab.)	(ISRM Field-determined)
Aphanitic Basalt	2.81	35.3±2.08	33.2±3.4	188.1±26.3	R5 (100-250)	
Limestone Marble (from Saraburi)	2.57	34.3±0.58	21.3±4.4	78.7±14.6	R4 (50-100)	
Limestone Marble (from Lopburi)	2.72	35.7±1.53	28.7±2.4	74.4±12.6	R4 (50-100)	
Quartz Syenite	2.62	18.3±1.53	34.5±4.3	138.1±18.9	R5 (100-250)	
Plagiogranite	2.62	24.7±0.58	32.4±4.6	119.4±8.8	R5 (100-250)	
Quartz Monzonite	2.64	25.7±0.58	34.0±8.0	119.3±18.3	R5 (100-250)	
Calcareous Lithic Sandstone	2.53	33.7±1.53	12.2±0.7	72.8±5.7	R4 (50-100)	
Quartz Sandstone	2.27	31.7±2.31	18.4±1.1	72.4±8.5	R4 (50-100)	
White Quartz Sandstone	2.33	31.7±2.52	13.9±2.0	71.3±9.0	R4 (50-100)	
Arkosic Feldspathic Sandstone	2.33	30.7±3.21	11.5±0.5	67.5±4.6	R5 (100-250)	

The field-identification of the UCS follows the ISRM suggested method given by Brown (1981). Two engineers independently identify the grade for each rock type using mainly the geologic hammer and pocket knife. The nominal specimen sizes are 10 x 10 x 5 cm. The grades for the selected rock specimens are identified to be R4 and R5. Brown (1981) describes the detailed test method and the classification scheme. The strength results obtained by the two engineers coincide, and agree with the UCS's from the uniaxial compression test (Table 2). This suggests that the range of the

rock strength identified by the field method is sufficiently accurate and probably adequate for use as a joint wall strength parameter in Barton's criterion.

Determination of Basic Friction Angles

Direct shear testing is carried out on the saw cut surfaces of rock specimens to determine their basic friction angle. The test procedure follows as much as practical the ASTM D5607 standard practice. Three specimens are tested for each

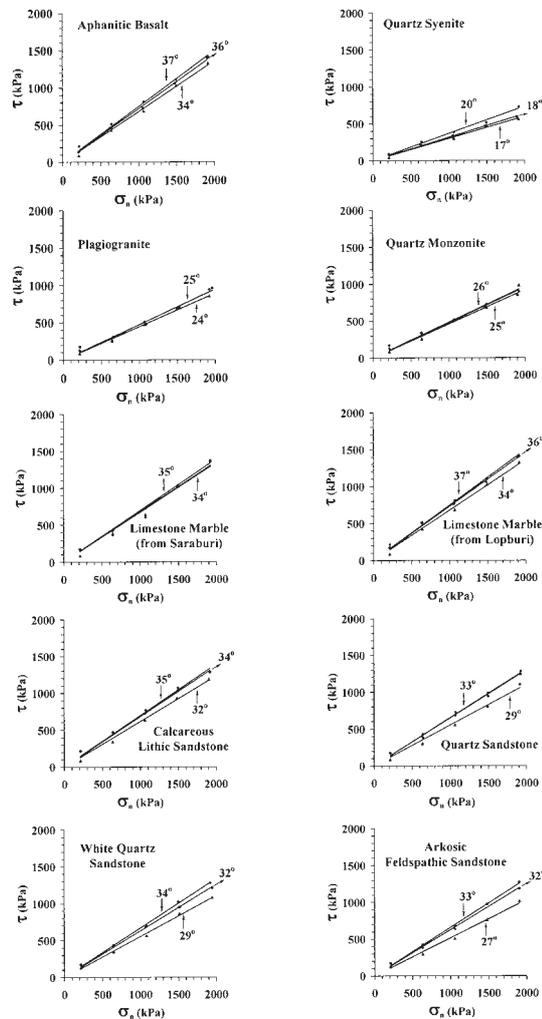


Figure 3. Results of direct shear testing on saw cut surfaces of ten rock types

rock type. The tested fracture area is 10 x 10 cm. Figure 2 shows some rock specimens prepared for this test. The direct shear machine (SBEL D44) with a maximum shear load of 30,000 lbs and maximum normal load of 10,000 lbs is used. Pre-defined normal loads are maintained constant during the test (CNL testing - Indraratna and Haque, 2000). Shear force is continuously applied and monitored until a total shear displacement of 1 cm is obtained. The shearing rate is about 1 mm/min. Each block specimen is sheared 3 times (forward-backward-forward) with the normal stresses increasing from 1.07, 1.92 to 2.29 MPa. The peak shear stress is calculated and plotted against the corresponding normal stress. Linear relationship between the shear and normal stresses is obtained for all tests (Figure 3). The basic friction angle (ϕ_b) is calculated from the shear-normal stress slopes. Table 2 lists the ϕ_b values for the ten rock types.

An attempt has been made here at correlating the ϕ_b with the intact rock strength. The UCS and ϕ_b for various rock types obtained elsewhere (Goodman, 1989; Grasselli and Egger, 2003; Hoek and Bray, 1981; Waltham, 1994) have been compiled and compared with the results obtained here. Surprisingly, publications

reporting both UCS and ϕ_b tested for the same rocks are very rare, particularly those providing detailed rock descriptions or the source locations.

Figure 4 plots ϕ_b as a function of UCS for the marble tested here and the marble and limestone tested elsewhere. From the available information, ϕ_b appears to be independent of UCS and grain size. The average ϕ_b is 35 ± 3 degrees.

For sandstones from all source locations, ϕ_b is averaged as 33 ± 8 degrees (Figure 5). The averaged ϕ_b for the quartz sandstones (pure sandstone) is 32 ± 3 degrees. The averaged ϕ_b for the arkosic feldspathic sandstone is slightly lower (31 ± 3 degrees), and for the calcareous lithic sandstone is slightly greater (34 ± 2 degrees). This suggests that for the tested fine-grained sandstones, the cementing materials may have some influence on ϕ_b . The UCS however may not be an appropriate indicator for ϕ_b of sandstones, as it shows a significantly high standard variation (over 10%).

The ϕ_b values for the tested quartz syenite, plagiogranite and quartz monzonite are 18 ± 2 , 25 ± 1 , and 26 ± 1 degrees, which are notably lower than those obtained for the granites elsewhere. Most granite and gneiss have ϕ_b about 30 degrees. This is probably due to the

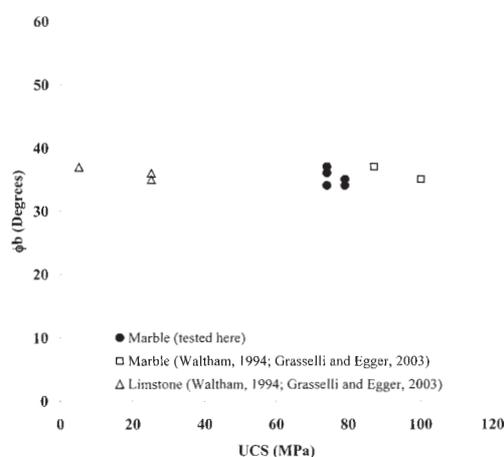


Figure 4. Basic friction angles as a function of UCS for marbles and limestone

fact that the saw-cut surfaces for the coarse-grained and very strong crystalline rocks (such as granites) are very smooth, even without polishing, and hence results in an unrealistically low ϕ_b from the direct shear testing. This also implies that the rock cutting process and equipment can govern the characteristics of the cut surfaces, and hence affect ϕ_b as well. The tested aphanitic basalt has ϕ_b equal to 35 ± 2 degrees which agrees well with those obtained elsewhere. The number and diversity of the basalt specimens are inadequate to determine the relationship between ϕ_b and the mineral or mechanical properties of the basalts.

Figure 6 plots ϕ_b as a function of UCS for various rock types, except sandstone, limestone

and marble. It seems that for strong rocks (ISRM-designated R4 & R5), ϕ_b increases with UCS. A liner fit shows a mathematical relation as

$$\phi_b = 0.077 \text{ UCS} + 25.2$$

where ϕ_b is in degree and UCS is in MPa.

Extreme care should be taken when applying the above equation for other rocks. The available data are widely scattered, and are not truly sufficient to support the dependency of ϕ_b on UCS, as reflected by the low coefficient of correlation ($R^2 = 0.474$). It is believed that ϕ_b does not always depend on the UCS. Other factors governing the ϕ_b for the crystalline rocks are probably the crystal size, mineral compositions, and the cutting process, and for

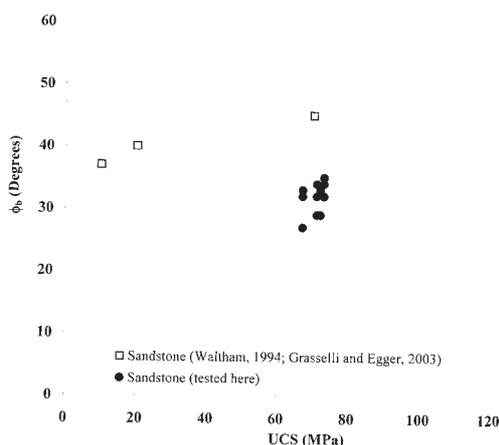


Figure 5. Basic friction angles as a function of UCS for sandstones

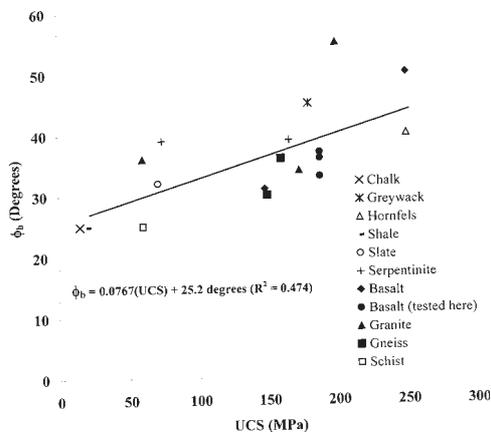


Figure 6. Basic friction angles as a function of UCS for various rock types obtained elsewhere (Waltham, 1994; Geasselli and Eggar, 2002)

the clastic rocks are the grain size and shape, and the strength of cementing materials.

For all rock types, no relationship has been found between ϕ_b and the elastic modulus of the rocks.

Direct Shear Testing on Rough Joints

In order to obtain the shear strength of rough joints, tensile fractures are induced in rock blocks with a dimension of 10 x 10 x 20 cm. Line load

Table 3. Predicted and actual shear strengths for 3 rough joints from each rock type

Rock Name	JRC range	Predicted Shear Strength (kPa)						Actual Shear Strength (kPa)	Normal stress (kPa)
		UCS (ASTM)		UCS Min. (ISRM)		UCS Max. (ISRM)			
		JRC Min.	JRC Max.	JRC Min.	JRC Max.	JRC Min.	JRC Max.		
Aphanitic Basalt	8-11	2,076	2,413	1,921	2,181	2,152	2,528	1,986	1,655
	8-11	3,483	3,961	3,225	3,590	3,607	4,145	3,185	2,986
	8-11	4,140	4,677	3,834	4,241	4,287	4,891	3,305	3,636
Limestone Marble (from Saraburi)	8-10	1,237	1,413	1,169	1,315	1,273	1,466	1,250	1,078
	12-14	1,738	1,959	1,644	1,826	1,788	2,032	2,146	1,588
	8-10	2,598	2,928	2,384	2,640	2,718	3,094	2,450	1,934
Limestone Marble (from Lopburi)	8-10	1,297	1,483	1,236	1,393	1,347	1,557	1,230	1,060
	10-12	1,709	1,951	1,607	1,806	1,793	2,074	1,589	1,255
	10-12	2,416	2,714	2,274	2,518	2,533	2,879	2,861	1,893
Quartz Syenite	10-12	868	1,008	826	950	951	1,123	3,204	1,068
	10-12	1,440	1,645	1,368	1,550	1,581	1,834	3,588	1,922
	8-10	1,477	1,688	1,414	1,602	1,598	1,854	4,150	2,378
Plagiogranite	12-14	1,237	1,434	1,198	1,380	1,421	1,670	1,555	1,051
	8-10	1,582	1,795	1,548	1,748	1,733	2,009	2,833	1,932
	10-12	2,074	2,338	2,020	2,266	2,322	2,678	4,581	2,291
Quartz Monzonite	12-14	1,310	1,519	1,269	1,461	1,507	1,805	2,938	1,080
	14-16	2,410	2,743	2,321	2,624	2,841	3,343	3,293	1,950
	14-16	2,818	3,182	2,714	3,046	3,315	3,863	3,557	2,386
Calcareous Lithic Sandstone	6-8	1,077	1,225	1,041	1,169	1,109	1,273	1,076	1,076
	6-8	1,532	1,721	1,480	1,644	1,577	1,788	1,367	1,282
	6-8	1,833	2,046	1,771	1,954	1,886	2,126	1,852	1,939
Quartz Sandstone	6-8	1,001	1,137	968	1,088	1,031	1,183	1,333	1,075
	6-8	1,180	1,333	1,141	1,275	1,215	1,387	1,545	1,288
	8-10	1,903	2,124	1,821	2,010	1,981	2,234	2,704	1,932
White Quartz Sandstone	6-8	1,002	1,138	971	1,091	1,034	1,186	1,380	1,078
	6-8	1,181	1,334	1,144	1,279	1,219	1,391	1,464	1,292
	8-10	1,911	2,132	1,832	2,021	1,992	2,246	2,075	1,945
Arkosic Feldspathic Sandstone	4-6	841	955	826	930	862	991	1,197	1,069
	6-8	1,118	1,261	1,088	1,217	1,160	1,324	1,356	1,271
	6-8	1,643	1,831	1,600	1,767	1,705	1,922	1,900	1,943

is applied at the mid-section of the specimen until splitting tensile failure occurs (Figure 7). This results in a clean, rough and perfectly matched fracture. Three pairs of specimens are prepared for each rock type. A shear direction is then pre-defined (Figure 8). Six engineers independently determine the JRC along the shear direction. Their results agree reasonably well; usually 5 out of 6 give the same range of JRC. Table 3 summarizes the JRC's for each pair of the rock specimens.

A series of direct shear tests are performed on the specimens with the tension-induced fracture. The selected normal stresses are 1.08, 1.29, and 1.95 MPa. Each specimen is sheared only once for each normal stress using a direct shear machine (SBEL D44). A constant shearing

rate of 1 mm/min is maintained. Shear force is continuously applied until a total shear displacement of 1 cm is reached. The peak and residual shear loads are monitored. Table 3 lists the peak shear stresses calculated for the ten rock types. As expected, the greater the normal stress applied, the greater the peak shear stress obtained.

Post-tested observation on the sheared off area indicates that the asperity areas that have been sheared off are very small; about 10 - 15% for sandstone and marble specimens and about 3 - 5% for granite and basalt specimens. Figure 9 shows post-tested fractures for a pair of arkosic feldspathic sandstone specimens. It seems that the larger the sheared off areas obtained, the greater the applied normal load, and the lower the rock strength.

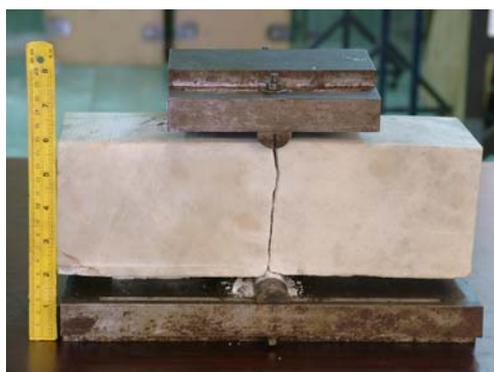


Figure 7. Line load applied on rock block to create tension-induced fracture

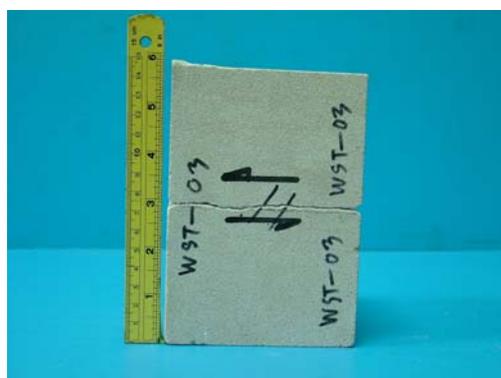


Figure 8. Quartz sandstone specimens with tension-induced fracture prepared for direct shear testing. Arrows indicate shear direction.

Prediction of Rough Joint Shear Strengths

Barton's criterion is used to calculate (predict) the shear strengths for the specimens with tension-induced fractures. The calculations use several combinations of the maximum and minimum values for the JRC, the UCS obtained from the ISRM field-identification, and the UCS determined by the ASTM standard method. For all calculations the actual ϕ_b is used. This is primarily to assess the predictive capability of the criterion, the adequacy of the field-identified UCS, and the sensitivity of the JRC and UCS on Barton's shear strength.

Table 3 compares the predicted shear strength with those actually tested for the rough joints. The criterion using field-identified parameters satisfactorily predicts the shear strength of the rough joints in marbles and sandstones from all source locations, and slightly over-predicts the shear strength in the basalt specimens. It drastically underestimates the shear strength of granite specimens from all locations. This is mainly due to the fact that ϕ_b from direct shear testing on the smooth saw-cut surfaces in granite is lower than the actual values.

Discussions and Conclusions

The sensitivity evaluation suggests that the Barton's shear strength is more sensitive to ϕ_b than to UCS and JRC. For all rock types, the

range of UCS from the ISRM field-identified method agrees well with the corresponding value determined by ASTM laboratory testing. Variations of the UCS by 25 MPa for weak and medium rocks (R2 and R3) and by 50 MPa for strong and very strong rocks (R4 and R5) do not significantly affect the predicted shear strengths. The range of JRC determined by six engineers, though it shows some subjectivity, appears to be adequate.

The basic friction angle for the tested fine-grained sandstones is averaged as 33 ± 8 degrees. The cementing materials may have some influence on ϕ_b . For the tested marbles and for the limestone recorded elsewhere, ϕ_b is averaged as 35 ± 5 degrees, and appears to be independent of UCS. For other strong rocks (ISRM-designated R4 & R5), ϕ_b apparently increases linearly with UCS. This relationship remains inconclusive due to insufficient information.

Based on the observation, the factors governing the ϕ_b for the crystalline rocks are probably the crystal size, mineral compositions, and the cutting process, and for the clastic rocks are the grain size and shape, and the bond strength of cementing materials. The number and diversity of the basalt and granite specimens are not adequate to determine the relationship between ϕ_b and the mineralogical variations, even if there is any. For some igneous or



Figure 9. Post-tested fractures of askosic feldspathic sandstone

metamorphic rocks in particular, e.g., granite, diorite and gneiss, it may be virtually impossible to determine the relationship between ϕ_b and their mineralogy due to the infinite combinations of the rock compositions and textures on the fracture surfaces.

By using the measured ϕ_b and the field-identified UCS and JRC, Barton's criterion satisfactorily predicts the shear strength of rough joints in marbles and sandstones from all source locations, and slightly over-predicts the shear strength in the basalt specimens. It can not describe the joint shear strengths for the granite specimens.

Even though some uncertainties remain, as described above, the findings from this research still provide a quick and useful approach for determining the shear strength of clean, tight and rough joints by using Barton's criterion and field-identified parameters. The information compiled in Figures 4 through 6 can be used as a guideline to estimate ϕ_b of medium and strong rocks, if applicable. The ISRM field-identification for UCS and JRC seems adequate for use in Barton's equation.

More testing is required. For clastic rocks, specimens with significantly different grain sizes and cementing materials are desirable. Application of a greater normal load may enhance the effect of cementing materials and UCS on the joint shear strength. For strong crystalline rocks, the effects of the cutting process on the surface roughness should be further investigated.

Acknowledgements

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