A COMPARISON OF EMPIRICAL CONTROLLER TUNING METHODS FOR THIRD ORDER INVERSE RESPONSE PROCESSES.

Adrian Flood[1*]

Abstract

Until recently the two most commonly used empirical methods for choosing feedback controller tuning constants were the Cohen - Coon (CC) method (based on the open loop 'process reaction curve'), and the Ziegler - Nichols (ZN) method (based on ultimate response of the closed loop process). Both of these methods have substantial limitations: the CC method is unsuitable for processes which are open loop unstable, and the ZN method generally produces closed - loop responses which are not sufficiently damped, particularly for chemical engineering processes. Recently a new empirical method was developed by Tyreus and Luyben (1992). The method is also based on the ultimate response of the closed - loop system, however instead of being optimized in terms of simple performance criteria it aims to produce a closed - loop response which has a maximum log modulus of ±2 dB. The objective of the current research was to compare these three empirical tuning methods on third order inverse response processes obtained as a result of competing first and second order processes in parallel. It was determined that the ZN method gave consistently better tuning constants than the other methods, based on both simple and integral performance criteria. The CC method suffered from the obvious inability for inverse response processes (particularly third order processes which may be oscillatory) to be approximated by a first order plus dead time response. The Tyreus - Luyben (TL) method did not achieve the stated objective of having a maximum closed - loop log modulus of ±2 dB in any case studied: in fact the ZN method appeared to follow this criteria more closely for these processes. It appears a major drawback of the TL method is that the tuning equations meet the ±2 dB criteria mainly for the processes the method was initially tested for, rather than in general. An improvement of the method would be to set equations for the integral and derivative time constants, and vary the controller gain in order to achieve the ±2 dB criteria for each process: however this method would greatly increase the difficulty of tuning, especially in the process plant.

Introduction

Despite the advances in research on advanced control systems, most processes in the chemical industry are still controlled using feedback controllers. The PID controller equation is given by:

$$c(t) = K_c \varepsilon(t) + \frac{K_c}{\tau_I}\int_0^t \varepsilon(t)dt + K_c \tau_D \frac{d\varepsilon}{dt} + c_s$$

In this equation c is the control action, ε is the "error" (the difference between the set-point and the controlled variable), and c_s is the controller

[1] Ph, D., Chemical Engineering, Faculty of Engineering , Suranaree University of Technology, Nakhon Ratchasima, 30000.

bias, or the initial value of the control action. The control parameters K_c, τ_I, and τ_D are set in order to provide suitable controlled response. The transfer function for the PID controller is:

$$G_c(s) = K_c\left(1 + \frac{1}{\tau_I s} + \tau_D s\right)$$

Note that these feedback control equations do not depend on a model of the system being controlled, although obviously the choice of the control constants will depend on the system. The integral control action may be taken out of the controller by making the integral time constant (τ_I) infinite (which gives PD control), and the derivative control action may be removed by making the derivation time constant (τ_D) equal to zero (which gives PI control).

In order to choose suitable tuning constants it is important to have some criteria for what "good" control is. Obviously we need the process to be bounded-input bounded-output (BIBO) stable, however a range of control constants can achieve this, and not all stable controllers give suitable responses. Another obvious choice is zero offset, where the final value of the controlled variable is equal to the set-point, and any controller with integral action will achieve this. In general the control performance is determined through the use of either simple performance criteria, which use only one or two points of the response in order to evaluate performance, or integral performance criteria, which use the entire response (from t = 0 to infinity) to determine the performance. Simple performance criteria are easier to use (especially on real processes), however the integral criteria are a more analytical evaluation of the control performance since they relate to the whole performance. This study will use both types of performance criteria. The definitions of the common performance criteria are given in Table 1, which refers to the control shown in Figure 1.

Empirical controller tuning

Controller tuning is the task of choosing suitable tuning constants. Empirical controller tuning methods for feedback controllers are important for choosing initial values of the PID tuning constants. In most cases these constants need further improvement using on - line tuning. The Cohen - Coon method (Cohen and Coon, 1953) requires the process to be approximated by a first-order-plus-dead time (FOPDT) process. Usually this is achieved by introducing a step change input to the process and finding best fits for the FOPDT parameters (the process gain K_p, the time constant τ_p, and the dead time t_d) from the response. The Cohen-Coon (CC) tuning constants (for P, PI, and PID controllers) may then be calculated from the following equations:

For proportional controllers:

$$K_c = \frac{\tau}{K_p t_d}\left(1 + \frac{t_d}{3\tau}\right)$$

For proportional - integral controllers:

$$K_c = \frac{\tau}{K_p t_d}\left(0.9 + \frac{t_d}{12\tau}\right), \text{and}\ \tau_I = t_d\left(\frac{30 + 3t_d/\tau}{9 + 20t_d/\tau}\right)$$

For proportional - integral - derivative controllers:

$$K_c = \frac{\tau}{K_p t_d}\left(\frac{4}{3} + \frac{t_d}{4\tau}\right), \tau_I = \tau_d\left(\frac{32 + 6t_d/\tau}{13 + 8t_d/\tau}\right),$$

$$\text{and}\ \tau_D = t_d\left(\frac{4}{11 + 2t_d/\tau}\right)$$

It is noticeable that the controller gain for PI control is made less than that for P control due to the destabilizing effect of the integral term, while the gain for the PID control is slightly higher than that for P control. Many chemical engineering processes have responses similar to FOPDT systems; they are in general higher order from series of first order processes (thus imitating FOPDT) and rarely display underdamped or inverse open loop responses. This shows that the CC method is suitable for most processes found in chemical engineering, however some processes do show inverse response (for instance boilers), and some elements do display inherently second order dynamics, and can thus be underdamped (for instance valves and transducers).

The Ziegler-Nichols (Ziegler and Nichols, 1942) and Tyreus-Luyben (Tyreus and Luyben, 1992; Luyben and Luyben, 1997; Luyben, 1998)

Table 1 : Definitions of performance criteria used in this study.

Criteria	Definition (see Figure 1 for details).
Overshot	(maximum value of response-steady state value)/steady state value × 100%. (B/A × 100%).
Rise time	Time required to reach the steady state value for the first time.
Decay ratio	Ratio of height of neighboring peaks in the response (C/B).
Settling time	Time required for the response to maintain new steady state within ±5%
IAE	The integral of the absolute value of the error, evaluated from t = 0 to ∞.
ISE	The integral of the error squared, evaluated from t = 0 to ∞.
ITAE	The integral of (the absolute value of the error × t), evaluated from t = 0 to ∞.

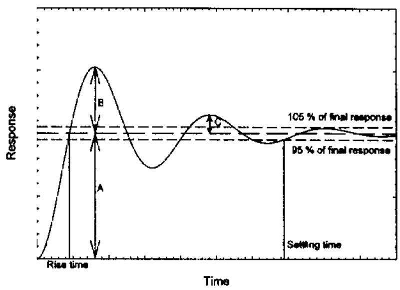


Figure 1 : A typical closed loop response showing details of simple performance criteria

methods both use the process controlled with a proportional only feedback controller to characterize the system in terms of its frequency response. The system undergoes a step change in its input (in this case the set-point), and the gain of the closed loop response is varied (by varying K_c) until the system reaches the crossover frequency (ω_{co}). This point is characterized by a response which oscillates at constant amplitude. Once this point is known the control constants are calculated from the ultimate period (P_u) and ultimate gain (K_u), which are defined by:

$$P_U = 2\pi / \omega_{co} \text{ and } K_U = M^{-1}$$

where M is the amplitude ratio of the response at the ultimate frequency. (Note that the ultimate gain is the value of the controller gain needed to achieve the ultimate response). Once these two parameters are known the controller tuning constants for the Ziegler -Nichols (ZN) method may be calculated using the equations in Table 2, and those for the Tyreus-Luyben (TL) method may be calculated using the equations in Table 3. The TL method does not predict tuning constants for proportional only controllers.

Table 2: Ziegler-Nichols tuning constants from the ultimate period (P_u) and ultimate gain (K_u).

Controller type	K_c	τ_I	τ_D
Proportional	$K_u/2.0$	-	-
Proportional-Integral	$K_u/2.2$	$P_u/1.2$	-
Proportional-Integral-Derivative	$K_u/1.7$	$P_u/2.0$	$P_u/8.0$

Table 3: Tyreus-Luyben tuning constants from the ultimate period (P_u) and ultimate gain (K_u).

Controller type	K_c	τ_I	τ_D
Proportional-Integral	$K_u/3.2$	$2.2P_u$	-
Proportional-Integral-Derivative	$K_u/2.2$	$2.2P_u$	$P_u/6.3$

Note that open loop unstable processes controlled by proportional controllers may have two values where the system oscillates at constant amplitude. At low controller gain the system may pass through a transition from being unstable to being stable, while at a higher value of the controller gain the system will move back to being unstable. The higher value of the controller gain is used for the ultimate gain (and therefore the related ultimate period) to select tuning constants: if the lower value is used the chosen tuning constants will probably result in an unstable closed loop system. In cases where the gain required to produce a stable process is more than half the ultimate gain, neither gain may result in a stable controller when used to predict tuning constants. It is likely these cases are very rare.

It should be noted that the criteria that Ziegler and Nichols used to determine their equations were largely simple performance criteria such as the decay ratio, and rise time. The more recent method of Tyreus and Luyben has the stated aim of obtaining a closed loop response which has maximum log modulus of +2 dB. This was claimed to be an improvement on the ZN method, which often produces closed loop response which are too underdamped for most chemical engineering processes.

Inverse response processes

Inverse response is where the initial response of the process occurs in the opposite direction to the final value of the response. This occurs when the slope of the initial response is opposite in sign to the slope of the response as the process reaches equilibrium. Inverse response may occur when two competing processes (which have gains of opposite signs) occur in parallel. Although this is not necessarily the reason for inverse response in all cases, it certainly helps to visualize what occurs in inverse response. The easiest way to recognize processes which will have inverse response is to examine their transfer functions: processes displaying inverse response will have a positive root in the *numerator* of their transfer function, or in other words, a positive zero. Note that this is different to unstable processes, which have a root with a positive real part in the *denominator* of their transfer function, or a positive pole. When processes have multiple zeros, inverse response will occur when there is an odd number of zeros (Rosenbrock, 1970). Inverse response processes are difficult to control because the controller initially obtains signals showing the process moving in the direction opposite to the final steady state. This problem led to the development of controller designs particularly for this type of process, such as inverse response compensators. However Scali and Rachid (1998) have shown that the commercial PID controller equation will be able to give control equivalent to compensators for typical values of plant/model mismatch.

Most research in the chemical engineering field on inverse response has investigated the control of second-order processes with inverse response (SOPIR) or their transfer functions. These are characterized by being equivalent to two competing first order systems in parallel. It has often been stated that these processes are typical of inverse response in general (see for example Scali and Rachid, 1988), however it should be noted that these processes will never be oscillatory (as they can never have complex roots in their transfer functions) whereas higher order inverse response processes certainly may be oscillatory.

This study will focus on third order processes with inverse response (TOPIR) and particularly those which result from competing first and second order processes. This distinction is important: all TOPIR will have a third order polynomial in the denominator of their transfer function, however the criteria of having one (or more) positive zeros means that TOPIR transfer functions must have numerators of order one or higher. (Usually processes in chemical engineering will have strictly proper transfer functions, and hence the numerator for TOPIR will normally be either first or second order). TOPIR resulting from competing first and second order processes have a second order polynomial in the numerator of their transfer function. The block diagram of the processes considered is shown in Figure 2, and a typical response is shown in Figure 3. The transfer function is:

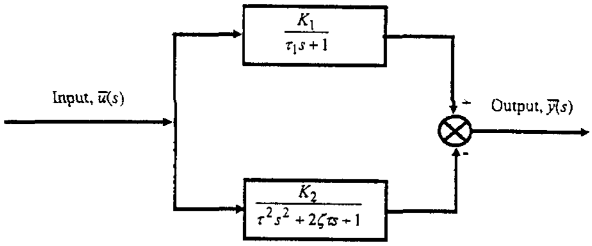


Figure 2: Block diagram of a third order inverse response process (TOPIR) as described by parallel first and second order process.

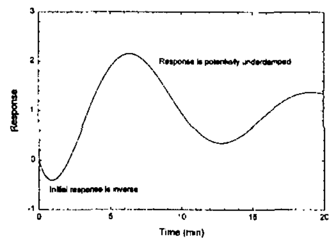


Figure 3: Typical response of an uncontrolled third order inverse response process (TOPIR).

$$G(s)=\frac{K_2}{\tau^2 s^2+2\xi\tau s+1}-\frac{K_1}{\tau_1 s+1}$$

$$=\frac{-(K_1\tau^2)+(K_2\tau_1-2K_1\xi\tau)s+(K_2-K_1)}{(\tau^2 s^2+2\xi\tau s+1)(\tau_1 s+1)}$$

This transfer function will display inverse response if $K_2 > K_1$.

Methods

Controller tuning constants were chosen based on the Cohen-Coon (CC), Ziegler-Nichols (ZN), or Tyreus-Luyben (TL) methods. The stability limit required for the ZN and TL methods were calculated analytically from the Routh-Hurwitz matrix of the closed-loop feedback transfer function. The first-order-plus-dead time fit for the CC method was found from the process response (in the time domain) using the curve fitting feature of SigmaPlot 5.0 (SPSS Inc., 1999). Once the controller constants had been determined the controlled process was studied using the Matlab Simulink 1.2c program (The MathWorks, Inc., 1993) using $4^{th}/5^{th}$ order Runge-Kutta integration method. A tolerance of 10^{-5} was used for all simulations, and the stop time for the simulation was chosen based on the speed of the process response. Both open-loop and closed-loop Bode plots were prepared analytically from the transfer functions ($G(i\omega)$) of the systems.

A performance criteria which has not (to this authors knowledge) been noted in the literature, but is of obvious suitability for control of inverse responses is the percent inverse response, which is defined as the magnitude of the inverse response divided by the magnitude of the final response $\times$ 100 %. Obviously a good controller should minimize this criteria. The inverse response, and the other performance criteria,were calculated numerically from the results of the Matlab simulation.

Results and Discussion

Characteristics of the response of third order inverse response systems.

Initially a study was made of the uncontrolled response of third order processes with inverse response. This showed some interesting characteristics of these processes, and hence will be discussed here. The processes considered consist of competing first and second order systems, which allow for variation in several parameters. The second order system is modeled in terms of the process gain (K_2), natural period of oscillation (τ), and damping coefficient (ξ). In the current work the time constant (τ_1) for the first order system is set at one minute for all simulations, as the relative speeds of the first and second order systems are more informative than actual values. The gain of the first order system (K_1) is set at negative one, which when combined with a gain of two on the second order system, will result in an ultimate value of (plus) one for the open loop response. The damping coefficient for the second order system was varied between 0.5 and 2.0, and the natural period of oscillation was varied between 0.5 and 2.0 minutes. The damping coefficients were chosen so that the response of the inverse response process with underdamped, critically damped, and overdamped second order systems was studied. The natural period of oscillation values were chosen so that the second order system would have speeds different to, but of the same order of magnitude, as the first order part of the system. The input to the process was a step change with a magnitude of one.

The results of the open loop simulation with the second order system having a natural period of oscillation of 1.0, and damping coefficients of 0.5, 1.0, and 2.0 are shown in Figure 4. The response of the TOPIR process is not oscillatory for damping coefficients of 1.0 and 2.0, but it is oscillatory when the damping coefficient is 0.5. This result is not unexpected: when the damping coefficient is less than one the second order system is underdamped, and this will be seen in the response of the TOPIR process.

The results of the open loop simulations with the second order system having a damping coefficient of 1.0, and natural period of oscillation of 0.5, 1.0, and 2.0 are shown in Figure 5. The responses should not be oscillatory, as the transfer function for the proce ss will have real poles for all of these cases, while oscillatory response is characteristic of processes which have complex conjugate poles. For example the transfer function of the case with a natural period of oscillation of 0.5, and a damping coefficient of 1.0 is:

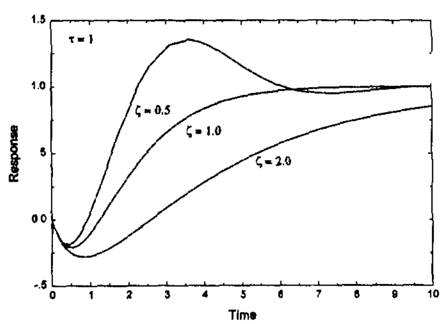


Figure 4: Uncontrolled response of TOPIR with $\tau_1 = 1, K_1 = 1, \tau = 1$ and $\zeta = 0.5, 1.0,$ and 2.0

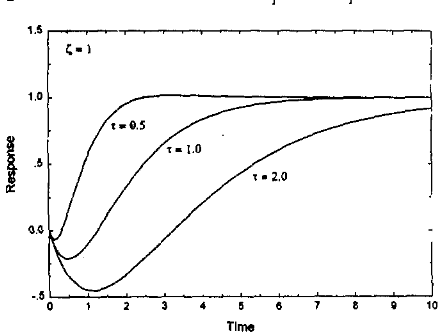


Figure 5: Uncontrolled response of TOPIR with $\tau_1 = 1, K_1 = 1, \zeta = 1,$ and $\tau = 0.5, 1.0,$ and $2.0.$

$$G(s) = \frac{-s^2 + 4s + 4}{(s+2)^2(s+1)}$$

which has poles at -2 (repeated twice), and -1. However the response of this process has a maximum at t = 3.2617 minutes, where y = 1.0162. When the natural period of oscillation is less than 0.5 this maximum value is larger. The steady state value of the response is one, so the maxima before the steady state may suggest oscillatory behavior to those unaware of the poles. To find the truth we may look at the time domain response of the transfer function above with a unit step change input. This is:

$$y(t) = 1 + e^{-t} - 2e^{-2t}(1 + 2t)$$

Clearly this is not a sine function. The maxima and minima of the function may be found by setting the time derivative of the function to zero:

$$\frac{dy}{dt} = -e^{-t} + 8te^{-2t} = 0$$

This shows a minima at 0.1444 minutes (having y = -0.066) due to the inverse response, and the maxima at 3.2617 minutes. Clearly this maxima is due to the differing speeds of the first and second order response at different times: initially the second order response is slower than the first response of the inverse response process. The speeds of the two competing processes are compared in Figure 6. In this case the gain of the first order process has been changed to +2.0 to make visual comparison easier.

Control of third order inverse response processes using Ziegler-Nichols and Tyreus-Luyben tuning.

The TL and ZN methods are discussed together since they both use the ultimate response as a means of determining tuning constants. The experiments performed investigate the tuning constants chosen, and controlled response, as the damping of the second order response and the *relative* speeds of the first and second order responses are varied. For this reason the gains of the first and second order processes are not varied (they are set at -1.0 and +2.0 respectively), and the time constant of the first order process is also not varied (it is set at 1.0 minute). The natural period of oscillation (τ) of the second order process is varied, having values of 0.5, 1.0, and 2.0 minute. These values are chose so that the second order process is faster than the first order process for some simulations, and slower for others. The damping coefficient of the second

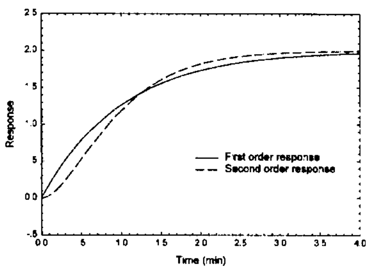


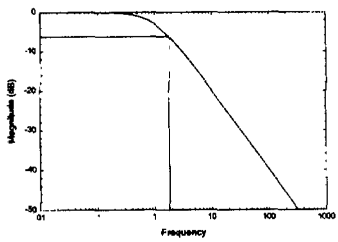
Figure 6: A comparison of the speed of a first order response $\left(\tau_1 = 1\right)$ and a second order response $\left(\tau = 0.5, \zeta = 1\right)$

order response, since second order responses are initially sluggish, which gives the inverse response. After a certain period of time however, the second order response becomes quicker than the first order response due to the natural period of oscillation being much less than the time constant of the first order process. The second order system approaching its ultimate value of 2.0 before the first order system reaches its ultimate value of -1.0 causes a maximum in the order process is also varied, (having values of 0.5, 1.0, and 2.0 also). These values were chosen so that the second order process is underdamped for some simulations, and overdamped for others.

For each process, values for the ultimate gain and ultimate period were calculated from the Routh stability criteria, and then confirmed with the frequency response, $G(i\omega)$: the ZN and TL controller constants were then evaluated from

Table 4: Controller tuning constants for the Ziegler-Nichols and Tyreus-Luyben methods.

			Ziegler-Nichols			Tyreus-Luyben		
ζ and τ(min)	K_u	P_u	K_c	τ_I(min)	τ_D(min)	Kc	τ_I(min)	τ_D(min)
1. $\zeta=1.0$, $\tau=1.0$	2.00	3.70	1.18	1.85	0.462	0.909	8.14	0.587
2. $\zeta=1.0$, $\tau=0.5$	4.16	1.27	2.45	0.635	0.159	1.89	2.79	0.202
3. $\zeta=1.0$, $\tau=2.0$	1.18	7.72	0.692	3.86	0.965	0.535	17.0	1.22
4. $\zeta=0.5$, $\tau=1.0$	1.30	1.82	0.766	1.73	0.432	0.592	7.61	0.549
5. $\zeta=2.0$, $\tau=1.0$	2.00	6.28	1.18	3.14	0.785	0.909	13.8	0.997



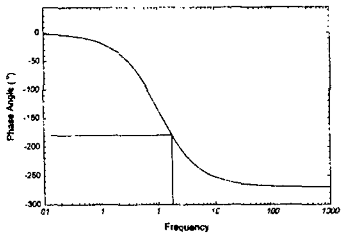


Figure 7: Bode plot of the open loop TOPIR process with $\tau = 1, \zeta = 1$ showing the critical frequency $\left(\omega_{co} \approx 1.7\text{min}^{-1}\right)$ and the magnitude at the critical frequency $(M \approx -6.1\text{dB})$.

the results. The values of the ultimate gain, ultimate period, and control tuning constants for each process studied are given in Table 4. Figure 7 shows a Bode plot of system 1 in Table 4, and is shown as an example of the open loop frequency response. This shows the value of the crossover frequency is 1.7 (which gives an ultimate period of 3.7 minutes), and the value of the magnitude at the crossover frequency is M = - 6.0 dB, which results in an ultimate gain of 2.0. Figure 8 shows a simulation of the closed loop response of system 1 controlled with a proportional controller with a controller gain of 2.0. This confirms the values of the ultimate gain (since the peaks are constant amplitude) and ultimate period, which is labeled T on the graph.

Figure 9 shows an example closed loop response using controller constants predicted by the ZN and TL methods (along with the Cohen-Coon method which will be discussed later). The TL method displays very slow response, while the ZN method produces an acceptable closed loop response. The open loop Bode plots for the system plus controller (Figure 10) show why the ZN controlled process is faster: it displays higher amplitude ratios for all frequencies, as would be expected since the ZN method results in higher controller gains and lower integral time constants than the TL method. This is further shown by the Bode plot of the closed loop response of the controlled process (Figure 11): the ZN method displays a maximum magnitude of almost 2.5 dB (at a frequency of 1.3), where the maximum for the TL method is the low frequency asymptote of 0 dB. The response of the ZN controlled process almost meets the stated aim of the TL tuning (having a maximum in the closed loop log modulus of +2 dB), while the TL tuning does not come close to achieving this aim.

Analysis of the performance criteria reinforce the discussion above. The ZN responses are sometimes too damped, but the TL responses are always far too damped, and in fact always have zero overshoot. It must be noted that the definitions of overshoot and decay ratio (as used in this study) are slightly deceptive. The response may still be slightly underdamped and yet have no decay ratio or overshoot because the initial peak occurs before the system has reached the ultimate response, or in other words before the rise time. The ZN method also gives closed loop response with much shorter rise

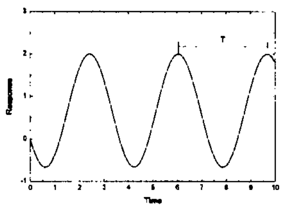


Figure 8: Closed loop response of a Topir process with $\tau = 1, \zeta = 1$ controlled with a proportional controller of gain 2.0. The system critically stable with an ultimate period of 3.7 minutes, and an amplitude ratio of 2.0.

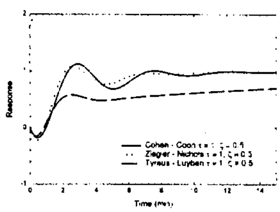


Figure 9: Closed loop response of a Topir process with $\tau = 1, \zeta = 0.5$ controlled with PID control using Cohen-Coon, Ziegler-Nichols, and Tyreus-Luyben control constants.

Figure 10: Open loop Bode plot of the TOPIR process with $\tau = 1, \zeta = 1$ controlled with PID control using Cohen-Coon, Ziegler-Nichols, and Tyreus-Luyben control constants.

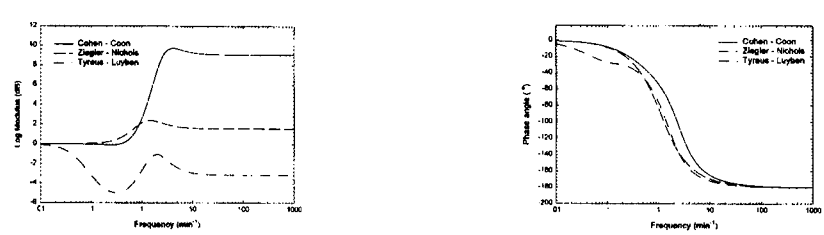


Figure 11: Closed loop Bode plot of the TOPIR process with $\tau = 1, \zeta = 1$ controlled with PID control using Cohen-Coon, Ziegler-Nichols, and Tyreus-Luyben control constants.

times and settling times. It can be seen the integral performance criteria also suggest the ZN constants are an improvement over the TL constants: in essentially all cases the IAE, ISE, and ITAE are all much higher for the TL tuning.

There is not much to be gained from a complete analysis of plant/model mismatch since the ZN method gives closed loop response far enough away from the ultimate response that they are unlikely to become unstable at reasonable values of plant/model errors, and the TL method gives responses which are far too damped. However a few examples of response where the plant (or model of the plant) used to determine the ultimate gain and ultimate period is significantly different from the actual plant.

Figure 12 shows the response of the process in system 1 (Table 4) controlled with both the ZN and TL methods, when the values of both the natural period of oscillation and damping coefficient are modeled as 25 % less than the actual value. (In this case the actual values for the system are = 1.25 minute and ξ= 1.25, however values for the ultimate gain and ultimate period were based on a model where τ = 1.0 minute and ξ = 1.0). The reason for attempting this is that chemical processes are potentially highly time dependent: the tuning constants may be chosen based on a model of the system at startup, however the actual process may change significantly as equipment becomes clogged or scaled. The response of the controlled process (for both TL and ZN constants) is not much different than the process which uses accurate values of the ultimate period and ultimate gain. If both the natural period of oscillation and the damping coefficient were overestimated by 25% in the process model, the controlled response (based on either ZN or TL tuning) becomes significantly more damped, in fact the closed loop processes become overdamped for both TL and ZN control in many cases. The results are shown in Figure 13. The larger values of the damping coefficient and the natural period of oscillation used in the calculation of the ultimate gain and ultimate period has resulted in values of the controller gain which are much lower than necessary for the process, and values of the integral time constant being chosen much higher than necessary.

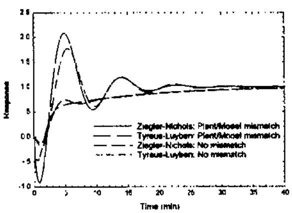


Figure 12: Response of TOPIR processes controlled with PID controllers. For response with plant model mismatch: model constants are $\tau = 1, \zeta = 1$, plant constants are $\tau = 1.25, \zeta = 1.25$. For response without mismatch both plant and model are $\tau = 1, \zeta = 1$

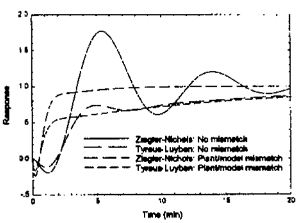


Figure 13: Response of TOPIR processes controlled with PID controllers. For response with plant model mismatch: model constants are $\tau = 1, \zeta = 1$, plant constants are $\tau = 1.25, \zeta = 1.25$. For response without mismatch both plant and model are $\tau = 1, \zeta = 1$

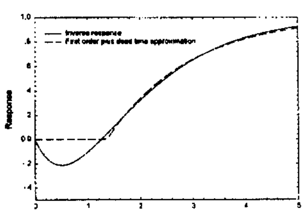


Figure 14: First order plus dead time (FOPDT) fit to a TOPIR with $\tau = 1, \zeta = 1$

Table 5: Performance criteria for Ziegler-Nichols controller tuning.

Criteria	$\zeta=1.0, \tau=1.0$	$\zeta=1.0, \tau=0.5$	$\zeta=1.0, \tau=2.0$	$\zeta=0.5, \tau=1.0$	$\zeta=2.0, \tau=1.0$
Overshoot (%)	17.0	45.3	5.4	8.9	32.2
Rise time (min)	2.41	0.64	6.70	2.14	4.61
Decay ratio	0	0.022	0	0	0.007
Setting time (min)	5.71	1.67	9.00	6.62	12.8
Inverse response (%)	47.8	25.0	65.9	22.2	70.6
IAE	2.43	0.867	5.94	2.38	6.16
ISE	2.31	0.630	7.18	1.64	5.84
ITAE	3.43	0.573	14.2	6.32	24.1

Table 6: Performance criteria for Tyreus-Luyben controller tuning.

Criteria	$\zeta=1.0, \tau=1.0$	$\zeta=1.0, \tau=0.5$	$\zeta=1.0, \tau=2.0$	$\zeta=0.5, \tau=1.0$	$\zeta=2.0, \tau=1.0$
Overshoot (%)	0	0	0	0	0
Rise time (min)	∞	∞	∞	∞	∞
Decay ratio	0	0	0	0	0
Setting time (min)	35.5	1.80	114.0	50.0	57.6
Inverse response (%)	29.5	16.5	41.2	14.6	43.3
IAE	8.93	1.48	31.4	12.8	15.2
ISE	3.63	0.639	13.7	4.67	6.97
ITAE	120.6	4.26	1230	245.9	318

Table 7: Controller tuning constants for the Cohen-Coon method.

	FOPDT approximation			Cohen-Coon		
ζ and τ (min)	K_p	τ_p (min)	τ_d (min)	K_p	τ_p (min)	τ_d (min)
1. $\zeta=1.0, \tau=1.0$	1.00	1.518	1.369	1.73	2.53	0.428
2. $\zeta=1.0, \tau=0.5$	1.00	0.601	0.457	2.00	0.876	0.146
3. $\zeta=1.0, \tau=2.0$	1.00	2.870	3.325	1.40	5.82	0.999
4. $\zeta=0.5, \tau=1.0$	1.00	0.368	1.237	0.647	1.62	0.279
5. $\zeta=2.0, \tau=1.0$	1.00	3.948	2.671	2.22	5.23	0.865

Table 8: Performance criteria for Cohen-Coon controller tuning.

Criteria	$\zeta=1.0, \tau=1.0$	$\zeta=1.0, \tau=0.5$	$\zeta=1.0, \tau=2.0$	$\zeta=0.5, \tau=1.0$	
$\zeta=2.0, \tau=1.0$	Overshoot (%)	12.3 5.45	UNSTABLE	12.5	UNSTABLE
Rise time (min)	1.52	0.89	-	2.31	-
Decay ratio	0	0	-	0.058	-
Setting time (min)	2.70	5.28	-	10.6	-
Inverse response (%)	91.8	16.2	-	17.7	-
IAE	1.77	1.17	-	2.71	-
ISE	2.38	0.605	-	1.80	-
ITAE	1.30	2.63	-	8.00	

Control of third order inverse response processes using Cohen-Coon tuning.

The ability of the first order plus dead time (FOPDT) model to fit the inverse response process is (as expected) not very good. The FOPDT model cannot model the inverse response except to give it as pure dead time. In fact the model prediction of the dead time is in all cases very close to the time period of the negative response. The positive part of the response is fitted acceptably well by the FOPDT model if the second order part of the process is overdamped, but the fit is significantly worse if the second order part is underdamped. An example of the FOPDT fit is given in Figure 14. Once the results of the fit are known then it is possible to use the CC equations to determine the tuning constants. Both the FOPDT fitting parameters and the CC tuning constants are shown in Table 7. Examples of the open and closed loop Bode plots of the CC controlled system are shown in Figures 10 and 11 while an example response is shown in Figure 9. From the response of the controlled process we may calculate the performance criteria and these are shown in Table 8. It is obvious that the CC method cannot give adequate estimates for tuning constants for TOPIR processes. In two of the five cases studied the controlled response becomes unstable. In the other three cases the CC method gives performance criteria which compare well to the ZN method and are much better than the TL method, however this is little compensation for the possibility that in other cases the response is unstable, and therefore totally unsuitable.

It is noticeable that the closed loop performance of the CC method occurs when the second order process is slow, either when the natural period of oscillation is large, or when the damping coefficient is large. This may result because the period of the inverse response is longer, and possibly the magnitude of the inverse response is larger. Because the FOPDT model cannot model the inverse part of the response, it is probable that when the inverse response is very large, the tuning constants chosen will be poor. A slightly more surprising result is that the CC method did reasonably well for the process which was underdamped! It appears that the method has more trouble to choose constants for inverse response processes than for underdamped ones.

The performance criteria for the CC method are not very informative since two of the five cases studied have unstable closed loop responses. The other three cases show reasonable performance criteria, but obviously in the case of TOPIR processes the CC method produces closed loop responses which are too oscillatory. No attempt was made to study plant/model mismatch for the CC tuning since the method produces unsuitable results for no plant/model mismatch.

Conclusions

The Ziegler-Nichols method clearly gives the best settings of the three controller tuning methods for the third order inverse response processes studied here. The Cohen-Coon method suffered from the inability of the first order plus dead time model in approximating the inverse response process, particularly those which are very slow and thus have long periods of inverse response. The Cohen-Coon method produced closed loop responses which were very underdamped, and in some cases the response was unstable. This is clearly unacceptable even for choosing initial tuning constants for on-line improvement. The Tyreus-Luyben method produced closed loop response which were too damped, and in some cases overdamped. Although this is better than producing an unstable response, it is clearly not optimal. The Tyreus-Luyben method was never close to achieving its stated aim of producing closed loop systems with a maximum log modulus of +2 dB: the Ziegler-Nichols method, although not designed for this criteria, achieved the aim far better. It must be noted that the conclusions of this paper only relate to control of third order inverse response processes: no conclusions should be made relating to the suitability (or unsuitability) of the tuning methods on other types of process.

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