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Attached documents :
1) Tabulated wiggler field distribution
2) Fortran source code LEID.f for calculations of linear effects of ID
3) Fortran source code FIAW.f for calculations of field integrals and wiggler focusing parameter
4) Output file from FIAW.f

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Linear effects of the planned 7.5 Tesla superconducting wiggler on the beam parameters of the Siam Photon Source are investigated. The effects arising from focusing properties of the wiggler field are taken into account via effective thin-lense focusing matrix of the wiggler. Analytical expressions for the tune shift, stopband and betatron shift are derived and calculated. Dependence of the linear effects on the original betatron functions and the wiggler peak field are investigated. Matching condition to reduce the perturbation is also suggested.

1. Wiggler focusing matrix

Due to longitudinal variation of the wiggler field and the deflection of the electron beam in the horizontal plane a wiggler acts as a focusing device in the vertical plane. The averaged focusing parameter is given by $K_y = \langle 1/\rho^2 \rangle$, where ρ is the radius of curvature. For a wiggler with length L a transfer matrix for such focusing element is therefore [1]

$$\mathbf{M} = \begin{bmatrix} \cos\theta & \beta_K \sin\theta \\ -\frac{1}{\beta_K}\sin\theta & \cos\theta \end{bmatrix} \tag{1}$$

where $\theta = \sqrt{K_y}L$ and $\beta_K = 1/\sqrt{K_y}$. The effective thin-lens focusing matrix $\mathbf{M}_{\mathrm{eff}}$ acting at the center of the wiggler may be constructed by multiplying the wiggler focussing matrix from the left and from the right by an inverse of the straight section matrix [2]

$$\mathbf{M}_{\mathrm{eff}} = \begin{bmatrix} 1 & -L/2 \\ 0 & 1 \end{bmatrix} M \begin{bmatrix} 1 & -L/2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta^* & \beta^* \sin\theta^* \\ -\frac{1}{\beta^*}\sin\theta^* & \cos\theta^* \end{bmatrix} \tag{2}$$

where

$$\cos\theta^* = \cos\theta + \frac{\theta\sin\theta}{2} \quad , \tag{3}$$

$$\beta^* = L\left(\frac{1}{\theta^2} - \frac{\cot\theta}{\theta} - \frac{1}{4}\right)^{1/2} \tag{4}$$

and

$$\sin\theta^* = \frac{\beta^*}{\beta_K}\sin\theta \quad . \tag{5}$$

2. Tune shift

The linear tune shift due to the wiggler field can be calculated by perturbation method [3,4]. The unperturbed, i.e. wiggler-off, one-turn matrix is

$$\mathbf{M}_{\mathbf{s\to s+C}} = \begin{bmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{bmatrix} \quad . \tag{6}$$

where μ is the unperturbed one-turn phase-advance and α, β and γ are the unperturbed betatron functions. The perturbed, i.e. wiggler-on, one-turn matrix is then

$$\mathbf{M}_{\mathbf{W}} = \begin{bmatrix} \cos\mu_W + \alpha_W\sin\mu_W & \beta_W\sin\mu_W \\ -\gamma_W\sin\mu_W & \cos\mu_W - \alpha_W\sin\mu_W \end{bmatrix} \quad . \tag{7}$$

where the the subscripts W indicates the perturbed parameters. This perturbed one-turn matrix can also be constructed by multiplying $\mathbf{M}_{\mathbf{s\to s+C}}$ by the wiggler focusing matrix $\mathbf{M}_{\mathbf{eff}}$. This gives

$$\mathbf{M}_{\mathbf{W,eff}} = \mathbf{M}_{\mathbf{s\to s+C}}\mathbf{M}_{\mathbf{eff}}$$
$$= \begin{bmatrix} (\cos\mu + \alpha\sin\mu)\cos\theta^* - \frac{\beta}{\beta^*}\sin\mu\sin\theta^* & (\beta^*\cos\mu + \alpha\beta^*\sin\mu)\sin\theta^* + \beta\sin\mu\cos\theta^* \\ -\gamma\sin\mu\cos\theta^* - (\cos\mu + \alpha\sin\mu)\frac{\sin\theta^*}{\beta^*} & (\cos\mu - \alpha\sin\mu)\cos\theta^* - \beta^*\gamma\sin\mu\sin\theta^* \end{bmatrix} \tag{8}$$

Comparing the two perturbed one-turn matrices the perturbed one-turn phase-advance can then be obtained from

$$\cos\mu_W = \frac{1}{2}\mathrm{Tr}\left[\mathbf{M}_{\mathbf{W,eff}}\right] = \cos\mu\cos\theta^* - \frac{\sin\mu\sin\theta^*}{2}\left(\gamma\beta^* + \frac{\beta}{\beta^*}\right) \tag{9}$$

where β is the unperturbed betatron function at the wiggler position. The tune-shift is then given by

$$\Delta\upsilon = \frac{\Delta\mu}{2\pi} = \frac{\mu_W - \mu}{2\pi} \quad . \tag{10}$$

3. Stopband width

The stopband is produced when $|\cos\mu_W| > 1$, which can readily be evaluated from (9). We rewrite (9) in the form

$$\cos\mu_W = a\cos\mu - b\sin\mu \tag{11}$$

where

$$a = \cos\theta^* \qquad \text{and} \qquad b = \frac{\sin\theta^*}{2}\left(\gamma\beta^* + \frac{\beta}{\beta^*}\right) \quad .$$

With $|\cos\mu_W| > 1$ the equation for the stopband may be written, from (11),

$$(a\cos\mu - b\sin\mu)^2 > 1 \tag{12}$$

From the above equation the stopband width can then be evaluated from the quadratic equation

$$(b^2 - 1)\tan^2\mu - 2ab\tan\mu + (a^2 - 1) > 0 \quad . \tag{13}$$

This gives the equation for the stopband boundary points

$$\tan\mu = \frac{ab \pm \sqrt{b^2 + a^2 - 1}}{b^2 - 1} \quad . \tag{14}$$

4. Change in betatron function

The focusing property of the wiggler causes a change in the betatron function around the ring. To calculate the change in betatron function we adopt the perturbation methods of Refs. [1] and [2] as the following.

Assume the ring has at least two-fold symmetry with the symmetry points at the wiggler position and the position opposite the wiggler. Starting at the symmetry point, in which $\alpha = 0$, opposite the wiggler the one-turn matrix is

$$\mathbf{M}_{0W0} = \begin{bmatrix} \cos\frac{\mu}{2} & \beta_0\sin\frac{\mu}{2} \\ -\gamma\sin\frac{\mu}{2} & \cos\frac{\mu}{2} \end{bmatrix} \begin{bmatrix} \cos\theta^* & \beta^*\sin\theta^* \\ -\frac{1}{\beta^*}\sin\theta^* & \cos\theta^* \end{bmatrix} \begin{bmatrix} \cos\frac{\mu}{2} & \beta_0\sin\frac{\mu}{2} \\ -\gamma\sin\frac{\mu}{2} & \cos\frac{\mu}{2} \end{bmatrix} . \tag{15}$$

We can then obtain the perturbed betatron function at the point opposite the wiggler from the m_{12} element of the above one-turn matrix

$$\beta_W \sin\mu_W = \beta_0\sin\mu\cos\theta^* + \frac{\sin\theta^*}{\beta^*}\left(\beta^{*2}\cos^2\frac{\mu}{2} - \beta_0^2\sin^2\frac{\mu}{2}\right), \tag{16}$$

where β_0 and β_W are the unperturbed and the perturbed betatron functions, respectively, at the point opposite the wiggler. Value of $\sin\mu_W$ can be found from Eq.

(9). We note however that Eq.(9) does not give the sign of the perturbed phase advance. The sign of $\sin\mu_W$, and hence μ_W, can be determined by the requirement that the betatron function is always positive.

To find the perturbed betatron function at other points we write down the beam matrix at some starting point [4]

$$\mathbf{A}(0)=\begin{bmatrix}\beta & -\alpha\\ -\alpha & \gamma\end{bmatrix}. \tag{17}$$

We may then write for the perturbed beam matrix

$$\begin{bmatrix}\beta_W & -\alpha_W\\ -\alpha_W & \gamma_W\end{bmatrix}=\begin{bmatrix}\beta_0 & -\alpha_0\\ -\alpha_0 & \gamma_0\end{bmatrix}+\begin{bmatrix}\beta_W-\beta_0 & -(\alpha_W-\alpha_0)\\ -(\alpha_W-\alpha_0) & \gamma_W-\gamma_0\end{bmatrix}. \tag{18}$$

For a transformation matrix $\mathbf{T}(s)$,

$$\mathbf{T}(s)=\begin{bmatrix}t_{11} & t_{12}\\ t_{21} & t_{22}\end{bmatrix}\quad , \tag{19}$$

which transforms the beam from the starting point to point s, the beam matrix at s is then given by [4]

$$\mathbf{A}(s)=\mathbf{T}\ \mathbf{A}(0)\ \mathbf{T}^T\quad , \tag{19}$$

where $\mathbf{T}^T$ is the transpose of the matrix $\mathbf{T}$. We can therefore obtain the change in the beam matrix from, taking into account the fact that $\alpha=0$ at symmetry points,

$$\begin{bmatrix}\Delta\beta(s) & -\Delta\alpha(s)\\ -\Delta\alpha(s) & \Delta\gamma(s)\end{bmatrix}=\begin{bmatrix}t_{11} & t_{12}\\ t_{21} & t_{22}\end{bmatrix}\begin{bmatrix}\beta_W-\beta_0 & 0\\ 0 & \gamma_W-\gamma_0\end{bmatrix}\begin{bmatrix}t_{11} & t_{21}\\ t_{12} & t_{22}\end{bmatrix}\quad . \tag{20}$$

Evaluating the m_{11} element of the above matrix gives

$$\Delta\beta(s)=(\beta_W-\beta_0)\left(t_{11}^2-\frac{t_{12}^2}{\beta_W\beta_0}\right)\quad . \tag{21}$$

The transformation matrix which transforms the beam from a symmetry point to point s is given by [4]

$$\mathbf{T}_{0\to s}=\begin{bmatrix}\sqrt{\dfrac{\beta(s)}{\beta_0}}\cos\mu(s) & \sqrt{\beta_0\beta(s)}\sin\mu(s)\\ -\dfrac{\sin\mu(s)+\alpha(s)\cos\mu(s)}{\sqrt{\beta_0\beta(s)}} & \sqrt{\dfrac{\beta_0}{\beta(s)}}\left[\cos\mu(s)-\alpha(s)\sin\mu(s)\right]\end{bmatrix}\quad . \tag{22}$$

Together with (21) we obtain the change in the betatron function

$$\frac{\Delta\beta(s)}{\beta(s)} = \frac{(\beta_W - \beta_0)}{\beta_0}\left[1 - \sin^2\mu(s)\left(1 + \frac{\beta_0}{\beta_W}\right)\right] \quad . \tag{23}$$

From (23) the maximum change of the beatraton function depends on the relative values of β_W and β_0. In the case of $\beta_W > \beta_0$ the maximum betatron change is reached when the quantity in the square bracket is reaching the value 1. In the case of $\beta_0 > \beta_W$ the betatron change reaches the maximum when the quantity in the square bracket is reaching $-\beta_0 / \beta_W$. The maximum changes of the betatron function due to the wiggler field may therefore be summarized as the following

$$\left(\frac{\Delta\beta(s)}{\beta(s)}\right)_{\max} = \begin{cases} \dfrac{\beta_W - \beta_0}{\beta_0} & , \beta_W > \beta_0 \\ \\ \dfrac{\beta_0 - \beta_W}{\beta_W} & , \beta_W < \beta_0 \end{cases} \quad . \tag{24}$$

5. The planned 7.5 T superconducting wiggler for SPS storage ring

For the planned superconducting wiggler we calculate the wiggler focusing parameter numerically. The focusing parameter is then

$$K_y = \left\langle \frac{1}{\rho^2} \right\rangle = \frac{1}{L}\left(\frac{0.3}{E}\right)^2 \int_{-L/2}^{L/2} B^2(s)\, ds \quad . \tag{25}$$

The integration limits extend over the wiggler magnetic length. The limits are then chosen at the points where the integrated focusing strength, $K_y L$, converges. The wiggler magnetic field is simulated from a modeled three-pole wiggler magnet, reported in [5]. The wiggler magnetic length used is $L = 0.5\, m$. This gives the following parameters for the wiggler

$$K_y = 0.846, \quad \theta = 0.46, \quad \beta_K = 1.09, \quad \beta^* = 0.15, \quad \cos\theta^* = 0.998 \quad . \tag{26}$$

From these parameters we obtain the calculated betatron shift, tune shift and stopband width using Eqs. (9), (10), (14), (16) and (24). The calculated values are $(\Delta\beta / \beta)_{\max} = 1.31$, $\Delta\upsilon = 0.13$ and $\Delta\upsilon_{stopband} = 0.23$. The residual tune value used in the calculations is 0.18 and the original betatron function is 4.29 *m*.

6. Matching

From the above results it can be seen that installation of the superconducting wiggler could cause serious effects on the SPS storage ring operation. This undesirable result is to be expected from installation of such high field wiggler in such low energy machine. Nevertheless, the effects can be substantially reduced by various correction methods. One of the methods is retuning the storage ring to find suitable operation point for operation with the wiggler on. This matching condition may be considered from the linear effects obtained above.

For the wiggler located at a symmetery point, i.e. $\gamma = 1/\beta$, Eq.(9) can be rewritten, using $\cos(\mu+\theta^*) = \cos\mu\cos\theta^* - \sin\mu\sin\theta^*$,

$$\cos\mu_W = \cos(\mu+\theta^*) - \sin\mu\sin\theta^* \frac{(\beta^* - \beta)^2}{2\beta\beta^*} \quad . \tag{27}$$

We then consider the betatron function in Eq.(16). Using the identities $\sin(\mu+\theta^*) = \sin\mu\cos\theta^* + \cos\mu\sin\theta^*$ and $\cos\mu = \cos^2\mu/2 - \sin^2\mu/2$ we can rewrite Eq.(16) as

$$\beta_W \sin\mu_W = \beta_0\left[\sin(\mu+\theta^*) + (\beta^* - \beta_0)\sin\theta^*\left(\frac{\cos^2\mu/2}{\beta_0} + \frac{\sin^2\mu/2}{\beta^*}\right)\right]. \tag{28}$$

From (27) and (28) one matching condition becomes apparent [2]. It can be seen that if the condition

$$\beta_0 = \beta^* \tag{29}$$

is fulfilled we immediately obtain the results,

$$\mu_W = \mu + \theta^*, \qquad \beta_W = \beta_0 = \beta^* \quad . \tag{30}$$

The betatron shift of Eq.(24) is therefore zero. Moreover, from these results we also see from Eq.(11) that $b = \sin\theta^*$, and hence $b^2 + a^2 = 1$. The stopband width, Eq.(14), therefore becomes zero.

With the matching condition (29) the betatron shift and the stopband width can therefore be minimized. The storage ring can then be operated without lattice function change, though the tune shift still remains. It is to be noted, however, that the perfect match is dependent on β^*, which in turn depends on the focusing parameter. It can therefore be realized for only one field value of the wiggler.

7. Betatron function dependence

As clearly seen that the linear effects of the wiggler on the ring parameters are dependent on the betatron function at the wiggler position. These effects are shown as a function of storage ring's unperturbed betatron function at the wiggler position in Figures 1a, 1b and 1c.

It can be seen that the betatron shift and the stopband width are zero at the matched value of betatron function. The tune shift is minimum but nonzero at this betatron function. To reduce the perturbation from the wiggler it should then be preferable to retune the storage ring to make the betatron function close to the matched value. However, though this matching method is apparently helpful it may not be practical in many cases. Since the betatron function has to be matched to β^*, which is usually very small. The matching attempt will put the storage ring operation under a heavy constraint, which may not be realizable in practice

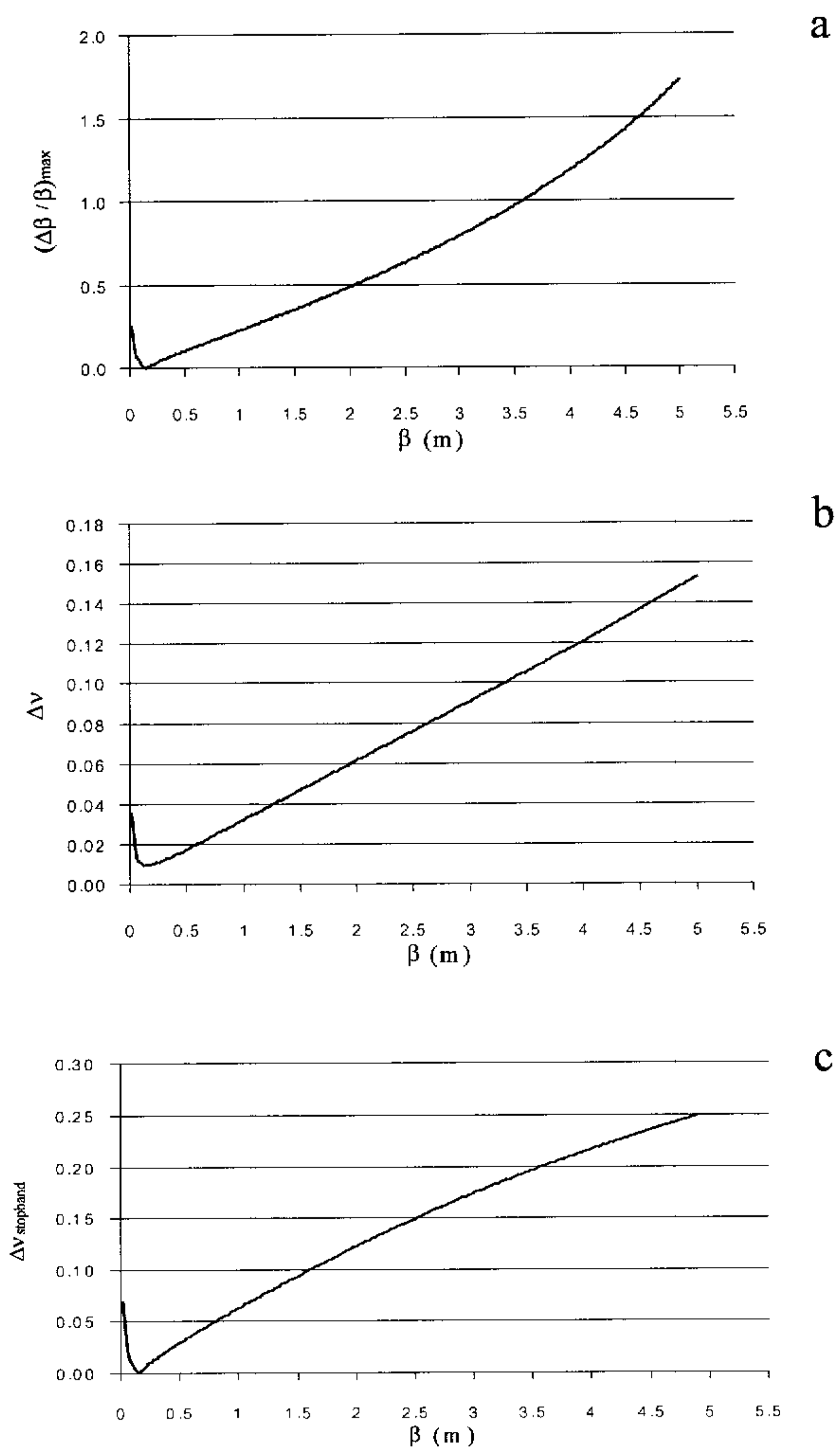


Figure 1 : Calculated maximum betatron shift (a), vertical tune shift (b) and stopband width (c) caused by the 7.5 T superconducting wiggler as a function of unperturbed betatron function at the wiggler position. The residual tune is 0.18.

8. Perturbation as a function of wigger peak field

It is expected that the aimed operating peak field of 7.5 Tesla of the wiggler in the 1 GeV storage ring will be difficult. To estimate the practicality of operating the wiggler we investigate here how the perturbation scales with the wiggler peak field.

For simplicity in the following calculations we will assume the sinusoidal field distribution for the wiggler and use the focusing parameter averaged over the wiggler period. With the beam energy of 1 GeV the focusing parameter of the planned 7.5 T superconducting wiggler is therefore,

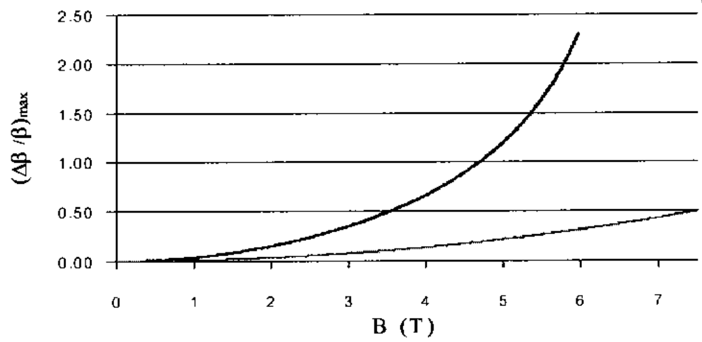
$$K_y = \left\langle \frac{1}{\rho^2} \right\rangle = \frac{1}{2\rho_0^2} = \frac{0.045\, B_0^2}{E^2} = 2.53 \quad . \tag{31}$$

With an assumed magnetic length of $L = 0.35\, m$, this gives the following parameter for the wiggler

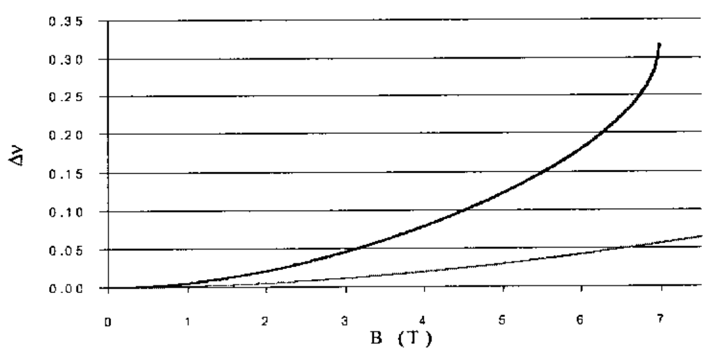
$$\theta = 0.56, \quad \beta_K = 0.63, \quad \beta^* = 0.11, \quad \cos\theta^* = 0.996 \quad . \tag{32}$$

Figures 2a, 2b and 2c show the betatron shift, the tune shift and the stopband width as a function of wiggler peak field. The calculations are for two values of the unperturbed vertical betatron function, 4.29 m for the present operation and 1.00 m to see effectiveness of the matching.

a



b



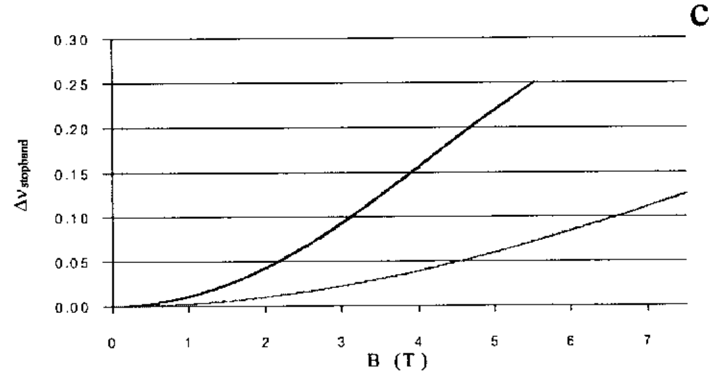


Figure 2 : Calculated maximum betatron shift (a), vertical tune shift (b) and stopband width (c) caused by the superconducting wiggler as a function of wiggler peak magnetic field. The blue lines are for the unperturbed betatron function of 4.29 m and the red lines are for 1m. The residual tune is 0.18.

From the calculations it is seen that effects of the wiggler on the storage ring will be serious at high operating peak field. The effects are especially large for large betatron function. It is noted here that for the betatron function of 4.29 m the betatron shift and the tune shift cannot be calculated when the wiggler peak field is higher than about 6 Tesla and 7 Tesla, respectively. In the same case, the quadratic equation for the stopband, Eq.(13), becomes upright when the wiggler peak field is higher than about 5.5 Tesla. This indicates instability of operation at such condition. However, the calculations are based on an assumed sinusoidal wiggler field. The focusing parameter in this case is larger than that calculated from simulated wiggler field. The results obtained in these calculations are therefore exaggerated.

Nevertheless, it is seen that the effects increase rapidly with increasing wiggler peak field. It is also clearly seen that the linear effects decrease substantially in case of very low betatron function. If such low betatron function can be achieved it should be reasonable to expect practical operation of the wiggler at maximum field of 7.5 Tesla. The remaining perturbation will have to be corrected by some effective correction methods.

9. Conclusions

From the calculations the focusing parameter of the planned wiggler is calculated from simulated wiggler field to be $K_y = 0.846 \quad m^{-2}$, with the integrated focusing strength of $K_y L = 0.423 \quad m^{-1}$. The betatron shift, tune shift and stopband width are calculated for the original betatron function of 4.29 m and the operating residual tune of 0.18. The calculated values are $(\Delta\beta/\beta)_{\max} = 1.31$, $\Delta\upsilon = 0.13$ and $\Delta\upsilon_{stopband} = 0.23$. The matching condition is found for the betatron function of $\beta = 0.15 \quad m$ at the wiggler position.

Results of the calculations suggest very large perturbation caused by the wiggler. Possibility of retuning the storage ring to adjust the betatron function at the wiggler position to match the above matching value has to be further investigated. Effective correction method to correct the remaining perturbations will have to be sought.

References

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[3] E. D. Courant and H. S. Snyder, *Theory of the Alternating-Gradient Synchrotron*, Ann. Phys. 3(1958)1.
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