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Effects of slurry concentration and powder filling on the net mill power of a laboratory ball mill

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Abstract

The tests covered a range of slurry concentrations from 30 to 55 vol.% solid and fractional interstitial bed filling (U) from 0.3 to 1.75, at a fixed ball load (30% of mill volume) and 70% of critical speed, using batch grinding of a feed of -30 mesh (0.6 mm) quartz. At a fixed slurry concentration, the net mill power versus U went through a maximum, and both the optimum value of U for maximum power and the maximum power varied with slurry concentration. A slurry concentration of about 40 to 45 vol.% solid and a U of approximately 1 gave the maximum power. An empirical equation is given that fits the data reasonably well. It was concluded that the fraction of the slurry that rotates with the balls, the inclination of the resulting rotating charge, and the expansion of the rotating charge are all factors that affect the power; the data were too limited to obtain precise descriptions of these effects. At a constant powder load ($U=1$), the specific rates of breakage of the quartz were linearly proportional to the net mill power for the different slurry concentrations. However, this was not true if the mill power was changed by using water–glycerine carrier liquids to vary the viscosity, at a fixed solid concentration (45%). Thus, mill power alone does not define the breakage action.

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Keywords: Mill power; Ball milling; Wet grinding; Slurry density; Powder filling

1. Introduction

Correlations between mill power and the size and operating conditions of the mill are essential elements in the design of tumbling media mills. Attempts to develop fundamentally based expressions for the calculation of mill power date from Davis [1] in 1919 to the present, but Cilliers et al. [2] have concluded from their own studies and those of others [3–7] that a fundamentally based prediction of mill power is still not available. There are, of course, equations that are used for mill design [8], but these are generally based on very simple mechanical models that are used to codify a large amount of empirical experience. For example, the equation of Bond [9] applies for the calculation of the mill power of large industrial (cylindrical) ball mills, where the mass of the media in the mill is much larger than that of the material being ground. A similar treatment exists for the conical (Hardinge) type of ball mill [10], and Austin [11] and Morrell [12] have extended the method to industrial semi-autogenous (SAG) mills, where the mass and size of the material being ground are comparable to those of the media. Shoji et al. [13] and Austin et al. [14] give a power equation applicable for smaller ball mills of 0.2 to 2 m in diameter.

None of these treatments includes any term for the influence of the rheology of the slurry in wet grinding, even though the movement of the charge, and hence the mill power, must be affected by the rheological behavior of the slurry. The rheological properties of a slurry are determined [15] primarily by the viscosity of the liquid medium and the volume fraction of solid in the slurry, with a less noticeable effect of the particle size distribution (providing the grinding is not extremely fine). It is likely that slurry concentration does not occur explicitly in these empirical power equations because the liquid is always water, the solid concentration is always in a narrow range (not excessively high or low) known from experience, and the equations only apply for normal product size distributions and normal materials.

In addition, the amount of powder in a mill relative to the amount of media is usually defined by "The fractional powder filling (U) is the fraction of the interstices of the mill bed (at rest in a packed bed condition) that is filled with

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Table 1
Mill and test conditions

Mill	
Inside diameter (m)	0.195
Length (m)	0.175
Speed (rpm)	70
Fraction of critical speed (ϕ_c) (–)	0.70
Size of lifters (mm in diameter)	20
Number of lifters (–)	6
Grinding media	
Material	chrome alloy steel
Diameter (m)	0.0254 (1 in.)
Density (kg/m^3)	7800
Weight (kg)	7.34
Ball bed (J_b), fraction of mill volume (–)	0.30
Feed solid	
Material (mm)	– 30 mesh (0.6 mm) crystalline quartz
Density (kg/m^3)	2650
Powder filling (U), fraction of ball interstitial volume (–)	0.30–1.75
Slurry concentration (vol.% solid)	30–55
Slurry density (kg/m^3)	1495–1908

powder, calculated using a bulk density of the powder and assuming that the porosity of the bed is 0.4 for a bed of balls." This definition is easy to appreciate for dry grinding, but in wet grinding, it is the slurry that fills the interstices, so the amount of solid in the active breakage regions will depend on the slurry concentration as well as U.

This paper will present some results on the effect of slurry concentration and powder filling on mill power for a small laboratory ball mill, with a constant ball load and rotational speed of the mill. Also given is a correlation of the net mill power with the rates of breakage of the test material (crystalline quartz) at a fixed value of solid mass in the mill (i.e., at constant U).

2. Experimental techniques

The test mill was a cylindrical steel mill, with horizontal lifters of semicircular cross section, rotated on a roller table.

Table 2
Particle size distribution of the – 30 mesh quartz feed for mill power measurement

Sieve size (mesh)	Particle size (mm)	Cumulative wt.% less than size
30	0.60	100.0
40	0.42	44.7
50	0.30	26.3
70	0.21	16.8
100	0.15	11.5
140	0.11	7.6
200	0.075	4.8
270	0.053	3.2
400	0.038	2.1

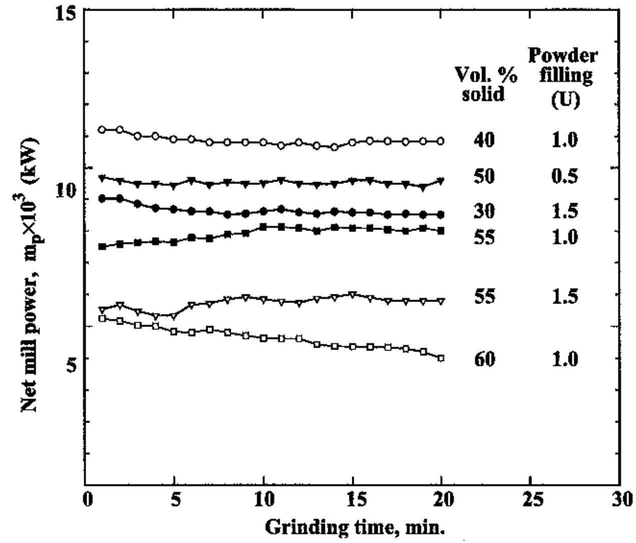

Fig. 1. Typical variations of net mill power with grinding time (– 30 mesh quartz feed, J_b = 0.3, ϕ_c = 0.70).

Table 1 gives the mill characteristics and test conditions. Mill power was measured during batch ball milling of a feed of – 30 mesh quartz in water (see Table 2) as a function of slurry concentration (as vol.% solid in the slurry) and powder filling level. The mill power was measured by a torque transducer (model 1404-200, Lebow Associates) connected between the drive motor and the shaft of the roller table using a flexible coupling. The transducer output was monitored by a torque indicator, and a chart recorder connected to the output of the torque indicator recorded the torque signal during the grinding time. Due to the random fluctuation of the torque reading caused by the ball motion, a 300-μF capacitor was connected in parallel with the recorder to average the signal.

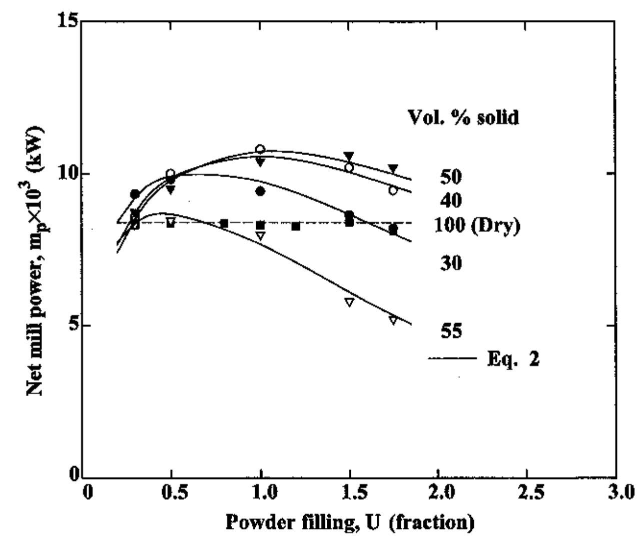

Fig. 2. Effect of slurry concentration and powder filling on mean net mill power (– 30 mesh quartz feed, J_b = 0.3, ϕ_c = 0.70).

Table 3
Experimental results for variable powder filling U

Volume fraction of solid in slurry (C)	Powder filling (U)	Net mill power	
		$m_p \times 10^3$ (kW)	m_p/M_b (kW/ton balls)
0.3	0.3	9.33	1.27
	0.5	9.81	1.34
	1.0	9.42	1.28
	1.5	8.64	1.18
	1.75	8.21	1.12
0.4	0.3	8.59	1.17
	0.5	10.0	1.36
	1.0	10.8	1.47
	1.5	10.2	1.39
	1.75	9.45	1.29
0.5	0.3	8.74	1.19
	0.5	9.50	1.29
	1.0	10.4	1.42
	1.5	10.6	1.44
	1.75	10.2	1.39
0.55	0.3	8.33	1.13
	0.5	8.45	1.15
	1.0	7.99	1.09
	1.5	5.80	0.79
	1.75	5.21	0.71
1.0 (dry)	0.3	8.30	1.13
	0.5	8.36	1.14
	0.8	8.36	1.14
	1.0	8.31	1.13
	1.2	8.27	1.13
	1.5	8.39	1.14
	1.75	8.11	1.10
	2.0	8.02	1.09

To obtain the net torque, hence, the net mill power to the mill, the no-load torque for a given weight of the running mill was subtracted from the measured torque to eliminate the bearing friction losses. The no-load loss was measured by running either one or two empty mills on the roller table in order to vary the applied weights. The no-load torque was linearly proportional to the weight on the roller, enabling the no-load torque to be estimated for any total weight of mill, media, and charge. The net mill power was then calculated.

3. Experimental results

Fig. 1 shows typical variations of net mill power during the grinding time. For slurry concentrations ranging from 30 to 50 vol.% solid, the net mill power decreased in the first 7 min of batch grinding, then remained constant for the rest of the 20-min grinding period. At 55 vol.% solid, the net mill power increased in the first 7 min then stabilized. Thus, for each slurry, the tumbling movement of the ball charge is not much affected by further increase in the slurry viscosity as more fines are produced. However, tests at slurry concentrations greater than 55 vol.% solid showed a continuous decrease in mill power as grinding proceeded, resulting from the sticking of grinding media to each other and to the mill wall.

Table 4
Interpolated optimum value of U for maximum net mill power (see Fig. 2)

Slurry concentration (vol.% solid)	Optimum value of U (U_{op}) (–)	Max. power, $m_{p,\max} \times 10^3$ (kW)
30	0.5	9.8
40	1.0	10.8
50	1.5	10.6
55	0.5	8.5
100 (dry)	–	8.4

Table 5
Estimated specific rates of breakage of quartz (20 × 30 mesh feed) in wet grinding (water): $J_b = 0.3$, $U = 1.0$, $\phi_c = 0.70$ ($S_i = a(x_i/x_o)^\alpha$, $\alpha = 0.8$)

Feed	Slurry concentration (vol.% solid)	Net mill power, $m_p \times 10^3$ (kW)	Breakage rate constant, a (min^{-1})	Ratio, a/m_p (min^{-1} kW^{-1})
Quartz (20 × 30 mesh)	20	8.3	0.34	40.9
	30	9.4	0.36	38.3
	40	10.8	0.38	35.2
	45	11.0	0.39	35.4
	50	10.4	0.23	22.1
	54	8.1	0.21	25.9
	60	6.3	0.20	31.7
	65	6.7	0.24	35.8
	100 (dry)	8.4	0.24	28.6
Quartz + 30% fines	40	11.5	0.40	34.8
Quartz + 50% fines	40	11.7	0.45	38.5

Fig. 2 and Table 3 show the effects of slurry concentration and powder filling on the average net mill power plus the values for dry grinding under the same conditions. For dry grinding, the net mill power was independent of powder filling, at least for interstitial filling fraction (U) varying from 0.3 to 2.0. The net mill power depended on both the slurry

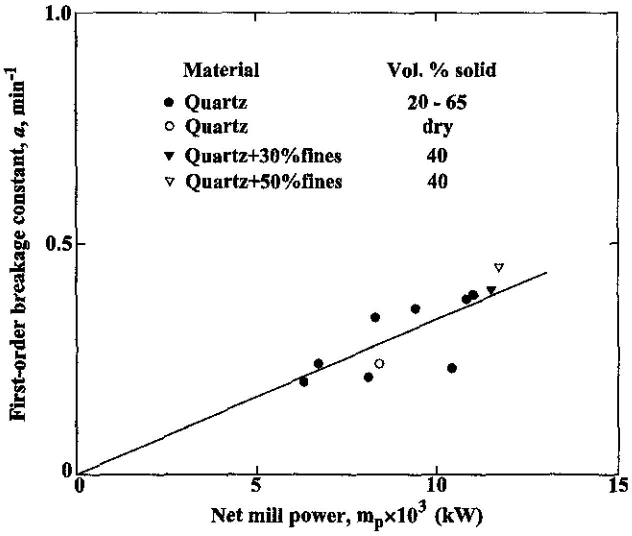

Fig. 3. Correlation between net mill power and breakage rate parameter (a) for grinding 20 × 30 mesh quartz at $J_b = 0.3$, $U = 1.0$, and $\phi_c = 0.70$ ($S_i = a(x_i/x_o)^\alpha$, $\alpha = 0.8$).

concentration and the powder filling, with the existence of an optimum powder filling for each slurry concentration that gave a maximum in mill power. This optimum U value shifts to a higher value as the slurry concentration is increased from 30 to 50 vol.% solid, then decreases at the 55% slurry concentration (see Table 4). Grinding at about 40 vol.% solid and $U \approx 1$ gave the maximum in net mill power.

Specific rates of breakage, S_i [16], of the same quartz were estimated in the same mill at the same milling conditions, with ball load J_b, fraction of critical speed ϕ_c, and powder filling U held constant, so that slurry concentration was the only variable [17] (see Table 5). The starting feed was generally 20 × 30 mesh (0.85 × 0.60 mm) material, but in two tests, the feed consisted partly of this material and partly of − 400 mesh quartz. The results were expressed in the form $S_i = a(x_i/x_o)^{\alpha}$, where x_o is a standard size of 1 mm, x_i is the upper size of a $\sqrt{2}$ screen interval (in mm), and α is a constant that is characteristic of the material. The α value of 0.8 was used for the quartz and taken to be constant for all grinding conditions. Since U is constant for this data, the value of a is representative of the rates of breakage of the quartz under the test conditions.

In agreement with previous findings [18], the maximum specific rates of breakage (maximum a value) occurred at 40–45 vol.% solid. This was also the condition of maximum mill power. Fig. 3 shows the relation of the breakage rate constant (a) to net mill power (m_p) and, since the point (0, 0) is a valid point, the results can be fitted by the equation:

$$a = 33.7m_p, \qquad (1)$$

where the net mill power is in kW and a has units of $\min^{-1}$. In agreement with the results of Shoji et al. [13], the addition of a significant proportion of fines to the starting feed gave increased first-order breakage rates, with a increasing from 0.38 to 0.40 to 0.45 $\min^{-1}$ as the proportion of fines was increased from 0 to 30 to 50 wt.%. Within the scatter of the results, the increased rates were matched by corresponding increases in mill power (see Fig. 3).

Table 6 shows the results of grinding the 20 × 30 mesh quartz using water–glycerine solutions as the carrier fluid, at fixed powder load and slurry concentrations. It is known that increasing the viscosity of the carrier fluid gives a proportionate increase in the slurry viscosity [15].

Table 6
Experimental values of net mill power for grinding 20 × 30 mesh quartz at $C = 0.45$, $J_b = 0.3$, $U = 1.0$, and $\phi_c = 0.70$ using water–glycerine solutions

Weight percent glycerine (wt.%)	Net mill power, $m_p \times 10^3$ (kW)	Breakage rate constant, a ($\min^{-1}$)	Liquid viscosity (mPa·s)	Ratio, a/m_p ($\min^{-1}$ kW^{-1})
0	11.0	0.39	1	35.4
20	10.5	0.28	2	26.7
40	10.4	0.27	4	26.0
60	9.6	0.24	12	25.0
70	9.1	0.21	30	23.1
80	8.0	0.14	80	17.5

4. Discussion of results

The changes in net mill power during the first 7 min of grinding are probably the result of generation of extra fines and possibly the generation of heat in the mill. This "conditioning effect" was small enough (less than ± 5%) that its presence does not significantly change the other conclusions of the study.

The relationship between mill power and slurry concentration and powder filling can be described by the following empirical equation:

$$m_p/M_b = a_1 U^{a_2} \exp(-a_3 U), \quad \text{kW/ton balls}; \quad 0.3 \le U \le 1.75 \qquad (2)$$

where M_b is the mass of balls in the mill (7.34 kg) and a_2 and a_3 are functions of the volume fraction of solid in the slurry (C). A nonlinear regression using the minimization of sum-of-squares error gave

$$a_1 = 2.23$$

$$a_2 = 0.14 + 0.7C - 190C^{11.9}, \qquad 0.3 \le C \le 0.55$$

$$a_3 = 0.79 - 0.97C + 0.22C^2 + (8.1 \times 10^4)C^{20.3}, \quad 0.3 \le C \le 0.55$$

These are only applicable for the mill conditions of $J_b = 0.3$ and $\phi_c = 0.70$. The form of Eq. (2) was obtained from a similar effect of U on the net production rate of particles finer than 53 μm, as reported by Katzer et al. [19].

Differentiating Eq. (2) with respect to U and setting to zero gives the optimum value of U that gives maximum mill power at a given C value as:

$$U_{op} = a_2/a_3, \qquad (3)$$

Substituting into Eq. (2) gives

$$(m_p/M_b)_{max} = a_1 \left(\frac{a_2}{a_3}\right)^{a_2} \exp(-a_2), \qquad (4)$$

The value of C that maximized this expression was $C = 0.45$, corresponding to $U_{op} = 1.09$ and $m_{p,\max} = 10.94 \times 10^{-3}$ kW.

Eq. (2) was used to prepare the curves through the data points in Fig. 2 and gave reasonable fits except at $C = 0.5$. However, it must be recognized that the movement of a charge of 25-mm balls in a mill of less than 200 mm in diameter cannot be equated with that in larger diameter mills. For example, the measured net specific mill power calculated from the Shoji et al. [13] equation for small mills for these conditions is 2.38 kW/ton media. Again, the Bond [9] mill power equation states that a dry grinding mill requires 10% more power than a wet grinding mill, all other factors being the same. The data in Table 3 show significantly higher power for wet grinding than for dry grinding at the normal conditions of $C=0.4$ and $U=1.0$. It is known that tests of the effect of ball diameter on breakage rates in such laboratory-scale mills show negligible effect, whereas tests in mill of 0.6 m and larger show a very clear effect. Adjustment of ball size to match the size of the particles being ground is common in industrial practice . Similarly, Austin and Tangsathitkulchai [20] showed that the Herbst–Fuerstenau [21] method of sizing ball mills, which scales from small test mill data to a full-scale mill using an energy-based scaling procedure, worked fairly well using test data from a 0.56-m diameter mill but failed using data from a 0.20-m diameter test mill.

On the other hand, there is ample evidence to show that breakage results from tests in small laboratory mills (e.g. the Bond Work Index test mill [9] or the mill used by Austin et al. [16]) give good predictions of full-scale mill behavior provided appropriate scale-up procedures are used. Thus, although Eq. (2) cannot give correct predictions of the specific power of full-scale mills, the way that it predicts that the power varies with mill conditions may be an indication, in some circumstances, of the effects of slurry concentration in larger mills.

The easiest result to explain is that for slurry concentrations greater than 55%. For these slurry concentrations, close to a packed bed containing all water in the interstices, it is expected that the slurry will be extremely viscous. As stated previously, this leads to coating of the slurry and balls on the mill walls. The higher the amount of slurry, the more coating occurs, so the mill power decreases with higher U and increased grinding time. These effects would not be so evident in large mills because

(i) the higher ratio of charge volume to the wall surface area would remove a smaller proportion of charge,
(ii) a layer of media adhering to the walls would not much reduce the effective mill diameter for a mill of large diameter, and
(iii) the larger charge mass would generate much higher shear to strip adherent layers from the wall as each section of the wall traveled upward.

Results from tests using highly viscous slurries in small mills are largely meaningless.

The slurry rheology can influence the net mill power in several ways, but it is also necessary to consider the simple effects of increase of the mass rotating in the mill and possible expansion of the charge due to the added volume of solid and water.

Assuming that a fraction f (by volume) of the slurry rotates with the ball charge, the mass of rotating mill charge per unit mill volume, M_c, can be derived as follows:

$$M_c = (\text{Mass of balls/mill volume}) + (\text{Mass of rotating slurry/mill volume}) \tag{5}$$

Since

$$\text{Mass of balls/mill volume} = (1-\varepsilon_b)J_b\rho_b \tag{6}$$

and

$$\text{Volume of slurry/mill volume} = \varepsilon_b J_b(1-\varepsilon_s)U\left[1+\left(\frac{1-C}{C}\right)\right] \tag{7}$$

Multiplying Eq. (7) with f to obtain the volume of rotating slurry per unit mill volume:

$$V_s = f\varepsilon_b J_b(1-\varepsilon_s)U\left[1+\left(\frac{1-C}{C}\right)\right] \tag{8}$$

The density of slurry is

$$\rho' = C\rho_s + (1-C)\rho_w \tag{9}$$

Multiplying Eq. (8) with $\rho,'$ we obtain

$$\frac{\text{Mass of rotating slurry}}{\text{unit mill volume}} = V_s\rho' = f\varepsilon_b J_b(1-\varepsilon_s)U \times \left[1+\left(\frac{1-C}{C}\right)\right][C\rho_s+(1-C)\rho_w] \tag{10}$$

Substituting Eqs. (6) and (10) into Eq. (5) gives

$$M_c = (1-\varepsilon_b)J_b\rho_b + f\varepsilon_b J_b U(1-\varepsilon_s)\rho_s \times \left[1+\left(\frac{1-C}{C}\right)\left(\frac{\rho_w}{\rho_s}\right)\right]$$

or

$$M_c = (1-\varepsilon_b)J_b\rho_b\left[1+f\varepsilon_b U\left(\frac{1-\varepsilon_s}{1-\varepsilon_b}\right)\left(\frac{\rho_s}{\rho_b}\right) \times\left(1+\left(\frac{1-C}{C}\right)\left(\frac{\rho_w}{\rho_s}\right)\right)\right] \tag{11}$$

The expanded volume of rotating charge (balls + slurry) per unit mill volume, J, is

$$J = J_b, \quad \text{if } \frac{fV'}{V_o} < 1 \tag{12a}$$

and

$$J = (1-\varepsilon_b)J_b + V_s, \quad \text{if } \frac{fV'}{V_o} > 1 \tag{12b}$$

where V' and V_o are volume of slurry charge and void volume of the ball bed, respectively. If fV'/V_o is equal to unity, it indicates that the void space of the ball bed is just completely filled with the slurry. The term fV'/V_o, which is

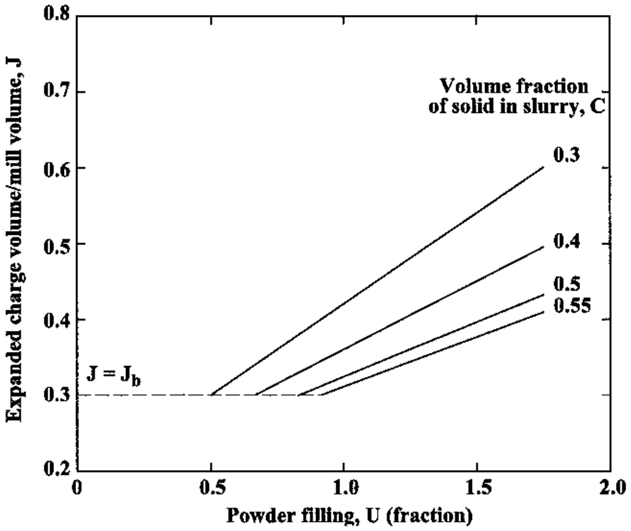

Fig. 4. Fraction of mill volume filled by total contents (balls, solid, water), J, as a function of U.

used in conjunction with Eqs. (12a) and (12b), can be transformed to the parameter U (powder filling) by the following manipulation:

$$\frac{fV'}{V_o} = 1 \tag{13}$$

The void volume of ball bed is

$$V_o = J_b \varepsilon_b V \tag{14}$$

Substituting Eq. (14) into Eq. (13) gives

$$V' = \frac{J_b \varepsilon_b V}{f} \tag{15}$$

By definition:

$$C = \frac{V_p}{V'}, \tag{16}$$

where V_p is the volume of solid in slurry.

Eliminating V' from Eqs. (15) and (16), we obtain

$$V_p = \frac{J_b \varepsilon_b C V}{f} \tag{17}$$

Since V_P is related to U by

$$V_p = J_b \varepsilon_b U (1 - \varepsilon_s) V, \tag{18}$$

equating Eqs. (17) and (18) gives

$$U = \frac{C}{(1 - \varepsilon_s) f} \tag{19}$$

Therefore, the expressions for J in Eqs. (12a) and (12b) can be rewritten with U as the criteria parameter as:

$$J = J_b, \quad \text{if } U < \frac{C}{(1 - \varepsilon_s) f} \tag{20a}$$

and

$$J = (1 - \varepsilon_b) J_b + V_s, \quad \text{if } U > \frac{C}{(1 - \varepsilon_s) f} \tag{20b}$$

The mean density of the rotating charge ρ_c is obtained from Eqs. (11), (20a), and (20b) by

$$\rho_c = M_c / J \tag{21}$$

Figs. 4 and 5 show the values of J and ρ_c as functions of U for the four test values of slurry concentration, assuming all material is included in the charge, i.e., $f = 1$.

Now consider the form of mill power equation proposed by Bond [9] for horizontal cylindrical ball mills:

$$m_p = KJ(1 - AJ)\rho_c V, \tag{22a}$$

where K is a constant for a given mill diameter and rotational speed and V is the mill volume. This can also be put as:

$$m_p = K(1 - AJ) M_c V \tag{22b}$$

This equation is based on assuming that a vertical cross section through the mill charge is a simple segment inclined at an angle to the horizontal. Although this model is too simple to be realistic [22], it contains three features that have been amply verified experimentally. First, any change to the mill charge that increases the effective angle of the bed will increase K at a fixed rotational speed. Second, as J is increased, the mass of the charge will increase, but the length of the torque arm of the center of gravity of the

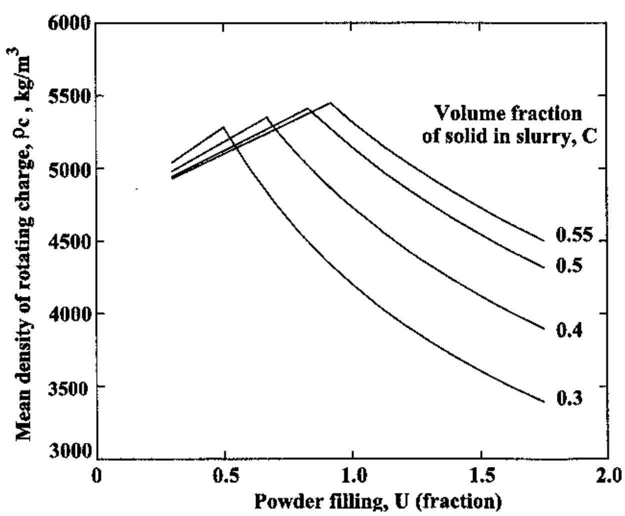

Fig. 5. Average charge density as a function of U.

charge to the mill center will decrease. Third, any change to the shape of the charge will affect A by changing the location of the center of gravity of the charge.

If all factors are constant except J, the maximum in mill power occurs at a value of J of $J_{op} = 1/2A$. For the value of $A = 0.937$ used by Bond, this gives $J_{op} = 0.534$, but tests in small mills [14] typically give the J for maximum power as close to 0.45. However, in the tests being considered here, the density of the rotating charge (ρ_c) decreases as J increases above J_b, and because of the low density of added slurry as compared to steel, the effect of the decrease in the length of the torque arm dominates over the increase in mass. Thus, the predicted maximum in power occurs as soon as there is sufficient volume of slurry to fill the interstices of the ball bed, i.e., when

$$U \geq \frac{C}{(1-\varepsilon_s)f} \tag{23}$$

Assuming f, A, and K to be constant, the predicted net mill power would increase until this value is reached then start to decrease, with a sharp change in slope at this point (see Fig. 6).

It is clear that the experimental data for $C = 0.3$, 0.4, and 0.5 show some similarity to this pattern, but the result for $C = 0.55$ cannot be explained by an expansion of charge holding A, K, and f constant. Since each experimental point at each C and U may be influenced by changes in the three "constants", it is not possible to get the three functions $K(C, U)$, $A(C, U)$, and $f(C, U)$ from such limited data. Similarly, although the limited data in Table 6 indicate that an increase in medium viscosity by a factor of 80 decreased the net mill power at $U = 1$ and $C = 0.45$, from 11 to 8 W, the results in Fig. 2 cannot be explained solely by changes in viscosity.

At constant ball load and powder filling ($U = 1.0$), the specific breakage rate factor a was linearly proportional to the net mill power as the slurry concentration was varied.

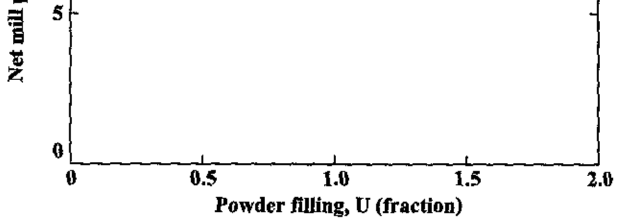

Fig. 6. Predicted variation of net mill power with increased U at $C = 0.4$, assuming K, A, and f to be constant ($A = 0.937$, $K = 1.72$, and $f = 0.8$).

However, this does not prove that the same proportionality will apply as J_b, ϕ_c, and U are changed. For example, it is known that low ball loads are more efficient in producing breakage [23]. Also, the ratio of the specific breakage factor a to net mill power for the tests with water–glycerine solutions showed an obvious decrease as the viscosity of the carrier liquid increased (see Table 6), and the values were significantly lower than those given in Table 5. This shows that mill power alone does not define the specific breakage rates.

5. Conclusions

The dependence of net mill power on the slurry concentration and powder filling was investigated in laboratory batch ball milling of quartz of natural size distribution, at fixed ball loading ($J_b = 0.3$) and mill speed (70% of critical). For slurry concentrations ranging from 30 to 55 vol.% solid, the net mill power was almost constant over a test time of 20 min. For each slurry concentration, the net mill power increased with increasing powder filling and then decreased after an optimum value of powder filling that gave a maximum in net mill power, with the optimum being a function of slurry concentration. Grinding at $\approx$ 40 vol.% solid concentration and $U \approx 1$ gave the maximum power of about 1.5 kW/ton of balls. An empirical equation describing the effects of slurry concentration and powder filling on net mill power was developed for these conditions. It was clear that both bed expansion and changes in bed inclination were involved, but the data were too limited to obtain a satisfactory explanation of the results.

A linear relation between the first-order breakage rate parameter (a value) and the net mill power was found for wet grinding of a fixed mass of quartz under these varying slurry concentrations. However, varying slurry viscosity by using water–glycerine solutions as the suspending media in a 45 vol.% solid slurry gave a values that increased as mill power increased but not linearly.

Nomenclature

a_1	constant in Eq. (2) (kW/ton balls)
a_2	constant in Eq. (2) (–)
a_3	constant in Eq. (2) (–)
a	breakage rate parameter defined by $S_i = a(x_i/x_o)^{\alpha}$ (min^{-1})
A	constant in Bond equation, Eqs. (22a) and (22b) (–)
C	slurry concentration, volume fraction of solid (–)
f	fraction of slurry (by volume) in rotating mill charge (–)
i	an integer indexing a $\sqrt{2}$ screen interval (–)
J	the fraction of mill volume filled by the rotating charge (–)
J_b	fraction of mill volume filled by the ball bed assuming a bed porosity of 0.4 (–)
J_{op}	value of J that gives maximum net mill power (–)
K	constant in Bond equation, Eqs. (22a) and (22b) (–)

m_p net mill power (kW)
M_b mass of balls in the mill (kg)
M_c mass of rotating charge per unit mill volume (kg/m^3)
S_i specific rate of breakage of material in the $\sqrt{2}$ size interval indexed by i (min^{-1})
U fraction of the interstices of the ball bed filled by powder assuming $\varepsilon_s = \varepsilon_b = 0.4$ (–)
U_{op} value of U that gives maximum net mill power (–)
V mill volume (m^3)
V_o void volume of ball bed (m^3)
V_p volume of solid in the slurry (m^3)
V_s volume of rotating slurry per unit mill volume (–)
V' volume of slurry (m^3)
x_i the top size of the $\sqrt{2}$ screen interval indexed by i (mm)
x_o a standard screen size of 1 mm (mm)
α constant in $S_i = a(x_i/x_o)^{\alpha}$ (–)
ε_b formal fractional porosity of the ball bed at rest (–)
ε_s formal fractional porosity of dry powder bed (–)
ϕ_c mill rotational speed as a fraction of the critical speed (–)
ρ_b the density of the steel balls (kg/m^3)
ρ_c the mean density of the rotating charge (kg/m^3)
ρ_s the density of the solid being ground (kg/m^3)
ρ_w the density of water (kg/m^3)
ρ' the density of slurry (kg/m^3)

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