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Numerical approach to loss minimization in an induction motor ☆

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Abstract

This paper describes a numerical approach to power-loss minimization in a fractional hp induction motor driven by a voltage-source inverter. The motor parameters are obtained from a genetic algorithm search. Optimum voltage and frequency excitations are arranged as a table for an energy-saving controller. The proposed method is useful under variable-torque load conditions. Simulation and experimental results are presented.

Keywords: Loss minimization; Induction motor

1. Introduction

Voltage-source-inverters (VSIs) are widely used for driving induction motors. The drive is capable of saving energy to some extent. A greater amount of energy saving can still be obtained by the minimization of the motors' power losses. Some researchers attempted to find the stator exciting frequency to minimize losses [1]. They assumed constant motor parameters as well as equal rotor and stator frequencies. This condition is true only when the slip is equal to unity. An insertion of external impedance was proposed for slip-ring induction motors [2]. The method introduces

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Nomenclature

f	stator frequency (Hz)
I_r	rotor current (A_{rms})
I_s	stator current (A_{rms})
L_m	core inductance (H)
L_r	rotor inductance (H)
L_s	stator inductance (H)
N	number of data
N_m	motor speed (rpm)
N_s	synchronous speed (rpm)
P_{ag}	air-gap power loss (W)
R_c	core resistance (Ω)
R_r	rotor resistance (Ω)
R_s	stator resistance (Ω)
s	slip
T	load torque (N m)
V	supply voltage (V_{rms})
V_m	air-gap voltage (V_{rms})
ω_e	supply radian frequency (rad/s)
ω_s	synchronous speed (rad/s)

harmonics into the system. The work [3] took account of core saturation, skin effect and source harmonics. These factors are very difficult to determine in practice and bar the concept from practicality. Finding an optimum air-gap flux to minimize losses was also introduced [4].

Our work considers induction motors of a few kilowatts rating. We show that the motor losses are expressed as a function of torque, speed, voltage and frequency. The approach is realistic because it is referred to the motor performance and true motor parameters. It is also easy to implement.

2. Experimental

Fig. 1 represents our hardware system using a PC-based controller with 12-bit resolution converters. The PC-based controller produces numerical switching commands sent to two microcontrollers performing real-time switching functions to drive the chopper and the inverter. We tested the system under varied load conditions in the range of 10–50%, as well as constant and non-constant speed demands. The closed-loop speed control uses a simple rule-based controller to maintain a constant speed. It contains three rules as follows:

if the magnitude of the speed error is greater than 200 rpm then increase or decrease the motor phase voltage by 30 V_{rms};

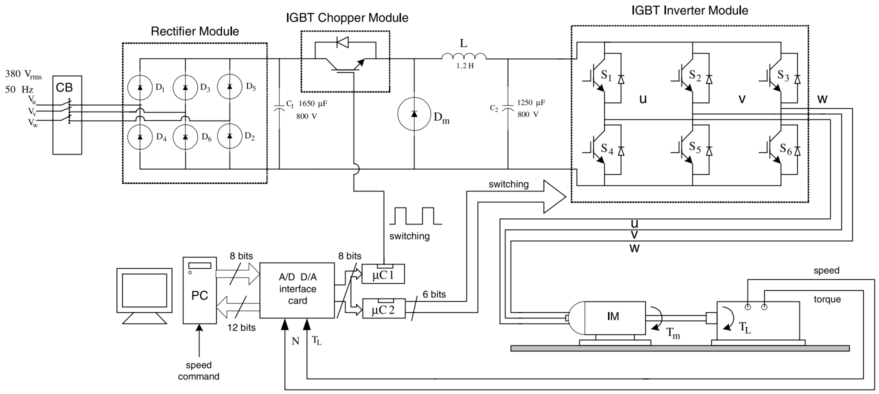

Fig. 1. Experimental system.

if the magnitude of the speed error is between 100 and 200 rpm then increase or decrease the motor phase voltage by 10 V_{rms};

if the magnitude of the speed error is between 20 and 100 rpm then increase or decrease the motor phase voltage by 1 V_{rms}.

This rule-based controller achieves a steady-state speed error within ±1.2%. Tables 1–3 give details of the experimental results in which the input power (P_{in}) and

Table 1
Experimental results of the system without the energy-saving controller

Load (%)	Speed (rpm)	P_{out} (W)	P_{in} (W)	Efficiency (%)	Power factor	Line-to-line voltage (V_{rms})	f (Hz)
10	1497	99.86	200	49.93	0.3198	374.12	50
20	1487	194.18	280	69.35	0.4445	365.12	50
30	1475	295.17	380	77.68	0.5502	357.50	50
40	1462	390.10	490	79.61	0.6621	348.49	50
50	1450	483.61	600	80.60	0.7194	339.14	50

Table 2
Experimental results of the system with the controller

Load (%)	Speed (rpm)	P_{out} (W)	P_{in} (W)	Efficiency (%)	Power factor	Line-to-line voltage (V_{rms})	f (Hz)
10	1497	99.86	125	79.89	0.7838	162.81	52.2
20	1487	194.18	245	79.26	0.8177	208.19	52.1
30	1475	295.17	370	79.78	0.8261	242.14	51.7
40	1462	390.10	480	81.27	0.8207	274.01	51.1
50	1450	483.61	590	81.97	0.8078	305.19	50.5

Table 3
Experimental results of the system with the controller and speed kept constant (1200 rpm)

Load (%)	Speed (rpm)	P_{out} (W)	P_{in} (W)	Efficiency (%)	Power factor	Line-to-line voltage (V_{rms})	f (Hz)
10	1200	80.05	100	80.05	0.8107	126.09	42.7
20	1200	160.09	195	82.10	0.7836	179.79	42.2
30	1200	240.14	310	77.47	0.8048	209.23	42.2
40	1200	320.18	400	80.05	0.8005	237.29	42.1
50	1200	400.23	490	81.68	0.8088	255.65	42.1

the power factor (p.f.) were measured at the motor terminals by using FLUKE™ 41B.

3. Motor model

3.1. Motor's equivalent-circuit

This plays an important role in loss minimization because most losses arise during steady-state operation [5,6]. Conventionally, the parameters of the circuit in Fig. 2 are obtained from no-load and block-rotor tests. To conduct on-line and real-time estimations of stator and rotor impedances during operation is difficult and time-consuming. The task would be costly due to the requirement of either transducers or sophisticated estimation algorithms. To keep the problem tractable, the genetic algorithm (GA) has been used to identify the motor's parameters from its steady-state characteristics.

3.2. Genetic algorithm

This is an efficient search method based on the principle of natural selection [7]. To apply GA, the problem must be converted firstly into a criterion function called "fitness function" representing the system's performance. A higher fitness value indicates a better performance. GA firstly starts with a randomly selected population from the population set. The retained population must pass the fitness evaluation

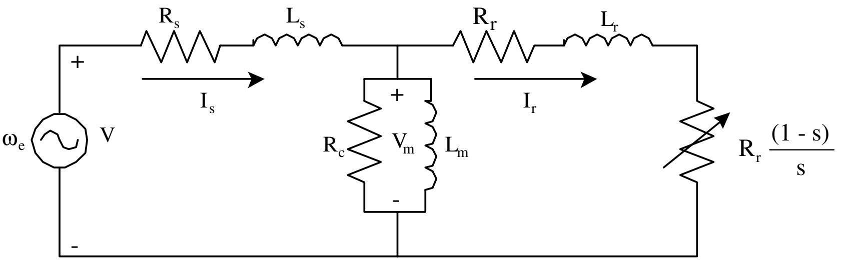

Fig. 2. Motor's equivalent-circuit.

and the rest is discarded. These retained members are parenting to produce offspring. All the parents and offspring have to go through the process of fitness evaluation again and only the strong ones are retained. Strong members are used as replacements to the previous population. Afterward, parenting occurs and the process is repeated until the most fitness member or optimum solution is found. A brief summary of the construction of the GA is as follows:

1. *Define chromosome*. The searching parameters are defined and coded as binary or real and termed chromosomes.
2. *Define the fitness function*. It is designed in accordance with the problem's requirement, e.g., error, convergent rate, etc.
3. *Generate initial population*. The initial population of N sets is generated randomly with the size of N chosen arbitrarily.
4. *Generate the next generation or stop*. To generate the next generation, the GA uses the operations of reproduction, crossover and mutation. A stop criterion must be defined such as the number of repetitive loops, acceptable error, etc.

3.3. Identification

The proposed method requires steady-state characteristics measured by using a common machine test-bed. Various line voltages were fed to the motor and the speed-torque characteristics were recorded. Some of the characteristics are illustrated in Fig. 3, in which the noisy curves represent the measured motor characteristics. The smooth curves are obtained by calculation using the equivalent circuit. The motor parameters were estimated from the measured characteristics using the GA. R_s, R_r, L_s

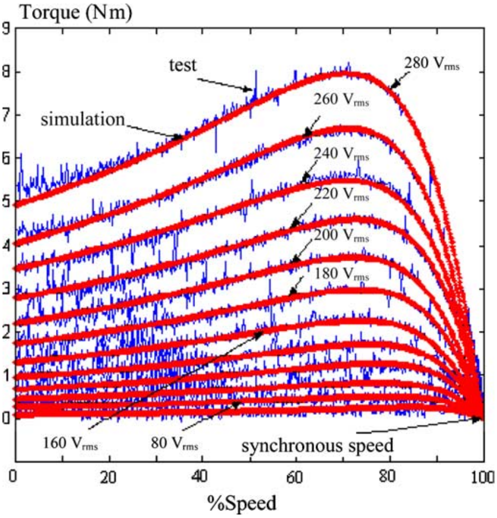

Fig. 3. Motor's characteristics.

and L_r were defined as chromosomes. The conventional test of the motor yielded $R_s = 6.4333\ \Omega$, $R_r = 4.1178\ \Omega$, $L_s = L_r = 0.0289$ H, which were used as initial guessed solutions for the GA search. The resolution of each parameter was 30 bits. These parameters were concatenated to form a single chromosome of 120-bit resolution.

The fitness function is given by

$$\text{Fitness function} = \frac{1}{\varepsilon}, \tag{1}$$

where

$$\varepsilon = \frac{\sum_{i=1}^{N} e(i)}{N} \tag{2}$$

The error term $e(i)$ is defined by $[\hat{T}(i) - T_a(i)]^2$. T_a is the actual torque obtained from the measurement. $\hat{T}$ is the estimated torque expressed by

$$\hat{T} = \frac{V^2}{\omega_s} \frac{\hat{R}_r/s}{(\hat{R}_s + \hat{R}_r/s)^2 + (\hat{X}_s + \hat{X}_r)^2}, \tag{3}$$

where $\hat{X}_s = 2\pi f L_s$, $\hat{X}_r = 2\pi f L_r$ and any symbols with ^ represent estimated values.

At each trial of the search, the torque was estimated according to Eq. (3) and the error was calculated. The search took 3000 trials. Fig. 3 illustrates a good agreement between the experimental and the model-based results.

4. Loss expression and minimization

Stator, rotor and core losses dominate the overall power loss [6]. Stray, friction, windage and converter losses are small enough to be negligible. Eq. (4) describes the power losses.

$$\begin{aligned} P_{\text{loss,total}} &= |I_s|^2 R_s + |I_r|^2 R_r + \frac{|V_m|^2}{R_c} \\ &= V^2 \left[\left| \frac{Z_2 + Z_m}{Z_T} \right|^2 R_s + \left| \frac{Z_m}{Z_T} \right|^2 R_r + \left| \frac{Z_2 Z_m}{Z_T} \right|^2 \Big/ R_c \right], \end{aligned} \tag{4}$$

where

$$Z_1 = R_s + \mathrm{j}2\pi f L_s, \tag{5}$$

$$Z_2 = \frac{R_r}{s} + \mathrm{j}2\pi f L_r, \tag{6}$$

$$Z_m = \frac{\mathrm{R_c j}2\pi f L_m}{R_c + \mathrm{j}2\pi f L_m}, \tag{7}$$

$$Z_T = Z_1 Z_2 + Z_1 Z_m + Z_2 Z_m, \tag{8}$$

$$\text{slip } s = \frac{N_s - N_m}{N_s}. \tag{9}$$

The torque of an induction motor can be described by

$$T = \frac{P_{\mathrm{ag}}}{\omega_{\mathrm{s}}} = V^2 \left| \frac{Z_{\mathrm{m}}}{Z_{\mathrm{T}}} \right|^2 \frac{R_{\mathrm{r}}}{s} \frac{1}{\omega_{\mathrm{s}}} \tag{10}$$

From Eqs. (4) and (10), one can realize that

$$P_{\mathrm{loss,total}} = T\omega_{\mathrm{s}} \frac{s}{R_{\mathrm{r}}} \left[\left| \frac{Z_2 + Z_{\mathrm{m}}}{Z_{\mathrm{m}}} \right|^2 R_{\mathrm{s}} + R_{\mathrm{r}} + \frac{|Z_2|^2}{R_{\mathrm{c}}} \right] \tag{11}$$

Eq. (11) is useful for assessing loss minimization. It means that the total power losses depend on load torque, synchronous speed and exciting voltage. With suitable substitution of the impedances in Eq. (11), one can compute the power losses. Figs. 4(a)–(d) illustrate an example of the loss surfaces. They show clearly that some particular frequencies yield minimum loss lines. These frequencies can be computed

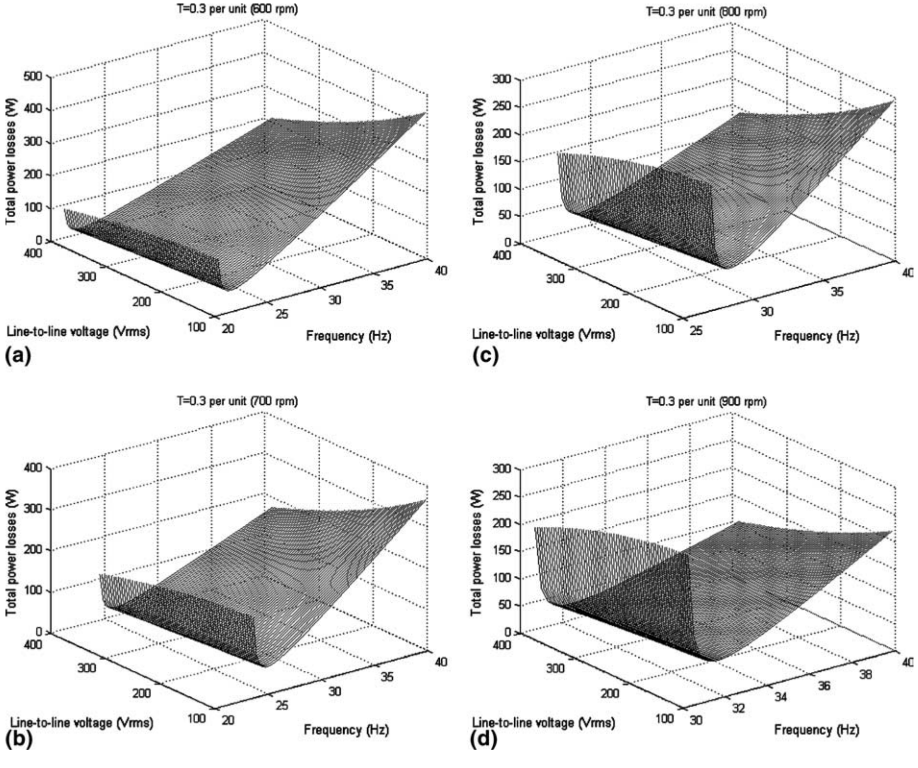

Fig. 4. Power losses in an induction motor having 0.3 per unit load and speed of (a) 600 rpm, (b) 700 rpm, (c) 800 rpm and (d) 900 rpm.

off-line and further used as a table. This computing approach is more attractive than the conventional differentiation to find the minimum point because the conventional method results in a very high-order polynomial.

5. Results and discussion

5.1. Simulation results

Referring to Fig. 4 for the case of 0.3 per unit load, the loss surfaces reveal the possibility of minimum loss attainment. Similarity in the shape of the surfaces can be assumed for different loads. The rpm values in the figure represent the steady-state speed demanded. The amount of total power losses varies according to the levels of voltage excitation. It is observed that the minimum loss lines can be found for some specific exciting frequencies.

Referring to Eq. (11), the load torque T is known from measurement, the motor parameters are obtained from identification and the synchronous speed is also known from motor performance. In terms of implementation, real-time computing using a high-performance processor to obtain the optimal exciting voltage and frequency is possible. The optimal excitation will result in the minimum power loss according to an individual speed command. Off-line calculation with a look-up table approach is also an attractive option.

The usefulness and limitation of the proposed method are disclosed by the simulation results shown in Figs. 5(a)–(c) that provide a comparison of three drive schemes in terms of total power losses. These schemes are constant ratio of voltage-to-frequency ("v/f constant" in the figure), fixed voltage with varied frequency and the proposed method, respectively. The ratings of the motor under test are 1500 rpm, 6.37 N m, 380 V_{rms} and 1.5 hp. Fig. 5(a) shows that the proposed method is the most efficient approach when the load torque is low. When the load torque is up to half-rated, the proposed method is still efficient as can be seen from Fig. 5(b). From Fig. 5(c), with high load (above half rated), the proposed method and the fixed voltage with varied frequency method result in nearly equal total power losses for the speed range of 60–80% rated speed. Note that at 80% rated speed, the proposed method requires about the rated voltage supplied to the motor corresponding to the exciting frequency. The exciting voltage and frequency have to be satisfied simultaneously to achieve the minimum loss. Running the motor beyond this limit under the proposed method should be avoided because an excessive voltage can cause damaging stress to the motor's insulation. Nonetheless, it is still efficient for the low-speed range as can be observed from Fig. 5(c).

5.2. Experimental results

Table 1 summarizes the results of the system without the energy-saving controller. The motor speed and the line voltage drop naturally when the load increases. The motor power factor is considerably lower in the load range between 10 and 50%. The

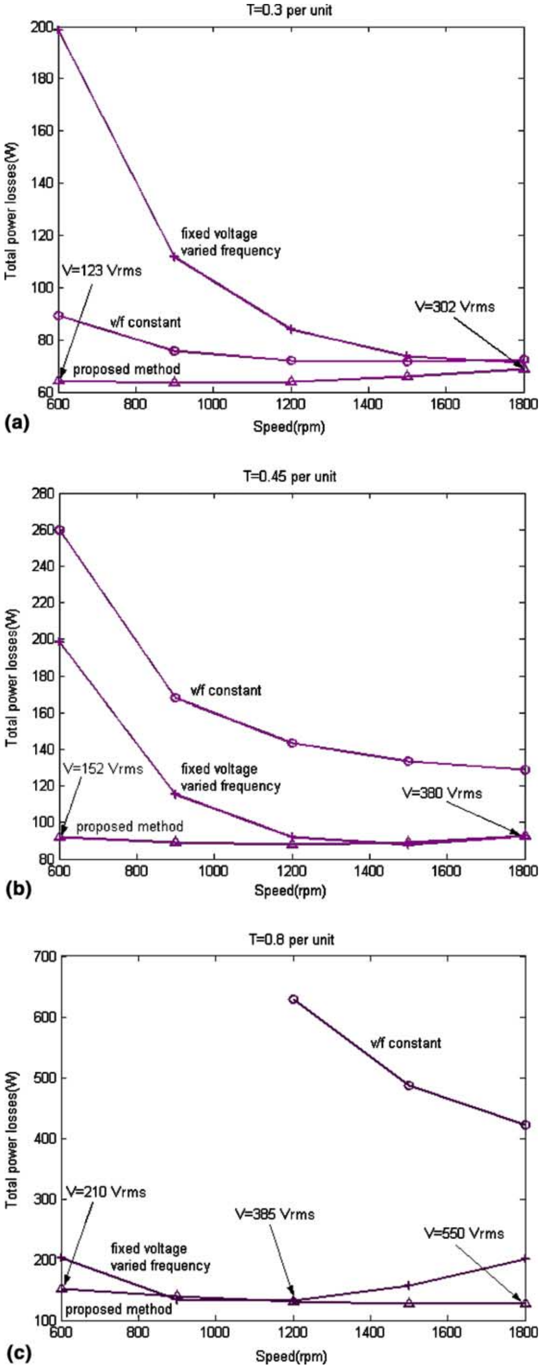

Fig. 5. Calculated results compare the total power-losses of various drive schemes: (a) 0.3 per unit load, (b) 0.45 per unit load and (c) 0.8 per unit load.

efficiency is lower than 80%, when the controller is used under the same condition as for the previous case. Table 2 summarizes the results. It reveals that our proposed method can maintain the efficiency around 80% and the power factor about 0.8–0.83 for the considered load-range without any volt–ampere-reactive compensation. The proposed method is effective for 0–30% load in terms of input power savings. The amount of energy savings ranges from 3 to 60%. Table 3 reveals the power consumed by driving the motor at a constant speed, e.g., 1200 rpm, with the energy-saving controller. The efficiency can be maintained at around 80%. The power factor at the motor terminals is maintained around 0.8 without any volt–ampere-reactive compensation. The exciting frequency is acceptably constant at 42.2 Hz. Our simulation and experimental results agree and confirm the effectiveness of the approach.

6. Conclusions

We have presented a numerical approach to power-loss minimization in induction motors driven by VSIs. The method relies on the motor's equivalent circuit, whose parameters can be readily identified from the measured characteristics. The loss model incorporates the variation of the exciting voltage, frequency and load torque as major factors. The computing results show that the proposed method is effective when the load torque ranges from zero to half-rated. Above a half-rated load, the method is attractive for the low-speed range. The simulation and experimental results agree and confirm the claim.

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