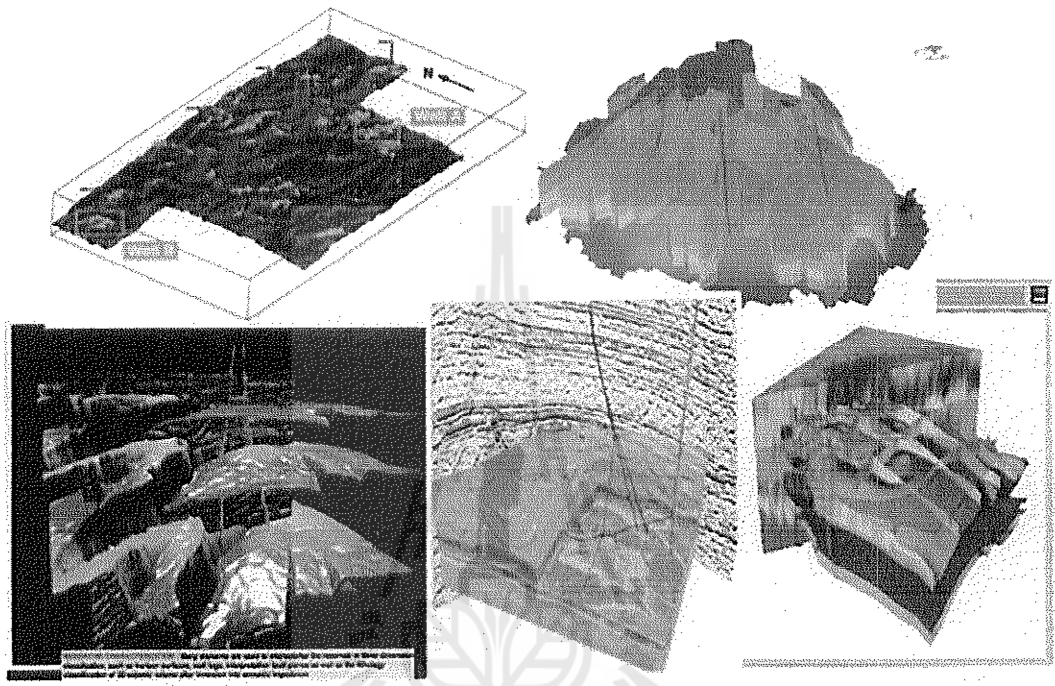
Lecture Note and Document

434620

ADVANCED RESERVOIR ENGINEERING



Prepared by

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Petroleum Engineering
School of Geotechnology
Institute of Engineering

434619,534619 ADVANCED RESERVOIR ENGINEERING
2/2556(4 credits(4-0-8), September 23,- Dec.26, 2013.)

Course Contents

1. Optimization of Material Balance Equations (6 hrs.)
2. Saturation and Relative Permeability Calculations (4 hrs.)
3. Steady State Radial Flow (4 hrs.)
4. Unsteady Steady State Radial Flow (4 hrs.)
5. Pseudo-steady State Flow and Superposition (4 hrs.)
6. Well Testing Pressure Drawdown and Build Up (3 hrs.)
7. Interference Test and Type Curve Analysis(3hrs)
8. Displacement Efficiency(4 hrs)
9. Potential flow and Streamlines (4 hrs.)
10. Dynamics of Water Drive Reservoir. (6 hrs.)
11. Water and Gas Coning (4hrs.)
12. Multi-Phase Flow and Introduction to Reservoir Simulation (2 hrs.)
13. Enhance Oil Recovery(2 hrs)

GRADING

Homework	25%,	Quiz I,II	25%
Mid-term	25%,	Final Exam	25%

TEXTS

1. CRAFT, B.C. and Hawkins, M.F. : ***Applies Reservoir Engineering,@1996 revised***, PRENTICE-HALL, INC., ENGLEWOOD, N.J.
2. 2. Henry B, Crichlow. : ***Advanced Reservoir Engineering, @1974*** , THE UNIVERSITY OF OKLAHOMA, OK., USA.

REFERENCES

1. Tarek Ahmed : ***Reservoir Engineering*** Handbook; Gulf Publishing Company 2000, Houston, Texas, USA.
2. Tarek Ahmed : ***Reservoir Engineering*** Handbook; Third Edition, ELSEVIER 2006, NEW YORK, USA.
3. Tarek Ahmed, Paul D. McKINNET : ***Advanced Reservoir Engineering*** ELSEVIER 2006, NEW YORK, USA.
4. J.S. ARCHER & C.G. WALL ,: ***Petroleum Engineering,*** Kluwer Academic Publishers 1996, Boston, USA.
5. Dr. CHARLES R. SMITH & G.M. TRACY : ***Applies Reservoir Engineering Manual,*** OGCI, INC., Tulsa, Oklahoma, USA.
6. M.A. MIAN : ***Petroleum Engineering*** Handbook for the Practicing Engineer. PennWell Books 1995, Tulsa, Oklahoma, USA.

ADVANCED RESERVOIR ENGINEERING

by

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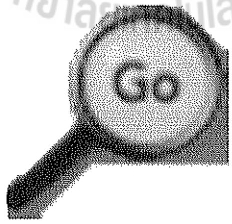
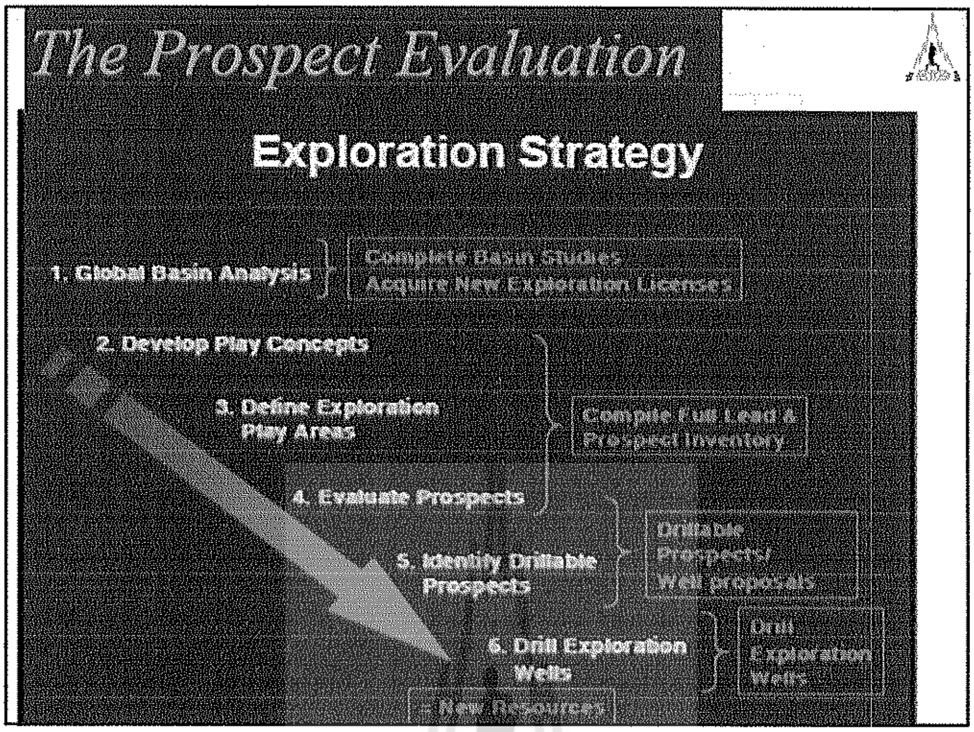
Suranaree University of Technology

Chapter 1.1
Reserve Calculation



CHAPTER 1.2
STRAIGHT LINE MATERIAL BALANCE


The Prospect Evaluation
Exploration Strategy
1. Global Basin Analysis
Complete Basin Studies
Acquire New Exploration Licenses
2. Develop Play Concepts
3. Define Exploration Play Areas
Compile Full Lead & Prospect Inventory
4. Evaluate Prospects
5. Identify Drillable Prospects
Drillable Prospects/ Well proposals
6. Drill Exploration Wells
Drill Exploration Wells
= New Resources

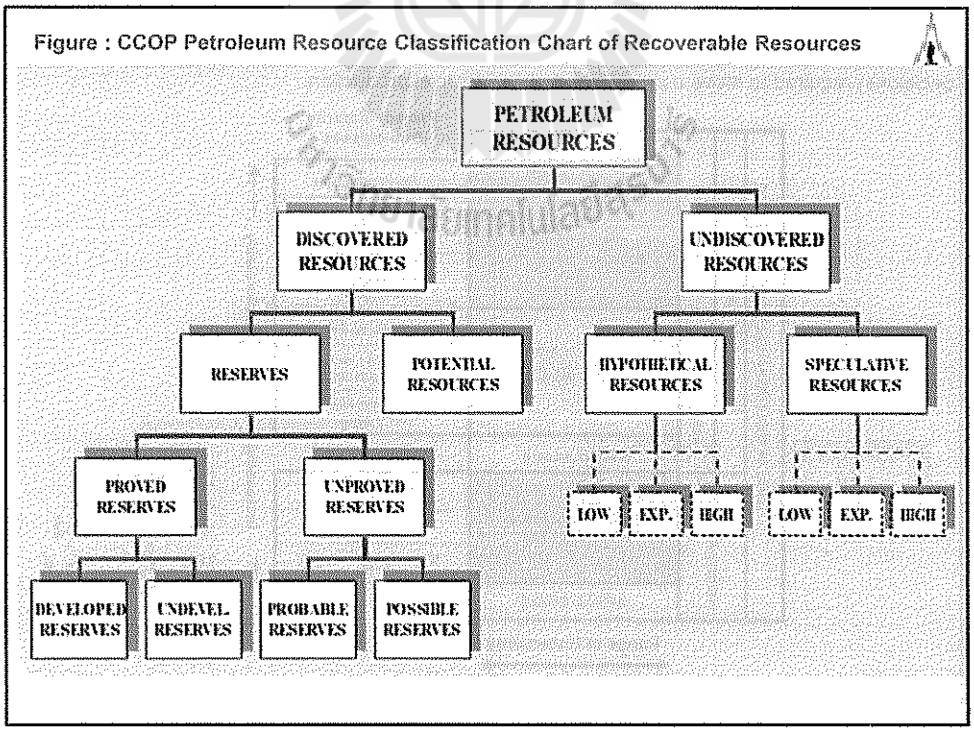


Figure : CCOP Petroleum Resource Classification Chart of Recoverable Resources
PETROLEUM RESOURCES
DISCOVERED RESOURCES
UNDISCOVERED RESOURCES
RESERVES
POTENTIAL RESOURCES
HYPOTHETICAL RESOURCES
SPECULATIVE RESOURCES
PROVED RESERVES
UNPROVED RESERVES
LOW
EXP.
HIGH
LOW
EXP.
HIGH
DEVELOPED RESERVES
UNDEVEL. RESERVES
PROBABLE RESERVES
POSSIBLE RESERVES

Figure 1-1 is a graphical representation of the SPE/WPC/AAPG/SPEE resources classificat system. The system defines the major recoverable resources classes: Production, Reserv Contingent Resources, and Prospective Resources, as well as Unrecoverable petroleum.

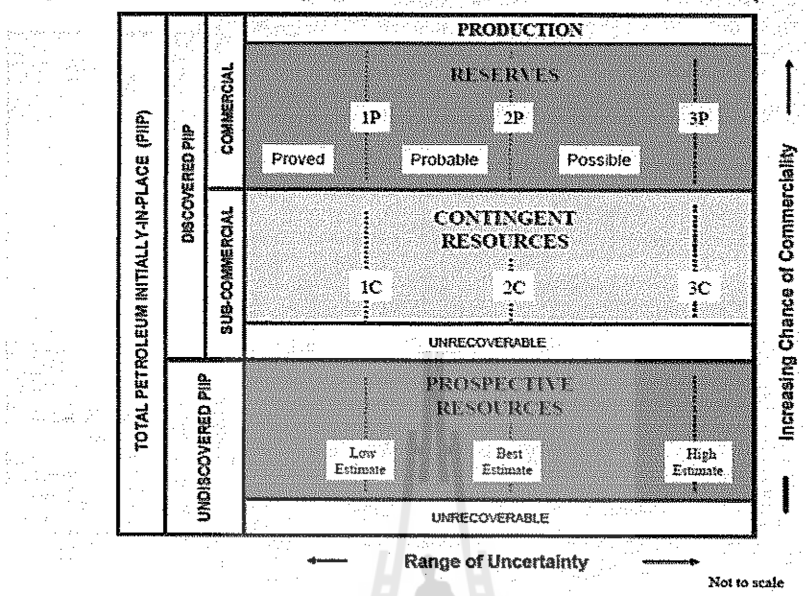
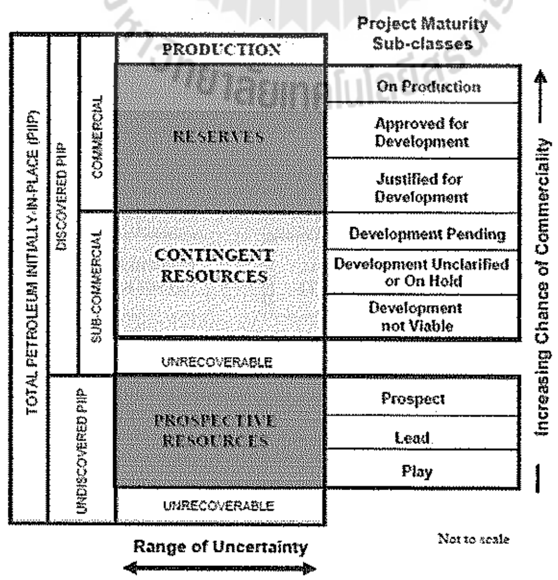


Figure 1-1: Resources Classification Framework.

As illustrated in Figure 2-1, development projects (and their associated recoverable quantities) may be sub-classified according to project maturity levels and the associated actions (business decisions) required to move a project toward commercial production.



แหล่ง
โครงสร้าง
เป้าประสงค์

Figure 2-1: Sub-classes based on Project Maturity.

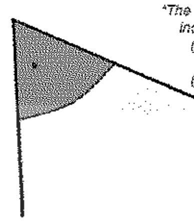
PROVED RESERVE

- Commercially Recoverable
  - Geological, Engineering And Economic Data, Rule & Regulation
  - Operating Methods
  - Developed/ Undeveloped
  - Price, Cost, Time Span Of Development
  - Deterministic (High Degree Of Confidence)
  - Probabilistic (90% Probability >= The Estimate)

2009 SEC Regulations: Proved Reserves

"Reserves are estimated remaining quantities of oil and gas and related substances anticipated to be economically producible, as of a given date, by application of development projects to known accumulations."

Source: 210.4-10 (a)(26) pg. 2192



"The area of the reservoir considered as proved includes:
(A) The area identified by drilling and limited by fluid contacts, if any, and
(B) Adjacent undrilled portions of the reservoir that can, with reasonable certainty, be judged to be continuous with it and contain economically producible oil or gas on the basis of available geoscience and engineering data."

Source: 210.4-10 (a)(22)(i) pg. 2191

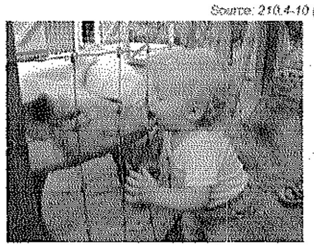
PROBABLE RESERVE

- More Likely Than Not To Be Recoverable
- Step-Out Drilling/ Inadequate Subsurface Control
- Well Log/No Core Test/No Analogous To Proved Or Producing Area
- In Fill Which Can Be Proved If Approved
- Improved Recovery/Adjacent to Proved Area/Workover/Incremental From Volumetric Estimation
- 50-90% Confidence

2009 SEC Regulations: Probable Reserves

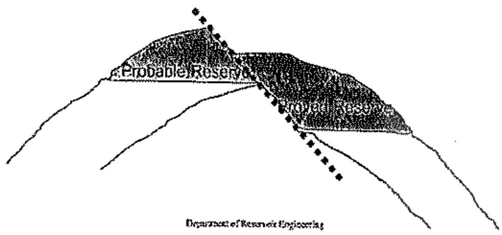
"Probable reserves are those additional reserves that are less certain to be recovered than proved reserves but which, together with proved reserves, are as likely as not to be recovered."

Source: 210.4-10 (a)(18) pg. 2191



PROBABLE RESERVE

- Separated From Proved Area By Faults And In The Higher Structure

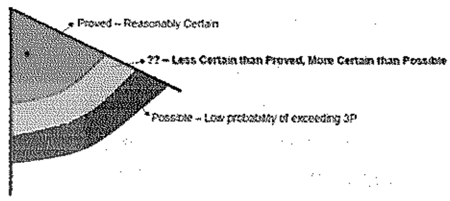


Department of Resources Engineering

Probable Reserves: Can you book downdip of Proved?

RYDER SCOTT

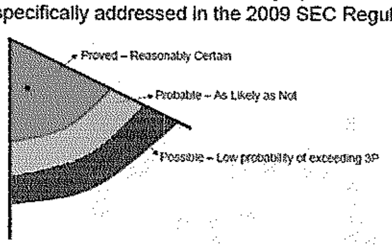
Reserves structurally lower than the Proved area are not addressed in the 2009 SEC Regulations



Probable and Possible Reserves: Are Proved Reserves Required?

RYDER SCOTT

The case where Proved Reserves alone are not economically producible, but the Proved plus Probable reserve case is economically producible, is not specifically addressed in the 2009 SEC Regulations



POSSIBLE RESERVE

- Less Likely To Be Recoverable
- 10-50% Confidence
- Supported By Geo/ Eng/ Eco Data But Beyond Probable Areas
- Log /Core Not Productive @ Commercial Rate
- Infill Drilling Subject To Uncertainty
- Improved Recovery

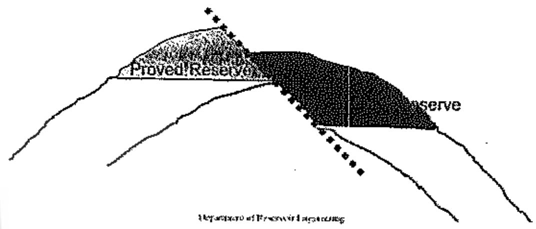
2009 SEC Regulations: Possible Reserves

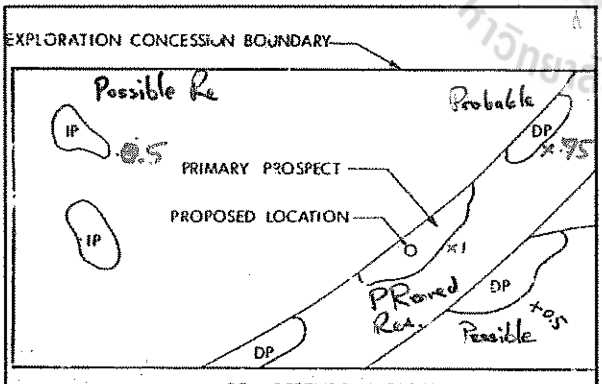
"Possible reserves are those additional reserves that are less certain to be recovered than probable reserves."

Source: 210.4-10 (a)(17) pg. 2191

POSSIBLE RESERVE

- Adjacent To Proved Areas, Separated By Faults But In Lower Structure





1 Volumetric METHOD

2 Material Balance METHOD

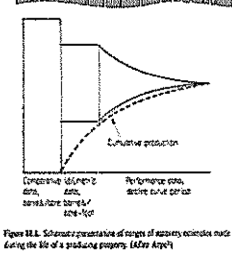
3 Monte Carlo Simulations

4 Stochatic(Statistic) & Deterministic

5 Reservoir Performances

5.1 Decline Curves

5.2 Reservoir Simulations



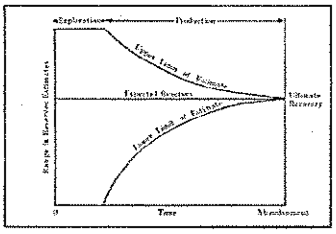


Figure 1: Magnitude of uncertainty in reserves estimates

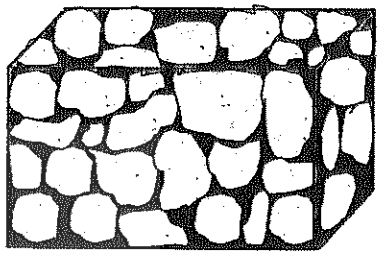
The oil and gas reserves estimation methods can be grouped into the following categories:

1. Analogy.
2. Volumetric.
3. Decline analysis.
4. Material balance calculations for oil reservoirs.
5. Material balance calculations for gas reservoirs.
6. Reservoir simulation.

Reserves Estimation Methodology

Volumetric Calculation

Reserves = Bulk Volume . ϕ . *So* . *Bo* . *Rf*



ϕ = Porosity

So = *Saturation*

Bo = *Volume Factor*

Rf = *Recovery Factor*

The gas volumetric calculation can be performed as follows:

(a) Determine volume of rock containing gas(or hydrocarbons) from the structural and isopach maps.
(b) Determine void space in rock; average effective ((porosity)
(c) Determine volume percentage containing gas(or hydrocarbons) fluid saturation $S_g = 1\text{-}S_w$
(d) Determine recoverable gas (or hydrocarbons) by multipling with recovery factor (F_R)
(e) Determine reserve volume in standard condition by dividing with gas formation volume factor (B_g).

Reserves (SCF) = $A \times h \times \phi \times (1\text{-}S_w) \times F_R \times (1/B_g)$

- $A \times h$: Bulk Reservoir Volume
- $A \times h \times \phi$: Pore Volume
- $A \times h \times \phi \times (1\text{-}S_w)$: Gas Volume in Reservoir
- $\times F_R$: Recoverable Gas Volume
- $\times (1/B_g)$: Recoverable Gas Volume in SCF

SCF = Standard Cubic Foot (at standard condition)
$A \times h$ = reservoir volume in cubic feet
ϕ = porosity expressed as a fraction of bulk volume
S_w = water saturation expressed as a fraction of void space
F_R = Recovery factor expressed as a fraction of total in-place
B_g = formation volume factor in cu.ft reservoir/ SCF surface

If A (h is expressed in Acre-ft. the above formula will be

Reserves(SCF) = $43{,}560\ A \times h \times \phi \times (1\text{-}S_w) \times F_R \times (1/B_g)$

Gas Volumetric

OIL Reserve= $7758Ah\phi(1\text{-}S_W)B_oF_r$

Estimation of hydrocarbons in place

The tools and methods employed are indicated below:

RESERVOIR BOUNDARIES

CAP ROCK

Geology, geophysics, drilling

BASE(S) OF ACCUMULATION(S): HC/W and O/G interface(s)

Well tests, logs, analysis of capillary mechanisms in cores

FRACTION OF ROCK VOLUME OCCUPIED BY FLUIDS (porosity)

Core analysis, logs

DISTRIBUTION OF FLUIDS IN THE PORES (saturation)

Analysis of capillary mechanisms in cores, logs

DOWNHOLE / SURFACE RELATIONSHIP (HC volumes in initial reservoir conditions/HC volumes in reference surface conditions)

PVT laboratory analysis of representative samples of fluids

Pore volume V_p

Volume occupied by oil + gas or gas $V_p(1 - S_{wi})$

Volume of storage oil and/ or standard gas $V_p(1 - S_{wi})/B$

Volumetric Estimates

- Reserves = Reservoir Volume x Porosity x Oil Saturation x Recovery Factor x Shrinkage to Surface Conditions
- In oilfield units:

Reserves = [7758 x A x h x ø x (1-S_w) x R]/Bo

where Or 43560 for gas or Bg

7757	= bbls/acre-ft
A	= area (sq. ft) or Acre
h	= net thickness (ft)
ø	= porosity (fraction)
S_w	= water saturation (fraction)
R	= recovery factor (fraction)
Bo	= formation volume factor
Bg	= Gas formatiom volume factor

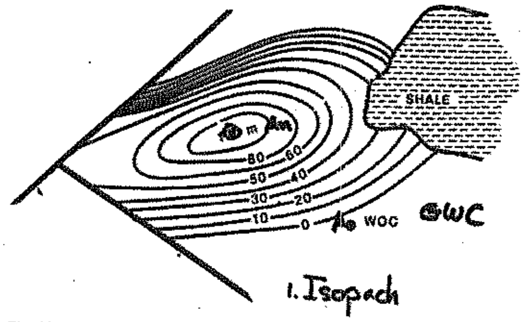
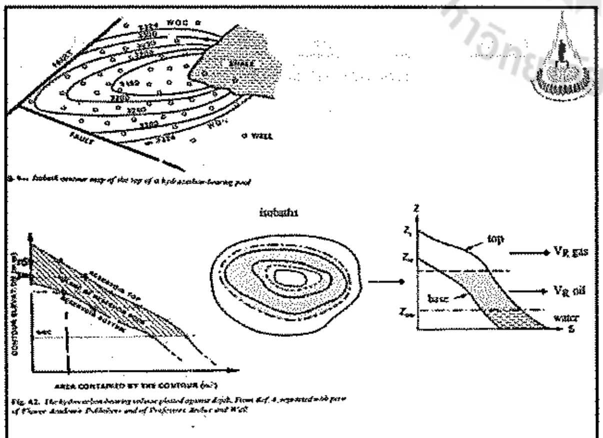


Fig. 4.3 Isopay map for a hydrocarbon-bearing pool



HOW TO CALCULATE FORMATION VOLUME

Simpson's rule(if the numbers of contours are even)

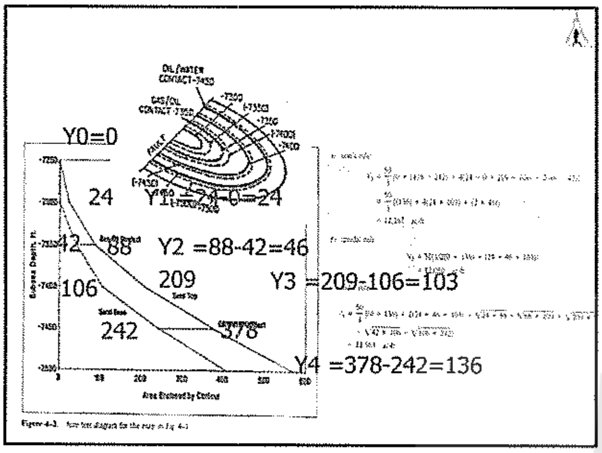
$$V_B = h/3[(y_0+y_n)+4(y_1+y_3+....+y_{n-1})+2(y_2+y_4+....+y_{n-2})]$$

Trapezoidal rule(with some what less accuracy)

$$V_B = h[1/2(y_0+y_n)+y_1+y_2+....+y_{n-1}]$$

Pyramidal rule(if $A_{n-1}/A_n<0.5$)

$$V_B = h/3(A_n+A_{n+1}+\sqrt{A_nA_{n+1}})$$



...son's rule: $V_B = h/3[(y_0+y_n)+4(y_1+y_3+....+y_{n-1})+2(y_2+y_4+....+y_{n-2})]$

$$V_B = \frac{50}{3}[0 + (378 - 242) + 4(24 - 0 + 209 - 106) + 2(88 - 42)]$$

$$= \frac{50}{3}[(136) + 4(24 + 103) + (2 \times 46)]$$

$$= 12{,}267 \text{ ac-ft}$$

Y0=0 Y1 =24 Y2 =46 Y3 =103 Y4 =136

...zoidal rule: $V_B = h[1/2(y_0+y_n)+y_1+y_2+....+y_{n-1}]$

$$V_B = 50[1/2(0 + 136) + (24 + 46 + 103)]$$

$$= 12{,}050 \text{ ac-ft}$$

...idal rule: $V_B = h/3(A_n+A_{n+1}+\sqrt{A_nA_{n+1}})$

$$V_B = \frac{50}{3}[(0 + 136) + 2(24 + 46 + 103) + \sqrt{24 \times 88} + \sqrt{88 \times 209} + \sqrt{209 \times 378} - \sqrt{42 \times 106} - \sqrt{106 \times 242}]$$

$$= 11{,}963 \text{ ac-ft}$$

Material Balance for Gas Reservoirs

Material Balance Equations

$GB_g = (G - G_p)B_g$ (5-145)

$GB_g = (G - G_p)B_g + (W_e - W_p)$ (5-146)

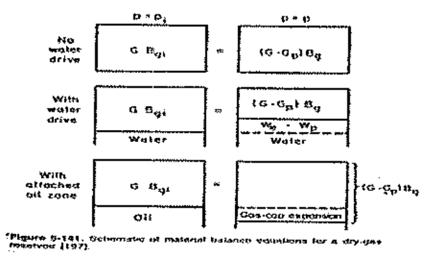
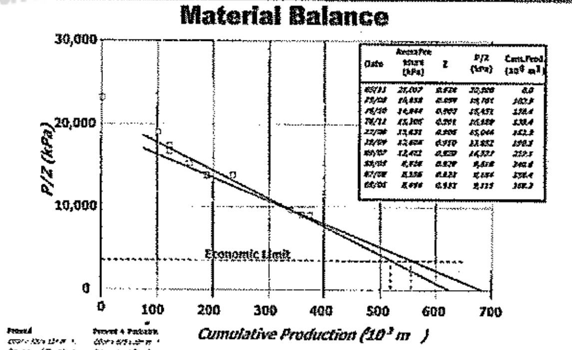


Figure 5-141. Schematic of material balance equations for a dry-gas reservoir (197).

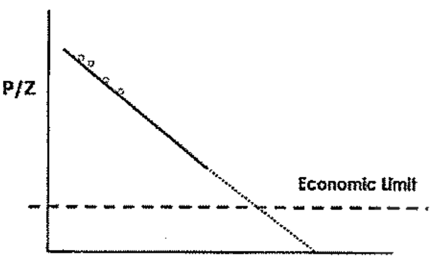
PRODUCTION FORCEAST TECHNIQUES

Material Balance



Reserves Estimation Methodology

Material Balance



Initial reservoir oil volume = NB_{oi}

Oil volume at time t and pressure p = $(N - N_p)B_o$

① Change in oil volume = $NB_{oi} - (N - N_p)B_o$ (2.1)

Change in Oil Volume

Change in Free Gas Volume:

$$\begin{bmatrix}\text{Ratio of initial free gas}\\ \text{to initial oil volume}\end{bmatrix} = m = \frac{GB_{gi}}{NB_{oi}} = \frac{V_g}{V_o}$$

Initial free gas volume = $GB_{gi} = NmB_{oi}$

$$\begin{bmatrix}\text{SCF free}\\ \text{gas at } t\end{bmatrix} = \begin{bmatrix}\text{SCF initial gas,}\\ \text{free and dissolved}\end{bmatrix} - \begin{bmatrix}\text{SCF gas}\\ \text{produced}\end{bmatrix} - \begin{bmatrix}\text{SCF remaining}\\ \text{in solution}\end{bmatrix}$$

$$G_f = \left[\frac{NmB_{oi}}{B_{gi}} + NR_{si}\right] - [N_pR_p] - [(N - N_p)R_{so}]$$

$$\begin{bmatrix}\text{Reservoir free gas}\\ \text{volume at time } t\end{bmatrix} = \left[\frac{NmB_{oi}}{B_{gi}} + NR_{si} - N_pR_p - (N - N_p)R_{so}\right]B_g$$

② $$\begin{bmatrix}\text{Change in free}\\ \text{gas volume}\end{bmatrix} = NmB_{oi} - \left[\frac{NmB_{oi}}{B_{gi}} + NR_{si} - N_pR_p - (N - N_p)R_{so}\right]B_g \quad (2.2)$$

Change in Water Volume:

Initial reservoir water volume = W
Cumulative water produced at t = W_p
Reservoir volume of cumulative produced water = B_wW_p
Volume of water encroached at t = W_e

Initial Final

③ $$\begin{bmatrix}\text{Change in}\\ \text{water volume}\end{bmatrix} = W - (W + W_e - B_wW_p + Wc_w\Delta p) = -W_e + B_wW_p - Wc_w\Delta p \quad (2.3)$$

Change in the Void Space Volume:

Initial void space volume = V_f V_f

④ $$\begin{bmatrix}\text{Change in void}\\ \text{space volume}\end{bmatrix} = V_f - [V_f - V_fc_f\Delta p] = V_fc_f\Delta p$$

..., because the change in void space volume is the negative of the change in rock volume:

$$\begin{bmatrix}\text{Change in}\\ \text{rock volume}\end{bmatrix} = -V_fc_f\Delta p \quad (2.4)$$

Combining the changes in water and rock volumes into a single term, yields the following:

$$= -W_e + B_wW_p - Wc_w\Delta p - V_fc_f\Delta p$$

Recognizing that $W = V_fS_{wi}$ and that $V_f = \frac{NB_{oi} + NmB_{oi}}{1 - S_{wi}}$ and substituting, the following is obtained:

$$= -W_e + B_wW_p - \left[\frac{NB_{oi} + NmB_{oi}}{1 - S_{wi}}\right](c_w S_{wi} + c_f)\Delta p$$

or

$$= -W_e + B_wW_p - (1 + m)NB_{oi}\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta p \quad (2.5)$$

Equating the changes in the oil and free gas volumes to the negative of the changes in the water and rock volumes and expanding all terms

$$NB_{oi} - NB_o + N_pB_o + NmB_{oi} - \left[\frac{NmB_{oi}B_g}{B_{gi}}\right] - NR_{si}B_g + N_pR_pB_g$$
$$+ NB_gR_{so} - N_pB_gR_{so} = W_e - B_wW_p + (1 + m)NB_{oi}\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta p$$

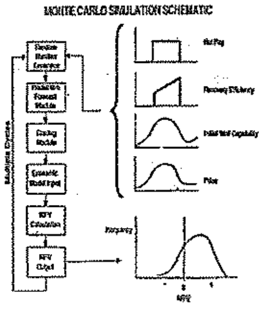
Now adding and subtracting the term $N_pB_gR_{si}$

$$NB_{oi} - NB_o + N_pB_o + NmB_{oi} - \left[\frac{NmB_{oi}B_g}{B_{gi}}\right] - NR_{si}B_g + N_pR_pB_g + NB_gR_{so}$$
$$- N_pB_gR_{so} + N_pB_gR_{si} - N_pB_gR_{si} = W_e - B_wW_p + (1 + m)NB_{oi}\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta p$$

Then grouping terms:

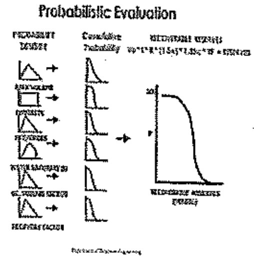
$$NB_{oi} + NmB_{oi} - N[B_o + (R_{si} - R_{so})B_g] + N_p[B_o + (R_{si} - R_{so})B_g]$$
$$+ (R_p - R_{si})B_gN_p - \left[\frac{NmB_{oi}B_g}{B_{gi}}\right] = W_e - B_wW_p + (1 + m)NB_{oi}\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta p$$

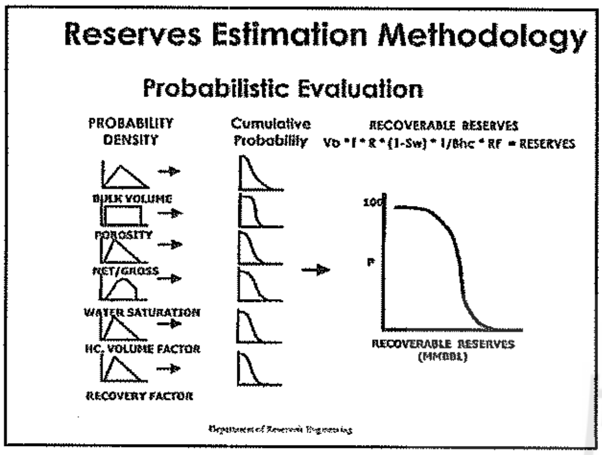
FIGURE 8.1
MONTE CARLO SIMULATION SCHEMATIC



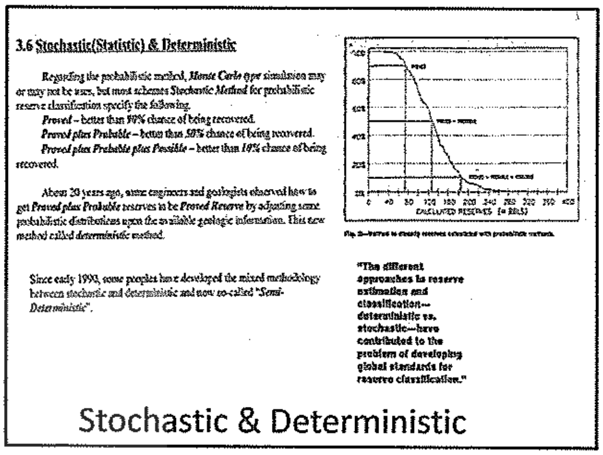
Reserves Estimation Methodology

Probabilistic Evaluation

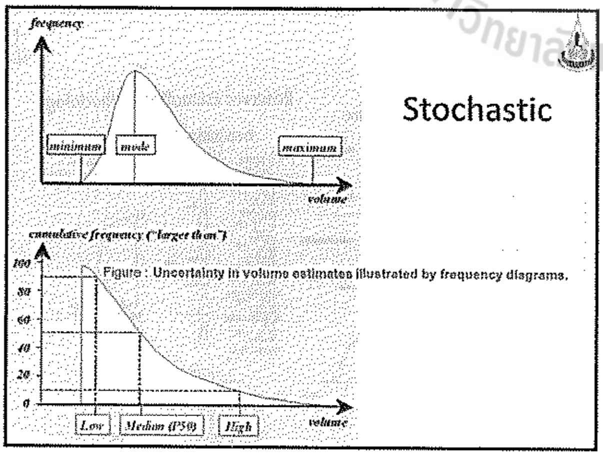




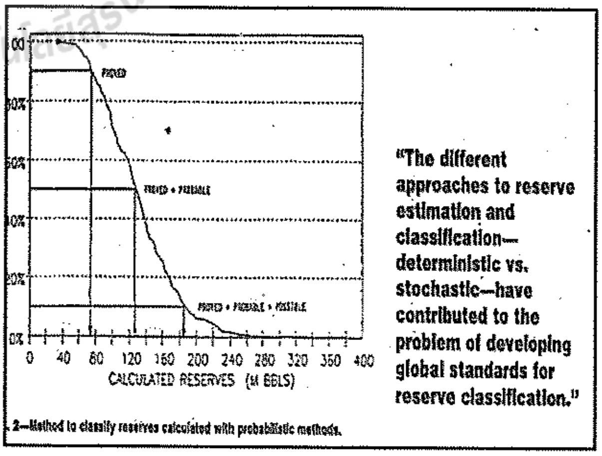
Reserves Estimation Methodology
Probabilistic Evaluation
PROBABILITY DENSITY
Cumulative Probability
RECOVERABLE RESERVES
Vb * f * R * (1-Sw) * 1/Bhc * RF = RESERVES
BULK VOLUME
POROSITY
NET/GROSS
WATER SATURATION
HC. VOLUME FACTOR
RECOVERY FACTOR
100
P
RECOVERABLE RESERVES (MMBBL)



3.6 Stochastic(Statistic) & Deterministic
Regarding the probabilistic method, Monte Carlo type simulation may or may not be uses, but most schemes Stochastic Method for probabilistic reserve classification specify the following.
Proved – better than 90% chance of being recovered.
Proved plus Probable – better than 50% chance of being recovered.
Proved plus Probable plus Possible – better than 10% chance of being recovered.
About 20 years ago, some engineers and geologists observed how to get Proved plus Probable reserves to be Proved Reserve by adjusting some probabilistic distributions upon the available geologic information. This new method called deterministic method.
Since early 1990, some peoples have developed the mixed methodology between stochastic and deterministic and now so-called "Semi-Deterministic".
"The different approaches to reserve estimation and classification—deterministic vs. stochastic—have contributed to the problem of developing global standards for reserve classification."
Stochastic & Deterministic



frequency
minimum
mode
maximum
volume
cumulative frequency ("larger than")
100
80
60
40
20
0
Low
Medium (P50)
High
volume
Stochastic
Figure : Uncertainty in volume estimates illustrated by frequency diagrams.



PROVED
PROVED + PROBABLE
PROVED + PROBABLE + POSSIBLE
0%
20%
40%
60%
80%
0
40
80
120
160
200
240
280
320
360
400
CALCULATED RESERVES (M BBLS)
"The different approaches to reserve estimation and classification—deterministic vs. stochastic—have contributed to the problem of developing global standards for reserve classification."
2—Method to classify reserves calculated with probabilistic methods.

Possible Reserves: Deterministic vs. Probabilistic

RYDER SCOTT

Deterministic Methods:

"When deterministic methods are used, the total quantities ultimately recovered from a project have a low probability of exceeding proved plus probable plus possible reserves."

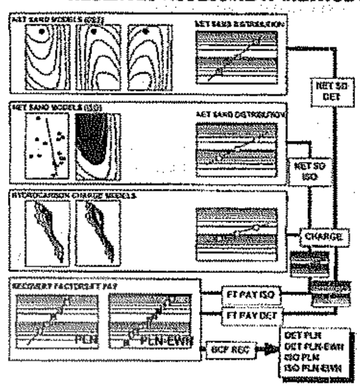
Source: 210.4-10 (a)(17)(i) pg. 2191

Probabilistic Methods:

"When probabilistic methods are used, there should be at least a 10% probability that the total quantities ultimately recovered will equal or exceed the proved plus probable plus possible reserves estimates."

Source: 210.4-10 (a)(17)(i) pg. 2191

RESERVES ASSESSMENT METHODOLOGY



Net Sand
x
Charge
=
Ft of Pay

Ft of Pay
x
Recovery Factor
=
Reserve

GOMIN 'A' PLATFORM

Well	Unrisked Reserves(BCF)	POS (%)	Risked Reserves(BCF)	
Gomin-3	4.87	1.00	4.87	
Gomin A-10	3.73	0.71	2.66	omit from economics
Gomin A-5	3.73	0.81	3.02	
Gomin A-2	3.73	0.90	3.36	
Gomin A-9	3.73	0.77	2.85	
Gomin A-6	3.73	0.81	3.01	
Gomin A-12	3.73	0.70	2.61	
Gomin A-4	2.80	0.90	2.52	
Gomin A-13	2.80	0.80	2.24	
Gomin A-8	1.87	0.80	1.49	
Gomin A-7	3.43	0.81	2.77	omit from economics
Gomin A-3	3.43	0.81	2.77	omit from economics
Gomin A-11	3.43	0.72	2.48	omit from economics
total	45.03		36.67	

GOMIN 'B' PLATFORM

Well	Unrisked Reserves(BCF)	POS (%)	Risked Reserves(BCF)	
Gomin-5	4.10	1.00	4.10	
G875A S.	4.36	0.70	3.05	omit from economics
G875A M.	4.36	0.60	2.62	omit from economics
G875A N.	4.36	0.60	2.62	omit from economics
G860A N.	2.18	0.90	1.96	
G865A S.	4.36	0.90	3.92	
G865A S.M.	4.36	0.90	3.92	
G865A N.M	4.36	0.85	3.71	
G865A N.	4.36	0.85	3.71	
G855A N.	4.36	0.86	3.73	
G855A M.	4.36	0.81	3.52	
G855A S.	4.36	0.81	3.52	
total	49.87		40.38	

DEFINITIONS

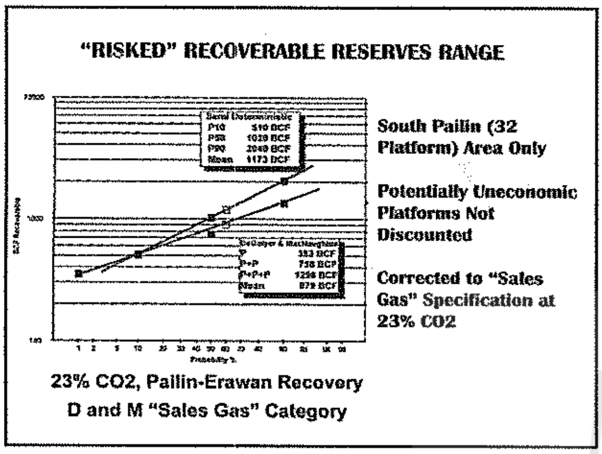
Swanson's Mean (SM) - a quick approximate method to estimate the mean value of a log-normal distribution which can be used instead of the statistical mean if the ratio of the reserve at the P90 level over the reserve at the P50 level is less than five.

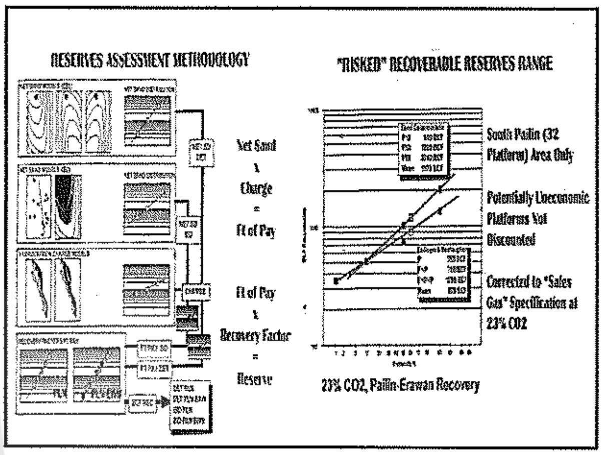
$$SM = (0.30 \times P10) + (0.4 \times P50) + (0.3 \times P90)$$

where,

P10 = the level for which there is a 10% probability of occurrence.
P50 = the level for which there is a 50% probability of occurrence.
P90 = the level for which there is a 90% probability of occurrence.

Probability of Success (POS) - the possibility of getting onto the reserves distribution or range of outcomes being used in the evaluation, and is a function of the risks (geologic, mechanical, etc.) of the project.


"RISKED" RECOVERABLE RESERVES RANGE
South Pailin (32 Platform) Area Only
Potentially Uneconomic Platforms Not Discounted
Corrected to "Sales Gas" Specification at 23% CO2
23% CO2, Pailin-Erawan Recovery
D and M "Sales Gas" Category


RESERVES ASSESSMENT METHODOLOGY
Net Sand
x
Charge
=
Ft of Pay
Ft of Pay
x
Recovery Factor
=
Reserve
"RISKED" RECOVERABLE RESERVES RANGE
South Pailin (32 Platform) Area Only
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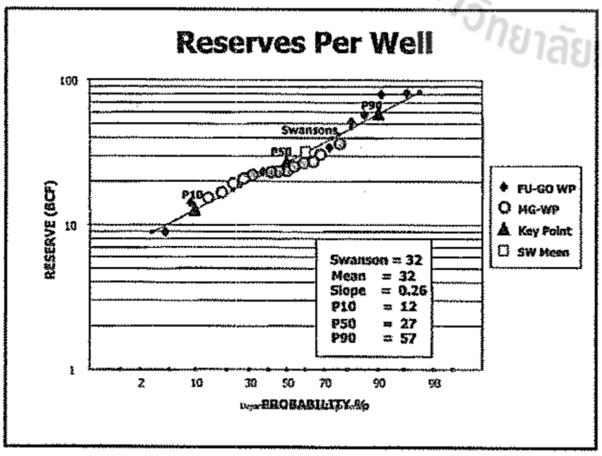
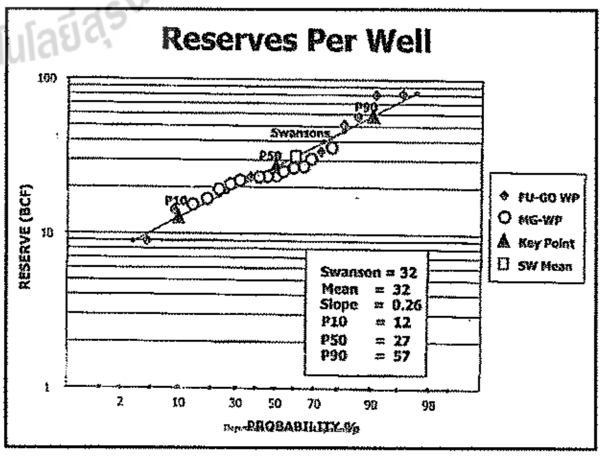
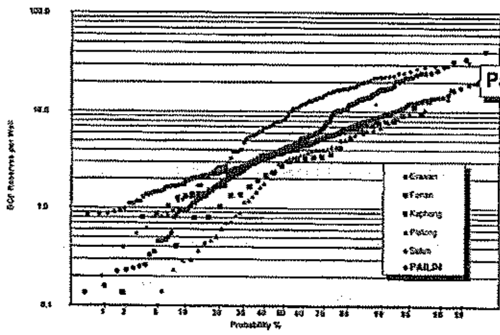
23% CO2, Pailin-Erawan Recovery


Reserves Per Well
RESERVE (BCF)
PROBABILITY %
P10
P50
P90
Swansons
FU-GO WP
MG-WP
Key Point
SW Mean
Swanson = 32
Mean = 32
Slope = 0.26
P10 = 12
P50 = 27
P90 = 57


Reserves Per Well
RESERVE (BCF)
PROBABILITY %
P10
P50
P90
Swansons
FU-GO WP
MG-WP
Key Point
SW Mean
Swanson = 32
Mean = 32
Slope = 0.26
P10 = 12
P50 = 27
P90 = 57

RESERVES/WELL vs PRODUCING FIELDS

Higher at Low Probabilities; Possibly Related to Massive Sands Reservoirs



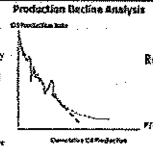
3.7 Reservoir Performances

Prior to drilling, the reserve may be estimated by reservoir structure size comparison with the previous discovered reservoir, but after exploratory drilling the volumetric method will be applied. After production begins, the reservoir performance will be collected and the more accurate methods will be applied to determine gas(or petroleum) reserve.

3.7.1 Decline Curves

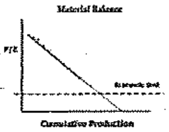
When the production data has been collected for some time (cumulative production about 5-10% of the gas in place) then the "Decline Curve" method can be applied to give the more accurate results. Specially the gas, P/Z V.S. G_p (cumulative production from material balance equation) plot can be projected as a straight line to the economic limit and get the reserve.

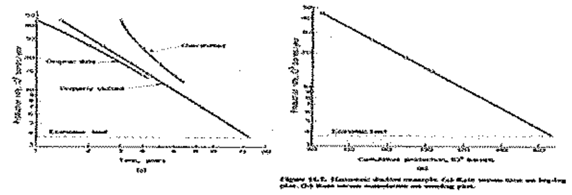
Production Decline Analysis



Reserves Estimation Method

Material Balance





3.7.2 Reservoir Simulations

What Simulation is

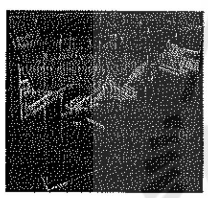
- *Simulate* = to give the appearance of as a reservoir
- *Simulation* involves the utilization of a model to obtain some insight into the behavior of a physical process.
- *Simulation* has long been recognized in many applied science disciplines as the final resort; as Wagner aptly says: " When all else fails,... simulate.
- In operations research, extensive use has been made of simulation studies; some examples are:
  1. Transportation model networks
  2. Stock market performance
  3. Telephone system design
  4. Supermarket checkout counter
  5. Engine Simulation
  6. Other Simulations

What Reservoir Simulation is

- A *RESERVOIR Simulation* is a mathematical model that represents the physical phenomena of a reservoir

Reserves Estimation Methodology

Reservoir Simulation



Reservoir Simulation

What is Simulation?

☑ Imitation

☑ An attempt to duplicate the features, appearance, and characteristics of a real system

1. To imitate a real-world situation mathematically
2. To study its properties and operating characteristics
3. To draw conclusions and make action decisions based on the results of the simulation

---

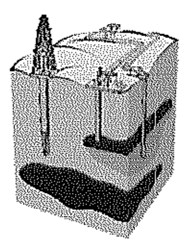
Simulation-New Definition

- A simulation is a computer-based model used to run experiments on a real system
  - Typically done on a computer
  - Determines reactions to different operating rules or change in structure
  - A given system is copied and the variables and constants associated with it are manipulated in that artificial environment to examine the behaviour of the system

---

What is Simulation?

As used here, the words *petroleum reservoir system* include the reservoir rock and fluids, aquifer, and the surface and subsurface facilities.



Fluid Flow Equations

Governing Equations

- **Mass Conservation (Material Balances)**
- **Darcy's Law**
  - *Conservation of Momentum (Navier Stoke Equation)*
  - *Conservation of Energy (First Law of Thermodynamics)*
- **Equation of State**

Darcy's Law

$$Q = \frac{kA}{\mu}\frac{\Delta P}{\Delta x}$$

or if we assume pressure gradient reducing in negative sign and we rewrite in a differential equation form.

$$v = -\frac{k}{\mu}\frac{\partial P}{\partial x}$$

Substitute v in Mass Balance Equation (consider only x-direction)

$$\frac{\partial\left(-\frac{k}{\mu}\frac{\partial P}{\partial x}\rho\right)}{\partial x} = -\phi\frac{\partial\rho}{\partial t}$$

What is reservoir simulation?

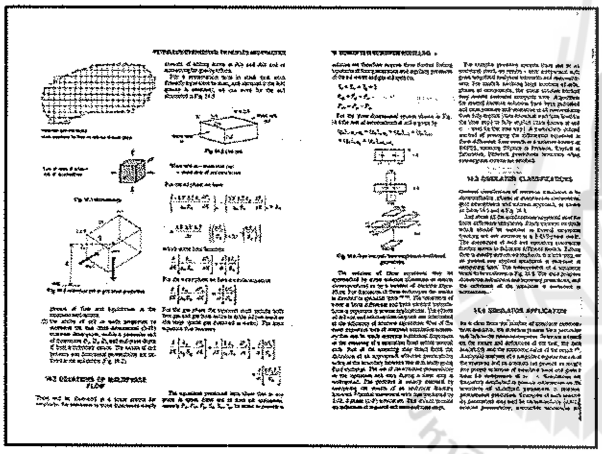
- **A reservoir simulation is a mathematical model that represent the physical phenomena of a reservoir.**

Why do we need a Reservoir Simulator?

- **Because a reservoir simulator is a powerful tool and not expensive . We can predict what is going in the reservoir and a amount of production from alternative operations.**

Types of Reservoir Simulation

- **Black Oil Simulation**
- **Compositional Black Oil Simulation**
- **Coupled Fluid Flow /Geomechanic Simulation**



The equations presented here show that at any point in space there are at least six unknowns, namely P_o, P_w, P_g, S_o, S_w, S_g. In order to provide a solution we therefore require three further linking equations defining saturation and capillary pressures of the oil-water and gas-oil system

$$S_o + S_w + S_g = 1 \quad , \quad P_{cg} = P_g - P_o \quad , \quad P_{cw} = P_o - P_w$$

For three-dimensional system shown in Fig.3.26

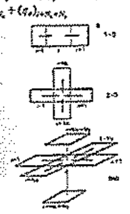


Figure 3.26 Flow in the cell i from neighbors in different geometries.

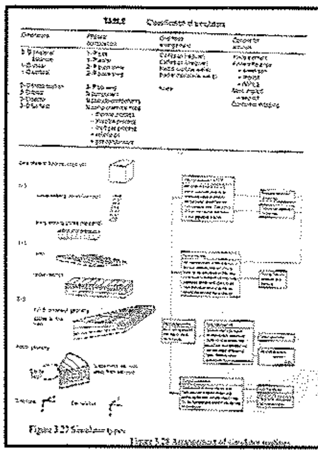


Figure 3.27 Simulator types

Figure 3.28 Arrangement of simulator modules

RESERVOIR DESCRIPTION IN MODELLING

Whether a complex of simple reservoir model is being applied, a number of steps in analysis and data requirements are common, and illustrated in Fig. 3.29. The validity of the initialisation reservoir model is largely dependent on geological model and the flow performance is linked to reservoir and production engineering description.

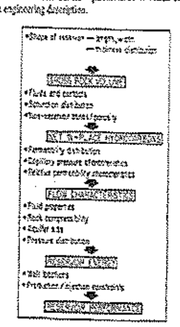


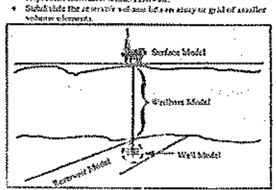
Figure 3.29 Steps needed to build a reservoir model.

Comprehensive Reservoir Management Model

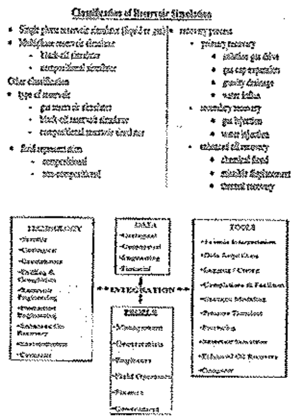
- Reservoir Model
- Well Model
- Wellbore Model
- Surface Model

Reservoir Model

- Represent fluid flow within reservoir.
- Subdivide the reservoir volume into an array or grid of smaller volume elements.



Reservoir management system.



Modeling Methods

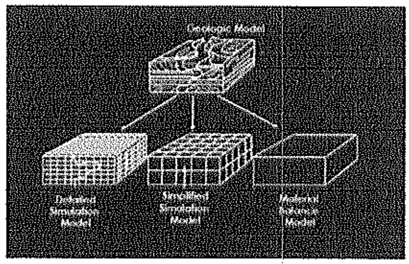
All mathematical techniques are simply an application of the three fundamental equations of reservoir engineering

- Darcy's Law
  \+
- Material Balance Equation
  \+
- Fluid Properties **(PVT or EOS)**

with varying boundary conditions

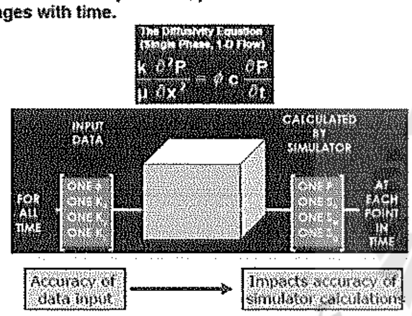
Modeling Methods
Finite Difference Process

Divide the reservoir into numerous blocks and represents it with a mesh of points or grid blocks.



Modeling Methods
Finite Difference 3 Step Process

Solve mathematical equations for each cell by numerical methods to obtain pressure, production and saturation changes with time.



3-4th WEEK (June 18-28, 2013) Outline

- Reserve Calculation
- **Volumetric**

The gas volumetric calculation can be performed as follows:

(a) Determine volume of rock containing gas(or hydrocarbons) from the structural and isopach maps.
(b) Determine void space in rock; average effective ((porosity)
(c) Determine volume percentage containing gas(or hydrocarbons) fluid saturation $S_g = 1-S_w$
(d) Determine recoverable gas (or hydrocarbons) by multipling with recovery factor (F_R)
(e) Determine reserve volume in standard condition by dividing with gas formation volume factor (B_g).

$$\text{Reserves (SCF)} = \underbrace{\underbrace{\underbrace{\underbrace{\underbrace{A \times h}_{\text{Bulk Reservoir Volume}} \times \phi}_{\text{Pore Volume}} \times (1-S_w)}_{\text{Gas Volume in Reservoir}} \times F_R}_{\text{Recoverable Gas Volume}} \times (1/B_g)}_{\text{Recoverable Gas Volume in SCF}}$$

SCF = Standard Cubic Foot (at standard condition)
$A \times h$ = reservoir volume in cubic feet
ϕ = porosity expressed as a fraction of bulk volume
S_w = water saturation expressed as a fraction of void space
F_R = Recovery factor expressed as a fraction of total in-place
B_g = formation volume factor in cu ft reservoir/ SCF surface

If A (h is expressed in Acre-ft, the above formula will be

$$\text{Reserves(SCF)} = 43{,}560\, A \times h \times \phi \times (1-S_w) \times F_R \times (1/B_g)$$

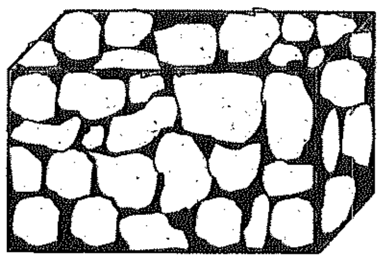
Gas Volumetric

OIL Reserve= $7758Ah\phi(1-S_w)B_oF_r$

Reserves Estimation Methodology

Volumetric Calculation

Reserves = Bulk Volume . ϕ . So . Bo . Rf



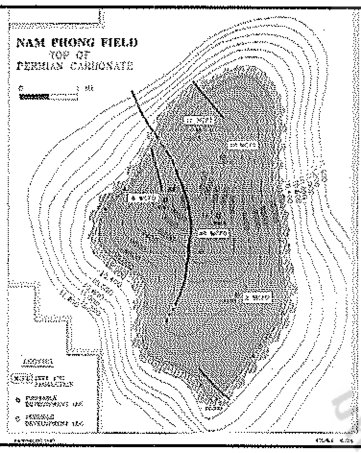
ϕ = Porosity
So = Saturation
Bo = Volume Factor
Rf = Recovery Factor

Volumetric Estimates

- Reserves = Reservoir Volume x Porosity x Oil Saturation x Recovery Factor x Shrinkage to Surface Conditions
- In oilfield units:

Reserves = [7758 x A x h x ø x (1-S_w) x R]/Bo
where   Or 43560 for gas   or Bg

7757	= bbls/acre-ft
A	= area (sq. ft) or Acre
h	= net thickness (ft)
ø	= porosity (fraction)
S_w	= water saturation (fraction)
R	= recovery factor (fraction)
Bo	= formation volume factor
Bg	= Gas formatiom volume factor



The magnitude of uncertainty, however, decreases with time until the economic limit is reached and the ultimate recovery is realized, see Figure 1.

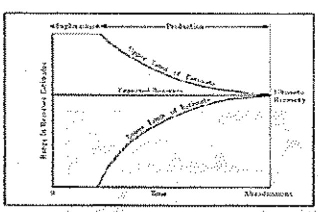


Figure 1: Magnitude of uncertainty in reserves estimates

The oil and gas reserves estimation methods can be grouped into the following categories:

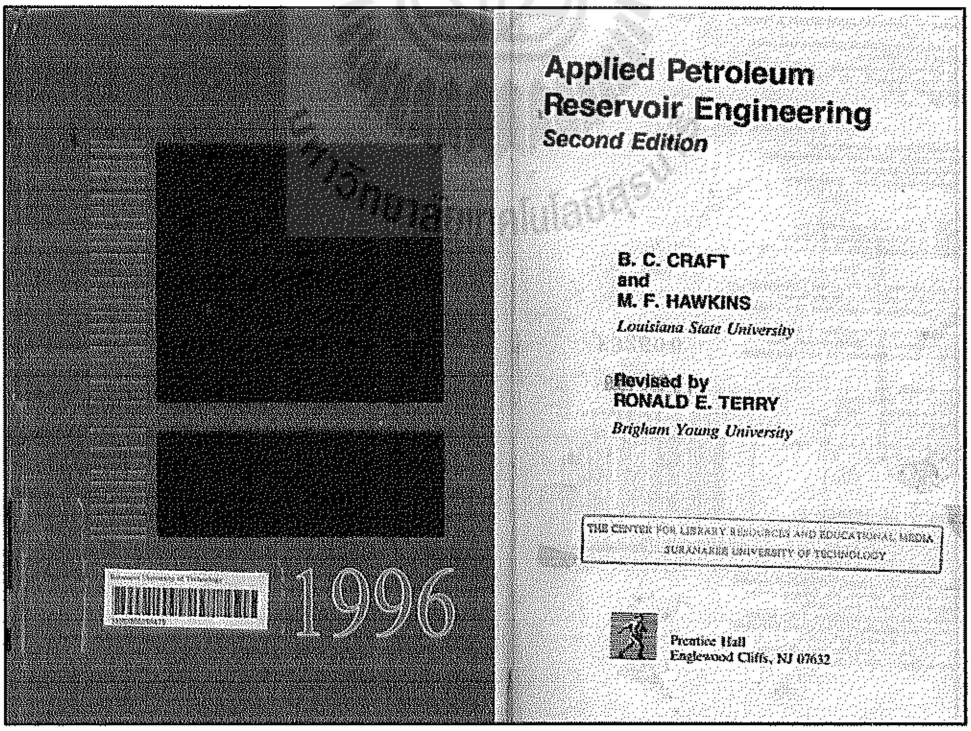
1. Analogy,
2. Volumetric,
3. Decline analysis,
4. Material balance calculations for oil reservoirs,
5. Material balance calculations for gas reservoirs,
6. Reservoir simulation.

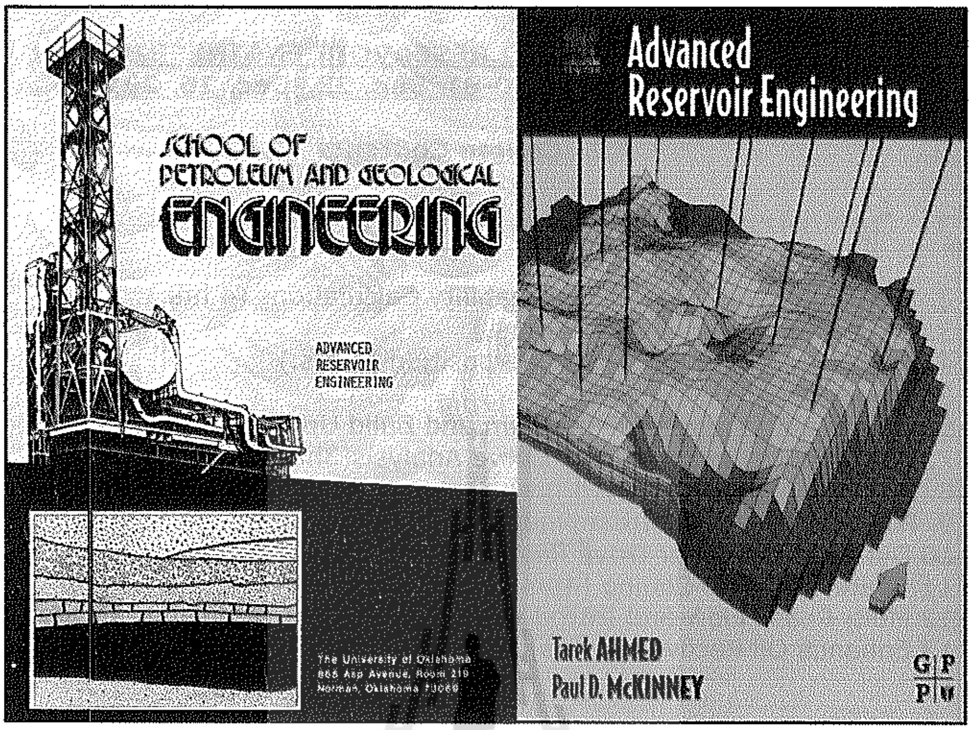
534619 ADVANCED RESERVOIR ENGINEERING
2/2556(4 credits(4-0-8), Sep. 23,-Dec. 26, 2013.)

Course Contents

1. **Optimization of Material Balance Equations (6 hrs.)**
2. Saturation and Relative Permeability Calculations (4 hrs.)
3. Steady State Radial Flow (4 hrs.)
4. Unsteady State Flow and Superposition (4 hrs.)
5. Pseudo-steady State
6. Well Testing Pressure Drawdown and Build Up (3 hrs.)
7. Interference Test and Type Curve Analysis(3hrs)
8. Displacement Efficiency(4 hrs)
9. Potential flow and Streamlines (4 hrs.)

10 Dynamics of Water Drive Reservoir. (6 hrs.)
11. Water and Gas Coning (4hrs.)
12 Multi-Phase Flow and Introduction to Reservoir Simulation (4 hrs.)
13 Enhance Oil Recovery(2 hrs)




SCHOOL OF
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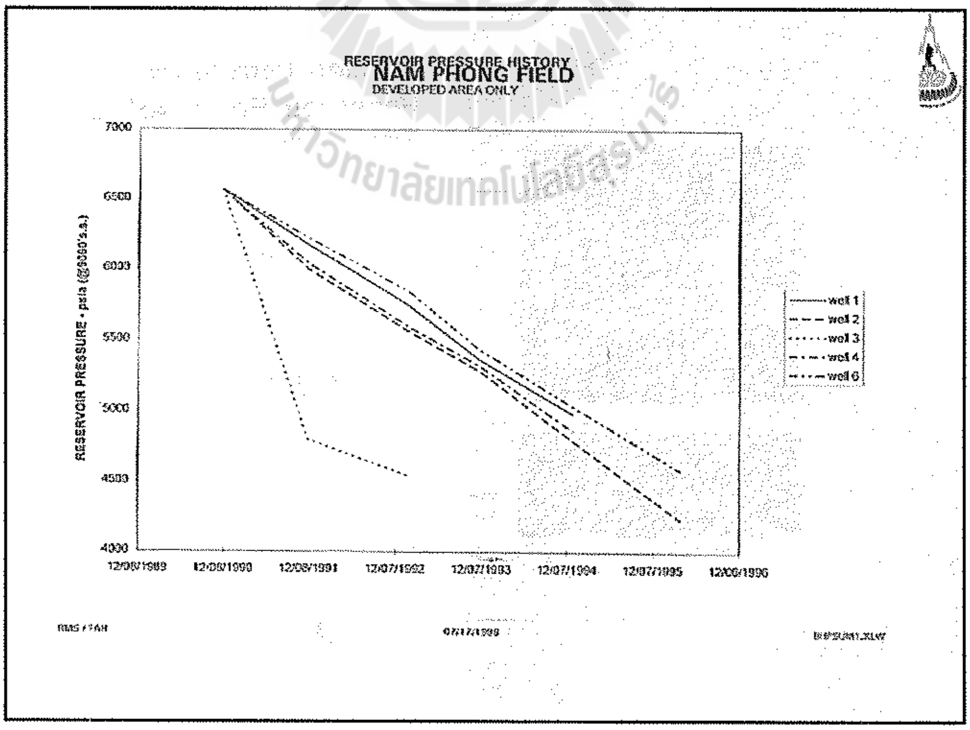
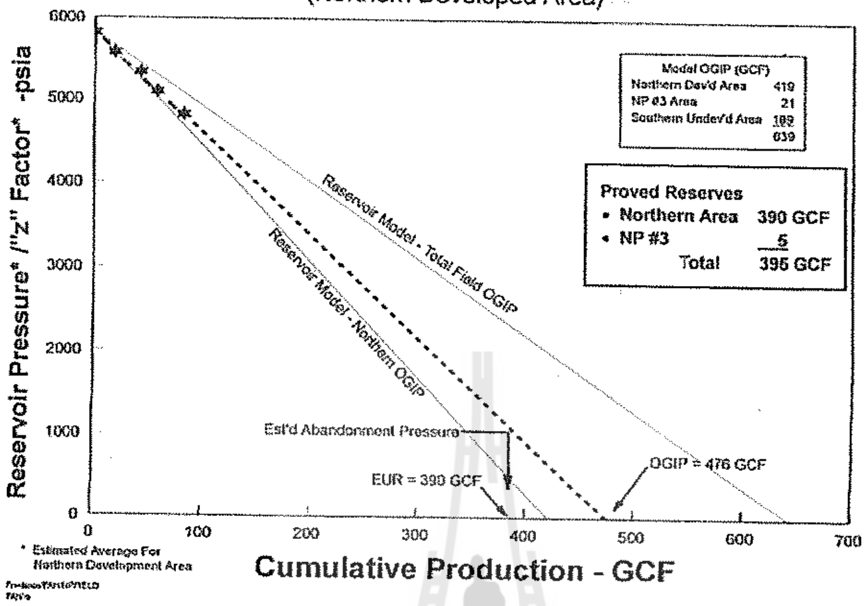
The University of Oklahoma
865 Asp Avenue, Room 210
Norman, Oklahoma 73069
Advanced
Reservoir Engineering
Tarek AHMED
Paul D. McKINNEY


RESERVOIR PRESSURE HISTORY
NAM PHONG FIELD
DEVELOPED AREA ONLY
7000
6500
6000
5500
5000
4500
4000
12/08/1989
12/08/1990
12/08/1991
12/07/1992
12/07/1993
12/07/1994
12/07/1995
12/06/1996
well 1
well 2
well 3
well 4
well 6

Nam Phong Field

PRESSURE vs PRODUCTION
(Northern Developed Area)



DISSOLVES GAS DRIVE

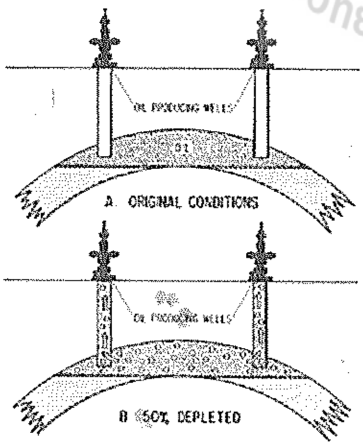


Fig. 61—*Dissolved gas drive reservoir.*

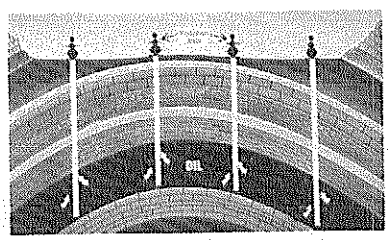


Fig. 1.3—Dissolved gas drive.

GAS CAP DRIVE

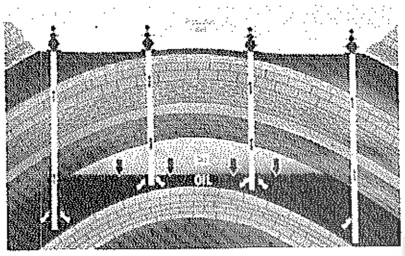


Fig. 1.4—Gas-cap drive.

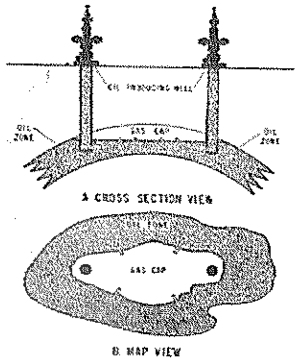


Fig. 63—Gas cap drive reservoir.

WATER DRIVE

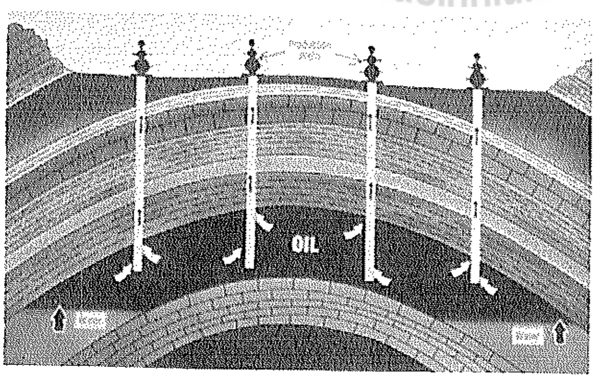


Fig. 1.2—Water-drive reservoir.

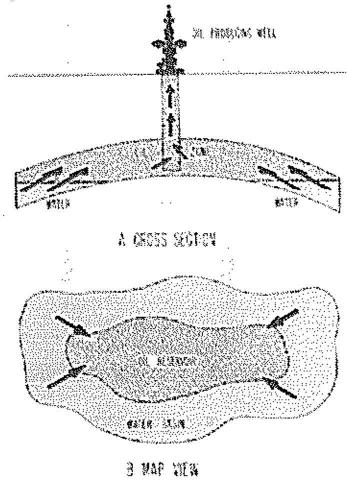


Fig. 65—Water drive reservoir.

COMBINATION DRIVE

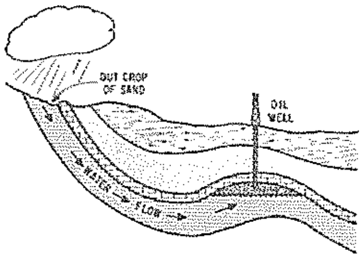


Fig. 68—Reservoir having artesian water drive.

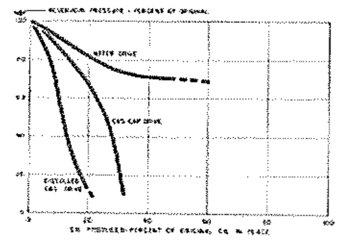


Fig. 69—Reservoir pressure trends for reservoirs under various drives. (Courtesy API, Drilling and Production Practices—1945.)

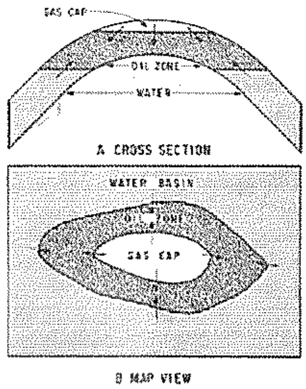
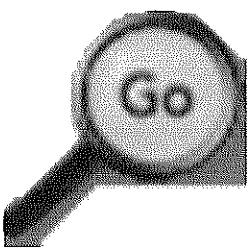


Fig. 71—Combination drive reservoir.

1-2th WEEK (Sep.25-October6 2013) Outline



- Reserve Calculation
- Material Balance

MATERIAL BALANCES IN GAS RESERVOIRS

$$V_f = V_i - W_e + B_w W_p$$

Conservation of Mass.

$$\begin{bmatrix}\text{Wt. of Gas} \\ \text{Produced}\end{bmatrix} = \begin{bmatrix}\text{Initial wt.} \\ \text{in the Reservoir}\end{bmatrix} - \begin{bmatrix}\text{Remaining} \\ \text{wt.}\end{bmatrix}$$

$$n_p = n_i - n_f$$

$$\frac{P_{sc} G_p}{T_{sc}} = \frac{P_i V_i}{z_i T} - \frac{P_f V_f}{z_f T}$$

$$= \frac{P_i V_i}{z_i T} - \frac{P_f (V_i - W_e + B_w W_p)}{z_f T}$$

Volumetric Reservoir $V_i = V_f$

$$\frac{P_{sc} G_p}{T_{sc}} = \frac{P_i V_i}{z_i T} - \frac{P_f V_i}{z_f T}$$

$$G_p = b - m\left(\frac{P_f}{z_f}\right) \quad \Big| \quad b = \frac{P_i V_i T_{sc}}{z_i P_{sc} T}, \quad m = V_i T_{sc} / P_{sc} T$$

$$\frac{P_{sc} G_p}{T_{sc}} = \frac{P_i V_i}{z_i T} - \frac{P_f V_i}{z_f T}$$

$$\frac{P_{sc}}{T_{sc}} \cdot \frac{z_f T}{P_f} G_p = \frac{P_i}{z_i T} \cdot \frac{z_f T}{P_f} \cdot V_i - V_i$$

แทนค่า $B_{gf} = \frac{P_{sc} z_f T}{T_{sc} P_f}$ (Eq. 1.16), $V_i \Rightarrow B_{gi} G$

และ $$\frac{B_{gf}}{B_{gi}} = \frac{\frac{P_{sc} z_f T}{T_{sc} P_f}}{\frac{P_{sc} z_i T}{T_{sc} P_i}} = \frac{P_i}{z_i T} \cdot \frac{z_f T}{P_f}$$

ดังนั้น Eq.(1) becomes

$$B_{gf} G_p = \frac{B_{gf}}{B_{gi}} \cdot B_{gi} G - B_{gi} \cdot G$$

$$B_{gf} G_p = B_{gf} G - B_{gi} G.$$

$$G B_{gi} = (G - G_p) B_{gf}$$

$$= (G - G_p) B_g$$

$$G(B_g - B_{gi}) = G_p B_g$$

For most gas reservoirs, the gas compressibility term is much greater than the formation and water compressibilities, and the second term on the left-hand side of Eq. (2.10) becomes negligible

$$G B_{gi} \left[\frac{C_w S_{wi} + C_f}{1 - S_{wi}}\right] + G(B_g - B_{gi}) + W_e = G_p B_g + B_w W_p \qquad (3.15)$$

When reservoir pressures are abnormally high, this term is not negligible and should not be ignored. This situation is discussed in a later section of this chapter.

When there is neither water encroachment into nor water production from a reservoir of interest, the reservoir is said to be *volumetric*. For a volumetric gas reservoir, Eq. (3.15) reduces to:

$$G(B_g - B_{gi}) = G_p B_g \qquad (3.16)$$

Using Eq. (1.16) and substituting expressions for B_g and B_{gi} into Eq. (3.16), the following is obtained:

$$G\left(\frac{p_{sc} z T}{T_{sc} p}\right) - G\left(\frac{p_{sc} z_i T_i}{T_{sc} p_i}\right) = G_p\left(\frac{p_{sc} z T}{T_{sc} p}\right) \qquad (3.17)$$

Noting that production is essentially an isothermal process (i.e., the reservoir temperature remains constant), then Eq. (3.17) is reduced to:

$$G\left(\frac{z}{p}\right) - G\left(\frac{z_i}{p_i}\right) = G_p\left(\frac{z}{p}\right)$$

Rearranging: $y = ax + b$

$$\frac{p}{z} = -\frac{p_i}{z_i G} G_p + \frac{p_i}{z_i} \qquad (3.18)$$

Because p_i, z_i, and G are constants for a given reservoir, Eq. (3.18) suggests that a plot of p/z as the ordinate versus G_p as the abscissa would yield a straight line with:

$$\text{slope} = -\frac{p_i}{z_i G}$$

$$y \text{ intercept} = \frac{p_i}{z_i}$$

This plot is shown in Fig. 3.6.

Table B1 - Gas-in-Place Estimates

Δp (psi)	Cf (1/psi)	GIP (Gscf)	Comments
7.70	6x10^{-6}	433	No communication wit Nam Phong 2
4.47	6x10^{-6}	744	90% confidence interval
10.93	6x10^{-6}	304	90% confidence interval
7.70	6x10^{-5}	249	High formation compressibilit
6.00	6x10^{-6}	565	Communication with Nam Phong Includes gas volume of bot wells

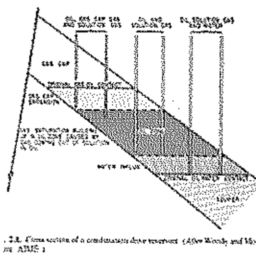
Values for other variables used in the equation:

Gp	: 0.3267 Gscf
z	: 1.140082
Δz/Δp	: 0.0000707/psi
S_w	: 0.25
c_w	: 0.0000034/psi

time t. The change in the rock volume is expressed as a chang space volume, which is simply the negative of the change in the In the development of the general material balance equation, terms are used:

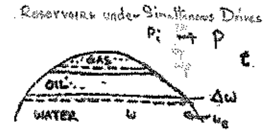
- N Initial reservoir oil, STB
- B_{oi} Initial oil formation volume factor, bbl/STB @ P_i >1.0
- N_p Cumulative produced oil, STB
- B_o oil formation volume factor, bbl/STB @ P
- G_i Initial reservoir gas, SCF @ P_i
- B_{gi} Initial gas formation volume factor, bbl/SCF @ P_i
- G_f Amount of free gas in the reservoir, SCF @ P
- R_{si} Initial solution gas-oil ratio, SCF/STB @ P_i
- R_p Cumulative produced gas-oil ratio, SCF/STB $\frac{G_p}{N_p}$
- R_{so} Solution gas-oil ratio, SCF/STB @ P
- B_g Gas formation volume factor, bbl/SCF @ P
- W Initial reservoir water, bbl
- W_p Cumulative produced water, STB
- B_w Water formation volume factor, bbl/STB >1.0
- W_e Water influx into reservoir, bbl
- c_w Water isothermal compressibility, psi^{-1}
- $\Delta\bar{p}$ Change in average reservoir pressure, psia $P_i \to P$
- S_{wi} Initial water saturation
- V_f Initial void space, bbl
- c_f Formation isothermal compressibility, psi^{-1}

C_o
C_g



2.2. Cross section of a combination drive reservoir. (After Woody and Moscrip; ... AIME.)

Reservoirs under Simultaneous Drives



① Change in Oil Volume

Change in the Oil Volume:

Initial reservoir oil volume = NB_{oi}
Oil volume at time t and pressure p = $(N - N_p)B_o$
Change in oil volume = $NB_{oi} - (N - N_p)B_o$ (2.1)

Change in Free Gas Volume:

$$\begin{bmatrix}\text{Ratio of initial free gas}\\ \text{to initial oil volume}\end{bmatrix} = m = \frac{GB_{gi}}{NB_{oi}}$$

gas cap / initial oil — produce

Initial free gas volume = $GB_{gi} = NmB_{oi}$

$$\begin{bmatrix}\text{SCF free}\\ \text{gas at } t\end{bmatrix} = \begin{bmatrix}\text{SCF initial gas,}\\ \text{free and dissolved}\end{bmatrix} - \begin{bmatrix}\text{SCF gas}\\ \text{produced}\end{bmatrix} - \begin{bmatrix}\text{SCF remaining}\\ \text{in solution}\end{bmatrix}$$

$$G_f = \left[\frac{NmB_{oi}}{B_{gi}} + NR_{soi}\right] - [N_pR_p] - [(N - N_p)R_{so}]$$

$$\begin{bmatrix}\text{Reservoir free gas}\\ \text{volume at time } t\end{bmatrix} = \left[\frac{NmB_{oi}}{B_{gi}} + NR_{soi} - N_pR_p - (N - N_p)R_{so}\right]B_g$$

$$\begin{bmatrix}\text{Change in free}\\ \text{gas volume}\end{bmatrix} = NmB_{oi} - \left[\frac{NmB_{oi}}{B_{gi}} + NR_{soi} - N_pR_p - (N - N_p)R_{so}\right]B_g \quad (2.2)$$

$G B_{gi} = V_{gi}$ — $G_f B_g = V_g$

Change in Water Volume:

Initial reservoir water volume = W
Cumulative water produced at t = W_p
Reservoir volume of cumulative produced water = B_wW_p
Volume of water encroached at t = W_e

$$\begin{bmatrix}\text{Change in}\\ \text{water volume}\end{bmatrix} = W - (W + W_e - B_wW_p + Wc_w\Delta\bar{p}) = -W_e + B_wW_p - Wc_w\Delta\bar{p} \quad (2.3)$$

$c_w = \frac{1}{V_w}\frac{dV_w}{dp}$

Change in the Void Space Volume:

Initial void space volume = V_f

$c_f = \frac{1}{V_f}\frac{dV_f}{dp}$

$$\begin{bmatrix}\text{Change in void}\\ \text{space volume}\end{bmatrix} = V_f - [V_f - V_fc_f\Delta\bar{p}] = V_fc_f\Delta\bar{p}$$

Or, because the change in void space volume is the negative of the change in rock volume:

$$\begin{bmatrix}\text{Change in}\\ \text{rock volume}\end{bmatrix} = -V_fc_f\Delta\bar{p} \quad (2.4)$$

Combining the changes in water and rock volumes into a single term, yields the following:

$$= -W_e + B_wW_p - Wc_w\Delta\bar{p} - V_fc_f\Delta\bar{p}$$

Recognizing that $W = V_fS_{wi}$ and that $V_f = \frac{NB_{oi} + NmB_{oi}}{1 - S_{wi}}$ and substituting, the following is obtained:

$$= -W_e + B_wW_p - \left[\frac{NB_{oi} + NmB_{oi}}{1 - S_{wi}}\right](c_wS_{wi} + c_f)\Delta\bar{p}$$

or

$$= -W_e + B_wW_p - (1 + m)NB_{oi}\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta\bar{p} \quad (2.5)$$

Equating the changes in the oil and free gas volumes to the negative of the changes in the water and rock volumes and expanding all terms

$$NB_{oi} - NB_o + N_pB_o + NmB_{oi} - \left[\frac{NmB_{oi}B_g}{B_{gi}}\right] - NR_{soi}B_g + N_pR_pB_g + NB_gR_{so} - N_pB_gR_{so} = W_e - B_wW_p + (1 + m)NB_{oi}\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta\bar{p}$$

Now adding and subtracting the term $N_pB_gR_{soi}$

$$NB_{oi} - NB_o + N_pB_o + NmB_{oi} - \left[\frac{NmB_{oi}B_g}{B_{gi}}\right] - NR_{soi}B_g + N_pR_pB_g + NB_gR_{so} - N_pB_gR_{so} + N_pB_gR_{soi} - N_pB_gR_{soi} = W_e - B_wW_p + (1 + m)NB_{oi}\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta\bar{p}$$

Then grouping terms:

$$NB_{oi} + NmB_{oi} - N[B_o + (R_{soi} - R_{so})B_g] + N_p[B_o + (R_{soi} - R_{so})B_g] + (R_p - R_{soi})B_gN_p - \left[\frac{NmB_{oi}B_g}{B_{gi}}\right] = W_e - B_wW_p + (1 + m)NB_{oi}\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta\bar{p}$$

Now writing $B_{oi} = B_{ti}$ and $[B_o + (R_{soi} - R_{so})B_g] = B_t$, where B_t is the two phase formation volume factor, as defined by Eq. 1.78:

$$N(B_t - B_{ti}) + N_p[B_t + (R_p - R_{soi})B_g] + NmB_{ti}\left(1 - \frac{B_g}{B_{gi}}\right) = W_e - B_wW_p + (1 + m)NB_{ti}\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta\bar{p} \quad (2.6)$$

This is the general volumetric material balance equation. It can be rearranged into the following form that is useful for discussion purposes.

$$N(B_t - B_{ti}) + \frac{NmB_{ti}}{B_{gi}}(B_g - B_{gi}) + (1 + m)NB_{ti}\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta\bar{p} + W_e = N_p[B_t + (R_p - R_{soi})B_g] + B_wW_p \quad (2.7)$$

Each term on the left-hand side of Eq. (2.7) accounts for a method of fluid production, and each term on the right-hand side represents an amount of hydrocarbon or water production. The first two terms on the left-hand side account for the expansion of any oil and/or gas zones that might be present. The third term accounts for the change in void space volume, which is the expansion of the formation and connate water. The fourth term is the amount of water influx that has occurred into the reservoir. On the right-hand side, the first term represents the production of oil and gas and the second term represents the water production.

Equation (2.7) can be arranged to apply to any of the different reservoir types discussed in Chapter 1. Without eliminating any terms, Eq. (2.7) is used for the case of a saturated oil reservoir with an associated gas cap. These reservoirs are discussed in Chapter 6. When there is no original free gas, such as in an undersaturated oil reservoir (discussed in Chapter 5), $m = 0$ and Eq. (2.7) reduces to:

$$N(B_t - B_{ti}) + NB_{ti}\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta\bar{p} + W_e = N_p[B_t + (R_p - R_{soi})B_g] + B_wW_p \quad (2.8)$$

For gas reservoirs, Eq. (2.7) can be modified by recognizing that $N_pR_p = G_p$ and that $NmB_{ti} = GB_{gi}$ and substituting these terms into Eq. (2.7):

$$N(B_t - B_{ti}) + G(B_g - B_{gi}) + (NB_{ti} + GB_{gi})\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta\bar{p} + W_e = N_pB_t + (G_p - N_pR_{soi})B_g + B_wW_p \quad (2.9)$$

When working with gas reservoirs, there is no initial oil amount, therefore, N and N_p are equal to zero. The general material balance equation for a gas reservoir can then be obtained

$$G(B_g - B_{gi}) + GB_{gi}\left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta\bar{p} + W_e = G_pB_g + B_wW_p \quad (2.10)$$

This equation is discussed in conjunction with gas and gas condensate reservoirs in Chapters 3 and 4.

In the study of reservoirs that are produced simultaneously by the three major mechanisms of depletion drive, gas cap drive, and water drive, it is of practical interest to determine the relative magnitude of each of these mechanisms that contribute to the production. Pirson rearranged the material balance Equation (2.7) as follows to obtain three fractions, whose sum is one, which he called the depletion drive index (DDI), the segregation (gas cap) index (SDI), and the water-drive index (WDI).[7]

When all three drive mechanisms are contributing to the production of oil and gas from the reservoir, the compressibility term in Eq. (2.7) is negligible and can be ignored. Moving the water production term to the left-hand side of the equation, the following is obtained:

$$N(B_t - B_{ti}) + \frac{NmB_{ti}}{B_{gi}}(B_g - B_{gi}) + (W_e - B_wW_p) = N_p[B_t + (R_p - R_{soi})B_g]$$

Dividing through by the term on the right-hand side of the equation:

$$\frac{N(B_t - B_{ti})}{N_p[B_t + (R_p - R_{soi})B_g]} + \frac{\frac{NmB_{ti}}{B_{gi}}(B_g - B_{gi})}{N_p[B_t + (R_p - R_{soi})B_g]} + \frac{(W_e - B_wW_p)}{N_p[B_t + (R_p - R_{soi})B_g]} = 1 \quad (2.11)$$

The numerators of these three fractions that result on the left-hand side of Eq. (2.11) are the expansion of the initial oil zone, the expansion of the initial gas zone, and the net water influx, respectively. The common denominator is the reservoir volume of the cumulative gas and oil production expressed at the lower pressure, which evidently equals the sum of the gas and oil zone expansions plus the net water influx. Then, using Pirson's abbreviations:

$$DDI + SDI + WDI = 1$$

Calculations are performed in Chapter 6 to illustrate how these drive indexes are used

$$N_p[B_t + (R_p - R_{soi})B_g] + B_wW_p - W_i - G_iB_{ig} = N\left[(B_t - B_{ti}) + B_{ti}(1 + m)\left(\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right)\Delta\bar{p} + \frac{mB_{ti}}{B_{gi}}(B_g - B_{gi})\right] + W_e \quad (2.12)$$

E_o, $E_{f,w}$, E_g

The terms W_i, cumulative water injection, G_i, cumulative gas injection, and B_{ig}, formation volume factor of the injected gas have been added to Eq. (2.7).

MATERIAL BALANCE AS A STRAIGHT LINE (HAVLENA + ODEH)

of the compressibilities of the formation and connate water in the gas cap portion of the reservoir. That is, in their development the compressibility term is multiplied by N and not by $N(1 + m)$. In Eq. (2.12), the compressibility term is multiplied by $N(1 + m)$ for completeness. You may choose to ignore $(1 + m)$ multiplier in particular applications. Havlena and Odeh defined the following terms and rewrote Eq. (2.12) as:

$$F = N_p[B_t + (R_p - R_{soi})B_g] + B_wW_p - W_i - G_iB_{ig}$$

$$E_o = B_t - B_{ti}$$

$$E_{f,w} = \left[\frac{c_wS_{wi} + c_f}{1 - S_{wi}}\right]\Delta\bar{p}$$

$$E_g = B_g - B_{gi}$$

$F' = F - B_wW_p$

$$F = NE_o + N(1 + m)B_{ti}E_{f,w} + \left[\frac{NmB_{ti}}{B_{gi}}\right]E_g + W_e \quad (2.13)$$

$F = NE_o + Nm\frac{B_{ti}}{B_{gi}}E_g + W_e$ (No $E_{f,w}$)

In Eq. (2.13) F represents the net production from the reservoir. E_o, $E_{f,w}$, and E_g represent the expansion of oil, formation and water, and gas, respectively. [illegible]

1. No GAS CAP, N_{g} [illegible] compressibility $E_{f,w} = 0$

$$F = NE_o + W_e$$

$$\frac{F}{E_o} = N + \frac{W_e}{E_o}$$

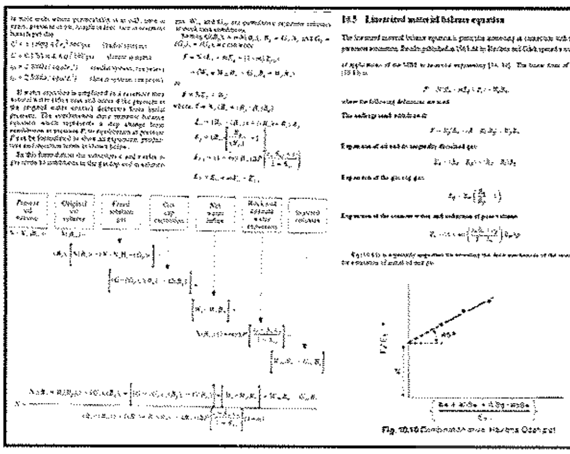
[illegible]

6. THE HAVLENA AND ODEH METHOD OF APPLYING THE MATERIAL BALANCE EQUATION

[illegible]

$$F = NE_o \quad (2.14)$$

[illegible]



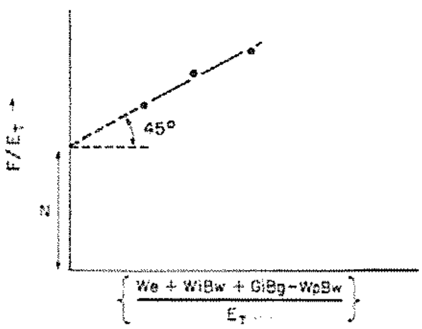
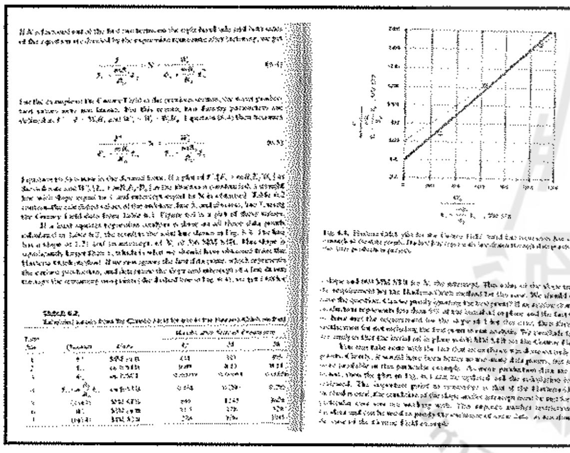
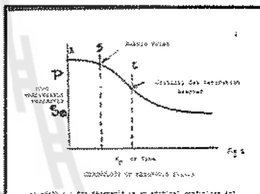


Fig. 10.10 Combination drive, Havlena-Odeh plot.





Solution:

Now that we have examined the chronology of events which occur in the depletion of the reservoir, lets examine the alternative solution schemes. The solution process can be visualized as being split into two schemes depending on the parameter being sought:

(a) Determination of reservoir parameters.

(b) Determination of prediction performance.

In case (a) above the reservoir parameters as enumerated earlier can be determined from historical data by either:

(1) Point by point solution of the MBE at every data point then using some averaging process.

(2) Dynamic type solution using the MBE as a straight line and solving graphically.

In case (b) the performance prediction is calculated by any of the MBE's listed earlier, but depending on the stage of depletion, several auxiliary equations must be used in addition to the MBE. These equations are:

(1) Producing Gas Oil Ratio Equation

(2) Saturation Equation

The Material Balance Equation

I. Introduction and Derivation

A petroleum reservoir is a complex system in which many forces are interacting to produce flow of fluids in the rock. No matter how complex the system however the basic laws of physics still apply. One physical principle which is utilized in petroleum engineering is the Law of Conservation of Mass. The quantitative application of the above principle to petroleum systems is referred to as the Material Balance Equation.

The uses of the Material Balance equation are as follows:

I. Evaluation of Reservoir Parameters - the MBE is used to evaluate

1. Original Oil in place N
2. Gas cap size m, N
3. Water Influx We

II. As a predictive tool -

1. Oil production Np
2. Gas production Gp
3. Reservoir pressure P

III. Reservoir Energy

1. Magnitude of each Recovery Mechanism
2. Effects of supplemental energies

5

Sources of Error and Assumptions:

The Material Balance Equation method of reservoir analysis is based on several assumptions which must be recognized along with the effects of these assumptions in limiting the applicability of these equations.

The major assumptions are:

1. Implication of Thermodynamic Equilibrium is established instantaneously throughout the reservoir at all times.
   - uniform properties
2. PVT data used in solving the MBE obtained from processes which duplicate the field process.

Errors can be due to:

1. The correctness of the average bottom hole pressure. This enters all PVT data so it has significant effect on the parameters calculated.
2. The value of m = $\frac{\text{initial free gas}}{\text{initial oil volume}}$
   - this quantity usually obtained from log data may be in error.
3. Cumulative Production
   - the quantity of oil produced is generally well known.
   - gas production is susceptible to errors in reporting
   - water production figures are generally pretty

The effects of these errors can be minimized plotting the data and smoothing it graphically.

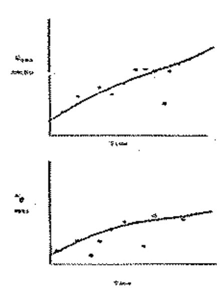


Fig.3

7

Derivation of GMBE

The Material Balance principle (Mass Conservation) can be applied to:

a. The complete Reservoir System
b. Any Fraction of the system
c. To any component of the system

It is applicable between any two sequential time periods.

The basic equation can be written either in a form suitable for analysis of the system or part there of or for the analysis at a point; in either:

1. Finite Form
2. Differential Form

Since the reservoir volume available to the reservoir fluid is a "constant" the sum total of all volume changes should be zero: In other words:

FLUID VOIDAGE ; EXPANSION OF ROCK POREVOLUME AND FLUIDS

The Voidage originates from several sources

1. Production
   a. oil production
   b. gas production
   c. water production
2. Influx
   a. Water injection
   b. Water influx
   c. Gas Injection

P. 7

1. Fini Form
2. Differential Form

Since the reservoir volume available to the reservoir fluid is a "constant" the sum total of all volume changes should be zero: In other words:

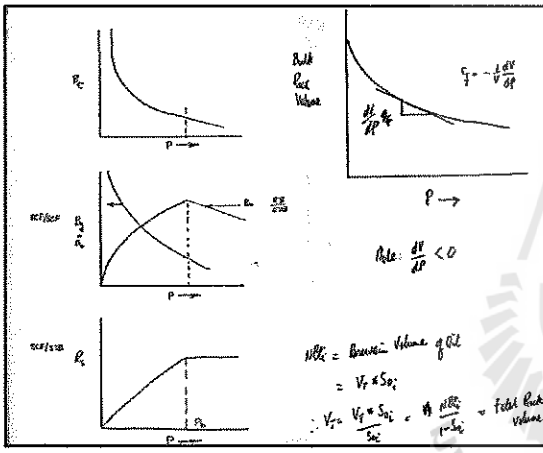
FLUID VOIDAGE ; EXPANSION OF ROCK POREVOLUME AND FLUIDS

The Voidage originates from several sources

1. Production
   a. oil production
   b. gas production
   c. water production
2. Influx
   a. Water injection
   b. Water influx
   c. Gas Injection

The Expansion originates from the following sources:

1. Oil zone components
2. Gas Zone components
3. Connate Water
4. Rock Matrix



Note: $\frac{dV}{dP} < 0$

NB_{ti} = Reservoir Volume of Oil

$= V_f \times S_{oi}$

$\therefore V_f = \frac{V_f \times S_{oi}}{S_{oi}} = \frac{NB_{ti}}{1-S_{wi}}$ = Total Rock Bulk Volume

Consider a reservoir as shown below

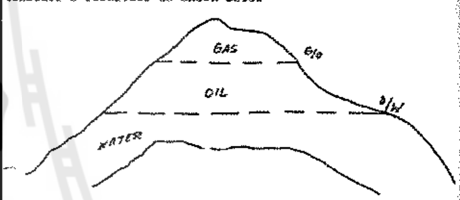


Fig 3

Between any two times when the pressure goes from P_i (original) to P we can write:

Expansion Terms : in Reservoir Bbls

a. Oil zone expansion with its associated gas:

$\Delta V = NB_t - NB_{ti}$

where B_t = total formation volume factor

$G = \frac{mNB_{ti}}{B_{gi}}$

(b) Gas Cap Expansion:

$\Delta V = \underbrace{mNB_{ti}\frac{B_g}{B_{gi}}}_{present} - \underbrace{mNB_{ti}}_{original}$

$\frac{mNB_{ti} \cdot B_g}{B_{gi}} - \frac{mNB_{ti} B_{gi}}{B_{gi}}$

$\frac{mNB_{ti}}{B_{gi}}[B_g - B_{gi}]$

$= mNB_{ti}[\frac{B_g}{B_{gi}} - 1]$

(c) Interstitial Water:

This interstitial water is present in the gas cap as well as in the oil zone

Gas Cap Water = $\frac{mNB_{ti}}{1-S_{wi}} S_{wi}$

$c_w = -\frac{dV}{V dP}$

Oil Zone Water = $\frac{NB_{ti}}{1-S_{wi}} S_{wi}$

$\Delta V = NB_{ti}(1+m)(\frac{S_{wi}}{1-S_{wi}}) c_w \Delta P$

where ΔP is the pressure drop between the two times

(d) Rock Expansion:

Void volume = $\frac{HPV}{\text{Hydrocarbon fraction}}$

void in gas cap = $\frac{mNB_{ti}}{1-S_{wi}}$

$c_f = -\frac{dV}{V_f dP}$

void in oil zone = $\frac{NB_{ti}}{1-S_{wi}}$

Δvoid = - Δrock expansion per unit volume = $c_f \Delta P$

$\Delta V = (\frac{mNB_{ti}}{1-S_{wi}} + \frac{NB_{ti}}{1-S_{wi}}) c_f \Delta P$

$\Delta V = \frac{NB_{ti}}{1-S_{wi}}(1+m) c_f \Delta P$

10

11

VOIDAGE TERMS

1. $= \underbrace{N_p B_t}_{\text{oil and associated gas}} + \underbrace{N_p(R_p - R_{si})B_g}_{\text{free gas}}$ (5)

Water production = $W_p B_w$ (6)

2. Influx:
   1. Water Injection = $- W_i B_{w_{inj}}$
   2. Water Influx = $- W_e$
   3. Gas Injected = $- G_i B_{g_{inj}}$

(7)

Equating the voidage terms to the expansion terms:

$\sum_{i=1}^{3}\{\text{voidage}\}_i = \sum_{i=1}^{4}\{\text{expansion}\}_i$

We obtain by summing equations (1) - (4) and equating to (5) - (7):

$N(B_t - B_{ti}) + mNB_{ti}(\frac{B_g}{B_{gi}} - 1) + \frac{NB_{ti}}{(1-S_{wi})}((1+m)(S_{wi}c_w + c_f))\Delta P$ or $\frac{mNB_{ti}}{B_{gi}}[B_g - B_{gi}]$

$= N_p B_t + N_p(R_p - R_{si})B_g + W_p B_w - (W_i B_{w_{inj}} + W_e + G_i B_{inj})$ (8)

$N(B_t - B_{ti}) + mNB_{ti}(\frac{B_g}{B_{gi}} - 1) + \frac{NB_{ti}}{1-S_{wi}}(1+m)(S_{wi}c_w + c_f)\Delta P$

$= N_p[B_t + (R_p - R_{si})B_g] + W_p B_w - (W_i B_{w_{inj}} + W_e + G_i B_{inj})$

12

Equation 8 is the generalized material balance and can be written as:

$$N(B_t - B_{ti}) + mNB_{ti}\left(\frac{B_g}{B_{gi}} - 1\right) + \frac{NB_{ti}}{1 - S_{wi}}(1+m)(S_w C_w + C_f)\Delta P$$

$$= N_p[B_t + (R_p - R_{si})B_g] + W_p B_w - (W_i B_{w_{inj}} + W_e + G_i B_{inj}) \qquad (8)$$

There are several conditions under which the above equation can be simplified:

1. No gas cap m=0
2. No influx $W_e = 0$
3. No production of water $W_p = 0$
4. No injection of gas $G_i = 0$

The appropriately simplified equation is then solved.

The General Material Balance equation can be rearranged to plot one variable vs another group of variables. The variable group selected will depend on the particular mechanism being studied and the particular data available. The material balance as a straight line involves the selection of the variables in such a manner that a straight line results, the significance of the approach is that the sequence of plotting is important and if the actual data plotted deviates from this straight line there is some reason for it.

Because of the many terms in the equation and the numerous possible drive mechanisms in the reservoir the engineer has quite a variation of plotting possibilities. In the ensuing pages we will try to indicate the methodology for determining

Determination of Reservoir Parameters

(a) The Point by Point method involves utilizing the appropriate MBE between any two points and rearranging it to solve for the required parameters. This is illustrated in example 3.

(b) The Havlena solution requires the forecasting of the sequence of data points and treats ALL the data by using a graphical plot to obtain the desired parameters. This approach uses the material balance equation as a straight line. Recall the GMBE:

This can be abbreviated by grouping terms together:

where E_o is oil expansion term,

E_g is gas expansion term,

E_{fw} is rock and connate water expansion,

F is net production term.

In this form the equation is more readily simplified and the

Variables for the Straight Line Material Balance Equation

Variable	Description
E_o	$(B_t - B_{ti})$
E_g	$E_g = B_{oi}\,[(B_g/B_{gi}) - 1]$
E_{fw}	$(S_w C_w + C_f)\Delta P$
F	$N_p[B_t + (R_p - R_{si})B_g] - (W_i B_{winj} + W_e + G_i B_{inj})$
m	gas zone size / oil zone size
N	Original oil in place
W_p	Water production term
W_e	Water influx term
W_i	Water injection term

10.5 Linearized material balance equation

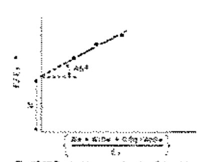


Fig. 10.10 Combination drive, Havlena-Odeh plot

Gas + Saturated Reservoir, of P Fabric

Neglecting rock + connate water Expansion

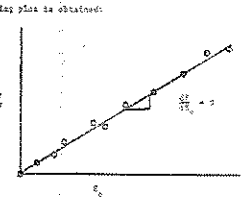
Applications: Neglecting rock and connate water expansion.

(1) No Water Drive → $W_e = 0$ $W_e \to 0$

No Gas Cap → $m = 0$ $m = 0$

∴ $F = NE_o$ (19)

The following plot is obtained:



This graph should go through the origin since (0,0) is a valid point and have a uniform slope of N. Any deviation from normal scatter indicates other mechanisms are operating and should be examined further.

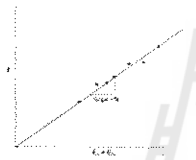


Figure 11-17. Underground withdrawal vs. $E_o + E_{fw}$

Applying the above conditions on Equation 11-32 gives:

$$F = N(E_o + E_{f,w}) \qquad (11\text{-}33)$$

or

$$N = \frac{F}{E_o + E_{f,w}} \qquad (11\text{-}34)$$

where N = initial oil in place, STB

$$F = N_p B_o + W_p B_w \qquad (11\text{-}35)$$

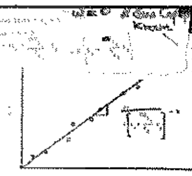
$$E_o = B_o - B_{oi} \qquad (11\text{-}36)$$

$$E_{f,w} = B_{oi}\left[\frac{c_w S_w + c_f}{1 - S_{wi}}\right]\Delta p \qquad (11\text{-}37)$$

$\Delta p = p_i - \bar{p}$

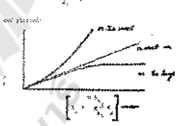
p_i = initial reservoir pressure

$\bar{p}$ = volumetric average reservoir pressure



We = 0

Unknown Gas Cap.



be considered negligible. Under these conditions, the Havlena-Odeh material balance can be expressed as:

$$F = N[E_o + mE_g] \qquad (11\text{-}39)$$

where E_g is defined by Equation 11-30 as:

$$E_g = B_{oi}\,[(B_g/B_{gi}) - 1]$$

The way in which Equation 11-39 can be used depends on the number of unknowns in the equation. There are three possible unknowns in Equation 11-39:

- N is unknown, m is known
- m is unknown, N is known
- N and m are unknown

a. Unknown N, known m:

Equation 11-39 indicates that a plot of F versus $(E_o + m E_g)$ on a Cartesian scale would produce a straight line through the origin with a slope of N, as shown in Figure 11-19. In making the plot, the underground withdrawal F can be calculated at various times as a function of the production terms N_p and R_p.

Conclusion: N = Slope

b. Unknown m, known N:

Equation 11-39 can be rearranged as an equation of straight line, to give:

$$\left(\frac{F}{N} - E_o\right) = mE_g \qquad (11\text{-}40)$$

The above relationship shows that a plot of the term $(F/N - E_o)$ versus E_g would produce a straight line with a slope of m. One advantage of this particular arrangement is that the straight line must pass through the origin which, therefore, acts as a control point. Figure 11-20 shows an illustration of such a plot.

be considered negligible. Under these conditions, the Havlena-Odeh material balance can be expressed as:

$$F = N[E_o + mE_g] \qquad (11\text{-}39)$$

where E_g is defined by Equation 11-30 as:

$$E_g = B_{oi}\,[(B_g/B_{gi}) - 1]$$

The way in which Equation 11-39 can be used depends on the number of unknowns in the equation. There are three possible unknowns in Equation 11-39:

- N is unknown, m is known
- m is unknown, N is known
- N and m are unknown

The practical use of Equation 11-39 in determining the three possible unknowns is presented below:

a. Unknown N, known m:

Equation 11-39 indicates that a plot of F versus $(E_o + m E_g)$ on a Cartesian scale would produce a straight line through the origin with a slope of N, as shown in Figure 11-19. In making the plot, the underground withdrawal F can be calculated at various times as a function of the production terms N_p and R_p.

Conclusion: N = Slope

b. Unknown m, known N:

Equation 11-39 can be rearranged as an equation of straight line, to give:

$$\left(\frac{F}{N} - E_o\right) = mE_g \qquad (11\text{-}40)$$

The above relationship shows that a plot of the term $(F/N - E_o)$ versus E_g would produce a straight line with a slope of m. One advantage of this particular arrangement is that the straight line must pass through the origin which, therefore, acts as a control point. Figure 11-20 shows an illustration of such a plot.

Conclusion: m = Slope

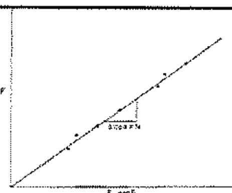


Figure 11-19. F vs. $E_o + mE_g$

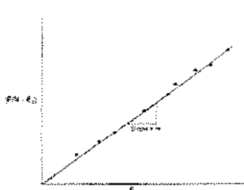
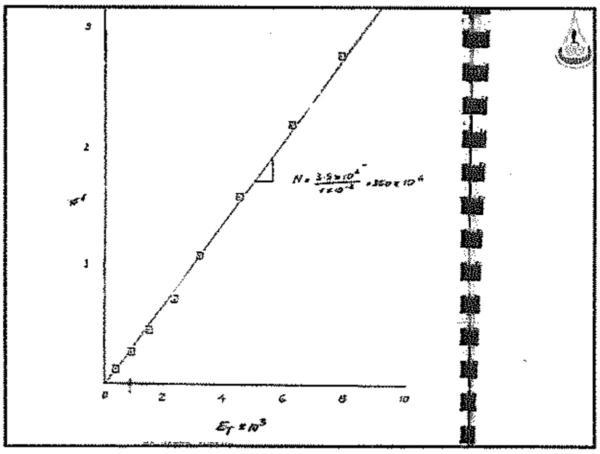


Figure 11-20. $(F/N - E_o)$ vs. E_g

Material Balance as a Straight Line C. [illegible]ations

1	2	3	4	5	6	7	8
	P_{avg}	N_p	W_p	$F = N_p B_o + W_p$	$(S_o c_o + S_w c_w + c_f)/(1-S_w)$	ΔP	E_t
1	3680	20.48	0	26.83 × 10	18.685 × 10	5	93 × 10
2	3676	34.75	0	45.536	18.694	9	168
3	3669	78.557	0	103.93	18.704	18	337
4	3664	101.89	0	133.46	18.715	21	393
5	3640	215.88	0	282.73	18.795	45	846
6	3605	364.61	0	478.72	18.795	80	1504
7	3567	542.68	.16	712.66	18.844	118	2224
8	3515	587.59	.80	866.64	18.925	170	3217
9	3448	1375.53	2.57	1814.72	19.015	237	4506
10	3360	1611.10	5.00	2233.84	19.149	325	6223
11	3275	2125.07	6.50	2806.73	19.264	410	7898
12	3188	2575.35	8.80	3399.20	19.524	497	9703

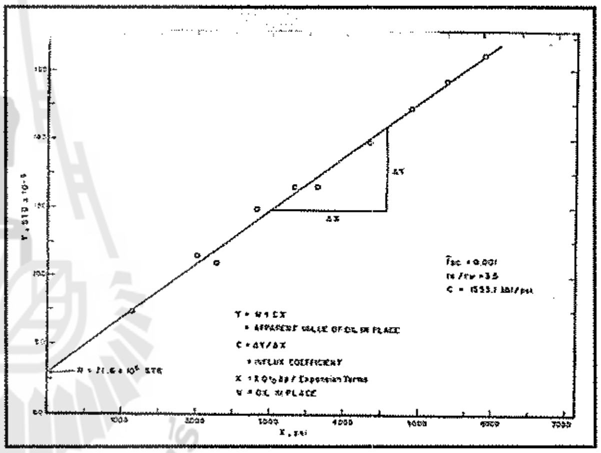
$\times 10^{-6}$

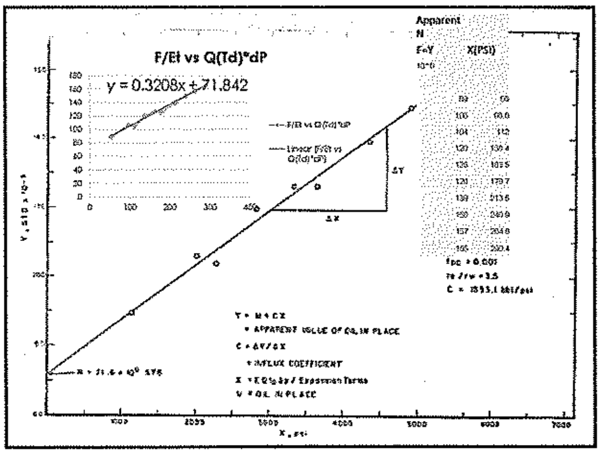


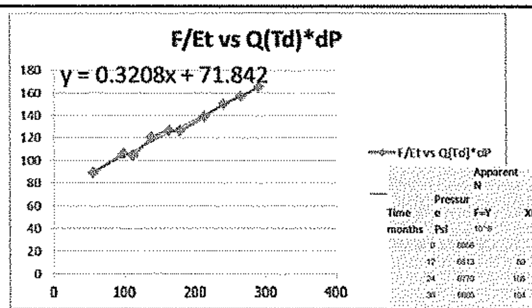
$$Y = \frac{N_p B_o}{B_{oi} \Delta P \times \left(\frac{S_o c_o + S_w c_w + c_f}{1 - S_w}\right)} = N + C\frac{\Sigma Q(t_D)\Delta P}{X}$$

Initial Pressure	= 6655.5	r_e/r_w	= 3.5	
Bubble Point Pressure	= 2111.5	t_{Dc}	= 0.001	
Initial Solution GOR	= 552.1	C	= 1595.1 bbl/psi	
Initial Water Sat.	= 0.4284			
Compressibility, Vol/Vol/Psi X 10^{-6}	=	Oil in Place		
Oil	= 18.77	Wtd. Avg. "Apparent N"	= 144,065,144 STB	
Water	= 2.29	Wtd. Avg. N W/Influx	= 71,417,061 STB	
Rock	= 3.16	Least Squares Solution	= 71,676,917 STB	

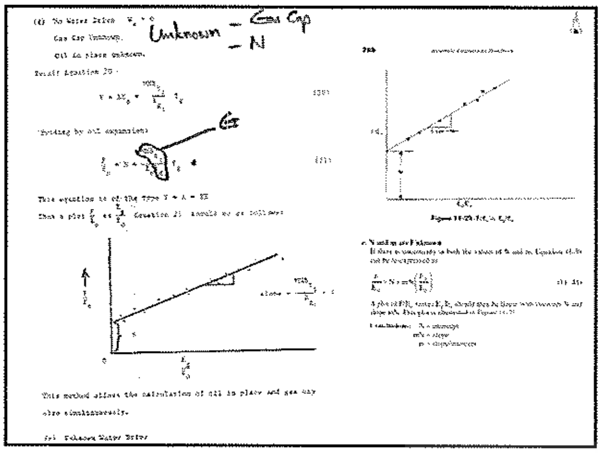
Time Mo.	Pressure Psi	Bo BBL/STB	Np STB	"Apparent N" STB (Y)	X, Psi Σ Q(t) Δp / Expan. Terms (X)	Influx, U.S.S. BBL.	N, Calc. W/Influx STB
0	6655.5	1.52200	0				
12	6517.9	1.52293	95,760	89,659,590	56.4	46,863	71,217,168
24	6770.3	1.52386	222,248	105,631,970	96.6	101,666	74,655,452
36	6631.9	1.52571	455,584	103,032,819	112.8	231,017	67,217,807
48	6599.5	1.52757	791,517	119,589,896	138.4	416,060	71,935,759
60	6514.1	1.52961	1,257,117	128,107,942	161.5	574,178	73,615,987
72	6409.4	1.53190	1,497,568	126,434,171	176.7	962,839	69,757,136
84	6343.5	1.53324	1,741,795	139,114,521	213.6	1,111,770	70,470,816
96	6295.6	1.53456	2,078,751	149,511,545	240.9	1,656,561	71,835,928
108	6229.7	1.53564	2,494,870	157,422,229	264.6	2,082,126	72,161,714
120	6187.0	1.53662	2,708,203	165,414,914	290.4	2,351,470	71,693,368

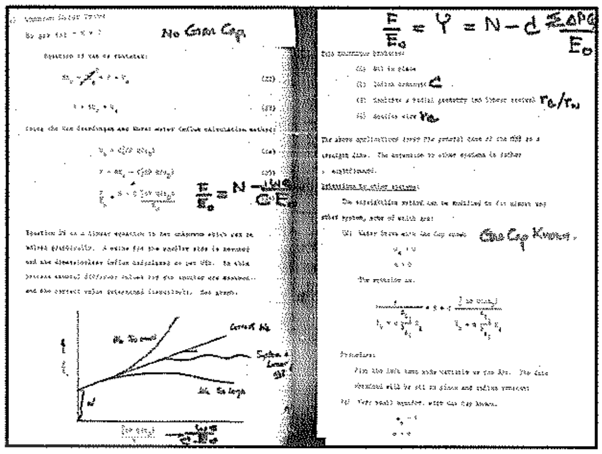


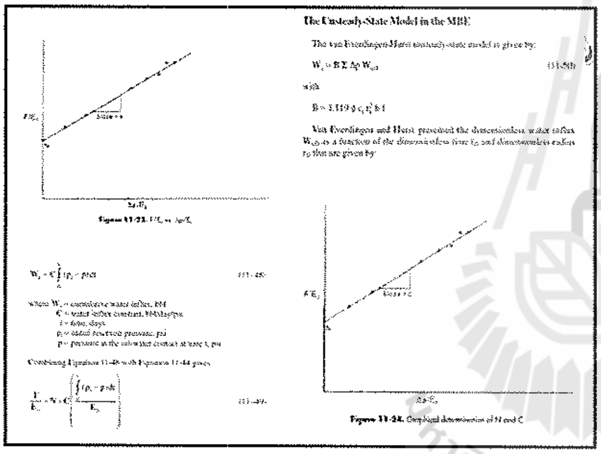


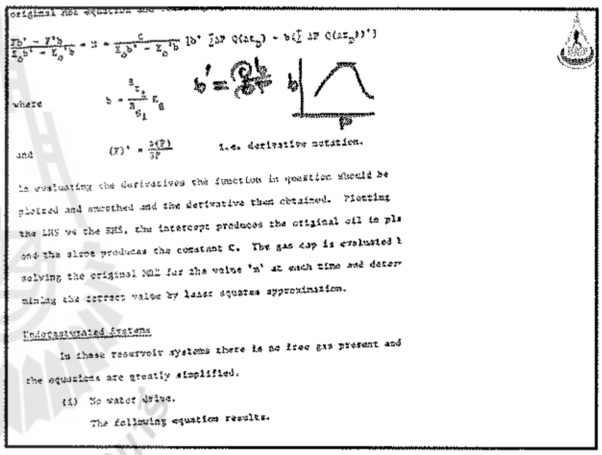


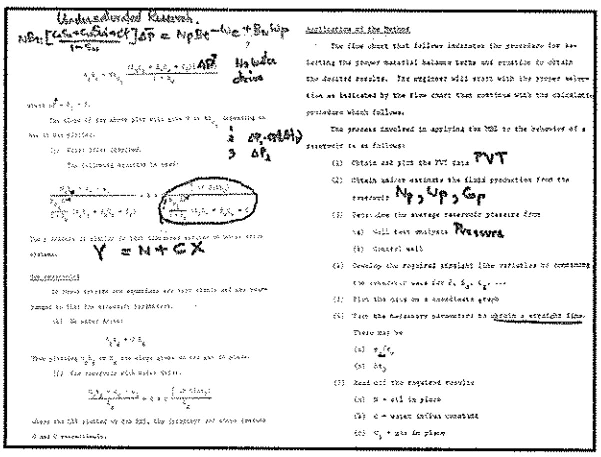
Time months	Pressure Psi	Apparent N F=Y 10^6	X(PSI)
0	6656		
12	6613	89	56
24	6770	106	96.6
36	6600	104	112
48	6600	120	138.4
60	6514	128	161.5
72	6400	126	176.7
84	6344	139	213.6
96	6287	150	240.9
108	6233	157	264.6
120	6187	165	290.4


Unknown
Gas Cap
N
This method allows the calculation of oil in place and gas cap also simultaneously.


No Gas Cap
Gas Cap Known
Procedure:


The Unsteady-State Model in the MBE
Figure 11-24. Graphical determination of N and C


where
and
i.e. derivative notation.
In evaluating the derivatives the function in question should be plotted and smoothed and the derivative then obtained. Plotting the LHS vs the RHS, the intercept produces the original oil in pl
and the slope produces the constant C. The gas cap is evaluated b
solving the original MBE for the value 'm' at each time and deter
mining the correct value by least squares approximation.
Undersaturated Systems
In these reservoir systems there is no free gas present and the equations are greatly simplified.
(i) No water drive.
The following equation results.


Undersaturated Reservoir.
No water drive
Y = N + CX
Application of the Method
PVT
Pressure
obtain a straight line

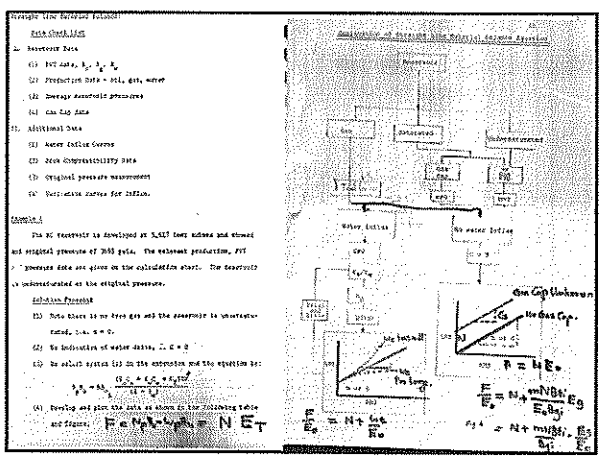

Data Check List
Gas Cap Unknown
No Gas Cap

TABLE 6.1.
Material balance calculation of water influx or oil in place for oil reservoirs below the bubble-point pressure

For Conroe Field: B_{oi} = 7.37 cu ft/STB; B_{gi} = 0.00637 cu ft/SCF (14.4 psia and 60°F); $m = \frac{181{,}225 \text{ ac-ft}}{810{,}000 \text{ ac-ft}} = 0.224$; mB_{oi}/B_{gi} = 259 SCF/STB; R_{soi} = 600 SCF/STB

Line No.	Quantity	Units	12	18	24	30	36
			Months after Start of Production				
1	N_p	MM STB	9.070	22.34	32.03	40.18	48.24
2	R_p	SCF/STB	1634	1180	1070	1025	995
3	p	psig	2143	2108	2098	2087	2091
4	B_g	cu ft/SCF	0.00676	0.00687	0.00691	0.00694	0.00693
5	B_t	cu ft/STB	7.46	7.51	7.51	7.53	7.52
6	N_pR_p	MM SCF	14,800	26,361	34,400	41155	48,100
7	R_p-R_{soi}	SCF/STB	1030	550	470	425	395
8	$(R_p-R_{soi})B_g$	cu ft/STB	6.95	3.98	3.24	2.95	2.74
9	(5) + (8)	cu ft/STB	14.41	11.49	10.75	10.48	10.26
10	B_g-B_{gi}	cu ft/SCF	0.00039	0.0005	0.00054	0.00057	0.00056
11	(10) × (mB_{oi}/B_{gi})	cu ft/STB	0.101	0.1395	0.137	0.148	0.145
12	B_t-B_{oi}	cu ft/STB	0.09	0.140	0.14	0.16	0.15
13	(11) + (12)	cu ft/STB	0.191	0.2795	0.277	0.306	0.295
14	(1) × (9)	MM cu ft	131	256.7	345	421	495
15	W_e-W_p	MM cu ft	51.5	115	178	250	320
16	(14) − (15)	MM cu ft	79.5	141.7	167	171	175
17	N = (16)/(13)	MM STB	415	525	602	555	594
18	DDI	Fraction	0.265	0.34	0.244	0.41	0.180
19	SDI	Fraction	0.320	0.26	0.239	0.19	0.174
20	WDI	Fraction	0.395	0.45	0.516	0.60	0.646

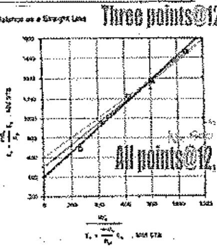
6.1

Two points@24 and 36 months N= 600 STB

TABLE 6.2.
Tabulated values from the Conroe Field for use in the Havlena-Odeh method

Three points@12, 24 and 36 months N= 400 STB

3. Material Balance as a Straight Line

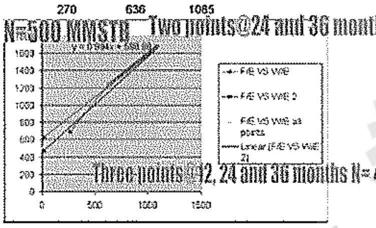


All points@12, 18, 24, 30 and 36 months N= 500 STB

Fig. 6.6. Havlena-Odeh plot for the Conroe Field.

F'	131	257	345	421	495
Eo	0.090	0.140	0.140	0.160	0.150
Eg	0.00039	0.00050	0.00054	0.00057	0.00056
Eo+mBti*Eg/Bgi	0.191	0.270	0.280	0.308	0.295
Y=(1)/(4)	685	953	1231	1369	1677
W'e	52	90	176	240	320
X=(6)/(4)	270	334	636	780	1085
	685	1231	1677		
	270	636	1085		

ALL POINTS N=500 MMSTB   Two points@24 and 36 months N= 600 STB



Three points@12, 24 and 36 months N= 400 STB

Example 11-3

Solution

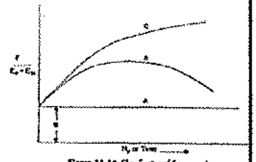
Basic Assumptions in the MBE

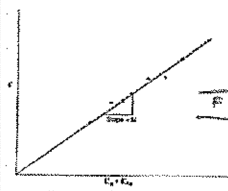
The MBE as an Equation of a Straight Line

The Straight-Line Solution Method to the MBE

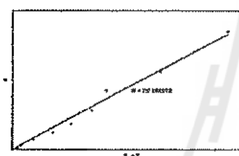
CASE I

Case 1. Volumetric Undersaturated Oil Reservoirs

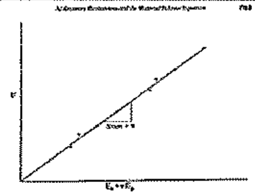


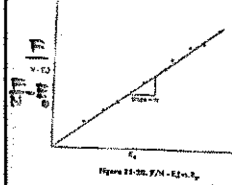


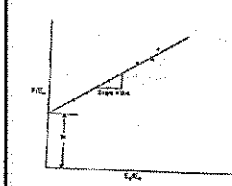
CASE 2



Case 3







3.9. GASCAP DRIVE

If a reservoir has a gas cap the material balance can be stated as:

$$F = N(E_o + mE_g) + W_e B_w \qquad (3.49)$$

in which it is assumed that, in the presence of high-compressibility free gas, the water and pore compressibility term E_{fw} is negligible (an assumption that must always be checked). The inclusion of the gas cap component raises the level of complication in trying to solve the equation by a considerable amount. Havlena and Odeh [3] acknowledged that they met with only "limited success" in its solution and stated that the reason for this was because:

> "Whenever a gas cap is to be solved for, an exceptional degree of accuracy of basic data, mainly pressures, is required."

The statement is perfectly correct but the problems are more acute than that. In the first place the right-hand side of equation 3.49 now contains three major, potential unknowns, N, W_e and m, which adds to the lack of mathematical uniqueness in its solution referred to in section 3.4. Secondly comes the difficulty mentioned by Havlena and Odeh of the great sensitivity to the quality of the input data, particularly the pressure. What they did not refer to, however, is that in "gassy" reservoirs the measurement of well pressures is extremely difficult. If there is a gas

If there is no water influx ($W_e = 0$) and both N and m are unknowns, the material balance is reduced to:

$F = N(E_o + mE_g)$ (3.50)

which can be also expressed as:

$\frac{F}{E_o} = N + mN\frac{E_g}{E_o}$ (3.51)

These equations offer two methods of solution for m and N. The first is to plot F as a function of $E_o + mE_g$ for an assumed value of m, as shown in Fig. 3.13a. If the value of m has been chosen correctly, the points will lie on a straight line with slope N that passes through the origin. The second method is to plot F/E_o versus E_g/E_o (Fig. 3.13b). The data points should plot on a straight line with slope mN whose intercept on the ordinate gives the STOIIP, N. Havlena and Odeh [3] suggest that the plotting method shown in Fig. 3.13a is the more powerful because the line is constrained to pass through the origin but suggest that both methods should be applied for checking purposes. The technique is illustrated with an example in reference 2.

In the event that there is a finite water influx, the technique recommended by Havlena and Odeh [3] is to differentiate equation 3.49 with respect to pressure. The resulting equation is then manipulated with equation 3.49 so as to eliminate m which results in the expression

$\frac{FE_g' - F'E_g}{E_oE_g' - E_o'E_g} = N + \frac{W_eE_g' - W_e'E_g}{E_oE_g' - E_o'E_g}$ (3.52)

in which the primes denote derivatives with respect to pressure. A plot of the left-hand side of the equation versus the second term on the right for a selected

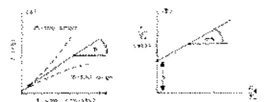


Fig. 3.13. Havlena and Odeh plots for gas cap drive reservoirs, no water influx, m and N are unknowns.

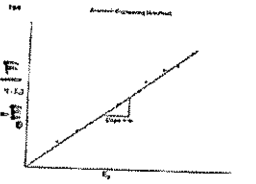


Figure 11-20. F/(1 + E_o) vs. E_g.

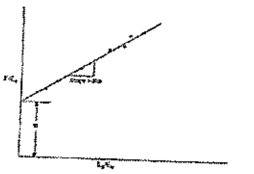


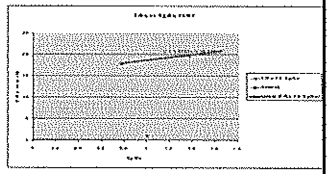
Figure 11-21. F/E_o vs. E_g/E_o.

$E_g = B_{oi}\left[(B_g/B_{gi}) - 1\right]$

Date	P	Np MSTB	Gp MMSCF	Bt bbl/STB	Bg bbl/SCF	Rp=Gp/Np SCF/STB	F MMSTB	Eo	Eg	F/Eo MMMSTB	Eg/Eo
5/1/1989	4415	0		1.6291	0.00077						
1/1/1991	3875	460	700	1.68384	0.00079	1522	873.3	0.05480	0.04231	17.76	0.77
1/1/1992	3315	1015.7	2409.6	1.7835	0.00087	2372	3048.3	0.1544	0.211571	19.73	1.37
1/1/1993	2845	1500	4500	1.911	0.00099	3000	5873.6	0.2819	0.465457	20.84	1.65

$F = N_p\left[B_t + (R_p - R_{si})B_g\right] + W_pB_w$

$E_o = B_t - B_{ti}$



Step 4. Plot (F/E_o) versus (E_g/E_o) as shown in Figure 11-22 to give:

- Intercept = N = 15.175 MMSTB
- Slope = Nm = 3.3 × 107

Step 5. Calculate m:

m = 3.3 × 107/15.175*(10⁶) = 2.22

Step 6. Calculate initial gas in place G:

$m = \frac{GB_{gi}}{NB_{oi}}$

$G = \frac{(2.22)(15.175\cdot10^6)(1.6291)}{0.00077} = 71.3$ MMMscf

Step 4. Plot (F/E_o) versus (E_g/E_o) as shown in Figure 11-22 to give:

- Intercept = N = 9 MMSTB
- Slope = N m = 3.1 × 107

Step 5. Calculate m:

m = 3.1 × 10⁷/(9 × 10⁶) = 3.44

Step 6. Calculate initial gas in place G:

$m = \frac{GB_{gi}}{NB_{oi}}$

$G = \frac{(3.44)(9\times10^6)(1.6291)}{0.00077} = 65.5$ MMMscf

Case 4. Water-Drive Reservoirs

In a water-drive reservoir, identifying the type of the aquifer and characterizing its properties are perhaps the most challenging tasks involved in conducting a reservoir engineering study. Yet, without an accurate

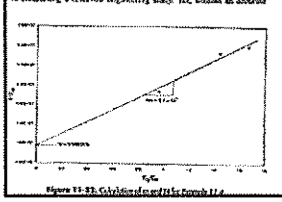


Figure 11-22. Calculation of m and N for Example 11-4.

description of the aquifer, future reservoir performance and management cannot be properly evaluated.

The full MBE can be expressed again as:

$F = N(E_o + mE_g + E_{f,w}) + W_e$

Dake (1978) points out that the term $E_{f,w}$ can frequently be neglected in water-drive reservoirs. This is not only for the usual reason that the water and pore compressibilities are small, but also because a water influx helps to maintain the reservoir pressure and, therefore, the Δp appearing in the $E_{f,w}$ term is reduced, or

$F = N(E_o + mE_g) + W_e$ (11-42)

If, in addition, the reservoir has initial gas cap, then Equation 11-42 can be further reduced to:

$F = NE_o + W_e$ (11-43)

Dake (1978) points out that in attempting to use the above two equations to match the production and pressure history of a reservoir, the greatest uncertainty is always the determination of the water influx W_e. In fact, in order to calculate the influx the engineer is confronted with what is inherently the greatest uncertainty in the whole subject of reservoir engineering. The reason is that the calculation of W_e requires a mathematical model which itself relies on the knowledge of aquifer properties. These, however, are seldom measured since wells are not deliberately drilled into the aquifer to obtain such information.

For a water-drive reservoir with no gas cap, Equation 11-43 can be rearranged and expressed as:

$\frac{F}{E_o} = N + \frac{W_e}{E_o}$ (11-44)

Several water influx models have been described in Chapter 10, including the:

- Pot-aquifer model
- Schilthuis steady-state method
- Van Everdingen-Hurst model

The use of these models in connection with Equation 11-44 to simultaneously determine N and W_e is described below.

The Pot-Aquifer Model in the MBE

Assume that the water influx could be properly described using the simple pot aquifer model given by Equation 10-5 as:

$W_e = (c_w + c_f)W_if(p_i - p)$ (11-45)

$f = \frac{(\text{encroachment angle})^\circ}{360^\circ} = \frac{\theta}{360^\circ}$

$W_i = \left[\frac{\pi(r_a^2 - r_e^2)h\phi}{5.615}\right]$

where
r_a = radius of the aquifer, ft
r_e = radius of the reservoir, ft
h = thickness of the aquifer, ft
φ = porosity of the aquifer
θ = encroachment angle
c_w = aquifer water compressibility, psi⁻¹
c_f = aquifer rock compressibility, psi⁻¹
W_i = initial volume of water in the aquifer, bbl

Since the aquifer properties c_w, c_f, h, r_a, and θ are seldom available, it is convenient to combine these properties and treat as one unknown K. Equation 11-45 can be rewritten as:

$W_e = K\,\Delta p$ (11-46)

Combining Equation 11-46 with Equation 11-44 gives:

$\frac{F}{E_o} = N + K\left(\frac{\Delta p}{E_o}\right)$ (11-47)

Equation 11-47 indicates that a plot of the term (F/E_o) as a function of ($\Delta p/E_o$) would yield a straight line with an intercept of N and slope of K, as illustrated in Figure 11-23.

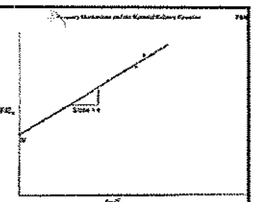


Figure 11-23. F/E_o vs. $\Delta p/E_o$.

The Steady-State Model in the MBE

The steady-state aquifer model as proposed by Schilthuis (1936) is given by:

$W_e = C\int_0^t (p_i - p)\,dt$ (11-48)

where
W_e = cumulative water influx, bbl
C = water influx constant, bbl/day/psi
t = time, days
p_i = initial reservoir pressure, psi
p = pressure at the oil-water contact at time t, psi

Combining Equation 11-48 with Equation 11-44 gives:

$\frac{F}{E_o} = N + C\left(\frac{\int_0^t (p_i - p)\,dt}{E_o}\right)$ (11-49)

Plotting (F/E_o) versus $\int_0^t (p_i - p)\,dt/E_o$ results in a straight line with an intercept that represents the initial oil in place N and a slope that describes the water influx C as shown in Figure 11-24.

The Unsteady-State Model in the MBE

The van Everdingen-Hurst unsteady-state model is given by:

$W_e = B\sum\Delta p\,W_{eD}$ (11-50)

with

$B = 1.119\,\phi\,c_t\,r_e^2\,h\,f$

van Everdingen and Hurst presented the dimensionless water influx W_{eD} as a function of the dimensionless time t_D and dimensionless radius r_D that are given by:

$t_D = 6.328\times10^{-3}\frac{kt}{\phi\mu_w c_t r_e^2}$

$r_D = \frac{r_a}{r_e}$

$c_t = c_w + c_f$

where t = time, days

r_a = radius of the aquifer, ft
r_e = radius of the reservoir, ft
c_w = compressibility of the water, psi⁻¹

Combining Equation 11-50 with Equation 11-44 gives:

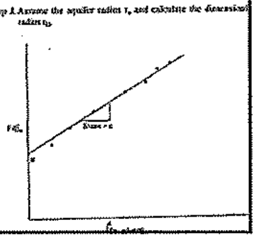
$\frac{F}{E_o} = N + B\left(\frac{\sum\Delta p\,W_{eD}}{E_o}\right)$ (11-51)

The proper methodology of solving the above linear relationship is summarized in the following steps.

Step 1. From the field past production and pressure history, calculate underground withdrawal F and oil expansion E_o.

Step 2. Assume an aquifer configuration, i.e., linear or radial.

Step 3. Assume the aquifer radius r_a and calculate the dimensionless radius r_D.



Step 4. Plot (F/E_o) versus ($\sum\Delta p\,W_{eD}$)/E_o on a Cartesian scale. If the assumed aquifer parameters are correct, the plot will be a straight line with N being the intercept and the water influx constant B being the slope. It should be noted that four other different plots might result. These are:

- Complete random scatter of the individual points, which indicates that the calculation and/or the basic data are in error.
- A systematically upward curved line, which suggests that the assumed aquifer radius (or dimensionless radius) is too small.
- A systematically downward curved line, indicating that the selected aquifer radius (or dimensionless radius) is too large.
- An s-shaped curve indicates that a better fit could be obtained if a linear water influx is assumed.

Figure 11-25 shows a schematic illustration of Havlena-Odeh (1963) methodology in determining the aquifer fitting parameters.

Example 11-5

The material balance parameters, the underground withdrawal F, and oil expansion E_o of a saturated oil reservoir (i.e., m = 0) are given below:

p	F	E_o
3500	—	—
3488	2.04 × 10⁶	0.0548
3162	8.77 × 10⁶	0.1540
2782	17.05 × 10⁶	0.2820

Assuming that the rock and water compressibilities are negligible, calculate the initial oil in place.

Solution

Step 1. The most important step in applying the MBE is to verify that no water influx exists. Assuming that the reservoir is volumetric, calculate the initial oil in place N by using every individual production data point in Equation 11-31, or:

$N = F/E_o$

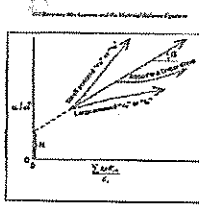


Figure 11-25. Havlena and Odeh straight-line plot. (Source: Havlena and Odeh, 1963.)

F	E_o	$N = F/E_o$
2.04 × 10⁶	0.0548	37 MMSTB
8.77 × 10⁶	0.1540	57 MMSTB
17.05 × 10⁶	0.2820	60 MMSTB

Step 2. The above calculations show the calculated values of the initial oil in place are increasing (as shown graphically in Figure 11-26), which indicates a water encroachment, i.e., water-drive reservoir.

Step 3. For simplicity, select the pot-aquifer model to represent the water encroachment calculations in the MBE as given by Equation 11-47, or:

$\frac{F}{E_o} = N + K\left(\frac{\Delta p}{E_o}\right)$

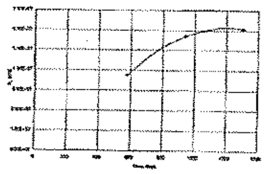


Figure 11-26. Selection of water influx.

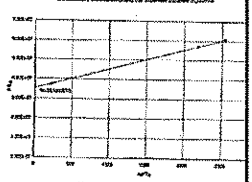


Figure 11-27. F/E_o versus Δp/E_o.

Step 4. Calculate the terms (F/E_o) and $(\Delta p/E_o)$ of Equation 11-42.

Step 5. Plot (F/E_o) versus $(\Delta p/E_o)$, as shown in Figure 11-27, and determine the intercept and the slope.

Intercept = N = 35 MMSTB

Slope = K = 9511

Tracy's Form of the Material Balance Equation

Neglecting the formation and water compressibilities, the general material balance equation as expressed by Equation 11-13 can be reduced to the following:

Tracy (1955) suggested that the above relationship can be rearranged into a more usable form as:

$$N = N_p \Phi_o + G_p \Phi_g + (W_p B_w - W_e) \Phi_w$$

where Φ_o, Φ_g and Φ_w are considered PVT related properties that are functions of pressure and defined by:

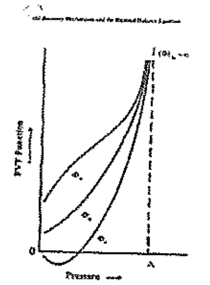


Figure 11-28. Tracy's PVT functions.

Example 11-6

Solution

PROBLEMS

REFERENCES

9-4 The General Material Balance for Oil Reservoirs

Havlena and Odeh (1963, 1964) introduced the application of material balance to oil reservoirs that have initial oil-in-place N and a ratio m relating the initial hydrocarbon volume in the gascap to the initial hydrocarbon volume in the oil zone.

9-4.1 The Generalized Expression

The generalized expression as presented by Dake (1978) (and ignoring water influx and production) is

$$N_p[B_o + (R_p - R_s)B_g] = NB_{oi}\left[\frac{(B_o - B_{oi}) + (R_{si} - R_s)B_g}{B_{oi}} + m\left(\frac{B_g}{B_{gi}} - 1\right) + (1+m)\left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\Delta p\right] \quad (9\text{-}13)$$

In Eq. (9-13), N_p is the cumulative oil production, R_p is the producing gas-oil ratio, and S_{wc} is the interstitial water saturation. All other variables are the usual thermodynamic and physical properties of a two-phase system. Groups of variables, as defined below, correspond to important components of production.

- Underground withdrawal:

$$F = N_p[B_o + (R_p - R_s)B_g] \quad (9\text{-}14)$$

A graph of F/E_o versus E_g/E_o would result in a straight line with N the intercept and Nm the slope.

Undersaturated reservoirs. In undersaturated reservoirs, since $m=0$ and $R_p=R_s$, Eq. (9-13) becomes

$$N_p B_o = NB_{oi}\left[\frac{B_o - B_{oi}}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\Delta p\right] \quad (9\text{-}24)$$

Equation (9-6) can be written in terms of the formation volume factors and simplified as a Taylor series dropping all terms with powers of 2 and larger:

$$B_o = B_{oi} e^{c_o \Delta p} \approx B_{oi}(1 + c_o \Delta p)$$

Therefore

$$\frac{B_o - B_{oi}}{B_{oi}} = c_o \Delta p$$

Multiplying and dividing by S_o results in

$$\frac{B_o - B_{oi}}{B_{oi}} = \frac{c_o S_o \Delta p}{S_o}$$

Since $S_o = 1 - S_{wc}$, then also

$$\frac{B_o - B_{oi}}{B_{oi}} = \frac{c_o S_o \Delta p}{1 - S_{wc}}$$

and therefore Eq. (9-24) becomes

$$N_p B_o = NB_{oi}\left[\frac{c_o S_o \Delta p}{1 - S_{wc}} + \frac{c_w S_{wc} + c_f}{1 - S_{wc}}\Delta p\right] \quad (9\text{-}29)$$

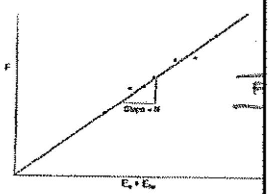


Figure 11-17. Underground withdrawal vs. $E_o + E_{fw}$.

From the definition of the total compressibility, $c_t = c_o S_o + c_w S_{wc} + c_f$, this is

$$N_p B_o = \frac{NB_{oi} c_t \Delta p}{1 - S_{wc}} \quad (9\text{-}30)$$

A graph of $N_p B_o$ versus $B_{oi} c_t \Delta p/(1 - S_{wc})$ would result in a straight line with a slope equal to N.

EXAMPLE 9-3

Material balance for saturated reservoirs[1]

An oil field produced as in the schedule in Table 9-2. Fluid properties are also given. The reservoir pressure in the oil and gas zones varied somewhat, therefore, fluid properties are given

Table 9-2

Production and Fluid Data for Example 9-3

Date	N_p (STB)	G_p (MSCF)
5/1/89	—	—
1/1/91	492,500	751,300
1/1/92	1,015,700	2,409,600
1/1/93	1,322,500	3,901,600

Date	$\bar{p}_o$ (psia)	$\bar{p}_g$ (psia)	B_t at $\bar{p}_o$ (res bbl/STB)	B_g at $\bar{p}_g$ (res ft³/SCF)
5/1/89	4415	4245	1.6291	0.00431
1/1/91	3875	4025	1.6839	0.00445
1/1/92	3315	3505	1.7835	0.00490
1/1/93	2845	2985	1.9110	0.00556

p_b = 4290 psia

R_{si} = 975 SCF/STB

B_{ob} = 1.6330 res bbl/STB

B_{oi} = 1.6291 res bbl/STB

for the simultaneous pressures in the two zones. Calculate the best value of the initial oil in place (STB) and the initial *total* gas in place (MM SCF).

The oil zone thickness is 21 ft, the porosity is 0.17, and the water saturation is 0.31. Calculate the areal extent of the initial gas cap and the reservoir pore volume.

Reservoir simulation has shown that a good time for a waterflood start is when 16% of the original oil-in-place is produced. When will this happen if a constant flow rate is maintained?

Since B_t data are given in this example, the already-developed equations for F, E_o and E_g are developed below in terms of B_t. Because

$$B_t = B_o + (R_{si} - R_s)B_g \tag{9-31}$$

then

$$B_o = B_t - (R_{si} - R_s)B_g \tag{9-32}$$

and therefore

$$F = N_p[B_t + (R_p - R_{si})B_g] \tag{9-33}$$

$$E_o = B_t - B_{oi} \tag{9-34}$$

$$E_g = B_{oi}\left(\frac{B_t}{B_{ti}} - 1\right) \tag{9-35}$$

Recovery Mechanisms and the Material Balance Equation 765

Example 11-4[1]

The production history and the PVT data of a gas-cap-drive reservoir are given below:

Date	p psi	N_p MSTB	G_p Mscf	B_t bbl/STB	B_g bbl/scf
5/1/89	4415	—	—	1.6291	0.00077
1/1/91	3875	492.5	751.3	1.6839	0.00079
1/1/92	3315	1015.7	2409.6	1.7835	0.00087
1/1/93	2845	1322.5	3901.6	1.9110	0.00099

The initial gas solubility R_{si} is 975 scf/STB. Estimate the initial oil and gas in place.

R $\frac{SCF}{STB} \cdot \frac{bbl}{SCF}$

Solution

Step 1. Calculate the cumulative produced gas-oil ratio R_p

p	G_p Mscf	N_p MSTB	$R_p = G_p/N_p$ scf/STB
4415	—	—	—
3875	751.3	492.5	1525
3315	2409.6	1015.7	2372
2845	3901.6	1322.5	2950

Step 2. Calculate F, E_o, and E_g:

p	F	E_o	E_g
3875	2.04×10^6	0.0548	0.0529
3315	8.77×10^6	0.1540	0.2220
2845	17.05×10^6	0.2820	0.4720

Step 3. Calculate F/E_o and E_g/E_o

p	F/E_o	E_g/E_o
3875	3.72×10^7	0.96
3315	5.69×10^7	1.44
2845	6.00×10^7	1.67

[1]After Economides, M., and Hill, D., *Petroleum Production Systems*, Prentice Hall, 1993.

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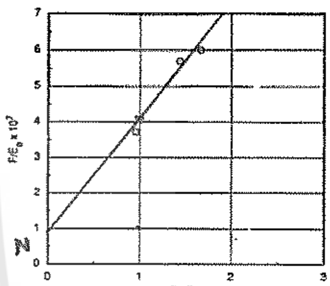
Solution Table 9-3 contains the calculated R_p (= G_p/N_p) and the variables F, E_o, and E_g as given by Eqs. (9-33) through (9-35). Also, in the manner suggested by Eq. (9-23), the variables F/E_o and E_g/E_o are listed.

Table 9-3

Calculated Variables for the Material Balance Straight Line of Example 9-3

Date	R_p (SCF/STB)	F	E_o	E_g	F/E_o	E_g/E_o
1/1/91	1525	2.04×10^6	5.48×10^{-2}	5.29×10^{-2}	3.72×10^7	0.96
1/1/92	2372	8.77×10^6	1.54×10^{-1}	2.22×10^{-1}	5.69×10^7	1.44
1/1/93	2950	17.05×10^6	2.82×10^{-1}	4.72×10^{-1}	6.0×10^7	1.67

Figure 9-4 is a plot of the results, providing an intercept of 9×10^6 STB (= N, the initial oil-in-place) and a slope equal to 3.1×10^7 (= Nm). This leads to $m = 3.44$. [If there were no initial gas cap, the slope would be equal to zero. In general, the use of Eq. (9-23) instead of Eq. (9-21) is recommended, since the latter is included in the former.]



$m = \frac{3.1 \times 10^7}{9 \times 10^6} = 3.44$

Figure 9-4
Material balance calculation for a two-phase reservoir (Example 9-3).

The pore volume of oil is then

$$V_{po} = NB_{oi} = (9 \times 10^6)(1.6291) = 1.47 \times 10^7 \text{ res bbl} \quad (9\text{-}36)$$

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Solution Table 9-3 contains the calculated R_p (= G_p/N_p) and the variables F, E_o, and E_g as given by Eqs. (9-33) through (9-35). Also, in the manner suggested by Eq. (9-23), the variables F/E_o and E_g/E_o are listed.

Table 9-3

Calculated Variables for the Material Balance Straight Line of Example 9-3

Date	R_p (SCF/STB)	F	E_o	E_g	F/E_o	E_g/E_o
1/1/91	1525	2.04×10^6	5.48×10^{-2}	5.29×10^{-2}	3.72×10^7	0.96
1/1/92	2372	8.77×10^6	1.54×10^{-1}	2.22×10^{-1}	5.69×10^7	1.44
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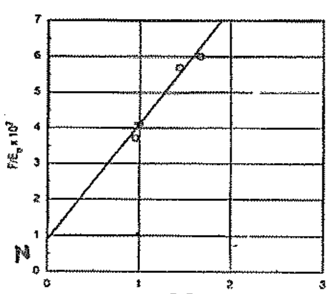
Figure 9-4 is a plot of the results, providing an intercept of 9×10^6 STB (= N, the initial oil-in-place) and a slope equal to 3.1×10^7 (= Nm). This leads to $m = 3.44$. [If there were no initial gas cap, the slope would be equal to zero. In general, the use of Eq. (9-23) instead of Eq. (9-21) is recommended, since the latter is included in the former.]



$m = \frac{3.1 \times 10^7}{9 \times 10^6} = 3.44$

Figure 9-4
Material balance calculation for a two-phase reservoir (Example 9-3).

The pore volume of oil is then

$$V_{po} = NB_{oi} = (9 \times 10^6)(1.6291) = 1.47 \times 10^7 \text{ res bbl} \quad (9\text{-}36)$$

and the pore volume of the gas cap is

$$V_{pg} = mV_{po} = (3.44)(1.47 \times 10^7) = 5.04 \times 10^7 \text{ res bbl} \quad (9$$

The drainage area of the oil zone in acres is

$$A = \frac{V_{po}}{7758\, \phi\, h\, (1 - S_w)} \quad (9$$

and therefore

$$A = \frac{(1.47 \times 10^7)}{(7758)(0.17)(21)(1 - 0.31)} = 770 \text{ acres} \quad (9$$

If 16% of initial oil-in-place is to be produced, then

$$N_p = (0.16)(9 \times 10^6) = 1.44 \times 10^6 \text{ STB} \quad (9$$

From Table 9-2 the flow rate between 1/1/92 and 1/1/93 is approximately 840 ST From 1/1/93 until the required date for water flooding, there will be a need for an addi 1.12 × 10⁵ STB of production. With the same production rate it would require 140 d additional production. Thus water flooding should begin at about May 1993.

$$\frac{F}{E_o} = N + Nm\left(\frac{E_g}{E_o}\right) \quad (9\text{-}23)$$

HW NO 1; Due Date: Friday 11 Oct. 2013

1. The Lone Star Oil Co. producer 5-B gave the following pressure-production data:

Date	Avg. Press.	Cum. STB (N_p)	Cum. Gas MCF (G_p)
Initial	2840	0	0
7/65	2660	36933	37851
5/66	2364	65465	74137
6/67	2338	75629	91910
7/68	2375	85544	115256
7/69	2305	96100	148200

The PVT data is as follows:

$$E_g = B_{oi}\,[(B_g/B_{gi}) - 1]$$

Press	B_t (B/B)	B_g (Vol/Vol)	R_s (SCF/B)
2840	1.528	-	827
2660	1.563	0.00618	772
2364	1.636	0.00680	680
2338	1.648	0.00691	675
2375	1.634	0.00674	685
2305	1.655	0.00702	665

/5.615

Determine the original oil in place by N

(1) using material balance as a straight line.

(2) using the last measured production.

Assume: no gas cap or water influx.

Applying the above conditions on Equation 11-32 gives:

$$F = N(E_o + E_{f,w})$$

$$F = N_p\,[B_t + (R_p - R_{si})\,B_g] + W_p\,B_w$$

$$E_o = B_t - B_{ti}$$

3. The MBE for a gas reservoir can be stated as:

$$G_pB_g + W_pB_w - W_e = GB_g - GB_{gi}$$

Using the above equation set up the st. line equation necessary to obtain:

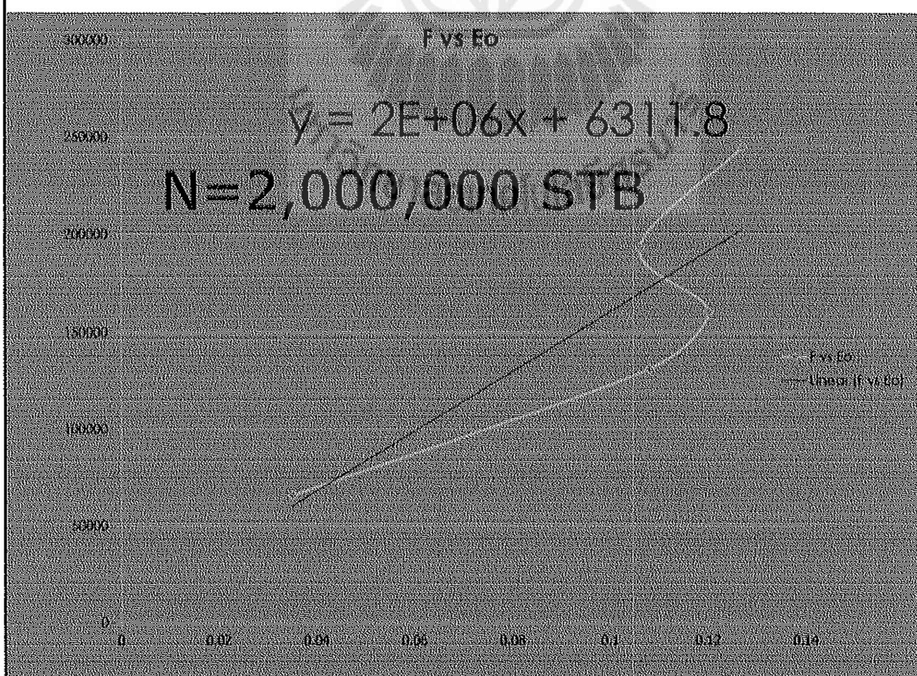
(1.) Gas in place

(2.) Gas in place and influx constant

under (a) no water flux and no water production

(b) water influx and water production.

Sketch the relationship you will expect!



(2) A secondary gas injection project was initiated after 4 years in a volumetric oil reservoir. The following data describes the reservoir:

$P_b = 4290$ $R_{s_b} = 975$ $m = 0.075$

$B_{o_b} = 1.6330$ $B_{t_i} = 1.6291$ $B_w = 1.055$

$Y = 1.75 + 0.435 * 10^{-3}\ p$

* $= (P_b - P)/[P(V_t/V_b) - 1]$

Assuming no water influx, using MBE as a st. line:

1. Calculate the original oil in place
2. Original gas in place
3. Using Pirson's form of $F = N_p[B_t + (R_p - R_{si})B_g] + W_p B_w - G_i B_{ginj}$ the following at 1/65

   (i) Depletion Drive Index

   (ii) Gas Drive Index

* Pg 384 ABW.

Data follows:

DATE (1)	N_p bbl. (2)	W_p bbl. (3)	G_p MCF (4)	G_i MCF (5)	$\bar{P}_o$ psia (6)	$\bar{P}_g$ psia (7)	B_t @ $\bar{P}_o$ (8)	B_g @ $\bar{P}_o$ (9)	B_{ig} @ $\bar{P}_g$ (10)
[illegible]	-	-			4,415	4,415	1.6291	0.00432	
[illegible]	452,300	7,950	752,300		3,875	4,025	1.6839	0.00445	
[illegible]	1,617,700	25,320	2,409,700		3,315	3,505	1.7835	0.00490	
[illegible]	1,329,500	42,340	3,901,600	0	2,845	2,955	1.9110	0.00558	
[illegible]	1,444,600	51,080	4,437,300	557,400	2,695	3,050	1.8949	0.00545	0.00670
[illegible]	1,532,200	60,350	4,937,700	1,309,400	2,975	3,240	1.8700	0.00539	0.00585

/5.615

/5.615

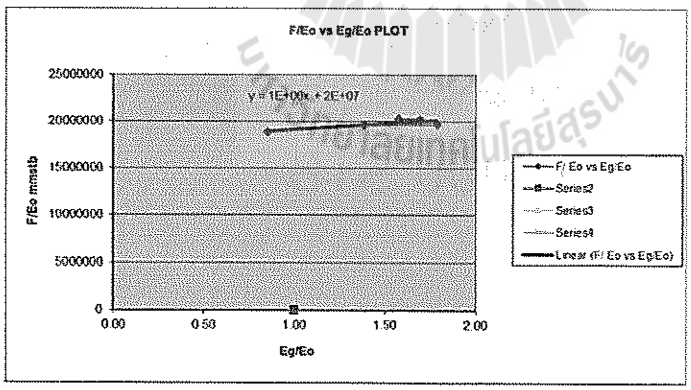
$$F = N_p [B_t + (R_p - R_{si}) B_g] + W_p B_w$$

$$E_o = B_t - B_{ti}$$

$$E_g = B_{oi} [(B_g/B_{gi}) - 1]$$

$$\frac{F}{E_o} = N + mN\left(\frac{E_g}{E_o}\right)$$

$$F = N [E_o + m E_g]$$



Step 4. Plot (F/E_o) versus (E_g/E_o) as shown in Figure 11-22 to give:

- Intercept = N = 18 MMSTB
- Slope = N m = 1,350,000

Step 5. Calculate m:

m = 1350000/18000000=0.075

Step 6. Calculate initial gas in place G:

$$G = \frac{m N B_{oi}}{B_{gi}} = 0.075*18*1.6291/0.00077 = 2.856\text{MMMSCF}$$

3. The MBE for a gas reservoir can be stated as:

$$G_p B_g + W_p B_w - W_e = G B_g - G B_{gi}$$

Using the avove equation set up the st. line equation necessary to obtain:

(1.) Gas in place

(2.) Gas in place and Influx constant

under (a) no water flux and no-water production

(b) water influx and water production.

Sketch the relationship you will expect!

The cumulative gas-oil ratio at 1302 psi is recorded at 953 scf/STB. Calculate:

a. Oil saturation at 1302 psia

b. Volume of the free gas in the reservoir at 1302 psia

c. Relative permeability ratio (k_g/k_o) at 1302 psia

4. The Nameless Field is an undersaturated-oil reservoir. The crude oil system and rock type indicates that the reservoir is highly compressible. The available reservoir and production data are given below:

$S_{wi} = 0.25$ $\phi = 20\%$ Area = 1,000 acres

$h = 70'$ $T = 150°F$

Bubble-point pressure = 3500 psia

	Original condition	Current conditions
Pressure, psi	5000	4500
B_o, bbl/STB	1.905	1.920
R_s, scf/STB	700	700
NP, MSTB	0	610.9

Calculate the cumulative oil production at 3900 psi. The PVT data show that the oil formation volume factor is equal to 1.938 bbl/STB at 3900 psia.

4 5. The following data[7] is available on a gas-cap-drive reservoir:

Pressure (psi)	N_p (MMSTB)	R_p (scf/STB)	B_o (RB/STB)	R_s (scf/STB)	B_g (RB/scf)
3,330			1.2511	510	0.00087
3,150	3.295	1,050	1.2353	477	0.00092
3,000	5.903	1,060	1.2222	450	0.00096
2,850	8.852	1,160	1.2122	425	0.00101
2,700	11.503	1,235	1.2022	401	0.00107
2,550	14.513	1,265	1.1922	375	0.00113
2,400	17.730	1,300	1.1822	352	0.00120

Calculate the initial oil and free gas volumes.

6. The Wildcat Reservoir was discovered in 1980. This reservoir had an initial reservoir pressure of 3,000 psia, and laboratory data indicated a bubble-point pressure of 2,500 psi. The following … available:

Area	= 700 acres
Thickness	= 35 ft
Porosity	= 20%
Temperature	= 150°F
API gravity	= 50°
Specific gravity of gas	= 0.72
Initial water saturation	= 25%

Average isothermal oil compressibility above the … 10^{-6} psi^{-1}

Calculate the volume of oil initially in place at 3,… in STB.

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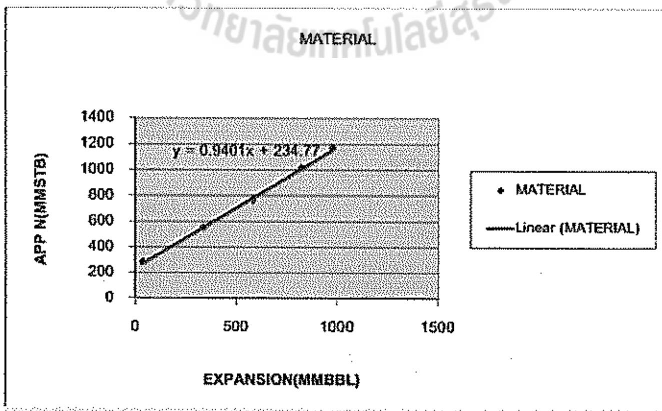
HW NO 1; Due Date: Friday 12 Oct. 2012

4. A saturated oil reservoir is believed to contain an initial gas cap with m = 0.3. The initial pressure at the gas-oil contact, which is also a convenient datum, is 5,000 psi. The PVT properties of the system at the reservoir conditions are as follows:

Pressure psia	R_{so} SCF/STB	B_o RB/STB	B_g RB./SCF
5,000	500	1.3050	0.00065
4,300	338	1.2280	0.00075
4,250	325	1.2200	0.00076
4,200	310	1.215	0.00077
4,150	295	1.205	0.00078
4100	280	1.2	0.00079

The uniform initial water saturation is 30% and water (C_w)and pore volume(C_f) compressibility are each $3x10^{-6}$ psi^{-1}. The oil, gas and water productions start at 1/1/2001 as follows. Water and gas injection started at a constant rate of 25 MMSTB/year and 10 MMMSCF/year on 1/1/2002.(12 months later). Assume B_w for injection and produced water are 1.00 RB/STB., and B_{gi} for injection gas is 0.00076 RB/SCF. And B_{wp} for producing water is also 1.00 RB/STB. The cumulative gas, water and reservoir production pressure has been reported, and water invasion has been estimated as follows:

D/M/Y	P PSI	Np MMSTB	GP MMMSCF	Wp MMBBL	We MMBBL	WI MMBBL	Gi MMMSCF
1/1/2001	5000	0					
31/12/2001	4300	25	10	1	5	0	0
31/12/2002	4250	50	25	2	11	25	10
31/12/2003	4200	75	40	6	20	50	20
31/12/2004	4150	100	60	10	30	75	30
31/12/2005	4100	125	80	15	40	100	40



N= 235 MM STB

G=NmBti/Bgi= 141409.8MMSCF

434620,505653 ADVANCED RESERVOIR ENGINEERIN
2/2556(4 credits(4-0-8), SEP.23-Dec.26, 2013.)

Course Contents

1. Optimization of Material Balance Equations (6 hrs.)
2. Saturation and Relative Permeability Calculations (4 hrs.)
3. Steady State Radial Flow (4 hrs.)
4. Pseudo-steady State Flow and Superposition (4 hrs.)
5. Well Testing Pressure Drawdown and Build Up (3 hrs.)
6. Interference Test and Type Curve Analysis(3hrs)
7. Displacement Efficiency(4 hrs)
8. Potential flow and Streamlines (4 hrs.)
9. Dynamics of Water Drive Reservoir. (6 hrs.)
10. Water and Gas Coning (4hrs.)
11. Multi-Phase Flow and Introduction to Reservoir Simulation (4 h

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1. Porosity

A. Absolute porosity

B. Effective porosity

1. Primary porosity
2. Secondary porosity

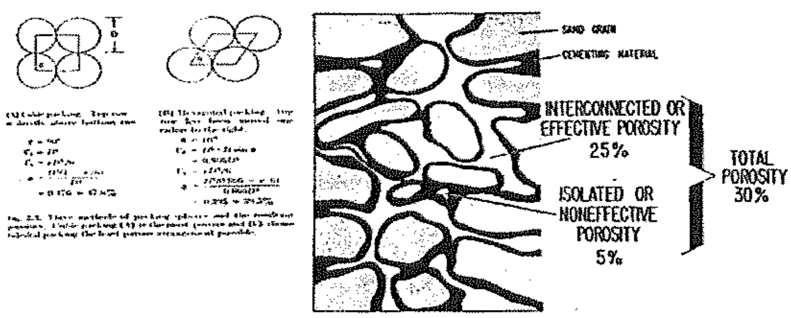

Figure 1.18 Effective, non-effective and total porosity

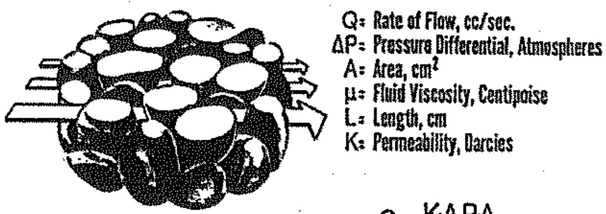

Figure 1.24 **Permeability** $Q = \frac{K \cdot \Delta P \cdot A}{\mu \cdot L}$

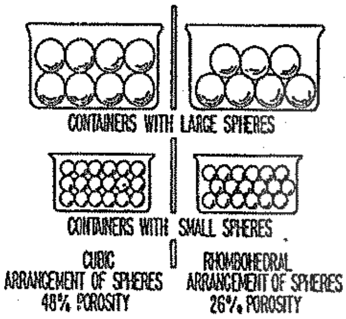

Fig. 21—Effects of size and arrangement of spheres on porosity.

Size + shape
Arrangement
Compaction
Cementation
Grain kind

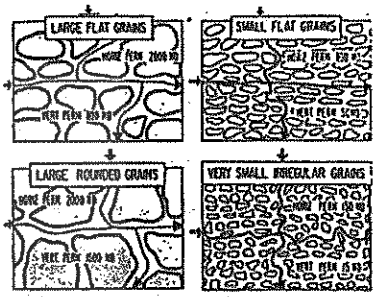

Fig. 18—Effects of shape and size of sand grains on permeability.

C. PERMEABILITY $K = \frac{Q\mu\Delta L}{A\Delta P}$

- Absolute one fluid phase in pore = k = 413 md ; k_w @ S_w = 1.0 or k_o @ S_o = 1.0
- Effective 2-3 phases in pore; k_{ew} @ S_w = 0.7, k_{ew} = 248 md, k_{eo} =50 md.
- Relative: $Krw = \frac{k_{ew}}{k}$. $Kro = \frac{k_{eo}}{k}$

D. SATURATION AND WETTABILITY

- Water wet
- Oil wet

E. FORMATIOM COMPRESSIBILITY

$Cf = 1.87 \times 10^{-6} \times \phi^{-0.415}$ by Hall Humble

$= -\left(\frac{1}{V}\right)\left(\frac{dV}{dp}\right)$

F. FORMATION FACTORS

$F = Ro/Rw = a/\phi^m$

ABSOLUTE PERMEABILITY

a single fluid is called the *absolute permeability* of the rock. If a core sample 0.00215 ft² in cross section and 0.1 ft long flows a 1.0 cp brine with a formation volume factor of 1.0 bbl/STB at the rate of 0.30 STB/day under a 30 psi pressure differential, it has an absolute permeability of

Absolute K

K_w

$$k = \frac{q_w B_w \mu_w L}{0.001127 A_c \Delta p} = \frac{0.30(1.0)(0.1)}{0.001127(0.00215)(30)} = 413 \text{ md}$$

If the water is replaced by an oil of 3.0 cp viscosity and 1.2 bbl/STB formation volume factor, under the same pressure differential the flow rate will be 0.0834 STB/day, and again the absolute permeability is

K_o

$$k = \frac{q_o B_o \mu_o L}{0.001127 A_c \Delta p} = \frac{0.0834(1.2)(3.0)(0.1)}{0.001127(0.00215)(30)} = 413 \text{ md}$$

If the same core is maintained at 70% water saturation ($S_w = 70\%$) and 30% oil saturation ($S_o = 30\%$), and at these and only these saturations and

Microscopic Displacement Efficiency, Ed

under the same pressure drop it flows 0.18 STB/day of the brine and 0.01 STB/day of the oil, then the effective permeability to water is

$$k_w = \frac{q_w B_w \mu_w L}{0.001127 A_c \Delta p} = \frac{0.18(1.0)(1.0)(0.1)}{0.001127(0.00215)(30)} = 248 \text{ md}$$

and the effective permeability to oil is

$$k_o = \frac{q_o B_o \mu_o L}{0.001127 A_c \Delta p} = \frac{0.01(1.2)(3.0)(0.1)}{0.001127(0.00215)(30)} = 50 \text{ md}$$

The effective permeability, then, is the permeability of a rock to a particular fluid when that fluid has a pore saturation of less than 100%. As noted in the foregoing example, the sum of the effective permeabilities (i.e., 298 md) is always less than the absolute permeability, 413 md.

When two fluids, such as oil and water, are present, their relative rates of flow are determined by their relative viscosities, their relative formation volume factors, and their relative permeabilities. Relative permeability is the ratio of effective permeability to the absolute permeability. For the previous example, the relative permeabilities to water and to oil are

$$k_{rw} = \frac{k_w}{k} = \frac{248}{413} = 0.60$$

$$k_{ro} = \frac{k_o}{k} = \frac{50}{413} = 0.12$$

The flowing water-oil ratio at reservoir conditions depends on the viscosity ratio and the effective permeability ratio (i.e., on the mobility ratio), or connate water saturation is 20% for this particular rock, then the recovery from the portion of the reservoir invaded by high-pressure water influx is

$$\text{Recovery} = \frac{\text{initial-final}}{\text{initial}} = \frac{0.80 - 0.15}{0.80} = 81\%$$

Experiments show that essentially the same relative permeability curves are obtained for a gas-water system as for the oil-water system, which also means that the critical, or residual, gas saturation will be the same. Furthermore, it has been found that if both oil and free gas are present, the residual *hydrocarbon* saturation (oil and gas) will be about the same, in this case 15%. Suppose, then, that the rock is invaded by water at a pressure below saturation pressure so that free gas is present. If, for example, the residual free gas saturation behind the flood front is 10%, then the oil saturation is 5%, and neglecting small changes in the formation volume factors of the oil, the recovery is increased to:

$$\text{Recovery} = \frac{0.80 - 0.05}{0.80} = 94\%$$

Returning to Fig. 9.1, as the water saturation decreases further, the relative permeability to water continues to decrease and the relative permeability to oil increases. At 20% water saturation, the (connate) water is immobile, and the relative permeability to oil is quite high. This explains why some rocks may contain as much as 50% connate water and yet produce water-free oil. Most reservoir rocks are preferentially water wet—that is, the water phase and not the oil phase is next to the walls of the pore spaces. Because of this, at 20% water saturation the *water* occupies the *least favorable* portions of the pore spaces—that is, as thin layers about the sand grains, as thin layers on the walls of the pore cavities, and in the smaller crevices and capillaries. The oil, which occupies 80% of the pore space, is in the *most favorable* portions of the pore spaces, which is indicated by a relative permeability of 93%. The curves further indicate that about 10% of the pore spaces contribute *nothing* to the permeability, for at 10% water saturation, the relative permeability to oil is essentially 100%. Conversely, on the other end of the curves, 15% of the pore spaces contribute 40% of the permeability, for an increase in oil saturation from zero to 15% reduces the relative permeability to water from 100% to 60%.

In describing two-phase flow mathematically, it is always the relative permeability ratio that enters the equations. Figure 9.2 is a plot of the relative permeability ratio, k_o/k_w, versus water saturation for the same data of Fig. 9.1. Because of the wide range of k_o/k_w values, the relative permeability ratio is usually plotted on the log scale of semilog paper. Like many relative permeability ratio curves, the central or main portion of the curve is quite linear. As

a straight line on semilog paper, the relative permeability ratio may be expressed as a function of the water saturation by:

$$\frac{k_o}{k_w} = ae^{-bS_w} \qquad (9.3)$$

The constants a and b may be determined from the graph shown in Fig. 9.2, or they may be determined from simultaneous equations. At $S_w = 0.30$, $k_o/k_w = 25$, and at $S_w = 0.70$, $k_o/k_w = 0.14$. Then

$$25 = ae^{-0.30b} \quad \text{and} \quad 0.14 = ae^{-0.70b}$$

Solving simultaneously, the intercept $a = 1220$, and the slope $b = 13.0$. Equation (9.3) indicates that the relative permeability ratio for a rock is a function of only the relative saturations of the fluids present. Although it is true that the viscosities, the interfacial tensions, and other factors have some effect on

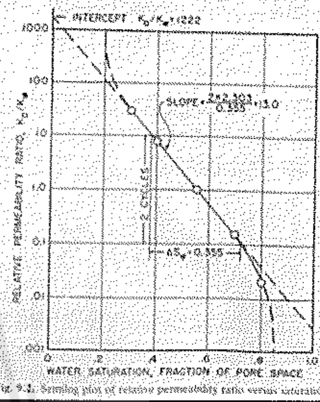

Fig. 9.2. Semilog plot of relative permeability ratio versus saturation.

C H A P T E R 5

RELATIVE PERMEABILITY CONCEPTS

Numerous laboratory studies have concluded that the effective permeability of any reservoir fluid is a function of the reservoir fluid saturation and the wetting characteristics of the formation. It becomes necessary, therefore, to specify the fluid saturation when stating the effective permeability of any particular fluid in a given porous medium. Just as k is the accepted universal symbol for the absolute permeability, k_o, k_g, and k_w are the accepted symbols for the effective permeability to oil, gas, and water, respectively. The saturations, i.e., S_o, S_g, and S_w, must be specified to completely define the conditions at which a given effective permeability exists.

Effective permeabilities are normally measured directly in the laboratory on small core plugs. Owing to many possible combinations of saturation for a single medium, however, laboratory data are usually summarized and reported as relative permeability.

The absolute permeability is a property of the porous medium and is a measure of the capacity of the medium to transmit fluids. When two or more fluids flow at the same time, the relative permeability of each phase at a specific saturation is the ratio of the effective permeability of the phase to the absolute permeability, or:

$$k_{ro} = \frac{k_o}{k}$$

$$k_{rg} = \frac{k_g}{k}$$

$$k_{rw} = \frac{k_w}{k}$$

where k_{ro} = relative permeability to oil
k_{rg} = relative permeability to gas
k_{rw} = relative permeability to water
k = absolute permeability
k_o = effective permeability to oil for a given oil saturation
k_g = effective permeability to gas for a given gas saturation
k_w = effective permeability to water at some given water saturation

For example, if the absolute permeability k of a rock is 200 md and the effective permeability k_o of the rock at an oil saturation of 80 percent is 60 md, the relative permeability k_{ro} is 0.30 at $S_o = 0.80$.

Since the effective permeabilities may range from zero to k, the relative permeabilities may have any value between zero and one, or:

$$0 \leqq k_{rw}, k_{ro}, k_{rg} \leqq$$

It should be pointed out that when three phases are present the *sum of the relative permeabilities* ($k_{ro} + k_{rg} + k_{rw}$) *is both variable and always less than or equal to unity.* An appreciation of this observation and of its physical causes is a prerequisite to a more detailed discussion of two- and three-phase relative permeability relationships.

It has become a common practice to refer to the relative permeability curve for the nonwetting phase as k_{nw} and the relative permeability for the wetting phase as k_w.

TWO-PHASE RELATIVE PERMEABILITY

When a wetting and a nonwetting phase flow together in a reservoir rock, each phase follows separate and distinct paths. The distribution of the two phases according to their wetting characteristics results in characteristic wetting and nonwetting phase relative permeabilities. Since the wetting phase occupies the smaller pore openings at small saturations, and these pore openings do not contribute materially to flow, it follows that the presence of a small wetting phase saturation will affect the nonwetting phase permeability only to a limited extent. Since the nonwetting phase occupies the central or larger pore openings which contribute materially to fluid flow through the reservoir, however, a small nonwetting phase saturation will drastically reduce the wetting phase permeability.

Figure 5-1 presents a typical set of relative permeability curves for a water-oil system with the water being considered the wetting phase. Figure 5-1 shows the following four distinct and significant points:

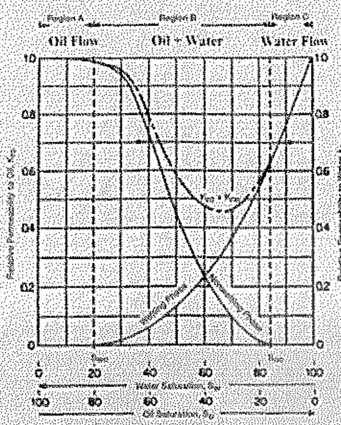

Figure 5-1. Typical two-phase flow behavior.

- **Point 1**

Point 1 on the wetting phase relative permeability shows that a small saturation of the nonwetting phase will drastically reduce the relative permeability of the wetting phase. The reason for this is that the nonwetting phase occupies the larger pore spaces, and it is in these large pore spaces that flow occurs with the least difficulty.

- **Point 2**

Point 2 on the nonwetting phase relative permeability curve shows that the nonwetting phase begins to flow at the relatively low saturation of the nonwetting phase. The saturation of the oil at this point is called *critical oil saturation* S_{oc}.

- **Point 3**

Point 3 on the wetting phase relative permeability curve shows that the wetting phase will cease to flow at a relatively large saturation. This is because the wetting phase preferentially occupies the smaller pore spaces, where capillary forces are the greatest. The saturation of the water at this point is referred to as the *irreducible water saturation* S_{wi} or *connate water saturation* S_{wc}—both terms are used interchangeably.

- **Point 4**

Point 4 on the nonwetting phase relative permeability curve shows that, at the lower saturations of the wetting phase, changes in the wetting phase saturation have only a small effect on the magnitude of the nonwetting phase relative permeability curve. The reason for the phenomenon at Point 4 is that at the low saturations the wetting phase fluid occupies the small pore spaces which do not contribute materially to flow, and therefore changing the saturation in these small pore spaces has a relatively small effect on the flow of the nonwetting phase.

This process could have been visualized in reverse just as well. It should be noted that this example portrays oil as nonwetting and water as wetting. The curve shapes shown are typical for wetting and nonwetting phases and may be mentally reversed to visualize the behavior of an oil-wet system. Note also that the total permeability to both phases, $k_{rw} + k_{ro}$, is less than 1, in regions B and C.

The above discussion may be also applied to gas-oil relative permeability data, as can be seen for a typical set of data in Figure 5-2. Note that this might be termed gas-liquid relative permeability since it is plotted versus the liquid saturation. This is typical of gas-oil relative permeability data in the presence of connate water. Since the connate (irreducible) water normally occupies the smallest pores in the presence of oil

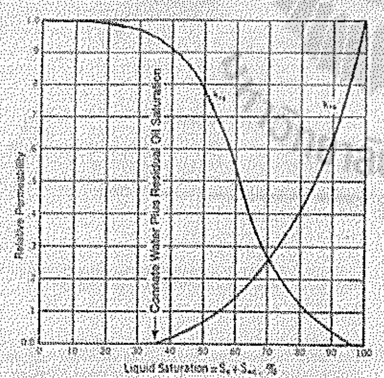

Figure 5-2. Gas-oil relative permeability curves.

and gas, it appears to make little difference whether water or oil that would also be immobile in these small pores occupies these pores. Consequently, in applying the gas-oil relative permeability data to a reservoir, the total liquid saturation is normally used as a basis for evaluating the relative permeability to the gas and oil.

Note that the relative permeability curve representing oil changes completely from the shape of the relative permeability curve for oil in the water-oil system. In the water-oil system, as noted previously, oil is normally the nonwetting phase, whereas in the presence of gas the oil is the wetting phase. Consequently, in the presence of water only, the oil relative permeability curve takes on an S shape whereas in the presence of gas the oil relative-permeability curve takes on the shape of the wetting phase, or is concave upward. Note further that the critical gas saturation S_{gc} is generally very small.

Another important phenomenon associated with fluid flow throug porous media is the concept of residual saturations. As when one immi cible fluid is displacing another, it is impossible to reduce the saturatio of the displaced fluid to zero. At some small saturation, which is pre sumed to be the saturation at which the displaced phase ceases to be con tinuous, flow of the displaced phase will cease. This saturation is ofte referred to as the *residual* saturation. This is an *important* concept as determines the maximum recovery from the reservoir. Conversely, a flui must develop a certain minimum saturation before the phase will begi to flow. This is evident from an examination of the relative permeabilit curves shown in Figure 5-1. The saturation at which a fluid will jus begin to flow is called the *critical* saturation.

Theoretically, the critical saturation and the residual saturation shoul be exactly equal for any fluid; however, they are not identical. **Critica saturation is measured in the direction of increasing saturation, whil irreducible saturation is measured in the direction of reducing satura tion.** Thus, the saturation histories of the two measurements are different.

As was discussed for capillary-pressure data, there is also a saturatio history effect for relative permeability. The effect of saturation history o relative permeability is illustrated in Figure 5-3. If the rock sample is ini tially saturated with the wetting phase (e.g., water) and relative-perme ability data are obtained by decreasing the wetting-phase saturation whil flowing nonwetting fluid (e.g., oil) in the core, the process is classified a *drainage or desaturation*.

If the data are obtained by increasing the saturation of the wettin phase, the process is termed *imbibition or resaturation*. The nomencla ture is consistent with that used in connection with capillary pressure This difference in permeability when changing the saturation history i called *hysteresis*. Since relative permeability measurements are subject t hysteresis, it is important to duplicate, in the laboratory, the saturatio history of the reservoir.

Drainage Process

It is generally agreed that the pore spaces of reservoir rocks were orig inally filled with water, after which oil moved into the reservoir, displac ing some of the water, and reducing the water to some residual satura tion. When discovered, the reservoir pore spaces are filled with a connat water saturation and an oil saturation. If gas is the displacing agent, the gas moves into the reservoir, displacing the oil.

This same history must be duplicated in the laboratory to eliminate the effects of hysteresis. The laboratory procedure is to first saturate the core with water, then displace the water to a residual, or connate, water saturation with oil after which the oil in the core is displaced by gas. This flow process is called the gas drive, or drainage, depletion process. In the gas drive depletion process, the nonwetting phase fluid is continuously increased, and the wetting phase fluid is continuously decreased.

Imbibition Process

The *imbibition process* is performed in the laboratory by first saturating the core with the water (wetting phase), then displacing the water to its irreducible (connate) saturation by injection oil. This "drainage" procedure is designed to establish the original fluid saturations that are found when the reservoir is discovered. The wetting phase (water) is reintroduced into the core and the water (wetting phase) is continuously increased. This is the imbibition process and is intended to produce the relative permeability data needed for water drive or water flooding calculations.

Figure 5-3 schematically illustrates the difference in the drainage and imbibition processes of measuring relative permeability. It is noted that the imbibition technique causes the nonwetting phase (oil) to lose its mobility at higher values of water saturation than does the drainage process. The two processes have similar effects on the wetting phase (water) curve. The drainage method causes the wetting phase to lose its mobility at higher values of wetting-phase saturation than does the imbibition method.

Two-phase Relative Permeability Correlations

In many cases, relative permeability data on actual samples from the reservoir under study may not be available, in which case it is necessary to obtain the desired relative permeability data in some other manner. Field relative permeability data can usually be calculated, and the procedure will be discussed more fully in Chapter 6. The field data are unavailable for future production, however, and some substitute must be devised. Several methods have been developed for calculating relative permeability relationships. Various parameters have been used to calculate the relative permeability relationships, including:

- Residual and initial saturations
- Capillary pressure data

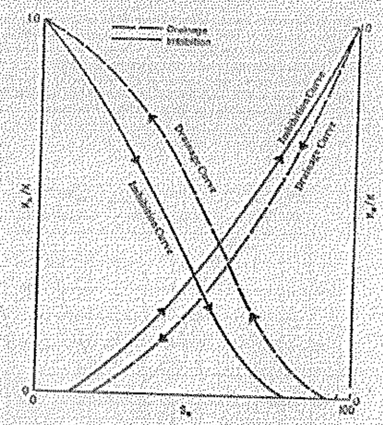

Figure 5-3. Hysteresis effect in relative permeability.

In addition, most of the proposed correlations use the effective phase saturation as a correlating parameter. The effective phase saturation is defined by the following set of relationships:

$$S_o^* = \frac{S_o}{1 - S_{wc}} \quad (5\text{-}1)$$

$$S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc}} \quad (5\text{-}2)$$

$$S_g^* = \frac{S_g}{1 - S_{wc}} \quad (5\text{-}3)$$

where S_o^*, S_w^*, S_g^* = effective oil, water, and gas saturation, respectively
S_o, S_w, S_g = oil, water and gas saturation, respectively
S_{wc} = connate (irreducible) water saturation

1. Wyllie and Gardner Correlation

Wyllie and Gardner (1958) observed that, in some rocks, the relationship between the reciprocal capillary pressure squared ($1/P_c^2$) and the effective water saturation S_w^* is linear over a wide range of saturation. Honapour et al. (1988) conveniently tabulated Wyllie and Gardner correlations as shown below:

Drainage Oil-Water Relative Permeabilities

Type of formation	k_{ro}	k_{rw}	Equation
Unconsolidated sand, well sorted	$(1 - S_w^*)^3$	$(S_w^*)^3$	(5-4)
Unconsolidated sand, poorly sorted	$(1 - S_w^*)^2 (1 - S_w^{*1.5})$	$(S_w^*)^{3.5}$	(5-5)
Cemented sandstone, oolitic limestone	$(1 - S_w^*)^2 (1 - S_w^{*2})$	$(S_w^*)^4$	(5-6)

Drainage Gas-Oil Relative Permeabilities

Type of formation	k_{ro}	k_{rg}	Equation
Unconsolidated sand, well sorted	$(S_o^*)^3$	$(1 - S_o^*)^3$	(5-7)
Unconsolidated sand, poorly sorted	$(S_o^*)^{3.5}$	$(1 - S_o^*)^2 (1 - S_o^{*1.5})$	(5-8)
Cemented sandstone, oolitic limestone, rocks with vugular porosity	$(S_o^*)^4$	$(1 - S_o^*)^2 (1 - S_o^{*2})$	(5-9)

Wyllie and Gardner have also suggested the following two expressions that can be used when one relative permeability is available:

- **Oil-water system**

$$k_{rw} = (S_w^*)^2 - k_{ro}\left[\frac{S_w^*}{1 - S_w^*}\right] \quad (5\text{-}10)$$

- **Gas-oil system**

$$k_{ro} = (S_o^*)^2 - k_{rg}\left[\frac{S_o^*}{1 - S_o^*}\right] \quad (5\text{-}11)$$

2. Torcaso and Wyllie Correlation

Torcaso and Wyllie (1958) developed a simple expression to determine the relative permeability of the oil phase in a gas-oil system. The expression permits the calculation of k_{ro} from the measurements of k_{rg}. The equation has the following form:

$$k_{ro} = k_{rg}\left[\frac{(S_o^*)^4}{(1 - S_o^*)^2 (1 - (S_o^*)^2)}\right] \quad (5\text{-}12)$$

The above expression is very useful since k_{rg} measurements are easily made and k_{ro} measurements are usually made with difficulty.

3. Pirson's Correlation

From petrophysical considerations, Pirson (1958) derived generalized relationships for determining the wetting and nonwetting phase relative permeability for both imbibition and drainage processes. The generalized expressions are applied for water-wet rocks.

For the water (wetting) phase

$$k_{rw} = \sqrt{S_w^*}\, S_w^3 \quad (5\text{-}13)$$

The above expression is valid for both the imbibition and drainage processes.

For the nonwetting phase

- **Imbibition**

$$(k_r)_{nonwetting} = \left[1 - \left(\frac{S_w - S_{wc}}{1 - S_{wc} - S_{nw}}\right)\right]^2 \quad (5\text{-}14)$$

• **Drainage**

$$(k_r)_{nonwetting} = (1 - S_w^*)\left[1 - (S_w^*)^{0.25}\sqrt{S_w}\right]^{0.5} \qquad (5\text{-}15)$$

where S_{nw} = saturation of the nonwetting phase
S_w = water saturation
S_w^* = effective water saturation as defined by Equation 5-2

Example 5-1

Generate the drainage relative permeability data for an unconsolidated well-sorted sand by using the Wyllie and Gardner method. Assume the following critical saturation values:

$S_{oc} = 0.3, \quad S_{wc} = 0.25, \quad S_{gc} = 0.05$

Solution

Generate the oil-water relative permeability data by applying Equation 5-4 in conjunction with Equation 5-2, to give:

S_w	$S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc}}$	$k_{ro} = (1 - S_w^*)^3$	$K_{rw} = (S_w^*)^3$
0.25	0.0000	1.000	0.0000
0.30	0.0667	0.813	0.0003
0.35	0.1333	0.651	0.0024
0.40	0.2000	0.512	0.0080
0.45	0.2667	0.394	0.0190
0.50	0.3333	0.296	0.0370
0.60	0.4667	0.152	0.1017
0.70	0.6000	0.064	0.2160

Apply Equation 5-7 in conjunction with Equation 5-1 to generate relative permeability data for the gas-oil system.

S_g	$S_o = 1 - S_g - S_{wc}$	$S_o^* = \frac{S_o}{1 - S_{wc}}$	$k_{ro} = (S_o^*)^3$	$k_{rg} = (1 - S_o^*)^3$
0.05	0.70	0.933	0.813	—
0.10	0.65	0.867	0.651	0.002
0.20	0.55	0.733	0.394	0.019
0.30	0.45	0.600	0.216	0.064
0.40	0.35	0.467	0.102	0.152
0.50	0.25	0.333	0.037	0.296
0.60	0.15	0.200	0.008	0.512
0.70	0.05	0.067	0.000	0.813

Example 5-2

Resolve Example 5-1 by using Pirson's correlation for the water-oil system.

Solution

S_w	$k_{rw} = \frac{k_w}{k}$	$k_{rw} = \sqrt{S_w^*}\,S_w^3$	$k_{ro} = (1 - S_w^*)\left[1 - (S_w^*)^{.25}\sqrt{S_w}\right]^{0.5}$
0.25	0.0000	0.000	1.000
0.30	0.0667	0.007	0.793
0.35	0.1333	0.016	0.695
0.40	0.2000	0.029	0.608
0.45	0.2667	0.047	0.528
0.50	0.3333	0.072	0.454
0.60	0.4667	0.148	0.320
0.70	0.6000	0.266	0.205

4. Corey's Method

Corey (1954) proposed a simple mathematical expression for generating the relative permeability data of the gas-oil system. The approximation is good for drainage processes, i.e., gas-displacing oil.

$$k_{ro} = (1 - S_g^*)^4 \qquad (5\text{-}16)$$

$$k_{rg} = (S_g^*)(2 - S_g^*) \qquad (5\text{-}17)$$

where the effective gas saturation S_g^* is defined in Equation 5-3.

Example 5-3

Use Corey's approximation to generate the gas-oil relative permeability for a formation with a connate water saturation of 0.25.

Solution

S_g	$S_g^* = \frac{S_g}{1 - S_{wc}}$	$k_{ro} = (1 - S_g^*)^4$	$k_{rg} = (S_g^*)^3(2 - S_g^*)$
0.05	0.0667	0.759	0.001
0.10	0.1333	0.564	0.004
0.20	0.2667	0.289	0.033
0.30	0.4000	0.130	0.102
0.40	0.5333	0.047	0.222
0.50	0.6667	0.012	0.395
0.60	0.8000	0.002	0.614
0.70	0.9333	0.000	0.867

5. Relative Permeability from Capillary Pressure Data

Rose and Bruce (1949) showed that capillary pressure p_c is a measure of the fundamental characteristics of the formation and could also be used to predict the relative permeabilities. Based on the concepts of tortuosity, Wyllie and Gardner (1958) developed the following mathematical expression for determining the drainage water-oil relative permeability from capillary pressure data:

$$k_{rw} = \left(\frac{S_w - S_{wc}}{1 - S_{wc}}\right)^2 \frac{\int_{S_{wc}}^{S_w} dS_w/p_c^2}{\int_{S_{wc}}^{1} dS_w/p_c^2} \qquad (5\text{-}18)$$

$$k_{ro} = \left(\frac{1 - S_w}{1 - S_{wc}}\right)^2 \frac{\int_{S_w}^{1} dS_w/p_c^2}{\int_{S_{wc}}^{1} dS_w/(p_c)^2} \qquad (5\text{-}19)$$

Wyllie and Gardner also presented two expressions for generating the oil and gas relative permeabilities in the presence of the connate water saturation. The authors considered the connate water as part of the rock matrix to give:

$$k_{ro} = \left(\frac{S_o - S_{or}}{1 - S_{or}}\right)^2 \frac{\int_0^{S_o} dS_o/p_c^2}{\int_0^1 dS_o/p_c^2} \qquad (5\text{-}20)$$

$$k_{rg} = \left(1 - \frac{S_o - S_{or}}{S_g - S_{gc}}\right)^2 \frac{\int_{S_o}^{1} dS_o/p_c^2}{\int_0^1 dS_o/p_c^2} \qquad (5\text{-}21)$$

where S_{gc} = critical gas saturation
S_{wc} = connate water saturation
S_{or} = residual oil saturation

Example 5-4

The laboratory capillary pressure curve for a water-oil system, between the connate water saturation and a water saturation of 100% is represented by the following linear equation:

$P_c = 22 - 20\,S_w$

The connate water saturation is 30%. Using Wyllie and Gardner methods, generate the relative permeability data for the oil-water system.

Solution

Step 1. Integrate the capillary pressure equation, to give:

$$I = \int_a^b \frac{dS_w}{(22 - 20S_w)^2} = \left[\frac{1}{440 - 400b}\right] - \left[\frac{1}{440 - 400a}\right]$$

Step 2. Evaluate the above integral at the following limits:

$$\bullet \int_{0.3}^{1} \frac{dS_w}{(22-20S_w)^2} = \left[\frac{1}{440-400(1)} - \frac{1}{440-400(0.3)}\right] = 0.02188$$

$$\bullet \int_{.3}^{S_w} \frac{dS_w}{(22-20S_w)^2} = \left[\frac{1}{440-400S_w} - 0.00313\right]$$

$$\bullet \int_{S_w}^{1} \frac{dS_w}{(22-20S_w)^2} = \left[0.025 - \frac{1}{440-400S_w}\right]$$

Step 3. Construct the following working table:

S_w	k_{rw} Equation 5-18	k_{ro} Equation 5-19
0.3	0.0000	1.0000
0.4	0.0004	0.7195
0.5	0.0039	0.4858
0.6	0.0157	0.2985
0.7	0.0466	0.1574

6. Relative Permeability from Analytical Equations

Analytical representations for individual-phase relative permeabilities are commonly used in numerical simulators. The most frequently used functional forms for expressing the relative-permeability and capillary-pressure data are given below:

Oil-Water Systems:

$$k_{ro} = (k_{ro})_{S_{wc}} \left[\frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}}\right]^{n_o} \quad (5\text{-}22)$$

$$k_{rw} = (k_{rw})_{S_{orw}} \left[\frac{S_w-S_{wc}}{1-S_{wc}-S_{orw}}\right]^{n_w} \quad (5\text{-}23)$$

$$p_{cwo} = (p_c)_{S_{wc}} \left(\frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}}\right)^{n_p} \quad (5\text{-}24)$$

Gas-Oil Systems:

$$k_{ro} = (k_{ro})_{S_{gc}} \left[\frac{1-S_g-S_{lc}}{1-S_{gc}-S_{lc}}\right]^{n_{go}} \quad (5\text{-}25)$$

$$k_{rg} = (k_{rg})_{S_{wc}} \left[\frac{S_g-S_{gc}}{1-S_{lc}-S_{gc}}\right]^{n_g} \quad (5\text{-}26)$$

$$p_{cgo} = (p_c)_{S_{lc}} \left[\frac{S_g-S_{gc}}{1-S_{lc}-S_{gc}}\right]^{n_{pg}} \quad (5\text{-}27)$$

with

$$S_{lc} = S_{wc} + S_{org}$$

where S_{lc} = total critical liquid saturation
$(k_{ro})_{S_{wc}}$ = oil relative permeability at connate water saturation
$(k_{ro})_{S_{gc}}$ = oil relative permeability at critical gas saturation
S_{orw} = residual oil saturation in the water-oil system
S_{org} = residual oil saturation in the gas-oil system
S_{gc} = critical gas saturation
$(k_{rw})_{S_{orw}}$ = water relative permeability at the residual oil saturation
n_o, n_w, n_g, n_{go} = exponents on relative permeability curves
p_{cwo} = capillary pressure of water-oil systems

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Step 2. Evaluate the above integral at the following limits:

$$\bullet \int_{0.3}^{1} \frac{dS_w}{(22-20S_w)^2} = \left[\frac{1}{440-400(1)} - \frac{1}{440-400(0.3)}\right] = 0.02188$$

$$\bullet \int_{.3}^{S_w} \frac{dS_w}{(22-20S_w)^2} = \left[\frac{1}{440-400S_w} - 0.00313\right]$$

$$\bullet \int_{S_w}^{1} \frac{dS_w}{(22-20S_w)^2} = \left[0.025 - \frac{1}{440-400S_w}\right]$$

Step 3. Construct the following working table:

S_w	k_{rw} Equation 5-18	k_{ro} Equation 5-19
0.3	0.0000	1.0000
0.4	0.0004	0.7195
0.5	0.0039	0.4858
0.6	0.0157	0.2985
0.7	0.0466	0.1574

6. Relative Permeability from Analytical Equations

Analytical representations for individual-phase relative permeabilities are commonly used in numerical simulators. The most frequently used functional forms for expressing the relative-permeability and capillary-pressure data are given below:

Oil-Water Systems:

$$k_{ro} = (k_{ro})_{S_{wc}} \left[\frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}}\right]^{n_o} \quad (5\text{-}22)$$

Relative Permeability Concepts

$$k_{rw} = (k_{rw})_{S_{orw}} \left[\frac{S_w-S_{wc}}{1-S_{wc}-S_{orw}}\right]^{n_w}$$

$$p_{cwo} = (p_c)_{S_{wc}} \left(\frac{1-S_w-S_{orw}}{1-S_{wc}-S_{orw}}\right)^{n_p}$$

Gas-Oil Systems:

$$k_{ro} = (k_{ro})_{S_{gc}} \left[\frac{1-S_g-S_{lc}}{1-S_{gc}-S_{lc}}\right]^{n_{go}}$$

$$k_{rg} = (k_{rg})_{S_{wc}} \left[\frac{S_g-S_{gc}}{1-S_{lc}-S_{gc}}\right]^{n_g}$$

$$p_{cgo} = (p_c)_{S_{lc}} \left[\frac{S_g-S_{gc}}{1-S_{lc}-S_{gc}}\right]^{n_{pg}}$$

with

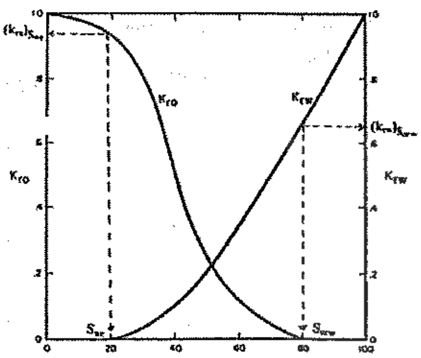
$$S_{lc} = S_{wc} + S_{org}$$

where S_{lc} = total critical liquid saturation
$(k_{ro})_{S_{wc}}$ = oil relative permeability at connate water satur
$(k_{ro})_{S_{gc}}$ = oil relative permeability at critical gas saturati
S_{orw} = residual oil saturation in the water-oil system
S_{org} = residual oil saturation in the gas-oil system
S_{gc} = critical gas saturation
$(k_{rw})_{S_{orw}}$ = water relative permeability at the residual oil saturation
n_o, n_w, n_g, n_{go} = exponents on relative permeability curves
p_{cwo} = capillary pressure of water-oil systems

$(p_c)_{S_{wc}}$ = capillary pressure at connate water saturation
n_p = exponent of the capillary pressure curve for the oil-water system
p_{cgo} = capillary pressure of gas-oil system
n_{pg} = exponent of the capillary pressure curve in gas-oil system
$(p_c)_{S_{Lc}}$ = capillary pressure at critical liquid saturation.

The exponents and coefficients of Equations 5-22 through 5-26 are usually determined by the least-squares method to match the experimental or field relative permeability and capillary pressure data.

Figures 5-4 and 5-5 schematically illustrate the key critical saturations and the corresponding relative permeability values that are used in Equations 5-22 through 5-27.



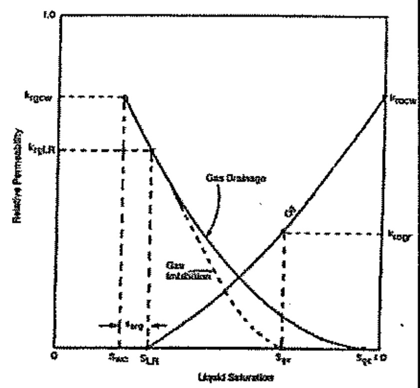


Figure 5-5. Gas-oil relative permeability curves.

Example 5-5

Using the analytical expressions of Equations 5-22–5-27, generate the relative permeability and capillary pressure data. The following information on the water-oil and gas-oil systems is available:

$S_{wc} = 0.25$	$S_{orw} = 0.35$	$S_{gc} = 0.05$	$S_{org} = .23$
$(k_{ro})_{S_{wc}} = 0.85$	$(k_{rw})_{S_{orw}} = 0.4$	$(P_c)_{S_{wc}} = 20$ psi	
$(k_{ro})_{S_{gc}} = 0.60$	$(k_{rg})_{S_{wc}} = 0.95$		
$n_o = 0.9$	$n_w = 1.5$	$n_p = 0.71$	
$n_{go} = 1.2$	$n_g = 0.6$	$(p_c)_{S_{Lc}} = 30$ psi	
$n_{pg} = 0.51$			

Solution

Step 1. Calculate residual liquid saturation S_{Lc}.

$$S_{Lc} = S_{wc} + S_{org} = 0.25 + 0.23 = 0.48$$

Step 2. Generate relative permeability and capillary pressure data for oil-water system by applying Equations 5-22 through 5-24.

S_w	k_{ro} Equations 5-22	k_{rw} Equation 5-23	P_c Equation 5-24
0.25	0.850	0.000	20.00
0.30	0.754	0.018	18.19
0.40	0.557	0.092	14.33
0.50	0.352	0.198	9.97
0.60	0.131	0.327	4.57
0.65	0.000	0.400	0.00

Step 3. Apply Equations 5-25 through 5-27 to determine the relative permeability and capillary data for the gas-oil system.

S_g	k_{ro} Equation 5-25	k_{rg} Equation 5-26	P_c Equation 5-27
0.05	0.600	0.000	0.000
0.10	0.524	0.248	9.56
0.20	0.378	0.479	16.76
0.30	0.241	0.650	21.74
0.40	0.117	0.796	25.81
0.52	0.000	0.95	30.00

RELATIVE PERMEABILITY RATIO

Another useful relationship that derives from the relative permeability concept is the relative (or effective) permeability ratio. This quantity lends itself more readily to analysis and to the correlation of flow performances than does relative permeability itself. The relative permeability ratio expresses the ability of a reservoir to permit flow of one fluid as related to its ability to permit flow of another fluid under the same circumstances. The two most useful permeability ratios are k_{rg}/k_{ro} the relative permeabili-

ty to gas with respect to that to oil and k_{rw}/k_{ro} the relative permeability to water with respect to that to oil, it being understood that both quantities in the ratio are determined simultaneously on a given system. The relative permeability ratio may vary in magnitude from zero to infinity.

In describing two-phase flow mathematically, it is always the relative permeability ratio (e.g., k_{rg}/k_{ro} or k_{ro}/k_{rw}) that is used in the flow equations. Because the wide range of the relative permeability ratio values, the permeability ratio is usually plotted on the log scale of semilog paper as a function of the saturation. Like many relative permeability ratio curves, the central or the main portion of the curve is quite linear.

Figure 5-6 shows a plot of k_{rg}/k_{ro} versus gas saturation. It has become common usage to express the central straight-line portion of the relationship in the following analytical form:

$$\frac{k_{rg}}{k_{ro}} = a\,e^{bS_g} \qquad (5\text{-}28)$$

The constants a and b may be determined by selecting the coordinate of two different points on the straight-line portion of the curve and substituting in Equation 5-28. The resulting two equations can be solved simultaneously for the constants a and b. To find the coefficients of Equation 5-28 for the straight-line portion of Figure 5-6, select the following two points:

Point 1: at $S_g = 0.2$, the relative permeability ratio $k_{rg}/k_{ro} = 0.07$
Point 2: at $S_g = 0.4$, the relative permeability ratio $k_{rg}/k_{ro} = 0.70$

Imposing the above points on Equation 5-28, gives:

$$0.07 = a\,e^{0.2b}$$
$$0.70 = a\,e^{0.4b}$$

Solving simultaneously gives:

- The intercept a = 0.0070
- The slope b = 11.513

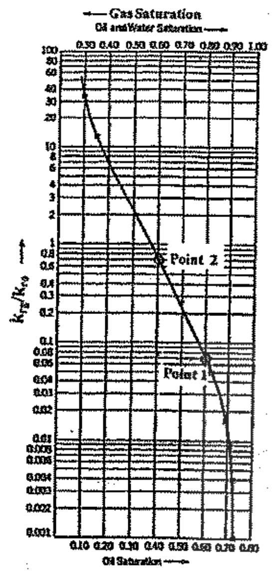


Figure 5-6. k_{rg}/k_{ro} as a function of saturation.

or

$$\frac{k_{rg}}{k_{ro}} = 0.0070\, e^{11.513 S_g}$$

In a similar manner, Figure 5-7 shows a semilog plot of k_{ro}/k_{rw} versus water saturation.

The middle straight-line portion of the curve is expressed by a relationship similar to that of Equation 5-28

$$\frac{k_{ro}}{k_{rw}} = a\, e^{bS_w} \qquad (5\text{-}29)$$

where the slope b has a negative value.

DYNAMIC PSEUDO-RELATIVE PERM... BILITIES

For a multilayered reservoir with each layer as described by a set of relative permeability curves, it is possible to treat the reservoir by a single layer

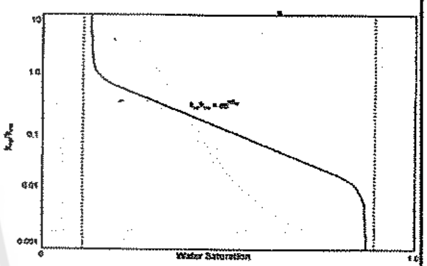


Figure 5-7. Semilog plot of relative permeability ratio vs. saturation.

connate water saturation is 20% for this particular rock, then the recovery from the portion of the reservoir invaded by high-pressure water influx is

$$\text{Recovery} = \frac{\text{initial} - \text{final}}{\text{initial}} = \frac{0.80 - 0.15}{0.80} = 81\%$$

Experiments show that essentially the same relative permeability curves are obtained for a gas-water system as for the oil-water system, which also means that the critical, or residual, gas saturation will be the same. Furthermore, it has been found that if both oil and free gas are present, the residual *hydrocarbon* saturation (oil and gas) will be about the same, in this case 15%. Suppose, then, that the rock is invaded by water at a pressure below saturation pressure so that free gas is present. If, for example, the residual free gas saturation behind the flood front is 10%, then the oil saturation is 5%, and neglecting small changes in the formation volume factors of the oil, the recovery is increased to:

$$\text{Recovery} = \frac{0.80 - 0.05}{0.80} = 94\%$$

Returning to Fig. 9.1, as the water saturation decreases further, the relative permeability to water continues to decrease and the relative permeability to oil increases. At 20% water saturation, the (connate) water is immobile, and the relative permeability to oil is quite high. This explains why some rocks may contain as much as 50% connate water and yet produce water-free oil. Most reservoir rocks are preferentially water wet—that is, the water phase and not the oil phase is next to the walls of the pore spaces. Because of this, at 20% water saturation the *water* occupies the *least favorable* portions of the pore spaces—that is, as thin layers about the sand grains, as thin layers on the walls of the pore cavities, and in the smaller crevices and capillaries. The *oil*, which occupies 80% of the pore space, is in the *most favorable* portions of the pore spaces, which is indicated by a relative permeability of 93%. The curves further indicate that about 10% of the pore spaces contribute nothing to the permeability, for at 10% water saturation, the relative permeability to oil is essentially 100%. Conversely, on the other end of the curves, 15% of the pore spaces contribute 40% of the permeability, for an increase in oil saturation from zero to 15% reduces the relative permeability to water from 100% to 60%.

In describing two-phase flow mathematically, it is always the relative permeability ratio that enters the equations. Figure 9.2 is a plot of the relative permeability ratio, k_o/k_w, versus water saturation for the same data of Fig. 9.1. Because of the wide range of k_o/k_w values, the relative permeability ratio is usually plotted on the log scale of semilog paper. Like many relative permeability ratio curves, the central or main portion of the curve is quite linear. As a straight line on semilog paper, the relative permeability ratio may be expressed as a function of the water saturation by:

$$\frac{k_o}{k_w} = a e^{-bS_w} \qquad (9.3)$$

The constants a and b may be determined from the graph shown in Fig. 9.2, or they may be determined from simultaneous equations. At $S_w = 0.30$, $k_o/k_w = 25$, and at $S_w = 0.70$, $k_o/k_w = 0.14$. Then

$$25 = ae^{-0.30b} \quad \text{and} \quad 0.14 = ae^{-0.70b}$$

Solving simultaneously, the intercept $a = 1220$, and the slope $b = 13.0$. Equation (9.3) indicates that the relative permeability ratio for a rock is a function of only the relative saturations of the fluids present. Although it is true that the viscosities, the interfacial tensions, and other factors have some effect on

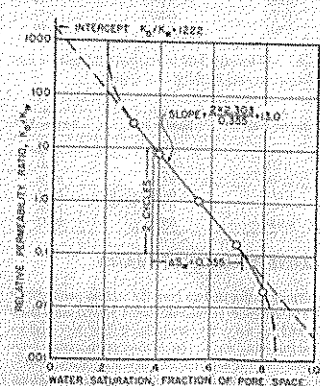


Fig. 9.2. Semilog plot of relative permeability ratio versus saturation.

that is characterized by a weighted-average porosity, absolute permeability, and a set of dynamic pseudo-relative permeability curves. These averaging properties are calculated by applying the following set of relationships:

Average Porosity

$$\phi_{avg} = \frac{\sum_{i=1}^{N} \phi_i h_i}{\sum h_i} \quad (5\text{-}30)$$

Average Absolute Permeability

$$k_{avg} = \frac{\sum_{i=1}^{N} k_i h_i}{\sum h_i} \quad (5\text{-}31)$$

Average Relative Permeability for the Wetting Phase

$$\bar{k}_{rw} = \frac{\sum_{i=1}^{N} (k\,h)_i (k_{rw})_i}{\sum_{i=1}^{N} (k\,h)_i} \quad (5\text{-}32)$$

Average Relative Permeability for the Nonwetting Phase

$$\bar{k}_{rnw} = \frac{\sum_{i=1}^{N} (k\,h)_i (k_{rnw})_i}{\sum_{i=1}^{N} (k\,h)_i} \quad (5\text{-}33)$$

The corresponding average saturations should be determined by using Equations 4-16 through 4-18. These equations are given below for convenience:

Average Oil Saturation

$$\bar{S}_o = \frac{\sum_{i=1}^{n} \phi_i h_i S_{oi}}{\sum_{i=1}^{n} \phi_i h_i}$$

Average Water Saturation

$$\bar{S}_w = \frac{\sum_{i=1}^{n} \phi_i h_i S_{wi}}{\sum_{i=1}^{n} \phi_i h_i}$$

Average Gas Saturation

$$\bar{S}_g = \frac{\sum_{i=1}^{n} \phi_i h_i S_{gi}}{\sum_{i=1}^{n} \phi_i h_i}$$

where n = total number of layers
h_i = thickness of layer i
k_i = absolute permeability of layer i
$\bar{k}_{rw}$ = average relative permeability of the wetting phase
$\bar{k}_{rnw}$ = average relative permeability of the nonwetting phase

In Equations 5-22 and 5-23, the subscripts *w* and n_w represent wetting and *nonwetting*, respectively. The resulting dynamic pseudo-relative permeability curves are then used in a single-layer model. The objective of the single-layer model is to produce results similar to those from the multilayered, cross-sectional model.

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that is characterized by a weighted-average porosity, absolute permeability, and a set of dynamic pseudo-relative permeability curves. These averaging properties are calculated by applying the following set of relationships:

Average Porosity

$$\phi_{avg} = \frac{\sum_{i=1}^{N} \phi_i h_i}{\sum h_i} \quad (5\text{-}30)$$

Average Absolute Permeability

$$k_{avg} = \frac{\sum_{i=1}^{N} k_i h_i}{\sum h_i} \quad (5\text{-}31)$$

Average Relative Permeability for the Wetting Phase

$$\bar{k}_{rw} = \frac{\sum_{i=1}^{N} (k\,h)_i (k_{rw})_i}{\sum_{i=1}^{N} (k\,h)_i} \quad (5\text{-}32)$$

Average Relative Permeability for the Nonwetting Phase

$$\bar{k}_{rnw} = \frac{\sum_{i=1}^{N} (k\,h)_i (k_{rnw})_i}{\sum_{i=1}^{N} (k\,h)_i} \quad (5\text{-}33)$$

The corresponding average saturations should be determined by using Equations 4-16 through 4-18. These equations are given below for convenience:

Average Oil Saturation

$$\bar{S}_o = \frac{\sum_{i=1}^{n} \phi_i h_i S_{oi}}{\sum_{i=1}^{n} \phi_i h_i}$$

Average Water Saturation

$$\bar{S}_w = \frac{\sum_{i=1}^{n} \phi_i h_i S_{wi}}{\sum_{i=1}^{n} \phi_i h_i}$$

Average Gas Saturation

$$\bar{S}_g = \frac{\sum_{i=1}^{n} \phi_i h_i S_{gi}}{\sum_{i=1}^{n} \phi_i h_i}$$

where n = total number of layers
h_i = thickness of layer i
k_i = absolute permeability of layer i
$\bar{k}_{rw}$ = average relative permeability of the wetting phase
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In Equations 5-22 and 5-23, the subscripts *w* and n_w represent and *nonwetting*, respectively. The resulting dynamic pseudo-rela meability curves are then used in a single-layer model. The obj the single-layer model is to produce results similar to those from tilayered, cross-sectional model.

NORMALIZATION AND AVERAGING RELATIVE PERMEABILITY DATA

Results of relative permeability tests performed on several core samples of a reservoir rock often vary. Therefore, it is necessary to average the relative permeability data obtained on individual rock samples. Prior to usage for oil recovery prediction, the relative permeability curves should first be normalized to remove the effect of different initial water and critical oil saturations. The relative permeability can then be de-normalized and assigned to different regions of the reservoir based on the existing critical fluid saturation for each reservoir region.

The most generally used method adjusts all data to reflect assigned end values, determines an average adjusted curve and finally constructs an average curve to reflect reservoir conditions. These procedures are commonly described as normalizing and de-normalizing the relative permeability data.

To perform the normalization procedure, it is helpful to set up the calculation steps for each core sample i in a tabulated form as shown below:

Relative Permeability Data for Core Sample i

(1)	(2)	(3)	(4)	(5)	(6)
S_w	k_{ro}	k_{rw}	$S^*_w = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{oc}}$	$k^*_{ro} = \frac{k_{ro}}{(k_{ro})_{S_{wc}}}$	$k^*_{rw} = \frac{k_{rw}}{(k_{rw})_{S_{oc}}}$

The following normalization methodology describes the necessary steps for a water-oil system as outlined in the above table.

Step 1. Select several values of S_w starting at S_{wc} (column 1), and list the corresponding values of k_{ro} and k_{rw} in columns 2 and 3.

Step 2. Calculate the normalized water saturation S^*_w for each set of relative permeability curves and list the calculated values in column 4 by using the following expression:

$$S^*_w = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{oc}} \tag{5-34}$$

where S_{oc} = critical oil saturation
S_{wc} = connate water saturation
S^*_w = normalized water saturation

Step 3. Calculate the normalized relative permeability for the oil phase at different water saturation by using the relation (column 5):

$$k^*_{ro} = \frac{k_{ro}}{(k_{ro})_{Swc}} \tag{5-35}$$

where k_{ro} = relative permeability of oil at different S_w
$(k_{ro})_{S_{wc}}$ = relative permeability of oil at connate water saturation
k^*_{ro} = normalized relative permeability of oil

Step 4. Normalize the relative permeability of the water phase by applying the following expression and document results of the calculation in column 6

$$k^*_{rw} = \frac{k_{rw}}{(k_{rw})_{S_{oc}}} \tag{5-36}$$

where $(k_{rw})_{S_{oc}}$ is the relative permeability of water at the critical oil saturation.

Step 5. Using regular Cartesian coordinate, plot the normalized k^*_{ro} and k^*_{rw} versus S^*_w for *all* core samples on the same graph.

Step 6. Determine the average normalized relative permeability values for oil and water as a function of the normalized water saturation by select arbitrary values of S^*_w and calculate the average of k^*_{ro} and k^*_{rw} by applying the following relationships:

$$(k^*_{ro})_{avg} = \frac{\sum_{i=1}^{n} (h\,k\,k^*_{ro})_i}{\sum_{i=1}^{n} (h\,k)_i} \tag{5-37}$$

$$(k^*_{rw})_{avg} = \frac{\sum_{i=1}^{n} (h\,k\,k^*_{rw})_i}{\sum_{i=1}^{n} (h\,k)_i} \tag{5-38}$$

where n = total number of core samples
h_i = thickness of sample i
k_i = absolute permeability of sample i

Step 7. The last step in this methodology involves de-normalizing the average curve to reflect actual reservoir and conditions of S_{wc} and S_{oc}. These parameters are the most critical part of the methodology and, therefore, a major effort should be spent in determining representative values. The S_{wc} and S_{oc} are usually determined by averaging the core data, log analysis, or correlations, versus graphs, such as: $(k_{ro})_{S_{wc}}$ vs. S_{wc}, $(k_{rw})_{S_{oc}}$ vs. S_{oc}, and S_{oc} vs. S_{wc} which should be constructed to determine if a significant correlation exists. Often, plots of S_{wc} and S_{or} versus log $\sqrt{k/\phi}$ may demonstrate a reliable correlation to determine end-point saturations as shown schematically in Figure 5-8. When representative end values have been estimated, it is again convenient to perform the de-normalization calculations in a tabular form as illustrated below:

(1)	(2)	(3)	(4)	(5)	(6)
S^*_w	$(k^*_{ro})_{avg}$	$(k^*_{rw})_{avg}$	$S_w = S^*_w(1 - S_{wc} - S_{oc}) + S_{wc}$	$k_{ro} = (k^*_{ro})_{avg}(\bar{k}_{ro})_{S_{wc}}$	$k_{rw} = (k^*_{rw})_{avg}(\bar{k}_{rw})_{S_{oc}}$

Where $(\bar{k}_{ro})_{S_{wc}}$ and $(\bar{k}_{rw})_{S_{oc}}$ are the average relative permeability of oil and water at connate water and critical oil, respectively, and given by:

$$(\bar{k}_{ro})_{S_{wc}} = \frac{\sum_{i=1}^{n} \left[h\,k\,(k_{ro})_{S_{wc}}\right]_i}{\sum_{i=1}^{n} (h\,k)_i} \tag{5-39}$$

$$(\bar{k}_{rw})_{S_{oc}} = \frac{\sum_{i=1}^{n} \left[h\,k\,(k_{rw})_{S_{oc}}\right]_i}{\sum_{i=1}^{n} (h\,k)_i} \tag{5-40}$$

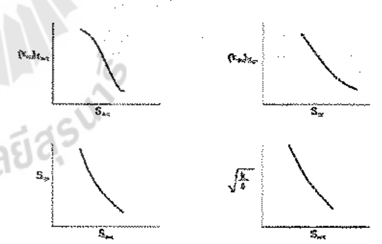


Figure 5-8. Critical saturation relationships.

Example 5-6

Relative permeability measurements are made on three core samples. The measured data are summarized below:

	Core Sample #1		Core Sample #2		Core Sample #3	
	h = 1 ft k = 100 md S_{oc} = 0.35 S_{wc} = 0.25		h = 1 ft k = 80 md S_{oc} = 0.28 S_{wc} = 0.30		h = 1 ft k = 150 md S_{oc} = 0.35 S_{wc} = 0.20	
S_w	k_{ro}	k_{rw}	k_{ro}	k_{rw}	k_{ro}	k_{rw}
0.20	—	—	—	—	1.000*	0.000
0.25	0.850*	0.000	—	—	0.872	0.008
0.30	0.754	0.018	0.800	0	0.839	0.027
0.40	0.557	0.092	0.593	0.077	0.663	0.088
0.50	0.352	0.198	0.393	0.191	0.463	0.176
0.60	0.131	0.327	0.202	0.323	0.215	0.286
0.65	0.000	0.400*	0.111	0.394	0.000	0.350*
0.72	—	—	0.000	0.500*	—	—

*Values at critical saturations

It is believed that a connate water saturation of 0.27 and a critical oil saturation of 30% better describe the formation. Generate the oil and water relative permeability data using the new critical saturations.

Solution

Step 1. Calculate the normalized water saturation for each core sample by using Equation 5-36.

S^*_w	Core Sample #1 S^*_w	Core Sample #2 S^*_w	Core Sample #3 S^*_w
0.20	—	—	0.000
0.25	0.000	—	0.111
0.30	0.125	0.000	0.222
0.40	0.375	0.238	0.444
0.50	0.625	0.476	0.667
0.60	0.875	0.714	0.889
0.65	1.000	0.833	1.000
0.72	—	1.000	—

Step 2. Determine relative permeability values at critical saturation for each core sample.

	Core 1	Core 2	Core 3
$(k_{ro})S_{wc}$	0.850	0.800	1.000
$(k_{rw})S_{or}$	0.400	0.500	0.35

Step 3. Calculate $(\bar{k}_{ro})_{S_{wc}}$ and $(\bar{k}_{rw})_{S_{or}}$ by applying Equations 5-39 and 5-40 to give:

$$(\bar{k}_{ro})_{S_{wc}} = 0.906$$

$$(\bar{k}_{rw})_{S_{wc}} = 0.402$$

Step 4. Calculate the normalized k^*_{ro} and k^*_{rw} for all core samples:

	Core 1			Core 2			Core 3		
S_w	S^*_w	k^*_{ro}	k^*_{rw}	S^*_w	k^*_{ro}	k^*_{rw}	S^*_w	k^*_{ro}	k^*_{rw}
0.20	—	—	—	—	—	—	0.000	1.000	0
0.25	0.000	1.000	0	—	—	—	0.111	0.872	0.023
0.30	0.125	0.887	0.045	0.000	1.000	0	0.222	0.839	0.077
0.40	0.375	0.655	0.230	0.238	0.741	0.154	0.444	0.663	0.251
0.50	0.625	0.414	0.495	0.476	0.491	0.382	0.667	0.463	0.503
0.60	0.875	0.154	0.818	0.714	0.252	0.646	0.889	0.215	0.817
0.65	1.000	0.000	1.000	0.833	0.139	0.788	1.000	0.000	1.000
0.72	—	—	—	1.000	0.000	1.000	—	—	—

Step 5. Plot the normalized values of k^*_{ro} and k^*_{rw} versus S^*_w for each core on a regular graph paper as shown in Figure 5-9.

Step 6. Select arbitrary values of S^*_w and calculate the average k^*_{ro} and k^*_{rw} by applying Equations 5-37 and 5-38.

S^*_w	k^*_{ro}			$(k^*_{ro})_{avg}$	k^*_{rw}			$(k^*_{rw})_{avg}$
	Core 1	Core 2	Core 3		Core 1	Core 2	Core 3	
0.1	0.91	0.88	0.93	0.912	0.035	0.075	0.020	0.038
0.2	0.81	0.78	0.85	0.821	0.100	0.148	0.066	0.096
0.3	0.72	0.67	0.78	0.735	0.170	0.230	0.134	0.168
0.4	0.63	0.51	0.70	0.633	0.255	0.315	0.215	0.251
0.5	0.54	0.46	0.61	0.552	0.360	0.405	0.310	0.348
0.6	0.44	0.37	0.52	0.459	0.415	0.515	0.420	0.442
0.7	0.33	0.27	0.42	0.356	0.585	0.650	0.550	0.585
0.8	0.23	0.17	0.32	0.256	0.700	0.745	0.680	0.702
0.9	0.12	0.07	0.18	0.135	0.840	0.870	0.825	0.833

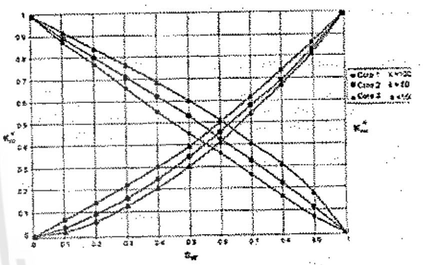


Figure 5-9. Averaging relative permeability data.

Step 7. Using the desired formation S_{oc} and S_{wc} (i.e., $S_{oc} = 0.30$, $S_{wc} = 0.27$), de-normalize the data to generate the required relative permeability data as shown below:

S^*_w	$(k^*_{ro})_{avg}$	$(k^*_{rw})_{avg}$	$S_w = S^*_w(1 - S_{wc} - S_{oc}) + S_{wc}$	$k_{ro} = 0.906\ (k^*_{ro})_{avg}$	$k_{rw} = 0.402\ (k^*_{rw})_{avg}$
0.1	0.912	0.038	0.313	0.826	0.015
0.2	0.821	0.096	0.356	0.744	0.039
0.3	0.735	0.168	0.399	0.666	0.068
0.4	0.633	0.251	0.442	0.573	0.101
0.5	0.552	0.368	0.485	0.473	0.140
0.6	0.459	0.442	0.528	0.416	0.178
0.7	0.356	0.585	0.571	0.323	0.235
0.8	0.256	0.702	0.614	0.232	0.282
0.9	0.135	0.833	0.657	0.122	0.335

It should be noted that the proposed normalization procedure for water-oil systems as outlined above could be extended to other systems, i.e., gas-oil or gas-water.

THREE-PHASE RELATIVE PERMEABILITY

The relative permeability to a fluid is defined as the ratio of effective permeability at a given saturation of that fluid to the absolute permeability at 100% saturation. Each porous system has unique relative permeability characteristics, which must be measured experimentally. Direct experimental determination of three-phase relative permeability properties is extremely difficult and involves rather complex techniques to determine the fluid saturation distribution along the length of the core. For this reason, the more easily measured two-phase relative permeability characteristics are experimentally determined.

In a three-phase system of this type it is found that the relative permeability to water depends only upon the water saturation. Since the water can flow only through the smallest interconnect pores that are present in the rock and able to accommodate its volume, it is hardly surprising that the flow of water does not depend upon the nature of the fluids occupying the other pores. Similarly, the gas relative permeability depends only upon the gas saturation. This fluid, like water, is restricted to a particular range of pore sizes and its flow is not influenced by the nature of the fluid or fluids that fill the remaining pores.

The pores available for flow of oil are those that, in size, are larger than pores passing only water, and smaller than pores passing only gas. The number of pores occupied by oil depends upon the particular size distribution of the pores in the rock in which the three phases coexist and upon the oil saturation itself.

In general, the relative permeability of each phase, i.e., water, gas, and oil, in a three-phase system is essentially related to the existing saturation by the following functions:

$$k_{rw} = f(S_w) \qquad (5\text{-}41)$$

$$k_{rg} = f(S_g) \qquad (5\text{-}42)$$

$$k_{ro} = f(S_w, S_g) \qquad (5\text{-}43)$$

Function 5-43 is rarely known and, therefore, several practical approaches are proposed and based on estimating the three-phase relative permeability from two sets of two-phase data:

Set 1: Oil-Water System

$$k_{row} = f(S_w)$$

$$k_{rw} = f(S_w)$$

Set 2: Oil-Gas System

$$k_{rog} = f(S_g)$$

$$k_{rg} = f(S_g)$$

where k_{row} and k_{rog} are defined as the relative permeability to oil in the water-oil two-phase system and similarly k_{rog} is the relative permeability of oil in the gas-oil system. The symbol k_{ro} is reserved for the oil relative permeability in the three-phase system.

The triangular graph paper is commonly used to illustrate the changes in the relative permeability values when three phases are flowing simultaneously, as illustrated in Figures 5-10 and 5-11. The relative permeability data are plotted as lines of constant percentage relative permeability (oil, water, and gas isoperms). Figures 5-10 and 5-11 show that the relative permeability data, expressed as isoperms, are dependent on the saturation values for all three phases in the rock.

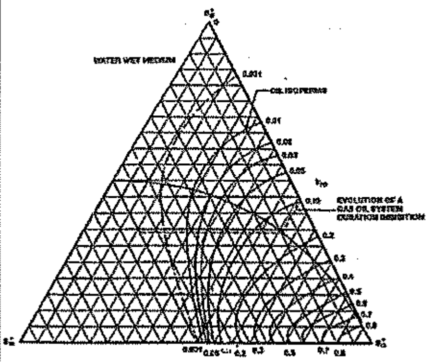


Figure 5-10. Three-phase relative permeability imbibition. (After Honarpour et al., 1988.)

Three-Phase Relative Permeability Correlations

Honarpour, Keoderitz, and Harvey (1988) provided a comprehensive treatment of the two- and three-phase relative permeabilities. The authors listed numerous correlations for estimating relative permeabilities. The simplest approach to predict the relative permeability to the oil phase in a three-phase system is defined as:

$$k_{ro} = k_{row} k_{rog} \qquad (5\text{-}44)$$

There are several practical and more accurate correlations that have developed over the years, including:

- Wyllie's Correlations
- Stone's Model I
- Stone's Model II
- The Hustad-Holt Correlation

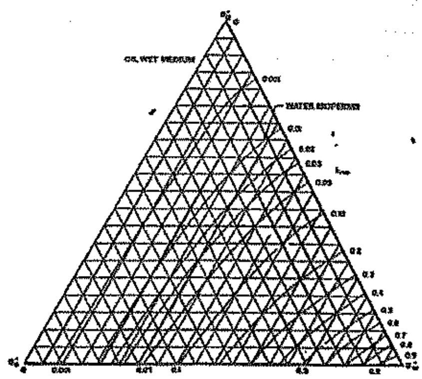


Figure 5-11. Three-phase drainage. (After Honarpour et al., 1988.)

Wyllie's Correlations

Wyllie (1961) proposed the following equations for three-phase relative permeabilities in a water-wet system:

In a cemented sandstone, Vugular rock, or oolitic limestone:

$$k_{rg} = \frac{S_g^2 \left[(1 - S_{wc})^2 - (S_w + S_o - S_{wc})^2\right]}{(1 - S_{wc})^4} \qquad (5\text{-}45)$$

$$k_{ro} = \frac{S_o^3 (2S_w + S_o - 2S_{wc})}{(1 - S_{wc})^4} \qquad (5\text{-}46)$$

$$k_{rw} = \left(\frac{S_w - S_{wc}}{1 - S_{wc}}\right)^4 \qquad (5\text{-}47)$$

In unconsolidated, well-sorted sand:

$$k_{rw} = \left(\frac{S_w - S_{wc}}{1 - S_{wi}}\right)^3 \qquad (5\text{-}48)$$

$$k_{ro} = \frac{(S_o)^3}{(1 - S_{wc})^3} \qquad (5\text{-}49)$$

$$k_{rg} = \frac{(S_g)^3 (2S_w + S_g - 2S_{wc})^4}{(1 - S_{wi})^4} \qquad (5\text{-}50)$$

Stone's Model I

Stone (1970) developed a probability model to estimate three-phase relative permeability data from the laboratory-measured two-phase data. The model combines the channel flow theory in porous media with probability concepts to obtain a simple result for determining the relative permeability to oil in the presence of water and gas flow. The model accounts for hysteresis effects when water and gas saturations are changing in the same direction of the two sets of data.

The use of the channel flow theory implies that water-relative permeability and water-oil capillary pressure in the three-phase system are functions of water saturation alone, irrespective of the relative saturations of oil and gas. Moreover, they are the same function in the three-phase system as in the two-phase water-oil system. Similarly, the gas-phase relative permeability and gas-oil capillary pressure are the same functions of gas saturation in the three-phase system as in the two-phase gas-oil system.

Stone suggested that a nonzero residual oil saturation, called *minimum oil saturation*, S_{om} exists when oil is displaced simultaneously by water and gas. It should be noted that this minimum oil saturation S_{om} is different than the critical oil saturation in the oil-water system (i.e., S_{orw}) and the residual oil saturation in the gas-oil system, i.e., S_{org}. Stone introduced the following normalized saturations:

$$S_o^* = \frac{S_o - S_{om}}{(1 - S_{wc} - S_{om})}, \text{ for } S_o \geq S_{om}$$ (

$$S_w^* = \frac{S_w - S_{wc}}{(1 - S_{wc} - S_{om})}, \text{ for } S_w \geq S_{wc}$$ (

$$S_g^* = \frac{S_g}{(1 - S_{wc} - S_{om})}$$ (

The oil-relative permeability in a three-phase system is then defin

$$k_{ro} = S_o^* \beta_w \beta_g$$ (

The two multipliers β_w and β_g are determined from:

$$\beta_w = \frac{k_{row}}{1 - S_w^*}$$ (5

$$\beta_g = \frac{k_{rog}}{1 - S_g^*}$$ (5

where S_{om} = minimum oil saturation
k_{row} = oil relative permeability as determined from the oil-w two-phase relative permeability at S_w
k_{rog} = oil relative permeability as determined from the gas- two-phase relative permeability at S_g

The difficulty in using Stone's first model is selecting the minimu saturation S_{om}. Fayers and Mathews (1984) suggested an expressi determining S_{om}:

$$S_{om} = \alpha S_{orw} + (1 - \alpha) S_{org}$$ (5

with

$$\alpha = 1 - \frac{S_g}{1 - S_{wc} - S_{org}}$$ (5

where S_{orw} = residual oil saturation in the oil-water relative permeability system
S_{org} = residual oil saturation in the gas-oil relative permeability system

Aziz and Sattari (1979) pointed out that Stone's correlation could give k_{ro} values greater than unity. The authors suggested the following normalized form of Stone's model:

$$k_{ro} = \frac{S_o^*}{(1-S_w^*)(1-S_g^*)}\left(\frac{k_{row}\,k_{rog}}{(k_{ro})_{S_{wc}}}\right) \qquad (5\text{-}59)$$

where $(k_{ro})_{S_{wc}}$ is the value of the relative permeability of the oil at the connate water saturation as determined from the oil-water relative permeability system. It should be noted that it is usually assumed that k_{rg} and krog curves are measured in the presence of connate water.

Stone's Model II

It was the difficulties in choosing S_{om} that led to the development of Stone's Model II. Stone (1973) proposed the following normalized expression:

$$k_{ro} = (k_{ro})_{S_{wc}}\left[\left(\frac{k_{row}}{(k_{ro})_{S_{wc}}} + k_{rw}\right)\left(\frac{k_{rog}}{(k_{ro})_{S_{wc}}} + k_{rg}\right) - (k_{rw} + k_{rg})\right] \qquad (5\text{-}60)$$

This model gives a reasonable approximation to the three-phase relative permeability.

The Hustad-Holt Correlation

Hustad and Holt (1992) modified Stone's Model I by introducing an exponent term n to the normalized saturations to give:

$$k_{ro} = \left[\frac{k_{row}\,k_{rog}}{(k_{ro})_{S_{wc}}}\right](\beta)^n \qquad (5\text{-}61)$$

where

$$\beta = \frac{S_o^*}{(1-S_w^*)(1-S_g^*)} \qquad (5\text{-}62)$$

$$S_o^* = \frac{S_o - S_{om}}{1 - S_{wc} - S_{om} - S_{gc}} \qquad (5\text{-}63)$$

$$S_g^* = \frac{S_g - S_{gc}}{1 - S_{wc} - S_{om} - S_{gc}} \qquad (5\text{-}64)$$

$$S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{om} - S_{gc}} \qquad (5\text{-}65)$$

The β term may be interpreted as a variable that varies between zero and one for low- and high-oil saturations, respectively. If the exponent n is one, the correlation is identical to Stone's first model. Increasing n above unity causes the oil isoperms at low oil saturations to spread from one another. n values below unity have the opposite effect.

Example 5.7

Two-phase relative permeability tests were conducted on core sample to generate the permeability data for oil-water and oil-gas systems. The following information is obtained from the test:

$S_{gc} = 0.10$ $S_{wc} = 0.15$
$S_{orw} = 0.15$ $S_{org} = 0.05$
$(k_{ro})S_{wc} = 0.88$

At the existing saturation values of $S_o = 40\%$, $S_w = 30\%$, and $S_g = 30\%$ the two-phase relative permeabilities are listed below:

$k_{row} = 0.403$
$k_{rw} = 0.030$

$k_{rg} = 0.035$
$k_{rog} = 0.175$

Estimate the three-phase relative permeability at the existing saturations by using:

a. Stone's Model I
b. Stone's Model II

Solution

a. Stone's Model I

Step 1. Calculate S_{om} by applying Equations 5-58 and 5-57, to give:

$$\alpha = 1 - \frac{0.3}{1-0.15-0.05} = 0.625$$

$$S_{om} = (0.625)(0.15) + (1-0.625)(0.05) = 0.1125$$

Step 2. Calculate the normalized saturations by applying Equations 5-51 through 5-53.

$$S_o^* = \frac{0.4-0.1125}{1-0.15-0.1125} = 0.3898$$

$$S_w^* = \frac{0.30-0.15}{1-0.15-0.1125} = 0.2034$$

$$S_g^* = \frac{0.3}{1-0.15-0.1125} = 0.4068$$

Step 3. Estimate k_{ro} by using Equation 5-59.

$$k_{ro} = \frac{0.3898}{(1-0.2034)(1-0.4068)}\left[\frac{(0.406)(0.175)}{0.88}\right] = 0.067$$

b. Stone's Model II

Apply Equation 5-60 to give:

$$k_{ro} = 0.88\left[\left(\frac{0.406}{0.88}+0.03\right)\left(\frac{0.175}{0.88}+0.035\right) - (0.03+0.035)\right] = 0.044$$

PROBLEMS

1. Given:
   - $S_{wc} = 0.30$ $S_{gc} = 0.06$ $S_{oc} = 0.35$
   - unconsolidated-well sorted sand

   Generate the drainage relative permeability data by using:

   a. The Wyllie-Gardner correlation
   b. Pirson's correlation
   c. Corey's method

2. The capillary pressure data for an oil-water system are given below:

S_w	p_c, psi
0.25	35
0.30	16
0.40	8.5
0.50	5
1.00	0

   a. Generate the relative permeability data for this system.
   b. Using the relative permeability ratio concept, plot k_{ro}/k_{rw} versus S_w on a semi-log scale and determine the coefficients of the following expression:

   $k_{ro}/k_{rw} = a\,e^{bS_w}$

3. Using the relative permeability data of Example 5-5, generate the relative permeability values for a layer in the reservoir that is characterized by the following critical saturations:

   $S_{oc} = 0.25$ $S_{wc} = 0.25$

4. Prepare a k_{rg}/k_{ro} versus S_g plot for the following laboratory data:

k_{rg}/k_{ro}	S_g
1.9	0.50

5.Calculate the three phase relative permeability ~~for the data in example 5.7~~
At the exiting value of So= 45%, Sw=30% and Sg= 25%

PROBLEMS

HW. NO 2; Due date Friday 25 October 2013

1. Given:
   - $S_{wc} = 0.30$ $S_{gc} = 0.06$ $S_{oc} = 0.35$
   - unconsolidated-well sorted sand

   Generate the drainage relative permeability data by using:

   a. The Wyllie-Gardner correlation
   b. Pirson's correlation
   c. Corey's method

2. The capillary pressure data for an oil-water system are given below:

S_w	p_c, psi
0.25	35
0.30	16
0.40	8.5
0.50	5
1.00	0

   a. Generate the relative permeability data for this system.
   b. Using the relative permeability ratio concept, plot k_{ro}/k_{rw} versus S_w on a semi-log scale and determine the coefficients of the following expression:

   $k_{ro}/k_{rw} = \alpha e^{bS_w}$

3. Using the relative permeability data of Example 5-5, generate the relative permeability values for a layer in the reservoir that is characterized by the following critical saturations:

   $S_{oc} = 0.25$ $S_{wc} = 0.25$

4. Prepare a k_{rg}/k_{ro} versus S_g plot for the following laboratory data:

k_{rg}/k_{ro}	S_g
1.9	0.50
0.109	0.30

5. Calculate the three phase relative permeability for the data in example 5.7At the exiting value of So= 45%, Sw=30% and Sg= 25% the two phase relative Permeability are list below

K_{row}=0.43, Krw=0.02, Krg= 0.03, and Krog= 0.20

PROBLEMS

HW. NO 2; Due date Friday 25 October 2013

1. Given:
   - $S_{wc} = 0.30$ $S_{gc} = 0.06$ $S_{oc} = 0.35$
   - unconsolidated-well sorted sand

   Generate the drainage relative permeability data by using:

   a. The Wyllie-Gardner correlation
   b. Pirson's correlation
   c. Corey's method

2. The capillary pressure data for an oil-water system are given below:

S_w	p_c, psi
0.25	35
0.30	16
0.40	8.5
0.50	5
1.00	0

   a. Generate the relative permeability data for this system.
   b. Using the relative permeability ratio concept, plot k_{ro}/k_{rw} versus S_w on a semi-log scale and determine the coefficients of the following expression:

   $k_{ro}/k_{rw} = \alpha e^{bS_w}$

3. Using the relative permeability data of Example 5-5, generate the relative permeability values for a layer in the reservoir that is characterized by the following critical saturations:

   $S_{oc} = 0.25$ $S_{wc} = 0.25$

4. Prepare a k_{rg}/k_{ro} versus S_g plot for the following laboratory data:

k_{rg}/k_{ro}	S_g
1.9	0.50
0.109	0.30

- 5.Calculate the three phase relative permeability
- for the data in example 5.7At the exiting value of
- So= 45%, Sw=30% and Sg= 25% the two phase relative
- Permeability are list below

K_{row}=0.43, Krw=0.02, Krg= 0.03, and Krog= 0.20

Example 5-5

Using the analytical expressions of Equations 5-22–5-27, generate the relative permeability and capillary pressure data. The following information on the water-oil and gas-oil systems is available:

$S_{wc} = 0.25$ $S_{orw} = 0.35$ $S_{gc} = 0.05$ $S_{org} = .23$
$(k_{ro})_{S_{wc}} = 0.85$ $(k_{rw})_{S_{orw}} = 0.4$ $(P_c)_{S_{wc}} = 20$ psi
$(k_{ro})_{S_{gc}} = 0.60$ $(k_{rg})_{S_{wc}} = 0.95$
$n_o = 0.9$ $n_w = 1.5$ $n_p = 0.71$
$n_{go} = 1.2$ $n_g = 0.6$ $(p_c)_{S_{gc}} = 30$ psi
$n_{pg} = 0.51$

ADVANCED RESERVOIR ENGINEERING
2/2556(4 credits(4-0-8), 2013)

Course Contents

1. Optimization of Material Balance Equations (6 hrs.)
2. Saturation and Relative Permeability Calculations (4 hrs.)
3. Steady Flow (4 hrs)
4. Unsteady State and Superposition (6 hrs.)
5. Pseudo-steady State Flow (4 hrs.)
6. Interference Test and Type Curve Analysis(3hrs)
7. Displacement Efficiency(4 hrs)
8. Potential flow and Streamlines (4 hrs.)
9. Dynamics of Water Drive Reservoir. (6 hrs.)
10. Water and Gas Coning (4hrs.)
11. Multi-Phase Flow and Introduction to Reservoir Simulation (4 hrs.)
12. Enhance Oil Recovery(2 hrs)

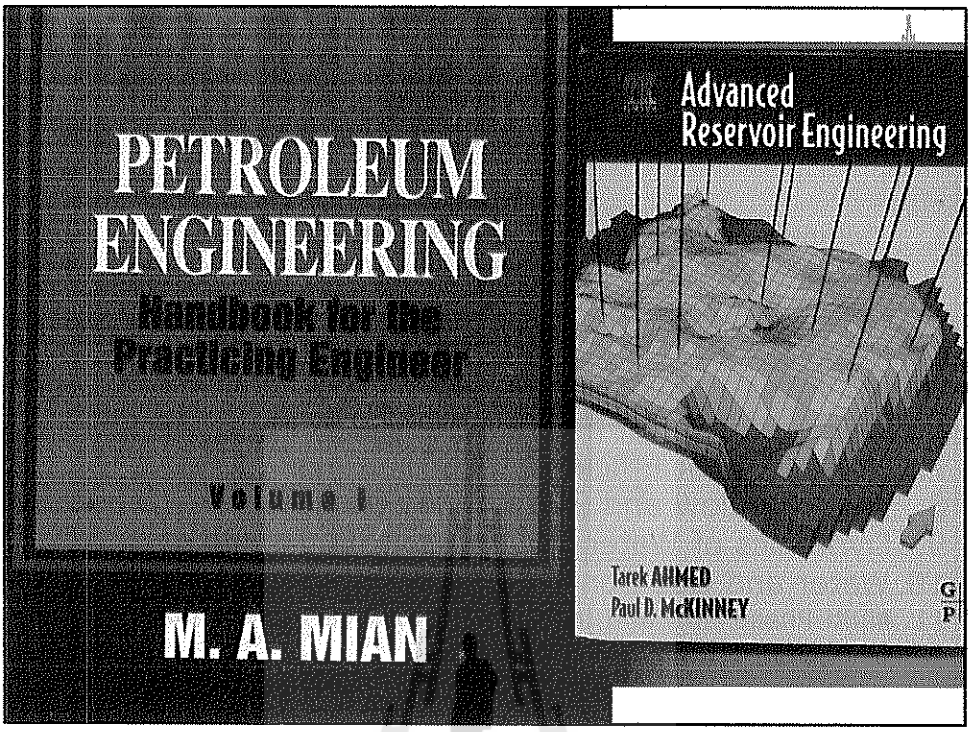
434619,534619 ADVANCED RESERVOIR ENGINEERING
2/2556(4 credits(4-0-8), September 23,- Dec.26, 2013.)

Course Contents

1. Optimization of Material Balance Equations (6 hrs.)
2. Saturation and Relative Permeability Calculations (4 hrs.)
3. Steady State Radial Flow (4 hrs.)
4. Unsteady Steady State Radial Flow (4 hrs.)
5. Pseudo-steady State Flow and Superposition (4 hrs.)
6. Well Testing Pressure Drawdown and Build Up (3 hrs.)
7. Interference Test and Type Curve Analysis(3hrs)
8. Displacement Efficiency(4 hrs)
9. Potential flow and Streamlines (4 hrs.)
10. Dynamics of Water Drive Reservoir. (6 hrs.)
11. Water and Gas Coning (4hrs.)
12. Multi-Phase Flow and Introduction to Reservoir Simulation (2 hrs.)
13. Enhance Oil Recovery(2 hrs)

GRADING

Homework	25%,	Quiz I,II	25%
Mid-term	25%,	Final Exam	25%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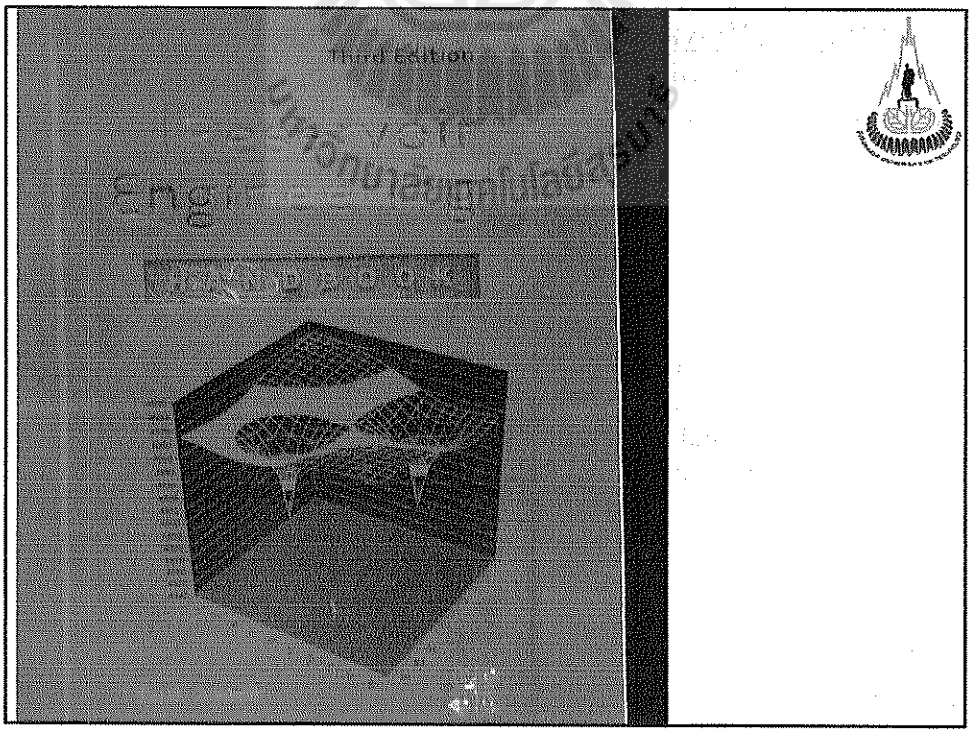
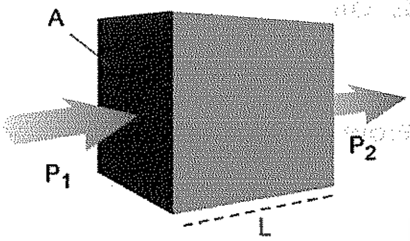

Third Edition

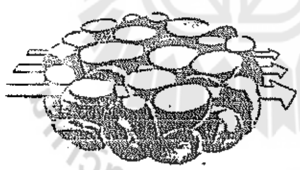
Definition of 1 Darcy



$$k = \frac{Q\mu * L}{A * (P_1 - P_2)}$$

(k: 1 Darcy; Q: cm³/sec; μ: cp; L: cm; A: cm²; P₁-P₂: atm)

5



Q = Rate of Flow, cc/sec.
ΔP = Pressure Differential, Atmospheres
A = Area, cm²
μ = Fluid Viscosity, Centipoise
L = Length, cm
K = Permeability, Darcies

$$Q = \frac{K \cdot \Delta P \cdot A}{\mu \cdot L}$$

υ = the apparent velocity

$$v = \frac{q}{A} = \frac{Q}{A} = \frac{K \cdot \Delta P}{\mu \cdot L}$$

$$q\ bbl/d \times \frac{159{,}00}{24 \times 3600\ \sec/d}$$

Or then:

$$= \frac{2\pi\left(h\ ft \times 30.48\frac{cm}{ft} . rft \times 30.48\frac{cm}{ft}\right)k.md \times \frac{0.001.d}{md}\left[psi \times \frac{1}{14.7}\frac{atm}{psi}\right]dp}{\mu\ .dr.ft \times 30.48\frac{cm}{ft}}$$

$$q_{gs} = \frac{0.001127(2.\pi.rh)k}{\mu} \times \frac{dp}{dr_s}$$

$$v = -0.001127\frac{k}{\mu}\left[\frac{dp}{ds}\right]$$

(4.23)

The Classification of Reservoir Flow System

The fluids flow in reservoir may be categorized upon the characteristics as follows.

• TYPES OF FLUID

1. **Incompressible Fluids (Water).**
2. **Slightly Compressible Fluids (Oil).**
3. **Compressible Fluids (Gas).**

FLOW REGIMES (TIME DEPENDENCE)

1. **1.Stead-State Flow**
2. **2.Unstead-State (Transient) Flow.**
3. **3.Pseudostead-State Flow**

• RESERVOIR GEOMETRY

1. **Linear Flow**
2. **Radial Flow**
3. **Spherical and Hemispherical Flow**

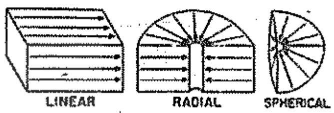


Fig. 7.1. Common flow geometries.

• NUMBER OF FLUIDS IN RESERVOIR

1. **1.Single-phase Flow (oil, water, or gas)**
2. **2.Two-phase Flow (oil-water, oil-gas, or gas-water).**
3. **3.Three-phase Flow (oil-water-gas).**

WELL TESTING ANALYSIS 1/3

1. Stead-State Flow
2. Unstead-State (Transient) Flow.
3. Pseudostead-State Flow

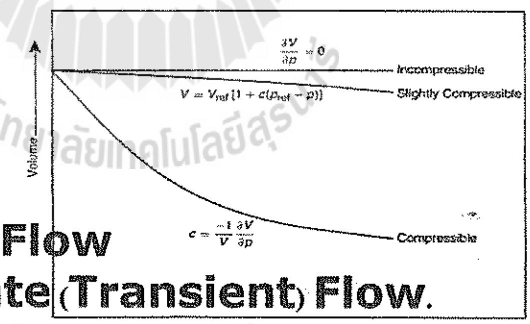


Figure 1.1 ...-volume relationship.

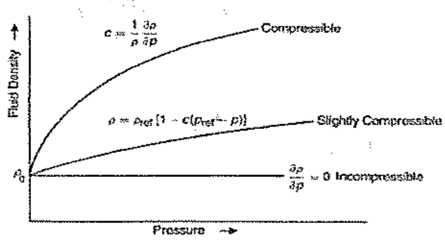


Figure 1.2 Fluid density versus pressure for different fluid types.

Steady-state flow ... a function of both position *i* and time *t*, thus:
The flow regime is identified as a steady-state flow if the pres...

Pseudosteady-State Flow

When the pressure at different locations in the reservoir is declining linearly as a function of time, i.e., at a constant declining rate, the flowing condition is characterized as the pseudosteady-state flow. Mathematically, this definition states that the rate of change of pressure with respect to time at every position is constant, or

$$\left(\frac{\partial p}{\partial t}\right)_i = \text{constant} \qquad (4\text{-}3)$$

It should be pointed out that the pseudosteady-state flow is commonly referred to as semisteady-state flow and quasisteady-state flow.

Figure shows a schematic comparison of the pressure declines as a function of time of the three flow regimes.

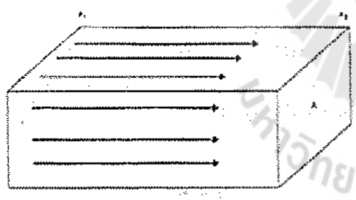


Figure.4.3 Linear flow

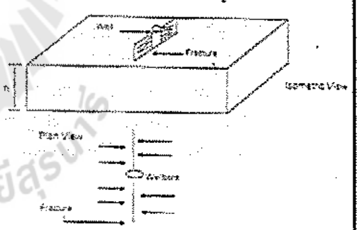


Figure.4.4 Ideal linear flow into vertical fracture

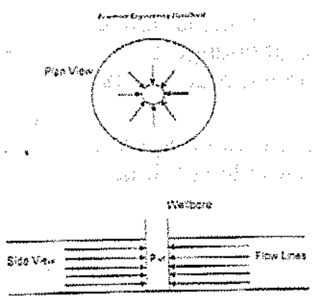


Figure.4.5 Ideal radial flow into a wellbore

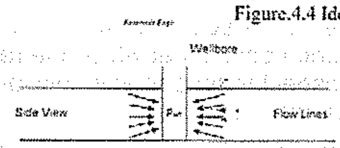


Figure.4.6 Spherical flow

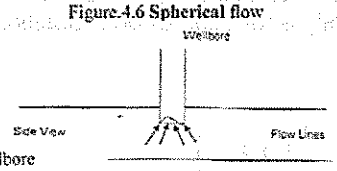


Figure.4.7Hemispherical flow

1

FLOW REGIMES

There are three flow regimes:

- Steady-state flow
- Unsteady-state flow
- Pseudosteady-state flow

Steady-State Flow

The flow regime is identified as a steady-state flow if the pressure at every location in the reservoir remains constant, i.e., does not change with time. Mathematically, this condition is expressed as:

$$\left(\frac{\partial p}{\partial t}\right)_i = 0 \qquad (4\text{-}1)$$

The above equation states that the rate of change of pressure p with respect to time t at any location i is zero. In reservoirs, the steady-state flow condition can only occur when the reservoir is completely recharged and supported by strong aquifer or pressure maintenance operations.

Unsteady-State Flow

The unsteady-state flow (frequently called *transient flow*) is defined as the fluid flowing condition at which the rate of change of pressure with respect to time at any position in the reservoir is not zero or constant.

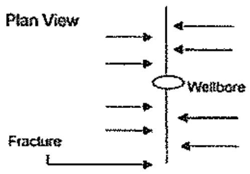
This definition suggests that the pressure derivative with respect to time is essentially a function of both position i and time t, thus

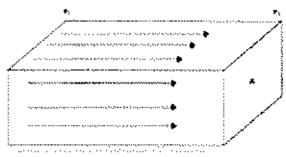
$$\left(\frac{\partial p}{\partial t}\right) = f(i, t) \qquad (4\text{-}2)$$

Linear Flow

Linear flow occurs when flow paths are parallel and the fluid flows in a single direction. In addition, the cross sectional area to flow must be constant. Figure 4-2 shows an idealized linear flow system.

Figure 4-2 Ideal linear flow into vertical fracture





Spherical and Hemispherical Flow

Depending upon the type of wellbore completion configuration, it is possible to have a spherical or hemispherical flow near the wellbore. A well with a limited perforated interval could result in spherical flow in the vicinity of the perforations as illustrated in Figure 4-3. A well that only partially penetrates the pay zone, as shown in Figure 4-4, could result in hemispherical flow. The condition could arise where coning of bottom water is important.

Figure 4-3 Spherical flow due to limited entry

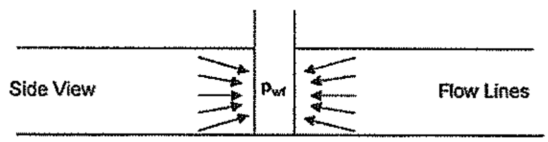
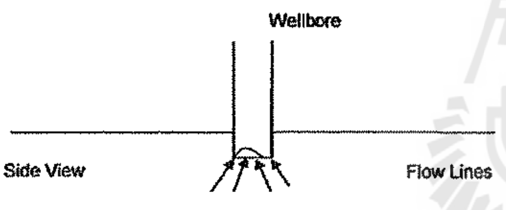


Figure 4-4 Hemispherical flow in a partially penetrating well



A slightly compressible fluid, which is the description of nearly all liquids, is sometimes defined as one whose volume change with pressure is quite small and expressible by the equation:

$$V = V_R e^{c(p_R - p)} \quad (7.2)$$

where

R = reference conditions. Slightly Compressible.

Equation (7.2) may be derived by integrating Eq. (1.1), which defines compressibility between limits, assuming an average compressibility c, as

$$\int_{p_R}^{p} -c\,dp = \int_{V_R}^{V} \frac{dV}{V} \qquad c = -\frac{1}{V}\frac{dV}{dP}$$

$$c(p_R - p) = \ln\left(\frac{V}{V_R}\right)$$

$$e^{c(p_R - p)} = \frac{V}{V_R}$$

$$V = V_R\left(e^{c(p_R - p)}\right) \qquad = \ln\left(\frac{\rho_R}{\rho}\right)$$

But e^x may be represented by a series expansion as

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \frac{x^n}{n!}$$

Where x is small, the first two terms, $1 + x$, suffice, and where the exponent $c(p_R - p)$, the equation may be written as

$$V = V_R[1 + c(p_R - p)] + c^2(p_R - p)^2 + \ldots \quad (7.3)$$

A compressible fluid is one in which the volume has a strong dependence on pressure. All gases are in this category. In Chapter 1, the real gas law was used to describe how gas volumes vary with pressure:

$$V = \frac{znRT}{p} \quad (2.7)$$

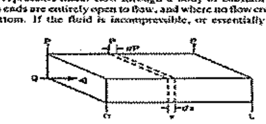
Unlike the case of the slightly compressible fluids, the gas isothermal com-

4. STEADY-STATE FLOW SYSTEMS

Now that Darcy's law has been reviewed and the classification of flow system has been discussed, the actual models that relate flow rate to reservoir pres

4.1. Linear Flow of Incompressible Fluids, Steady State

Figure 7.3 represents linear flow through a body of constant cross section where both ends are entirely open to flow, and where no flow crosses the sides top, or bottom. If the fluid is incompressible, or essentially so for all en



-ting purposes, then the velocity is the same at all points, as is the total rate across any cross section, so that,

$$v = \frac{qB}{A_c} = -0.001127\frac{k}{\mu}\frac{dp}{dx}$$

-ating variables and integrating over the length of the porous body:

$$\frac{qB}{A_c}\int_0^L dx = -0.001127\frac{k}{\mu}\int_{p_1}^{p_2} dp$$

$$q = 0.001127\frac{kA_c(p_1 - p_2)}{B\mu L} \quad (7.7)$$

example, under a pressure differential of 100 psi for a permeability of 250 a fluid viscosity of 2.5 cp, a formation volume factor of 1.127 bbl/STB, a th of 450 ft, and a cross-sectional area of 45 sq ft, the flow rate is

$$q = 0.001127\frac{(250)(45)(100)}{(1.127)(2.5)(450)} = 1.0 \text{ STB/day}$$

4.2. Linear Flow of Slightly Compressible Fluids, Steady State $qB = q_R[1 + c(p_R - p)]$

equation for flow of slightly compressible fluids is modified from what was derived in the previous section since the volume of slightly compressible written for this case, variables separated, and the resulting equation integrated over the length of the porous body, the following is obtained:

$$\frac{q_R}{A_c}\int_0^L dx = -0.001127\frac{k}{\mu}\int_{p_1}^{p_2}\frac{dp}{1 + c(p_R - p)}$$

$$q_R = \frac{0.001127kA_c}{\mu Lc}\ln\left[\frac{1 + c(p_R - p_2)}{1 + c(p_R - p_1)}\right] \quad (7.9)$$

This integration assumes a constant compressibility over the entire pressure . For example, under a pressure differential of 100 psi for a permeability of 250 md, a fluid viscosity of 2.5 cp, a length of 450 ft, a cross-sectional area of 45 sq ft, a constant compressibility of 65(10⁻⁶) psi⁻¹, and choosing p_1 as the reference pressure, the flow rate is

$$q_1 = \frac{(0.001127)(250)(45)}{(2.5)(450)(65 \times 10^{-6})}\ln\left[\frac{1 + 65 \times 10^{-6}(100)}{1}\right] = 1.125 \text{ bbl/day}$$

When compared with the flow rate calculation in the preceding section, q_1 is found to be different due to the assumption of a slightly compressible fluid in the calculation rather than an incompressible fluid. Note also, that the flow rate is not in STB units because the calculation is being done at a reference pressure that is not the standard pressure. If p_2 is chosen to be the reference pressure, then the result of the calculation will be q_2, and the value of the calculated flow rate will be different still because of the volume dependence on the reference pressure:

$$q_2 = \frac{(0.001127)(250)(45)}{(2.5)(450)(65 \times 10^{-6})}\ln\left[\frac{1}{1 + 65 \times 10^{-6}(-100)}\right] = 1.131 \text{ bbl/day}$$

The calculations show that q_1 and q_2 are not largely different, which confirms what was discussed earlier: the fact that volume is not a strong function of pressure for slightly compressible fluids.

4.3 Linear Flow of Compressible Fluid (Steady State)

Fig. 7.3.

Substituting in Darcy's law: $v = \frac{q\beta}{A_c} = -0.001127 \frac{k}{\mu}\frac{dp}{dx}$

$$\frac{qp_{sc}Tz}{5.615T_{sc}pA_c} = -0.001127\frac{k}{\mu}\frac{dp}{dx}$$

Separating variables and integrating:

$$\frac{qp_{sc}Tz\mu}{(5.615)(0.001127)kT_{sc}A_c}\int_0^L dx = -\int_{p_1}^{p_2} p\,dp = \frac{1}{2}(p_1^2 - p_2^2)$$

Finally

$$q = \frac{0.003164T_{sc}A_c k(p_1^2 - p_2^2)}{p_{sc}TzL\mu} \qquad (7.10)$$

For example, where $T_{sc} = 60°F$, $A_c = 45$ sq ft, $k = 125$ md, $p_1 = 1000$ psia, $p_2 = 500$ psia, $p_{sc} = 14.7$ psia, $T = 140°F$, $z = 0.92$, $L = 450$ ft, and $\mu = 0.015$ cp

$$q = \frac{0.003164(520)(45)(125)(1000^2 - 500^2)}{14.7(600)(0.92)(450)(0.015)} = 126.7 \text{ M SCF/day}$$

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[illegible] psia, depending on the properties of the flowing gas. Above this pressure range, it would be more accurate to assume that the product $\mu z/p$ is constant. For the case of $\mu z/p$ constant, the following is obtained:

$$\frac{qp_{sc}T(z\mu/p)}{(5.615)(0.001127)kT_{sc}A_c}\int_0^L dx = -\int_{p_1}^{p_2} dp = p_1 - p_2$$

$$q = \frac{0.006328kT_{sc}A_c(p_1 - p_2)}{p_{sc}T(z\mu/p)L} \qquad (7.11)$$

In [illegible]

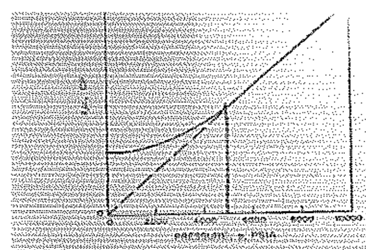


Fig. 7.4. [illegible]

4.4. Permeability Variations in Linear Systems

Consider two or more beds of equal cross section but of unequal lengths and permeabilities (Fig. 7.5) in which the same linear flow rate q exists, assuming an incompressible fluid. Obviously the pressure drops are additive, and

$$(p_1 - p_4) = (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4)$$

Substituting the equivalents of these pressure drops from Equation (7.7),

$$\frac{q_tB\mu L_t}{0.001127k_{avg}A_c} = \frac{q_1B\mu L_1}{0.001127k_1A_{c1}} + \frac{q_2B\mu L_2}{0.001127k_2A_{c2}} + \frac{q_3B\mu L_3}{0.001127k_3A_{c3}}$$

$$\frac{L_t}{K_{avg}} = \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}$$

$$K_{avg} = \frac{\Sigma L_i}{\Sigma L_i/K_i}$$

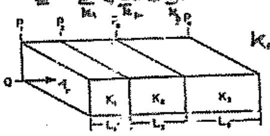


Fig. 7.5. Series flow in linear beds.

4.4. Permeability Variations in Linear Systems

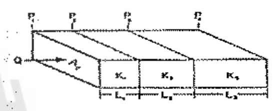


Fig. 7.5. Series flow in linear beds.

$$(p_1^2 - p_4^2) = (p_1^2 - p_2^2) + (p_2^2 - p_3^2) + (p_3^2 - p_4^2)$$

$$\frac{qP_{sc}T_{sc}L_t\mu}{0.003164T_{sc}A_cK} = \frac{qP_{sc}T_{sc}\mu}{O}\frac{L_1}{K_1} + \frac{qP_{sc}T_{sc}\mu}{O}\frac{L_2}{K_2} + \frac{qP_{sc}T_{sc}\mu}{O}\frac{L_3}{K_3}$$

$$\frac{L_t}{k_{avg}} = \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}$$

$$k_{avg} = \frac{L_t}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}} = \frac{\Sigma L_i}{\Sigma L_i/k_i} \qquad (7.12)$$

The average permeability of 10 md, 50 md, and 1000 md beds, which [illegible] 18 ft, and 40 ft in length, respectively, but of equal cross section, w[illegible] [illegible]ced in series is

$$k_{avg} = \frac{\Sigma L_i}{\Sigma L_i/k_i} = \frac{6 + 18 + 40}{6/10 + 18/50 + 40/1000} = 64 \text{ md}$$

$$\frac{L_t}{k_{avg}} = \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}$$

$$k_{avg} = \frac{L_t}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}} = \frac{\Sigma L_i}{\Sigma L_i/k_i} \qquad (7.12)$$

[illegible] average permeability as defined by Eq. (7.12) is that permeability [illegible] [illegible] a number of beds of various geometries and permeabilities could be [illegible]aged, and yet the same total flow rate under the same applied pressure [illegible] could be obtained.

[illegible]uation (7.12) was derived using the incompressible fluid equation. [illegible] the permeability is a property of the rock and not of the fluids flowing [illegible]ugh it, except for gases at low pressure, the average permeability must be [illegible]ally applicable to gases. This requirement may be demonstrated by observ[illegible] that for pressures below 1500 to 2000 psia: use Eq. 7

$$(p_1^2 - p_4^2) = (p_1^2 - p_2^2) + (p_2^2 - p_3^2) + (p_3^2 - p_4^2)$$

[illegible]stituting the equivalents from Eq. (7.10), the same Eq. (7.12) is obtained.

The average permeability of 10 md, 50 md, and 1000 md beds, which are [illegible], 18 ft, and 40 ft in length, respectively, but of equal cross section, when [illegible]ced in series is

$$k_{avg} = \frac{\Sigma L_i}{\Sigma L_i/k_i} = \frac{6 + 18 + 40}{6/10 + 18/50 + 40/1000} = 64 \text{ md}$$

Consider two or more beds of equal length but unequal cross sections [illegible]d permeabilities flowing the same fluid in linear flow under the same pres[illegible]re drop (p_1 to p_2) as shown in Fig. 7.6.

[illegible]viously the total flow is the sum of the individual flows, or

$$q_t = q_1 + q_2 + q_3$$

[illegible]nd

$$\frac{k_{avg}A_{ct}(p_1 - p_2)}{B\mu L} = \frac{k_1A_{c1}(p_1 - p_2)}{B\mu L} + \frac{k_2A_{c2}(p_1 - p_2)}{B\mu L} + \frac{k_3A_{c3}(p_1 - p_2)}{B\mu L}$$

$q_t =$

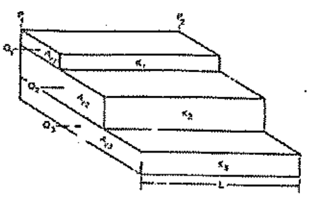


Fig. 7.6. Parallel flow in linear beds.

$q_t = q_1 + q_2 + q_3$

Cancelling

$$k_{avg}A_{ct} = k_1A_{c1} + k_2A_{c2} + k_3A_{c3}$$

$$k_{avg} = \frac{\Sigma k_jA_{cj}}{\Sigma A_{cj}} = \frac{\Sigma k_i h_i \times \phi}{\Sigma h_i \times \phi} \qquad (7.13)$$

And where all beds are of the same width, so that their areas are proportional to their thicknesses,

$$k_{avg} = \frac{\Sigma k_jh_j}{\Sigma h_j} \qquad (7.14)$$

4.4. Permeability Variations in Linear Systems

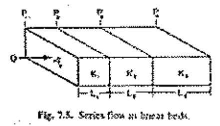


Fig. 7.5. Series flow in linear beds.

$$(p_1^2 - p_4^2) = (p_1^2 - p_2^2) + (p_2^2 - p_3^2) + (p_3^2 - p_4^2)$$

$$\frac{q P_{sc} T \bar{z} L \mu}{0.003164\, T_{sc} A\, k} = \frac{q P_{sc} T \bar{z}\mu}{\bigcirc}\frac{L_1}{k_1} + \frac{q P_{sc} T \bar{z}\mu}{\bigcirc}\frac{L_2}{k_2} + \frac{q P_{sc} T \bar{z}\mu}{\bigcirc}\frac{L_3}{k_3}$$

$$\frac{L_t}{k_{avg}} = \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}$$

$$k_{avg} = \frac{L_t}{\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}} = \frac{\Sigma L_i}{\Sigma L_i/k_i} \qquad (7.12)$$

The average permeability of 10 md, 50 md, and 1000 md beds, which are 6 ft, 18 ft, and 40 ft in length, respectively, but of equal cross section, when placed in series is

$$k_{avg} = \frac{\Sigma L_i}{\Sigma L_i/k_i} = \frac{6 + 18 + 40}{6/10 + 18/50 + 40/1000} = 64 \text{ md}$$

4.5. Flow Through Capillaries and Fractures

Although the pore spaces within rocks seldom resemble straight, smooth-walled capillary tubes of constant diameter, it is often convenient and instructive to treat these pore spaces as if they were composed of bundles of parallel capillary tubes of various diameters. Consider a capillary tube of length L and inside radius r_c which is flowing an incompressible fluid of μ viscosity in ... or viscous flow under a pressure difference of $(p_1 - p_2)$. From fluid dynamics, Poiseuille's law, which describes the total flow rate through the capillary, can be written as:

$$q = 1.30(10)^{8}\frac{\pi r_c^4 (p_1 - p_2)}{B\mu L} \quad \text{Capillary} \qquad (7.15)$$

Darcy's law for the linear flow of incompressible fluids in permeable beds, Eq. (7.7), and Poiseuille's law for incompressible fluid capillary flow, Eq. (7.15), are quite similar:

$$q = 0.001127\frac{kA_c(p_1 - p_2)}{B\mu L} \quad \text{Darcy} \qquad (7.7)$$

Writing $A_c = \pi r_c^2$ for area in Eq. (7.7), and equating it to Eq. (7.15):

$$k = 1.15(10)^{11} r_c^2 \qquad (7.16)$$

Thus the permeability of a rock composed of closely packed capillaries, each having a radius of $4.17(10)^{-5}$ foot (0.0005 in.), is about 200 md. And if only 25% of the rock consists of pore channels (i.e., it has 25% porosity), the permeability is about one-quarter as large, or about 50 md.

An equation for the viscous flow of incompressible wetting fluids through smooth fractures of constant width may be obtained as:

$$k = 7.7(10)^{12} W^2$$

$$q = 8.7(10)^{9}\frac{W^2 A_c(p_1 - p_2)}{B\mu L} = 0.001127\frac{kA_c(p_1 - p_2)}{B\mu L} \qquad (7.17)$$

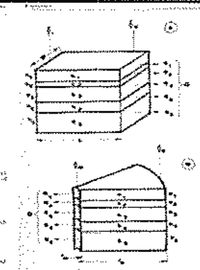
$$k = 7.7(10)^{12} W^2 \qquad (7.18)$$

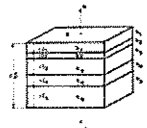
The permeability of a fracture only $8.33(10)^{-5}$ ft wide (0.001 in.) is 53,500 md. Fractures and solution channels account for economic production rates in many dolomite, limestone, and sandstone rocks, which could not be produced economically if such openings did not exist. Consider, for example, rock of very low primary or matrix permeability, say 0.01 md, but which contains on the average a fracture $4.17(10)^{-4}$ ft wide and 1 ft in lateral extent per square foot of rock. Assuming the fracture is in the direction in which flow is desired, the law of parallel flow, Eq. (7.13) will apply, and

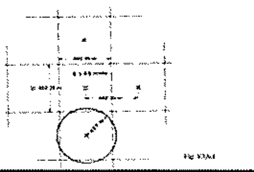
$$k_{avg} = \frac{0.01[1 - (1)(4.17(10)^{-4})] + (7.7(10)^{12}(4.17(10)^{-4})^2[1(4.17(10)^{-4})]}{1}$$

$$k_{avg} = 558 \text{ md}$$

$$\frac{\Sigma k_i h_i}{\Sigma h_i}$$







4.6. Radial Flow of Incompressible Fluid, Steady State

Consider radial flow toward a vertical wellbore of radius r_w in a horizontal stratum of uniform thickness and permeability, as shown in Fig. 7.7. If the fluid is incompressible, the flow across any circumference is a const... be the pressure maintained in the wellbore when the well is flowing and a pressure p_e is maintained at the external radius r_e. Let the ... any radius r be p. Then at this radius r

$$v = \frac{qB}{A_c} = \frac{qB}{2\pi rh} = -0.001127\frac{k\,dp}{\mu\,dr}$$

where positive q is in the positive r direction. Separ... grating between any two radii, r_1 and r_2, where the ... respectively

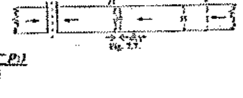


Fig. 7.7.

$$\int_{r_1}^{r_2}\frac{qB\,dr}{2\pi rh} = -0.001127\int_{p_1}^{p_2}\frac{k}{\mu}\,dp$$

$$q = -\frac{0.00708\,kh\,(p_2 - p_1)}{\mu B \ln(r_2/r_1)}$$

The minus sign is usually dispensed with, for where p_2 is greater than p_1, the flow is known to be negative—that is, in the negative r direction, or toward the wellbore:

$$q = \frac{0.00708\,kh\,(p_2 - p_1)}{\mu B \ln(r_2/r_1)}$$

4.7. Radial Flow of Slightly Compressible Fluids, Steady State

Equation (7.3) is again used to express the volume dependence on pressure for slightly compressible fluids. If this equation is substituted into the radial form of Darcy's law, the following is obtained:

$$qB = \frac{q_R[1 + c(p_R - p)]}{2\pi rh} = -0.001127\frac{k}{\mu}\frac{dp}{dr}$$

Separating the variables, assuming a constant compressibility over the entire pressure drop, and integrating over the length of the porous medium,

$$q_R = \frac{0.00708\ kh}{\mu c\ \ln(r_2/r_1)}\ln\left[\frac{1 + c(p_R - p_2)}{1 + c(p_R - p_1)}\right] \qquad (7.20)$$

4.8. Radial Flow of Compressible Fluids, Steady State

Substituting in the radial form of Darcy's law

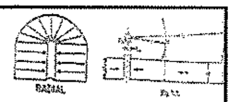
Separating variables and integrating

or

Finally

$$q = \frac{0.01988 \times (60+460)}{14.7} \cdot \frac{kh\,(P_e^2 - P_w^2)}{T\bar{z}\bar{\mu}\ln\left(\frac{r_e}{r_w}\right)}$$

$$q = \frac{0.703\,kh\,(p_e^2 - p_w^2)}{\bar{\mu}\bar{z}\,T_f \ln r_e/r_w} \qquad Q_g = \frac{kh\,(p_e^2 - p_{wf}^2)}{1422\,T\,(\mu_g z)_{avg}\ln\left(\frac{r_e}{r_w}\right)}$$



$$q = \frac{0.7032\,kh\,(p_e^2 - p_w^2)}{\bar{\mu}\bar{z}\,T_f \ln r_e/r_w} \qquad Q_g = \frac{kh\,(p_e^2 - p_{wf}^2)}{1422\,T\,(\mu_g z)_{avg}\ln\left(\frac{r_e}{r_w}\right)}$$

where Q_g = gas flow rate, Mscf/day; k = permeability, md

k = 10 md

11,633 M

$$q = \frac{0.7032 \times 10 \times 20 \times (3000^2 - 1000^2)}{0.02 \times 0.9 \times 580 \times \ln\frac{2850}{0.25}}$$

$= 11,633,000$ SCF/day

Radial Flow of Gas in Pseudopressure Form

For more accuracy, the radial gas flow equation may be expressed in pseudopressure form. Gas pseudopressure, $\psi(p)$, is defined by the integral

$$\psi(p) = 2\int_{p_b}^{p} \frac{p}{\mu z}dp$$

Where p_b is some arbitrary low base pressure.

Rearrange Eq.(4.25)

$$\left(\frac{p_{sc}}{5.615T_{sc}}\right)\left(\frac{zT}{p}\right)Q_g = -0.001127(2\pi rh)\frac{k}{\mu}\frac{dp}{dr}$$

Assuming that $T_{sc} = 520°R$ and $p_{sc} = 14.7$ psia

$$\left(\frac{TQ_g}{kh}\right)\frac{dr}{r} = 0.703\left(\frac{2p}{\mu_g z}\right)dp \qquad (4.28)$$

Integrating Eq.(4.28) from the wellbore conditions (r_w and p_{wf}) to any point in the reservoir (r and p)

$$\int_{r_w}^{r}\left(\frac{TQ_g}{kh}\right)\frac{dr}{r} = 0.703\int_{p_{wf}}^{p}\left(\frac{2p}{\mu_g z}\right)dp \qquad (4.29)$$

The term $\int_{p_{wf}}^{p}\left(\frac{2p}{\mu_g z}\right)dp$ can be expanded to give:

$$\int_{p_{wf}}^{p}\left(\frac{2p}{\mu_g z}\right)dp = \int_{0}^{p}\left(\frac{2p}{\mu_g z}\right)dp - \int_{0}^{p_{wf}}\left(\frac{2p}{\mu_g z}\right)dp$$

Combine the above relationships Eq.(4.29) yields:

$$\left(\frac{TQ_g}{kh}\right)\ln\left(\frac{r}{r_w}\right) = 0.703\left[\int_0^{p}\left(\frac{2p}{\mu_g z}\right)dp - \int_0^{p_{wf}}\left(\frac{2p}{\mu_g z}\right)dp\right] \qquad (4.30)$$

The integral $\int_0^{p}\left(\frac{2p}{\mu_g z}\right)dp$ is called the real gas potential or real gas pseudopressure and it is usually represented by m(p) or $\psi = \int_0^{p}\left(\frac{2p}{\mu_g z}\right)dp$

Equation (4.30) can be written in terms of the real gas potential to give:

$$\left(\frac{TQ_g}{kh}\right)\ln\left(\frac{r}{r_w}\right) = 0.703\,(\psi - \psi_w)$$

or

$$\psi = \psi_w + \left(\frac{TQ_g}{0.703kh}\right)\ln\left(\frac{r}{r_w}\right)$$

or

$$Q_g = \frac{0.703kh(\Psi - \Psi_w)}{T\ln\frac{r}{r_w}} \qquad (4.31)$$

The gas flow rate is commonly expressed in MSCF/D and $r = r_e$ then

$$Q_g = \frac{kh(\Psi - \Psi_w)}{1422T\ln\frac{r_e}{r_w}} \qquad (4.32)$$

Example 4.1 The following PVT data from a gas well in the Anaconda Gas Field is given below:

p (psi)	μ_g (cp)	z
0	0.01270	1.000
400	0.01286	0.937
800	0.01390	0.882
1200	0.01530	0.832
1600	0.01680	0.794
2000	0.01840	0.770
2400	0.02010	0.763
2800	0.02170	0.775
3200	0.02340	0.797
3600	0.02500	0.827
4000	0.02660	0.860
4400	0.02831	0.896

The well is producing at a stabilized bottom-hole flowing pressure of 3600 psi. The wellbore radius is 0.3 ft. The following additional data is available:

k = md   h = 15 ft   T = 600°R

p_e = 4400 psi r_e = 1000 ft

Calculate the gas flow rate in Mscf/day.

Solution.

Step1. Calculate the term $\left(\frac{2p}{\mu_g z}\right)$ for each pressure as shown below:

p (psi)	μ_g (cp)	z	$\left(\frac{2p}{\mu_g z}\right)\left(\frac{psia}{cp}\right)$
0	0.01270	1.000	00
400	0.01286	0.937	66391
800	0.01390	0.882	130508
1200	0.01530	0.832	188537
1600	0.01680	0.794	239894
2000	0.01840	0.770	282326
2400	0.02010	0.763	312983
2800	0.02170	0.775	332986
3200	0.02340	0.797	343167
3600	0.02500	0.827	348247
4000	0.02660	0.860	349711
4400	0.02831	0.896	346924

The well is producing at a stabilized bottom-hole flowing pressure of 3600 psi. The wellbore radius is 0.3 ft. The following additional data is available:

k = md   h = 15 ft   T = 600°R

p_e = 4400 psi r_e = 1000 ft

Calculate the gas flow rate in Mscf/day

5.1

Solution

Step 1. Calculate the term for each pressure as show below.

$$m(p) = \left(\frac{2p}{\mu_g z}\right) * dp$$

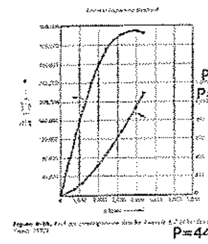
p (psi)	μ_g (cp)	z	$\frac{2p}{\mu_g z}\left(\frac{psia}{cp}\right)$	
0	0.01270	1.000	0	
400	0.01286	0.937	66391	13278200
800	0.01390	0.882	130508	52658000
1200	0.01530	0.832	188537	116467000
1600	0.01680	0.794	239894	202153200
2000	0.01840	0.770	282326	306597200
2400	0.02010	0.763	312983	425659000
2800	0.02170	0.775	332986	554852800
3200	0.02340	0.797	343167	690083400
3600	0.02500	0.827	348247	828366200
4000	0.02660	0.860	349711	967057800
4400	0.02831	0.896	346924	1107284800

$$m(p) = ((0 + 66391) * 400/2) =$$
$$m(p) = 13278200 + ((66391 + 130508) * 400/2) =$$
$$m(p) = 52658000 + ((130508 + 138537) * 400/2) =$$

Step 2. Plot the term $\left(\frac{2p}{\mu_g z}\right)$ versus pressure as shown in Figure 6-16.

Step 3. Calculate numerically the area under the curve for each value of p. These areas correspond to the real gas potential ψ at each pressure. These ψ values are tabulated below

ψ versus p is also plotted in the figure



P=400psia, M(P)= 13278200
P=800psia, M(P)= 52658000
P=1200psia, M(P)= 116467000
P=1600psia, M(P)= 202153200
306597200
425659000
554852800
690083400
828366200
967957800
P=4400psia, M(P)= 1107284800

Step 4. Calculate the flow rate by applying Equation

$$Q = \frac{0.703kh(\psi - \psi_w)}{T \ln \frac{r}{r_w}}$$

$$p_w = 816 \times 10^6 \qquad p_e = 1089 \times 10^6$$

$$Q_g = \frac{(65)(15)(1089 - 816)10^6}{(1422)(600)\ln(1000/0.25)}$$

=37,614 Mscf/day

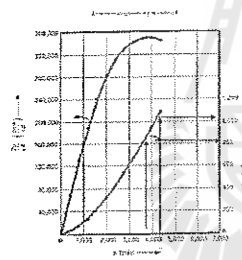


Figure 6-10.

Approximation of the Gas Flow Rate

The Eq.4.29 can be approximated by removing the term $\left(\frac{2}{\mu_g z}\right)$ outside the integral as a constant. It should be pointed out that the z is considered constant only under a pressure range of <2000psia.

$$\left(\frac{TQ_g}{kh}\right)\ln\left(\frac{r}{r_w}\right) = 0.703\left(\frac{2}{\mu_g z}\right)\int_{p_{wf}}^{p_r} p\,dp \qquad (4.33)$$

Perform the integration and let Q_g = Mscf/d the equation becomes

$$Q_g = \frac{kh(p_e^2 - p_{wf}^2)}{1422T(\mu_g z)_{avg}\ln(r_e/r_w)} \qquad (4.34)$$

The term $(\mu_g z)_{avg}$ is evaluated at an average pressure $\bar{p}$

$$\bar{p} = \sqrt{\frac{p_{wf}^2 + p_e^2}{2}}$$

Example 4.2 Using the data given in Ex.4.1, resolve for the gas flow rate by using the pressure-square method. Compare with the exact method(i.e. real gas potential ψ solution)

Solution.

Step1. Calculate the arithmetic average pressure.

$$\bar{p} = \left[\frac{4400^2 + 3600^2}{2}\right]^{.5} = 4020$$

Step2. Determine gas viscosity and gas compressibility factor at 4020 psi.

$\mu_g = 0.0267$

$z = 0.862$

Step3. Apply Equation 4.34

$$Q_g = \frac{(65)(15)(4400^2 - 3600^2)}{(1422)(600)(0.0267)(0.862)\ln(1000/0.25)}$$

= 38,314 Mscf/day

Step4. Results show that the pressure-squared method approximates the exact solution of 37,614 with an absolute error of 1.86%. This error is due to the limited applicability of the pressure-squared method to a pressure rang of <2000 psi.

4.9 Permeability Variations

Many producing formations are composed of strata or stringers that may vary widely in permeability and thickness, as illustrated in Fig. 7.8. If these strata are producing fluid to a common wellbore under the same drawdown and from the same drainage radius, then

$$q_t = q_1 + q_2 + q_3 \cdots + q_n$$

$$\frac{0.00708\,k_{avg}h_t(p_e - p_w)}{\mu B \ln(r_e/r_w)} = \frac{0.00708\,k_1h_1(p_e - p_w)}{\mu B \ln(r_e/r_w)} + \frac{0.00708\,k_2h_2(p_e - p_w)}{\mu B \ln(r_e/r_w)} + \text{etc.}$$

Then cancelling

$$k_{avg}h_t = k_1h_1 + k_2h_2 + \cdots + k_nh_n$$

$$k_{avg} = \frac{\Sigma k_ih_i}{\Sigma h_i} \qquad (7.23)$$

Radial flow in parallel beds

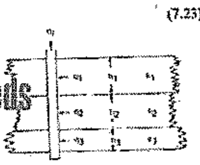


Fig. 7.8. Radial flow in parallel beds.

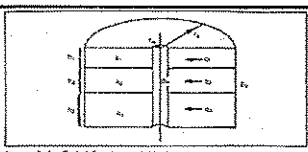


Figure 6-4 Radial flow in parallel beds

Then

$$q_t = q_1 + q_2 + q_3 = \frac{707.8\,(p_e^2 - p_w^2)}{\bar{\mu}\bar{z}T_f \ln(r_e/r_w)}\left[k_1h_1 + k_2h_2 + k_3h_3\right] \quad (6\text{-}10)$$

$$q = \text{[illegible]}\ \frac{707.8\,(p_e^2 - p_w^2)}{\mu \bar{z} T_f \ln \frac{r_e}{r_w}} \quad \text{— 6-9/1}$$

or

$$\bar{k} = \frac{(k_1h_1 + k_2h_2 + k_3h_3)}{(h_1 + h_2 + h_3)} \quad (6\text{-}12)$$

where the bar over the symbol indicates an average value. In a more general form, the result is

$$\bar{k} = \frac{\sum_{j=1}^{n} k_j h_j}{\sum_{j=1}^{n} h_j} \quad (6\text{-}11)$$

Radial Flow in Series

We [illegible] with a permeability of [illegible] radius r_w and an [al]tered permeability k_s between the radius r_s and the wellbore radius r_w as [sh]own in Fig. 7.9. The pressure drops are additive, and

$$(p_e - p_w) = (p_e - p_s) + (p_s - p_w)$$

[T]hen from Eq. (7.19)

$$\frac{q\mu B \ln(r_e/r_w)}{0.00708\,k_{avg}h} = \frac{q\mu B \ln(r_s/r_w)}{0.00708\,k_s h} + \frac{q\mu B \ln(r_e/r_s)}{0.00708\,k_e h}$$

[C]ancelling and solving for k_{avg}

$$k_{avg} = \frac{k_s k_e \ln(r_e/r_w)}{k_s \ln(r_e/r_s) + k_e \ln(r_s/r_w)} \quad (7.24)$$

[Equa]tion (7.24) may be extended to include three or more zones in series. [This] equation is important in studying the effect of a decrease or increase of [perm]eability in the zone about the wellbore on the well productivity.

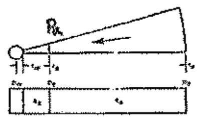


Fig. 7.9. Radial flow in beds in series

RADIAL FLOW OF GAS IN SERIES BEDS

Referring to Figure 6-5 and by applying equation 6-6, the result is

$$q = \frac{707.8 k_s h\,(p_s^2 - p_w^2)}{\bar{\mu}\bar{z}_s T_f \ln(r_s/r_w)} = \frac{707.8 k_e h\,(p_e^2 - p_s^2)}{\bar{\mu}\bar{z}_e T_f \ln(r_e/r_s)}$$

$$= \frac{707.8 \bar{k} h\,(p_e^2 - p_w^2)}{\bar{\mu}\bar{z} T_f \ln(r_e/r_w)}$$

But [illegible handwritten equation]

$$(p_e^2 - p_w^2) = (p_e^2 - p_s^2) + (p_s^2 - p_w^2)$$

and [illegible handwritten equation]

$$\bar{k} = \frac{\bar{\mu}\bar{z}\ln(r_e/r_w)}{\frac{\bar{\mu}\bar{z}_s}{k_s}\ln(r_s/r_w) + \frac{\bar{\mu}\bar{z}_e}{k_e}\ln(r_e/r_s)} \quad (6\text{-}13)$$

$$\bar{k} = \frac{k_1 k_2 \ln \frac{r_e}{r_w}}{\text{[illegible]}}$$

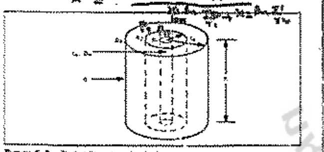


Figure 6-5 Radial flow in series beds

RADIAL FLOW OF GAS IN PARALLEL BEDS

If one considers radial flow in parallel beds as illustrated in Figure 4.11, the flow through each of the separate beds is as follows:

$$q_1 = \frac{0.703hk(p_e^2 - p_w^2)}{\bar{\mu}\bar{z}T_f \ln(r_e/r_w)}$$

$$q_2 = \frac{0.703hk(p_e^2 - p_w^2)}{\bar{\mu}\bar{z}T_f \ln(r_e/r_w)}$$

$$q_3 = \frac{0.703hk(p_e^2 - p_w^2)}{\bar{\mu}\bar{z}T_f \ln(r_e/r_w)}$$

Then

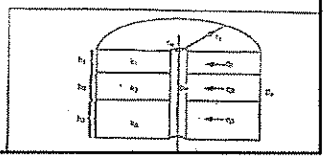
$$Q_t = q_1 + q_2 + q_3 = \frac{0.703(p_e^2 - p_w^2)}{\bar{\mu}\bar{z}T_f \ln(r_e/r_w)}\left[k_1h_1 + k_2h_2 + k_3h_3\right] \quad (4.35)$$

or

$$\bar{K} = \frac{k_1h_1 + k_2h_2 + k_3h_3}{(h_1 + h_2 + h_3)}$$

or

$$\bar{K} = \frac{\sum_{i=1}^{n} k_i h_i}{\sum_{i=1}^{n} h_i}$$



RADIAL FLOW OF GAS IN SERIES BEDS

If one considers radial flow in series beds as illustrated in Figure 4.12, the flow through each bed is as follows:

$$Q_t = q_1 = \frac{0.703hk_1(p_e^2 - p_1^2)}{\bar{\mu}_1\bar{z}_1T_f \ln(r_e/r_1)} = q_2 = \frac{0.703hk_2(p_1^2 - p_w^2)}{\bar{\mu}_2\bar{z}_2T_f \ln(r_1/r_w)}$$

$$= q = \frac{0.703h\bar{k}(p_e^2 - p_w^2)}{\bar{\mu}\bar{z}T_f \ln(r_e/r_w)}$$

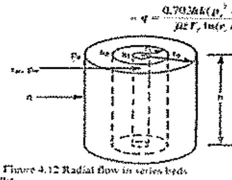


Figure 4.12 Radial flow in series beds

But

$$(p_e^2 - p_1^2) + (p_1^2 - p_w^2) = (p_e^2 - p_w^2) \quad (4.37)$$

Then

$$\frac{q\bar{\mu}_1\bar{z}_1T_f \ln(r_e/r_1)}{0.703\,hk_1} + \frac{q\bar{\mu}_2\bar{z}_2T_f \ln(r_1/r_w)}{0.703\,hk_2} = \frac{q\bar{\mu}\bar{z}T_f \ln(r_e/r_w)}{0.703\,h\bar{k}}$$

and

$$\bar{k} = \frac{\bar{\mu}\bar{z}\ln(r_e/r_w)}{\frac{\bar{\mu}_1\bar{z}_1}{k_1}\ln(r_e/r_1) + \frac{\bar{\mu}_2\bar{z}_2}{k_2}\ln(r_1/r_w)}$$

In more general form the result is:

$$\bar{K} = \frac{\bar{\mu}\bar{z}\ln(r_e/r_w)}{\sum_{i=1}^{n} \frac{\bar{\mu}_i\bar{z}_i}{k_i}\ln(r_i/r_{i-1})}$$

Steady State Flow

The steady-state flow equations—Darcy's equations accounting for specific geometries—are [illegible] here.

For linear geometry: $q_o = \frac{0.001127kA(p_1 - p_2)}{\mu L}$ (6.1)

For radial geometry: $q_o = \frac{0.00708kh(p_e - p_w)}{\mu \ln(r_e/r_w)}$ (6.2)

For hemispherical geometry: [illegible] (6.3)

5-spot waterflood: [illegible] (6.4)

7-spot waterflood: [illegible] (6.5)

For gas reservoirs:

Linear (using p): [illegible] (6.6)

Linear (using p^2): [illegible] (6.7)

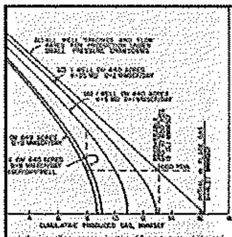
Linear (using $\psi(p)$): [illegible] (6.8)

Radial (using p): [illegible] (6.9)

Radial (using p^2): [illegible] (6.10)

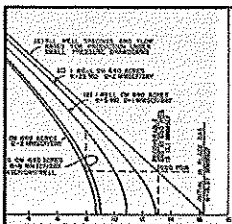
Radial (using $\psi(p)$): [illegible] (6.11)

The correct value of $(1/\mu z)_{avg}$ to use in the equations for gas is [illegible].



PROBLEMS

12/04/57



PROBLEMS

434620,505653 ADVANCED RESERVOIR ENGINEERING
2/2550(4 credits(4-0-8), SEPT.19,-DEC 23, 2011)
Course Contents

1. Optimization of Material Balance Equations (6 hrs.)
2. Saturation and Relative Permeability Calculations (4 hrs.)

4. **Unsteady State Flow (4 Hrs)**
5. Pseudo-steady State Flow and Superposition (4 hrs.)
6. Well Testing Pressure Drawdown and Build Up (3 hrs.)
7. Interference Test and Type Curve Analysis(3hrs)
8. Displacement Efficiency(4 hrs)
9. Potential flow and Streamlines (4 hrs.)
10. Dynamics of Water Drive Reservoir. (6 hrs.)
11. Water and Gas Coning (4hrs.)
12. Multi-Phase Flow and Introduction to Reservoir Simulation (4 hr

UNSTEADY STATE AND SUPERPOSITION

OIL OR GAS OR WATER

The mass entering the volume element during Δt is given by: $qB = Av$

Mass Entering $(qB\rho)_{r+\Delta r} = 2\pi(r+\Delta r)h(\rho v(5.615/24))_{r+\Delta r}$ (7.25)

The mass leaving the volume element during Δt is given by:

Mass Leaving $(qB\rho)_r = 2\pi rh(\rho v(5.615/24))_r$ (7.26)

The rate at which mass accumulates during the interval Δt is given by:

Mass Acc. $\dfrac{2\pi r\Delta rh[(\phi\rho)_{t+\Delta t} - (\phi\rho)_t]}{\Delta t}$ (7.27)

Combining Eqs. (7.25), (7.26), and (7.27), as suggested by the word "equation" written above,

MASS ENTER − MASS LEAVING = MASS ACC.

$$2\pi(r+\Delta r)h(\rho v(5.615/24))_{r+\Delta r} - 2\pi rh(\rho v(5.615/24))_r = \frac{2\pi r\Delta rh[(\phi\rho)_{t+\Delta t} - (\phi\rho)_t]}{\Delta t}$$

$2\pi rh(\rho v \cdot 0.234)_{r+\Delta r} - 2\pi rh(\rho v \cdot 0.234)_r$

If both sides of this equation are divided by $2\pi r\Delta rh$ and the limit is taken in each term as Δr and Δt approach zero, the following is obtained:

$$\frac{\partial}{\partial r}(0.234\,\rho v) + \frac{1}{r}(0.234\,\rho v) = \frac{\partial}{\partial t}(\phi\rho)$$

or $2\pi rh(\rho v)0.234_{r+\Delta r} + 2\pi\Delta rh(\rho v \cdot 0.234) - 2\pi rh(\rho v \cdot 0.234)_r$ =

$$\frac{0.234}{r}\frac{\partial}{\partial r}(r\rho v) = \frac{\partial}{\partial t}(\phi\rho) \qquad (7.28)$$

Equation (7.28) is the continuity equation and is valid for any flow system of

reservoir pressure:

$$v = -0.001127\,\frac{k}{\mu}\frac{\partial p}{\partial r}$$

Realiz the sign convention for fluid flow in porous media and substituting Darcy's 'y's equation because equation into Eq. (7.28):

$$\frac{0.234}{r}\frac{\partial}{\partial r}\left(0.001127\,\frac{k}{\mu}\rho r\frac{\partial p}{\partial r}\right) = \frac{\partial}{\partial t}(\phi\rho) \qquad (7.29)$$

he porosity from the partial derivative term on the right-hand side is eliminated by expanding the right-hand side by taking the indicated derivatives:

$$\frac{\partial}{\partial t}(\phi\rho) = \phi\frac{\partial\rho}{\partial t} + \rho\frac{\partial\phi}{\partial t} \qquad (7.30)$$

can be shown that porosity is related to the formation compressibility by the following:

$$c_f = \frac{1}{\phi}\frac{\partial\phi}{\partial p} \Rightarrow \frac{\partial\phi}{\partial p} = c_f\phi \qquad (7.31)$$

pplying the chain rule of differentiation to $\partial\phi/\partial t$,

$$\frac{\partial\phi}{\partial t} = \frac{\partial\phi}{\partial p}\frac{\partial p}{\partial t}$$

bstituting Eq. (7.31) into this equation,

$$\frac{\partial\phi}{\partial t} = \phi c_f\frac{\partial p}{\partial t}$$

nally, substituting this equation into Eq. (7.30) and the result into Eq. (7.29),

$$\frac{0.234}{r}\frac{\partial}{\partial r}\left(0.001127\,\frac{k}{\mu}\rho r\frac{\partial p}{\partial r}\right) = \rho\phi c_f\frac{\partial p}{\partial t} + \phi\frac{\partial\rho}{\partial t} \qquad (7.32)$$

CHOOSING THE BEST PRESSURE FUNCTIONS[4]

We have discussed the dimensionless pressure solution to the diffusivity equation, the exponential (E_i function) solution to the diffusivity equation, the log approximation to the E_i function solu... n pseudosteady-state solution to the diffusivity equation. Each of these solutions was based on assumptions and limitations. Because of these limitations, it is often confusing to determine which solutions will provide the appropriate analysis.

To avoid using an incorrect solution, a decision flow chart is presented in Figure 6–12. This flow ...ill help you decide on the appropriate solution.

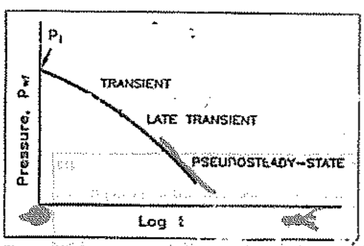


Figure 6–11. Plot of bottomhole flowing pressure (p_{wf}) v... ...s log of producing time t.

Unsteady State q ไม่ Constant.

...e well, the movement of pressure begins to occur away from the we... ...ovement of pressure is a diffusion phenomenon and is modeled ...iffusivity equation (see Sect. 5). The pressure moves at a rate propo... ...the formation diffusivity, η,

$dp \propto \eta$

$$\eta = \frac{k}{\phi \mu c_t}$$

...here k is the effective permeability of the flowing phase, ϕ is the tota... ...v... porosity, μ is the fluid viscosity of the flowing phase, and c_t is t... ...mpressibility. The total compressibility is obtained by weighting th... ...ressibility of each phase by its saturation and adding the formatio... ...ressibility, i.e.:

$$c_t = c_g S_g + c_o S_o + c_w S_w + c_f$$

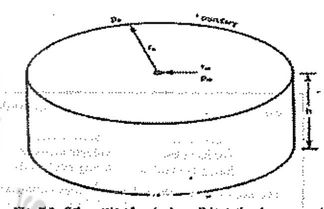


Fig. 7.2. Schematic of a single well in a circular reservoir.

...ter boundary of the reservoir and before the pressure behavior has ... ...stabilize in the reservoir. In this region, the pressure no longer travels at a ...te proportional to η. It is very difficult to describe the pressure behavior ...uring this period.

The fourth period, the pseudosteady-state, is the period after the pressure behavior has stabilized in the reservoir. During this period, the pressure at every point throughout the reservoir is changing at a constant rate and as a linear function of time. This period has been incorrectly referred to as the ...teady-state period. $\frac{dp}{dt}$ = Constant,

An estimation for the time when a flow system of the type shown in Figure 7.2 reaches pseudosteady-state can be made from the following equation

$$t_{pss} = \frac{1200 r_e^2}{\eta} = \frac{1200 \phi \mu c_t r_e^2}{k} \qquad (7.6)$$

where t_{pss} is the time to reach pseudosteady-state expressed in hours.[7] For a well producing an oil with a reservoir viscosity of 1.5 cp and a total compressibility of 15×10^{-6} psi^{-1}, from a circular reservoir of 1000-ft radius with a permeability of 100 md and a total effective porosity of 20%:

$$t_{pss} = \frac{1200(0.2)(1.5)(15(10)^{-6})(1000^2)}{100} = 54 \text{ hr}$$

This means that approximately 54 hours, or 2.25 days, is required for the flow in this reservoir to reach pseudosteady-state conditions after a well located in ...enter is opened to flow, or following a change in the well flow rate. It also means that if the well is shut in, it will take approximately this time for the pressure to equalize throughout the drainage area of the well, so that the measured subsurface pressure equals the average drainage area pressure of the well.

This same criterion may be applied approximately to gas reservoirs but with less certainty because the gas is more compressible. For a gas viscosity of 0.015 cp and a compressibility of 400×10^{-6} psi^{-1},

$$t_{pss} = \frac{1200(0.2)(0.015)(400(10)^{-6})(1000^2)}{100} = 14.4 \text{ hr}$$

Thus, under somewhat comparable conditions (i.e., the same r_e and k), gas reservoirs reach pseudosteady-state conditions more rapidly than do oil reser...

Tight Sand ...

t_{pss} = (1200 × 0.2 × 0.015 × ... (10^-6) × 1000^2) / ... = ... hr

6. DEVELOPMENT OF THE RADIAL DIFFERENTIAL EQUATION

The radial differential equation, which is the general differential equation used to model time-dependent flow systems, is now developed. Consider the volume element shown in Fig. 7.10. The element has a thickness Δr and is located r distance from the center of the well. Mass is allowed to flow into and out of the volume element during a period Δt. The volume element is in a reservoir of constant thickness and constant properties. Flow is allowed in only the radial direction. The following nomenclature, which is the same nomenclature defined previously, is used:

- q = volume flow rate, STB/day for incompressible and slightly compressible fluids and SCF/day for compressible fluids
- ρ = density of flowing fluid at reservoir conditions, lb/ft³
- r = distance from wellbore, ft
- h = formation thickness, ft
- v = velocity of flowing fluid, bbl/day-ft²
- t = hours
- ϕ = porosity, fraction
- k = permeability, md
- μ = flowing fluid viscosity, cp

With these assumptions and definitions, a mass balance can be written around the volume element over the time interval Δt. In word form, the mass balance is written as:

Fig. 7.10. Volume element used in the development of the radial differential equation.

The mass entering the volume element during Δt is given by: $qB = \Delta V$

Mass Entering $(qB\rho)_{r+\Delta r} = 2\pi(r+\Delta r)h(\rho v(5.615/24))_{r+\Delta r}$ (7.25)

The mass leaving the volume element during Δt is given by:

Mass Leaving $(qB\rho)_r = 2\pi rh(\rho v(5.615/24))_r$ (7.26)

The rate at which mass accumulates during the interval Δt is given by:

Mass Acc. $$\frac{2\pi r\Delta rh[(\phi\rho)_{t+\Delta t} - (\phi\rho)_t]}{\Delta t} \quad (7.27)$$

Combining Eqs. (7.25), (7.26), and (7.27), as suggested by the word "equation" written above,

MASS ENTER − MASS LEAVING = MASS ACC.

$$2\pi(r+\Delta r)h(\rho v(5.615/24))_{r+\Delta r} - 2\pi rh(\rho v(5.615/24))_r = \frac{2\pi r\Delta rh[(\phi\rho)_{t+\Delta t} - (\phi\rho)_t]}{\Delta t}$$

$2\pi rh(\rho v \cdot 0.234)_{r+\Delta r} - 2\pi rh(\rho v \cdot 0.234)_r$

If both sides of this equation are divided by $2\pi r\Delta rh$ and the limit is taken in each term as Δr and Δt approach zero, the following is obtained:

$$\frac{\partial}{\partial r}(0.234\rho v) + \frac{1}{r}(0.234\rho v) = \frac{\partial}{\partial t}(\phi\rho)$$

or $2\pi rh(\rho v)0.234) + 2\pi\Delta rh(\rho v \cdot 0.234) - 2\pi rh(\rho v\, 0.234)_r$

$$\frac{0.234}{r}\frac{\partial}{\partial r}(r\rho v) = \frac{\partial}{\partial t}(\phi\rho) \quad (7.28)$$

Equation (7.28) is the continuity equation and is valid for any flow system of

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Mass entering volume element during interval Δt − Mass leaving volume element during interval Δt = Rate at which mass accumulates during interval Δt

The mass entering the volume element during Δt is given by:

$$(qB\rho)_{r+\Delta r} = 2\pi(r+\Delta)rh(\rho v(5.615/24))_{r+\Delta r} \quad (7.25)$$

The mass leaving the volume element during Δt is given by:

$$(qB\rho)_r = 2\pi rh(\rho v(5.615/24))_r \quad (7.26)$$

The rate at which mass accumulates during the interval Δt is given by:

$$\frac{2\pi r\Delta rh[(\phi\rho)_{t+\Delta t} - (\phi\rho)_t]}{\Delta t} \quad (7.27)$$

Combining Eqs. (7.25), (7.26), and (7.27), as suggested by the word "equation" written above,

$$2\pi(r+\Delta r)h(\rho v(5.615/24))_{r+\Delta r} - 2\pi rh(\rho v(5.615/24))_r = \frac{2\pi r\Delta rh[(\phi\rho)_{t+\Delta t} - (\phi\rho)_t]}{\Delta t}$$

If both sides of this equation are divided by $2\pi r\Delta rh$ and the limit is taken in each term as Δr and Δt approach zero, the following is obtained:

$$\frac{\partial}{\partial r}(0.234\rho v) + \frac{1}{r}(0.234\rho v) = \frac{\partial}{\partial t}(\phi\rho)$$

or

$$\frac{0.234}{r}\frac{\partial}{\partial r}(r\rho v) = \frac{\partial}{\partial t}(\phi\rho) \quad (7.28)$$

Equation (7.28) is the continuity equation and is valid for any flow system of radial geometry. To obtain the radial differential equation that will be the basis for time-dependent models, pressure must be introduced and ϕ eliminated from the partial derivative term on the right-hand side of Eq. (7.28). To do this, Darcy's equation must be introduced to relate the fluid flow rate to reservoir pressure:

$$v = -0.001127\frac{k}{\mu}\frac{\partial p}{\partial r}$$

Realizing that the minus sign can be dropped from Darcy's equation because

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of the sign convention for fluid flow in porous media and substituting Darcy's equation into Eq. (7.28):

$$\frac{0.234}{r}\frac{\partial}{\partial r}\left(0.001127\frac{k}{\mu}\rho r\frac{\partial p}{\partial r}\right) = \frac{\partial}{\partial t}(\phi\rho) \quad (7.29)$$

The porosity from the partial derivative term on the right-hand side is eliminated by expanding the right-hand side by taking the indicated derivatives:

$$\frac{\partial}{\partial t}(\phi\rho) = \phi\frac{\partial\rho}{\partial t} + \rho\frac{\partial\phi}{\partial t} \quad (7.30)$$

It can be shown that porosity is related to the formation compressibility by the following:

$$c_f = \frac{1}{\phi}\frac{\partial\phi}{\partial p} \quad (7.31)$$

Applying the chain rule of differentiation to $\partial\phi/\partial t$,

$$\frac{\partial\phi}{\partial t} = \frac{\partial\phi}{\partial p}\frac{\partial p}{\partial t}$$

Substituting Eq. (7.31) into this equation,

$$\frac{\partial\phi}{\partial t} = \phi c_f\frac{\partial p}{\partial t}$$

Finally, substituting this equation into Eq. (7.30) and the result into Eq. (7.29),

$$\frac{0.234}{r}\frac{\partial}{\partial r}\left(0.001127\frac{k}{\mu}\rho r\frac{\partial p}{\partial r}\right) = \rho\phi c_f\frac{\partial p}{\partial t} + \phi\frac{\partial\rho}{\partial t} \quad (7.32)$$

Equation 7.32 is the general partial differential equation used to describe the flow of any fluid flowing in a radial direction in porous media. In addition to the initial assumptions, Darcy's equation has been added, which implies that the flow is laminar. Otherwise, the equation is not restricted to any type of fluid or any particular time region.

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By applying appropriate boundary and initial conditions, particular solutions to the differential equation derived in the preceding section can be discussed. The solutions obtained pertain to the transient and pseudosteady-state flow reservoir pressure:

$$v = -0.001127\frac{k}{\mu}\frac{\partial p}{\partial r}$$

Realiz the sign convention for fluid flow in porous media and substituting Darcy's y's equation because quation into Eq. (7.28):

$$\frac{0.234}{r}\frac{\partial}{\partial r}\left(0.001127\frac{k}{\mu}\rho r\frac{\partial p}{\partial r}\right) = \frac{\partial}{\partial t}(\phi\rho) \quad (7.29)$$

he porosity from the partial derivative term on the right-hand side is elimi- ated by expanding the right-hand side by taking the indicated derivatives:

$$\frac{\partial}{\partial t}(\phi\rho) = \phi\frac{\partial \rho}{\partial t} + \rho\frac{\partial \phi}{\partial t} \quad (7.30)$$

can be shown that porosity is related to the formation compressibility by the llowing:

$$c_f = \frac{1}{\phi}\frac{\partial \phi}{\partial p} \Rightarrow \frac{\partial \phi}{\partial p} = c_f\phi \quad (7.31)$$

pplying the chain rule of differentiation to $\partial\phi/\partial t$,

$$\frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial p}\frac{\partial p}{\partial t}$$

bstituting Eq. (7.31) into this equation,

$$\frac{\partial \phi}{\partial t} = \phi c_f \frac{\partial p}{\partial t}$$

nally, substituting this equation into Eq. (7.30) and the result into Eq. 29),

$$\frac{0.234}{r}\frac{\partial}{\partial r}\left(0.001127\frac{k}{\mu}\rho r\frac{\partial p}{\partial r}\right) = \rho\phi c_f\frac{\partial p}{\partial t} + \phi\frac{\partial \rho}{\partial t} \quad (7.32)$$

6. TRANSIENT FLOW SYSTEM

6.1. Radial Flow of Slightly Compressible Fluids, Transient Flow

$d\rho = c\rho\, dp$

If Eq. (7.2) is expressed in terms of density, ρ, which is the inverse of specific volume, then the following is obtained:

$$\rho = \rho_R e^{c(p-p_R)} \quad (7.33)$$

where p_R is some reference pressure and ρ_R is the density at that reference pressure. Inherent in this equation is the assumption that the compressibility of the fluid is constant. This is nearly always a good assumption over the pressure range of a given application. Substituting Eq. (7.33) into Eq. (7.32),

$$\frac{0.234}{r}\frac{\partial}{\partial r}\left(0.001127\frac{k}{\mu}[\rho_R e^{c(p-p_R)}]r\frac{\partial p}{\partial r}\right) = [\rho_R e^{c(p-p_R)}]\phi c_f\frac{\partial p}{\partial t} + \phi\frac{\partial}{\partial t}[\rho_R e^{c(p-p_R)}]$$

To simplify this equation, one must make the assumption that k and μ are constant over the pressure, time, and distance ranges in our applications. This is rarely true about k. However, if k is assumed to be a volumetric average permeability over these ranges, then the assumption is good. In addition, it has been found that viscosities of liquids do not change significantly over typical pressure ranges of interest. Making this assumption allows k/μ to be brought outside the derivative. Taking the necessary derivatives and simplifying,

$\frac{k}{\mu} \approx$ constant.

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + c\left[\frac{\partial p}{\partial r}\right]^2 = \frac{\phi\mu}{0.0002637k}(c_f + c)\frac{\partial p}{\partial t}$$

or

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + c\left[\frac{\partial p}{\partial r}\right]^2 = \frac{\phi\mu c_t}{0.0002637k}\frac{\partial p}{\partial t} \quad (7.34)$$

$$\frac{\partial^2 P}{\partial r^2}+\frac{1}{r}\frac{\partial P}{\partial r}=\frac{\phi\mu c_t}{0.0002637k}\frac{\partial P}{\partial t} \quad 7.35$$

or.

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial P}{\partial r}\right]=\frac{1}{0.0002637}\cdot\frac{1}{\eta}\frac{\partial P}{\partial t} \quad (7.35)$$

Assume $S=\frac{\phi\mu c r^2}{4kt\times 0.0002637}$ = Boltzmann Transformation

$$\frac{\partial S}{\partial r}=\frac{\phi\mu c r}{2kt\times 0.0002637}=\frac{2S}{r}\times \quad \left(\eta=\frac{k}{\phi\mu c_t}\right)$$

$$\frac{\partial S}{\partial t}=-\frac{\phi\mu c}{k}\cdot\frac{r^2}{4t^2\times}=-\frac{S}{t}\times\frac{1}{0.0002637}$$

Eq. 7.35 can be Transformed to

$$\frac{1}{r}\frac{\partial}{\partial s}\left(r\frac{\partial P}{\partial s}\cdot\frac{\partial s}{\partial r}\right)\frac{\partial s}{\partial r}=\frac{1}{0.0002637\eta}\cdot\frac{\partial P}{\partial s}\cdot\frac{\partial s}{\partial t}$$

$$\frac{1}{r}\frac{\partial}{\partial s}\left(r\frac{\partial P}{\partial s}\cdot\frac{2s}{r}\right)\frac{\phi\mu c r}{2kt\times 0.0002637}=\frac{1}{0.0002637\eta}\cdot\frac{S}{t}\frac{\partial P}{\partial s}$$

$$\frac{d}{ds}\left(s\frac{dP}{ds}\right)=-S\frac{dP}{ds}$$

$$S\frac{d}{ds}\left(\frac{dP}{ds}\right)+\frac{dP}{ds}=-S\frac{dP}{ds}$$

Let $\bar{P}=\frac{dP}{ds}$ [illegible]

$$S\frac{d\bar{P}}{ds}+\bar{P}=-S\bar{P}$$

$$\frac{d\bar{P}}{\bar{P}}+\frac{ds}{s}=-ds$$

Rearranged and then integrated:

$$-\int\frac{1}{\bar{P}}d\bar{P}=\int\left(1+\frac{1}{S}\right)ds+c$$

$$-\ln\bar{P}=s+\ln s+\ln c$$

$$\ln\bar{P}+\ln s+\ln c=-S$$

$$\ln s\bar{P}c=-S$$

$$c.s\bar{P}=e^{-s}$$

$$s\bar{P}=Ce^{-s} \quad \text{— (A)}$$

First boundary $r=r_w$ at $t=0$ applied

Darcy's eq.

$$q=-0.001127\frac{kh}{B\mu}2\pi r\left(\frac{dP}{dr}\right)_{r=r_w}$$

$$\left(\frac{dP}{dr}\right)_{r=r_w}=-\frac{qB\mu}{0.001127\times 2\pi r\times kh}=$$

$$\bar{P}=\frac{dP}{ds}=\frac{dP}{dr}\cdot\frac{dr}{ds}=\frac{r}{2s}\frac{dP}{dr}$$

$$\therefore\ s\bar{P}=s\cdot\frac{r}{2s}\cdot\frac{dP}{dr}=\frac{r}{2}\frac{dP}{dr}=$$

$$Ce^{-s}=\frac{r}{2}\cdot\left(-\frac{qB\mu}{0.001127\times 2\pi r kh}\right)=-\frac{70.6qB\mu}{kh}$$

$$c\bar{e}^{s} = \frac{-70.6\, qB\mu}{kh} \quad \text{second boundary}$$

$$\left(c = \frac{-70.6\, qB\mu}{kh}\, e^{+s} \right) \begin{matrix} t = \infty \\ s \to 0 \\ e^{+s} = 1 \end{matrix}$$

$$s\bar{p} = c\bar{e}^{s}$$

$$\bar{p} = c\,\frac{\bar{e}^{s}}{s}$$

$$\frac{dp}{ds} = c\,\frac{\bar{e}^{s}}{s}$$

$$\int_{p}^{p_e} dp = c \int_{\infty}^{s} \frac{e^{-s}}{s}\, ds$$

$$= \frac{70.6\, qB\mu}{kh} \int_{s}^{\infty} \frac{e^{-s}}{s}\, ds$$

$$p_e - p = \frac{70.6\, qB\mu}{kh}\left[-Ei(-s)\right]$$

$$p = p_e - \frac{70.6\, qB\mu}{kh}\left[-Ei\left(-\frac{\phi\mu c_t r^2}{0.00105\, kt}\right)\right]$$

Ei = Exponential Integral Function Solution

x	$-E_i(-x)$	x	$-E_i(-x)$	x	$-E_i(-x)$
0.1	1.82292	4.3	0.00263	8.5	0.00002
0.2	1.22265	4.4	0.00234	8.6	0.00002
0.3	0.90568	4.5	0.00207	8.7	0.00002
0.4	0.70238	4.6	0.00184	8.8	0.00002
0.5	0.55977	4.7	0.00164	8.9	0.00001
0.6	0.45438	4.8	0.00145	9.0	0.00001
0.7	0.37377	4.9	0.00129	9.1	0.00001
0.8	0.31060	5.0	0.00115	9.2	0.00001
0.9	0.26018	5.1	0.00102	9.3	0.00001
1.0	0.21938	5.2	0.00091	9.4	0.00001
1.1	0.18599	5.3	0.00081	9.5	0.00001
1.2	0.15841	5.4	0.00072	9.6	0.00001
1.3	0.13545	5.5	0.00064	9.7	0.00001
1.4	0.11622	5.6	0.00057	9.8	0.00001
1.5	0.10002	5.7	0.00051	9.9	0.00000
1.6	0.08631	5.8	0.00045	10.0	0.00000
1.7	0.07465	5.9	0.00040		
1.8	0.06471	6.0	0.00036		
1.9	0.05620	6.1	0.00032		
2.0	0.04890	6.2	0.00029		
2.1	0.04261	6.3	0.00026		
2.2	0.03719	6.4	0.00023		
2.3	0.03250	6.5	0.00020		
2.4	0.02844	6.6	0.00018		
2.5	0.02491	6.7	0.00016		
2.6	0.02185	6.8	0.00014		
2.7	0.01918	6.9	0.00013		
2.8	0.01686	7.0	0.00012		
2.9	0.01482	7.1	0.00010		
3.0	0.01305	7.2	0.00009		
3.1	0.01149	7.3	0.00008		
3.2	0.01013	7.4	0.00007		
3.3	0.00894	7.5	0.00007		
3.4	0.00789	7.6	0.00006		
3.5	0.00697	7.7	0.00005		
3.6	0.00616	7.8	0.00005		
3.7	0.00545	7.9	0.00004		
3.8	0.00482	8.0	0.00004		
3.9	0.00427	8.1	0.00003		
4.0	0.00378	8.2	0.00003		
4.1	0.00335	8.3	0.00003		
4.2	0.00297	8.4	0.00002		

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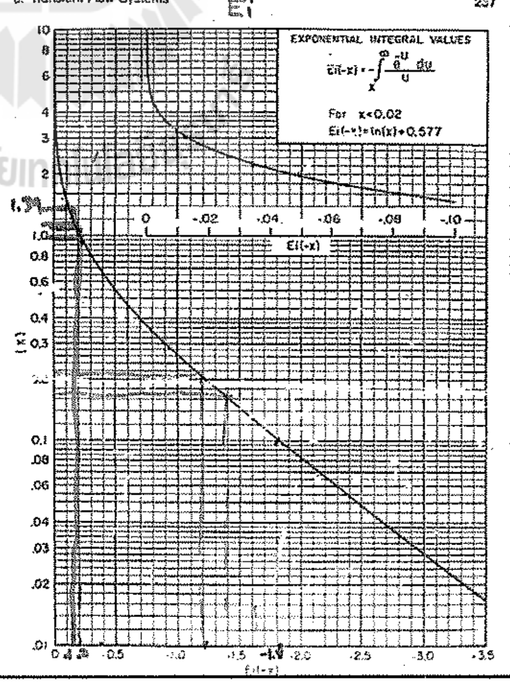

5 Radial Flow Through Porous Media: Slightly Compressible Fluids

5.1 Introduction

The flow pattern in the strata in the vicinity of a well producing oil or gas can be considered as essentially horizontal radial (if we limit the discussion initially to near vertical wells and non-dipping formations). An understanding of horizontal radial flow is therefore important if we are to explain or predict the productive performance of a well.

The behaviour of *slightly* compressible fluids (undersaturated oil, water), whose properties are little affected by changes in pressure; and *highly* compressible fluids (dry and wet gas, condensates), which are very sensitive to the pressure, will be treated separately.

In this chapter, we will examine the bottom-hole pressure behaviour observed during the flow of the fluid through the formation towards the well, or vice versa, (production or injection tests); and during the period following the shutting in of the well – i.e. the termination of flow (buildup or fall off tests).

5.2 Equation for Single Phase Radial Flow

The basic assumptions made in the theory of single phase horizontal radial flow are:

- the porous medium is homogeneous in ϕ and k, and is horizontal and of uniform thickness; the permeability is isotropic ($k_r = k_\theta$),
- there are no saturation gradients: in the case of the flow of oil, its saturation is constant and equal to $(1 - S_{wi})$ everywhere, the water phase being immobile; in the case of the flow of water, its saturation is everywhere $\geq (1 - S_{or})$, and the oil phase (if present) is immobile,
- the pressure throughout the reservoir is above the bubble point of the fluid,
- the fluid properties are uniform over the thickness of the reservoir, so that gravitational effects can be ignored,
- the well has been perforated over the entire reservoir thickness, so that inflow is horizontal radial.

Consider the segment of a cylindrical reservoir shown in Fig. 5.1. An incremental volume (shaded) is defined by an arc at a distance r from the axis of the well, and is of thickness dr, with fluid flowing across it as indicated. Applying the principle of the conservation of mass to this volume:

(mass flow rate in) − (mass flow rate out) = (rate of change of mass in the incremental volume).

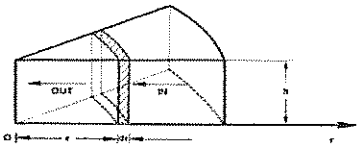

Fig. 5.1. *Diagram showing the conservation of mass in radial flow*

Therefore,

$$(q\rho)_{r+dr} - (q\rho)_r = 2\pi rh\,dr \frac{\partial}{\partial t}(\phi\rho)\,. \tag{5.1}$$

where $(2\pi rh\,dr)$ is the volume of the increment, ρ is the density of the mobile fluid, ϕ is the porosity. As was explained in Sect. 3.4.3, ϕ is pressure-dependent, given that the geostatic stress σ remains constant.

Recalling that:

$$(q\rho)_{r+dr} = (q\rho)_r + \frac{\partial(q\rho)}{\partial r}dr\,.$$

equation (5.1) becomes:

$$\frac{\partial(q\rho)}{\partial r} = 2\pi rh\frac{\partial(\phi\rho)}{\partial t}\,. \tag{5.2}$$

For a cross-sectional area to flow of $2\pi rh$, and a pressure gradient of $\partial p/\partial r$, Darcy's law can be written as

$$q = \frac{2\pi rhk}{\mu}\frac{\partial p}{\partial r}\,. \tag{5.3}$$

Furthermore, we have:

$$\frac{\partial}{\partial t}(\phi\rho) = \frac{d(\phi\rho)}{dp}\frac{\partial p}{\partial t} \tag{5.4a}$$

with:

$$\frac{d(\phi\rho)}{dp} = \phi\frac{d\rho}{dp} + \rho\frac{d\phi}{dp} = \rho\phi\left[\frac{1}{\rho}\frac{d\rho}{dp} + \frac{1}{\phi}\frac{d\phi}{dp}\right]. \tag{5.4b}$$

If we define the compressibility of the pore fluid as:

$$c = \frac{1}{\rho}\frac{d\rho}{dp}\,, \tag{5.4c}$$

we have, by substituting from Eq. (3.8c):

$$\frac{d(\rho\phi)}{dp} = \rho\phi(c + c_f) = c_t\rho\phi\,. \tag{5.5a}$$

Here, c_t represents the *total system compressibility* (rock + pore fluid). If the pores contain only water ($S_w = 1$), $c = c_w$, and we have:

$$c_t = c_w + c_f \quad (S_w = 1)\,. \tag{5.5b}$$

If, on the other hand, there is oil in the presence of irreducible (and non-mobile) water, we can write the following reasonable approximation:

$$c = c_oS_o + c_wS_{iw} = c_o + (c_w - c_o)S_{iw}\,.$$

From these two cases we can derive a more general form for c_t:

$$c_t = c_oS_o + c_wS_w + c_f\,. \tag{5.5c}$$

From Eqs. (5.2), (5.3), (5.4a) and (5.5a) it follows that:

$$\frac{\partial}{\partial r}\left(\rho\frac{2\pi rhk}{\mu}\frac{\partial p}{\partial r}\right) = 2\pi rh\rho\phi c_t\frac{\partial p}{\partial t}\,,$$

which simplifies to:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{k}{\mu}\rho r\frac{\partial p}{\partial r}\right) = \rho\phi c_t\frac{\partial p}{\partial t}\,, \tag{5.6}$$

where ρ and μ are the density and viscosity of the mobile single phase fluid.

Equation (5.6) describes the pressure at any time and at any point in the porous medium for a single phase fluid in horizontal radial flow. It is called the *general diffusivity equation*.

This is a *non-linear* partial differential equation – non-linear because ϕ, k, c_t, ρ and μ are all pressure-dependent. As a consequence, no analytical solution is possible without "linearising" the diffusivity equation so as to remove the pressure dependence of the parameters.

5.3 Linearisation of the Diffusivity Equation for Horizontal Radial Flow – Case Where the Rock-Fluid Diffusivity is Independent of the Pressure

Equation (5.6) can be expanded as:

$$\frac{1}{r}\left[\rho r\frac{d}{dp}\left(\frac{k}{\mu}\right)\left(\frac{\partial p}{\partial r}\right)^2 + \frac{k}{\mu}r\frac{d\rho}{dp}\left(\frac{\partial p}{\partial r}\right)^2 + \frac{k}{\mu}\rho\frac{\partial p}{\partial r} + \frac{k}{\mu}\rho r\frac{\partial^2 p}{\partial r^2}\right] = \rho\phi c_t\frac{\partial p}{\partial t}\,. \tag{5.7}$$

Since $\partial p/\partial r$ is small,* we can assume its square term to be negligible:

$$\left(\frac{\partial p}{\partial r}\right)^2 \ll \frac{\partial p}{\partial r}, \frac{\partial^2 p}{\partial r^2}\,. \tag{5.8}$$

Equation (5.7) then reduces to:

$$\frac{k}{\mu}\left[\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r}\right] = \phi c_t\frac{\partial p}{\partial t}\,. \tag{5.9}$$

* This assumption is not necessarily valid close to the wellbore, where neglecting the term $(\partial p/\partial r)^2$ may cause errors in the local estimate of $p(r,t)$.

Rearranging:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = \frac{\phi\mu c_t}{k}\frac{\partial p}{\partial t}\,. \tag{5.10}$$

where:

$$\frac{k}{\phi\mu c_t} = \eta = \text{hydraulic diffusivity} \tag{5.11}$$

is assumed to be constant for small changes of pressure, since k, ϕ and μ increase with pressure, while c_t decreases. The dimensions of η are $[L^2t^{-1}]$.

Equation (5.10) has now been linearised (provided the hydraulic diffusivity can be considered constant), and is referred to in this form as the *radial diffusivity equation*.

This can now be solved analytically. One approach is to take advantage of the similarity between it and the thermal diffusivity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{K}\frac{\partial T}{\partial t}\,, \tag{5.12}$$

where K = thermal diffusivity (m^2/s),
T = temperature (K).

Solutions to Eq. (5.12) for a wide variety of initial and boundary conditions have been published in the definitive work by Carslaw and Jäger, *Conduction of Heat in Solids*.[2]

Dranchuk and Quon[3] have shown that the linearisation process just described is only valid when:

$$c_t p \ll 1\,. \tag{5.13}$$

5.4 Dimensionless Form of the Radial Diffusivity Equation

The usefulness of a dimensionless version of the radial diffusivity equation will become apparent in Chaps. 6 and 7.

In fact, any dimensionless analytical solution for a particular set of initial and boundary conditions can be converted immediately into practical units to suit the real case in question.

The dimensionless variables are defined as follows:

Dimensionless radius: $$r_D = \frac{r}{r_w}\,, \tag{5.14a}$$

Dimensionless time: $$t_D = \frac{k}{\phi\mu c_t r_w^2}t = \frac{\eta}{r_w^2}t\,, \tag{5.14b}$$

Dimensionless pressure: $$p_D(r_D, t_D) = \frac{2\pi kh}{q\mu}(p_i - p_{r,t})\,, \tag{5.14c}$$

where p_i is the initial static reservoir pressure; $p_{r,t}$ is the pressure at time t and radius r; q is the flow rate *at reservoir conditions*; and r_w is the wellbore radius.

Using the terms defined in Eq. (5.14), Eq. (5.10) can be rewritten:

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial p_D}{\partial r_D}\right)=\frac{\partial p_D}{\partial t_D}, \quad (5.15)$$

which is the *dimensionless form of the radial diffusivity equation.*

If p_{wf} is the bottom hole flowing pressure ($r = r_w$, $r_D = 1$), we have:

$$p_D(1, t_D) = p_{D,w}(t_D) = \frac{2\pi kh}{q\mu}(p_i - p_{wf}) \quad (5.16a)$$

and, in the absence of any additional pressure drop due to skin effect (to be covered later):

$$p_{wf} = p_i - \frac{q\mu}{2\pi kh}p_{D,w}(t_D). \quad (5.16b)$$

This equation shows clearly the importance of having available $p_D(t_D)$ solutions for the flow of undersaturated oil towards the wellbore, with various initial and boundary conditions.

5.5 Behaviour with Time Under Flowing Conditions

Equations such as (5.10) or (5.15) are always associated with a set of initial and boundary conditions.

We will first consider qualitatively the idealised case of a cylindrical reservoir of uniform thickness h, with a sealing ("no-flow") external boundary of radius r_e. It is initially in a static condition, with a uniform pressure $p = p_i$ throughout.

There is a well of radius r_w at its centre. From time $t = 0$, oil is flowed from the well at a constant rate q (at reservoir conditions).

The assumptions listed in Sect. 5.2 regarding the nature of the well and reservoir are applied here.

The conclusions we shall draw about this single well will also be valid for the case where there are *other* wells present in the reservoir, provided their production rates – which may be different – do not change with time. In this situation, there is an area around each well within which the fluid movement is towards that well. This is referred to as the *drainage area*, A. Its shape and size depend on a number of factors, and the radius of the *equivalent circular area* is the *drainage radius* r_e of the well.

If the production rates from any of the wells change with time, their drainage areas (and consequently those of neighbouring wells) will expand or contract, and the pressure behaviour will differ from that predicted for a constant rate case.

We will use the dimensionless form of the radial diffusivity equation for convenience.

Consider a circular drainage area, with production starting at time $t_D = 0$, and continuing until:

$$t_{D,ei} \cong 0.06\left(\frac{r_e}{r_w}\right)^2. \quad (5.17a)$$

From the definition of t_D, this corresponds to a real time of:

$$t_{ei} \cong 0.06\frac{\phi\mu c_t}{k}r_e^2. \quad (5.17b)$$

Up to this time, the pressure disturbance invoked by flowing the well is contained within the drainage area of radius r_e (Fig. 5.2).

During this period, since $p_D(r_D, t_D)$ has not yet been affected by the non-permeable outer boundary at $r = r_e$, the reservoir is effectively infinite in size as far as the pressure is concerned. This is referred to as *infinite acting.*

For t_D between zero and $t_{D,ei}$, the flow regime is said to be *transient* or *infinite acting.*

Still in the same circular geometry, let us now look at the period:

$$0.06\left(\frac{r_e}{r_w}\right)^2 \le t_D \le 0.1\left(\frac{r_e}{r_w}\right)^2. \quad (5.17c)$$

This is a transition period between early transient and the next mode, and the flow regime is referred to as *late transient.*

Note that the t_D period during which late transient flow occurs is different – in both start time and duration – for different drainage area geometries; Fig. 5.7 lists the relevant values for non-circular shapes.

In a circular geometry, when $t_D > 0.1\,(r_e/r_w)^2$, the pressure decreases at a constant rate at all points in the drainage area:

$$\frac{\partial p_D}{\partial t_D} = \text{constant for any } r_D \text{ or } t_D. \quad (5.18a)$$

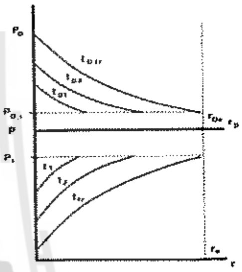

Fig. 5.2. *Outward diffusion of a pressure disturbance around a well: early transient period*

At the outer boundary of the drainage area (no-flow condition) for any t_D we have:

$$\left(\frac{\partial p_D}{\partial r_D}\right)_{r_{eD}} = 0. \quad (5.18b)$$

In other words, the pressure profile $p_D(r_D)$ keeps the same *shape*, and the whole profile declines at a steady rate with increasing time. Figure 5.3 shows a series of these parallel profiles corresponding to different times, in both real and dimensionless terms. The flow regime is now *pseudo-steady state* (also called *semi-steady state*).

In order for mass to be conserved at any radius r:

$$\frac{dp}{dt} = -\frac{q}{\pi r_e^2 h\phi c_t}, \quad (5.19a)$$

which, in dimensionless form is:

$$\frac{dp_D}{dt_D} = 2\left(\frac{r_w}{r_e}\right)^2. \quad (5.19b)$$

Equation (5.19) allows us to calculate the pressure distribution $p(r)$ in the reservoir for any value of $t > t_{ss}$, provided we have one pressure profile, also measured at a $t > t_{ss}$, to start from.

The variation of the bottom hole flowing pressure p_{wf} during these three flow regime periods is shown schematically in Fig. 5.4.

Pseudo-steady state flow occurs when we have a "no-flow" (i.e. non-permeable, or closed) outer boundary condition. An alternative condition is the "constant pressure boundary" (i.e. fluid is replenished across the boundary, so that the pressure p_e remains constant). In this case the flow regime goes to *steady state* after the late transient period.

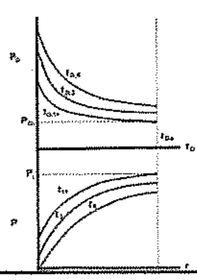

Fig. 5.3. *Pseudo-steady state pressure behaviour in the drainage area of a producing well with a no-flow boundary at $r = r_e$*

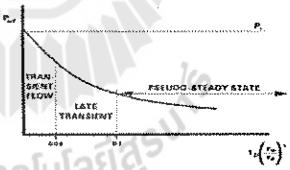

Fig. 5.4. *Bottom hole flowing pressure p_{wf} versus time*

From Darcy's equation:

$$q = \frac{2\pi rhk}{\mu}\frac{dp}{dr}, \quad (5.20a)$$

where $2\pi rh$ is the cross-sectional area to flow.

Integrating Eq. (5.20a) from r_w to r:

$$p - p_w = \frac{q\mu}{2\pi hk}\ln\frac{r}{r_w}, \quad (5.20b)$$

which is, in dimensionless form, for any value of t:

$$p_D(1) - p_D(r_D) = \ln r_D, \quad (5.20c)$$

where $r_D = 1$ is by definition [Eq. (5.14a)] the dimensionless wellbore radius.

5.6 Solutions to the Radial Diffusivity Equation for a Fluid of Constant Compressibility

5.6.1 Transient Flow

5.6.1.1 Treatment for an Ideal Well

Earlier in this chapter it was shown that during a certain initial period when the well is put on production ($t \le t_{ei}$), the reservoir is *infinite acting* and the flow regime is *transient.*

The initial conditions are:

$$p = p_i \quad \text{at } t = 0 \text{ for all } r \quad (5.21a)$$

The boundary conditions are:

$$p = p_i \quad \text{at } r = \infty \text{ for all } t \quad (5.21b)$$

$$q = \frac{2\pi khr_w}{\mu}\left(\frac{\partial p}{\partial r}\right)_{r_w} = \text{constant for all } t. \quad (5.21c)$$

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If we assume that r_w is negligibly small, Eq. (5.21c) simplifies to:

$$\lim_{r\to 0}\left(r\frac{\partial p}{\partial r}\right)=\frac{q\mu}{2\pi kh}=\text{constant}\,. \qquad (5.21d)$$

The solution to the radial diffusivity equation under these conditions is referred to as the *line source solution for constant terminal rate*.

Using the Boltzmann transform

$$s=\frac{r^2}{4(k/\phi\mu c_t)t}=\frac{\phi\mu c_t r^2}{4kt} \qquad (5.22a)$$

so that

$$\frac{\partial s}{\partial r}=\frac{\phi\mu c_t r}{2kt} \qquad (5.22b)$$

and:

$$\frac{\partial s}{\partial t}=-\frac{\phi\mu c_t r^2}{4kt^2}\,, \qquad (5.22c)$$

the radial diffusivity equation:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right)=\frac{\phi\mu c_t}{k}\frac{\partial p}{\partial t} \qquad (5.10)$$

becomes, through transforming the independent variables (r,t) to the independent variable s:

$$\frac{1}{r}\frac{d}{ds}\left(r\frac{dp}{ds}\frac{\partial s}{\partial r}\right)\frac{\partial s}{\partial r}=\frac{\phi\mu c_t}{k}\frac{dp}{ds}\frac{\partial s}{\partial t}\,, \qquad (5.23a)$$

where $p=p(r,t)$ is now only a function of $s=s(r,t)$.

Substituting from Eqs. (5.22b) and (5.22c), Eq. (5.23a) becomes:

$$\frac{1}{r}\frac{\phi\mu c_t r}{2kt}\frac{d}{ds}\left(\frac{\phi\mu c_t r^2}{2kt}\frac{dp}{ds}\right)=-\frac{\phi\mu c_t}{k}\frac{\phi\mu c_t r^2}{4kt^2}\frac{dp}{ds} \qquad (5.23b)$$

which, substituting from Equation (5.22a)

$$\frac{d}{ds}\left(s\frac{dp}{ds}\right)=-s\frac{dp}{ds} \qquad (5.23c)$$

or:

$$\frac{dp}{ds}+s\frac{d}{ds}\left(\frac{dp}{ds}\right)=-s\frac{dp}{ds}\,. \qquad (5.23d)$$

Equation (5.10), which is a partial differential equation in r and t, has been converted by means of Boltzmann's transform to an ordinary differential equation which is easy to solve. If we write:

$$\frac{dp}{ds}=p'\,. \qquad (5.24a)$$

equation (5.23d) is now:

$$p'+s\frac{dp'}{ds}=-sp' \qquad (5.24b)$$

or:

$$\frac{dp'}{p'}=-ds-\frac{ds}{s}\,, \qquad (5.24c)$$

which, when integrated, results in the general equation:

$$\ln p'=-s-\ln s+C_1 \qquad (5.25a)$$

so that:

$$p'=C_2\frac{e^{-s}}{s}\,, \qquad (5.25b)$$

where $C_2=e^{C_1}$.

For the calculation of C_2, we need to refer to the boundary conditions at the well. We already have that:

$$r\frac{\partial p}{\partial r}=r\frac{dp}{ds}\frac{\partial s}{\partial r}=r\frac{2\phi\mu c_t r}{4kt}\frac{dp}{ds}=2s\frac{dp}{ds}=2sC_2\frac{e^{-s}}{s}=2C_2e^{-s} \qquad (5.26)$$

and, from Eq. (5.21d):

$$\lim_{s\to 0}(2C_2e^{-s})=2C_2=\frac{q\mu}{2\pi kh}\,, \qquad (5.27)$$

since when $r\to 0$, $s\to 0$ as well.

Substituting from Eq. (5.25b) into Eq. (5.27) we now have:

$$p'=\frac{q\mu}{4\pi kh}\frac{e^{-s}}{s}\,. \qquad (5.28)$$

Equation (5.28) is integrated, at radius r, between the pressures $p_i(t=0)$ and $p(r,t)$ at time t. The corresponding values of s are [Eq. (5.22a)]:

$$s(t=0)=\infty$$

and

$$s(r,t)=\frac{\phi\mu c_t r^2}{4kt}=x\,.$$

Therefore:

$$\int_{p_i}^{p(r,t)}dp=\frac{q\mu}{4\pi kh}\int_{\infty}^{x}\frac{e^{-s}}{s}ds \qquad (5.29)$$

from which:

$$\frac{4\pi kh}{q\mu}[p_i-p(r,t)]=\int_{x}^{\infty}\frac{e^{-s}}{s}ds\,. \qquad (5.30)$$

The integral in the right hand term of Eq. (5.30) is the well-known *exponential integral* $ei(x)$, whose behaviour is shown in Fig. 5.5.

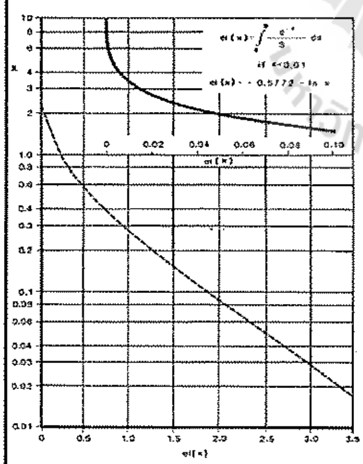

Fig. 5.5. *The exponential integral function ei(x), for the range x = 0.01 to x = 10*

$ei(x)$ can be developed as a series:

$$ei(x)=-0.57721-\ln x-\sum_{n=1}^{\infty}\frac{(-1)^n x^n}{n\,n!}\,. \qquad (5.31a)$$

For $x<0.01$, ignoring the summation introduces only a very small error,[11] and the equation reduces to:

$$ei(x)=-0.57721-\ln x \quad (x<0.01)\,. \qquad (5.31b)$$

where the 0.57721 is *Euler's constant*, and

$$e^{0.57721}=1.781=\gamma\,.$$

We can express Eq. (5.31b) alternatively as:

$$ei(x)=-\ln(\gamma x) \quad \text{for } x<0.01\,. \qquad (5.31c)$$

Substituting this into Eq. (5.30) we obtain:

$$p_i-p(r,t)=-\frac{q\mu}{4\pi kh}\ln\frac{\gamma\phi\mu c_t r^2}{4kt}=\frac{q\mu}{4\pi kh}\ln\frac{4kt}{\gamma\phi\mu c_t r^2}\,. \qquad (5.32a)$$

In dimensionless terms, this is:

$$p_D=\frac{1}{2}\ln\left[\frac{4}{\gamma}\left(\frac{r_w}{r}\right)^2 t_D\right]=\frac{1}{2}(\ln t_D-2\ln r_D+0.809)\,, \qquad (5.32b)$$

where:

$$\ln\frac{4}{\gamma}=\ln 2.246=0.809\,. \qquad (5.32c)$$

At the well, where $r_D=1$ $(r=r_w)$, the relationship between pressure and time is given by:

$$p_D(1,t_D)=0.5(\ln t_D+0.809)\,, \qquad (5.33a)$$

which, in real terms, is:

$$p_{wf}=p_i-\frac{q\mu}{4\pi kh}\left(\ln\frac{kt}{\phi\mu c_t r_w^2}+0.809\right)\,. \qquad (5.33b)$$

It is important to realise that Eqs. (5.33a) and (5.33b) are based on an approximation [Eq. (5.31b)] which is only valid when x (or its equivalent in the terms of the equations) <0.01, i.e.:

$$\frac{\phi\mu c_t r_w^2}{4kt}<0.01 \qquad (5.34a)$$

otherwise stated as

$$\frac{kt}{\phi\mu c_t r_w^2}>25\,. \qquad (5.34b)$$

For values of t less than that required to satisfy Eq. (5.34), the solution containing the full exponential integral should be used.

In most cases, Eq. (5.34) is usually satisfied after a few minutes, or even seconds, after a well starts flowing.

5.6.1.2 *Treatment for a Real Well – Skin Effect*

Among the initial assumptions upon which we have based the derivation of the above equations, two are of particular importance:

- the porous medium is homogeneous and isotropic in permeability,
- the well is open to production over the entire thickness of the reservoir.

These conditions are *not* met in the following situations:

- damage to the near-wellbore formation by mud filtrate invasion, causing a reduction in permeability from k to k_s out to a distance r_s,
- well only drilled through a portion of the reservoir thickness ("partial penetration"),

- well completed with casing and cement, but only perforated over a portion of the reservoir interval.
- the reservoir interval consists of a number of layers of different permeabilities; this situation is further complicated if the well is only perforated in one or some of the layers.
- presence of fractures which intersect the well, resulting in a local increase in permeability.

Each of these factors will modify the flow pattern around the wellbore, so that Eq. (5.33) will not be a true representation of the situation (Fig. 5.6). Their influence is accounted for by introducing a quantity S, the *skin factor*, into the equation:

$$p_D(1, t_D) = 0.5(\ln t_D + 0.809) + S \qquad (5.35)$$

from which we derive the following classical equations:

$$p_D(1, t_D) = 0.5(\ln t_D + 0.809 + 2S)\,, \qquad (5.36a)$$

$$p_{wf} = p_i - \frac{q\mu}{4\pi kh}\left(\ln \frac{kt}{\phi\mu c_t r_w^2} + 0.809 + 2S\right). \qquad (5.36b)$$

S is a dimensionless parameter, and therefore *just a number*. Depending on conditions, the additional pressure drop Δp_s caused by the skin:

$$\Delta p_s = \frac{q\mu}{2\pi kh} S \qquad (5.37)$$

may be *positive* [$(p_i - p_{wf})$ in the real well is *greater* than $(p_i - p_{wf})$ in the ideal well], or *negative* [$(p_i - p_{wf})$ in the real well is *less* than $(p_i - p_{wf})$ in the ideal well].

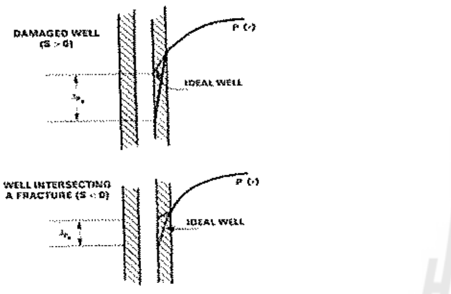

Fig. 5.6. The pressure profile close to the wellbore in the case of formation damage (S > 0), and in the case of improved productivity due to fracturing (S < 0)

Δp_s is positive in the case of blockage by mud filtrate,[10] partial penetration of the reservoir interval,[1,9] and partial perforation of the casing.[9]

In the case of filtrate damage, we have:

$$\Delta p_s = \frac{q\mu}{2\pi kh}\,\frac{k - k_s}{k_s}\,\ln\frac{r_s}{r_w}\,, \qquad (5.38a)$$

which means that:

$$S = \frac{k - k_s}{k_s}\,\ln\frac{r_s}{r_w}\,. \qquad (5.38b)$$

This damage can be reduced by means of surfactant treatment, which facilitates the removal of the filtrate.

In a reservoir section consisting of zones of differing permeabilities, only some of which have been perforated, Δp_s may be positive or negative[3] depending on whether the zones open to flow have an average permeability which is lower or higher than the average of the entire interval.

Δp_s is always negative where the well is intercepted by fractures.[4]

Where the well has been drilled through an oil-bearing formation and an underlying aquifer, the section will obviously be only partially perforated, so as to avoid producing water. We might therefore expect the skin to be positive ($\Delta p_s > 0$).

However, we have two fluids present: oil (which flows towards the well), and water (which, although it does not flow, still transmits the pressure disturbance).

If the oil has a higher viscosity than the water, Chierici[3] has shown that it is possible to find a negative skin factor (Δp_s).

We shall return to the skin factor, and its estimation from well tests, in Chap. 6.

5.6.2 Pseudo-Steady State Flow

Once the pressure disturbance has reached the outer limits of the drainage area of the well, and after a relatively short "late transient" period which follows this, the flow regime enters pseudo-steady state, during which for q = constant, dp/dt is constant at all points in the drainage area (see Sect. 5.5).

To satisfy the conservation of mass

(mass of fluid leaving the well per unit time) = (change per unit time of mass of fluid in the drainage area of the well)

we have the following relationship, which also allows for the rock compressibility:

$$q\rho = -\pi r_e^2 h \frac{\partial}{\partial t}(\rho\phi) = -\pi r_e^2 h\,\frac{d(\rho\phi)}{dp}\,\frac{\partial p}{\partial t} = -\pi r_e^2 h\phi\rho c_t \frac{\partial p}{\partial t}\,, \qquad (5.39a)$$

from which it follows that:

$$\frac{\partial p}{\partial t} = -\frac{q}{\pi r_e^2 h\phi c_t}\,, \qquad (5.39b)$$

From Eqs. (5.10) and (5.39b) we have:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = -\frac{q\mu}{2\pi r_e^2 hk} \qquad (5.40)$$

and, after first integrating with respect to r:

$$r\frac{\partial p}{\partial r} = -\frac{q\mu}{2\pi kh}\left(\frac{r}{r_e}\right)^2 + C_1\,. \qquad (5.41)$$

At $r = r_e$, $(\partial p/\partial r)_{r_e} = 0$, so that:

$$C_1 = \frac{q\mu}{2\pi kh}\,. \qquad (5.42)$$

We therefore have:

$$\frac{\partial p}{\partial r} = \frac{q\mu}{2\pi kh}\left(\frac{1}{r} - \frac{r}{r_e^2}\right). \qquad (5.43)$$

which when integrated between r_w and r (equivalent to integration between p_{wf} and p) gives:

$$p - p_{wf} = \frac{q\mu}{2\pi kh}\left(\ln\frac{r}{r_w} - \frac{r^2 - r_w^2}{2r_e^2}\right). \qquad (5.44a)$$

The term $(r_w/r_e)^2$ is negligibly small, so that we can write, after introducing the skin factor S (see Sect. 5.6.1.2):

$$p_e - p_{wf} = \frac{q\mu}{2\pi kh}\left(\ln\frac{r_e}{r_w} - \frac{1}{2} + S\right). \qquad (5.44b)$$

This equation would enable us to calculate the bottom hole flowing pressure p_{wf} in a producing well at any time, given the outer boundary pressure p_e at the same instant. This latter term is not usually known, and it is more useful to write the equation in terms of the *average* drainage area pressure $\bar{p}$, since this can usually be estimated.

From the radial symmetry of the system we have:

$$\bar{p} = \frac{\int_{r_w}^{r_e} 2\pi rhp\,dr}{\pi(r_e^2 - r_w^2)h} \qquad (5.45a)$$

or, ignoring the r_w^2 term (negligible relative to r_e^2) in the denominator:

$$\bar{p} = \frac{2}{r_e^2}\int_{r_w}^{r_e} pr\,dr\,. \qquad (5.45b)$$

Substituting for p from Eq. (5.44a):

$$\bar{p} = \frac{2}{r_e^2}\left\{p_{wf}\int_{r_w}^{r_e} r\,dr + \frac{q\mu}{2\pi kh}\left[\int_{r_w}^{r_e} r\ln\frac{r}{r_w}dr - \frac{1}{2r_e^2}\int_{r_w}^{r_e} r^3\,dr\right]\right\}, \qquad (5.45c)$$

which, when we ignore r_w^2, becomes:

$$\bar{p} = p_{wf} + \frac{q\mu}{2\pi kh}\frac{2}{r_e^2}\left(\frac{r_e^2}{2}\ln\frac{r_e}{r_w} - \frac{r_e^2}{4} - \frac{r_e^2}{8}\right). \qquad (5.45d)$$

Finally, after simplifying and adding the skin factor we get:

$$\bar{p} = p_{wf} + \frac{q\mu}{2\pi kh}\left(\ln\frac{r_e}{r_w} - \frac{3}{4} + S\right). \qquad (5.46)$$

Now, Eq. (5.46) can be used to calculate the average pressure in a circular drainage area in pseudo-steady state flow. Other geometries can be handled by rewriting Eq. (5.46) in a more general form:

$$\bar{p} = p_{wf} + \frac{q\mu}{2\pi kh}\left[\frac{1}{2}\ln\left(\frac{r_e}{r_w}\right)^2 - \frac{1}{2}\ln(e^{3/2}) + S\right]$$

$$= p_{wf} + \frac{q\mu}{4\pi kh}\left[\ln\frac{\pi r_e^2}{\pi e^{3/2} r_w^2} + 2S\right]$$

$$= p_{wf} + \frac{q\mu}{4\pi kh}\left(\ln\frac{A}{31.62 r_w^2} + 0.809 + 2S\right), \qquad (5.47)$$

where A is the area of a circle of radius r_e, and the quantity 31.62 is the value of the *Dietz shape factor* C_A for a circular geometry.

We now assume that A represents any drainage area, regardless of shape. The geometry is taken into account via the shape factor; some of the values of C_A calculated by Dietz[8] for a wide range of geometries are listed in Fig. 5.7. In the same table there are also reported values of $t_D r_w^2/A = t_{DA}$, the dimensionless time at which, for a given geometry, pseudo-steady state flow can be assumed to have developed.

With Eq. (5.47) we are able to calculate $\bar{p}$ in fields where well placement leads to non-circular drainage areas, provided of course that we know S in the well under test (see Chap. 6).

For any distribution of wells in a field, once pseudo-steady state flow conditions are established, the size of the drainage area of each well is proportional to its production offtake per unit pay thickness, q/h. This assumes that there is no variation in permeability between the wells.

Consider the case of two wells a distance d apart (Fig. 5.8):

$$\frac{r_1}{q_1/h_1} = \frac{r_2}{q_2/h_2} \qquad (5.48a)$$

and

$$r_1 + r_2 = d\,, \qquad (5.48b)$$

$$r_1 = \frac{q_1 h_2}{q_1 h_2 + q_2 h_1} d\,, \qquad (5.49a)$$

$$r_2 = \frac{q_2 h_1}{q_1 h_2 + q_2 h_1} d\,. \qquad (5.49b)$$

The average pressure of the reservoir $\bar{p}_R$ can be calculated as the volume weighted mean of the average pressures $\bar{p}_i$ of the drainage areas of the wells, the volume being the volume of oil $V_{R,i}$ present in each area:

$$\bar{p}_R = \frac{\sum_i \bar{p}_i V_{R,i}}{\sum_i V_{R,i}}\,. \qquad (5.50a)$$

To a first approximation, the production rate q_i from the ith well is proportional to the volume $V_{R,i}$ contained in its drainage area, so that we can simplify Eq. (5.50a)

	C_A	$\ln C_A$	Exact for $t_{DA}>$	Less than 1% error for $t_{DA}>$	Use infinite system solution with less than 1% error for $t_{DA}<$		C_A	$\ln C_A$	Exact for $t_{DA}>$	Less than 1% error for $t_{DA}>$	Use infinite system solution with less than 1% error for $t_{DA}<$
	31.62	3.4538	0.1	0.06	0.10		10.8374	2.3830	0.4	0.15	0.025
	31.6	3.4532	0.1	0.06	0.10		4.5141	1.5072	1.5	0.50	0.06
	27.6	3.3178	0.2	0.07	0.09		2.0769	0.7309	1.7	0.50	0.02
	27.1	3.2995	0.2	0.07	0.09		3.1573	1.1497	0.4	0.15	0.005
	21.9	3.0865	0.4	0.12	0.08		0.5813	-0.5425	2.0	0.60	0.02
	0.098	-2.3227	0.9	0.60	0.015		0.1109	-2.1991	3.0	0.60	0.005
	30.8828	3.4302	0.1	0.05	0.09		5.3790	1.6825	0.8	0.30	0.01
	12.9851	2.5638	0.7	0.25	0.03		2.6896	0.9894	0.8	0.30	0.01
	4.5132	1.5070	0.6	0.30	0.025		0.2318	-1.4619	4.0	2.00	0.03
	3.3351	1.2045	0.7	0.25	0.01		0.1155	-2.1585	4.0	2.00	0.01
	21.8369	3.0836	0.3	0.15	0.025		2.3606	0.8589	1.0	0.40	0.025

Fig. 5.7. *Values of the shape factor for different drainage area shapes, and the time limits to the validity of Eqs. (5.36) and (5.47). From Ref. 6. 1965, Society of Petroleum Engineers of AIME, reprinted by permission of the SPE*

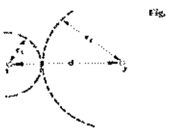

Fig. 5.8. *Drainage areas of two adjacent wells*

to:

$$\bar{p}_R = \frac{\sum_i \bar{p}_i q_i}{\sum_i q_i}\,. \qquad (5.50b)$$

5.6.3 Steady State Flow

Steady state flow conditions will occur in a reservoir only when there is an influx of water from a flanking aquifer sufficient to maintain the outer boundary pressure p_e constant with time; or when the reservoir is developed with injection of water or other fluid in such a way that the total volume of fluid (oil + injected fluid) is kept constant.

The boundary conditions applicable at the end of the transient period are:

$$p = p_e = \text{constant} \quad \text{at } r = r_e \qquad (5.51a)$$

$$\frac{\partial p}{\partial t} = 0 \quad \text{for all } r \text{ and } t \qquad (5.51b)$$

When these conditions apply, Eq. (5.10) becomes:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dp}{dr}\right) = 0\,, \qquad (5.52a)$$

$$r\frac{dp}{dr} = \frac{q\mu}{2\pi kh} = \text{const}\,. \qquad (5.52b)$$

Integrating, and introducing the skin term:

$$p - p_{wf} = \frac{q\mu}{2\pi kh}\left(\ln\frac{r}{r_w} + S\right). \qquad (5.53)$$

Following the same procedure as for pseudo-steady state flow in Sect. 5.6.2, we obtain the following equation:

$$\bar{p} = p_{wf} + \frac{q\mu}{2\pi kh}\left(\ln\frac{r_e}{r_w} - \frac{1}{2} + S\right). \qquad (5.54)$$

Table 5.1 summarises the radial inflow equations for steady and pseudo-steady state flow introduced in the preceding pages.

The C term on the right-hand side is the units conversion coefficient.

If we adhere strictly to the SI units convention, with p in MPa, μ in mPa s, then the value of C would be $10^{-7}/2\pi$. For practical metric and oilfield units systems, the values of C are listed in Table 5.1, along with the units for each parameter.

Table 5.1. *Radial inflow equations*

	Flow regime	
	Steady state	Pseudo-steady state
General equation	$p - p_{wf} = C\frac{q\mu}{kh}\left(\ln\frac{r}{r_w} + S\right)$	$p - p_{wf} = C\frac{q\mu}{kh}\left(\ln\frac{r}{r_w} - \frac{r^2}{2r_e^2} + S\right)$
Equation in terms of p_e	$p_e - p_{wf} = C\frac{q\mu}{kh}\left(\ln\frac{r_e}{r_w} + S\right)$	$p_e - p_{wf} = C\frac{q\mu}{kh}\left(\ln\frac{r_e}{r_w} - \frac{1}{2} + S\right)$
Equation in terms of $\bar{p}$	$\bar{p} - p_{wf} = C\frac{q\mu}{kh}\left(\ln\frac{r_e}{r_w} - \frac{1}{2} + S\right)$	$\bar{p} - p_{wf} = C\frac{q\mu}{kh}\left(\ln\frac{r_e}{r_w} - \frac{3}{4} + S\right)$

Parameter	Symbol	Units			
		Practical metric		Oilfield (US)	
Pay thickness	h	metres	(m)	feet	(ft)
Permeability	k	millidarcy	(md)	millidarcy	(md)
Downhole flow rate	q	cubic metres/day	(m^3/d)	barrels/day	(bbl/d)
Radii	r, r_e, r_w	metres	(m)	feet	(ft)
Viscosity	μ	centipoise	(cP)	centipoise	(cP)
Pressure	$p, p_e, p_{wf}, \bar{p}$	kilogram/square cm	(kg/cm^2)	pounds/square inch	(psi)
Constant	C	19.03		141.2	

5.7 The Principle of Superposition Applied to the Solution of the Diffusivity Equation

The linearised diffusivity equation, Eq. (5.10), and indeed all differential equations with constant coefficients, are amenable to Duhamel's theorem, which states that "a linear combination of the solutions of a differential equation is itself a solution to that equation".

In other words, if we combine – in a linear fashion – solutions to the diffusivity equation corresponding to different initial and boundary conditions, we will obtain another solution to the diffusivity equation. This is the so-called *principle of superposition*, which is so widely used in mathematical physics.

Consider the case of a well which is initially shut in ($q = 0$ at $t = 0$), with a constant pressure p_i over the whole of its drainage area.

The well is now put on production at a rate q, which is varied with time (Fig. 5.9) instead of being held constant. The flow schedule is as follows:

Flow rate	*Duration*
q_1	$t_1 - 0$
q_2	$t_2 - t_1$
q_3	$t_3 - t_2$
...	...
q_{j-1}	$t_{j-1} - t_{j-2}$
q_j	$t_j - t_{j-1}$
...	...
q_n	$t_n - t_{n-1}$

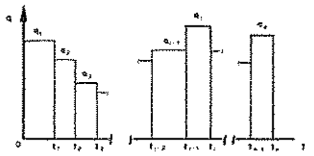

Fig. 5.9. *Schematic presentation of flow rate variations with time for use with the principle of superposition*

The same information can be presented in a different way:

Flow rate	*Duration*
q_1	$t_n - 0$
$q_2 - q_1$	$t_n - t_1$
$q_3 - q_2$	$t_n - t_2$
...	...
$q_j - q_{j-1}$	$t_n - t_{j-1}$
...	...
$q_n - q_{n-1}$	$t_n - t_{n-1}$

Using the general solution of diffusivity equation [Eq. (5.16)], written for convenience in dimensionless form, we have:

$$\frac{2\pi kh}{\mu}[p_i - p_{wf}(t_n)] = (q_1 - 0)[p_{wD}(t_{Dn} - 0) + S]$$
$$+ (q_2 - q_1)[p_{wD}(t_{Dn} - t_{D1}) + S]$$
$$+ (q_3 - q_2)[p_{wD}(t_{Dn} - t_{D2}) + S]$$
$$\vdots$$
$$+ (q_j - q_{j-1})[p_{wD}(t_{Dn} - t_{D,j-1}) + S]$$
$$\vdots$$
$$+ (q_n - q_{n-1})[p_{wD}(t_{Dn} - t_{D,n-1}) + S] \qquad (5.55a)$$

or:

$$p_{wf}(t_n) = p_i - \frac{\mu}{2\pi kh}\left\{\sum_{j=1}^{n}[(q_j - q_{j-1})p_{wD}(t_{Dn} - t_{D,j-1})] + q_n S\right\}. \qquad (5.55b)$$

Equation (5.55b) provides a generalised (and theoretically rigorous) means of interpreting the bottom hole flowing pressure response when the well is producing at any rate, or when it has been shut in after a period of production. It is also

applicable (by using a *negative q*) to the equivalent injectivity and pressure fall-off tests in wells where water is being injected for pressure maintenance or water flooding.

Equation (5.55) can therefore be used to evaluate kh, S and the drainage radius (or the distance to an impermeable barrier) from the bottom-hole flowing/shut-in pressure response and a knowledge of the rate variations.

Chapter 6 will deal exclusively with this topic. By way of an introduction to this, we will round off the present chapter with a summary of the most common types of production and injection tests in current use.

5.8 Testing Production and Injection Wells

Production wells are almost invariably tested under flowing conditions ($q > 0$). Only if the reservoir pressure is low enough for production to require artificial lift (downhole pump, gas lift), might it be preferable to conduct an injection test ($q < 0$). This would be in order to avoid mechanical difficulties associated with having the pressure gauge in the well while the artificial lift equipment is still operational, and is generally achieved by pumping down diesel or reservoir crude.

Water injection wells are usually tested by injection, although very occasionally production (back flow) tests may be performed.

Starting from stabilised conditions ($p = p_i$ over the whole drainage area at $t = 0$; $q = 0$), a well test will involve putting the well on production or injection either at a constant rate q (at downhole conditions) or, as operating conditions often dictate, at a sequence of rates $q_1, q_2, \ldots, q_n$ for time periods $t_1, (t_2 - t_1), \ldots, (t_n - t_{n-1})$.

The former case is referred to as a *constant rate test*, the latter as a *multi-rate test*.

With a well on production, the measurement of the bottom hole flowing pressure $p_{wf}(t)$ – a decreasing trend – constitutes a *pressure drawdown test*; while in the case of injection, the measurement – in this case an increasing trend – constitutes an *injectivity test*.

The well is usually shut in ($q = 0$) at the end of the production or injection period, and the bottom hole pressure measurement is continued.

When a producing well is shut in, the bottom hole pressure $p_{ws}(t)$ rises, and the pressure measurement is referred to as a *buildup test* (or PBU); while the declining pressure response which follows the shutting in of an injection well is called a *pressure fall off test* (or PFO).

The methods of obtaining kh, S and r_e or the distance to a permeability boundary from pressure measurements during production or injection (p_{wf}), and during shut-in (p_{ws}), are described in detail in Chap. 6.

It is also common practice to calculate the following three additional parameters from the test data:

Well productivity index, J:

$$J = \frac{q}{\bar{p} - p_{wf}}. \qquad (5.56a)$$

Flow efficiency (FE), *or well completion factor* (CF):

$$CF = \frac{J\ \text{real}(S \neq 0)}{J\ \text{ideal}(S = 0)} \qquad (5.56b)$$

$$= \frac{\dfrac{q}{\bar{p} - p_{wf}}}{\dfrac{q}{\bar{p} - p_{wf} - \Delta p_s}} = 1 - \frac{\Delta p_s}{\bar{p} - p_{wf}}. \qquad (5.56b')$$

Damage ratio, F_s:

$$F_s = 1 - CF = \frac{\Delta p_s}{\bar{p} - p_{wf}}. \qquad (5.56c)$$

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EXERCISES

Exercise 5.1

An oil well has been put on production at a rate $q_o = 300\ m^3/d$, measured under standard conditions. The reservoir characteristics are:

– thickness:	h	$= 20\ m$,
– porosity:	ϕ	$= 0.25$,
– permeability:	k	$= 350\ md$,
– water saturation:	S_w	$= 0.15$,
– gas saturation:	S_g	$= 0.00$,
– well radius:	r_w	$= 6''$,
– static pressure:	p_i	$= 250\ kg/cm^2$.

The reservoir fluid properties (under reservoir conditions) were as follows:

– oil volume factor:	B_o	$= 1.8$,
– oil viscosity:	μ_o	$= 0.5\ cP$,
– oil compressibility:	c_o	$= 2.5 \times 10^{-4}\ cm^2/kg$,
– water compressibility:	c_w	$= 3.0 \times 10^{-5}\ cm^2/kg$.

In addition:

– pore compressibility: $c_f = 1.4 \times 10^{-4}\ cm^2/kg$.

Calculate the time at which the approximation

$$ei(x) \simeq -\ln(\gamma x) \qquad (5.31c)$$

becomes valid for the calculation of the wellbore flowing pressure, and the pressures at a distance of 50, 100 and 150 m from the well.

Derive the flowing pressure response during the first day of production, assuming the absence of any skin factor or interference from neighbouring wells.

Solution

It was explained in Sect. 5.6.1.1 that the logarithmic approximation to the exponential integral [Eq. (5.31c)] is valid when $x < 0.01$, where x is the argument of the "exponential integral".

In this example:

$$x = \frac{\phi\mu_o c_t r^2}{4kt}$$

and therefore the condition for validity of Eq. (5.31c) is:

$$\frac{\phi\mu_o c_t r^2}{4kt} < 0.01$$

or, in other words:

$$t > 25\frac{\phi\mu_o c_t r^2}{k}.$$

All quantities are expressed in SI units in these equations. Converting to practical metric units, we have:

$$t(\text{min}) \times 60 > 25\frac{\phi[\mu_o(\text{cP}) \times 10^{-3}]\left[\dfrac{c_t(\text{cm}^2/\text{kg})}{9.80665 \times 10^4}\right]r^2(\text{m}^2)}{k(\text{md}) \times 9.86923 \times 10^{-16}},$$

which gives:

$$t(\text{min}) > 4.305 \times 10^{3}\frac{\phi\mu_o(\text{cP})c_t(\text{cm}^2/\text{kg})r^2(\text{m}^2)}{k(\text{md})}.$$

Using Eq. (5.5c) for total compressibility:

$$c_t = c_o S_o + c_w S_w + c_f$$
$$= 2.5 \times 10^{-4}(1 - 0.15) + 3.0 \times 10^{-5} \times 0.15 + 1.4 \times 10^{-4}$$
$$= 3.57 \times 10^{-4}\ \text{cm}^2/\text{kg}$$

Therefore,

$$t(\text{min}) > 4.305 \times 10^{3}\frac{[0.25 \times 0.5 \times 3.57 \times 10^{-4} \times r^2(\text{m}^2)]}{350}$$

i.e.:

$$t(\text{min}) > 0.549\,r^2(\text{m}^2).$$

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For a well radius of 6" (0.1524 m), the onset of the validity of the logarithmic approximation will be as follows:

r(m)	t(min)	
$0.1524 = r_w$	1.3×10^{-2}	= 0.78 s
50	1373	≅ 1 day
100	5490	≅ 4 days
200	21960	≅ 15 days

To calculate the bottom hole flowing pressure in the absence of skin and interference, and assuming transient radial flow, we use Eq. (5.33b):

$$p_{wf} = p_i - \frac{q\mu_o}{4\pi kh}\left(\ln\frac{kt}{\phi\mu_o c_t r_w^2} + 0.809\right).$$

Converted from SI to practical metric units, the diffusivity term is:

$$\frac{k(m^2)t(s)}{\phi\mu_o(Pa\,s)c_t(Pa^{-1})r_w^2(m^2)} = \frac{[k(md)\times 9.869233\times 10^{-16}]\times[t(min)\times 60]}{\phi[\mu_o(cP)\times 10^{-3}]\left[\frac{c_t(cm^2/kg)}{9.80665\times 10^4}\right]r_w^2(m^2)}$$

$$= 5.807\times 10^{-6}\frac{k(md)t(min)}{\phi\mu_o(cP)c_t(cm^2/kg)r_w^2(m^2)},$$

while:

$$\frac{q(m^3/s)\mu_o(Pa\,s)}{4\pi k(m^2)h(m)} = \frac{\left[\frac{q(m^3/d)}{86\,400}\right][\mu_o(cP)\times 10^{-3}]}{4\pi[k(md)\times 9.869233\times 10^{-16}][h(m)]}$$

$$= 933\,240\frac{q(m^3/d)\mu_o(cP)}{k(md)h(m)}.$$

Using the relationship:

$$1\,kg/cm^2 = 9.80665\times 10^4\,Pa,$$

Equation (5.33) becomes:

$$p_{wf}(kg/cm^2) = p_i(kg/cm^2) - 9.516\frac{q_o(m^3/d)B_o\mu_o(cP)}{k(md)h(m)}$$

$$\times\left(\ln 5.807\times 10^{-6}\frac{k(md)t(min)}{\phi\mu_o(cP)c_t(cm^2/kg)r_w^2(m^2)} + 0.809\right).$$

Here, we have introduced the volume factor B_o to convert the surface oil production rate q_o to a downhole rate. Implementing the units coefficients, and changing from the natural logarithm to $\log_{10}$, the equation can now be written:

$$p_{wf} = p_i - 21.912\frac{q_o B_o\mu_o}{kh}\left[\log\frac{kt}{\phi\mu_o c_t r_w^2} - 4.885\right].$$

For the well in this exercise, we have:

$$p_{wf} = 250 - 21.912\frac{300\times 1.8\times 0.5}{350\times 20}\left[\log\frac{350\cdot t}{0.25\times 0.5\times 3.57\times 10^{-4}\times 0.1524^2} - 4.885\right],$$

so that:

$$p_{wf}(kg/cm^2) = 250 - 0.8451[\log(3.38\times 10^8 t) - 4.885].$$

At various times, this works out as follows:

t	p_{wf}(kg/cm²)
1'	246.92
10'	246.08
30'	245.67
1 h	245.42
3 h	245.01
6 h	244.76
12 h	244.51
18 h	244.36
24 h	244.25

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Exercise 5.2

A second well is drilled 200 m from the well of Ex. 5.1.

Geological evidence has lead to some doubt as to whether the reservoir is continuous between the two wells; there is the possibility that a sealing fault is preventing hydraulic communication.

An extended production test is performed in the *first* (Ex. 5.1) well, at a surface rate of 300 m³/d. The second well is shut in, and its bottom hole pressure is recorded to ascertain whether any pressure signal can be detected from the producing well, indicating at least some degree of communication between the two.

Assuming the pressure gauge used has a resolution of 0.2 kg/cm², and that the properties of the reservoir rock are constant, estimate how long the test must be run in order to determine that there is (perfect) hydraulic continuity between the wells.

Solution

From Eq. (5.30):

$$\frac{4\pi kh}{q\mu_o}[p_i - p(r,t)] = \int_x^\infty \frac{e^{-s}}{s}ds = ei(x),$$

we have, after converting from SI to practical metric units:

$$\frac{4\pi[k(md)\times 9.869233\times 10^{-16}]h(m)}{\frac{q_o(m^3/d)B_o}{86\,400}}[p_i - p(r,t)](kg/cm^2)\times 9.80665\times 10^4 = ei(x),$$

which gives:

$$0.1051\frac{k(md)h(m)}{q_o(m^3/d)B_o\mu_o(cP)}[p_i - p(r,t)](kg/cm^2) = ei(x).$$

We want to see a pressure decrease of 0.2 kg/cm² at a distance of $r = 200$ m from the producing well.

In other words:

$$p_i - p(200,t) = 0.2\,kg/cm^2$$

Using the data from Ex. 5.1, we therefore have:

$$ei(x) = 0.1051\frac{350\times 20\times 0.2}{300\times 1.8\times 0.5} = 0.545.$$

From Fig. 5.5 we have, for $ei(x) = 0.545$,

$$x = 0.51$$

and, from Eq. (5.22a),

$$x = y = \frac{\phi\mu_o c_t r^2}{4kt},$$

which, in practical metric units is:

$$x = \frac{\phi[\mu_o(cP)\times 10^{-3}]\left[\frac{c_t(cm^2/kg)}{9.80665\times 10^4}\right]r^2(m^2)}{4[k(md)\times 9.869233\times 10^{-16}][t(min)\times 60]}$$

$$= 43\,051\frac{\phi\mu_o(cP)c_t(cm^2/kg)r^2(m^2)}{k(md)t(min)},$$

so that:

$$x = 0.51 = 43\,051\left[\frac{0.25\times 0.5\times 3.57\times 10^{-4}\times 200^2}{350\,t(min)}\right].$$

Therefore:

$$t = 430\,min \cong 7\,h$$

If there is perfect hydraulic continuity between the two wells, we should measure a decrease of 0.2 kg/cm² in the shut in well after 7 h of production from the other well. In practical terms, we should have a measurable signal exceeding gauge error in under 12 h.

Referring to Ex. 5.1, note that this time is considerably less than that required for the logarithmic approximation to be valid at $r = 200$ m.

If instead of Eq. (5.30) we had written:

$$\frac{4\pi kh}{q\mu_o}[p_i - p(r,t)] = \ln\frac{kt}{\phi\mu_o c_t r^2} + 0.809,$$

we would therefore have had a serious error in our calculated time.

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Exercise 5.3

A field has been developed by drilling wells in a regular pattern, such that each well lies at the centre of a rectangle measuring 100 m by 400 m.

The drainage area of each well is therefore rectangular in shape, (100 × 400) m², with the well at the centre.

The oil from this field has the following properties:

- bubble point pressure (at reservoir T): $p_b = 160\,kg/cm^2$,
- volume factor at current average pressure: $B_o = 1.15$,
- viscosity at current average pressure: $\mu_o = 35\,cP$.

A production test is run in one of the wells, completed with a 9-⅝" casing perforated across the entire reservoir interval. In this well, the formation characteristics are:

- thickness: $h = 120\,m$,
- permeability: $k = 850\,md$.

There is a certain amount of mud filtrate damage to the near-wellbore formation, which has reduced the local permeability to $k_s = 200\,md$ out to a radius of 1 m.

The well test, an extended drawdown in pseudo-steady state flow, gave the following results:

- oil flow rate (surface): $q_o = 150\,m^3/d$ (standard conditions),
- bottom hole flowing pressure: $p_{wf} = 185\,kg/cm^2$.

Calculate the productivity index J, the completion factor CF, and the damage ratio F_s.

Solution

The first thing to note is that the bottom hole flowing pressure is higher than the bubble point, so we have single phase flow of undersaturated oil.

The skin damage effect is quantified as (Eq. (5.38b)):

$$S = \frac{k - k_s}{k_s}\ln\frac{r_s}{r_w}.$$

Since

$$r_w = 0.5(9\tfrac{5}{8}") = 0.122\,m,$$

we have,

$$S = \frac{850 - 200}{200}\ln\frac{1}{0.122} = 6.84.$$

With the rate expressed at reservoir conditions, Eq. (5.47) is:

$$\bar{p} - p_{wf} = \frac{qB_o\mu_o}{4\pi kh}\left[\ln\frac{A}{C_A r_w^2} + 0.809 + 2S\right].$$

Converting from SI units to practical metric:

$$[\bar{p} - p_{wf}](kg/cm^2)\times 9.80665\times 10^4 = \frac{\frac{q(m^3/d)}{86\,400}B_o[\mu_o(cP)\times 10^{-3}]}{4\pi[k(md)\times 9.869233\times 10^{-16}]h(m)}$$

$$\times\left[\ln\frac{A}{C_A r_w^2} + 0.809 + 2S\right],$$

which, replacing the natural log by $\log_{10}$, is:

$$[\bar{p} - p_{wf}](kg/cm^2) = 21.912\frac{q(m^3/d)B_o\mu_o(cP)}{k(md)h(m)}\left[\log\frac{A}{C_A r_w^2} + 0.35134 + 0.8686\,S\right].$$

Referring to Fig. 5.7, the Dietz shape factor for the drainage area (4:1 rectangle) with the well in the centre is:

$$C_A = 5.379.$$

Therefore:

$$\bar{p} - p_{wf} = 21.912\frac{150\times 1.15\times 35}{850\times 120}\left[\log\frac{400\times 100}{5.379\times 0.122^2} + 0.35134 + 0.8686\times 6.84\right] = 15.56\,kg/cm^2,$$

so that the average pressure of the drainage area is:

$$\bar{p} = p_{wf} + 15.56 = 185 + 15.56$$

$$= 200.56\,kg/cm^2.$$

The pressure drop Δp_s caused by the skin damage zone is defined in Eq. (5.37). In practical metric units, this is:

$$\Delta p_s(kg/cm^2) = 19.031\frac{q(m^3/d)B_o\mu_o(cP)}{k(md)h(m)}S$$

1

so that, in this example:

$$\Delta p_s = 19.033 \frac{(150 \times 1.15 \times 35)}{850 \times 120} 6.84 = 7.71\ \text{kg/cm}^2 .$$

Finally, we can calculate:

Productivity index:

$$J = \frac{q}{\bar{p} - p_{wf}} = \frac{150}{15.56} = 9.64 \frac{\text{m}^3}{\text{d} \times \text{kg/cm}^2} .$$

Completion factor:

$$\text{CF} = 1 - \frac{\Delta p_s}{\bar{p} - p_{wf}} = 1 - \frac{7.71}{15.56} = 50.4\% .$$

Damage ratio:

$$F_s = 1 - \text{CF} = 49.6\% .$$

Note that the productivity index could be almost *doubled* by eliminating the skin damage (e.g. surfactant treatment of the damaged zone). As a consequence, *twice the production rate could be obtained for the same bottom-hole flowing pressure.*

Equation (7.36) can be used to find the pressure drop $(p_i - p)$ that will have occurred at any radius about a flowing well after the well has flowed at a rate, q, for some time, t. For example, consider a reservoir where oil is flowing and $\mu_o = 0.72$ cp; $B_o = 1.475$ bbl/STB; $k = 100$ md; $h = 15$ ft; $c_t = 15 \times 10^{-6}$ psi^{-1}; $\phi = 23.4\%$; $p_i = 3000$ psia. After a well is produced at 200 STB/day for 10 days, the pressure at a radius of 1000 ft will be:

$$p = 3000 - \frac{70.6(200)(0.72)(1.475)}{100(15)}\left[-E_i\left(-\frac{0.234(0.72)15(10)^{-6}(1000)^2}{0.00105(100)(10)(24)}\right)\right]$$

Then

$$p = 3000 + 10.0\, E_i(-0.10)$$

From Fig. 7.11, $E_i(-0.10) = -1.82$. Therefore

$$p = 3000 + 10.0\,(-1.82) = 2981.8 \text{ psia}$$

$$p(r,t) = p_i - \frac{70.6 q \mu B}{kh}\left[-E_i\left(-\frac{\phi \mu c_t r^2}{0.00105\, kt}\right)\right]$$

Figure 7.12 shows this pressure plotted on the 10-day curve and shows the pressure distributions at 0.1, 1.0, and 100 days for the same flow conditions.

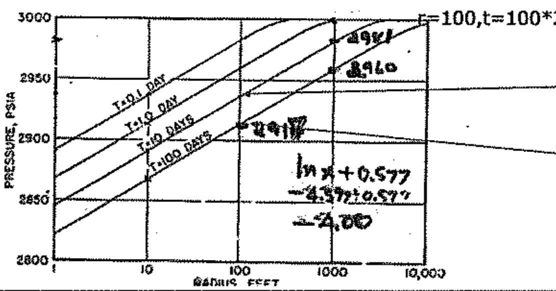
It has been shown that for values of the E_i function argument less than 0.01 the following approximation can be made:

$$-E_i(-x) = -\ln(x) - 0.5772$$

r=1000,t=100*24, x=0.001,E= -4.027970186

r=100,t=10*24, x=0.001,E= -6.330555279

r=100,t=100*24, x=0.0001,E= -8.633140372



2937=3000-63

2914=3000-86

$$\frac{}{0.00105kt} < 0.01$$

rearranging the equation and solving for t, the time required to make this approximation valid for the pressure determination 1000 ft from the producing well can be found:

$$t > \frac{0.234(0.72)15(10)^{-6}(1000)^2}{0.00105(100)(0.01)} = 2400 \text{ hr} = 100 \text{ days}$$

To determine if the approximation to the E_i function is valid when calculating the pressure at the sandface of a producing well, it is necessary to assume a wellbore radius, r_w, (0.25 ft) and to calculate the time that would make the approximation valid. The following is obtained:

$$t > \frac{0.234(0.72)15(10)^{-6}(0.25)^2}{0.00105(100)(0.01)} = 0.0002 \text{ hours}$$

It is apparent from these calculations that whether the approximation can be used is a strong function of the distance from the pressure disturbance to the point at which the pressure determination is desired or, in this case, from the producing well. For all practical purposes, the assumption is valid when considering pressures at the point of the disturbance. Therefore, at the wellbore and wherever the assumption is valid, Eq. (7.36) can be rewritten as:

$$p(r,t) = p_i - \frac{70.6q\mu B}{kh}\left[-\ln\left(\frac{\phi\mu c_t r^2}{0.00105kt}\right) - 0.5772\right]$$

Substituting the log base 10 into this equation for the ln term, rearranging and simplifying, one gets:

$$p(r,t) = p_i - \frac{162.6\,q\mu B}{kh}\left[\log\left(\frac{kt}{\phi\mu c_t r^2}\right) - 3.23\right] \quad (7.37)$$

6.2. Radial Flow of Compressible Fluids, Transient Flow

In Sect. 5, Eq. (7.32)

$$\frac{0.234}{r}\frac{\partial}{\partial r}\left(0.001127\frac{k}{\mu}\rho r\frac{\partial p}{\partial r}\right) = \rho\phi c_f\frac{\partial p}{\partial t} + \phi\frac{\partial \rho}{\partial t} \quad (7.32)$$

was developed to describe the flow of any fluid flowing in a radial geometry in porous media. To develop a solution to Eq. (7.32) for the compressible fluid, or gas, case, two additional equations are required: (1) an equation of state, usually the real gas law, which is Eq. (1.7); and (2) Eq. (1.19), which describes how the gas isothermal compressibility varies with pressure:

$$pV = znR'T \quad (1.7)$$

$$c_g = \frac{1}{p} - \frac{1}{z}\frac{dz}{dp} \quad (1.19)$$

These three equations can be combined to yield

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{p}{\mu z}\frac{\partial p}{\partial r}\right) = \frac{\phi c_t p}{0.0002637kz}\frac{\partial p}{\partial t} \quad (7.38)$$

Al-Hussainy, Ramey, and Crawford and Russel, Goodrich, Perry, and Bruskotter introduced a transformation of variables to obtain a solution to Eq. (7.38).[7,8] The transformation involves the real gas pseudopressure, $m(p)$, which has units of psia²/cp in standard field units and is defined as:

$$m(p) = 2\int_{p_b}^{p}\frac{p}{\mu z}dp \quad (7.39)$$

where p_R is a reference pressure, usually chosen to be 14.7 psia, from which the function is evaluated. Since μ and z are only functions of pressure for a given reservoir system, which we have assumed to be isothermal, Eq. (7.39) can be differentiated and the chain rule of differentiation applied to obtain the following relationships:

$$\frac{\partial m(p)}{\partial p} = \frac{2p}{\mu z} \tag{7.40}$$

$$\frac{\partial m(p)}{\partial r} = \frac{\partial m(p)}{\partial p}\frac{\partial p}{\partial r} \tag{7.41}$$

$$\frac{\partial m(p)}{\partial t} = \frac{\partial m(p)}{\partial p}\frac{\partial p}{\partial t} \tag{7.42}$$

Substituting Eq. (7.40) into Eq. (7.41) and (7.42) yields

$$\frac{\partial p}{\partial r} = \frac{\mu z}{2p}\frac{\partial m(p)}{\partial r} \tag{7.43}$$

$$\frac{\partial p}{\partial t} = \frac{\mu z}{2p}\frac{\partial m(p)}{\partial t} \tag{7.44}$$

Combining Eq. (7.43), (7.44), and (7.38) yields

$$\frac{\partial^2 m(p)}{\partial r^2} + \frac{1}{r}\frac{\partial m(p)}{\partial r} = \frac{\phi \mu c_t}{0.0002637k}\frac{\partial m(p)}{\partial t} \tag{7.45}$$

Equation (7.45) is the diffusivity equation for compressible fluids, and it has

Equation (7.45) is still a nonlinear differential equation because of the dependence of μ and c_t on pressure or the real gas pseudopressure. Thus, there is no analytical solution for Eq. (7.45). Al-Hussainy and Ramey, however, used finite difference techniques to obtain an approximate solution to Eq. (7.45).[9] The result of their studies for pressures at the wellbore (i.e., where the logarithm approximation to the E_i function can be made) is the following equation:

$$m(p_{wf}) = m(p_i) - \frac{1637(10)^3 qT}{kh}\left[\log\left(\frac{kt}{\phi \mu_i c_{ti} r_w^2}\right) - 3.23\right] \tag{7.46}$$

interest, p_1. The value of $m(p_1)$ that corresponds with pressure, p_1, is given by:

$$m(p_1) = 2\,(\text{area}_1)$$

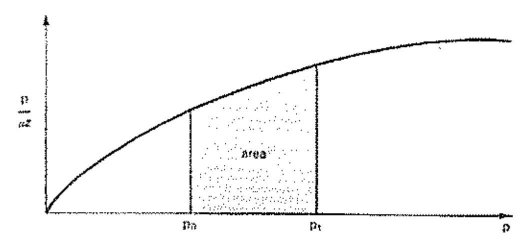


Fig. 7.13. Graphical determination of $m(p)$.

where

$$\text{area}_1 = \int_{p_R}^{p_1} \frac{p}{\mu z}\,dp$$

TABLE 7.2
Shape factors for various single-well drainage areas (after Earlougher[?])

In Bounded Reservoirs	C_A	$\ln C_A$	$\frac{1}{2}\ln\left(\frac{2.2458}{C_A}\right)$	Exact for t_{DA} >	Less than 1% Error for t_{DA} >	Use Infinite System Solution with Less than 1% Error for t_{DA} <
	31.62	3.4538	−1.3224	0.1	0.06	0.10
	31.6	3.4532	−1.3220	0.1	0.06	0.10
	27.6	3.3178	−1.2544	0.2	0.07	0.09
	27.1	3.2995	−1.2452	0.2	0.07	0.09
	21.9	3.0865	−1.1387	0.4	0.12	0.08
	0.098	−2.3223	1.5659	0.9	0.07	0.015
	30.8829	3.4302	−1.3106	0.1	0.05	0.09
	12.9851	2.5638	−0.8774	0.7	0.25	0.03
	4.5132	1.5070	−0.3490	0.6	0.30	0.025
	3.3351	1.2045	−0.1977	0.7	0.25	0.01
	21.8369	3.0836	−1.1373	0.3	0.15	0.025
	10.8374	2.3830	−0.7870	0.4	0.15	0.025
	4.5141	1.5072	−0.3495	1.5	0.50	0.06
	2.0769	0.7309	0.0391	1.7	0.50	0.02
	3.1573	1.1497	−0.1703	0.4	0.15	0.005

7. PSEUDOSTEADY-STATE FLOW SYSTEMS

7.1. Radial Flow of Slightly Compressible Fluids, Pseudosteady-State Flow

condition used to find a solution to the radial diffusivity equation is that the outer boundary of the reservoir is a no-flow boundary. In mathematical terms,

$$\frac{\partial p}{\partial r} = 0 \quad \text{at } r = r_e$$

Eq. 7.35

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu c_t}{0.0002637k}\frac{\partial p}{\partial t} \qquad (7.35)$$

Continuity Eq.

Applying these conditions to Eq. (7.35), the solution for the pressure at the wellbore becomes

$$p_{wf} = p_i - \frac{162.6 q \mu B}{kh} \log\left[\frac{4A}{1.781 C_A r_w^2}\right] - \frac{0.2339 qBt}{Ah\phi c_t} \qquad (7.47)$$

where A is the drainage area of the well in square feet and C_A is a reservoir shape factor. Values of the shape factor are given in Table 7.2 for several

After reaching pseudosteady-state flow, the pressure at every point in the reservoir is changing at the same rate, which suggests that the average reservoir pressure is also changing at the same rate. The volumetric average reservoir pressure, which is usually designated as $\bar{p}$ and is the pressure used to calculate fluid properties in material balance equations, is defined as:

$$\bar{p} = \frac{\sum_{j=1}^{n} \bar{p}_j V_j}{\sum_{j=1}^{n} V_j} \qquad (7.48)$$

where p_j is the average pressure in the jth drainage volume and V_j is the volume of the jth drainage volume. It is useful to rewrite Eq. (7.47) in terms of the average reservoir pressure, $\overline{p}$:

$$p_{wf} = \overline{p} - \frac{162.6 q \mu B}{kh} \log\left[\frac{4A}{1.781 C_A r_w^2}\right] \quad (7.49)$$

For a well in the center of a circular reservoir with a distance to the outer boundary of r_e, Eq. (7.49) reduces to:

$$p_{wf} = \overline{p} - \frac{70.6 q \mu B}{kh}\left[\ln\left(\frac{r_e^2}{r_w^2}\right) - 1.5\right] = \overline{p} - \frac{141.2 q \mu B}{kh}\left[\ln \frac{r_e}{r_w} - 0.75\right]$$

If this equation is rearranged and solved for q,

$$q = \frac{0.00708kh}{\mu B}\left[\frac{\overline{p} - p_{wf}}{\ln(r_e/r_w) - 0.75}\right] \quad (7.50)$$

7.2. Radial Flow of Compressible Fluids, Pseudosteady-State Flow

The differential equation for the flow of compressible fluids in terms of the real gas pseudopressure was derived in Eq. (7.45). When the appropriate boundary conditions are applied to Eq. (7.45), the pseudosteady-state solution rearranged and solved for q yields Eq. (7.51):

$$q = \frac{19.88(10)^{-6} khT_{sc}}{Tp_{sc}}\left[\frac{m(\overline{p}) - m(p_{wf})}{\ln(r_e/r_w) - 0.75}\right] \quad (7.51)$$

$10^6 \times 14.7 = 1422$

4.5 GAS FLOW EQUATION SUMMARY

Writing $C = 0.703\, kh/[\mu Tz\ln(r_e/r_w)]$, and the power n is turbulent flow effect in steady state gas flow, Equation (4.27) becomes

$$Q_t = C(p_e^2 - p_{wf}^2)^n \quad (4.44)$$

This equation is normally called the empirical back pressure equation. The power n is the turbulent effect which is ranging from low flow rate = 1.00 to absolutely turbulent = 0.500. The equation is not especially helpful in predicting reservoir characteristics or in analyzing the components of pressure drops, although it may be useful in characterizing well performance.

When multirate data are available it is more useful to revert to one of the basis flow equations in field units:

$$\overline{p}_r^2 - P_{wf}^2 = \left(\frac{1422 Q_g \overline{\mu z} T}{kh}\right)\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right] \quad (4.45)$$

Or

$$\overline{p}_r^2 - P_{wf}^2 = \left(\frac{1422 Q_g \overline{\mu z} T}{kh}\right)\left[\ln\left(\frac{0.472 r_e}{r_w}\right) + S\right] + BQ^2 \quad (4.46)$$

(semi-steady state)

Or

$$\overline{p}_r^2 - P_{wf}^2 = \left(\frac{1422 Q_g \overline{\mu z} T}{kh}\right)\left[\frac{1}{2}\ln(t_D + 0.809) + S\right] + BQ^2 \quad (4.47)$$

(transient)

$\Delta(p^2) = AQ + BQ^2$

Where

$$A = \left(\frac{1422 Q_g \overline{\mu z} T}{kh}\right)\left[\ln\left(\frac{0.472 r_e}{r_w}\right) + S\right] \text{(semi-steady state)}$$

$$= \left(\frac{1422 Q_g \overline{\mu z} T}{kh}\right)\left[\frac{1}{2}\ln\left(t_D + 0.809\right) + S\right] \text{(transient)}$$

And B = non-Darcy coefficient.

It is more appropriate to use the real gas pseudo-pressure *m(p)* equation:

$$m(\bar{p}_r) - m(p_{wf}) = \left(\frac{1422 Q_g T}{kh}\right)\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right] + FQ^2$$

$$= \left(\frac{1422 Q_g T}{kh}\right)\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S + DQ\right] \quad (4.48)$$

Where

$$D = \left(\frac{Fkh}{1422T}\right)$$

And DQ is known as the rate dependent skin factor

dp/dt at all points. If the pressure is declining at the same rate, then the wellbore flow rate is

$$q_w = c_e \times \pi r_e^2 h\phi \times \frac{dp}{dt}$$

The flow crossing any circumference of radius r will be proportional to the fluid volume between r and r_e, or

$$q = c_e \times \pi(r_e^2 - r^2)h\phi \times \frac{dp}{dt}$$

Dividing and solving for q in terms of q_w

$$q = q_w\left(1 - \frac{r^2}{r_e^2}\right)$$

Substituting this value in Darcy's law and writing $A = 2\pi rh$,

$$q_w\left(1 - \frac{r^2}{r_e^2}\right) = -1.127\frac{k}{\mu}(2\pi rh)\frac{dp}{dr}$$

Separating variables and integrating between any two radii r_1 and r_2 where the pressures are p_1 and p_2,

$$\int_{r_1}^{r_2}\left(1 - \frac{r^2}{r_e^2}\right)\frac{dr}{r} = -\frac{1.127\,k2\pi h}{\mu q_w}\int_{p_1}^{p_2} dp$$

$$q_w = \frac{7.08\,kh(p_2 - p_1)}{\mu[\ln(r_2/r_1) - r_2^2/2r_e^2 + r_1^2/2r_e^2]}$$

For $r_1 = r_w$ and $r_2 = r_e$ and $q_w = q_o B_o$, for $r_e \gg r_w$

$$q_o = \frac{7.08\,kh(p_e - p_w)}{\mu B_o[\ln(r_e/r_w) - \frac{1}{2}]} \quad (6.25)$$

Figure 6.17 shows the comparison between the pressure distributions given by Eqs. (6.24) and (6.25) for the same wellbore pressure. Near the well bore there is little difference between the two. A larger *drawdown*, i.e., $(p_e - p_w)$, exists for the case of flow across the external boundary, Eq. (6.24), because all of the fluid must travel from the external radius to the well bore. It should be realized that the difference between Eqs. (6.24) and (6.25) is not that of incompressible fluid versus compressible liquid, but the difference between flow and no flow across the external boundary. As pointed out previously, the incompressible fluid was used in Eq. (6.24) because it gives a much simpler form than a similar equation for the compressible liquid, and the difference in results between the two is negligible.

Conditions of strict applicability of Eqs. (6.24) and (6.25) are seldom met in petroleum reservoirs; however, they are commonly used as approxi-

engineering purposes. Since most of the pressure drop occurs in t vicinity of the well bore, where the flow is always quite close to radial, conditions at or near the external boundaries are of less importance. addition, many uses involve a study of the "before" and "after" conditi of particular wells or between wells similarly situated in the same reserv so that comparative results are more significant than absolute results. T use of the equations may be extended, with reduced accuracy, to flow bel the bubble point. They also apply equally well to the injection of fluid. this case the pressure will be higher in the well than at the drainage radi

The concept of the *drainage radius* is somewhat vague and is sometim misused in connection with these radial flow equations. Equation (6. implies either an infinite drainage radius, or, what is equivalent, maintenance of the pressure p_e at some external radius r_e. The use of t equation to demonstrate effective or efficient drainage of a given area is, itself, therefore of limited validity; for actually it implies that one well drain an infinite area. Both Eqs. (6.24) and (6.25) do indicate the establi ment of a pressure gradient at any radius, however distant, but the eff of the pressure gradient on flow and multiphase displacement is not together obvious.

There is another aspect of the drainage radius concept which will come apparent in the study of unsteady-state radial flow. When a sin well is placed on production, a drainage radius which increases with ti results. After a period of time, the pressure distribution about the w approaches that given by Eq. (6.24) where the reservoir pressure is reas ably near stabilization, or Eq. (6.25) for closed or bounded reservoirs. the case of water-drive reservoirs, the drainage radius extends out into aquifer until an impermeable boundary is reached.

13. Average Pressures in Radial Flow Systems and the Readju ment Time. The volumetric average pressure for a radial system may expressed by the integral

$$p_{avg} = \frac{\int p\,dV}{V} = \frac{\int_{r_w}^{r_e} p \times 2\pi rh\phi\,dr}{\pi r_e^2 h\phi} = \frac{2\int_{r_w}^{r_e} pr\,dr}{r_e^2}$$

For the incompressible system, the pressure at any radius is given by (6.24) as

$$p = p_w + \frac{\mu q_o B_o \ln(r/r_w)}{7.08\,kh}$$

Then

Integrating between limits,

$$p_{avg} = \frac{2\,p_w}{r_e^2}\left[\frac{r_e^2}{2} - \frac{r_w^2}{2}\right] + \frac{2\,\mu q_w B_o}{7.08\,khr_e^2}\left[\frac{r_e^2}{2}\ln\left(\frac{r_e}{r_w}\right) - \frac{r_e^2}{4} + \frac{r_w^2}{4}\right]$$

But $r_w^2 \ll r_e^2$, therefore the average pressure of a radial incompressible fluid flow system is closely approximated by

$$p_{avg} = p_w + \frac{\mu q_w B_o}{7.08\,kh}\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2}\right] \qquad (6.26)$$

The average pressure of a bounded compressible liquid radial flow system may be found similarly by using the pressure from Eq. (6.25), or

$$p = p_w + \frac{\mu q_w B_o}{7.08\,kh}\left[\ln\left(\frac{r}{r_w}\right) - \frac{r^2}{2r_e^2}\right]$$

The average pressure is found from

$$p_{avg} = \frac{2\,p_w}{r_e^2}\int_{r_w}^{r_e} r\,dr + \frac{2\,\mu q_w B_o}{7.08\,khr_e^2}\left[\int_{r_w}^{r_e}\ln\left(\frac{r}{r_w}\right)r\,dr + \int_{r_w}^{r_e}\frac{r^3 dr}{2\,r_e^2}\right]$$

Integrating between limits

$$p_{avg} = p_w + \frac{\mu q_w B_o}{7.08\,kh}\left[\ln\frac{r_e}{r_w} - \frac{3}{4}\right] \qquad (6.27)$$

Thus the difference between the average pressure of an incompressible and the bounded compressible liquid radial flow systems, Eq. (6.26) minus Eq. (6.27) is

$$\Delta p_{avg} = \frac{\mu q_w B_o}{28.32\,kh} \qquad (6.28)$$

The difference for a 2.0 cp oil with a volume factor of 1.25 bbl/STB, flowing at the rate of 200 BOPD through a formation of 200 md permeability and 25 ft thickness, is

$$\Delta p_{avg} = \frac{2.0 \times 200 \times 1.25}{28.32 \times 0.200 \times 25} = 3.5 \text{ psi}$$

Unsteady-state *compressible* liquid studies show that when a well is first placed on production at a constant rate, a pressure transient travels outward into the reservoir as indicated in Fig. 6.39. Further, the pressure distribution at any time is closely represented by Eq. (6.24), which is, however, for the *incompressible* fluid. At any time and corresponding drainage radius, the amount of fluid which has been removed to develop the pressure distribution is proportional to (a) the liquid compressibility, (b) the volume of fluid contained in the drainage area, and (c) the drop in

$$\Delta V = cV\Delta p$$

$$= c\left(\frac{\pi r_e^2 h\phi}{5.615}\right)(p_e - p_{avg}), \text{ barrels}$$

But $\qquad (p_e - p_{avg}) = (p_e - p_w) - (p_{avg} - p_w)$

Substituting from Eqs. (6.24) and (6.26)

$$(p_e - p_{avg}) = \frac{q_w\mu B_o \ln (r_e/r_w)}{7.08\,kh} - \frac{q_w\mu B_o[\ln (r_e/r_w) - \frac{1}{2}]}{7.08\,kh}$$

$$= \frac{q_w\mu B_o}{14.16\,kh}$$

Therefore,

$$\Delta V = c \times \frac{\pi r_e^2 h\phi}{5.615} \times \frac{q_w\mu B_o}{14.16\,kh}$$

The *readjustment time* t_R in days is the time required to create a logarit pressure distribution out to any transient drainage radius r_e. It is obta by dividing the volume ΔV, produced at the well by liquid expansion, b reservoir flow rate $q_w B_o$ barrels per day, or

$$t_R = \frac{\Delta V}{q_w B_o} = \frac{0.04\,\mu c\phi r_e^2}{k} \qquad ($$

This equation implies that t_R days will be required to establish a logarit pressure distribution as expressed by Eq. (6.24), between the well and any transient radius r_e, for a compressible liquid system in which a is first opened to flow at any constant rate. The readjustment tin evidently independent of the rate. The equation does not prove that pressure distribution at any time is closely represented by Eq. (6. however, studies to be presented in Sec. 20 indicate that this is the

If the average pressure for a bounded system Eq. (6.27) is used in p of Eq. (6.26) to derive the equation for the readjustment time, the s Eq. (6.29) is obtained except that the constant is 0.02 instead of (Since the equations are only used for *order of magnitude* estimates classifying radial flow systems as steady-state or unsteady-state, and s the form with the constant 0.04 appears more often in the literature, used in Sec. 4 and other places in this text.

14. Productivity Index. The ratio of the rate of production, expre in stock tank barrels per day, to the *pressure drawdown* at the midpoi the producing interval is called the *productivity index*, symbol J.

are not so susceptible to mathematical calculation. Many of these can be handled successfully by means of an electric analog model which is based on the analogy between Ohm's law and Darcy's law. Since electricity in straight resistance circuits behaves as an incompressible fluid, the results, strictly speaking, apply only to incompressible fluid flow in rocks. The electrical resistance R of a linear body is proportional to its length and inversely proportional to its cross section, the constant of proportionality being the resistivity ρ. Then, since the electrical current I is the voltage drop ΔE divided by the resistance R,

$$I = \frac{\Delta E}{R} = \frac{\Delta E}{\rho L/A} = \frac{A\Delta E}{\rho L} \qquad (6.37)$$

Darcy's law for linear flow is

$$q = \frac{kA\Delta p}{\mu L}$$

An inspection of these equations shows the analogy between flow rate and current, pressure drop and voltage drop, the fluid mobility (k/μ) and the reciprocal of the resistivity $(1/\rho)$, i.e., the electrical conductivity. The electrical *conductance*, $A/\rho L$, which is the reciprocal of the *resistance* is analogous to the *permeance*, $(kA/\mu L)$, of the fluid system. The term permeance is not generally used in reservoir engineering, but rather its equivalent $(q/\Delta p)$, the productivity (index) of a system. Thus in the use of steady-state electric models, the analogy is between the electrical conductance $(1/R)$ and the productivity $(q/\Delta p)$. Although the analogy has been shown only for linear systems it will hold for any other geometry where the electrical model is geometrically scaled to the portion of a reservoir it represents. The analogy between radial flow systems is apparent from a comparison of Eq. (6.24) and the similar equation for radial flow of electricity,

$$I = \frac{2\,\pi h\Delta E}{\rho \ln (r_e/r_w)} \qquad (6.38)$$

Let us now illustrate the use of the electrical model to the somewhat complex problem of well perforation. It is apparent that the productivity index of a perforated completion depends upon the number of perforations, the diameter of the perforations, the depth of penetration of the perforations, and also the distribution of the perforations. It should also be apparent that a mathematical treatment of the problem is rather complex. Suppose, however, we fill a plastic, i.e., electrically nonconductive tank of 36-inch diameter to a depth of 12 inches with a liquid of resistivity ρ, as shown in Fig. 6.22. Let us place a two-inch diameter copper electrode vertically at the center, and a 36-inch diameter copper electrode at the periphery of the tank. This system then represents the standard well

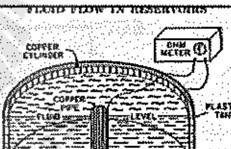


Fig. 6.22. Radial flow electric model for well productivity studies.

system, i.e., an open hole completion. Suppose the measured resistance between the center electrode and the outer electrode is 10.0 ohms. Now let us replace the central electrode by a two-inch diameter plastic (nonconductive) pipe through which extend 24 copper electrodes one-eighth inch in diameter and extending one-half inch beyond the plastic pipe, as shown in Fig 6.23, to represent a cased hole completed by the perforation

Fig. 6.23. Well electrode simulating perforated casing for electric model studies.

technique. With all the copper electrodes (perforations) connected together electrically inside the pipe, let us again measure the resistance between the perforations and the outer electrode. Suppose, with the same electrolyte at the same temperature, we measure 16.9 ohms.

From the above data it might be concluded that the productivity ratio of the perforated well is 10/16.9 or 0.592. However, it should be realized that the 36-inch model is not sufficiently large to represent any reasonable drainage radius, and, further, that it is impractical to do so. It is sufficiently accurate to assume that had the model been 3960 inches in diameter, the flow, even with the perforated well electrode, would be essentially radial from the 18-inch radius to a 1980-inch radius. Then, using Eq. (6.38), we can calculate the additional resistance between 18 inches and 1980 inches from the 10.0 ohms measured between the one-inch radius and the 18-inch radius

8. PRODUCTIVITY INDEX (PI)

19.84 × 520

The ratio of the rate of production, expressed in STB/day for liquid flow, to the pressure drawdown at the midpoint of the producing interval, is called the *productivity index*, symbol *J*.

$$J = \frac{q}{\bar{p} - p_{wf}} \qquad (7.52)$$

In some wells, the PI remains constant over a wide variation in flow rate such that the flow rate is directly proportional to the bottom-hole pressure drawdown. In other wells, at higher flow rates the linearity fails, and the PI index declines, as shown in Fig. 7.14. The cause of this decline may be (a) turbulence at increased rates of flow, (b) decrease in the permeability to oil due to presence of free gas caused by the drop in pressure at the well bore, (c) the increase in oil viscosity with pressure drop below bubble point, and/or (d) reduction in permeability due to formation compressibility.

In depletion reservoirs, the productivity indexes of the wells decline as depletion proceeds, owing to the increase in oil viscosity as gas is released from solution and to the decrease in the permeability of the rock to oil as the oil saturation decreases. Since each of these factors may change from a few to several-fold during depletion, the PI may decline to a small fraction of the initial value. Also, as the permeability to oil decreases, there is a corresponding increase in the permeability to gas, which results in rising gas-oil ratios.

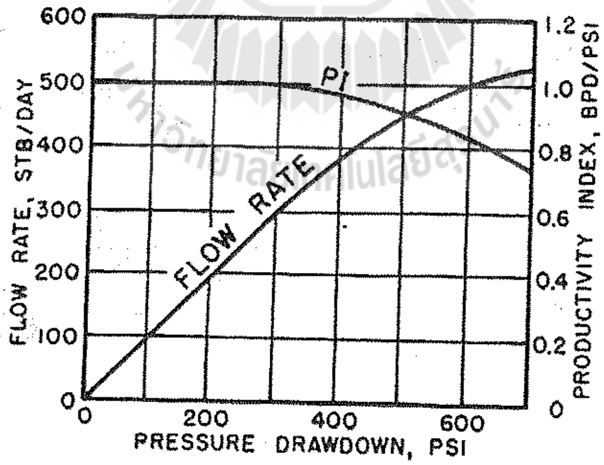


Fig. 7.14. Decline in productivity index at higher flow rates.

The *injectivity index* is used with salt water disposal wells and with *injection wells* for secondary recovery or pressure maintenance. It is the ratio of the injection rate in STB per day to the excess pressure above reservoir pressure that causes that injection rate, or

Injectivity

$$\text{Injectivity index} = I = \frac{q}{p_{wf} - \bar{p}} \text{ STB/day/psi} \qquad (7.53)$$

there is a variation in net productive thickness but when the other factors affecting the productivity index are essentially the same, the specific productivity index J_s is sometimes used, which is the productivity index divided by the net feet of pay, or,

$$q = \frac{0.00708kh(\bar{P}-P_w)}{\mu B(\ln \frac{r_e}{r_w} - 0.75)}$$

$$\text{Specific productivity index} = J_s = \frac{J}{h} = \frac{q}{h(\bar{p} - p_{wf})} \text{ STB/day/psi/ft} \quad (7.54)$$

P.I

$$J = \frac{q}{\bar{P} - P_{wf}}$$

8.1. Productivity Ratio (PR)

In evaluating well performance, the standard usually referred to is the productivity index of an open hole that completely penetrates a circular formation normal to the strata, and in which no alteration in permeability has occurred in the vicinity of the wellbore. Substituting Eq. (7.50) into Eq. (7.52) we get

$$J = 0.00708\frac{kh}{\mu B(\ln(r_e/r_w) - 0.75)} \quad (7.55)$$

The PR then is the ratio of the PI of a well in any condition to the PI of this *standard* well:

$$PR = \frac{J}{J_w} = \frac{\frac{0.00708kh}{\mu B(\ln \frac{r_e}{0.667} - 0.75)}}{\frac{0.00708kh}{\mu B(\ln \frac{r_e}{0.333} - 0.75)}} \quad (7.56)$$

Thus, the productivity ratio may be less than one, greater than one, or equal to one. Although the productivity index of the standard well is generally unknown, the relative effect of certain changes in the well system may be evaluated from theoretical considerations, laboratory models, or well tests. For example, the theoretical productivity ratio of a well reamed from an 8-in. borehole diameter to 16 in. is derived by Eq. (7.55):

$$PR = \frac{J_{16}}{J_8} = \frac{\ln(r_e/0.333) - 0.75}{\ln(r_e/0.667) - 0.75}$$

Assuming $r_e = 660$ ft,

$$PR = \frac{\ln(660/0.333) - 0.75}{\ln(660/0.667) - 0.75} = 1.11$$

9. SUPERPOSITION

SUPERPOSITION

$$\Delta p_t = \Delta p_1 + \Delta p_2$$

Each of the individual Δp terms is given by Eq. (7.36), or:

$$\Delta p = p_i - p(r, t) = \frac{70.6q\mu B}{kh}\left[-E_i\left(-\frac{\phi\mu c_t r^2}{0.00105kt}\right)\right]$$

To apply the method of superposition, pressure drops or changes are added. It is not correct simply to add or subtract individual pressure terms. It is obvious that if there are more than two flowing wells in the reservoir system, the procedure is the same, and the total pressure drop is given by the following.

$$\Delta p_t = \sum_{j=1}^{N} \Delta p_j \quad (7.57)$$

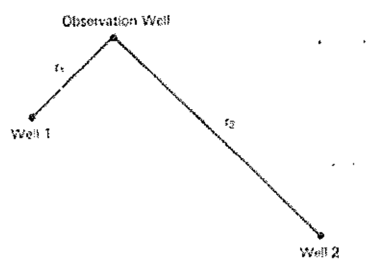


Fig. 7.15. Two flowing-well reservoir system to illustrate the principle of superposition.

Example 7.1. For the well layout shown in Fig. 7.16, calculate the total pressure drop as measured in the observation well (well 3) caused by the four flowing wells (wells 1, 2, 4, and 5) after 10 days. The wells were shut in for a long time before opening them to flow.

Given: The following data apply to the reservoir system:

oil viscosity = 0.40 cp — B_o = 1.50 bbl/STB
k = 47 md — formation thickness = 50 ft
porosity = 11.2% — $c_t = 15 \times 10^{-6}$ psi^{-1}

Well	Flow Rate (STB/day)	Distance to Observation well (feet)
1	265	700
2	270	1920
4	287	1870
5	260	1690

SOLUTION: The individual pressure drops can be calculated with Eq. (7.36), and the total pressure drop is given by Eq. (7.57). For well 1

$$\Delta p_1 = \frac{70.6(265)(0.40)(1.5)}{(47)(50)}\left[-E_i\left(-\frac{(.112)(0.40)(15\times10^{-6})(1700)^2}{0.00105(47)(240)}\right)\right]$$

$$\Delta p_1 = 4.78[-E_i(-0.164)]$$

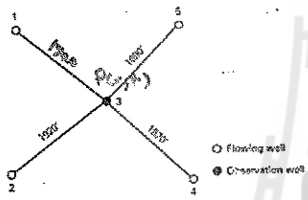
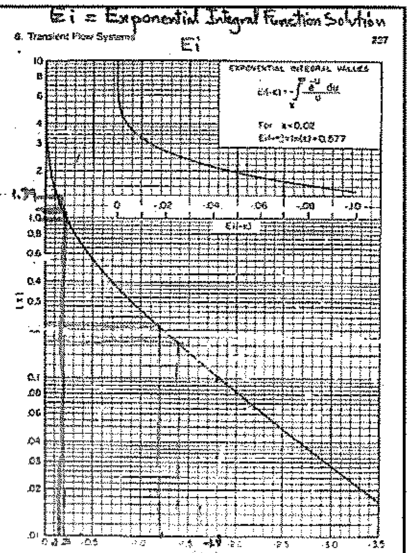


Fig. 7.16. Well layout for Ex. Prob. 7.1.



From Fig. 7.11

$$-E_i(-0.164) = 1.39$$

Therefore

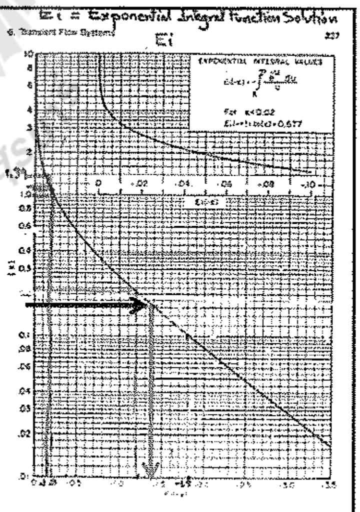
$$\Delta p_1 = 4.78(1.39) = 6.6 \text{ psi}$$

Similarly, for wells 2, 4, and 5

$$\Delta p_2 = 4.87[-E_i(-0.209)] = 5.7 \text{ psi}$$

$$\Delta p_4 = 5.14[-E_i(-0.198)] = 6.4 \text{ psi}$$

$$\Delta p_5 = 4.69[-E_i(-0.162)] = 6.6 \text{ psi}$$



Using Eq. (7.57) to find the total pressure drop at the observation well, Well 3, the individual pressure drops are added together to give the total:

$$\Delta p_t = \Delta p_1 + \Delta p_2 + \Delta p_4 + \Delta p_5$$

or

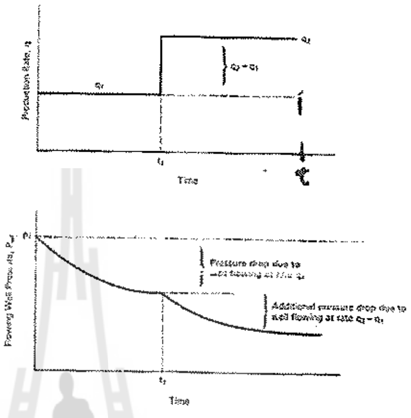
$$\Delta p_t = 6.6 + 5.7 + 6.4 + 6.6 = 25.3 \text{ psi}$$

at two flow rates. The change in the flow rate from q_1 to q_2 occurred at time t_1. Figure 7.17 shows that the total pressure drop is given by the sum of the pressure drop caused by the flow rate q_1 and the pressure drop caused by the change in flow rate $q_2 - q_1$. This new flow rate, $q_2 - q_1$, has flowed for time $t - t_1$.

The pressure drop for this flow rate, $q_2 - q_1$, is given by

$$\Delta p = p_i - p(r, t) = \frac{70.6(q_2 - q_1)\mu B}{kh}\left[-E_i\left(\frac{\phi\mu c_t r^2}{0.00105k(t - t_1)}\right)\right]$$

As in the case of the multiwell system just described, superposition can also be applied to multirate systems as well as the two rate examples depicted in Fig. 7.17.



250 Single-Phase Fluid Flow in Reservoirs

where N equals the number of flowing wells in the system. Example 7.1 illustrates the calculations involved when more than one well affects the pressure of a point in a reservoir.

Example 7.1. For the well layout shown in Fig. 7.16, calculate the total pressure drop as measured in the observation well (well 3) caused by the four flowing wells (wells 1, 2, 4, and 5) after 10 days. The wells were shut in for a long time before opening them to flow.

Given: The following data apply to the reservoir system:

oil viscosity = 0.40 cp
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$c_t = 15 \times 10^{-6}$ psi^{-1}

Well	*Flow Rate (STB/day)*	*Distance to Observation well (feet)*
1	265	1700
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SOLUTION: The individual pressure drops can be calculated with Eq. (7.36), and the total pressure drop is given by Eq. (7.57). For well 1

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$$\Delta p_1 = 4.78[-E_i(-0.164)]$$

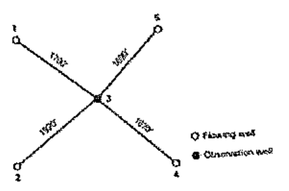

Fig. 7.16. Well layout for Ex. Prob. 7.1.

9. Superposition 251

From Fig. 7.11

$$-E_i(-0.164) = 1.39$$

Therefore

$$\Delta p_1 = 4.78(1.39) = 6.6 \text{ psi}$$

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Using Eq. (7.57) to find the total pressure drop at the observation well, Well 3, the individual pressure drops are added together to give the total:

$$\Delta p_t = \Delta p_1 + \Delta p_2 + \Delta p_4 + \Delta p_5$$

or

$$\Delta p_t = 6.6 + 5.7 + 6.4 + 6.6 = 25.3 \text{ psi}$$

The superposition principle can also be applied in the time dimension, as is illustrated in Fig. 7.17. In this case, one well (which means the position where the pressure disturbances occur remains constant) has been produced at two flow rates. The change in the flow rate from q_1 to q_2 occurred at time t_1. Figure 7.17 shows that the total pressure drop is given by the sum of the pressure drop caused by the flow rate q_1 and the pressure drop caused by the change in flow rate $q_2 - q_1$. This new flow rate, $q_2 - q_1$, has flowed for time $t - t_1$.

The pressure drop for this flow rate, $q_2 - q_1$, is given by

$$\Delta p = p_i - p(r, t) = \frac{70.6(q_2 - q_1)\mu B}{kh}\left[-E_i\left(\frac{\phi\mu c_t r^2}{0.00105k(t - t_1)}\right)\right]$$

As in the case of the multiwell system just described, superposition can also be applied to multirate systems as well as the two rate examples depicted in Fig. 7.17.

9.1. Superposition in Bounded or Partially Bounded Reservoirs

Although Eq. (7.36) applies to infinite reservoirs, it may be used in conjunction with the superposition principle to simulate boundaries of closed or partially closed reservoirs. The effect of boundaries is always to cause greater

434620,505653 ADVANCED RESERVOIR ENGINEERING 2/2556(4 credits(4-0-8), Sep. 19-Dec. 23, 2013)

Course Contents

1. Optimization of Material Balance Equations (6 hrs.)
2. Saturation and Relative Permeability Calculations (4 hrs.)
3. Steady State Radial Flow (4 hrs.)
4. Unsteady State Radial Flow (4 hrs.)
5. **Pseudo-steady State Flow and Superposition (4 hrs.)**
6. Well Testing Pressure Drawdown and Build Up (3 hrs.)
7. Interference Test and Type Curve Analysis(3hrs)
8. Displacement Efficiency(4 hrs)
9. Potential flow and Streamlines (4 hrs.)
10. Dynamics of Water Drive Reservoir. (6 hrs.)
11. Water and Gas Coning (4hrs.)

PSEUDO-STEADY STATE FLOWS

OIL OR GAS OR WATER

often very short in length. As soon as the pressure disturbance reaches all drainage boundaries, it ends the transient (unsteady-state) flow regime. A different flow regime begins that is called **pseudosteady (semisteady)-state flow.** It is necessary at this point to impose different boundary conditions on the diffusivity equation and drive an appropriate solution to this flow regime.

Consider Figure 6-23, which shows a well in radial system that is producing at a constant rate for a long enough period that eventually affects the entire drainage area. During this semisteady-state flow, the change in pressure with time becomes the same throughout the drainage area. Section B in Figure 6-23 shows that the pressure distributions become paralleled at successive time periods. Mathematically, this important condition can be expressed as:

$$\left(\frac{dp}{dt}\right)_r = \text{constant}$$

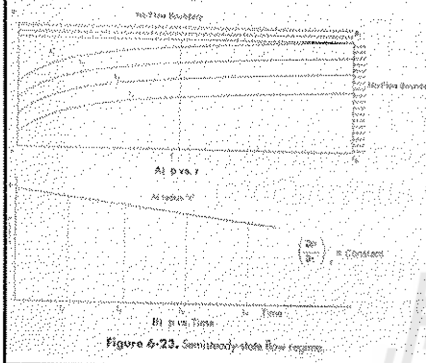

Figure 6-23. Semisteady-state flow regime.

The constant referred to in the above equation can be obtained from a simple material balance using the definition of the compressibility, thus:

$$c = \frac{-1}{V}\frac{dV}{dp}$$

Arranging:

$$cVdp = -dV$$

Differentiating with respect to time t:

$$cV\frac{dp}{dt} = -\frac{dV}{dt} = q$$

or

$$\frac{dp}{dt} = -\frac{q}{cV}$$

Expressing the pressure decline rate dp/dt in the above relation in psi/hr gives:

$$\frac{dp}{dt} = -\frac{q}{24cV} = -\frac{Q_o B_o}{24cV} \tag{6-126}$$

where q = flow rate, bbl/day
Q_o = flow rate, STB/day
dp/dt = pressure decline rate, psi/hr
V = pore volume, bbl

For a radial drainage system, the pore volume is given by:

$$V = \frac{\pi r_e^2 h\phi}{5.615} = \frac{Ah\phi}{5.615} \tag{6-127}$$

where A = drainage area, ft^2

Combining Equation 6-127 with Equation 6-126 gives:

$$\frac{dp}{dt} = \frac{-0.23396q}{c_t \pi r_e^2 h\phi} = \frac{-0.23396q}{c_t Ah\phi} \tag{6-128}$$

Examination of the above expression reveals the following important characteristics of the behavior of the pressure decline rate dp/dt during the semisteady-state flow:

- The reservoir pressure declines at a higher rate with an increase in the fluids production rate
- The reservoir pressure declines at a slower rate for reservoirs with higher total compressibility coefficients
- The reservoir pressure declines at a lower rate for reservoirs with larger pore volumes

Example 6-16

An oil well is producing at a constant oil flow rate of 1200 STB/day under a semisteady-state flow regime. Well testing data indicate that the pressure is declining at a constant rate of 4.655 psi/hr. The following additional data is available:

$h = 25$ ft $\phi = 15\%$ $B_o = 1.3$ bbl/STB
$c_t = 12 \times 10^{-6}$ psi^{-1}

Calculate the well drainage area.

Solution

- $q = Q_o B_o$
- $q = (1200)(1.3) = 1560$ bbl/day
- Apply Equation 6-128 to solve for A.

$$-4.655 = -\frac{0.23396\,(1560)}{(12\times10^{-6})(A)(25)(0.15)}$$

$A = 1{,}742{,}400$ ft^2

or

$A = 1{,}742{,}400 / 43{,}560 = 40$ acres

Matthews, Brons, and Hazebroek (1954) pointed out that once the reservoir is producing under the *semisteady-state condition*, each well will drain from within its own no-flow boundary independently of the other wells. For this condition to prevail, the pressure decline rate dp/dt must be approximately constant throughout the entire reservoir, otherwise flow would occur across the boundaries causing a readjustment in their positions. Because the pressure at every point in the reservoir is changing at the same rate, it leads to the conclusion that the average reservoir pressure is changing at the same rate. This average reservoir pressure is essentially set equal to the volumetric average reservoir pressure $\bar{p}_r$. It is the pressure that is used to perform flow calculations during the semisteady state flowing condition. In the above discussion, $\bar{p}_r$ indicates that, in principal, Equation 6-128 can be used to estimate by replacing the pressure decline rate dp/dt with $(p_i - \bar{p}_r)/t$, or:

$$p_i - \bar{p}_r = \frac{0.23396qt}{c_t Ah\phi}$$

or

$$\bar{p}_r = p_i - \frac{0.23396qt}{c_t Ah\phi} \tag{6-129}$$

where t is approximately the elapsed time since the end of the transient flow regime to the time of interest.

It should be noted that when performing material balance calculations, the volumetric average pressure of the entire reservoir is used to calculate the fluid properties. This pressure can be determined from the individual well drainage properties as follows:

$$\bar{p}_r = \frac{\sum_i \bar{p}_{ri} V_i}{\sum_i V_i} \tag{6-130}$$

in which V_i = pore volume of the ith drainage volume
$\bar{p}_i$ = volumetric average pressure within the ith drainage volume.

Figure 6-24 illustrates the concept of the volumetric average pressure. In practice, the V_i's are difficult to determine and, therefore, it is common to use the flow rate q_i in Equation 6-129.

$$\bar{p}_r = \frac{\sum_i (\bar{p}_i q_i)}{\sum_i q_i} \qquad (6\text{-}131)$$

The flow rates are measured on a routing basis throughout the lifetime of the field thus facilitating the calculation of the volumetric average reservoir pressure $\bar{p}_r$.

The practical applications of using the pseudosteady-state flow condition to describe the flow behavior of the following two types of fluids are presented below:

- Radial flow of slightly compressible fluids
- Radial flow of compressible fluids

$$-\frac{887.22q\mu}{(\pi r_e^2)hk}\left(\frac{r^2}{2}\right)+c_1 = -\frac{141.2q\mu}{r_e^2hk}\left(\frac{r^2}{\quad}\right)+c_1$$

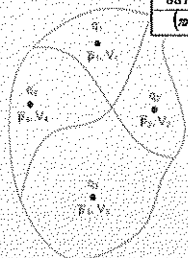

Figure 6-24. Volumetric average reservoir pressure.

Radial Flow of Slightly Compressible Fluids

The diffusivity equation as expressed by Equation 6-73 for the transient flow regime is:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \left(\frac{\phi\mu c_t}{0.000264k}\right)\frac{\partial p}{\partial t}$$

For the semisteady-state flow, the term $(\partial p/\partial t)$ is constant and is expressed by Equation 6-128. Substituting Equation 6-128 into the diffusivity equation gives:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \left(\frac{\phi\mu c_t}{0.000264k}\right)\left(\frac{-0.23396q}{c_t Ah\phi}\right)$$

or

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{-887.22q\mu}{Ahk} \qquad (6\text{-}132)$$

Equation 6-132 can be expressed as:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p}{\partial r}\right) = -\frac{887.22q\mu}{(\pi r_e^2)hk}$$

Integrating the above equation gives:

$$r\frac{\partial p}{\partial r} = -\frac{887.22q\mu}{(\pi r_e^2)hk}\left(\frac{r^2}{2}\right) + c_1$$

Where c_1 is the constant of the integration and can be evaluated by imposing the outer no-flow boundary condition [i.e., $(\partial p/\partial r)_{r_e} = 0$] on the above relation to give:

$$c_1 = \frac{141.2q\mu}{\pi hk} \qquad c_1 = \frac{141.2q\mu}{r_e^2 hk}\left(\frac{r_e^2}{\quad}\right) = \frac{141.2q\mu}{hk}$$

Combining the above two expressions gives:

$$\frac{\partial p}{\partial r} = \frac{141.2q\mu}{hk}\left(\frac{1}{r} - \frac{r}{r_e^2}\right)$$

Integrating again:

$$\int_{p_{wf}}^{p_i} dp = \frac{141.2q\mu}{hk}\int_{r_w}^{r_e}\left(\frac{1}{r} - \frac{r}{r_e^2}\right)dr$$

Performing the above integration and assuming (r_w^2/r_e^2) is negligible gives:

$$(p_i - p_{wf}) = \frac{141.2q\mu}{kh}\left[\ln\left(\frac{r_e}{r_w}\right) - \frac{1}{2}\right] \qquad (6\text{-}133)$$

A more appropriate form of the above is to solve for the flow rate, to give:

$$Q = \frac{0.00708\,kh\,(p_i - p_{wf})}{\mu B\left[\ln\left(\frac{r_e}{r_w}\right) - 0.5\right]} \qquad (6\text{-}134)$$

where Q = flow rate, STB/day
B = formation volume factor, bbl/STB
k = permeability, md

The volumetric average reservoir pressure $\bar{p}_r$ is commonly used in calculating the liquid flow rate under the semisteady-state flowing condition. Introducing the $\bar{p}_r$ into Equation 6-134 gives:

$$Q = \frac{0.00708\,kh\,(\bar{p}_r - p_{wf})}{\mu B\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right]} \qquad (6\text{-}135)$$

Note that:

$$\ln\left(\frac{0.471r_e}{r_w}\right) = \ln\left(\frac{r_e}{r_w}\right) - 0.75$$

The above observation suggests that the volumetric average pressure $\bar{p}_r$ occurs at about 47% of the drainage radius during the semisteady-state condition.

It is interesting to notice that the dimensionless pressure p_D solution to the diffusivity equation can be used to derive Equation 6-135. The p_D function for a bounded reservoir was given previously by Equation 6-96 for a bounded system as:

$$p_D = \frac{2t_D}{r_{eD}^2} + \ln(r_{eD}) - 0.75 \qquad p_D = \frac{2t_D}{r_{eD}^2} + \ln(r_{eD}) - 0.75$$

where the above three dimensionless parameters are given by Equations 6-86 through 6-88 as:

$$p_D = \frac{(p_i - p_{wf})}{\frac{QB\mu}{0.00708\,kh}}$$

$$t_D = \frac{0.000264\,kt}{\phi\mu c_t r_w^2}$$

$$r_{eD} = \frac{r_e}{r_w}$$

Combining the above four relationships gives:

$$p_{wf} = p_i - \frac{QB\mu}{0.00708\,kh}\left[\frac{0.0005274\,kt}{\phi\mu c_t r_e^2} + \ln\left(\frac{r_e}{r_w}\right) - 0.75\right]$$

Solving Equation 6-130 for the time t gives:

$$t = \frac{c_t A h \phi (p_i - \bar{p}_r)}{0.23396\, Q B} = \frac{c_t (\pi r_e^2) h \phi (p_i - \bar{p}_r)}{0.23396\, Q B}$$

Combining the above two equations and solving for the flow rate Q yields:

$$Q = \frac{0.00708\, k h (\bar{p}_r - p_{wf})}{\mu B \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right]}$$

It should be pointed out that the pseudosteady-state flow occurs regardless of the geometry of the reservoir. Irregular geometries also reach this state when they have been produced long enough for the entire drainage area to be affected.

Rather than developing a separate equation for each geometry, Ramey and Cobb (1971) introduced a correction factor that is called the **shape factor**, C_A, which is designed to account for the deviation of the drainage area from the ideal circular form. The shape factor, as listed in Table 6-4, accounts also for the location of the well within the drainage area. Introducing C_A into Equation 6-132 and performing the solution procedure gives the following two solutions:

- *In terms of the volumetric average pressure* $\bar{p}_r$:

$$p_{wf} = \bar{p}_r - \frac{162.6\, Q B \mu}{kh} \log\left[\frac{4A}{1.781\, C_A r_w^2}\right] \qquad (6\text{-}136)$$

- *In terms of the initial reservoir pressure* p_i:

Recalling Equation 6-129 which shows the changes of the average reservoir pressure as a function of time and initial reservoir pressure p_i:

$$\bar{p}_r = p_i - \frac{0.23396\, q\, t}{c_t A h \phi}$$

Combining the above equation with Equation 6-136 gives:

$$p_{wf} = \left[p_i - \frac{0.23396\, Q B t}{A h \phi c_t}\right] - \frac{162.6\, Q B \mu}{kh} \log\left[\frac{4A}{1.781\, C_A r_w^2}\right] \qquad (6\text{-}137)$$

where
- k = permeability, md
- A = drainage area, ft²
- C_A = shape factor
- Q = flow rate, STB/day
- t = time, hr
- c_t = total compressibility coefficient, psi⁻¹

Equation 6-136 can be arranged to solve for Q to give:

$$Q = \frac{kh(\bar{p}_r - p_{wf})}{162.6\, B \mu \log\left[\frac{4A}{1.781\, C_A r_w^2}\right]} \qquad (6\text{-}138)$$

It should be noted that if Equation 6-138 is applied to a circular reservoir of a radius r_e, then:

$$A = \pi r_e^2$$

and the shape factor for a circular drainage area as given in Table 6-3 is:

$$C_A = 31.62$$

Substituting in Equation 6-138, it reduces to:

$$p_{wf} = \bar{p}_r - \left(\frac{Q B \mu}{0.00708\, kh}\right)\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right]$$

The above equation is identical to that of Equation 6-135.

(text continued on page 416)

Table 6-4
Shape Factors for Various Single-Well Drainage Areas
(After Earlougher, R., Advances in Well Test Analysis, permission to publish by the SPE, copyright SPE, 1977)

In Bounded Reservoirs	C_A	$\ln C_A$	$S_a^* = \frac{S_a}{1 - S_{wc}}$	Exact for t_{DA} >	Less than 1% Error for t_{DA} >	Use Infinite System Solution with Less than 1% Error for t_{DA} <
	31.62	3.4538	−1.3224	0.1	0.06	0.10
	31.6	3.4532	−1.3220	0.1	0.06	0.10
	27.6	3.3178	−1.2544	0.2	0.07	0.09
60°	27.1	3.2995	−1.2452	0.2	0.07	0.09
1/3	21.9	3.0865	−1.1387	0.4	0.12	0.08
	0.098	−2.3227	+1.5659	0.9	0.60	0.015
	30.8828	3.4302	−1.3106	0.1	0.05	0.09
	12.9851	2.5638	−0.8774	0.7	0.25	0.03
	4.5132	1.5070	−0.3490	0.6	0.30	0.025
	3.3351	1.2045	−0.1977	0.7	0.25	0.01
	21.8369	3.0836	−1.1373	0.3	0.15	0.025
	10.8374	2.3830	−0.7870	0.4	0.15	0.025
	4.5141	1.5072	−0.3491	1.5	0.50	0.06
	2.0769	0.7309	+0.0391	1.7	0.50	0.02
	3.1573	1.1497	−0.1703	0.4	0.15	0.005

In Bounded Reservoirs	C_A	$\ln C_A$	$S_a^* = \frac{S_a}{1 - S_{wc}}$	Exact for t_{DA} >	Less than 1% Error for t_{DA} >	Use Infinite System Solution with Less than 1% Error for t_{DA} <
	0.5813	−0.5425	+0.6758	2.0	0.60	0.02
	0.1109	−2.1991	+1.5041	3.0	0.60	0.005
	5.3790	1.6825	−0.4367	0.8	0.30	0.01
	2.6896	0.9894	−0.0902	0.8	0.30	0.01
	0.2318	−1.4619	+1.1355	4.0	2.00	0.03
	0.1155	−2.1585	+1.4838	4.0	2.00	0.01
	2.3606	0.8589	−0.0249	1.0	0.40	0.025
In vertically fractured reservoirs	Use $(x_e/x_f)^2$ in place of A/r_w^2 for fractured systems					
0.1 x_f/x_e	2.6541	0.9761	−0.0835	0.175	0.08	cannot use
0.2	2.0348	0.7104	+0.0493	0.175	0.09	cannot use
0.3	1.9986	0.6924	+0.0583	0.175	0.09	cannot use
0.5	1.6620	0.5080	+0.1505	0.175	0.09	cannot use
0.7	1.3127	0.2721	+0.2685	0.175	0.09	cannot use
1.0	0.7887	−0.2374	+0.5232	0.175	0.09	cannot use
In water-drive reservoirs						
	19.1	2.95	−1.07	—	—	—
In reservoirs of unknown production character						
	25.0	3.22	−1.20	—	—	—

(not continued from page 415)

Example 6-16

An oil well is developed on the center of a 40-acre square drilling pattern. The well is producing at a constant flow rate of 800 STB/day under a semisteady-state condition. The reservoir has the following properties:

$\phi = 15\%$	$h = 30$ ft	$k = 200$ md
$\mu = 1.5$ cp	$B_o = 1.2$ bbl/STB	$c_t = 25 \times 10^{-6}$ psi-1
$p_i = 4500$ psi	$r_w = 0.25$ ft	$A = 40$ acres

a. Calculate and plot the bottom-hole flowing pressure as a function of time.
b. Based on the plot, calculate the pressure decline rate. What is the decline in the average reservoir pressure from t = 10 to t = 200 hr?

Solution

a. p_{wf} calculations:

Step 1. From Table 6-3, determine C_A:

$$C_A = 30.8828$$

Step 2. Convert the area A from acres to ft²:

$$A = (40)(43{,}560) = 1{,}742{,}400 \text{ ft}^2$$

Step 3. Apply Equation 6-137:

$$p_{wf} = \left[p_i - \frac{0.23396\,Q\,B\,t}{A h \phi c_t}\right] - \frac{162.6\,Q\,B\mu}{kh}\log\left[\frac{4A}{1.781 C_A r_w^2}\right]$$

$$p_{wf} = 4500 - 1.719\,t - 58.536 \log(2{,}027{,}436)$$

or

$$p_{wf} = 4493.69 - 1.719\,t$$

Step 4. Calculate p_{wf} at different assumed times.

t, hr	$p_{wf} = 44369 - 1.719\,t$
10	4476.50
20	4459.31
50	4407.74
100	4321.79
200	4149.89

Step 5. Present the results of Step 4 in a graphical form as shown in Figure 6-25.

b. It is obvious from Figure 6-25 and the above calculation that the bottom-hole flowing pressure is declining at a rate of 1.719 psi/hr, or:

$$\frac{dp}{dt} = -1.719 \text{ psi/hr}$$

The significance of this example is that the rate of pressure decline during the pseudosteady state is the same throughout the drainage area. This means that the *average reservoir pressure*, $\bar{p}_r$, is declining at the same rate of 1.719 psi, therefore the change in $\bar{p}_r$ from 10 to 200 hours is:

$$\Delta\bar{p}_r = (1.719)(200 - 10) = 326.6 \text{ psi}$$

Example 6-17

An oil well is producing under a constant bottom-hole flowing pressure of 1500 psi. The current average reservoir pressure $\bar{p}_r$ is 3200 psi.

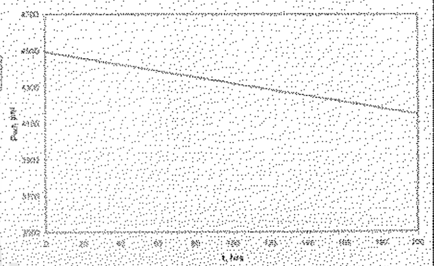

Figure 6-25. Bottom-hole flowing pressure as a function of time.

The well is developed in the center of a 40-acre square drilling pattern. Given the following additional information:

$\phi = 16\%$	$h = 15$ ft	$k = 50$ md
$\mu = 26$ cp	$B_o = 1.15$ bbl/STB	$c_t = 10 \times 10^{-6}$ psi^{-1}
$r_w = 0.25$ ft		

calculate the flow rate.

Solution

$$Q = \frac{kh(\bar{p}_r - p_{wf})}{162.6 B\mu \log\left[\frac{4A}{1.781 C_A r_w^2}\right]}$$

Because the volumetric average ... rate by applying Equation 6-138:

$$Q = \frac{(50)(15)(3200 - 1500)}{(162.6)(1.15)(2.6)\log\left[\frac{(4)(40)(43{,}560)}{1.781(30.8828)(0.25^2)}\right]}$$

$$= 416 \text{ STB/day}$$

Radial Flow of Compressible Fluids (Gases)

The radial diffusivity equation as expressed by Equation 6-106 was developed to study the performance of compressible fluid under unsteady-state conditions. The equation has the following form:

$$\frac{\partial^2 m(p)}{\partial r^2} + \frac{1}{r}\frac{\partial m(p)}{\partial r} = \frac{\phi\mu c_t}{0.000264\,k}\frac{\partial m(p)}{\partial t}$$

For the semisteady-state flow, the rate of change of the real gas pseudopressure with respect to time is constant, i.e.,

$$\frac{\partial m(p)}{\partial t} = \text{constant}$$

Using the same technique identical to that described previously for liquids gives the following exact solution to the diffusivity equation:

$$Q_g = \frac{kh[m(\bar{p}_r) - m(p_{wf})]}{1422\,T\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75\right]} \tag{6-139}$$

where Q_g = gas flow rate, Mscf/day
T = temperature, °R
k = permeability, md

Two approximations to the above solution are widely used. These approximations are:

- Pressure-squared approximation
- Pressure-approximation

Pressure-Squared Approximation Method

As outlined previously, the method provides us with compatible results to that of the exact solution approach when p < 2000. The solution has the following familiar form:

$$Q_g = \frac{kh(\bar{p}_r^2 - p_{wf}^2)}{1422\,T\bar{\mu}\bar{z}\left(\ln\frac{r_e}{r_w} - 0.75\right)} \tag{6-140}$$

The gas properties $\bar{z}$ and $\bar{\mu}$ are evaluated at:

$$\bar{p} = \sqrt{\frac{(\bar{p}_r)^2 + p_{wf}^2}{2}}$$

Pressure-Approximation Method

This approximation method is applicable at p > 3000 psi and has the following mathematical form:

$$Q_g = \frac{kh(\bar{p}_r - p_{wf})}{1422\,\bar{\mu}\bar{B}_g\left(\ln\frac{r_e}{r_w} - 0.75\right)} \tag{6-141}$$

with the gas properties evaluated at:

$$\bar{p} = \frac{\bar{p}_r + p_{wf}}{2}$$

where Q_g = gas flow rate, Mscf/day
k = permeability, md
$\bar{B}_g$ = gas formation volume factor at average pressure, bbl/scf

The gas formation volume factor is given by the following expression:

$$\bar{B}_g = 0.00504\,\frac{\bar{z}\,T}{\bar{p}}$$

In deriving the flow equations, the following two main assumptions were made:

- Uniform permeability throughout the drainage area
- Laminar (viscous) flow

Before using any of the previous mathematical solutions to the flow equations, the solution must be modified to account for the possible deviation from the above two assumptions. Introducing the following two correction factors into the solution of the flow equation can eliminate the above two assumptions:

- Skin factor
- Turbulent flow factor

Skin Factor

It is not unusual for materials such as mud filtrate, cement slurry, or clay particles to enter the formation during drilling, completion or workover operations and reduce the permeability around the wellbore. This effect is commonly referred to as a *wellbore damage* and the region of altered permeability is called the *skin zone*. This zone can extend from a few inches to several feet from the wellbore. Many other wells are stimulated by acidizing or fracturing which in effect increase the permeability near the wellbore. Thus, the permeability near the wellbore is always different from the permeability away from the well where the formation has not been affected by drilling or stimulation. A schematic illustration of the skin zone is shown in Figure 6-26.

Those factors that cause damage to the formation can produce additional localized pressure drop during flow. This additional pressure drop is commonly referred to as Δp_{skin}. On the other hand, well stimulation techniques will normally enhance the properties of the formation and increase the permeability around the wellbore, so that a decrease in pres-

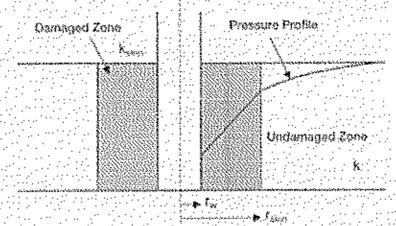

Figure 6-26. Near wellbore skin effect.

sure drop is observed. The resulting effect of altering the permeabili[ty] around the well bore is called the *skin effect*.

Figure 6-27 compares the differences in the skin zone pressure dr[op] for three possible outcomes:

- First Outcome:
  $\Delta p_{skin} > 0$, indicates an additional pressure drop due to wellbore dam[-]age, i.e., $k_{skin} < k$.
- Second Outcome:
  $\Delta p_{skin} < 0$, indicates less pressure drop due to wellbore improveme[nt], i.e., $k_{skin} > k$.
- Third Outcome:
  $\Delta p_{skin} = 0$, indicates no changes in the wellbore condition, i.e., $k_{skin} =$ [k].

Hawkins (1956) suggested that the permeability in the skin zone, i[.e.,] k_{skin}, is uniform and the pressure drop across the zone can be appro[xi]mated by Darcy's equation. Hawkins proposed the following approach:

$$\Delta p_{skin} = \left[\begin{array}{l}\Delta p \text{ in skin zone}\\ \text{due to } k_{skin}\end{array}\right] - \left[\begin{array}{l}\Delta p \text{ in the skin zone}\\ \text{due to } k\end{array}\right]$$

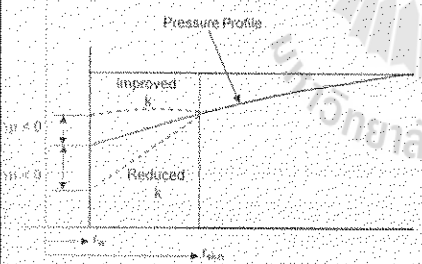

Figure 6-27. Representation of positive and negative skin effects.

Applying Darcy's equation gives:

$$\Delta p_{skin} = \left[\frac{Q_o B_o \mu_o}{0.00708\,h\,k_{skin}}\right]\ln\left(\frac{r_{skin}}{r_w}\right) - \left[\frac{Q_o B_o \mu_o}{0.00708\,h\,k}\right]\ln\left(\frac{r_{skin}}{r_w}\right)$$

or

$$\Delta p_{skin} = \left(\frac{Q_o B_o \mu_o}{0.00708\,kh}\right)\left[\frac{k}{k_{skin}} - 1\right]\ln\left(\frac{r_{skin}}{r_w}\right)$$

where k = permeability of the formation, md
k_{skin} = permeability of the skin zone, md

The above expression for determining the additional pressure drop in the skin zone is commonly expressed in the following form:

$$\Delta p_{skin} = \left[\frac{Q_o B_o \mu_o}{0.00708\,kh}\right]s = 141.2\left[\frac{Q_o B_o \mu_o}{kh}\right]s \qquad (6\text{-}142)$$

where s is called the skin factor and defined as:

$$s = \left[\frac{k}{k_{skin}} - 1\right]\ln\left(\frac{r_{skin}}{r_w}\right) \qquad (6\text{-}143)$$

Equation 6-143 provides some insight into the physical significance of the sign of the skin factor. There are only three possible outcomes in evaluating the skin factor s:

- **Positive Skin Factor, $s > 0$**
  When a damaged zone near the wellbore exists, k_{skin} is less than k and hence s is a positive number. The magnitude of the skin factor increases as k_{skin} decreases and as the depth of the damage r_{skin} increases.
- **Negative Skin Factor, $s < 0$**
  When the permeability around the well k_{skin} is higher than that of the formation k, a negative skin factor exists. This negative factor indicates an improved wellbore condition.
- **Zero Skin Factor, $s = 0$**
  Zero skin factor occurs when no alternation in the permeability around the wellbore is observed, i.e., $k_{skin} = k$.

Equation 6-143 indicates that a negative skin factor will result in a negative value of Δp_{skin}. This implies that a stimulated well will require less pressure drawdown to produce at rate q than an equivalent well with uniform permeability.

The proposed modification of the previous flow equation is based on the concept that the actual total pressure drawdown will increase or decrease by an amount of Δp_{skin}. Assuming that $(\Delta p)_{ideal}$ represents the pressure drawdown for a drainage area with a uniform permeability k, then:

$$(\Delta p)_{actual} = (\Delta p)_{ideal} + (\Delta p)_{skin}$$

or

$$(p_i - p_{wf})_{actual} = (p_i - p_{wf})_{ideal} + \Delta p_{skin} \qquad (6\text{-}144)$$

The above concept as expressed by Equation 6-144 can be applied to all the previous flow regimes to account for the skin zone around the wellbore as follows:

Steady-State Radial Flow

Substituting Equations 6-27 and 6-142 into Equation 6-144 gives:

$$(p_i - p_{wf})_{actual} = \left[\frac{Q_o B_o \mu_o}{0.00708 kh}\right] \ln\left(\frac{r_e}{r_w}\right) + \left[\frac{Q_o B_o \mu_o}{0.00708 kh}\right] s$$

or

$$Q_o = \frac{0.00708 kh (p_i - p_{wf})}{\mu_o B_o \left[\ln \frac{r_e}{r_w} + s\right]} \quad (6\text{-}145)$$

where Q_o = oil flow rate, STB/day
k = permeability, md
h = thickness, ft
s = skin factor
B_o = oil formation volume factor, bbl/STB
μ_o = oil viscosity, cp
p_i = initial reservoir pressure, psi
p_{wf} = bottom hole flowing pressure, psi

Unsteady-State Radial Flow

• For Slightly Compressible Fluids:
Combining Equations 6-83 and 6-142 with that of Equation 6-144 yields:

$$p_i - p_{wf} = 162.6 \left(\frac{Q_o B_o \mu_o}{kh}\right)\left[\log \frac{kt}{\phi \mu c_t r_w^2} - 3.23\right] + 141.2 \left(\frac{Q_o B_o \mu_o}{kh}\right) s$$

or

$$p_i - p_{wf} = 162.6 \left(\frac{Q_o B_o \mu_o}{kh}\right)\left[\log \frac{kt}{\phi \mu c_t r_w^2} - 3.23 + 0.87 s\right] \quad (6\text{-}146)$$

• For Compressible Fluids:
A similar approach to that of the above gives:

$$m(p_{wf}) = m(p_i) - \frac{1637 Q_g T}{kh}\left[\log \frac{kt}{\phi \mu c_{ti} r_w^2} - 3.23 + 0.87 s\right] \quad (6\text{-}147)$$

and, in terms of the pressure-squared approach, gives:

$$p_{wf}^2 = p_i^2 - \frac{1637 Q_g T \bar{z} \bar{\mu}}{kh}\left[\log \frac{kt}{\phi \mu_i c_{ti} r_w^2} - 3.23 + 0.87 s\right] \quad (6\text{-}148)$$

Pseudosteady-State Flow

• For Slightly Compressible Fluids:
Introducing the skin factor into Equation 6-135 gives:

$$Q_o = \frac{0.00708 kh (\bar{p}_r - p_{wf})}{\mu_o B_o \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]} \quad (6\text{-}149)$$

• For Compressible Fluids:

$$Q_g = \frac{kh[m(\bar{p}_r) - m(p_{wf})]}{1422\, T \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]} \quad (6\text{-}150)$$

or, in terms of the pressure-squared approximation, gives:

$$Q_g = \frac{kh(\bar{p}_r^2 - p_{wf}^2)}{1422\, T \bar{\mu} \bar{z} \left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s\right]} \quad (6\text{-}151)$$

where Q_g = gas flow rate, Mscf/day
k = permeability, md
T = temperature, °R
$(\bar{\mu}_g)$ = gas viscosity at average pressure $\bar{p}$, cp
$\bar{z}_g$ = gas compressibility factor at average pressure $\bar{p}$

Example 6-18

Calculate the skin factor resulting from the invasion of the drilling fluid to a radius of 2 feet. The permeability of the skin zone is estimated at 20 md as compared with the unaffected formation permeability of 60 md. The wellbore radius is 0.25 ft.

Solution

Apply Equation 6-143 to calculate the skin factor:

$$s = \left[\frac{60}{20} - 1\right] \ln\left(\frac{2}{0.25}\right) = 4.16$$

Matthews and Russell (1967) proposed an alternative treatment to the skin effect by introducing the **effective or apparent wellbore radius** rwa that accounts for the pressure drop in the skin. They define r_{wa} by the following equation:

$$r_{wa} = r_w e^{-s} \quad (6\text{-}152)$$

All of the ideal radial flow equations can be also modified for the skin by simply replacing wellbore radius r_w with that of the apparent wellbore radius r_{wa}. For example, Equation 6-146 can be equivalently expressed as:

$$p_i - p_{wf} = 162.6 \left(\frac{Q_o B_o \mu_o}{kh}\right)\left[\log \frac{kt}{\phi \mu c_t r_{wa}^2} - 3.23\right] \quad (6\text{-}153)$$

Turbulent Flow Factor

All of the mathematical formulations presented so far are based on the assumption that laminar flow conditions are observed during flow. During radial flow, the flow velocity increases as the wellbore is approached. This increase in the velocity might cause the development of a turbulent flow around the wellbore. If turbulent flow does exist, it is most likely to occur with gases and causes an additional pressure drop similar to that caused by the skin effect. The term *non-Darcy flow* has been adopted by the industry to describe the additional pressure drop due to the turbulent (non-Darcy) flow.

Referring to the additional real gas pseudopressure drop due to non-Darcy flow as Δψ non-Darcy, the total (actual) drop is given by:

$$(\Delta\psi)_{actual} = (\Delta\psi)_{ideal} + (\Delta\psi)_{skin} + (\Delta\psi)_{non\text{-}Darcy}$$

Wattenbarger and Ramey (1968) proposed the following expression for calculating $(\Delta\psi)_{non\text{-}Darcy}$:

$$(\Delta\psi)_{non\text{-}Darcy} = 3.161 \times 10^{-12}\left[\frac{\beta T \gamma_g}{\mu_{gw} h^2 r_w}\right] Q_g^2 \quad (6\text{-}154)$$

The above equation can be expressed in a more convenient form as:

$$(\Delta\psi)_{non\text{-}Darcy} = F Q_g^2 \quad (6\text{-}155)$$

where F is called the *non-Darcy flow coefficient* and is given by:

$$F = 3.161 \times 10^{-12}\left[\frac{\beta T \gamma_g}{\mu_{gw} h^2 r_w}\right] \quad (6\text{-}156)$$

where Q_g = gas flow rate, Mscf/day
μ_{gw} = gas viscosity as evaluated at p_{wf}, cp
γ_g = gas specific gravity
h = thickness, ft
F = non-Darcy flow coefficient, $psi^2/cp/(Mscf/day)^2$
β = turbulence parameter

Jones (1987) proposed a mathematical expression for estimating the turbulence parameter β as:

$$\beta = 1.88\,(10^{-10})\,(k)^{-1.47}\,(\phi)^{-0.53} \quad (6\text{-}157)$$

where k = permeability, md
φ = porosity, fraction

The term $F Q_g^2$ can be included in all the compressible gas flow equations in the same way as the skin factor. This non-Darcy term is interpreted as being a *rate-dependent skin*. The modification of the gas flow equations to account for the turbulent flow condition is given below:

Unsteady-State Radial Flow

The gas flow equation for an unsteady-state flow is given by Equation 6-147 and can be modified to include the additional drop in the real gas potential as:

$$m(p_i) - m(p_{wf}) = \left(\frac{1637 Q_g T}{kh}\right)\left[\log \frac{kt}{\phi \mu_i c_{ti} r_w^2} - 3.23 + 0.87s\right] + F Q_g^2 \quad (6\text{-}158)$$

Equation 6-158 is commonly written in a more convenient form as:

$$m(p_i) - m(p_{wf}) = \left(\frac{1637 Q_g T}{kh}\right) \times \left[\log \frac{kt}{\phi \mu_i c_{ti} r_w^2} - 3.23 + 0.87s + 0.87 D Q_g\right] \quad (6\text{-}159)$$

where the term DQ_g is interpreted as the rate dependent skin factor. The coefficient D is called the **inertial or turbulent flow factor** and given by:

$$D = \frac{Fkh}{1422 T} \quad (6\text{-}160)$$

The true skin factor s which reflects the formation damage or stimulation is usually combined with the non-Darcy rate dependent skin and labeled as the **apparent or total skin factor:**

$$s' = s + D Q_g \quad (6\text{-}161)$$

or

$$m(p_i) - m(p_{wf}) = \left[\frac{1637 Q_g T}{kh}\right] \times \left[\log \frac{kt}{\phi \mu_i c_{ti} r_w^2} - 3.23 + 0.87s'\right] \quad (6\text{-}162)$$

Equation 6-162 can be expressed in the pressure-squared approximation form as:

$$p_i^2 - p_{wf}^2 = \left[\frac{1637 Q_g T \bar{z} \bar{\mu}}{kh}\right]\left[\log \frac{kt}{\phi \mu_i c_{ti} r_w^2} - 3.23 + 0.87s'\right] \quad (6\text{-}163)$$

where Q_g = gas flow rate, Mscf/day
t = time, hr
k = permeability, md
μ_i = gas viscosity as evaluated at p_i, cp

Semisteady-State Flow

Equations 6-150 and 6-151 can be modified to account for the non-Darcy flow as follows:

$$Q_g = \frac{kh[m(\bar{p}_r) - m(p_{wf})]}{1422 T\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + DQ_g\right]} \quad (6\text{-}164)$$

or in terms of the pressure-squared approach:

$$Q_g = \frac{kh(\bar{p}_r^2 - p_{wf}^2)}{1422 T \bar{\mu} \bar{z}\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + s + DQ_g\right]} \quad (6\text{-}165)$$

where the coefficient D is defined as:

$$D = \frac{Fkh}{1422 T} \quad (6\text{-}166)$$

Steady-State Flow

Similar to the above modification procedure, Equations 6-44 and 6-45 can be expressed as:

$$Q_g = \frac{kh[m(p_e) - m(p_{wf})]}{1422 T\left[\ln \frac{r_e}{r_w} - 0.5 + s + DQ_g\right]} \quad (6\text{-}167)$$

$$Q_g = \frac{kh(p_e^2 - p_{wf}^2)}{1422 T \bar{\mu} \bar{z}\left[\ln \frac{r_e}{r_w} - 0.5 + s + DQ_g\right]} \quad (6\text{-}168)$$

where D is defined by Equation 6-166.

Example 6-19

A gas well has an estimated wellbore damage radius of 2 feet and an estimated reduced permeability of 30 md. The formation has a permeability and porosity of 55 md and 12%. The well is producing at a rate of 20 MMscf/day with a gas gravity of 0.6. The following additional data is available:

$r_w = 0.25$, h = 20', T = 140°F, $\mu_{gw} = 0.013$ cp

Calculate the apparent skin factor.

Solution

Step 1. Calculate skin factor from Equation 6-143

$$s = \left[\frac{55}{30} - 1\right]\ln\left(\frac{2}{0.25}\right) = 1.732$$

Step 2. Calculate the turbulence parameter β by applying Equation 6-155:

$$\beta = 1.88\,(10)^{10}\,(55)^{-1.47}\,(0.12)^{-0.53} = 159.904 \times 10^6$$

Step 3. Calculate non-Darcy flow coefficient from Equation 6-156:

$$F = 3.1612 \times 10^{-12}\left[\frac{159.904 \times 10^6\,(600)\,(0.6)}{(0.013)\,(20)^2\,(0.25)}\right] = 0.14$$

Step 4. Calculate the coefficient D from Equation 6-160:

$$D = \frac{(0.14)\,(55)\,(20)}{(1422)\,(600)} = 1.805 \times 10^{-4}$$

Step 5. Estimate the apparent skin factor by applying Equation 6-161:

$$s' = 1.732 + (1.805 \times 10^{-4})\,(20{,}000) = 5.342$$

PRINCIPLE OF SUPERPOSITION

The solutions to the radial diffusivity equation as presented earlier in this chapter appear to be applicable only for describing the pressure distribution in an infinite reservoir that was caused by a constant production from a single well. Since real reservoir systems usually have several wells that are operating at varying rates, a more generalized approach is needed to study the fluid flow behavior during the unsteady state flow period.

The principle of superposition is a powerful concept that can be applied to remove the restrictions that have been imposed on various forms of solution to the transient flow equation. Mathematically the superposition theorem states that any sum of individual solutions to the diffusivity equation is also a solution to that equation. This concept can be applied to account for the following effects on the transient flow solution:

- Effects of multiple wells
- Effects of rate change
- Effects of the boundary
- Effects of pressure change

Slider (1976) presented an excellent review and discussion of the practical applications of the principle of superposition in solving a wide vari

7. PSEUDOSTEADY-STATE FLOW SYSTEMS

7.1. Radial Flow of Slightly Compressible Fluids, Pseudosteady-State Flow

condition used to find a solution to the radial diffusivity equation is that the outer boundary of the reservoir is a no-flow boundary. In mathematical terms,

$$\frac{\partial p}{\partial r} = 0 \qquad \text{at } r = r_e$$

Eq. 7.35

Applying these conditions to Eq. (7.35), the solution for the pressure at the wellbore becomes

$$p_{wf} = p_i - \frac{162.6 q \mu B}{kh} \log\left[\frac{4A}{1.781 C_A r_w^2}\right] - \frac{0.2339 q B t}{Ah\phi c_t} \qquad (7.47)$$

where A is the drainage area of the well in square feet and C_A is a reservoir shape factor. Values of the shape factor are given in Table 7.2 for several

After reaching pseudosteady-state flow, the pressure at every point in the reservoir is changing at the same rate, which suggests that the average reservoir pressure is also changing at the same rate. The volumetric average reservoir pressure, which is usually designated as $\bar{p}$ and is the pressure used to calculate fluid properties in material balance equations, is defined as:

$$\bar{p} = \frac{\sum_{j=1}^{n} \bar{p}_j V_j}{\sum_{j=1}^{n} V_j} \qquad (7.48)$$

where p_j is the average pressure in the jth drainage volume and V_j is the volume of the jth drainage volume. It is useful to rewrite Eq. (7.47) in terms of the average reservoir pressure, $\bar{p}$:

$$p_{wf} = \bar{p} - \frac{162.6 q \mu B}{kh} \log\left[\frac{4A}{1.781 C_A r_w^2}\right] \qquad (7.49)$$

$\ln\left(0.22 \frac{r_e^2}{r_w^2}\right)$

For a well in the center of a circular reservoir with a distance to the outer boundary of r_e, Eq. (7.49) reduces to:

$$p_{wf} = \bar{p} - \frac{70.6 q \mu B}{kh}\left[\ln\left(\frac{r_e^2}{r_w^2}\right) - 1.5\right] = \bar{p} - \frac{141.2 q \mu B}{kh}\left[\ln \frac{r_e}{r_w} - 0.75\right]$$

If this equation is rearranged and solved for q,

$$q = \frac{0.00708 kh}{\mu B}\left[\frac{\bar{p} - p_{wf}}{\ln(r_e/r_w) - 0.75}\right] \qquad (7.50)$$

7.2. Radial Flow of Compressible Fluids, Pseudosteady-State Flow

The differential equation for the flow of compressible fluids in terms of the real gas pseudopressure was derived in Eq. (7.45). When the appropriate boundary conditions are applied to Eq. (7.45), the pseudosteady-state solution rearranged and solved for q yields Eq. (7.51):

$$q = \frac{19.88(10)^{-6} kh T_{sc}}{T p_{sc}}\left[\frac{m(\bar{p}) - m(p_{wf})}{\ln(r_e/r_w) - 0.75}\right] \qquad (7.51)$$

$10^{6} \times 14.7 = 1422$

TABLE 7.2.
Shape factors for various single-well drainage areas (after Earlougher.[3])

In Bounded Reservoirs	C_A	$\ln C_A$	$1/2 \ln\left(\frac{2.2458}{C_A}\right)$	Exact for t_{DA} >	Less than 1% Error for t_{DA} >	Use Infinite System Solution with Less than 1% Error for t_{DA} <
	31.62	3.4538	−1.3224	0.1	0.06	0.10
	31.6	3.4532	−1.3220	0.1	0.06	0.10
	27.6	3.3178	−1.2544	0.2	0.07	0.09
	27.1	3.2995	−1.2452	0.2	0.07	0.09
	21.9	3.0865	−1.1387	0.4	0.12	0.08
	0.098	−2.3227	1.5659	0.9	0.60	0.015

4.5 GAS FLOW EQUATION SUMMARY

Writing $C = 0.703\ kh/[\mu Tz \ln(r_e/r_w)]$, and the power n is turbulent flow effect in steady state gas flow, Equation (4.27) becomes

$$Q_g = C(p_r^2 - p_{wf}^2)^n \qquad (4.44)$$

This equation is normally called the empirical back pressure equation. The power n is the turbulent effect which is ranging from low flow rate = 1.00 to absolutely turbulent = 0.500. The equation is not especially helpful in predicting reservoir characteristics or in analyzing the components of pressure drops, although it may be useful in characterizing well performance.

When multirate data are available it is more useful to revert to one of the basis flow equations in field units:

$$\bar{p}_r^2 - P_{wf}^2 = \left(\frac{1422 Q_g \mu z T}{kh}\right)\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right] \qquad (4.45)$$

Or

$$\bar{p}_r^2 - P_{wf}^2 = \left(\frac{1422 Q_g \bar{\mu} \bar{z} T}{kh}\right)\left[\ln\left(\frac{0.472 r_e}{r_w}\right) + S\right] + BQ^2 \qquad (4.46)$$

(semi-steady state)

Or

$$\bar{p}_r^2 - P_{wf}^2 = \left(\frac{1422 Q_g \overline{\mu z} T}{kh}\right)\left[\frac{1}{2}\ln(t_D + 0.809) + S\right] + BQ^2 \qquad (4.47)$$

(transient)

$$\Delta(p^2) = AQ + BQ^2$$

Where

$$A = \left(\frac{1422 Q_g \overline{\mu z} T}{kh}\right)\left[\ln\left(\frac{0.472 r_e}{r_w}\right) + S\right] \text{(semi-steady state)}$$

$$= \left(\frac{1422 Q_g \overline{\mu z} T}{kh}\right)\left[\frac{1}{2}\ln\left(t_D + 0.809\right) + S\right] \text{(transient)}$$

And B = non-Darcy coefficient,

It is more appropriate to use the real gas pseudo-pressure $m(p)$ equation:

$$m(\overline{p}_r) - m(p_{wf}) = \left(\frac{1422 Q_g T}{kh}\right)\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S\right] + FQ^2$$

$$= \left(\frac{1422 Q_g T}{kh}\right)\left[\ln\left(\frac{r_e}{r_w}\right) - 0.75 + S + DQ\right] \quad (4.48)$$

Where

$$D = \left(\frac{Fkh}{1422 T}\right)$$

And DQ is known as the rate dependent skin factor

8. PRODUCTIVITY INDEX (PI)

The ratio of the rate of production, expressed in STB/day for liquid flow, to the pressure drawdown at the midpoint of the producing interval, is called the *productivity index*, symbol *J*.

$$J = \frac{q}{\overline{p} - p_{wf}} \quad (7.52)$$

In some wells, the PI remains constant over a wide variation in flow rate such that the flow rate is directly proportional to the bottom-hole pressure drawdown. In other wells, at higher flow rates the linearity fails, and the PI index declines, as shown in Fig. 7.14. The cause of this decline may be (a) turbulence at increased rates of flow, (b) decrease in the permeability to oil due to presence of free gas caused by the drop in pressure at the well bore, (c) the increase in oil viscosity with pressure drop below bubble point, and/or (d) reduction in permeability due to formation compressibility.

In depletion reservoirs, the productivity indexes of the wells decline as depletion proceeds, owing to the increase in oil viscosity as gas is released from solution and to the decrease in the permeability of the rock to oil as the oil saturation decreases. Since each of these factors may change from a few to several-fold during depletion, the PI may decline to a small fraction of the initial value. Also, as the permeability to oil decreases, there is a corresponding increase in the permeability to gas, which results in rising gas-oil ratios.

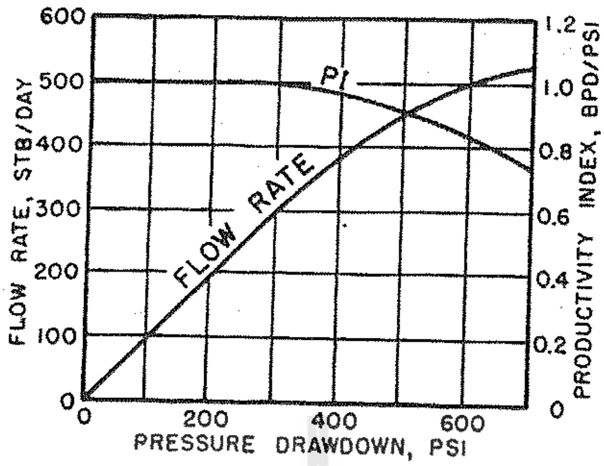

Fig. 7.14. Decline in productivity index at higher flow rates.

The *injectivity index* is used with salt water disposal wells and with *injection wells* for secondary recovery or pressure maintenance. It is the ratio of the injection rate in STB per day to the excess pressure above reservoir pressure that causes that injection rate, or

Injectivity

$$\text{Injectivity index} = I = \frac{q}{p_{wf} - \bar{p}} \text{ STB/day/psi} \quad (7.53)$$

there is a variation in net productive thickness but when the other factors affecting the productivity index are essentially the same, the specific productivity index J_s is sometimes used, which is the productivity index divided by the net feet of pay, or,

$$q = \frac{0.00708\,kh(\bar{p} - p_{wf})}{\mu B(\ln\frac{r_e}{r_w} - 0.75)}$$

$$\text{Specific productivity index} = J_s = \frac{J}{h} = \frac{q}{h(\bar{p} - p_{wf})} \text{ STB/day/psi/ft} \quad (7.54)$$

P.I $\quad J = \dfrac{q}{\bar{p} - p_{wf}}$

8.1. Productivity Ratio (PR)

In evaluating well performance, the standard usually referred to is the productivity index of an open hole that completely penetrates a circular formation normal to the strata, and in which no alteration in permeability has occurred in the vicinity of the wellbore. Substituting Eq. (7.50) into Eq. (7.52) we get

$$J = 0.00708 \frac{kh}{\mu B(\ln(r_e/r_w) - 0.75)} \quad (7.55)$$

The PR then is the ratio of the PI of a well in any condition to the PI of this *standard* well:

$$PR = \frac{J}{J_{sw}} = \frac{\dfrac{0.00708\,kh}{\mu B(\ln\frac{r_e}{0.667} - 0.75)}}{\dfrac{0.00708\,kh}{\mu B(\ln\frac{r_e}{0.333} - 0.75)}} \quad (7.56)$$

Thus, the productivity ratio may be less than one, greater than one, or equal to one. Although the productivity index of the standard well is generally unknown, the relative effect of certain changes in the well system may be evaluated from theoretical considerations, laboratory models, or well tests. For example, the theoretical productivity ratio of a well reamed from an 8-in. borehole diameter to 16 in. is derived by Eq. (7.55):

$$PR = \frac{J_{16}}{J_8} = \frac{\ln(r_e/0.333) - 0.75}{\ln(r_e/0.667) - 0.75}$$

Assuming $r_e = 660$ ft,

$$PR = \frac{\ln(660/0.333) - 0.75}{\ln(660/0.667) - 0.75} = 1.11$$

HW NO 3.1: DO THE PROBLEM 6.7, 6.19, 6.23, 6.26 6.36 AND NO 4 DUE DATE: FRIDAY 1 November 2013

6.7. Two wells are located 2500 ft apart. The static well pressure at the top of perforations (9332 ft subsea) in well A is 4365 psia and at the top of perforations (9672 ft subsea) in well B is 4372 psia. The reservoir fluid gradient is 0.25 psi/ft, reservoir permeability is 245 md, and reservoir fluid viscosity is 0.63 cp.

(a) Correct the two static pressures to a datum level of 9100 ft subsea. *Ans:* $p_A = 4307$, $p_B = 4229$ psia.

(b) In what direction is the fluid flowing between the wells? *Ans:* A to B.

(c) What is the average effective pressure gradient between the wells? *Ans:* 0.0309 psi/ft.

(d) What is the fluid velocity? *Ans:* 0.0758 ft/day.

(e) Is this the total velocity or only the component of the velocity in the direction between the two wells?

(f) Show that the same fluid velocity is obtained using Eq. (6.5). *Ans:* 0.0758 ft/day.

6.19. A sand body is 1500 feet long, 300 feet wide, and 12 feet thick. It has a uniform permeability of 345 md to oil at 17 per cent connate water saturation. The porosity is 32 per cent. The oil has a reservoir viscosity of 3.2 cp and an FVF of 1.25 at bubble point.

(a) If flow takes place above saturation pressure, what pressure drop will cause 100 reservoir barrels per day to flow through the sand body, assuming the fluid behaves essentially as an incompressible fluid? What for 200 reservoir BPD? *Ans:* 344; 688 psi.

(b) What is the apparent velocity of the oil in feet per day at the 100 BPD flow rate? *Ans:* 0.156 ft/day.

(c) What is the actual average velocity? *Ans:* 0.587 ft/day.

(d) What time will be required for complete displacement of the oil from the sand? *Ans:* 2555 days.

(e) What pressure gradient exists in the sand? *Ans:* 0.229 psi/ft.

(f) What will be the effect of raising both the upstream and downstream pressures by say 1000 psi?

(g) Considering the oil as a fluid with a very high compressibility of 65×10^{-6} psi^{-1}, how much greater is the flow rate at the downstream end than the upstream at 100 BPD? *Ans:* 2.23 BPD.

(h) Derive an equation for the linear flow of compressible liquids. *Suggestion:* The flow rate at any point will be $q = q_1[1 + c_o(p_1 - p)]$, where q_1 is the upstream flow rate, p_1 the upstream pressure, and c_o the effective average oil compressibility.

6.26. (a) A limestone formation has a matrix (primary or intergranular) permeability of less than one millidarcy. However, it contains ten solution channels per square foot, each 0.02 inches in diameter. If the channels lie in the direction of fluid flow, what is the permeability of the rock? *Ans:* 176 md.

(b) If the porosity of the matrix rock is 10 per cent, what percentage of the fluid is stored in the primary pores, and what in the secondary pores (vugs, fractures, etc.)? *Ans:* 99.978 and 0.022.

(c) If the secondary pore system is well connected throughout a reservoir, what conclusions must be drawn concerning the probable result of gas or water drive on the recovery of either oil, gas, or gas-condensate? What then are the means of recovering the hydrocarbons from the primary pores?

6.36. A producing formation consists of two strata, one 15 ft thick and 150 md in permeability; the other 10 ft thick and 400 md in permeability.

(a) What is the average permeability? *Ans:* 250 md.

(b) What is the capacity of the formation? *Ans:* 6250 md-ft. Kh

(c) If during well workover the 150 md strata permeability is reduced to 25 md out to a radius of 4 ft, and the 400 md strata is reduced to 40 md out to an 8-ft radius, what is the average permeability after well workover, assuming no cross-flow between beds? Use $r_e = 500$ ft and $r_w = 0.5$ ft. *Ans:* 71 md.

(d) To what per cent of the original productivity index will the well be reduced? *Ans:* 28.4.

(e) What is the capacity of the damaged formation? *Ans:* 1775 md-ft. Kh

4. An oil well is producing a crude oil system at 1000 STB/day and 2000 psi of bottom-hole flowing pressure. The pay zone and the producing well have the following characteristics:

h = 35 ft	r_w = 0.25 ft	drainage area = 40 acres
API = 45°	γ_g = 0.72	R_s = 700 scf/STB
k = 80 md	T = 100°F	

Assuming steady-state flowing conditions, calculate and plot the pressure profile around the wellbore.

6.26. (a) A limestone formation has a matrix (primary or intergranular) permeability of less than one millidarcy. However, it contains ten solution channels per square foot, each 0.02 inches in diameter. If the channels lie in the direction of fluid flow, what is the permeability of the rock? *Ans:* 176 md.

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h = 35 ft	r_w = 0.25 ft	drainage area = 40 acres
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k = 80 md	T = 100°F	

Assuming steady-state flowing conditions, calculate and plot the pressure profile around the wellbore.

HW NO 3 3.1 Steady state flow

RESERVOIR I TEXT; 6.7, 6.19, 6.23, 6.36

RESERVOIR ENG. HAND BOOK; 4

HW NO 3; DO THE PROBLEM 6.7, 6.19, 6.23, 6.36 AND NO 4

DUE DATE; FRIDAY 22 OCTOBER 2010

3.2 Unsteady state flow

4. PSEUDO Steady state flow and Superposition

DUE DATE; FRIDAY 29 OCTOBER 2010

HW NO 4

RESERVOIR I TEXT; 7.22

RESERVOIR ENG. HAND BOOK; 9,

DUE DATE; FRIDAY 29 OCTOBER 2010

HW NO 3; DO THE PROBLEM 6.7, 6.19, 6.23, 6.26 6.36 AND NO 4

DUE DATE; FRIDAY 1 November 2013

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(g) Considering the oil as a fluid with a very high compressibility of 65×10^{-6} psi^{-1}, how much greater is the flow rate at the downstream end than the upstream at 100 BPD? *Ans:* 2.23 BPD.

(h) Derive an equation for the linear flow of compressible liquids. *Suggestion:* The flow rate at any point will be $q = q_1[1 + c_o(p_1 - p)]$, where q_1 is the upstream flow rate, p_1 the upstream pressure, and c_o the effective average oil compressibility.

6.23. A horizontal pipe 10 cm I.D. and 3000 cm long is filled with a sand of 20 per cent porosity. It has a connate water saturation of 30 per cent, and, at that water saturation, a permeability to oil of 200 md. The viscosity of the oil is 0.65 cp and the water is immobile.

(a) What is the apparent velocity of the oil under a 100 psi pressure differential? *Ans:* 0.0007 cm/sec.

(b) What is the flow rate? *Ans:* 0.055 cu cm/sec.

(c) Calculate the oil contained in the pipe, and the time to displace it at the rate of 0.055 cu cm/sec. *Ans:* 7 days.

(d) From this actual time and the length of the pipe calculate the actual average velocity. *Ans:* 0.005 cm/sec.

(e) Calculate the actual average velocity from the apparent velocity, porosity, and connate water. *Ans:* 0.005 cm/sec.

(f) Which velocity is used to calculate flow rates and which is used to calculate displacement times?

(g) If the oil is displaced with water so that 20 per cent unrecoverable (or residual) oil saturation is left behind the water flood front, what are the apparent and actual average velocities in the watered zone behind the flood front if the oil production rate is maintained at 0.055 cu cm/sec? Assume piston-like displacement of the oil by the water. *Ans:* 0.0007 and 0.007 cm/sec.

(h) What is the rate of advance of the flood front? *Ans:* 0.007 cm/sec.

(i) How long will it take to obtain all of the recoverable oil, and how much will be recovered? *Ans:* 430,000 sec; 23,570 cu cm.

(j) If the permeability to water behind the flood front is 123 md at 80 per cent water saturation, and the viscosity of the water is 0.80 cp, what are the water and oil mobilities, and what is the mobility ratio of the water behind the flood front to the oil ahead of it? *Ans:* 0.154; 0.308; 0.500.

(k) How much pressure drop will be required to produce oil at the rate of 0.055 cu cm/sec when the water flood front is at the midpoint of the pipe? *Ans:* 150 psi.

7.19 (a) Plot pressure versus radius on both linear and semilog paper at 0.1, 1.0, 10, and 100 days for $p_e = 2500$ psia, $q = 300$ STB/day; $B_o = 1.32$ bbl/STB; $\mu = 0.44$ cp; $k = 25$ md; $h = 43$ ft; $c_t = 18 \times 10^{-6}$ psi^{-1}; $\phi = 0.16$.

(b) Assuming that a pressure drop of 5 psi can be easily detected with a pressure gauge, how long must the well be flowed to produce this drop in a well located 1200 ft away?

(c) Suppose the flowing well is located 200 ft due east of a north-south fault. What pressure drop will occur after 10 days of flow, in a shut-in well located 600 ft due north of the flowing well?

(d) What will the pressure drop be in a shut-in well 500 ft from the flowing well when the flowing well has been shut in for one day following a flow period of 5 days at 300 STB/day?

7.20 A shut-in well is located 500 ft from one well and 1000 ft from a second well. The first well flows for 3 days at 250 STB/day, at which time the second well begins to flow at 400 STB/day. What is the pressure drop in the shut-in well when the second well has been flowing for 5 days (i.e., the first has been flowing a total of 8 days)? Use the reservoir constants of Prob. 7.19.

3.2 Unsteady state;
Problem 7.20 and 7.24
8 and 13
Due date; 8 Nov. 2013

7.24 Develop an equation to calculate and then calculate the pressure at well 1, illustrated in Fig. 7.23, if the well has flowed for 5 days at a flow rate of 200 STB/day.

$\phi = 25\%$	$h = 20$ ft
$c_t = 30(10)^{-6}$ psi^{-1}	$\mu = 0.5$ cp
$k = 50$ md	$B_o = 1.32$ bbl/STB
$r_w = 0.33$ ft	$p_i = 4000$ psia

13. A well is flowing under unsteady-state flowing conditions for 5 days at 300 STB/day. The well is located at 350 ft and 420 ft distance from two sealing faults. Given:

$\phi = 17\%$	$c_t = 16 \times 10^{-6}$ psi^{-1}	$k = 80$ md
$p_i = 3000$ psi	$B_o = 1.3$ bbl/STB	$\mu_o = 1.1$ cp
$r_w = 0.25$ ft	$h = 25$ ft	

Calculate the pressure in the well after 5 days.

8. An oil well is producing at a constant flow rate of 800 STB/day under a transient flow condition. The following data are available:

$B_o = 1.2$ bbl/STB	$\mu_o = 3$ cp	$c_t = 15 \times 10^{-6}$ psi^{-1}
$k = 100$ md	$h = 25$ ft	$\phi = 15\%$
$r_w = 0.5$	$p_i = 4000$ psi	$r_e = 1000$ ft

Using the E_i-function approach and the p_D-method, calculate the bottom-hole flowing pressure after 1, 2, 3, 5, and 10 hr. Plot the results on a semi-log scale and Cartesian scale.

7.22 The following data pertain to a volumetric gas reservoir:

- Net formation thickness = 15 ft
- Hydrocarbon porosity = 20%
- Initial reservoir pressure = 6000 psia
- Reservoir temperature = 190°F
- Gas viscosity = 0.020 cp
- Casing diameter = 6 in.
- Average formation permeability = 6 md

HW no 4 Pseudo steady state;
Problem 7.22 and 9
Due date; 8 Nov. 2013

centage of recovery from a 640 ac unit for a producing rate of 4.00 MM SCF/day when the flowing well pressure reaches 500 psia.

(b) If the average reservoir permeability had been 60 md instead of 6 md, what recovery would be obtained at 4.00 MM SCF/day and a flowing well pressure of 500 psia?

(c) Recalculate part (a) for a production rate of 2.00 MM SCF/day.

(d) Suppose four wells are drilled on the 640 ac unit, and each is produced at 4.00 MM SCF/day. For 6 md and 500 psia minimum flowing well pressure, calculate the recovery.

9. A well is flowing under a drawdown pressure of 350 psi and produces at constant flow rate of 300 STB/day. The net thickness is 25 ft. Given:

$r_e = 660$ ft $\quad r_w = 0.25$ ft $\quad \mu_o = 1.2$ cp $\quad B_o = 1.25$ bbl/STB

Calculate:
- Average permeability
- Capacity of the formation

centage of recovery from a 640 ac unit for a producing rate of 4.00 MM SCF/day when the flowing well pressure reaches 500 psia.

(b) If the average reservoir permeability had been 60 md instead of 6 md, what recovery would be obtained at 4.00 MM SCF/day and a flowing well pressure of 500 psia?

(c) Recalculate part (a) for a production rate of 2.00 MM SCF/day.

(d) Suppose four wells are drilled on the 640 ac unit, and each is produced at 4.00 MM SCF/day. For 6 md and 500 psia minimum flowing well pressure, calculate the recovery.

7.23 A sandstone reservoir, producing well above its bubble-point pressure, contains only one producing well, which is flowing only oil at a constant rate of 175 STB/day. Ten weeks after this well began producing, another well was completed 660 ft away in the same formation. On the basis of the reservoir properties that follow, estimate the initial formation pressure that should be encountered by the second well at the time of completion.

$\phi = 15\%$ $\quad h = 30$ ft

$c_o = 18(10)^{-6}$ psi^{-1} $\quad \mu = 2.9$ cp

$c_w = 3(10)^{-6}$ psi^{-1} $\quad k = 35$ md

$c_f = 4.3(10)^{-6}$ psi^{-1} $\quad r_w = 0.33$ ft

$S_w = 33\%$ $\quad p_i = 4300$ psia

$B_o = 1.25$ bbl/STB

7.24 Develop an equation to calculate and then calculate the pressure at well 1, illustrated in Fig. 7.23, if the well has flowed for 5 days at a flow rate of 200 STB/day.

$\phi = 25\%$ $\quad h = 20$ ft

$c_t = 30(10)^{-6}$ psi^{-1} $\quad \mu = 0.5$ cp

$k = 50$ md $\quad B_o = 1.32$ bbl/STB

$r_w = 0.33$ ft $\quad p_i = 4000$ psia

7.25 A pressure drawdown test was conducted on the discovery well in a new reservoir to estimate the drainage volume of the reservoir. The well was flowed at a constant rate of 125 STB/day. The bottom-hole pressure data, as well as other rock and fluid property data, follow. What are the drainage volume of the well and the average permeability of the drainage volume? The initial reservoir pressure was 3900 psia.

$B_o = 1.1$ bbl/STB $\quad \mu_o = 0.80$ cp

$\phi = 20\%$ $\quad h = 22$ ft

$S_o = 80\%$ $\quad S_w = 20\%$

$c_o = 10(10)^{-6}$ psi^{-1} $\quad c_w = 3(10)^{-6}$ psi^{-1}

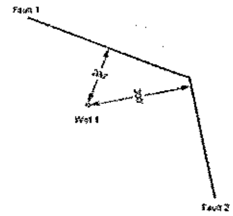

Fig. 7.23. Well layout for Prob. 7.24.

Time in Hours	p_{wf}, psi
0.5	3657
1.0	3639
1.5	3629
2.0	3620
3.0	3612
5.0	3598
7.0	3591
10.0	3583
20.0	3565
30.0	3551
40.0	3548
50.0	3544
60.0	3541
70.0	3537
80.0	3533
90.0	3529
100.0	3525
120.0	3518
150.0	3505

7.26 The initial average reservoir pressure in the vicinity of a new well was 4150 psia. A pressure drawdown test was conducted while the well was flowed at a constant oil flow rate of 550 STB/day. The oil had a viscosity of 3.3 cp and a formation volume factor of 1.55 bbl/STB. Other data, along with the bottom-hole pressure data recorded during the drawdown test, follow. Assume that wellbore storage considerations may be neglected, and determine the following:

(a) The permeability of the formation around the well.

is 18%. The flowing fluid has a formation volume factor of 1.12 bbl/STB and a viscosity of 1.5 cp.

(a) Calculate the flow rate in STB/day.

(b) Calculate the pressure in the reservoir at a distance of 300 ft from the center of the wellbore.

7.11 (a) A limestone formation has a matrix (primary or intergranular) permeability of less than 1 md. However, it contains 10 solution channels per square foot, each 0.02 in. in diameter. If the channels lie in the direction of fluid flow, what is the permeability of the rock?

(b) If the porosity of the matrix rock is 10%, what percentage of the fluid is stored in the primary pores, and what in the secondary pores (vugs, fractures, etc.)?

(c) If the secondary pore system is well connected throughout a reservoir, what conclusions must be drawn concerning the probable result of gas or water drive on the recovery of either oil, gas, or gas-condensate? What then are the means of recovering the hydrocarbons from the primary pores?

7.12 During a gravel rock operation the 6 in. I.D. liner became filled with gravel, and a layer of mill scale and dirt accumulated to a thickness of 1 in. on top of the gravel within the pipe. If the permeability of the accumulation is 1000 md, what additional pressure drop is placed on the system when pumping a 1 cp fluid at the rate of 100 bbl/hr?

7.13 One hundred capillary tubes of 0.02 in. ID and 50 capillary tubes of 0.04 in. ID, all of equal length, are placed inside a pipe of 2 in. inside diameter. The space between the tubes is filled with wax so that flow is only through the capillary tubes. What is the permeability of this "rock"?

7.14 Suppose, after cementing, an opening 0.01 in. wide is left between the cement and an 8 in. diameter hole. If this circular fracture extends from the producing formation through an impermeable shale 20 ft thick to an underlying water sand, at what rate will water enter the producing formation (well) under a 100 psi pressure drawdown? The water contains 60,000 ppm salt and the bottom-hole temperature is 150°F.

7.15 A high water-oil ratio is being produced from a well. It is thought that the water is coming from an underlying aquifer 20 ft from the oil producing zone. In between the aquifer and the producing zone is an impermeable shale zone. Assume that the water is coming up through an incomplete cementing job that left an opening 0.01 in. wide between the cement and the 8 in. hole. The water has a viscosity of 0.5 cp. Determine the rate at which water is entering the well at the producing formation level if the pressure in the aquifer is 150 psi greater than the pressure in the well at the producing formation level.

7.16 Derive the equation for the steady-state, semispherical flow of an incompressible fluid.

7.17 A well has a shut-in bottom-hole pressure of 2300 psia and flows 215 bbl/day of oil under a drawdown of 500 psi. The well produces from a formation of 36 ft net productive thickness. Use $r_w = 6$ in.; $r_e = 660$ feet; $\mu = 0.88$ cp; $B_o = 1.32$ bbl/STB.

(a) What is the productivity index of the well?

(b) What is the average permeability of the formation?

(c) What is the capacity of the formation?

7.18 A producing formation consists of two strata: one 15 ft thick and 150 md in permeability; the other 10 ft thick and 400 md in permeability.

(a) What is the average permeability?

(b) What is the capacity of the formation?

(c) If during a well workover the 150 md stratum permeability is reduced to 25 md out to a radius of 4 ft, and the 400 md stratum is reduced to 40 md out to an 8 ft radius, what is the average permeability after the workover, assuming no cross-flow between beds? Use $r_e = 500$ ft and $r_w = 0.5$ ft.

(d) To what percentage of the original productivity index will the well be reduced?

(e) What is the capacity of the damaged formation?

7.19 (a) Plot pressure versus radius on both linear and semilog paper at 0.1, 1.0, 10, and 100 days for $p_i = 2500$ psia, $q = 300$ STB/day; $B_o = 1.32$ bbl/STB; $\mu = 0.44$ cp; $k = 25$ md; $h = 43$ ft; $c_t = 18 \times 10^{-6}$ psi^{-1}; $\phi = 0.16$.

(b) Assuming that a pressure drop of 5 psi can be easily detected with a pressure gauge, how long must the well be flowed to produce this drop in a well located 1200 ft away?

(c) Suppose the flowing well is located 200 ft due east of a north-south fault. What pressure drop will occur after 10 days of flow, in a shut-in well located 600 ft due north of the flowing well?

(d) What will the pressure drop be in a shut-in well 500 ft from the flowing well when the flowing well has been shut in for one day following a flow period of 5 days at 300 STB/day?

7.20 A shut-in well is located 500 ft from one well and 1000 ft from a second well. The first well flows for 3 days at 250 STB/day, at which time the second well begins to flow at 400 STB/day. What is the pressure drop in the shut-in well when the second well has been flowing for 5 days (i.e., the first has been flowing a total of 8 days)? Use the reservoir constants of Prob. 7.19.

7.21 A well is opened to flow at 200 STB/day for 1 day. The second day its flow is increased to 400 STB/day and the third to 600 STB/day. What is the pressure drop caused in a shut-in well 500 ft away after the third day? Use the reservoir constants of Prob. 7.19.

7.22 The following data pertain to a volumetric gas reservoir:

Net formation thickness = 15 ft

Hydrocarbon porosity = 20%

Initial reservoir pressure = 6000 psia

Reservoir temperature = 190°F

Gas viscosity = 0.020 cp

Casing diameter = 6 in.

Average formation permeability = 6 md

(a) Assuming ideal gas behavior and uniform permeability, calculate the per-

HW NO 3: DO THE PROBLEM 6.7, 6.19, 6.23, 6.26 6.36 AND NO 4 DUE DATE: FRIDAY 1 November 2013

1. lowing properties.

$L = 2500$ ft $h = 30$ ft width $= 500$ ft
$k = 50$ md $\phi = 17\%$ $\mu = 2$ cp
inlet pressure $= 2100$ psi $Q = 4$ bbl/day $\rho = 45$ lb/ft^3

Calculate and plot the pressure profile throughout the linear system.

2. Assume the reservoir linear system as described in problem 1 is tilted with a dip angle of 7°. Calculate the fluid potential through the linear system.
3. A 0.7 specific gravity gas is flowing in a linear reservoir system at 150°F. The upstream and downstream pressures are 2000 and 1800 psi, respectively. The system has the following properties:

$L = 2000$ ft $W = 300$ ft $h = 15$ ft
$k = 40$ md $\phi = 15\%$

Calculate the gas flow rate.

4. An oil well is producing a crude oil system at 1000 STB/day and 2000 psi of bottom-hole flowing pressure. The pay zone and the producing well have the following characteristics:

$h = 35$ ft $r_w = 0.25$ ft drainage area $= 40$ acres
API $= 45°$ $\gamma_g = 0.72$ $R_s = 700$ scf/STB
$k = 80$ md $T = 100°F$

Assuming steady-state flowing conditions, calculate and plot the pressure profile around the wellbore.

5. Assuming steady-state flow and incompressible fluid, calculate the oil flow rate under the following conditions:

$p_e = 2500$ psi $p_{wf} = 2000$ psi $r_e = 745$ ft
$r_w = 0.3$ ft $\mu_o = 2$ cp $B_o = 1.4$ bbl/STB
$h = 30$ ft $k = 60$ md

Fundamentals of Reservoir Fluid Flow **477**

6. A gas well is flowing under a bottom-hole flowing pressure of 900 psi. The current reservoir pressure is 1300 psi. The following additional data are available:

$T = 140°F$ $\gamma_g = 0.65$ $r_w = 0.3$ ft
$k = 60$ md $h = 40$ ft $r_e = 1000$ ft

Calculate the gas flow rate by using a:
- Real gas pseudo-pressure approach
- Pressure-squared method.

7. An oil well is producing a stabilized flow rate of 500 STB/day under a transient flow condition. Given:

$B_o = 1.1$ bbl/STB $\mu_o = 2$ cp $c_t = 15 \times 10^{-6}$ psi^{-1}
$k_o = 50$ md $h = 20$ ft $\phi = 20\%$
$r_w = 0.3$ ft $p_i = 3500$ psi

Calculate and plot the pressure profile after 1, 5, 10, 15, and 20 hours.

8. An oil well is producing at a constant flow rate of 800 STB/day under a transient flow condition. The following data are available:

$B_o = 1.2$ bbl/STB $\mu_o = 3$ cp $c_t = 15 \times 10^{-6}$ psi^{-1}
$k_o = 100$ md $h = 25$ ft $\phi = 15\%$
$r_w = 0.5$ $p_i = 4000$ psi $r_e = 1000$ ft

Using the E_i-function approach and the p_D-method, calculate the bottom-hole flowing pressure after 1, 2, 3, 5, and 10 hr. Plot the results on a semi-log scale and Cartesian scale.

9. A well is flowing under a drawdown pressure of 350 psi and produces at constant flow rate of 300 STB/day. The net thickness is 25 ft. Given:

$r_e = 660$ ft $r_w = 0.25$ ft $\mu_o = 1.2$ cp $B_o = 1.25$ bbl/STB

Calculate:
- Average permeability
- Capacity of the formation

7.20 A shut-in well is located 500 ft from one well and 1000 ft from a second well. The first well flows for 3 days at 250 STB/day, at which time the second well begins to flow at 400 STB/day. What is the pressure drop in the shut-in well when the second well has been flowing for 5 days (i.e., the first has been flowing a total of 8 days)? Use the reservoir constants of Prob. 7.19.

7.24 **Develop an equation to calculate and then calculate the pressure at well 1, illustrated in Fig. 7.23, if the well has flowed for 5 days at a flow rate of 200 STB/day.**

$\phi = 25\%$ $h = 20$ ft
$c_t = 30(10)^{-6}$ psi^{-1} $\mu = 0.5$ cp
$k = 50$ md $B_o = 1.32$ bbl/STB
$r_w = 0.33$ ft $p_i = 4000$ psia

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An oil well is producing from the center of 40-acre square drilling pattern. Given:

$\phi = 20\%$ | $h = 15$ ft | $k = 60$ md
$\mu_o = 1.5$ cp | $B_o = 1.4$ bbl/STB | $r_w = 0.25$ ft
$p_e = 2000$ psi | $p_{wf} = 1500$ psi

Calculate the oil flow rate.

A shut-in well is located at a distance of 700 ft from one well and 1100 ft from a second well. The first well flows for 5 days at 180 STB/day, at which time the second well begins to flow at 280 STB/day. Calculate the pressure drop in the shut-in well when the second well has been flowing for 7 days. The following additional data are given:

$p_i = 3000$ psi | $B_o = 1.3$ bbl/STB | $\mu_o = 1.2$ cp | $h = 60$ ft
$c_t = 15 \times 10^{-6}$ psi^{-1} | $\phi = 15\%$ | $k = 45$ md

2. A well is opened to flow at 150 STB/day for 24 hours. The flow rate is then increased to 360 STB/day and lasted for another 24 hours. The well flow rate is then reduced to 310 STB/day for 16 hours. Calculate the pressure drop in a shut-in well 700 ft away from the well given:

$\phi = 15\%$ | $h = 20$ ft | $k = 100$ md
$\mu_o = 2$ cp | $B_o = 1.2$ bbl/STB | $r_w = 0.25$ ft
$p_i = 3000$ psi | $c_t = 12 \times 10^{-6}$ psi^{-1}

3. A well is flowing under unsteady-state flowing conditions for 5 days at 300 STB/day. The well is located at 350 ft and 420 ft distance from two sealing faults. Given:

$\phi = 17\%$ | $c_t = 16 \times 10^{-6}$ psi^{-1} | $k = 80$ md
$p_i = 3000$ psi | $B_o = 1.3$ bbl/STB | $\mu_o = 1.1$ cp
$r_w = 0.25$ ft | $h = 25$ ft

Calculate the pressure in the well after 5 days.

14. A drawdown test was conducted on a new well with results as given below:

t, hr	p_{wf}, psi
1.50	2978
3.75	2949
7.50	2927
15.00	2904
37.50	2876
56.25	2863
75.00	2848
112.50	2810
150.00	2790
225.00	2763

Given:

$p_i = 3400$ psi | $h = 25$ ft | $Q = 300$ STB/day
$c_t = 18 \times 10^{-6}$ psi^{-1} | $\mu_o = 1.8$ cp | $B_o = 1.1$ bbl/STB
$r_w = 0.25$ ft | $\phi = 12\%$

Assuming no wellbore storage, calculate:

- Average permeability
- Skin factor

15. A drawdown test was conducted on a discovery well. The well was flowed at a constant flow rate of 175 STB/day. The fluid and reservoir data are given below:

$S_{wi} = 25\%$ | $\phi = 15\%$ | $h = 30$ ft | $c_t = 18 \times 10^{-6}$ psi^{-1}
$r_w = 0.25$ ft | $p_i = 4680$ psi | $\mu_o = 1.5$ cp | $B_o = 1.25$ bbl/STB

centage of recovery from a 640 ac unit for a producing rate of 4.00 MM SCF/day when the flowing well pressure reaches 500 psia.

(b) If the average reservoir permeability had been 60 md instead of 6 md, what recovery would be obtained at 4.00 MM SCF/day and a flowing well pressure of 500 psia?

(c) Recalculate part (a) for a production rate of 2.00 MM SCF/day.

(d) Suppose four wells are drilled on the 640 ac unit, and each is produced at 4.00 MM SCF/day. For 6 md and 500 psia minimum flowing well pressure, calculate the recovery.

7.23 A sandstone reservoir, producing well above its bubble-point pressure, contains only one producing well, which is flowing only oil at a constant rate of 175 STB/day. Ten weeks after this well began producing, another well was completed 660 ft away in the same formation. On the basis of the reservoir properties that follow, estimate the initial formation pressure that should be encountered by the second well at the time of completion.

$\phi = 15\%$ | $h = 30$ ft
$c_o = 18(10)^{-6}$ psi^{-1} | $\mu = 2.9$ cp
$c_w = 3(10)^{-6}$ psi^{-1} | $k = 35$ md

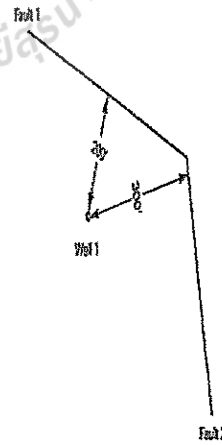

Fig. 7.23. Well layout for Prob. 7.24.

2. Saturation and Relative Permeability Calculations (4 hrs.)
3. Steady State Radial Flow (4 hrs.)
4. Unsteady State Radial Flow (4 hrs.)
5. Pseudo-steady State Flow and Superposition (4 hrs.)
6. Well Testing Pressure Drawdown and Build Up (3 hrs.)
7. Interference Test and Type Curve Analysis(3hrs)
8. Displacement Efficiency(4 hrs)
9. Potential flow and Streamlines (4 hrs.)
10. Dynamics of Water Drive Reservoir. (6 hrs.)
11. Water and Gas Coning (4hrs.)
12. Multi-Phase Flow and Introduction to Reservoir Simulation (4 hrs.)
13. Enhance Oil Recovery(2 hrs)

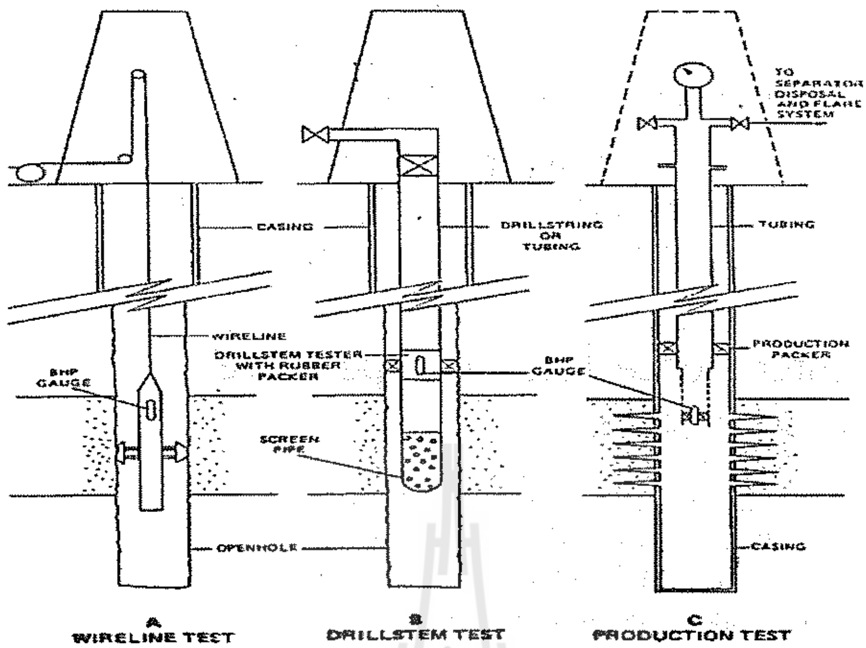

Fig.10.1 Test Techniques

PRODUCTION TESTING

1. <u>EXPENSIVE</u>
2. <u>SLOW</u>
3. <u>EXCELLENT RESULTS AND INFORMATION</u>

<u>4.OPTIMUM WELL SAFETY</u>

(PERMANENT) PACKER IS SET IN CASING

TUBING STRING SELECTED FOR HIGH GAS PRESSURE AND H2S SERVICE

2. Saturation and Relative Permeability Calculations (4 hrs.)
3. Steady State Radial Flow (4 hrs.)
4. Unsteady State Radial Flow (4 hrs.)
5. Pseudo-steady State Flow and Superposition (4 hrs.)
6. **Well Testing Pressure Drawdown and Build Up (3 hrs.)**
7. **Interference Test and Type Curve Analysis(3hrs)**
8. Displacement Efficiency(4 hrs)
9. Potential flow and Streamlines (4 hrs.)
10. Dynamics of Water Drive Reservoir. (6 hrs.)
11. Water and Gas Coning (4hrs.)
12. Multi-Phase Flow and Introduction to Reservoir Simulation (4 hrs.)
13. Enhance Oil Recovery(2 hrs)

10. INTRODUCTION TO PRESSURE TRANSIENT TESTING

Pressure transient testing is an important diagnostic tool that can provide valuable information for the reservoir engineer. A transient test is initiated by creating a disturbance at a wellbore, (i.e., a change in the flow rate) and then monitoring the pressure as a function of time. An efficiently conducted test that yields good data can provide information such as average permeability, drainage volume, wellbore damage or stimulation, and reservoir pressure.

A pressure transient test does not always yield a unique solution. There are often anomalies associated with the reservoir system that yield pressure data that could lead to multiple conclusions. In these cases, the strength of transient testing is realized only when the procedure is used in conjunction with other diagnostic tools or other information.

In the next two subsections, the two most popular tests (i.e., the drawdown and buildup tests) are introduced. However, notice that the material is intended to be only an introduction. You must be aware of many other considerations in order to conduct a proper transient test. We refer you to some excellent books in this area by Earlougher, Matthews and Russell, and Lee.[3,6,15]

10.1. Introduction to Drawdown Testing

The drawdown test consists of flowing a well at a constant rate following a shut-in period. The shut-in period should be sufficiently long for the reservoir pressure to have stabilized. The basis for the drawdown test is found in Eq. (7.37),

SHORTEN TERM

- Initial Reservoir Pressure/Temperature
- Permeability
- Active Pay thickness
- Skin Factor
- Fluid Type and properties
- Type of Flow System
- Production Rate/Problems

LONG TERM

1. Volume proving
2. Extended Investigation Radius
3. Reservoir Limits
4. Relative Reservoir shape
5. Boundary Type

Drill Stem Test

- RELATIVELY CHEAP AND FAST
- RESULTS MAY BE ADEQUATE
- WELL SAFETY NOT OPTIMUM

PACKER PERFORMANCE IN OPEN HOLE NOT RELIABLE (CASED HOLE FLOATERS)

DRILLSTRING UNSUITABLE FOR HIGH GAS PRESSURE AND H_2S SERVICE

OFFSHORE VESSEL MOVEMENT INDUCES STRING MOVEMENT

- NO DST FROM FLOATERS WITH OPEN HOLE PACKERS
- HARDLY EVER USED IN GROUP OPCO'S

10.4.1 Well Testing Objective

SHORT TERM
- Initial Reservoir Pressure/Temperature
- Permeabilities
- Active Pay Thickness
- Skin Factor
- Fluid Type and Properties
- Type of Flow System
- Production Rate/Problems

LONG TERM
- Volume Proving
- Extended Investigation Radius
  - Reservoir Limits
  - Relative Reservoir Shape
  - Boundary Type

10-10

10.1. Introduction to Drawdown Testing

$$p(r,t) = p_i - \frac{162.6\,q\mu B}{kh}\left[\log\frac{kt}{\phi\mu c_t r^2} - 3.23\right] \tag{7.37}$$

which predicts the pressure at any radius, r, as a function of time for a given reservoir flow system during the transient period. If $r = r_w$, then $p(r, t)$ will be the pressure at the wellbore. For a given reservoir system, p_i, q, μ, B, k, h, ϕ, c_t, and r_w are constant, and Eq. (7.37) can be written as

$$p_{wf} = b + m\log(t) \tag{7.58}$$

where,

p_{wf} = flowing well pressure in psia

b = constant

t = time in hrs

$$m = \text{constant} = -\frac{162.6\,q\mu B}{kh} \tag{7.59}$$

$$m = -\frac{162.6\,q\mu B}{kh}$$

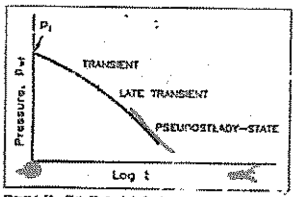

Figure 6-11. Plot of bottomhole flowing pressure (p_{wf}) versus log of producing time t.

1. Laminar, horizontal flow in a homogeneous reservoir.
2. Reservoir and fluid properties, k, ϕ, h, c_t, μ, and B are independent of pressure.
3. Single-phase liquid flow in the transient time region.
4. Negligible pressure gradients.

The expression for the slope, Eq. (7.59), can be rearranged to solve for the capacity, kh, of the drainage area of the flowing well. If the thickness is known, then the average permeability can be obtained by Eq. (7.60):

$$k = -\frac{162.6\,q\mu B}{mh} \tag{7.60}$$

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$$p(r,t) = p_i - \frac{162.6\,q\mu B}{kh}\left[\log\frac{kt}{\phi\mu c_t r^2} - 3.23\right] \tag{7.37}$$

which predicts the pressure at any radius, r, as a function of time for a given reservoir flow system during the transient period. If $r = r_w$, then $p(r, t)$ will be the pressure at the wellbore. For a given reservoir system, p_i, q, μ, B, k, h, ϕ, c_t, and r_w are constant, and Eq. (7.37) can be written as

$$p_{wf} = b + m\log(t) \tag{7.58}$$

where,

p_{wf} = flowing well pressure in psia

b = constant

t = time in hrs

$$m = \text{constant} = -\frac{162.6\,q\mu B}{kh} \tag{7.59}$$

Equation (7.59) suggests that a plot of p_{wf} versus t on semilog graph paper would yield a straight line with slope m through the early time data that correspond with the transient time period. This is providing that the assumptions inherent in the derivation of Eq. (7.37) are met. These assumptions are the following:

1. Laminar, horizontal flow in a homogeneous reservoir.
2. Reservoir and fluid properties, k, ϕ, h, c_t, μ, and B are independent of pressure.
3. Single-phase liquid flow in the transient time region.
4. Negligible pressure gradients.

The expression for the slope, Eq. (7.59), can be rearranged to solve for the capacity, kh, of the drainage area of the flowing well. If the thickness is known, then the average permeability can be obtained by Eq. (7.60):

$$k = -\frac{162.6\,q\mu B}{mh} \tag{7.60}$$

If the drawdown test is conducted long enough for the pressure transients to reach the pseudosteady-state period, then Eq. (7.47) is used to describe the pressure behavior:

$$p_{wf} = p_i - \frac{162.6\,q\mu B}{kh}\log\left[\frac{4A}{1.781 C_A r_w^2}\right] - \frac{0.2339\,qBt}{Ah\phi c_t} \tag{7.47}$$

Again grouping together the terms that are constant for a given reservoir system, Eq. (7.47) becomes

$$p_{wf} = b' + m't \tag{7.61}$$

where

b' = constant

$$m' = \text{constant} = -\frac{0.2339\,qB}{Ah\phi c_t} \tag{7.62}$$

Now a plot of pressure versus time on regular cartesian graph paper yields a straight line with slope equal to m' through the late time data that correspond to the pseudosteady-state period. If Eq. (7.62) is rearranged, an expression for the drainage volume of the test well can be obtained:

$$Ah\phi = -\frac{0.2339\,qB}{m'c_t} \tag{7.63}$$

The drawdown test can also yield information about damage that may have occurred around the wellbore during the initial drilling or during subsequent production. We now develop an equation that allows the calculation of a damage factor, using information from the transient flow region.

A damage zone yields an additional pressure drop because of the reduced permeability in that zone. Van Everdingen and Hurst developed an expression for this pressure drop and defined a dimensionless damage factor, S, called the skin factor[16,17]:

$$\Delta p_{skin} = \frac{141.2\,q\mu B}{kh}S \tag{7.64}$$

or

$$\Delta p_{skin} = -0.87mS \tag{7.65}$$

From Eq. (7.65), a positive value of S causes a positive pressure drop and therefore represents a damage situation. A negative value of S causes a negative pressure drop that represents a stimulated condition like a fracture. Notice that these pressure drops caused by the skin factor are compared to the pressure drop that would normally occur through this affected zone as predicted by Eq. (7.37). Combining Eq. (7.37) and (7.65), the following expression is obtained for the pressure at the wellbore:

$$p_{wf} = p_i - \frac{162.6\,q\mu B}{kh}\left[\log\frac{kt}{\phi\mu c_t r_w^2} - 3.23 + 0.87S\right] \tag{7.66}$$

- If the drawdown test is conducted long enough for the pressure transients to reach the pseudosteady-state period, then Eq. (7.47) is used to describe the pressure behavior:

Pseudosteady State

$$p_{wf} = p_i - \frac{162.6 q \mu B}{kh} \log\left[\frac{4A}{1.781 C_A r_w^2}\right] - \frac{0.2339 qBt}{Ah\phi c_t} \qquad (7.47)$$

$b' + m't$

10. Introduction to Pressure Transient Testing

Drawdown Test

Again grouping together the terms that are constant for a given reservoir system, Eq. (7.47) becomes

Limited Test

$$p_{wf} = b' + m't \qquad (7.61)$$

where

Cartesian

$b' =$ constant

$$m' = \text{constant} = -\frac{0.2339 qB}{Ah\phi c_t} \qquad (7.62)$$

Now a plot of pressure versus time on regular cartesian graph paper yields a straight line with slope equal to m' through the late time data that correspond to the pseudosteady-state period. If Eq. (7.62) is rearranged, an expression for the drainage volume of the test well can be obtained:

To Drainage Volume

$$Ah\phi = -\frac{0.2339 qB}{m' c_t} \qquad (7.63)$$

A damage zone yields an additional pressure drop because of the reduced permeability in that zone. Van Everdingen and Hurst developed an expression for this pressure drop and defined a dimensionless damage factor, S, called the skin factor[16,17]:

$$\Delta p_{skin} = \frac{141.2 q \mu B}{kh} S \qquad (7.64)$$

or

$$\Delta p_{skin} = -0.87 m S \qquad (7.65)$$

From Eq. (7.65), a positive value of S causes a positive pressure drop and therefore represents a damage situation. A negative value of S causes a negative pressure drop that represents a stimulated condition like a fracture. Notice that these pressure drops caused by the skin factor are compared to the pressure drop that would normally occur through this affected zone as predicted by Eq. (7.37). Combining Eq. (7.37) and (7.65), the following expression is obtained for the pressure at the wellbore:

$$p_{wf} = p_i - \frac{162.6 q \mu B}{kh}\left[\log\frac{kt}{\phi \mu c_t r_w^2} - 3.23 + 0.87S\right] \qquad (7.66)$$

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This equation can be rearranged and solved for the skin factor, S:

$$S = 1.151\left[\frac{p_i - p_{wf}}{\frac{162.6 q \mu B}{kh}} - \log\frac{kt}{\phi \mu c_t r_w^2} + 3.23\right]$$

The value of p_{wf} must be obtained from the straight line in the transient flow region. Usually a time corresponding to 1 hr is used, and the corresponding pressure is given by the designation, p_{1hr}. Substituting m into this equation and recognizing that the denominator of the first term within the brackets is actually $-m$,

$$S = 1.151\left[\frac{p_{1hr} - p_i}{m} - \log\frac{k}{\phi \mu c_t r_w^2} + 3.23\right] \qquad (7.67)$$

This equation can be rearranged and solved for the skin factor, S:

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Eq. (7.67) can be used to calculate a value for S from the slope of the transient flow region and the value of p_{1hr} also taken from the straight line in the transient period.

A drawdown test is often conducted during the initial production from a well since the reservoir has obviously been stabilized at the initial pressure, p_i. The difficult aspect of the test is maintaining a constant flow rate, q. If the flow rate is not kept constant during the length of the test, then the pressure behavior will reflect this varying flow rate and the correct straight line regions on the semilog and regular cartesian plots may not be identified. Other factors such as wellbore storage (shut-in well) or unloading (flowing well) can interfere with the analysis. Wellbore storage and unloading are phenomena that occur in every well to a certain degree and cause anomalies in the pressure behavior. Wellbore storage is caused by fluid flowing into the wellbore after a well has been shut in on the surface and by the pressure in the wellbore changing as the height of the fluid in the wellbore changes. Wellbore unloading in a flowing well will lead to more production at the surface than what actually occurs down hole. The effects of wellbore storage and unloading can be so dominating that they completely mask the transient time data. If this happens and if the engineer does not know how to analyze for these effects, the pressure data may be misinterpreted and errors in calculated values of permeability, skin, and the like may occur. Wellbore storage and unloading effects are discussed in detail by Earlougher.[7] These effects should always be taken into consideration when you evaluate pressure transient data.

The following example problem illustrates the analysis of drawdown test data.

Example 7.2. A drawdown test was conducted on a new oil well in a large reservoir. At the time of the test, the well was the only well that had been developed in the reservoir. Analysis of the data indicates that wellbore storage does not affect the pressure measurements. Use the data to calculate the average permeability of the area around the well the skin factor, and the drainage area of the well.

Given:

$p_i = 4000$ *psia* formation thickness = 20 ft

$q = 500$ STB/day $c_t = 30 \times 10^{-6}$ psia^{-1}

$\mu_o = 1.5$ cp porosity = 25%

$B_o = 1.2$ bbl/STB $r_w = 0.333$ ft

Flowing pressure, p_{wf} psia	*time, t hrs*
3503	2
3469	5
3443	10
3417	20
3383	50
3368	75
3350	100
3306	150
3282	200
3250	300

SOLUTION: Figure 7.19 contains a semilog plot of the pressure data. The slope of the early time data, which are in the transient time region, was found to be −86 psi/cycle and the value of P_{1hr} was found to be 3526 psia. Equation (7.60) can now be used to calculate the permeability:

$$k = -\frac{162.6(500)(1.5)(1.2)}{-86(20)} = 85.1 \text{ md}$$

The skin factor is found from Eq. (7.67):

$$S = 1.151\left[\frac{3526 - 4000}{-86} - \log\left(\frac{85.1}{(0.25)(1.5)(30\times10^{-6})(0.333)^2}\right) + 3.23\right]$$

$$S = 1.04$$

This positive value for the skin factor suggests the well is slightly damaged.

From the slope of a plot of P versus time on regular cartesian graph paper, shown in Fig. 7.20, and using Eq. (7.63), an estimate for the drainage area of the well can be obtained. From the semilog plot of pressure versus time, the first six data points fell on the straight line region indicating they

Example 7.2. A drawdown test was conducted on a new oil well in a large reservoir. At the time of the test, the well was the only well that had been developed in the reservoir. Analysis of the data indicates that wellbore storage does not affect the pressure measurements. Use the data to calculate the

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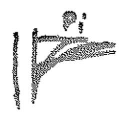

Flowing pressure, p_{wf} psia	*time, t hrs*
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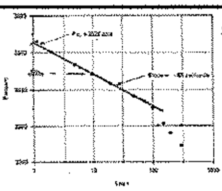


Fig. 7.19. Plot of pressure versus log time for the data of Ex. 7.2.

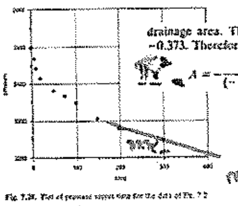


Fig. 7.20. Plot of pressure versus time for the data of Ex. 7.2

drainage area. The slope of a line drawn through the last three points is −0.373. Therefore,

$$A = -\frac{0.2339(500)(1.2)}{(-.373)(30\times10)^{-6}(20)(0.25)} = 2{,}508{,}000\ ft^2 = 57.6\ ac$$

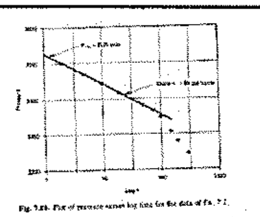


Fig. 7.19. Plot of pressure versus log time for the data of Ex. 7.2.

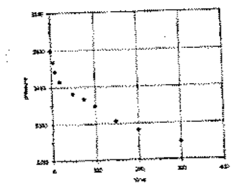


Fig. 7.20. Plot of pressure versus time for the data of Ex. 7.2

...are in the transient time period. Therefore, the last two to three points of the data are in the pseudosteady-state period and can be used to calculate the drainage area. The slope of a line drawn through the last three points is −0.373. Therefore,

$$A = -\frac{0.2339(500)(1.2)}{(-.373)(30\times10)^{-6}(20)(0.25)} = 2{,}508{,}000\ ft^2 = 57.6\ ac$$

7.8.2. Introduction to Buildup Testing

The buildup test is the most popular transient test used in the industry. It is conducted by shutting in a producing well that has obtained a stabilized pressure in the pseudosteady-state time region by flowing the well at a constant rate for a sufficiently long period. The pressure is then monitored during the length of the shut-in period. The primary reason for the popularity of the buildup test is the fact that it is easy to maintain the flow rate constant at zero during the length of the test. The main disadvantage of the buildup test over that of the drawdown test is that there is no production during the test and therefore no subsequent income.

A pressure buildup test is simulated mathematically by using the principle of superposition. Before the shut-in period, a well is flowed at a constant flow rate, q. At the time corresponding to the point of shut-in, t_p, a second well, superimposed over the location of the first well, is opened to flow at a rate equal to $-q$, while the first well is allowed to continue to flow at rate q. The time that the second well is flowed is given the symbol of Δt. When the effects of the two wells are added, the result is that a well has been allowed to flow at rate q for time t_p and then shut in for time Δt. This simulates the actual test procedure, which is shown schematically in Fig. 7.21. The time corresponding to the point of shut-in, t_p, can be estimated from the following equation,

$$t_p = \frac{N_p}{q} \quad (7.66)$$

where, N_p = cumulative production that has occurred during the time before shut-in that the well was flowed at the constant flow rate q.

Equations (7.37) and (7.57) can be used to describe the pressure behavior of the shut-in well.

$$p_{ws} = p_i - \frac{162.6 q\mu B}{kh}\left[\log\frac{k(t_p+\Delta t)}{\phi\mu c_t r_w^2} - 3.23\right] - \frac{162.6(-q)\mu B}{kh}\left[\log\frac{k\Delta t}{\phi\mu c_t r_w^2} - 3.23\right]$$

Expanding this equation and cancelling terms,

$$p_{ws} = p_i - \frac{162.6 q\mu B}{kh}\left[\log\frac{t_p+\Delta t}{\Delta t}\right] \quad (7.67)$$

...ssure buildup;
...ll drawdown;
...nterference;
...test;
...ill stem (DST);
...off;
...ectivity;
...p rate.

...hould be pointed out that when the flow rate is changed ...e pressure response is recorded in the same well, the ...s called a "single-well" test. Drawdown, buildup, injec... falloff, and step-rate tests are examples of a single-well ...When the flow rate is changed in one well and the pres... response is measured in another well(s), the test is ... a "multiple-well" test.

...eral of the above listed tests are briefly described in ...llowing sections.

...as long been recognized that the pressure behavior ...eservoir following a rate change directly reflects the ...etry and flow properties of the reservoir. Some of the ...mation that can be obtained from a well test includes:

...rawdown tests	Pressure profile Reservoir behavior Permeability Skin Fracture length Reservoir limit and shape
...uildup tests	Reservoir behavior Permeability Fracture length Skin Reservoir pressure Boundaries

- John Lee, *Well Testing* (1982);
- M. A. Sabet, *Well Test Analysis* (1991);
- Roland Horn, *Modern Well Test Analysis* (1995).

1.3.1 Drawdown test

A pressure drawdown test is simply a series of bottom-hole pressure measurements made during a period of flow at constant producing rate. Usually the well is shut in prior to the flow test for a period of time sufficient to allow the pressure to equalize throughout the formation, i.e., to reach static pressure. A schematic of the ideal flow rate and pressure history is shown in Figure 1.32.

The fundamental objectives of drawdown testing are to obtain the average permeability, k, of the reservoir rock within the drainage area of the well, and to assess the degree of damage of stimulation induced in the vicinity of the wellbore through drilling and completion practices. Other objectives are to determine the pore volume and to detect reservoir inhomogeneities within the drainage area of the well.

When a well is flowing at a constant rate of Q_o under the unsteady-state condition, the pressure behavior of the well will act as if it exists in an infinite-size reservoir. The pressure behavior during this period is described by Equation 1.2.134 as:

$$p_{wf} = p_i - \frac{162.6 Q_o B_o \mu}{kh}\left[\log\left(\frac{kt}{\phi\mu c_t r_w^2}\right) - 3.23 + 0.87s\right]$$

where:

k = permeability, md
t = time, hours
r_w = wellbore radius, ft
s = skin factor

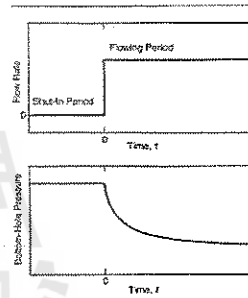


Figure 1.32 Idealized drawdown test.

The above expression can be written as:

$$p_{wf} = p_i - \frac{162.6 Q_o B_o \mu}{kh} \times \left[\log(t) + \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) - 3.23 + 0.87s\right] \quad [1.3.1]$$

This relationship is essentially an equation of a straight line and can be expressed as:

$$p_{wf} = a + m\log(t)$$

The skin effect can be obtained by rearranging Equation 1.3.1 as:

$$s = 1.151\left[\frac{p_i - p_{wf}}{|m|} - \log t - \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) + 3.23\right]$$

or, more conveniently, if selecting $p_{wf} = p_{1\,hr}$ which is found on the extension of the straight line at $t = 1$ hr, then:

$$s = 1.151\left[\frac{p_i - p_{1\,hr}}{|m|} - \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) + 3.23\right] \quad [1.3.3]$$

where $|m|$ is the absolute value of the slope m.

In Equation 1.3.3, $p_{1\,hr}$ must be obtained from the semilog straight line. If the pressure data measured at 1 hour does not fall on that line, the line must be extrapolated to 1 hour and the extrapolated value of $p_{1\,hr}$ must be used in Equation 1.3.3. This procedure is necessary to avoid calculating an incorrect skin by using a wellbore-storage-influenced pressure. Figure 1.33 illustrates the extrapolation to $p_{1\,hr}$.

Note that the additional pressure drop due to the skin was expressed previously by Equation 1.2.130 as:

$$\Delta p_{skin} = 141.2\left(\frac{Q_o B_o \mu_o}{kh}\right)s$$

This additional pressure drop can be equivalently written in terms of the semilog straight-line slope m by combining the above expression with that of Equation 1.3.3 to give:

$$\Delta p_{skin} = 0.87|m|s$$

Another physically meaningful characterization of the skin factor is the flow coefficient E as defined by the ratio of the well actual or observed productivity index J_{actual} and its ideal productivity index J_{ideal}. The ideal productivity index J_{ideal} is the value obtained with no alteration of permeability around the wellbore. Mathematically, the flow coefficient is given by:

$$E = \frac{J_{actual}}{J_{ideal}} = \frac{\bar{p} - p_{wf} - \Delta p_{skin}}{\bar{p} - p_{wf}}$$

where $\bar{p}$ is the average pressure in the well drainage area.

If the drawdown test is long enough, the bottom-hole pressure will deviate from the semilog straight line and make the

$$p_{wf} = p_i - \frac{162.6 Q_o B_o \mu}{kh} \times \left[\log(t) + \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) - 3.23 + 0.87s\right] \quad [1.3.1]$$

This relationship is essentially an equation of a straight line and can be expressed as:

$$p_{wf} = a + m\log(t)$$

where:

$$a = p_i - \frac{162.6 Q_o B_o \mu}{kh}\left[\log\left(\frac{k}{\phi\mu c_t r_w^2}\right) - 3.23 + 0.87s\right]$$

and the slope m is given by:

$$-m = \frac{-162.6 Q_o B_o \mu_o}{kh} \quad [1.3.2]$$

Equation 1.3.1 suggests that a plot of p_{wf} versus time t on semilog graph paper would yield a straight line with a slope m in psi/cycle. This semilog straight-line portion of the drawdown data, as shown in Figure 1.33, can also be expressed in another convenient form by employing the definition of the slope:

$$m = \frac{p_{wf} - p_{1\,hr}}{\log(t) - \log(1)} = \frac{p_{wf} - p_{1\,hr}}{\log(t) - 0}$$

or:

$$p_{wf} = m\log(t) + p_{1\,hr}$$

Notice that Equation 1.3.2 can also be rearranged to determine the capacity kh of the drainage area of the well. If the thickness is known, then the average permeability is given by:

$$k = \frac{162.6 Q_o B_o \mu_o}{|m| h}$$

where:

k = average permeability, md
$|m|$ = absolute value of slope, psi/cycle

Clearly, kh/μ_o or k/μ_o may also be estimated.

...the value obtained with no alteration of permeability around the wellbore. Mathematically, the flow coefficient is given by:

$$E = \frac{J_{actual}}{J_{ideal}} = \frac{\bar{p} - p_{wf} - \Delta p_{skin}}{\bar{p} - p_{wf}}$$

where $\bar{p}$ is the average pressure in the well drainage area.

If the drawdown test is long enough, the bottom-hole pressure will deviate from the semilog straight line and make the transition from infinite acting to pseudosteady state. The rate of pressure decline during the pseudosteady-state flow is defined by Equation 1.2.116 as:

$$\frac{dp}{dt} = \frac{-0.23396q}{c_t(\pi r_e^2)h\phi} = \frac{-0.23396q}{c_t(A)h\phi} = \frac{-0.23396q}{c_t(\text{pore volume})}$$

Under this condition, the pressure will decline at a constant rate at any point in the reservoir including the bottom-hole flowing pressure p_{wf}. That is:

$$\frac{dp_{wf}}{dt} = m' = \frac{-0.23396q}{c_t Ah\phi}$$

This expression suggests that during the semisteady-state flow, a plot of p_{wf} vs. t on a Cartesian scale would produce a straight line with a negative slope of m' that is defined by:

$$-m' = \frac{-0.23396q}{c_t Ah\phi}$$

where:

m' = slope of the *Cartesian straight line* during the pseudosteady state, psi/hr
q = flow rate, bbl/day
A = drainage area, ft²

Example 1.24[a] Estimate the oil permeability and skin factor from the drawdown data of Figure 1.34.

[a]This example problem and the solution procedure are given in Earlougher, R. Advances in Well Test Analysis, Monograph Series, SPE, Dallas (1977).

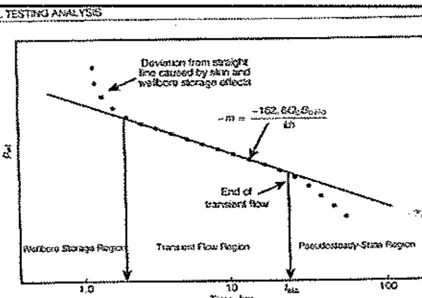
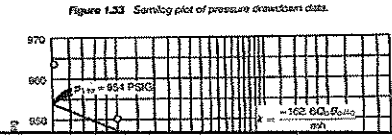


Figure 1.33 Semilog plot of pressure drawdown data.



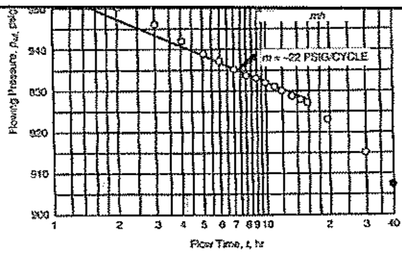


Figure 1.34 Earlougher's semilog data plot for the drawdown test (Permission to publish by the SPE, copyright SPE, 1977).

The following reservoir data are available:

$h = 130$ ft, $\phi = 20\%$, $r_w = 0.25$ ft,

$p_i = 1154$ psi, $Q_o = 348$ STB/D, $m = -22$ psi/cycle

$B_o = 1.14$ bbl/STB, $\mu_o = 3.93$ cp, $c_t = 8.74 \times 10^{-6}$ psi^{-1}

Assuming that the wellbore storage effect is not significant, calculate:

- the permeability;
- the skin factor;
- the additional pressure drop due to the skin.

Solution

Step 1. From Figure 1.34, calculate $p_{1\,hr}$:

$$p_{1\,hr} = 954 \text{ psi}$$

Step 2. Determine the slope of the transient flow line:

$$m = -22 \text{ psi/cycle}$$

Step 3. Calculate the permeability by applying Equation 1.3.2:

$$k = \frac{-162.6 Q_o B_o \mu_o}{mh}$$

$$= \frac{-(162.6)(348)(1.14)(3.93)}{(-22)(130)} = 89 \text{ md}$$

Step 4. Solve for the skin factor s by using Equation 1.3.3:

$$s = 1.151\left[\frac{p_i - p_{1\,hr}}{|m|} - \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) + 3.23\right]$$

$$= 1.151\left[\left(\frac{1154 - 954}{22}\right) - \log\left(\frac{89}{(0.2)(3.93)(8.74 \times 10^{-6})(0.25)^2}\right) + 3.2275\right] = 4.6$$

Step 5. Calculate the additional pressure drop:

$$\Delta p_{skin} = 0.87|m|s = 0.87(22)(4.6) = 88 \text{ psi}$$

It should be noted that for a *multiphase flow*, Equations 1.3.1 and 1.3.3 become:

$$p_{wf} = p_i - \frac{162.6 q_t}{\lambda_t h}\left[\log(t) + \log\left(\frac{\lambda_t}{\phi c_t r_w^2}\right) - 3.23 + 0.87s\right]$$

$$s = 1.151\left[\frac{p_i - p_{1\,hr}}{|m|} - \log\left(\frac{\lambda_t}{\phi c_t r_w^2}\right) + 3.23\right]$$

with:

$$\lambda_t = \frac{k_o}{\mu_o} + \frac{k_w}{\mu_w} + \frac{k_g}{\mu_g}$$

$$q_t = Q_o B_o + Q_w B_w + (Q_g - Q_o R_s)B_g$$

or equivalently in terms of GOR as:

$$q_t = Q_o B_o + Q_w B_w + (GOR - R_s)Q_o B_g$$

where:

q_t = total fluid voidage rate, bbl/day
Q_o = oil flow rate, STB/day
Q_w = water flow rate, STB/day
Q_g = total gas flow rate, scf/day
R_s = gas solubility, scf/STB
B_g = gas formation volume factor, bbl/scf
λ_t = total mobility, md/cp
k_o = effective permeability to oil, md
k_w = effective permeability to water, md
k_g = effective permeability to gas, md

The above drawdown relationships indicate that a plot of p_{wf} vs. t on a semilog scale would produce a straight line with a slope m that can be used to determine the total mobility λ_t from:

$$\lambda_t = \frac{162.6 q_t}{mh}$$

Perrine (1956) showed that the effective permeability of each phase, i.e., k_o, k_w, and k_g, can be determined as:

$$k_o = \frac{162.6 Q_o B_o \mu_o}{mh}$$

$$k_w = \frac{162.6 Q_w B_w \mu_w}{mh}$$

$$k_g = \frac{162.6(Q_g - Q_o R_s)B_g \mu_g}{mh}$$

If the drawdown pressure data is available during both the unsteady-state flow period and the pseudosteady-state flow period, it is possible to estimate the drainage shape and the drainage area of the test well. The transient semilog plot is used to determine its slope m and $p_{1\,hr}$; the Cartesian straight-line plot of the pseudosteady-state data is used to determine its slope m' and its intercept p_{int}. Earlougher (1977) proposed the following expression to determine the shape factor C_A:

$$C_A = 5.456\left(\frac{m}{m'}\right)\exp\left[\frac{2.303(p_{1\,hr} - p_{int})}{m}\right]$$

where:

m = slope of transient semilog straight line, psi/log cycle
m' = slope of the semisteady-state Cartesian straight line
$p_{1\,hr}$ = pressure at $t = 1$ hour from transient semilog straight line, psi
p_{int} = pressure at $t = 0$ from pseudosteady-state Cartesian straight line, psi

The calculated shape factor from applying the above relationship is compared with those values listed in Table 1.4 to select the geometry of well drainage with a shape factor closest to the calculated value. When extending the drawdown test time with the objective of reaching the drainage boundary of the test well, the test is commonly called the "reservoir limit test."

The reported data of Example 1.24 was extended by Earlougher to include the pseudosteady-state flow period and used to determine the geometry of the test well drainage area as shown in the following example.

Example 1.25 Use the data in Example 1.24 and the Cartesian plot of the pseudosteady-state flow period, as shown in Figure 1.35, to determine the geometry and drainage area of the test well.

Solution

Step 1. From Figure 1.35, determine the slope m' and intercept p_{int}:

$$m' = -0.8 \text{ psi/hr}$$

$$p_{int} = 940 \text{ psi}$$

Step 2. From Example 1.24:

$$m = -22 \text{ psi/cycle}$$

$$p_{1\,hr} = 954 \text{ psi}$$

Step 3. Calculate the shape factor C_A from Earlougher's equation:

$$C_A = 5.456\left(\frac{m}{m'}\right)\exp\left[\frac{2.303(p_{1\,hr} - p_{int})}{m}\right]$$

$$= 5.456\left(\frac{-22}{-0.8}\right)\exp\left[\frac{2.303(954 - 940)}{-22}\right]$$

$$= 34.6$$

Step 4. From Table 1.4, $C_A = 34.6$ corresponds to a well in the center of a circle, square, or hexagon:

For a circle: $C_A = 31.62$
For a square: $C_A = 30.88$
For a hexagon: $C_A = 31.60$

Step 5. Calculate the pore volume and drainage area from Equation 1.2.116:

$$\frac{dp}{dt} = m' = \frac{-0.23396(Q_o B_o)}{c_t(A)h\phi} = \frac{-0.23396(Q_o B_o)}{c_t(\text{pore volume})}$$

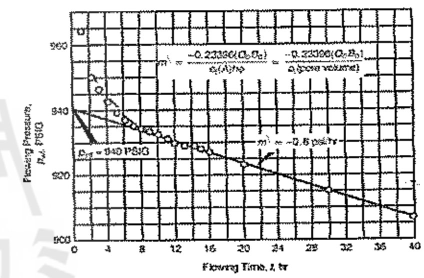


Figure 1.35 Cartesian plot of the drawdown test data (Permission to publish by the SPE, copyright SPE, 1977).

Solving for the pore volume gives:

$$\text{Pore volume} = \frac{-0.23396 q}{c_t m'} = \frac{-0.23396(348)(1.14)}{(8.74 \times 10^{-6})(-0.8)}$$

$$= 2.37 \text{ MMbbl}$$

and the drainage area:

$$A = \frac{2.37 \times 10^6(5.615)}{43560(0.2)(130)} = 11.7 \text{ acres}$$

The above example indicates that the measured bottom-hole flowing pressures are 88 psi more than they would be in the absence of the skin. However, it should be pointed out that when the concept of positive skin factor $+s$ indicates formation damage, whereas a negative skin factor $-s$ suggests formation stimulation, this is essentially a misleading interpretation of the skin factor. The skin factor as determined from any transient well testing analysis represents the composite "total" skin factor that includes the following other skin factors:

- skin due to wellbore damage or stimulation s_d;
- skin due to partial penetration and restricted entry s_r;
- skin due to perforations s_p;
- skin due to turbulence flow s_t;
- skin due to deviated well s_{dw}.

That is:

$$s = s_d + s_r + s_p + s_t + s_{dw}$$

where s is the skin factor as calculated from transient flow analysis. Therefore, to determine if the formation is damaged or stimulated from the skin factor value s obtained from well test analysis, the individual components of the skin factor in the above relationship must be known, to give:

$$s_d = s - s_r - s_p - s_t - s_{dw}$$

There are correlations that can be used to separately estimate these individual skin quantities.

Wellbore storage

Basically, well test analysis deals with the interpretation of the wellbore pressure response to a given change in the flow rate (from zero to a constant value for a drawdown test, or from a constant rate to zero for a buildup test). Unfortunately, the producing rate is controlled at the surface, not at the sand face. Because of the wellbore volume, a constant surface flow rate does not ensure that the entire rate is being produced from the formation. This effect is due to wellbore storage. Consider the case of a drawdown test. When the well is first open to flow after a shut-in period, the pressure in the wellbore drops. This drop in pressure causes the following two types of wellbore storage:

(1) a wellbore storage effect caused by fluid expansion;
(2) a wellbore storage effect caused by changing fluid level in the casing–tubing annulus.

As the bottom-hole pressure drops, the wellbore fluid expands and, thus, the initial surface flow rate is not from the formation, but basically from the fluid that had been stored in the wellbore. This is defined as the *wellbore storage due to fluid expansion*.

The second type of wellbore storage is due to a change in the annulus fluid level (falling level during a drawdown test, rising level during a drawdown test, and rising fluid level during a pressure buildup test). When the well is open to flow during a drawdown test, the reduction in pressure causes the fluid level in the annulus to fall. This annulus fluid production joins that from the formation and contributes to the total flow from the well. The falling fluid level is generally able to contribute more fluid than that by expansion.

The above discussion suggests that part of the flow will be contributed by the wellbore instead of the reservoir. That is:

$$q = q_f + q_{wb}$$

where:

q = surface flow rate, bbl/day
q_f = formation flow rate, bbl/day
q_{wb} = flow rate contributed by the wellbore, bbl/day

During this period when the flow is dominated by the wellbore storage, the measured drawdown pressures will not produce the ideal semilog straight-line behavior that is expected during transient flow. This indicates that the pressure data collected during the duration of the wellbore storage effect cannot be analyzed by using conventional methods. As production time increases, the wellbore contribution decreases and the formation rate increases until it eventually equals the surface flow rate, i.e., $q = q_f$, which signifies the *end of the wellbore storage effect*.

The effect of fluid expansion and changing fluid level can be quantified in terms of the *wellbore storage factor C* which is defined as:

$$C = \frac{\Delta V_{wb}}{\Delta p}$$

where:

C = wellbore storage coefficient, bbl/psi
ΔV_{wb} = change in the volume of fluid in the wellbore, bbl

The above relationship can be applied to mathematically represent the individual effect of wellbore fluid expansion and falling (or rising) fluid level, to give:

Wellbore storage effect caused by fluid expansion

$$C_{FE} = V_{wb} c_{wb}$$

where:

C_{FE} = wellbore storage coefficient due to fluid expansion, bbl/psi
V_{wb} = total wellbore fluid volume, bbl
c_{wb} = average compressibility of fluid in the wellbore, psi^{-1}

Wellbore storage effect due to changing fluid level

$$C_{FL} = \frac{144 A_a}{5.615\rho}$$

with:

$$A_a = \frac{\pi[(ID_C)^2 - (OD_T)^2]}{4(144)}$$

where

C_{FL} = wellbore storage coefficient due to changing

period of the test and is expressed by:

$$p_D = t_D / C_D \qquad [1.3.5]$$

where:

p_D = dimensionless pressure during wellbore storage domination time
t_D = dimensionless time

Taking the logarithm of both sides of this relationship gives:

$$\log(p_D) = \log(t_D) - \log(C_D)$$

This expression has a characteristic that is diagnostic of wellbore storage effects. It indicates that a plot of p_D vs. t_D on a log–log scale will yield a straight line of a *unit slope, i.e., a straight line with a 45° angle*, during the wellbore storage-dominated period. Since p_D is proportional to pressure drop Δp and t_D is proportional to time t, it is convenient to plot $\log(p_i - p_{wf})$ versus $\log(t)$ and observe where the plot has a slope of one cycle in pressure per cycle in time. This unit slope observation is of major value in well test analysis.

The log–log plot is a valuable aid for recognizing wellbore storage effects in transient tests (e.g., drawdown or buildup tests) when early-time pressure recorded data is available. It is recommended that this plot be made a part of the transient test analysis. As wellbore storage effects become less severe, the formation begins to influence the bottom-hole pressure more and more, and the data points on the log–log plot fall below the unit-slope straight line and signify the end of the wellbore storage effect. At this point, wellbore storage is no longer important and standard semilog data-plotting analysis techniques apply. As a rule of thumb, the time that indicates the end of the wellbore storage effect can be determined from the log–log plot by moving 1 to 1½ cycles in time after the plot starts to deviate from the unit slope and reading the corresponding time on the x-axis. This time may be estimated from:

$$t_D > (60 + 3.5s)C_D$$

$$C_{FL} = \frac{144A_a}{5.615\rho}$$

with:

$$A_a = \frac{\pi[(ID_C)^2 - (OD_T)^2]}{4(144)}$$

where:

C_{FL} = wellbore storage coefficient due to changing fluid level, bbl/psi
A_a = annulus cross-sectional area, ft²
OD_T = outside diameter of the production tubing, inches
ID_C = inside diameter of the casing, inches
ρ = wellbore fluid density, lb/ft³

This effect is essentially small if a packer is placed near the producing zone. The total storage effect is the sum of both coefficients. That is:

$$C = C_{FE} + C_{FL}$$

It should be noted during oil well testing that the fluid expansion is generally insignificant due to the small compressibility of liquids. For gas wells, the primary storage effect is due to gas expansion.

To determine the duration of the wellbore storage effect, it is convenient to express the wellbore storage factor in a dimensionless form as:

$$C_D = \frac{5.615C}{2\pi h\phi c_t r_w^2} = \frac{0.8936C}{\phi h c_t r_w^2} \quad [1.3.4]$$

where:

C_D = dimensionless wellbore storage factor
C = wellbore storage factor, bbl/psi
c_t = total compressibility coefficient, psi⁻¹
r_w = wellbore radius, ft
h = thickness, ft

Horn (1995) and Earlougher (1977), among other authors, have indicated that the wellbore pressure is directly proportional to the time during the wellbore storage-dominated techniques apply. As a rule of thumb, the time that indicates the end of the wellbore storage effect can be determined from the log–log plot by moving 1 to $1\frac{1}{2}$ cycles in time after the plot starts to deviate from the unit slop and reading the corresponding time on the x axis. This time may be estimated from:

$$t_D > (60 + 3.5s)C_D$$

or:

$$t > \frac{(200\,000 + 12\,000s)C}{(kh/\mu)}$$

where:

t = total time that marks the end of the wellbore storage effect and the beginning of the semilog straight line, hours
k = permeability, md
s = skin factor
μ = viscosity, cp
C = wellbore storage coefficient, bbl/psi

In practice, it is convenient to determine the wellbore storage coefficient C by *selecting a point on the log–log unit-slope straight line* and reading the coordinate of the point in terms of t and Δp, to give:

$$C = \frac{qt}{24\Delta p} = \frac{QBt}{24\Delta p}$$

where:

t = time, hours
Δp = pressure difference $(p_i - p_{wf})$, psi
q = flow rate, bbl/day
Q = flow rate, STB/day
B = formation volume factor, bbl/STB

It is important to note that the volume of fluids stored in the wellbore distorts the early-time pressure response and controls the duration of wellbore storage, especially in deep wells with large wellbore volumes. If the wellbore storage

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effects are not minimized or if the test is not continued beyond the end of the wellbore storage-dominated period, the test data will be difficult to analyze with current conventional well testing methods. To minimize wellbore storage distortion and to keep well tests within reasonable lengths of time, it may be necessary to run tubing, packers, and bottom-hole shut-in devices.

Example 1.26 The following data is given for an oil well that is scheduled for a drawdown test:

- volume of fluid in the wellbore = 180 bbl
- tubing outside diameter = 2 inches
- production oil density in the wellbore = 7.675 inches
- average oil density in the wellbore = 45 lb/ft³

$h = 50$ ft, $\phi = 15\%$,
$r_w = 0.25$ ft, $\mu_o = 2$ cp
$k = 30$ md, $s = 0$
$c_t = 20 \times 10^{-6}$ psi⁻¹, $c_o = 10 \times 10^{-6}$ psi⁻¹

If this well is placed under a constant production rate, calculate the dimensionless wellbore storage coefficient C_D. How long will it take for wellbore storage effects to end?

Solution

Step 1. Calculate the cross-sectional area of the annulus A_a:

$$A_a = \frac{\pi[(ID_C)^2 - (OD_T)^2]}{4(144)} = \frac{\pi[(7.675)^2 - (2)^2]}{(4)(144)} = 0.2995 \text{ ft}^2$$

Step 2. Calculate the wellbore storage factor caused by fluid expansion:

$$C_{FE} = V_{wb}c_{wb}$$

$$= (180)(10 \times 10^{-6}) = 0.0018 \text{ bbl/psi}$$

Step 3. Determine the wellbore storage factor caused by the falling fluid level:

$$C_{FL} = \frac{144A_a}{5.615\rho} = \frac{144(0.2995)}{(5.615)(45)} = 0.1707 \text{ bbl/psi}$$

Step 4. Calculate the total wellbore storage coefficient:

$$C = C_{FE} + C_{FL} = 0.0018 + 0.1707 = 0.1725 \text{ bbl/psi}$$

The above calculations show that the effect of fluid expansion C_{FE} can generally be neglected in crude oil systems.

Step 5. Calculate the dimensionless wellbore storage coefficient from Equation 1.3.4:

$$C_D = \frac{0.8936C}{\phi h c_t r_w^2} = \frac{0.8936(0.1707)}{0.15(50)(20 \times 10^{-6})(0.25)^2} = 16\,271$$

Step 6. Approximate the time required for wellbore storage influence to end from:

$$t = \frac{(200\,000 + 12\,000s)C\mu}{kh} = \frac{(200\,000 + 0)(0.1725)(2)}{(30)(50)} = 46 \text{ hours}$$

The straight-line relationship as expressed by Equation 1.3.2 is only valid during the infinite-acting behavior of the well. Obviously, reservoirs are not infinite in extent, so the infinite-acting radial flow period cannot last indefinitely. Eventually the effects of the reservoir boundaries will be felt at the well being tested. The time at which the boundary effect is felt is dependent on the following factors:

- permeability k;
- total compressibility c_t;
- porosity ϕ;
- viscosity μ;
- distance to the boundary;
- shape of the drainage area.

Earlougher (1977) suggested the following mathematical expression for estimating the duration of the infinite-acting period:

$$t_{eia} = \left[\frac{\phi\mu c_t A}{0.0002637k}\right](t_{DA})_{eia}$$

where:

t_{eia} = time to the end of infinite-acting period, hours
A = well drainage area, ft²
c_t = total compressibility, psi⁻¹
$(t_{DA})_{eia}$ = dimensionless time to the end of the infinite-acting period

This expression is designed to predict the time that marks the end of transient flow in a drainage system of any geometry by obtaining the value of t_{DA} from Table 1.4. *The last three columns of the table* provide with values of t_{DA} that allow the engineer to calculate:

- the maximum elapsed time during which a reservoir is infinite acting;
- the time required for the pseudosteady-state solution to be applied and predict pressure drawdown within 1% accuracy;
- the time required for the pseudosteady-state solution (equations) to be exact and applied.

As an example, for a well centered in a circular reservoir, the maximum time for the reservoir to remain as an infinite-acting system can be determined using the entry in the final column of Table 1.4 to give $(t_{DA})_{eia} = 0.1$, and accordingly:

$$t_{eia} = \left[\frac{\phi\mu c_t A}{0.0002637k}\right](t_{DA})_{eia} = \left[\frac{\phi\mu c_t A}{0.0002637k}\right]0.1$$

or:

$$t_{eia} = \frac{380\phi\mu c_t A}{k}$$

For example, for a well that is located in the center of a 40 acre circular drainage area with the following properties:

$k = 60$ md, $c_t = 6 \times 10^{-6}$ psi⁻¹, $\mu = 1.5$ cp, $\phi = 0.12$

the maximum time, in hours, for the well to remain in an infinite-acting system is:

$$t_{eia} = \frac{380\phi\mu c_t A}{k} = \frac{380(0.12)(1.4)(6 \times 10^{-6})(40 \times 43560)}{60} = 11.1 \text{ hours}$$

Similarly, the pseudosteady-state solution can be applied any time after the semisteady-state flow begins at t_{pss} as estimated from:

$$t_{pss} = \left[\frac{\phi\mu c_t A}{0.0002637k}\right](t_{DA})_{pss}$$

where $(t_{DA})_{pss}$ can be found from the entry in the fifth column of the table.

Hence, the specific steps involved in a drawdown test analysis are:

(1) Plot $p_i - p_{wf}$ vs. t on a log–log scale.

effects are not minimized or if the test is not continued beyond the end of the wellbore storage-dominated period, the test data will be difficult to analyze with current conventional well testing methods. To minimize wellbore storage distortion and to keep well tests within reasonable lengths of time, it may be necessary to run tubing, packers, and bottom-hole shut-in devices.

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- average oil density in the wellbore = 45 lb/ft³

$h = 50$ ft, $\phi = 15\%$, $r_w = 0.25$ ft, $\mu_o = 2$ cp, $k = 30$ md, $s = 0$, $c_t = 20 \times 10^{-6}$ psi⁻¹, $c_o = 10 \times 10^{-6}$ psi⁻¹

If this well is placed under a constant production rate, calculate the dimensionless wellbore storage coefficient C_D. How long will it take for wellbore storage effects to end?

Solution

Step 1. Calculate the cross-sectional area of the annulus A_a:

$$A_a = \frac{\pi[(ID_C)^2 - (OD_T)^2]}{4(144)} = \frac{\pi[(7.675)^2 - (2)^2]}{(4)(144)} = 0.2995 \text{ ft}^2$$

Step 2. Calculate the wellbore storage factor caused by fluid expansion:

well. Obviously, reservoirs are not infinite in extent, so the infinite-acting radial flow period cannot last indefinitely. Eventually the effects of the reservoir boundaries will be felt at the well being tested. The time at which the boundary effect is felt is dependent on the following factors:

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- total compressibility c_t;
- porosity ϕ;
- viscosity μ;
- distance to the boundary;
- shape of the drainage area.

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$$t_{eia} = \left[\frac{\phi\mu c_t A}{0.0002637k}\right](t_{DA})_{eia}$$

where:

t_{eia} = time to the end of infinite-acting period, hours
A = well drainage area, ft²
c_t = total compressibility, psi⁻¹
$(t_{DA})_{eia}$ = dimensionless time to the end of the infinite-acting period

This expression is designed to predict the time that marks the end of transient flow in a drainage system of any geometry by obtaining the value of t_{DA} from Table 1.4. The last three columns of the table provide with values of t_{DA} that allow the engineer to calculate:

- the maximum elapsed time during which a reservoir is infinite acting;
- the time required for the pseudosteady-state solution to be applied and predict pressure drawdown within 1% accuracy;

Step 2. Calculate the wellbore storage factor caused by fluid expansion:

$$C_{FE} = V_{wb}c_{wb} = (180)(10 \times 10^{-6}) = 0.0018 \text{ bbl/psi}$$

Step 3. Determine the wellbore storage factor caused by the falling fluid level:

$$C_{FL} = \frac{144A_a}{5.615\rho} = \frac{144(0.2995)}{(5.615)(45)} = 0.1707 \text{ bbl/psi}$$

Step 4. Calculate the total wellbore storage coefficient:

$$C = C_{FE} + C_{FL} = 0.0018 + 0.1707 = 0.1725 \text{ bbl/psi}$$

The above calculations show that the effect of fluid expansion C_{FE} can generally be neglected in crude oil systems.

Step 5. Calculate the dimensionless wellbore storage coefficient from Equation 1.3.4:

$$C_D = \frac{0.8936C}{\phi h c_t r_w^2} = \frac{0.8936(0.1707)}{0.15(50)(20 \times 10^{-6})(0.25)^2} = 16\,271$$

Step 6. Approximate the time required for wellbore storage influence to end from:

$$t = \frac{(200\,000 + 12\,000s)C\mu}{kh} = \frac{(200\,000 + 0)(0.1725)(2)}{(30)(50)} = 46 \text{ hours}$$

The straight-line relationship as expressed by Equation 1.3.2 is only valid during the infinite-acting behavior of the

be applied and predict pressure drawdown within 1% accuracy;
- the time required for the pseudosteady-state solution (equations) to be exact and applied.

As an example, for a well centered in a circular reservoir, the maximum time for the reservoir to remain as an infinite-acting system can be determined using the entry in the final column of Table 1.4 to give $(t_{DA})_{eia} = 0.1$, and accordingly:

$$t_{eia} = \left[\frac{\phi\mu c_t A}{0.0002637k}\right](t_{DA})_{eia} = \left[\frac{\phi\mu c_t A}{0.0002637k}\right]0.1$$

or:

$$t_{eia} = \frac{380\phi\mu c_t A}{k}$$

For example, for a well that is located in the center of a 40 acre circular drainage area with the following properties:

$k = 60$ md, $c_t = 6 \times 10^{-6}$ psi⁻¹, $\mu = 1.5$ cp, $\phi = 0.12$

the maximum time, in hours, for the well to remain in an infinite-acting system is:

$$t_{eia} = \frac{380\phi\mu c_t A}{k} = \frac{380(0.12)(1.4)(6 \times 10^{-6})(40 \times 43560)}{60} = 11.1 \text{ hours}$$

Similarly, the pseudosteady-state solution can be applied any time after the semisteady-state flow begins at t_{pss} as estimated from:

$$t_{pss} = \left[\frac{\phi\mu c_t A}{0.0002637k}\right](t_{DA})_{pss}$$

where $(t_{DA})_{pss}$ can be found from the entry in the fifth column of the table.

Hence, the specific steps involved in a drawdown test analysis are:

(1) Plot $p_i - p_{wf}$ vs. t on a log–log scale.

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(2) Determine the time at which the unit-slope line ends.
(3) Determine the corresponding time at $1\frac{1}{2}$ log cycle, ahead of the observed time in step 2. This is the time that marks the end of the wellbore storage effect and the start of the semilog straight line.
(4) Estimate the wellbore storage coefficient from:

$$C = \frac{qt}{24\Delta p} = \frac{QBt}{24\Delta p}$$

where t and Δp are values read from a point on the log–log unit-slope straight line and q is the flow rate in bbl/day.
(5) Plot p_{wf} vs. t on a semilog scale.
(6) Determine the start of the straight-line portion as suggested in step 3 and draw the best line through the points.
(7) Calculate the slope of the straight line and determine the permeability k and skin factor s by applying Equations 1.3.2 and 1.3.3, respectively:

$$k = \frac{-162.6Q_oB_o\mu_o}{mh}$$

$$s = 1.151\left[\frac{p_{1\,hr} - p_i}{|m|} - \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) + 3.23\right]$$

(8) Estimate the time to the end of the infinite-acting (transient flow) period, i.e., t_{eia}, which marks the beginning of the pseudosteady-state flow.
(9) Plot all the recorded pressure data after t_{eia} as a function of time on a regular Cartesian scale. This data should form a straight-line relationship.
(10) Determine the slope of the pseudosteady-state line, i.e., dp/dt (commonly referred to as m') and use Equation 1.2.116 to solve for the drainage area A:

$$A = \frac{-0.23396QB}{c_t h\phi(dp/dt)} = \frac{-0.23396QB}{c_t h\phi m'}$$

where:

- permeability;
- fluid viscosity;
- total compressibility.

As time t increases, more of the reservoir is influenced by the well and the radius of drainage, or investigation, increases as given by:

$$r_{inv} = 0.0325\sqrt{\frac{kt}{\phi\mu c_t}}$$

where:

t = time, hours
k = permeability, md
c_t = total compressibility, psi⁻¹

It should be pointed out that the equations developed for slightly compressible liquids can be extended to describe the behavior of real gases by replacing the pressure with the real-gas pseudopressure $m(p)$, as defined by:

$$m(p) = \int_0^p \frac{2p}{\mu Z}dp$$

with the transient pressure drawdown behavior as described by Equation 1.2.151, or:

$$m(p_{wf}) = m(p_i) - \left[\frac{1637Q_gT}{kh}\right] \times \left[\log\left(\frac{kt}{\phi\mu_i c_{ti} r_w^2}\right) - 3.23 + 0.87s\right]$$

Under constant gas flow rate, the above relation can be expressed in a linear form as:

$$m(p_{wf}) = \left\{m(p_i) - \left[\frac{1637Q_gT}{kh}\right]\right.$$

Example 7.3. Calculation of permeability and skin from a pressure buildup test.

$k = \frac{162.6 \times 280 \times 2.0 \times 1.31}{-170 \times 40}$

Given:

flow rate before shut in period = 280 STB/day

N_p during constant rate period before shut in = 2682 STB

p_{wf} at the time of shut-in = 1123 psia P_{ws} at $\Delta t = 0$

From the foregoing data and Eq. (7.68), t_p can be calculated

$$t_p = \frac{N_p}{q} = \left(\frac{2682}{280}\right)24 = 230 \text{ hours}$$

11.5 md

Other given data are

S = 3.71

$B_o = 1.31$ bbl/STB $\mu_o = 2.0$ cp

$h = 40$ ft $c_t = 15 \times 10^{-6}$ psi^{-1}

$\phi = 0.10$ $r_w = 0.333$ ft

(230+2)/2

time after shut in, Δt hours	Pressure, p_{ws} psia	$\frac{t_p + \Delta t}{\Delta t}$
2	2290	116.0
4	2514	58.5
8	2584	29.8
12	2612	20.2
16	2632	15.4
20	2643	12.5
24	2650	10.6
30	2658	8.7

SOLUTION: The slope of the straight line region (notice the difficulty in identifying the straight line region) of the Horner plot in Fig. 7.22 is ~170 psi/cycle. From Eq. (7.70)

$$k = -\frac{162.6(280)(2.0)(1.31)}{(-170)(40)} = 17.5 \text{ md}$$

Again from the Horner plot, p_{1hr} is 2435 psia and from Eq. (7.71)

$$S = 1.151\left[\frac{1123 - 2435}{-170} - \log\left(\frac{17.5}{(0.1)(2.0)(15\times10^{-6})(0.333)^2}\right) + 3.23\right]$$

$S = 3.71$

∴ difficulty in identifying the straight line of the transient time region results

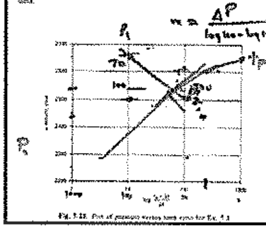


Fig. 7.21. Plot of pressure versus time ratio for Ex. 7.3.

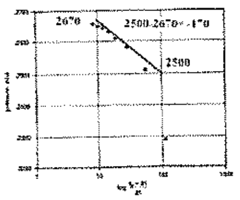


Fig. 7.22. Plot of pressure versus time ratio for Ex. 7.3.

SOLUTION: The slope of the straight line region (notice the difficulty in identifying the straight line region) of the Horner plot in Fig. 7.22 is ~170 psi/cycle. From Eq. (7.70)

$$k = -\frac{162.6(280)(2.0)(1.31)}{(-170)(40)} = 17.5 \text{ md}$$

Again from the Horner plot, p_{1hr} is 2435 psia and from Eq. (7.71)

$$S = 1.151\left[\frac{1123 - 2435}{-170} - \log\left(\frac{17.5}{(0.1)(2.0)(15\times10^{-6})(0.333)^2}\right) + 3.23\right]$$

$S = 3.71$

The difficulty in identifying the straight line of the transient time region results from the two earliest data points being dominated by wellbore storage effects. The time ratio $(t_p + \Delta t)/\Delta t$ decreases as Δt increases. Therefore, the early time data are on the right of Fig. 7.22. This difficulty in identifying the proper straight line region points out the importance of a thorough understanding of wellbore storage and other anomalies that could affect the pressure transient data.

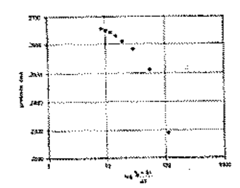


Fig. 7.22. Plot of pressure versus time ratio for Ex. 7.3.

PROBLEMS

$s' = 1.151\left[\frac{\bar{p}_i - \bar{p}_{1hr}}{|m|} - \log\left(\frac{k}{\phi(\mu c_t)_i r_w^2}\right) + 3.23\right]$

If the duration of the drawdown test of the gas well is long enough to reach its boundary, the pressure behavior during the boundary-dominated period (pseudosteady-state condition) is described by an equation similar to that of Equation 1.2.135 as:

For pseudopressure approach:

$$\frac{m(p_i) - m(p_{wf})}{q} = \frac{\Delta m(p)}{q} = \frac{711T}{kh}\left(\ln\frac{4A}{1.781C_A r_{wa}^2}\right) + \left[\frac{2.356T}{\phi(\mu_g c_g)_i Ah}\right]t$$

and as a linear equation by:

$$\frac{\Delta m(p)}{q} = b_{pss} + m' t$$

This relationship indicates that a plot of $\Delta m(p)/q$ vs. t will form a straight line with:

Intercept: $b_{pss} = \frac{711T}{kh}\left(\ln\frac{4A}{1.781C_A r_{wa}^2}\right)$

Slope: $m' = \frac{2.356T}{(\mu_g c_g)_i(\phi hA)} = \frac{2.356T}{(\mu_g c_g)_i(\text{pore volume})}$

For pressure-squared approach:

$$\frac{p_i^2 - p_{wf}^2}{q} = \frac{\Delta(p^2)}{q} = \frac{711\bar{\mu}\bar{Z}T}{kh}\left(\ln\frac{4A}{1.781C_A r_{wa}^2}\right) + \left[\frac{2.356\bar{\mu}\bar{Z}T}{\phi(\mu_g c_g)_i Ah}\right]t$$

and in a linear form as:

$$\frac{\Delta(p^2)}{q} = b_{pss} + m' t$$

any of the liquid analysis equations. However, care should be exercised when replacing the liquid flow rate with the gas flow rate. It should be noted that in all transient flow equations when applied to the oil phase, the flow rate is expressed as the product of Q_oB_o in bbl/day; that is, in reservoir bar rels/day. Therefore, when applying these equations to the gas phase, the product of the gas flow rate and gas formation volume factor Q_gB_g should be given in bbl/day. For example, if the gas flow rate is expressed in scf/day, the gas formation volume factor must be expressed in bbl/scf. The recorded pressure and time are then simply replaced by the normalized pressure and normalized time to be used in all the traditional graphical techniques, including pressure buildup.

1.3.2 Pressure buildup test

The use of pressure buildup data has provided the reservoir engineer with one more useful tool in the determination of reservoir behavior. Pressure buildup analysis describes the buildup in wellbore pressure with time after a well has been shut in. One of the principal objectives of this analysis is to determine the static reservoir pressure without waiting weeks or months for the pressure in the entire reservoir to stabilize. Because the buildup in wellbore pressure will generally follow some definite trend, it has been possible to extend the pressure buildup analysis to determine:

- the effective reservoir permeability;
- the extent of permeability damage around the wellbore;
- the presence of faults and to some degree the distance to the faults;
- any interference between producing wells;
- the limits of the reservoir where there is not a strong water drive or where the aquifer is no larger than the hydrocarbon reservoir.

Certainly all of this information will probably not be available from any given analysis, and the degree of usefulness of any of this information will depend on the experience in the area

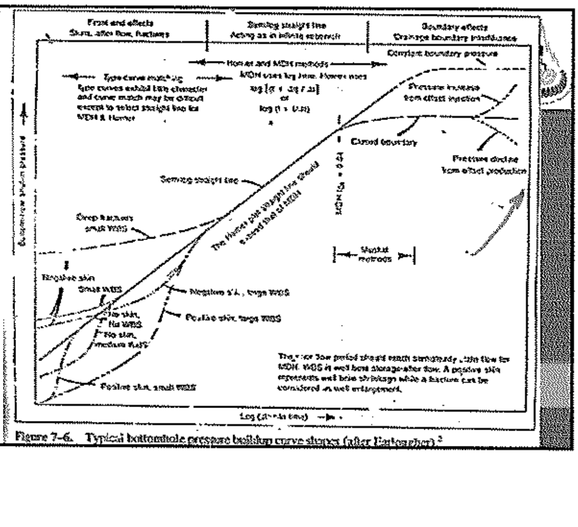
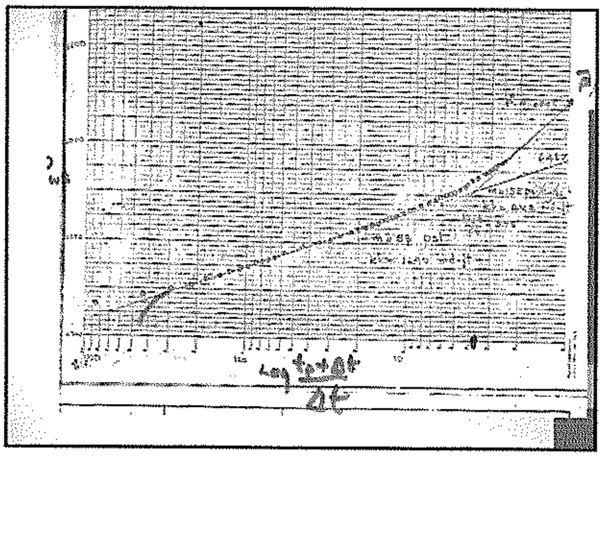
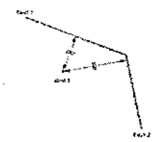


Figure 7-6. Typical bottomhole pressure buildup curve shapes (after Earlougher).[2]





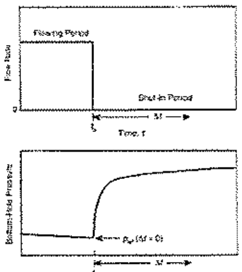


Figure 1.26 Idealized pressure buildup test.

and the amount of other information available for correlation purposes.

The general formulas used in analyzing pressure buildup data come from a solution of the diffusivity equation. In pressure buildup and drawdown analyses, the following assumptions, as regards the reservoir, fluid, and flow behavior, are usually made:

1.3.3 Horner plot

A pressure buildup test is described mathematically by using the principle of superposition. Before the shut-in, the well is allowed to flow at a constant flow rate of Q_o STB/day for t_p days. At the end of the flowing period, the well is shut in with a corresponding change in the flow rate from the "old" rate of Q_o to the "new" flow rate of $Q^{new} = 0$, i.e., $Q^{new} - Q^{old} = -Q_o$.

Calculation of the total pressure change which occurs at the sand face during the shut-in time is basically the sum of the pressure changes that are caused by:

- flowing the well at a stabilized flow rate of Q^{old}, i.e., the flow rate before shut-in Q_o, and is in effect over the entire time of $t_p + \Delta t$;
- the net change in the flow rate from Q_o to 0 and is in effect over Δt.

The composite effect is obtained by adding the individual constant-rate solutions at the specified rate–time sequence, as:

$$p_i - p_{ws} = (\Delta p)_{total} = (\Delta p)_{due\ to\ (Q_o - 0)} + (\Delta p)_{due\ to\ (0 - Q_o)}$$

where:

p_i = initial reservoir pressure, psi
p_{ws} = wellbore pressure during shut-in, psi

The above expression indicates that there are two contributions to the total pressure change at the wellbore resulting from the two individual flow rates.

The first contribution results from increasing the rate from 0 to Q_o and is in effect over the entire time period $t_p + \Delta t$, thus:

$$(\Delta p)_{Q_o - 0} = \left[\frac{162.6(Q_o - 0)B_o\mu_o}{kh}\right] \times \left[\log\left(\frac{k(t_p + \Delta t)}{\phi\mu_o c_t r_w^2}\right) - 3.23 + 0.87s\right]$$

The general formulas used in analyzing pressure buildup data come from a solution of the diffusivity equation. In pressure buildup and drawdown analyses, the following assumptions, as regards the reservoir, fluid, and flow behavior, are usually made:

- Reservoir: homogeneous; isotropic; horizontal of uniform thickness.
- Fluid: single phase; slightly compressible; constant μ_o and B_o.
- Flow: laminar flow; no gravity effects.

Pressure buildup testing requires shutting in a producing well and recording the resulting increase in the wellbore pressure as a function of shut-in time. The most common and simplest analysis techniques require that the well produce at a constant rate for a flowing time of t_p, either from startup or long enough to establish a stabilized pressure distribution, before shut in. Traditionally, the shut-in time is denoted by the symbol Δt. Figure 1.26 schematically shows the stabilized constant flow rate before shut-in and the ideal behavior of the pressure increase during the buildup period. The pressure is measured immediately before shut-in and is recorded as a function of time during the shut-in period. The resulting pressure buildup curve is then analyzed to determine reservoir properties and the wellbore condition.

Stabilizing the well at a constant rate before testing is an important part of a pressure buildup test. If stabilization is overlooked or is impossible, standard data analysis techniques may provide erroneous information about the formation.

Two widely used methods are discussed below; these are:

(1) the Horner plot;
(2) the Miller–Dyes–Hutchinson method.

$$(\Delta p)_{Q_o - 0} = \left[\frac{162.6(Q_o - 0)B_o\mu_o}{kh}\right] \times \left[\log\left(\frac{k(t_p + \Delta t)}{\phi\mu_o c_t r_w^2}\right) - 3.23 + 0.87s\right]$$

The second contribution results from decreasing the rate from Q_o to 0 at t_p, i.e., shut-in time, thus:

$$(\Delta p)_{0 - Q_o} = \left[\frac{162.6(0 - Q_o)B_o\mu_o}{kh}\right] \times \left[\log\left(\frac{k\Delta t}{\phi\mu_o c_t r_w^2}\right) - 3.23 + 0.87s\right]$$

The pressure behavior in the well during the shut-in period is then given by:

$$p_i - p_{ws} = \frac{162.6Q_o\mu_o B_o}{kh}\left[\log\frac{k(t_p + \Delta t)}{\phi\mu_o c_t r_w^2} - 3.23\right] - \frac{162.6(-Q_o)\mu_o B_o}{kh}\left[\log\frac{k\Delta t}{\phi\mu_o c_t r_w^2} - 3.23\right]$$

Expanding this equation and canceling terms gives:

$$p_{ws} = p_i - \frac{162.6Q_o\mu_o B_o}{kh}\left[\log\left(\frac{t_p + \Delta t}{\Delta t}\right)\right] \qquad (1.3.6)$$

where:

p_i = initial reservoir pressure, psi
p_{ws} = sand face pressure during pressure buildup, psi
t_p = flowing time before shut-in, hours
Q_o = stabilized well flow rate before shut-in, STB/day
Δt = shut-in time, hours

The pressure buildup equation, i.e., Equation 1.3.6 was introduced by Horner (1951) and is commonly referred to as the Horner equation.

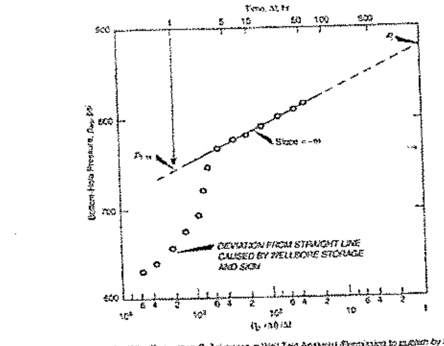


Figure 1.37 *Horner plot (After Earlougher, R. Advances in Well Test Analysis) (Permission to publish by the SPE, copyright SPE, 1977).*

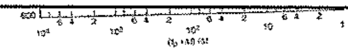


Figure 1.37 *Horner plot (After Earlougher, R. Advances in Well Test Analysis) (Permission to publish by the SPE, copyright SPE, 1977).*

Equation 1.3.6 is basically an equation of a straight line that can be expressed as:

$$p_{ws} = p_i - m\left[\log\left(\frac{t_p+\Delta t}{\Delta t}\right)\right] \qquad (1.3.7)$$

This expression suggests that a plot of p_{ws} vs. $(t_p + \Delta t)/\Delta t$ on a semilog scale would produce a straight line relationship with intercept p_i and slope m, where:

$$m = \frac{162.6 Q_o B_o \mu_o}{kh} \qquad (1.3.8)$$

or

$$k = \frac{162.6 Q_o B_o \mu_o}{mh}$$

and where:

m = slope of straight line, psi/cycle
k = permeability, md

This plot, commonly referred to as the Horner plot, is illustrated in Figure 1.37. Note that on the Horner plot, the scale of time ratio $(t_p + \Delta t)/\Delta t$ increases from right to left. It is observed from Equation 1.3.6 that $p_{ws} = p_i$ when the time ratio is unity. Graphically this means that the initial reservoir pressure, p_i, can be obtained by extrapolating the Horner plot straight line to $(t_p + \Delta t)/\Delta t = 1$.

The time corresponding to the point of shut-in, t_p can be estimated from the following equation:

$$t_p = \frac{24 N_p}{Q_o}$$

where:

N_p = well cumulative oil produced before shut in, STB
Q_o = stabilized well flow rate before shut in, STB/day
t_p = total production time, hours

Earlougher (1977) pointed out that a result of using the superposition principle is that the skin factor, s, does not appear in the general pressure buildup equation, Equation 1.3.6. That means the Horner plot slope is not affected by the skin factor; however, the skin factor still does affect the shape of the pressure buildup data. In fact, an early-time deviation from the straight line can be caused by the skin factor as well as by wellbore storage, as illustrated in Figure 1.36. The deviation can be significant for the large negative skins that occur in hydraulically fractured wells. The skin factor does affect flowing pressure before shut-in and its value may be estimated from the buildup test data plus the flowing pressure immediately before the buildup test, as given by:

$$s = 1.151\left[\frac{p_{1\,hr} - p_{wf\ at\ \Delta t=0}}{|m|} - \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) + 3.23\right] \qquad (1.3.9)$$

with an additional pressure drop across the altered zone of:

$$\Delta p_{skin} = 0.87|m|s$$

where:

$p_{wf\ at\ \Delta t=0}$ = bottom-hole flowing pressure immediately before shut-in, psi

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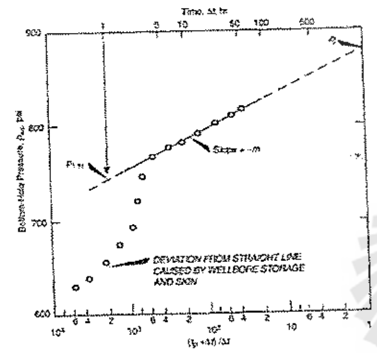

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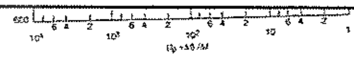

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or

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and where:

m = slope of straight line, psi/cycle
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$$s = 1.151\left[\frac{p_{1\,hr} - p_{wf\ at\ \Delta t=0}}{|m|} - \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) + 3.23\right] \qquad (1.3.9)$$

with an additional pressure drop across the altered zone of:

$$\Delta p_{skin} = 0.87|m|s$$

where:

$p_{wf\ at\ \Delta t=0}$ = bottom-hole flowing pressure immediately before shut-in, psi

s = skin factor
$|m|$ = absolute value of the slope in the Horner plot, psi/cycle
r_w = wellbore radius, ft

The value of $p_{1\,hr}$ must be taken from the Horner straight line. Frequently, the pressure data does not fall on the straight line at 1 hour because of wellbore storage effects or large negative skin factors. In that case, the semilog line must be extrapolated to 1 hour and the corresponding pressure is read.

It should be noted that *for a multiphase flow*, Equations 1.3.6 and 1.3.9 become:

$$p_{ws} = p_i - \frac{162.6 q_t}{\lambda_t h}\left[\log\left(\frac{t_p+\Delta t}{\Delta t}\right)\right]$$

$$s = 1.151\left[\frac{p_{1\,hr} - p_{wf\ at\ \Delta t=0}}{|m|} - \log\left(\frac{\lambda_t}{\phi c_t r_w^2}\right) + 3.23\right]$$

with:

$$\lambda_t = \frac{k_o}{\mu_o} + \frac{k_w}{\mu_w} + \frac{k_g}{\mu_g}$$

$$q_t = Q_o B_o + Q_w B_w + (Q_g - Q_o R_s) B_g$$

or equivalently in terms of GOR as:

$$q_t = Q_o B_o + Q_w B_w + (GOR - R_s) Q_o B_g$$

where:

q_t = total fluid voidage rate, bbl/day
Q_o = oil flow rate, STB/day
Q_w = water flow rate, STB/day
Q_g = gas flow rate, scf/day
R_s = gas solubility, scf/STB
B_g = gas formation volume factor, bbl/scf
λ_t = total mobility, md/cp
k_o = effective permeability to oil, md

For pressure-squared approach:

$$m = \frac{1637 Q_g \bar{Z} \bar{\mu}_g}{kh}$$

$$s = 1.151\left[\frac{p^2_{1\,hr} - p^2_{wf\ at\ \Delta t=0}}{|m|} - \log\left(\frac{k}{\phi\mu_i c_{ti} r_w^2}\right) + 3.23\right]$$

where the gas flow rate Q_g is expressed in Mscf/day.

It should be pointed out that when a well is shut in for a pressure buildup test, the well is usually closed at the surface rather than the sand face. Even though the well is shut in, the reservoir fluid continues to flow and accumulates in the wellbore until the well fills sufficiently to transmit the effect of shut-in to the formation. This "afterflow" behavior is caused by the wellbore storage and it has a significant influence on pressure buildup data. During the period of wellbore storage effects, the pressure data points fall below the semilog straight line. The duration of these effects may be estimated by making the log–log data plot described previously of $\log(p_{ws} - p_{wf})$ vs. $\log(\Delta t)$ with p_{wf} as the value recorded immediately before shut-in. When wellbore storage dominates, that plot will have a unit-slope straight line; as the semilog straight line is approached, the log–log plot bends over to a gently curving line with a low slope.

The wellbore storage coefficient C is, by *selecting a point on the log–log unit-slope straight line and reading the coordinate of the point in terms of Δt and Δp*:

$$C = \frac{q\Delta t}{24\Delta p} = \frac{QB\Delta t}{24\Delta p}$$

where

Δt = shut-in time, hours

where:

q_t = total fluid voidage rate, bbl/day
Q_o = oil flow rate, STB/day
Q_w = water flow rate, STB/day
Q_g = gas flow rate, scf/day
R_s = gas solubility, scf/STB
B_g = gas formation volume factor, bbl/scf
λ_t = total mobility, md/cp
k_o = effective permeability to oil, md
k_w = effective permeability to water, md
k_g = effective permeability to gas, md

The regular Horner plot would produce a semilog straight line with a slope m that can be used to determine the total mobility λ_t from:

$$\lambda_t = \frac{162.6 q_t}{mh}$$

Perrine (1956) showed that the effective permeability of each phase, i.e., k_o, k_w, and k_g, can be determined as:

$$k_o = \frac{162.6 Q_o B_o \mu_o}{mh}$$

$$k_w = \frac{162.6 Q_w B_w \mu_w}{mh}$$

$$k_g = \frac{162.6 (Q_g - Q_o R_s) B_g \mu_g}{mh}$$

For gas systems, a plot of $m(p_{ws})$ or p^2_{ws} vs. $(t_p + \Delta t)/\Delta t$ on a semilog scale would produce a straight line relationship with a slope of m and apparent skin factor s as defined by:

For pseudopressure approach:

$$m = \frac{1637 Q_g T}{kh}$$

$$s = 1.151\left[\frac{m(p_{1\,hr}) - m(p_{wf\ at\ \Delta t=0})}{|m|} - \log\left(\frac{k}{\phi\mu_i c_{ti} r_w^2}\right) + 3.23\right]$$

The wellbore storage coefficient C is, by selecting a *point on the log–log unit-slope straight line* and reading the coordinate of the point in terms of Δt and Δp:

$$C = \frac{q\Delta t}{24\Delta p} = \frac{QB\Delta t}{24\Delta p}$$

where

Δt = shut-in time, hours
Δp = pressure difference $(p_{ws} - p_{wf})$, psi
q = flow rate, bbl/day
Q = flow rate, STB/day
B = formation volume factor, bbl/STB

with a dimensionless wellbore storage coefficient as given by Equation 1.3.4 as:

$$C_D = \frac{0.8936 C}{\phi h c_t r_w^2}$$

In all the pressure buildup test analyses, the log–log data plot should be made before the straight line is chosen on the semilog data plot. This log–log plot is essential to avoid drawing a semilog straight line through the wellbore storage-dominated data. The beginning of the semilog line can be estimated by observing when the data points on the log–log plot reach the slowly curving low-slope line and adding 1 to 1½ cycles in time after the end of the unit-slope straight line. Alternatively, the time to the beginning of the semilog straight line can be estimated from:

$$\Delta t > \frac{170000\, C e^{0.14 s}}{(kh/\mu)}$$

where:

C = calculated wellbore storage coefficient, bbl/psi
k = permeability, md
s = skin factor
h = thickness, ft

Table 1.5 *Earlougher's pressure buildup data (Permission to publish by the SPE, copyright SPE, 1977.)*

Δt (hr)	$t_p + \Delta t$ (hr)	$\frac{t_p + \Delta t}{\Delta t}$	p_{ws} (psig)
0.0	–	–	2761
0.10	310.10	3101	3057
0.21	310.21	1477	3153
0.31	310.31	1001	3234
0.52	310.52	597	3249
0.63	310.63	493	3256
0.73	310.73	426	3260
0.84	310.84	370	3263
0.94	310.94	331	3266
1.05	311.05	296	3267
1.15	311.15	271	3268
1.36	311.36	229	3271
1.68	311.68	186	3274
1.99	311.99	157	3276
2.51	312.51	125	3280
3.04	313.04	103	3283
3.46	313.46	90.6	3286
4.08	314.08	77.0	3289
5.03	315.03	62.6	3293
5.97	315.97	52.9	3297
6.07	316.07	52.1	3297
7.01	317.01	45.2	3300
8.06	318.06	39.5	3303
9.00	319.00	35.4	3305
10.05	320.05	31.8	3306
13.09	323.09	24.7	3310
16.02	326.02	20.4	3313
20.00	330.00	16.5	3317
26.07	336.07	12.9	3320
31.03	341.03	11.0	3322
34.98	344.98	9.9	3323
37.54	347.54	9.3	3323

Step 3. Calculate the average permeability by using Equation 1.3.8:

$$k = \frac{162.6 Q_o B_o \mu_o}{mh} = \frac{(162.6)(4900)(1.55)(0.22)}{(40)(482)} = 12.8 \text{ md}$$

Step 4. Determine $p_{1\,hr}$ after 1 hour from the straight-line portion of the curve:

$$p_{1\,hr} = 3266 \text{ psi}$$

Step 5. Calculate the skin factor by applying Equation 1.3.9:

$$s = 1.151\left[\frac{p_{1\,hr} - p_{wf(\Delta t=0)}}{m} - \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) + 3.23\right]$$

$$= 1.151\left[\frac{3266 - 2761}{40} - \log\left(\frac{12.8}{(0.09)(0.20)(22.6 \times 10^{-6})(0.354)^2}\right) + 3.23\right]$$

$$= 8.6$$

Step 6. Calculate the additional pressure drop by using:

$$\Delta p_{skin} = 0.87\, m\, s$$

$$= 0.87(40)(8.6) = 299.3 \text{ psi}$$

It should be pointed out that Equation 1.3.6 assumes the reservoir to be infinite in size, i.e., $r_e = \infty$, which implies that at some point in the reservoir the pressure would be always equal to the initial reservoir pressure p_i and the Horner straight-line plot will always extrapolate to p_i. However, reservoirs are finite and soon after production begins, fluid removal will cause a pressure decline everywhere in the reservoir system. Under these conditions, the straight line will not extrapolate to the initial reservoir pressure p_i but, instead, the pressure obtained will be a false pressure as denoted by p^*. The false pressure, as illustrated by Matthews

31.03 341.03 11.0 3322
34.98 344.98 9.9 3323
37.54 347.54 9.3 3323

Example 1.27* Table 1.5 shows the pressure buildup data from an oil well with an estimated drainage radius of 2640 ft. Before shut-in, the well had produced at a stabilized rate of 4900 STB/day for 310 hours. Known reservoir data is:

depth = 10 476 ft, r_w = 0.354 ft, c_t = 22.6 × 10⁻⁶ psi⁻¹
Q_o = 4900 STB/D, h = 482 ft, $p_{wf}(\Delta t = 0)$ = 2761 psig
μ_o = 0.20 cp, B_o = 1.55 bbl/STB, ϕ = 0.09
t_p = 310 hours, r_e = 2640 ft

Calculate:

- the average permeability k;
- the skin factor;
- the additional pressure drop due to skin.

Solution

Step 1. Plot p_{ws} vs. $(t_p + \Delta t)/\Delta t$ on a semilog scale as shown in Figure 1.38.

Step 2. Identify the correct straight-line portion of the curve and determine the slope m:

$$m = 40 \text{ psi/cycle}$$

*This example problem and the solution procedure are given in Earlougher, R. *Advanced Well Test Analysis*, Monograph Series, SPE, Dallas (1977).

fluid removal will cause a pressure decline everywhere in the reservoir system. Under these conditions, the straight line will not extrapolate to the initial reservoir pressure p_i but, instead, the pressure obtained will be a false pressure as denoted by p^*. The false pressure, as illustrated by Matthews
and Russell (1967) in Figure 1.39, has no physical meaning but it is usually used to determine the average reservoir pressure $\bar{p}$. It is clear that p^* will only equal the initial (original) reservoir pressure p_i when a new well in a newly discovered field is tested. Using the concept of the false pressure p^*, Horner expressions as given by Equations 1.3.6 and 1.3.7 should be expressed in terms of p^* instead of p_i as:

$$p_{ws} = p^* - \frac{162.6 Q_o \mu_o B_o}{kh}\left[\log\left(\frac{t_p + \Delta t}{\Delta t}\right)\right]$$

and:

$$p_{ws} = p^* - m\left[\log\left(\frac{t_p + \Delta t}{\Delta t}\right)\right] \qquad [1.3.10]$$

Bossie-Codreanu (1989) suggested that the well drainage area can be determined from the Horner pressure buildup plot or the MDH plot, discussed next, by selecting the coordinates of any three points located on the semilog straight-line portion of the plot to determine the slope of the pseudosteady-state line m_{pss}. The coordinates of these three points are designated as:

- shut-in time Δt_1 and with a corresponding shut-in pressure p_{ws1};
- shut-in time Δt_2 and with a corresponding shut-in pressure p_{ws2};
- shut-in time Δt_3 and with a corresponding shut-in pressure p_{ws3}.

The selected shut-in times satisfy $\Delta t_1 < \Delta t_2 < \Delta t_3$. The slope of the pseudosteady-state straight-line m_{pss} is then

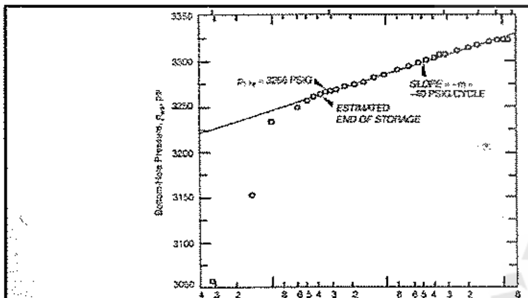


Figure 1.38 Earlougher's semilog data plot for the buildup test (Permission to publish by the SPE, copyright SPE, 1977).

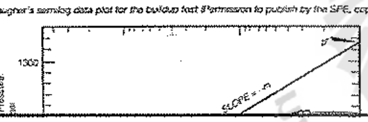


Figure 1.38 Earlougher's semilog data plot for the buildup test (Permission to publish by the SPE, copyright SPE, 1977).

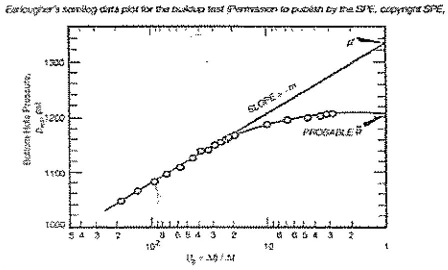


Figure 1.39 Typical pressure buildup curve for a well in a finite (After Earlougher, R. Advances in Well Test Analysis) (Permission to publish by the SPE, copyright SPE, 1977).

approximated by:

$$m_{pss} = \frac{(p_{ws3} - p_{ws1})\log(\Delta t_2/\Delta t_1) - (p_{ws2} - p_{ws1})\log(\Delta t_3/\Delta t_1)}{(\Delta t_3 - \Delta t_1)\log(\Delta t_2/\Delta t_1) - (\Delta t_2 - \Delta t_1)\log(\Delta t_3/\Delta t_1)} \qquad [1.3.11]$$

The well drainage area can be calculated from Equation 1.2.116:

$$m' = m_{pss} = \frac{0.23396 Q_o B_o}{c_t A h \phi}$$

Solving for the drainage area gives:

$$A = \frac{0.23396 Q_o B_o}{c_t m_{pss} h \phi}$$

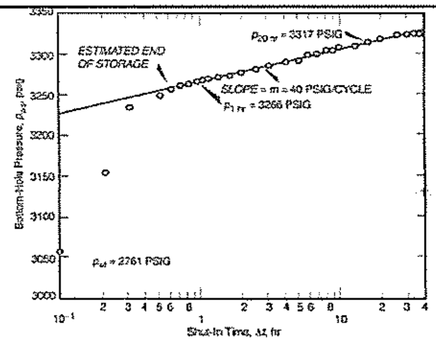


Figure 1.40 Miller-Dyes-Hutchinson plot for the buildup test (After Earlougher, R. Advances in Well Test Analysis) (Permission to publish by the SPE, copyright SPE, 1977).

where:

m_{pss} or m' = slope of straight line during the pseudosteady-state flow, psi/hr

Q_o = flow rate, bbl/day

A = well drainage area, ft²

Figure 1.40 shows a positive slope of m = 40 psi/cycle that is identical to the value obtained in Example 1.26 with $p_{1\,hr}$ = 3266 psig.

1.3.4 Miller-Dyes-Hutchinson method

The Horner plot may be simplified if the well has been producing long enough to reach a pseudosteady state. Assuming that the production time t_p is much greater than the total shut-in time Δt, i.e., $t_p \gg \Delta t$, the term $t_p + \Delta t \cong t_p$ and:

$$\log\left(\frac{t_p + \Delta t}{\Delta t}\right) \cong \log\left(\frac{t_p}{\Delta t}\right) = \log(t_p) - \log(\Delta t)$$

Applying the above mathematical assumption to Equation 1.3.10, gives:

$$p_{ws} = p^* - m[\log(t_p) - \log(\Delta t)]$$

or:

$$p_{ws} = [p^* - m\log(t_p)] + m\log(\Delta t)$$

This expression indicates that a plot of p_{ws} vs. $\log(\Delta t)$ would produce a semilog straight line with a positive slope of $+m$ that is identical to that obtained from the Horner plot. The slope is defined mathematically by Equation 1.3.8 as:

$$m = \frac{162.6 Q_o B_o \mu_o}{kh}$$

The semilog straight-line slope m has the same value as of the Horner plot. This plot is commonly called the Miller–Dyes–Hutchinson (MDH) plot. The false pressure p^* may be estimated from the MDH plot by using:

$$p^* = p_{1\,hr} + m\log(t_p + 1) \qquad [1.3.12]$$

where $p_{1\,hr}$ is read from the semilog straight-line plot at Δt = 1 hour. The MDH plot of the pressure buildup data given in Table 1.5 in terms of p_{ws} vs. $\log(\Delta t)$ is shown in Figure 1.40.

$p_{1\,hr}$ = 3266 psig.

As in the Horner plot, the time that marks the beginning of the MDH semilog straight line may be estimated by making the log–log plot of $(p_{ws} - p_{wf})$ vs. Δt and observing when the data points deviate from the 45° angle (unit slope). The exact time is determined by moving 1 to 1½ cycles in time after the end of the unit-slope straight line.

The observed pressure behavior of the test well following the end of the transient flow will depend on:

- shape and geometry of the test well drainage area;
- the position of the well relative to the drainage boundaries;
- length of the producing time t_p before shut-in.

If the well is located in a reservoir with no other wells, the shut-in pressure would eventually become constant (as shown in Figure 1.38) and equal to the *volumetric average reservoir pressure* $\bar{p}_r$. This pressure is required in many reservoir engineering calculations such as:

- material balance studies;
- water influx;
- pressure maintenance projects;
- secondary recovery;
- degree of reservoir connectivity.

Finally, in making future predictions of production as a function of $\bar{p}_r$, pressure measurements throughout the reservoir's life are almost mandatory if one is to compare such a prediction to actual performance and make the necessary adjustments to the predictions. One way to obtain this pressure is to shut in all wells producing from the reservoir for a period of time that is sufficient for pressures to equalize throughout the system to give $\bar{p}_r$. Obviously, such a procedure is not practical.

Pressure Build Up

FLOW EFFICIENCY AND DAMAGE RATIO

These two factors also give indication of wellbore damage and improvement. These two terms translate s into a physically meaningful characterization. Flow efficiency is denoted by FE and damage ratio by DR. Flow efficiency is defined as the ratio of the actual productivity index to the ideal productivity index. Productivity index obtained on a well with unaltered permeability (i.e. zero skin) at the wellbore is called ideal PI. Mathematically, flow efficiency is defined by the following equation:

$$FE = \frac{J_{actual}}{J_{ideal}} = \frac{\bar{p} - p_{wf} - (\Delta p)_{skin}}{\bar{p} - p_{wf}} \quad (7.21)$$

Equation (7.21) can be written in an approximate form as

$$FE = \frac{p^* - p_{wf} - (\Delta p)_{skin}}{p^* - p_{wf}} \quad (7.22)$$

where p^* is the intercept of the straight-line portion (MTR) of the Horner plot at infinite time (i.e. at $(t_p + \Delta t)/\Delta t = 1$). Flow efficiency is 1.0 (100%) for a well that is neither damaged nor stimulated. FE is less than 1.0 for a damaged well and greater than 1.0 for a stimulated well. If FE is 2.0 the well is producing twice as much for a given drawdown than if the well had not been stimulated.

Damage ratio is the inverse of the FE. Damage ratio is also a relative indicator of wellbore condition.

Example 7-6. The data in Table 7-1 were obtained in a pressure buildup test on an oil well in Indonesia. The well was produced for an effective time of 15 hours at the final rate. Other data include

$q_o = 5{,}535$ STB/D	$\mu_o = 0.89$ cp
$c_o = 9.5 \times 10^{-6}$ 1/psi	$B_o = 1.31$ RB/STB
$c_w = 3 \times 10^{-6}$ 1/psi	$h = 130$ ft
$c_f = 1 \times 10^{-6}$ 1/psi	$d = 8.681$ in
$S_w = 36\%$	$r_w = 0.362$ ft
$\phi = 23\%$	$k_H = k_v$
BH dia = 12.25 in	Casing ID = 8.681 in

Calculate:

(a) Total isothermal compressibility
(b) Permeability, k
(c) Skin factor, s
(d) Pressure drop due to skin $(\Delta p)_{skin}$
(e) Effective wellbore radius r_{wa}
(f) Flow efficiency, FE using p^*
(g) Damage ratio, DR using p^*
(h) Productivity index, PI
(i) Radius of investigation at $\Delta t = 5$ hours
(j) ...

TABLE 7-1. Oil well pressure buildup test data for Example 7-6

Δt (hours)	$(t_p + \Delta t)/\Delta t$	p_{ws} (psia)	$\Delta p = p_{ws} - p_{wf}$ (psia)
[illegible]	[illegible]	[illegible]	[illegible]

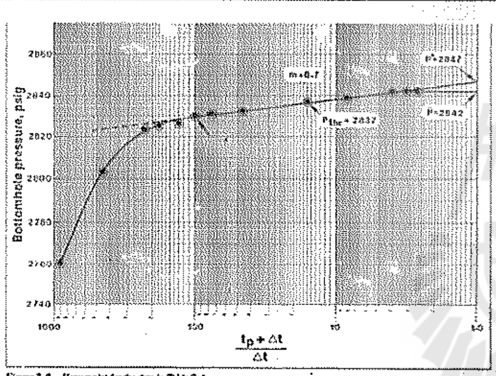


Figure 7-9. Horner plot for the data in Table 7-1.

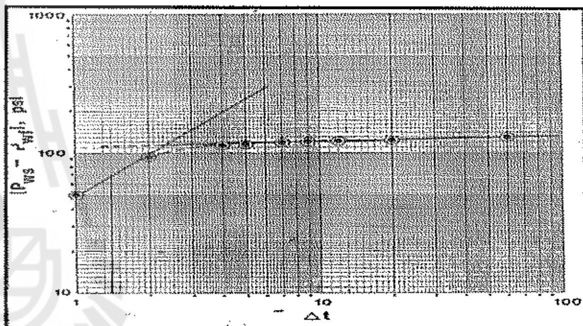


Figure 7-10. Log-log plot of Δp vs. Δt for the data in Table 7-1.

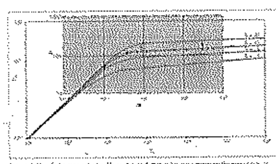
The well is damaged. Skin is also confirmed from the match in Figure 7-11.

(d) $(\Delta p)_{skin} = 0.869ms$ 8.35

$= (0.869)(8.7)(0.35)$

$= 70.77$ psi

(e) $r_{wa} = r_w e^{-s}$



Estimating Distance to noflow Boundary

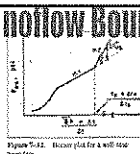


Figure 7-12. Horner plot for a well near boundary

$$L = \left(\frac{0.00148k\Delta t_x}{\phi\mu c_t} \right)^{0.5} \quad (7.24)$$

$$(7.25)$$

$$(7.26)$$

TABLE 7-2. Pressure buildup data for the well in Example 7-7

Δt (hours)	p_{ws} (psia)	$t_p+\Delta t/\Delta t$	Δt (hours)	p_{ws} (psia)	$t_p+\Delta t/\Delta t$
0	[illegible]	[illegible]	5.00	[illegible]	[illegible]
0.17	[illegible]	[illegible]	6.00	2,174	470
0.50	[illegible]	[illegible]	7.00	2,179	403
0.67	[illegible]	[illegible]	8.00	2,186	353
0.83	[illegible]	[illegible]	9.00	2,188	314
1.00	[illegible]	[illegible]	10.00	2,192	282
1.17	[illegible]	[illegible]	12.00	2,196	235
1.33	[illegible]	[illegible]	14.00	2,201	202
1.50	[illegible]	[illegible]	16.00	2,206	177
1.67	[illegible]	[illegible]	20.00	2,217	142
1.83	[illegible]	[illegible]	24.00	2,227	118
2.00	[illegible]	[illegible]	28.00	2,234	101
3.00	[illegible]	[illegible]	32.00	2,241	89
4.00	[illegible]	[illegible]	35.50	2,245	80

... = 10.51 The well is damaged

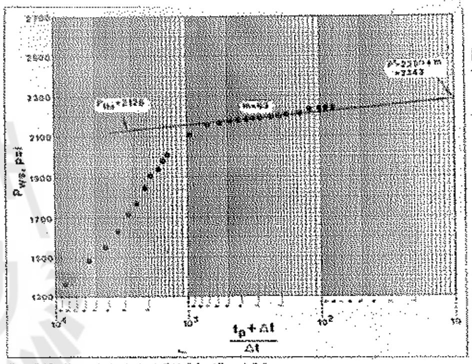


Figure 7-13. Horner plot for the data in Table 7-2 and Example 7-7.

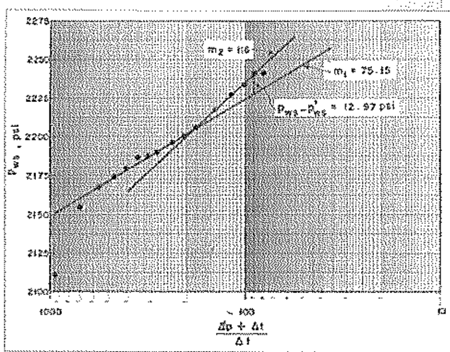


Figure 7-14. Horner plot for the data in Table 7-2.

(c)
$$(\Delta p)_{skin} = 0.869\, ms$$
$$= (0.869)(75.15)(10.51)$$
$$= 687.15 \text{ psi}$$

(d) In Figure 7-13, the second-slope line is not clear. Therefore, selected data points (late-time) from Table 7-2 are replotted using a wider pressure scale, as shown in Figure 7-14. From this figure we can see the two straight lines of slope m_1 = 75.15 psi/cycle and m_2 = 116 psi/cycle.

(e) To confirm the presence of a fault and estimate its distance, we use p_{ws} and Δt from the line of slope m_2 and the corresponding p^*_{ws} on the extrapolated portion of the line with slope m_1.

Δt (hours)	$t_p+\Delta t/\Delta t$	P_{ws} (psi)	P^*_{ws} (psi)	$\Delta p^*_{ws}=p_{ws}-p^*_{ws}$ (psi)	$E_i(-x)$	From Table 6-6 -x
20	142	2,217	2,212.77	4.23	0.130	1.30
24	118	2,227	2,218.78	8.22	0.252	0.99
28	101	2,234	2,223.89	10.11	0.310	0.79
32	89	2,241	2,228.03	12.97	0.398	0.67

We will use Equation (7.26) and Table 6-6 to calculate the values $E_i(-x)$ and $-x$ in the previous table. A sample calculation is

$$\Delta p^*_{ws} = p_{ws} - p^*_{ws} = 70.6 \frac{q_o \mu_o B_o}{kh} \left[-E_i \left(\frac{-3.792 \phi \mu_o c_t L^2}{k \Delta t} \right) \right]$$

$$4.23 = 70.6 \frac{(64)(1.4)(0.55)}{(4.27)(25)} \left\{ -E_i \left[\frac{(-3.792)(0.35)(0.55)(10.9 \times 10^{-7})(L^2)}{(4.27)(\Delta t)} \right] \right\}$$

$$-E_i \left(\frac{-0.00186 L^2}{\Delta t} \right) = 0.130$$

From Table 6-6, $-x$ = 1.30 for $E_i(-x)$ = 0.130

$$L^2 = \frac{(1.30)(\Delta t)}{0.00186}$$
$$= \frac{(1.30)(20)}{0.00186}$$
$$L = 118.23 \text{ ft}$$

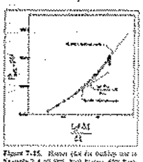


Figure 7.15. Horner plot for buildup test in Example 7.4.

PRESSURE BUILDUP TESTS-II

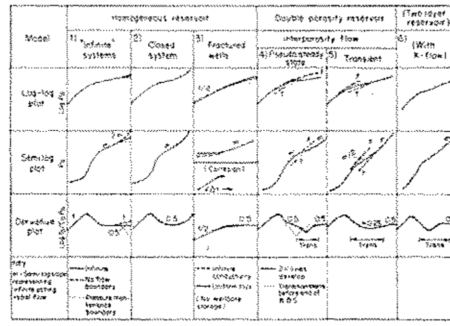


Fig. 9.5 Summary of well reservoir model responses in different reservoir systems (after [42]).

TABLE 2.5 – BUILDUP DATA FOR WELL NEAR BOUNDARY

Δt (hours)	p_{ws} (psia)	Δt (hours)	p_{ws} (psia)	Δt (hours)	p_{ws} (psia)
0	3,103	8	4,085	30	4,614
1	3,453	10	4,172	36	4,700
2	3,613	12	4,240	42	4,720
3	3,760	14	4,298	48	4,827
4	3,861	16	4,353	54	4,882
5	3,936	20	4,435	60	4,931
6	3,990	24	4,520	66	4,975

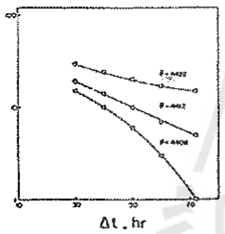


Fig. 2.19 – Modified Muskat method applied to example buildup test.

we stop recording pressures at $\Delta t = 72$ hours.

We make Table 2.5 for three trial values of $\bar{p}$. From plots of $(\bar{p}-p_{ws})$, we see that $\bar{p}=4,412$ psi is clearly the best choice; we also note the sensitivity of this method. There is a noticeable curvature for $\bar{p}=4,408$ psi and $\bar{p}=4,422$ psi (Fig. 2.19).

This example application of the modified Muskat method is intended to illustrate only the mechanics of using the method. The pressure had built up to within 5 psi of its static value at a shut-in time of 72 hours; in such a case, there is little value in applying either this method or the MBH p^* method. Both methods are of value only when the final pressure is a significant distance from stabilization.

2.10 Reservoir Limits Test

In this section, we deal briefly with techniques for estimating reservoir size and distance to boundaries. These comments are introductory only, and deal only with the simplest cases. The intent is to illustrate an approach to these problems rather than the state of the art. The techniques presented are based on buildup-test data analysis. Much technology based on drawdown test analysis has been developed and is summarized by Earlougher.[13]

We demonstrate that a single boundary near a well causes the slope of a buildup curve to double, and then develop a method for estimating distance from a tested well to a single boundary.

In Chap. 1, when we illustrated application of the superposition principle, we showed that the flowing pressure in a well a distance L from a no-flow boundary (such as a sealing fault) is given by Eq. 1.50. Rearranged, it becomes

$$p_i - p_{wf} = -70.6\frac{qB\mu}{kh}\left[\ln\left(\frac{1{,}688\,\phi\mu c_t r_w^2}{kt_p}\right)-2s\right]$$

$$-70.6\frac{qB\mu}{kh}Ei\left(\frac{-948\,\phi\mu c_t(2L)^2}{kt_p}\right).$$

We can develop an equation describing a buildup test for such a well. Note that

$$p_i - p_{ws} = -70.6\frac{qB\mu}{kh}\left\{\ln\left[\frac{1{,}688\,\phi\mu c_t r_w^2}{k(t_p+\Delta t)}\right]-2s\right\}$$

$$-70.6(-q)\frac{B\mu}{kh}\left\{\ln\left[\frac{1{,}688\,\phi\mu c_t r_w^2}{k\Delta t}\right]-2s\right\}$$

$$-70.6\frac{qB\mu}{kh}Ei\left[\frac{-3{,}792\,\phi\mu c_t L^2}{k(t_p+\Delta t)}\right]$$

$$-70.6(-q)\frac{B\mu}{kh}Ei\left(\frac{-3{,}792\,\phi\mu c_t L^2}{k\Delta t}\right).$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.25)$$

For a shut-in time sufficiently large that the logarithmic approximation is accurate for the Ei functions, the equation becomes

$$p_i - p_{ws} = 70.6\frac{qB\mu}{kh}\left[\ln\left(\frac{t_p+\Delta t}{\Delta t}\right)+\ln\left(\frac{t_p+\Delta t}{\Delta t}\right)\right]$$

$$=141.2\frac{qB\mu}{kh}\ln\left(\frac{t_p+\Delta t}{\Delta t}\right).$$

This can be written as

$$p_{ws} = p_i - 325.2\frac{qB\mu}{kh}\log[(t_p+\Delta t)/\Delta t].\quad\ldots(2.26)$$

Two observations can be made: (1) for a well near a single boundary, such as a sealing fault, Eq. 2.26 shows that the slope of a buildup curve will eventually double (compare Eq. 2.26 with Eq. 2.1) and (2) the time required for the slope to double can be long – specifically, $3{,}792\ \phi\mu c_t L^2/k\Delta t < 0.02$, or $\Delta t > 1.9\times10^5\ \phi\mu c_t L^2/k$. For large values of L or small values of permeability, shut-in time required for the logarithmic approximation to be valid can be longer than the time ordinarily available for a buildup test. For this reason, awaiting a doubling in slope on a buildup test is not necessarily a

satisfactory method of identifying a no-flow boundary near a well and estimating distance from the well to the boundary. Consequently, some analysts prefer to use Eq. 2.25 more directly, noting that for $t_p \gg \Delta t$ it may be rearranged as

$$p_{ws} = p_i - 162.6\frac{qB\mu}{kh}\left\{\log\left(\frac{t_p+\Delta t}{\Delta t}\right)\right.$$

$$\left.-0.434\,Ei\left(\frac{-3{,}792\,\phi\mu c_t L^2}{kt_p}\right)\right\}$$

$$-70.6\frac{qB\mu}{kh}Ei\left(\frac{-3{,}792\,\phi\mu c_t L^2}{k\Delta t}\right).\quad\ldots(2.27)$$

Reasons for arranging the equation in this form are as follows.

1. The term $162.6\ (qB\mu/kh)\{\log[(t_p+\Delta t)/\Delta t] - 0.434\ Ei(-3{,}792\ \phi\mu c_t L^2/kt_p)\}$ determines the position of the middle-time line. Note that the Ei function is a constant; thus, it affects only the position of the MTR and has no effect on slope.

2. At earliest shut-in times in a buildup test, $Ei(-3{,}792\ \phi\mu c_t L^2/k\Delta t)$ is negligible. Physically, this means that the radius of investigation has not yet encountered the no-flow boundary and, mathematically, that the late-time region in the buildup test has not yet begun.

These observations suggest a method for analyzing the buildup test (Fig. 2.20):

1. Plot p_{ws} vs. $\log(t_p+\Delta t)/\Delta t$.
2. Establish the middle-time region.

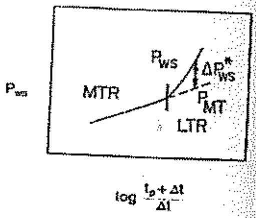


Fig. 2.20 – Buildup test graph for well near reservoir boundary.

drilled well. To confirm this fault and to estimate distance from it, we run a pressure buildup test. Data from the test are given in Table 2.6. Well and reservoir data include the following.

$\phi = 0.15,$
$\mu_o = 0.6$ cp,

3. Extrapolate the MTR into the LTR.
4. Tabulate the differences, Δp^*_{ws}, between the buildup curve and extrapolated MTR for several points $(\Delta p^*_{ws} = p_{ws} - p_{MT})$.
5. Estimate L from the relationship implied by Eq. 2.28:

$$\Delta p^*_{ws} = 70.6\frac{qB\mu}{kh}\left[-Ei\left(\frac{-3{,}792\,\phi\mu c_t L^2}{k\Delta t}\right)\right].$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.28)$$

L is the only unknown in this equation, so it can be solved directly. Remember, though, that accuracy of this equation requires that $\Delta t \ll t_{pi}$; when this condition is not satisfied, a computer history match using Eq. 2.25 in its complete form is required to determine L.

This calculation implied in Eq. 2.28 should be made for several values of Δt. If the apparent value of L tends to increase or to decrease systematically with time, there is a strong indication that the model does not describe the reservoir adequately (i.e., the well is not behaving as if it were in a reservoir of uniform thickness and porosity, and much nearer one boundary than any others).

The following example illustrates this computational technique.

Example 2.8 – Estimating Distance to a No-Flow Boundary

Problem. Geologists suspect a fault near a newly

$c_t = 17\times10^{-6}$ psi^{-1},
$r_w = 0.5$ ft,
$A_{wb} = 0.00545$ sq ft,
$\rho_o = 54.8$ lbm/cu ft,
$q_o = 1,221$ STB/D,
$B_o = 1.310$ RB/STB, and
$h = 8$ ft.

The well produced only oil and dissolved gas. Before shut-in, a total of 14,206 STB oil had been produced.

Analysis of these data show that afterflow distorts none of the data recorded at shut-in times of 1 hour or more. Based on the slope (650 psi/cycle) of the earliest straight line, permeability, k, appears to be 30 md. Depth of investigation at a shut-in time of 1 hour is 144 ft, lending confidence to the choice of the middle-time line. Pseudoproducing time, t_p, is 279.2 hours.

From these data, determine whether the buildup test data indicate that the well is behaving as if it were near a single fault and estimate distance to an apparent fault from buildup data at several times in the LTR.

Solution. Our attack is to plot p_{ws} vs. $(t_p+\Delta t)/\Delta t$; extrapolate the middle-time line on into the LTR; read pressures, p_{MT}, from this extrapolated line; subtract these pressures from observed values of p_{ws} in the LTR $(\Delta p^*_{ws} = p_{ws} - p_{MT})$; estimate values of L from Eq. 2.28; and assume that, if calculated values of L are fairly constant, the well is indeed near a single sealing fault.

From Fig. 2.21, we obtain the data in Table 2.7. We now estimate L from Eq. 2.28. Note that

$$\frac{3{,}792\,\phi\mu c_t}{k}=\frac{(3{,}792)(0.15)(0.6)(1.7\times10^{-5})}{30}$$

$$=1.934\times10^{-4}.$$

We first estimate L at $\Delta t=10$ hours, which assumes that the approximation $t_p \approx t_p+\Delta t$ is adequate in this case.

$$\Delta p_{ws}^{*}=52=-70.6\frac{qB\mu}{kh}\,Ei\left(\frac{-3{,}792\,\phi\mu c_t L^2}{k\Delta t}\right)$$

$$=\frac{-(70.6)(1{,}221)(1.310)(0.6)}{(30)(8)}\cdot Ei\left(\frac{-3{,}792\,\phi\mu c_t L^2}{k\Delta t}\right).$$

$$-Ei\left(\frac{-1.934\times10^{-4}L^2}{10}\right)=0.184.$$

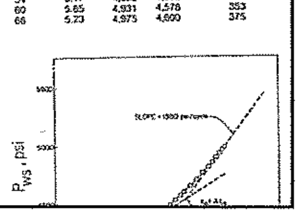
$$L^2=\frac{(1.107)(10)}{1.934\times10^{-4}}=5.72\times10^4,$$

or

$L=239$ ft.

TABLE 2.7 – ANALYSIS OF DATA FROM WELL NEAR BOUNDARY

Δt (hours)	$(t_p+\Delta t)/\Delta t$	p_{ws} (psi)	p_{wT} (psi)	$(p_{ws}-p_{wT})=\Delta p_{ws}^{*}$ (psi)
6	47.5	3,996	3,980	16
8	35.9	4,085	4,051	34
10	28.9	4,172	4,120	52
12	24.3	4,240	4,170	70
14	20.9	4,298	4,210	88
16	18.5	4,353	4,250	103
20	15.0	4,435	4,300	135
24	12.6	4,520	4,355	165
30	10.3	4,614	4,410	204
36	8.70	4,700	4,455	245
42	7.65	4,770	4,495	275
48	6.82	4,827	4,525	302
54	6.17	4,882	4,552	330
60	5.65	4,931	4,578	353
66	5.23	4,975	4,600	375



$$L^2=\frac{(1.107)(10)}{1.934\times10^{-4}}=5.72\times10^4,$$

or

$L=239$ ft.

For larger values of shut-in time, the approximation $t_p \approx t_p+\Delta t$ becomes decreasingly accurate, and no term in Eq. 2.25 can be neglected, but $L=240$ ft satisfies the equation for all values of shut-in time.

For the case in which the slope of the buildup test has time to double, estimation of distance from well to boundary is easier. From the buildup tests plot, we find the time, Δt_x, at which the two straight-line sections intersect (Fig. 2.22). Gray[14] suggests that the distance L from the well to the fault can be calculated from

$$L=\sqrt{\frac{0.000148\,k\Delta t_x}{\phi\mu c_t}}.\ \ \ (2.29)$$

In Fig. 2.21, the slope did double, and the figure shows that $(t_p+\Delta t_x)/\Delta t_x=17$, from which $\Delta t_x=17.45$ hours. Eq. 2.29 then shows that $L=225$ ft, in reasonable agreement with our previous calculation.

The results of pressure buildup tests sometimes can be used to estimate reservoir size. The basic idea is to compare average static reservoir pressure before and after production of a known quantity of fluid from a closed, volumetric reservoir, with constant compressibility, c_t. If V_R is the reservoir volume (barrels), ΔN_p is the stock-tank barrels of oil produced between Times 1 and 2, and $\bar p_1$ and $\bar p_2$ are the average reservoir pressures before and after oil production, then a material balance on the reservoir shows that

$$\bar p_2=\bar p_1-\frac{(\Delta N_p)(B_o)}{V_R c_t\phi}.$$

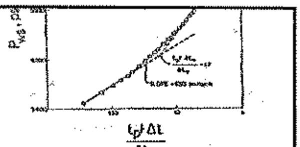


Fig. 2.21 – Estimating distance to a no-flow boundary.

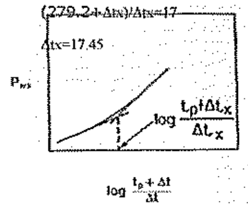


Fig. 2.22 – Distance to boundary from slope doubling.

or

$$V_R=\frac{(\Delta N_p)(B_o)}{(\bar p_1-\bar p_2)c_t\phi}.\qquad c_t=\frac{1}{V_R}\frac{dV}{dP}=\frac{\Delta N_p B_o}{V_R\phi(\bar P_1-\bar P_2)}$$

Example 2.9 – Estimating Reservoir Size

Problem. Two pressure buildup tests are run on the only well in a closed reservoir. The first test indicates an average pressure of 3,000 psi, the second indicates 2,100 psi. The well produced an average of 150 STB/D of oil in the year between tests. Average oil formation volume factor, B_o, is 1.3 RB/STB; total compressibility, c_t, is 10×10^{-6} psi^{-1}; porosity, ϕ, is 22%; and average sand thickness, h, is 10 ft. Estimate area, A_R, of the reservoir in acres.

Solution. From Eq. 2.30,

$$V_R=\frac{\Delta N_p B_o}{(\bar p_1-\bar p_2)c_t\phi}$$

$$=\frac{(qt)B_o}{(\bar p_1-\bar p_2)(c_t\phi)}$$

$$=\frac{(150\text{ STB/D})(365\text{ days})(1.3\text{ RB/STB})}{(3{,}000-2{,}100)\text{psi}(10\times10^{-6}\text{ psi}^{-1})(0.22)}.$$

Thus,

$V_R=35.9\times10^6$ bbl

pressed in thousands of standard cubic feet per day (Mscf/D), and gas formation volume factor, B_g, is then expressed in reservoir barrels per thousand standard cubic feet (RB/Mscf), so that the product q_gB_g is in reservoir barrels per day (RB/D) as in the analogous equation for slightly compressible liquids.

2. All gas properties (B_g, μ, and c_t) are evaluated at original reservoir pressure, p_i. (More generally, these properties should be evaluated at the uniform pressure in the reservoir before initiation of flow.) In Eq. 2.31,

1000/5.615=178.1

$$B_{gi}=\frac{178.1\,z_iTp_{sc}}{p_iT_{sc}}\ \text{(RB/Mscf)},$$

$$c_{ti}=c_{gi}S_g+c_wS_w+c_f\approx c_{gi}S_g.$$

3. The factor D is a measure of non-Darcy or turbulent pressure loss (i.e., a pressure drop in addition to that predicted by Darcy's law). It cannot be calculated separately from the skin factor from a single buildup or drawdown test; thus, the concept of apparent skin factor, $s'=s+Dq_g$, is sometimes convenient since it can be determined from a single test.

For many cases at pressures below 2,000 psi, flow in an infinite acting reservoir can be modeled by

P<2000 psi

$$p_{wf}^2=p_i^2+\frac{1{,}637\,q_g\mu_iz_iT}{kh}\left[\log\left(\frac{1{,}688\,\phi\mu c_{ti}r_w^2}{kt_p}\right)-\frac{(s+Dq_g)}{1.151}\right].\ \ (2.32)$$

$$=\frac{43{,}560\,A_Rh}{5.615},$$

and

$$A_R=\frac{(35.9\times10^6\text{ bbl})(5.615\text{ cu ft/bbl})}{(10\text{ ft})(43.56\times10^3\text{ sq ft/acre})}$$

$=463$ acres.

2.11 Modifications for Gases

This section presents modifications of the basic drawdown and buildup equations so that they can be applied to analysis of gas reservoirs. These modifications are based on results obtained with the gas pseudopressure,[15] although a more complete discussion of that subject is left to Chap. 5.

Wattenbarger and Ramey[16] have shown that for some gases at pressures above 3,000 psi, flow in an infinite-acting reservoir can be modeled accurately by the equation

$$p_{wf}=p_i+\frac{162.6\,q_gB_{gi}\mu_i}{kh}\left[\log\left(\frac{1{,}688\,\phi\mu_ic_{ti}r_w^2}{kt_p}\right)-\frac{(s+Dq_g)}{1.151}\right].\ \ (2.31)$$

This equation has the same form as the equation for a slightly compressible liquid, but there are some important differences:

1. Gas production rate, q_g, is conveniently ex-

Using these basic drawdown equations, we can use superposition to develop equations describing a buildup test for gas wells.

For $p>3{,}000$ psi, P>3000 psi

$$p_{ws}=p_i-162.6\frac{q_gB_{gi}\mu_i}{kh}\left[\log\left(\frac{t_p+\Delta t}{\Delta t}\right)\right],\ \ (2.33)$$

and

$$s'=s+D(q_g)=1.151\left[\frac{(p_{1hr}-p_{wf})}{m}-\log\left(\frac{k}{\phi\mu_ic_{ti}r_w^2}\right)+3.23\right],\ \ (2.34)$$

For $p<2{,}000$ psi, P<2000 psi

$$p_{ws}^2=p_i^2-1{,}637\frac{q_g\mu_iz_iT}{kh}\log\left(\frac{t_p+\Delta t}{\Delta t}\right),\ \ (2.35)$$

and

$$s'=s+D(q_g)=1.151\left[\frac{(p_{1hr}^2-p_{wf}^2)}{m'}-\log\left(\frac{k}{\phi\mu_ic_{ti}r_w^2}\right)+3.23\right],\ \ (2.36)$$

where m' is the slope of the plot p_{ws}^2 vs. $\log[(t_p+\Delta t)/\Delta t]$, which is $1{,}637\,q_g\mu_iz_iT/kh$.

An obvious question is, what technique should be used to analyze gas reservoirs with pressures in the range $2{,}000<p<3{,}000$ psi? One approach is to use equations written in terms of the gas pseudopressure instead of either pressure or pressure squared. This is at least somewhat inconvenient, so an alternative approach is to use equations written in terms of either p_{ws} or p_{ws}^2 and accept the resultant inaccuracies, which, in real, heterogeneous reservoirs, may be far from the most significant oversimplification on which the test analysis procedure is based. The smaller the pressure drawdown during the test, the less the inaccuracy in this approach.

2000<P<3000 psi use pseudopressure

Example 2.10 – Gas Well Buildup Test Analysis

Problem. A gas well is shut in for a pressure buildup test. Test data include the following.

q_g = 5,256 Mscf/D,
T = 181°F = 641°R,
h = 28 ft,
μ_i = 0.028 cp,
S_w = 0.3,
ϕ = 0.18,
z_i = 0.85,
r_w = 0.3 ft,
c_{gi} = 0.344×10^{-3} psi^{-1},
p_i = 2,906 psia, and
p_{wf} = 1,801 psia.

Most of the test data fall in the intermediate pressure range, $2{,}000<p<3{,}000$ psia. On a plot of p_{ws} vs. log

$$c_{ti}\approx c_{gi}S_g=(3.44\times10^{-4})(0.7)$$

$$=2.41\times10^{-4}\text{ psi}^{-1},$$

and

$$s'=s+D(q_g)=1.151\left[\frac{(p_{1hr}-p_{wf})}{m}-\log\left(\frac{k}{\phi\mu_ic_{ti}r_w^2}\right)+3.23\right]$$

$$=1.151\left\{\frac{(2{,}525-1{,}801)}{81}-\log\left[\frac{9.96}{(0.18)(0.028)(2.41\times10^{-4})(0.3)^2}\right]+3.23\right\}=4.84.$$

From results of the p_{ws}^2 plot,

$$k=1{,}637\frac{q_g\mu_iz_iT}{m'h}$$

$$=\frac{(1{,}637)(5{,}256)(0.028)(0.85)(641)}{(4.8\times10^5)(28)}$$

$=9.77$ md,

and

$c_{gi}=0.344\times10^{-3}$ psi^{-1},
p_i = 2,906 psia, and
p_{wf} = 1,801 psia.

Most of the test data fall in the intermediate pressure range, $2{,}000<p<3{,}000$ psia. On a plot of p_{ws} vs. log $(t_p+\Delta t)/\Delta t$, the MTR had a slope, m, of 81 psi/cycle; on this plot, p_{1hr} was found to be 2,525 psia. Alternatively, on a plot of p_{ws}^2 vs. log $(t_p+\Delta t)/\Delta t$, the MTR had a slope of 0.48×10^6 psi^2/cycle and p_{1hr}^2 of 7.29×10^6 psi^2.

From these data, estimate apparent values of k and s' (1) based on characteristics of the p_{ws} plot and (2) based on characteristics of the p_{ws}^2 plot.

Solution. From results of the p_{ws} plot, for standard conditions of 14.7 psia and 60°F,

$$B_{gi}=178.1\frac{z_iTp_{sc}}{p_iT_{sc}}$$

$$=\frac{(178.1)(0.85)(641)(14.7)}{(2{,}906)(520)}$$

$=0.944$ RB/Mscf,

$$k=162.6\frac{q_gB_{gi}\mu_i}{mh}$$

$$=\frac{(162.6)(5{,}256)(0.944)(0.028)}{(81)(28)}$$

$=9.96$ md,

$$(4.8\times10^5)(28)$$

$=9.77$ md,

and

$$s'=1.151\left[\frac{(p_{1hr}^2-p_{wf}^2)}{m'}-\log\frac{k}{\phi\mu_ic_{ti}r_w^2}+3.23\right]$$

$$=1.151\left\{\frac{[7.29\times10^6-(1{,}801)^2]}{4.8\times10^5}-\log\left[\frac{9.77}{(0.18)(0.028)(2.41\times10^{-4})(0.3)^2}\right]+3.23\right\}$$

$=4.27.$

Neither set of results (k and s') is necessarily more accurate than the other in the *general* case; as in this *particular* case, use of an analysis procedure based on gas pseudopressure can be used to improve accuracy if disagreement in results from p_{ws} and p_{ws}^2 plots is unacceptably large.

2.12 Modifications for Multiphase Flow

Basic buildup and drawdown equations can be modified to model multiphase flow.[17,18] For an infinite-acting reservoir, the drawdown equation becomes

$$p_{wf}=p_i+162.6\frac{q_{Rt}}{\lambda_th}\left[\log\left(\frac{1{,}688\,\phi c_tr_w^2}{\lambda_tt}\right)-\frac{s}{1.151}\right].\ \ (2.37)$$

and the buildup equation becomes

$$p_{ws} = p_i - 162.6 \frac{q_{Rt}}{\lambda_t h} \log\left(\frac{t_p+\Delta t}{\Delta t}\right). \quad \ldots\ldots (2.38)$$

In these equations, the total flow rate q_{Rt} is in reservoir barrels per day (neglecting solution gas liberated from produced water).

$$q_{Rt} = q_o B_o + \left(q_g - \frac{q_o R_s}{1,000}\right) B_g + q_w B_w, \quad \ldots\ldots (2.39)$$

and total mobility, λ_t, is

$$\lambda_t = \frac{k_o}{\mu_o} + \frac{k_w}{\mu_w} + \frac{k_g}{\mu_g}. \quad \ldots\ldots (2.40)$$

Total compressibility, c_t, was defined in Eq. 1.4.

These equations imply that it is possible to determine λ_t from the slope m of a buildup test run on a well that produces two or three phases simultaneously:

$$\lambda_t = 162.6 \frac{q_{Rt}}{mh}. \quad \ldots\ldots (2.41)$$

Perrine[17] has shown that it is also possible to estimate the permeability to *each phase* flowing from the same slope, m:

$$k_o = 162.6 \frac{q_o B_o \mu_o}{mh}, \quad \ldots\ldots (2.42)$$

$$k_g = 162.6 \frac{\left(q_g - \frac{q_o R_s}{1,000}\right) B_g \mu_g}{mh}, \quad \ldots\ldots (2.43)$$

Example 2.12 – Multiphase Buildup Test Analysis

Problem. A buildup test is run in a well that produces oil, water, and gas simultaneously. Well, rock, and fluid properties evaluated at average reservoir pressure during the test include the following.

S_o = 0.58,
S_g = 0.08,
S_w = 0.34,
c_w = 3.6×10^{-6} psi^{-1},
c_f = 3.5×10^{-6} psi^{-1},
c_g = 0.39×10^{-3} psi^{-1},
μ_o = 1.5 cp,
μ_w = 0.7 cp,
μ_g = 0.03 cp,
B_o = 1.3 RB/STB,
B_w = 1.02 RB/STB,
B_g = 1.480 RB/Mscf,
R_s = 685 scf/STB,
ϕ = 0.17,
r_w = 0.3 ft, and
h = 38 ft.

From plots of B_o vs. p and R_s vs. p at average pressure in the buildup test,

$$\frac{dR_s}{dp} = 0.0776 \text{ scf/STB/psi},$$

and

$$\frac{dB_o}{dp} = 2.48\times10^{-6} \text{ RB/STB/psi}.$$

$$k_g = 162.6 \frac{\left(q_g - \frac{q_o R_s}{1,000}\right) B_g \mu_g}{mh}, \quad \ldots\ldots (2.43)$$

and

$$k_w = 162.6 \frac{q_w B_w \mu_w}{mh}. \quad \ldots\ldots (2.44)$$

The term $(q_g - q_o R_s/1,000)B_g$, which appears in Eqs. 2.39 and 2.43, is the *free* gas flow rate in the reservoir. It is found by subtracting the dissolved gas rate $(q_o R_s/1,000)$ from the total surface gas rate (q_g) and converting to a reservoir-condition basis.

Simultaneous solution of Eqs. 2.37 and 2.38 results in the following expression for the skin factor s.

$$s = 1.151\left[\frac{p_{1\,hr} - p_{wf}}{m} - \log\left(\frac{\lambda_t}{\phi c_t r_w^2}\right) + 3.23\right]. \quad \ldots\ldots (2.45)$$

Static drainage-area pressure, $\bar{p}$, is calculated just as for a single-phase reservoir. In use of the MBH charts to determine $\bar{p}$ (and in the Horner plot itself), the effective production time t_p is best estimated by dividing cumulative *oil* production by the *oil* production rate just before shut-in.

An important assumption required for accurate use of these equations for multiphase flow analysis is that saturations of each phase remain essentially uniform throughout the drainage area of the tested well.

$$\frac{dR_s}{dp} = 0.0776 \text{ scf/STB/psi},$$

and

$$\frac{dB_o}{dp} = 2.48\times10^{-6} \text{ RB/STB/psi}.$$

The production rates prior to the buildup test were q_o = 245 STB/D, q_w = 38 STB/D, and q_g = 489 Mscf/D.

A plot of p_{ws} vs. $\log(t_p+\Delta t)/\Delta t$ shows that the slope of the MTR, m, is 78 psi/cycle and that $p_{1\,hr}$ = 2,466 psia. Flowing pressure, p_{wf}, at the instant of shut-in was 2,028 psia.

From these data, estimate λ_t, k_o, k_w, k_g, and s.

Solution. Permeabilities to each phase can be determined from the slope m of the MTR:

$$k_o = 162.6 \frac{q_o B_o \mu_o}{mh}$$

$$= \frac{(162.6)(245)(1.5)(1.3)}{(78)(38)} = 26.2 \text{ md}.$$

$$k_w = 162.6 \frac{q_w B_w \mu_w}{mh}$$

$$= \frac{(162.6)(38)(0.7)(1.02)}{(78)(38)} = 1.49 \text{ md}.$$

$$k_g = 162.6 \frac{\left(q_g - \frac{q_o R_s}{1,000}\right) B_g \mu_g}{mh}$$

$$= 162.6 \frac{\left[489 - \frac{(245)(685)}{(1,000)}\right](1.480)(0.03)}{(78)(38)}$$

$$= 0.782 \text{ md}.$$

To calculate total mobility, λ_t, we first need total flow rate, q_{Rt}:

$$q_{Rt} = q_o B_o + \left(q_g - \frac{q_o R_s}{1,000}\right) B_g + q_w B_w$$

$$= (245)(1.3) + \left[489 - \frac{(245)(685)}{1,000}\right]$$

$$\cdot (1.480) + (38)(1.02)$$

$$= 833 \text{ RB/D}.$$

Then,

$$\lambda_t = 162.6 \frac{q_{Rt}}{mh} = \frac{(162.6)(833)}{(78)(38)}$$

$$= 45.7 \text{ md/cp}.$$

To calculate skin factor, s, we first need c_o and c_t:

$$c_o = \frac{B_g}{B_o}\frac{dR_s}{dp} - \frac{1}{B_o}\frac{dB_o}{dp}$$

Exercises

2.1. In Example 2.1, what error arises because […] used Eq. 2.4 to calculate skin factor instead of […] more exact Eq. 2.3? What difference would it […] made in the value of s had we used a shut-in tim[e …] 10 hours in Eq. 2.3 and the corresponding valu[e …] p_{ws}? What assumption have we made about dista[nce] from tested well to reservoir boundaries in Exam[ple] 2.1?

2.2. Prove that the slope of a plot of shut-in B[…] vs. $\log (t_p+\Delta t)/\Delta t$ is, as asserted in the text, […] difference in pressure at two points one cycle ap[art]. Also prove that, for $\Delta t \ll t_p$, we obtain the same s[lope] on a plot of p_{ws} vs. $\log \Delta t$. Finally, prove that […] plot of p_{ws} vs. $\log \Delta t$, we obtain the same s[lope] regardless of the units used for shut-in time, Δt […] the plot (i.e., that Δt can be expressed in minu[tes], hours, or days without affecting the slope of […] plot).

2.3. A well producing only oil and dissolved […] has produced 12,173 STB. The well has not b[een] stimulated, nor is there any reason to believe […] there is a significant amount of formation damag[e …] pressure buildup test is run with the primary [ob]jective of estimating static drainage-area press[ure]. During buildup, there is a *rising* liquid level in […] wellbore. Well and reservoir data are:

ϕ = 0.14,
μ = 0.55 cp,
c_t = 16×10^{-6} psi^{-1},

$$\frac{B_g}{B_o}\frac{dR_s}{dp} \quad \frac{1}{B_o}\frac{dB_o}{dp}$$

$$= \frac{(1.480 \text{ RB/Mscf})(0.0776 \text{ scf})}{(1.3 \text{ RB/STB})(\text{STB-psi})}$$

$$\cdot \frac{1 \text{ Mscf}}{1,000 \text{ scf}} - \frac{(2.48\times10^{-6})}{1.3 \text{ (psi)}}$$

$$= 86.4\times10^{-6} \text{ psi}^{-1}.$$

Then,

$$c_t = S_o c_o + S_g c_g + S_w c_w + c_f$$

$$= (0.58)(86.4\times10^{-6}) + (0.08)(0.39\times10^{-3})$$

$$+ (0.34)(3.6\times10^{-6}) + 3.5\times10^{-6}$$

$$= 86.0\times10^{-6} \text{ psi}^{-1},$$

and

$$s = 1.151\left[\frac{p_{1\,hr} - p_{wf}}{m} - \log\left(\frac{\lambda_t}{\phi c_t r_w^2}\right) + 3.23\right]$$

$$= 1.151\left[\frac{2,466 - 2,028}{78} - \log\left[\frac{45.7}{(0.17)(86.0\times10^{-6})(0.3)^2}\right] + 3.23\right]$$

$$= 1.50.$$

c_t = 16×10^{-6} psi^{-1},
r_w = 0.5 ft,
A_{wb} = 0.0218 sq ft,
r_e = 1,320 ft (well centered in cylindrical drainage area),
ρ = 54.8 lbm/cu ft,
q = 988 STB/D,
B = 1.126 RB/STB, and
h = 7 ft.

Data recorded during the buildup test are given in Table 2.8. Plot p_{ws} vs. $(t_p+\Delta t)/\Delta t$ on semilog paper and $(p_{ws} - p_{wf})$ vs. Δt_e on log-log paper, and estimate the time at which afterflow ceased distorting the buildup test data.

2.4. Consider the buildup test described in Problem 2.3. Locate the MTR and estimate formation permeability.

2.5. Consider the buildup test described in Problems 2.3 and 2.4. Calculate skin factor, s; pressure drop across the altered zone, $(\Delta p)_s$; flow efficiency, E; and effective wellbore radius, r_{wa}.

2.6. Prove that in a buildup test for a well near a single fault, the technique suggested in the text (extrapolating the late-time line to infinite shut-in time) is the proper method for estimating original reservoir pressure. Comment on the possible errors in original reservoir pressure estimates in these cases: (1) some LTR data were obtained, but final straight line was not established; and (2) no LTR data were obtained.

2.7. Consider the buildup test described in Problems 2.3 and 2.4. Estimate static drainage-area pressure for this well (1) using the p^* method, and (2)

TABLE 2.8 – PRESSURE BUILDUP TEST DATA

Δt (hours)	p_{ws} (psia)	Δt (hours)	p_{ws} (psia)
0	709	19.7	4,198
1.97	3,169	24.6	4,245
2.95	3,508	29.6	4,279
3.94	3,672	34.5	4,306
4.92	3,772	39.4	4,327
5.91	3,873	44.4	4,343
7.88	3,963	49.3	4,356
9.85	4,026	59.1	4,375
14.8	4,133		

using the modified Muskat method.

2.8. In Example 2.7, explain how we could have applied the modified Muskat method to estimate static drainage-area pressure if we had not had estimates of $k/\phi\mu c_t$ or r_e.

2.9. Estimate formation permeability and skin factor from the following data available from a gas well pressure buildup test.

T = 199°F = 659°R,
h = 34 ft,
μ_i = 0.023 cp,
S_w = 0.33 (water is immobile),
c_{gi} = 0.000315 psi^{-1},
ϕ = 0.22,
z_i = 0.87, and
r_w = 0.3 ft.

The well produced 6,068 Mcf/D before the test. A plot of BHP, p_{ws}, vs. $\log (t_p+\Delta t)/\Delta t$ gave a middle-time line with a slope of 66 psi/cycle. Analysis of the buildup curve showed that static drainage-area pressure, $\bar{p}$, was 3,171 psia. Pressure on the middle-time line at Δt = 1 hour, $p_{1\,hr}$, was 2,745 psia; flowing pressure at shut-in, p_{wf}, was 2,486 psia.

TABLE 2.9 – BUILDUP TEST DATA FOR WELL NEAR FAULT

Δt (hours)	p_{ws} (psi)	Δt (hours)	p_{ws} (psi)
20	1,373	300	2,225
30	1,467	500	2,360
40	1,533	1,000	2,434
50	1,585	1,500	2,543
100	1,752	2,000	2,616
200	1,940		

2.10. Estimate total mobility, λ_t, oil, water and gas permeabilities, and skin factor for a well that produced oil, water, and gas simultaneously before a pressure buildup test. Production rates before the test were q_o = 276 STB/D, q_w = 68 STB/D, and q_g = 689 Mcf/D. A plot of p_{ws} vs. $\log (t_p+\Delta t)/\Delta t$ showed that the slope m of the middle-time line was 59 psi/cycle. Flowing pressure at shut-in, p_{wf}, was 1,581 psia; the pressure on the middle-time straight line at Δt = 1 hour, $p_{1\,hr}$, was 1,744 psia. Plots of B_o vs. p and R_s vs. p showed that dR_s/dp = 0.263 scf/STB/psi and that dB_o/dp = 0.248×10^{-3} RB/STB/psi. Rock, fluid, and well properties include the following.

S_o = 0.56,
S_g = 0.09,
S_w = 0.35,
c_w = 3.5×10^{-6} psi^{-1},
c_f = 3.5×10^{-6} psi^{-1},
c_g = 0.48×10^{-3} psi^{-1},
μ_o = 1.1 cp,
μ_w = 0.6 cp,
μ_g = 0.026 cp,
B_o = 1.28 RB/STB,
B_w = 1.022 RB/STB,
B_g = 1.122 RB/Mscf,

R_s = 748 scf/STB,
ϕ = 0.18,
r_w = 0.3 ft, and
h = 33 ft.

2.11. A pressure buildup test was run on an oil well believed to be near a sealing fault in an otherwise infinite-acting reservoir. Estimate the distance to the fault, given the well, rock, and fluid properties below and the buildup data in Table 2.9.

q = 940 STB/D,
μ = 50 cp,
ϕ = 0.2,
c_t = 78×10^{-6} psi^{-1},
h = 195 ft,
p_i = 2,945 psi,
N_p = 84,500 STB,
B_o = 1.11 RB/STB, and
t_{wbs} < 20 hours.

2.12. A well flowed for 10 days at 350 STB/D; it was then shut in for a pressure buildup test. Rock, fluid, and well properties include the following.

B_o = 1.13 RB/STB,
p_i = 3,000 psi,
μ = 0.5 cp,
k = 25 md,
s = 0,
h = 50 ft,
c_t = 20×10^{-6} psi^{-1},
ϕ = 0.16, and
r_w = 0.333 ft.

(a) Determine and plot the pressure distribution in the reservoir for shut-in times of 0, 0.1, 1, and 10 days. (Assume an infinite acting reservoir.)

(b) Calculate the radius of investigation at 0.1, 1, and 10 days. Compare r_i with the depth to which the transient appears to have moved on the plots prepared in Part a.

2.13. In Example 2.6, $\bar{p}$ was determined to be 4,411 psi. Both the Horner plot and the abscissa of the MBH chart used t_p = 13,630 hours. It can be shown that for a well centered in a square drainage area, the time required to reach semisteady state is $t_{pss} = (\phi\mu c_t A/0.000264\ k)(t_{DA})_{pss}$ and that $(t_{DA})_{pss}$ = 0.1. Show that if t_{pss} is used instead of t_p in both the Horner plot and in the abscissa of the MBH chart, the resulting estimate of $\bar{p}$ is essentially unchanged. Buildup data (from the MTR only) are given in Table 2.10. Other data include:

TABLE 2.10 – MTR DATA FROM BUILDUP TEST

Δt (hours)	p_{ws} (psi)
8	4,354
12	4,360
16	4,376
20	4,382
24	4,388

TABLE 2.11 – PRESSURE BUILDUP TEST DATA

Δt (hours)	p_{ws} (psia)	Δt (hours)	p_{ws} (psia)
0	2,752	10	4,272
0.3	3,464	12	4,280
0.5	3,640	14	4,287
1	3,852	16	4,297
2	4,055	20	4,303
3	4,153	24	4,308
4	4,207	30	4,313
5	4,244	36	4,317
6	4,251	42	4,320
8	4,263	50	4,322

q = 250 STB/D,
B = 1.136 RB/STB,
μ = 0.8 cp,
h = 69 ft,
ϕ = 0.039,
c_t = 17×10^{-6} psi^{-1},
r_e = 1,320 ft, and
k = 7.65 md.

2.14. A well producing only oil and dissolved gas has produced 13,220 STB. To characterize the severe damage believed present, the well is shut in for a buildup test. Well and reservoir data are given below.

ϕ = 0.17,
μ = 0.6 cp,
c_t = 18×10^{-6} psi^{-1},
r_e = 1,320 ft, well centered in square drainage area (160 acres),
r_w = 0.5 ft,
A_{wb} = 0.016 sq ft

presented at the SPE-AIME 49th Annual Fall Meeting, Houston, Oct. 6-9, 1974. An abridged version appears in J. Pet. Tech. (Aug. 1975) 991-996; Trans., AIME, 259.
3. Miller, C.C., Dyes, A.B., and Hutchinson, C.A. Jr.: "Estimation of Permeability and Reservoir Pressure From Bottom-Hole Pressure Build-Up Characteristics," Trans., AIME (1950) 189, 91-104.
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8. Matthews, C.S., Brons, F., and Hazebroek, P.: "A Method for Determination of Average Pressure in a Bounded Reservoir," Trans., AIME (1954) 201, 182-191.

WELL TESTING

TABLE 3.10 – DATA FOR EXAMPLE TWO-RATE FLOW TEST

t (hours)	p_{wf} at 600 STB/D (psi)	p_{wf} at 300 STB/D (psi)	t (hours)	p_{wf} at 600 STB/D (psi)	p_{wf} at 300 STB/D (psi)
0	5,000	3,633	3.03	4,012	4,324
0.114	4,710	3,978	3.64	4,002	4,330
0.136	4,665	4,000	4.37	3,992	4,334
0.164	4,615	4,025	5.24	3,982	4,339
0.197	4,563	4,051	6.29	3,972	4,343
0.236	4,507	4,079	7.54	3,963	4,346
0.283	4,449	4,108	9.05	3,953	4,350
0.340	4,390	4,138	10.9	3,944	4,353
0.408	4,332	4,167	13.0	3,935	4,356
0.490	4,277	4,194	15.6	3,926	4,359
0.587	4,227	4,219	18.8	3,918	4,361
0.705	4,182	4,242	22.5	3,909	4,363
0.846	4,144	4,261	27.0	3,900	4,364
1.02	4,112	4,278	32.4	3,891	4,365
1.22	4,087	4,289	38.9	3,883	4,365
1.46	4,067	4,299	46.7	3,874	4,364
1.75	4,050	4,307	56.1	3,865	4,362
2.11	4,038	4,314	67.3	3,855	4,359
2.53	4,024	4,319	80.7	3,845	4,354
			96.9	3,833	4,348

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434 619 ADV. RESERVOIR ENGINEERING
4(4-0-8)-@ 2011(2/2554)
HW NO 6.1; Due date: Friday, November 4, 2011.

Pressure Draw Down

Do the Exercises 3.1 and 14

Exercises

3.1. A constant-rate drawdown test was run in a well with the following characteristics:

q = 500 STB/D (constant),
ϕ = 0.2,
μ = 0.8 cp,
c_t = 10×10^{-6} psi^{-1},
r_w = 0.3 ft,
h = 56 ft,
B_o = 1.2 RB/STB,
A_{wb} = 0.022 sq ft,
ρ = 50 lb/cu ft, and
liquid/gas interface is in well.

From the test data in Table 3.6, estimate formation permeability, skin factor, and area (in acres) drained by the well.

3.2. A drawdown test in which the rate decreased continuously throughout the test was run in a well with the following characteristics:

ϕ = 0.2,
μ = 1.0 cp,
c_t = 18×10^{-6} psi^{-1},
r_w = 0.25 ft,
h = 100 ft,
B_o = 1.1 RB/STB,
A_{wb} = 0.0218 sq ft,
ρ = 55 lb/cu ft, and
liquid/gas interface is in well.

From the test data in Table 3.9, estimate formation permeability and skin factor.

3.3. A constant-rate drawdown test was run on the well described in Problem 3.2. Rate was held constant at 600 STB/D. After 96.9 hours, the surface rate was changed abruptly to 300 STB/D. Data for the time before and after the rate change are given in Table 3.10. From the data obtained with q = 600 STB/D, estimate k, s, and area of the reservoir. From the data obtained in the two-rate flow test after q changed to 300 STB/D, confirm the estimates of k and s, and calculate the current value of p*.

14. For the multirate flow test described in Example 3.6, (a) calculate the value of the ...

TABLE 3.6 – DATA FOR EXAMPLE CONSTANT-RATE DRAWDOWN TEST

TABLE 3.9 – DATA FOR EXAMPLE VARIABLE-RATE DRAWDOWN TEST

434 619 ADV. RESERVOIR ENGINEERING
4(4-0-8) @ 2012(2/2555)
HW NO 6.1; Due date: Friday, November 2, 2012.

3.1. A constant-rate drawdown test was run in a well with the following characteristics:

Pressure Draw Down

Do the Exercises 3.1 and 14

q = 500 STB/D (constant),
ϕ = 0.2,
μ = 0.8 cp,
c_t = 10×10^{-6} psi^{-1},
r_w = 0.3 ft,
h = 56 ft,
B_o = 1.2 RB/STB,
A_{wb} = 0.022 sq ft,
ρ = 50 lb/cu ft, and
liquid/gas interface is in well.

From the test data in Table 3.6, estimate formation permeability, skin factor, and area (in acres) drained by the well.

results as given below:

t (hr)	p_{wf} (psi)
1.50	2978
3.75	2949
7.50	2927
15.00	2904
37.50	2876
56.25	2863
75.00	2848
112.50	2810
150.00	2790
225.00	2763

TABLE 3.6 – DATA FOR EXAMPLE CONSTANT-RATE DRAWDOWN TEST

t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)
0	3,000	0.491	2,302	32.8	1,543
0.0109	2,976	0.546	2,256	38.2	1,533
0.0164	2,964	1.09	1,952	43.7	1,525
0.0218	2,953	1.65	1,828	49.1	1,517
0.0273	2,942	2.18	1,768	54.6	1,511
0.0328	2,930	2.73	1,724	65.5	1,500
0.0382	2,919	3.28	1,713	87.4	1,487
0.0437	2,908	3.82	1,696	109.2	1,468
0.0491	2,897	4.37	1,684	163.8	1,440
0.0546	2,886	4.91	1,674	218.4	1,416
0.109	2,785	5.46	1,665	273.0	1,393
0.164	2,693	6.55	1,651	327.6	1,373
0.218	2,611	8.74	1,630		
0.273	2,536	10.9	1,587		
0.328	2,469	16.4	1,569		
0.437	2,352	27.3	1,564		

Given:

p_i = 3400 psi, h = 25 ft, Q = 300 STB/day
c_t = 18×10^{-6} psi^{-1}, μ_o = 1.8 cp,
B_o = 1.1 bbl/STB, r_w = 0.25 ft, ϕ = 12%,

and assuming no wellbore storage, calculate:

(a) the average permeability;
(b) the skin factor.

2.3. A well producing only oil and dissolved gas has produced 12,173 STB. The well has not been stimulated, nor is there any reason to believe that there is a significant amount of formation damage. A pressure buildup test is run with the primary objective of estimating static drainage-area pressure. During buildup, there is a rising liquid level in the wellbore. Well and reservoir data are:

ϕ = 0.14,
μ = 0.55 cp,
c_t = 16×10^{-6} psi^{-1},
r_w = 0.5 ft,
A_{wb} = 0.0218 sq ft,
r_e = 1,320 ft (well centered in cylindrical drainage area),
ρ = 54.8 lbm/cu ft,
q = 988 STB/D,
B = 1.126 RB/STB, and
h = 7 ft.

Data recorded during the buildup test are given in Table 2.8. Plot p_{ws} vs. $(t_p + \Delta t)/\Delta t$ on semilog paper and $(p_{ws} - p_{wf})$ vs. Δt, on log-log paper, and estimate the time at which afterflow ceased distorting the buildup test data.

2.4. Consider the buildup test described in Problem 2.3. Locate the MTR and estimate formation permeability.

2.5. Consider the buildup test described in Problems 2.3 and 2.4. Calculate skin factor, s, pressure drop across the altered zone, $(\Delta p)_s$, flow efficiency, E, and effective wellbore radius, r_{wa}.

TABLE 2.8 – PRESSURE BUILDUP TEST DATA

Δt (hours)	p_{ws} (psia)	Δt (hours)	p_{ws} (psia)
0	709	19.7	4,198
1.97	3,169	24.6	4,245
2.95	3,508	29.6	4,279
3.94	3,672	34.5	4,306
4.92	3,772	39.4	4,327
5.91	3,873	44.4	4,343
7.88	3,963	49.3	4,356
9.86	4,026	59.1	4,376
14.8	4,133		

16. A pressure buildup test was conducted on a well that had been producing at 146 STB/day for 53 hours.

B_o = 1.29 bbl/STB, μ_o = 0.85 cp,
c_t = 12×10^{-6} psi^{-1}, ϕ = 10%, p_{wf} = 1426.9 psig
A = 20 acres

The buildup data is as follows.

Time	p_{ws} (psig)
0.167	1451.5
0.333	1476.0
0.500	1498.6
0.667	1520.1
0.833	1541.5
1.000	1561.3
1.167	1581.9
1.333	1599.7
1.500	1617.9
1.667	1635.3
2.000	1665.7
2.333	1691.8
2.667	1715.3
3.000	1736.3
3.333	1754.7
3.667	1770.1
4.000	1785.0
4.500	1800.7
5.000	1812.8
5.500	1822.4
6.000	1830.7
6.500	1837.2
7.000	1841.1
7.500	1844.5
8.000	1846.7
8.500	1849.6
9.000	1850.4
10.000	1852.7
11.000	1853.5
12.000	1854.0
12.667	1854.0
14.620	1855.0

Calculate:

(a) the average reservoir pressure;
(b) the skin factor;
(c) the formation capacity;
(d) an estimate of the drainage area and compare the given value.

Pressure Build Up

Do the Exercises 2.5 and 16
Due 9 Nov.2012

Pressure Build-Up

Do the Exercises 2.5, and 16

12/04/57

TABLE 2.8 – PRESSURE BUILDUP TEST DATA

Δt (hours)	p_{ws} (psia)	Δt (hours)	p_{ws} (psia)
0	709	19.7	4,198
1.97	3,169	24.6	4,245
2.95	3,508	29.6	4,279
3.94	3,672	34.5	4,306
4.92	3,772	39.4	4,327
5.91	3,873	44.4	4,343
7.88	3,963	49.3	4,356
9.86	4,026	59.1	4,375
14.8	4,133		

using the modified Muskat method.

2.8. In Example 2.7, explain how we could have applied the modified Muskat method to estimate static drainage-area pressure if we had not had estimates of $k/\phi\mu c_t$ or r_e.

2.9. Estimate formation permeability and skin factor from the following data available from a gas well pressure buildup test.

T = 199°F = 659°R,
h = 34 ft,
μ_i = 0.023 cp,
S_w = 0.33 (water is immobile),
c_{gi} = 0.000315 psi^{-1},
ϕ = 0.22,
z_i = 0.87, and
r_w = 0.3 ft.

The well produced 6,064 Mcf/D before the test. A

TABLE 2.9 – BUILDUP TEST DATA FOR WELL NEAR FAULT

Δt (hours)	p_{ws} (psi)	Δt (hours)	p_{ws} (psi)
20	1,373	500	2,225
30	1,487	800	2,350
40	1,535	1,000	2,434
50	1,585	1,500	2,545
100	1,752	2,000	2,615
200	1,940		

R_s = 748 scf/STB,
ϕ = 0.18,
r_w = 0.3 ft, and
h = 33 ft.

2.11. A pressure buildup test was run on an oil well believed to be near a sealing fault in an otherwise infinite-acting reservoir. Estimate the distance to the fault, given the well, rock, and fluid properties below and the buildup data in Table 2.9.

q = 940 STB/D,
μ = 50 cp,
ϕ = 0.2,
c_t = 78×10^{-6} psi^{-1},
h = 195 ft,
p_i = 2,945 psi,
N_p = 84,500 STB,
B_o = 1.31 RB/STB, and

TABLE 5.14 – ISOCHRONAL DELIVERABILITY TEST DATA

q (Mscf/D)	$\bar{p}$ (psia)	p_{wf} (psia)	t (hours)
983	352.4	344.7	0.5
977		342.4	1.0
970		339.5	2.0
965		337.6	3.0
2,631	352.3	329.5	0.5
2,588		322.9	1.0
2,533		315.4	2.0
2,500		310.5	3.0
3,654	351.0	318.7	0.5
3,565		309.5	1.0
3,453		298.5	2.0
3,390		291.9	3.0
4,782	349.5	305.5	0.5
4,625		293.6	1.0
4,438		279.6	2.0
4,318		270.5	3.0

WELL TESTING

Exercises

5.1. The data in Table 5.13 (from Ref. 6) were obtained on a well believed to be stabilized at each rate. Using equations in p^2 (strictly speaking, not applicable in this pressure range), estimate the AOF using (a) the empirical method and (b) the theoretical method.

Gas Well Test

Also, do the following: (c) plot the theoretical deliverability curve on the same graph paper as the empirical curve; (d) since p^2 equations are not accurate at this pressure level, develop and outline a theoretical method based on equations in p; and (e) apply equations in p to these data; in particular, calculate the AOF.

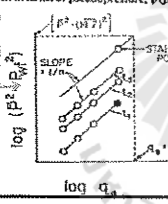
Do the Exercise 5.2 and 5.4

5.2. Cullender[7] presented data from an isochronal test and from an earlier, longer test that

In the 214-hour test, the rate was 1,156 Mscf/D, the shut-in pressure was 441.6 psia, and the flowing BHP was 401.4 psia. Using the data in Table 5.14, (a) determine the AOF with both empirical and theoretical methods, and (b) establish plots (on the same graph paper) of the empirical and theoretical stabilized deliverability curves.

5.3. Confirm $\psi(p)$ results stated in Example 5.4 for pressures in the range $450 \le p \le 3{,}150$ psia.

5.4. The well discussed in Example 5.5 was produced at 2,000 Mscf/D for 90 days and then shut in for a pressure buildup test. Data obtained in the buildup test are given in Table 5.15. Determine formation permeability and apparent skin factor using an analysis procedure based on equations written in terms of pseudopressure, $\psi(p)$.



CHAPTER 7

- Type Curve Analysis
- INTERFERENCE TESTS

2. Saturation and Relative Permeability Calculations (4 hrs.)
3. Steady State Radial Flow (4 hrs.)
4. Unsteady State Radial Flow (4 hrs.)
5. Pseudo-steady State Flow and Superposition (4 hrs.)
6. Well Testing Pressure Drawdown and Build Up (3 hrs.)
7. Interference Test and Type Curve Analysis(3hrs)
8. Displacement Efficiency(4 hrs)
9. Potential flow and Streamlines (4 hrs.)
10. Dynamics of Water Drive Reservoir. (6 hrs.)
11. Water and Gas Coning (4hrs.)
12. Multi-Phase Flow and Introduction to Reservoir Simulation (4 hrs.)
13. Enhance Oil Recovery(2 hrs)

CHAPTER 7

- Type Curve Analysis
- INTERFERENCE TESTS

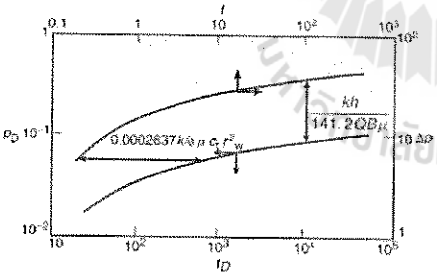


Figure 1.46 Concept of type curves.

where:

t = time, hours
c_t = total compressibility coefficient, psi^{-1}
ϕ = porosity

Hence, a graph of $\log(\Delta p)$ vs. $\log(t)$ will have an *identical shape* (i.e., parallel) to a graph of $\log(p_D)$ vs. $\log(t_D)$, although the curve will be shifted by $\log[kh/(141.2QB\mu)]$ vertically in pressure and $\log[0.0002637k/(\phi\mu c_t r_w^2)]$ horizontally in time. This concept is illustrated in Figure 1.46.

Not only do these two curves have the same shape, but if they are *moved relative to each other until they coincide or "match"*, the vertical and horizontal displacements required to achieve the match are related to these constants in Equations 1.4.3 and 1.4.4. Once these constants are determined from the vertical and horizontal displacements, it is possible

and:

$$\left(\frac{t_D/r_D^2}{t}\right)_{MP} = \frac{0.0002637k}{\phi\mu c_t r^2} \qquad [1.4.7]$$

The subscript "MP" denotes a match point.

A more practical solution then to the diffusivity equation is a plot of the dimensionless p_D vs. t_D/r_D^2 as shown in Figure 1.47 that can be used to determine the pressure at any time and radius from the producing well. Figure 1.47 is basically a type curve that is mostly used in interference tests when analyzing pressure response data in a shut-in observation well at a distance r from an active producer or injector well.

In general, the type curve approach employs the flowing procedure that will be illustrated by the use of Figure 1.47:

Step 1. Select the proper type curve, e.g., Figure 1.47.
Step 2. Place tracing paper over Figure 1.47 and construct a log–log scale having the same dimensions as those of the type curve. This can be achieved by tracing the major and minor grid lines from the type curve to the tracing paper.
Step 3. Plot the well test data in terms of Δp vs. t on the tracing paper.
Step 4. Overlay the tracing paper on the type curve and slide the actual data plot, keeping the x and y axes of both graphs parallel, until the actual data point curve coincides or matches the type curve.
Step 5. Select any arbitrary point match point MP, such as an intersection of major grid lines, and record $(\Delta p)_{MP}$ and $(t)_{MP}$ from the actual data plot and the corresponding values of $(p_D)_{MP}$ and $(t_D/r_D^2)_{MP}$ from the type curve.
Step 6. Using the match point, calculate the properties of

• Type Curve Analysis

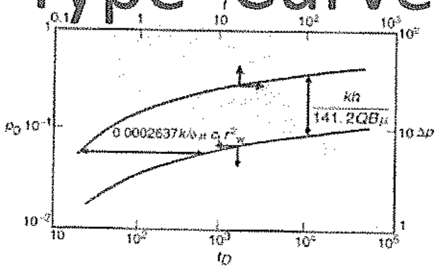


Figure 1.46 Concept of type curves.

where:

t = time, hours
c_t = total compressibility coefficient, psi^{-1}
ϕ = porosity

Hence, a graph of log(Δp) vs. log(t) will have an *identical shape* (i.e., parallel) to a graph of log(p_D) vs. log(t_D), although the curve will be shifted by log[$kh/(141.2QB\mu)$] vertically in pressure and log[$0.0002637k/(\phi\mu c_t r_w^2)$] horizontally in time. This concept is illustrated in Figure 1.46.

Not only do these two curves have the same shape, but if they are *moved relative to each other until they coincide or "match"*, the vertical and horizontal displacements required to achieve the match are related to these constants in Equations 1.4.3 and 1.4.4. Once these constants are determined from the vertical and horizontal displacements, it is possible

and:

$$\left(\frac{t_D/r_D^2}{t}\right)_{MP} = \frac{0.0002637k}{\phi\mu c_t r^2} \qquad [1.4.7]$$

The subscript "MP" denotes a match point.

A more practical solution then to the diffusivity equation is a plot of the dimensionless p_D vs. t_D/r_D^2 as shown in Figure 1.47 that can be used to determine the pressure at any time and radius from the producing well. Figure 1.47 is basically a type curve that is mostly used in interference tests when analyzing pressure response data in a shut-in observation well at a distance r from an active producer or injector well.

In general, the type curve approach employs the flowing procedure that will be illustrated by the use of Figure 1.47:

Step 1. Select the proper type curve, e.g., Figure 1.47.
Step 2. Place tracing paper over Figure 1.47 and construct a log–log scale having the same dimensions as those of the type curve. This can be achieved by tracing the major and minor grid lines from the type curve to the tracing paper.
Step 3. Plot the well test data in terms of Δp vs. t on the tracing paper.
Step 4. Overlay the tracing paper on the type curve and slide the actual data plot, keeping the x and y axes of both graphs parallel, until the actual data point curve coincides or matches the type curve.
Step 5. Select any arbitrary point match point MP, such as an intersection of major grid lines, and record $(\Delta p)_{MP}$ and $(t)_{MP}$ from the actual data plot and the corresponding values of $(p_D)_{MP}$ and $(t_D/r_D^2)_{MP}$ from the type curve.
Step 6. Using the match point, calculate the properties of

to achieve the match are related to these constants in Equations 1.4.3 and 1.4.4. Once these constants are determined from the vertical and horizontal displacements, it is possible to estimate reservoir properties such as permeability and porosity. This process of matching two curves through the vertical and horizontal displacements and determining the reservoir or well properties is called type curve matching.

As shown by Equation 1.2.83, the solution to the diffusivity equation can be expressed in terms of the dimensionless pressure drop as:

$$p_D = -\frac{1}{2}\text{Ei}\left(-\frac{r_D^2}{4t_D}\right)$$

Equation 1.2.84 indicates that when $t_D/r_D^2 > 25$, p_D can be approximated by:

$$p_D = \frac{1}{2}\left[\ln\left(t_D/r_D^2\right) + 0.080907\right]$$

Notice that:

$$\frac{t_D}{r_D^2} = \left(\frac{0.0002637k}{\phi\mu c_t r^2}\right)t$$

Taking the logarithm of both sides of this equation, gives:

$$\log\left(\frac{t_D}{r_D^2}\right) = \log\left(\frac{0.0002637k}{\phi\mu c_t r^2}\right) + \log(t) \qquad [1.4.5]$$

Equations 1.4.3 and 1.4.5 indicate that a graph of log(Δp) vs. log(t) will have an *identical shape* (i.e., parallel) to a graph of log(p_D) vs. log(t_D/r_D^2), although the curve will be shifted by log(kh141.2/$QB\mu$) vertically in pressure and log($0.0002637k/\phi\mu c_t r^2$) horizontally in time. When these two curves are moved relative to each other until they coincide or "match," the vertical and horizontal movements, in mathematical terms, are given by:

$$\left(\frac{p_D}{\Delta p}\right)_{MP} = \frac{kh}{141.2QB\mu} \qquad [1.4.6]$$

sponding values of $(p_D)_{MP}$ and $(t_D/r_D^2)_{MP}$ from the type curve.
Step 6. Using the match point, calculate the properties of the reservoir.

The following example illustrates the convenience of using the type curve approach in an interference test for 48 hours followed by a falloff period of 100 hours.

Example 1.31[a] During an interference test, water was injected at a 170 bbl/day for 48 hours. The pressure response in an observation well 119 ft away from the injector is given below:

t (hrs)	p (psig)	$\Delta p_{ws} = p_i - p$(psi)
0	$p_i = 0$	0
4.3	22	−22
21.6	82	−82
28.2	95	−95
45.0	119	−119
48.0	injection ends	
51.0	109	−109
69.0	55	−55
73.0	47	−47
93.0	32	−32
142.0	16	−16
148.0	15	−15

Other given data includes:

$p_i = 0$ psi, $B_w = 1.00$ bbl/STB

[a]This example problem and the solution procedure are given in Earlougher, R. *Advanced Well Test Analysis*, Monograph Series, SPE, Dallas (1977).

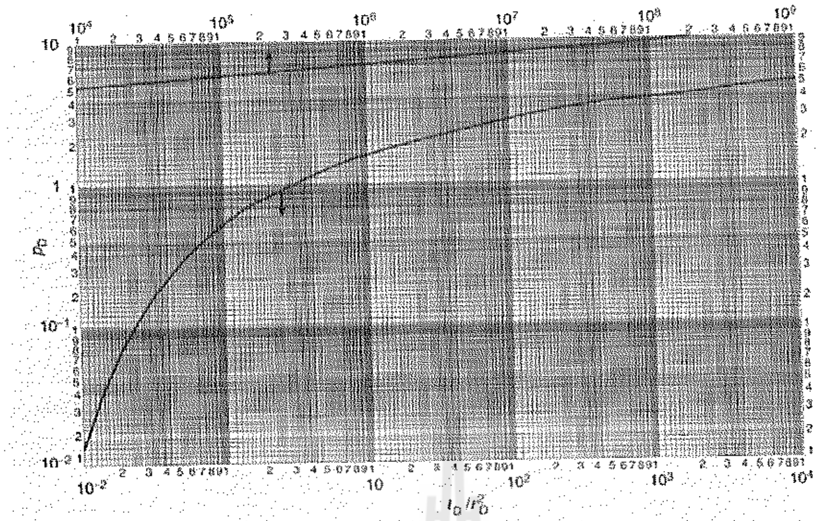


Figure 1.47 Dimensionless pressure for a single well in an infinite system, no wellbore storage, no skin. Exponential-integral solution (After Earlougher, R. Advances in Well Test Analysis) (Permission to publish by the SPE, copyright SPE, 1977).

$$c_t = 9.0 \times 10^{-6}\ \text{psi}^{-1}, \quad h = 45\ \text{ft}$$
$$\mu_w = 1.3\ \text{cp}, \quad q = -170\ \text{bbl/day}$$

and:

$$\phi = \frac{0.0002637k}{\mu c_t r^2 [(t_D/r_D^2)/t]_{MP}}$$

Figure 1.47 Dimensionless pressure for a single well in an infinite system, no wellbore storage, no skin. Exponential-integral solution (After Earlougher, R. Advances in Well Test Analysis) (Permission to publish by the SPE, copyright SPE, 1977).

$$c_t = 9.0 \times 10^{-6}\ \text{psi}^{-1}, \quad h = 45\ \text{ft}$$
$$\mu_w = 1.3\ \text{cp}, \quad q = -170\ \text{bbl/day}$$

Calculate the reservoir permeability and porosity.

Solution

Step 1. Figure 1.48 show a plot of the well test data during the injection period, i.e., 48 hours, in terms of Δp vs. t on tracing paper with the same scale dimensions as in Figure 1.47. Using the overlay technique with the vertical and horizontal movements, find the segment of the type curve that matches the actual data.

Step 2. Select any point on the graph that will be defined as a match point MP, as shown in Figure 1.48. Record $(\Delta p)_{MP}$ and $(t)_{MP}$ from the actual data plot and the corresponding values of $(p_D)_{MP}$ and $(t_D/r_D^2)_{MP}$ from the type curve, to give:
Type curve match values:

$$(p_D)_{MP} = 0.96, \quad (t_D/r_D^2)_{MP} = 0.94$$

Actual data match values:

$$(\Delta p)_{MP} = -100\ \text{psig}, \quad (t)_{MP} = 10\ \text{hours}$$

Step 3. Using Equations 1.4.6 and 1.4.7, solve for the permeability and porosity:

$$k = \frac{141.2QB\mu}{h}\left(\frac{p_D}{\Delta p}\right)_{MP}$$
$$= \frac{141.2(-170)(1.0)(1.0)}{45}\left(\frac{0.96}{-100}\right)_{MP} = 5.1\ \text{md}$$

and:

$$\phi = \frac{0.0002637k}{\mu c_t r^2 [(t_D/r_D^2)/t]_{MP}}$$
$$= \frac{0.0002637(5.1)}{(1.0)(9.0 \times 10^{-6})(119)^2[0.94/10]_{MP}} = 0.11$$

Equation 1.2.83 shows that the dimensionless pressure is related to the dimensionless radius and time by:

$$p_D = -\frac{1}{2}\text{Ei}\left(-\frac{r_D^2}{4t_D}\right)$$

At the wellbore radius where $r = r_w$, i.e., $r_D = 1$, and $p(r, t) = p_{wf}$, the above expression is reduced to:

$$p_D = -\frac{1}{2}\text{Ei}\left(\frac{-1}{4t_D}\right)$$

The log approximation as given by Equation 1.2.80 can be applied to the above solution to give:

$$p_D = \frac{1}{2}[\ln(t_D) + 0.80901]$$

and, to account for the skin s, by:

$$p_D = \frac{1}{2}[\ln(t_D) + 0.80901] + s$$

or:

$$p_D = \frac{1}{2}[\ln(t_D) + 0.80901 + 2s]$$

Notice that the above expressions assume zero wellbore storage, i.e., dimensionless wellbore storage $C_D = 0$. Several authors have conducted detailed studies on the effects and duration of wellbore storage on pressure drawdown and buildup data. Results of these studies were presented in the type curve format in terms of the dimensionless pressure as

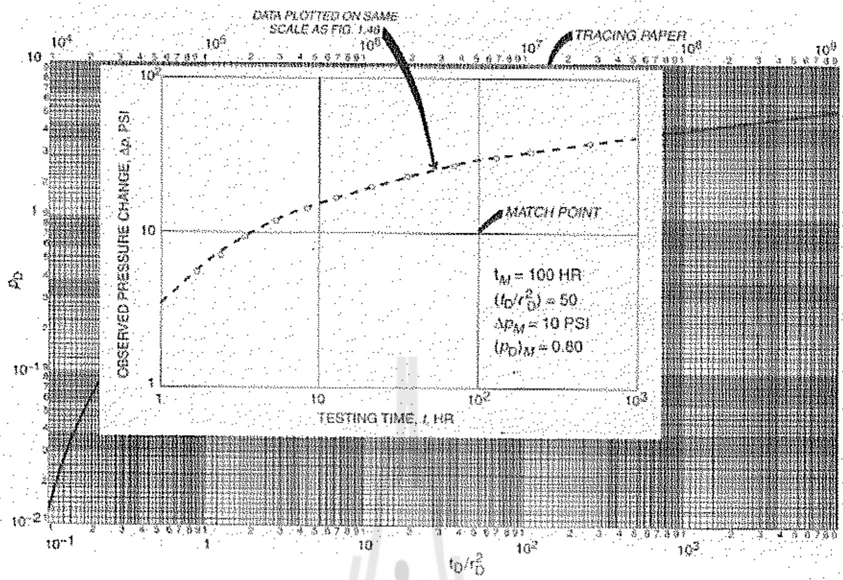


Figure 1.48 *Illustration of type curve matching for an interference test using the type curve (After Earlougher, R. Advances in Well Test Analysis) (Permission to publish by the SPE, copyright SPE, 1977).*

t_D/r_D^2

Figure 1.48 *Illustration of type curve matching for an interference test using the type curve (After Earlougher, R. Advances in Well Test Analysis) (Permission to publish by the SPE, copyright SPE, 1977).*

a function of dimensionless time, radius, and wellbore storage, i.e., $p_D = f(t_D, r_D, C_D)$. The following two methods that utilize the concept of the type curve approach are briefly introduced below:

(1) the Gringarten type curve;
(2) the pressure derivative method

1.4.1 Gringarten type curve

During the *early-time period* where the flow is dominated by the wellbore storage, the wellbore pressure is described by Equation 1.3.5 as:

$$p_D = \frac{t_D}{C_D}$$

or:

$$\log(p_D) = \log(t_D) - \log(C_D)$$

This relationship gives the characteristic signature of wellbore storage effects on well testing data which indicates that a plot of p_D vs. t_D on a log–log scale will yield a straight line of a *unit slope*. At the end of the storage effect, which signifies the beginning of the infinite-acting period, the resulting pressure behavior produces the usual straight line on a semilog plot as described by:

$$p_D = \frac{1}{2}[\ln(t_D) + 0.80901 + 2s]$$

It is convenient when using the type curve approach in well testing to include the dimensionless wellbore storage coefficient in the above relationship. Adding and subtracting $\ln(C_D)$ inside the brackets of the above equation gives:

$$p_D = \frac{1}{2}[\ln(t_D) - \ln(C_D) + 0.80901 + \ln(C_D) + 2s]$$

or, equivalently:

$$p_D = \frac{1}{2}\left[\ln\left(\frac{t_D}{C_D}\right) + 0.80907 + \ln(C_D e^{2s})\right] \quad [1.4.8]$$

where:

p_D = dimensionless pressure
C_D = dimensionless wellbore storage coefficient
t_D = dimensionless time
s = skin factor

Equation 1.4.8 describes the pressure behavior of a well with a wellbore storage and a skin in a homogeneous reservoir during the transient (infinite-acting) flow period. Gringarten et al. (1979) expressed the above equation in the graphical type curve format shown in Figure 1.49. In this figure, the dimensionless pressure p_D is plotted on a log–log scale versus dimensionless time group t_D/C_D. The resulting curves, characterized by the dimensionless group $C_D e^{2s}$, represent different well conditions ranging from damaged wells to stimulated wells.

Figure 1.49 shows that all the curves merge, in early time, into a unit-slope straight line corresponding to pure wellbore storage flow. At a later time with the end of the wellbore storage-dominated period, curves correspond to infinite-acting radial flow. The end of wellbore storage and the start of infinite-acting radial flow are marked on the type curves of Figure 1.49. There are three dimensionless

RAMEY'S TYPE CURVES

This method was published in 1970. Four different dimensionless variables are related:

(1) Dimensionless pressure drop, P_d.

(2) Dimensionless time, T_d.
(3) Skin effect, s, and
(4) Dimensionless wellbore storage coefficient, $\bar{C}$.

The definition of dimensionless pressure drop is dependent on whether the test well is an oil well or a gas well:

$$P_d = \frac{K\ H\ (\Delta P)}{141.2\ Q\ \mu\ B} \qquad \text{Oil Wells}$$

$$P_d = \frac{K\ H\ (\Delta P^2)}{1424\ Q\ \mu\ T\ z} \qquad \text{Gas Wells}$$

It should be noted that in both of these definitions, the right hand side is made up of terms that are regarded to be constants for a given well test, except for the pressure drop factor.

The dimensionless time is defined on the basis of the wellbore radius:

$$T_d = \frac{2.64 \times 10^{-4}\ K\ T}{\emptyset\ \mu\ c_e\ R_w^2}$$

Only the time (hours) varies on the righthand side.

The skin effect, s, is the same as previously discussed, and is a measure of the amount of flow resistance near the wellbore (due to damage or stimulation).

RAMEY'S TYPE CURVES

The dimensionless time is defined on the basis of the wellbore radius:

$$T_d = \frac{2.64 \times 10^{-4}\ K\ T}{\emptyset\ \mu\ c_e\ R_w^2}$$

Only the time (hours) varies on the righthand side.

The skin effect, s, is the same as previously discussed, and is a measure of the amount of flow resistance near the wellbore (due to damage or stimulation).

The dimensionless storage coefficient is defined to be:

$$\bar{C}_d = \frac{5.615\ C}{2\ \pi\ H\ \emptyset\ R_w^2\ c_e}$$

where: C = the wellbore storage coefficient, Bbls/psi,
H = the net pay thickness, feet,
Ø = the porosity, fraction,
R_w = wellbore radius, feet, and
c_e = total reservoir compressibility, psi^{-1}.

The wellbore storage coefficient is evaluated in several different ways depending on the types of fluids produced. This term actually represents the product of the average compressibility of the fluids in the wellbore and the wellbore volume available for fill up; i.e.:

$$C = (V_w)(c_{avg})$$ Volume Well bore

Therefore, for a gas well, the value of C is calculated by multiplying the volume of the wellbore times the gas compressibility:

... except for the pressure drop factor.

The dimensionless time is defined on the basis of the wellbore radius:

$$T_d = \frac{2.64 \times 10^{-4}\, K\, T}{\emptyset\, \mu\, c_e\, R_w^2}$$

Only the time (hours) varies on the righthand side.

The skin effect, s, is the same as previously discussed, and is a measure of the amount of flow resistance near the wellbore (due to damage or stimulation).

The dimensionless storage coefficient is defined to be:

$$\bar{C} = \frac{5.615\, C}{2\pi\, H\, \emptyset\, R_w^2\, c_e}$$

where: C = the wellbore storage coefficient, Bbls/psi,
H = the net pay thickness, feet,
$\emptyset$ = the porosity, fraction,
R_w = wellbore radius, feet, and
c_e = total reservoir compressibility, psi^{-1}.

The wellbore storage coefficient is evaluated in several different ways depending on the types of fluids produced. This term actually represents the product of the average compressibility of the fluids in the wellbore and the wellbore volume available for fill up; i.e.:

$$C = (V_w)(c_{Avg})$$ Volume Well bore

Therefore, for a gas well, the value of C is calculated by multiplying the volume of the wellbore times the gas compressibility:

W. S. coefficient. $q \propto P$

$$C = (V_w)(c_g) = (V_w)\left(\frac{2}{P_{BHP} + P_{SURF}}\right)$$

For oil wells producing significant quantities of water and/or free gas, the value of C can be calculated by using a producing rate weighted average of the compressibilities of the different fluids:

$$c_{avg} = \frac{B_o c_o + (GOR^* - R_s) B_g c_g + (WOR) c_w}{B_o + (GOR - R_s) B_g + WOR}$$ — *

Then, the wellbore storage coefficient is:

$$C = (c_{avg})(V_w)$$ $- C_{avg} \cdot V_w$ $\quad \frac{1}{V_w}\frac{dV}{dP} \cdot V_w = \frac{dV}{dP}$

For clean oil wells, the wellbore storage coefficient is normally calculated through the use of data collected during the log-log plot unit slope. If the correlation of the pressure change ($P_{ws} - P_{wf}$) is linear with shutin time, ΔT, then a plot of log ΔP versus log ΔT will yield a unit slope. Therefore, any corroborated points along the unit slope line will yield the same quotient, ($\Delta P/\Delta T$). Consequently, any point ($\Delta P, \Delta T$) on the unit slope line may be used in the following formula for wellbore storage coefficient:

$$C = \frac{Q_o B_o}{(24)(\Delta P/\Delta T)_{unit\ slope}}$$ $\quad \frac{dV/dt}{dP/dt} = \frac{q_o b_o}{24 \frac{\Delta P}{\Delta T}}$

where: ...

$C = (V_{avg})(V_u)$ $-C_{avg} V_u$ $\frac{1}{V_u}\frac{dV}{dP}$, $V_u = \frac{dV}{dP}$

For clean oil wells, the wellbore storage coefficient is normally calculated through the use of data collected during the log-log plot unit slope. If the correlation of the pressure change $(P_{ws} - P_{wf})$ is linear with shutin time, ΔT, then a plot of log ΔP versus log ΔT will yield a unit slope. Therefore, any corroborated points along the unit slope line will yield the same quotient, $(\Delta P/\Delta T)$. Consequently, any point $(\Delta P, \Delta T)$ on the unit slope line may be used in the following formula for wellbore storage coefficient:

$$C = \frac{Q_o B_o}{(24)(\Delta P/\Delta T)_{unit\ slope}} \qquad \frac{dV/dt}{dP/dt} = \frac{q_o B_o}{24\,\Delta P/\Delta T}$$

where: Q_o = oil rate just prior to shutin, STB/D,
B_o = oil formation volume factor, Bbl/STB,
$(\Delta P/\Delta T)$ = ratio of the pressure change to the shutin time from some point on the unit slope line, psi/hr.

$$C_s = \frac{qB}{24}\left(\frac{\Delta t}{\Delta p}\right)_{unit\ slope\ line}$$

Note that, in a well with a liquid/gas interface in the wellbore,

$$C_s = \frac{25.65\,A_{wb}}{\rho}\ \text{bbl/psi} \qquad (4.1)$$

and for a wellbore filled with a single-phase liquid or with gas,

$$C_s = c_{wb} V_{wb}\ \text{bbl/psi} \qquad (4.2)$$

and

$$C_{sD} = 0.894\,C_s/(\phi c_t h r_w^2) \qquad (4.3)$$

[illegible], draw the major grids such as:

3. Label the vertical log axis on the transparency as ΔP (psi). Ignore the scale on the type curve and use whatever scale is convenient for your data. Label the horizontal log axis as shutin time in hours. Once again, use whatever scale is convenient for your data.

4. Plot the test data (ΔP vs. ΔT) on the log-log transparency.

5. Calculate the dimensionless storage coefficient, $\bar{C}$. (This allows you to know which bundle of curves on the Agarwal type curve to try to match.) As discussed earlier, the first task here is to compute the wellbore storage coefficient. Depending on whether the well is an oil well or gas well, one of the equations discussed earlier is used. For a clean oil well, the recommended equation uses data from the unit slope line:

$$C = \frac{Q_o B_o}{(24)(\Delta P/\Delta T)_{unit\ slope}}$$

Then, the dimensionless wellbore storage coefficient can be calculated:

$$\bar{C} = \frac{0.894\,C}{\phi\,h\,C_t\,R_w^2}$$

...llbore storage effects.

Then, considering the producing time before the buildup test, a dimensionless producing time is calculated:

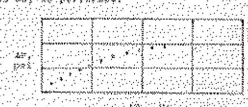
$$T_{pd} = \frac{0.000264\,K\,T}{\phi\,u\,c_t\,R_w^2}$$

Then, for the "equivalent drawdown time" method to be valid, the dimensionless producing time must satisfy:

$$T_{pd} \geq (60 + 3.5\,S)\,\bar{C}$$

where: $\bar{C} = \frac{5.615\,C}{2\pi\,\phi\,h\,c_t\,R_w^2}$

Thus, by using ΔT_e, instead of ΔT, the Agarwal et. al. type curve analysis may be performed.



Further, instead of using a Horner plot for a short producing time buildup test, an MDH type semilog plot can be made with shutin pressure plotted on the linear vertical axis and the equivalent drawdown time (hours) plotted on the log horizontal scale. Notice that at infinite shutin time, the value of equivalent drawdown time is equal to the producing time before shutin. Thus, the semilog straight line can still be extrapolated (to an equivalent drawdown time equal to the producing time) to obtain P^*.

31

...the shift to obtain a match is more in the horizontal direction than in the vertical. Remember that in the moving process the transparency and type curve axes must remain parallel.

7. After you have made a successful match, and the curves are overlaying each other, note the skin effect labeled on the matched type curve. Then, pick any point from your plot on the transparency such as the intersection of two major grid lines, $(\Delta T, \Delta P)$. This point will be referred to as the match point. Then, note the corresponding point on the type curve grid, (T_d, P_d). This is also called a match point.

8. Calculate the flow capacity.

$$(Kh)_o = 141.2\,Q_o\,\mu_o\,B_o\left(\frac{P_d}{\Delta P}\right)_{match\ point}$$

This result is usually of better quality than any of the other answers obtained from this analysis. However, the value of skin, s, is also a pretty good estimate.

9. Calculate the porosity-thickness product:

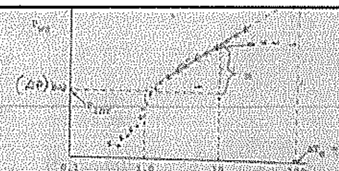
$$(\phi h) = \frac{2.64 \times 10^{-4}(K\,h)_o}{\mu_o\,c_t\,R_w^2}\left(\frac{\Delta T}{T_d}\right)_{match\ point}$$

This is usually the poorest quality result from the Agarwal et. al. type curve analysis.

10. Calculate pressure drop due to skin:

$$\Delta P_{skin} = s\left(\frac{\Delta P}{P_d}\right)_{match\ point}$$

Although used most of the time to analyze buildup tests, the Agarwal et. al. type curves were actually constructed for drawdown tests. The reason for this is simple: superposition is



ΔT_e ← (Δt) change to [illegible]

$$K\,h = \frac{162.6\,Q\,\mu\,B}{m}$$

$$s = 1.151\left[\left(\frac{P_{1hr}(\Delta T_e) - P_{wf}}{m}\right) - \log\left(\frac{k}{\phi\mu c_t r_w^2}\right) + 3.23\right]$$

If $\Delta T_e = \frac{T_p \cdot \Delta T}{T_p + \Delta T}$ $\xrightarrow{\lim} \Delta T \to \infty$

$\Delta T_e = T_p$

$$\Delta T_e = \frac{\Delta T}{1 + \frac{\Delta T}{T_p}}$$

...we need to check if we have chosen proper range of MTR.

$$C_s = 25.65\frac{A_{wb}}{\rho}$$
$$= 25.65\frac{0.0218}{53} = 0.0106\ \text{bbl/psi}$$
$$t_{wbs} = \frac{(200{,}000 + 12{,}000s)C_s}{kh/\mu}$$
$$= \frac{[200{,}000 + (12{,}000)(6.18)]0.0106}{(7.6)(69)/(0.80)}$$
$$= 4.43\ \text{hours}$$

...since the PF = 7.9. This confirms our choice of the start of the MTR line.

ANALYSIS OF WELL TESTS USING TYPE CURVES

Type Curve Analysis

In this section we will discuss the quantitative use of some type curves in well test analysis. The ... of using type curves ... enable ... test ... curves that frequently appear in the literature.

As previously mentioned, some type curves are used to help identify the MTR, while other type curves are used to determine formation permeability, skin factor, fracture length, etc. The type curves are used in situations where the conventional methods fail to properly analyze a test or to double check the results obtained with conventional methods with those obtained with type curve matching.

Fundamentally, a type curve is a preplotted family of pressure drawdown curves generated by obtaining solutions to the diffusivity equation with specified initial and boundary conditions. Some of these solutions are analytical; others are based on finite-difference approximations generated by computer reservoir simulators.

The type curves discussed in this section include

Ramey et al.'s type curves for buildup and constant-rate drawdown tests
McKinley's type curves for the same applications and curves for when wellbore storage effects dominate the entire test duration
Gringarten et al.'s type curves for vertically fractured wells with uniform flux.

RAMEY'S TYPE CURVES

The use of this type curve to help identify the MTR on a Horner plot was discussed in Chapter 6. ...-13. To use the type curve for estimating permeability and skin factor, the following steps should ...lowed:

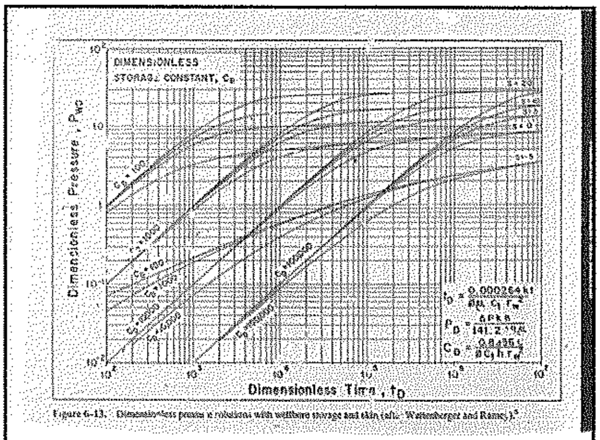
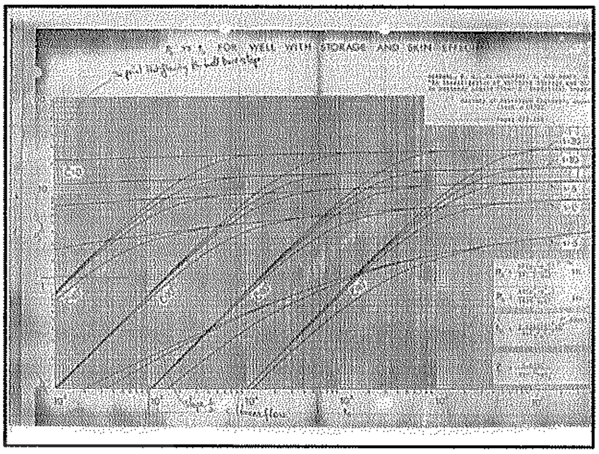


Figure 6-13. Dimensionless pressure solutions with wellbore storage and skin (after Wattenbarger and Ramey).



RAMEY'S TYPE CURVES

The use of this type curve to help identify the MTR on a Horner plot was discussed in Chapter 6, Figure 6-13. To use the type curve for estimating permeability and skin factor, the following steps should be employed:

1. Plot $\Delta p = (p_i - p_{wf})$ vs. t (drawdown test) or $\Delta p = (p_{ws} - p_{wf})$ vs. $\Delta t_e = \Delta t/(1 + \Delta t/t_p)$ (buildup test) on a tracing paper, using the grid of the type curve in Figure 6-13 as a plotting aid. For buildup tests, if $\Delta t < 0.1 t_p$, then $\Delta t \approx \Delta t_e$. Therefore, plot Δp vs. Δt.
2. If the test has the early-time unit slope line, use this line and Equation (6.47) to estimate the wellbore storage constant C_s. If the unit slope line is not present, C_s must be estimated using Equations (6.44) or (6.45).
3. Using C_s estimated in Step 2 and Equation (6.46), estimate the dimensionless wellbore storage constant C_{sD}.
4. Slide the tracing paper (with test data plotted) on Figure 6-13 vertically and horizontally, keeping the grids parallel until you find the curve that most nearly matches the majority of the plotted data. A match should be obtained with the curve with a C_{sD} value close to the value estimated in Step 3.
5. After the match is completed, the skin factor is read directly from the matched curve. To calculate permeability, pick a convenient match point on the traced grid, note the value of $(\Delta p)_{MP}$ and t_{MP} or $(\Delta p)_{MP}$ and $(\Delta t_e)_{MP}$ and the corresponding value of p_{DMP} and t_{DMP} from the type curve. The subscript *MP* designates *match point*.
6. The permeability is estimated using the match points and the following equation:

$$k = 141.2 \frac{q\mu B}{h} \frac{(p_D)_{MP}}{(\Delta p)_{MP}} \qquad (7.88)$$

If the value of C_{sD} cannot be estimated with confidence because a match could be found with several type curves, a ϕc_t product should be calculated and compared with the actual ϕc_t product (if available). The ϕc_t product can be calculated using the following equation:

$$\phi c_t = \left(\frac{0.0002644 k}{\mu r_w^2}\right) \frac{(\Delta t)_{MP}}{(t_D)_{MP}} \qquad (7.89)$$

Example 7-23. Using Ramey's type curves, analyze the pressure buildup data of Example 7-6 for skin factor and permeability.

Solution: From Example 7-6,

$q_o = 1{,}515$ STB/D, $\phi = 0.23$
$\mu_o = 0.89$ cp, $r_w = 0.302$ ft
$h = 110$ ft, $c_t = 8.03 \times 10^{-6}$ 1/psi
$k = 1{,}096.45$ md, $s = 9.33$
$B_o = 1.31$ RB/STB

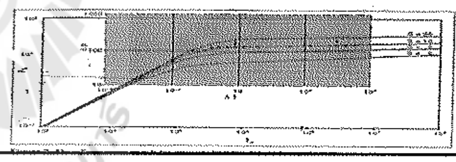
From Figure 7-21, the match points are

$\Delta t = 100$, $t_D = 9 \times 10^{5}$
$\Delta p = 10$, $p_D = 1.35$

From the pressure match point,

$$k = 141.2 \frac{q_o \mu_o B_o}{h} \frac{(p_D)_{MP}}{(\Delta p)_{MP}}$$
$$= 141.2 \frac{(1{,}515)(0.89)(1.31)(1.35)}{(110)(10)}$$
$$= 1{,}101.72 \text{ md}$$

The permeability is also in good agreement with $k = 1{,}096.45$ md calculated in Example 7-6.



Determine:

(a) Total isothermal compressibility
(b) Permeability k
(c) Skin factor s
(d) Pressure drop due to skin $(\Delta p)_{skin}$
(e) Effective wellbore radius r_{wa}
(f) Flow efficiency, FE using p^*
(g) Damage ratio, DR using p^*
(h) Productivity index, PI
(i) Radius of investigation at $\Delta t = 5$ hours
(j) It is initially assumed that the well is draining from the center of a circle. Is it valid to equate $\bar{p}$ to p^*?

TABLE 7-1. Oil well pressure buildup test data for Example 7-6

Δt (min)	$t_p + \Delta t/\Delta t$	p_{ws} (psia)	$\Delta p = p_{ws} - p_{wf}$ (psia)
0	0	2,710(p_{wf})	0
1	901.0	2,760	50
2	451.0	2,803	93
4	226.0	2,823	113
[illegible]	181.0	2,839	[illegible]
7	129.6	2,834	[illegible]
[illegible]	101.0	2,835	[illegible]
12	76.0	2,831	[illegible]
20	45.9	2,832	[illegible]
40	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
300	4.0	[illegible]	[illegible]
420	[illegible]	2,842	132
560	2.6	2,842	132

Solution. To solve this example, refer to the Horner Plot in Figure 7-9 and the log-log plot of Δp vs. Δt in Figure 7-10.

(a)
$$c_t = c_o S_o + c_w S_w + c_f$$
$$= (8.5 \times 10^{-6})(1 - 0.30) + (3.0 \times 10^{-6})(0.30) + 10^{-6}$$
$$= 8.03 \times 10^{-6} \text{ 1/psi}$$

(b)
$$k = 162.6 \frac{q_o \mu_o B_o}{mh}$$
$$= 162.6 \frac{(1{,}515)(0.89)(1.31)}{(8.7)(110)}$$
$$= 1{,}096.45 \text{ md}$$

(c)
$$s = 1.151 \left[\frac{p_{1hr} - p_{wf}}{m} - \log\left(\frac{k}{\phi \mu_o c_t r_w^2}\right) + 3.23\right]$$
$$= 1.151 \left[\frac{2{,}827 - 2{,}710}{8.7} - \log\left(\frac{1{,}096.45}{(0.23)(0.89)(8.03 \times 10^{-6})(0.302)^2}\right) + 3.23\right]$$
$$= 1.151(14.4 - 9.7 + 3.23)$$
$$= 9.33$$

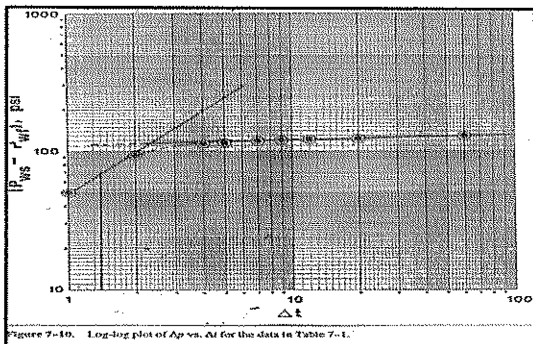


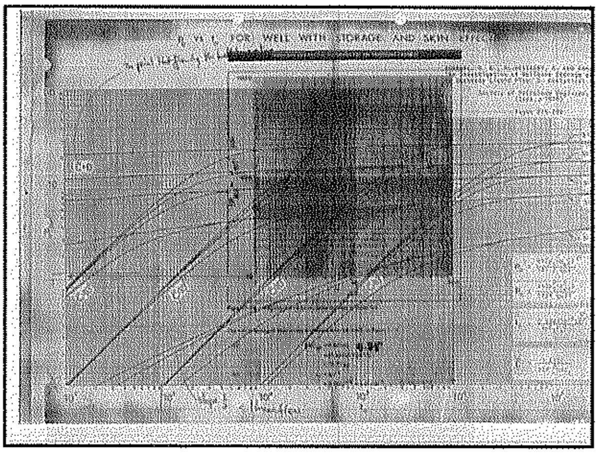
Figure 7-10. Log-log plot of Δp vs. Δt for the data in Table 7-1.

The well is damaged. Skin is also confirmed from the match in Figure 7-11.

(d)
$$(\Delta p)_{skin} = 0.869 ms$$
$$= (0.869)(8.7)(9.35)$$
$$= 70.77 \text{ psi}$$

9.35

(e)
$$r_{wa} = r_w e^{-s}$$
$$= 0.302 e^{-9.33} = 3.15 \times 10^{-5}$$



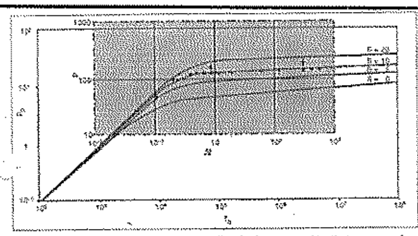


Figure 7-11. Estimating end of wellbore storage distortion by curve matching the type curves of Figure 6-13.

(f)

$$FE = \frac{p^* - p_{wf} - (\Delta p)_{skin}}{p^* - p_{wf}} = \frac{2{,}847 - 2{,}710 - 70.77}{2{,}847 - 2{,}710} \times 100 = 48.34\%$$

This means that the well is producing at approximately 48% of the rate it would have been producing if the well was not damaged.

(g)

$$DR = 1/FE = 1/0.4834 = 2.07$$

Although the type curves were developed for slightly compressible liquids, they can also be used to analyze gas well tests. In the section on gas well buildup test analysis, we discussed the use of p_{ws}, p_{ws}^2, and $\psi(p_{ws})$. The following equations can be used to calculate permeability using the different pressure vs. time plots:

For the plot of Δp vs. Δt, or Δp vs. t:

$$k = 141.2 \frac{q_g \mu_g B_g}{h} \frac{(p_D)_{MP}}{(\Delta p)_{MP}} \quad (7.90)$$

where B_g is in RB/MSCF.

For the plot of $(p_i^2 - p_{wf}^2)$ vs. t or $(p_{ws}^2 - p_{wf}^2)$ vs. Δt:

$$k = 1{,}422 \frac{q_g \mu_g z T_R}{h} \frac{(p_D)_{MP}}{(p_i^2 - p_{wf}^2)_{MP}} \quad (7.91)$$

For the plot of $[\psi(p_i) - \psi(p_{wf})]$ vs. t or $[\psi(p_{ws}) - \psi(p_{wf})]$ vs. Δt:

$$k = 50{,}300 \frac{q_g T_R p_{sc}}{h T_{sc}} \frac{(\psi_D)_{MP}}{[\psi(p_i) - \psi(p_{wf})]_{MP}} \quad (7.92)$$

McKinley Type Curve

Using the early time data, calculate the wellbore storage factor:

$$F = \left[\left(\frac{\Delta p F}{Q}\right)_{curve}\right]\left[\frac{Q_0 B_0}{(\Delta p)_{match\ point,\ early\ data}}\right]$$

Then, the value of transmissibility near the well is computed:

$$(T)_{near\ well} = \left[\left(\frac{T}{F}\right)_{early\ match}\right][F] = (kh/\mu)_{near\ well}$$

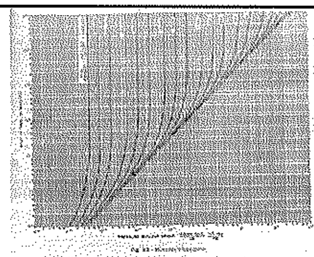
Now the late time data (after 30 minutes) is matched. Once again, the transparency should be shifted to the left or right only with no vertical "slippage" relative to the type curve. Move the transparency until the upper portion of the plot (late time data) matches one of the type curves. Using the matched curve's value of (T/F), together with the value of the storage factor (F) computed earlier, the value of the transmissibility away from the wellbore can be computed.

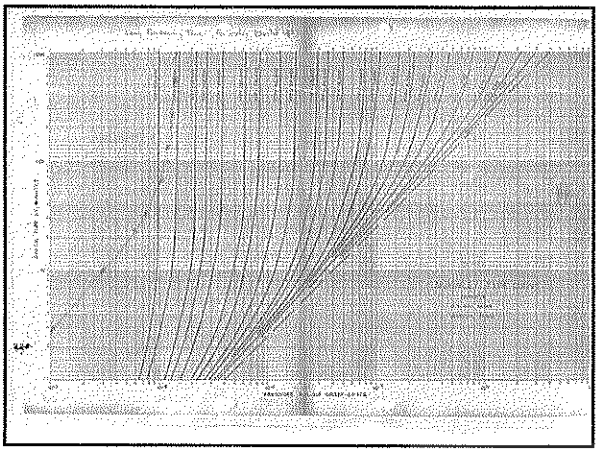
$$(T)_{interwell} = \left[\left(\frac{T}{F}\right)_{late\ match}\right][F] = (kh/\mu)_{interwell}$$

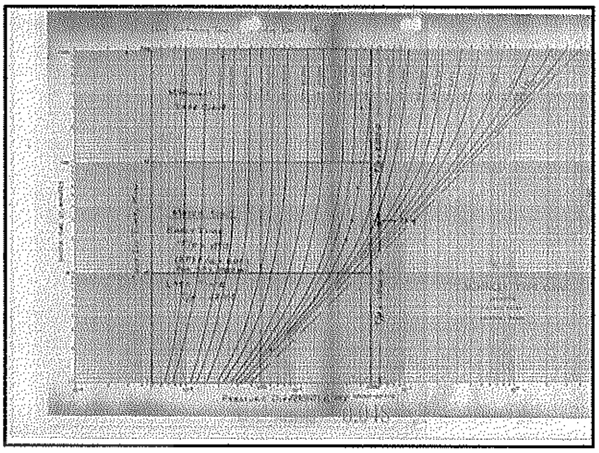
Comparing the calculated transmissibility away from the wellbore with that computed near the wellbore will give a qualitative indication of positive or negative skin effect:

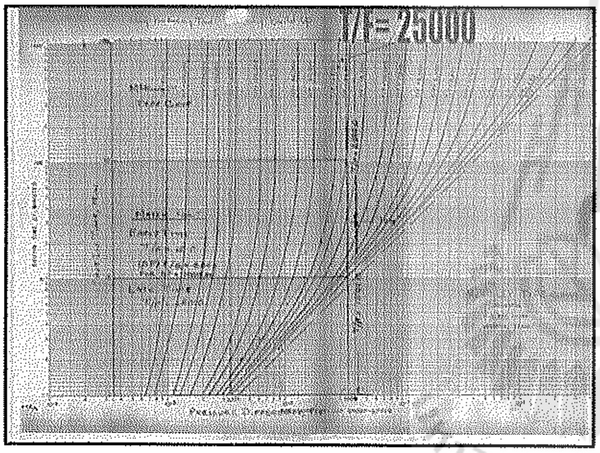
$(T)_{near\ well} < (T)_{interwell}$, $s = +$

$(T)_{near\ well} > (T)_{interwell}$, $s = -$









Then calculate the dimensionless wellbore storage constant:

$$C_{sD} = \frac{0.894\,C_s}{\phi c_t h r_w^2}.$$

Note that estimates of ϕ and c_t are required at this point – with implications that are discussed later.

If a unit-slope line is not present, C_s and C_{sD} must be calculated from wellbore properties, and inaccuracies may result if these properties do not describe actual test behavior.

3. Using type curves with C_{sD} as calculated in Step 2, find the curve that most nearly fits all the plotted data. This curve will be characterized by some skin factor, s; record its value. Interpolation between curves should improve the precision of the analysis, but may prove difficult. Even for fixed C_{sD} from the unit-slope curve, the analyst may experience difficulty in determining that one value of s provides a better fit than another, particularly if all data are distorted by wellbore storage or if the "scatter" or "noise" that characterizes much actual field data is present. If C_{sD} is not known with certainty, the possible ambiguity in finding the best fit is even more pronounced.

4. With the actual test data plot placed in the position of best fit, record corresponding values of $(p_i - p_{wf}, p_D)$ and (t, t_D) from any convenient match point.

5. Calculate k and ϕc_t (Eqs. 4.4 and 4.5):

$$k = 141.2\frac{qB\mu}{h}\left(\frac{p_D}{p_i - p_{wf}}\right)_{MP},$$

$$\phi c_t = \frac{0.0002637\,k}{\mu r_w^2}\left(\frac{t}{t_D}\right)_{MP}.$$

Eq. 4.5 does not establish ϕc_t based on test performance unless it is possible to establish C_{sD} without assuming values for ϕc_t – it simply reproduces those values assumed in Step 2.

In summary, the procedure outlined in Steps 1 through 5 provides estimates of k, s, and C_s.

q = 500 STB/D,
ϕ = 0.2,
μ = 0.8 cp,
c_t = 10 × 10⁻⁶ psi⁻¹,
r_w = 0.3 ft,
h = 56 ft,
B_o = 1.2 RB/STB, and
p_i = 3,000 psia.

Solution. We must first prepare the data for plotting (Table 4.2). The data are plotted in Fig. 4.5.

From the unit-slope line (on which the data lie for $t \leq 0.0218$ hour),

$$C_s = \frac{qB}{24}\left[\frac{t}{(p_i - p_{wf})}\right]_{\text{point on unit-slope line}}$$

$$= \frac{(500)(1.2)}{(24)} \times \frac{(0.0218)}{(37)}$$

$$= 0.0136 \text{ RB/psi}.$$

Then,

$$C_{sD} = \frac{0.894\,C_s}{\phi c_t h r_w^2}$$

$$= \frac{(0.894)(0.0136)}{(0.2)(1\times10^{-5})(56)(0.3)^2}$$

$$= 1.01 \times 10^3$$

$$\approx 1 \times 10^3.$$

For $C_{sD} = 10^3$, the best-fitting type curve is for $s = 5$. A time match point is $t = 1$ hour, when $t_D = 1.93 \times 10^4$. A pressure match point is $(p_i - p_{wf}) = 100$ psi, when $p_D = 0.81$.

From the match, we also note that wellbore storage distortion ends at $t \approx 5.0$ hours (i.e., the type curve for $C_{sD} = 10^3$ becomes identical to the type curve for $C_{sD} = 0$).

From the pressure match point,

$$k = 141.2\frac{qB\mu}{h}\left(\frac{p_D}{p_i - p_{wf}}\right)_{MP}$$

...

4.4 McKinley's Type Curves

...

Use of McKinley's Type Curves

...

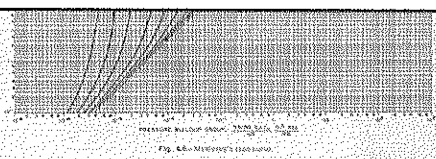


Fig. 4.6—McKinley's type curve.

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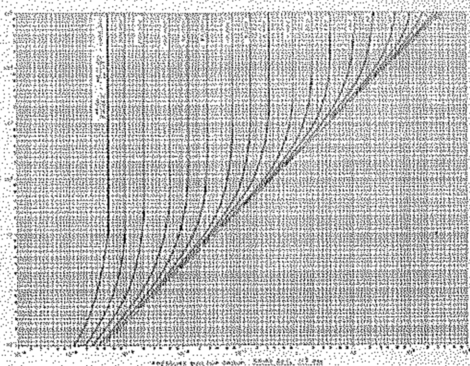


Fig. 4.6 – McKinley's type curve.

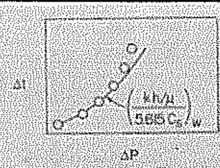


Fig. 4.7 – Early data fit on McKinley's type curve.

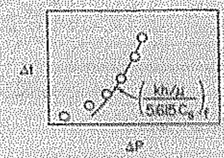


Fig. 4.8 – Later data fit on McKinley's type curve.

the McKinley type curves in the following manner (Fig. 4.9):

1. Δp^* is the vertical asymptote approached by Δp in the McKinley plot.

2. Δp_s can be calculated from Δp_d, the time at which the actual test data depart from the earliest-fitting type curve. Picking a time of departure is subjective, so no great accuracy is assured for this reason alone.

McKinley[8] states that Δp_s and Δp_d are related by

$$\Delta p_s = \left(1 - \frac{k_{wb}}{k_f}\right)\Delta p_d.$$

3. Thus, E can be calculated:

$$E = \frac{\Delta p^* - \Delta p_s}{\Delta p^*}.$$

Example 4.2 – Drawdown Test Analysis Using McKinley's Type Curves

Problem. Estimate near-well and formation permeability and flow efficiency using the data presented in Example 4.1 from a drawdown test on an oil well.

Solution. We first prepare those data, most of which lie in the time range 1 minute < t < 1,000 minutes for plotting as Δt (minutes) vs. $\Delta p = p_i - p_{wf}$ (Table 4.3). From the data plot (Fig. 4.10) and the match with the best-fitting McKinley curve for the early data,

$$\left(\frac{kh}{\mu}\right)_{wb} \times \frac{1}{5.615\,C_s} = 5{,}000.$$

We also note that the data depart from the best-fitting curve (early time) at $\Delta t_d = 100$ minutes; here, $\Delta p_d = 1{,}180$ psi.

presented in Example 4.1 from a drawdown test on an oil well.

Solution. We first prepare those data, most of which lie in the time range 1 minute < t < 1,000 minutes for plotting as Δt (minutes) vs. Δp = p_i − p_wf (Table 4.3). From the data plot (Fig. 4.10) and the match with the best-fitting McKinley curve for the early data,

(kh/μ)_wb × 1/(5.615 C_s) = 5,000.

We also note that the data depart from the best-fitting curve (early time) at Δt_d = 100 minutes; here, Δp_d = 1,180 psi.

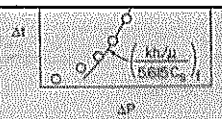


Fig. 4.8 – Later data fit on McKinley's type curve.

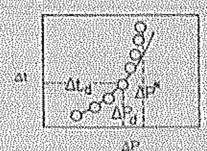


Fig. 4.9 – Data for flow efficiency calculation from McKinley's type curve.

A match point for the early fit is $\Delta p_{MP} = 107$ psi when $5.615\,\Delta p C_s / qB = 0.010$. The best fit of the later data is for $(kh/\mu)_f (1/5.615\,C_s) = 10{,}000$. Thus, from the match-point data,

$$C_s = \left(\frac{5.615\,\Delta p C_s}{qB}\right)_{MP} \left(\frac{1}{\Delta p}\right)_{MP} \times \frac{qB}{5.615}$$

$$= (0.010)\left(\frac{1}{107}\right)\left(\frac{500 \times 1.2}{5.615}\right)$$

$$= 0.01 \text{ RB/psi}.$$

The near-well transmissibility and apparent permeability, k_{wb}, are then

$$\left(\frac{kh}{\mu}\right)_{wb} = \left(\frac{kh/\mu}{5.615\,C_s}\right)_{wb} (5.615\,C_s)$$

$$= (5{,}000)(5.615)(0.010)$$

$$= 281 \text{ md-ft/cp}.$$

Then,

$$k_{wb} = \frac{(281)(0.8)}{[illegible]} = 4.01 \text{ md}.$$

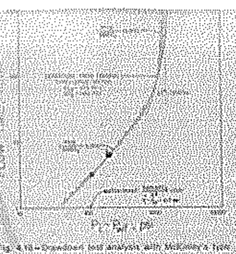


Fig. 4.10 – Drawdown test analysis with McKinley's type curve.

TABLE 4.3 – DRAWDOWN DATA FOR McKINLEY CURVE ANALYSIS

[illegible]

The formation transmissibility and permeability are

$$\left(\frac{kh}{\mu}\right)_f = \frac{(kh/\mu \times 5.615\,C_s)_f}{(kh/\mu \times 5.615\,C_s)_{wb}} \times \left(\frac{kh}{\mu}\right)_{wb}$$

$$= \frac{10{,}000}{5{,}000} \times 281$$

$$= 562 \text{ md-ft/cp},$$

and

$$k_f = \left(\frac{kh}{\mu}\right)_f \times \frac{\mu}{h}$$

$$= (562)\left(\frac{0.8}{56}\right)$$

$$= 8.03 \text{ md}.$$

Flow efficiency becomes

$\Delta p^* = 1{,}500$ psi (Fig. 4.10),

$$\Delta p_s = \left(1 - \frac{k_{wb}}{k_f}\right)\Delta p_d$$

$$\Delta p_s = \left(1 - \frac{4.01}{8.03}\right)(1{,}180) = 590 \text{ psi},$$

$$E = \frac{1{,}500 - 590}{1{,}500} = 0.607.$$

4.5 Gringarten *et al.*[7] Type Curves for Fractured Wells

Gringarten *et al.*[7] developed type curves for hydraulically fractured wells in which vertical fractures with two equal-length wings were created. The curves discussed in this section assume *uniform flux* into the fracture (same flow rate per unit cross-sectional area of fracture from wellbore to fracture tip). High fracture conductivity is required to achieve uniform flux, but this is not identical to an infinitely conductive fracture (no pressure drop from fracture tip to wellbore), as Gringarten *et al.* demonstrated.

The study was made for finite reservoirs (i.e., boundary effects become important at later times in the test). The reservoir is assumed to be at uniform pressure, p_i, initially. The type curve (Fig. 4.11), developed for a constant-rate drawdown test for a slightly compressible liquid, also can be used for buildup tests (for $\Delta t_{max} \le 0.1\,t_p$) and for gas wells* using the modifications discussed earlier. Wellbore storage effects are ignored.

All the dimensionless variables and parameters considered important are taken into account in Fig. 4.11, which is a log-log plot of p_D vs. $t_D r_w^2/L_f^2$ with parameter x_e/L_f. In these parameters, L_f is the fracture half-length and x_e is the distance from the well to the side of the square drainage area in which it is assumed to be centered. Dimensionless pressure has the usual definition,

$$p_D = \frac{kh(p_i - p_{wf})}{141.2\,qB\mu} \text{ (drawdown test)},$$

and

$$\frac{t_D r_w^2}{L_f^2} = \frac{0.000264\,kt}{\phi\mu c_t L_f^2} = t_{DL_f}. \quad (4.17)$$

Several features of Fig. 4.11 are of interest.

1. The slope of the log-log plot is 1/2 up to $t_{DL_f} \approx 0.16$ for $x_e/L_f > 1$. This is linear flow. We have shown that, in linear flow,

*[illegible]

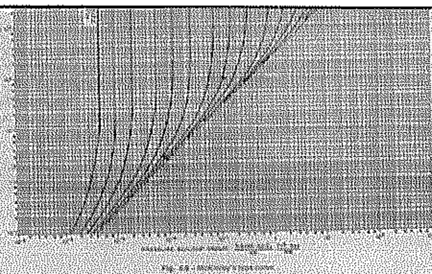


Fig. 4.6 – McKinley's type curve.

1 to 1,000 minutes or 0.01 to 10 minutes or 10^{1} to 10^{4} [illegible]

2. Match the time axis of the test data plot with one of McKinley's. Move the data along the plot horizontally (no vertical shifting allowed) until the earliest data fall along one of the type curves.

3. Record the parameter value $(kh/\mu)/5.615\,C_s$ for the matched type curve.

4. Choose a data match point (any Δp from the test graph paper) and the corresponding value of $5.615\,\Delta p C_s / qB$ from the type curves.

5. Determine the wellbore storage constant, C_s, from values of Δp and $5.615\,\Delta p C_s / qB$ at the match point: [illegible]

[illegible] shift the data plot horizontally to find another type curve that better fits the later data. A shift to a higher value of $(kh/\mu)/5.615\,C_s$ indicates damage; a shift to a lower value indicates stimulation.

6. Calculate formation transmissibility:

[illegible]

Note that we do not need a new pressure match point to redetermine C_s; C_s is found once and for all in Step 5. In fact, if only data reflecting formation transmissibility (after wellbore storage distortion has disappeared) are analyzed, error will result using the McKinley method. (However, conventional methods work here, so no problem arises.) The match point [illegible]

GRINGARTEN ET AL. TYPE CURVES FOR FRACTURED WELLS

Gringarten et al. developed type curves for hydraulically fractured wells in which vertical fractures with two equal-length wings were created.[12] The type curves were initially developed for a constant-rate drawdown test and a slightly compressible fluid; however, they also can be used for buildup tests when $\Delta t_{max} \le 0.1 t_p$ and for gas wells (as discussed in the previous sections) with the exception of some gas wells that exhibit time-dependent non-Darcy flow in the fracture. Gringarten et al. type curves are presented in Figures 7–20 through 7–22.

A match of actual test data with the appropriate curves in Figures 7–20 through 7–22 will show that portion of the data which can be analyzed for reservoir properties using conventional Horner analysis.

The procedure for matching the actual test data with the type curves is the same as previously discussed. Once the data are matched and the match points $[(p_D)_{MP}, (p_i - p_{wf})_{MP}]$ and $[(t_{Df})_{MP}, (\Delta t)_{MP}]$ obtained, then the following equations are used to estimate permeability and fracture length:

$$k = 141.2 \frac{Q \mu B}{h} \frac{(p_D)_{MP}}{(\Delta p)_{MP}} \quad (7.92)$$

$$x_f = \left[\frac{0.0002637 k (\Delta t)_{MP}}{\phi \mu c_t (t_{Df})_{MP}}\right]^{0.5} \quad (7.93)$$

Note that as mentioned before, if all test data fall on the half-slope line on the log-log plot of Δp vs. Δt (or the straight line in the Δp vs. $\sqrt{\Delta t}$ plot), then type curve matching or Horner analysis will not be valid. In such cases, as previously discussed, only the upper limit of the permeability and lower limit of the fracture length can be calculated.

The reservoir properties obtained from curve matching can sometimes be verified by (1) the conventional methods, if a radial flow regime appears (before the data deviate from the $x_e/x_f = \infty$ curve, i.e., before boundary effects become important), and (2) using the slope (m_{vf}) of this line and Equation (7.37) to calculate $x_f \sqrt{k}$ (if the straight line portion on a plot of Δp vs. $\sqrt{\Delta t}$ is present) which should agree with the result from the type curve analysis.

Example 7–24. Pressure buildup test data for a well believed to be vertically fractured are given in Table 7–13. Other pertinent well and reservoir data are given. Use Gringarten et al.'s type curves to calculate permeability and fracture length. Compare your results with the values calculated using Horner method and the Δp vs. $\sqrt{t}$ plot.

TABLE 7–13. Pressure buildup data for a vertically fractured well

Δt (hours)	p_{ws} (psi)	$(t_p+\Delta t)/\Delta t$	Δp (psi)	$\sqrt{\Delta t}$ (hours)$^{1/2}$
0	3,420			
0.083	3,431	235,364.6	11	0.289
0.167	3,435	46,707.58	15	0.409
0.250	3,438	31,201.00	18	0.500
0.500	3,444	15,601.00	24	0.707
0.750	3,449	10,401.00	29	0.866
1.000	3,452	7,801.00	32	1.000
2.000	3,463	3,901.00	43	1.414
3.000	3,471	2,601.00	51	1.732
4.000	3,477	1,951.00	57	2.000
5	3,482	1,561.00	62	2.236
6	3,486	1,301.00	66	2.449
7	3,490	1,115.28	70	2.646
8	3,495	976.00	75	2.828
9	3,498	867.60	78	3.000
10	3,500	781.00	80	3.162
12	3,506	651.00	86	3.464
24	3,528	326.00	108	4.899
36	3,544	217.66	124	6.000
48	3,555	163.50	135	6.928
60	3,563	131.00	143	7.746
72	3,570	109.33	150	8.485
96	3,582	82.25	162	9.798
120	3,590	66.00	170	10.954
144	3,600	55.16	180	12.000
192	3,610	41.62	190	13.856
240	3,620	33.50	200	15.492

$B_o = 1.26$ RB/STB $A = 1,600$ ac
$q_o = 419$ STB/D $\phi = 12\%$
$\mu_o = 0.65$ cp $r_w = 0.28$ ft
$c_t = 21 \times 10^{-6}$ 1/psi $h = 82$ ft
$t_p = 7,800$ hours

Solution: The first steps in analyzing this type of problem are

1. Prepare a plot of Δp vs. $\sqrt{\Delta t}$ as shown in Figure 7–55. The plot shows a straight line portion with a slope $m_{vf} = 23.63$.
2. Prepare a log-log plot of Δp vs. Δt as shown in Figure 7–56. The plot shows some data falling on half-slope line confirming linear flow.
3. Prepare a Horner plot as shown in Figure 7–57.
4. Prepare a plot (on tracing paper) for type curve matching, obtain a match and the match points. The matched data are shown in Figure 7–58.

The match points from Figure 7–58 are

$(\Delta t)_{MP} = 1.0$ $(p_D)_{MP} = 13.4$
$(\Delta p)_{MP} = 1,000$ $(t_{Df})_{MP} = 0.050$

$$k = 141.2 \frac{q_o \mu_o B_o}{h} \frac{(p_D)_{MP}}{(\Delta p)_{MP}}$$
$$= 141.2 \frac{(419)(0.65)(1.26)(13.4)}{(82)(1,000)}$$
$$= 7.92 \text{ md}$$

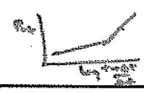
$$x_f = \left[\frac{0.0002637 k (\Delta t)_{MP}}{\phi \mu_o c_t (t_{Df})_{MP}}\right]^{0.5}$$
$$= \left[\frac{(0.0002637)(7.92)(1.0)}{(0.12)(0.65)(21 \times 10^{-6})(0.050)}\right]^{0.5} = 159.74 \text{ ft}$$

$x_f \sqrt{k} = 449.66$

Using a Horner plot, the characteristics of the straight line portion of the data are

Slope $m = 92.82$ psi/cycle
Extrapolated pressure $p^* = 3,760.37$ psi
Pressure at $\Delta t = 1$ hr $p_{1hr} = 3,399.12$ psi

$$k_{est} = 162.6 \frac{q_o \mu_o B_o}{mh}$$
$$= 162.6 \frac{(419)(0.65)(1.26)}{(92.82)(82)}$$
$$= 7.331 \text{ md}$$



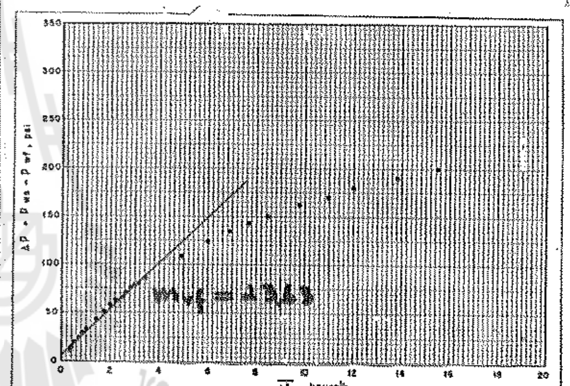


Figure 7–55. Plot of Δp vs. $\sqrt{\Delta t}$ for data in Table 7–13 for Example 7–24.

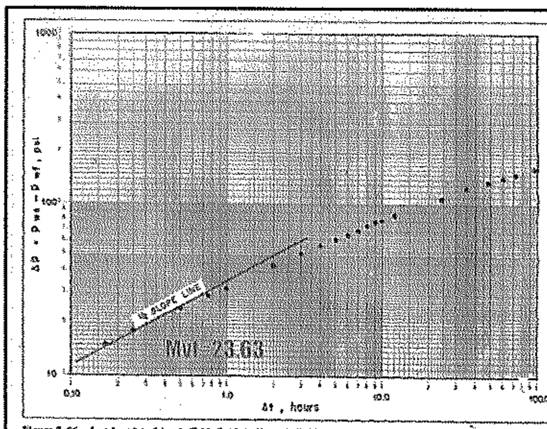


Figure 7–56. Log-log plot of data in Table 7–13 for Example 7–24.

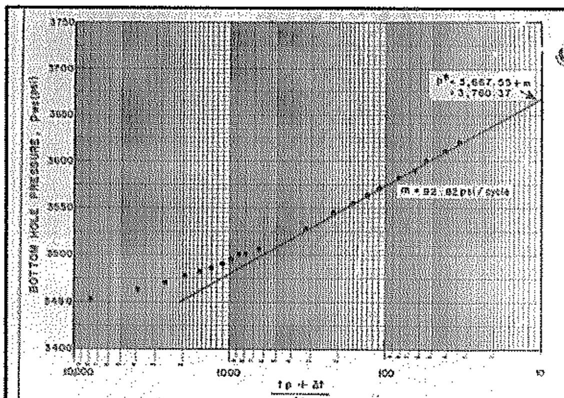


Figure 7–57. Horner plot of data in Table 7–13 for Example 7–24.

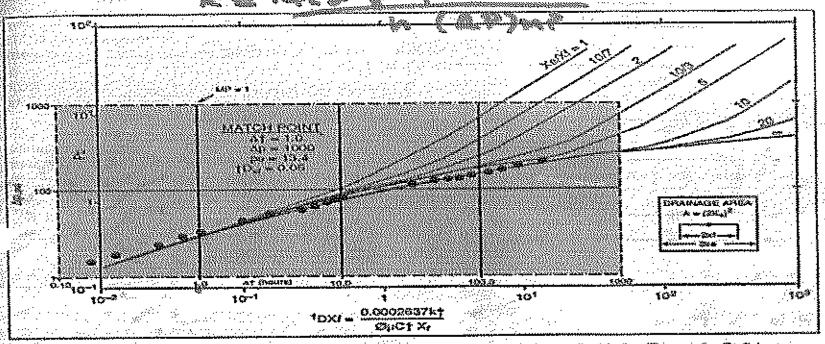


Figure 7–58. Gringarten et al. type curve with matched data of Table 7–13 for Example 7–24.

F_{cor}=0.82 from Figure 7–19, $k = k_{cor}F_{cor}$ = 7.33 × 0.82=6 md.

$$x_f = \frac{0.3187}{m_{vf}}\sqrt{\frac{mq_oB_o}{\phi c_t h F_{cor}}}$$

$$= \frac{0.3187}{23.63}\left[\frac{(92.82)(419)(1.26)}{(0.12)(21\times10^{-6})(82)(0.82)}\right]^{0.50}$$

$$= 229.36 \text{ ft}$$

$$x_f\sqrt{k} = \frac{4.046q_oB_o}{hm_{vf}}\sqrt{\frac{\mu}{\phi c_t}}$$

$$= \frac{(4.064)(419)(1.26)}{(82)(23.63)}\left[\frac{0.65}{(0.12)(21\times10^{-6})}\right]^{0.50}$$

$$= 562.36$$

Horner

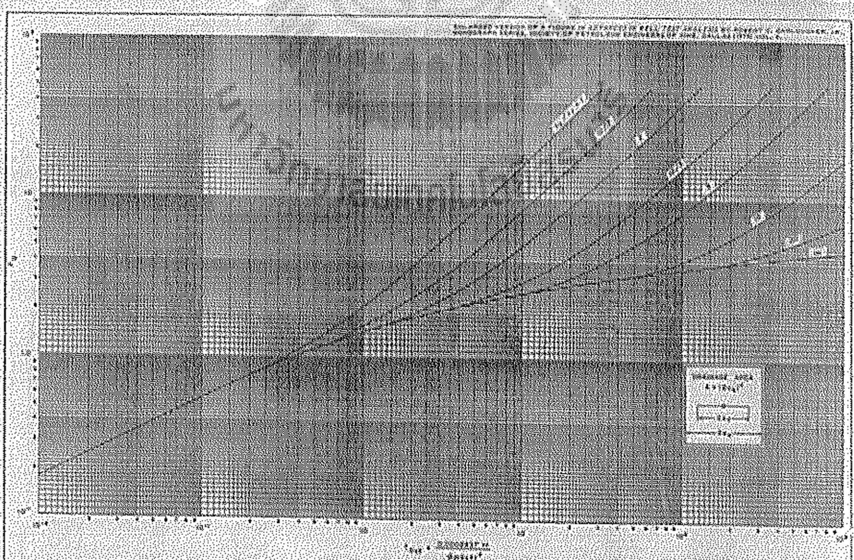


Figure 7-21. Dimensionless pressure for vertically fractured well in the center of a closed square, no wellbore storage, uniform-flux fracture (© SPE, SPE Monograph, Vol. 5, 1977).[14]

$$p_i - p_{wf} = ct^{1/2},$$

or

$$p_D = c' t_{DL_f}^{1/2}.$$

Then,

$$\log p_D = \log c' + 1/2 \log t_{DL_f}.$$

2. Although not apparent on a log-log plot, a semilog plot of the data in Fig. 4.11 (p_D vs. log t_{DL_f}) is a straight line, signifying radial flow when, for $x_e/L_f > 5$, $t_{DL_f} \approx 2$. The straight-line terminates, of course, when boundary effects become important, but a match of actual test data with Fig. 4.11 can show the amount of data in the radial flow region (and, thus, can be analyzed for permeability by the conventional p_{wf} vs. log t or Horner plots). Fig. 4.11 thus combines, in a single graph, the linear flow and radial flow regions (and a region with neither), boundary effects, and the effect of various fracture lengths. *If* fracture conductivity is high and constant throughout the test and *if* wellbore storage has negligible effect on earliest data, then this figure allows a rather complete analysis of a hydraulically fractured well – specifically, estimation of fracture length, L_f, and formation permeability, k. The method is frequently superior to the nontype-curve methods discussed in Chap. 2.

Steps in use of Fig. 4.11 as a type curve for test analysis include the following.

1. Plot $(p_i - p_{wf})$ (drawdown test) or $(p_{ws} - p_{wf})$ (buildup test) on the ordinate vs. t (drawdown test) or Δt_e (buildup test) on the abscissa on a 3×5 cycle log-log paper if an undistorted version of the type curve is available. Otherwise, use tracing paper.

2. Select the best match by sliding the actual test data plot both horizontally and vertically.

3. Note the value of the match points $[(p_D)_{MP}, (p_i - p_{wf})_{MP}]$ and $[(t_{DL_f})_{MP}, t_{MP}]$.

4. Estimate formation permeability from the pressure match point:

$$k = 141.2 \frac{qB\mu}{h} \frac{(p_D)_{MP}}{(p_i - p_{wf})_{MP}}.$$

5. Estimate fracture length from the time match point:

$$L_f = \left[\frac{0.000264\, k t_{MP}}{\phi \mu c_t (t_{DL_f})_{MP}} \right]^{1/2}. \quad \text{..........(4.18)}$$

Three useful checks are sometimes possible:

1. If a half-slope (linear flow) region appears on the test data plot, replot data from the region as p_{wf} (or p_{ws}) vs. $\sqrt{t}$ (or $\sqrt{\Delta t}$); from the slope m_L and linear flow theory,

$$L_f \sqrt{k} = \frac{4.064\, qB}{h m_L} \sqrt{\frac{\mu}{\phi c_t}},$$

which should agree with the result from the type-curve analysis.

2. If a radial flow region appears (before boundary effects become important – that is, before the data deviate from the $x_e/L_f = \infty$ curve), a plot of p_{wf} vs. log t [p_{ws} vs. log Δt or log $(t_p + \Delta t)/\Delta t$] should show that $k = 162.6\, qB\mu/mh$, in agreement with type-curve analysis.

3. If a well proves to be in a finite-acting reservoir, it may be possible to estimate x_e from a matching parameter, x_e/L_f, to compare with the known (or assumed) value of x_e to check the quality of the match.

Example 4.3 – Buildup Test Analysis for a Vertically Fractured Well

Problem. Gringarten *et al.*[13] presented buildup test data for a well believed to be fractured vertically. From these data, presented below and in Table 4.4, estimate fracture length and formation permeability. Producing time, t_p, was significantly greater than maximum shut-in time, so that $\Delta t \cong \Delta t_e$.

q = 2,750 STB/D,
μ = 0.23 cp,
B = 1.76 RB/STB,
h = 230 ft,
ϕ = 0.3, and
c_t = 30×10^{-6} psi^{-1}.

Solution. Fig. 4.12 is a plot of $\Delta p = p_{ws} - p_{wf}$ vs. Δt. An adequate fit is characterized by the match points (t=0.062 hour, t_{DL_f}=0.01) and (Δp=15.2 psi, p_D=0.1). From the pressure match point,

$$k = 141.2 \frac{qB\mu}{h} \frac{(p_D)_{MP}}{(\Delta p)_{MP}}$$

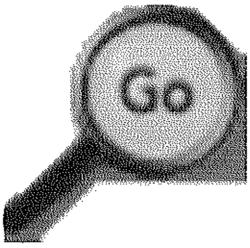
$$= \frac{(141.2)(2{,}750)(1.76)(0.23)(0.1)}{(230)(15.2)}$$

$$= 4.5 \text{ md}.$$

TABLE 4.5 – CONSTANT-RATE DRAWDOWN TEST DATA

t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)	t (hours)	p_{wf} (psi)
0.0109	2,976	0.218	2,611	3.28	1,712	32.8	1,543
0.0164	2,964	0.273	2,536	3.82	1,696	38.2	1,533
0.0218	2,953	0.328	2,469	4.37	1,684	43.7	1,525
0.0273	2,942	0.382	2,408	4.91	1,674	49.1	1,517
0.0328	2,930	0.437	2,352	5.46	1,665	54.6	1,511
0.0382	2,919	0.491	2,302	6.55	1,651	65.5	1,500
0.0437	2,908	0.546	2,256	8.74	1,630	87.4	1,482
0.0491	2,897	1.09	1,952	10.9	1,614	109.2	1,468
0.0546	2,886	1.64	1,828	16.4	1,587	163.8	1,440
0.109	2,785	2.18	1,768	21.8	1,568	218.4	1,416
0.164	2,693	2.73	1,734	27.3	1,554	273.0	1,393
						327.6	1,370

CHAPTER 7



- INTERFERENCE TESTS

ANALYSIS OF WELL INTERFERENCE TESTS

$$p(r,t) = p_i - \frac{70.6 q\mu B}{kh}\left[-E_i\left(-\frac{\phi\mu c_t r^2}{0.00105\,kt}\right)\right] \quad (7.36)$$

7.1 Reasons for Interference Tests

When one well is closed in and its pressure is measured while others in the reservoir are produced, the test is termed an interference test. The name comes from the fact that the pressure drop caused by the producing wells at the closed-in observation well "interferes with" the pressure at the observation well. This type of test can give information on reservoir properties which cannot be obtained from ordinary pressure build-up or drawdown tests. First of all, one can determine reservoir connectivity. Is the portion of the reservoir at this well location being drained by other wells? How rapidly? An interference test can determine this.[1-4] Another important use of interference tests is to determine directional reservoir flow patterns. This is done by selectively opening wells surrounding the shut-in well.

In addition to this qualitative information, it is possible to obtain a quantitative estimation of connected porosity from such a test. Porosity cannot be estimated from a pressure buildup test alone. Elkins[4] has also used interference tests to determine the nature and magnitude of an anisotropic directional reservoir permeability. Groundwater hydrologists[5,6] have made much more use of interference tests than have petroleum engineers. Muskat[7] shows an example of their work.

7.2 Equations for Pressure Interference

The mathematical basis for interference tests was first presented by Theis[8] in 1935.* The following method uses the same basic equations but differs in treatment and method of analysis. This method is based on superposition of the effects of each of the producing wells at

$$p_{ws} = p^* - 162.6\,\frac{q\mu B}{kh}\log\left(\frac{t+\Delta t}{\Delta t}\right) + 70.6\,\frac{q\mu B}{kh}$$
$$\left[\sum_{j=1}^{NW}\frac{q_j}{q}\left\{Ei\left(\frac{-\phi\mu c a_j^2}{0.00105k\,(t_j+\Delta t_j)}\right) - Ei\left(\frac{-\phi\mu c a_j^2}{0.00105kt_j}\right)\right\}\right] \quad (7.1)^{**}$$

where

q = the production rate at our observation well before it was closed in
q_j = the rate of production at Well j
t_j = producing time of j^{th} interfering well prior to shut-in of the observation well
Δt_j = producing time interval of the j^{th} interfering well subsequent to shut-in of the observation well
NW = number of interfering wells
a_j = distance of the j^{th} interfering well from the observation well.

All the quantities in Eq. 7.1 are in oilfield units.

The log term in Eq. 7.1 gives the effect of producing and shutting in the observation well itself. The Ei terms give the pressure drop at the observation well caused by production at Wells 1, 2, 3, . . . at distances a_1, a_2, a_3 . . . .

If a reservoir boundary is close by, it may be taken into account by the method of images to be discussed in Chapter 10. The "image terms" are exactly like the Ei-function terms in Eq. 7.1, there being one such term for each "image" well. The distance a_j in this case is the distance from the image well to the observation well.

For use in Eq. 7.1 we obtain times t, t_j, Δt_j, etc., by

7.1 Reasons for Interference Tests

When one well is closed in and its pressure is measured while others in the reservoir are produced, the test is termed an interference test. The name comes from the fact that the pressure drop caused by the producing wells at the closed-in observation well "interferes with" the pressure at the observation well. This type of test can give information on reservoir properties which cannot be obtained from ordinary pressure buildup or drawdown tests. First of all, one can determine reservoir connectivity. Is the portion of the reservoir at this well location being drained by other wells? How rapidly? An interference test can determine this.[1-3] Another important use of interference tests is to determine directional reservoir flow patterns. This is done by selectively opening wells surrounding the shut-in well.

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The mathematical basis for interference tests was first presented by Theis[8] in 1935.* The following method uses the same basic equations but differs in treatment and method of analysis. This method is based on superposition of the effects of each of the producing wells at the shut-in observation well. Using the *Ei*-function solution of Chapter 2 (Eq. 2.31), we find that the pressure at the closed-in observation well due to continued production at Wells 1, 2, 3, etc., is given by:

$$p_{ws} = p^* - 162.6\,\frac{q\mu B}{kh}\log\left(\frac{t+\Delta t}{\Delta t}\right) + 70.6\,\frac{q\mu B}{kh}$$

$$\left[\sum_{i=1}^{NW}\frac{q_i}{q}\left\{Ei\left(\frac{-\phi\mu c a_i^2}{0.00105k\,(t_i+\Delta t_i)}\right)\right.\right.$$

$$\left.\left.-Ei\left(\frac{-\phi\mu c a_i^2}{0.00105 k t_i}\right)\right\}\right] \quad (7.1)^{**}$$

where

- q = the production rate at our observation well before it was closed in
- q_i = the rate of production at Well i
- t_i = producing time of i^{th} interfering well prior to shut-in of the observation well
- Δt_i = producing time interval of the i^{th} interfering well subsequent to shut-in of the observation well
- NW = number of interfering wells
- a_i = distance of the i^{th} interfering well from the observation well.

All the quantities in Eq. 7.1 are in oilfield units.

The log term in Eq. 7.1 gives the effect of producing and shutting in the observation well itself. The *Ei* terms give the pressure drop at the observation well caused by production at Wells 1, 2, 3, . . . at distances a_1, a_2, a_3 . . .

If a reservoir boundary is close by, it may be taken into account by the method of images to be discussed in Chapter 10. The "image terms" are exactly like the *Ei*-function terms in Eq. 7.1, there being one such term for each "image" well. The distance a_i in this case is the distance from the image well to the observation well.

For use in Eq. 7.1 we obtain times t, t_i, Δt_i, etc., by the same type of equation as in Chapter 3.

$$t = \frac{\text{cumulative production at observation well}}{\text{rate of production just before closing in}}$$

*In this same paper Theis also developed a pressure buildup equation similar to Eq. 3.4. However, pressure buildup methods never gained great popularity with groundwater hydrologists.

**Eqs. 7.1 through 7.3 apply to a more general case than the corresponding equations in the original monograph. The authors are indebted to Raj K. Prasad for the derivation.

$$t_1 = \frac{\text{cumulative production at Well 1 prior to shut-in of observation well}}{\text{average rate of production } q_1 \text{ during interference test}}$$

$$\Delta t_1 = \frac{\text{incremental production at Well 1 subsequent to shut-in of observation well}}{\text{average rate of production } q_1 \text{ during interference test}}$$

and similarly for t_2, Δt_2, etc. These equations for t_1, Δt_1, etc., apply best when the rates of production at these wells are reasonably constant during the interference test. We have already implied that this is the case by using only one *Ei* term to represent each producing well in Eq. 7.1. If the rate at a producing well varies considerably during the test, a series of *Ei* functions should be used in Eq. 7.1 to represent the rate at that well; that is, the method of superposition (see Chapter 2, Section 2.8) should be employed rather than the above equations for t_i and Δt_i. Eq. 7.1 is written for single-phase flow conditions above the bubble point. For two-phase flow below bubble point, total mobilities and compressibilities should be used as in pressure buildup examples 2A and 2.

Using Eq. 7.1, it is possible to determine the quantity $(\phi\mu c/k)$. The value of this quantity which, by trial and error, gives the best fit between observed and calculated values of the pressure drop at the observation well, best represents these quantities in the reservoir between the interfering wells.

An example interference test is shown in Fig. 7.1. The dotted line in this figure, called "Extrapolated Buildup Pressure", was obtained by extrapolating the linear portion of the log plot as shown on Fig. 7.2. From Eq. 7.1 we see that the difference between this extrapolated curve and the observed pressure is the sum of the *Ei* functions, or

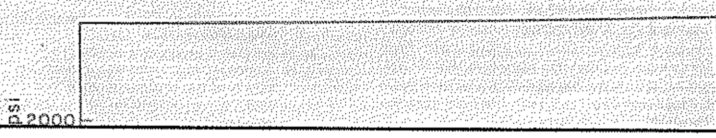
$$\left(p^* - 162.6\,\frac{q\mu B}{kh}\log\frac{t+\Delta t}{\Delta t}\right) - p_{ws}$$

$$= -70.6\,\frac{q\mu B}{kh}\left[\sum_{i=1}^{NW}\right.$$

$$\cdot\frac{q_i}{q}\left\{Ei\left(\frac{-\phi\mu c a_i^2}{0.00105k\,(t_i+\Delta t_i)}\right)\right.$$

$$\left.\left.-Ei\left(\frac{-\phi\mu c a_i^2}{0.00105 k t_i}\right)\right\}\right] \quad (7.2)$$

Since the first two terms on the left side of the equation represent the straight-line extrapolation on Fig. 7.2 and the third term, p_{ws}, represents the observed pressure, this may be rewritten as



and error, gives the best fit between observed and calculated values of the pressure drop at the observation

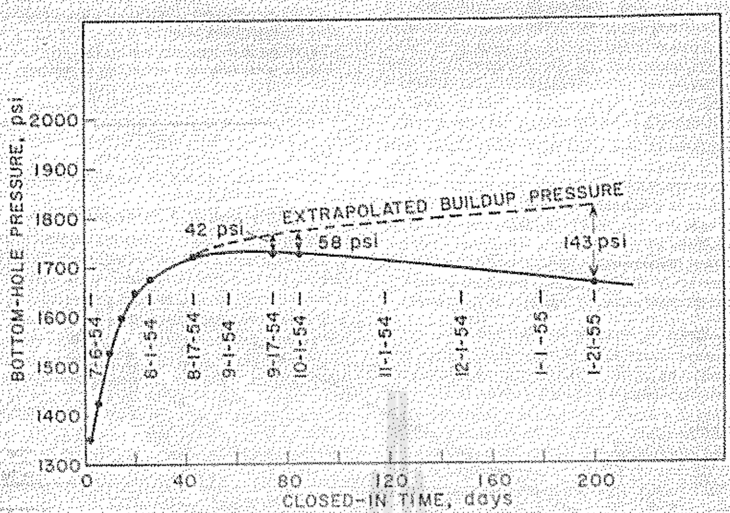


Fig. 7.1 Interference test in a low-permeability reservoir.

$$p_{ws} - p_{obs} = \frac{-m}{2.303}\left[\sum_{i=1}^{NW}\frac{q_i}{q}\left\{Ei\left(\frac{-\phi\mu c a_i^2}{0.00105k(t_i+\Delta t_i)}\right) - Ei\left(\frac{-\phi\mu c a_i^2}{0.00105kt_i}\right)\right\}\right] \quad (7.3)$$

The terms on the right-hand side of this equation represent the calculated pressure drop at the observation well due to production at Wells 1, 2, 3, . . . This will be made clear by the following example calculation.

7.3 Example Calculation, Interference Test

Calculate the pressure drop at the observation well caused by production at Well 1 at the time when incremental production at Well 1 subsequent to shut-in is 23,050 bbl. There is no production prior to shut-in and thus $t_i = 0$.

Data

q = 140 B/D, rate at observation well prior to shut-in,

m = 270 psi/cycle, for buildup curve in observation well (from Fig. 7.2),

B = 1.1, formation volume factor,

$c = 6.9 \times 10^{-6}$ psi^{-1}, compressibility,

q_i = 180 B/D, average rate at Well 1 during test,

$$\Delta t_i = \frac{\text{incremental production at Well 1 subsequent to shut-in}}{\text{average rate at Well 1}} = \frac{23{,}050}{180} = 128 \text{ days} = 3{,}070 \text{ hours},$$

a_i = 1,835 ft,

and assume that $\phi\mu c/k = 10^{-6}$.

Calculation

The calculated pressure drop at the observation well caused by production at Well 1 is, from Eq. 7.3,

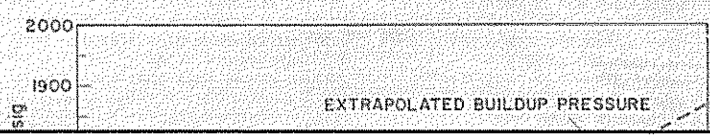
$$\Delta p = \frac{-270}{2.303}\left[\frac{180}{140}Ei\left(\frac{-10^{-6}(1835)^2}{0.00105(3070)}\right)\right],$$

$$= -117.2\,[1.285\,Ei\,(-1.042)],$$

$$= -117.2\,[1.285\,(-0.21)],$$

$$= 32 \text{ psi}.$$

Similarly, one can calculate the effect of production at other wells—Wells 2, 3, 4—as a function of time and for several values of $\phi\mu c/k$. Fig. 7.3 shows results calculated as above for the effect of four wells surrounding the observation well. Wells 1 and 2 began producing at the time the observer was closed in. Note that their influence was not appreciably felt at rates of 125 to 180 B/D until 60 days had passed. Wells 3 and 4 began producing at rates of 20 to 40 B/D 80 days



q = 140 B/D, rate at observation well prior to shut-in,

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B = 1.1, formation volume factor,

c = 6.9 × 10^{-6} psi^{-1}, compressibility,

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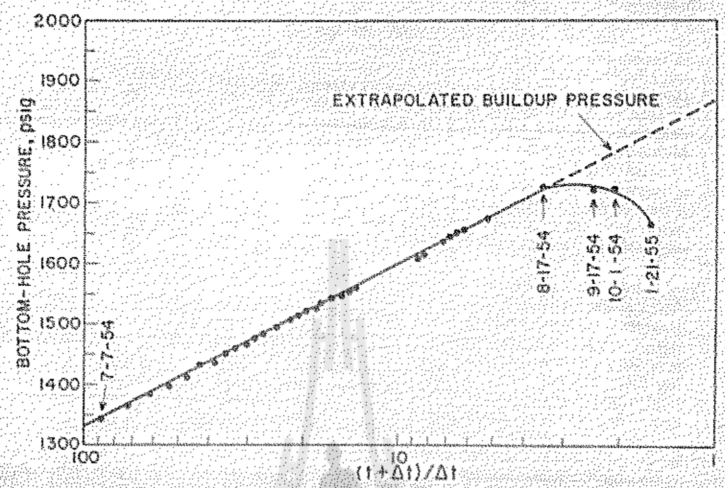


Fig. 7.2 Interference test in a low-permeability reservoir, log-time plot.

after the observer was closed in. Their influence was not felt until about 80 days after they began producing and, because of their low rates, their influence was small.

The assumed value of $\phi\mu c/k$ which gave the agreement shown is $\phi\mu c/k = 3.5 \times 10^{-7}$. From Eq. 3.5 we have

$$\frac{kh}{\mu} = \frac{162.6\,qB}{m} = \frac{162.6\,(140)\,1.1}{270} = 92.6.$$

Then

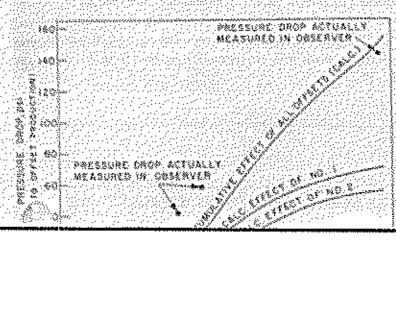
$$\phi h = \frac{\phi\mu c}{k}\left(\frac{kh}{\mu}\right)\frac{1}{c} = 3.5\times10^{-7}\,(92.6)\,\frac{1}{6.9\times10^{-6}}$$
$$= 4.70.$$

For

h = 43 ft,

ϕ = 0.11.

Note from this example that a very long closed-in time may be required to see the effects of interference. A minimum of two months was necessary in this case where the permeability is a few millidarcies. Note also that it was not necessary to find k or μ to find ϕh.



7.4 Least-Squares Methods

In the above example the "best" value of $\phi\mu c/k$ was the value which gave the best fit to the three plotted points when judged by eye. A more precise method of obtaining $\phi\mu c/k$ is to use the method of least squares.

To use this method, first measure $p_{ext} - p_{obs}$ (from a plot such as that of Fig. 7.2) for each data point such as the three shown in Fig. 7.3. Call this difference Δp_{obs}. Then calculate $p_{ext} - p_{obs}$ at each point for several values of $\phi\mu c/k$, using Eq. 7.3 as in the above example, and summing the effects of all the producers as in Fig. 7.3. Call this total calculated pressure drop caused by all wells at the observer Δp_{calc}. Compute $(\Delta p_{obs} - \Delta p_{calc})^2$ for each measured point such as the three in Fig. 7.3. Plot a curve of $\Sigma(\Delta p_{obs} - \Delta p_{calc})^2$ vs $\phi\mu c/k$. The value of $\phi\mu c/k$ which gives a minimum in this curve is the least-squares choice for $\phi\mu c/k$.

Thus far in this chapter we have assumed that a value for μ/kh was available from a buildup test in our observation well. If this is not so, it is possible to obtain both μ/kh and $\phi\mu c/k$ from an interference test, as shown by Morris and Tracy.[4] Their paper should be consulted for details.

7.5 Other Methods for Computing Interference

The methods presented thus far in this chapter assume a homogeneous reservoir. A more sophisticated method has been developed by Jacquard and Jain[8] (see also Jahns[18]), which takes into account variations in permeability in the reservoir. The method requires use of a digital computer to determine, by successive approximations, the modification in permeability distribution required to give a best fit to observed pressures at wells. Single-phase flow is assumed. Thomere and Arthur[19] have developed a method for computing an interference "function" and a well "function"

$$p_i - p_r = -70.6 \frac{qB\mu}{kh} Ei\left(-948 \frac{\phi \mu c_t r^2}{kt}\right). \quad \ldots (6.1)$$

$$\frac{p_i - p_r}{\left(141.2 \frac{qB\mu}{kh}\right)} = -\frac{1}{2} Ei \left[\left(-\frac{1}{4}\right) \left(\frac{\phi \mu c_t r_w^2}{0.000264\, kt}\right) \left(\frac{r}{r_w}\right)^2 \right],$$

or

$$p_D = -\frac{1}{2} Ei \left(\frac{-r_D^2}{4 t_D}\right), \quad \ldots \ldots \ldots \ldots (6.2)$$

where

$$p_D = \frac{(p_i - p_r)\, kh}{141.2\, qB\mu}, \quad \text{and}$$

$$r_D = r/r_w, \qquad t_D = \frac{0.000264\, kt}{\phi \mu c_t r_w^2}.$$

expected from a w

INTERFERENCE TESTING

Interference tests are used (1) to determine whether two or more wells are in pressure communication and (2) to provide estimates of permeability in the vicinity of the tested well when communication exists. Interference tests are also used in vertical-well testing to determine vertical permeability and vertical pressure communication between two reservoirs.

For interference testing, at least two wells are required. The test is conducted by producing from or injecting into one of these wells (*active well*) and the pressure response is observed in the other well (*observation well*). The active well starts producing at uniform pressure at time zero, and the pressure response in the observation well at a distance r from active well begins after some time lag.

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Interference tests are not affected by skin factor as long as the skin is concentrated directly around the active well. However, the observation well's response can be affected by a large negative skin or a fracture. The most common method for interference test analysis is using type curves. Figure 7–63 illustrates the type curve that is used for interference test analysis. The following steps are used to analyze erference test:

1. Plot pressure drawdown in an observation well, $\Delta p = p_i - p_R$ vs. elapsed time t on tracing paper using the grid of Figure 7–63.
2. Slide the plotted test data over the type curve (vertically and horizontally) until a match is found.
3. Record the pressure and time match points, $(p_D)_{MP}$, Δp_{MP}, and $[(t_D/r_D^2)_{MP}, t_{MP}]$.
4. Calculate permeability in the test region using the pressure match points and the following equation:

$$k = 141.2 \frac{q\mu B}{h} \frac{(p_D)_{MP}}{(\Delta p)_{MP}} \quad (7.114)$$

5. Calculate ϕc_t from the time match points using the following equation:

$$\phi c_t = \left(\frac{0.0000264 k}{\mu r^2}\right) \left[\frac{t_{MP}}{(t_D/r_D^2)_{MP}}\right] \quad (7.115)$$

where r is the distance between the two wells.

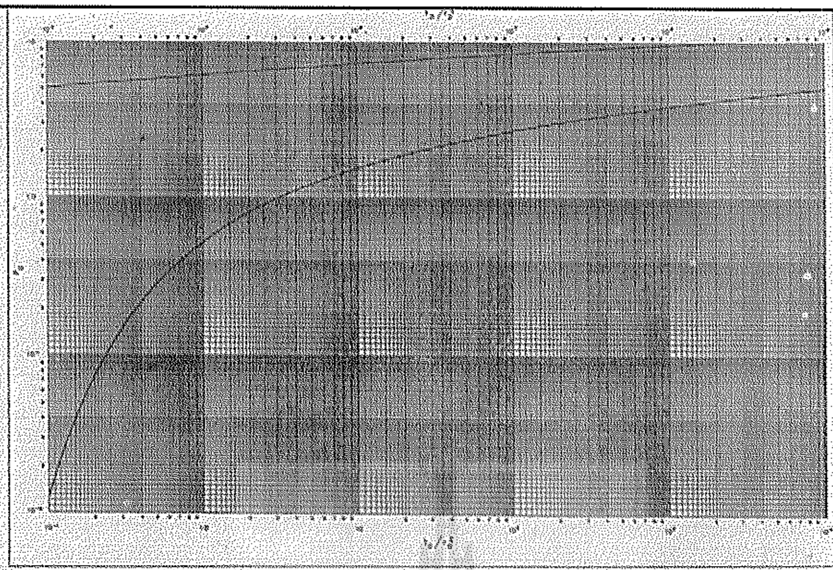


Figure 7–63. Dimensionless pressure for a single well in an infinite system, no wellbore storage, no skin. Exponential integral solution (© SPE, SPE Monograph, Vol. 5, 1977).[2]

Problem

After the analyses are complete, another technique can be used to check if the match was satisfactory; this technique utilizes the pressure changes at the observation well after the active well is shut in at time t_1. After the change in rate, the difference between the extrapolated matched type curve pressure, $p_{w_{ext}}$, and the actual data points, $\Delta p_{\Delta t}$, is determined from the data plot; Δt is the time from the change in rate at the active well to the time the pressure is recorded at the observation well. The $\Delta p_{\Delta t}$ is then plotted vs. Δt on the same data plot. If the points fall on the matched curve, the analyses are correct.

If they do not, then either the original match was incorrect or there is some other problem that is influencing the test.

Example 7–30.[2] During an interference test, water was injected into Well *A* for 48 hours. The pressure response in Well *B* 119 feet away was observed for 148 hours. The observed pressure data are given in Table 7–20, and the known reservoir properties are given. Calculate the permeability and porosity of the formation between the two tested wells.

$q_w = -170$ STB/D	$B_w = 1.0$ RB/STB
$h = 45$ ft	$\mu_w = 1.0$ cp
$p_i = 0$ psig	$r = 119$ ft
$t_1 = 48$ hours	$c_t = 9.0 \times 10^{-6}$ 1/psi

$q_w = 170$ STB/D

TABLE 7–20. Interference test data for observation well in Example 7–30[2]

t (hours)	p_w (psig)	$\Delta p = p_i - p_w$ (psig)	$\Delta t = t - 48$ (hours)	$\Delta p_{w_{ext}}$ (psi)	$\Delta p_{\Delta t} = \Delta p_{w_{ext}} - \Delta p_w$ (psi)
0	$0 = p_i$				
4.3	22	−22			

Example 7-30.[2] During an interference test, water was injected into Well A for 48 hours. The pressure response in Well B 119 feet away was observed for 148 hours. The observed pressure data are given in Table 7-20, and the known reservoir properties are given. Calculate the permeability and porosity of the formation between the two tested wells.

$$q_w = -170 \text{ STB/D} \qquad B_w = 1.0 \text{ RB/STB}$$
$$h = 45 \text{ ft} \qquad \mu_w = 1.0 \text{ cp}$$
$$p_i = 0 \text{ psig} \qquad r = 119 \text{ ft}$$
$$t_1 = 48 \text{ hours} \qquad c_t = 9.0\times10^{-6} \text{ 1/psi}$$

TABLE 7-20. Interference test data for observation well in Example 7-30[2]

t (hours)	p_w (psig)	$\Delta p = p_i - p_w$ (psig)	$\Delta t = t - 48$ (hours)	$\Delta p_{w_{ext}}$ (psi)	$\Delta p_M = \Delta p_{w_{ext}} - \Delta p_w$ (psi)
0	$0 = p_i$				
4.3	22	−22			
21.6	82	−82			
28.2	95	−95			
45.0	119	−119			
48.0	Injection		Ends		
51.0	109	−109	3	125	16
69.0	55	−55	21	139	84
73.0	47	−47	25	141	94
93.0	32	−32	45	157	125
142.0	16	−16	94	180	164
148.0	15	−15	100	181	166

Solution: The data in Table 7-20 are matched with the type curve in Figure 7-63. The match is shown in Figure 7-64 with the match points as follows:

$$t_{MP} = 100 \qquad (p_D)_{MP} = 0.92$$
$$(\Delta p)_{MP} = 100 \qquad (t_D/r_D^2)_{MP} = 8.80$$

$$k = 141.2\frac{q_w \mu_w B_w}{h}\frac{(p_D)_{MP}}{(\Delta p)_{MP}}$$
$$= 141.2\frac{(170)(1.0)(0.92)(1.0)}{(45)(100)}$$
$$= 4.91 \text{ md}$$

$$\phi c_t = \left(\frac{0.000264k}{\mu_w r^2}\right)\left[\frac{t_{MP}}{(t_D/r_D^2)_{MP}}\right]$$
$$= \frac{(0.000264)(4.91)(100)}{(1.0)(119)^2(8.8)}$$
$$= 1.04\times10^{-6}$$
$$\phi = \frac{\phi c_t}{c_t} = \frac{1.04\times10^{-6}}{9\times10^{-6}}$$
$$= 11.6\%$$

The accuracy of the above analysis is estimated by plotting $(p_{w_{ext}} - p_w)$ vs. Δt points on Δp vs. t plot as shown in Figure 7-64. Since the $(p_{w_{ext}} - p_w)$ vs. Δt (plus symbol) data fall on the curve, the analyses are correct.

PULSE TESTING

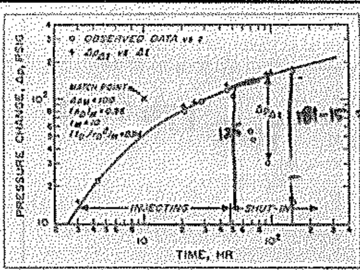


Figure 7-64. Type curve match of interference test data for Example 7-30 (© SPE, SPE Monograph, Vol. 5, 1977).

Interference Test

1. Plot pressure drawdown in an observation well, $\Delta p = p_i - p_r$, vs. elapsed time t on the same size log-log paper as the full-scale, type-curve version of Fig. 6.3 using an undistorted curve (the reader can prepare such a curve easily).

2. Slide the plotted test data over the type curve until a match is found. (Horizontal and vertical sliding both are required.)

3. Record pressure and time match points, $(p_D)_{MP}$, Δp_{MP} and $[(t_D/r_D^2)_{MP}, t_{MP}]$.

4. Calculate permeability k in the test region from the pressure match point:

$$k = 141.2 \frac{qB\mu}{h} \frac{(p_D)_{MP}}{(\Delta p)_{MP}}.$$

5. Calculate ϕc_t from the time match point:

$$\phi c_t = \left(\frac{0.000264\,k}{\mu r^2}\right)\left[\frac{t_{MP}}{(t_D/r_D^2)_{MP}}\right].$$

Example 6.1 – Interference Test in Water Sand

Problem. An interference test was run in a shallow-water sand. The active well, Well 13, produced 466 STB/D water. Pressure response in shut-in Well 14, which was 99 ft from Well 13, was measured as a function of time elapsed since the drawdown in Well 13 began. Estimated rock and fluid properties include $\mu_w = 1.0$ cp, $B_w = 1.0$ RB/STB, $h = 9$ ft, $r_w = 3$ in., and $\phi = 0.3$. Total compressibility is unknown. Pressure readings in Well 14 were as given in Table 6.1. Estimate formation permeability and total compressibility.

Solution. We assume that the aquifer is homogeneous, isotropic, and infinite-acting; we use the *Ei*-function type curves to estimate k and c_t. Data to be plotted are presented in Table 6.2. The data fit the *Ei*-function type curve well. A pair of match points are ($\Delta t = 128$ minutes, $t_D/r_D^2 = 10$) and ($\Delta p = 5.1$ psi, $p_D = 1.0$). (See Fig. 6.4.) Thus,

$$k = 141.2 \frac{qB\mu}{h} \frac{(p_D)_{MP}}{(\Delta p)_{MP}}$$

$$= \frac{(141.2)(466)(1.0)(1.0)}{(9.0)} \frac{(1.0)}{(5.1)}$$

$$= 1{,}433 \text{ md},$$

and

$$c_t = \frac{0.000264}{\phi r^2} \frac{k}{\mu} \frac{(t_{MP}/60)}{(t_D/r_D^2)_{MP}}$$

$$= \frac{(0.000264)(1{,}433)(128/60)}{(0.3)(99)^2(1.0)(10)}$$

$$= 2.74 \times 10^{-5} \text{ psi}^{-1}.$$

TABLE 6.1 – PRESSURE/TIME DATA FROM INTERFERE[...]

Δt (minutes)
0
5
25
40
50
100
200
300
400
500

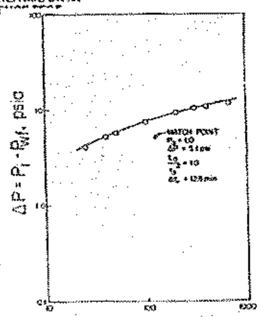


Fig. 6.4 – Interference test data from water reservoir.

TABLE 6.2 – INTERFER[...] FOR LOG-LO[...]

Δt (minutes)	
0	
5	
25	
40	
50	
100	
200	
300	10.22
400	10.93
500	11.00

Example 7-31.[3] A pulse test was run in a reservoir. The test information and reservoir properties are given. In the test following rate stabilization, the active well was shut in for 2 hours, then produced for 2 hours, shut in for 2 hours, and so on.

$r = 933$ ft	$q_o = 425$ STB/D
$\mu_o = 0.8$ cp	$B_o = 1.26$ RB/STB
$h = 26$ ft	$\Delta p_4 = 0.629$ psi
$\phi = 0.08$	$t_L = 0.4$ hour

Using the data above, estimate k and ϕc_t.

Solution:

$$F' = \frac{\Delta t_p}{\Delta t_c}$$

$$= \frac{2}{(2+2)} = 0.50$$

$$\frac{t_L}{\Delta t_c} = \frac{0.4}{4} = 0.10$$

Since we have information for the fourth (even) pulse, use Figures 7-72 and 7-73 to determine $\Delta p_D [t_L/\Delta t_c]^2 = 0.0021$ and $(t_L)_D/r_D^2 = 0.091$, respectively.

$q_o \mu_o B_o (t_L/\Delta t_c)^2$ [...]

Figure 7–72. Relation between time lag and response amplitude for all even pulses after the first (© SPE, SPE Monograph, Vol. 5, 1977).[2]

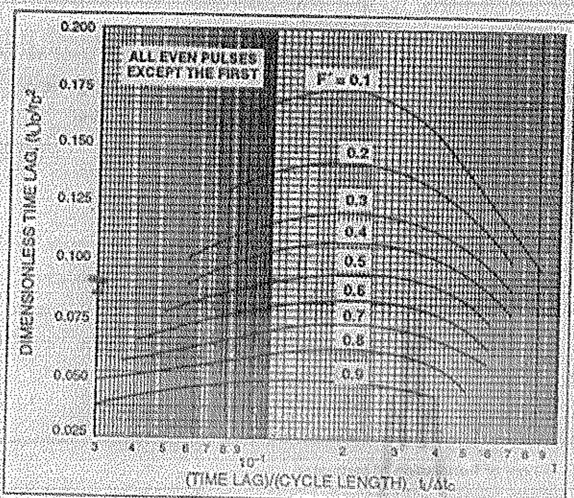


Figure 7–73. Relation between time lag and cycle length for all even pulses after the first (© SPE, SPE Monograph, Vol. 5, 1977).[2]

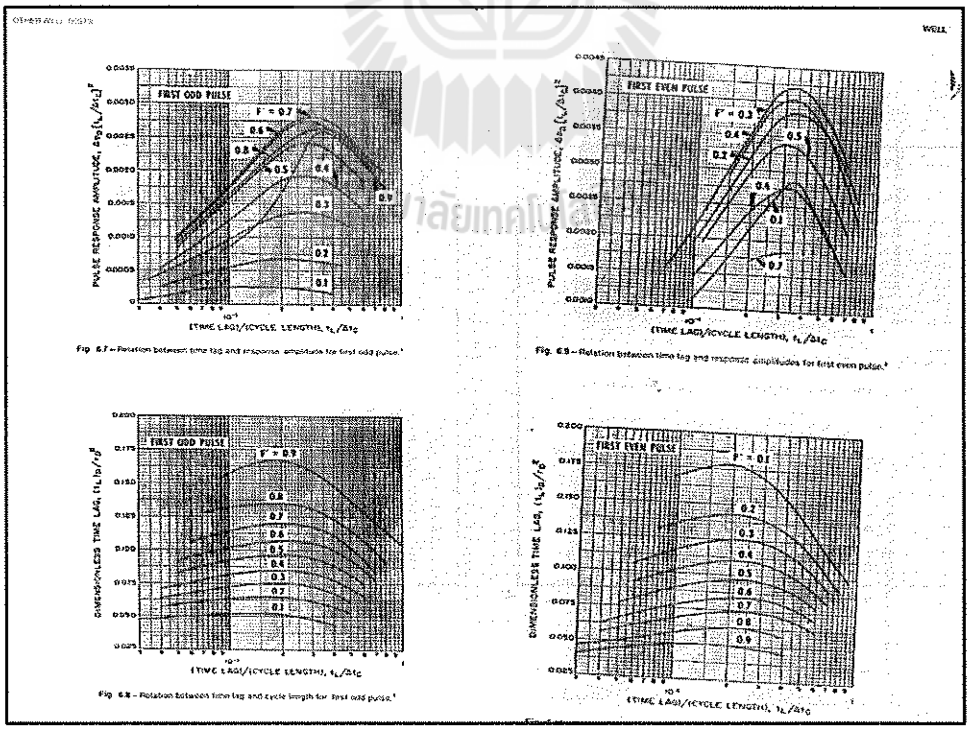
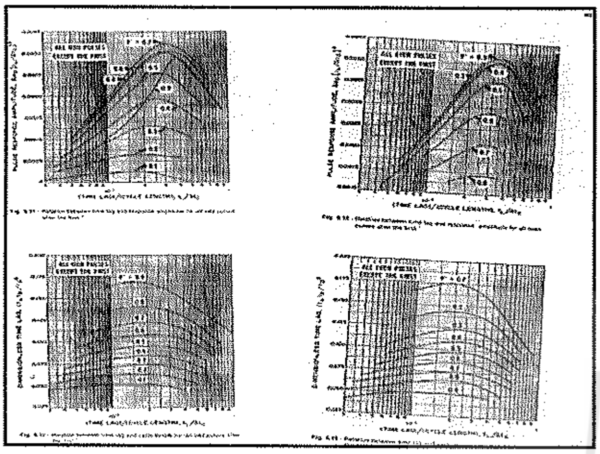
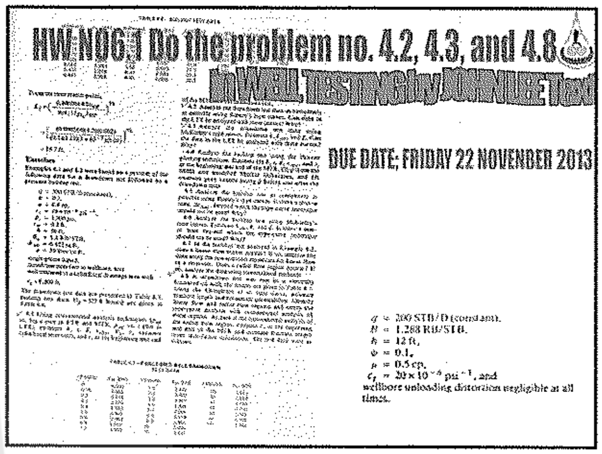


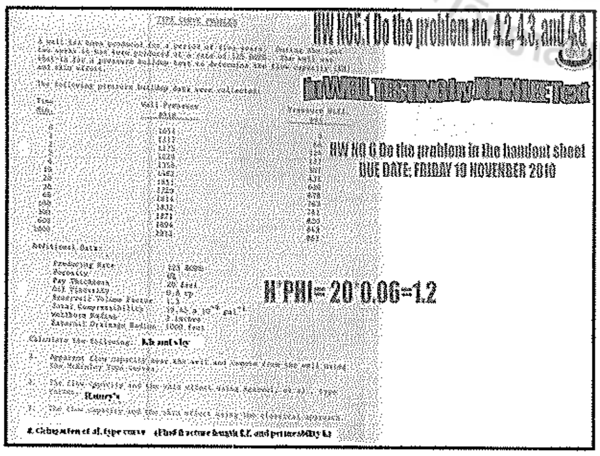
Fig. 6.7 – Relation between time lag and response amplitude for first odd pulse.[1]

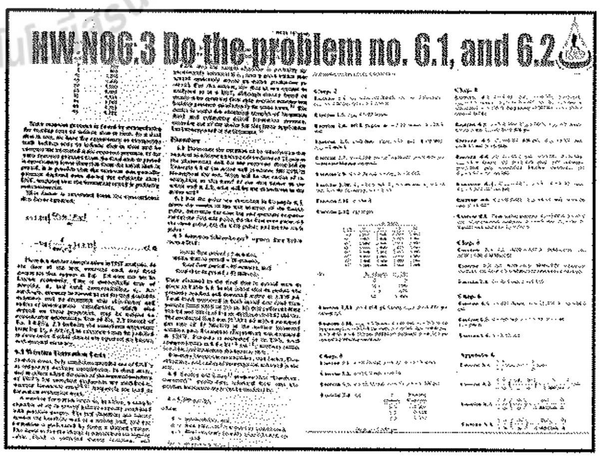
Fig. 6.9 – Relation between time lag and response amplitudes for first even pulse.[1]

Fig. 6.8 – Relation between time lag and cycle length for first odd pulse.[1]




HW NO6.1 Do the problem no. 4.2, 4.3, and 4.8
DUE DATE; FRIDAY 22 NOVEMBER 2013
q = 200 STB/D (constant),
B = 1.288 RB/STB,
h = 12 ft,
φ = 0.1,
μ = 0.5 cp,
wellbore unloading distortion negligible at all times.


HW NO5.1 Do the problem no. 4.2, 4.3, and 4.8
HW NO 6 Do the problem in the handout sheet
DUE DATE: FRIDAY 19 NOVEMBER 2010
H*PHI= 20*0.06=1.2


HW NO6.3 Do the problem no. 6.1, and 6.2

CHAPTER 8 Chapter 8

11th week 15-18 AUGUST 2013

WATER INFLUX

Water Influx

. INTRODUCTION

lany reservoirs are bounded on a portion or all of their peripheries by water-earing rocks called *aquifers* (Latin:aqua—water, ferre—to bear). The aqui-ers may be so large compared with the reservoirs they adjoin as to appear finite for all practical purposes, and they may range down to those so small s to be negligible in their effect on reservoir performance. The aquifer itself nay be entirely bounded by impermeable rock so that the reservoir and quifer together form a closed, or volumetric, unit (Fig. 8.1). On the other and, the reservoir may outcrop at one or more places where it may be eplenished by surface waters (Fig. 8.2). Finally, an aquifer may be essentially orizontal with the reservoir it adjoins, or it may rise, as at the edge of tructural basins, considerably above the reservoir to provide some artesian ind of flow of water to the reservoir.*

In response to a pressure drop in the reservoir, the aquifer reacts to ffset, or retard, pressure decline by providing a source of water influx or ncroachment by (a) expansion of the water; (b) expansion of other known or

TYPES OF AQUIFERS

Water drives based on the geometry and location of aquifer relative to the reservoir.

Peripheral water drive: Aquifer encircles the reservoir fully or partially.

Edge water drive: Aquifer feeds only one side or flank of the reservoir.

Bottom water drive: Aquifer underlies the reservoir and feeds it from the bottom.

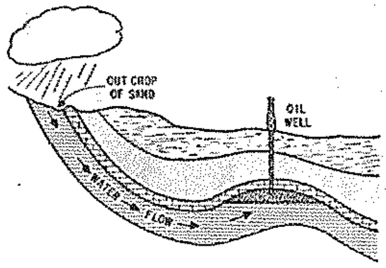


Fig. 61—Reservoir having artesian water drive.

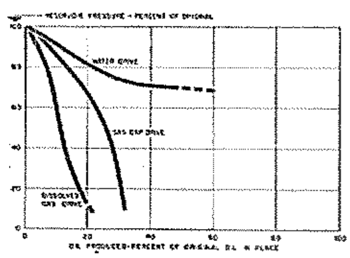


Fig. 69—Reservoir pressure trends for reservoirs under various drives. (Courtesy API, Drilling and Production Practices—194_.)

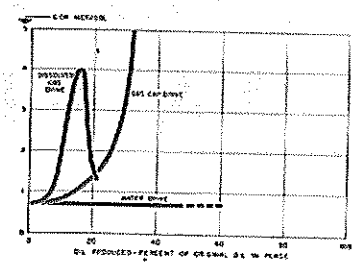
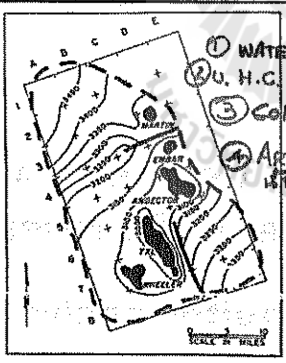


Fig. 70—Reservoir gas-oil ratio trends for reservoirs under various drives. (Courtesy API, Drilling and Production Practices—1945.)



① WATER EXPANSION
② u. H.C. EXPANSION
③ COMPRESSIBILITY
④ ARTESIAN FLOW

Fig. 8.1. A reservoir analyzer study of five fields completed in a closed aquifer in the Ellenburger formation in West Texas. (After Moore and Truby.[1])

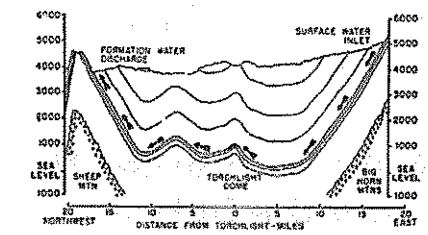
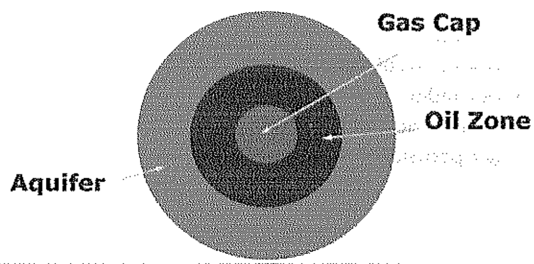
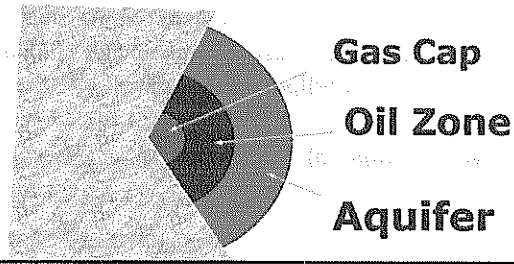


Fig. 8.2. Geologic cross section through the Torchlight Tensleep Reservoir, Wyoming. (After Stewart, Callaway, and Gladfelter.[2])

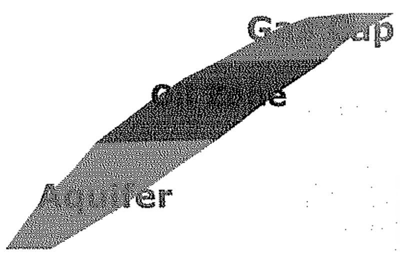
- SCHEMATIC DIAGRAM OF PERIPHERAL WATER DRIVE



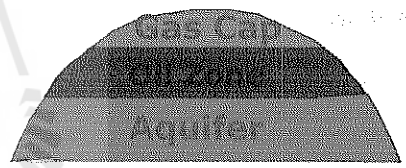
SCHEMATIC DIAGRAM OF PERIPHERAL PARTIAL WATER DRIVE



SCHEMATIC DIAGRAM OF EDGE WATER DRIVE



SCHEMATIC DIAGRAM OF BOTTOM WATER DRIVE



KEY AQUIFER PROPERTIES

The following aquifer properties are important for water influx modeling and characterization

- 1. Size and shape
  2. Permeability
  3. Porosity
  4. Water compressibility
  5. Formation compressibility
  6. Water viscosity

MATHEMATICAL WATER INFLUX MODELS USED IN THE PETROLEUM INDUSTRY

- POT AQUIFER;
- SCHILTHUIS STEADY STATE;
- HURST MODIFIED STEADY STATE;
- VAN EVERDINGEN AND HURST UNSTEADY STATE:
  - EDGE-WATER DRIVE;
  - BOTTOM-WATER DRIVE;
- CARTER–TRACY UNSTEADY STATE;
- FETKOVICH METHOD:
  - RADIAL AQUIFER;
  - LINEAR AQUIFER.

- simplest model used to estimate the water influx into a gas or oil reservoir
- Based on the basic definition of compressibility.
- Water influx = (aquifer compressibility) × (initial volume of water)(pressure drop)
- or:
- *We = ctWi(pi − p)*

Initial Volume Of Water

- knowledge of aquifer dimensions and properties.

$$W_i = \left[\frac{\pi\,(r_a^2 - r_e^2)\,h\phi}{5.615}\right]$$

- where:
- *ra* = radius of the aquifer, ft
- *re* = radius of the reservoir, ft
- *h* = thickness of the aquifer, ft
- *φ* = porosity of the aquifer

Encroachment angle *f*

- *We = (cw + cf)Wi f (pi − p)*
- where the fractional encroachment angle *f* is defined by:

$$f = \frac{(\text{encroachment angle})^\circ}{360^\circ} = \frac{\theta}{360^\circ}$$

The Schilthuis steady-state model

- Rate of water influx "ew" can be determined by applying Darcy's equation

$$\frac{dW_e}{dt} = e_w = \left[\frac{0.00708\,kh}{\mu_w \ln(r_a/r_e)}\right](p_i - p)$$

- *dWe/dt = ew = C(pi − p)*
- where:
- *ew* = rate of water influx, bbl/day
- *k* = permeability of the aquifer, md
- *h* = thickness of the aquifer, ft
- *ra* = radius of the aquifer, ft
- *re* = radius of the reservoir, ft
- *t* = time, days

2. STEADY-STATE MODELS

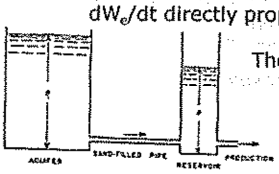
The simplest model we discuss is the Schilthuis steady-state model, in which the rate of water influx, dW_e/dt, is directly proportional to $(p_i - p)$, where the pressure, p, is measured at the original oil-water contact.[7] This model assumes that the pressure at the external boundary of the aquifer is maintained at the

Steady State Model (Schilthuis)

The simplest method for characterizing water influx is due to Schilthuis [3]. It is often a good idea to try this model first since the calculations are considerably less involved than with either of the other two methods.

The assumptions inherent in the Schilthuis model are many. First, it is assumed that the aquifer is gigantic and highly permeable. In fact, the aquifer is taken to have permeability so high that the pressure gradient across the aquifer itself is negligible. And the aquifer is so huge that the pressure within the aquifer never declines; i.e., the initial pressure, P_i, always exists at all locations within the aquifer. Consider the hydraulic analog to the Schilthuis steady state model shown below.

dW_e/dt directly propertional to $(p_i$-p)



Then $dW_e/dt = k^*(p_i$-p)

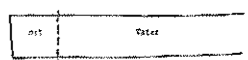
Hydraulic analog of steady-state water influx into a reservoir (from Craft & Hawkins[4]).

WATER INFLUX

Aquifer Description:

The aquifer can be either of several types depending on the geometrical configuration:

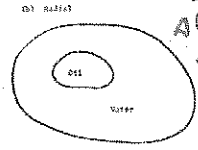
(a) Linear:



Schilthuis Steady State (Darcy)

$\frac{\Delta We}{\Delta t} = k'(P_i - P)$ or $W_e = k\int_0^t (P_i$-

(b) Radial



Hurst

$\frac{\Delta We}{\Delta t} = \frac{C'(P_i - P)}{\log at}$

or it may be characterized depending on how the water approaches.

STEADY STATE MODEL
$dW/dt \propto (p_i - p)$

Schilthuis Model.

$$\frac{F}{E_o} = N + \frac{W_e}{E_o}$$

2. STEADY-STATE MODELS

The simplest model we discuss is the Schilthuis steady-state model, in which the rate of water influx, dW_e/dt, is directly proportional to $(p_i - p)$, where the initial value p_i and that flow in the reservoir is, by Darcy's Law, proportional to the pressure differential, assuming the water viscosity, average permeability, and aquifer geometry remain constant.

$$W_e = k' \int_0^t (p_i - p)\,dt \qquad (8.1)$$

$$\frac{dW_e}{dt} = k'(p_i - p) \qquad (8.2)$$

where k' is the water influx constant in barrels per day per pounds per square inch and $(p_i - p)$ is the boundary pressure drop in pounds per square inch. If the value of k' can be found, then the value of the cumulative water influx W_e can be found from Eq. (8.1), from a knowledge of the pressure history of the reservoir. If during any reasonably long period the rate of production and reservoir pressure remain substantially constant, it is obvious that the volumetric withdrawal rate, or reservoir voidage rate, must equal the water influx rate, or

$$\frac{dW_e}{dt} = \begin{bmatrix}\text{Rate of active}\\ \text{oil volumetric}\\ \text{voidage}\end{bmatrix} + \begin{bmatrix}\text{Rate of free}\\ \text{gas volumetric}\\ \text{voidage}\end{bmatrix} + \begin{bmatrix}\text{Rate of water}\\ \text{volumetric}\\ \text{voidage}\end{bmatrix}$$

In terms of single-phase oil volume factors,

$$\frac{dW_e}{dt} = B_o\frac{dN_p}{dt} + (R - R_{so})\frac{dN_p}{dt}B_g + \frac{dW_p}{dt}B_w \qquad (8.3)$$

where dN_p/dt is the daily oil rate in STB/day and $(R - R_{so})dN_p/dt$ is the daily free gas rate in SCF/day. The solution gas-oil ratio R_{so} is subtracted from the net daily or current gas-oil ratio R because the solution gas R_{so} is accounted for in the oil volume factor B_o of the oil voidage term. Equation (8.3) may be converted to an equivalent one using two-phase volume factors by adding and subtracting the term $R_{soi}B_g dN_p/dt$, and grouping as

$$\frac{dW_e}{dt} = [B_o + (R_{soi} - R_{so})B_g]\frac{dN_p}{dt} + (R - R_{soi})B_g\frac{dN_p}{dt} + B_w\frac{dW_p}{dt}$$

and since $[B_o + (R_{soi} - R_{so})B_g]$ is the two-phase volume factor B_t:

$$\frac{dW_e}{dt} = B_t\frac{dN_p}{dt} + (R - R_{soi})B_g\frac{dN_p}{dt} + B_w\frac{dW_p}{dt} \qquad (8.4)$$

When dW_e/dt has been obtained in terms of the voidage rates by Eqs. (8.3) or

Then $dW_e/dt = k^*(p_i - p)$

B. Water production during the period was negligible. Example 8.1 ... the calculation of the water influx constant k' for the Conroe Field from ... this period of stabilized pressure. If the pressure stabilizes and the ... withdrawal rates are not reasonably constant, the water influx for the period of stabilized pressure may be obtained from the total oil, gas, and water ... for the period.

P stabilize ได้ดี

$$\Delta W_e = B_t\Delta N_p + (\Delta G_p - R_{soi}\Delta N_p)B_g + B_w\Delta W_p$$

... ΔG_p, ΔN_p, and ΔW_p are the gas, oil, and water produced during the ... in surface units. The influx constant is obtained by dividing ΔW_e by the ... of the days in the interval and the stabilized pressure drop $(p_i - p_s)$.

$$k' = \frac{\Delta W_e}{\Delta t(p_i - p_s)}$$

$$\Delta w = k(\Delta P)\Delta t$$

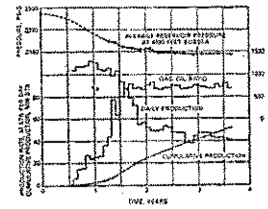


Fig. 8.3. Reservoir pressure and production data, Conroe Field. (After Schilthuis.[1])

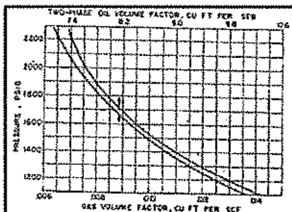


Fig. 8.4. Pressure volume relations for Conroe Field oil and original complement of dissolved gas. (After Schilthuis.[1])

Example 8.1. Calculating the water influx constant when reservoir pressure stabilizes.

Given:

The PVT data for Conroe Field, Fig. 8.4:

$p_i = 2275$ psig

$p = 2090$ psig (stabilized pressure)

$B_t = 7.520$ cu ft/STB at 2090 psig

$B_g = 0.00693$ cu ft/SCF at 2090 psig

$R_{soi} = 600$ SCF/STB (initial solution gas)

$R = 825$ SCF/STB, from production data

$dN_p/dt = 44{,}100$ STB/day, from production data

$dW_p/dt = 0$

SOLUTION: At 2090 psig by Eq. (8.4) the daily voidage rate ...

Eq. 8.4 $\frac{dW_e}{dt} = B_t\frac{dN_p}{dt} + (R - R_{soi})B_g\frac{dN_p}{dt} + B_w\frac{dW_p}{dt}$

$\frac{dW_e}{dt} = 7.520 \times 44{,}100 + (825 - 600)\,0.00693 \times 44{,}100 + 0$

$= 401{,}000$ ft³/day

Since this must equal the water influx rate at stabilized pressure conditions, by Eq. (8.2)

$p_i - p_s$

$\frac{dW_e}{dt} = 401{,}000 = k'(2275 - 2090)$

$k' = 2170$ cu ft/day/psi

A water influx constant of 2170 cu ft/day/psi means that if the reservoir pressure suddenly drops from an initial pressure of 2275 to, say, 2265 psig (i.e., $\Delta p = 10$ psi) and remains there for 10 days, during this period the water influx will be

$K \times \Delta P \times \Delta t$ $\qquad \Delta W_e = k'\Delta t(p_i - p)$

$\Delta W_e = 2170 \times 10 \times 10 = 217{,}000$ cu ft

If at the end of 10 days it drops to, say, 2255 psi ($\Delta p = 20$ psi) and remains there for 20 days, the water influx during this second period will be

$\Delta W_e = 2170 \times 20 \times 20 = 868{,}000$ cu ft

There is four times the influx in the second period because the influx rate was twice as great (because the pressure drop was twice as great) and because the interval was twice as long. The cumulative water influx at the end of 30 days, then, is

$k'\int_0^t \Delta P\,dt + \frac{\Delta P}{\Delta t}$

$W_e = k'\int_0^t (p_i - p)\,dt = k'\sum_0^t (p_i - p)\Delta t$

$= 2170[(2275 - 2265) \times 10 + (2275 - 2255) \times 20]$

$= 1{,}085{,}000$ cu ft $\quad = 4170 K$

In Fig. 8.5 the $\int (p_i - p)\,dt$ is represented by the area beneath the curve of pressure drop, $(p_i - p)$, plotted versus time, or it represents the area above the curve of pressure versus time. The areas may be found by graphical integration.

One of the problems associated with the Schilthuis steady-state model is that as the water is drained from the aquifer, the distance that the water has to travel to the reservoir increases. Hurst suggested a modification to the Schilthuis equation by including a logarithmic term to account for this increasing distance. The Hurst method has met with limited application ...

Hurst modified steady-state equation

- Hurst (1943) proposed the "apparent" aquifer radius *ra* would increase with time,
- Dimensionless radius *ra/re* may be replaced with a *time-dependent function* as given below:
- *ra/re = at*
- Substituting in schilthuis equation

$$e_w = \frac{dW_e}{dt} = \frac{0.00708\,kh(p_i - p)}{\mu_w \ln(at)}$$

Hurst modified steady-state

- It can be written in a more simplified form as,

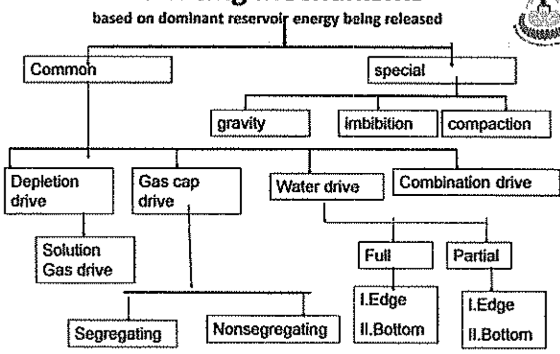
$$e_w = \frac{dW_e}{dt} = \frac{C(p_i - p)}{\ln(at)}$$

- C=0.00708kh/Uw
- **In terms of the cumulative water influx:**

$$W_e = C\int_0^t \left[\frac{p_i - p}{\ln(at)}\right]dt$$

Producing mechanisms

based on dominant reservoir energy being released



Water influx Rate ew

During long production period rate and reservoir pressure remain reasonably constant,

Reservoir voidage rate = the water influx rate:

$$\begin{bmatrix}\text{water influx}\\ \text{rate}\end{bmatrix} = \begin{bmatrix}\text{oil flow}\\ \text{rate}\end{bmatrix} + \begin{bmatrix}\text{free gas}\\ \text{flow rate}\end{bmatrix} + \begin{bmatrix}\text{water}\\ \text{production}\\ \text{rate}\end{bmatrix}$$

or:

$$e_w = Q_o B_o + Q_g B_g + Q_w B_w$$

Cont'

$$e_w = \frac{dW_e}{dt} = B_o \frac{dN_p}{dt} + (GOR - R_s)\frac{dN_p}{dt}B_g + \frac{dW_p}{dt}B_w$$

where:

W_e = cumulative water influx, bbl
t = time, days
N_p = cumulative oil production, STB
GOR = current gas–oil ratio, scf/STB
R_s = current gas solubility, scf/STB
B_g = gas formation volume factor, bbl/scf
W_p = cumulative water production, STB
dN_p/dt = daily oil flow rate Q_o, STB/day
dW_p/dt = daily water flow rate Q_w, STB/day
dW_e/dt = daily water influx rate e_w, bbl/day
$(GOR - R_s)dN_p/dt$ = daily free gas rate, scf/day

Example for Influx rate

- *Example: Calculate the water influx rate ew in a reservoir whose pressure is stabilized at 3000 psi.*
- Given:
- Initial reservoir pressure = 3500 psi,
- *dNp/dt = 32 000 STB/day*
- *Bo = 1.4 bbl/STB, GOR = 900 scf/STB, Rs = 700 scf/STB*
- *Bg =0.00082 bbl/scf, dWp/dt =0, Bw 1.0 bbl/STB*

$$e_w = \frac{dW_e}{dt} = B_o \frac{dN_p}{dt} + (GOR - R_s)\frac{dN_p}{dt}B_g + \frac{dW_p}{dt}B_w$$

- = (1. 4)(32 000) + (900 - 700)(32 000)(0. 00082) + 0
- 50048 bbl/day

Water influx calculation

- Calculate the cumulative water influx that result from a pressure drop of 200 psi at the oil–water contact with an encroachment angle of 80. The reservoir–aquifer system is characterized by the following properties:

	Reservoir	*Aquifer*
radius, ft	2600	10 000
porosity	0.18	0.12
c_f, psi^{-1}	4×10^{-6}	3×10^{-6}
c_w, psi^{-1}	5×10^{-6}	4×10^{-6}
h, ft	20	25

- Step-01 Calculate initial water volume Wi=?
- Step-02 cumulative water influx by eq. *We = ct Wi(pi – p)*

Cont'

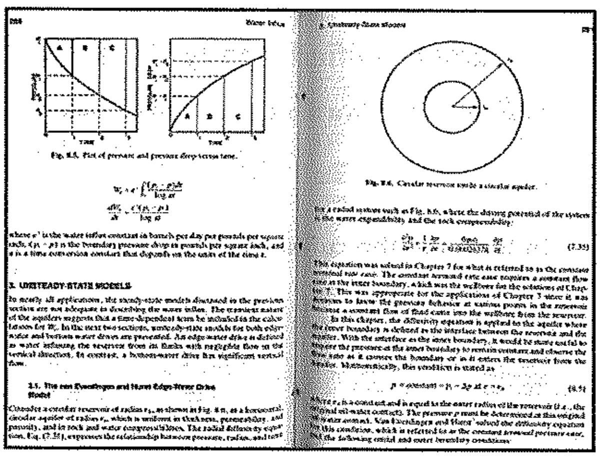
- Initial volume of the water in the aquifer

$$W_i = \left[\frac{\pi(r_a^2 - r_e^2)h\phi}{5.615}\right]$$

$$\left[\frac{\pi(10000^2 - 2600^2)(25)(0.12)}{5.615}\right] = 156.5\ \text{MMbbl}$$

- Cum $W_e = (c_w + c_f) W_i f (p_i - p)$

- =(4.0+3.0)10-6(156.5x106) 80/360(200)=48689 bbl



van everdingen and hurst unsteady-state model

- Van Everdingen and Hurst (1949) solved the diffusivity equation for the aquifer-reservoir system

$$\frac{\partial^2 P_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial P_D}{\partial r_D} = \frac{\partial P_D}{\partial t_D}$$

Estimated water influx in the following systems:
Edge-water drive system (radial system);
Bottom-water drive system;
Linear-water drive system

- Van Everdingen and Hurst assumed that the aquifer is characterized by:
  - uniform thickness;
  - constant permeability;
  - uniform porosity;
  - constant rock compressibility;
  - constant water compressibility

3. UNSTEADY-STATE MODELS

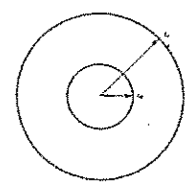


Fig. 6.6. Circular reservoir inside a circular aquifer.

3. UNSTEADY-STATE MODELS

In nearly all applications, the steady-state models discussed in the previous section are not adequate in describing the water influx. The transient nature of the aquifers suggests that a time-dependent term be included in the calculations for W_e. In the next two sections, unsteady-state models for both edge-water and bottom-water drives are presented. An edge-water drive is defined as water influxing the reservoir from its flanks with negligible flow in the vertical direction. In contrast, a bottom-water drive has significant vertical flow.

3.1. The van Everdingen and Hurst Edge-Water Drive Model

Consider a circular reservoir of radius r_R, as shown in Fig. 6.6, in a horizontal circular aquifer of radius r_a, which is uniform in thickness, permeability, and porosity, and in rock and water compressibilities. The radial diffusivity equation, Eq. (7.35), expresses the relationship between pressure, radius, and time

3.1. The van Everdingen and Hurst Edge-Water Drive Model

for a radial system such as Fig. 6.6, where the driving potential of the system is the water expandability and the rock compressibility:

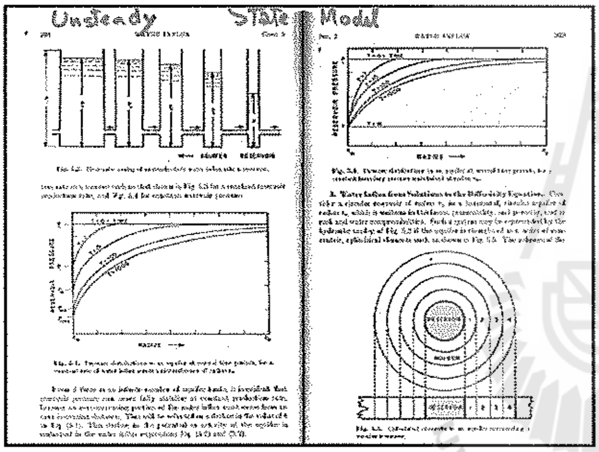
$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu c_t}{0.0002637k}\frac{\partial p}{\partial t} \qquad (7.35)$$

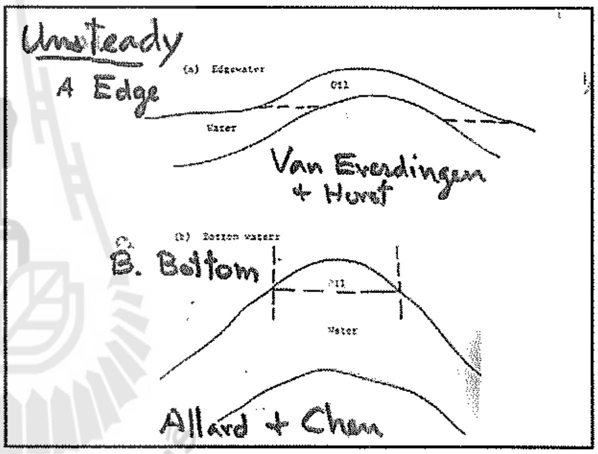
This equation was solved in Chapter 7 for what is referred to as the *constant terminal rate case*. The constant terminal rate case requires a constant flow rate at the inner boundary, which was the wellbore for the solutions of Chapter 7. This was appropriate for the applications of desirous to know the pressure behavior at various because a constant flow of fluid came into the well

In this chapter, the diffusivity equation is app the inner boundary is defined as the interface betw aquifer. With the interface as the inner boundary, it ...

$$p = \text{constant} = p_i - \Delta p \text{ at } r = r_R \qquad (8.5)$$

where r_R is a constant and is equal to the outer radius of the reservoir (i.e., the original oil-water contact). The pressure p must be determined at this original oil-water contact. Van Everdingen and Hurst[2] solved the diffusivity equation for this condition, which is referred to as the *constant terminal pressure case*. ...





Initial condition:

$p = p_i$ for all values of r

Outer boundary conditions

For an infinite aquifer:

$p = p_i$ at $r = \infty$

For a finite aquifer:

$\frac{\partial p}{\partial r} = 0$ at $r = r_a$

this point, we rewrite the diffusivity equation in terms of the following dimensionless parameters:

$$\text{Dimensionless time: } t_D = 0.0002637\frac{kt}{\phi \mu c_t r_R^2} \qquad (8.6)$$

$$\text{Dimensionless radius: } r_D = \frac{r}{r_R}$$

$$\text{Dimensionless pressure: } p_D = \frac{p_i - p}{p_i - p_{wf}}$$

where k = average aquifer permeability, md; t = time, hours; ϕ = aquifer porosity, fraction; μ = water viscosity, cp; c_t = aquifer compressibility, psi^{-1}; r_R = reservoir radius, feet. With these dimensionless parameters, the diffusivity equation becomes:

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D} \qquad (8.7)$$

van Everdingen and Hurst converted their solutions to dimensionless, cumulative water influx values and made the results available in a convenient form here given in Tables 8.1 and 8.2 for various ratios of aquifer to reservoir size, expressed by the ratio of their radii, r_a/r_R. Figures 8.7 to 8.10 are plots of some of the tabular values. The data are given in terms of dimensionless time, t_D, and dimensionless water influx, W_{eD}, so that one set of values suffices for all aquifers whose behavior can be represented by the radial form of the diffusivity equation. The water influx is then found by using Eq. (8.8):

$$W_e = B'\Delta p W_{eD} \qquad (8.8)$$

- The water influx is then given by:

$$W_e = B \Delta p W_{eD}$$

$$B = 1.119 \phi c_t r_e^2 h$$

- where:
- *We* = cumulative water influx, bbl
- *B* = water influx constant, bbl/psi
- *p* = pressure drop at the boundary, psi
- *WeD* = dimensionless water influx

$W_e = B'\Delta P W_{eD}$

where

$$B' = 1.119\phi c_t h r_o^2 \frac{\theta}{360} \qquad (8.9)$$

B' is the water influx constant in barrels per pounds per square inch and θ is the angle subtended by the reservoir circumference, (i.e., for a full circle, θ = 360° and for a semicircular reservoir against a fault, θ = 180°). c_t is in psi^{-1} and r_o and h are in feet.

Example 8.1 shows the use of Eq. (8.8) and the values of Tables 8.1 and 8.2 to calculate the cumulative water influx at successive periods for the case of a constant reservoir boundary pressure. The infinite aquifer values may be used for small time values even though the aquifer is limited in size.

Example 8.1. Calculate the water influx after 100 days, 200 days, 400 days, and 800 days into a reservoir the boundary pressure of which is suddenly lowered and held at 2724 psia (p_i = 2734 psia).

Given:

ϕ = 0.20	k = 83 md
c_t = 8(10)$^{-6}$ psi^{-1}	r_o = 5000 ft
r_e = 20,000 ft	μ = 0.62 cp
θ = 360°	h = 40 ft

SOLUTION: From Eq. (8.6):

$$t_D = \frac{0.0002637(83)t}{0.20(0.62)(8(10)^{-6})(5000)^2} = 0.00245t \times 100$$

From Eq. (8.9)

$$B' = 1.119(0.20)(8(10)^{-6})(5000)^2(40)\left(\frac{360}{360}\right) = 644.5 \quad 661/psi$$

At 100 days t_D = 0.00245(100)(24) = 5.88 dimensionless time units. From the r_e/r_o = 10 curve of Fig. 8.8 find corresponding to t_D = 5.88, W_{eD} = 5.07 dimensionless influx units. This same value may also be found by interpolation of Table 8.1, since below t_D = 15 the aquifer behaves essentially as if it were infinite, and no values are given in Table 8.2. Since Δp = 2734 − 2724 = 10 psi, and water influx at 100 days from Eq. (8.8) is

$$W_e = B'\Delta p W_{eD} = 644.5(10)(5.07) = 32,680 \text{ bbl}$$

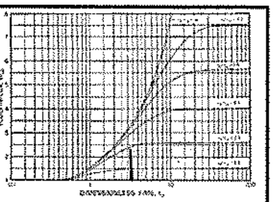


Fig. 8.7. Limited aquifer values of dimensionless influx W_{eD} for values of dimensionless time t_D and aquifer limits given by the ratio r_e/r_o.

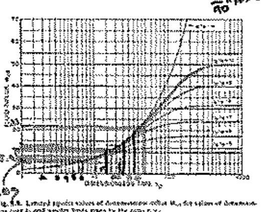


Fig. 8.8. Limited aquifer values of dimensionless influx W_{eD} for values of dimensionless time t_D and aquifer limits given by the ratio r_e/r_o.

TABLE 8.1.
Infinite aquifer values of dimensionless water influx W_{eD} for values of dimensionless time t_D

Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}
0.00	0.000	79	35.697	455	150.249	1190	340.843	3250	816.090	35.000	6780.247
0.01	0.112	80	36.058	460	151.640	1200	343.308	3300	827.088	40.000	7650.096
0.05	0.278	81	36.418	465	153.029	1210	345.779	3350	838.067	50.000	9363.099
0.10	0.404	82	36.777	470	154.416	1220	348.230	3400	849.028	60.000	11,047.299
0.15	0.520	83	37.136	475	155.801	1225	349.466	3450	859.974	70.000	12,708.358
0.20	0.606	84	37.494	480	157.184	1230	350.688	3500	870.903	75.000	13,531.457
0.25	0.689	85	37.851	485	158.565	1240	353.144	3550	881.816	80.000	14,350.121
0.30	0.758	86	38.207	490	159.945	1250	355.597	3600	892.712	90.000	15,975.389
0.40	0.898	87	38.563	495	161.322	1260	358.048	3650	903.594	100.000	17,586.284
0.50	1.020	88	38.919	500	162.698	1270	360.496	3700	914.459	125.000	21,560.732
0.60	1.140	89	39.272	510	165.444	1275	361.720	3750	925.309	1.5(10)^5	2.538(10)^4
0.70	1.251	90	39.626	520	168.183	1280	362.942	3800	936.144	2.0"	3.308"
0.80	1.359	91	39.979	525	169.549	1290	365.386	3850	946.966	2.5"	4.066"
0.90	1.469	92	40.331	530	170.914	1300	367.828	3900	957.773	3.0"	4.817"
1	1.569	93	40.684	540	173.639	1310	370.267	3950	968.566	4.0"	6.267"
2	2.447	94	41.034	550	176.357	1320	372.704	4000	979.344	5.0"	7.699"
3	3.202	95	41.385	560	179.069	1325	373.922	4050	990.108	6.0"	9.113"
4	3.893	96	41.735	570	181.774	1330	375.139	4100	1000.858	7.0"	1.051(10)^5
5	4.539	97	42.084	575	183.124	1340	377.572	4150	1011.595	8.0"	1.189"
6	5.153	98	42.433	580	184.473	1350	380.003	4200	1022.318	9.0"	1.326"
7	5.743	99	42.781	590	187.166	1360	382.432	4250	1033.028	1.0(10)^6	1.462"
8	6.314	100	43.129	600	189.852	1370	384.859	4300	1043.724	1.5"	2.126"
9	6.869	105	44.858	610	192.533	1375	386.070	4350	1054.409	2.0"	2.781"
10	7.411	110	46.574	620	195.208	1380	387.283	4400	1065.082	2.5"	3.427"
11	7.940	115	48.277	625	196.544	1390	389.705	4450	1075.743	3.0"	4.064"
12	8.457	120	49.968	630	197.878	1400	392.125	4500	1086.390	4.0"	5.313"
13	8.964	125	51.648	640	200.542	1410	394.543	4550	[illegible]	[illegible]	[illegible]

TABLE 8.1.
(Cont'd.)

Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}	Dimensionless time t_D	Fluid influx W_{eD}
42	21.701	270	97.081	880	263.073	1950	522.521	7000	1603.418	2.0"	1.880"
43	22.101	275	98.571	890	265.629	1975	528.337	7100	1625.729	2.5"	2.328"
44	22.500	280	100.057	900	268.181	2000	534.145	7200	1646.011	3.0"	2.771"
45	22.897	285	101.540	910	270.729	2025	539.945	7300	1666.265	4.0"	3.645"
46	23.291	290	103.019	920	273.274	2050	545.737	7400	1686.490	5.0"	4.510"
47	23.684	295	104.495	925	274.545	2075	551.522	7500	1706.688	6.0"	5.368"
48	24.076	300	105.968	930	275.815	2100	557.299	7600	1726.859	7.0"	6.220"
49	24.466	305	107.437	940	278.353	2125	563.068	7700	1747.002	8.0"	7.066"
50	24.855	310	108.904	950	280.888	2150	568.830	7800	1767.120	9.0"	7.909"
51	25.244	315	110.367	960	283.420	2175	574.585	7900	1787.212	1.0(10)^{10}	8.747"
52	25.633	320	111.827	970	285.948	2200	580.332	8000	1807.278	1.5"	1.288(10)^9
53	26.020	325	113.284	975	287.211	2225	586.072	8100	1827.319	2.0"	1.697"
54	26.406	330	114.738	980	288.473	2250	591.806	8200	1847.336	2.5"	2.103"
55	26.791	335	116.189	990	290.995	2275	597.532	8300	1867.329	3.0"	2.505"
56	27.174	340	117.638	1000	293.514	2300	603.252	8400	1887.298	4.0"	3.299"
57	27.555	345	119.083	1010	296.030	2325	608.965	8500	1907.243	5.0"	4.087"
58	27.935	350	120.526	1020	298.543	2350	614.672	8600	1927.166	6.0"	4.868"
59	28.314	355	121.966	1025	299.799	2375	620.372	8700	1947.065	7.0"	5.643"
60	28.691	360	123.403	1030	301.053	2400	626.066	8800	1966.942	8.0"	6.414"
61	29.068	365	124.838	1040	303.560	2425	631.755	8900	1986.796	9.0"	7.183"
62	29.443	370	126.720	1050	306.065	2450	637.437	9000	2006.628	1.0(10)^{11}	7.948"
63	29.818	375	127.699	1060	308.567	2475	643.113	9100	2026.438	1.5"	1.17(10)^{10}
64	30.192	380	129.126	1070	311.066	2500	648.781	9200	2046.227	2.0"	1.55"
65	30.565	385	130.550	1075	312.314	2550	660.093	9300	2065.996	2.5"	1.92"

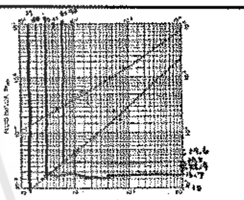


Fig. 8.9. Infinite aquifer values of dimensionless influx W_{eD} for values of dimensionless time t_D.

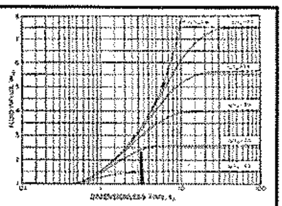


Fig. 8.7. Limited aquifer values of dimensionless influx W_{eD} for values of dimensionless time t_D and aquifer limits given by the ratio r_e/r_o.

t_D = 0.00245 × 800 × 24 = 11.76

$W_e = B'\Delta P \times W_{eD}$

644.5 × 10 ×

Similarly at

t = 100 days	200 days	400 days	800 days
t_D = 5.88	11.76	23.52	47.04
W_{eD} = 5.07	8.43	[illegible]	23.75
W_e = 32,680	54,330	[illegible]	148,460

For aquifers 10 times as large as the reservoirs they surround, or r_e/r_o = 10, this means that the effects of the aquifer limits are negligible for dimensionless time values under 15, and that it is some time before the aquifer limits affect the water influx appreciably. This is also illustrated by the coincidence of the curves of Figs. 8.7 and 8.8 with the infinite aquifer curve for the smaller time values. It should also be noted that, unlike a steady state system, the values of water influx calculated in Ex. 8.1 fail to double for a doubling of the time.

While water is entering the reservoir from the aquifer at a declining rate, in response to the first pressure signal $\Delta p_1 = p_i - p_1$, let a second, sudden pressure drop $\Delta p_2 = p_1 - p_2$ (not $p_i - p_2$) be imposed at the reservoir boundary at a time t_1. This is an application of the principle of superposition, which was discussed in Chapter 7. The total or net effect is the sum of the two as illustrated in Fig. 8.11 [illegible]. The upper

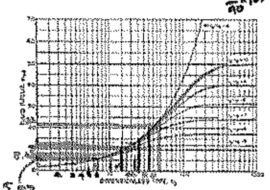


Fig. 8.8. Limited aquifer values of dimensionless influx W_{eD} for values of dimensionless time t_D and aquifer limits given by the ratio r_e/r_o.

and lowest curves represent the pressure distributions in the aquifer in response to the first signal alone, at times t_1 and t_2, respectively. The upper curve may also be used to represent the pressure distribution for the second pressure signal alone at time t_2 because in this simplified case $\Delta p_1 = \Delta p_2$, and $\Delta t_1 = \Delta t_2$. The lower curve, then, is the sum of the upper and middle curves. Mathematically, this means that Eq. (8.10) can be used to calculate the cumulative water influx:

$$W_e = B'\Sigma\Delta p W_{eD} \qquad (8.10)$$

This calculation is illustrated in Ex. Prob. 8.2.

Example 8.2. Suppose in Ex. 8.1 at the end of 100 days the reservoir boundary pressure suddenly drops to p_2 = 2704 psia (i.e., $\Delta p_2 = p_1 - p_2$ = 20 psi, *not* $p_i - p_2$ = 30 psi). Calculate the water influx at 400 days total time.

The water influx due to the *first* pressure drop Δp_1 = 10 psi for 400 days was calculated in Example 8.1 to be 89,590 bbl. This will be the same even though a second pressure drop occurs at 100 days and continues to 400 days. This second drop will have acted for 300 days, or a dimensionless time of t_D = 0.0588 × 300 = 17.6. From Fig. 8.8 or Table 8.2 for r_e/r_o = 10 W_{eD} = 11.14 for t_D = 17.6 and the water influx is

$$\Delta W_{e2} = B' \times \Delta p_2 \times W_{eD2} = 644.5 \times 20 \times 11.14 = 143,600 \text{ bbl}$$

$$W_e = \Delta W_{e1} + \Delta W_{e2} = B' \times \Delta p_1 \times W_{eD1} + B' \times \Delta p_2 \times W_{eD2} = B'\Sigma\Delta p W_{eD}$$

$$= 644.5(10 \times 13.90 + 20 \times 11.14)$$

$$= 89,590 + 143,600 = 233,190 \text{ bbl}$$

Example 8.2 illustrates the calculation of water influx when a second pressure drop occurs 100 days after the first drop in Example 8.1. A continuation of this method may be used to calculate the water influx into reservoirs for which boundary pressure histories are known, and also for which sufficient information is known about the aquifer to calculate the constant B' and the dimensionless time t_D.

The history of the reservoir boundary pressure may be approximated as closely as desired by a series of step-by-step pressure reductions (or increases), as illustrated in Fig. 8.12. The best approximation of the pressure history is made as shown by making the pressure step at any time equal to half of the drop in the previous interval of time plus half of the drop in the succeeding period of time.[4] When reservoir boundary pressures are not known, average reservoir pressures may be substituted with some reduction in the accuracy of the results. In addition, for best accuracy, the average boundary pressure

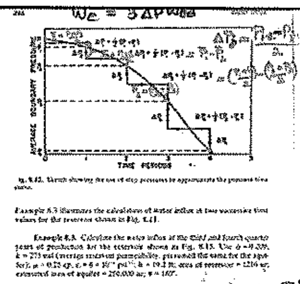


Fig. 8.12. Sketch showing the use of step pressures to approximate the pressure time curve.

Example 8.3 illustrates the calculation of water influx at two successive time values for the reservoir shown in Fig. 8.13.

Example 8.3. Calculate the water influx at the third and fourth quarter years of production for the reservoir shown in Fig. 8.13. Use ϕ = 0.209, k = 275 md (average reservoir permeability, presumed the same for the aquifer), μ = 0.25 cp, c_t = 6 × 10^{-6} psi^{-1}, h = 19.2 ft, area of reservoir = 1216 ac, estimated area of aquifer = 250,000 ac, θ = 180°.

SOLUTION: Since the reservoir is against a fault, $A = \frac{1}{2}\pi r_o^2$

$$r_o^2 = \frac{1216 \times 43,560}{0.5 \times 3.1416}$$

$$r_o = 5807 \text{ ft}$$

For t = 91.3 days (one-quarter year or one period):

$$t_D = \frac{6.323 \times 10^{-3} \times 275 \times 91.3}{0.209 \times 0.25 \times 6 \times 10^{-6} \times (5807)^2} = 15.0$$

$$B' = 1.119 \times 0.209 \times 6 \times 10^{-6} \times (5807)^2 \times 19.2 \times (180°/360°)$$

$$= 455 \text{ bbl/psi}$$

Since the aquifer is 250,000/1216 = 206 times the area of the reservoir, for a considerable time the infinite aquifer values may be used. Table 8.1

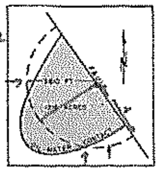


Fig. 8.11. Sketch showing the equivalent radius of a reservoir

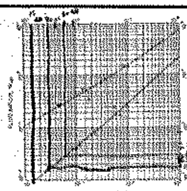


Fig. 8.9.

$\frac{A}{\frac{\theta}{360}} = \frac{\pi r^2}{2}$

$\Delta P_j = \frac{P_{j-1} - P_j}{2}$

Fig 8.9

TABLE 8.3.
Boundary step pressures and W_{eD} values for Ex. 8.3

Time Period j	Time in Days t	Dimensionless Time, t_D	W_{eD}	$\bar{p}_r$ Avg. Reservoir Pressure, psia	p_b Avg. Boundary Pressure, psia	Δp_j Step Pressure, psi
0	0	0	0.0	3793	3793	0.0
1	91.3	15	[illegible]	3788	3788	2.5
2	182.6	30	[illegible]	3774	3774	9.5
3	273.9	45	[illegible]	3747	3756	20.0
4	365.2	60	26.7	3733	3728	32.5
5	456.5	75	34.3	3697	3697	34.0
6	547.9	90	39.6	3653	3649	35.0

*Infinite aquifer ... 8.5 of Table 8.1.

$W_e(T) = B' \sum_{j=0}^{n-1} \Delta P_j \cdot W_{eD}(T_D - t_{Dj})$

shows the values of boundary step pressures and the W_{eD} values for the first six periods. The calculation of the step pressures Δp is illustrated in Fig. 8.12. For example,

$\Delta p_3 = \frac{1}{2}(p_2 - p_4) = \frac{1}{2}(3788 - 3748) = 20.0$ psi

Tables 8.4 and 8.5 show the calculation of $\Sigma \Delta p \times W_{eD}$ at the end of the third

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$\Delta P = 2.5$ Water influx ... 8.3
$\Delta P = 9.5$ ... 8.3
... 8.3

TABLE 8.4.
Water influx at the end of the third quarter for Example 8.3

t_D	W_{eD}	Δp	$\Delta p \times W_{eD}$
45	22.9	2.5	57.3
30	16.7	9.5	158.7
15	10.0	20.0	200.0

416

TABLE 8.5.
Water influx at the end of the fourth quarter for Example 8.3

t_D	W_{eD}	Δp	$\Delta p \times W_{eD}$
60	28.7	2.5	71.8
45	22.9	9.5	217.6
30	16.7	20.0	334.0
15	10.0	32.5	325.0

948.4

and fourth periods, the values being 416.0 and 948.4, respectively. Then the corresponding water influx at the end of these periods is

W_e (3rd quarter) $= B' \Sigma \Delta p \times W_{eD} = 455 \times 416.0 = 189{,}300$ bbl

W_e (4th quarter) $= B' \Sigma \Delta p \times W_{eD} = 455 \times 948.4 = 431{,}500$ bbl

In calculating the water influx in Ex. 8.3 at the end of the third quarter, it should be carefully noted in Table 8.4 that since the first pressure drop, $\Delta p_1 = 2.5$ psi, had been operating for the full three quarters ($t_D = 45$), it was multiplied by $W_{eD} = 22.9$, which corresponds to $t_D = 45$. Similarly, for the fourth quarter calculation in Table 8.5, the 2.5 psi was multiplied by $W_{eD} = 28.7$, which is the value for $t_D = 60$. Thus the W_{eD} values are inverted so that the one corresponding to the longest time is multiplied by the first pressure drop, and vice versa. Also, in calculating each successive value of $\Sigma \Delta p \times W_{eD}$, it is not simply a matter of adding a new $\Delta p \times W_{eD}$ term to the former summation, but a complete recalculation, as shown in Tables 8.4 and 8.5.

You should show that the water influx values at the end of the fifth and sixth quarters are 773,100 and 1,201,600 bbl, respectively.

Ex 8.3

At the end of fifth quarter

t_D	Wed	ΔP	ΔPxWed
75	34.3	2.5	85.75
60	28.7	9.5	272.65
45	22.9	20.0	458.00
30	16.7	32.5	542.75
15	10.0	34.0	340.00
			1699.15

W_e (5th quarter) $= B' \cdot \Sigma \Delta P \times W_{ed}$

$= 455 \times 1699.15 = 773{,}100$ bbl

At the end of sixth quarter

t_D	Wed	ΔP	ΔPxWed
90	39.6	2.5	99.0
75	34.3	9.5	325.85
60	28.7	20.0	574
45	22.9	32.5	744.25
30	16.7	34.0	567.8
15	10.0	35.0	350

$\Sigma = 2640.9$

W_e (6th quarter) $= B' \cdot \Sigma \Delta P W_{ed}$

$= 455 \times 2640.9 = 1{,}201{,}600$ bbl

3.2. Bottom-Water Drive

The van Everdingen and Hurst model discussed in the previous section is based on the radial diffusivity equation written without a term describing vertical flow from the aquifer. In theory, this model should not be used when there is significant movement of water into the reservoir from a bottom-water drive. To account for the flow of water in a vertical direction, Coats and later Allard and Chen, added a term to Eq. (7.35) to yield the following:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} + F_k\frac{\partial^2 p}{\partial z^2} = \frac{\phi \mu c_t}{0.0002637k}\frac{\partial p}{\partial t} \quad (8.10)$$

where F_k is the ratio of vertical to horizontal permeability.[12,14] $\frac{K_v}{K_h}$

Using the definitions of dimensionless time, radius, and pressure and introducing a second dimensionless distance, z_D, Eq. (8.10) becomes Eq. (8.11):

$$z_D = \frac{z}{r_R F_k^{1/2}} \qquad F_k = \frac{K_v}{K_h}$$

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_D}{\partial r_D} + \frac{\partial^2 p_D}{\partial z_D^2} = \frac{\partial p_D}{\partial t_D} \quad (8.11)$$

Coats solved Eq. (8.11) for the terminal rate case for infinite aquifers.[13] Allard and Chen used a numerical simulator to solve the problem for the terminal pressure case.[14] They defined a water influx constant, B', and a dimensionless water influx, W_{eD} analogous to those defined by van Everdingen and Hurst except that B' does not include the angle θ:

$$B' = 1.119\phi h c_t r_R^2 \quad (8.12)$$

The actual values of W_{eD} will be different from those of the van Everdingen and Hurst model because W_{eD} for the bottom-water drive is a function of the vertical permeability. Because of this functionality, the solutions presented by Allard and Chen, found in Tables 8.6 to 8.10, are functions of two dimen-

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Using the definitions of dimensionless time, radius, and pressure and introducing a second dimensionless distance, z_D, Eq. (8.10) becomes Eq. (8.11):

$$z_D = \frac{z}{r_R F_k^{1/2}} \qquad F_k = \frac{K_v}{K_h}$$

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_D}{\partial r_D} + \frac{\partial^2 p_D}{\partial z_D^2} = \frac{\partial p_D}{\partial t_D} \quad (8.11)$$

Coats solved Eq. (8.11) for the terminal rate case for infinite aquifers.[13] Allard and Chen used a numerical simulator to solve the problem for the terminal pressure case.[14] They defined a water influx constant, B', and a dimensionless water influx, W_{eD} analogous to those defined by van Everdingen and Hurst except that B' does not include the angle θ:

$$B' = 1.119\phi h c_t r_R^2 \quad (8.12)$$

The actual values of W_{eD} will be different from those of the van Everdingen and Hurst model because W_{eD} for the bottom-water drive is a function of the vertical permeability. Because of this functionality, the solutions presented by Allard and Chen, found in Tables 8.6 to 8.10, are functions of two dimensionless parameters, r'_D and z'_D.

$W_e = B' \Sigma \Delta P W_{ed}$

$$r'_D = \frac{r_e}{r_R} \quad (8.13)$$

$$z'_D = \frac{h}{r_R F_k^{1/2}} \qquad F_k = \frac{K_v}{K_h} \quad (8.14)$$

The method of calculating water influx from the dimensionless values obtained from these tables follows exactly the method illustrated in Exs. 8.1 to 8.3. The procedure is shown in Ex. 8.4, which is a problem taken from Allard and Chen.[14]

Example 8.4. Calculate the water influx as a function of time for the reservoir data and boundary pressure data that follow:

Given:

$r_R = 2000$ ft $\qquad r_e = \infty$

$h = 200$ ft $\qquad k = 50$ md

$F_k = 0.04$ $\qquad \phi = 0.10$ $\qquad \frac{k_v}{k_h}$

$\mu = 0.395$ cp $\qquad c_t = 8 \times 10^{-6}$ psi^{-1}

t_D	Wed	Time in Days (t)	ΔP	Average Boundary Pressure, psia (p_b)
30	14	0		3000
22.5	11.4	30	41.5	2956
15.0	8.4	60	39.5	2917
7.5	5.0	90		2877
		120	35.5	2844
		150		2811
		180		2791
		210		2773
		240		2755

Ex. 8.4 Advanced Reservoir.

① water influx at the end of the third Period

t_D	Wed	ΔP	ΔPxWed
22.5	11.414	22	251.1
15.0	8.389	41.5	348.1
7.5	5.038	39.5	199

$\Sigma = 798.2$

$W_e = B' \Sigma \Delta P W_{ed}$

$= 716 \times 798.2 = 572{,}000$ bbl

② At the end of fourth Period

t_D	Wed	ΔP	ΔPxWed
30	14.263	22	314
22.5	11.414	41.5	474
15	8.389	39.5	331
7.5	5.038	36.5	184

$\Sigma = 1303$

$W_e = 716 \times 1303 = 933$ Mbbl

③ At the end of fifth Period

37.5	16.996	22	374
30	14.263	41.5	592
22.5	11.414	39.5	451
15	8.389	36.5	306
7.5	5.038	33	166

$= 1889$

$W_e = 1889 \times 716$

$= 1353$ Mbbl

End of Period 2
$W_e = B' \sum \Delta P \, W_{eD}$
(41.5×5.038 + 22.0×8.389)

SOLUTION:

$r_D' = \infty$

$z_D' = \frac{200}{2000(0.040)^{1/2}} = 0.5$

$t_D = \frac{0.0002637(50)}{0.10(0.395)8(10)^{-6}2000^2} = 0.0104t$ (where t is in hours)

$B' = 1.119(0.10)(?)(10)^{-6}(?)2000^2 = 716$ bbl/psi

$\frac{3000 - 2956}{2}$

Time in Days (t)	Dimensionless Time (t_D)	W_{eD}	Average Boundary Pressure, psia (p_B)	Step Pressure (Δp)	Water Influx, M bbl (W_e)
0	0	0	3000 P_0	0	0
30	7.5	5.038	2956 P_1	22.0	79
60	15.0	8.389	2917 P_2	41.5	282
90	22.5	11.414	2877 P_3	39.5	572
120	30.0	14.263	2844 P_4	36.5	933
150	37.5	16.994	2811	33.0	1353
180	45.0	19.641	2791	26.5	1820
210	52.5	22.214	2773	19.0	2284
240	60.0	24.728	2755	18.0	2782

Using the definitions of dimensionless time, radius, and pressure and introducing a second dimensionless distance, z_D, Eq. (8.10) becomes Eq. (8.11):

$z_D = \frac{z}{r_e\sqrt{F_k}}$ $F_k = \frac{K_v}{K_h}$

$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D}\frac{\partial p_D}{\partial r_D} + \frac{\partial^2 p_D}{\partial z_D^2} = \frac{\partial p_D}{\partial t_D}$ (8.11)

Coats solved Eq. (8.11) for the terminal rate case for infinite aquifers.[?] Allard and Chen used a numerical simulator to solve the problem for the terminal pressure case.[?] They defined a water influx constant, B', and a dimensionless water influx, W_D, analogous to those defined by van Everdingen and Hurst except that B' does not include the angle θ.

$B' = 1.119\phi h c_t r_e^2$ (8.12)

The actual values of W_D will be different from those of the van Everdingen and Hurst model because W_D for the bottom-water drive is a function of the vertical permeability. Because of this functionality, the solutions presented by Allard and Chen, found in Tables 8.6 to 8.10, are functions of two dimensionless parameters, r_D' and z_D'.

$W_e = B' \sum \Delta P W_{eD}$

$r_D' = \frac{r_a}{r_e}$ (8.13)

$z_D' = \frac{h}{r_e\sqrt{F_k}}$ $F_k = \frac{K_v}{K_h}$ (8.14)

The method of calculating water influx from the dimensionless values obtained from these tables follows exactly the method illustrated in Exs. 8.3 to 8.5. The procedure is shown in Ex. 8.6, which is a problem taken from Allard and Chen.[?]

End of Period 2
$W_e = B' \sum \Delta P \, W_{eD}$
(41.5×5.038 + 22.0×8.389)

SOLUTION:

$r_D' = \infty$

$z_D' = \frac{200}{2000(0.040)^{1/2}} = 0.5$

$t_D = \frac{0.0002637(50)}{0.10(0.395)8(10)^{-6}2000^2} = 0.0104t$ (where t is in hours)

$B' = 1.119(0.10)(?)(10)^{-6}(?)2000^2 = 716$ bbl/psi

$\frac{3000 - 2956}{2}$

TABLE 8.6 (annotations): → 5.038, → 8.389, → 14.9[illegible]

we= 1809831

TABLE 8.6.
(Cont'd.)

t_D	z_D' 0.05	0.1	0.3	0.5	0.7	0.9	1.0
48	22.006	21.958	21.491	20.678	19.648	18.524	[illegible]
49	22.365	22.317	21.844	21.021	19.978	18.840	[illegible]
50	22.722	22.674	22.196	21.363	20.307	19.154	[illegible]
51	23.081	23.032	22.547	21.704	20.635	19.467	[illegible]
52	23.436	23.387	22.897	22.044	20.962	19.779	[illegible]
53	23.791	23.741	23.245	22.383	21.288	20.091	[illegible]
54	24.145	24.094	23.593	22.721	21.613	20.401	[illegible]
55	24.498	24.446	23.939	23.058	21.937	20.711	[illegible]
56	24.849	24.797	24.285	23.393	22.260	21.020	[illegible]
57	25.200	25.147	24.629	23.728	22.583	21.328	[illegible]
58	25.549	25.496	24.973	24.062	22.904	21.636	[illegible]
59	25.898	25.844	25.315	24.393	23.225	21.942	[illegible]
60	26.246	26.191	25.657	24.728	23.545	22.248	[illegible]
61	26.592	26.537	25.998	25.059	23.864	22.553	[illegible]
62	26.938	26.883	26.337	25.390	24.182	22.857	[illegible]
63	27.283	27.227	26.676	25.719	24.499	23.161	[illegible]
64	27.627	27.570	27.015	26.048	24.816	23.464	[illegible]
65	27.970	27.913	27.352	26.376	25.132	23.766	[illegible]
66	28.312	28.255	27.688	26.704	25.447	24.068	[illegible]
67	28.653	28.596	28.024	27.030	25.762	24.369	[illegible]
68	28.994	28.936	28.359	27.356	26.075	24.669	[illegible]
69	29.334	29.275	28.693	27.681	26.389	24.969	[illegible]
70	29.673	29.614	29.026	28.006	26.701	25.268	[illegible]
71	30.011	29.951	29.359	28.329	27.013	25.566	[illegible]
72	30.349	30.288	29.691	28.652	27.324	25.864	[illegible]

500	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
600	49.84	49.81	49.81	49.74	49.65	49.50	49.41
700	49.91	49.91	49.90	49.87	49.82	49.74	49.67
800	49.94	49.94	49.93	49.92	49.90	49.85	49.83
900	49.96	49.96	49.94	49.94	49.93	49.91	49.90
1,000	49.96	49.96	49.95	49.96	49.94	49.93	49.93
1,200	49.96	49.96	49.96	49.96	49.96	49.96	49.96

4. PSEUDOSTEADY-STATE MODELS ของ Fetkovich

The edge-water and bottom-water, unsteady-state methods discussed in Sect. 3 provide correct procedures for calculating water influx in nearly any reservoir application. However, the calculations tend to be somewhat cumbersome, and therefore there have been various attempts to simplify the calculations. The most popular and seemingly accurate method is one developed by Fetkovich using an aquifer material balance and an equation that describes the flow rate from the aquifer.[?] The equations for flow rate used by Fetkovich are similar to the productivity index equation defined in Chapter 7. The productivity index required pseudosteady-state flow conditions. Thus, this method neglects the effects of the transient period in the calculations of water influx, which will obviously introduce errors into the calculations. However, the method has been found to give results similar to those of the van Everdingen and Hurst model in many applications.

Fetkovich first wrote a material balance equation on the aquifer for constant water and rock compressibilities as

$\bar{p} = -\left(\frac{p_i}{W_{ei}}\right)W_e + p_i$ (8.15) $W_e = (P_i - \bar{P})\frac{W_{ei}}{P_i}$

$\frac{dW_e}{dt} = -\frac{W_{ei}}{P_i}\frac{d\bar{P}}{dt}$

where $\bar{p}$ is the average pressure in the aquifer after the removal of W_e bbl of water, p_i is the initial pressure of the aquifer, and W_{ei} is the initial encroachable water in place at the initial pressure. Fetkovich next defined a generalized rate equation as

$q_w B_w = J(\bar{p} - p_R)^n = \frac{dW_e}{dt}$ (8.16)

where $q_w B_w$ is the flow rate of water from the aquifer, J is the productivity index of the aquifer and is a function of the aquifer geometry, p_R is the

$-\frac{W_{ei}}{P_i}\frac{d\bar{P}}{dt} = J(\bar{P} - P_R)$

Fetkovich Pseudosteady-State Model

หลัก Compressibility
$W_e \propto \Delta P$ or $P_i - \bar{P}$

$W_e = \frac{W_{ei}}{P_i}(P_i - \bar{P})$ —①

Definition of Productivity

$q_w B_w = J(\bar{P} - P_R) = \frac{dW_e}{dt}$ —

diff. Integrate eq ①

$\frac{dW_e}{dt} = \frac{W_{ei}}{P_i}\left(-\frac{d\bar{P}}{dt}\right)$ แทนค่าใน (

$\frac{dW_e}{dt} = -\frac{W_{ei}}{P_i}\cdot\frac{d\bar{P}}{dt} = J(\bar{P} - P_R)$ —③

Integrate Eq.(3)

$-\frac{W_{ei}}{P_i}\int_{P_i}^{\bar{P}}\frac{d\bar{P}}{(\bar{P} - P_R)} = J\int_0^t dt$

$-\frac{W_{ei}}{P_i}\ln\frac{(\bar{P} - P_R)}{(P_i - P_R)} = Jt$

$\ln\frac{\bar{P} - P_R}{P_i - P_R} = -\frac{Jt P_i}{W_{ei}}$

$\frac{(\bar{P} - P_R)}{(P_i - P_R)} = e^{-\frac{Jt P_i}{W_{ei}}}$

$\bar{P} - P_R = (P_i - P_R)e^{-\frac{Jt P_i}{W_{ei}}}$

$-\left(\frac{P_i}{W_{ei}}\right)W_e + P_i - P_R = (P_i - P_R)e^{-\frac{Jt P_i}{W_{ei}}}$

$\frac{P_i}{W_{ei}}W_e = (P_i - P_R) - (P_i - P_R)e^{-\frac{Jt P_i}{W_{ei}}}$

$W_e = \frac{W_{ei}}{P_i}(P_i - P_R)\left(1 - e^{-\frac{Jt P_i}{W_{ei}}}\right)$

Compressibility

$c_t = -\frac{1}{V}\frac{dV}{dP} = +\frac{1}{V_b \times \phi}\cdot\frac{W_{ei}}{P_i}$

$\therefore W_{ei} = c_t P_i \cdot V_b \cdot \phi\left(\frac{\theta}{360}\right)$

226 $\ln\frac{\bar{P} - P_R}{P_i - P_R} = -\frac{Jt P_i}{W_{ei}}$ Water Influx

pressure at the reservoir-aquifer boundary, and n, is equal to 1 for Darcy flow during the pseudosteady-state flow region. Equations (8.15) and (8.16) can be combined to yield the following equation (see References 15 and 18 for the complete derivation):

$W_e = \frac{W_{ei}}{p_i}(p_i - p_R)\left(1 - e^{-\frac{p_i J t}{W_{ei}}}\right)$ (8.17)

This equation was derived for constant pressures at both the reservoir-aquifer boundary, p_R, and the average pressure in the aquifer, $\bar{p}$. At this point, to apply the equation to a typical reservoir application where both of these pressures are changing with time, it would normally be required to use the principle of superposition. Fetkovich showed that by calculating the water influx for a short time period, Δt, with a corresponding average aquifer pressure, $\bar{p}$, and an average boundary pressure, $\bar{p}_R$, and then starting the calculation over again for a new period and new pressures, superposition was not needed. The following equations are used in the calculation for water influx with this method:

$\Delta W_{en} = \frac{W_{ei}}{p_i}(\bar{p}_{n-1} - \bar{p}_{Rn})\left(1 - e^{-\frac{p_i J \Delta t_n}{W_{ei}}}\right)$ (8.18)

$\bar{p}_{n-1} = p_i\left(1 - \frac{W_{e,n-1}}{W_{ei}}\right)$ จาก Eq 8.15 (8.19)

$\bar{p}_{Rn} = \frac{p_{R,n-1} + p_{Rn}}{2}$ (8.20)

$W_{ei} = c_t\left(\frac{\theta}{360}\right)V_b \times \phi \times P_i$ $c_t = \frac{1}{V}\frac{dV}{dP}$

where n represents a particular interval, $\bar{p}_{n-1}$ is the average aquifer pressure at the end of the $n-1$ time interval, $\bar{p}_{Rn}$ is the average reservoir-aquifer boundary pressure during interval n, and W_e is the total, or cumulative, water influx and is given by

$W_e = \Sigma \Delta W_{en}$ (8.21)

The productivity index, J, used in the calculation procedure is a function of the geometry of the aquifer. Table 8.11 contains several aquifer productivity indexes as presented by Fetkovich.[?] When you use the equations for the condition of a constant pressure outer aquifer boundary, the average aquifer pressure in Eq. (8.15) will always be equal to the initial outer boundary pressure, which is usually p_i. Example 8.5 illustrates the use of the Fetkovich method.

Example 8.5. Repeat the water influx calculations for the reservoir in Ex. 8.3 using the Fetkovich approach.

TABLE 8.11.
Productivity indices for radial and linear aquifers (taken from reference 15) Fetkovich

Type of Outer Aquifer Boundary	Radial Flow[a]	Linear Flow[b]
Finite—no flow	$J = \dfrac{0.00708kh\left(\frac{\theta}{360}\right)}{\mu[\ln(r_e/r_R) - 0.75]}$	$J = \dfrac{0.003381kwh}{\mu L}$
Finite—constant pressure	$J = \dfrac{0.00708kh\left(\frac{\theta}{360}\right)}{\mu[\ln(r_e/r_R)]}$	$J = \dfrac{0.001127kwh}{\mu L}$

[a] Units are in normal field units with k in millidarcies.
[b] w is width and L is length of linear aquifer.

SOLUTION:

area of aquifer $= \frac{1}{2}\pi r_e^2$ or $r_e = \left[\dfrac{250{,}000(43560)}{0.5\pi}\right]^{1/2} = 83{,}263$ ft

area of reservoir $= \frac{1}{2}\pi r_R^2$ or $r_R = \left[\dfrac{1216(43560)}{0.5\pi}\right]^{1/2} = 5807$ ft

$$W_{ei} = \frac{c_t\left(\frac{\theta}{360}\right)\pi(r_e^2 - r_R^2)h\phi p_i}{5.615}$$

$$W_{ei} = \frac{6(10)^{-6}\left(\frac{180}{360}\right)\pi(83{,}263^2 - 5807^2)19.2(0.209)3793}{5.615} = 176.3(10)^6 \text{ bbl}$$

$$J = \frac{0.00708kh\left(\frac{\theta}{360}\right)}{\mu[\ln(r_e/r_R - 0.75]} = \frac{0.00708(275)(19.2)\left(\frac{180}{360}\right)}{0.25\left[\ln\left(\frac{83{,}263}{5807}\right) - 0.75\right]} = 39.08$$

$$\Delta W_{en} = \frac{W_{ei}}{p_i}(\bar{p}_{n-1} - \bar{p}_{Rn})\left(1 - e^{-\frac{Jp_i\Delta t_n}{W_{ei}}}\right)$$

$$= \frac{176.3\,(10)^6}{3793}(\bar{p}_{n-1} - \bar{p}_{Rn})\left(1 - e^{-\frac{39.08(3793)(91.3)}{176.3(10)^6}}\right)$$

$$\Delta W_{en} = 3435(\bar{p}_{n-1} - \bar{p}_{Rn}) \tag{8.22}$$

$$\bar{p}_{n-1} = p_i\left(1 - \frac{\Sigma\Delta W_{en}}{W_{ei}}\right)$$

$$\bar{p}_{n-1} = 3793\left(1 - \frac{\Sigma\Delta W_{en}}{176.3(10)^6}\right) \tag{8.23}$$

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Solving Eqs. (8.22) and (8.23), we get Table 8.12.

TABLE 8.12.

Time	p_R	$\bar{p}_{Rn}$	$\bar{p}_{n-1} - \bar{p}_{Rn}$	ΔW_e	W_e	$\bar{p}_n$
0	3793	3793	0	0	0	3793
1	3788	3790.5	2.5	8,600	8,600	3792.8
2	3774	3781	11.8	40,500	49,100	3791.9
3	3748	3761	30.9	106,100	155,200	3789.7
4	3709	3728.5	61.2	210,000	365,300	3785.1
5	3680	3694.5	90.6	311,200	676,500	3778.4
6	3643	3661.5	116.9	401,600	1,078,100	3769.8

The water influx values calculated by the Fetkovich method agree fairly closely with those calculated by the van Everdingen and Hurst method used in Ex. 8.3. The Fetkovich method consistently gives water influx values smaller than the values calculated by the van Everdingen and Hurst method for this problem (Fig. 8.14). This result could be because the Fetkovich method does not apply to an aquifer that remains in the transient time flow. It is apparent from observing the values of p_{n-1}, which are the average pressure values in the aquifer, that the pressure in the aquifer is not dropping very fast, which would indicate that the aquifer is very large and that the water flow from it to the reservoir could be transient in nature.

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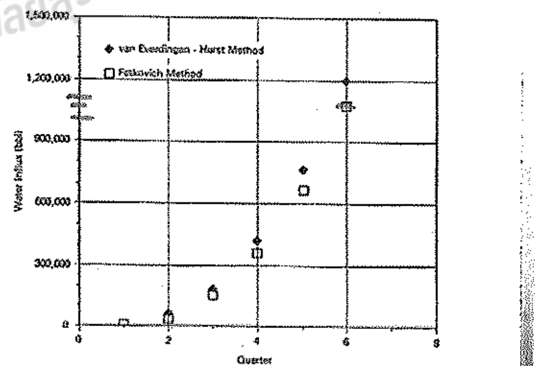


Fig. 8.14 Results of water influx calculations from Ex. 8.5.

Material Balance Equation:

The various forms of the Water Influx equation can be used to determine the term W_e which is part of the MBE. This additional information is incorporated into one form of MBE equation as follows:

$$F = N_p B_o + G_p B_g + W_p B_w - W_e B_w$$

$$= k\int(P_i - P)\,dt\;B_w \quad \text{steady}$$

$$= \frac{k\int(P_i - P)\,dt\;B_w}{\log at}$$

$$= U\sum \Delta P\, Q(t)\, B_w \quad \text{Unsteady}$$

$$\frac{F}{E_o} = N + \frac{W_e}{E_o} \quad \text{Oil No gas Cap}$$

$$\frac{F}{E_g} = N + \frac{W_e}{E_g} \quad \text{Gas Reservoir.}$$

$$\frac{F}{E_o + m\frac{B_{ti}}{B_{gi}}E_g} = N + C\,\frac{\sum \Delta P\, Q(\Delta t_D)}{E_o + m\frac{B_{ti}}{B_{gi}}E_g} \quad W_{eD}$$

PROBLEMS

REFERENCES

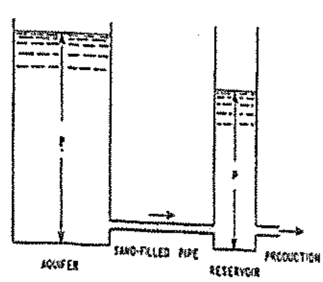
CHAPTER 8-2

Dynamic of Water Drive Reservoir

Steady State Model (Schilthuis)

The simplest method for characterizing water influx is due to Schilthuis [3]. It is often a good idea to try this model first since the calculations are considerably less involved than with either of the other two methods.

The assumptions inherent in the Schilthuis model are many. First, it is assumed that the aquifer is gigantic and highly permeable. In fact, the aquifer is taken to have permeability so high that the pressure gradient across the aquifer itself is negligible. And the aquifer is so huge that the pressure within the aquifer never declines; i.e., the initial pressure, P_i, always exists at all locations within the aquifer. Consider the hydraulic analog to the Schilthuis steady state model shown below.

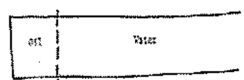


Hydraulic analog of steady-state water influx into a reservoir (from Craft & Hawkins[4]).

Aquifer Description:

The aquifer can be either of several types depending on the geometrical configuration:

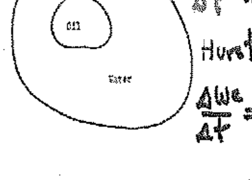
(a) Linear:



Schilthuis Steady State (Darcy)

$\frac{\Delta W_e}{\Delta t} = k'(P_i - P)$ or $W_e = k'\int_0^t (P_i - P)$

(b) Radial



Hurst

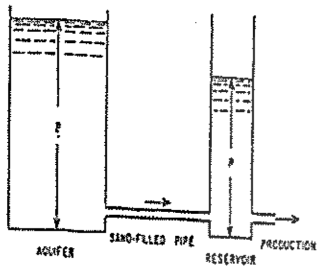
$\frac{\Delta W_e}{\Delta t} = \frac{C'(P_i - P)}{\log at}$

... be characterized according to how the water encroaches.

Steady State Model (Schilthuis)

The simplest method for characterizing water influx is due to Schilthuis [3]. It is often a good idea to try this model first since the calculations are considerably less involved than with either of the other two methods.

The assumptions inherent in the Schilthuis model are many. First, it is assumed that the aquifer is gigantic and highly permeable. In fact, the aquifer is taken to have permeability so high that the pressure gradient across the aquifer itself is negligible. And the aquifer is so huge that the pressure within the aquifer never declines; i.e., the initial pressure, P_i, always exists at all locations within the aquifer. Consider the hydraulic analog to the Schilthuis steady state model shown below.



Hydraulic analog of steady-state water influx into a reservoir (from Craft & Hawkins[4]).

Aquifer Description:

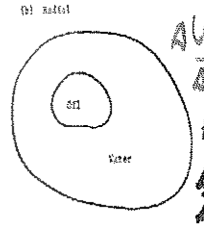
The aquifer can be either of several types depending on the geometrical configuration:

(a) Linear:

Schilthuis Steady State (Darcy)

$\frac{\Delta W_e}{\Delta t} = K'(P_i - P)$ or $W_e = k\int_0^t (P_i - P)$

(b) Radial:



Hurst

$\frac{\Delta W_e}{\Delta t} = \frac{C'(P_i - P)}{\log at}$

... depending on how the water encroaches.

where: $(W_e)_n$ = the cumulative water influx at time T_n, Bbls,
P_i = the initial pressure, psi,
P_j = the static reservoir pressure at T_j, psi,
ΔT_j = the time interval between T_{j-1} and T_j.

It is interesting to note that this model contains no water influx rate dampening features at all. A change in rate in the field has an instantaneous effect on the aquifer influx. Essentially, this means that there is no compressibility in the system between the reservoir and the aquifer. There is an immediate aquifer reaction to any pressure change. This is not entirely realistic, but this model is easy to use.

Actually, the aim here is to use this aquifer model in conjunction with material balance to do two things:

(1) Calculate the original oil-in-place, N, and

(2) Solve for the steady-state aquifer constant (Schilthuis constant), C_s. This constant actually relates the rate of water influx per psi of pressure drop across the o/w contact.

Recall the following material balance equation:

$$N_a = N + \frac{W_e}{D} \quad \text{or} \quad \frac{F}{E_{o,g}} = N + \frac{W_e}{E_{o,g}}$$

Thus, the Schilthuis equation can be substituted into the above equation with the result:

$$N_a = N + C_s\left(\frac{\Sigma \Delta P \ \Delta T}{D}\right) \quad \text{Steady}$$

where: N_a = the apparent oil-in-place (calculated by assuming that W_e is zero), STB,
N = the actual original oil-in-place, STB.

entirely realistic, but this model is easy to use.

Actually, the aim here is to use this aquifer model in conjunction with material balance to do two things:

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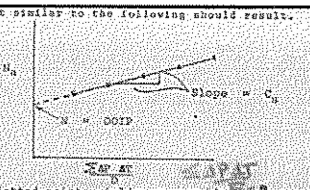
$$N_a = N + C_s\left(\frac{\Sigma \Delta P\ \Delta T}{D}\right)$$

where: N_a = the apparent oil-in-place (calculated by assuming that W_e is zero), STB,
N = the actual original oil-in-place, STB,
$\Sigma \Delta P\ \Delta T$ = the integral of pressure drop across the original oil-water contact with respect to time, and
D = the denominator of the material balance equation, given earlier.

Assuming that N and C_s are constants, the above equation is in the form of a straight line: y = mx + b. Thus, if the calculated N_a's are plotted versus:

$$\left(\frac{\Sigma \Delta P\ \Delta T}{D}\right)$$

...a plot similar to the following should result.



...the plotted points yield a reasonably straight line, then extrapolate the straight line back to the "Y-intercept" to get N. Further, the slope of the straight line is equal to C_s.

However, if the points do not plot as a straight line, but are nonlinear, then the aquifer-reservoir system is not behaving according to the assumptions inherent in the Schilthuis model. In this case, it is not wise to attempt to use the Schilthuis model. If the points are only slightly nonlinear, then the plot may be able to be extrapolated to obtain N without significant error. However, it would not be a good idea to use the C_s to attempt to predict future aquifer performance. If the plot is extremely nonlinear, then do not attempt to use this model to obtain N or C_s; one of the other models will need to be used.

The steady state model is used with the same logic for a gas reservoir as for an oil reservoir. A different material balance denominator, D, is used:

$$D = B_g - B_{gi}$$

The apparent gas in place (setting W_e = 0) at different times in

...return to the material balance equation arranged to solve for original hydrocarbon in place (N for an oil reservoir; G for a gas reservoir). To illustrate, consider the material balance equation once again for an oil reservoir without an initial gas cap:

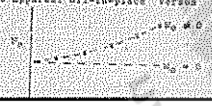
$$N = \frac{N_p\left[B_o + B_g(R_p - R_s)\right] + W_pB_w - W_e}{B_g(R_{si} - R_s) - (B_{oi} - B_o)}$$

The initial purpose is to determine whether or not a water drive exists. Obviously, then the function, W_e, in the above equation is unknown. To begin, W_e is set equal to zero, and the resulting N that is calculated is referred to as the apparent oil in place, N_a:

$$N_a = \frac{N_p\left[B_o + B_g(R_p - R_s)\right] + W_pB_w}{B_g(R_{si} - R_s) - (B_{oi} - B_o)}$$

Now, assuming that at least 4 or 5 years of production history and the corresponding static pressures are available, then N_a can be calculated using data corresponding to the end of each of the years of production history. More credence should be given to the later values of N_a, since more data is available; and small differences (or errors) in PVT property values will not influence the calculations as much. (The denominator of the above material balance equation is quite small at early times; hence errors are magnified.)

If these calculations are performed, then we should have an N_a for each year of production history: N_{a1}, N_{a2}, N_{a3}, . . . etc. Then, a graph is prepared of the apparent oil-in-place versus time or cumulative production.



...In this case, it is not wise to attempt to use the Schilthuis model. If the points are only slightly nonlinear, then the plot may be able to be extrapolated to obtain N without significant error. However, it would not be a good idea to use the C_s to attempt to predict future aquifer performance. If the plot is extremely nonlinear, then do not attempt to use this model to obtain N or C_s; one of the other models will need to be used.

The steady state model is used with the same logic for a gas reservoir as for an oil reservoir. A different material balance denominator, D, is used:

$$D = B_g - B_{gi}$$

The apparent gas in place (setting W_e = 0) at different times in the performance history is:

$$G_A = \frac{G_pB_g + W_pB_w}{B_g - B_{gi}}$$

Then, G_A is plotted versus $\Sigma \Delta P\ \Delta T$ / D. Once again, the plot (if linear) is extrapolated to get G, and the slope is measured to get C_s.

Whether the reservoir is oil or gas, there are two basic methods to obtain the "best" straight line through the plotted points:

(1) Eyeball and

(2) Least squares.

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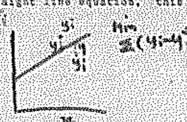
In the past, most engineers have traditionally used the first method (assuming that their eyes were better than their mathematics). Today, however, the least squares approach is receiving more and more attention.

Development of Least Squares Equations

If there are more data points than unknowns, then the least squares method may be used to determine the "best" values of the unknowns. As the Schilthuis aquifer model - material balance combination equation is actually a straight line equation, this is the equation that will be considered.

$$Y_i = A + B\,X_i$$



where: $Y_i = N_{ai}$
$X_i = \left(\frac{\Sigma \Delta P\ \Delta T}{D}\right)_i$
$A = N$
$B = C_s$

In order to find the best constants A and B, the following residual function will be minimized:

$$R = \Sigma(Y_i - A - B\,X_i)^2$$

The conditions for minimizing this function with respect to A and B are:

$$\frac{\partial R}{\partial A} = 0 \quad \text{and} \quad \frac{\partial R}{\partial B} = 0$$

Equating these derivatives and simplifying:

A = N
B = C_s

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$$R = \Sigma(Y_i - A - B\,X_i)^2$$

The conditions for minimizing this function with respect to A and B are:

$$\frac{\partial R}{\partial A} = 0 \quad \text{and} \quad \frac{\partial R}{\partial B} = 0$$

Equating these derivatives and simplifying:

$$\frac{\partial R}{\partial A} = 2\Sigma Y_i - 2nA - 2B\Sigma X_i = 0$$

So $\Sigma Y_i = nA + B\Sigma X_i$

where: n = number of data points Y_i and X_i

$$\frac{\partial R}{\partial B} = 2\Sigma Y_i X_i - 2A\Sigma X_i - 2B\Sigma X_i^2 = 0$$

So $\Sigma Y_i X_i = A\Sigma X_i + B\Sigma X_i^2$

Solving for A:

$$N = A = \frac{(\Sigma Y_i X_i)(\Sigma X_i) - (\Sigma X_i^2)(\Sigma Y_i)}{(\Sigma X_i)(\Sigma X_i) - n\Sigma X_i^2} = N$$

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If there is no water influx occurring, then the calculated N_a's will be fairly constant as shown on the previous diagram. If water encroachment is occurring, then the N_a's will be increasing with time or cumulative production.

If we return to the material balance equation, it is easily seen that:

$$N_a = N + \frac{W_e}{D}$$

where: D = denominator of the material balance equation. Also called the "expansibility" of the reservoir system. Physically, it is the expansion associated with one STB of original oil-in-place (from the original pressure to the present static pressure).

For an oil reservoir without an initial gas cap,

$$D = B_g(R_{si} - R_s) + (B_{oi} - B_o)$$

Aquifer Models

Normally, we have very little data, if any concerning the aquifer of a hydrocarbon reservoir. So, the usual approach to describe an aquifer is to use some sort of implicit method or conceptual model.

Considering the type of trap is usually helpful. In blanket sands, with no indication of faulting, we would expect large (possibly capable of being regarded as infinite) aquifer sizes. On the other hand, many stratigraphic traps have limited water drives due to limited aquifer extent. So, studying the type of trap should shed some light on what model to choose.

Hopefully, it is already apparent that water influx depends more on the properties of the aquifer than on the characteristics of

---

$$\text{Expansion} = N\left[B_g(R_{si} - R_s) - (B_{oi} - B_o) + m\frac{B_{oi}}{B_{gi}}(B_g - B_{gi})\right]$$

where: $m = \dfrac{G\,B_{gi}}{N\,B_{oi}}$

G = initial gas cap size, (MCF or SCF), and

B_{gi} = gas formation volume factor at P_i, (Bbl/MCF or Bbl/SCF, such that $G \times B_{gi}$ = Bbl).

For an undersaturated oil reservoir (where the entire reservoir history has been above the bubble point):

$$\text{Expansion} = N\,c_{oe}\,B_{oi}(P_i - \bar{P})$$

where: c_{oe} = the effective oil compressibility (based on the oil volume, but takes into account the effects of the expanding rock and water), psi^{-1}

$$c_{oe} = c_o + \frac{c_w S_w}{1 - S_w} + \frac{c_f}{1 - S_w}$$

where: c_f = the pore volume compressibility, psi^{-1}, or

$$c_f = \left(\frac{1}{\text{pore volume}}\right)\frac{\Delta(\text{pore volume})}{\Delta(\text{pressure})} \quad \text{psi}^{-1}$$

Please carefully note that the effective oil compressibility defined above (used in the material balance equation for an undersaturated oil reservoir) is not the same as the effective or total compressibility used in various well testing equations.

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Solving for B:

$$B = \frac{(\sum X_i)(\sum Y_i) - n(\sum Y_i X_i)}{(\sum X_i)(\sum X_i) - n\sum(X_i^2)} = C_s$$

Illustrative Example I

Calculate water influx, W_e, using the steady state (Schilthuis) model. The basic data include:

C_s = 1000 Bbl/Month/psi

P_i = 2500 psia

Reservoir Static Pressure Data:

Time, Months	Pressure, Psia
0	2500
12	2480
24	2470
36	2464
48	2460

Recall the steady state aquifer equation:

$$W_e = C_s \sum \Delta P\,\Delta T$$

where: $\Delta P = P_i - P_{avg}$

In order to evaluate P_{avg} and the $\sum \Delta P\,\Delta T$ function, it is helpful

---

C_s = 1000 Bbl/Month/psi

P_i = 2500 psia

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24	2470
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48	2460

Recall the steady state aquifer equation:

$$W_e = C_s \sum \Delta P\,\Delta T$$

where: $\Delta P = P_i - P_{avg}$

In order to evaluate P_{avg} and the $\sum \Delta P\,\Delta T$ function, it is helpful to use a tabular calculation procedure:

Time Mo.	ΔT Mo.	Pressure psia	Average Pressure	ΔP psi	ΔP ΔT	Σ ΔP ΔT	W_e M Bbls
0		2500	2500	0	0	0	
12	12	2480	2490	10	120	120	120
24	12	2470	2475	25	300	420	420
36	12	2464	2467	33	396	816	816
48	12	2460	2462	38	456	1272	1272

pressure @ Oil-Water Contact

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Steady State

$$\frac{F}{E_o} = N + \frac{C_s \sum \Delta P \Delta T}{E_o}$$

Illustrative Example II -- Steady State Model

Calculations of OOIP (N) and Water Influx Constant (C_s)

Basic Data:

Porosity	:	18 %
Connate Water Saturation	:	25 %
Oil Compressibility	:	10 x 10^{-6} psi^{-1}
Water Compressibility	:	3 x 10^{-6} psi^{-1}
Formation Compressibility	:	4 x 10^{-6} psi^{-1}
Water Form. Vol. Factor	:	1.0 Bbl/STB
Bubble Point Pressure	:	2150 psia
Initial Pressure	:	3000 psia

$$\frac{F}{E_o} = N + C_s\frac{\sum \Delta P \Delta T}{E_o}$$

Pressure -- Production Data

Time Years	Oil Production MB	Water Production MB	Pressure psia	B_o
0	0	0	3000	1.3100
1	80.2	0	2870	1.3117
2	221.4	20	2810	1.3125
3	393.5	60	2760	1.3131
4	586.1	130	2720	1.3137
5	806.9	200	2690	1.3141

All of the static pressures are above the bubble point pressure, so this system can be analyzed as an undersaturated reservoir. Therefore, we need to calculate the effective oil compressibility:

$$c_{oe} = c_o + \frac{c_w S_w}{S_o} + \frac{c_f}{S_o} = \frac{c_o S_o + c_w S_w + c_f}{1 - S_w}$$

Pressure -- Production Data

Time Years	Oil Production MB	Water Production MB	Pressure psia	B_o
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$$= \left[10 + \frac{(3)(0.25)}{0.75} + \frac{4}{0.75}\right](10^{-6})$$

$$= 16.33 \times 10^{-6}\ \text{psi}^{-1}$$

$$= \frac{1}{V}\frac{dV}{dP}$$

Calculations of Expansibility, "D"

The expansibility (D) is the denominator of the material balance equation when solved for "N". Thus, for an undersaturated oil reservoir we have:

$$\text{Expansibility} = D = (c_{oe})(B_{oi})(P_i - P) = E_o$$

$$= (16.33)(10^{-6})(1.3100)(P_i - P)$$

$$= (21.392)(10^{-6})(P_i - P)$$

Thus, with each static pressure, an expansibility can be calculated:

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3. UNSTEADY-STATE MODELS

3.1. The van Everdingen and Hurst Edge-Water Drive Model

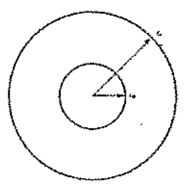
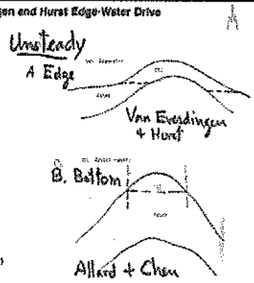


Fig. 8.6. Circular reservoir inside a circular aquifer.



for a radial system such as Fig. 8.6, where the driving potential of the s[illegible] is the water expandability and the rock compressibility:

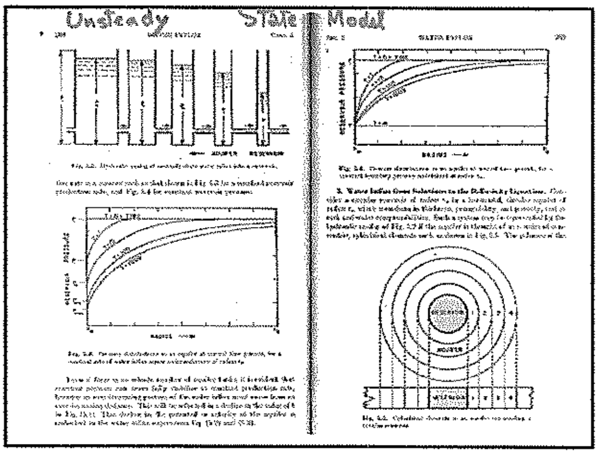
$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi\mu c_t}{0.0002637k}\frac{\partial p}{\partial t} \qquad (7.35)$$

This equation was solved in Chapter 7 for what is referred to as the *constant terminal rate case*. The constant terminal rate case [illegible] rate at the inner boundary, which was the wellbore fo[illegible] ter 7. This was appropriate for the applications of[illegible] desirous to know the pressure behavior at various[illegible] because a constant flow of fluid came into the wellb[illegible]

In this chapter, the diffusivity equation is appli[illegible] the inner boundary is defined as the interface betwe[illegible] aquifer. With the interface as the inner boundary, it would be more useful to require the pressure at the inner boundary to remain constant and observe the

$$p = \text{constant} = p_i - \Delta p \text{ at } r = r_o \qquad (8.5)$$

where r_o is a constant and is equal to the outer radius of the reservoir (i.e., the original oil-water contact). The pressure p must be determined at this original oil-water contact. Van Everdingen and Hurst[4] solved the diffusivity equation for this condition, which is referred to as the *constant terminal pressure case*, [illegible] and outer boundary conditions.



Time, Years	Pressure Psia	$P_i - P$ Psi	Expansibility E_o
0	3000	0	—
1	2870	130	0.00278
2	2810	190	0.004064
3	2760	240	0.005134
4	2720	280	0.005990
5	2690	310	0.006682

Calculations of Withdrawals

Withdrawals = $N_p B_o + W_p$ = F (B_w = 1.0)

$N_a = \frac{\text{Withdrawals}}{\text{Expansibility}} = \frac{F}{E_o}$

Time Yr.	N_p MB	B_o	W_p MB	$N_p B_o$	Withdrawals MB	N_a MMB (F/E_o)
0	0	1.3100	0	0	0	
1	80.7	1.3117	0	105.85	105.85	38.076
2	221.4	1.3125	20	290.59	310.59	76.425
3	395.5	1.3131	60	519.33	579.33	112.842
4	586.1	1.3137	130	769.96	899.96	150.244
5	806.9	1.3141	200	1060.35	1260.35	190.407
						567.994

Calculation of $(\sum \Delta P \Delta T)/D$

Time Yrs.	Pressure Psia	P_{avg} Psia	$P_i - P_{avg}$ Psi	ΔT Mo.	$\Delta P \Delta T$	$\sum \Delta P \Delta T$	$(\sum \Delta P \Delta T)/D$
0	3000	3000	0				
1	2870	2935	65	12	780	780	280.58 x 10^3
2	2810	2840	160	12	1920	2700	664.37 x 10^3
3	2760	2785	215	12	2580	5280	1028.44 x 10^3
4	2720	2740	260	12	3120	8400	1402.34 x 10^3
5	2690	2705	295	12	3540	11940	1800.36 x 10^3
							5.17609 x 10^6

$\frac{\sum \Delta P \Delta T}{E_o}$

Calculation of N and C_v (Least Squares Method)

$Y = N_a$

$\sum Y = 567.994$ MMB

$X = \sum \Delta P \Delta T / D$

$\sum X = 5.17609 \times 10^6$

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Recall the least squares formulas:

$$N = \frac{(\sum X_i Y_i)(\sum X_i) - (\sum X_i^2)(\sum Y_i)}{(\sum X_i)(\sum X_i) - n(\sum X_i^2)}$$

$$C_v = \frac{(\sum X_i)(\sum Y_i) - n(\sum X_i Y_i)}{(\sum X_i)(\sum X_i) - n\sum (X_i^2)}$$

One more table is needed:

Time	X	X^2	Y, MMB	(Y)(X)
1	280.58(10^3)	0.0787(10^{12})	38.076	10.683(10^6)
2	664.37(10^3)	0.4414(10^{12})	76.425	50.774(10^6)
3	1028.44(10^3)	1.0577(10^{12})	112.842	116.051(10^6)
4	1402.34(10^3)	1.9666(10^{12})	150.244	210.693(10^6)
5	1800.36(10^3)	3.2413(10^{12})	190.407	342.801(10^6)
	5.17609(10^6)	6.7857(10^{12})	567.994	731.002(10^6)

Therefore:

$$N = \frac{(731.002)(5.17609)(10^{12}) - (6.7857)(567.994)(10^{12})}{(5.17609)(5.17609)(10^{12}) - (5)(6.7857)(10^{12})}$$

$= 9.339$ MMSTB

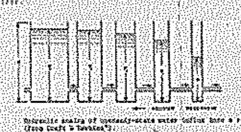
$$C_v = \frac{(5.17609)(567.994)(10^6) - (5)(731.002)(10^6)}{(5.17609)(5.17609)(10^{12}) - (5)(6.7857)(10^{12})}$$

$= 106.2 \times 10^{-6}$ MMB/month/psi

C_v = 106.2 barrels/~~month~~/psi (year)

Unsteady-State Flow Aquifer Model -- van Everdingen and Hurst

The figure below provides a hydraulic analog of the van Everdingen and Hurst aquifer model using a series of tanks interconnected with sand-filled pipes. [illegible]

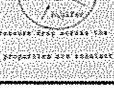


Hydraulic analog of unsteady-state water influx into a reservoir (from Craft & Hawkins).

[illegible]

The van Everdingen and Hurst mathematical model for flow of water through the aquifer involves the following assumptions:

(1) There is radial flow of water through the aquifer as shown below:



(2) Constant pressure drop across the aquifer throughout all finite time.

(3) The aquifer properties are constant and uniform.

Based on these assumptions, the van Everdingen and Hurst aquifer equation is:

$$W_e = 2\pi \alpha h \phi c_e R_f^2 \left[\Delta P\, Q(t_d)\right]$$

where: α = a fraction between 0 and 1 that represents the extent to which the aquifer surrounds the reservoir. In the figure on the last page, α = 1.0. (Aquifer completely surrounds the reservoir.)
W_e = cumulative water influx, cm^3,
h = the net aquifer thickness, cm,
ϕ = aquifer porosity, fraction,
c_e = effective compressibility of the aquifer, atm^{-1},
R_f = radius of the oil or gas reservoir, cm,
ΔP = constant pressure drop across the aquifer, atm,
$Q(t_d)$ = cumulative influx function (developed by van Everdingen and Hurst),
t_d = dimensionless time based on the reservoir radius; i.e.:

$$t_d = \frac{K\,t}{\phi \mu_w c_e R_f^2}$$

K = permeability of the aquifer, darcies,
t = time, seconds.

This aquifer equation, written in Darcy units, is sometimes written as:

$$W_e = C_v\, \Delta P\, Q(t_d)$$

where: $C_v = 2\pi \alpha h \phi c_e R_f^2$

ΔP = constant pressure drop across the aquifer, atm,
$Q(t_d)$ = cumulative influx function (developed by van Everdingen and Hurst),
t_d = dimensionless time based on the reservoir radius; i.e.:

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K = permeability of the aquifer, darcies,
t = time, seconds.

This aquifer equation, written in Darcy units, is sometimes written as:

$$W_e = C_v\, \Delta P\, Q(t_d)$$

where: $C_v = 2\pi \alpha h \phi c_e R_f^2$

In so-called "practical oil-field" units, the water influx equation is:

$$W_e = \underbrace{1.119\, \phi h c_e R_f^2 \alpha}_{C_v} \Delta P\, Q(t_d)$$

or

$$W_e = C_v\, \Delta P\, Q(t_d)$$

where: α = as defined with the Darcy units equation,
W_e = cumulative water influx, Res. Bbls,
h = effective aquifer thickness, feet,
ϕ = aquifer porosity, fraction,
c_e = effective compressibility of the aquifer, psi^{-1}.

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R_f = radius of the oil or gas reservoir, feet,
ΔP = constant pressure drop across the aquifer, psi,
$Q(t_d)$ = cumulative influx function,
t_d = dimensionless time; i.e.:

$$t_d = \frac{0.00633\, K\, t}{\phi \mu_w c_e R_f^2}$$

where: K = aquifer permeability, md, and
t = time, days.

Of the assumptions implicit in this model, only the one requiring a constant value of ΔP needs to be modified to allow practical use of this theory. The superposition principle will be used to apply the theory to handle real situations. Consider the basic differential equation that describes radial flow through an aquifer:

$$\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r} = \frac{\phi \mu_w c_e}{K}\frac{\partial p}{\partial t}$$

where: P = pressure at any radial position, r, and time, t, atm,
r = radial position from center of reservoir, cm,
t = time, seconds.

Application of Superposition Principle

The above equation is a linear partial differential equation; hence, the solutions are additive. Thus, the principle of superposition can be used. To apply the superposition principle, consider a representative pressure-time profile at the original

[illegible]

ΔP_1 = [illegible]

ΔP_2 = [illegible]

ΔP_3 = [illegible]

ΔP_4 = [illegible]

[illegible]

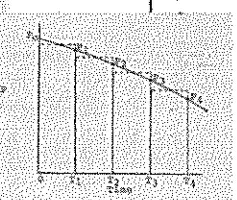
ΔP_j = [illegible]

[illegible]

ΔP_1 = [illegible]

[illegible]

$$W_e = C_v \sum_{j=1}^{n} \Delta P_j\, Q[\ldots]$$



where:

for Darcy units: $C_v = 2\pi h \alpha \phi c_e R_f^2$

for practical oil field units: $C_v = 1.119\, h \phi \alpha c_e R_f^2$

The constant "A" found in the previous water encroachment equation is a multiplier which converts real time into dimensionless time. Using practical oil field units with time in days,

$$A = \frac{0.00633\, K}{\phi \mu_w c_e R_f^2}$$

where: K = aquifer permeability, md,
μ_w = water viscosity, cp,
ϕ = porosity, fraction,
R_f = reservoir external radius or aquifer internal radius, ft,
c_e = effective aquifer compressibility, psi^{-1},
t = time, days.

If the time steps are of equal length, then the practical evaluation of the water influx equation becomes easier. Let the time step size be "Δt". Then, the water influx equation can be written as follows:

$$W_e = C_v \sum_{j=1}^{n} \Delta P_j\, Q\left[A(n - j + 1)(\Delta t)\right]$$

Recall the following material balance equation developed earlier in the chapter:

$$W_e = N - \frac{W_e}{D}$$

Substituting the equal time step van Everdingen and Hurst aquifer

the water influx function, $Q(t_d)$. To have a better feeling for the water influx -- material balance equation, a discussion of the nature of the $Q(t_d)$ function is in order.

Cumulative Influx Function, Q(td)

Recall that the outer radius of the hydrocarbon (oil or gas) reservoir is R_f, which is the original o/w contact. This is also the inner radius of the aquifer. Aquifers may be divided into two categories: finite and infinite. Of course, a finite aquifer will have an outer boundary or radius. There are no true infinite aquifers; however, there are some that are large enough to be "infinite-acting" during the producing life of the reservoir.

Figures 7 and 8 contain the graphical presentation of the influx function, $Q(t_d)$. Figure 7 is for a limited (finite) aquifer; while, Figure 8 describes an infinite aquifer. The value of $Q(t_d)$ is a function of dimensionless time.

Consider Figure 7. Notice that there is one curve for the infinite aquifer and several other curves relating to finite aquifers. The finite aquifers are characterized by their radius ratio: outside radius divided by inside radius. Note that every finite aquifer begins as an infinite-acting aquifer; i.e., its curve begins on the infinite aquifer curve. Then at a certain time, depending on the radius ratio, the finite aquifer behavior departs from the infinite aquifer response. At large time, each finite aquifer curve flattens and finally becomes a constant (unchanging) value. The maximum value (the flat part) of each finite aquifer curve is a function of its radius ratio:

$$\left[Q(t_d)\right]_{max} = 0.5\left[\left(\frac{R_a}{R_f}\right)^2 - 1\right]$$

where: R_a = outside radius of limited aquifer, feet,
R_f = radius of the hydrocarbon reservoir, or inside radius of the aquifer, feet.

Calculation of Original Oil-in-Place and Aquifer Constants

The procedure for using reservoir history (performance data) to calculate the original oil-in-place and the water influx constants (C_v and A) is summarized below:

1. Assume that the aquifer is infinite-acting over the time of the reservoir data used in the calculation. Therefore, the values of $Q(t_d)$ should be chosen from the infinite table: Table 1.

2. Calculate an approximate value of "A." To do this, values of permeability and porosity from the aquifer are needed. This data is normally not in hand, so average values from the hydrocarbon reservoir are typically used. Aquifer water viscosity is also needed. This is usually determined on the basis of reservoir temperature. Also needed are the approximate radius of the reservoir and the effective compressibility of the aquifer. The effective compressibility of the aquifer is simply equal to the water compressibility plus the pore volume compressibility.

$$A = \frac{0.00633\, K}{\phi\, \mu_w\, c_e\, R_f^2}$$

where: K = aquifer permeability, md,
ϕ = porosity, fraction,
μ_w = aquifer water viscosity, cp,
c_e = effective aquifer compressibility, psi^{-1} (i.e.,

$$c_e = c_w + c_f$$

R_f = radius of the hydrocarbon reservoir, ft.

Much of the data used in the above equation are normally guessed or assumed, so the resulting "A" is really an assumed value.

Choose a time step size, Δt. This is normally determined on the basis of the frequency of measurement of static pressure data for the reservoir. Typically, this is on the order of 365 days.

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Much of the data used in the above equation are normally guessed or assumed, so the resulting "A" is really an assumed value.

Choose a time step size, Δt. This is normally determined on the basis of the frequency of measurement of static pressure data for the reservoir. Typically, this is on the order of 365 days.

3. Evaluate $(j)(A)(\Delta t)$ where the j's are successive integers from one to the number of time steps to be evaluated.

4. Evaluate the cumulative withdrawal function, $Q[(j)(A)(\Delta t)]$, for each of the values of $(j)(A)(\Delta t)$ of step 3.

5. Evaluate the values of ΔP_j for each pressure change (each new time step).

$\Delta P_j = (P_{j-2} - P_j)/2$ for all $j > 1$

$\Delta P_1 = (P_0 - P_1)/2$ for $j = 1$.

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6. Evaluate the summation: $\sum \Delta P_j\, Q[(n-j+1)(A)(\Delta t)]$

This is conveniently done with the aid of a "Superposition Work Sheet" which will be discussed shortly.

7. Divide the value of the summation (Step 6) by the expansibility factor "D". That is, calculate:

$$\frac{\sum \Delta P_j\, Q[(n-j+1)(A)(\Delta t)]}{D}$$

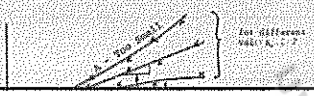
Recall that the expansibility was discussed earlier in this chapter. Basically, it is the denominator of the material balance equation when solved for "N." Because "D" is comprised of fluid properties, it is a strict function of pressure. Therefore, a new "D" must be calculated for each new time step or static pressure.

8a. Calculate the apparent hydrocarbon in place (N_a or G_a) at the end of each time step. Recall that apparent oil-in-place (or apparent gas-in-place) is calculated with the material balance equation and assuming that $W_e = 0$.

8b. Plot apparent hydrocarbon in place versus

$$\frac{\sum \Delta P_j\, Q[(n-j+1)(A)(\Delta t)]}{D}$$

on Cartesian coordinate paper (both scales linear). This plot is illustrated below for an oil reservoir.



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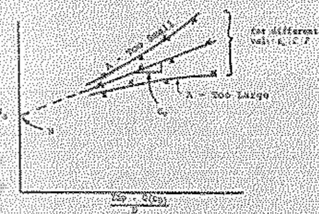

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of least squares. An example of this procedure will be given later.

Illustrative Example III

The following data are available from a water drive oil reservoir:

Aquifer and reservoir area:	28,850 acres
Oil reservoir area:	451 acres
Porosity:	0.22
Effective aquifer thickness:	60 feet
Formation compressibility:	4.0×10^{-6} psi^{-1}
Aquifer permeability:	100 md
Water viscosity:	0.3 cp
Water compressibility:	3.0×10^{-6} psi^{-1}
Water saturation (oil reservoir):	0.28

The system is concentric and circular.

(a) Calculate the effective radii of the oil reservoir and the aquifer. Calculate the radius ratio, R_a/R_f.

$$R_a = \sqrt{[(28,850)(43,560)/\pi]} = 20,000 \text{ feet}$$

$$R_f = \sqrt{[(451)(43,560)/\pi]} = 2,500 \text{ feet}$$

$$R_a/R_f = 20,000 / 2,500 = 8$$

(b) Calculate the theoretical time conversion, A:

$$A = \frac{0.00633\, K}{\phi\, \mu_w\, c_e\, R_f^2} = \frac{(0.00633)(100)}{(0.22)(0.3)(7\times 10^{-6})(2500)^2}$$

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$A = 0.219$

So, $t_d = (A)(t)$ (with "t" in days)

(c) Calculate the theoretical value of C_v.

$$C_v = 1.119\, \phi\, h\, c_e\, R_f^2\, \alpha \qquad (\alpha = 1.0)$$

$$= (1.119)(0.22)(60)(3.0 + 4.0)(10^{-6})(2500)^2$$

$$C_v = 646.2 \text{ bbl/psi}$$

(d) Calculate water influx at 100, 200, 400 and 800 days if the reservoir boundary pressure is lowered to 3450 psia from the initial pressure of 3500 psia.

t days	t_d 0.219t	$Q(t_d)$*	ΔP, psi	$Q(t_d) \times \Delta P$	W_e, bbl $C_v \times Q(t_d) \times \Delta P$
100	21.9	12.91	50	645.5	417,100
200	43.8	19.95	50	997.5	644,600
400	87.6	27.04	50	1352.0	873,700
800	175.2	30.83	50	1541.5	996,100

* from Table 1 or Figure 7, using $R_a/R_f = 8$.

(e) Given the data in columns (1) and (2) below, calculate the cumulative water influx at 500 days.

(1) t, days	(2) P, psia	t_d	$Q(t_d)$	ΔP	$Q(t_d)$ Inverted	ΔP x $Q(t_n - t_{j-1})$
0	3500	0	0	0		
100	3490	21.9	12.91	5	28.68	143.4
200	3476	43.8	19.95	12	27.04	324.5
300	3458	65.7	24.30	16	24.30	388.8

(d) Calculate water influx at 100, 200, 400 and 800 days if the reservoir boundary pressure is lowered to 3450 psia from the initial pressure of 3500 psia.

t days	t_D 0.219t	$Q(t_D)$*	ΔP, psi	$Q(t_D) \times \Delta P$	W_e, bbl $C_v \times Q(t_D) \times \Delta P$
100	21.9	12.91	50	645.5	417,300
200	43.8	19.95	50	997.5	644,700
400	87.6	27.04	50	1352.0	873,700
800	175.2	30.93	50	1541.5	996,100

* from Table 1 or Figure 7, using $R_a/R_r = 8$.

(e) Given the data in columns (1) and (2) below, calculate the cumulative water influx at 500 days.

(1) t, days	(2) P_r psia	t_D	$Q(t_D)$	ΔP	$Q(t_D)$ Inverted	ΔP x $Q(t_n - t_{j-1})$
0	3500	0	0	0		
100	3490	21.9	12.91	5	28.68	143.4
200	3476	43.8	19.95	12	27.04	324.5
300	3458	65.7	24.30	16	24.30	388.8
400	3444	87.6	27.04	16	19.95	319.2
500	3420	109.5	28.68	19	12.91	245.3
					Sum =	1421.2

Therefore,

$$W_e = (646.2)(1421.2) = 918,400 \text{ bbl}$$

Discussion

The calculation of the radius ratio that was performed in part (a) would normally not be able to be done in this manner. It is highly unlikely that the total area of aquifer and reservoir would be available. However, if this information were known,

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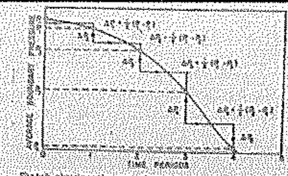

Sketch showing the use of step pressures to approximate the pressure-time curve (from Craft & Hawkins*).

On the next page is located a "Superposition Worksheet" that is tied to this problem. This worksheet is convenient because it helps the reservoir analyst go through the superposition steps by automatically matching the appropriate ΔP with the proper $Q(t_D)$. However, it is only valid to use this sheet when the time steps are all equal.

The ΔP's are entered into the left-hand column. Notice that there is a row associated with each ΔP. And each row has an upper part and a lower part. In the upper part of each row, starting with the first block on the left that is not crosshatched, the $Q(t_D)$'s are written from left to right; i.e., Q(1Δt), Q(2Δt), Q(3Δt), etc. After this is accomplished, then attention is turned to the lower part of each row. Into each of these blocks is entered the product of the ΔP over on the left and the $Q(t_D)$ just above it.

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SUPERPOSITION WORKSHEET

ΔP_j — Q(jΔt) and $\Delta P_j Q(j\Delta t)$

ΔP_j								
5	12.91 64.55	19.95 99.75	24.30 121.50	27.04 135.20	28.68 143.4			
12		12.91 154.92	19.95 239.40	24.30 291.6	27.04 324.48			
16			12.91 206.56	19.95 319.20	24.30 388.80			
16				12.91 206.56	19.95 319.20			
19					12.91 245.29			
$\sum \Delta P_j Q(n\Delta t)$	64.55	254.67	567.46	952.56	1421.17			
W_e, bbls	41712	164563	366683	615545	918360			

SUPERPOSITION WORKSHEET

ΔP_j — Q(jΔt) and $\Delta P_j Q(j\Delta t)$

ΔP_1	Q(1Δt) ΔP_1Q(1Δt)	Q(2Δt) ΔP_1Q(2Δt)	Q(3Δt) ΔP_1Q(3Δt)	Q(4Δt) ΔP_1Q(4Δt)	Q(5Δt) ΔP_1Q(5Δt)	Q(6Δt) ΔP_1Q(6Δt)	Q(7Δt) ΔP_1Q(7Δt)	Q(8Δt) ΔP_1Q(8Δt)
ΔP_2		Q(1Δt) ΔP_2Q(1Δt)	Q(2Δt) ΔP_2Q(2Δt)	Q(3Δt) ΔP_2Q(3Δt)	Q(4Δt) ΔP_2Q(4Δt)	Q(5Δt) ΔP_2Q(5Δt)	Q(6Δt) ΔP_2Q(6Δt)	Q(7Δt) ΔP_2Q(7Δt)
ΔP_3			Q(1Δt) ΔP_3Q(1Δt)	Q(2Δt) ΔP_3Q(2Δt)	Q(3Δt) ΔP_3Q(3Δt)	Q(4Δt) ΔP_3Q(4Δt)	Q(5Δt) ΔP_3Q(5Δt)	Q(6Δt) ΔP_3Q(6Δt)
ΔP_4				Q(1Δt) ΔP_4Q(1Δt)	Q(2Δt) ΔP_4Q(2Δt)	Q(3Δt) ΔP_4Q(3Δt)	Q(4Δt) ΔP_4Q(4Δt)	Q(5Δt) ΔP_4Q(5Δt)
ΔP_5					Q(1Δt) ΔP_5Q(1Δt)	Q(2Δt) ΔP_5Q(2Δt)	Q(3Δt) ΔP_5Q(3Δt)	Q(4Δt) ΔP_5Q(4Δt)
ΔP_6						Q(1Δt) ΔP_6Q(1Δt)	Q(2Δt) ΔP_6Q(2Δt)	Q(3Δt) ΔP_6Q(3Δt)
ΔP_7							Q(1Δt) ΔP_7Q(1Δt)	Q(2Δt) ΔP_7Q(2Δt)
ΔP_8								Q(1Δt) ΔP_8Q(1Δt)
$\sum \Delta P_j Q(n\Delta t)$								

Then, sums are performed on each column, adding the lower blocks in each row only. This calculation yields the

$$\sum \Delta P_j \, Q[(n-j+1)(A)(\Delta t)]$$

associated with the time at the end of the time step under consideration. Notice that the first column corresponds with the first time step, and the second column with the second time step, etc. Then to obtain W_e at the particular time under consideration, merely multiply the above sum by "C_v".

Illustrative Example IV

This problem will illustrate the calculation of water influx, W_e, using the van Everdingen and Hurst radial aquifer model. An infinite aquifer is assumed.

Basic Data

Porosity:	0.21
Aquifer permeability:	500 md
Aquifer effective thickness:	50 feet
Reservoir radius:	2000 feet
Water viscosity:	0.3 cp
Water compressibility:	3.0×10^{-6} psi^{-1}
Formation compressibility:	3.6×10^{-6} psi^{-1}
α:	1.0

$$c_e = c_w + c_f = (3.0 + 3.6)(10^{-6}) = 6.6 \times 10^{-6} \text{psi}^{-1}$$

Calculation of the time coefficient, A, for time in months:

$$A = \frac{(0.00633)(30.42)\,K}{\phi\,\mu_w\,c_e\,R_r^2} = \frac{0.193\,K}{\phi\,\mu_w\,c_e\,R_r^2}$$

$$A = \frac{(0.193)(500)}{(0.21)(0.3)(6.6)(10^{-6})(2000)^2} = 58.02 \text{ mo.}^{-1}$$

infinite aquifer is assumed.

Basic Data

Porosity:	0.21
Aquifer permeability:	500 md
Aquifer effective thickness:	50 feet
Reservoir radius:	2000 feet
Water viscosity:	0.3 cp
Water compressibility:	3.0×10^{-6} psi^{-1}
Formation compressibility:	3.6×10^{-6} psi^{-1}
α:	1.0

$$c_e = c_w + c_f = (3.0 + 3.6)(10^{-6}) = 6.6 \times 10^{-6} \text{psi}^{-1}$$

Calculation of the time coefficient, A, for time in months:

$$A = \frac{(0.00633)(30.42)\,K}{\phi\,\mu_w\,c_e\,R_r^2} = \frac{0.193\,K}{\phi\,\mu_w\,c_e\,R_r^2}$$

$$A = \frac{(0.193)(500)}{(0.21)(0.3)(6.6)(10^{-6})(2000)^2} = 58.02 \text{ mo.}^{-1}$$

Calculation of transmission constant, C_v:

$$C_v = 1.119\,h\,\phi\,\alpha\,c_e\,R_r^2$$

$$C_v = (1.119)(50)(0.21)(6.6)(10^{-6})(2000)^2$$

$$C_v = 310.187 \text{ barrels/month/psi}$$

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Field Pressure – Time Data

Time, Months	Pressure at Original Oil-Water Contact, psi
0	2500
12	2480
24	2470
36	2464
48	2460

Calculation of $\Delta\bar{P}_j$: P_j (Pj-1+Pj)/2 ΔPj=(Pj-2-Pj)/2

Time Step, j	P	$\bar{P} = P_{avg}$	$\Delta\bar{P}_j$
0	2500	2500	0
1	2480	2490	10
2	2470	2475	15
3	2464	2467	8
4	2460	2462	5

Example: For j = 1, $\Delta\bar{P}_1 = (P_0 - P_1)/2 = (2500 - 2480)/2$

$\Delta\bar{P}_1 = 10$

For j = 2, $\Delta\bar{P}_j = (P_{j-2} - P_j)/2 = (2500 - 2470)/2$

$\Delta\bar{P}_2 = 15$

Calculation of the Cumulative Influx Function, $Q[(j)(A)(\Delta t_j)]$:

$\Delta\bar{P}_1 = 10$

For j = 2, $\Delta\bar{P}_j = (P_{j-2} - P_j)/2 = (2500 - 2470)/2$

$\Delta\bar{P}_2 = 15$

Calculation of the Cumulative Influx Function, $Q[(j)(A)(\Delta t_j)]$:

A = 58.02
Δt = 12 months
(A)(Δt) = 696.24 td Qtd

j	(j)(A)(Δt)	Q[(j)(A)(Δt)]
1	696.24	215.427
2	1392.48	390.305
3	2088.72	554.692
4	2784.96	713.914

Table 2 was used to obtain the $Q(t_d)$ values in column 3 above based on the dimensionless time in column 2. Notice that these dimensionless time values fall in between the table values. Linear interpolation was used to determine the appropriate influx function value.

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Field Pressure – Time Data

Time, Months	Pressure at Original Oil-Water Contact, psi
0	2500
12	2480
24	2470
36	2464
48	2460

Calculation of $\Delta\bar{P}_j$: P_j $\bar{P}_j = \frac{P_{j-1}+P_j}{2}$ $\Delta P_j = \frac{P_{j-2}-P_j}{2}$

Time Step, j	P	$\bar{P} = P_{avg}$	$\Delta\bar{P}_j$
0	2500	2500	0
1	2480	2490	10
2	2470	2475	15
3	2464	2467	8
4	2460	2462	5

Example: For j = 1, $\Delta\bar{P}_1 = (P_0 - P_1)/2 = (2500 - 2480)/2$

$\Delta\bar{P}_1 = 10$

For j = 2, $\Delta\bar{P}_j = (P_{j-2} - P_j)/2 = (2500 - 2470)/2$

$\Delta\bar{P}_2 = 15$

Example: For j = 1, $\Delta\bar{P}_1 = (P_0 - P_1)/2 = (2500 - 2480)/2$

$\Delta\bar{P}_1 = 10$

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A = 58.02
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Table 2 was used to obtain the $Q(t_d)$ values in column 3 above based on the dimensionless time in column 2. Notice that these dimensionless time values fall in between the table values. Linear interpolation was used to determine the appropriate influx function value.

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Example calculation:

For $(j)(A)(\Delta t) = 696.24$,

$Q(690) = 213.784$ and $Q(700) = 216.417$

Hence: $Q(696.24) = 213.784 + \left[\frac{696.24-690}{700-690}\right]\left[216.417-213.784\right]$

$= 215.427$

For $(j)(A)(\Delta t) = 1392.48$,

$Q(1390) = 389.705$ and $Q(1400) = 392.125$

Hence: $Q(1392.48) = 389.705 + \left[\frac{1392.48-1390}{1400-1390}\right]\left[392.125-389.705\right]$

$= 390.305$

Now, that the ΔP's, $Q(t_d)$'s, and C_v have been calculated, W_e may be determined at the end of each time step. This is accomplished with the worksheet on the next page. The use of this calculation sheet was explained in the last example.

After the $\sum(\Delta P)[Q(t_d)]$ calculation is accomplished in the next to last row at the bottom of the worksheet, W_e is obtained by multiplying by C_v. Recall that C_v was determined to be 510.187.

Unsteady State

Illustrative Example V

This problem illustrates the calculation of N and C_v using the van Everdingen and Hurst model (assuming infinite radial aquifer). The solution technique is the least squares method developed by Mr. G. W. Tracy.

Basic data:

Porosity:	0.16
Connate water saturation:	0.25
Oil compressibility:	10×10^{-6} psi^{-1}
Water compressibility:	3.0×10^{-6} psi^{-1}
Formation compressibility:	4.0×10^{-6} psi^{-1}
Bubble point pressure:	2100 psia
Initial pressure:	3000 psia
Water formation vol. factor:	1.0 Res. bbls / STB

Pressure-Production Data:

Time Yrs.	Pressure Psia	B_o	Cum. Oil Prod., N_p, MB	Water Prod., W_p, MB
0	3000	1.3100	0.0	0
1	2870	1.3117	932.92	10
2	2810	1.3125	2346.80	37
3	2760	1.3131	3919.25	153
4	2720	1.3137	5687.65	372

In this problem, the pressure and cumulative production data will be used in conjunction with material balance and the van Everdingen and Hurst aquifer equation to calculate the original oil-in-place (N) and the aquifer constants (A and C_v). Note that all of the above pressure data is above the bubble point pressure. Therefore, the appropriate material balance formulation is that for an undersaturated oil reservoir.

Pressure-Production Data:

Time Yrs.	Pressure Psia	B_o	Cum. Oil Prod., N_p, MB	Water Prod., W_p, MB
0	3000	1.3100	0.0	0
1	2870	1.3117	952.92	10
2	2810	1.3125	2346.80	37
3	2760	1.3131	3919.25	153
4	2720	1.3137	5687.65	272

In this problem, the pressure and cumulative production data will be used in conjunction with material balance and the van Everdingen and Hurst aquifer equation to calculate the original oil-in-place (N) and the aquifer constants (A and C_v). Note that all of the above pressure data is above the bubble point pressure. Therefore, the appropriate material balance formulation is that for an undersaturated oil reservoir.

Recall that the material balance equation can be written as:

$$N_a = N + \frac{W_e}{D}$$

For an undersaturated oil reservoir, the expansibility, "D" is:

$$D = c_{oe} B_{oi} (P_i - P)$$

Hence, the effective oil compressibility is needed:

$$c_{oe} = c_o + \frac{c_w S_w}{S_o} + \frac{c_f}{S_o} = \left[10 + \frac{(3)(.25)}{(0.75)} + \frac{4}{.75}\right](10^{-6})$$

$$= 16.33 \times 10^{-6} \text{ psi}^{-1}$$

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The effective compressibility of the aquifer is:

$$c_{ae} = c_w + c_f = (3.0 + 4.0)(10^{-6})$$

$$= 7.0 \times 10^{-6} \text{ psi}^{-1}$$

Calculation of expansibility, "D":

$$\text{Expansibility} = D = c_{oe} B_{oi} (P_i - P)$$

$$= (16.33)(10^{-6})(1.3100)(P_i - P)$$

$$= 21.392 \times 10^{-6} (P_i - P)$$

Time Yrs.	Pressure Psia	$P_i - P$ Psi	Expansibility "D"
0	3000	-	-
1	2870	130	0.002781
2	2810	190	0.004064
3	2760	240	0.005134
4	2720	280	0.005990

The apparent oil in place for an oil reservoir is:

$$N_a = (\text{cumulative withdrawals}) / (\text{expansibility})$$

Of course, an undersaturated oil reservoir has no free gas; therefore, the expression for cumulative withdrawals is:

$$\text{Cumulative withdrawals} = N_p B_o + W_p B_w \quad (B_w = 1.0)$$

Time Yrs.	N_p MB	$N_p B_o$ MB	W_p MB	With-drawals, MB	N_a MMB
0	0.0	0.0	0	0.0	-
1	952.92	1249.93	10	1259.93	453.056
2	2346.80	3080.18	37	3117.18	767.023
3	3919.25	5146.37	153	5299.37	1032.211
4	5687.65	7471.87	272	7743.87	1292.800

Time step size = 12 months

Calculation of $\Delta\bar{P}_j$

$$\bar{P}_j = \frac{P_{j-1} + P_j}{2} \qquad \Delta P_j = \frac{P_{j-1} - P_j}{2}$$

Time Step, j	Time, Mo.	P	$\bar{P}$	$\Delta\bar{P}_j$
0	0	3000	3000	0
1	12	2870	2935	65
2	24	2810	2840	95
3	36	2760	2785	55
4	48	2720	2740	45

Van Everdingen & Hurst Aquifer Model

Least Squares Solution

by G. W. Tracy

The material balance equation may be written as:

$$N = \frac{\text{Withdrawals} - W_e}{D}$$

Then, Withdrawals $= (N)(D) + W_e$

Substituting in the van Everdingen and Hurst Equation:

$$\text{Withdrawals} = (N)(D) + C_v \sum \Delta\bar{P}_j\, Q(t_D)$$

Now, let: z_i = cumulative withdrawals at P_i

x_i = expansibility at P_i, and

$y_i = \sum \Delta\bar{P}_j\, Q(t_D)$ at P_i

The material balance equation may be written as:

$$N = \frac{\text{Withdrawals} - W_e}{D}$$

Then, Withdrawals $= (N)(D) + W_e$

Substituting in the van Everdingen and Hurst Equation:

$$\text{Withdrawals} = (N)(D) + C_v \sum \Delta\bar{P}_j\, Q(t_D)$$

Now, let: z_i = cumulative withdrawals at P_i

x_i = expansibility at P_i, and

$y_i = \sum \Delta\bar{P}_j\, Q(t_D)$ at P_i

Then, the least squares equations are:

$$N = \frac{(\sum xz)(\sum y^2) - (\sum xy)(\sum yz)}{(\sum x^2)(\sum y^2) - (\sum xy)(\sum xy)}$$

$$C_v = \frac{(\sum x^2)(\sum yz) - (\sum xy)(\sum xz)}{(\sum x^2)(\sum y^2) - (\sum xy)(\sum xy)}$$

Then, a deviation function is used as a measure of goodness of fit:

$$\text{Dev.} = \frac{1}{n}\sqrt{\sum (z_i - Nx_i - C_v y_i)^2}$$

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This deviation function can be considered to be a function of "A," the time coefficient, or a function of (A)(Δt). Recall that Δt is the time step size. In this problem, months are used as the time unit, and the time step size is: Δt = 12 months.

The least squares procedure is to assume an "A," and then calculate N, C_v, and Dev. Then, choose a different "A" and repeat the calculations. Continue investigating different "A's" until a minimum of the Dev. function is found.

Case 1:

To begin, let (A)(Δt) = (10)(12) = 120.

Therefore,

Time Yrs.	Time Step, i	(i)(A)(Δt)	Q[(i)(A)(Δt)]
0	0	-	-
1	1	120	49.968
2	2	240	88.082
3	3	360	123.403
4	4	480	157.184

The $\sum \Delta\bar{P}_j\, Q[(n-j+1)(A)(\Delta t)]$ calculation is performed using the work sheet on the next page.

Recall that:

x_i = the expansibility at P_i (end of time step "i"),

$y_i = \sum \Delta\bar{P}_j\, Q(t_D)$ at the end of time step "i,"

z_i = cumulative withdrawals at the end of time step "i"

0	0	-	-
1	1	120	49.968
2	2	240	88.082
3	3	360	123.403
4	4	480	157.184

The $\sum \Delta\bar{P}_j\, Q[(n-j+1)(A)(\Delta t)]$ calculation is performed using the work sheet on the next page.

Recall that:

x_i = the expansibility at P_i (end of time step "i"),

$y_i = \sum \Delta\bar{P}_j\, Q(t_D)$ at the end of time step "i,"

z_i = cumulative withdrawals at the end of time step "i"

i	x_i	x_i^2	y_i	y_i^2
0	-	-	-	-
1	0.002781	$(7.734)(10^{-6})$	3,247.9	10,548,854.6
2	0.004064	$(16.516)(10^{-6})$	10,471.0	109,641,841.0
3	0.005134	$(26.358)(10^{-6})$	16,133.3	266,159,706.1
4	0.005990	$(35.880)(10^{-6})$	29,032.3	842,874,443.3
		$(86.488)(10^{-6})$		1,229,224,844.8

i	z_i	$(x_i)(z_i)$	$(y_i)(z_i)$	$(x_i)(y_i)$
0	-	-	-	-
1	1259.93	3.5039	4,092,191.6	9.0324
2	3117.10	12.6684	32,639,991.8	42.5541
3	5299.37	27.2070	101,405,034.8	98.2406
4	7743.87	46.3858	224,832,357.0	173.9038
		89.7649	362,970,575.2	323.7306

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Q (ΔAT) and Δq Q (ΔAT)

Δq								
65	49.960 / 3247.9	88.022 / 5187.0	124.903 / 8081.2	157.184 / 10217.0				
95		49.968 / 4147.0	88.022 / 8166.9	124.903 / 11723.3				
55			49.968 / 2748.2	88.022 / 4840				
45				49.968 / 2248.6				
$\sum \Delta q\, Q(\Delta AT)$	3247.9	10471.9	19136.3	29032.3				
ω_e								

Therefore,

$$N = \frac{(\sum xz)(\sum y^2) - (\sum xy)(\sum yz)}{(\sum x^2)(\sum y^2) - (\sum xy)(\sum xy)}$$

$$= \frac{(89.7649)(1.32922)(10^9) - (323.731)(3.62960)(10^9)}{(86.488)(10^{-9})(1.32922)(10^9) - (323.731)(323.731)}$$

N = 178,733 MSTB

$$C_v = \frac{(\sum x^2)(\sum yz) - (\sum xy)(\sum xz)}{(\sum x^2)(\sum y^2) - (\sum xy)(\sum xy)}$$

$$= \frac{(86.488)(10^{-9})(3.62960)(10^9) - (323.731)(89.7649)}{(86.488)(10^{-9})(1.32922)(10^9) - (323.731)(323.731)}$$

C_v = 0.2295 MB/psi

$$Dev. = \frac{1}{n}\sqrt{\sum(z_i - Nx_i - C_v y_i)^2}$$

i	$(z_i - Nx_i - C_v y_i)$	$(z_i - Nx_i - C_v y_i)^2$
0	-	-
1	17.5005	306.267
2	-12.2854	150.931
3	- 9.7966	95.973
4	10.3465	107.050
		660.221

Dev. = $\frac{1}{4}\sqrt{660.22}$

Dev

$$= \frac{(86.488)(10^{-9})(3.62960)(10^9) - (323.731)(89.7649)}{(86.488)(10^{-9})(1.32922)(10^9) - (323.731)(323.731)}$$

C_v = 0.2295 MB/psi

$$Dev. = \frac{1}{n}\sqrt{\sum(z_i - Nx_i - C_v y_i)^2}$$

i	$(z_i - Nx_i - C_v y_i)$	$(z_i - Nx_i - C_v y_i)^2$
0	-	-
1	17.5005	306.267
2	-12.2854	150.931
3	- 9.7966	95.973
4	10.3465	107.050
		660.221

Dev. = $\frac{1}{4}\sqrt{660.22}$

Dev. = 6.424

Dev — ΔAt

The calculation details have all be reported for this first case where (A)(Δt) = 120.

For the other cases investigated, only the superposition work sheets and major results will be given.

41

Δq 10, ΔA 50, ANS

[illegible]

ΔAT 600 — SUPER POSITION WORKSHEET

Q (ΔAT) and Δq Q (ΔAT)

Δq								
65	184.972 / 12340.4	343.308 / 22317.0	497.477 / 31683.4	636.416 / 40249.3				
95		184.972 / 18057.9	343.308 / 32614.1	497.497 / 47268.6				
55			184.972 / 10441.9	343.308 / 18881.9				
45				184.972 / 8543.5				
$\sum \Delta q\, Q(\Delta AT)$	12340.4	40760.9	74915.2	115711.0				
ω_e								

ΔAT 1200 — SUPER POSITION WORKSHEET

Q (ΔAT) and Δq Q (ΔAT)

Δq								
65	343.308 / 22315.0	626.066 / 40697.3	892.712 / 58026.3	1150.062 / 74750.0				
95		343.308 / 32614.9	626.066 / 59476.3	892.712 / 84807.6				
55			343.308 / 18881.9	626.066 / 34433.6				
45				343.308 / 15448.9				
$\sum \Delta q\, Q(\Delta AT)$	22315.0	73306.6	136880.5	209590.9				
ω_e								

Case VI: (A)(at) = 720

Results:

Σ(x)(y) = [illegible]
Σ(y)(x) = [illegible] x 10^6
Σ(x)(x) = [illegible]
Σx² = [illegible] x 10^6
Σy² = [illegible] x 10^7

Then, N = 200,043 MSTB
C_v = 0.04883 MB/psi
Dev. = [illegible]

Since the Dev. function increased in value over that computed in Case IV, the best (at) should be in the interval:

600 < [(A)(at)]$_{best}$ < 700

Case VII: (A)(at) = 680

Results:

Σ(x)(y) = [illegible]
Σ(y)(x) = [illegible] x 10^7
Σ(x)(x) = [illegible]
Σx² = [illegible] x 10^-6
Σy² = [illegible] x 10^7

Then, N = 199,510 MSTB
C_v = 0.05134 MB/psi
Dev. = [illegible]

Case VI yielded the smallest deviation of all of the values of (A)(at) considered. Additional cases could be investigated, but notice that N and C_v did not change appreciably from the values calculated in Case IV. So, we have convergence, and

N = 199.5 MMSTB

Cv = 0.05134 MB/psi

A = 680/12 = 56.67 mo.$^{-1}$

ΔaT = 720 — SUPER POSITION VI WORKSHEET

ΔQ — Q(Σaat) and ΔQ Q(Σaat)

ΔQ				
65	221.670	461.716	571.192	734.138
	[illegible]	[illegible]	[illegible]	[illegible]
95		221.670	461.716	571.192
		[illegible]	[illegible]	[illegible]
55			221.670	461.716
			[illegible]	[illegible]
45				221.670
				[illegible]
ΣΔQ Q(Σaat)	[illegible]	[illegible]	[illegible]	[illegible]
W_e				

ΔaT = 680 — SUPER POSITION V WORKSHEET

ΔQ — Q(Σaat) and ΔQ Q(Σaat)

ΔQ				
65	211.145	441.492	543.420	697.313
	[illegible]	[illegible]	[illegible]	[illegible]
95		211.145	441.492	543.420
		[illegible]	[illegible]	[illegible]
55			211.145	441.492
			[illegible]	[illegible]
45				211.145
				[illegible]
ΣΔQ Q(Σaat)	[illegible]	[illegible]	[illegible]	[illegible]
W_e				

various dimensionless times, depending upon the value of r_e/r_w, the infinite and limited aquifer values coincide. [illegible]

While the previous discussion has been limited to oil reservoirs, it should be evident that where the initial content of gas reservoirs is known, the water influx at successive time periods may be calculated [illegible] for oil reservoirs.

4. Simultaneous Calculation of Initial Oil in Place and Water Influx. For convenience the general material-balance expression Eq. (1.8) may be rewritten as follows: [illegible]

influx. The method consists of writing a series of Eqs. (3.14) at successive production intervals and determining the initial oil and water influx constant(s) from this set of linear equations by the method of least squares.

Example 3.5. Simultaneous calculation of the water influx constants and the initial oil in place.

[illegible]

[illegible]

Table 3.10. Standard Deviation Calculations for N and k for Example 3.5 — Std. Dev. = $\sqrt{\frac{\Sigma\Delta^2}{(n-1)}}$, where n = Number of Data — Avg. Dev. = $\frac{\Sigma\Delta}{n}$

	k = 43.6			N = 183.88			k = 33.5			N = 227.4		
	N	ΔN	ΔN²	k	Δk	Δk²	N	ΔN	ΔN²	k	Δk	Δk²
	182.17	−3.71	13.76	42.2	−1.4	1.96	215.46	−11.91	141.85	29.2	−4.0	16.00
	190.92	5.04	25.40	47.2	1.6	2.56	229.10	1.73	2.99	34.0	0.8	0.64
	186.13	0.25	0.06	43.7	0.1	0.01	229.91	2.54	6.45	34.2	1.0	1.00
	181.29	−1.59	2.51	43.2	−0.4	0.16	235.02	7.65	30.52	35.3	2.1	4.41
	741.51	10.59	41.73	182.3	3.5	4.69	909.49	23.83	201.81	132.7	7.9	22.05
Average and Average Deviation:	185.88	±2.65		43.6	±0.88		227.37	±5.96		31.2	±1.98	
Average per cent Deviation:		±1.43			±1.91			±2.62			±5.96	
Standard Numerical Deviation:			±3.73			±1.25			±8.36			±2.71
Standard per cent Deviation:			±2.01			±2.74			±3.68			±8.16

15/04/57

Table 5.10. Standard Deviation Calculations for N and k for Example 5.5 — Std. Dev. = $\sqrt{\frac{\Sigma \Delta^2}{(n-1)}}$ where n = Number of Data — Avg. Dev. = $\frac{\Sigma \Delta}{n}$

Estimated — Volumetric

	k = 45.6			N = 185.88			k = 33.5			N = 227.4		
	N	ΔN	ΔN²	k	Δk	Δk²	N	ΔN	ΔN²	k	Δk	Δk²
	182.17	−3.71	13.76	44.2	−1.4	1.96	215.46	−11.91	141.85	29.3	−4.0	16.00
	190.92	5.04	25.40	47.2	1.6	2.56	229.10	1.73	2.99	34.0	0.5	0.64
	186.13	0.25	0.06	45.7	0.1	0.01	229.91	2.54	6.45	34.2	1.0	1.00
	184.29	−1.59	2.53	45.2	−0.4	0.16	235.07	7.66	58.52	35.3	2.1	4.41
	743.51	10.59	41.75	182.3	3.5	4.69	909.49	23.85	209.81	132.7	7.9	22.05
Average and Average Deviation:	185.88	±2.65		45.6	±0.88		227.37	±5.96		33.2	±1.98	
Average per cent Deviation:		±1.43			±1.93			±2.62			±5.96	
Standard Numerical Deviation:			±3.73			±1.25			±8.36			±2.71
Standard per cent Deviation:			±2.01			±2.74			±3.68			±8.16

Example 5.5 in

$K = 45.6$; $N = 185.88$

	ΔN
N = 307.62 − 2.751 × 45.6 = 182.22	−3.65
N = 334.53 − 3.156 × 45.6 = 190.92	5.04
N = 351.11 − 3.614 × 45.6 = 186.13	0.25
N = 375.45 − 4.192 × 45.6 = 184.29	−1.59
Σ = 743.56	Σ|ΔN| = 10.53

$\Sigma(\Delta N)^2 = 41.31$

$$\text{STD Dev.} = \sqrt{\frac{\Sigma(\Delta N)^2}{(n-1)}}$$

$$= \sqrt{\frac{41.31}{(4-1)}} = 3.7$$

Average Deviation $= \frac{\Sigma|\Delta N|}{n} = \frac{10.53}{4} = 2.63$

∴ Average N = 185.88 ± 2.63

∴ % deviation $= \frac{2.63}{185.88} \times 100 = 1.41\%$

S percent deviation $= \frac{3.7}{185.88} \times 100 = 2\%$

The use of these models in connection with Equation 11-44 to simultaneously determine N and W_e is described below.

The Pot-Aquifer Model in the MBE

$$W_e = (c_w + c_f) W_i f (p_i - p) \quad (11\text{-}45)$$

$$f = \frac{\text{(encroachment angle)}^\circ}{360^\circ} = \frac{\theta}{360^\circ}$$

$$W_i = \left[\frac{\pi (r_a^2 - r_e^2) h \phi}{5.615}\right]$$

$$W_e = K \Delta p \quad (11\text{-}46)$$

$$\frac{F}{E_o} = N + K\left(\frac{\Delta p}{E_o}\right) \quad (11\text{-}47)$$

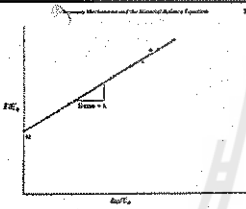


Figure 11-23. F/E₀ vs. Δp/E₀.

The Steady-State Model in the MBE

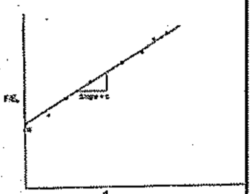
$$W_e = C \int_0^t (p_i - p)\, dt$$

$$\frac{F}{E_o} = N + C\left(\frac{\int_0^t (p_i - p)\, dt}{E_o}\right) \quad (11\text{-}49)$$

Unsteady-State Model in the MBE

$$W_e = B \sum \Delta p\, W_{eD} \quad (11\text{-}50)$$

$$\frac{F}{E_o} = N + B\left(\frac{\sum \Delta p\, W_{eD}}{E_o}\right)$$



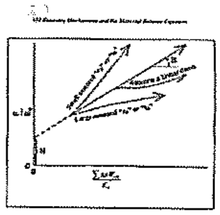


Figure 11-25. Havlena and Odeh straight-line plot.

Example 11-5

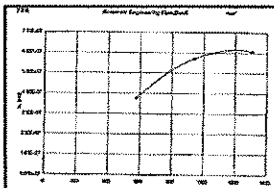


Figure 11-26. Indication of water influx.

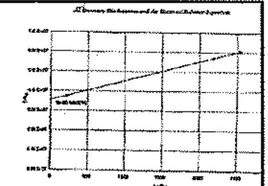


Figure 11-27. F/E₀ versus Δp/E₀.

Tracy's Form of the Material Balance Equation

Material Balance Equation:

The various form of the Water Influx equation can be used to determine the term W_e which is part of the MBE. This additional information is incorporated into one form of MBE equation as follows:

Steady

Unsteady

$$\frac{F}{E_o} = N + \frac{We}{E_o} \quad \text{Oil No gas Cap}$$

$$\frac{F}{E_g} = N + \frac{We}{E_g} \quad \text{Gas Reservoir.}$$

$$\frac{F}{E_o + m\frac{B_{ti}}{B_{gi}}E_g} = N + C\frac{\sum \Delta P Q(\Delta t_D)}{E_o + m\frac{B_{ti}}{B_{gi}}E_g}$$

HW no 8

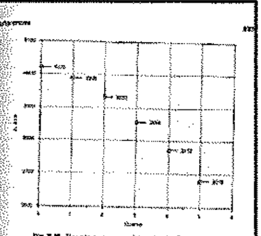
(1.)8.9 inCraft(new)

(2.) 5.9and 5.10 in Craft(old)

(3.) Illuatrate ExampleII, and V By straight line technique plotting

(4) Handout sheet

Due Date; 20 Dec. 2011



435619 ADVANCED RESERVOIR ENGINEERING

HW NO 8 of 2/2554(2011) Due date: Friday 9December/2011

1. Do the problems 8.9 in Applied PETROLEUM RESERVOIR ENGINEERING by B.C. CRAFT and M.F. HAWKINS O 1991.
2. Do the problems 5.9 and 5.10 in Applied PETROLEUM RESERVOIR ENGINEERING by B.C. CRAFT and M.F. HAWKINS O 1959 (handout sheets).
3. Do the Illustrate Example II, and V by straight line plotting technique.
4. A saturated oil reservoir is believed to contain an initial gas cap with m = 0.5. The initial pressure at the gas-oil contact, which is also a convenient datum, is 5,000 psi. The PVT properties of the system at the reservoir conditions are as follows:

Pressure psia	R_{so} SCF/STB	B_o RB/STB	B_g RB/SCF
5,000	500	1.3050	0.00065
4,300	338	1.2280	0.00075
4,250	325	1.2200	0.00076
4,200	310	1.215	0.00077
4,150	295	1.205	0.00078
4100	280	1.2	0.00079

The uniform initial water saturation is 30% and water(C_w)and pore volume(C_f) compressibilities are each $3x10^{-6}$ psi^{-1}. The oil, gas and water productions start at 1/1/2001 as follows. Water and gas injection started at a constant rate of 25 MMSTB/year and 10 MMMSCF/year on 1/1/2002 (12 months later). Assume B_w for injection and produced water are 1.00 RB/STB., and B_g for injection gas is 0.00076 RB/SCF. And B_{wp} for producing water is also 1.00 RB/STB. The cumulative gas, water and reservoir production pressure has been reported as follows:

D/M/Y	P PSI	Np MMSTB	GP MMMSCF	Wp MMBBL	We MMBBL	Wi MMBBL	Gi MMMSCF
1/1/2001	5000	0					
31/12/2001	4300	25	10	1		0	0
31/12/2002	4250	50	25	2		25	10
31/12/2003	4200	75	40	6		50	20
31/12/2004	4150	100	60	10		75	30
31/12/2005	4100	125	80	15		100	40

Estimate initial oil and gas inplace and cumulative water influx by

- CHAPTER 9
- WATER AND GAS CONING (4 hrs.)

434620,505653 ADVANCED RESERVOIR ENGINEERIN
2/2556(4 credits(4-0-8), Sep. 20,-Dec 22, 2013)
Course Contents

1. Optimization of Material Balance Equations (6 hrs.)
2. Saturation and Relative Permeability Calculations (4 hrs.)
3. Steady State Radial Flow (4 hrs.)
4. Pseudo-steady State Flow and Superposition (4 hrs.)
5. Well Testing Pressure Drawdown and Build Up (3 hrs.)
6. Interference Test and Type Curve Analysis(3hrs)
7. Displacement Efficiency(6 hrs)
8. Dynamics of Water Drive Reservoir. (6 hrs.)
9. Water and Gas Coning (4hrs.)
10. Multi-Phase Flow and Introduction to Reservoir Simulation (4 h
11. Enhance Oil Recovery(2 hrs)

GAS AND WATER CONING

1. COSTLY ADDED WATER AND GAS HANDLING

2. GAS PRODUCTION WITHOUT OBTAINING THE DISPLACEMENT EFFECT

3. REDUCED EFFICIENCY OF THE DEPLETION MECHANISM

4. CORROSION DUE TO WATER

5. EARLY ABANDONMENT

6. LOSS OF OVERALL RECOVERY

CONING

Coning is primarily the result of movement of reservoir fluids in the direction of least resistance, balanced by a tendency of the fluids to maintain gravity equilibrium. The analysis may be made with respect to either gas or water. Let the original condition of reservoir fluids exist as shown schematically in Figure 9-1, water underlying oil and gas overlying oil. For the purposes of discussion, assume that a well is partially penetrating the formation (as shown in Figure 9-1) so that the production interval is halfway between the fluid contacts.

Production from the well would create pressure gradients that tend to lower the gas-oil contact and elevate the water-oil contact in the immediate vicinity of the well. Counterbalancing these flow gradients is the tendency of the gas to remain above the oil zone because of its lower density and of the water to remain below the oil zone because of its higher density. These counterbalancing forces tend to deform the gas-oil and water-oil contacts into a bell shape as shown schematically in Figure 9-2.

There are essentially three forces that may affect fluid flow distributions around the well bores. These are:

- Capillary forces
- Gravity forces
- Viscous forces

Forces affect fluid flow
- Capillary
- Gravity
- Viscous

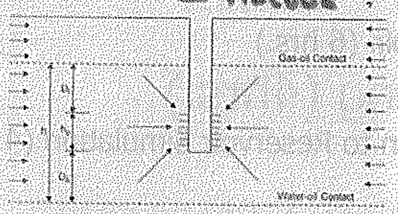


Figure 9-1. Original reservoir static condition.

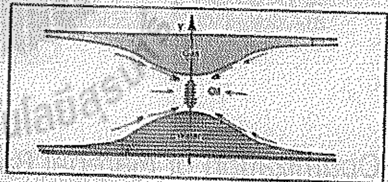


Figure 9-2. Gas and water coning.

Capillary forces usually have negligible effect on coning and will be neglected. Gravity forces are directed in the vertical direction and arise from fluid density differences. The term *viscous forces* refers to the pressure gradients associated fluid flow through the reservoir as described by Darcy's Law. Therefore, at any given time, there is a balance between gravitational and viscous forces at points on and away from the well completion interval. When the dynamic (viscous) forces at the wellbore exceed gravitational forces, a "cone" will ultimately break into the well.

We can expand on the above basic visualization of coning by introducing the concepts of:

- Stable cone
- Unstable cone
- Critical production rate

Stable & constant
Unstable
Critical q_o

If a well is produced at a constant rate and the pressure gradients in the drainage system have become constant, a steady-state condition is reached. If at this condition the dynamic (viscous) forces at the well are less than the gravity forces, then the water or gas cone that has formed will not extend to the well. Moreover, the cone will neither advance nor recede, thus establishing what is known as a *stable cone*. Conversely, if the pressure in the system is an unsteady-state condition, then an *unstable cone* will continue to advance until steady-state conditions prevail.

If the flowing pressure drop at the well is sufficient to overcome the gravity forces, the unstable cone will grow and ultimately break into the

- Chaney et al.
- Chaperson
- Schols

Chaney
Chapperson
Schols

The practical applications of these correlations in predicting the critical oil flow rate are presented over the following pages.

The Meyer-Garder Correlation

Meyer and Garder (1954) suggest that coning development is a result of the radial flow of the oil and associated pressure sink around the wellbore. In their derivations, Meyer and Garder assume a homogeneous system with a uniform permeability throughout the reservoir, i.e., $k_h = k_v$. It should be pointed out that the ratio k_h/k_v is the most critical term in evaluating and solving the coning problem. They developed three separate correlations for determining the critical oil flow rate:

- Gas coning
- Water coning
- Combined gas and water coning

Gas coning

GAS CONING

Consider the schematic illustration of the gas-coning problem shown in Figure 9-3.

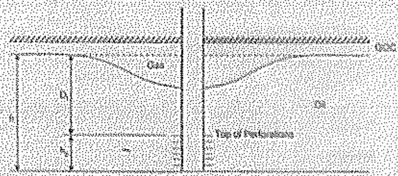


Figure 9-3. Gas coning.

Meyer and Garder correlated the critical oil rate required to achieve a stable gas cone with the following well penetration and fluid parameters:

- Difference in the oil and gas density
- Depth D_t from the original gas-oil contact to the top of the perforations
- The oil column thickness h

The well perforated interval h_p, in a gas-oil system, is essentially defined as

$$h_p = h - D_t$$

Meyer and Garder propose the following expression for determining the oil critical flow rate in a gas-oil system:

$$Q_{oc} = 0.246 \times 10^{-4}\left[\frac{\rho_o - \rho_g}{\ln(r_e/r_w)}\right]\left(\frac{k_o}{\mu_o B_o}\right)\left[h^2 - (h - D_t)^2\right] \quad (9\text{-}1)$$

where Q_{oc} = critical oil rate, STB/day
ρ_g, ρ_o = density of gas and oil, respectively, lb/ft^3
k_o = effective oil permeability, md
r_e, r_w = drainage and wellbore radius, respectively, ft
h = oil column thickness, ft
D_t = distance from the gas-oil contact to the *top* of the perforations, ft

Water coning

Meyer and Garder propose a similar expression for determining the critical oil rate in the water coning system shown schematically in Figure 9-4. The proposed relationship has the following form:

$$Q_{oc} = 0.246 \times 10^{-4}\left[\frac{\rho_w - \rho_o}{\ln(r_e/r_w)}\right]\left(\frac{k_o}{\mu_o B_o}\right)\left(h^2 - h_p^2\right) \quad (9\text{-}2)$$

where ρ_w = water density, lb/ft^3
h_p = perforated interval, ft

WATER

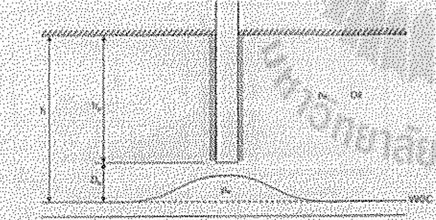


Figure 9-4. Water coning.

Simultaneous gas and water coning

If the effective oil-pay thickness h is comprised between a gas cap and a water zone (Figure 9-5), the completion interval h_p must be such as to permit maximum oil-production rate without having gas and water simultaneously produced by coning, gas breaking through at the top of the interval and water at the bottom.

This case is of particular interest in the production from a thin column underlaid by bottom water and overlaid by gas.

For this combined gas and water coning, Pirson (1977) combined Equations 9-1 and 9-2 to produce the following simplified expression for determining the maximum oil-flow rate without gas and water coning:

$$Q_{oc} = 0.246 \times 10^{-4}\left[\frac{k_o}{\mu_o B_o}\right]\frac{h^2 - h_p^2}{\ln(r_e/r_w)} \times \left[(\rho_w - \rho_o)\left(\frac{\rho_o - \rho_g}{\rho_w - \rho_g}\right)^2 + (\rho_o - \rho_g)\left(1 - \frac{\rho_o - \rho_g}{\rho_w - \rho_g}\right)^2\right] \quad (9\text{-}3)$$

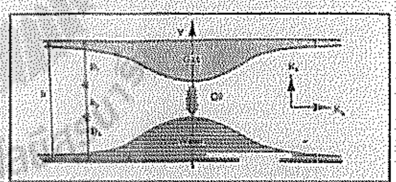


Figure 9-5. The development of gas and water coning.

Example 9-1

A vertical well is drilled in an oil reservoir overlaid by a gas cap. The related well and reservoir data are given below:

horizontal and vertical permeability, i.e., k_h, k_v	= 110 md
oil relative permeability, k_{ro}	= 0.85
oil density, ρ_o	= 47.5 lb/ft^3
gas density, ρ_g	= 5.1 lb/ft^3
oil viscosity, μ_o	= 0.73 cp
oil formation volume factor, B_o	= 1.1 bbl/STB
oil column thickness, h	= 40 ft
perforated interval, h_p	= 15 ft
depth from GOC to top of perforations, D_t	= 25 ft
wellbore radius, r_w	= 0.25 ft
drainage radius, r_e	= 660 ft

Using the Meyer and Garder relationships, calculate the critical oil flow rate.

Solution

The critical oil flow rate for this gas-coning problem can be determined by applying Equation 9-1. The following two steps summarize Meyer-Garder methodology:

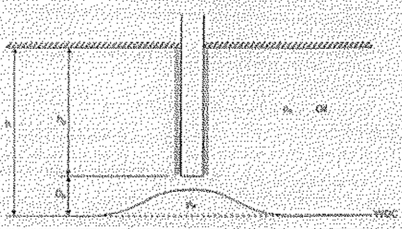


Figure 9-4. Water coning.

Simultaneous gas and water coning

If the effective oil-pay thickness h is comprised between a gas cap and a water zone (Figure 9-5), the completion interval h_p must be such as to permit maximum oil-production rate without having gas and water simultaneously produced by coning, gas breaking through at the top of the interval and water at the bottom.

This case is of particular interest in the production from a thin column underlaid by bottom water and overlaid by gas.

For this combined gas and water coning, Pirson (1977) combined Equations 9-1 and 9-2 to produce the following simplified expression for determining the maximum oil-flow rate without gas and water coning:

$$Q_{oc} = 0.246 \times 10^{-4} \left[\frac{k_o}{\mu_o B_o}\right] \frac{h^2 - h_p^2}{\ln(r_e/r_w)}$$

$$\times \left[(\rho_w - \rho_o)\left(\frac{\rho_o - \rho_g}{\rho_w - \rho_g}\right)^2 + (\rho_o - \rho_g)\left(1 - \frac{\rho_o - \rho_g}{\rho_w - \rho_g}\right)^2\right] \qquad (9\text{-}3)$$

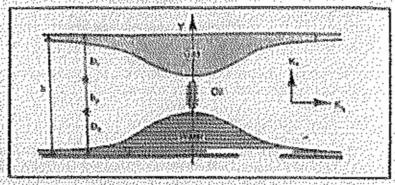


Figure 9-5. The development of gas and water coning.

Example 9-1

A vertical well is drilled in an oil reservoir overlaid by a gas cap. The related well and reservoir data are given below:

horizontal and vertical permeability, i.e., k_h, k_v	= 110 md
oil relative permeability, k_{ro}	= 0.85
oil density, ρ_o	= 47.5 lb/ft³
gas density, ρ_g	= 5.1 lb/ft³
oil viscosity, μ_o	= 0.73 cp
oil formation volume factor, B_o	= 1.1 bbl/STB
oil column thickness, h	= 40 ft
perforated interval, h_p	= 15 ft
depth from GOC to top of perforations, D_t	= 25 ft
wellbore radius, r_w	= 0.25 ft
drainage radius, r_e	= 660 ft

Using the Meyer and Garder relationships, calculate the critical oil flow rate.

Solution

The critical oil flow rate for this gas-coning problem can be determined by applying Equation 9-1. The following two steps summarize Meyer-Garder methodology:

Step 1. Calculate effective oil permeability k_o

$$k_o = k_{ro}\, k = (0.85)(110) = 93.5 \text{ md}$$

Step 2. Solve for Q_{oc} by applying Equation 9-1

$$Q_{oc} = 0.246 \times 10^{-4} \frac{47.5 - 5.1}{\ln(660/0.25)} \frac{93.5}{(0.73)(1.1)} [40^2 - (40 - 25)^2]$$

$$= 21.20 \text{ STB/day}$$

Example 9-2

Resolve Example 9-1 assuming that the oil zone is underlaid by bottom water. The water density is given as 63.76 lb/ft³. The well completion interval is 15 feet as measured from the top of the formation (no gas cap) to the bottom of the perforations.

Solution

The critical oil flow rate for this water-coning problem can be estimated by applying Equation 9-2. The equation is designed to determine the critical rate at which the water zone "touches" the bottom of the well to give

$$Q_{oc} = 0.246 \times 10^{-4} \left[\frac{(63.76 - 47.5)}{\ln(660/0.25)}\right] \left(\frac{93.5}{(0.73)(1.1)}\right) [40^2 - 15^2]$$

$$Q_{oc} = 8.13 \text{ STB/day}$$

The above two examples signify the effect of the fluid density differences on critical oil flow rate.

Example 9-3

A vertical well is drilled in an oil reservoir that is overlaid by a gas cap and underlaid by bottom water. Figure 9-6 shows an illustration of the simultaneous gas and water coning.

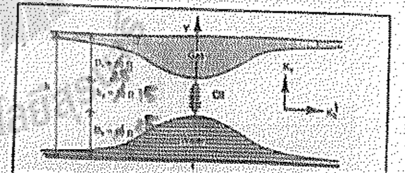


Figure 9-6. Gas and water coning problem (Example 9-3).

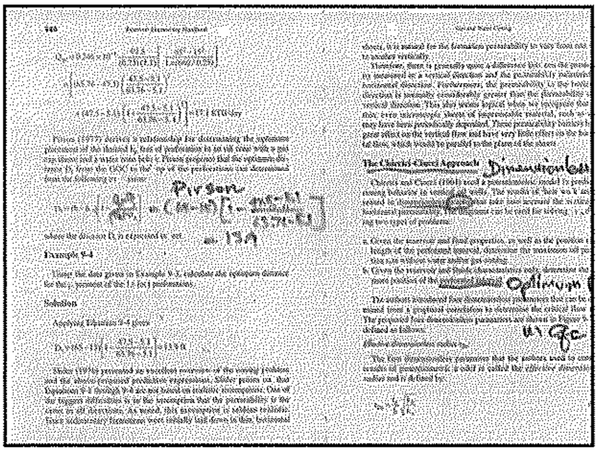
The following data are available:

oil density	ρ_o = 47.5 lb/ft³
water density	ρ_w = 63.76 lb/ft³
gas density	ρ_g = 5.1 lb/ft³
oil viscosity	μ_o = 0.73 cp
oil FVF	B_o = 1.1 bbl/STB
oil column thickness	h = 65 ft
depth from GOC to top of perforations	D_t = 25 ft
well perforated interval	h_p = 15 ft
wellbore radius	r_w = 0.25 ft
drainage radius	r_e = 660 ft
oil effective permeability	k_o = 93.5 md
horizontal and vertical permeability, i.e., k_h, k_v	= 110 md
oil relative permeability	k_{ro} = 0.85

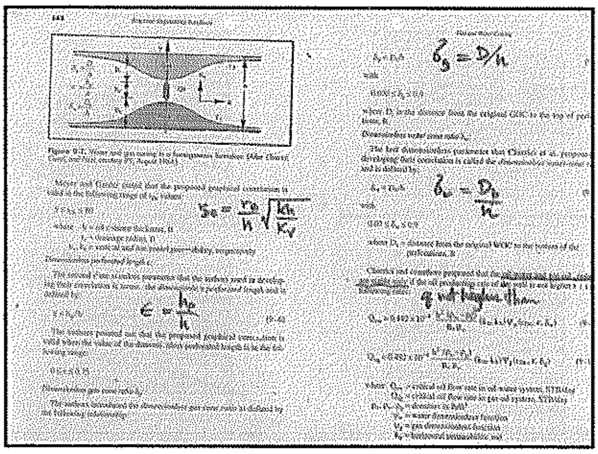
Calculate the maximum permissible oil rate that can be imposed to avoid cones breakthrough, i.e., water and gas coning.

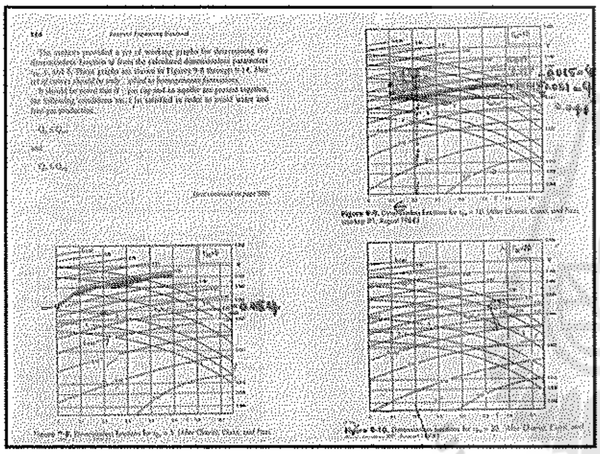
Solution

Apply Equation 9-3 to solve for the simultaneous gas- and water-coning problem, to give:


Example 9-4

Solution





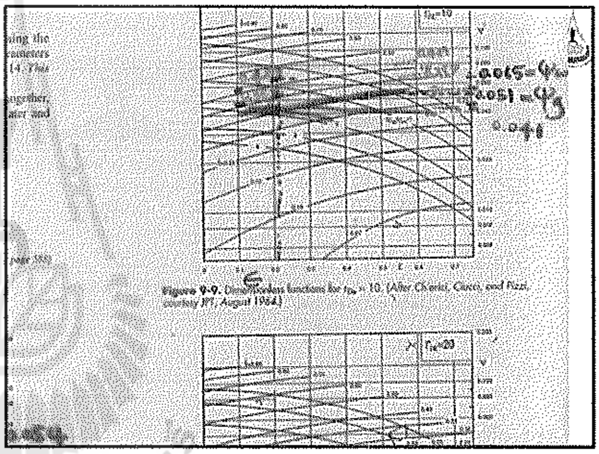
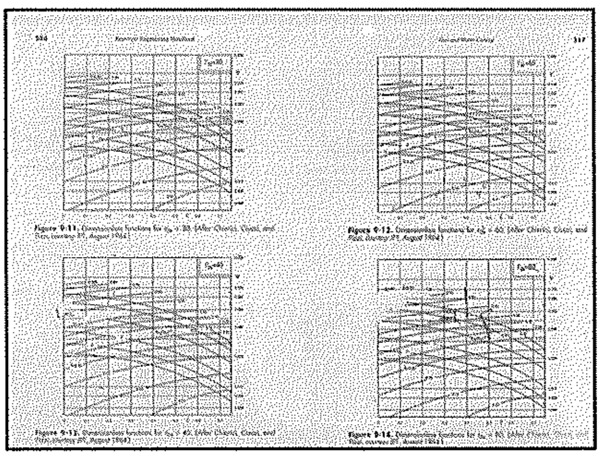
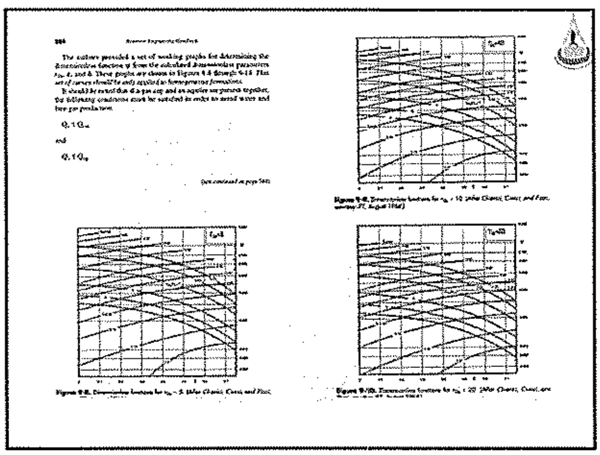
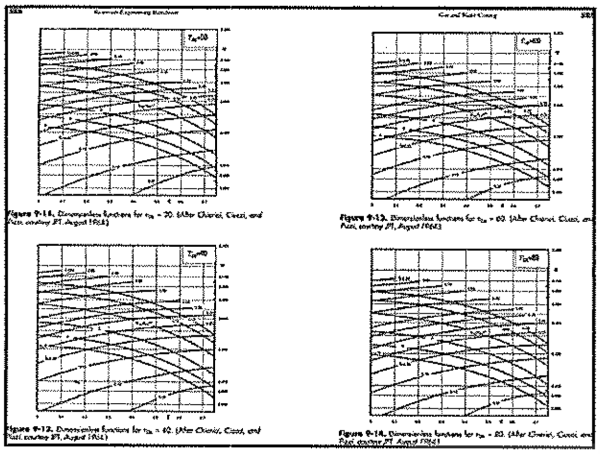
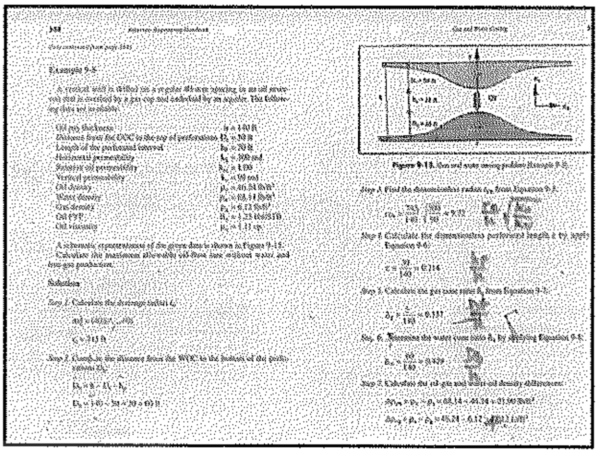


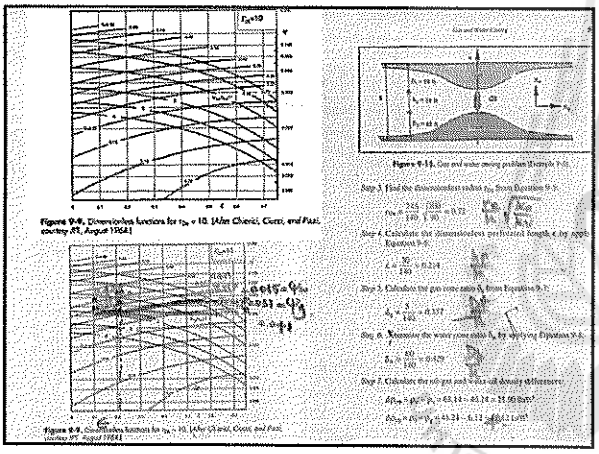
Figure 9-9. Dimensionless functions for r_{De} = 10. (After Chierici, Ciucci, and Pizzi, courtesy JPT, August 1964.)

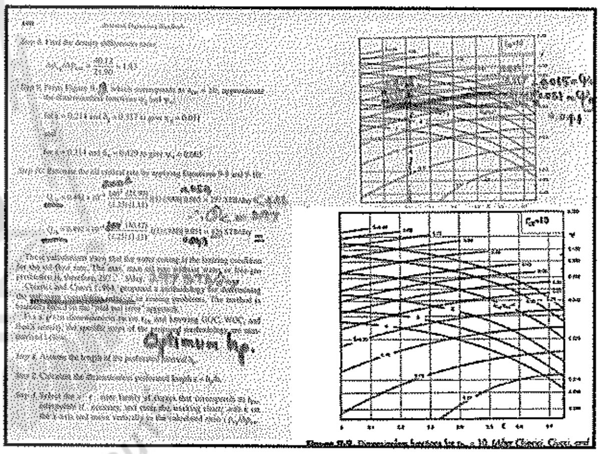


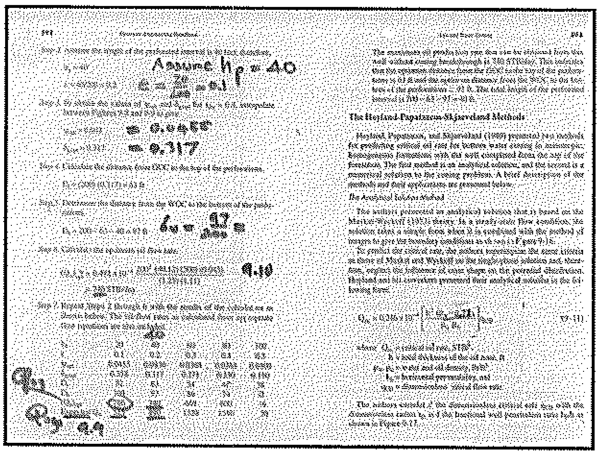


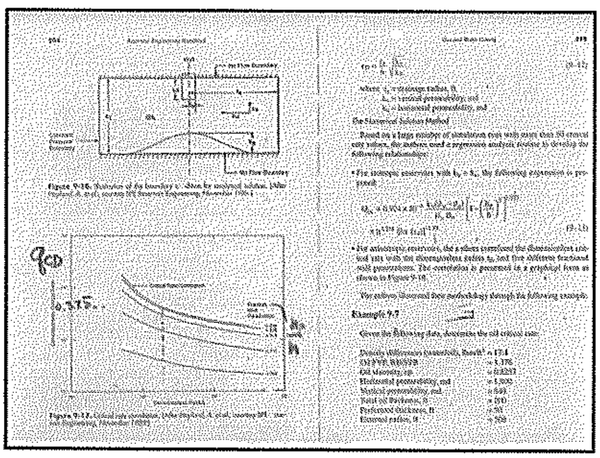


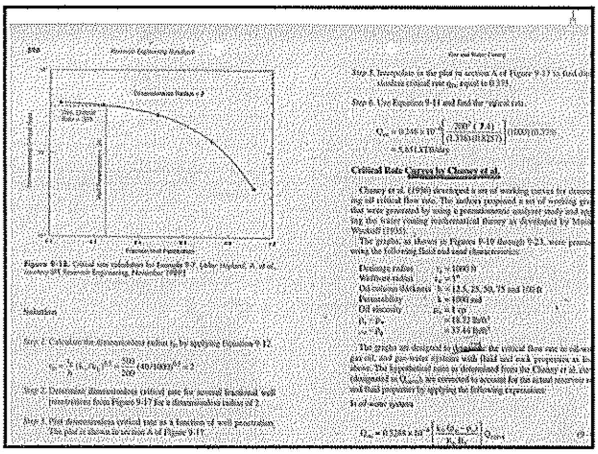




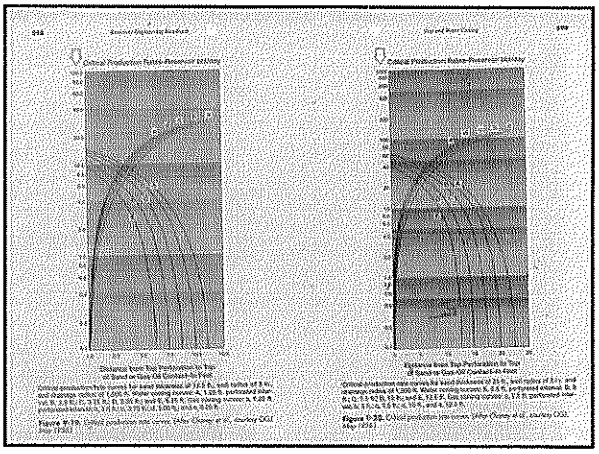


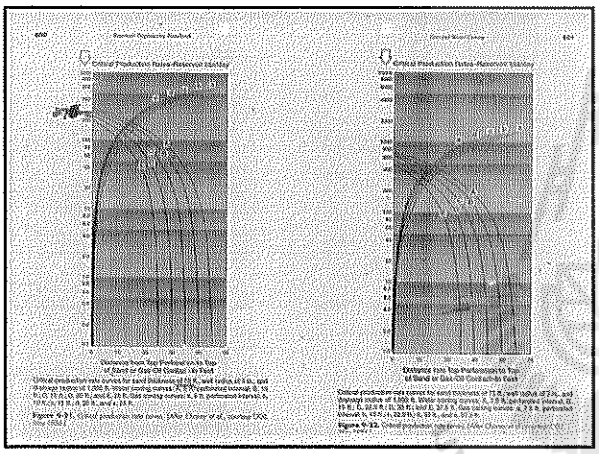


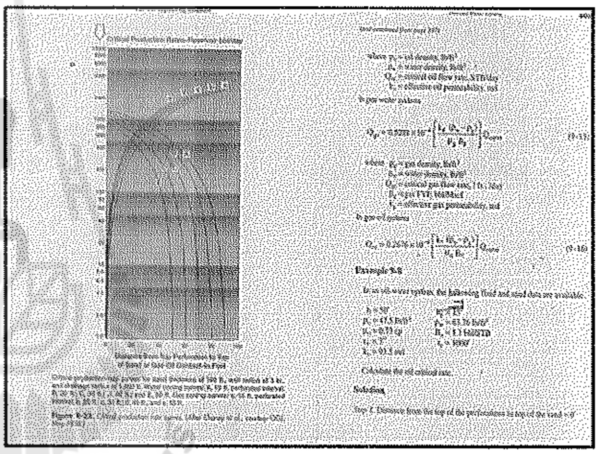




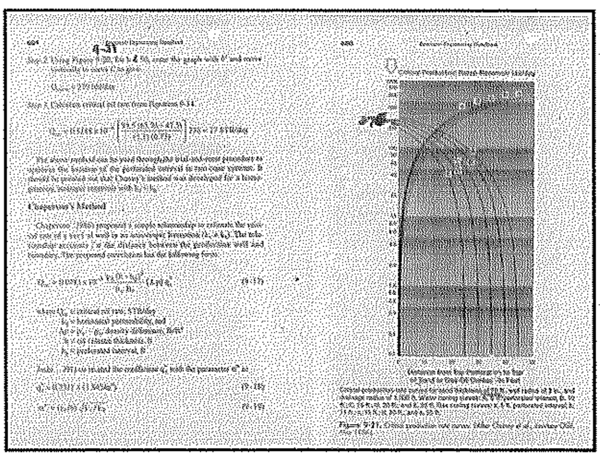
Critical Rate Curves by Chaney et al.



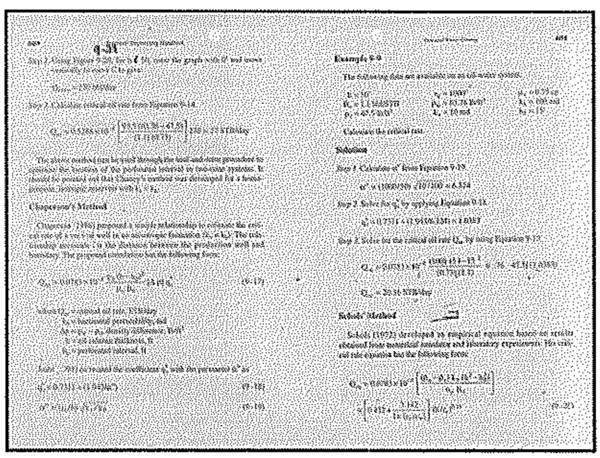




Example 9-8



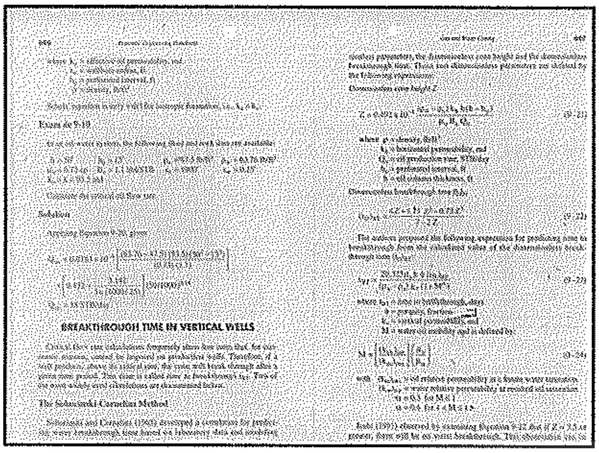
Chaperon's Method



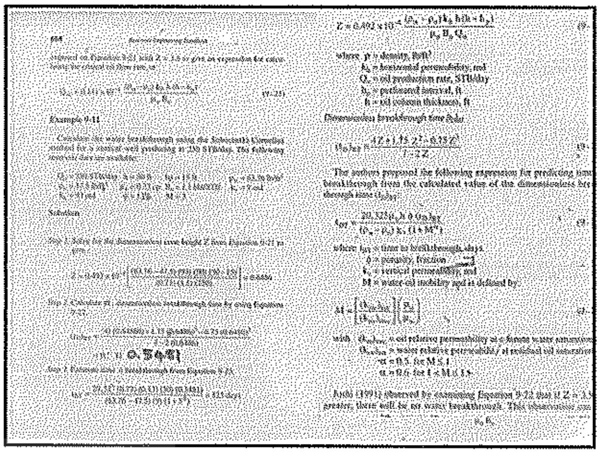
Example 9-9

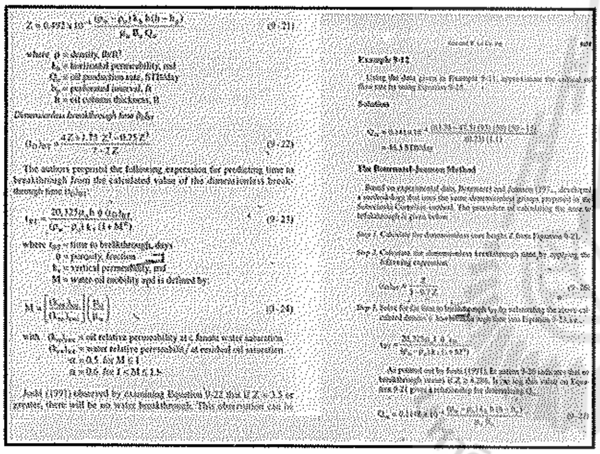
Chaperon's Method

Schols' Method


BREAKTHROUGH TIME IN VERTICAL WELLS

The Sobocinski-Cornelius Method


Example 9-11


where ρ = density, lb/ft³
k_h = horizontal permeability, md
Q_o = oil production rate, STB/day
h_p = perforated interval, ft
h = oil column thickness, ft

Dimensionless breakthrough time $(t_D)_{BT}$

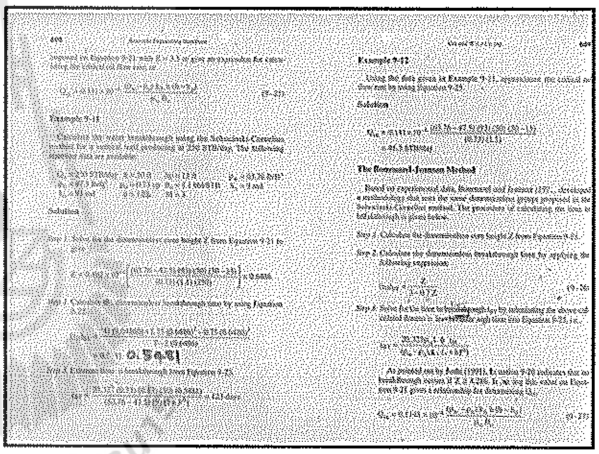
The authors proposed the following expression for predicting time to breakthrough from the calculated value of the dimensionless breakthrough time $(t_D)_{BT}$:

where t_{BT} = time to breakthrough, days
φ = porosity, fraction
k_v = vertical permeability, md
M = water-oil mobility and is defined by:

Example 9-12

Solution

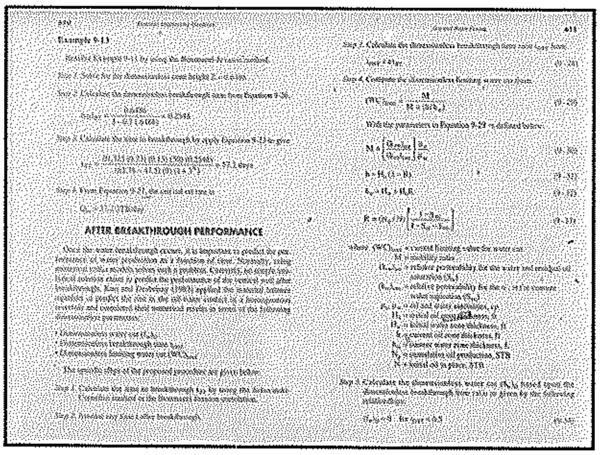
The Bournazel-Jeanson Method


Example 9-11

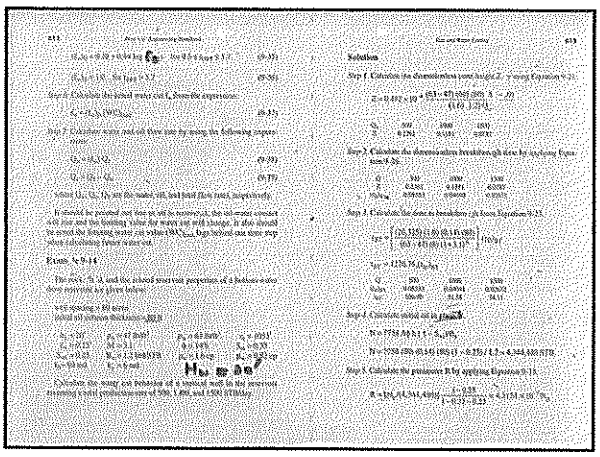
Example 9-12

Solution

The Bournazel-Jeanson Method


Example 9-13

AFTER BREAKTHROUGH PERFORMANCE


Solution

Solution

hw=22.05

CONING IN HORIZONTAL WELLS

Table 9-2
Results of Example 9-14
Total Production Rate is 1000 STB/day

Horizontal Well Critical Rate Correlations

Chaperon's Method

Table 9-3
Results of Example 9-14
Total Production Rate is 1500 STB/day

Example 9-15

Solution

Table 9-3
Results of Example 9-14
Total Production Rate in 1500 STB/day

Example 9-15

Solution

Chaperson's Method

Example 9-15

Solution

Example 9-16

Solution

Karcher's Method

Example 9-17

Joshi's Method

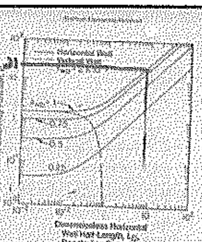
Solution

HORIZONTAL WELL BREAKTHROUGH TIME

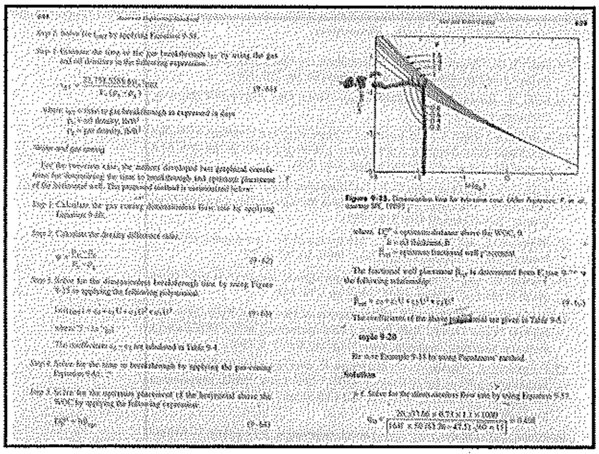
The Ozkan-Raghavan Method

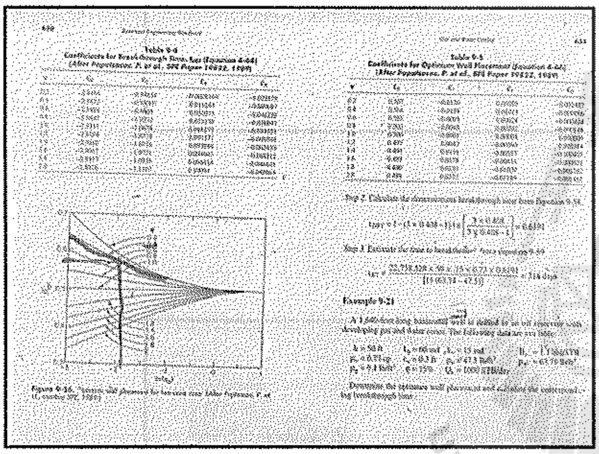
Example 9-19

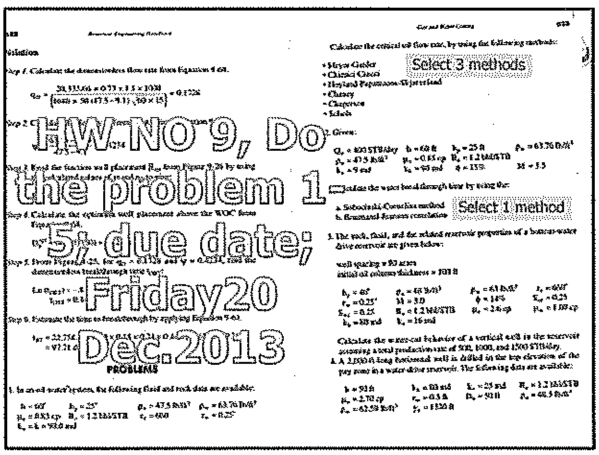
Solution



Papatzacos' Method






Select 3 methods
HW NO 9, Do the problem 1-5; due date; Friday20 Dec.2013
Select 1 method
PROBLEMS

Calculate the critical flow rate by using:

a. Chaperson's method
b. Efros' correlation
c. Karcher's equation
d. Joshi's method

Select 2 methods

5. A 2,000-foot-long horizontal well is producing at 1500 STB/day. The following data are available:

h = 60 ft	k_h = 80 md	k_v = 15 md	B_o = 1.2 bbl/STB
μ_o = 2.70 cp	r_w = 0.3 ft	ρ_o = 47.5 lb/ft^3	ρ_w = 63.76 lb/ft^3
z_{wD} = 1	ϕ = 15%	S_{wc} = 0.25	S_{or} = 0.25

Calculate the time to breakthrough by using the:

a. Ozkan-Raghavan method
b. Papatzacos' method

Select 1 method

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Calculate the critical flow rate by using:

a. Chaperson's method
b. Efros' correlation
c. Karcher's equation
d. Joshi's method

เลือก 2 ข้อ.

5. A 2,000-foot-long horizontal well is producing at 1500 STB/day. The following data are available:

h = 60 ft	k_h = 80 md	k_v = 15 md	B_o = 1.2 bbl/STB
μ_o = 2.70 cp	r_w = 0.3 ft	ρ_o = 47.5 lb/ft^3	ρ_w = 63.76 lb/ft^3
z_{wD} = 1	ϕ = 15%	S_{wc} = 0.25	S_{or} = 0.25

Calculate the time to breakthrough by using the:

a. Ozkan-Raghavan method
b. Papatzacos' method

เลือก 1 ข้อ.

REFERENCES

= 97.13 days

PROBLEMS

1. In an oil-water system, the following fluid and rock data are available:

h = 6[illegible]	h_p = 25'	ρ_o = 47.5 lb/ft^3	ρ_w = 63.76 lb/ft^3
μ_o = [illegible]5 cp	B_o = 1.2 bbl/STB	r_e = 660	r_w = 0.25'
k_v = k = 90.0 md			

Calculate the critical oil flow rate, by using the following methods:

- Meyer-Garder
- Chierici-Ciucci
- Hoyland-Papatzacos-Skjaeveland
- Chaney
- Chaperson
- Schols

เลือก 3 ข้อ.

2. Given:

Q_o = 400 STB/day	h = 60 ft	h_p = 25 ft	ρ_w = 63.76 lb/ft^3
ρ_o = 47.5 lb/ft^3	μ_o = 0.85 cp	B_o = 1.2 bbl/STB	
k_v = 9 md	k_h = 90 md	ϕ = 15%	M = 3.5

Calculate the water breakthrough time by using the:

a. Sobocinski-Cornelius method
b. Bournazel-Jeanson correlation

เลือก 1 ข้อ.

3. The rock, fluid, and the related reservoir properties of a bottom-water drive reservoir are given below:

well spacing = 80 acres
initial oil column thickness = 100 ft

h_p = 40'	ρ_o = 48 lb/ft^3	ρ_w = 63 lb/ft^3	r_e = 660'
r_w = 0.25'	M = 3.0	ϕ = 14%	S_{or} = 0.25
S_{wc} = 0.25	B_o = 1.2 bbl/STB	μ_o = 2.6 cp	[illegible] = 1.00 cp
k_h = 80 md	k_v = 16 md		

Calculate the water-cut behavior [illegible] a vertical well in [illegible] reservoir assuming a total production rate of [illegible], 1000, and 1500 S[illegible]/day.

4. A 2,000-ft-long horizontal we[illegible]l is drilled in the top elevation of the pay zone in a water-drive reservoir. The following data ar[illegible] table:

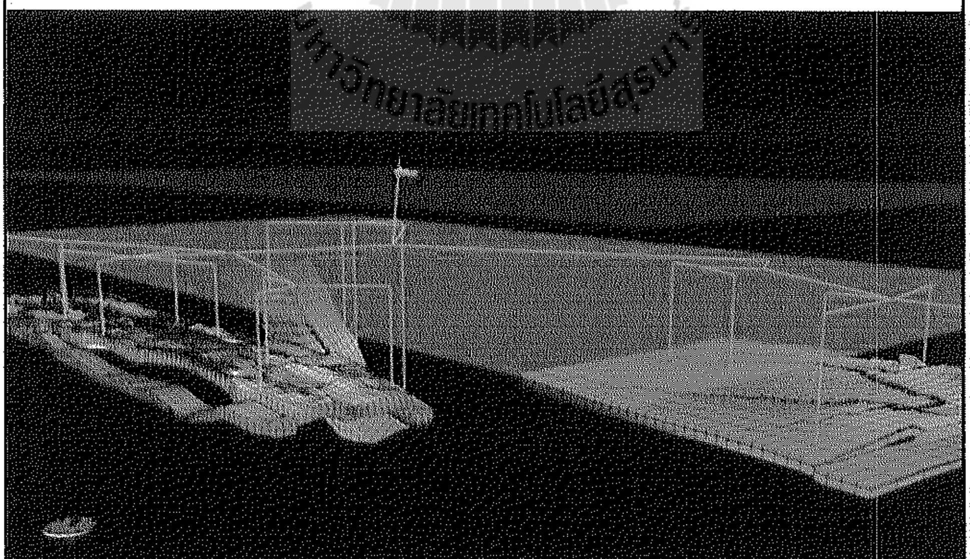
h = 50 ft	k_h = 80 md	k_v = 25 md	B_o = 1.2 bbl/STB
μ_o = 2.70 cp	r_w = 0.3 ft	D_v = 50 ft	ρ_o = 48.5 lb/ft^3
ρ_w = 62.50 lb/ft^3	y_e = 1320 ft		

Advanced Reservoir Engineering

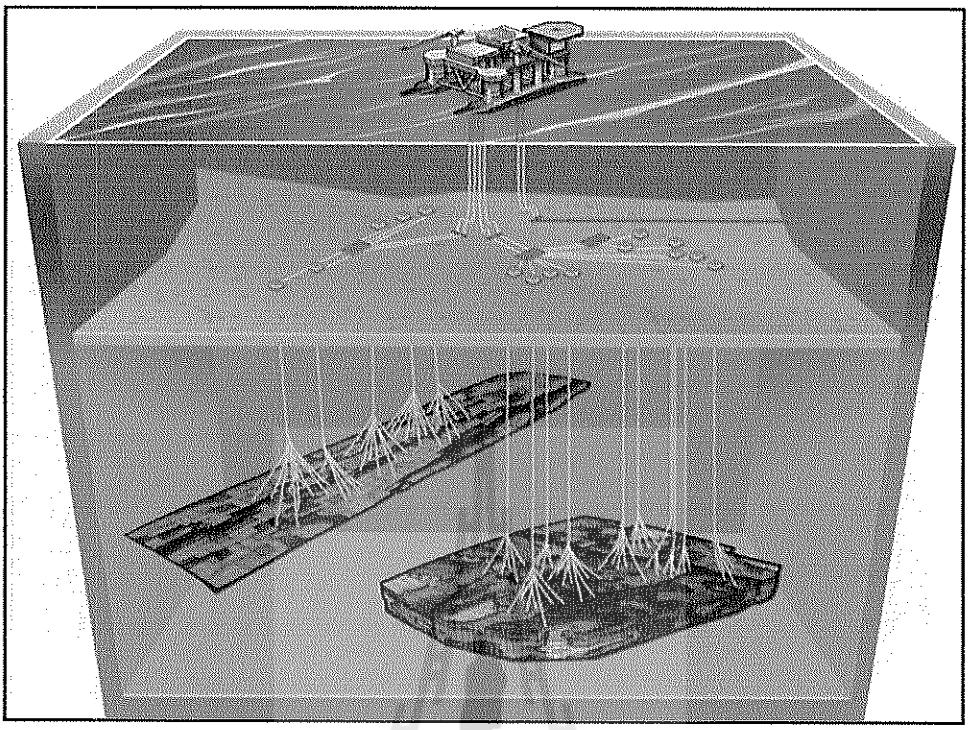
Chapter 10

Introduction to Reservoir Simulation

Overview of Reservoir Simulation



By Associate Prof.Kriangkrai Trisarn



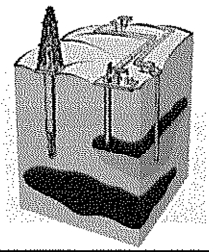
What is Simulation?

As applied to petroleum reservoirs, simulation can be stated as:

The process of mimicking or inferring the behavior of fluid flow in a <u>petroleum reservoir system</u> through the use of either physical or mathematical models

What is Simulation?

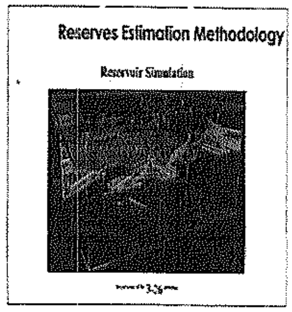
As used here, the words *petroleum reservoir system* include the reservoir rock and fluids, aquifer, and the surface and subsurface facilities.



3.7.2 Reservoir Simulations

What Simulation is
- *Simulate* = to give the appearance of m a reservoir
- *Simulation* involves the utilization of a model to obtain some insight into the behavior of a physical process.
- *Simulation* has long been recognized in many applied science disciplines as the final resort; as Wagner aptly says: " When all else fails,... simulate.
- In operations research, extensive use has been made of simulation studies; some examples are:
  1. Transportation model networks
  2. Stock market performance
  3. Telephone system design
  4. Supermarket checkout counter
  5. Engine Simulation
  6. Other Simulations

What Reservoir Simulation is
- A *RESERVOIR Simulation* is a mathematical model that represents the physical phenomena of a reservoir



Reservoir Simulation

What Simulation is

- ***Simulate*** = to give the appearance of m a reservoir
- ***Simulation*** involves the utilization of a model to obtain some insight into the behavior of a physical process.
- ***Simulation*** has long been recognized in many applied science disciplines as the final resort; as Wagner aptly says: " When all else fails,... simulate.
- In operations research, extensive use has been made of simulation studies; some examples are:
  1. Transportation model networks
  2. Stock market performance
  3. Telephone system design
  4. Supermarket checkout counter
  5. Engine Simulation
  6. Other Simulations

What Reservoir Simulation is

- A ***RESERVOIR Simulation*** is a mathematical model that represents the physical phenomena of a reservoir

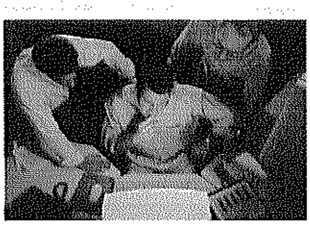
---

Why Do We Need Reservoir Simulation?

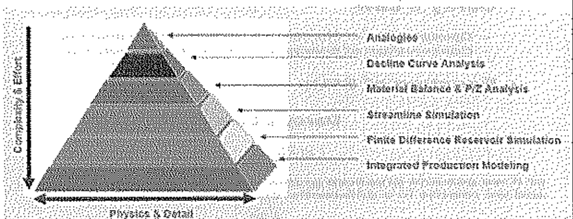
- What is the most efficient well spacing?
- What are the optimum production strategies?
- Where are the external boundaries located?
- What are the intrinsic reservoir properties?
- What is the predominant recovery mechanism?
- What and how should we employ infill drilling?
- When and which improved recovery technique should we implement?

Business Reasons for Using Reservoir Simulation

- Economics and timing of investments
- Credibility and Reliability
- Decision Making
- Arbitration, Unitization, & Regulation
- Performance Monitoring



Modeling Methods



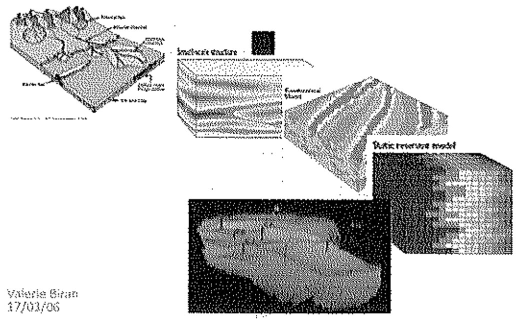
- Any problem is solvable if you can make *assumptions* - the key is determining the *right* assumptions.
- Not every question demands in-depth modeling of every detail.

Modeling Methods
Data Considered by Method

What are the assumptions for each method?

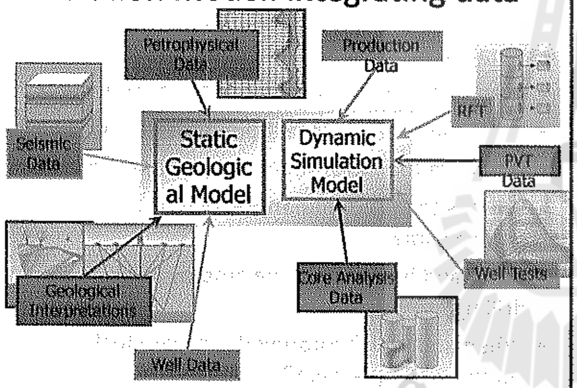
	Decline Curve	Material Balance	Numerical Simulation
Field Measurements			
Well Pressures		•	•
Oil, Water, Gas Production	•	•	•
Production Logs			•
Well Tests			•
Reservoir Description			
Geometry			•
Petrophysical Properties			•
OWC's, GOC's			•
Lab Measurements			
PVT Properties		•	•
Relative Permeability			•
Capillary Pressure			•
Well Descriptions			
Location			•
Completion Interval			•
Completion Changes			•
Stimulations			•

Reservoir Simulation



Valerie Biran
17/03/06

Simulation Model: Integrating data



What is reservoir simulation?

- A reservoir simulation is a mathematical model that represent the physical phenomena of a reservoir.

Why do we need a Reservoir Simulator?

- Because a reservoir simulator is a powerful tool and not expensive . We can predict what is going in the reservoir and a amount of production from alternative operations.

Types of Reservoir Simulation

- Black Oil Simulation
- Compositional Black Oil Simulation
- Coupled Fluid Flow /Geomechanic Simulation

Fluid Flow Equations
Governing Equations

- Mass Conservation (Material Balances)
- Darcy's Law
  - *Conservation of Momentum (Navier Stoke Equation)*
  - *Conservation of Energy (First Law of Thermodynamics)*
- Equation of State

Darcy's Law

$$Q = \frac{kA}{\mu}\frac{\Delta P}{\Delta x}$$

or if we assume pressure gradient reducing in negative sign and we rewrite in a differential equation form.

$$v = -\frac{k}{\mu}\frac{\partial P}{\partial x}$$

Substitute *v* in Mass Balance Equation (consider only x-direction)

$$\frac{\partial\left(-\frac{k}{\mu}\frac{\partial P}{\partial x}\rho\right)}{\partial x} = -\phi\frac{\partial \rho}{\partial t}$$

Modeling Methods

All mathematical techniques are simply an application of the three fundamental equations of reservoir engineering

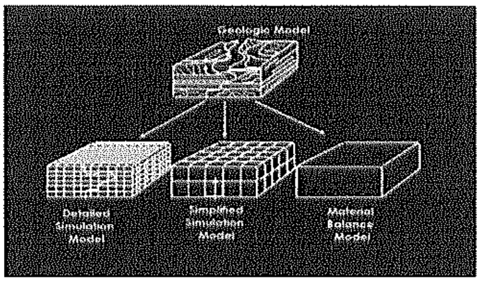
- Darcy's Law
- \+
- Material Balance Equation
- \+
- Fluid Properties **(PVT or EOS)**

with varying boundary conditions

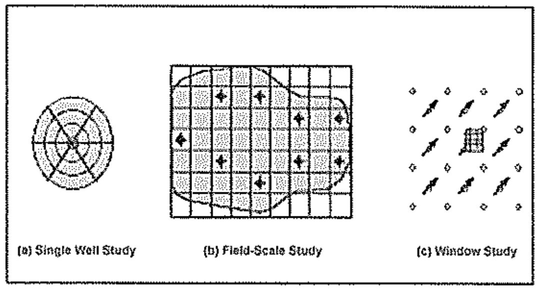
Modeling Methods

Finite Difference Process

Divide the reservoir into numerous blocks and represents it with a mesh of points or grid blocks.



Simulation Approaches



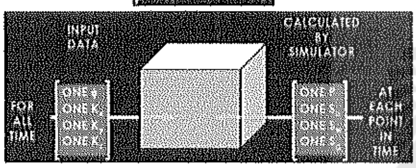
Modeling Methods

Finite Difference 3 Step Process

Solve mathematical equations for each cell by numerical methods to obtain pressure, production and saturation changes with time.

The Diffusivity Equation (Single Phase, 1-D Flow)

$$\frac{k}{\mu}\frac{\partial^2 P}{\partial x^2} = \phi c \frac{\partial P}{\partial t}$$



Accuracy of data input → Impacts accuracy of simulator calculations

CONCEPTS IN RESERVOIR MODELLING

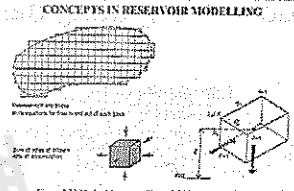


Figure 3.23 Methodology

Figure 3.24 Individual cell or grid block properties

EQUATION OF MULTIPHASE FLOW

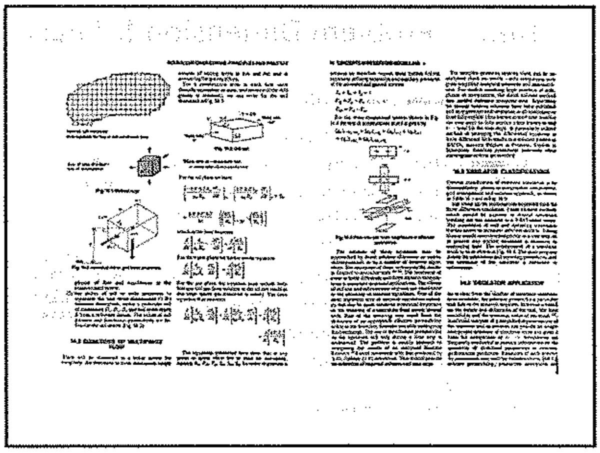
These will be illustrated in a linear system for simplicity. An extension to three dimensions simply consists of adding terms in ∂/∂y and ∂/∂z and account for gravity effects.

For a conservation term in stock tank units (broadly equivalent to mass, and identical if the API gravity is constant), we can write for the cell illustrated in Fig 3.25

Figure 3.25 Unit Cell

Mass rate in – Mass rate out = Mass rate of accumulation

For the oil phase we have



The equations presented here show that at any point in space there are at least six unknowns, namely P_o, P_w, P_g, S_o, S_w, S_g. In order to provide a solution we therefore require three further linking equations defining saturation and capillary pressures of the oil-water and gas-oil system

$$S_o + S_w + S_g = 1 \quad , \quad P_{cg} = P_g - P_o \quad , \quad P_{cw} = P_o - P_w$$

For three-dimensional system shown in Fig.3.26

$$(q_o)_{i-N_x-N_y} + (q_o)_{i-N_x} + (q_o)_{i-1} + (q_o)_{i+1} + (q_o)_{i+N_x} + (q_o)_{i+N_x+N_y}$$

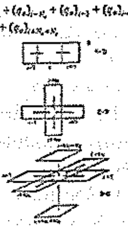
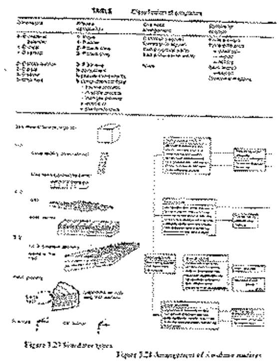


Figure 3.26 Flow in the cell *i* from neighbors in different geometries.



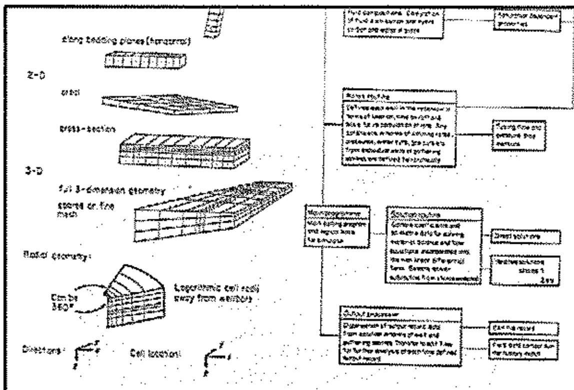
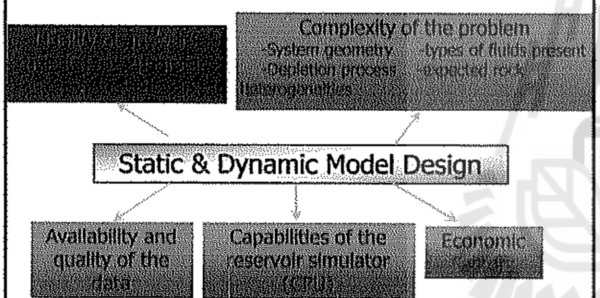


Figure 3.27 Simulator types

Figure 3.28 Arrangement of simulator routines

Simulation Model: Integrating data



Outline

- Reservoir Simulation Definition
- Integration of Multidisciplinary Data
- **Simulation Model Structure**
- Data QC
- Well Modeling
- Others:
  - Regions, Sectors and Flux Boundaries
  - ECLIPSE Geomechanics
  - HUTS

Simulation Model Structure

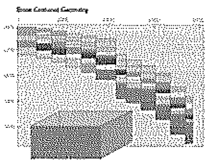
1. Define problem dimension and fluid phases
2. Define grid and rock properties: volume discretization
3. Define fluids properties and rock-fluids relationships
4. Specify initial conditions in the reservoir
5. Specify data to be outputted at each timestep
6. Specify operations schedule:
   - prod, inj, workover
   - controls, targets, constraints
   - timestepping (time discretization)

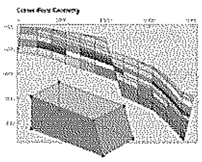
1. Define Problem Dimension & Phases

- Type of simulation model:
  - Black Oil: **3 immiscible phases as mass components (gas only can dissolve in water and oil, Rs solution GOR)**
    - Primary Recovery (Fluid Expansion, Solution Gas Drive)
    - Secondary Recovery (Waterflooding, Gas Inj, WAG)

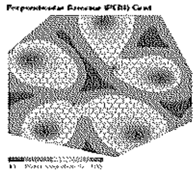
  - Compositional: **explicit description of oil and gas to account for mass transfer between each component**
    - CO2 flooding, gas injection in near critical reservoirs
    - Gas recycling in condensate reservoirs
    - Thermal: (steam injection recovery fro heavy oil etc)

1. Define Problem Dimension & Phas



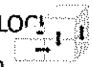


- Grid dimension (1, 2, 3D)
- Grid general geometry
  - Areal, radial
  - Cartesian, corner point, Pebi



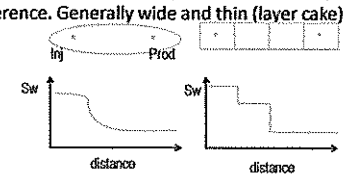
2. Define Grid and Rock Properties

- Define Discrete volume elements=GRIDBLOC



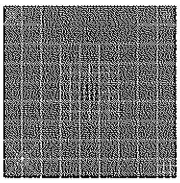
~ tanks of uniform properties (at a given timestep) with permeable sides.

Each block is connected to neighbors and fluids flow from one to another depending on permeabilities and pressure difference. Generally wide and thin (layer cake)



4 gridblock waterflood model

2. Define Grid and Rock Properties



- Grid fineness/coarseness:
  - how many grid blocks are needed
  - Local Grid Refinement :Enhance grid definition around areas of interest
  - Coarsening : Merge areas of little interest/activity into a single cell
- General Structure: Elevation, faults location, displacement, fractures
  → *seismic*
- Rock Properties: NTG thickness, Poro, Perm, Thickness → *core, well-logs*
- Rock distribution in space: Faults positions, isotropy etc
  → *deposit. environment, well-test, geol studies, analogs, geostats*

RESERVOIR DESCRIPTION IN MODELLING

Whether a complex of simple reservoir model is being applied, a number of steps in analysis and data requirements are common and illustrated in Fig. 3.29. The validity of the initialization reservoir model is largely dependent on geological model and the flow performance is linked to reservoir and production engineering description.

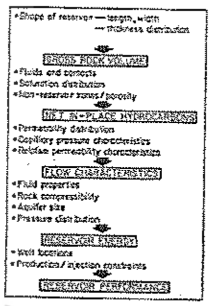
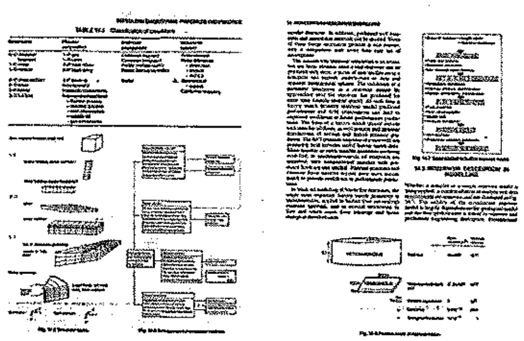


Figure 3.29 Steps needed to build a reservoir model.

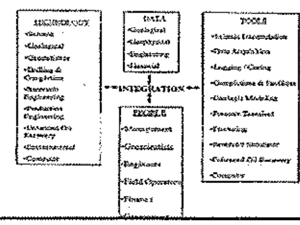


Classification of Reservoir Simulation

- Single phase reservoir simulator (liquid or gas)
- Multiphase reservoir simulator
  - black-oil simulator
  - compositional simulator

Other classification

- type of reservoir
  - gas reservoir simulator
  - black-oil reservoir simulator
  - compositional reservoir simulator
- fluid representation
  - compositional
  - non-compositional
- recovery process
  - primary recovery
    - solution gas drive
    - gas cap expansion
    - gravity drainage
    - water influx
  - secondary recovery
    - gas injection
    - water injection
  - enhanced oil recovery
    - chemical flood
    - miscible displacement
    - thermal recovery



Reservoir Simulation Concept

- Apply the concepts and techniques of mathematical modeling to the analysis of the behavior of petroleum reservoir systems.
- Commonly refer only to the dynamics of flow within the reservoir.
- Consist of the partial differential equations which govern the flows of all fluid phases in reservoir medium, numerical schemes, and algorithms needed to solve these equations.
- Reservoir simulation is a collection of computer program which implement the mathematical model on a particular computing machine.

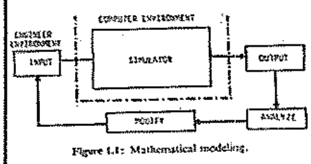
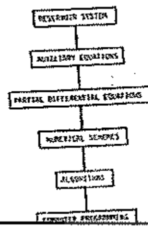
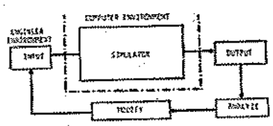


Figure 1.1: Mathematical modeling.





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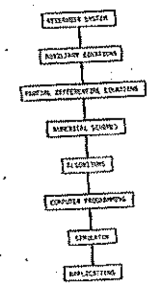


Mathematical Methods of Forecasting Reservoir Performance

- Material Balance Equation
- Decline Curve Analysis
- Statistical Approach
- Analytical Methods
- Numerical Reservoir Simulation

Introduction to Reservoir Managem...

Table 1-1
Why Simulate?

Corporate Impact
♦ Cash Flow Prediction
○ Need Economic Forecast of Hydrocarbon Price
Reservoir Management
♦ Coordinate Reservoir Management Activities
♦ Evaluate Project Performance
○ Interpret/Understand Reservoir Behavior
♦ Model Sensitivity to Estimated Data
○ Determine Need for Additional Data
♦ Estimate Project Life
♦ Predict Recovery vs Time
♦ Compare Different Recovery Processes
♦ Plan Development or Operational Changes
♦ Select and Optimize Project Design
○ Maximize Economic Recovery

1.1 Conventional Modeling

Reservoir modeling is the application of a computer simulation system ...the description of fluid flow in a reservoir [for example, see Peaceman, 1977; ...ds and Settari, 1979; Mattax and Dalton, 1990]. The computer simulation ...stem is usually just one or more computer programs. To minimize confusion ...this text, the computer simulation system is called the reservoir simulator, and ...e input data set is called the reservoir model.

Many different disciplines contribute to the preparation of the input data ...t. The information is integrated during the reservoir modeling process, and ...e concept of the reservoir is quantified in the reservoir simulator. Figure 1-1 ...ustrates the contributions different disciplines make to reservoir modeling.

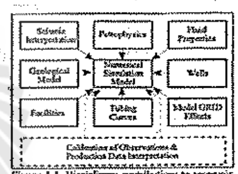


Figure 1-1. Disciplinary contributions to reservoir modeling (after H.H. Haldorsen and E. Damsleth, ©1993; reprinted by permission of the American Association of Petroleum Geologists).

The simulator is the point of contact between disciplines. It serves as a ...lter that selects from among all of the proposed descriptions of the reservoir.

Reservoir Modeling

- Physical Models
  - Scaled-down reproductions of the original.
  - Models constructed to duplicate processes which are physically similar to the original.
- Analog Models
  - Use similarities between the phenomenon of fluid flows through porous media and other physical phenomena to simulate reservoir performance.
- Mathematical Models
  - Systems of mathematical equations describing the physical behavior of the process under investigation.

Application of Reservoir Simulation

- Evaluate reservoir performance
  - Interpret/Understand reservoir behavior
- Predict hydrocarbon recovery vs time
- Estimate project life
- Model sensitivity to estimated data
  - Determine need for additional data
- Compare different recovery process
- Plan development or operational changes
- Select and optimize project design

Application of Reservoir Simulation

- Evaluate reservoir performance
  - Interpret/Understand reservoir behavior
- Predict hydrocarbon recovery vs time
- Estimate project life
- Model sensitivity to estimated data
  - Determine need for additional data
- Compare different recovery process
- Plan development or operational changes
- Select and optimize project design

Benefits of Reservoir Simulation

- Compile all data pertinent to a reservoir into one compact database.
- Provide opportunity to produce the reservoir before commencing actual production.
- Can produce the reservoir several times to examine alternatives.
- Can be utilized as a management tool for selecting development plan and operational changes
- Present a common ground between companies and regulatory agencies that deal with petroleum resources.

response of a new well in the field provides useful *history match* dat

RESERVOIR DESCRIPTION IN MODELLING

Whether a complex of simple reservoir model is being applied, a m steps in analysis and data requirements are common, and illustrate 3.29. The validity of the initialization reservoir model is largely d on geological model and the flow performance is linked to reser production engineering description.

- Shape of reservoir — length, width — thickness distribution

GROSS ROCK VOLUME

- Fluids and contacts
- Saturation distribution
- Non-reservoir zones / porosity

NET IN-PLACE HYDROCARBONS

- Permeability distribution
- Capillary pressure characteristics
- Relative permeability characteristics

FLOW CHARACTERISTICS

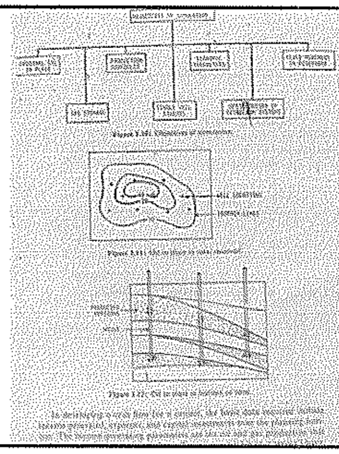
- Fluid properties
- Rock compressibility
- Aquifer size
- Pressure distribution

RESERVOIR ENERGY

- Well locations
- Production / injection constraints

RESERVOIR PERFORMANCE

Figure 3.29 Steps needed to build a reservoir model.



Comprehensive Reservoir Management Model

- Reservoir Model
- Well Model
- Wellbore Model
- Surface Model

Reservoir Model

- Represent fluid flow within reservoir.
- Subdivide the reservoir volume into an array or grid of smaller volume elements.

Reservoir Simulation Concept

- Apply the concepts and techniques of mathematical modeling to the analysis of the behavior of petroleum reservoir systems.
- Commonly refer only to the dynamics of flow within the reservoir.
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Comprehensive Reservoir Management Mode

- Reservoir Model
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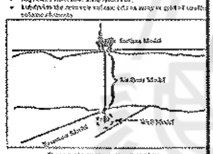
Reservoir Model



Reservoir management system.

A Black Oil Model

ECLIPSE 100

A Compositional Model

ECLIPSE 300

A Coupled Fluid Flow/ Geomechanic Model

HISTORY MATCHING

What is it?

Adjustment of input data which describes the elements of the reservoir to allow the simulator to reasonably calculate observed reservoir performance.

What is matched?

- Individual Well History
  - Shut-in Pressures (Build-ups)
  - Gas-Oil Ratio (GOR)
  - Water-Oil Ratio (WOR)
  - Temperature
  - Rates
  - Break Through
- Fluid Contact History
- Overall Reservoir Performance

What is adjusted?

Any parameters which describe the reservoir

- Permeability
- Porosity
- Thickness
- Net-to-Gross

- Water-Oil Ratio (WOR)
- Temperature
- Rates
- Break Through
- Fluid Contact History
- Overall Reservoir Performance

What is adjusted?

Any parameters which describe the reservoir

- Permeability
- Porosity
- Thickness
- Net-to-Gross

- Uncertain Areas of the Structure
- Faults
  - Location
  - Conductivity
- Shape and Endpoints of Saturation Functions
- Well Saturation
- Aquifer size, Strength and Location
- Vertical Flow Parameters, i.e., flow barriers
- Rock Compressibility and Distribution
- Etc.

3 Formulation of Reservoir Simulation Equations

Consider a system represented by Fig. 3.1. This system consists of a portion of the universe which is separated from the rest by a definite boundary. The system exists in space (x-, y-, z-dimensions) and in time (t). This is a finite system. We can make several observations about this system.

1. Anything that enters or leaves the system must cross the boundary.
2. At some initial time the system could be described by some set of conditions.
3. The processes which occur within the system obey some known physical laws and consequently can be described by some set of conditions.

The above observations allow us to describe in abstract terms the behavior of the system. Observation 1 gives us the *boundary conditions*. These spell out

Figure 3.1: System.

the interaction between the problem domain and the rest of the world. It can

sical laws and consequently can be described by some set of conditions.

The above observations allow us to describe in abstract terms the behavior of the system. Observation 1 gives us the *boundary conditions*. These spell out

Figure 3.1: System.

the interaction between the problem domain and the rest of the world. It can be visualized from the following. Consider some independent parameter P of the system shown in Fig. 3.2.

Figure 3.2: Boundary conditions.

processes within the system to the best of our ability and in that we do not always have complete knowledge of the minutest working of the system. We sometimes hypothesize. Consider the system again as shown in Fig. 3.3.

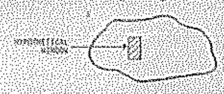


Figure 3.3: System processes.

If there were a hypothetical "window," we could look into the system at random locations and record exactly what we see and then try to associate these processes with the physical laws that apply. These laws may govern fluid flow, energy conservation, and the like. By defining the physical laws that apply, we can then formulate the mathematical equations which govern the processes within the system. These *governing equations* form part of the model of the system.

The complete mathematical model is then a combination of:

1. Governing equations
2. Boundary conditions
3. Initial conditions

3.2 DERIVATIONS OF EQUATIONS

To understand the flow of fluids in porous media we must be able to postulate some system of equations which govern the behavior of these fluids. Having developed such a system of equations, we can then analyze the effect of varying conditions on the flow behavior.

The basic equations are obtained by combining several physical principles, namely:

1. Conservation of mass
2. Conservation of momentum
3. Conservation of energy (first law of thermodynamics)
4. Rate equations—Darcy's law
5. Equations of state

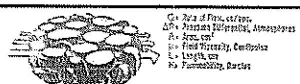
v = the apparent velocity $\quad Q = \frac{K A \Delta P}{\mu L}$

$$v = \frac{q}{A} = \frac{Q}{A} = \frac{K \Delta P}{\mu L}$$

As indicated in the previous section, the governing equations together with the necessary boundary conditions and initial conditions form the mathematical model for our system. To solve this mathematical model we need

Derivation Overview: The process involved in derivation of many of these equations consists of the following steps:

1. Select an elemental volume of the system (see Fig. 3.4).



v = the apparent velocity $\quad Q = \frac{K A \Delta P}{\mu L}$

$$v = \frac{q}{A} = \frac{Q}{A} = \frac{K \Delta P}{\mu L}$$

The following conversion factors are needed:

1 darcy = 1000 millidarcys $\quad$ 1 barrel = 159,000 cc

1 ft = 30.48 cm

1 atm = 14.7 psi

v = the apparent velocity, bbl/day-ft^2

k = permeability, millidarcies (md)

μ = fluid viscosity, cp

p = pressure, psia

s = distance along flow path in ft

Then:

$$\frac{q\ bbl/d \times \frac{159{,}000\ cc/bbl}{24 \times 3600\ sec/d}}{A\ ft^2 \left(30.48 \frac{cm}{ft}\right)^2} = \frac{\frac{k\ md}{1000\ md/darcy}\ dp \left[psi \times \frac{1}{14.7}\frac{atm}{psi}\right]}{\mu\ ds \left(30.48 \frac{cm}{ft}\right)}$$

$$v = -0.001127 \frac{k}{\mu}\left[\frac{dp}{ds}\right]$$

Derivation Overview: The process involved in derivation of many of these equations consists of the following steps:

1. Select an elemental volume of the system (see Fig. 3.4).

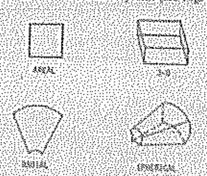
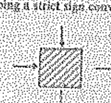


Figure 3.4: Elemental volumes.

2. Write all the fluxes into and out of the elemental volume over an interval of time, keeping a strict sign convention (Fig. 3.5).



4. $\Delta t \to 0$, $\Delta x \to 0$

5. Derrive Dif. Eq.

Figure 3.5: Flux directions.

3. Equate the fluxes to the changes within the element during this time...

SEC. 3.2 / Derivation of Equations

The total process involved in setting up the simulation equations is summarized in Fig. 3.6.

PROCESS OVERVIEW

DARCY'S LAW

CONSERVATION OF MASS

THREE PHASE FLOW

PRESSURE EQUATION

SATURATION EQUATION

Figure 3.6: Process overview.

Single-Phase Flow

The equation governing the single-phase flow of a fluid through a porous medium is developed by combining the following:

1. Conservation of Mass
2. Rate equation Darcy's law
3. Equation of State

Conservation of Mass: Consider an element of a reservoir through which a single phase is flowing in the x-direction (Fig. 3.7). Than at any instant:

Mass rate in − Mass rate out = Mass rate of accumulation

$$(v_x\rho_x\,\Delta y\,\Delta z) - (v_{x+\Delta x}\rho_{x+\Delta x}\,\Delta y\,\Delta z) = (\Delta x\,\Delta y\,\Delta z)\phi\frac{(\rho_{t+\Delta t}-\rho_t)}{\Delta t} \qquad (3.1)$$

The equation governing the single-phase flow of a fluid through a porous medium is developed by combining the following:

1. Conservation of Mass
2. Rate equation Darcy's law
3. Equation of State

Conservation of Mass: Consider an element of a reservoir through which a single phase is flowing in the x-direction (Fig. 3.7). Than at any instant:

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$$(v_x\rho_x\,\Delta y\,\Delta z) - (v_{x+\Delta x}\rho_{x+\Delta x}\,\Delta y\,\Delta z) = (\Delta x\,\Delta y\,\Delta z)\phi\frac{(\rho_{t+\Delta t}-\rho_t)}{\Delta t} \qquad (3.1)$$

Dividing Eq. (3.1) by $\Delta x, \Delta y\, \Delta z$:

$$-\frac{(v_{x+\Delta x}\rho_{x+\Delta x})-(v_x\rho_x)}{\Delta x} = \frac{\phi(\rho_{t+\Delta t}-\rho_t)}{\Delta t} \qquad (3.2)$$

Take the limit as $\begin{bmatrix}\Delta x\\ \Delta t\end{bmatrix}$ go to zero simultaneously:

$$\frac{\partial(v\rho)}{\partial x} = -\phi\frac{\partial\rho}{\partial t} \qquad (3.3)$$

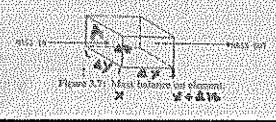

Figure 3.7: Mass balance on element.

This is the *continuity equation* in a linear system. Similarly:

$$\frac{\partial(v\rho)}{\partial y} = -\phi\frac{\partial\rho}{\partial t} \qquad (3.4)$$

$$\frac{\partial(v\rho)}{\partial z} = -\phi\frac{\partial\rho}{\partial t} \qquad (3.5)$$

Then for three-dimensional flow:

$$\frac{\partial(v\rho)}{\partial x} + \frac{\partial(v\rho)}{\partial y} + \frac{\partial(v\rho)}{\partial z} = -\phi\frac{\partial\rho}{\partial t} \qquad (3.6)$$

Rate Equation: Darcy's law relates the velocity to the pressure gradient:

$$v = -\frac{k}{\mu}\frac{\partial P}{\partial x} \qquad (3.7)$$

Then, substituting Eq. (3.7) into Eq. (3.3):

$$\frac{\partial\left(-\frac{k}{\mu}\frac{\partial P}{\partial x}\rho\right)}{\partial x} = -\phi\frac{\partial\rho}{\partial t} \qquad (3.8)$$

Equation of State: The equation of state is needed to express the density in terms of pressure. Most oil field liquid systems are considered to be slightly compressible. In this case, the equation of state is:

$$\rho = \rho_0 e^{c(P-P_0)} \qquad (3.9)$$

where

Then, substituting Eq. (3.7) into Eq. (3.3):

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$$\rho = \rho_0 e^{c(P-P_0)} \qquad (3.9)$$

where

ρ = density at pressure P

ρ_0 = density at pressure P_0

c = isothermal compressibility factor

$$c = -\frac{1}{V}\left(\frac{dV}{dP}\right)_T$$

Equation 3.8 can be written as follows by expanding the left-hand side:

$$-\left(\frac{k}{\mu}\frac{\partial^2 P}{\partial x^2}\rho + \frac{k}{\mu}\frac{\partial P}{\partial x}\frac{\partial\rho}{\partial x}\right) = -\phi\frac{\partial\rho}{\partial t} \qquad (3.10)$$

Note that:

$$\frac{\partial\rho}{\partial x} = \frac{\partial\rho}{\partial P}\frac{\partial P}{\partial x}$$

Chain Rule

and

$$\frac{\partial\rho}{\partial t} = \frac{\partial\rho}{\partial P}\frac{\partial P}{\partial t}$$

$\rho = \frac{M}{V}$

Therefore,

$$-\left(\frac{k}{\mu}\frac{\partial^2 P}{\partial x^2}\rho + \frac{k}{\mu}\frac{\partial P}{\partial x}\frac{\partial P}{\partial x}\frac{\partial\rho}{\partial P}\right) = -\phi\frac{\partial\rho}{\partial P}\frac{\partial P}{\partial t} \qquad (3.11)$$

$$-\left[\frac{k}{\mu}\frac{\partial^2 P}{\partial x^2}\rho + \frac{k}{\mu}\frac{\partial\rho}{\partial P}\left(\frac{\partial P}{\partial x}\right)^2\right] = -\phi\frac{\partial\rho}{\partial P}\frac{\partial P}{\partial t} \qquad (3.12)$$

Neglecting the $(\partial P/\partial x)^2$ term, since we are going to assume small pressure gradients, Eq. (3.12) becomes, by multiplication through by (−1):

$$\frac{k}{\mu}\frac{\partial^2 P}{\partial x^2}\rho = \phi\frac{\partial\rho}{\partial P}\frac{\partial P}{\partial t} \qquad (3.13)$$

Dividing both sides by density:

$$\frac{k}{\mu}\frac{\partial^2 P}{\partial x^2} = \phi\frac{1}{\rho}\frac{\partial\rho}{\partial P}\frac{\partial P}{\partial t} \qquad (3.14)$$

By definition, the compressibility is as follows:

$$c = \frac{1}{\rho}\frac{\partial\rho}{\partial P}$$

$c = -\frac{1}{V}\frac{dV}{dP} = -\frac{1}{\frac{M}{\rho}}\frac{d\frac{M}{\rho}}{dP} = -\frac{1}{\rho}\frac{(\frac{1}{\rho})\partial\rho}{\partial P}$

This is indicated in the graph of ρ versus P in Fig. 3.8. Then:

$$\frac{k}{\mu}\frac{\partial^2 P}{\partial x^2} = \phi c\frac{\partial P}{\partial t} \qquad (3.15)$$

Since k/μ was considered independent of spatial dimension:

$$\frac{k}{\mu}\frac{\partial^2 P}{\partial x^2} = \phi\frac{1}{\rho}\frac{\partial\rho}{\partial P}\frac{\partial P}{\partial t} \qquad (3.14)$$

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This is indicated in the graph of ρ versus P in Fig. 3.8. Then:

$$\frac{k}{\mu}\frac{\partial^2 P}{\partial x^2} = \phi c\frac{\partial P}{\partial t} \qquad (3.15)$$

Since k/μ was considered independent of spatial dimension:

$$\frac{\partial^2 P}{\partial x^2} = \frac{\phi\mu c}{k}\frac{\partial P}{\partial t} \qquad (3.16)$$

If k/μ were a function of the spatial dimension, then:

$$\frac{\partial\left(\frac{k}{\mu}\frac{\partial P}{\partial x}\right)}{\partial x} = \phi c\frac{\partial P}{\partial t} \qquad (3.17)$$

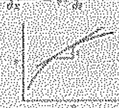


Figure 3.8: ρ versus P.

80

Equation (3.16) is generally called the *diffusivity equation* because of its resemblance to the diffusivity equation for heat transfer:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (3.18)$$

$$\frac{\partial^2 P}{\partial x^2} = \frac{\phi\mu c}{k}\frac{\partial P}{\partial t}$$

Other Coordinate Systems:

$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{\phi\mu c}{k}\frac{\partial P}{\partial t} \qquad \text{Radial flow}$$

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = \frac{\phi\mu c}{k}\frac{\partial P}{\partial t} \qquad \text{Two-dimensional} \qquad (3.19)$$

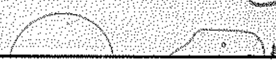
$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = \frac{\phi\mu c}{k}\frac{\partial P}{\partial t} \qquad \text{Three-dimensional}$$

The typical reservoir configurations for the above equations are shown in Fig. 3.9.

* H.W #3

Differential Eq

Integration



$$\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{\partial P}{\partial r} = \frac{\phi\mu c}{k}\frac{\partial P}{\partial t} \qquad \text{Radial flow}$$

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} = \frac{\phi\mu c}{k}\frac{\partial P}{\partial t} \qquad \text{Two-dimensional} \qquad (3.19)$$

$$\frac{\partial^2 P}{\partial x^2} + \frac{\partial^2 P}{\partial y^2} + \frac{\partial^2 P}{\partial z^2} = \frac{\phi\mu c}{k}\frac{\partial P}{\partial t} \qquad \text{Three-dimensional}$$

The typical reservoir configurations for the above equations are shown in Fig. 3.9.

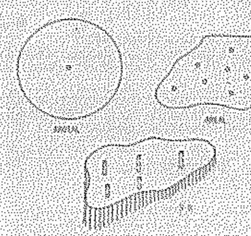


Figure 3.9: Radial, areal, and three-dimensional systems.

MULTIPHASE FLOW

3.3 DERIVATION OF MULTIPHASE FLOW EQUATIONS

The flow for each phase is developed identically to that scheme outlined for a single-phase fluid.

Oil: Starting with an element of the reservoir, the basic equation for oil flow is derived by combining the continuity equation, the Darcy flow equation, and equation of state (see Fig. 3.10). Using a balance on the STB oil

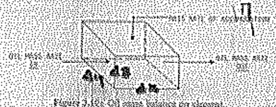


Figure 3.10: Oil mass balance on element.

flowing in a linear system:

Mass rate in − Mass rate out = Mass rate of accumulation

Thus:

$$(v_x\rho_x\,\Delta y\,\Delta z) - (v_{x+\Delta x}\rho_{x+\Delta x}\,\Delta y\,\Delta z) = (\Delta x\,\Delta y\,\Delta z)\phi\frac{(\rho_{t+\Delta t}-\rho_t)}{\Delta t} \qquad (3.1)$$

Dividing Eq. (3.1) by $\Delta x\,\Delta y\,\Delta z$:

$$-\frac{(v_{x+\Delta x}\rho_{x+\Delta x}) - (v_x\rho_x)}{\Delta x} = \frac{\phi(\rho_{t+\Delta t}-\rho_t)}{\Delta t} \qquad (3.2)$$

Figure 3.10: Oil mass balance on element.

flowing in a linear system:

Mass rate in − Mass rate out = Mass rate of accumulation

Thus:

$$\left(-A\frac{k_o}{\mu_o B_o}\frac{\partial P}{\partial x}\right)_x - \left(-A\frac{k_o}{\mu_o B_o}\frac{\partial P}{\partial x}\right)_{x+\Delta x} = V\left[\frac{\left(\frac{\phi S_o}{B_o}\right)^{t+1} - \left(\frac{\phi S_o}{B_o}\right)^t}{\Delta t}\right] \qquad (3.20)$$

where

$$A = \Delta y\,\Delta z$$
$$V = \Delta x\,\Delta y\,\Delta z$$

Equation (3.20) becomes in the limit:

$$A\,\frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o B_o}\frac{\partial P}{\partial x}\right) = \frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) V \qquad (3.21)$$

For a radial system the equivalent system is:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{k_o}{\mu_o B_o}\frac{\partial P}{\partial r}\right) = \frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) \qquad (3.22)$$

Gas: A mass balance on the gas phase must include all possible sources of gas (Fig. 3.11). For a linear system we can write:

Mass rate in − Mass rate out = Mass rate of accumulation

Each of the sources of gas as indicated in Fig. 3.11 is incorporated in the

$$\frac{\partial}{\partial x}\left[\left(\frac{k_g}{\mu_g B_g} + \frac{R_{so}k_o}{\mu_o B_o} + \frac{R_{sw}k_w}{\mu_w B_w}\right)\frac{\partial P}{\partial x}\right] = \frac{\partial}{\partial t}\left[\phi\left(\frac{S_g}{B_g} + \frac{R_{so}S_o}{B_o} + \frac{R_{sw}S_w}{B_w}\right)\right] \qquad (3.24)$$

For a radial system the following equation is obtained:

$$B_g\,\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\frac{k_g}{\mu_g B_g} + \frac{R_{so}k_o}{\mu_o B_o} + \frac{R_{sw}k_w}{\mu_w B_w}\right)\frac{\partial P}{\partial r}\right] = \frac{\partial}{\partial t}\left[\phi\left(\frac{S_g}{B_g} + \frac{R_{so}S_o}{B_o} + \frac{R_{sw}S_w}{B_w}\right)\right] B_g \qquad (3.25)$$

Water: The water phase is essentially the same as the oil phase. For a linear system:

$$\frac{\partial}{\partial x}\left[\frac{k_w}{\mu_w B_w}\frac{\partial P}{\partial x}\right] = \frac{\partial}{\partial t}\left[\phi\frac{S_w}{B_w}\right] \qquad (3.26)$$

For a radial system:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{k_w}{\mu_w B_w}\frac{\partial P}{\partial r}\right] = \frac{\partial}{\partial t}\left[\phi\frac{S_w}{B_w}\right] \qquad (3.27)$$

Expansion in Radial Form

The generalized multiphase flow equation for the unsteady-state flow of oil, gas, and water in a porous medium is developed by combining the three single-phase flow equations into one basic equation. To do this several other observations are made. First, for all phases the following is true:

$$S_o + S_g + S_w = 1 \qquad (3.28)$$

Thus:

$$\frac{\partial}{\partial t}[S_o + S_g + S_w] = 0 \qquad (3.29)$$

Pressure gradients are assumed small and the square of this term is neglected:

$$\left(\frac{\partial P}{\partial r}\right)^2 \approx 0 \qquad (3.30)$$

The derivation is as follows in radial coordinates. Multiply the oil equation (Eq. 3.22) by B_o and expand by differentiation:

$$\frac{B_o}{r}\left[r\frac{k_o}{\mu_o B_o}\frac{\partial^2 P}{\partial r^2} + r\frac{\partial P}{\partial r}\frac{k_o}{\mu_o}\left(-\frac{1}{B_o^2}\right)\frac{\partial B_o}{\partial P}\frac{\partial P}{\partial r} + \frac{k_o}{\mu_o B_o}\frac{\partial P}{\partial r}\right] = \phi B_o\left(\frac{1}{B_o}\frac{\partial S_o}{\partial t} + \frac{S_o}{-B_o^2}\frac{\partial B_o}{\partial P}\frac{\partial P}{\partial t}\right) \qquad (3.31)$$

Thus:

$$\frac{k_o}{\mu_o}\frac{\partial^2 P}{\partial r^2} - \frac{k_o}{\mu_o B_o}\frac{\partial B_o}{\partial P}\left(\frac{\partial P}{\partial r}\right)^2 + \frac{1}{r}\frac{k_o}{\mu_o}\frac{\partial P}{\partial r} = \phi\left(\frac{\partial S_o}{\partial t} - \frac{S_o}{B_o}\frac{\partial B_o}{\partial P}\frac{\partial P}{\partial t}\right) \qquad (3.32)$$

Neglecting $(\partial P/\partial r)^2$ terms, Eq. (3.32) becomes

$$\frac{k_o}{\mu_o}\frac{\partial^2 P}{\partial r^2} + \frac{1}{r}\frac{k_o}{\mu_o}\frac{\partial P}{\partial r} = \phi\left(\frac{\partial S_o}{\partial t} - \frac{S_o}{B_o}\frac{\partial B_o}{\partial P}\frac{\partial P}{\partial t}\right) \qquad (3.33)$$

fluid phase in a one-dimensional system:

$$A_x\frac{\partial}{\partial x}\left(\frac{k_o}{\mu_o B_o}\frac{\partial \Phi_o}{\partial x}\right)+q_o=V_R\frac{\partial}{\partial t}\left(\frac{\phi S_o}{B_o}\right) \quad \text{Oil} \tag{3.47}$$

$$A_x\frac{\partial}{\partial x}\left(\frac{k_w}{\mu_w B_w}\frac{\partial \Phi_w}{\partial x}\right)+q_w=V_R\frac{\partial}{\partial t}\left(\frac{\phi S_w}{B_w}\right) \quad \text{Water} \tag{3.48}$$

$$A_x\frac{\partial}{\partial x}\left(\frac{k_g}{\mu_g B_g}\frac{\partial \Phi_g}{\partial x}+\frac{R_{so}k_o}{\mu_o B_o}\frac{\partial \Phi_o}{\partial x}+\frac{R_{sw}k_w}{\mu_w B_w}\frac{\partial \Phi_w}{\partial x}\right)+q_g = V_R\frac{\partial}{\partial t}\left[\phi\left(\frac{S_g}{B_g}+\frac{R_{so}S_o}{B_o}+\frac{R_{sw}S_w}{B_w}\right)\right] \quad \text{Gas} \tag{3.49}$$

We can combine these to obtain the equations for flow in a reservoir. In order to do this, however, we need to express some accessory conditions.

The potential terms are defined as:[4]

$$\Phi_o = P_o + \rho_o gh \tag{3.50}$$
$$\Phi_g = P_g + \rho_g gh \tag{3.51}$$
$$\Phi_w = P_w + \rho_w gh \tag{3.52}$$

The capillary pressure terms are:[5]

$P_c = P_o - P_{cw}$

$$P_{cw} = P_o - P_w \tag{3.53}$$
$$P_{cg} = P_g - P_o \tag{3.54}$$

Equations (3.47) through (3.54) can be combined using, in addition, the

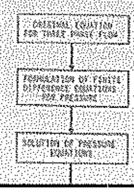
saturation equation (Eq. 3.29) to obtain:

$$A_x\frac{\partial}{\partial x}\left(\lambda_T\frac{\partial P_o}{\partial x}\right)+A_x\frac{\partial}{\partial x}\left(\lambda_g\frac{\partial P_{cg}}{\partial x}-\lambda_w\frac{\partial P_{cw}}{\partial x}\right)+A_x\frac{\partial}{\partial x}\left[\lambda_o\frac{\partial(\rho_o gh)}{\partial x}+\lambda_w\frac{\partial(\rho_w gh)}{\partial x}+\lambda_g\frac{\partial(\rho_g gh)}{\partial x}\right]=\beta_1\frac{\partial P_o}{\partial t}+\beta_2 \tag{3.55}$$

where the λ-variables are mobility terms, β_1-variables are functions of PVT (pressure-volume-temperature) terms, and β_2-variables are production terms.

For two-dimensional flow, Eq. (3.55) is expanded to include the y-coordinate terms.

Solution Outline: The two basic methods to solve the simulator equations are covered in more detail in Chapter 5. A brief outline of one method is presented in Fig. 3.12 to introduce the engineer to the procedure.



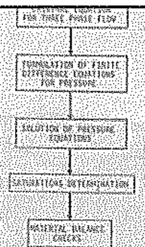


Figure 3.12: Procedure outline for solution of flow equation.

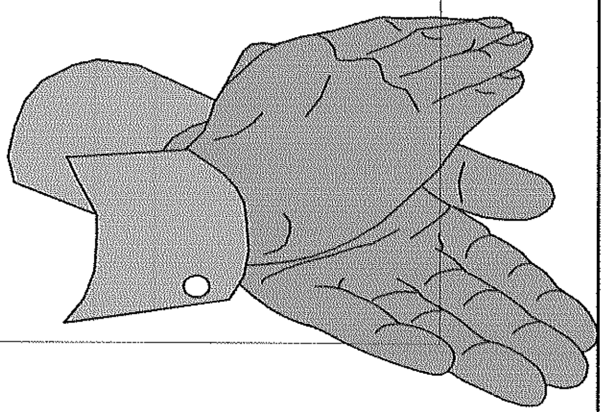
3.4 MULTICOMPONENT SYSTEMS[6,7]

In some hydrocarbon systems there is considerable mass transfer between the flowing phases. This mass transfer complicates the already complex system, since a mass balance must be made on every flowing fraction instead of on each phase. In a reservoir system there are generally several species of chemical compounds. These components vary in concentration in different phases, while each phase flows at a different rate.

Consider an element of the reservoir as shown in Fig. 3.13. There are N species of chemical compounds flowing into the reservoir element in three

End of the Day:

THANK YOU



Advanced Reservoir Engineering

Chapter 11 Introduction to

ENHANCED OIL RECOVERY

434620,505653 ADVANCED RESERVOIR ENGINEERING
2/2556(4 credits(4-0-8), Sept.-December , 2013.)

Course Contents

1. Optimization of Material Balance Equatio
2. Saturation and Relative Permeability Calc rs.)
3. Steady State Radial Flow (4 hrs.)
4. Pseudo-steady State Flow and Super tion (4 hrs.)
5. Well Testing Pressure Drawdown and Build Up (3 hrs.)
6. Type Curve Analysis AND Interference Test (3hrs)
7. Displacement Efficiency(4 hrs)
8. Dynamics of Water Drive Reservoir. (6 hrs.)
9. Water and Gas Coning (4hrs.)
10. Multi-Phase Flow and Introduction to Reservoir Simulation (4 h
11. Enhanced Oil Recovery(2 hrs)

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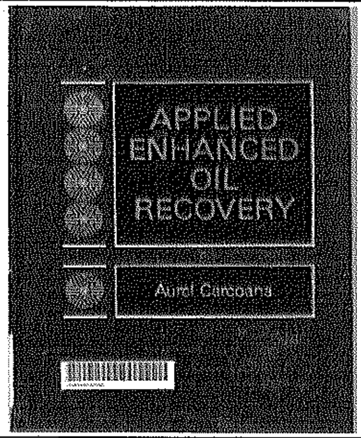

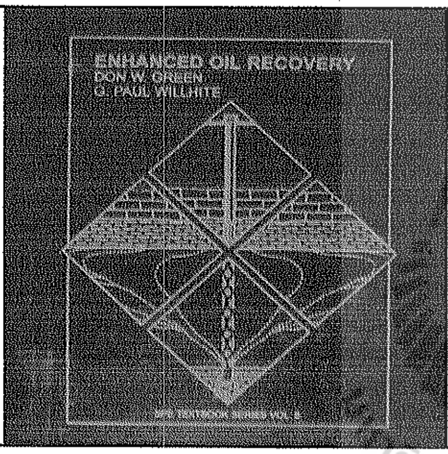

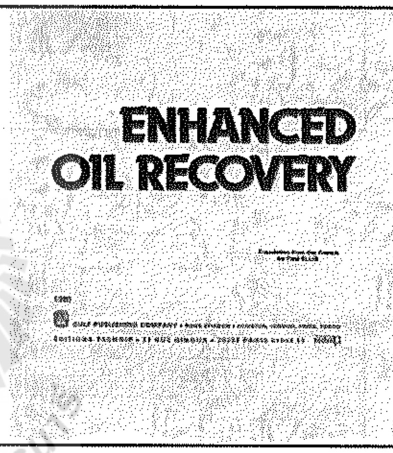

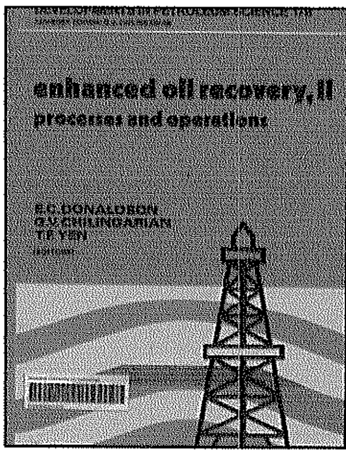

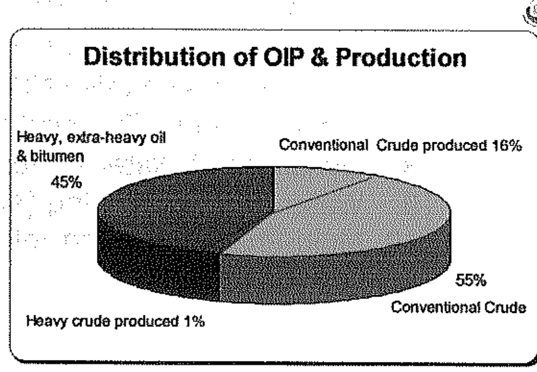

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Tertiary Recovery

- Producing the oil that remain in the part of the reservoir already swept by the displacing.
- A) Increasing the displacement efficiency
  (Part of the reservoir that was already swept in secondary recovery)
- B) Increasing the sweep efficiency
  (producing oil that remains in the part of the reservoir not swept by displacing fluid)
- C) Increasing both displacement and sweep efficiencies

Definition of EOR

EOR refers to any method used to recover more oil from a reservoir than would be produced by primary recovery

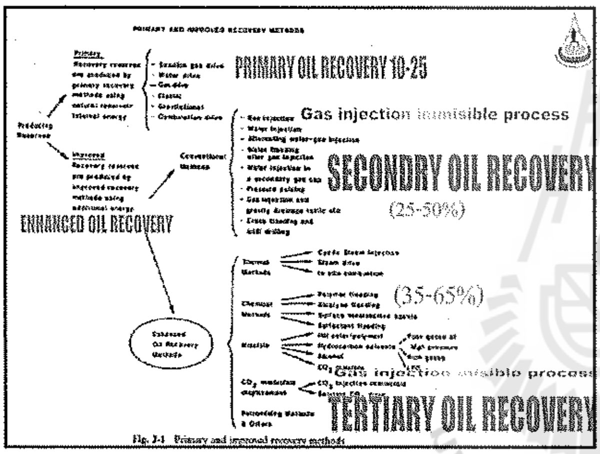

Fig. 1-1 Primary and improved recovery methods

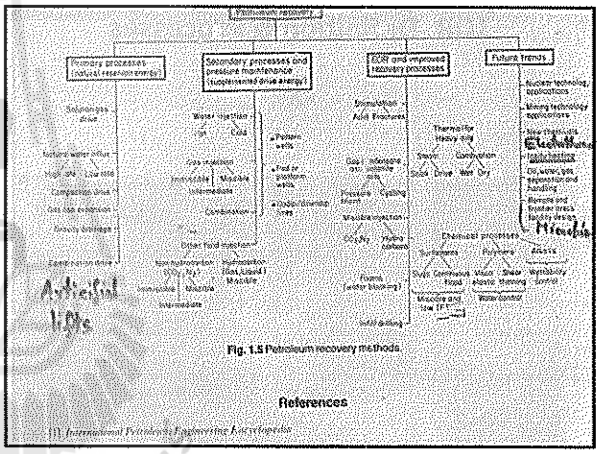

Fig. 1.5 Petroleum recovery methods.

References

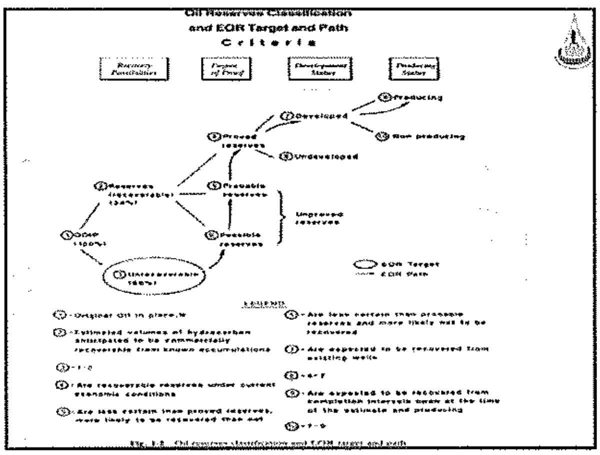

Fig. 1-2 Oil reserves classification and EOR target and path

TWO STAGES OF OIL RECOVERY

1. PRIMARY RECOVERY

2. IOR (Improved Oil Recovery)

Definition of IOR

IOR refers to any process which enhances the production or recovers more oil from a reservoir during the life of the reservoir

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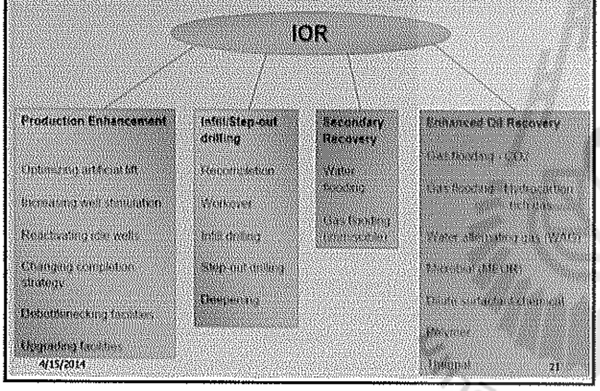
IOR Definition & Processes



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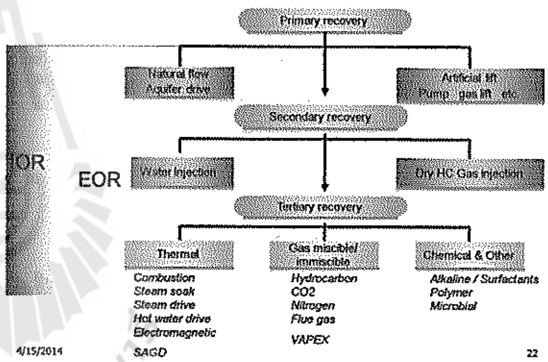
Definition of terms



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Enhanced Oil Recovery (EOR) Processes

Enhanced oil recovery (EOR) processes include all methods that use external sources of energy and/or materials to recover oil that cannot be produced, economically by conventional means.

EOR methods include:

- Water flooding
- Thermal methods: steam stimulation, steam flooding, hot water drive, and in-situ combustion
- Chemical methods:polymer,surfactant,caustic,and miscellar /polymer flooding.
- Miscible methods: hydrocarbon gas,CO2,and nitrogen (flue gas and partial miscible/immiscible gas injection may also be considered)

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Waterflood	Thermal	Chemical	Miscible gas
Maintains reservoir pressure &physically displaces oil with water moving through the reservoir from injector to producer.	Reduce Sorw by steam distillation and reduces oil viscosity.	Reduces Sorw by lowering water-oil interfacial tension, and increases volumetric sweep efficiency by reducing the water-oil mobility ratio.	Reduces Sorw by developing miscibility with the oil through a vaporizing or condensing gas drive process.

The goal of any enhanced oil recovery process is to mobilize"remaining"oil.

This is achieved by enhancing oil displacement and volumetric sweep efficiencies.

- Oil displacement efficiency is improved by reducing oil viscosity (e.g., thermal floods) or by reducing capillary forces or interfacial tension (e.g., miscible floods).
- Volumetric sweep efficiency is improved by developing a more favorable mobility ratio between the injectant and the remaining oil-in-place (e.g., polymer floods, water alternating-gas processes).

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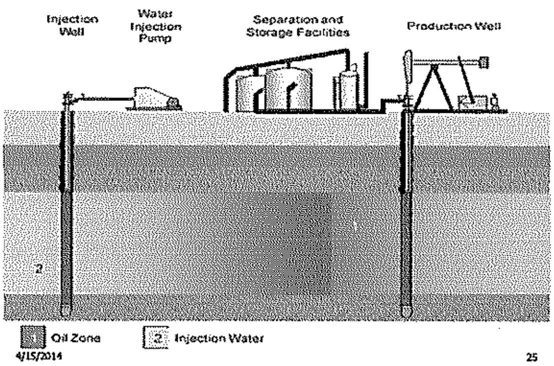
WATER FLOODING



Challenges

Compatibility between the injected water and the reservoir may cause formation damage.

Screening Parameters

Gravity >25 API Viscosity <30cp
Composition not critical Oil saturation >10% mobile oil
Formation type sandstone/carbonate Net thickness not critical Average permeability not critical Transmissibility not critical Depth not critical
Temperature not critical

Note: *Most EOR screening values are approximations based on successful north American project.*

E_A = Sweep Efficiency

TABLE 2-1. Methods of Enhanced Recovery

	Methods Used	Basic Principle
Chemical methods	Polymer-augmented waterflooding; surfactant flooding; alkaline flooding; CO_2-augmented waterflooding; immiscible CO_2 displacement.	Improvement of sweep efficiency; improvement of displacement efficiency.
Miscible methods	Miscible fluid displacement using CO_2, nitrogen, alcohol, LPG or rich gas, dry gas.	Improvement of displacement efficiency.
Thermal methods	Cyclic steam injection; steam drive; in situ combustion	Improvement of both sweep efficiency and displacement efficiency.

and is used in few large-scale projects. Alkaline flooding has been used only in those reservoirs containing specific types of high-acid-number crude oils.

Miscible methods have their greatest potential for enhanced recovery of low-viscosity oils. Among these methods, CO_2 miscible flooding on a large scale is expected to make the greatest contribution to miscible ...

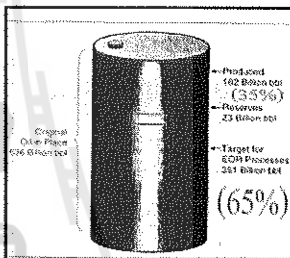


Fig. 1.1—U.S. oil barrel showing oil produced, proven reserves, and target for EOR at end of 1993 (after U.S. DOE[1]).

EOR processes often involve the injection of more than one fluid. ... a typical case, a relatively small volume of an expensive chemical (primary slug) is injected to mobilize the oil. This primary slug ... displaced with a larger volume of a relatively inexpensive chemical (secondary slug). The purpose of the secondary slug is to displace the primary slug efficiently with as little deterioration as possible of the primary slug. In some cases, additional fluids of even lower ... cost are injected after a secondary slug to reduce expenses. In such a case of multiple fluid injection, all injected fluids are considered to be part of the EOR process, even though the final chemical slug might be water or dry gas that is injected solely to displace economically the fluids injected earlier in the process.

A parallel difficulty is that the oil resource exists in reservoirs of widely varying characteristics. Oil reservoir types range from very thick carbonate reef formations at significant depths to relatively shallow, thin sandstone bodies. Subsequent chapters will show that reservoir rock type and structure have an effect on most EOR processes and are important variables. Willhite[2] describes the role of geology and its significance for displacement processes.

Finally, the saturation, distribution, and physical state of the oil in a reservoir as a result of past production operations are important factors in the implementation of an EOR process. Typically, a reservoir will undergo primary production followed by waterflooding. Recovery by these processes in individual reservoirs might have approached 35 to 50% OOIP when the waterflood reached an economic limit. The residual oil in the part of the reservoir swept by the waterflood remains largely as isolated, trapped droplets (or ganglia) in the pores or films around the rock particles, depending on the rock wettability. Residual saturation typically is about 20% to 35% in swept regions. An EOR process must be able to mobilize the droplets or oil blobs and to create an oil bank that can be efficiently propagated to producing wells.

In other circumstances, an EOR process might be applied after primary production—i.e., as a secondary recovery operation in a chronological sense. In this case, the oil may exist at a relatively high saturation, perhaps about 50 to 60%, and may still exist in a connected state with a relative permeability to oil significantly greater than zero. A gas saturation might also be present, depending on the primary recovery mechanism. In this case, the EOR process would be expected to build an oil bank, much in the manner of a waterflood. The displacement efficiency, however, must be better than for a waterflood in that only small amounts of trapped oil should be left behind. That is, the residual oil saturation following the displacement must be low relative to waterflooding because the economic attractiveness of an EOR process applied as a secondary operation normally is compared with the alternative of waterflooding—i.e., recovery (and costs) above that expected from a standard waterflood.

In some instances, EOR processes are implemented as the initial or primary production stage. The usual situation is a viscous ...

EOR CLASSIFICATION

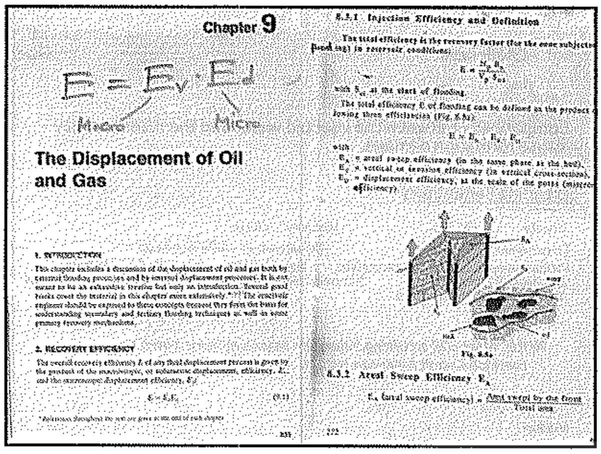
1. MOBILITY-CONTROL PROCESS
2. CHEMICAL PROCESS
3. MISCIBLE PROCESSES
4. THERMAL PROCESSES
5. OTHER PROCESSES

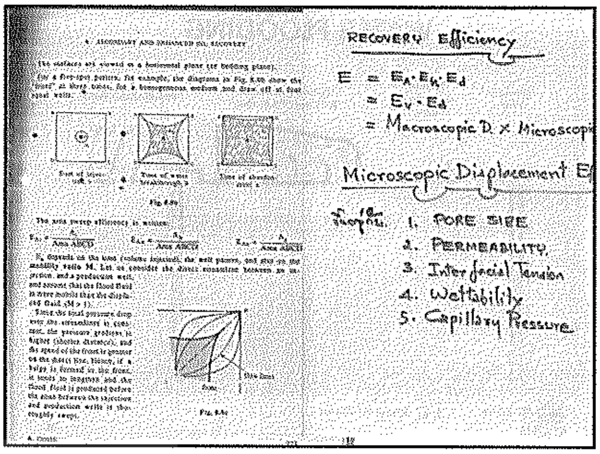
RECOVERY Efficiency

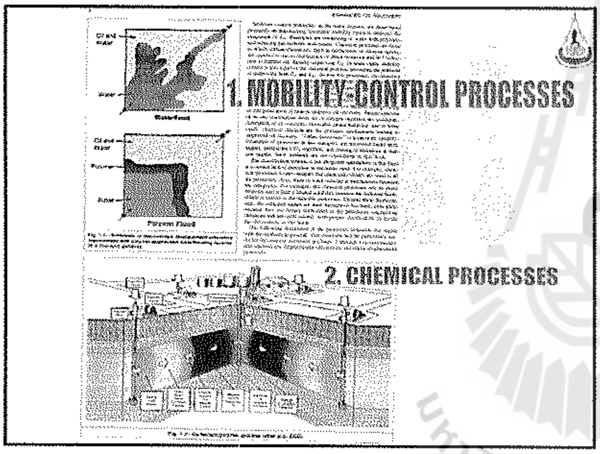
$$E = E_A \cdot E_h \cdot E_d$$
$$= E_v \cdot E_d$$
$$= \text{Macroscopic D.} \times \text{Microscopic D.}$$

Microscopic Displacement Eff.

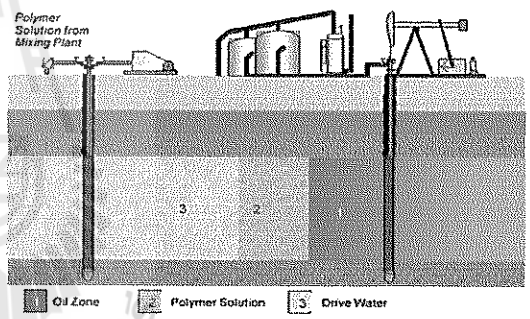
ขึ้นอยู่กับ
1. PORE SIZE
2. PERMEABILITY
3. Interfacial Tension
4. Wettability
5. Capillary Pressure







Polymer Flooding



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Description

Waterflooding consists of adding water soluble polymers to the water before it is injected into the reservoir.

Mechanisms That Improve Polymer augment Recovery Efficiency

Mobility control(improves volumetric sweep efficiency)

Limitations

- High oil viscosities require a higher polymer concentration.
- Results are normally better if the polymer flood is started before the water–oil ratio becomes excessively high.
- Clays increase polymer adsorption.
- Some heterogeneity is acceptable ,but avoid extensive fractures. if fractures are present, the crosslinked or gelled polymer techniques may be applicable.

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Challenges

Lower injectivity than with water can adversely affect oil production rates in the early stages of the polymer flood. Acrylamide-type polymers loose viscosity due to shear degradation, or it increases in salinity and divalent ions.

Screening Parameters

Gravity >18 API Viscosity <200cp
Composition Not Critical Oil saturation >10% PV mobile oil
Formation type sandstone /carbonate Net thickness not critical
Average permeability >20md Transmissibility not critical Depth <9000ft Temperature <225

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Polymers Commonly used are

Polyacrylamides & Polysaccharides

General Properties

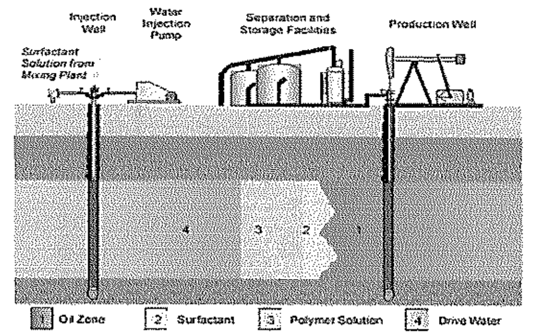
PA:

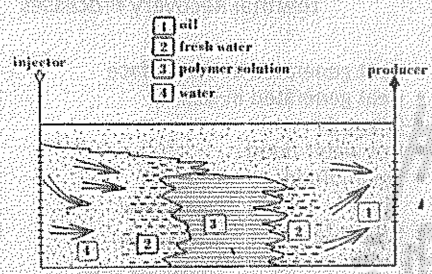
Shear thinning
Shear sensitive (degradable)
High adsorption/retention
Brine Sensitive
Cheap

PS:

Shear thinning
Less shear Sensitive
Less retention/adsorption
Less retention to brine
Sensitive to bacteria
More expensive

Surfactant/Polymer Flooding





Description

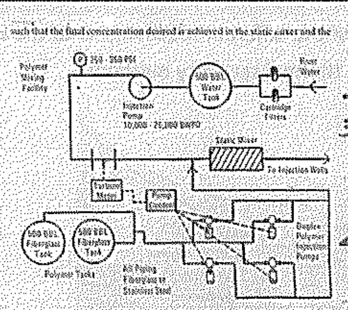
Surfactant/polymer flooding consists of injecting a slug that contains water ,surfactant, electrolyte (salt), usually a co-solvent (alcohol), and possibly a hydrocarbon (oil), followed by polymer-thickened water.

Mechanisms That Improve Recovery

- Interfacial tension reduction (improves displacement sweep efficiency)
- Mobility control

Limitations

- An areal sweep of more than 50% for waterflood is desired.
- Relatively homogeneous formation.
- High amounts of anhydrite, gypsum, or clays are undesirable.
- Available systems provide optimum behavior within a narrow set of conditions.
- Water chlorides should be <20000 ppm and divalent ions<500ppm



Chemical Method
1. Polymer
2. Alcalyne
3. Surface Tension Active
4. Surfactant

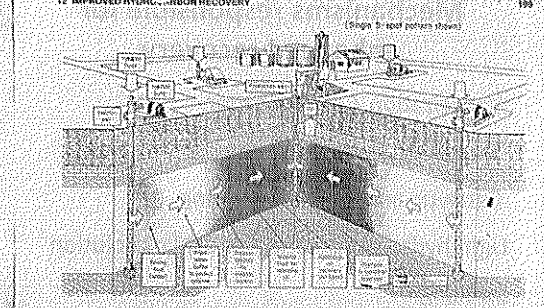


Fig. 12.10 Chemical flooding, micellar polymer

3: MISCIBLE PROCESSES

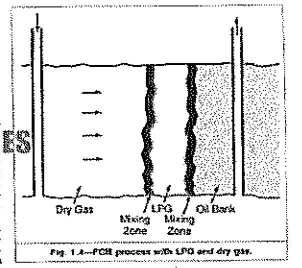
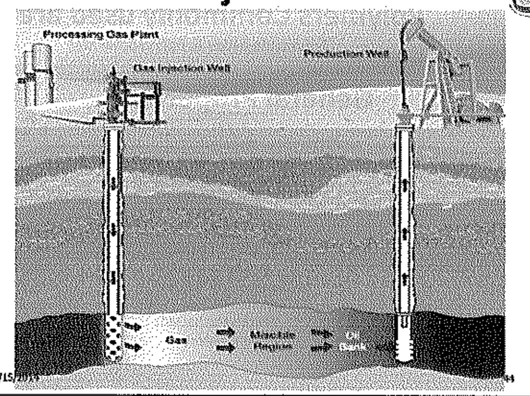


Fig. 1.4—FCM process with LPG and dry gas.

MCM(multiple-contact miscible process)

Gas Injection



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CO2 under unfavorable mobility ratio process)

WAG will be applied to overcome this difficulty

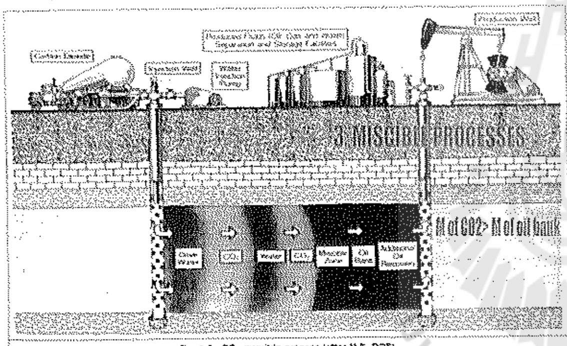

Fig. 1.6—CO2 miscible process (after U.S. DOE).

Thermal Recovery Processes

- Heat generated at the surface
- Heat generated in-situ.
- Group 1:
- Hot water flood
- Steam flood

Continuous

Huff and Puff

Steam/Cold water

- Group 2:
- In-situ combustion
- Forward (Dry or Wet)
- Reverse
- Enriched air

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Mechanisms responsible for enhanced recovery

Vaporization / condensation

Steam distillation

Catalytic and thermal cracking

Light hydrocarbon and / or CO2 dissolution

Swelling

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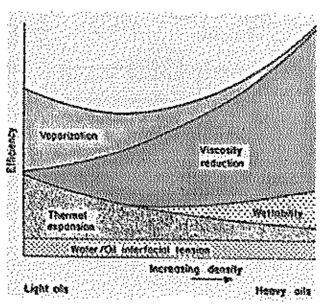
Contributions of the different mechanisms to the EOR by thermal recovery methods



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Steam Injection Modes

- Hot Water Injection
- Huff and puff
- Continuous steam injection

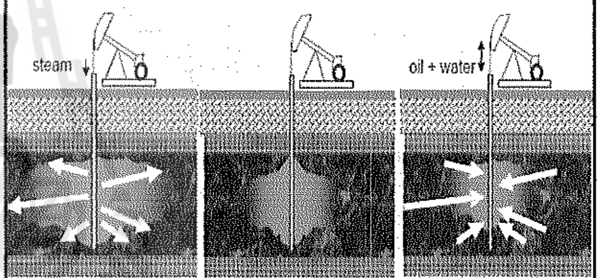
Hot Water Injection

- Same principle as Water flood
- Higher heat transport capacity

Cyclic Steam Stimulation

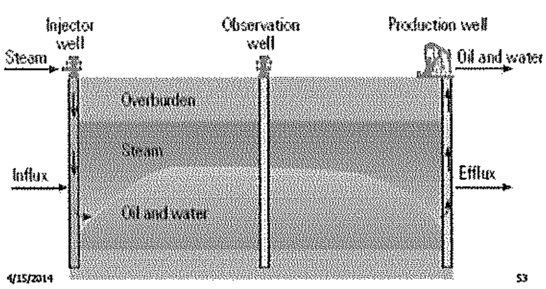
("Huff-and-Puff")
(A well-stimulation method)

HUFF-SOAK-PUFF

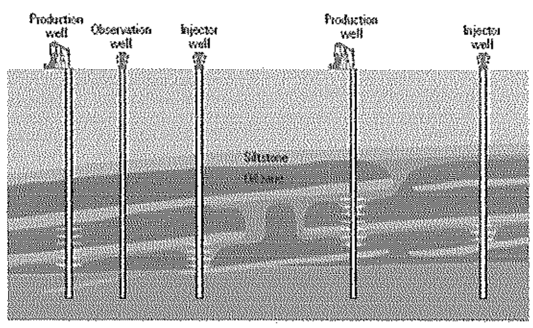


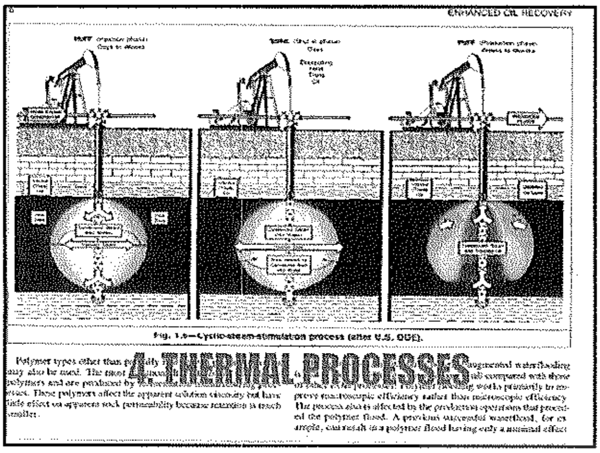
Steam Injection Process

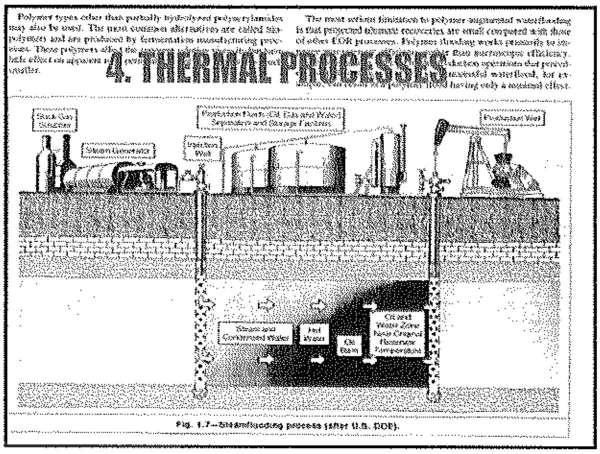
- In the flooding process, steam is injected continuously into one or more wells and oil is driven to separate production wells.

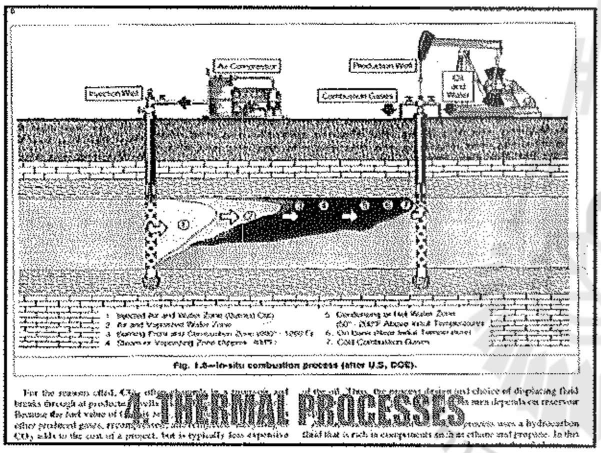


Well Configuration in Steam Injection




ENHANCED OIL RECOVERY
4. THERMAL PROCESSES
Fig. 1.6—Cyclic-steam-stimulation process (after U.S. DOE).


4. THERMAL PROCESSES
Production Well
Injection Well
Steam and Condensed Water
Hot Water
Oil Bank
Fig. 1.7—Steamflooding process (after U.S. DOE).


Air Compressor
Production Well
Injection Well
Combustion Gases
Oil and Water
4. THERMAL PROCESSES
Fig. 1.8—In-situ combustion process (after U.S. DOE).

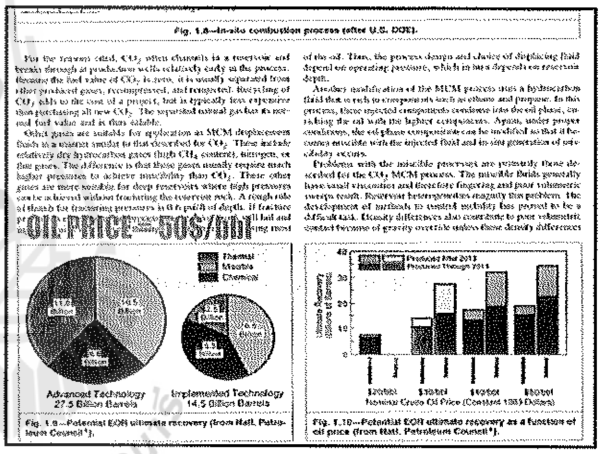
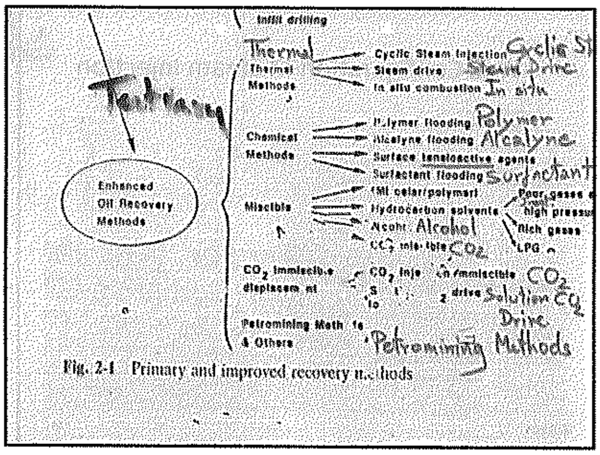
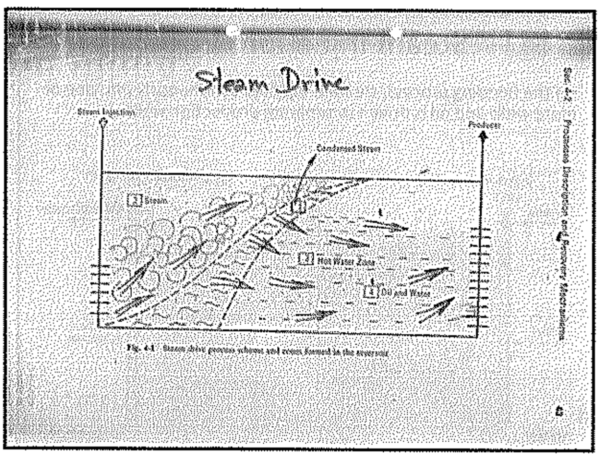
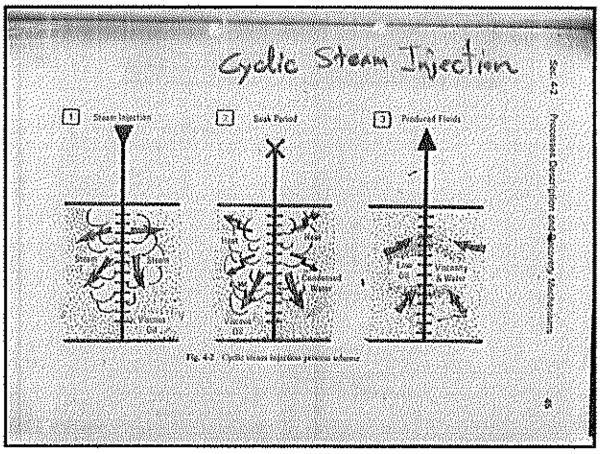
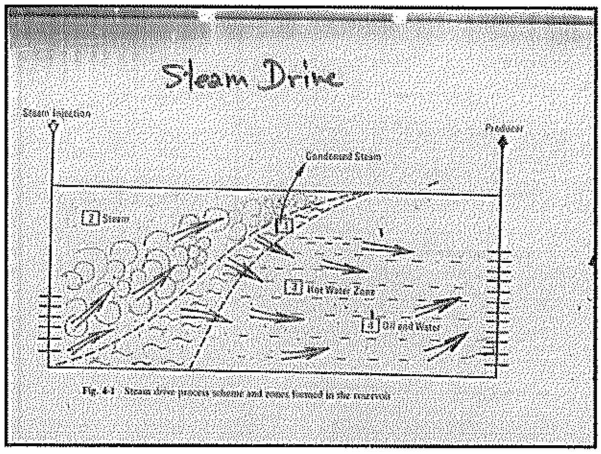


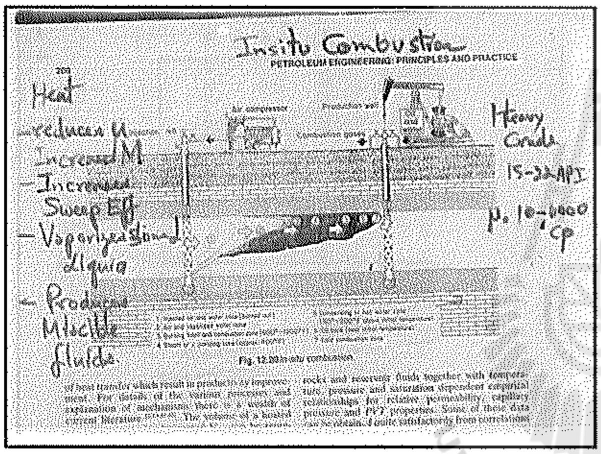
Fig. 1.8—In-situ combustion process (after U.S. DOE).
OIL PRICE = 50$/bbl
Thermal
Miscible
Chemical
Advanced Technology 27.5 Billion Barrels
Implemented Technology 14.5 Billion Barrels
Fig. 1.9—Potential EOR ultimate recovery (from Natl. Petroleum Council).
Nominal Crude Oil Price (Constant 1983 Dollars)
Fig. 1.10—Potential EOR ultimate recovery as a function of oil price (from Natl. Petroleum Council).

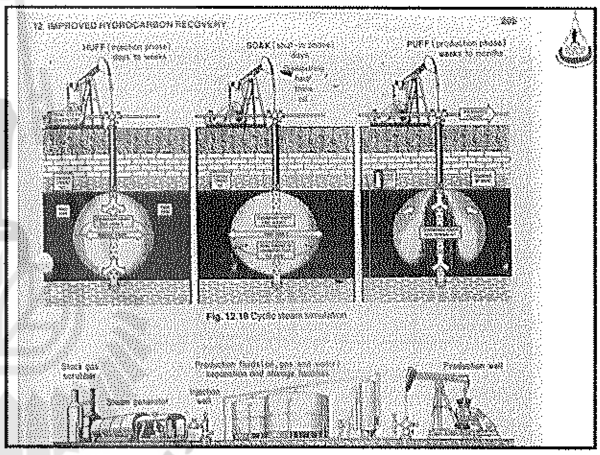

Infill drilling
Tertiary
Enhanced Oil Recovery Methods
Thermal Methods
Cyclic Steam Injection
Steam drive
In situ combustion
Chemical Methods
Polymer flooding
Alcalyne flooding
Surface tensioactive agents
Surfactant flooding
(Micelar/polymer)
Miscible
Hydrocarbon solvents
Alcohol
Poor gases at high pressure
Rich gases
LPG
CO2 immiscible displacement
Solution CO2 drive
Petromining Methods & Others
Fig. 2-1 Primary and improved recovery methods


Steam Drive
Steam Injection
Producer
Condensed Steam
Steam
Hot Water Zone
Oil and Water


Cyclic Steam Injection
1 Steam Injection
2 Soak Period
3 Produced Fluids


Steam Drive
Steam Injection
Producer
Condensed Steam
2 Steam
3 Hot Water Zone
4 Oil and Water


Insitu Combustion
PETROLEUM ENGINEERING: PRINCIPLES AND PRACTICE
Heat
- reduces μ
- Increased M
- Increased Sweep Eff
- Vaporized liquid
- Produced Miscible fluids
Heavy Crude
15-22 API
μ 10-6000 cp
Air compressor
Production well
Fig. 12.20 In-situ combustion


12 IMPROVED HYDROCARBON RECOVERY
HUFF (injection phase) days to weeks
SOAK (shut-in phase) days
PUFF (production phase) weeks to months
Fig. 12.18 Cyclic steam stimulation
Production well

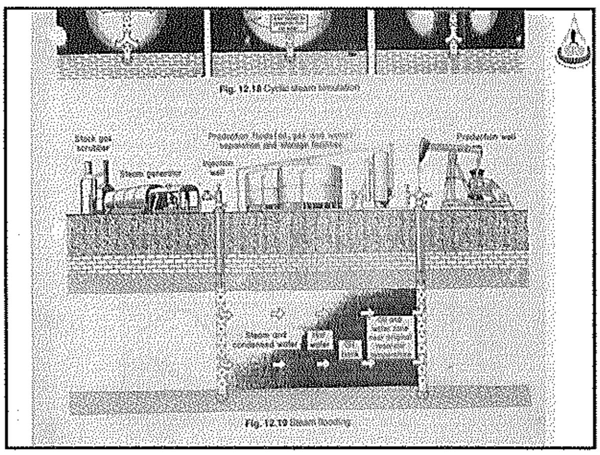
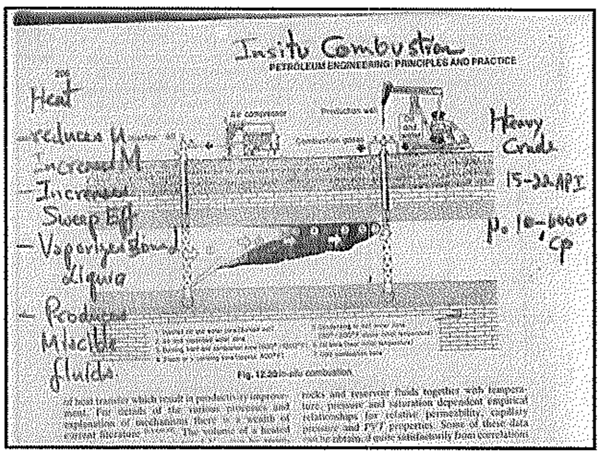


Fig. 12.18 Cyclic steam stimulation
Stack gas scrubber
Steam generator
Injection well
Production fluids (oil, gas and water) separation and storage facilities
Production well
Steam and condensed water
Fig. 12.19 Steam flooding


Insitu Combustion
PETROLEUM ENGINEERING: PRINCIPLES AND PRACTICE
Heat
- reduces μ
- Increased M
- Increased Sweep Eff
- Vaporized liquid
- Produced Miscible fluids
Heavy Crude
15-22 API
μ 10-6000 cp
Air compressor
Production well
Fig. 12.20 In-situ combustion

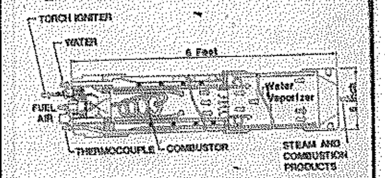


Fig. 3-4 Schematic of flow patterns for [illegible]

Reservoir Heat Loss to Adjacent Formations

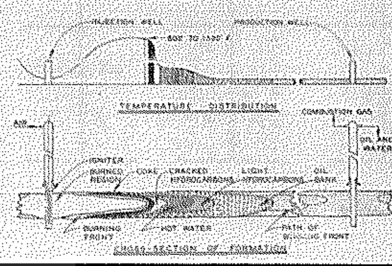
Before the steam reaches the heat front, heat is also lost to adjacent formations. When steam is injected continuously (steam drive) for a long period of time, the heated area of the reservoir rock is large. Therefore, the cumulative heat loss to adjacent formations is also large. The reservoir volume heated by a sustained steam injection project is determined by the heat lost to the base and to the cap rock and by the amount of steam required to heat the formation.

Details regarding the heat lost to adjacent strata are given in Chapter 4, where oil displacement by a steam mechanism and the evaluation of a heat injection project are presented.

3-8 THE HEAT EFFECT ON RESERVOIR OIL VISCOSITY

The mobility of a fluid in the porous space of an oil reservoir is expressed as the ratio of the effective permeability to fluid viscosity. By reducing the viscosity a higher mobility is developed and the fluid flows much more easily ... displaced through the reservoir. Oil production is also inversely

Fig. 170—Insitu combustion project, West Newport field, Orange County, Calif. From left to right are transformers, cooling tower, water storage tank, heat exchange manifold and 2,580-hp air compression installation. (Courtesy G. E. Kadane & Sons.)



5-4 WET COMBUSTION

The wet combustion process, also known as COFCAW (combination of forward combustion and waterflood) (Parish and Craig, 1969), transfers the accumulated thermal energy forward (upstream) from the combustion front. Water is the transferring agent because of its high heat capacity and its latent heat of vaporization.

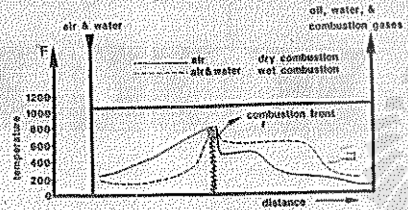


Fig. 5-5 Temperature profiles in dry and wet combustion

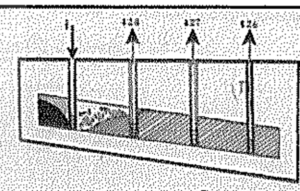


Fig. 5-18a First producer row reached by combustion front (From Carcoana, 1990)

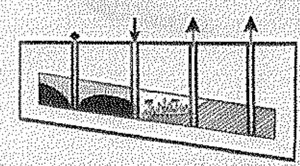


Fig. 5-18b Transfer of air injection from well I3 to well 428 (From Carcoana, 1990)

...tional effect on the flow of fluids.

In Figure 5-9a, Well 1 will produce more oil at higher rates because of a slightly isobatic difference. In Figure 5-9b, the combustion gas segregation upward is accentuated as the formation dip increases and air is injected in the higher regions. Also, the air's tendency to override the top of the formation is diminished and the oil flows downdip where it can be produced at a lower air-oil ratio and at higher rates.

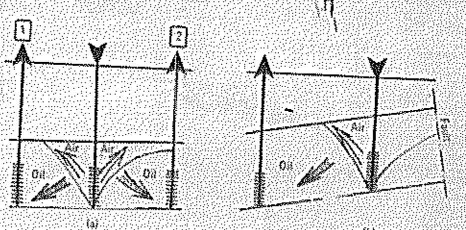


Fig. 5-9 Gravitational effect on the fluids flow in an in situ combustion process

5-5 REVERSE COMBUSTION

When oil is too viscous to flow under reservoir conditions but the reservoir has an adequate air permeability, it is possible to produce oil by reverse combustion. In this case, the combustion front moves counter to the air flow (Figure 5-6). After ignition, the well is put into production and another well is used for injection. The front moves in the same way in which a cigar is consumed—by expelling the air instead of inhaling it (Crawford, 1971). The process has limited use and was tested in a tar sand reservoir in Bellamy Field (Trantham and Marx, 1965).

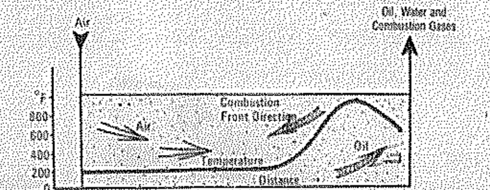
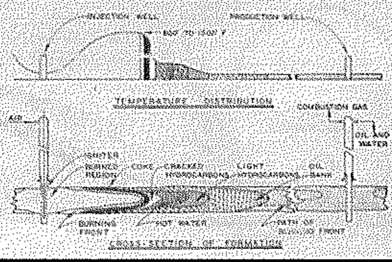


Fig. 5-6 Temperature profile in reverse combustion

Fig. 170—In-situ combustion project, West Newport field, Orange County, Calif. From left to right are transformers, cooling tower, water storage tank, heat exchange manifold and 2,500-hp air compression installation. (Courtesy G. E. Kadane & Sons.)



5-4 WET COMBUST`ON

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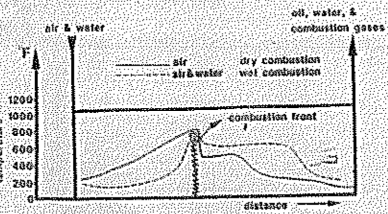


Fig. 5-5 Temperature profiles in dry and wet combustion

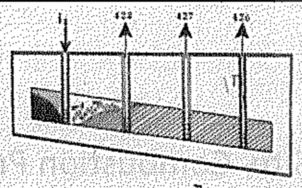


Fig. 5-18a First producer row reached by combustion front (From Carcoana, 1990)

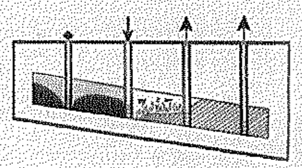


Fig. 5-18b Transfer of air injection from well 14 to well 426 (From Carcoana, 1990)

tional effect on the flow of fluids.

In Figure 5-9a, Well 1 will produce more oil at higher rates because of a slightly isobatic difference. In Figure 5-9b, the combustion gas segregation upward is accentuated as the formation dip increases and air is injected in the higher regions. Also, the air's tendency to override the top of the formation is diminished and the oil flows downdip where it can be produced at a lower air-oil ratio and at higher rates.

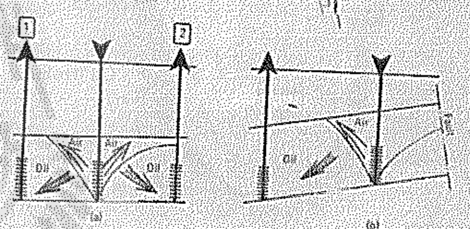


Fig. 5-9 Gravitational effect on the fluids flow in an in situ combustion process

5-5 REVERSE COMBUSTIO`I

When oil is too viscous to flow under reservoir conditions but the reservoir has an adequate air permeability, it is possible to produce oil by reverse combustion. In this case, the combustion front moves counter to the air flow (Figure 5-6). After ignition, the well is put into production and another well is used for injection. The front moves in the same way in which a cigar is consumed—by expelling the air instead of inhaling it (Crawford, 1971). The process has limited use and was tested in a tar sand reservoir in Bellamy Field (Trantham and Marx, 1965).

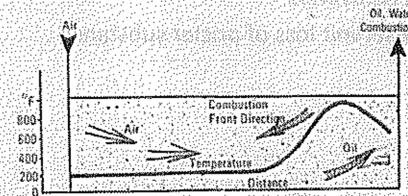
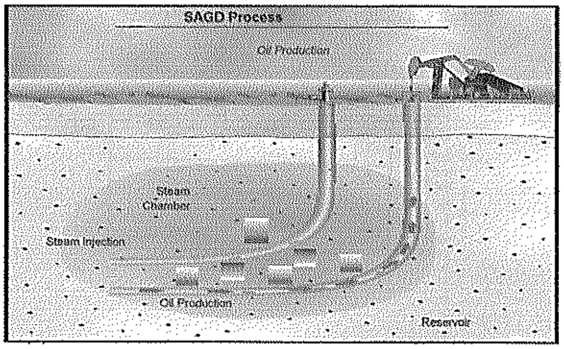


Fig. 5-6 Temperature profile in reverse combustion

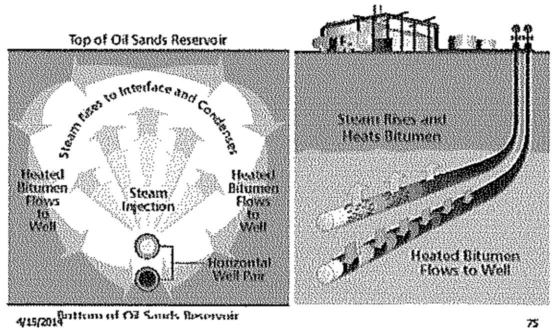
TABLE 5-2 Oil Recovery Factor

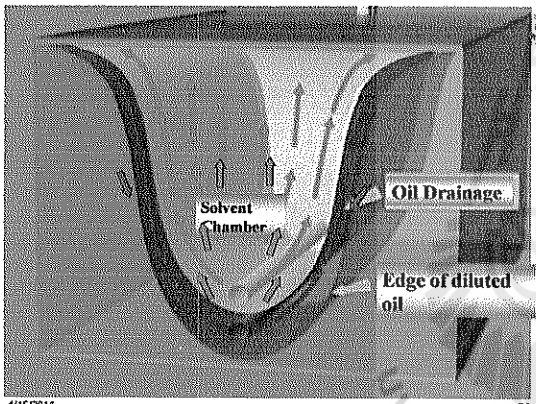
Oil in Place (MM bbl)		Oil Produced (MM bbl)		Oil Recovery: % Oil in Place at Start of Project	Oil Recovery: By …
Original	10	By primary method	0.6	0.6/10 × 100 = 6%	6% Primary
After primary (start waterflood)	9.4	By waterflood	1.1	1.1/9.4 × 100 = 11.7%	[illegible] × 100 = [illegible] water F
After waterflood (start combustion)	8.3	By in situ combustion	3.2	3.2/8.3 × 100 = 38.5%	[illegible] × 100 = [illegible] in situ
Total oil produced			4.9	E_{R} = 4.9/10 × 100 = 49%	

SAGD Process



SAGD – Going commercial after 20 years and some 30 pilot projects





In-Situ Combustion Process

Process Variations

- Dry
- Normal Wet
- Incomplete Wet
- Super wet (Quenched)
- Reverse
- Enriched Air

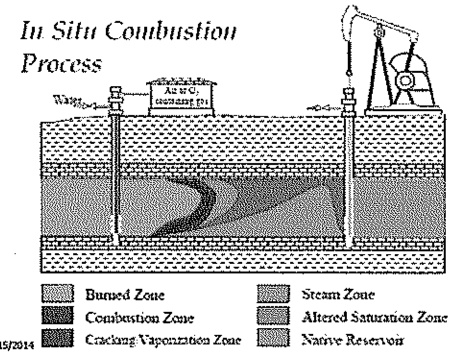
In Situ Combustion

- In theory this is great!
- minimal fuel requirement
- high recoveries
- no reservoir loss of pricier substance

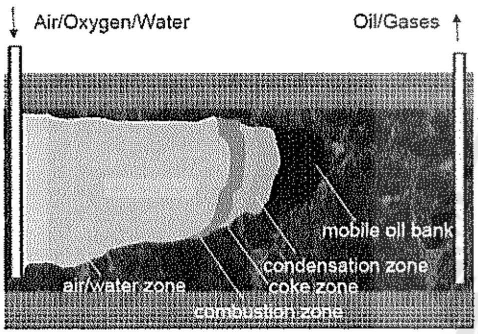
Why Should In Situ Combustion Be Considered?

- Availability of air.
- Reduced water requirement compared to steam.
- Applicable to a wide range of reservoirs and fluid characteristics.
- No theoretical pressure limitation.
- Can be applied to deep reservoirs where lifting costs make water flood unattractive.
- Can be applied as a follow-up to steam-based processes.
- Lack of obvious alternatives.

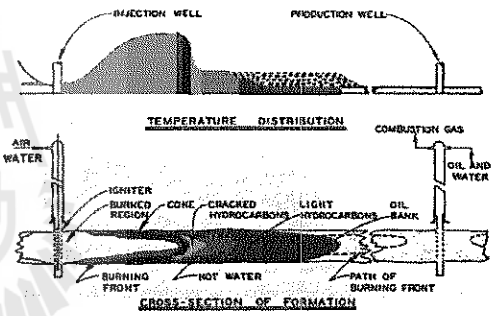
In Situ Combustion Process



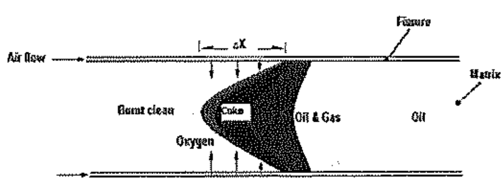
In Situ Combustion Process



In-situ Combustion Process

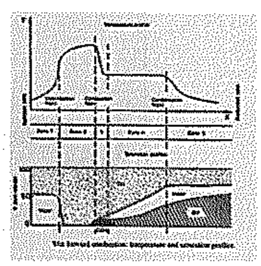


Distribution of Oil & Coke in A Typical Combustion Experiment



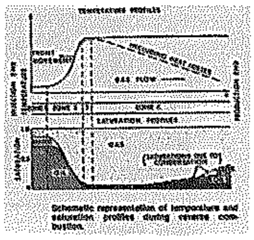
Wet Combustion

Zone 1 : swept zone- T below T_B of water
2: gas / vapor zone
3: combustion zone
4: vaporization/ condensation
5: high back pressure



Reservoir Combustion

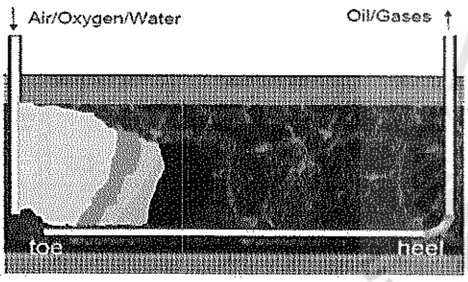
Zone 1: original formation
2: vaporization / distillation / cracking / coke formation
3: combustion zone
4: burned zone
condensation zone (if heat loss)



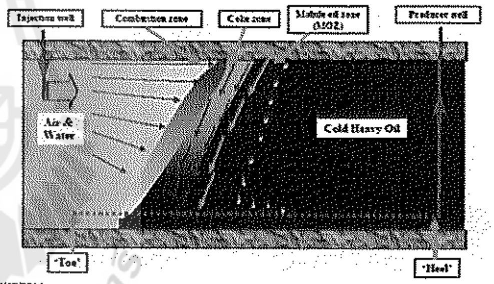
Toe-to Heel Air Injection (THAI)

- Toe-to-Heel Air Injection, or THAI, is a proposed method of recovery that combines a vertical air injection well with a horizontal production well.

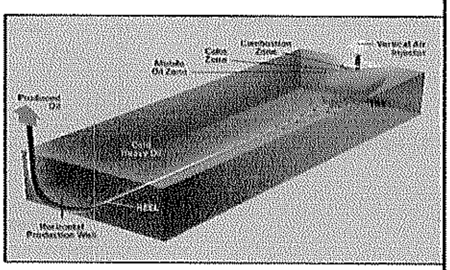
Toe-to Heel Air Injection



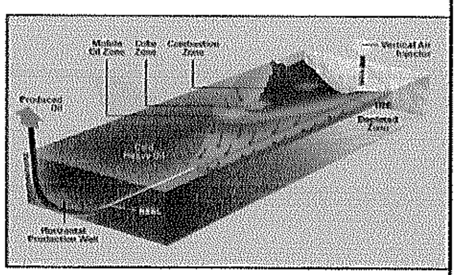
Toe-to Heel Air Injection



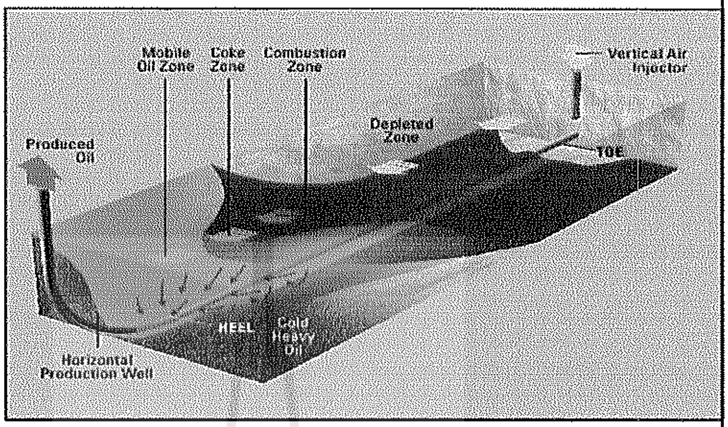
Start up:



Steady State:

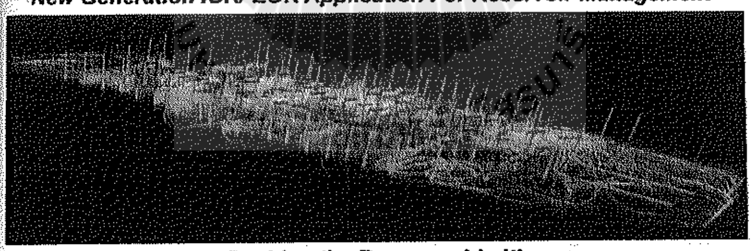


End Phase:



FlowStream Studio

New Generation IOR/ EOR Application For Reservoir Management



Pushing the Recovery Limit!