LARGE-N_c AND HEAVY-QUARK OPERATOR ANALYSIS OF 2-BODY MESON-BARYON INTERACTIONS WITH OPEN CHARM

Daris Samart

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Physics

Suranaree University of Technology

Academic Year 2011

การวิเคราะห์อันตรกิริยาระหว่างมีซอนกับแบริออนของวัตถุ 2 ก้อนที่มีชาร์มเปิดโดยใช้ตัวดำเนินการแบบ N_c ขนาดใหญ่และควาร์กหนัก

นายดริศ สามารถ

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรดุษฎีบัณฑิต

สาขาวิชาฟิสิกส์

มหาวิทยาลัยเทคโนโลยีสุรนารี

ปีการศึกษา 2554

LARGE-N_c AND HEAVY-QUARK OPERATOR ANALYSIS OF 2-BODY MESON-BARYON INTERACTIONS WITH OPEN CHARM

Suranaree University of Technology has approved this thesis submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy.

Thesis Examining Committee

(Asst. Prof. Dr. Chinorat Kobdaj)
Chairperson

(Prof. Dr. Yupeng Yan)
Member (Thesis Advisor)

(Priv. Doz. Dr. Matthias Lutz)
Member

(Dr. Kem Pumsa-ard)
Member

(Dr. Nopmanee Supanam)
Member

(Prof. Dr. Sukit Limpijumnong)
Vice Rector for Academic Affairs

(Assoc. Prof. Dr. Prapun Manyum)
Dean of Institute of Science

ดริศ สามารถ : การวิเคราะห์อันตรกิริยาระหว่างมีซอนกับแบริออนของวัตถุ 2 ก้อนที่มีชาร์มเปิดโดยใช้ตัวดำเนินการแบบ N_c ขนาดใหญ่และควาร์กหนัก (LARGE-N_c AND HEAVY-QUARK OPERATOR ANALYSIS OF 2-BODY MESON-BARYON INTERACTIONS WITH OPEN CHARM) อาจารย์ที่ปรึกษา : ศาสตราจารย์ ดร. ยูเปิ้ง แยน, 69 หน้า.

ไครัล SU(3) ลากรางเจียนที่มีซูโดสเกลาร์และเวกเตอร์ดีมีซอนกับออคเตทและดีคับเลทแบริออนได้ถูกพิจารณา เคาท์เตอร์เทอมของวัตถุสองก้อนที่เกี่ยวข้องกับสองแบริออนฟิลด์และสองดีมีซอนฟิลด์ได้ถูกสร้างขึ้นในส่วนของชาร์มเปิด โดยเทอมทั้งหมดมี 26 เทอม การรวมกันระหว่างการกระจายแบบผกผันของมวลควาร์กหนักและจำนวนของสีได้ทำให้มีกฎการรวมที่สามารถลดจำนวนของพารามิเตอร์เหลือเพียง 5 ตัวเท่านั้น ผลลัพธ์ที่ได้แสดงให้เห็นถึงความเป็นไปได้ในการคำนวณอย่างเป็นระบบของสเปกตรัมแบริออนที่มีชาร์มเปิดซึ่งอยู่บนพื้นฐานของพลวัตรแบบคับเปิลแชนแนล

สาขาวิชาฟิสิกส์ ลายมือชื่อนักศึกษา________________

ปีการศึกษา 2554 ลายมือชื่ออาจารย์ที่ปรึกษา________________

DARIS SAMART : LARGE-N_c AND HEAVY-QUARK OPERATOR ANALYSIS OF 2-BODY MESON-BARYON INTERACTIONS WITH OPEN CHARM. THESIS ADVISOR : PROF. YUPENG YAN, Ph.D. 69 PP.

CHIRAL LAGRANGIAN/LARGE-N_C LIMIT/HEAVY-QUARK SYMMETRY

The chiral SU(3) Lagrangian with pseudoscalar and vector D mesons and with the octet and decuplet baryons is considered. The leading two-body counter terms involving two baryon fields and two D meson fields are constructed in the open-charm sector. There are 26 terms. A combined expansion in the inverse of the charm quark mass and in the inverse of the number of colors provides sum rules that reduce the number of free parameters down to 5 only. Our result shows the feasibility of a systematic computation of the open-charm baryon spectrum based on coupled-channel dynamics.

School of Physics | Student's Signature ____________

Academic Year 2011 | Advisor's Signature ____________

ACKNOWLEDGEMENTS

I offer my gratitude to my supervisor, Prof. Dr. Yupeng Yan, who has supported me with his guidance and encouragement to do my thesis research at GSI with Priv. Doz. Dr. Matthias Lutz who is my co-supervisor and has supported me throughout my thesis with his great patience.

This work has been financially supported by CHE-PhD-SW-SUPV from the Commission of Higher Education, Thailand.

It is my pleasure to thank the thesis committee members Asst. Prof. Dr. Chinorat Kobdaj, Dr. Kem Pumsa-ard and Dr. Nopmanee Supanam.

I would like to thank all members of the theory group at GSI for sharing their experience and their warm welcome. I am especially grateful to Dr. Alexander Semke, who contributed to our project significantly. In addition, I would like to thank my friend Yonggoo Heo who helped me to deepen my understanding of quantum field theory.

Finally, I am grateful to my parents for emotional and moral support.

Daris Samart

CONTENTS

CONTENTS (Continued)

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

The modern theory of strong interactions is Quantum Chromodynamics (QCD), the quantum field theory (QFT) of quarks and gluons based on the non-abelian gauge group SU(3). Combined with the SU(2)×U(1) electroweak theory, QCD is part of the Standard Model of particle physics. QCD is well tested at high energies, where the strong coupling constant becomes small and perturbation theory applies. But, in the low-energy regime, QCD is a strongly-coupled theory with many aspects poorly understood. The thriving questions are: How can we bring order into the rich phenomena of low energy QCD? Are there effective degrees of freedom in terms of which we can understand the resonances and bound states of QCD in an efficient and systematic way? Does QCD generate exotic structures so far undiscovered?

FAIR at GSI with $\overline{P}ANDA$ (Anti<u>P</u>roton <u>A</u>nnihilation at <u>Da</u>rmstadt) will be in a promising position to provide answers to such important questions about non-perturbative QCD (Lutz et al., 2009). A major part of the physics program of $\overline{P}ANDA$ is designed to collect high-quality data that allow to scrutinize various concepts and approaches to non-perturbative QCD.

The spectrum of open-charm baryons is so far poorly understood. Recently the attention has turned towards studying resonances with charm degrees of freedom, motivated by the discovery of quite a few new and unexpected states, as reported by the CLEO, Belle and BaBar Collaborations (Artuso et al., 2001).

The main objective of the present study is to pave the way for systematic coupled-channel computations on open-charm baryon resonances. A first application of the chiral Lagrangian to s-wave baryon resonances considered the coupled-channel interaction of the Goldstone bosons with the ground-state baryons with open charm (Lutz and Kolomeitsev, 2004). A rich spectrum of chiral excitations was obtained. Such results are similar to findings on s-wave scattering of Goldstone bosons off the baryon octet and decuplet states [see e.g. (Kolomeitsev and Lutz, 2004a; Kaiser, Siegel and Weise, 1995; Oller and Meissner, 2001)]. Here the coupled-channel dynamics based on the leading order chiral Lagrangian generates s- and d-wave baryon resonances. Also, the chiral SU(3) Lagrangian with pseudoscalar and vector D-meson has been applied extensively in the literature [see e.g. (Georgi, 1990; Wise, 1992; Casalbuoni, Deandrea, Di Bartolomeo, Gatto, Feruglio and Nardulli, 1997; Mehen and Springer, 2004)]. Most exciting are recent studies on s-wave scattering of Goldstone boson off D-mesons (Hofmann and Lutz, 2004; Lutz and Soyeur, 2008; Guo, Hanhart, Krewald, and Meissner, 2008; Kolomeitsev and Lutz, 2004b). The leading order coupled-channel interaction leads to the formation of scalar and axial-vector D-mesons with properties compatible with the known empirical constraints.

The challenge of the open-charm baryon sector is the possibility of charm-exchange reactions, where the charm of the baryon is transferred to the meson. Thus a complete description requires the consideration of the interaction of D-mesons with the baryon octet and decuplet states. A first phenomenological case study modeled the coupled-channel force by a t-channel exchange of vector mesons in the static limit (Hofmann and Lutz, 2005; Hofmann and Lutz, 2006; Tolos, Schaffner-Bielich and Mishra, 2004). Such a t-channel force recovers the leading order predictions of chiral symmetry whenever Goldstone bosons are involved. It was shown in (Hofmann and Lutz, 2005; Hofmann and Lutz, 2006; Jimenez-Tejero, Ramos and Vidana, 2009) that a rich spectrum of s- and d-wave baryon resonances is generated dynamically based on such a schematic ansatz. Two types of resonances are generated. The first are formed predominantly by the interaction of the Goldstone bosons with the open-charm baryon ground states, and the second are a consequence of the coupled-channel interaction of the D-meson with baryon octet and decuplet baryons (Tolos et al., 2004; Guo et al., 2008; Jimenez-Tejero et al., 2009). For both type of resonances the exchange of the light vector mesons constitute the driving forces that generate the hadronic molecules. The exchange of charmed vector mesons lead to the typically small widths of the second type. While the interaction of Goldstone bosons with any hadron is dictated by chiral symmetry at the leading order, this is not true for the interaction of D-mesons. At leading order D-mesons interact with baryons via local counter terms that are undetermined by chiral symmetry. This resembles the situation encountered in chiral studies of the nucleon-nucleon force [see e.g. (Epelbaum, Hammer and Meissner, 2008)]. Only the long-range part of the interaction is controlled by chiral interactions, the short range part needs to be parameterized in terms of a priori unknown counter terms. Needless to state that a reliable coupled-channel computation requires the consideration of both short-range and long-range forces. The purpose of the present work is a systematic construction of the leading order counter terms for the interaction of the D-mesons with the baryon octet and decuplet states. Though chiral symmetry is not constraining such counter terms there are significant correlations amongst the counter terms implied by the heavy-quark symmetry and large-N_c QCD. In the limit of a large charm-quark mass the pseudoscalar and vector D-mesons form a spin multiplet, with degenerate properties (Georgi, 1990). Thus a systematic coupled-channel computation requires the simultaneous consideration of pseudo-scalar and vector D-mesons. This leads necessarily to the presence of long-range t-channel forces. The transition of a vector D-meson to a pseudo-scalar D-meson involves a Goldstone boson. A first attempt to consider pseudo-scalar and vector D-mesons on equal footing, however, assuming zero range forces only, can be found in (Gamermann et al., 2010). Similarly, in the limit of a large number of colors in QCD the baryon octet and decuplet states form a super multiplet (Gervais and Sakita, 1984). This asks for the simultaneous consideration of the octet and decuplet baryons.

In a recent study the implications of large-N_c QCD on the local counter terms of the Goldstone boson interaction with the baryon octet and decuplet states was worked out (Lutz and Semke, 2011). The technology developed in Luty and March-Russell (1994); Dashen, Jenkins and Manohar (1995); Lutz and Semke (2011) was applied. Matrix elements of current-current correlation functions were evaluated in the baryon octet and decuplet states. The latter was expanded in powers of $1/N_c$ applying the operator reduction rules of (Luty and March-Russell, 1994). This technology is well suited for an application to the open-charm sector. The implications of heavy-quark symmetry on the counter terms can be worked out using a suitable multiplet representation of the pseudoscalar and vector D-mesons (Wise, 1992; Casalbuoni et al., 1997; Mehen and Springer, 2004).

CHAPTER II

SYMMETRIES OF QCD

Quantum Chromodynamics (QCD) is the gauge theory of strong interactions, which is based on an exact SU(3) color symmetry. The fundamental matter and gauge fields are so-called "Quarks" and "Gluons", respectively. Quarks are particles with spin $\frac{1}{2}$ and gluons are massless gauge particles with spin 1. The full QCD Lagrangian is given by

$$\mathscr{L}_{QCD} = \sum_f \bar{q}_f^j \left(i\,\gamma^\mu\, D_\mu^{jk} - m_f\, \delta^{jk} \right) q_f^k - \frac{1}{4}\, G_{\mu\nu}^{(a)}\, G^{(a),\,\mu\nu}\,, \tag{2.1}$$

where q_f are the quark fields with flavors $f = u, d, s, c, b, t$ and m_f are the quark masses. We use Greek indices $(\mu, \nu, \cdots = 0, 1, 2, 3)$ for Lorentz spacetime indices, Latin indices $(a, b, c, \cdots = 1, \cdots, 8)$ for SU(3) color adjoint indices and Latin indices $(i, j, k, \cdots = 1, 2, 3)$ for SU(3) color indices in the fundamental representation. The quark fields form a color triplet as

$$q_f = \begin{pmatrix} q^1 \\ q^2 \\ q^3 \end{pmatrix}_f = \begin{pmatrix} q^R \\ q^G \\ q^B \end{pmatrix}_f . \tag{2.2}$$

One may label the quarks by using the colors (red, green, blue) instead of numbers as $R \to 1$, $G \to 2$ and $B \to 3$. The covariant derivative D_μ is defined by

$$D_\mu = \partial_\mu - i\, g_s\, A_\mu^{(a)}\, T^{(a)}\,, \tag{2.3}$$

where $A_\mu^{(a)}$ are the eight gluon fields, g_s is the strong interaction coupling constant and $T^{(a)} = \frac{1}{2}\,\lambda^{(a)}$ are the generators of SU(3) color group with $\lambda^{(a)}$ being the

Gell-Mann's matrices. The strength of the gauge fields $G^{(a)}_{\mu\nu}$ is defined as

$$G^{(a)}_{\mu\nu} = D_\mu\, A^{(a)}_\nu - D_\nu\, A^{(a)}_\mu = \partial_\mu\, A^{(a)}_\nu - \partial_\nu\, A^{(a)}_\mu + g_s\, f^{abc}\, A^{(b)}_\mu\, A^{(c)}_\nu\,, \tag{2.4}$$

where $f^{abc} = -f^{bac}$ are structure constants of the SU(3) color group.

The QCD Lagrangian is invariant under local SU(3) gauge symmetry transformation, which we parameterize by the functions $\Theta^{(a)}(x)$. The dynamical variables are transformed as

$$\begin{aligned}
&q_f \;\to\; U[\Theta(x)]\, q_f = e^{-\,i\,\Theta^{(a)}(x)\,T^{(a)}}\, q_f \\
&D_\mu\, q_f \;\to\; U[\Theta(x)]\, D_\mu\, q_f \\
&A_\mu \;\to\; U[\Theta(x)]\, A^{(a)}_\mu\, T^{(a)}\, U^{-1}[\Theta(x)] - \frac{i}{g_s}\,(\partial_\mu\, U[\Theta(x)])\, U^{-1}[\Theta(x)] \\
&G_{\mu\nu} \;\to\; U[\Theta(x)]\, G^{(a)}_{\mu\nu}\, T^{(a)}\, U^{-1}[\Theta(x)]\,.
\end{aligned} \tag{2.5}$$

In QCD not only quarks carry color charges but also gluons carry color charges. This is typical for a theory based on a non-abelian gauge symmetry. We can directly see from the QCD Lagrangian that there are vertices with three and four gluon fields.

The QCD Lagrangian is invariant under a number of additional transformations

- Lorentz transformations

$$S(\Lambda)\, \mathscr{L}(x')_{QCD}\, S(\Lambda)^{-1} = \mathscr{L}(\Lambda\, x)_{QCD}\,, \tag{2.6}$$

  where $x' = \Lambda\, x$ and Λ is a Lorentz transformation matrix.

- Parity and charge conjugation transformations

$$\begin{aligned}
\mathscr{P}\, \mathscr{L}(x)_{QCD}\, \mathscr{P}^{-1} &= \mathscr{L}(\tilde{x})_{QCD}\,, \\
\mathscr{C}\, \mathscr{L}(x)_{QCD}\, \mathscr{C}^{-1} &= \mathscr{L}(x)_{QCD}\,,
\end{aligned} \tag{2.7}$$

where $\tilde{x} = \left(x^0 , -\mathbf{x}\right)^t$ with t standing for the transpose, $\mathscr{P}$ is the parity transformation operator and $\mathscr{C}$ is the charge conjugation transformation operator.

Note that any local quantum field theory is invariant under successive applications of charge conjugation, parity and time reversal transformations. Thus, there is no need to discuss time-reversal transformations explicitly.

We have briefly surveyed general properties of the QCD Lagrangian in this section. In the next section, we will discuss additional symmetries of the QCD Lagrangian in the limit of massless quarks.

2.1 Chiral symmetry

Quarks can be grouped into light and heavy flavors according to their masses: the u, d, s quarks are much lighter than the c, b, t quarks (Nakamura and Group, 2010) as shown below

$$\begin{pmatrix} m_u = 0.005\,\mathrm{GeV} \\ m_d = 0.009\,\mathrm{GeV} \\ m_s = 0.175\,\mathrm{GeV} \end{pmatrix} \ll 1\,\mathrm{GeV} \le \begin{pmatrix} m_c = (1.15 - 1.35)\,\mathrm{GeV} \\ m_b = (4.0 - 4.4)\,\mathrm{GeV} \\ m_t = 174\,\mathrm{GeV} \end{pmatrix}, \tag{2.8}$$

The masses of the three light quarks are smaller than the typical hadronic scale as

$$m_{u,d,s} \ll 1 \text{ GeV} \approx \Lambda_{\text{hadronic}} \ . \tag{2.9}$$

Therefore, it is reasonable to approximate QCD by its limit of massless light quarks (i.e. $m_u = m_d = m_s = 0$), the so-called chiral limit. Theoretical results may be obtained by treating the light quark masses in perturbation theory. On the other hand, the c, b, t quarks can be treated as infinitely heavy at low energies. The only active degrees of freedom are those associated with the light u, d, s quarks.

In the chiral limit, the QCD Lagrangian takes the form

$$\mathscr{L}_{\rm QCD}^{(0)} = \sum_{f=u,d,s} i\,\bar{q}_f\,\gamma^\mu\,D_\mu\,q_f - \frac{1}{4}\,G_{\mu\nu}^{(a)}\,G^{(a),\,\mu\nu}\,, \tag{2.10}$$

where q_f represents the u, d and s quark fields. Now we are ready to explore additional symmetries of the QCD Lagrangian. For this purpose it is useful to introduce right- and left-handed quark fields

$$q_R = P_R\,q\,, \quad q_L = P_L\,q\,, \tag{2.11}$$

where the projection matrices P_R and P_L are defined by

$$P_R = \frac{1}{2}\,(1+\gamma_5)\,, \quad P_L = \frac{1}{2}\,(1-\gamma_5)\,. \tag{2.12}$$

The P_R and P_L matrices have the properties,

$$\begin{aligned} P_R + P_L &= 1\,, \\ P_R^2 = P_R\,, \quad P_L^2 &= P_L\,, \\ P_R\,P_L = P_L\,P_R &= 0\,. \end{aligned} \tag{2.13}$$

In application of (2.13) the QCD Lagrangian (2.10) takes the following form

$$\mathscr{L}_{\rm QCD}^{(0)} = \sum_{q=u,d,s} (i\,\bar{q}_L\,\gamma^\mu\,D_\mu\,q_L + i\,\bar{q}_R\,\gamma^\mu\,D_\mu\,q_R) - \frac{1}{4}\,G_{\mu\nu}^{(a)}\,G^{(a),\,\mu\nu}\,, \tag{2.14}$$

where we note the absence of terms involving right- and left-handed fields simultaneously. From this observation we deduce the presence a $\mathrm{SU}(3)_R \otimes \mathrm{SU}(3)_L \otimes \mathrm{U}(1)_R \otimes \mathrm{U}(1)_L$ symmetry. The right- and left-handed quark fields transform separately as follows

$$\begin{aligned} q_R &= \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix} \longrightarrow e^{-i\left(\Theta_R^{(a)}T^{(a)}+\Theta_R\right)} \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}, \\ q_L &= \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \longrightarrow e^{-i\left(\Theta_L^{(a)}T^{(a)}+\Theta_L\right)} \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix}, \end{aligned} \tag{2.15}$$

with the various group elements generated by flavor octet $\Theta^{(a)}_{R,L}$ and singlet $\Theta_{R,L}$ parameters. Here $T^{(a)} = \frac{1}{2}\,\lambda^{(a)}$ are the generators of SU(3) flavor group with $\lambda^{(a)}$ being Gell-Mann's matrices. In the limit of massless quarks QCD has a so-called chiral symmetry.

According to Noether's theorem, in the chiral limit we obtain 18 conserved currents from the QCD Lagrangian, which are given by

$$\begin{aligned} R^{(a)}_\mu &= \sum_{q\,=\,u,d,s} \bar{q}_R\,\gamma_\mu\,T^{(a)}\,q_R\,, \qquad & L^{(a)}_\mu &= \sum_{q\,=\,u,d,s} \bar{q}_L\,\gamma_\mu\,T^{(a)}\,q_L\,, \\ R_\mu &= \sum_{q\,=\,u,d,s} \bar{q}_R\,\gamma_\mu\,q_R\,, & L_\mu &= \sum_{q\,=\,u,d,s} \bar{q}_L\,\gamma_\mu\,q_L\,. \end{aligned} \tag{2.16}$$

For convenience, we may identify vector and axial-vector currents, V_μ and A_μ, with conventional properties under parity transformations. We find

$$\begin{aligned} V^{(a)}_\mu &= L^{(a)}_\mu + R^{(a)}_\mu = \bar{q}\,\gamma_\mu\,T^{(a)}\,q\,, \quad & A^{(a)}_\mu &= L^{(a)}_\mu - R^{(a)}_\mu = \bar{q}\,\gamma_5\,\gamma_\mu\,T^{(a)}\,q\,, \\ V_\mu &= L_\mu + R_\mu = \bar{q}\,\gamma_\mu\,q\,, & A_\mu &= L_\mu - R_\mu = \bar{q}\,\gamma_5\,\gamma_\mu\,q\,. \end{aligned} \tag{2.17}$$

The conservation of the currents implies the identities

$$\partial^\mu\,V^{(a)}_\mu = \partial^\mu\,A^{(a)}_\mu = \partial^\mu\,V_\mu = 0\,, \tag{2.18}$$

where we note the exceptional behavior of the singlet axial-vector current

$$\partial_\mu\,A^\mu = \frac{3\,g_s^2}{32\,\pi^2}\,\epsilon_{\mu\nu\rho\sigma}\,G^{(a),\,\mu\nu}\,G^{(a),\,\rho\sigma}\,, \tag{2.19}$$

which is broken by the so-called Adler-Bell-Jackiw anomaly (Adler, 1969; Bell and Jackiw, 1969; Itzykson and Zuber, 1980; Peskin and Schroeder, 1995). Each conserved current implies a conserved charge as given by

$$\begin{aligned} Q^{(a)}_V &= \int d^3\mathbf{x}\,V^{(a)}_0(t,\mathbf{x})\,, \\ Q^{(a)}_A &= \int d^3\mathbf{x}\,A^{(a)}_0(t,\mathbf{x})\,, \\ Q_V &= \int d^3\mathbf{x}\,V_0(t,\mathbf{x})\,. \end{aligned} \tag{2.20}$$

The Noether's charge operators commute with the massless QCD Hamiltonian $\mathscr{H}_{\mathrm{QCD}}$, i.e. it holds

$$[Q_V^{(a)}, \mathscr{H}_{\mathrm{QCD}}] = [Q_A^{(a)}, \mathscr{H}_{\mathrm{QCD}}] = [Q_V, \mathscr{H}_{\mathrm{QCD}}] = 0\,. \tag{2.21}$$

The $\mathrm{U}(1)_V$ symmetry in QCD is associated with the baryon-number conservation. Mesons and baryons are assigned the baryon-quantum numbers $B = 0$ and $B = 1$, respectively. This symmetry is analogous to the U(1) symmetry in QED, which leads to the conservation of the electrical charge.

The group $\mathrm{SU}(3)_V \otimes \mathrm{SU}(3)_A$ is isomorph to the group $\mathrm{SU}(3)_R \otimes \mathrm{SU}(3)_L$, that is,

$$\mathrm{SU}(3)_R \otimes \mathrm{SU}(3)_L = \mathrm{SU}(3)_V \otimes \mathrm{SU}(3)_A\,. \tag{2.22}$$

In principle, the conservation of the charge operators $Q_V^{(a)}$ and $Q_A^{(a)}$ implies the existence of chiral doublets in the spectrum of QCD, that is, there should exist two particles which have the same mass and spin but opposite parities. However, such a pattern is not observed in the hadronic spectrum (Nakamura and Group, 2010). For instance, the observed ground states of baryons with positive parity are classified by an approximate $\mathrm{SU}(3)_V$ symmetry. A corresponding symmetry pattern is not observed for baryons with negative parity. In other words, the ground state of QCD is not invariant under $\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L$ transformations. This implies

$$Q_A^{(a)}\,|\,0\rangle \neq 0\,, \tag{2.23}$$

where $|\,0\rangle$ is the vacuum state. Thus the chiral symmetry is spontaneously broken down to the $\mathrm{SU}(3)_V$ subgroup as

$$\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L \longrightarrow \mathrm{SU}(3)_V\,. \tag{2.24}$$

According to Goldstone's theorem (Nambu, 1960; Goldstone, 1961), there exists one massless boson for each charge operator $Q_A^{(a)}$ which does not annihilate the vacuum ground state. The Goldstone boson carries the quantum numbers of the charge operator $Q_A^{(a)}$. The pseudoscalar octet mesons $\pi^{\pm}$, π^0, $K^{\pm}$, K^0, $\bar{K}^0$, η have small masses compared to the typical hadronic scale, and hence one identifies them with the pseudo-Goldstone bosons of QCD. In the limit of massless quarks we expect the masses of the pseudoscalar mesons $\pi^{\pm}$, π^0, $K^{\pm}$, K^0 and η to vanish.

2.2 Green's function of QCD at low energy

In any field theory, the crucial objects are the Green's functions, which are the vacuum expectation value of time-ordered products of field operators. From the Green's functions physical transition amplitudes can be extracted. In this section, we will discuss a systematic method to extract such quantities from QCD.

We use the external source-field method following Gasser and Leutwyler (1984; 1985). Auxiliary c-number fields are coupled to the quark currents of QCD. This leads to an extended QCD Lagrangian of the from

$$\begin{aligned}\mathscr{L}_{QCD}\big(\bar{q},\, q,\, G_\mu\,;\, \mathcal{V},\, \mathcal{A},\, \mathcal{S},\, \mathcal{P}\big) &= \mathscr{L}_{QCD}^{(0)} + \bar{q}\,\gamma^\mu\,\mathcal{V}_\mu\, q \\ &+ \bar{q}\,\gamma^\mu\,\gamma_5\,\mathcal{A}_\mu\, q - \bar{q}\,\mathcal{S}\, q + \bar{q}\, i\,\gamma_5\,\mathcal{P}\, q\,, \end{aligned} \tag{2.25}$$

where $\mathscr{L}_{QCD}^{(0)}$ was introduced already in (2.10). The external source fields transform under parity transformations as a vector ($\mathcal{V}_\mu$), an axial vector ($\mathcal{A}$), a scalar ($\mathcal{S}$) and a pseudo-scalar ($\mathcal{P}$). The original QCD Lagrangian of the light quarks is recovered by setting $\mathcal{V}_\mu = \mathcal{A}_\mu = \mathcal{P} = 0$ and $\mathcal{S} = \mathcal{M}$ where $\mathcal{M}$ is light-quark mass matrix. The flavor structure of the external source fields is given by

$$\begin{aligned}\mathcal{V}_\mu &= \sum_{a=1}^{8} \mathcal{V}_\mu^{(a)}\, T^{(a)}\,, \qquad\qquad \mathcal{A}_\mu = \sum_{a=1}^{8} \mathcal{A}_\mu^{(a)}\, T^{(a)}\,, \\ \mathcal{S} &= \mathcal{S}^{(0)}\mathbb{1} + \sum_{a=1}^{8} \mathcal{S}^{(a)}\, T^{(a)}\,, \quad \mathcal{P} = \mathcal{P}^{(0)}\mathbb{1} + \sum_{a=1}^{8} \mathcal{P}^{(a)}\, T^{(a)}\,. \end{aligned} \tag{2.26}$$

where $\mathbb{1}$ is the 3-dimensional identity matrix and $T^{(a)} = \frac{1}{2}\,\lambda^a$ are the SU(3) generators.

The generating functional of QCD is given by (Gasser and Leutwyler, 1984; Gasser and Leutwyler, 1985)

$$
\begin{aligned}
e^{i\,Z[\mathcal{V},\mathcal{A},\mathcal{S},\mathcal{P}]} &= \int \mathcal{D}\,G_\mu\,\mathcal{D}\,\bar{q}\,\mathcal{D}\,q\,e^{i\int d^4x\,\mathscr{L}_{\rm QCD}\left(\bar{q},q,G_\mu\,;\,\mathcal{V},\mathcal{A},\mathcal{S},\mathcal{P}\right)} \\
&= \langle\,0\,|\,T\,e^{i\int d^4x\,(\bar{q}\,\gamma^\mu\,\mathcal{V}_\mu\,q+\bar{q}\,\gamma_5\,\gamma^\mu\,\mathcal{A}_\mu\,q+\bar{q}\,\mathcal{S}\,q+\bar{q}\,\gamma_5\,\mathcal{P})\,q}\,|\,0\,\rangle \\
&= \langle\,0_{\rm out}\,|\,0_{\rm in}\,\rangle_{\mathcal{V},\mathcal{A},\mathcal{S},\mathcal{P}}\,.
\end{aligned} \tag{2.27}
$$

In (2.27) we provide three equivalent representations of the generating functional. The first one in terms of a path integral, the second one in terms of field operators in the Heisenberg representation and third, in the interaction picture. While in the second line, $|\,0\,\rangle$, is the ground state in the presence of QCD's interaction, the state, $|\,0_{\rm in}\,\rangle_{\mathcal{V},\mathcal{A},\mathcal{S},\mathcal{P}}$, is the free vacuum state in the presence of the external fields.

The generating functional (2.27) has the local chiral symmetry $\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L$ (Gasser and Leutwyler, 1984; Gasser and Leutwyler, 1985; Krause, 1990; Scherer, 2003). The simultaneous transformations of the external fields

$$
\begin{aligned}
r_\mu(x) = \mathcal{V}_\mu(x) + \mathcal{A}_\mu(x) &\;\to\; U_R(x)\,r_\mu(x)\,U_R^\dagger(x) + i\,U_R(x)\,\partial_\mu\,U_R^\dagger(x)\,, \\
l_\mu(x) = \mathcal{V}_\mu(x) - \mathcal{A}_\mu(x) &\;\to\; U_L(x)\,l_\mu(x)\,U_L^\dagger(x) + i\,U_L(x)\,\partial_\mu\,U_L^\dagger(x)\,, \\
\mathcal{S}(x) + i\,\mathcal{P}(x) &\;\to\; U_R(x)\,(\mathcal{S}(x) + i\,\mathcal{P}(x))\,U_L^\dagger(x)\,, \\
\mathcal{S}(x) - i\,\mathcal{P}(x) &\;\to\; U_R(x)\,(\mathcal{S}(x) - i\,\mathcal{P}(x))\,U_L^\dagger(x)\;,
\end{aligned} \tag{2.28}
$$

does not change the generating functional. In (2.28) the local transformations are parameterized with $U_R(x) = \exp(-\,i\,\Theta_R^a(x)\,T^{(a)}) \in \mathrm{SU}(3)_R$ and $U_L(x) = \exp(-\,i\,\Theta_L^a(x)\,T^{(a)}) \in \mathrm{SU}(3)_L$. The invariance follows since the quark fields in the path-integral representation (2.27) may be transformed with

$$
\begin{aligned}
q_R(x) &\quad\to\quad U_R(x)\,q_R(x)\,, \\
q_L(x) &\quad\to\quad U_L(x)\,q_L(x)\,.
\end{aligned} \tag{2.29}
$$

In order to calculate matrix elements of the current operators one takes functional derivatives of the generating functional with respect to the external source fields (Gasser and Leutwyler, 1984; Gasser and Leutwyler, 1985; Leutwyler, 1994; Krause, 1990). Consider the four examples

$$\begin{aligned}
\langle 0\,|\,S^{(a)}(x)\,|\,0\rangle &= \frac{\delta}{i\,\delta\,\mathcal{S}^{(a)}(x)}\,e^{i\,Z[\mathcal{V},\mathcal{A},\mathcal{S},\mathcal{P}]}\Big|_{\mathcal{V}=\mathcal{A}=\mathcal{P}=0,\,\mathcal{S}=\mathcal{M}}\,, \qquad (2.30)\\
\langle 0\,|\,V^{(a)}_\mu(x)\,|\,0\rangle &= \frac{\delta}{i\,\delta\,\mathcal{V}^\mu_{(a)}(x)}\,e^{i\,Z[\mathcal{V},\mathcal{A},\mathcal{S},\mathcal{P}]}\Big|_{\mathcal{V}=\mathcal{A}=\mathcal{P}=0,\,\mathcal{S}=\mathcal{M}}\,,\\
\langle 0\,|\,T\,P^{(a)}(x)\,P^{(b)}(y)\,|\,0\rangle &= \frac{\delta}{i\,\delta\,\mathcal{P}^{(a)}(x)}\,\frac{\delta}{i\,\delta\,\mathcal{P}^{(b)}(y)}\,e^{i\,Z[\mathcal{V},\mathcal{A},\mathcal{S},\mathcal{P}]}\Big|_{\mathcal{V}=\mathcal{A}=\mathcal{P}=0,\,\mathcal{S}=\mathcal{M}}\,,\\
\langle 0\,|\,T\,A^{(a)}_\mu(x)\,A^{(b)}_\nu(y)\,|\,0\rangle &= \frac{\delta}{i\,\delta\,\mathcal{A}^\mu_{(a)}(x)}\,\frac{\delta}{i\,\delta\,\mathcal{A}^\nu_{(b)}(y)}\,e^{i\,Z[\mathcal{V},\mathcal{A},\mathcal{S},\mathcal{P}]}\Big|_{\mathcal{V}=\mathcal{A}=\mathcal{P}=0,\,\mathcal{S}=\mathcal{M}}\,,
\end{aligned}$$

with the vector and axial-vector currents already introduced in (2.17). The scalar and pseudo-scalar densities are

$$S^{(a)}(x) = \bar{q}(x)\,T^{(a)}\,q(x)\,, \qquad P^{(a)}(x) = i\,\bar{q}(x)\,\gamma_5\,T^{(a)}\,q(x)\,. \qquad (2.31)$$

The formulae in (2.30) are obtained from an expansion of the generating functional (2.27) in powers of the external fields, i.e.

$$\begin{aligned}
e^{i\,Z[\mathcal{V},\mathcal{A},\mathcal{S},\mathcal{P}]} &= 1\\
&+ \int d^4x\,\Big\{ i\,\mathcal{V}^{(a)}_\mu(x)\,\langle 0|V^\mu_{(a)}(x)|0\rangle + i\,\mathcal{A}^{(a)}_\mu(x)\,\langle 0|A^\mu_{(a)}(x)|0\rangle\\
&\qquad + i\,\mathcal{S}^{(a)}(x)\,\langle 0|S_{(a)}(x)|0\rangle + i\,\mathcal{P}^{(a)}(x)\,\langle 0|P_{(a)}(x)|0\rangle \Big\}\\
&- \frac{1}{2}\int d^4x\,d^4y\,\Big\{ \mathcal{V}^{(a)}_\mu(x)\,\mathcal{V}^{(b)}_\nu(y)\,\langle 0|T\,V^\mu_{(a)}(x)\,V^\nu_{(b)}(y)|0\rangle\\
&\qquad + \mathcal{A}^{(a)}_\mu(x)\,\mathcal{A}^{(b)}_\nu(y)\,\langle 0|T\,A^\mu_{(a)}(x)\,A^\nu_{(b)}(y)|0\rangle\\
&\qquad + \mathcal{S}^{(a)}(x)\,\mathcal{S}^{(b)}(y)\,\langle 0|T\,S_{(a)}(x)\,S_{(b)}(y)|0\rangle\\
&\qquad + \mathcal{P}^{(a)}(x)\,\mathcal{P}^{(b)}(y)\,\langle 0|T\,P_{(a)}(x)\,P_{(b)}(y)|0\rangle\\
&\qquad + \;\text{time ordered products of cross terms}\Big\}\\
&+ \;\ldots\,. \qquad (2.32)
\end{aligned}$$

CHAPTER III

CHIRAL LAGRANGIAN

In this chapter we briefly review some general properties of Chiral Perturbation Theory (ChPT) which is a systematic method to study low-energy QCD. More details of ChPT and its current progress may be found in Scherer (2010); Bernard (2008); Scherer (2003); Pich (1998); Bernard (1995); Leutwyler (1994b); Leutwyler (1994a). At the end of this chapter, we construct chiral Lagrangians with D-meson and baryon fields and evaluate baryon-matrix elements of charm-changing currents.

3.1 Generating functional with effective degrees of freedom

An effective field theory is a convenient and powerful tool to perform systematic calculations in terms of the relevant degrees of freedom. The basic idea is to identify the lightest particles as the relevant and active degrees of freedom at low energies. While the fundamental degrees of freedom in QCD are the quarks and the gluons, at low energies the Goldstone bosons dominate the dynamics. An effective field theory is based on an effective Lagrangian which is constructed in terms of the relevant degrees of freedom incorporating all important symmetries and symmetry-breaking patterns of the underlying fundamental theory (Weinberg, 1979; Donoghue, Golowich. and Holstein, 1995).

Chiral Perturbation Theory (ChPT) is the effective field theory of QCD at low energies. In this section, we will discuss and summarize the fundamental principles and building blocks of the chiral Lagrangian. A powerful method to construct the chiral Lagrangian is the external field method. Let U be an effective degree of freedom relevant for our effective field theory. Then we expect

$$e^{i\,Z[\mathcal{V},\mathcal{A},\mathcal{S},\mathcal{P}]} = \int \mathcal{D}\,U\,e^{i\int d^4x\,\mathscr{L}_{\rm eff}\left(U\,;\,\mathcal{V},\mathcal{A},\mathcal{S},\mathcal{P}\right)}\,, \tag{3.1}$$

to define a systematic approximation of QCD's generating functional (2.27) at low energies. Here $\mathscr{L}_{\rm eff}$ is the effective Lagrangian constructed as to keep the invariance properties of QCD's generating functional under local $\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L$ transformations in the external sources (2.28). This constraint provides the construction rules of the effective Lagrangian (Gasser and Leutwyler, 1984; Gasser and Leutwyler, 1985; Krause, 1990; Donoghue et al., 1995; Scherer, 2003).

We start with the transformation properties of the effective field $U(x)$. Under the local chiral symmetry group $\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L$, the field $U(x)$ transforms as follows

$$U(x) \to U'(x) = U_R(x)\,U(x)\,U_L^{-1}(x) = U_R(x)\,U(x)\,U_L^{\dagger}(x)\,, \tag{3.2}$$

where the transformation fields $U_L(x)$ and $U_R(x)$ were introduced already in (2.28). It is convenient to express the field $U(x) \in \mathrm{SU}(3)$ in terms of the Goldstone boson fields $\Phi(x)$ via the exponential representation

$$U(x) = \exp\left\{\frac{i}{f}\,\Phi(x)\right\}\,, \tag{3.3}$$

where f is a constant and

$$\Phi = \sqrt{2}\begin{pmatrix} \frac{1}{\sqrt{2}}\,\pi^0 + \frac{1}{\sqrt{6}}\,\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\,\pi^0 + \frac{1}{\sqrt{6}}\,\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\,\eta \end{pmatrix}\,, \tag{3.4}$$

is given in terms of the physical pion, kaon and eta fields. We introduce a covariant derivative of the effective field U as follows

$$\partial_\mu\,U \to \mathcal{D}_\mu\,U = \partial_\mu\,U - i\,l_\mu\,U + i\,U\,r_\mu\,, \tag{3.5}$$

such that under a local chiral transformation it transforms with

$$\mathcal{D}_\mu U \to U_R \, (\mathcal{D}_\mu U) \, U_L^\dagger \, . \tag{3.6}$$

It is convenient to identify combinations of the external fields, $R_{\mu\nu}$ and $L_{\mu\nu}$, that transform like the U field. We introduce

$$\begin{aligned} R_{\mu\nu} &= \partial_\mu \, r_\nu - \partial_\nu \, r_\mu - i \, [r_\mu, \, r_\nu]_- \\ L_{\mu\nu} &= \partial_\mu \, l_\nu - \partial_\nu \, l_\mu - i \, [l_\mu, \, l_\nu]_- \, , \end{aligned} \tag{3.7}$$

with

$$R_{\mu\nu}(x) \to U_R(x) \, R_{\mu\nu}(x) \, U_R^\dagger(x) \;, \qquad L_{\mu\nu} \to U_L(x) \, L_{\mu\nu}(x) \, U_L^\dagger(x) \, . \tag{3.8}$$

In this section, we have discussed the basic idea behind an effective Lagrangian. We derived the construction rules for the chiral Lagrangian and identified the transformation properties of the effective chiral field $U(x)$ and its covariant derivative $\mathcal{D}_\mu U$. Given such transformation rules it is straightforward to identify the infinite hierarchy of interaction terms in the chiral Lagrangian.

3.2 Heavy fields in the chiral Lagrangian

We proceed with a discussion about heavy fields in the chiral Lagrangian. How to generalize the construction rules of the previous section to the case where a baryon or D-meson field is involved? We first investigate the flavor SU(3) decomposition of the four fields of interest in our work. The flavor structure of the fields will play a crucial role in the construction of the Lagrangian.

The baryon octet field with $J^P = \frac{1}{2}^+$ may be represented by a 3×3 matrix

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\,\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix} , \tag{3.9}$$

where the various elements give the eight baryon fields of the octet, i.e. the proton (p), neutron (n), lambda (Λ), etc fields. The matrix structure is a consequence of co- and contra-variant flavor SU(3) indices, i.e. $B = B^i_j$. Here $i, j \cdots = 1, 2, 3$ are indices in the fundamental representation of the flavor SU(3) group. The baryon decuplet field with $J^P = \frac{3}{2}^+$ has three upper flavor indices $\Delta = \Delta^{ijk}$, which are symmetric under mutual exchanges. The ten fields are identified

$$
\begin{aligned}
&\Delta^{111} = \Delta^{++}\,, \quad \Delta^{112} = \Delta^{+}/\sqrt{3}\,, \quad \Delta^{122} = \Delta^{0}/\sqrt{3}\,, \quad \Delta^{222} = \Delta^{-}\,, \\
&\Delta^{113} = -\Sigma^{+}/\sqrt{3}\,, \quad \Delta^{123} = \Sigma^{0}/\sqrt{6}\,, \quad \Delta^{223} = \Sigma^{-}/\sqrt{3}\,, \\
&\Delta^{133} = \Xi^{0}/\sqrt{3}\,, \quad \Delta^{233} = \Xi^{-}/\sqrt{3}\,, \\
&\Delta^{333} = \Omega^{-}\,,
\end{aligned} \tag{3.10}
$$

with the various physical fields. There remain the open-charm fields with $J^P = 0^-$ and $J^P = 1^-$. Although the pseudo-scalar mesons composed of up, down and strange quarks are a consequence of spontaneous chiral symmetry breaking, the pseudo-scalar open-charm mesons are not to be identified with pseudo-Goldstone bosons. The masses of the open-charm mesons do not vanish in the chiral SU(3) limit since they involve a charm quark that remains heavy in that limit. For our considerations the open-charm fields have to be treated like any other heavy field. With respect to the flavor SU(3) group the open charm fields transform as antitriplets, i.e. they are characterized by one lower flavor index. We write

$$
D = \Big(D^0,\, -D^+,\, D_s^+\Big)\,, \qquad D_{\mu\nu} = (D^0_{\mu\nu}, -D^+_{\mu\nu}, D^+_{s,\mu\nu})\,, \tag{3.11}
$$

where we recall the quark content of the open-charm fields with $(\bar u\, c\,,\, -\bar d\, c\,,\, \bar s\, c\,)$. The corresponding antiparticles are described by the fields

$$
\bar D = \begin{pmatrix} \bar D^0 \\ -\bar D^+ \\ \bar D^+_s \end{pmatrix}\,, \qquad \bar D_{\mu\nu} = \begin{pmatrix} \bar D^0_{\mu\nu} \\ -\bar D^+_{\mu\nu} \\ \bar D^+_{s,\mu\nu} \end{pmatrix}\,, \tag{3.12}
$$

where we use $\bar{D} = D^\dagger$ for notational convenience. In this work, we use anti-symmetric tensor fields to interpolate the vector D mesons, i.e. it holds $D_{\mu\nu} = -D_{\nu\mu}$ [see e.g. (Jones, 1962; Kyriakopoulos, 1969; Kyriakopoulos, 1972; Ecker, Gasser, Pich and De Rafael, 1989; Nowakowski and Pilaftsis, 1993; Veltman, 1994; Lutz and Soyeur, 2008)].

We return to the construction of the chiral Lagrangian. The starting point is a generating functional

$$\mathcal{F}(\,\mathbf{p}'\,,\mathbf{p}\;;\,\mathcal{V},\,\mathcal{A},\,\mathcal{S},\,\mathcal{P}) = e^{i\,Z[\mathbf{p}',\mathbf{p}\;;\mathcal{V},\mathcal{A},\mathcal{S},\mathcal{P}]} = \langle\,\mathbf{p}'_{\rm out}\,|\,\mathbf{p}_{\rm in}\,\rangle_{\mathcal{V},\mathcal{A},\mathcal{S},\mathcal{P}}\,, \tag{3.13}$$

in the presence of $|\,\mathbf{p}_{\rm in}\,\rangle$ and $\langle\,\mathbf{p}'_{\rm out}\,|$ states that describe an incoming and outgoing heavy state with momentum $\mathbf{p}$ and $\mathbf{p}'$ respectively (Gasser et al., 1988; Krause, 1990). Like before the generating functional (3.13) is invariant under the local transformation of the external sources (2.28). We can calculate matrix elements of various current operators from $\mathcal{F}$ in the same way as we have done it in the vacuum case by taking functional derivatives.

We conclude that the effective Lagrangian that includes any heavy field must maintain the local $\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L$ symmetry. All what needs to be done is to work out the transformation properties of the various heavy fields. This is done in two steps. First we introduce a new field u with $u^2 = U$. According to (Georgi, 1984; Krause, 1990; Scherer, 2003; Scherer, 2010) the new field u field transforms as

$$\begin{aligned} u \;\longrightarrow\; \sqrt{U_L\,U\,U_R^\dagger} \;&=\; U_L\,u\,K^\dagger(U_L,\,U_R,U) \\ &=\; K(U_L,\,U_R,\,U)\,u\,U_R^\dagger\,, \end{aligned} \tag{3.14}$$

where $K(U_L,\,U_R,\,U) \in \mathrm{SU}(3)$ is a so-called compensator field which is a non-linear function of U_L, U_R, and U. For a $\mathrm{SU}(3)_V$ transformations with $U_L = U_R$ it follows $K(U_L,\,U_R,\,U) = U_L = U_R$ from (3.14).

The transformations K and $K^\dagger$ will play a central role in the construction of a chiral Lagrangian with heavy fields. It is useful to identify field combinations that transform exclusively with K and/or $K^\dagger$. A first example is the chiral axial vector, a so-called "vielbein"

$$U^\mu = i\big(u^\dagger(\partial^\mu - i\, r^\mu)u - u(\partial^\mu - i\, l^\mu)u^\dagger\big)\,, \tag{3.15}$$

which transforms according to

$$U^\mu \quad \stackrel{\mathrm{SU(3)}_R\times \mathrm{SU(3)}_L}{\longrightarrow} \quad K U^\mu K^\dagger\,. \tag{3.16}$$

It remains to construct the covariant derivative

$$\mathcal{D}_\mu\, U_\nu \;\; = \;\; \partial_\mu\, U_\nu + \Gamma_\mu\, U_\nu - U_\nu\, \Gamma_\mu\,, \tag{3.17}$$

where Γ^μ is the chiral connection. The request that $\mathcal{D}_\mu\, U_\nu$ transforms like the U_μ field determines the chiral connection

$$\Gamma^\mu = \frac{1}{2}\big(u^\dagger\,(\partial^\mu - i\, r^\mu)\, u + u\,(\partial^\mu - i\, l^\mu)\, u^\dagger\big)\,. \tag{3.18}$$

Our claim follows from the transformation property

$$\Gamma^\mu \longrightarrow K\, \Gamma^\mu\, K^\dagger - (\partial^\mu\, K)\, K^\dagger\,. \tag{3.19}$$

To this end we remark that we may construct the chiral Lagrangian with either $U, \mathcal{D}_\mu$ of (3.5) or $U_\mu, \mathcal{D}_\mu$ of (3.15, 3.17). The resulting two Lagrangians will be equivalent.

Now we are well prepared to consider any heavy field in the chiral Lagrangian. The central result we use here that the transformation rules depend on the number of co and contra variant flavor indices only (Georgi, 1984; Krause, 1990; Scherer, 2003; Scherer, 2010; Pich, 1998; Bernard, 2008). A simple example is the baryon octet field $B = B^i_{\,j}$, which has one upper and one lower index. While the upper index transforms with K, the lower index transforms with $K^\dagger$, i.e.

$$B \quad \stackrel{\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L}{\longrightarrow} \quad K B K^\dagger \,. \tag{3.20}$$

The baryon octet field transform like the U_μ field, which implies that their covariant derivatives are identical. The baryon decuplet field $\Delta_\mu = \Delta_\mu^{ijk}$ has three upper indices. As a consequence it transforms as

$$\Delta_\mu^{ijk} \quad \stackrel{\mathrm{SU}(3)_R \times \mathrm{SU}(3)_L}{\longrightarrow} \quad (K)^i_l \, (K)^j_m \, (K)^k_n \, \Delta_\mu^{lmn} \,. \tag{3.21}$$

As we have seen from the covariant derivative acting on the octet fields any upper or lower index comes with the chiral connection, however, with a sign and index contraction depending on the case. For the baryon octet and decuplet fields it follows

$$\begin{aligned}
\mathcal{D}_\mu \, B^i_{\,j} &= \partial_\mu \, B^i_{\,j} + (\Gamma_\mu)^i_{\,k} \, B^k_{\,j} - B^i_{\,k} \, (\Gamma_\mu)^k_{\,j} \,, \\
\mathcal{D}_\mu \, \Delta_\nu^{ijk} &= \partial_\mu \, \Delta_\nu^{ijk} + (\Gamma_\mu)^i_l \, \Delta_\nu^{lmn} + (\Gamma_\mu)^j_m \, \Delta_\nu^{lmn} + (\Gamma_\mu)^k_n \, \Delta_\nu^{lmn} \,.
\end{aligned} \tag{3.22}$$

Analogous results hold for the open-charm fields (Wise, 1992; Yan et al., 1992; Burdman and Donoghue, 1992; Casalbuoni et al., 1997). Given the transformation rules of U_μ and all the heavy fields it is straight forward to write down terms that respect the local $\mathrm{SU}(3)_R \otimes \mathrm{SU}(3)_L$ symmetry. The chiral Lagrangian consists of all terms one may write down. It is the task of so called power counting rules to order the terms according to their relevance (Gasser and Leutwyler, 1984; Gasser and Leutwyler, 1985; Gasser et al., 1988; Krause, 1990; Scherer, 2003; Fuchs et al., 2003).

3.3 Chiral Lagrangian with open-charm meson and baryon fields

The main focus of this work is to study the interaction of the open-charm mesons with the octet and decuplet baryons. We focus on the residual short-range interactions described by local two-body counter terms. Since we are interested in open-charm mesons and baryons with small 3-momentum we construct the two-body terms with the minimal number of derivatives. We discriminate terms with two baryon octet fields, $\mathscr{L}^{(c)}$, two baryon decuplet fields, $\mathscr{L}^{(d)}$, and the remaining mixed terms, $\mathscr{L}^{(e)}$. We write

$$\mathscr{L}_{\text{counter term}} = \mathscr{L}^{(c)} + \mathscr{L}^{(d)} + \mathscr{L}^{(e)} \,. \tag{3.23}$$

The chiral Lagrangian for the two octet-baryon fields is given by

$$\begin{aligned}
\mathscr{L}^{(c)} \;=\;& D\Big\{ c_1^{(S)}\,(\bar B\,B)_+ + c_2^{(S)}\,(\bar B\,B)_- + \frac{1}{2}\,c_3^{(S)}\,\mathrm{tr}\,\big(\bar B\,B\big)\Big\}\,\bar D \\
&- \frac{1}{2}\,D_{\mu\nu}\Big\{\tilde c_1^{(S)}\,(\bar B\,B)_+ + \tilde c_2^{(S)}\,(\bar B\,B)_- + \frac{1}{2}\,\tilde c_3^{(S)}\,\mathrm{tr}\,\big(\bar B\,B\big)\Big\}\,\bar D^{\mu\nu} \\
&+ \frac{i}{M_c}\,D_{\mu\nu}\Big\{c_1^{(A)}\,(\bar B\,\gamma_5\,\gamma^\mu\,B)_+ + c_2^{(A)}\,(\bar B\,\gamma_5\,\gamma^\mu\,B)_- \\
&\qquad\qquad + \frac{1}{2}\,c_3^{(A)}\,\mathrm{tr}\,\big(\bar B\,\gamma_5\,\gamma^\mu\,B\big)\Big\}\,(\partial^\nu\,\bar D) + \text{h.c.} \\
&+ \frac{1}{4\,M_c}\,\epsilon^{\mu\nu\alpha\beta}\,D_{\mu\nu}\Big\{\tilde c_1^{(A)}\,(\bar B\,\gamma_5\,\gamma_\alpha\,B)_+ + \tilde c_2^{(A)}\,(\bar B\,\gamma_5\,\gamma_\alpha\,B)_- \\
&\qquad\qquad + \frac{1}{2}\,\tilde c_3^{(A)}\,\mathrm{tr}\,\big(\bar B\,\gamma_5\,\gamma_\alpha\,B\big)\Big\}\,(\partial^\tau\,\bar D_{\tau\beta}) + \text{h.c.}\,,
\end{aligned} \tag{3.24}$$

where $\bar D = D^\dagger$ and M_c is the charm quark mass. In (3.24) we used the notation $(\bar B\,\Gamma\,B)_\pm$ with Γ any Dirac matrix and

$$\begin{aligned}
(\bar B\,\Gamma\,B)_\pm &= \frac{1}{2}\sum_{c\,,d}\bar b^{(d)}\,\Gamma\,b^{(c)}\,[\lambda^{(d)},\lambda^{(c)}]_\pm\,, \\
B &= \frac{1}{\sqrt 2}\,b^{(a)}\,\lambda^{(a)}\,.
\end{aligned} \tag{3.25}$$

We note that all derivatives in (3.24) should be replaced by appropriate covariant derivatives as to render the various interaction terms invariant under chiral rotations. Since such a replacement is unambiguous we keep the normal derivative for notational clarity. All structures in (3.24) describe s-wave scattering and therefore we assign them the chiral power Q^0 . There are thre independent flavor SU(3) structures which are parameterized by three coupling constants $c_1^{(X)}, c_2^{(X)}$ and $c_3^{(X)}$. It remains to understand the four spin structures in the chiral Lagrangians $\mathscr{L}^{(c)}$. There is one term involving the pseudo-scalar D mesons with coupling constants $c_1^{(S)}$. This follows since a two-body system of a spin zero and a spin one-half particle allows to form one s-wave state only. In contrast two s-wave states involving the vector D mesons are possible. The relevant coupling constants are introduced with $\tilde{c}_1^{(S)}$ and $\tilde{c}_1^{(A)}$. It remains the term parameterized with $c_1^{(A)}$ describing an s-wave transition from the pseudo-scalar to the vector D mesons.

We continue with the leading order terms involving two baryon decuplet fields. At the leading order we find the relevance of the following 10 terms

$$
\begin{aligned}
\mathscr{L}^{(d)} \;=\; & - D \left\{ d_1^{(S)}\, \bar{\Delta}^\sigma \cdot \Delta_\sigma + \frac{1}{2}\, d_2^{(S)} \,\mathrm{tr}\, \left(\bar{\Delta}^\sigma \cdot \Delta_\sigma \right) \right\} \bar{D} \\
& + \frac{1}{2}\, D_{\mu\nu} \left\{ \tilde{d}_1^{(S)}\, \bar{\Delta}^\sigma \cdot \Delta_\sigma + \frac{1}{2}\, \tilde{d}_2^{(S)} \,\mathrm{tr}\, \left(\bar{\Delta}^\sigma \cdot \Delta_\sigma \right) \right\} \bar{D}^{\mu\nu} \\
& + \frac{i}{4}\, \epsilon^{\mu\nu\alpha\beta}\, D_{\mu\nu} \left\{ d_1^{(E)}\, \bar{\Delta}_\alpha \cdot \Delta_\beta + \frac{1}{2}\, d_2^{(E)} \,\mathrm{tr}\, \left(\bar{\Delta}_\alpha \cdot \Delta_\beta \right) \right\} \bar{D} + \text{h.c.} \\
& + \frac{1}{2}\, D_{\beta\mu} \left\{ \tilde{d}_1^{(E)}\, \bar{\Delta}_\alpha \cdot \Delta^\beta + \frac{1}{2}\, \tilde{d}_2^{(E)} \,\mathrm{tr}\, \left(\bar{\Delta}_\alpha \cdot \Delta^\beta \right) \right\} \bar{D}^{\alpha\mu} \\
& - \frac{1}{2}\, D^{\alpha\mu} \left\{ \tilde{d}_3^{(E)}\, \bar{\Delta}_\alpha \cdot \Delta^\beta + \frac{1}{2}\, \tilde{d}_4^{(E)} \,\mathrm{tr}\, \left(\bar{\Delta}_\alpha \cdot \Delta^\beta \right) \right\} \bar{D}_{\beta\mu} \,,
\end{aligned}
\tag{3.26}
$$

where we used the notation

$$
(\bar{\Delta}^\mu \cdot \Delta_\mu)^a_b = \bar{\Delta}^\mu_{ijb}\, \Delta^{ija}_\mu \,, \tag{3.27}
$$

suggested in Lutz and Kolomeitsev (2002). The decuplet fields satisfy the constraint equations

$$
\gamma^\mu\, \Delta_\mu = \partial^\mu\, \Delta_\mu = 0 \,. \tag{3.28}
$$

The spin structures of the chiral Lagrangian (3.26) are readily understood. Again there is one term involving two pseudo-scalar D-meson fields with coupling constants $d_1^{(S)}$. Since a two-body system of a spin three-half and a spin one particle allows to form three s-wave states only, there are the three coupling constants only, $\tilde{d}_1^{(S)}$, $\tilde{d}_1^{(E)}$ and $\tilde{d}_3^{(E)}$. There remains the term parameterized by $d_1^{(E)}$ describing the s-wave transitions from a pseudo-scalar to a vector D meson.

We close the collection of our counter terms with structures involving a baryon octet and decuplet field. At the leading order we find the following 4 terms

$$\begin{aligned}\mathscr{L}^{(e)} \;=\;& \frac{i}{4}\,\epsilon^{\mu\nu\alpha\beta}\Big\{e_1^{(A)}\,D\,(\bar{\Delta}_\mu\cdot\gamma_5\,\gamma_\nu B)\,\bar{D}_{\alpha\beta}\,+\,e_2^{(A)}\,D_{\alpha\beta}\,(\bar{\Delta}_\mu\,\gamma_5\,\gamma_\nu\cdot B)\,\bar{D}\Big\}+\text{h.c.}\\ &-\frac{\tilde{e}_1^{(A)}}{2}\,D^{\tau\mu}\,(\bar{\Delta}_\mu\cdot\gamma_5\,\gamma_\nu\,B)\,\bar{D}_\tau{}^{\nu}+\text{h.c.}\\ &+\frac{\tilde{e}_2^{(A)}}{2}\,D_\tau{}^{\nu}\,(\bar{\Delta}_\mu\cdot\gamma_5\,\gamma_\nu\,B)\,\bar{D}^{\tau\mu}+\text{h.c.}\,,\end{aligned}\tag{3.29}$$

where we used the notation

$$(\bar{\Delta}^\mu\cdot B)^a_b=\epsilon^{ika}\,\bar{\Delta}^\mu_{ijb}\,B^j_k\,,\tag{3.30}$$

introduced in Lutz and Kolomeitsev (2002).

Altogether we have 26 coupling constants which all carry dimension $[\text{Mass}]^{-1}$. In the following chapters, we will use the heavy-quark symmetry and the large N_c operator analysis to correlate the coupling constants introduced here.

3.4 Correlation functions of charm-changing currents

In the previous section we constructed a chiral Lagrangian with D mesons. While from the construction it follows that the various terms are invariant under chiral rotations the relation to QCD's Greens function is less transparent. In order to connect more closely to QCD it is useful to consider charm-changing axial-vector and vector currents in the generating functional (2.25, 2.27). In turn the effective

Lagrangian and generating functional (3.1) will then depend on such external and charm-changing sources.

We introduce charm-changing axial-vector and vector currents

$$
\begin{aligned}
&A_\mu(x) = \Big(\bar{u}(x)\,,\ \bar{d}(x)\,,\ \bar{s}(x) \Big)\, \gamma_\mu\, \gamma_5\, c(x)\,,\\
&V_\mu(x) = \Big(\bar{u}(x)\,,\ \bar{d}(x)\,,\ \bar{s}(x) \Big)\, \gamma_\mu\, c(x)\,,\\
&\bar{A}_\mu(x) = \bar{c}(x)\,\gamma_\mu\,\gamma_5 \begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix}, \qquad \bar{V}_\mu(x) = \bar{c}(x)\ \gamma_\mu \begin{pmatrix} u(x) \\ d(x) \\ s(x) \end{pmatrix},
\end{aligned}
\tag{3.31}
$$

with the quark-field operators $u(x), d(x), s(x), c(x)$ of the up, down, strange and charm quarks. We note that $\bar{A}_\mu$, $\bar{V}_\mu$ and A_μ, V_μ form triplets and anti-triplets in SU(3) flavor space respectively. The $\bar{A}_\mu$ and $\bar{V}_\mu$ operators create pseudo-scalar and vector D mesons as

$$
\begin{aligned}
&\langle\, 0\,|\, \bar{A}_\mu(0)\,|\, D(q)\,\rangle = f_A\, q_\mu\,,\\
&\langle\, 0\,|\, \bar{V}_\mu(0)\,|\, D^*(q,\lambda)\,\rangle = f_V\, M_V\, \varepsilon_\mu(q,\lambda)\,,
\end{aligned}
\tag{3.32}
$$

where f_A and f_V are some coupling constants. With $\varepsilon_\mu(q,\lambda)$ the conventional wave function of a spin-one particle is denoted. In (3.32) and in the following we consider the flavor SU(3) limit for simplicity. The parameter M_V is the SU(3) limit value of the vector D meson masses. The chiral Lagrangian will involve the additional terms

$$
\mathscr{L}_{\rm ext}(x) = \mathscr{A}^\mu(x)\, \bar{A}_\mu(x) + A_\mu(x)\, \bar{\mathscr{A}}^\mu(x) + \mathscr{V}^\mu(x)\, \bar{V}_\mu(x) + V_\mu(x)\, \bar{\mathscr{V}}^\mu(x)\,. \tag{3.33}
$$

In this work, we consider baryon matrix elements of the following products of quark currents:

$$C^{AA}_{\mu\nu,a}(q) = i \int d^4x\, e^{-i\,q\cdot x}\, T\, A_\mu(0)\, \lambda^{(a)}\, \bar{A}_\nu(x)\,, \qquad \bar{A}_\mu(x) = A^\dagger_\mu(x)\,,$$
$$C^{VV}_{\mu\nu,a}(q) = i \int d^4x\, e^{-i\,q\cdot x}\, T\, V_\mu(0)\, \lambda^{(a)}\, \bar{V}_\nu(x)\,, \qquad \bar{V}_\mu(x) = V^\dagger_\mu(x)\,,$$
$$C^{VA}_{\mu\nu,a}(q) = i \int d^4x\, e^{-i\,q\cdot x}\, T\, V_\mu(0)\, \lambda^{(a)}\, \bar{A}_\nu(x)\,, \tag{3.34}$$

where the Gell-Mann matrices, $\lambda^{(a)}$, are supplemented with the singlet matrix $\lambda^{(0)} = \sqrt{2/3}\,\mathbf{1}_{3\times3}$. Therefore in (3.34) the index is $a = 0, \cdots, 8$. We are interested in such matrix elements, since they receive contributions from the chiral Lagrangian (3.23) and they define a rigorous basis for the derivation of large-N_c sum rules in the Chapter V.

In the following we derive and analyze the contributions of the chiral Lagrangian (3.23). Following Lutz and Semke (2011), the baryon octet and decuplet states

$$|p,\, \chi,\, a\rangle\,, \qquad\qquad |p,\, \chi,\, ijk\rangle\,, \tag{3.35}$$

are specified by the momentum p and the flavor indices $a = 1, \cdots, 8$ and $i, j, k = 1, 2, 3$ in the flavor SU(3) limit. The spin-polarization label is $\chi = 1, 2$ for the octet and $\chi = 1, \cdots, 4$ for the decuplet states.

According to the LSZ reduction formalism (Itzykson and Zuber, 1980; Peskin and Schroeder, 1995), baryon matrix elements of the correlation operators in (3.34) have contributions from the on-shell D-meson baryon scattering amplitudes. They are identified unambiguously by taking the residuum of poles generated by the incoming and outgoing D meson. For the sake of a more compact notation in what follows, it is convenient to introduce

$$\bar{C}^{XY}_{\mu\nu,\,a}(\bar{q}, q) = \frac{\bar{q}^2 - M_X^2}{f_X}\, C^{XY}_{\mu\nu,\,a}(\bar{q}, q)\, \frac{q^2 - M_Y^2}{f_Y}\,, \tag{3.36}$$

with $X, Y = V, A$ and M_A and M_V the masses of the pseudo-scalar and vector D mesons in the flavor SU(3) limit. In (3.36) we identify q_μ and $\bar{q}_\mu$ with the 4-momenta of the incoming and outgoing D mesons.

The baryon matrix elements of $\bar{C}^{AA}_{\mu\nu,\,a}(\bar{q}, q)$ are

$$
\begin{aligned}
\langle \bar{p}, \bar{\chi}, c \,|\, \bar{C}^{AA}_{\mu\nu,\,a}(\bar{q}, q) \,|\, p, \chi, b \rangle &= \bar{q}_\mu\, q_\nu\, \bar{u}(\bar{p}, \bar{\chi})\, u(p, \chi) \\
&\quad \times \begin{cases} \Big(2\sqrt{\tfrac{2}{3}}\, c_1^{(S)} + \sqrt{\tfrac{3}{2}}\, c_3^{(S)} \Big)\, \delta^{bc} & , \; a = 0 \\ 2\, c_1^{(S)}\, d^{abc} + 2\, c_2^{(S)}\, i\, f^{abc} & , \; a = 1, \cdots, 8 \end{cases} , \\
\langle \bar{p}, \bar{\chi}, nop \,|\, \bar{C}^{AA}_{\mu\nu,\,a}(\bar{q}, q) \,|\, p, \chi, b \rangle &= 0 \,, \\
\langle \bar{p}, \bar{\chi}, nop \,|\, \bar{C}^{AA}_{\mu\nu,\,a}(\bar{q}, q) \,|\, p, \, \chi, klm \rangle &= -\, \bar{q}_\mu\, q_\nu\, \bar{u}^\alpha(\bar{p}, \bar{\chi}) \cdot u_\alpha(p, \chi) \\
&\quad \times \begin{cases} \Big(\sqrt{\tfrac{2}{3}}\, d_1^{(S)} + \sqrt{\tfrac{3}{2}}\, d_2^{(S)} \Big)\, \delta^{nop}_{klm} & , \; a = 0 \\ d_1^{(S)}\, \Lambda^{a,\,rst}_{klm}\, \delta^{nop}_{rst} & , \; a = 1, \cdots, 8 \end{cases}
\end{aligned} \tag{3.37}
$$

where the upper row corresponds to the singlet component of the correlation function and the second row specifies the matrix elements for the octet components with $a = 1, \ldots 8$. We refer to the Appendix for the definition of the baryon octet and decuplet wave functions $u(p)$ and $u_\mu(p)$. Furthermore, we assume a flavor summation over the indices $r, s, t = 1, 2, 3$. The symmetric and antisymmetric structure constants d^{abc} and f^{abc} of the SU(3) flavor group are defined by

$$
\lambda^{(a)}\, \lambda^{(b)} = \frac{2}{3}\, \delta^{ab} + d^{abc}\, \lambda^{(c)} + i\, f^{abc}\, \lambda^{(c)} \,. \tag{3.38}
$$

The flavor SU(3) structures require additional notation

$$
\begin{aligned}
\Lambda^{klm}_{ab} &= \Big[\varepsilon_{ijk}\, \lambda^{(a)}_{li}\, \lambda^{(b)}_{mj} \Big]_{\mathrm{sym}(klm)} , \qquad & \delta^{\,klm}_{\,nop} &= \Big[\delta_{kn}\, \delta_{lo}\, \delta_{mp} \Big]_{\mathrm{sym}(nop)} , \\
\Lambda^{ab}_{klm} &= \Big[\varepsilon_{ijk}\, \lambda^{(a)}_{il}\, \lambda^{(b)}_{jm} \Big]_{\mathrm{sym}(klm)} , \qquad & \Lambda^{a,klm}_{nop} &= \Big[\lambda^{(a)}_{kn} \delta_{lo}\, \delta_{mp} \Big]_{\mathrm{sym}(nop)} ,
\end{aligned} \tag{3.39}
$$

which proved convenient in various derivations (Lutz and Semke, 2011). The symbol 'sym(nop)' in (3.39) asks for a symmetrization of the three indices nop, i.e. take the six permutations and divide out a factor 6.

We proceed with matrix elements of the vector currents. Here we will need the propagator of a vector meson in the tensor field representation. For a generic vector meson field $V_{\mu\nu}$ we recall the central result (Ecker et al., 1989; Lutz and Soyeur, 2008)

$$\begin{aligned}\langle\, 0\,|\, T\, V_{\mu\nu}(x)\, V_{\alpha\beta}(y)\,|\, 0\,\rangle &= i\int \frac{d^4k}{(2\pi)^4}\,\frac{e^{-i\,k\cdot(x-y)}}{k^2-M^2+i\,\epsilon}\, S^{\alpha\beta,\mu\nu}(k)\,,\\ S^{\alpha\beta,\mu\nu}(k) &= -\frac{1}{M^2}\,\big\{(M^2-k^2)\, g^{\alpha\mu}\, g^{\beta\nu} + g^{\alpha\mu}\, k^\beta\, k^\nu - g^{\alpha\nu}\, k^\beta\, k^\mu \\ &\qquad - (\mu\leftrightarrow\nu)\big\}\,. \end{aligned} \tag{3.40}$$

where M is the mass of the vector meson. Some algebra leads to

$$\begin{aligned}&\langle\bar p,\bar\chi,c\,|\,\bar C^{VV}_{\mu\nu,\,a}(\bar q,q)\,|\,p,\chi,b\rangle = \\ &\quad -\frac{1}{2\,M_V^2}\, S^{\alpha\beta}{}_{,\,\mu}(\bar q)\, S_{\alpha\beta,\nu}(q)\;\bar u(\bar p,\bar\chi)\, u(p,\chi) \\ &\qquad\times\begin{cases}\Big(2\sqrt{\tfrac{2}{3}}\,\tilde c_1^{(S)}+\sqrt{\tfrac{3}{2}}\,\tilde c_3^{(S)}\Big)\,\delta^{bc} & ,\ a=0\\ 2\,\tilde c_1^{(S)}\, d^{abc}+2\,\tilde c_2^{(S)}\, i\, f^{abc} & ,\ a=1,\cdots,8\end{cases} \\ &\quad +\frac{i}{4\,M_c\,M_V^2}\,\epsilon^{\alpha\beta\rho\sigma}\,\Big\{\,S_{\sigma,\,\mu}(\bar q)\, S_{\alpha\beta,\,\nu}(q)\,\bar u(\bar p,\bar\chi)\,\gamma_5\,\gamma_\rho\, u(p,\chi) \\ &\qquad\qquad -S_{\alpha\beta,\,\mu}(\bar q)\, S_{\sigma,\,\nu}(q)\,\bar u(\bar p,\bar\chi)\,\gamma_5\,\gamma_\rho\, u(p,\chi)\Big\} \\ &\qquad\times\begin{cases}\Big(2\sqrt{\tfrac{2}{3}}\,\tilde c_1^{(S)}+\sqrt{\tfrac{3}{2}}\,\tilde c_3^{(S)}\Big)\,\delta^{bc} & ,\ a=0\\ 2\,\tilde c_1^{(S)}\, d^{abc}+2\,\tilde c_2^{(S)}\, i\, f^{abc} & ,\ a=1,\cdots,8\end{cases}\,,\end{aligned}$$

$$\begin{aligned}&\langle\bar p,\bar\chi,nop\,|\,\bar C^{VV}_{\mu\nu,\,a}(\bar q,q)\,|\,p,\chi,b\rangle = \\ &\quad -\frac{1}{2\sqrt2\,M_V^2}\, S_{\beta\rho,\mu}(\bar q)\, S^{\beta\alpha}{}_{,\,\nu}\,(q)\,\bar u_\alpha(\bar p,\bar\chi)\,\gamma_5\,\gamma^\rho\, u(p,\chi) \\ &\qquad\times\begin{cases}0 & ,\ a=0\\ \tilde e_1^{(A)}\,\Lambda^{nop}_{ab} & ,\ a=1,\cdots,8\end{cases} \\ &\quad +\frac{1}{2\sqrt2\,M_V^2}\, S^{\beta\alpha}{}_{,\,\mu}(\bar q)\, S_{\beta\rho,\nu}\,(q)\,\bar u_\alpha(\bar p,\bar\chi)\,\gamma_5\,\gamma^\rho\, u(p,\chi)\end{aligned}$$

$$\times \begin{cases} 0 & , \; a = 0 \\ \tilde{e}_2^{(A)}\, \Lambda_{ab}^{nop} & , \; a = 1, \cdots, 8 \end{cases} ,$$

$$\langle \bar{p}, \bar{\chi}, nop \,|\, \bar{C}^{VV}_{\mu\nu,\,a}(\bar{q}, q) \,|\, p,\, \chi, klm \rangle =$$
$$\frac{1}{2\,M_V^2}\, S^{\alpha\beta}{}_{,\,\mu}(\bar{q})\, S_{\alpha\beta,\nu}(q)\; \bar{u}^{\sigma}(\bar{p}, \bar{\chi})\, u_{\sigma}(p, \chi)$$
$$\times \begin{cases} \left(\sqrt{\tfrac{2}{3}}\, \tilde{d}_1^{(S)} + \sqrt{\tfrac{3}{2}}\, \tilde{d}_2^{(S)}\right) \delta_{klm}^{nop} & , \; a = 0 \\ d_1^{(S)}\, \Lambda_{klm}^{a,\,rst}\, \delta_{rst}^{nop} & , \; a = 1, \cdots, 8 \end{cases}$$
$$+ \frac{1}{2\,M_V^2}\, S^{\alpha\rho}{}_{,\,\mu}(\bar{q})\, S_{\beta\rho,\,\nu}(q)\, \bar{u}_{\alpha}(\bar{p}, \bar{\chi})\, u^{\beta}(p, \chi)$$
$$\times \begin{cases} \left(\sqrt{\tfrac{2}{3}}\, \tilde{d}_1^{(E)} + \sqrt{\tfrac{3}{2}}\, \tilde{d}_2^{(E)}\right) \delta_{klm}^{nop} & , \; a = 0 \\ \tilde{d}_1^{(E)}\, \Lambda_{klm}^{a,\,rst}\, \delta_{rst}^{nop} & , \; a = 1, \cdots, 8 \end{cases}$$
$$- \frac{1}{2\,M_V^2}\, S_{\beta\rho,\,\mu}(\bar{q})\, S^{\alpha\rho}{}_{,\,\nu}(q)\, \bar{u}_{\alpha}(\bar{p}, \bar{\chi})\, u^{\beta}(p, \chi)$$
$$\times \begin{cases} \left(\sqrt{\tfrac{2}{3}}\, \tilde{d}_3^{(E)} + \sqrt{\tfrac{3}{2}}\, \tilde{d}_4^{(E)}\right) \delta_{klm}^{nop} & , \; a = 0 \\ \tilde{d}_3^{(E)}\, \Lambda_{klm}^{a,\,rst}\, \delta_{rst}^{nop} & , \; a = 1, \cdots, 8 \end{cases} , \quad (3.41)$$

where we use the compact notations of (Lutz and Soyeur, 2008) with

$$S^{\beta,\nu}(k) = k_{\alpha}\, S^{\alpha\beta,\mu\nu}(k)\, k_{\mu}\,, \qquad S^{\alpha\beta,\nu}(k) = S^{\alpha\beta,\mu\nu}(k)\, k_{\mu}\,,$$
$$S^{\beta,\mu\nu}(k) = k_{\alpha}\, S^{\alpha\beta,\mu\nu}(k)\,, \quad (3.42)$$

where $k_{\mu}\, k^{\mu} = M_V^2 = M^2$.

There remain the matrix elements of the mixed correlation function, for which we find

$$\langle \bar{p}, \bar{\chi}, c \,|\, \bar{C}^{VA}_{\mu\nu,\,a}(\bar{q}, q) \,|\, p, \chi, b \rangle =$$
$$- \frac{1}{2\,M_c\,M_V}\, S_{\alpha\beta,\,\mu}(\bar{q})\, q_{\nu}\, q^{\beta}\, \bar{u}(\bar{p}, \bar{\chi})\, \gamma_5\, \gamma^{\alpha}\, u(p, \chi)$$

$$\times \begin{cases} \left(2\sqrt{\frac{2}{3}}\, c_1^{(S)} + \sqrt{\frac{3}{2}}\, c_3^{(S)}\right) \delta^{bc} & ,\ a=0 \\ 2\, c_1^{(S)}\, d^{abc} + 2\, c_2^{(S)}\, i\, f^{abc} & ,\ a=1,\cdots,8 \end{cases},$$

$$\langle \bar{p}, \bar{\chi}, nop \,|\, \bar{C}^{VA}_{\mu\nu,\,a}(\bar{q}, q) \,|\, p, \chi, b\rangle =$$
$$+\frac{i}{4\sqrt{2}\, M_V}\, \epsilon^{\alpha\beta\rho\sigma}\, S_{\alpha\beta,\,\mu}(\bar{q})\, q_\nu\, \bar{u}_\alpha(\bar{p}, \bar{\chi})\, \gamma_5\, \gamma_\beta\, u(p, \chi)$$
$$\times \begin{cases} 0 & ,\ a=0 \\ e_1^{(A)}\, \Lambda^{nop}_{ab} & ,\ a=1,\cdots,8 \end{cases},$$

$$\langle \bar{p}, \bar{\chi}, nop \,|\, \bar{C}^{VA}_{\mu\nu,\,a}(\bar{q}, q) \,|\, p,\, \chi, klm\rangle =$$
$$-\frac{i}{4\, M_V}\, \epsilon^{\alpha\beta\rho\sigma}\, S_{\alpha\beta,\,\mu}(\bar{q})\, q_\nu\, \bar{u}_\sigma(\bar{p}, \bar{\chi})\, u_\rho(p, \chi)$$
$$\times \begin{cases} \left(\sqrt{\frac{2}{3}}\, d_1^{(E)} + \sqrt{\frac{3}{2}}\, d_2^{(E)}\right) \delta^{nop}_{klm} & ,\ a=0 \\ d_1^{(E)}\, \Lambda^{a,\,rst}_{klm}\, \delta^{nop}_{rst} & ,\ a=1,\cdots,8 \end{cases}. \qquad (3.43)$$

We close this chapter with a derivation of the non-relativistic limit of our results (3.37, 3.41, 3.43), which will be needed in Chapter V when deriving the large-N_c sum rules. For technical convenience we choose the center-of-momentum frame with $q_i = -p_i$ and $\bar{q}_i = -\bar{p}_i$. We focus on the space components of the correlation functions. The leading terms in the low-momentum expansion of the matrix elements of the product of the two currents are

$$\langle \bar{p}, \bar{\chi}, c|\, \bar{C}^{AA}_{ij,a}\, |p, \chi, b\rangle = \bar{p}_i\, p_j\, \delta_{\bar{\chi}\chi} \times \begin{cases} \left(2\,\sqrt{\frac{2}{3}}\, c_1^{(S)} + \sqrt{\frac{3}{2}}\, c_3^{(S)}\right) \delta^{bc} \ :\ a=0 \\ 2\, c_1^{(S)}\, d^{abc} + 2\, c_2^{(S)}\, i\, f^{abc} \ :\ a=1,\ldots,8 \end{cases},$$

$$\langle \bar{p}, \bar{\chi}, nop|\, \bar{C}^{AA}_{ij,a}\, |p, \chi, b\rangle = 0\,,$$

$$\langle \bar{p}, \bar{\chi}, nop|\, \bar{C}^{AA}_{ij,a}\, |p, \chi, klm\rangle = \bar{p}_i\, p_j\, \delta_{\bar{\chi}\chi} \times \begin{cases} \left(\sqrt{\frac{2}{3}}\, d_1^{(S)} + \sqrt{\frac{3}{2}}\, d_2^{(S)}\right) \delta^{nop}_{klm} \\ d_1^{(S)}\, \Lambda^{a,rst}_{klm}\, \delta^{nop}_{rst} \end{cases},$$

$$\langle \bar{p}, \bar{\chi}, c|\, \bar{C}^{VV}_{ij,a}\, |p, \chi, b\rangle = \delta_{ij}\, \delta_{\bar{\chi}\chi} \times \left\{ \begin{array}{r} \left(2\, \sqrt{\tfrac{2}{3}}\, \tilde{c}^{(S)}_1 + \sqrt{\tfrac{3}{2}}\, \tilde{c}^{(S)}_3\right) \delta^{bc} \\ 2\, \tilde{c}^{(S)}_1 d^{abc} + 2\, \tilde{c}^{(S)}_2 i f^{abc} \end{array} \right.$$

$$-i\varepsilon_{ijk}\, \sigma^{(k)}_{\bar{\chi}\chi} \times \left\{ \begin{array}{r} \left(2\, \sqrt{\tfrac{2}{3}}\, \tilde{c}^{(A)}_1 + \sqrt{\tfrac{3}{2}}\, \tilde{c}^{(A)}_3\right) \delta^{bc} \\ 2\, \tilde{c}^{(A)}_1 d^{abc} + 2\, \tilde{c}^{(A)}_2\, i\, f^{abc} \end{array} \right. ,$$

$$\langle \bar{p}, \bar{\chi}, nop|\, \bar{C}^{VV}_{ij,a}\, |p, \chi, b\rangle = -\frac{1}{2\sqrt{2}} \left(S_i \sigma_j\right)_{\bar{\chi}\chi} \times \left\{ \begin{array}{r} 0 \\ \tilde{e}^{(A)}_1 \Lambda^{nop}_{ab} \end{array} \right.$$

$$+\frac{1}{2\sqrt{2}} \left(S_j\, \sigma_i\right)_{\bar{\chi}\chi} \times \left\{ \begin{array}{r} 0 \\ \tilde{e}^{(A)}_2\, \Lambda^{nop}_{ab} \end{array} \right. ,$$

$$\langle \bar{p}, \bar{\chi}, nop|\, \bar{C}^{VV}_{ij,a}\, |p, \chi, klm\rangle = \delta_{ij}\, \delta_{\bar{\chi}\chi} \times \left\{ \begin{array}{r} \left(\sqrt{\tfrac{2}{3}}\, \tilde{d}^{(S)}_1 + \sqrt{\tfrac{3}{2}}\, \tilde{d}^{(S)}_2\right) \delta^{nop}_{klm} \\ \tilde{d}^{(S)}_1\, \Lambda^{a,rst}_{klm}\, \delta^{nop}_{rst} \end{array} \right.$$

$$+\frac{1}{2} \left(S_i S^\dagger_j\right)_{\bar{\chi}\chi} \times \left\{ \begin{array}{r} \left(\sqrt{\tfrac{2}{3}}\, \tilde{d}^{(E)}_1 + \sqrt{\tfrac{3}{2}}\, \tilde{d}^{(E)}_2\right) \delta^{nop}_{klm} \\ \tilde{d}^{(E)}_1 \Lambda^{a,rst}_{klm}\, \delta^{nop}_{rst} \end{array} \right.$$

$$-\frac{1}{2} \left(S_j\, S^\dagger_i\right)_{\bar{\chi}\chi} \times \left\{ \begin{array}{r} \left(\sqrt{\tfrac{2}{3}}\, \tilde{d}^{(E)}_3 + \sqrt{\tfrac{3}{2}}\, \tilde{d}^{(E)}_4\right) \delta^{nop}_{klm} \\ \tilde{d}^{(E)}_3\, \Lambda^{a,rst}_{klm}\, \delta^{nop}_{rst} \end{array} \right. ,$$

$$\langle \bar{p}, \bar{\chi}, c|\, \bar{C}^{VA}_{ij,a}\, |p, \chi, b\rangle = p_j\, \sigma^{(i)}_{\bar{\chi}\chi} \times \left\{ \begin{array}{r} \left(2\, \sqrt{\tfrac{2}{3}} c^{(A)}_1 + \sqrt{\tfrac{3}{2}}\, c^{(A)}_3\right) \delta^{bc} \\ 2\, c^{(A)}_1\, d^{abc} + 2\, c^{(A)}_2\, i\, f^{abc} \end{array} \right. ,$$

$$\langle \bar{p}, \bar{\chi}, nop|\, \bar{C}^{VA}_{ij,a}\, |p, \chi, b\rangle = -\frac{1}{2\sqrt{2}}\, p_j\, S^{(i)}_{\bar{\chi}\chi} \times \left\{ \begin{array}{r} 0 \\ e^{(A)}_1 \Lambda^{nop}_{ab} \end{array} \right. ,$$

$$\langle \bar{p}, \bar{\chi}, nop|\, \bar{C}^{VA}_{ij,a}\, |p, \chi, klm\rangle =$$

$$-\frac{1}{2}\, p_j \left(\vec{S} \sigma^{(i)}\, \vec{S}^\dagger\right)_{\bar{\chi}\chi} \times \left\{ \begin{array}{r} \left(\sqrt{\tfrac{2}{3}}\, d^{(E)}_1 + \sqrt{\tfrac{3}{2}}\, d^{(E)}_2\right) \delta^{nop}_{klm} \\ d^{(E)}_1 \Lambda^{a,rst}_{klm}\, \delta^{nop}_{rst} \end{array} \right. . \qquad (3.44)$$

The flavor transition tensors δ^{nop}_{klm}, Λ^{nop}_{ab} and $\Lambda^{c,nop}_{klm}$ used above, were already introduced in (3.39). The spin matrices $\vec{\sigma}$ and $\vec{S}$ are given in the Appendix B.

CHAPTER IV

HEAVY-QUARK SYMMETRY

In this chapter, we will explore another symmetry of QCD that arises in the limit of a large quark mass. We are primarily interested in properties of the D mesons, which were introduced already in the previous chapter. At the end of this chapter we will correlate the coupling constants of the chiral Lagrangians (3.23).

4.1 Heavy-quark expansion in QCD

The mass of a heavy-quark field is denoted by M_Q. In QCD a heavy-quark field may be decomposed with

$$Q(x) \;=\; e^{-i\,M_Q\,v\cdot x}\,h_v^{(+)}(x) + e^{+i\,M_Q\,v\cdot x}\,h_v^{(-)}(x)\,, \tag{4.1}$$

into a rapidly varying phase function and slowly varying residual fields $h_v^{(+)}$ and $h_v^{(-)}$ [see e.g. (Georgi, 1990)]. The latter are called upper and lower components and can be obtained from the original field by

$$\begin{aligned} h_v^{(+)}(x) &= e^{+i\,M_Q\,v\cdot x}\,\frac{1}{2}\,(1+\not{v})\,Q(x)\,, \\ h_v^{(-)}(x) &= e^{-i\,M_Q\,v\cdot x}\,\frac{1}{2}\,(1-\not{v})\,Q(x)\,, \end{aligned} \tag{4.2}$$

with the projection matrices

$$\frac{1}{2}\,(1\pm\not{v})\,. \tag{4.3}$$

The momentum p_μ associated with the quark-field operator is given by

$$p_\mu \simeq M_Q\,v_\mu\,, \qquad v_\mu\,v^\mu = 1\,, \tag{4.4}$$

in terms of the 4-velocity v_μ in the limit where the slow fields $h_v^{(\pm)}(x)$ are constants.

The projection matrices (4.3) have the properties (Manohar and Wise, 2000)

$$\left(\tfrac{1}{2}\,(1 \pm \not{v})\right)^2 = 1\,, \quad \tfrac{1}{2}\,(1+\not{v}) + \tfrac{1}{2}\,(1-\not{v}) = 1\,, \quad \tfrac{1}{2}\,(1 \pm \not{v})\,\tfrac{1}{2}\,(1 \mp \not{v}) = 0\,,$$
$$\not{v}\,h_v^{(+)} = +h_v^{(+)}\,, \qquad \tfrac{1}{2}\,(1+\not{v})\,\gamma^\mu = \gamma^\mu\,\tfrac{1}{2}\,(1-\not{v}) + v^\mu\,,$$
$$\not{v}\,h_v^{(-)} = -\,h_v^{(-)}\,, \qquad \tfrac{1}{2}\,(1-\not{v})\,\gamma^\mu = \gamma^\mu\,\tfrac{1}{2}\,(1+\not{v}) - v^\mu\,. \tag{4.5}$$

From the definitions of the residual fields in (4.2) it follows that $h_v^{(+)}$ destroys a heavy-quark with velocity v and $h_v^{(-)}$ creates an anti-heavy quark with velocity v.

Applying the decomposition of the heavy-quark fields to the QCD Lagrangian and considering only the quark in the infinite quark-mass limit, one obtains

$$\begin{aligned}
\mathscr{L}_{\mathrm{QCD}}^{(v)} &= \bar{Q}\,\{i\,\gamma^\mu\,D_\mu - M_Q\}\,Q \\
&= \bar{h}_v^{(+)}\left\{i\,v\cdot D + \frac{1}{2\,M_Q}\Big((i\,D)^2 - (i\,v\cdot D)^2\Big) + \frac{1}{4\,M_Q}\,g_s\,\sigma^{\mu\nu}\,G_{\mu\nu}\right\}h_v^{(+)} \\
&+ \bar{h}_v^{(-)}\left\{i\,v\cdot D + \frac{1}{2\,M_Q}\Big((i\,D)^2 - (i\,v\cdot D)^2\Big) - \frac{1}{4\,M_Q}\,g_s\,\sigma^{\mu\nu}\,G_{\mu\nu}\right\}h_v^{(-)} \\
&+ \mathcal{O}\left(\frac{1}{M_Q^2}\right),
\end{aligned} \tag{4.6}$$

where $D_\mu = \partial_\mu + i\,g_s\,A_\mu$ is the covariant derivative and A_μ are the gluon fields. To get the expression in (4.6), one needs to apply the identities in (4.5) together with the set of equations of motion of the $h_v^{(\pm)}$ fields (Riazuddin and Fayyazuddin, 1993). We derive

$$\begin{aligned}
&e^{-\,i\,M_Q\,v\cdot x}\,i\,\gamma^\mu\,D_\mu\,h_v^{(+)}(x) + e^{+\,i\,M_Q\,v\cdot x}\,\{i\,\gamma^\mu\,D_\mu - 2\,M_Q\}\,h_v^{(-)}(x) = 0\,, \\
&\bar{h}_v^{(-)}(x)\,i\,\gamma^\mu\,\overleftarrow{D}_\mu\,e^{-\,i\,M_Q\,v\cdot x} + \bar{h}_v^{(+)}(x)\left\{i\,\gamma^\mu\,\overleftarrow{D}_\mu + 2\,M_Q\right\}e^{+\,i\,M_Q\,v\cdot x} = 0\,, \\
&\qquad \rightarrow \\
&e^{+\,i\,M_Q\,v\cdot x}h_v^{(-)}(x) = \frac{e^{-\,i\,M_Q\,v\cdot x}}{2\,M_Q + i\,v\cdot\,D}\,\{i\,\gamma^\mu\,D_\mu - \,i\,v\cdot\,D\}\,h_v^{(+)}(x)\,,
\end{aligned} \tag{4.7}$$

$$\bar{h}_v^{(+)}(x)\, e^{+i M_Q v\cdot x} = -\,\bar{h}_v^{(-)}(x)\, \frac{1}{2\, M_Q + i\, v \cdot \overleftarrow{D}} \left\{ i\, \gamma^\mu \overleftarrow{D}_\mu + i\, v \cdot \overleftarrow{D} \right\} e^{-2\, i\, M_Q\, v\cdot x} \,,$$

after appropriate projections. We have kept terms only to the first order in $\frac{1}{M_Q}$. In the infinite heavy-quark mass limit it holds (Georgi, 1990; Neubert, 1994)

$$\mathscr{L}_{\mathrm{HQET}}^{(v)} \;=\; \bar{h}_v^{(+)}\, i\, v \cdot D\, h_v^{(+)} + \bar{h}_v^{(-)}\, i\, v \cdot D\, h_v^{(-)} + \mathcal{O}\left(\frac{1}{M_Q}\right) . \qquad (4.8)$$

This Lagrangian is the so-call heavy-quark effective theory (HQET). There are a number of reviews on the heavy-quark symmetry, for example, (Georgi, 1991; Wise, 1991; Wise, 1993; Riazuddin and Fayyazuddin, 1993; Neubert, 1994; Casalbuoni et al., 1997; Manohar and Wise, 2000) and works referred to therein.

The Lagrangian (4.8) shows that the $h_v^{(+)}$ and $h_v^{(-)}$ fields are decoupled to leading order in the large quark-mass expansion. This means that the quark field $h_v^{(+)}$ does not interact with the anti-quark field $h_v^{(-)}$. In particular there is no pair creation of heavy quarks possible at this leading order. This leads to the so-called heavy-quark spin symmetry. Formally the effective Lagrangian is invariant under a spin rotation of the heavy-quark fields (Georgi, 1990; Neubert, 1994)

$$\begin{aligned} & h_v^{(\pm)} \xrightarrow{\mathrm{SU}_v(2)} e^{-i\,\theta\cdot S}\, h_v^{(\pm)} , \\ & S^{\mu} = -\frac{1}{4}\,\epsilon^{\mu\nu\alpha\beta}\, v_\nu\, \sigma_{\alpha\beta} = \frac{1}{4}\,\gamma_5\, [\, \not{v}\, ,\, \gamma^\mu\,]_- \,, \end{aligned} \qquad (4.9)$$

where θ_μ are infinitesimal parameters of the $\mathrm{SU}_v(2)$ spin group. The vector S_μ is a Pauli-Lubanski vector. For the particular case with $\vec{v} = 0$ it follows

$$S^{(i)} \;=\; \frac{1}{4}\,\gamma_5\,[\,\gamma^0\,,\,\gamma^i\,]_- = \frac{1}{2}\,\gamma_5\,\gamma^0\,\gamma^i = \frac{1}{2}\begin{pmatrix} \sigma^i & 0 \\ 0 & \sigma^i \end{pmatrix} , \qquad (4.10)$$

where σ^i are Pauli's 2×2 spin matrices.

Heavy quarks moving with different velocities correspond to different strong interaction processes. Therefore, the four velocity of a heavy quark is a good quantum number in the infinite quark-mass limit of QCD. This is the velocity superselection rule discussed in (Georgi, 1990). The effective Lagrangian must be a sum over all different velocities

$$\mathscr{L}_{\mathrm{HQET}} = \int \frac{d^4v}{v^0}\, \mathscr{L}^{(v)}_{\mathrm{HQET}}\,, \tag{4.11}$$

where v^0 is the time-component of the four velocity of the heavy quark v^μ.

4.2 Open-charm mesons and the heavy-quark spin symmetry

In the last section we have derived the heavy-quark spin symmetry. What are the consequences of that symmetry for the open-charm spectrum and the interaction of D mesons with the baryons?

In a first step consider a 0^- meson with one heavy quark. In a quark-model picture the state is a bound state of one anti-light and one heavy quark, where the spins of the two quarks add up to zero. Given the heavy-quark spin symmetry we should be able to change the spin of the heavy quark without changing the energy of the system. Obviously under a flip of the heavy-quark spin the quantum number of the state must change into a 1^-. Thus we expect a spin multiplet with degenerate 0^- and 1^- mesons in the heavy-quark mass limit (Isgur and Wise, 1989; Isgur and Wise, 1990; Riazuddin and Fayyazuddin, 1993). This expectation is confirmed by the empirical open-charm spectrum. The mass differences of the 0^- and 1^- mesons are of the order of the pion mass and therewith much smaller than the masses of the D mesons (Nakamura and Group, 2010).

We limit our attention to the pseudo-scalar and vector D mesons, which both contain one charm quark. The states were introduced already in the previous chapter. We recall the two flavor antitriplets $(D^0\,,\,-D^+\,,\,D_s^+)$ and $(D^{*,0}\,,\,-D^{*,+}\,,\,D_s^{*,+})$.

In order to work out the consequences of the heavy-quark spin symmetry for the D mesons we have to identify an appropriate field combination of the pseudo-scalar and vector D meson fields, that reflects the spin symmetry of the charm quark. Here we use the conventions of (Manohar and Wise, 2000; Lutz and Soyeur, 2008). We introduce a super multiplet field

$$H_v = \frac{1}{2}\,(1 + \not{v}\,)\,(\gamma_\mu\,P_+^\mu + i\,\gamma_5\,P_+)\;, \qquad v_\mu\,P_+^\mu = 0\,, \tag{4.12}$$

where P_+ and P_+^μ destroy a pseudo-scalar and vector D meson with four velocity v^μ, respectively. The condition $v_\mu\,P_+^\mu = 0$ excludes the presence of longitudinal vector meson modes. Note that here the H_v field is a 4×4 valued matrix field. The conjugation of the H_v field is given by

$$\bar{H}_v = \gamma_0\,H_v^\dagger\,\gamma_0 = \left(\bar{P}_+^\mu\,\gamma_\mu + i\,\bar{P}_+\,\gamma_5\right)\frac{1}{2}\,(1 + \not{v}\,)\,, \tag{4.13}$$

where $\bar{P}_+^\mu = (P_+^\mu)^\dagger$ and $\bar{P}_+ = P_+^\dagger$.

Under a spin rotation the H_v field transform like the heavy-quark fields $h_v^{(\pm)}$ of the previous section (see 4.9). According to (Georgi, 1990; Manohar and Wise, 2000; Lutz and Soyeur, 2008) it holds

$$H_v(x) \quad \overset{\mathrm{SU}_v(2)}{\longrightarrow} \; e^{-\,i\,\theta_\mu\,S^\mu}\,H_v\,, \tag{4.14}$$

$$\bar{H}_v(x) \quad \overset{\mathrm{SU}_v(2)}{\longrightarrow} \; \gamma_0\,\left(e^{-\,i\,\theta_\mu\,S^\mu}\,H_v\right)^\dagger\,\gamma_0 = \bar{H}_v\,e^{i\,\theta_\mu\,S^\mu}\,, \tag{4.15}$$

where θ_μ characterizes the spin rotation in $\mathrm{SU}_v(2)$ and S_μ is the Pauli-Lubansky vector introduced in (4.9). We note $S_\mu^\dagger\,\gamma_0 = \gamma_0\,S_\mu\,.$

The H_v field transforms under a Lorentz transformation as a bispinor (Manohar and Wise, 2000; Lutz and Soyeur, 2008)

$$\begin{aligned} H_v(x) \quad &\longrightarrow \quad D(\Lambda)\,H_v(\Lambda^{-1}\,x)\,D(\Lambda)^{-1} \\ &= \quad e^{+i\,S_{\mu\nu}\,\omega^{\mu\nu}}\,H_v(\Lambda^{-1}\,x)\,e^{-\,i\,S_{\mu\nu}\,\omega^{\mu\nu}}\;, \end{aligned} \tag{4.16}$$

where the Lorentz transformation Λ is characterized by the anti-symmetric Lorentz tensor $\omega^{\mu\nu}$. The spinor part of the Lorentz transformation $D(\Lambda)$ is implied by

$$S_{\mu\nu} = \frac{i}{4}\left[\, \gamma_\mu \,,\, \gamma_\nu \,\right] . \tag{4.17}$$

The Lorentz transformation of the conjugate H_v field is given by

$$\bar{H}_v \quad \longrightarrow \quad e^{i\, S_{\mu\nu}\, \omega^{\mu\nu}}\, \bar{H}_v\, e^{-\, i\, S_{\mu\nu}\, \omega^{\mu\nu}} \,. \tag{4.18}$$

As the application of the heavy-quark spin symmetry in our work we construct spin symmetric interaction terms involving two H_v fields and two baryon fields. We find the following eleven terms

$$\begin{aligned} \mathscr{L}_H \;=\; & -\frac{1}{2}\,\mathrm{Tr}\, H_v \Big\{ f_1^{(S)}\, (\bar{B}\, B)_+ + f_2^{(S)}\, (\bar{B}\, B)_- + \frac{1}{2}\, f_3^{(S)}\, \mathrm{tr}\,(\bar{B}\, B) \\ & \qquad - f_4^{(S)}\, (\bar{\Delta}_\alpha \cdot \Delta^\alpha) - \frac{1}{2}\, f_5^{(S)}\, \mathrm{tr}\,(\bar{\Delta}_\alpha \cdot \Delta^\alpha) \Big\}\, \bar{H}_v \\ & -\frac{1}{2}\,\mathrm{Tr}\, H_v \Big\{ f_1^{(A)}\, (\bar{B}\, \gamma_5\, \gamma^\mu\, B)_+ + f_2^{(A)}\, (\bar{B}\, \gamma_5\, \gamma^\mu\, B)_- \\ & \qquad + \frac{1}{2}\, f_3^{(A)}\, \mathrm{tr}\,(\bar{B}\, \gamma_5\, \gamma^\mu\, B) \Big\}\, \gamma_5\, \gamma_\mu\, \bar{H}_v \\ & +\frac{i}{4}\,\mathrm{Tr}\, H_v \Big\{ f_1^{(T)}\, (\bar{\Delta}_\mu \cdot \Delta_\nu) + \frac{1}{2}\, f_2^{(T)}\, \mathrm{tr}\,(\bar{\Delta}_\mu \cdot \Delta_\nu) \\ & \qquad + f_3^{(T)}\, \mathrm{tr}\,(\bar{\Delta}_\mu \gamma_5 \gamma_\nu \cdot B - \bar{B}\, \gamma_5\, \gamma_\nu \cdot \Delta_\mu) \Big\}\, \sigma^{\mu\nu}\, \bar{H}_v \,, \end{aligned} \tag{4.19}$$

where Tr is the trace in Dirac space of the H_v-field and tr is the trace in flavor space. The notation $(\bar{B}\,\Gamma\, B)_\pm$ was introduced already in (3.25). Note that only combinations where a Dirac matrix is right to the field H_v or left to the field $\bar{H}_v$ are invariant under the spin group $\mathrm{SU}_v(2)$.

The QCD action depends linearly on the charm-quark mass. The effective Lagrangian (4.19) should therefore be proportional to M_c. All of the coupling constants above must then scale with the charm-quark mass.

We close the section by an illustration of the predictive power of the spin symmetry. Substituting the definition of the H_v field into the Lagrangians (4.19), we obtain

$$
\begin{aligned}
\mathcal{L} \;=\;& P_+ \Big\{ f_1^{(S)} (\bar{B}\, B)_+ + f_2^{(S)} (\bar{B}\, B)_- + \frac{1}{2}\, f_3^{(S)} \,\mathrm{tr}\, \big(\bar{B}\, B\big) \Big\}\, \bar{P}_+ \\
& - P_{+\mu} \Big\{ f_1^{(S)} (\bar{B}\, B)_+ + f_2^{(S)} (\bar{B}\, B)_- + \frac{1}{2}\, f_3^{(S)} \,\mathrm{tr}\, \big(\bar{B}\, B\big) \Big\}\, \bar{P}_+^{\mu} \\
& - P_+ \Big\{ f_4^{(S)}\, \bar{\Delta}_\alpha \cdot \Delta^\alpha + \frac{1}{2}\, f_5^{(S)} \,\mathrm{tr}\, \big(\bar{\Delta}_\alpha \cdot \Delta^\alpha\big) \Big\}\, \bar{P}_+ \\
& + P_{+\mu} \Big\{ f_4^{(S)}\, \bar{\Delta}_\alpha \cdot \Delta^\alpha + \frac{1}{2}\, f_5^{(S)} \,\mathrm{tr}\, \big(\bar{\Delta}_\alpha \cdot \Delta^\alpha\big) \Big\}\, \bar{P}_+^{\mu} \\
& + i\, P_{+\mu} \Big\{ f_1^{(A)} (\bar{B}\, \gamma_5\, \gamma^\mu\, B)_+ + f_2^{(A)} (\bar{B}\, \gamma_5\, \gamma^\mu\, B)_- \\
& \qquad\qquad + \frac{1}{2}\, f_3^{(A)} \,\mathrm{tr}\, \big(\bar{B}\, \gamma_5\, \gamma^\mu\, B\big) \Big\}\, \bar{P}_+ \\
& - i\, P_+ \Big\{ f_1^{(A)} (\bar{B}\, \gamma_5\, \gamma^\mu\, B)_+ + f_2^{(A)} (\bar{B}\, \gamma_5\, \gamma^\mu\, B)_- \\
& \qquad\qquad + \frac{1}{2}\, f_3^{(A)} \,\mathrm{tr}\, \big(\bar{B}\, \gamma_5\, \gamma^\mu\, B\big) \Big\}\, \bar{P}_{+\mu} \\
& + i\, v_\mu\, \epsilon^{\mu\nu\alpha\beta}\, P_{+\nu} \Big\{ f_1^{(A)} (\bar{B}\, \gamma_5\, \gamma_\alpha\, B)_+ + f_2^{(A)} (\bar{B}\, \gamma_5\, \gamma_\alpha\, B)_- \\
& \qquad\qquad + \frac{1}{2}\, f_3^{(A)} \,\mathrm{tr}\, \big(\bar{B}\, \gamma_5\, \gamma_\alpha\, B\big) \Big\}\, \bar{P}_{+\beta} \\
& - \frac{1}{2}\, v_\mu\, \epsilon^{\mu\nu\alpha\beta}\, P_{+\nu} \Big\{ f_1^{(T)}\, \bar{\Delta}_\alpha \cdot \Delta_\beta + \frac{1}{2}\, f_2^{(T)} \,\mathrm{tr}\, \big(\bar{\Delta}_\alpha \cdot \Delta_\beta\big) \Big\}\, \bar{P}_+ \\
& \quad - \frac{1}{2}\, v_\mu\, \epsilon^{\mu\nu\alpha\beta}\, P_+ \Big\{ f_1^{(T)}\, \bar{\Delta}_\beta \cdot \Delta_\alpha + \frac{1}{2}\, f_2^{(T)} \,\mathrm{tr}\, \big(\bar{\Delta}_\beta \cdot \Delta_\alpha\big) \Big\}\, \bar{P}_{+\nu} \\
& + \frac{1}{2}\, P_{+\beta} \Big\{ f_1^{(T)}\, \bar{\Delta}_\alpha \cdot \Delta^\beta + \frac{1}{2}\, f_2^{(T)} \,\mathrm{tr}\, \big(\bar{\Delta}_\alpha \cdot \Delta^\beta\big) \Big\}\, \bar{P}_+^{\alpha} \\
& - \frac{1}{2}\, P_+^{\alpha} \Big\{ f_1^{(T)}\, \bar{\Delta}_\alpha \cdot \Delta^\beta + \frac{1}{2}\, f_2^{(T)} \,\mathrm{tr}\, \big(\bar{\Delta}_\alpha \cdot \Delta^\beta\big) \Big\}\, \bar{P}_{+\beta} \\
& + \frac{1}{2}\, \epsilon^{\mu\nu\alpha\beta}\, v_\alpha\, P_+ \Big\{ (f_3^{(T)}\, \bar{\Delta}_\mu \cdot \gamma_5\, \gamma_\nu B) - f_3^{(T)}\, \bar{B}\, \gamma_5\, \gamma_\nu \cdot \Delta_\mu \Big\}\, \bar{P}_{+\beta} \\
& + \frac{1}{2}\, \epsilon^{\mu\nu\alpha\beta}\, v_\alpha\, P_{+\beta} \Big\{ (f_3^{(T)}\, \bar{B} \gamma_5\, \gamma_\nu \cdot \Delta_\mu) - f_3^{(T)}\, \bar{\Delta}_\mu \cdot\, \gamma_5\, \gamma_\nu B \Big\}\, \bar{P}_+ \\
& - \frac{1}{2}\, f_3^{(T)}\, P_+^{\mu}\, (\bar{\Delta}_\mu \cdot \gamma_5\, \gamma_\nu B)\, \bar{P}_+^{\nu} - \frac{1}{2}\, f_3^{(T)}\, P_+^{\nu}\, (\bar{B}\, \gamma_5\, \gamma_\nu \cdot \Delta_\mu)\, \bar{P}_+^{\mu} \\
& + \frac{1}{2}\, f_3^{(T)}\, P_+^{\nu}\, (\bar{\Delta}_\mu \cdot \gamma_5\, \gamma_\nu B)\, \bar{D}^{\mu} + \frac{1}{2}\, f_3^{(T)}\, P_+^{\mu}\, (\bar{B}\, \gamma_5\, \gamma_\nu \cdot \Delta_\mu)\, \bar{P}_+^{\nu}\,,
\end{aligned}
\tag{4.20}
$$

where we used the convention

$$
\mathrm{Tr}\, \big(\gamma_5\, \gamma_\mu\, \gamma_\nu\, \gamma_\alpha\, \gamma_\beta\big) = -\, 4\, i\, \epsilon_{\mu\nu\alpha\beta}\,. \tag{4.21}
$$

With (4.20) we obtain 26 distinct interaction terms that are parameterized by the eleven coupling constants of (4.19). It remains to match the terms in (4.20) with the effective chiral Lagrangian introduced in (3.23). Note that the number of independent terms in (3.23) and (4.20) coincide.

4.3 Sum rules from the heavy-quark spin symmetry

Now we are ready to work out the implications of the heavy-quark symmetry for the chiral Lagrangian (3.23) that we have already constructed before. In this section, we will perform a matching of (3.23) with the results of the previous section (4.19, 4.20).

The first task is to relate the velocity dependent D meson fields in (4.19) with the relativistic fields used in (3.23). In analogy to the decomposition (4.1) of a heavy-quark field we decompose the relativistic D meson fields into rapidly varying phase factors and slow varying residual fields (Lutz and Soyeur, 2008). We write

$$
\begin{aligned}
D(x) \;\; &= e^{-i\,(v\cdot x)\,M_c}\,P_+(x) + e^{+i\,(v\cdot x)\,M_c}\,P_-(x)\,, \qquad (4.22)\\
D^{\mu\nu}(x) &= i\,e^{-i\,(v\cdot x)\,M_c}\left\{v^\mu\,P_+^\nu(x) - v^\nu\,P_+^\mu(x) + \frac{i}{M_c}\Big(\partial^\mu P_+^\nu(x) - \partial^\nu P_+^\mu(x)\Big)\right\}\\
&\quad + i\,e^{+i\,(v\cdot x)\,M_c}\left\{v^\mu\,P_-^\nu(x) - v^\nu\,P_-^\mu(x) - \frac{i}{M_c}\Big(\partial^\mu P_-^\nu(x) - \partial^\nu P_-^\mu(x)\Big)\right\},
\end{aligned}
$$

where the four velocity v^μ is normalized as $v^2 = 1$. The charm-quark mass parameter M_c was already introduced in (3.24). The time and spatial derivatives of the fields, $P_\pm$ and $P_\pm^\mu$, in (4.22) are small compared to $M_c\,P_\pm$ and $M_c\,P_\pm^\mu$ since the charm-quark mass is close to the mass of the D mesons. In the limit $M_c \to \infty$ such terms can be neglected. We identify the fields P_+ and P_+^μ with the fields introduced in (4.12).

It is useful to illustrate some properties of the slowly varying fields. We begin with the pseudo-scalar fields. The equation of motion is

$$\begin{aligned} &\partial^\mu\,\partial^\mu\,D(x) + M_c^2\,D(x) = 0 \\ \to \qquad &\big\{\partial_\nu\,\partial^\nu \mp 2\,M_c\,v^\nu\,\partial_\nu\big\}\,P_\pm(x) = 0\,, \end{aligned} \tag{4.23}$$

where we approximated the D-meson mass by M_c. The second line in (4.23) follows since the phase factors $e^{\pm i\,(v\cdot x)\,M_c}$ are significantly more rapidly varying than any of the residual fields $P_\pm(x)$. The two fields $P_+(x)$ and $P_-(x)$ decouple in the limit of $M_c \to \infty$. Similarly, the equation of motion for the vector field is given by (Ecker et al., 1989; Lutz and Soyeur, 2008)

$$\partial^\mu\,\partial^\alpha\,D^{\alpha\nu} - \partial^\nu\,\partial^\alpha\,D^{\alpha\mu} + M_c^2\,D^{\mu\nu} = 0 \tag{4.24}$$

where we approximated the D-meson mass by M_c again. Together with the decomposition of the D-meson field in (4.22), we get

$$\begin{aligned} \big\{\partial_\nu\,\partial^\nu \mp 2\,M_c\,v^\nu\,\partial_\nu\big\}\,P^\mu_\pm &= 0\,, \\ \mp\, i\,M_c\,v_\mu\,P^\mu_\pm + \partial_\mu\,P^\mu_\pm &= 0\,. \end{aligned} \tag{4.25}$$

From the above results we conclude that $v_\mu\,P^\mu_\pm$ vanishes in the heavy-quark masses limit.

Substituting the expansions of the D and $D_{\mu\nu}$ fields into the chiral Lagrangians in (3.24), (3.26) and (3.29), we obtain

$$\begin{aligned} \mathscr{L}^{(c)}_{HQS} = \;& P_+\Big\{c_1^{(S)}\,(\bar B\,B)_+ + c_2^{(S)}\,(\bar B\,B)_- + \frac{1}{2}\,c_3^{(S)}\,\mathrm{tr}\,\big(\bar B\,B\big)\Big\}\,\bar P_+ \\ & - P_{+\mu}\Big\{\tilde c_1^{(S)}\,(\bar B\,B)_+ + \tilde c_2^{(S)}\,(\bar B\,B)_- + \frac{1}{2}\,\tilde c_3^{(S)}\,\mathrm{tr}\,\big(\bar B\,B\big)\Big\}\,\bar P_+^\mu \\ & + i\,P_{+\mu}\Big\{c_1^{(A)}\,(\bar B\,\gamma_5\,\gamma^\mu\,B)_+ + c_2^{(A)}\,(\bar B\,\gamma_5\,\gamma^\mu\,B)_- + \frac{1}{2}\,c_3^{(A)}\,\mathrm{tr}\,\big(\bar B\,\gamma_5\,\gamma^\mu\,B\big)\Big\}\,\bar P_+ \\ & - i\,P_+\Big\{c_1^{(A)}\,(\bar B\,\gamma_5\,\gamma^\mu\,B)_+ + c_2^{(A)}\,(\bar B\,\gamma_5\,\gamma^\mu\,B)_- + \frac{1}{2}\,c_3^{(A)}\,\mathrm{tr}\,\big(\bar B\,\gamma_5\,\gamma^\mu\,B\big)\Big\}\,\bar P_{+\mu} \\ & + i\,v_\mu\,\epsilon^{\mu\nu\alpha\beta}\,P_{+\nu}\Big\{\tilde c_1^{(A)}\,(\bar B\,\gamma_5\,\gamma_\alpha\,B)_+ + \tilde c_2^{(A)}\,(\bar B\,\gamma_5\,\gamma_\alpha\,B)_- \end{aligned}$$

$$+\frac{1}{2}\,\tilde{c}_3^{(A)}\,\mathrm{tr}\,\left(\bar{B}\,\gamma_5\,\gamma_\alpha\,B\right)\Big\}\,\bar{P}_{+\beta}\,+\,\cdots\,,$$

$$\begin{aligned}
\mathscr{L}^{(d)}_{HQS} = &- P_+\left\{d_1^{(S)}\,\bar{\Delta}^\sigma\cdot\Delta_\sigma + \frac{1}{2}\,d_2^{(S)}\,\mathrm{tr}\,\left(\bar{\Delta}^\sigma\cdot\Delta_\sigma\right)\right\}\bar{P}_+ \\
&+ P_{+\mu}\left\{\tilde{d}_1^{(S)}\,\bar{\Delta}^\sigma\cdot\Delta_\sigma + \frac{1}{2}\,\tilde{d}_2^{(S)}\,\mathrm{tr}\,\left(\bar{\Delta}^\sigma\cdot\Delta_\sigma\right)\right\}\bar{P}_+^\mu \\
&- \frac{1}{2}\,v_\mu\,\epsilon^{\mu\nu\alpha\beta}\,P_{+\nu}\left\{d_1^{(E)}\,\bar{\Delta}_\alpha\cdot\Delta_\beta + \frac{1}{2}\,d_2^{(E)}\,\mathrm{tr}\,\left(\bar{\Delta}_\alpha\cdot\Delta_\beta\right)\right\}\bar{P}_+ \\
&- \frac{1}{2}\,v_\mu\,\epsilon^{\mu\nu\alpha\beta}\,P_+\left\{d_1^{(E)}\,\bar{\Delta}_\beta\cdot\Delta_\alpha + \frac{1}{2}\,d_2^{(E)}\,\mathrm{tr}\,\left(\bar{\Delta}_\beta\cdot\Delta_\alpha\right)\right\}\bar{P}_{+\nu} \\
&+ \frac{1}{2}\,P_{+\beta}\left\{\tilde{d}_1^{(E)}\,\bar{\Delta}_\alpha\cdot\Delta^\beta + \frac{1}{2}\,\tilde{d}_2^{(E)}\,\mathrm{tr}\,\left(\bar{\Delta}_\alpha\cdot\Delta^\beta\right)\right\}\bar{P}_+^\alpha \\
&- \frac{1}{2}\,P_+^\alpha\left\{\tilde{d}_3^{(E)}\,\bar{\Delta}_\alpha\cdot\Delta^\beta + \frac{1}{2}\,\tilde{d}_4^{(E)}\,\mathrm{tr}\,\left(\bar{\Delta}_\alpha\cdot\Delta^\beta\right)\right\}\bar{P}_{+\beta} + \cdots\,,
\end{aligned}$$

$$\begin{aligned}
\mathscr{L}^{(e)}_{HQS} = &\frac{1}{2}\,\epsilon^{\mu\nu\alpha\beta}\,v_\alpha\,P_+\left\{e_1^{(A)}\,\bar{\Delta}_\mu\cdot\gamma_5\,\gamma_\nu B - e_2^{(A)}\,\bar{B}\,\gamma_5\,\gamma_\nu\cdot\Delta_\mu\right\}\bar{P}_{+\beta} \\
&+ \frac{1}{2}\,\epsilon^{\mu\nu\alpha\beta}\,v_\alpha\,P_{+\beta}\left\{e_1^{(A)}\,\bar{B}\gamma_5\,\gamma_\nu\cdot\Delta_\mu - e_2^{(A)}\,\bar{\Delta}_\mu\cdot\,\gamma_5\,\gamma_\nu B\right\}\bar{P}_+ \\
&- \frac{1}{2}\,\tilde{e}_1^{(A)}\,P_+^\mu\,(\bar{\Delta}_\mu\cdot\gamma_5\,\gamma_\nu B)\,\bar{P}_+^\nu - \frac{1}{2}\,\tilde{e}_1^{(A)}\,P_+^\nu\,(\bar{B}\,\gamma_5\,\gamma_\nu\cdot\Delta_\mu)\,\bar{P}_+^\mu \\
&+ \frac{1}{2}\,\tilde{e}_2^{(A)}\,P_+^\nu\,(\bar{\Delta}_\mu\cdot\gamma_5\,\gamma_\nu B)\,\bar{D}^\mu + \frac{1}{2}\,\tilde{e}_2^{(A)}\,P_+^\mu\,(\bar{B}\,\gamma_5\,\gamma_\nu\cdot\Delta_\mu)\,\bar{P}_+^\nu + \cdots\,.
\end{aligned} \tag{4.26}$$

In the above equations we show only terms with P_+ and P_+^μ. Corresponding terms with P_- and P_-^μ are redundant and do not lead to additional sum rules. Note that our H_v field contains the P_+ and P_+^μ fields only. To match the P_- and P_-^μ fields an additional H_v^- field had to be introduced. In (4.26) the ellipses stand also for additional terms involving derivatives on the slow fields. The latter are suppressed in the heavy-quark mass expansion.

Finally, matching the coupling constants of (4.26) and (4.20), we derive the following 15 sum rules

$$\begin{aligned}
&c_1^{(S)} = \tilde{c}_1^{(S)} = f_1^{(S)}\,, \quad c_2^{(S)} = \tilde{c}_2^{(S)} = f_2^{(S)}\,, \quad c_3^{(S)} = \tilde{c}_3^{(S)} = f_3^{(S)}\,, \\
&d_1^{(S)} = \tilde{d}_1^{(S)} = f_4^{(S)}\,, \quad d_2^{(S)} = \tilde{d}_1^{(S)} = f_5^{(S)}\,, \\
&c_1^{(A)} = \tilde{c}_1^{(A)} = f_1^{(A)}\,, \quad c_2^{(A)} = \tilde{c}_2^{(A)} = f_2^{(A)}\,, \quad c_3^{(A)} = \tilde{c}_3^{(A)} = f_3^{(A)}\,,
\end{aligned}$$

$$d_1^{(E)} = \tilde{d}_1^{(E)} = \tilde{d}_3^{(E)} = f_1^{(T)}\,, \quad d_2^{(E)} = \tilde{d}_2^{(E)} = \tilde{d}_4^{(E)} = f_2^{(T)}\,,$$
$$e_1^{(A)} = e_2^{(A)} = \tilde{e}_1^{(A)} = \tilde{e}_2^{(A)} = f_3^{(T)}\,. \tag{4.27}$$

Having implemented the heavy-quark symmetry, out of the independent 26 parameters from chiral Lagrangian in (3.24 , 3.26 , 3.29) there remain 11 independent parameters only.

CHAPTER V

LARGE-N_C QCD

In this chapter, we discuss QCD in the limit of large numbers of colors (N_c). We give a short introduction to the large-N_c expansion in QCD, where we follow the works (Manohar, 1998; Lebed, 1999; Jenkins, 1998; Matagne, 2006; Semke, 2010). The final goal of this chapter will be the derivation of sum rules for the parameters of the chiral Lagrangian introduced in (3.23).

5.1 Large-N_c counting in QCD

QCD is a non-abelian gauge field theory of the SU(3) color group that we discussed already in Chapter II. Large-N_c QCD is a generalization of QCD from the color group SU(3) to SU(N_c) where N_c is considered to be a parameter of the theory ('t Hooft, 1974a).

Consider the gluon polarization at leading order in perturbation theory with an arbitrary number of colors. The corresponding diagram is depicted in Figure

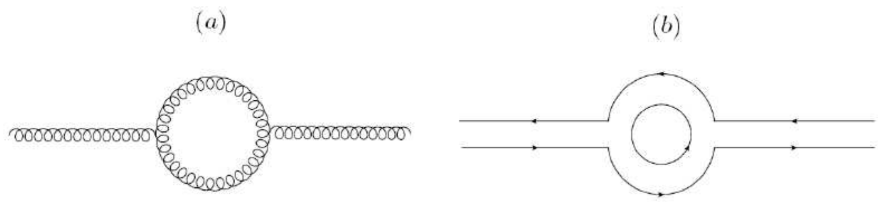

Figure 5.1 One-loop gluon self energy (a) quark-gluon representation (b) double-line representation.

$$\sum_a A_\mu^{(a)} \left(\lambda^{(a)}\right)^i_j \quad = \quad (A_\mu)^i_j$$

Figure 5.2 From the quark-gluon to the double-line representation.

5.1(a). Collecting all factors of N_c the diagram scales with $\sim g_s^2\, N_c$. In order to have a finite limit as $N_c \to \infty$ 't Hooft suggested to assume

$$g_s \sim \frac{1}{\sqrt{N_c}}\,. \tag{5.1}$$

In turn the gluon polarization approaches a finite value at $N_c \to \infty$.

To facilitate the derivation of the scaling power of a given diagram it is useful to represent Feynman diagrams in QCD by a double-line notation. As illustrated in Figure 5.2 a gluon-propagator line is replaced by a pair of effective quark-propagator lines. In such a diagram, the fundamental rather than the adjoint representation for the gluon fields is used. The color octet index a is replaced by the matric indices ij. A first example is given in Figure 5.1(b). Given the diagram in its double-line notation, the large-N_c scaling is determined by the number of closed lines and the number of gauge-coupling constants occurring in the original diagram. While each closed line gives a factor N_c, each coupling constant the

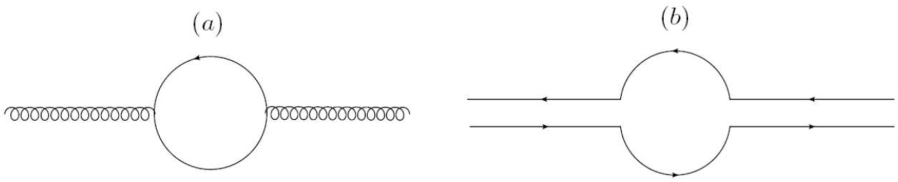

Figure 5.3 Quark-loop diagram for the gluon self energy. (a) quark-gluon representation (b) double-line representation.

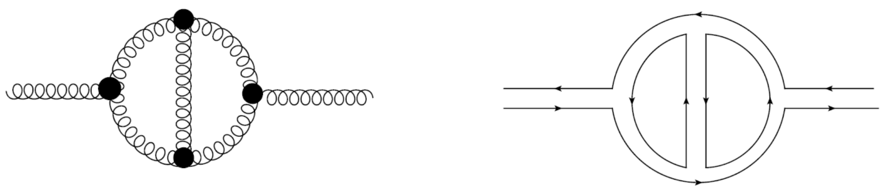

Figure 5.4 Two-loop contribution to the gluon-self energy. This planar diagram scales with N_c^0.

factor $1/\sqrt{N_c}$.

One can show that all leading order diagrams in the large-N_c expansion all planar. A planar diagram is a Feynman diagram that can be drawn in a 2-dimensional plane such that no lines are crossed. The large-N_c expansion is a non-perturbative method since there is an infinite class of quark-gluon diagrams contributing to a given order. In Figure 5.3 and Figure 5.4 we give two further examples of planar diagrams. The first figure shows the quark-loop contribution to the gluon propagator. From the coupling constant we obtain the suppression factor $1/N_c$. This is the final scaling power of the diagram since there is no closed loop in its double-line notation. The second figure shows a two-loop diagram contributing to the gluon propagator. The diagram in Figure (5.4) has four vertices with three

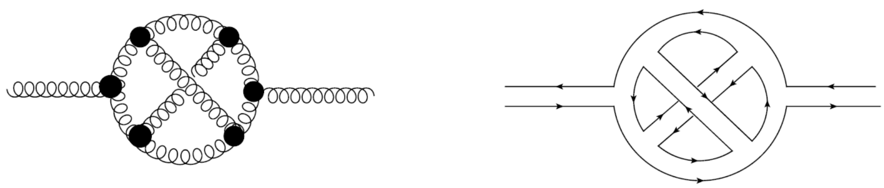

Figure 5.5 Two-loop contribution to the gluon-self energy. This non-planar diagram scales with N_c^{-2}.

gluons lines and two closed quark lines. Altogether it scales with $\left(1/\sqrt{N_c}\right)^4 N_c^2 = N_c^0$. This diagram survives the limit $N_c \to \infty$. An example for a non-planar diagram is given in Figure 5.5. The diagram is a two-loop contribution to the gluon propagator, where two gluon lines cross each other. It has six vertices with three gluon lines and one closed quark line. Thus it scales with $\left(1/\sqrt{N_c}\right)^6 N_c = N_c^{-2}$ and therefore is suppressed in the limit $N_c \to \infty$.

We close this section with a summary of the N_c-counting rules for diagram in large-N_c QCD:

- Three gluons vertices and quark-gluon vertices scale with $N_c^{-\frac{1}{2}}$.
- Four gluons vertices scale with N_c^{-1}.
- Any closed quark line of a Feynman diagram in its double-line representation implies the factor N_c.
- Non-planar diagrams are suppressed at least by the factor N_c^{-2}.

5.2 Mesons in Large-N_c in QCD

As a first simple application of the large-N_c counting rules presented in the previous section we consider mesonic systems as studied in ('t Hooft, 1974b) and (Witten, 1979). According to Witten and 't Hooft mesons are stable and non-interacting particles in large-N_c QCD ('t Hooft, 1974b; Witten, 1979). Meson masses take a smooth and finite value in the large-N_c limit. For a given quantum number there is an infinite tower of states.

Consider a quark bilinear operator J with

$$J(x) = \bar{q}(x)\,\Gamma\,q(x)\,, \tag{5.2}$$

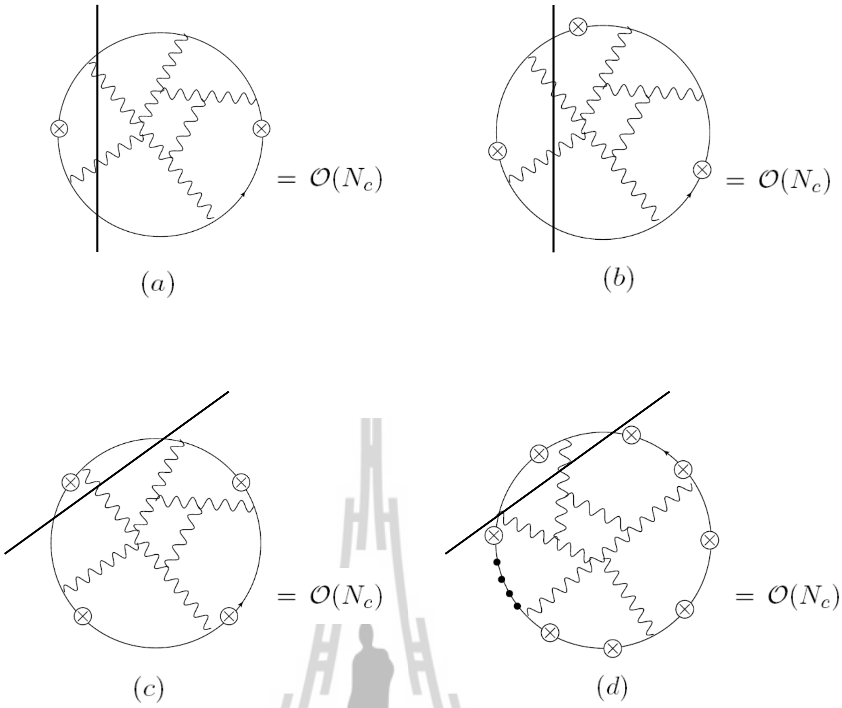

Figure 5.6 Typical planar diagrams for n-point functions. (a) two-point function, (b) three-point function, (c) four-point function, (d) n-point function.

where Γ is a Dirac and flavor matrix diagonal in color space. For an appropriate matrix Γ the operator J is an interpolating field for a meson state $|n\rangle$ of a given quantum number, i.e. it holds

$$\langle 0|\, J(0)\,|\, n\rangle \equiv f_n \neq 0\,. \tag{5.3}$$

How does the parameter f_n scale with N_c? In order to find out we study the two-point function

$$\langle 0\,|\, J(x)\, J(y)\,|\, 0\rangle = \sum_n e^{-i\,p_n\cdot(x-y)}\, |\langle 0\,|\, J(0)\,|\, n\rangle|^2\,, \tag{5.4}$$

where we insert a complete set of states. Due to the color confinement in QCD only mesonic states will contribute in (5.4). The time-ordered version of (5.4) may be represented by quark-gluon diagrams of QCD, for which we know their large-N_c

scaling. At leading order only planar diagrams contribute. An example is drawn in Figure 5.6(a). Applying the large-N_c counting rules we find the two-point function to scale with N_c at leading order. Analogous results hold for the n-point functions

$$\langle 0 \,|\, T\, J(x)\, J(y)\, J(z) \,|\, 0\rangle = \mathcal{O}(N_c)\,, \qquad \langle 0 \,|\, T\, J(x)\, J(y)\, J(z)\, J(w) \,|\, 0\rangle = \mathcal{O}(N_c)\,,$$
$$\langle 0 \,|\, T\, J(x_1)\, J(x_2)\, \cdots\, J(x_n) \,|\, 0\rangle = \mathcal{O}(N_c). \tag{5.5}$$

This is illustrated in Figures 5.6(b)–5.6(d).

Taking the Fourier transform of the two-point function it follows

$$\int d^4x\, e^{-i\,q\,x}\, \langle 0 \,|\, T\, J(x)\, J(0) \,|\, 0\rangle = \sum_n \frac{|f_n|^2}{q^2 - m_n^2} = \mathcal{O}(N_c)\,, \tag{5.6}$$

where f_n was introduced in (5.3) and m_n is the mass of the meson state $|n\rangle$. The result (5.6) is illustrated by the cut line in Figure 5.6(a). Right and left to the cut there can only be states in a color singlet. The sum of all diagrams must lead to intermediate mesonic states as claimed in (5.6). Since the meson masses scale with $m_n \sim N_c^0$ it follows

$$f_n = \langle 0 \,|\, J(0) \,|\, n\rangle = \mathcal{O}(\sqrt{N_c})\,. \tag{5.7}$$

The scaling result (5.7) is easily generalized for vertices with n number of meson fields. According to the LSZ reduction scheme (Itzykson and Zuber, 1980; Peskin and Schroeder, 1995) an n-body scattering amplitude, or a $2\,n$-body mesonic vertex, can be extracted from the vacuum expectation value of the time-ordered product of $2\,n$ interpolating fields as studied in (5.5) and in Figures 5.6(b)–5.6(d). The scattering amplitude follows by dividing out the $2\,n$ wave-function renormalization factors, which all scale with $\sqrt{N_c}$ [see (5.7)]. Therefore a vertex with n meson fields scales with

$$N_c^{1-\frac{n}{2}}\,. \tag{5.8}$$

5.3 Baryons in large-N_c in QCD

In last section, we have studied the N_c-scaling of mesons in large-N_c QCD. The first study of baryons in large-N_c has been by Witten (Witten, 1979). In the large-N_c picture, baryons are bound states of N_c valence quarks with a completely anti-symmetric color wave function. The baryon masses grow linear with N_c. A striking prediction of large-N_c QCD is the approximate degeneracy of the baryon octet and decuplet states (Witten, 1979). The octet and decuplet states with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$ form a super multiplet in the large-N_c limit. The spin flip of a valence quark inside the baryon does not cost any energy. According to Witten a vertex with two baryon fields and n meson field scales with

$$\sim \left(\frac{1}{\sqrt{N_c}}\right)^n N_c = N_c^{1-n/2}\,. \tag{5.9}$$

In this section, we follow the works of Luty and March-Russell (1994); Dashen et al. (1995); Lutz and Semke (2011), where a formalism how to systematically expand baryon-matrix elements of QCD quark currents in powers of $1/N_c$ was developed. Here we will use the main results and basic ideas.

The large-N_c operator analysis of Luty and March-Russell (1994); Dashen et al. (1995); Lutz and Semke (2011) rewrites matrix elements in the physical baryon states $|p,\,\chi\rangle$ in terms of auxiliary states $|\chi)$ that reflect the spin and flavor structure of the baryons only. Here we use $|p,\,\chi\rangle$ as a synonym for the octet, $|p,\,\chi,\,a\rangle$, and decuplet states, $|p,\,\chi,\,ijk\rangle$, introduced already in (3.35). All dynamical information is moved into effective operators. The $1/N_c$ expansion takes the generic form

$$\langle \bar p,\,\bar\chi|\,\bar C_{\mu\nu,a}(\bar q,q)\,|p,\,\chi\rangle = \sum_n c_n(\bar p,p)(\bar\chi|\,O^{(n)}_{\mu\nu,a}\,|\chi)\,, \tag{5.10}$$

where we assume all 4-momenta to be on-shell. The correlation operator $\bar C_{\mu\nu,a}(\bar q,q)$ was introduced already in (3.34, 3.36). In the the center-of-momentum frame the two three vectors $\bar{p}$ and p characterize the kinematics. The effective operators $O^{(n)}_{\mu\nu,a}$ are composed out of spin, flavor and spin-flavor operators, $J_i, T^{(a)}$ and G^a_i, which act on the auxiliary states $|\chi)$. For $N_c = 3$ we recall the results of Lutz and Kolomeitsev (2002); Lutz and Semke (2011). It holds

$$
\begin{aligned}
& J_i\,|a,\chi) = \tfrac{1}{2}\,\sigma^{(i)}_{\bar{\chi}\chi}\,|a,\bar{\chi})\,, \\
& T^{(a)}\,|b,\chi) = i\,f_{bca}\,|c,\chi)\,, \\
& G^a_i\,|b,\chi) = \sigma^{(i)}_{\bar{\chi}\chi}\left(\tfrac{1}{2}\,d_{bca} + \tfrac{i}{3}\,f_{bca}\right)|c,\bar{\chi}) + \tfrac{1}{2}\sqrt{2}\,S^{(i)}_{\bar{\chi}\chi}\,\Lambda^{klm}_{ab}\,|klm,\bar{\chi})\,, \\
& J_i\,|klm,\chi) = \tfrac{3}{2}\left(\vec{S}\,\sigma_i\,\vec{S}^\dagger\right)_{\bar{\chi}\chi}|klm,\bar{\chi}), \\
& T^{(a)}\,|klm,\chi) = \tfrac{3}{2}\,\Lambda^{a,nop}_{klm}\,|nop,\chi), \\
& G^a_i\,|klm,\chi) = \tfrac{3}{4}\left(\vec{S}\,\sigma_i\,\vec{S}^\dagger\right)_{\bar{\chi}\chi}\Lambda^{a,nop}_{klm}\,|nop,\bar{\chi}) + \tfrac{1}{2}\sqrt{2}\left(S^\dagger_i\right)_{\bar{\chi}\chi}\Lambda^{ab}_{klm}\,|b,\bar{\chi})\,,
\end{aligned}
\tag{5.11}
$$

with the Pauli matrices σ_i and the transition matrices S_i characterized by

$$
\begin{aligned}
& S^\dagger_i\,S_j = \delta_{ij} - \tfrac{1}{3}\,\sigma_i\sigma_j\,, \qquad S_i\,\sigma_j - S_j\,\sigma_i = -i\,\varepsilon_{ijk}\,S_k\,, \qquad \vec{S}\cdot\vec{S}^\dagger = \mathbf{1}_{\,(4\times 4)}\,, \\
& \vec{S}^\dagger\cdot\vec{S} = 2\,\mathbf{1}_{\,(2\times 2)}\,, \qquad \vec{S}\cdot\vec{\sigma} = 0\,, \qquad \epsilon_{ijk}\,S_i\,S^\dagger_j = i\,\vec{S}\,\sigma_k\,\vec{S}^\dagger\,.
\end{aligned}
\tag{5.12}
$$

In (5.11) we apply the notation introduced in (3.39).

We return to the expansion (5.10). There are infinitely many terms one may write down. At a given order in the $1/N_c$ expansion a finite number of terms is relevant only. The counting is intricate since there is a subtle balance of suppression and enhancement effects. An r-body operator consisting of the r products of any of the spin and flavor operators receives the suppression factor N_c^{-r}. On the other hand baryon matrix elements taken at $N_c \neq 3$ are enhanced by factors of N_c for the flavor and spin-flavor operators. The counting advocated in (Dashen et al., 1995) may be summarized by the effective scaling laws

$$
J_i \sim \frac{1}{N_c}\,, \qquad T^{(a)} \sim N_c^0\,, \qquad G^a_i \sim N_c^0\,. \tag{5.13}
$$

The counting rules (5.13) by themselves are insufficient to arrive at significant results. At a given order in the $1/N_c$ expansion there still is an infinite number of terms contributing. Taking higher products of flavor and spin-flavor operators does not reduce the N_c scaling power. The systematic $1/N_c$ expansion is implied by a set of operator identities (Dashen et al., 1995; Lutz and Semke, 2011) which leads to the two reduction rules:

- All operator products in which two flavor indices are contracted using δ^{ab}, f^{abc} or d^{abc} or two spin indices on G's are contracted using δ_{ij} or ε_{ijk} can be eliminated.

- All operator products in which two flavor indices are contracted using symmetric or antisymmetric combinations of two different d and/or f symbols can be eliminated. The only exception to this rule is the antisymmetric combination $f^{acg}\, d^{bch} - f^{bcg}\, d^{ach}$.

As a consequence the infinite tower of spin-flavor operators truncates at any given order in the $1/N_c$ expansion.

We turn to the $1/N_c$ expansion of the baryon matrix elements of the operators in (3.36). We focus on the space components of the correlation functions. In application of the operator reduction rules, the baryon matrix elements of the product of two quark currents are expanded in powers of the effective one-body operators according to (5.10). The ansatz for the momentum dependence of the expansion coefficients in (5.10) is provided by the structure of the leading order terms in the corresponding low-energy expansion stated in (3.44). In the course of the construction of the various structures, parity and time-reversal transformation properties are taken into account. To the subleading order in the $1/N_c$-expansion we find the relevance of the following 11 effective operators

$$\begin{aligned}
\langle \bar p, \bar\chi|\, \bar C^{AA}_{ij,a}\,|p,\chi\rangle &= \bar p_i\, p_j\, (\bar\chi|\, g_1^{AA}\, T^{(a)} + \tfrac{1}{2}\, g_2^{AA}\, \big[J_k,\, G^a_k\big]_+ |\chi)\,, \\
\langle \bar p, \bar\chi|\, \bar C^{VV}_{ij,a}\,|p,\chi\rangle &= \delta_{ij}\, (\bar\chi|\, g_1^{VV}\, T^{(a)} + \tfrac{1}{2}\, g_2^{VV}\, \big[J_k,\, G^a_k\big]_+ |\chi) \\
&\quad + i\,\epsilon_{ijk} (\bar\chi|\, g_3^{VV}\, G^a_k + \tfrac{1}{2}\, g_4^{VV}\, \big[J_k,\, T^{(a)}\big]_+ |\chi) \\
&\quad + (\bar\chi| \tfrac{1}{2}\, g_5^{VV}\, \big[J_i,\, G^a_j\big]_+ + \tfrac{1}{2}\, g_6^{VV}\, \big[J_j,\, G^a_i\big]_+ |\chi)\,, \\
\langle \bar p, \bar\chi|\, \bar C^{VA}_{ij,a}\,|p,\chi\rangle &= p_j\, (\bar\chi|\, g_1^{VA}\, G^a_i + \tfrac{1}{2}\, g_2^{VA}\, \big[J_i,\, T^{(a)}\big]_+ + \tfrac{1}{2}\, g_3^{VA}\, i\,\varepsilon_{ikl}\, \big[J_k,\, G^a_l\big]_+ |\chi)\,.
\end{aligned} \tag{5.14}$$

In contrast to (5.11), where the flavor index a takes the values $1, ..., 8$, the flavor index a in (5.14) runs from 0 to 8 with the identification*

$$T^{(0)} = \sqrt{\frac{1}{6}}\, \mathbb{1}\,, \qquad G^0_i = \sqrt{\frac{1}{6}}\, J_i\,. \tag{5.15}$$

The matrix elements of the operators in (5.14) are obtained from the identities (5.11). For the one-body operators our results follow from (5.11) directly. For the symmetric combinations of two one-body effective operators consecutive applications of (5.11) yield

$$\begin{aligned}
&(d,\bar\chi|\big[J_i,\, J_j\big]_+|c,\chi) = \tfrac{1}{2}\,\delta_{ij}\,\delta_{\bar\chi\chi}\,\delta_{cd}\,, \\
&(d,\bar\chi|\big[J_i,\, T^{(a)}\big]_+|c,\chi) = \sigma^{(i)}_{\bar\chi\chi}\, i\, f_{cda}\,, \\
&(d,\bar\chi|\big[J_i,\, G^a_j\big]_+|c,\chi) = \delta_{ij}\,\delta_{\bar\chi\chi}\left\{\tfrac{1}{2}\, d_{cda} + \tfrac{i}{3}\, f_{cda}\right\}, \\
&(nop,\bar\chi|\big[J_i,\, G^a_j\big]_+|c,\chi) = \tfrac{1}{4}\sqrt{2}\,\Big(3\, i\,\varepsilon_{ijk}\, S_k + S_i\,\sigma_j + S_j\,\sigma_i\Big)_{\bar\chi\chi}\,\Lambda^{nop}_{ac}\,, \\
&(nop,\bar\chi|\big[J_i,\, J_j\big]_+|klm,\chi) = \left\{\tfrac{9}{2}\,\delta_{ij}\,\delta_{\bar\chi\chi} - 3\,\Big(S_i\, S^\dagger_j + S_j\, S^\dagger_i\Big)_{\bar\chi\chi}\right\}\delta^{nop}_{klm}\,, \\
&(nop,\bar\chi|\big[J_i,\, T^{(a)}\big]_+|klm,\chi) = \tfrac{9}{4}\,\Big(\vec S\,\sigma_i\,\vec S^\dagger\Big)_{\bar\chi\chi}\,\delta^{nop}_{rst}\,\Lambda^{a,rst}_{klm}\,, \\
&(nop,\bar\chi|\big[J_i,\, G^a_j\big]_+|klm,\chi) = \left\{\tfrac{9}{4}\,\delta_{ij}\,\delta_{\bar\chi\chi} - \tfrac{3}{2}\,\Big(S_i\, S^\dagger_j + S_j\, S^\dagger_i\Big)_{\bar\chi\chi}\right\}\delta^{nop}_{rst}\,\Lambda^{a,rst}_{klm}\,.
\end{aligned} \tag{5.16}$$

We are now well prepared to evaluate the matrix elements in (5.14), where we present results for the singlet with $a = 0$ and octet case with $a \neq 0$ separately.

*The factor $1/\sqrt{6}$ in (5.15) follows from the normalization of $\lambda^{(0)}$ and from the definition of the effective operators $T^{(a)}$ and G^a_i.

Using (5.11) and (5.16) we obtain for the singlet case

$$
\begin{aligned}
\langle \bar{p}, \bar{\chi} \,|\, \bar{C}^{AA}_{ij,0} \,|\, p, \chi \rangle &= \bar{p}_i \, p_j \,\sqrt{\tfrac{1}{6}} \,\Big\{ (3\, g_1^{AA} + \tfrac{3}{4}\, g_2^{AA})\, \delta^{bc}\, \delta_{\bar{\chi}\chi} \\
&\quad + (3\, g_1^{AA} + \tfrac{15}{4}\, g_2^{AA})\, \delta^{nop}_{klm}\, \delta_{\bar{\chi}\chi} \Big\}\,, \\
\langle \bar{p}, \bar{\chi} \,|\, \bar{C}^{VV}_{ij,0} \,|\, p, \chi \rangle &= \delta_{ij} \,\sqrt{\tfrac{1}{6}} \,\Big\{ (3\, g_1^{VV} + \tfrac{3}{4}\, g_2^{VV} - \tfrac{1}{4}\, g_5^{VV} - \tfrac{1}{4}\, g_6^{VV})\, \delta^{bc}\, \delta_{\bar{\chi}\chi} \\
&\quad + (3\, g_1^{VV} + \tfrac{15}{4}\, g_2^{VV} - \tfrac{9}{4}\, g_5^{VV} - \tfrac{9}{4}\, g_6^{VV})\, \delta^{nop}_{klm}\, \delta_{\bar{\chi}\chi} \Big\} \\
&\quad + i\, \epsilon_{ijk} \,\sqrt{\tfrac{1}{6}} \,\Big\{ \tfrac{1}{2}\, (g_3^{VV} + 3\, g_4^{VV})\, \delta^{bc}\, \sigma^{(k)}_{\bar{\chi}\chi} \Big\} \\
&\quad - \tfrac{3}{2}\sqrt{\tfrac{1}{6}}\, (S_i\, S_j^\dagger)_{\bar{\chi}\chi} \,\Big\{ g_3^{VV} + g_4^{VV} + g_5^{VV} + g_6^{VV} \Big\}\, \delta^{nop}_{klm} \\
&\quad + \tfrac{3}{2}\sqrt{\tfrac{1}{6}}\, (S_j\, S_i^\dagger)_{\bar{\chi}\chi} \,\Big\{ g_3^{VV} + g_4^{VV} - g_5^{VV} - g_6^{VV} \Big\}\, \delta^{nop}_{klm}\,, \\
\langle \bar{p}, \bar{\chi} \,|\, \bar{C}^{VA}_{ij,0} \,|\, p, \chi \rangle &= p_j \,\sqrt{\tfrac{1}{6}} \,\Big\{ (\tfrac{1}{2}\, g_1^{VA} + \tfrac{3}{2}\, g_2^{VA})\, \delta^{bc}\, \sigma^{(i)}_{\bar{\chi}\chi} \\
&\quad + (\tfrac{3}{2}\, g_1^{VA} + \tfrac{9}{2}\, g_2^{VA})\, \delta^{nop}_{klm}\, (\vec{S}\, \sigma^{(i)}\, \vec{S}^\dagger)_{\bar{\chi}\chi} \Big\}\,.
\end{aligned}
\tag{5.17}
$$

For the octet case we find

$$
\begin{aligned}
\langle \bar{p}, \bar{\chi} \,|\, \bar{C}^{AA}_{ij,a} \,|\, p, \chi \rangle &= \bar{p}_i \, p_j \,\Big\{ (g_1^{AA} + \tfrac{1}{2}\, g_2^{AA})\, i\, f^{abc}\, \delta_{\bar{\chi}\chi} + \tfrac{3}{4}\, g_2^{AA}\, d^{abc}\, \delta_{\bar{\chi}\chi} \\
&\quad + (\tfrac{3}{2}\, g_1^{AA} + \tfrac{15}{8}\, g_2^{AA})\, \Lambda^{a,rst}_{klm}\, \delta^{nop}_{rst}\, \delta_{\bar{\chi}\chi} \Big\}\,, \\
\langle \bar{p}, \bar{\chi} \,|\, \bar{C}^{VV}_{ij,a} \,|\, p, \chi \rangle &= \delta_{ij} \,\Big\{ (\tfrac{3}{4}\, g_2^{VV} - \tfrac{1}{4}\, g_5^{VV} - \tfrac{1}{4}\, g_6^{VV})\, d^{abc}\, \delta_{\bar{\chi}\chi} \\
&\quad + (g_1^{VV} + \tfrac{1}{2}\, g_2^{VV} - \tfrac{1}{6}\, g_5^{VV} - \tfrac{1}{6}\, g_6^{VV})\, i\, f^{abc}\, \delta_{\bar{\chi}\chi} \\
&\quad + (\tfrac{3}{2}\, g_1^{VV} + \tfrac{15}{8}\, g_2^{VV} - \tfrac{9}{8}\, g_5^{VV} - \tfrac{9}{8}\, g_6^{VV})\, \Lambda^{a,rst}_{klm}\, \delta^{nop}_{rst}\, \delta_{\bar{\chi}\chi} \Big\} \\
&\quad + i\, \epsilon_{ijk} \,\Big\{ \tfrac{1}{2}\, g_3^{VV}\, d^{abc}\, \sigma^{(k)}_{\bar{\chi}\chi} + (\tfrac{1}{3}\, g_3^{VV} + \tfrac{1}{2}\, g_4^{VV})\, i\, f^{abc}\, \sigma^{(k)}_{\bar{\chi}\chi} \Big\} \\
&\quad + \tfrac{1}{2\sqrt{2}}\, (S_i\, \sigma_j)_{\bar{\chi}\chi} \,\Big\{ g_3^{VV} - \tfrac{1}{2}\, g_5^{VV} + g_6^{VV} \Big\}\, \Lambda^{nop}_{ab} \\
&\quad + \tfrac{1}{2\sqrt{2}}\, (S_j\, \sigma_i)_{\bar{\chi}\chi} \,\Big\{ - g_3^{VV} + g_5^{VV} - \tfrac{1}{2}\, g_6^{VV} \Big\}\, \Lambda^{nop}_{ab} \\
&\quad - \tfrac{3}{4}\, (S_i\, S_j^\dagger)_{\bar{\chi}\chi} \,\Big\{ g_3^{VV} + \tfrac{3}{2}\, g_4^{VV} + g_5^{VV} + g_6^{VV} \Big\}\, \Lambda^{a,rst}_{klm}\, \delta^{nop}_{rst} \\
&\quad + \tfrac{3}{4}\, (S_j\, S_i^\dagger)_{\bar{\chi}\chi} \,\Big\{ g_3^{VV} + \tfrac{3}{2}\, g_4^{VV} - g_5^{VV} - g_6^{VV} \Big\}\, \Lambda^{a,rst}_{klm}\, \delta^{nop}_{rst}\,, \\
\langle \bar{p}, \bar{\chi} \,|\, \bar{C}^{VA}_{ij,a} \,|\, p, \chi \rangle &= p_j \,\Big\{ \tfrac{1}{2}\, g_1^{VA}\, d^{abc}\, \sigma^{(i)}_{\bar{\chi}\chi} + (\tfrac{1}{3}\, g_1^{VA} + \tfrac{1}{2}\, g_2^{VA})\, i\, f^{abc}\, \sigma^{(i)}_{\bar{\chi}\chi} \\
&\quad + (\tfrac{1}{2\sqrt{2}}\, g_1^{VA} - \tfrac{3}{4\sqrt{2}}\, g_3^{VA})\, \Lambda^{nop}_{ab}\, S^{(i)}_{\bar{\chi}\chi}
\end{aligned}
$$

$$+\,(\tfrac{3}{4}\,g_1^{VA}+\tfrac{9}{8}\,g_2^{VA})\,\Lambda^{a,rst}_{klm}\,\delta^{nop}_{rst}\,(\vec{S}\,\sigma^{(i)}\,\vec{S}^\dagger)_{\bar\chi\chi}\Big\}\,. \tag{5.18}$$

We now match the large-N_c results (5.17, 5.18) with the results from the chiral Lagrangian (3.44). As a consequence, the parameters of the chiral Lagrangian (3.23) are expressed in terms of the large-N_c coupling constants introduced in (5.14). There are three type of matching schemes possible. The following results are obtained by matching either the expressions for the $\bar{C}^{VV}_{ij,\,a}$ or $\bar{C}^{AA}_{ij,\,a}$ or $\bar{C}^{VA}_{ij,\,a}$ correlation functions. We obtain the three matching results

$$\begin{aligned}
&c_1^{(S)}=\frac{3}{8}\,g_2^{AA}, \qquad c_2^{(S)}=\frac{1}{2}\,g_1^{AA}+\frac{1}{4}\,g_2^{AA}, \qquad c_3^{(S)}=g_1^{AA}-\frac{1}{4}\,g_2^{AA},\\
&d_1^{(S)}=\frac{3}{2}\,g_1^{AA}+\frac{15}{8}\,g_2^{AA}, \qquad d_2^{(S)}=0\,,
\end{aligned} \tag{5.19}$$

and

$$\begin{aligned}
&\tilde c_1^{(S)}=\frac{3}{8}\,g_2^{VV}+\frac{1}{4}\,g_+^{VV}, \qquad \tilde c_2^{(S)}=\frac{1}{2}\,g_1^{VV}+\frac{1}{4}\,g_2^{VV}+\frac{1}{6}\,g_+^{VV},\\
&\tilde c_3^{(S)}=g_1^{VV}-\frac{1}{4}\,g_2^{VV}-\frac{1}{6}\,g_+^{VV},\\
&\tilde c_1^{(A)}=-\frac{1}{4}\,g_3^{VV}, \qquad \tilde c_2^{(A)}=-\frac{1}{6}\,g_3^{VV}-\frac{1}{4}\,g_4^{VV}, \qquad \tilde c_3^{(A)}=\frac{1}{6}\,g_3^{VV}-\frac{1}{2}\,g_4^{VV},\\
&\tilde d_1^{(S)}=\frac{3}{2}\,g_1^{VV}+\frac{15}{8}\,g_2^{VV}+\frac{9}{4}\,g_+^{VV}, \qquad \tilde d_2^{(S)}=0,\\
&\tilde d_1^{(E)}=\frac{3}{2}\,g_3^{VV}+\frac{9}{4}\,g_4^{VV}-3\,g_+^{VV}, \qquad \tilde d_2^{(E)}=\frac{3}{2}\,g_4^{VV},\\
&\tilde d_3^{(E)}=\frac{3}{2}\,g_3^{VV}+\frac{9}{4}\,g_4^{VV}+3\,g_+^{VV}, \qquad \tilde d_4^{(E)}=\frac{3}{2}\,g_4^{VV},\\
&\tilde e_1^{(A)}=g_3^{VV}+\frac{3}{2}\,g_-^{VV}-\frac{1}{2}\,g_+^{VV}, \qquad \tilde e_2^{(A)}=g_3^{VV}+\frac{3}{2}\,g_-^{VV}+\frac{1}{2}\,g_+^{VV},
\end{aligned} \tag{5.20}$$

with $g_\pm^{VV}=\frac{1}{2}\,(g_5^{VV}\pm g_6^{VV})$, and

$$\begin{aligned}
&c_1^{(A)}=\frac{1}{4}\,g_1^{VA}, \qquad c_2^{(A)}=\frac{1}{6}\,g_1^{VA}+\frac{1}{4}\,g_2^{VA}, \qquad c_3^{(A)}=-\frac{1}{6}\,g_1^{VA}+\frac{1}{2}\,g_2^{VA},\\
&e_1^{(A)}=-g_1^{VA}+\frac{3}{2}\,g_3^{VA},\\
&d_1^{(E)}=-\frac{3}{2}\,g_1^{VA}-\frac{9}{4}\,g_2^{VA}, \qquad d_2^{(E)}=-\frac{3}{2}\,g_2^{VA}.
\end{aligned} \tag{5.21}$$

Note that the parameter $e_2^{(A)}$ does not occur in the presented results here. The large-N_c operator expansion for this parameter is based on the analysis of the matrix elements of $\bar{C}^{AV}$, which may be decomposed in terms of additional parameters $g^{AV}_{1,2,3}$ in analogy to the decomposition of the matrix elements of $\bar{C}^{VA}$ in (5.14). The corresponding results follow from (5.21) by the replacements $e^{(A)}_1 \to e^{(A)}_2$ and $g^{VA}_{1,2,3} \to g^{AV}_{1,2,3}$. As a consequence it follows that $g^{VA}_{1,2} = g^{AV}_{1,2}$, but not necessarily $g^{VA}_3 = g^{AV}_3$.

Altogether we find 12 large-N_c parameters relevant at leading order. Compared with the 26 chiral parameters we expect a set of 14 sum rules. We group the sum rules into three parts

$$c^{(S)}_3 = 2\left(c^{(S)}_2 - c^{(S)}_1\right), \qquad d^{(S)}_1 = 3\left(c^{(S)}_1 + c^{(S)}_2\right), \qquad d^{(S)}_2 = 0\,,$$

$$c^{(A)}_3 = 2\left(c^{(A)}_2 - c^{(A)}_1\right), \qquad d^{(E)}_1 = -9\,c^{(A)}_2\,, \qquad d^{(E)}_2 = 2\left(2\,c^{(A)}_1 - 3\,c^{(A)}_2\right),$$

$$\begin{aligned}
&\tilde{c}^{(S)}_3 = 2\left(\tilde{c}^{(S)}_2 - \tilde{c}^{(S)}_1\right), \qquad \tilde{d}^{(S)}_1 = 3\left(\tilde{c}^{(S)}_1 + \tilde{c}^{(S)}_2\right) + \left(\tilde{e}^{(A)}_2 - \tilde{e}^{(A)}_1\right), \\
&\tilde{d}^{(S)}_2 = 0\,, \qquad \tilde{c}^{(A)}_3 = 2\left(\tilde{c}^{(A)}_2 - \tilde{c}^{(A)}_1\right), \qquad \tilde{d}^{(E)}_1 = -9\,\tilde{c}^{(A)}_2 - 3\left(\tilde{e}^{(A)}_2 - \tilde{e}^{(A)}_1\right), \\
&\tilde{d}^{(E)}_2 = \tilde{d}^{(E)}_4 = 2\left(2\,\tilde{c}^{(A)}_1 - 3\,\tilde{c}^{(A)}_2\right), \qquad \tilde{d}^{(E)}_3 = -9\,\tilde{c}^{(A)}_2 + 3\left(\tilde{e}^{(A)}_2 - \tilde{e}^{(A)}_1\right),
\end{aligned} \tag{5.22}$$

where the first and the third parts correlate the coupling constants describing the interactions of pseudoscalar and vector D mesons, respectively. The second part provides the analogous relations for the transition interactions, i.e. terms with one pseudoscalar and one vector D meson field.

We observe that given the third set of the sum rules in (5.22), the first two parts are recovered by applying the results of the heavy-quark mass expansion as summarized in (4.27). This is a remarkable result demonstrating the consistency of a combined heavy-quark and large-N_c expansion.

CHAPTER VI

DISCUSSIONS AND CONCLUSIONS

We derived sum rules for the leading order two-body counter terms of the chiral Lagrangian as implied by a combined heavy-quark and large-N_c analysis. There are altogether 26 independent terms in the chiral Lagrangian with baryon octet and decuplet fields that contribute to the D and D^* meson baryon scattering process at chiral order Q^0.

At leading order in the heavy-quark expansion we find the relevance of 11 operators only. Additional sum rules were derived from the $1/N_c$ expansion. Combining both expansions, the number of unknown parameters is further reduced to 5. At present such sum rules can not be confronted directly with empirical information. They are useful constraints in establishing a systematic coupled-channel effective field theory for D meson baryon scattering beyond the threshold region.

REFERENCES

REFERENCES

Adler, S. L. (1969). Axial vector vertex in spinor electrodynamics. **Phys. Rev.** 177: 2426–2438.

Artuso, M., Ayad, R., Boulahouache, C., Bukin, K., Dambasuren, E., Karamov, S., Majumder, G., Moneti, G. C., Mountain, R., Schuh, S., and Skwarnicki, T. (2001). Observation of new states decaying into $\Lambda_c^+ \pi^- \pi^+$. **Phys. Rev. Lett.** 86: 4479–4482.

Bell, J. S. and Jackiw, R. (1969). A PCAC puzzle: $\pi^0 \to \gamma\gamma$ in the sigma model. **Nuovo Cim. A** 60: 47–61.

Bernard, V. (2008). Chiral perturbation theory and baryon properties. **Prog. Par. Nucl. Phys.** 60: 82–160.

Burdman, G. and Donoghue, J. F. (1992). Union of chiral and heavy quark symmetries. **Phys. Lett. B** 280: 287–291.

Casalbuoni, R., Deandrea, A., Di Bartolomeo, N., Gatto, R., Feruglio, F., and Nardulli, G. (1997). Phenomenology of heavy meson chiral Lagrangians. **Phys. Rept.** 281: 145–238.

Dashen, R. F., Jenkins, E. E., and Manohar, A. V. (1995). Spin-Flavor Structure of Large N Baryons. **Phys. Rev. D** 51: 3697–3727.

Donoghue, F. J., E., G., and Holstein, R. B. (1995). **Dynamics of the standard model**. New York: Cambridge University Press.

Ecker, G., Gasser, J., Pich, A., and De Rafael, E. (1989). The role resonances in chiral perturbation theory. **Nucl. Phys. B** 321: 311–342.

Fuchs, T., Schindler, M. R., Gegelia, J., and Scherer, S. (2003). Power counting in baryon chiral perturbation theory including vector mesons. **Phys. Lett. B** 575: 11–17.

Gamermann, D., Garcia-Recio, C., Nieves, J., Salcedo, L. L., and Tolos, L. (2010). Exotic dynamically generated baryons with negative charm quantum number. **Phys. Rev. D** 81: 094016.

Gasser, J. and Leutwyler, H. (1984). Chiral perturbation theory to one loop. **Annals Phys.** 158: 142–210.

Gasser, J. and Leutwyler, H. (1985). Chiral perturbation theory: expansions in the mass of the strange quark. **Nucl. Phys. B** 250: 465–516.

Gasser, J., Sainio, M. E., and Švarc, A. (1988). Nucleons with chiral loops. **Nucl. Phys. B** 307: 779–853.

Georgi, H. (1984). **Weak interactions and modern particle theory**. Menlo Park: Benjamin/cummings.

Georgi, H. (1990). An effective theory for heavy quarks at low-enenrgies. **Phys. Lett. B** 240: 447–450.

Georgi, H. (1991). Heavy Quark Effective Theory. **TASI Lecture**: 259–396. HUTP-91-A039.

Gervais, J.-L. and Sakita, B. (1984). Large N QCD Baryon Dynamics: Exact Results from Its Relation to the Static Strong Coupling Theory. **Phys. Rev. Lett.** 52: 87–89.

Goldstone, J. (1961). Field Theories with Superconductor Solutions. **Nuovo Cim.** 19: 154–164.

Guo, F.-K., Hanhart, C., Krewald, S., and Meissner, U.-G. (2008). Subleading contributions to the width of the $D_{s0}*(2317)$. **Phys. Lett. B** 666: 251–255.

Isgur, N. and Wise, M. B. (1989). Weak Decays of Heavy Mesons in the Static Quark Approximation. **Phys. Lett. B** 232: 113–117.

Isgur, N. and Wise, M. B. (1990). Weak transition form-factors between heavy mesons. **Phys. Lett. B** 237: 527–530.

Itzykson, C. and Zuber, J. B. (1980). **Quantum field theory**. New York: Mcgraw-hill.

Jenkins, E. E. (1998). Large-N_c Baryons. **Ann. Rev. Nucl. Part. Sci.** 48: 81–119.

Jimenez-Tejero, C. E., Ramos, A., and Vidana, I. (2009). Dynamically generated open charmed baryons beyond the zero range approximation. **Phys. Rev. C** 80: 055206.

Krause, A. (1990). Baryon matrix elements of the vector current in chiral perturbation theory. **Helv. Phys. Acta** 63: 3–70.

Lebed, R. F. (1999). Phenomenology of large-N_c QCD. **Czech. J. Phys.** 49: 1273–1306.

Leutwyler, H. (1994). Principles of chiral perturbation theory. **arXiv**: hep–ph/9406283. Lectures given at the Workshop : Hadrons 1994, Brasil.

Luty, M. A. and March-Russell, J. (1994). Baryons from quarks in the 1/N expansion. **Nucl. Phys. B** 426: 71–93.

Lutz, M. F. M., Erni, W., Keshelashvili, I., Krusche, B., Steinacher, M., Heng, Y., Liu, Z., Liu, H., Shen, X., Wang, O., Xu, H., Becker, J., Feldbauer, F., Heinsius, F. H., Held, T., Koch, H., Kopf, B., Pelizaeus, M., Schroeder, T., Steinke, M., Wiedner, U., Zhong, J., Bianconi, A., Bragadireanu, M., Pantea, D., Tudorache, A., Tudorache, V., De Napoli, M., Giacoppo, F., Raciti, G., Rapisarda, E., Sfienti, C., Bialkowski, E., Budzanowski, A., Czech, B., Kistryn, M., Kliczewski, S., Kozela, A., Kulessa, P., Pysz, K., Schaefer, W., Siudak, R., Szczurek, A., Czy.zycki, W., Domagala, M., Hawryluk, M., Lisowski, E., Lisowski, F., Wojnar, L., and Gil, D. (2009). Physics Performance Report for PANDA: Strong Interaction Studies with Antiprotons. **arXiv**: 0903.3905 [hep–ex].

Lutz, M. F. M. and Kolomeitsev, E. E. (2004). On charm baryon resonances and chiral symmetry. **Nucl. Phys. A** 730: 110–120.

Lutz, M. F. M. and Semke, A. (2011). Large-N_c operator analysis of 2-body meson-baryon counterterms in the chiral Lagrangian. **Phys. Rev. D** 83: 034008.

Lutz, M. F. M. and Soyeur, M. (2008). Radiative and isospin-violating decays of Ds mesons in the hadrogenesis conjecture. **Nucl. Phys. A** 813: 14–95.

Manohar, A. V. (1998). Large-N_c QCD. **arXiv**: hep–ph/9802419. Lectures at the 1997 Les Houches Summer School.

Manohar, A. V. and Wise, M. B. (2000). **Heavy quark physics**. New York: Cambridge University Press.

Matagne, N. (2006). Baryon resonances in large-N_c QCD. **arXiv**: hep–ph/0701061.

Mehen, T. and Springer, R. P. (2004). Heavy-quark symmetry and the electromagnetic decays of excited charmed strange mesons. **Phys. Rev. D** 70: 074014.

Nakamura, K. and Group, P. D. (2010). Review of particle physics. **J. Phys. G** 37: 075021.

Nambu, Y. (1960). Quasi-particles and gauge invariance in the theory of superconductivity. **Phys. Rev.** 117: 648–663.

Neubert, M. (1994). Heavy quark symmetry. **Phys. Rept.** 245: 259–396.

Peskin, M. and Schroeder, D. (1995). **An introduction to quantum field theory**. Perseus Books.

Pich, A. (1998). Effective field theory. **arXiv**: hep–ph/9806303. Lectures at the 1997 Les Houches Summer School.

Riazuddin and Fayyazuddin (1993). Heavy quark spin symmetry and old formalism. Conference on Highlights of Particle and Condensed Matter Physics (SALAMFEST), Trieste, Italy, 8-12 Mar 1993.

Scherer, S. (2003). Introduction to chiral perturbation theory. **Adv. Nucl. Phys.** 27: 277.

Scherer, S. (2010). Chiral Perturbation Theory: Introduction and Recent Results in the One-Nucleon Sector. **Prog. Part. Nucl. Phys.** 64: 1–60.

Semke, A. (2010). **On the quark-mass dependence of baryon ground-state masses**. TU Darmstadt.

't Hooft, G. (1974a). A planar diagram theory for strong interactions. **Nucl. Phys. B** 72: 461–473.

't Hooft, G. (1974b). A Two-Dimensional Model for Mesons. **Nucl. Phys. B** 75: 461–470.

Tolos, L., Schaffner-Bielich, J., and Mishra, A. (2004). Properties of D-mesons in nuclear matter within a self- consistent coupled-channel approach. **Phys. Rev. C** 70: 025203.

Wise, M. B. (1991). New symmetries of the strong interaction. Lectures given at Lake Louise Winter Inst., Lake Louise, Canada, Feb 17-23, 1991.

Wise, M. B. (1992). Chiral perturbation theory for hadrons containing a heavy quark. **Phys. Rev. D** 45: 2188–2191.

Wise, M. B. (1993). Combining chiral and heavy quark symmetry. **arXiv**: hep–ph/9306277. Lectures given at the CCAST Symposium on Particle Physics at the Fermi Scale May27-June4,1993.

Witten, E. (1979). Baryons in the $1/N$ Expansion. **Nucl. Phys. B** 160: 57–115.

Yan, T.-M., Cheng, H.-Y., Cheung, C.-Y., Lin, G.-L., Lin, Y. C., and Yu, H.-L. (1992). Heavy quark symmetry and chiral dynamics. **Phys. Rev. D** 46: 1148–1164.

APPENDICES

APPENDIX A

CHIRAL POWER COUNTING

In this appendix, we summarize about the chiral power counting scheme for heavy fields in Chiral Lagrangian. Such scheme is very useful to identify a low-energy expansion. The parameter Q plays an important role in this framework. The Q parameter has meaning as small external momenta or the quarks masses (Krause, 1990; Lutz and Semke, 2011; Semke, 2010). The field operators in this thesis are characterized by

- D-meson fields:

$$D,\ \bar{D},\ D_{\mu\nu},\ \bar{D}_{\mu\nu}\ \sim \mathcal{O}(Q^0) \tag{A.1}$$

- Baryon fields:

$$B,\ \bar{B},\ \Delta_\mu,\ \bar{\Delta}_\mu\ \sim \mathcal{O}(Q^0) \tag{A.2}$$

- The elements of Clifford's algebra:

$$\mathbf{1},\ \gamma_\mu,\ \gamma_5\,\gamma_\mu,\ \sigma_{\mu\nu}\ \sim \mathcal{O}(Q^0)\,, \qquad \gamma_5\ \sim \mathcal{O}(Q)\,. \tag{A.3}$$

One can use above counting rule to check chiral power of the chiral Lagrangians $(3.24, 3.26, 3.29)$ which introduced in Chapter III. According to the Chapter III, we found that D-mesons and baryons are not massless particles in the chiral limit. From this reason, the four momenta p_μ of such fields have dimensionless in the chiral power counting scheme. So, the chiral power counting of the four-partial derivative is assigned by $i\,\partial_\mu \sim \mathcal{O}(Q^0)\,$.

APPENDIX B

NON-RELATIVISTIC EXPANSION

In this appendix, we derive the non-relativistic expansion of the expressions

$$\bar{u}(\vec{p}\,', s')\,\Gamma\,u(\vec{p}, s)\,, \qquad \bar{u}^{\mu}(\vec{p}\,', s')\,\Gamma\,u^{\nu}(\vec{p}, s)\,, \qquad \bar{u}^{\mu}(\vec{p}\,', s')\,\Gamma\,u(\vec{p}, s)\,, \tag{B.1}$$

with a Dirac matrix

$$\Gamma \;\in\; \mathbf{1}_{4\times 4},\, \gamma_5,\, \gamma^{\mu},\, \gamma_5\,\gamma^{\mu},\, \sigma^{\mu\nu},\, \gamma_5\,\sigma^{\mu\nu}\,. \tag{B.2}$$

The non-relativistic expansion is an expansion in powers of $\vec{p}/M$ and $\vec{p}\,'/M$ where we assume a degenerate mass, M, for the spin 1/2 and 3/2 particles. We use the convention and notation of (Itzykson and Zuber, 1980) with

$$\gamma^0 = \begin{pmatrix} \mathbf{1}_{2\times 2} & 0 \\ 0 & -\mathbf{1}_{2\times 2} \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma_5 = \gamma^5 = \begin{pmatrix} 0 & \mathbf{1}_{2\times 2} \\ \mathbf{1}_{2\times 2} & 0 \end{pmatrix},$$

$$\sigma^{0j} = i \begin{pmatrix} 0 & \sigma^j \\ \sigma^j & 0 \end{pmatrix}, \quad \sigma^{ij} = \epsilon_{ijk} \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}, \tag{B.3}$$

where $\mathbf{1}_{n\times n}$ is the identity matrix in n dimension, σ^i the Pauli matrices and ϵ_{ijk} the antisymmetric tensor with $\epsilon_{123} = 1$.

We recall the wave function of a spin-$\frac{1}{2}$ particle with

$$u(\vec{p}, s) = N_p \begin{pmatrix} \chi_{1/2}(s) \\ \frac{\sigma\cdot\vec{p}}{E_p+M}\,\chi_{1/2}(s) \end{pmatrix}, \qquad \bar{u}(p, s) = u^{\dagger}(p, s)\,\gamma^0\,,$$

$$N_p = \sqrt{\frac{E_p + M}{2\,M}}\,, \qquad E_p = \sqrt{\vec{p}^{\,2} + M^2}\,, \tag{B.4}$$

where M is the baryon mass and $\chi_{1/2}(s)$ is the two-component spin wave function.

Table B.1 Non-relativistic reduction and expansion as defined in (B.7).

Γ	$\Gamma_{\rm eff}$	$\tilde{\Gamma}^{\rm nr}_{\rm eff}$
$\mathbf{1}$	$N_{p'}N_p\left(1-\frac{\vec\sigma\cdot\vec p\,'}{E_{p'}+M}\frac{\vec\sigma\cdot\vec p}{E_p+M}\right)$	$\mathbf{1}$
γ_5	$N_{p'}N_p\left(\frac{\vec\sigma\cdot\vec p}{E_p+M}-\frac{\vec\sigma\cdot\vec p\,'}{E_{p'}+M}\right)$	$\frac{1}{2\,M}\,\vec\sigma\cdot(\vec p-\vec p\,')$
γ^0	$N_{p'}N_p\left(1+\frac{\vec\sigma\cdot\vec p\,'}{E_{p'}+M}\frac{\vec\sigma\cdot\vec p}{E_p+M}\right)$	$\mathbf{1}$
γ^i	$N_{p'}N_p\left(\sigma^i\frac{\vec\sigma\cdot\vec p}{E_p+M}+\frac{\vec\sigma\cdot\vec p\,'}{E_{p'}+M}\,\sigma^i\right)$	$\frac{1}{2\,M}\left((p+p')^i+i\,(p-p')^j\,\sigma^k\,\epsilon^{ijk}\right)$
$\gamma_5\,\gamma^0$	$-N_{p'}N_p\left(\frac{\vec\sigma\cdot\vec p}{E_p+M}+\frac{\vec\sigma\cdot\vec p\,'}{E_{p'}+M}\right)$	$-\frac{1}{2\,M}\,\vec\sigma\cdot(\vec p+\vec p\,')$
$\gamma_5\,\gamma^i$	$-N_{p'}N_p\left(\sigma^i+\frac{\vec\sigma\cdot\vec p\,'}{E_{p'}+M}\sigma^i\frac{\vec\sigma\cdot\vec p}{E_p+M}\right)$	$-\sigma^i$
σ^{0j}	$N_{p'}N_p\,i\left(\sigma^j\frac{\vec\sigma\cdot\vec p}{E_p+M}-\frac{\vec\sigma\cdot\vec p\,'}{E_{p'}+M}\,\sigma^j\right)$	$\frac{i}{2\,M}\left((p-p')^j+i\,(p+p')^k\,\sigma^i\,\epsilon^{jki}\right)$
σ^{ij}	$N_{p'}N_p\,\epsilon^{ijk}\left(\sigma^k-\frac{\vec\sigma\cdot\vec p\,'}{E_{p'}+M}\sigma^k\frac{\vec\sigma\cdot\vec p}{E_p+M}\right)$	$\epsilon^{ijk}\sigma^k$
$\gamma_5\,\sigma^{0j}$	$N_{p'}N_p\,i\left(\sigma^j-\frac{\vec\sigma\cdot\vec p\,'}{E_{p'}+M}\sigma^j\frac{\vec\sigma\cdot\vec p}{E_p+M}\right)$	$i\,\sigma^j$
$\gamma_5\,\sigma^{ij}$	$N_{p'}N_p\,\epsilon^{ijk}\left(\sigma^k\frac{\vec\sigma\cdot\vec p}{E_p+M}-\frac{\vec\sigma\cdot\vec p\,'}{E_{p'}+M}\,\sigma^k\right)$	$\frac{1}{2\,M}\,\epsilon^{ijk}\left((p-p')^k+i\,(p+p')^l\,\sigma^m\,\epsilon^{klm}\right)$

For a spin-$\frac{3}{2}$ particle the spinor wave function is

$$u_\mu(\vec p,s)=N_p\begin{pmatrix} S^\dagger_\mu(p)\,\chi_{3/2}(s) \\ \frac{\sigma\cdot\vec p}{E_p+M}\,S^\dagger_\mu(p)\,\chi_{3/2}(s)\end{pmatrix}\,,\qquad \bar u_\mu(p,s)=u^\dagger_\mu(p,s)\,\gamma^0\,,\tag{B.5}$$

where $\chi_{3/2}(s)$ is the four-component spin wave function and

$$\begin{aligned} S^{0\,\dagger}(\vec p) &= \frac{|\vec p|}{M}\begin{pmatrix} 0 & \sqrt{\frac{2}{3}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{2}{3}} & 0\end{pmatrix}, \quad S^{1\,\dagger}(\vec p)=\begin{pmatrix} -\frac{1}{\sqrt2} & 0 & \frac{1}{\sqrt6} & 0 \\ 0 & -\frac{1}{\sqrt6} & 0 & \frac{1}{\sqrt2}\end{pmatrix}, \\ S^{2\,\dagger}(\vec p) &= \begin{pmatrix} -\frac{i}{\sqrt2} & 0 & -\frac{i}{\sqrt6} & 0 \\ 0 & \frac{i}{\sqrt6} & 0 & \frac{i}{\sqrt2}\end{pmatrix}, \quad S^{3\,\dagger}(\vec p)=\frac{E_p}{M}\begin{pmatrix} 0 & \sqrt{\frac{2}{3}} & 0 & 0 \\ 0 & 0 & \sqrt{\frac{2}{3}} & 0\end{pmatrix}. \end{aligned}\tag{B.6}$$

The non-relativistic expansion of the expressions in (B.1) goes in two steps.

First we provide the exact rewrite

$$\begin{aligned}
\bar{u}(\vec{p}\,', s')\,\Gamma\, u(\vec{p}, s) &= \chi^\dagger_{1/2}(s')\,\Gamma_{\text{eff}}\,\chi_{1/2}(s)\,,\\
\bar{u}^\mu(\vec{p}\,', s')\,\Gamma\, u^\nu(\vec{p}, s) &= \chi^\dagger_{3/2}(s')\left(S^\mu(\vec{p})\,\Gamma_{\text{eff}}\,S^{\nu\,\dagger}(\vec{p})\right)\chi_{3/2}(s)\,,\\
\bar{u}^\mu(\vec{p}\,', s')\,\Gamma\, u(\vec{p}, s) &= \chi^\dagger_{3/2}(s')\left(S^\mu(\vec{p})\,\Gamma_{\text{eff}}\right)\chi_{1/2}(s)\,,
\end{aligned} \tag{B.7}$$

in terms of effective 2×2 dimensional matrices Γ_{eff}. In a second step we compute the leading term in the non-relativistic expansion $\Gamma^{\text{nr}}_{\text{eff}}$. The results are displayed in Table B.1.

CURRICULUM VITAE

NAME	Mr. Daris Samart
DATE OF BIRTH	January 30, 1982
PLACE OF BIRTH	Kalasin, Thailand
PERSONAL ADDRESS	455/3, Mueng, Kalasin, 46000, Thailand
E-MAIL	jod_daris@yahoo.com

<u>EDUCATION</u>

Master of Science (2005 - 2007)

The Tah Poe Academia Institute (TPTP), Department of Physics, Naresuan University, Phitsanulok, Thailand

Supervisors: Dr. Burin Gumjudpai

Bachelor of Science (2000 - 2004)

Department of Physics, Rajabhat Institute Ubon Ratchathani, Ubon Ratchathani, Thailand

Supervisor: Assoc. Prof. Danai Wiroj-uraireung