DETERMINISTIC, STOCHASTIC AND ROBUST OPTIMIZATIONS OF DYNAMIC INTEGRATED NETWORK DESIGN AND TRAFFIC SIGNAL SETTING DESIGN PROBLEM: METAHEURISTIC APPROACH

Ampol Karoonsoontawong [1]

[1] *School of Transportation Engineering, Suranaree University of Technology, Nakhon Ratchasima, 30000, Thailand*
ampolk@gmail.com

ABSTRACT : The user-optimal dynamic traffic assignment (UODTA)-based network design problem (NDP), the signal setting design problem (SSD), and the integrated NDP and SSD problem (NDP-SSD) for the deterministic (DET), stochastic (SP) and robust (RO) optimizations are np-hard, so the metaheuristics are employed to solve the problems. The modifications of the simulated annealing (SA), genetic algorithm (GA) and reactive tabu search (RTS) for the nine problems (DET/SP/RO of NDP/SSD/NDP-SSD) are briefly discussed. In the experiment, SA, GA and RTS are compared for the nine problems to identify the best algorithms. The best algorithm for each problem is employed to find a best solution, which is then evaluated under stochastic condition. We find that the RO solutions yield the best robustness and lowest risk, whereas the DET solutions perform worst. The integrated robust approach is the most desirable, and the next best approach is the sequential robust approach. The deterministic approach can yield worse solutions than the do-nothing case.

KEYWORDS : *network design problem, traffic signal optimization, robust optimization, metaheuristic*

1. INTRODUCTION

The network design problem (NDP) determines an optimal budget allocation policy for expanding the capacity of pre-specified links while accounting for the user-optimal dynamic traffic assignment (UODTA) condition and the budget constraint. The signal setting design problem (SSD) determines an optimal signal setting strategy (cycle lengths, green times, time offsets and phase sequences) for the pre-specified signalized intersections while accounting for UODTA. The SSD in this paper is referred to the pre-timed signal setting design for the operational planning. The integrated NDP and SSD problem (NDP-SSD) simultaneously determines an optimal budget allocation policy and an optimal signal setting strategy while accounting for the UODTA condition and the budget constraint. These problems are bi-level in nature; i.e. the upper level problem (transport planner) minimizes the total system travel time (TSTT), and the lower-level problem observes the decisions made by the upper-level and behaves in the user-optimal manner (i.e. the UODTA condition). The three problems (NDP, SSD and NDP-SSD) can be formulated as deterministic, two-stage stochastic linear programming (SLP2), and robust optimization (RO) models. The deterministic model assumes all problem parameters are known and fixed. The SLP2 and RO models assume that the origin-destination (OD) demands are uncertain with known probability distributions. The SLP2 minimizes the expected TSTT, while the RO minimizes the expected sum of TSTT and risk.

[4] formulated the robust optimization model for the NDP-SSD (RO-BLPNDP-SSD), and showed that it can be reduced to eight related models in Table 1. These models embed Daganzo (1994)'s cell transmission model (CTM). These mathematical formulations are limited to only single-destination networks due to the underlying UODTA model, so the useful, practical application on multi-destination larger-size networks are prohibited. Moreover, the bi-level formulations are NP-hard, so there is not an efficient solution method that guarantees an optimal solution. To address these limitations, [3] applied the genetic algorithm (GA) and simulated annealing (SA) to efficiently solve the deterministic NDP, and [5] applied the reactive tabu search (RTS) to the deterministic SSD. GA, SA and RTS are metaheuristic techniques can efficiently find solutions beyond local optimality.

In this paper, we compare the performance of SA, GA and RTS on a test network for the nine problems in Table 1, and evaluate the solutions of the nine problems under the stochastic condition in terms of the expected robustness, the expected TSTT and the expected risk.

2. MODIFICATIONS OF METAHEURISTICS

Three popular metaheuristics are considered in this paper, namely, simulated annealing (SA), genetic algorithm (GA) and reactive tabu search (RTS). [3] developed the SA to solve the UODTA-based NDP, and also calibrated the SA parameters for the NDP. In this paper, we employ this SA algorithm with the calibrated set of SA parameters (assume that the parameters also work well for all nine problems in Table 1). We employ the GA developed by [3] to solve the nine problems. The parameters calibrated in [3] for the DTA-based NDP are used in this study. [5] proposed and compared three variations of the modified RTS (called RTS-MT0, RTS-MT1 and RTS-MT2) based on three different neighborhood definitions to solve the DTA-based SSD problem. RTS-MT2 outperforms RTS-MT0 and RTS-MT1. In this paper, we, thus, employ the RTS-MT2, and refer to it as RTS. Note that RTS does not have parameters for calibration.

The major modifications to the three metaheuristics for the nine problems include the solution representation, the associated encoding and decoding procedures, all of which can be found in [3]-[5]. The functional evaluations for the deterministic, stochastic and robust optimization models are proposed in this paper.

This study employs the UODTA module in the Visual Interactive System for Transport Algorithms (VISTA) [6] to evaluate different solutions for larger-size problems. The UODTA module in VISTA is a departure-time-based version of the simulation-based UODTA approach using a mesoscopic simulator based on an extension of the cell transmission model, to propagate traffic and satisfy capacity constraints. The objective of deterministic problems is to minimize the TSTT, whereas that of stochastic problem is to minimize the expected TSTT. The objective of robust problems is to minimize the expected robust objective. Specifically, for deterministic problems (DET-NDP, DET-SSD and DET-NDP-SSD), the UO DTA module in VISTA is called to evaluate the current solution *fr*, and the objective function value *z*(*fr*) is set to the TSTT:

$$z(fr) = TSTT \quad \text{for deterministic problems}$$

With the current solution *fr*, the functional evaluation procedures of both stochastic (SP-NDP, SP-SSD and SP-NDP-SSD) and robust (RO-NDP, RO-SSD and RO-NDP-SSD) problems involve running the UODTA for *N_d* times (where *N_d* is the number of demand scenarios); each UODTA run corresponds to each OD demand scenario generated in the initialization step of the metaheuristic algorithms. The objective function values for the stochastic and robust problems are:

$$z(fr) = \frac{\sum_{i=1}^{N\_d} TSTT_i}{N\_d} \quad \text{for stochastic problems}$$

$$z(fr) = \frac{\sum_{i=1}^{N\_d} TSTT_i}{N\_d} + \lambda \cdot \frac{\sum_{i=1}^{N\_d} \max^2(0, TSTT_i - TSTT^{TARGET})}{N\_d}$$

for robust problems

where $TSTT_i$ is the TSTT from the i^{th} run of UODTA for $i = 1,\ldots,N\_d$.

3. COMPUTATIONAL EXPERIMENT

The test network is first described, followed by the three parts of the experiment: Part 1- algorithm performance comparison, Part 2- metaheuristic search, and Part 3- solution robustness evaluation under stochastic environment. The outcome of Part 1 is the identification of the winner on each of the nine problems. These winner algorithms will be employed in Part 2 to search for a “good” or near-global solution in each problem. The outcome of Part 2 is the best solution found so far by the winner algorithm in each of the nine problems. These best solutions are evaluated in Part 3.

3.1 Test Network

The well-known Sioux Fall network in Figure 1a is employed in our experiment. All links are 2-lane with the capacity of 1200 vehicles per hours (vph) and the free flow speed of 49.5 miles per hour (mph). Nodes 6, 8, 9, 10, 16, 17 and 18 are signalized intersections. Ten links are candidates for capacity expansion: links (6,8), (8,6), (7,8), (8,7), (9,10), (10,9), (10,16), (16,10), (13,24) and (24,13). The study period is a peak hour with 33 O-D pairs[1]. All OD demands are determined from a normal random variable (the mean of 900 vph and standard deviation of 100 vph) multiplied by factors[1]. We employ the NDP parameters: B = 1500, χ =160 and ϕ=160. For SSD parameters,

- all red time = 2 seconds per movement phase,
- yellow time = 4 seconds per movement phase,
- Node 10 (a 5-leg intersection)
  - Min green times for movement phases 1, 2, 3, 4 and 5 = 10 seconds
  - Min and Max cycle length = 80 and 250 seconds
- Nodes 8 and 16 (4-leg intersections)
  - Min green times for movement phases 1, 2, 3, 4 = 10 seconds
  - Min and Max cycle length = 64 and 210 seconds
- Nodes 6, 9, 17 and 18 (3-leg intersections)
  - Min green times for movement phases 1, 2, 3 = 10 seconds
  - Min and Max cycle length = 48 and 180 seconds

In all three parts, for the deterministic problems, the three metaheuristic algorithms are performed using the average OD demand matrix, which is obtained by multiplying 900 vph to the factors[1]. In Part 1, the stochastic and robust problems are solved using a small number of demand realizations (say 3), so that the CPU time for each run is manageable. The small size of demand realizations is employed with the assumption that the relative performances of SA, GA and RTS remain the same regardless of the size of demand realizations. Furthermore, the same set of demand realizations is employed to perform all iterations of SA, GA and RTS, so that the results are comparable. Note that the OD demand realizations are generated by the Monte Carlo simulation in Parts 1, 2 and 3. The generated OD demand samples are independent and identically distributed from the distribution of OD demands.

In Part 2, the stochastic and robust problems are solved using a larger number (i.e. 20) of demand realizations[2]. The same set of demand realizations is employed in all nine problems using the corresponding winner algorithm. In Part 3, the best solution in each problem is evaluated under stochastic environment, using a new set of demand realizations with the sample size thirty[3]. We employ the new set of demand realizations, so that any possible evaluation bias can be eliminated.

3.2 Experimental Results

3.2.1 Part 1: Algorithm Performance Comparison

The performances of metaheuristics are compared according to solution quality and convergence speed. The CPU time is dominated by the functional evaluation time, so the CPU times of all metaheuristics for the same total trials are approximately the same. Especially for the stochastic and robust problems, a trial involves performing multiple UODTA runs corresponding to all OD demand realizations. The common stopping criterion of the three metaheuristics is the total trials. We employ 250 trials for DET-SSD and DET-NDP; 500 trials for DET-NDP-SSD; 150 trials for SP-SSD, SP-NDP, RO-SSD and RO-NDP; and 300 trials for SP-NDP-SSD and RO-NDP-SSD. Since the employed total trials are relatively small, especially for the stochastic and robust problems, the better solution quality also implies the better convergence speed. Thus, in this particular study, these two criteria may be considered the same, and we only discuss the comparison in terms of solution quality as this is of higher interest.

Due to the space limit, we show the convergence characteristics of the three algorithms for only RO-NDP-SSD in Figure 1b. We rank the three metaheuristics with respect to the solution quality for the deterministic, stochastic and robust problems on the test network in Figure 1c. Apparently, there is not the single winner that outperforms the others across all nine problems on the test network. For the deterministic problems, RTS appears best; GA the second best; and SA the third best. For the stochastic problems, RTS, GA and SA appear best, second best and third best, respectively. For the robust problems, there is not a clear order.

3.2.2 Part 2: Metaheuristic Search for Best Solutions

Based on the results in Figure 1c, the identified best metaheuristic algorithm for each problem is employed to find a best solution. Since we would like to evaluate and compare the deterministic, stochastic and robust solutions, the same number of total trials is employed. Specifically, we employ 75 total trials for the SSD and NDP problems and 150 total trials for the NDP-SSD problems. For the three SSD problems (DET-SSD, SP-SSD and RO-SSD), all metaheuristics optimize the SSD variables with the original link capacity to obtain the best signal settings. Then, the three best signal settings obtained from the deterministic, stochastic and robust SSD are fixed for running the respective deterministic, stochastic and robust NDP. This allows us to compare the sequential SSD and NDP against the combined NDP-SSD approach. For a fair comparison, the number of trials (i.e. the number of functional evaluations) for NDP-SSD is two times as many as that for SSD and NDP. That is, given the same total trials, the sequential approach is compared with the combined approach. For the robust problems, we set $TSTT^{TARGET}$ equals to the minimal *E[TSTT]* obtained from the sequential stochastic approach (SP-SSD & SP-NDP) and the combined approach (SP-NDP-SSD).

It is noted that the desirable λ value in robust problems depends on the preference of transportation planners. We arbitrarily use λ=1. It is also noted that we tradeoff the OD demand sample size (the larger sample size implies the better quality of solution) for the reasonable computational time. According to the rule of thumb in the Monte Carlo-based stochastic optimization, we use the sample size of 20. The larger sample size is indeed desirable; however, it costs too long computational time, given that we have to do multiple metaheuristic runs. Since DET-NDP, SP-NDP and RO-NDP employ different traffic signal settings, these three may not be comparable. As such, we employ the original signal settings[4] for NDP problems, and we denote the three NDP problems by NDP-D (where D stands for the default signal settings). Specifically, three additional NDP problems are solved (DET-NDP-D, SP-NDP-D and RO-NDP-D). All deterministic, stochastic and robust SSD and NDP-SSD solutions employ different time offsets and different cycle lengths for intersections on both networks. This means we benefit from relaxing the common cycle length assumption and from the traffic signal coordination such that the great number of vehicles is progressed through the network.

3.2.3 Part 3: Solution Robustness Evaluation under Stochastic Environment

To evaluate and compare all solutions from the twelve problems on the test network, we use a separate set of 30 OD demand realizations that are independent and identically distributed from the distribution of OD demands. Evaluating a solution involves running a DTA corresponding to an O-D demand realization. After all 30 DTA runs, we compute the expected value (*E*[.]), standard deviation (*SD*[.]), error (*Error*[.]) and two-sided 95% confidence interval (95%*CI*[.]) of *Robust_Obj(λ=1), TSTT* and *Risk*. Note that $Error[.] = t_{0.025,29} \cdot SD[.]/\sqrt{30}$; $95\%CI[.] = E[.] \pm Error[.]$; and *t*(0.025, 29)=2.045. Figures 2-4 depict the 95% CIs of *E[Robust_Obj(λ=1)]*, *E[TSTT]* and *E[Risk]* for the original network and all twelve problems (DET/SSD/RO of SSD/NDP/NDP-D/NDP-SSD). Since the expected risk in the objective is not the variance of TSTT, the pattern of *V[TSTT]* is not consistent with that of *E[Risk]*. However, the RO-NDP-SSD solution with the least *E[Risk]* also has the least V[TSTT].

The comparisons of point estimates are conducted in three dimensions. First, we compare among the deterministic, stochastic and robust solutions on the test network; that is, the solutions within the following three cases are compared against each other within the case: case 1 (DET-SSD, SP-SSD, RO-SSD), case 2 (DET-NDP-D, SP-NDP-D, RO-NDP-D) and case 3 (DET-NDP-SSD, SP-NDP-SSD, RO-NDP-SSD). We find that the RO solutions yield the least *E[Robust_Obj(λ=1)]* and *E[Risk]* as expected, whereas the DET solutions perform the worst with the highest *E[Robust_Obj(λ=1)]*, *E[TSTT]* and *E[Risk]*. Since the metaheuristics for robust problems are not designed to minimize the *V[TSTT]*, the solutions of RO problems do not always yield the least *V[TSTT]*. For example, in case 1 (DET-SSD, SP-SSD, RO-SSD), the SP-SSD solution yields the least *V[TSTT]*, whereas the RO-SSD solution yields the least *E[Risk]*.

Second, we compare the sequential SSD and NDP approaches against the integrated NDP-SSD approaches; i.e., DET-NDP versus DET-NDP-SSD; SP-NDP versus SP-NDP-SSD; and RO-NDP versus RO-NDP-SSD. In terms of *E[Robust_Obj(λ=1)]*, the integrated approach performs better than the sequential approach except the case of DET-NDP-SSD and DET-NDP. In terms of *E[TSTT]*, the integrated approach performs better than the sequential approach in all cases. In terms of *E[Risk]*, the integrated approach outperforms the sequential approach in all cases except the case of DET-NDP-SSD versus DET-NDP. Third, we compare the original network and the twelve problems altogether. Apparently, the RO-NDP-SSD solution outperforms the others on the test network in terms of *E[Robust_Obj(λ=1)]*, *E[TSTT]*, *E[Risk]* and *V[TSTT]*, whereas the sequential robust approach (i.e. RO-SSD & RO-NDP) performs the second best. The solutions from the twelve problems perform better than the original network in terms of *E[Robust_Obj(λ=1)]*, *E[TSTT]* and *E[Risk]* except DET-NDP-D. This shows that the deterministic approach can yield worse solutions than the do-nothing case when evaluated under stochastic condition. Thus, the integrated robust approach (RO-NDP-SSD) should be desirable whenever possible. Unlike the analytical result in [4], the sequential robust approach (RO-SSD & RO-NDP) performs the second best. This implies that for larger-size multi-destination networks where the analytical approach is not applicable, the integrated robust approach is still the most desirable; otherwise, the next best approach is the sequential robust approach.

It is noted that we have made rather weak comparisons that one solution is better than another in expected values. Apparently, one can observe the overlapping 95% confidence intervals of *E[TSTT]*, *E[Risk]* and *E[Robust_Obj(1)]* of many solutions. We can make a stronger comparison via paired *t* tests. We perform 36 paired *t* tests to test the null hypotheses that the RO-NDP-SSD solution outperforms the other 12 solutions in terms of the three measures (*E[TSTT], E[Risk]* and *E[Robust_Obj(1)]*). The minimal *p*-value from the 36 paired *t* tests is 0.7950, implying that on the test network we fail to reject the null hypothesis when the level of significance $\alpha < 0.7950$.

4. CONCLUSIONS AND FUTURE RESEARCH

The solutions of SSD and NDP-SSD problems imply that the common cycle length assumption should not be used, and the traffic signal coordination should be utilized. The RO solutions yield the best robustness and lowest risk as expected, whereas the DET solutions perform the worst. The integrated robust approach is the most desirable; otherwise, the next best approach is the sequential robust approach. The deterministic approach can yield worse solutions than the do-nothing case when evaluated under stochastic condition.

For the future research, alternative methods that can improve the performance of Monte Carlo simulation should be explored such as the Quasi-Monte Carlo simulation and the variance reduction techniques.

ENDNOTES

[1]Table F.2**.** Peak-Hour O-D Trip Matrix for the Modified Sioux Fall Network with 33 OD Pairs (page 244 in [2])

[2]Table F.5. 20 OD Demand Realizations for Metaheuristic Search for the Modified Sioux Fall Network with 33 OD Pairs (page 246 in [2])

[3]Table F.6. 30 OD Demand Realizations for Evaluation for the Modified Sioux Fall Network with 33 OD Pairs (pages 247-248 in [2])

[4]Table 6.10b. Original Traffic Signal Settings (page 198 in [2])

REFERENCES

[1] Daganzo, C. F. (1994) The Cell Transmission Model: A Simple Dynamic Representation of Highway Traffic Consistent with the Hydrodynamic Theory. *Transportation Research B 28*, No. 4, pp. 269-287.

[2] Karoonsoontawong, A. (2006) *Robustness Approach to the Integrated Network Design Problem, Signal Optimization and Dynamic Traffic Assignment Problem.* Ph.D. Dissertation, The University of Texas at Austin.

[3] Karoonsoontawong, A., S.T. Waller (2006) Dynamic Continuous Network Design Problem: Linear Bi-level Programming and Metaheuristics. *Journal of the Transportation Research Board 1964*, 104-117.

[4] Karoonsoontawong, A. and S.T. Waller. (2008) Integrated Network Capacity Expansion and Traffic Signal Optimization Problem: Robust Bi-level Dynamic Formulation. *Networks and Spatial Economics, DOI /10.1007s-9071-008-11067x.*

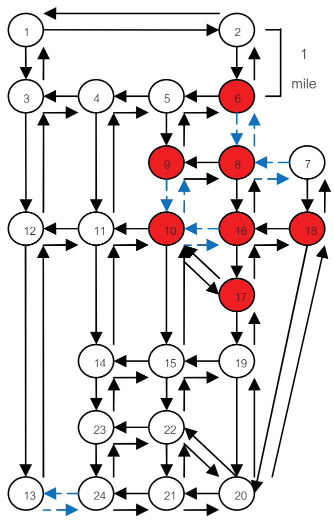
[5] Karoonsoontawong, A. and S.T. Waller (2009) "Combined Dynamic User Equilibrium and Traffic Signal Optimization Problem: Reactive Tabu Search Approach," *88th Transportation Research Board Meeting CD-ROM*, Washington, D.C., USA.

[6] Ziliaskopoulos, A.K. and S.T. Waller. (2000) An Internet-based geographic information system that integrates data, models and users for transportation applications. *Transportation Research Part C 8*, 427-444.

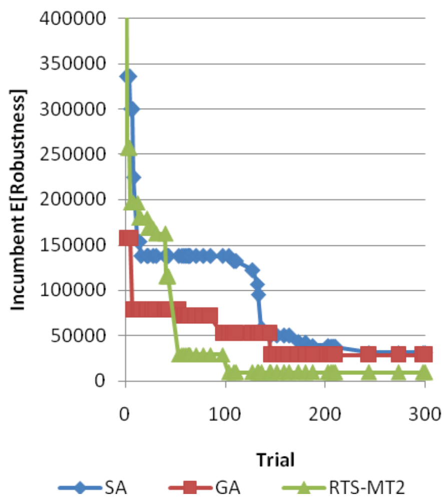
Table 1. Abbreviations and Descriptions of Nine Problems

Math Formulation	Problem Abbreviation	Problem Description
RO-BLPNDP-SSD	RO-NDP-SSD	Robust Bi-Level Integrated NDP, SSD and UODTA Model
SLP2-BLPNDP-SSD	SP-NDP-SSD	Two-Stage Stochastic Bi-Level Integrated NDP, SSD and UODTA Model
BLPNDP-SSD	DET-NDP-SSD	Deterministic Bi-Level Integrated NDP, SSD and UODTA Model
RO-BLPSSD	RO-SSD	Robust Bi-Level Combined SSD and UODTA Model
SLP2-BLPSSD	SP-SSD	Two-Stage Stochastic Bi-Level Combined SSD and UODTA Model
BLPSSD	DETSSD	Deterministic Bi-Level Combined SSD and UODTA Model
RO-BLPNDP	RO-NDP	Robust Bi-Level Combined NDP and UODTA Model
SLP2-BLPNDP	SP-NDP	Two-Stage Stochastic Bi-Level Combined NDP and UODTA Model
BLPNDP	DET-NDP	Deterministic Bi-Level Combined NDP and UODTA Model

a) Modified Sioux Falls network (Nodes 6, 8, 9, 10, 16, 17 and 18 are signalized intersections; and ten dashed links are candidates for capacity expansion.)



b) Algorithm Performance Comparison for RO-NDP-SSD



c) Results of Algorithm Performance Comparisons based on the Solution Quality

Rank	Deterministic			Stochastic			Robust		
	SSD	NDP	NDP-SSD	SSD	NDP	NDP-SSD	SSD	NDP	NDP-SSD
1	RTS	RTS	GA	RTS	RTS	RTS	RTS	GA	RTS
2	GA	GA	RTS	GA	GA	SA	GA	SA	GA
3	SA	SA	SA	SA	SA	GA	SA	RTS	SA

Figure 1. Test Network and Algorithm Performance Comparison Results

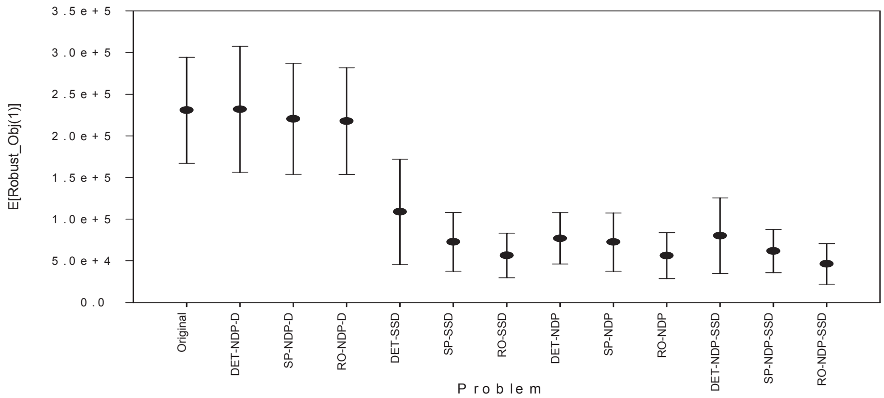


Figure 2. 95% Confidence Intervals of *E[Robust_Obj(□=1)]*

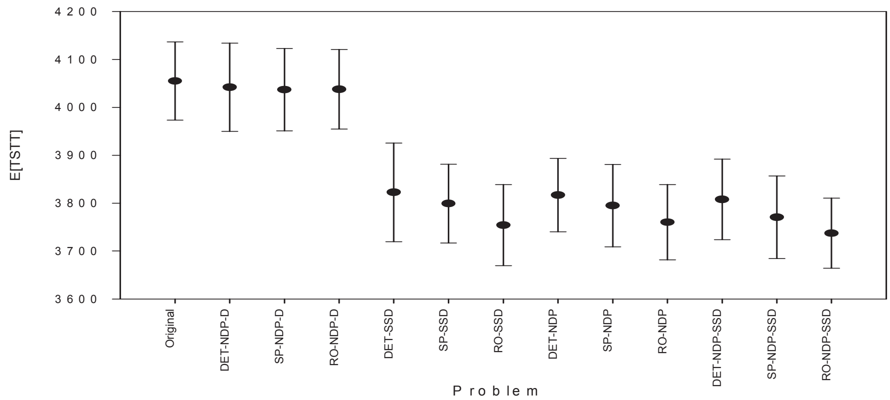


Figure 3. 95% Confidence Intervals of *E[TSTT]*

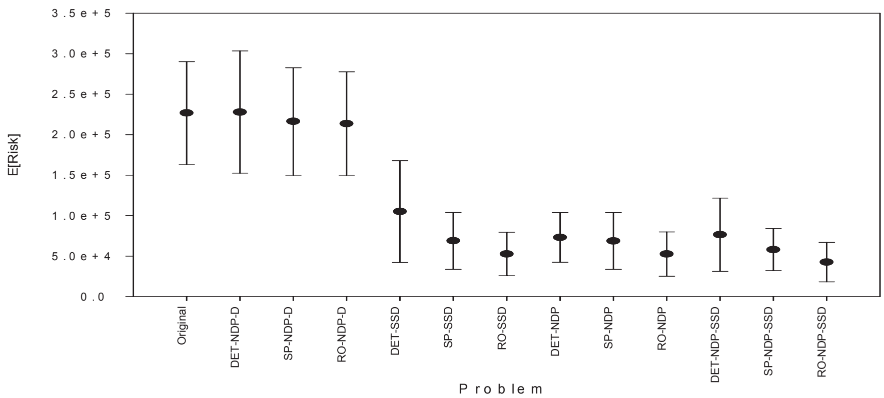


Figure 4. 95% Confidence Intervals of *E[Risk]*