Effect of Medium Viscosity on Breakage Parameters of Quartz in a Laboratory Ball-Mill

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The role and effect of liquid medium viscosity on breakage parameters (S and B values) of the batch grinding kinetic model was investigated by grinding a single feed fraction of quartz (20 × 30 mesh) in a batch ball-mill at fixed mill loadings and at 45 vol % solid concentration, using glycerol–water mixtures of different proportions. The primary breakage distributions (B values) were normalized and separated into three different groups for grinding in media viscosities of 0.001(water), 0.002–0.03, and 0.08–0.16 Pa·s, respectively, with higher viscosity range giving proportionally finer products. The disappearance grinding kinetics of the top size showed that the specific breakage rates (S values) depended both on the viscosity of the liquid medium and on the extent of grinding. The specific breakage rates of the top size increased with the fineness of grinding while the breakage of the smaller sizes proceeded approximately in a first-order manner as characterized by a characteristic breakage rate constant. The acceleration of the top-size breakage rates as grinding proceeded was represented by a parameter called *the acceleration factor*, which correlated reasonably well with the changing slurry effective viscosity. Alternatively, the acceleration factor can be presented in the form of a single generalized first-order plot for grinding at different medium viscosities. The first-order breakage rate constant decreased with increasing medium viscosity over the range from 0.001 to 0.16 Pa·s but not linearly. The proportional variation of net mill power did not match exactly with that of the breakage rate constant on increasing medium viscosity. On the basis of the analysis of grinding results, three grinding regimes were identified relative to the change in medium viscosity but their existence was not confirmed experimentally.

Introduction

It is generally observed that ball-milling in water gives higher breakage rates than dry grinding both in batch and continuous operations.[1–5] This difference in grinding efficiency should be attributed to the difference in the mode of material movement in the mill charge, that is, powder flow versus slurry suspension flow, which in turn affects the way particles are captured and fractured by the grinding media. Most mineral slurries in grinding are non-Newtonian in character and their rheological properties are dependent on such parameters as particle size and shape, slurry concentration, temperature, and degree of particle dispersion. The easiest and most practical means of examining the rheological influence on particle breakage is by varying the percentage of solid-to-water ratio in the slurry, with grinding studies often being conducted in a small-scale batch mill. Batch tests in a laboratory mill can focus solely on the factors that affect breakage without the complicating effect of mass transport and the experiments are easier and quicker to perform and can be closely controlled. However, there is no guarantee that results from a small-scale mill will be similar to those in a larger mill.

Tangsathitkulchai and Austin[4] studied in detail the rheological influence on the specific rates of breakage (S values) and the primary breakage distributions (B values) of the batch grinding kinetic model by grinding a single feed fraction (20 × 30 mesh) of quartz and copper ore in water as a function of slurry density at a fixed mill speed and constant ball and powder loadings. The results indicated an increase in S values of the feed size as fines built up in the mill for slurry densities less than 60 vol % solid and deceleration of breakage rates at higher slurry densities. However, the breakage of smaller sizes was essentially first order once a naturally broken size distribution accumulated in the mill. The derived first-order breakage constant increased with increasing slurry density and went through a maximum at 45 vol % solid. The proportional variation of the rate constant with slurry density matched reasonably with that of the net mill power, signifying an approximate correlation between breakage and energy being expended in the mill. The B values were constant and normalized for normal slurry densities of <45 vol % solid but changed to a different set of values at higher slurry densities, with proportionally finer products.

Later, the effect of slurry density on the dynamics and grinding behavior of a laboratory batch ball-mill was reported by Tangsathitkulchai and Austin.[6] It was discovered that slurry density determined the spatial distribution of solid charge within the mill, with migration of particles from the tumbling zone to the mill wall as slurry concentration was progressively increased above 45 vol % solid. The overall effect gave rise to the change in the circulation path of the ball charge. In addition, the interaction between grinding media and the slurry under varying slurry density resulted in various grinding characteristics relevant to wet grinding systems. Recently, the work has been extended by Tangsathitkulchai[7,8] to identify and correlate the slurry

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apparent viscosity with the acceleration of top-size breakage and the slowing down of breakage rates in fine wet grinding.

An extensive investigation into the effect of controlled changes in slurry rheology on the grinding characteristics of ore minerals and coals, both in laboratory batch ball-mills and large-scale tumbling media mills, has been reported in a series of papers by Klimpel.[9–12] By using the net production rate of material less than some specified screen size as a measure of grinding efficiency, along with the analysis of S and B values, it was found that there was a consistent pattern of change in breakage characteristics of both coal and mineral slurries as slurry density was changed. In the normal range of low density and low viscosity, the slurry exhibited dilatant character. Grinding in this region gave virtually no variation in mill production and the breakage followed the first-order hypothesis. Increasing slurry density caused a trend toward pseudoplastic behavior. If no yield stress was developed, first-order grinding was still maintained but with higher rates of breakage than in the dilatant system. This is the region which represents the most efficient wet grinding practice. When grinding was performed on a very dense slurry, the yield stress increased rapidly and led to the slowing down of breakage rates. The use of certain viscosity control chemicals (grinding additives) was also demonstrated to have an advantageous effect in reducing yield stress and overall viscosity of a dense slurry, thus allowing more efficient breakage interaction between grinding media and particles and hence increasing the rate of size reduction.[13,14]

Similar work was also performed by Shi and Napier-Munn[15] to determine the effect of slurry rheology on industrial grinding performance, evaluated based on the so-called "grinding index". The rheology of the mill discharge slurries was measured and the four operating variables of mill throughput, slurry density, slurry viscosity, and feed fines content were investigated. Statistical analysis of data from 16 industrial grinding mills in five sites confirmed that slurry rheology did influence the mill grinding efficiency. The trends of these rheological effects on grinding depended on whether the slurries were pseudoplastic and dilatant and whether they exibited high or low yield stresses.

In addition to slurry density, another property that can affect the rheological nature of the slurry, and hence the grinding performance, is the viscosity of the suspending liquid medium. The work reported here is a series of wet grinding studies dealing with the influence of suspending medium viscosity on the breakage behavior through the analysis of S and B values. A narrow-sized feed of quartz was batch-ground in a laboratory ball-mill using synthetic mixtures of water and glycerol. Glycerol was used mainly because of its similarity in density and surface tension to those of water, its Newtonian flow behavior, and its wide range of viscosity when mixed with water.

Batch Grinding Analysis

The analysis and interpretation of grinding data obtained in this work was based on the well-known size–mass balance batch grinding equation[16,17]

$$\mathrm{d}w_i(t)/\mathrm{d}t = -S_i w_i(t) + \sum_{j=1}^{i-1} b_{ij} S_j w_j(t), \quad n \geq i \geq j \geq 1 \quad (1)$$

where n is the number of size intervals including the sink interval, $w_i(t)$ is the weight fraction of particles of screen size i at grinding time t. S_i is a constant called the specific rate of breakage of size i, and b_{ij} is the weight fraction of broken products from size interval j, which appears in size interval i on primary fracture before rebreakage occurs. The values of S_i can be fitted to the expression[5]

$$S_i = a(x/x_o)^{\alpha} \quad (2)$$

where x_i is the upper screen size i (mm), x_0 is a reference size (1 mm), α is a constant being characteristic of the material, and a is the first-order breakage constant which varies with mill conditions. The primary breakage distribution function b_{ij} can be expressed in cumulative form as

$$B_{ij} = \sum_{k=n}^{i} b_{kj} \quad (3)$$

where the set of B values can be represented by the sum of two power functions in the form[5]

$$B_{ij} = \Phi_j(x_{i-1}/x_j)^{\gamma} + (1 - \Phi_j)(x_{i-1}/x_j)^{\beta} \quad (4)$$

The solution of eq 1 for this set of n differential equations gives a prediction of product size distributions at various grinding times, given a starting feed size distribution, $w_i(0)$, and the two breakage parameters (S_i and b_{ij}). The disappearance kinetics of the top-size material (size 1) can be evaluated by integrating eq 1, with S_1 being constant, to obtain

$$w_1(t)/w_1(0) = \exp[-S_1 t]$$

or

$$\log w_1(t) = \log w_1(0) - S_1 t/2.3 \quad (5)$$

A plot of log $w_1(t)$ versus t should give a straight line if grinding is first order. Any deviation from the linear relationship indicates non-first-order grinding, which can be either slowing down or acceleration of breakage rates.

Experimental Section

Batch grinding of 20 × 30 mesh quartz feed in glycerol–water mixtures at constant ball and powder filling and at a slurry concentration of 45 vol % solid was conducted in a steel laboratory ball-mill, fitted with lifters of semicircular cross section. Size distributions of ground products at various times were obtained using a nest of $\sqrt{2}$ interval screens and a Rotap sieve shaker. Details of material preparation and test procedures are given elsewhere.[7,8] Table 1 lists the mill and wet grinding conditions used. Some dry grinding tests under the same conditions were also performed for comparison purposes.

The mixtures of glycerol and distilled water were prepared by weight. The densities and viscosities of glycerol–water solutions were measured with a glass hydrometer and a glass capillary viscometer, respec-

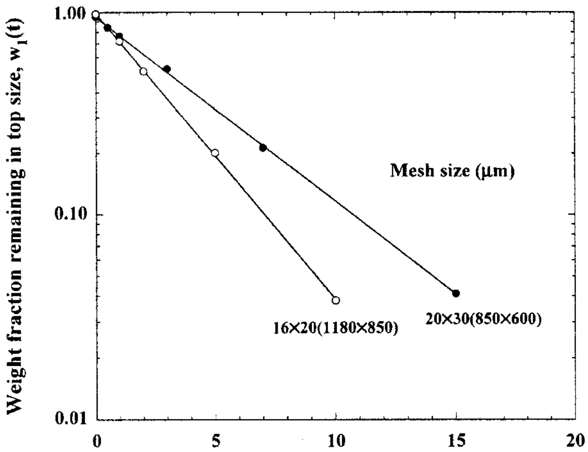

Figure 1. First-order plot for batch dry grinding of single feed fraction of quartz ($J = 0.3$, $U = 1.0$, 1-in. balls, $\phi_c = 0.70$).

Table 1. Ball Mill and Test Conditions

Mill	
internal diameter, m	0.20
length, m	0.175
running speed (n), rpm	76
critical speed (n_c), rpm	108
fraction of critical speed (ϕ_c)	0.70
lifter cross section	semicircular
lifter diameter, m	2×10^{-2}
number of lifters	6
Grinding Media	
material	alloy steel
diameter, m	2.54×10^{-2} (1 in.)
density, kg/m^3	7.8×10^3
ball filling (J), fraction of mill volume occupied by ball bed	0.3
total ball weight, kg	7.72
Solid Charge	
test material	white crystalline quartz
density, kg/m^3	2.65×10^3
particle size	20 × 30 mesh (850 × 600 µm)
total weight, kg	1.05
powder filling (U), fractional filling of void volume of ball bed	1.0
slurry concentration, vol % solid	45

Table 2. Physical Properties of Glycerol−Water Mixtures Used in the Batch Grinding Tests

percent glycerol in mixture (wt %)	density at 25 °C (kg/m^3)	viscosity at 25 °C (Pa·s)
0(water)	1.00×10^3	0.001
20	1.036×10^3	0.002
40	1.062×10^3	0.004
60	1.142×10^3	0.012
70	1.172×10^3	0.030
80	1.204×10^3	0.080
85	1.213×10^3	0.160

tively. Table 2 shows the physical properties of the liquid media used in the present study.

Experimental Results

Kinetics of Top-Size Breakage. Figure 1 shows the first-order disappearance plots for dry grinding of two feed fractions of quartz, 20 × 30 mesh (850 × 600 µm) and 16 × 20 mesh (1180 × 850 µm). It can be seen that the breakage kinetics obeys the first-order hypothesis; that is, increasing fineness of powder charge has no effect on the specific rate of breakage of the top-size feed. For the breakage of the 20 × 30 mesh feed fraction, the S value of this size was estimated from the slope of the straight line of Figure 1 to have the value of 0.21 min^{-1}. The first-order rate constant (a), required for generating S_i values, had a value of 0.24 min^{-1} calculated using eq 2 with $S_1 = 0.21$ min^{-1}, $x_1 = 0.85$ mm (850 µm), and $\alpha = 0.8$ for quartz material. Experimental B values for dry grinding were calculated using the BII procedure[18] and the corresponding primary breakage parameters were $\Phi = 0.68$, $\gamma = 1.18$, and $\beta = 4.0$.

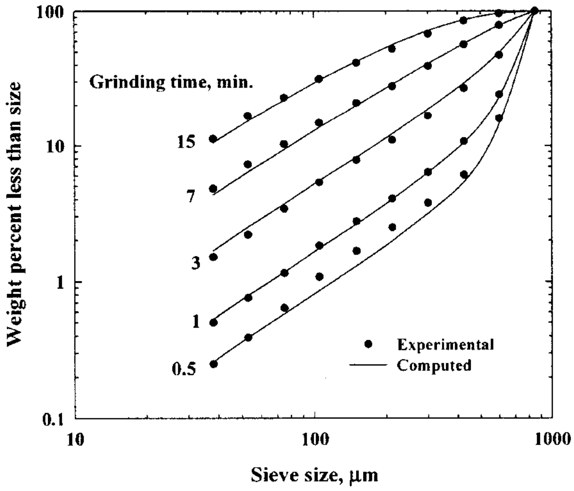

Figure 2. Comparison of experimental and computed size distributions for dry grinding of 20 × 30 mesh quartz feed ($J = 0.3$, $U = 1.0$, 1-in. balls, $\phi_c = 0.70$).

Figure 2 compares the experimental size distributions at various times of grinding with those computed from the solution of the size−mass balance equation, using the previously estimated breakage parameters and assuming normalized B values. The simulated and experimental results show good agreement, indicating that the estimated breakage parameters were reasonably correct, that dry grinding of quartz was first-order under the test conditions and that the primary breakage distributions were constant with time and are normalizable.

Figure 3 gives the results on the disappearance kinetics of 20 × 30 mesh quartz ground in glycerol−water mixtures at a normal slurry concentration of 45 vol % solid, with the viscosities of the liquid media varying in the range from 0.001 to 0.16 Pa·s. In comparison with dry grinding, it is clear that the breakage of top-size material in wet grinding deviates from the first-order behavior such that the specific rates of breakage increase as grinding proceeds to finer sizes. It is also observed that there is an upward shift of the curves as the medium viscosity is increased. This shift indicates that the rate of grinding of the top feed size decreases with increasing medium viscosity. This result is also reflected by a decrease in the production of fine material (as % minus 200 mesh in 15 min of grinding) and the coarser product size distributions, as shown in Figures 4 and 5, respectively. In addition, it is interesting to note from Figure 4 that the production of fines with respect to change in medium viscosity decreases linearly with medium viscosity up to the value of 0.03 Pa·s (70 wt % glycerol) and then falls with a steeper slope at higher viscosities. This observation suggests

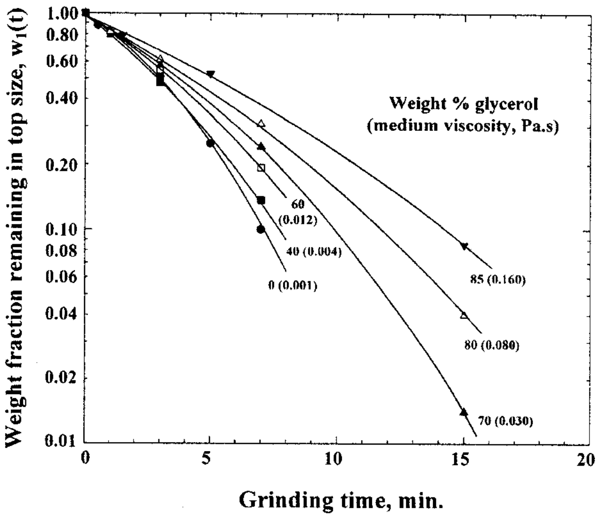

Figure 3. First-order plots for wet grinding of 20 × 30 mesh quartz in glycerol–water mixtures of various glycerol concentrations (45 vol % solid, $J = 0.3$, $U = 1.0$, 1-in. balls, $\phi_c = 0.7$).

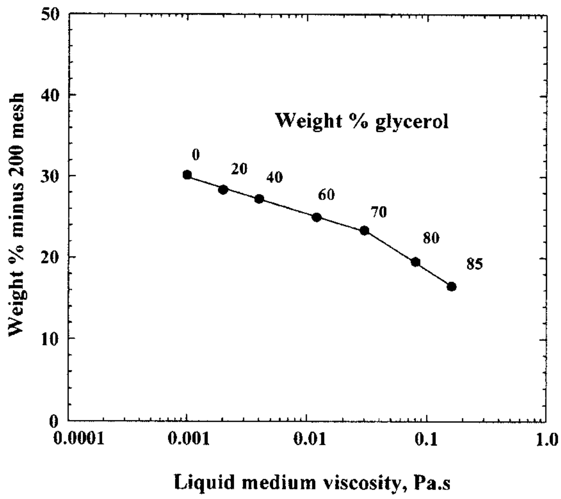

Figure 4. Effect of liquid medium viscosity on the production of minus 200 mesh fines in 15 min of batch grinding of a feed of 20 × 30 mesh quartz (45 vol % solid, $J = 0.3$, $U = 1.0$, 1-in. balls, $\phi_c = 0.7$).

that there may be basic differences in the breakage mechanism for grinding in these two ranges of suspending medium viscosities.

Primary Breakage Distributions. Figure 6 shows the effect of suspending liquid viscosity on the primary breakage distribution (BII method) for grinding quartz at 45 vol % solid in slurry. Within the limit of experimental variability, the B values fall into three different groups, that is, for media viscosities of 0.001(water), 0.002–0.03, and 0.08–0.16 Pa·s, respectively, with increasing viscosity range producing proportionally finer primary breakage products. The values of the primary breakage distribution parameters according to eq 4 are presented in Table 3. Since the all the shapes of the primary breakage distributions show the characteristic of impact type of breakage, it is probable that increased medium viscosity could dispose the particles in advantageous positions to receive even greater impact from colliding balls, hence producing a catastrophic fracture of particles with finer daughter fragments. With limited experimental data, it is not possible at this stage to verify the proposed hypothesis.

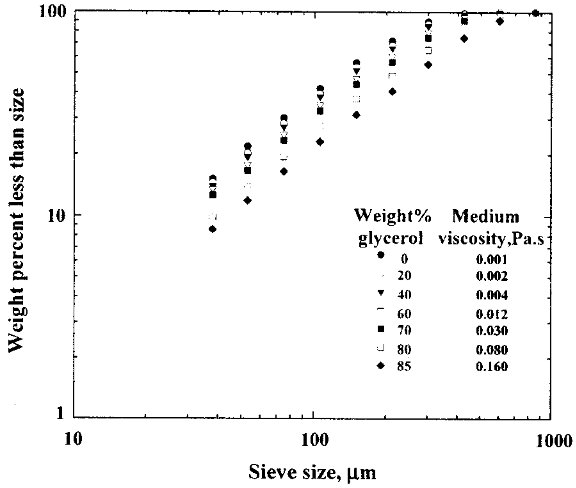

Figure 5. Effect of liquid medium viscosity on experimental product size distributions for 15 min of grinding of 20 × 30 mesh quartz feed (45 vol % solid, $J = 0.3$, $U = 1.0$, 1-in. balls, $\phi_c = 0.7$).

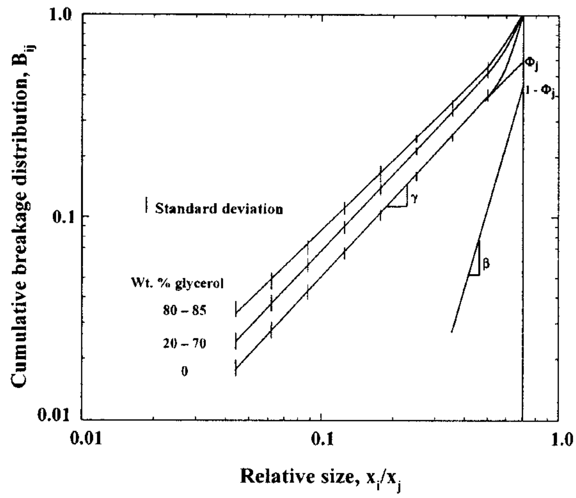

Figure 6. Cumulative primary breakage distribution function for wet grinding 20 × 30 mesh quartz at 45 vol % solid in various concentrations of glycerol–water mixtures ($J = 0.3$, $U = 1.0$, 1-in. balls, $\phi_c = 0.7$).

Table 3. Effect of Liquid Medium Viscosity on the Primary Breakage Distribution Parameters for Grinding 20 × 30 Mesh Quartz in Glycerol–Water Mixtures (45 vol % Solid, $J = 0.3$, $U = 1.0$, 1-in. Balls, $\phi_c = 0.7$)′

percent glycerol (wt %)	medium viscosity at 25 °C (Pa·s)	Φ	γ	β
0(water)	0.001	0.58	1.25	6.0
20	0.002	0.78	1.25	6.0
40	0.004	0.78	1.25	6.0
60	0.012	0.78	1.25	6.0
70	0.030	0.78	1.25	6.0
80	0.080	0.82	1.15	6.0
85	0.160	0.82	1.15	6.0

Breakage Rate Parameter. Because of the inherent non-first-order breakage in wet grinding, the forward computation of size distributions assuming a constant specific breakage rate for the top size, for example, the use of initial constant value of the first-order plot, will not give the correct predictive results. There is evidence

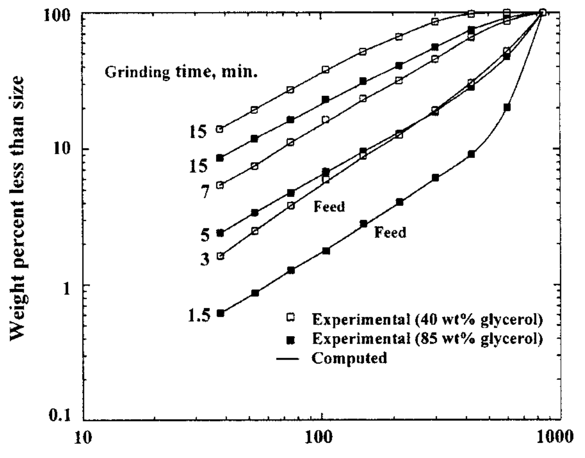

Figure 7. Comparison of experimental and computed size distributions for wet grinding of 20 × 30 mesh quartz at low and high glycerol concentrations (45 vol % solid, $J = 0.3$, $U = 1.0$, 1-in. balls, $\phi_c = 0.7$).

that this rate acceleration may occur predominantly with the top size and the degree of this effect tends to diminish as the next smaller size becomes the new top size in the mill charge.[4,7] It is therefore reasonable to simulate the grinding process at longer grinding times, where the amount of the top size present is relatively small, by letting the breakage of the top size accelerate but keeping the specific rates of breakage of smaller sizes constant.

On the basis of this discussion, the forward computation was performed by using the product at a short grinding time, for example, 1.5 min as a starting feed. Specific breakage rates of the top size were determined from the first-order kinetic plot of Figure 3 over the range from 1.5 min on, while for smaller size fractions their specific breakage rates followed the functional relation of eq 2. By using $\alpha = 0.8$ for quartz and normalized B values from eq 4 and Table 3, the batch grinding equation was solved by a trial-and-error search method to estimate the rate parameter a, which gave a one-point match for a percentage passing 200 mesh (75 μm) between the simulated and experimental size distributions. Figure 7 shows good agreement between the computed and experimental size distributions for grinding at low and high glycerol concentrations. The influence of B values, which produces flatter size distributions at the high glycerol concentration, is discernible. A similar approach in simulating product size distributions with acceleration phenomena in a laboratory overflow ball-mill was also reported by Austin and Klimpel,[19] by allowing the top three sizes to accelerate while keeping the specific rates of breakage of smaller sizes unchanged.

The effect of liquid medium viscosity on the first-order rate constant, a, is depicted in Figure 8 on a semilog plot. In general, the breakage rate constant decreases continuously as the medium viscosity is increased over the range from 0.001 to 0.16 Pa·s, indicating better advantage of grinding particles in a low-viscosity medium such as water. In accordance with the results of B values, the plot in Figure 8 also shows the existence of the three grinding regimes, corresponding to grinding in the three ranges of liquid medium viscosities: 0.001–0.002, 0.002–0.03, and 0.03–0.16 Pa·s, respectively.

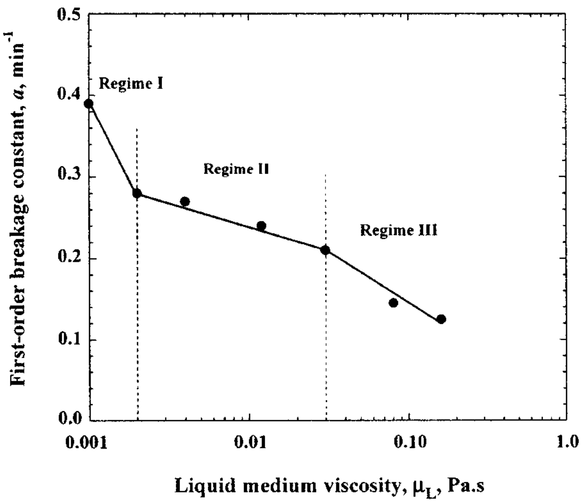

Figure 8. Effect of liquid medium viscosity on the first-order breakage constant for batch grinding of 20 × 30 mesh quartz (45 vol % solid, $J = 0.3$, $U = 1.0$, 1-in. balls, $\phi_c = 0.7$).

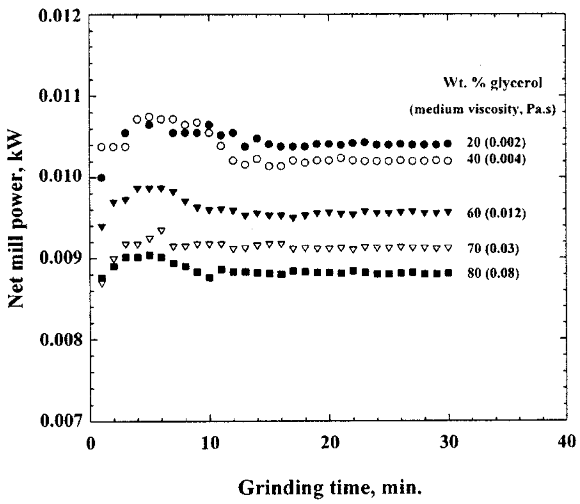

Figure 9. Variation of net mill power with grinding time during batch grinding of 20 × 30 mesh quartz in glycerol–water mixtures (45 vol % solid, $J = 0.3$, $U = 1.0$, 1-in. balls, $\phi_c = 0.7$).

Discussion of Results

It was clear that the decreased specific rates of breakage of the feed size at higher glycerol content, plus the increase in these specific rates of breakage as grinding proceeded with time, were not in themselves sufficient to explain the product size distributions. It was necessary to assume that the basic specific rates of breakage of all particle sizes decreased with higher glycerol content, in the same proportion as that of the feed size. Thus, there are two major effects of the viscosity of the water–glycerol carrier liquid. First, the higher viscosity of the solutions containing higher glycerol content caused a basic decrease of specific rates of breakage. Second, going from dry grinding to wet grinding, at the filling conditions of $J = 0.3$, $U = 1$, led to non-first-order grinding with rate acceleration.

Considering the first (major) effect, it is necessary to see if the net mill power was also changed by the use of water–glycerol media. A separate series of tests[20] gave the results shown in Figure 9. There is a small but consistent increase in net mill power in the first few

Table 4. Effect of Liquid Medium Viscosity on the Average Net Mill Power (m_p) and Breakage Rate Constant (a) for Grinding 20 × 30 Mesh Quartz (45 vol % Solid, $J = 0.3$, $U = 1.0$, $\phi_c = 0.7$)'

percent glycerol (wt %)	medium viscosity (Pa·s)	net mill power (m_p) (kW)	breakage constant (a) (min^{-1})	m_p/a (kW min)	$m_p/(m_p)_{water}$	$a/(a)_{water}$
0(water)	0.001	0.0110	0.39	0.028	1.00	1.00
20	0.002	0.0104	0.28	0.037	0.94	0.72
40	0.004	0.0102	0.27	0.039	0.93	0.68
60	0.012	0.00956	0.24	0.040	0.87	0.62
70	0.030	0.00910	0.21	0.043	0.83	0.54
80	0.080	0.00880	0.14	0.061	0.80	0.36

Table 5. Proportional Changes in the Average Net Mill Power and Breakage Rate Constant with Reference to the Optimum Condition at 45 vol % Solid Concentration, for Grinding 20 × 30 Mesh Quartz in Water at $J = 0.3$, $U = 1.0$, $\phi_c = 0.7$

vol % of solid	net mill power (m_p) (kW)	breakage constant (a) (min^{-1})	$m_p/(m_p)_{max}$	a/a_{max}
30	0.0106	0.36	0.96	0.92
40	0.0108	0.38	0.98	0.97
45	0.0110	0.39	1.0	1.0
50	0.00781	0.23	0.71	0.60
54	0.00682	0.22	0.62	0.56
60	0.00627	0.20	0.57	0.52
65	0.00649	0.24	0.59	0.61

minutes of grinding, followed by a decrease to almost constant values over long periods of grinding time. However, as shown in Table 4, the decrease in average net mill power as the glycerol content was increased was much less than the decrease in the specific rates of breakage. Also, for fixed mill loadings and feed size, the ratio m_p/a is directly related to specific grinding energy. These results show that an increase in the viscosity of the carrier liquid alone can reduce the grinding efficiency. It is known[6] that a decrease in net mill power can be caused by accumulating dense layer of slurry on the mill walls, but this was not observed in this study.

To understand the effects of viscosity on breakage and mill power, it is necessary to consider the three major factors that influence the effective viscosity of a slurry of irregularly shaped particles suspended in a liquid. These are (a) the viscosity of the carrier liquid (medium), (b) the volume concentration of solid in the slurry, and (c) the fineness of the particle size distribution in the slurry. It is well-known that the effective viscosity of a slurry at a given shear rate is directly proportional to the viscosity of the medium, and this was confirmed by Tangsathitkulchai and Austin[21] for a range of medium viscosities. It is also well-established that very large increases in effective viscosity occur as the volume fraction of solid in the slurry approaches that of a packed bed with liquid just filling the interstices. The work of Tangsathitkulchai and Austin[21] also gave a method of estimating the rheological properties of slurries formed with particles of the natural size distributions (approximately Rosin-Rammler distributions) produced by grinding and showed that yield stress and viscosity increased for finer size distributions. These three factors must be borne in mind when comparing the results obtained here with those from similar investigations, as follows.

Tangsathitkulchai and Austin,[4] as quoted above, showed an increase in the specific rates of breakage as the volume concentration of solid was increased up to about 45 vol % of solid. At higher solid concentration the specific rates of breakage decreased, but the changes in the specific rates of breakage were approximately proportional to the changes in net mill power, for both the increasing and decreasing regions (see Table 5). A similar correlation was found by Katzer, Klimpel, and Sewell,[22] using dispersion agents to modify the "fluidity" of the slurries. On the other hand, Austin and Klimpel[19] performed tests where the breakage kinetics were studied with a starting feed containing added fine (minus 200 mesh, 75 μm) material. The results showed increased specific rates of breakage with increased percentage of fine material (that would increase the effective viscosity of the slurry).

It can only be concluded from these apparently contradictory findings that it is not the effective viscosity alone that determines whether the specific breakage rates increase or decrease as viscosity is increased, nor can the resulting changes always be ascribed to changes in mill power that result from viscosity affecting how the ball charge moves. It seems likely that the different methods of influencing viscosity also cause differences in how particles flow into the active breakage regions of the bed, including retention of particles on the surfaces of colliding balls.

Returning to the other major effects seen in the results is the non-first-order kinetics of disappearance of the feed size. Although the viscosity effect on the grinding characteristics of slurry is not conclusively clear, an attempt was made in this study to try to correlate the rate acceleration of the feed fraction with progressive changes in the effective viscosity of the slurry during the course of batch grinding. The acceleration of specific breakage rates of the top size fraction can be quantitatively analyzed by defining a parameter called the acceleration factor, κ, as

$$\kappa = S_1(t)/S_1(N) \qquad (6)$$

where $S_1(t)$ is the instantaneous specific rate of breakage determined from the slope of the first-order plot, that is,

$$S_1(t) = -\frac{d}{dt}\left[\ln\frac{w_1(t)}{w_1(0)}\right] \qquad (7)$$

and $S_1(N)$ is the expected first-order S value derived from $S_1(N) = a(0.85/1)^{\alpha}$, where a is obtained from the forward computation procedure previously described and $\alpha = 0.8$ for the quartz.

Figure 10 shows on a logarithmic scale the dependence of the acceleration factor (κ) on the slurry apparent viscosity (μ_a) during the grinding of 20 × 30 mesh quartz in glycerol–water mixtures. The apparent effective viscosities of slurries were estimated from the rheological correlations developed by Tangsathitkulchai and Austin[21] for homogeneous slurries of particles produced by ball-milling. The range of these correlations covers $m = 0.4-1.2$, $k = 10-200$ μm, $C = 0.20-0.60$, and shear rate = 0–200 s^{-1}, where m and k are the distribution and size moduli of the fitted Rosin-Rammler size distribution and C is the solid volume fraction of the slurry. For comparative purposes, it was assumed in this study that the developed correlations are applicable to the coarse grinding of the closely sized feed of quartz and the shear rate is constant throughout the grinding charge. A shear rate of 100 s^{-1} was arbitrarily

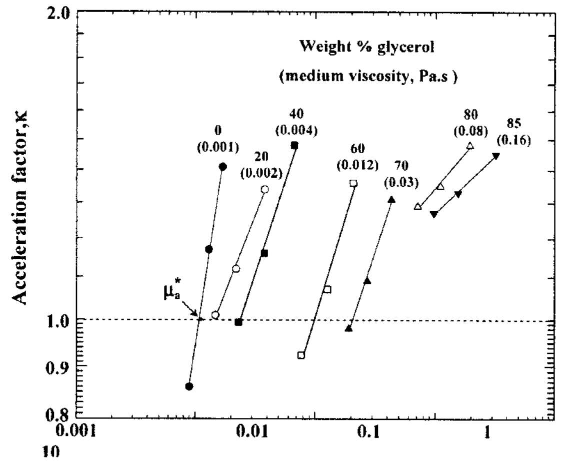

Figure 10. Effect of slurry apparent viscosity (at shear rate of 100 s^{-1}) on the acceleration factor of 20 × 30 mesh quartz feed ground in glycerol–water mixtures (45 vol % solid, $J = 0.3$, $U = 1.0$, 1-in. balls, $\phi_c = 0.7$).

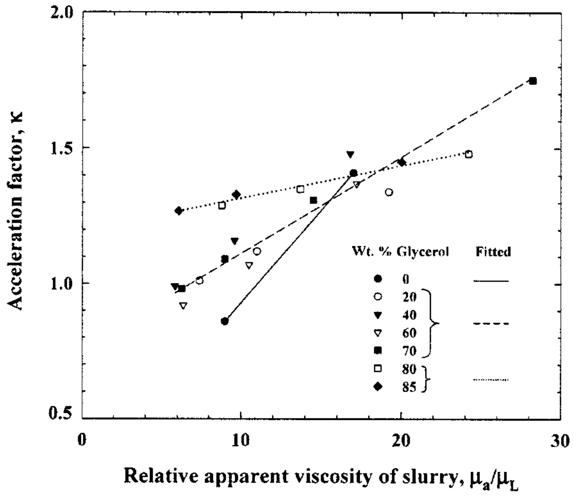

Figure 11. Variation of acceleration factor with relative apparent viscosity of slurry for batch grinding of 20 × 30 mesh quartz (45 vol % solid, $J = 0.3$, $U = 1.0$, 1-in. balls, $\phi_c = 0.7$).

chosen to represent the average shear level of slurry over the course of batch grinding.

Results in Figure 10 show that the acceleration factor starts from somewhat less than unity during the initial period of grinding and increases with slurry viscosity as grinding proceeds to finer sizes. The degree of increase (slope of the plot) roughly decreases with increasing medium viscosity and that can be separated into three viscosity ranges: 0.001(water), 0.002–0.03, and 0.08–0.16 Pa·s, respectively. Figure 11 shows the same plot of acceleration factor but with slurry viscosity expressed as a relative apparent viscosity, μ_a/μ_L where μ_L is the viscosity of a liquid medium. The data points tend to combine into one single group for each of the three liquid viscosity ranges but the influence of medium viscosity range is still very marked, again indicating the possible existence of the three grinding regimes as previously noted.

As pointed out by Tangsathitkulchai,[7] one of the possible explanations for increased breakage rates of the top size is due to the increased probability of these

Table 6. Values of Constants μ_a^* and b in eq 8 as a Function of Medium Viscosity for 20 × 30 Mesh Quartz Ground in Glycerol–Water Mixtures (45 vol % Solid, $J = 0.3$, $U = 1.0$, $\phi_c = 0.7$)'

percent glycerol (wt %)	medium viscosity at 25 °C (μ_L) (Pa·s)	critical slurry viscosity (μ_a^*) (Pa·s)	b	μ_a^*/μ_L
0(water)	0.001	0.011	0.78	10.8
20	0.002	0.015	0.30	7.3
40	0.004	0.024	0.37	6.0
60	0.012	0.098	0.39	8.2
70	0.030	0.205	0.36	6.8
80	0.080	0.116	0.14	1.5
85	0.160	0.116	0.11	0.7

larger size particles staying on the grinding surfaces as grinding proceeds. This was hypothesized to result from the fluid drag that tends to remove smaller particles from the ball surfaces due to their lower masses as the ball charge is moving up against the pool of slurry. This hydrodynamic effect may cause a partition of particles based on sizes between those residing at the grinding surfaces and in the suspension. The increase in suspending medium viscosity tends to counteract this effect since the flow of slurry becomes less turbulent. This effect leads to a more uniform distribution of particles throughout the mill charge, giving rise to the lowering in the degree of rate acceleration of the top-size breakage.

The functional dependence of the acceleration factor on slurry viscosity for $\kappa \geq 1$, as shown in Figure 10, can be fitted by a simple power-law equation,

$$\kappa = \left(\frac{\mu_a}{\mu_a^*}\right)^b, \quad \mu_a \geq \mu_a^* \tag{8}$$

where μ_a^* is the slurry viscosity that gives first-order grinding ($\kappa = 1$) and b is a measure of the degree of rate acceleration. Table 6 lists the values of μ_a^* and b as a function of liquid medium viscosity. Except for the data of 80 and 85 wt % glycerol contents, the onset of the acceleration effect starts when the slurry viscosity is about 6–11 times that of the medium viscosity, or on the average at $\mu_a = \mu_a^* \cong 8\mu_L$.

In analyzing the rate acceleration of the three top sizes of quartz (30 × 40, 40 × 50, and 50 × 70 mesh) for a given powder filling condition, Austin and Klimpel[19] found that all first-order kinetic plots fell on the same curve when the time scale (t) was represented by a dimensionless time parameter defined as $S_1(0)t$, with $S_1(0)$ being the initial slope of the usual first-order curve. This finding indicates that the acceleration effect appears to depend on the size distribution in the mill with respect to the top size only. However, a different set of the generalized kinetic curves existed for grinding at another powder filling level ($U = 0.9$ versus $U = 1.75$).

The same approach to the method of Austin and Klimpel[19] was adopted in this work to reanalyze the rate acceleration of the feed size by plotting the time scale of the first-order disappearance kinetics as $S_1(N)t$. The results are shown in Figure 12. It is seen that all first-order plots are combined into one single curve for grinding at different liquid medium viscosities (curve III), except the result of grinding in 0% glycerol content (water) (curve I). Also shown in Figure 12 for comparison are the results for the effect of slurry density (for grinding quartz in water) on the breakage kinetics of

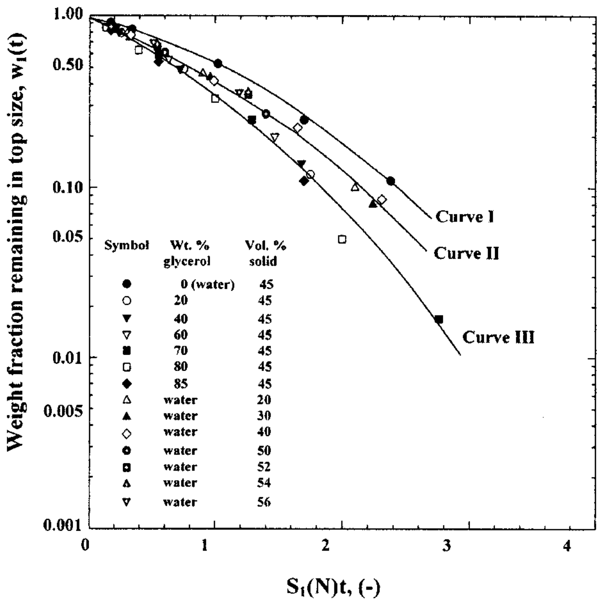

Figure 12. First-order kinetic plots on a dimensionless time basis for wet grinding of 20 × 30 mesh quartz ($J = 0.3$, $U = 1.0$, 1-in. balls, $\phi_c = 0.7$).

the same top-size feed under the same powder and ball loading conditions. A separate set of curves also exists which combines the results of grinding at different slurry concentrations (curve II), except again for grinding at 45 vol % solid. It is fair to say that the acceleration of top-size breakage rates as fines accumulate in the mill is a complex process, depending not only on the fineness of size distribution or the viscosity of the slurry content (the rheological effect) but also on other related factors such as the packing and distribution of particles in the mill charge and the way particles interact with the grinding media. It is noted that the presentation of breakage kinetics as shown in Figure 12 offers the advantage of containing all the acceleration factors (κ) of the top size in one single set since the slope of the curve represents the value of the acceleration factor. That is,

$$-\frac{\mathrm{d}\left[\ln\frac{w_1(t)}{w_1(0)}\right]}{\mathrm{d}[S_1(N)t]} = \frac{-\mathrm{d}\left[\ln\frac{w_1(t)}{w_1(0)}\right]}{S_1(N)\,\mathrm{d}t} = \frac{S_1(t)}{S_1(N)} = \kappa \qquad (9)$$

Since the acceleration effect was assumed in this study to occur only for the top size, the forward simulation for product size distributions under varying medium viscosity can be conveniently achieved from the generalized kinetic curves (Figure 12), which generates the values of $S_1(t)$ via eq 9 plus the knowledge of a from Table 4 for specific breakage rates of smaller sizes.

Conclusions

The effect of suspending liquid viscosity on the breakage characteristic of 20 × 30 mesh quartz was investigated in a batch laboratory ball mill, using mixtures of glycerol and water in varying proportions and at a fixed concentration of 45 vol % of solid in slurry. Over the medium viscosity range from 0.001 to 0.16 Pa·s, the expected first-order breakage constant (a values) was found to decrease continuously as the medium viscosity was increased, with the largest decrease occurring in a narrow range of viscosity from 0.001(water) to 0.002 Pa·s. The acceleration in the specific breakage rates (S values) of the feed size was also observed for different medium viscosities as grinding proceeded with time to finer sizes. The primary breakage distribution (B values) was normalized and separated into three different sets for media viscosities of 0.001(water), 0.002–0.03, and 0.08–0.16 Pa·s, with the higher viscosity range producing finer primary breakage products. Two approaches were used in correlating the rate acceleration of the top size with the extent of grinding: (a) by presenting the acceleration factor with reference to changes in slurry viscosity as grinding proceeded with time and (b) as a single-disappearance kinetics plot, containing information of the acceleration factor for grinding in different medium viscosities. Evidence from the grinding results also suggested that three different breakage regimes may exist corresponding to the change in medium viscosity range of 0.001–0.002, 0.002–0.003, and 0.003–0.16 Pa·s, respectively. It can be conferred that different means of influencing the slurry rheology, for example, by varying slurry density, adding fine material, using different viscosity of a carrier liquid, and modifying the viscous nature with chemical additives probably will have different effects on the grinding performance, although the resultant changes in the viscous properties of the slurry may be the same.

Further work should be directed toward elucidating the sharp decrease in the first-order breakage constant over the narrow increase of medium viscosity from 0.001 Pa·s (water) to 0.002 Pa·s (20 wt % glycerol) since the physical properties of the two media (density, viscosity, and surface tension) are not so different. To further understand the effect of medium viscosity on particle breakage, grinding tests with glycerol–water mixtures should be extended to cover different milling conditions including ball and powder filling, mill rotational speed, and solid content in the slurry.

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