A FUZZY SYSTEM AIDS VISIBILITY FORECASTING

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Abstract

This paper describes the innovative implementation of a fuzzy system that assists meteorologists in forecasting visibility. The forecast made is for a restricted area, Don Muang Air Base, in Bangkok. The subjective view of an experienced human forecaster was modelled verbally and a set of forecast rules and a fuzzy system could then be devisede. The system utilized the fuzzy inference mechanism and was coded in FORTRAN. The system produces relatively good results even though there are some errors due to the fuzzification/defuzzification processes and truncation in computational terms.

Key words: *Weather prediction, linguistic variables, fuzzy inference, human modelling.*

Today's science and engineering are mostly characterized by quantitative mathematical models of various types.

However, some scientific phenomena cannot be described precisely by such concrete terms. One of those is climatological prediction.

Predictions made by meteorologists are subjective judgment based on investigations of classes of data. Each prediction illustrates decisions and judgments belonging to a specific meteorologist on a particular occasion (Kandel 1978). Moreover, the weather data investigated by meteorologists have no clear boundaries. In such circumstances, human modelling and verbal models can be appropriately applied.

In human modelling, human's thought processes can be expressed by natural language.

Then, a non classical type of mathematical model, so called verbal models, can be constructed. The concept of fuzziness (Zadeh 1965) and linguistic variables (Zadeh 1973) play an important role in such models in this context. Appendix (i) reviews some basic properties of fuzzy sets and fuzzy logic.

A verbal model can be viewed as a system consisting of a set of condition-action rules with mathematical operators (Negoita 1985). These rules operate in cycles in the manner of a pattern invoked program. Quite often, rules are formulated in natural language having the form of "condition IMPLIES action", whose vagueness reflects the human expert's knowledge. When condition and action are expressed in terms of linguistic variables and inference mechanism used is the fuzzy inference type, the system is known as a fuzzy system.

The purpose of this article is to describe the application of fuzzy inference to the problem of weather prediction. In this, the task of prediction is limited to visibility forecast in a confined area in Bangkok. It is possible to use the implemented fuzzy system for making a prediction on a day-to-day basis and to give advice to inexperienced forecasters.

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Relevant Linguistic Variables

It is well known that poor visibility has severe effects on flight operation. Don Muang Air Base, where both civilian and military air-ports are situated, is often affected by poor visibility during the cold season (from December to February). Visibility forecasting is thus one of several important tasks necessary to support flight planning and operation.

Visibility problems are primarily concerned with fog and mist. During this particular time of the year in Don Muang Air Base, it is believed that the two major factors contributing to the occurrence of fog and mist are surface wind and relative humidity. Other factors, such as trough and wind direction, occur consistently during the cold season. Hence, this application considers surface wind and relative humidity as linguistic variables or fuzzy variables with various labels or descriptors. Visibility is another linguistic variable and has relationships with the other two in the form of a verbal model as follows:

IF [surface wind is rather calm] AND [relative humidity is high] THEN
[visibility is quite poor].

Meteorologists normally obtain wind speed and the degree of relative humidity from measuring instruments. However, meteorologists do not apply these values for their prediction in a pre-cise manner. In this application, fuzzy concepts for decision making (in an imprecise manner) are applied to handle this type of unconventional prediction (similar to meteorologists' subjective judgment). Descriptors are necessarily predefined for such prediction. Firstly, universes of discourse are defined as follows:

surface wind (knots) : [0 1 2 3 4 5 6 7 8 9 10],
- relative humidity (%) : [70 75 80 85 90 95 100], and
- visibility (miles): [0 1 2 3 4 5 6 7]

These variables are assigned their descriptors as follows:
- surface wind : rather calm, light, moderate and rather moderate,
- relative humidity: low, medium and high, and
- visibility: good, poor, quite good and quite poor,

the membership grades of which are shown in Tables 1, 2 and 3, respectively.

In order to obtain good results, forecast rules were grouped into four sets according to time intervals from 05.00 a.m. to 10.00 a.m. Appendix (ii) presents the collection of these rules.

Inference Rules

The fuzzy system of this application can be viewed as a decision support system. Such a system can be defined as an interactive computer system, which directly helps humans make decisions or supplies recommendations and conclusions. In most cases, the system has production rules with their structure in the form of :

IF [antecedent(s)] THEN [consequent(s)].

Normally, the inference mechanism, also known as rule interpreter, in these systems is an antecedent-driven type called modus ponens.

Fuzzy control systems have also been studied and their relevant theories on multivariable structure of open-loop and closed-loop systems have been developed (Gupta et al. 1986). Forecast rules with the structure mentioned in the previous section are similar to control rules of an open-loop system. It was suggested that fundamental rules of the open-loop type be of the form :

Table 1. Membership grades of descriptors of surface wind.

SW Universe (knots)	0	1	2	3	4	5	6	7	8	9	10
RC	0	1.0	1.0	0.6	0	0	0	0	0	0	0
LI	0	0.8	0.8	1.0	1.0	1.0	0.7	0.4	0.3	0.2	0
MO	0	0	0	0	0	0	0.2	1.0	1.0	1.0	1.0
RM	0	0	0	0	0	0	1.0	1.0	0.3	0	0

Table 2. Membership grades of descriptors of relative humidity.

RH Universe (%)	70	75	80	85	90	95	100
L	1.0	1.0	0.8	0.4	0	0	0
M	0	0.2	0.6	1.0	0.8	0.4	0
H	0	0	0.4	0.6	0.8	1.0	1.0

Table 3. Membership grades of descriptors of visibility.

VI Universe (miles)	0	1	2	3	4	5	6	7
G	0	0	0.3	0.5	0.6	0.7	1.0	1.0
P	1.0	1.0	1.0	0.5	0.4	0.3	0	0
QG	0	0	0	0.6	1.0	0.8	0	0
QP	0	0	0.8	1.0	0.6	0	0	0

IF $[X_{(1)}]$ THEN $[Y_{(1)}]$ ALSO

IF $[X_{(2)}]$ THEN $[Y_{(2)}]$ ALSO - - -

IF $[X_{(n)}]$ THEN $[Y_{(n)}]$

These rules can be represented by a fuzzy relational matrix R, where :

$$R = \bigvee_{i=1}^{n} [\wedge \{X_{(i)}, Y_{(i)}\}]$$

(v is a max operator, ∧ is a min operator).

An appropriate consequent Y can be inferred by applying the compositional rule of inference, which is :

$$Y = X \, o \, R \qquad \text{(o is a max - min operator).}$$

For a multivariable system, one possibility is to use the following linguistic description :

$\{(IF\ [X_{1(i)}\ and\ X_{2(i)}\ and...X_{k(i)}\ ]\ THEN$

$[Y_{1(i)}\ and\ Y_{2(i)}\ and...Y_{j(i)}\ ]), (ALSO)\ \}, i = 1, 2, ..., n.$

In this system, there are k Xs , j Ys and n rules. These rules are joined together by the connective ALSO. The fuzzy relational matrix R_{kj} representing these rules can be computed from:

$$R_{kj} = \bigvee_{i=1}^{n} [\wedge (X_{k(i)}, Y_{j(i)})], \; k = 1, 2, ..., \; j = 1, 2, ...$$

To infer Y_j , one could follow the computation :

$$Y_{kj} = X_k \, o \, R_{kj}, \; k = 1, 2, ... , \; j = 1, 2, ...$$

and

$$Y_j = \bigwedge_{k=1,2,...} Y_{kj}$$

Let consider the set of forecast rules for 05.00 - 06.00 a.m. as an example. Each rule is of the form :

IF [$X_{I(i)}$ *and* $X_{2(i)}$] *THEN* [$Y_{I(i)}$]. Thus, it is needed to compute 2 relational matrices namely

$$Q_{11} = \bigvee_{i=1}^{6} [\wedge \{\mu_{(i)} (RH), \mu_{(i)} (VI)\}], \text{ and}$$

$$Q_{21} = \bigvee_{i=1}^{6} [\wedge \{\mu_{(i)} (SW), \mu_{(i)} (VI)\}]$$

Q_{11} and Q_{21} are shown in appendix (iii). Computation to make a conclusion about visibility requires descriptors of RH and SW. Once the system knows such descriptors, the computation is as follows:

$$W_1 = \mu (RH) \ o \ Q_{11}$$

$$W_2 = \mu (SW) \ o \ Q_{21}$$

then, $$Y = \wedge (W_1, W_2)$$

For numerical example, lets consider the rule :
IF [surface wind is light and relative humidity is high] THEN [visibility is quite poor], where:

$$W_1 = \mu \ (RH, high) \ o \ Q_{11}$$

$$= [0.0\ 0.0\ 0.8\ 1.0\ 0.6\ 0.4\ 0.0\ 0.0], \text{ and}$$

$$W_2 = \mu \ (SW, high) \ o \ Q_{21}$$

$$= [0.0\ 0.0\ 0.8\ 1.0\ 1.0\ 0.8\ 0.0\ 0.0], \text{ therefore}$$

$$Y = \wedge [W_1, W_2] = [0.0\ 0.0\ 0.8\ 1.0\ 0.6\ 0.4\ 0.0\ 0.0].$$

If the α - cut principle or α - thereshold (Zadeh 1968) is applied with the value of α = 0.5, the final membership grades Y will be :

$$Y = [0.0\ 0.0\ 0.8\ 1.0\ 0.6\ 0.4\ 0.0\ 0.0].$$

The obtained membership grades show compatibility with the originally defined ones for "quite poor".

Computational Results and Discussions

The previous section exemplified the simplest case in computation. If the set of forecast rules for 05.00 - 06.00 a.m. is considered, it will be seen that the system produces three different sets of membership grades as follows:

- [0.0 0.0 0.6 0.6 1.0 0.8 0.0 0.0],
- [0.0 0.0 0.8 1.0 0.6 0.6 0.0 0.0], and
- [0.0 0.0 0.8 1.0 0.6 0.4 0.0 0.0].

When the α = cut principle with α = 0.5 is applied, only the third set is modified to be :
[0.0 0.0 0.8 1.0 0.6 0.0 0.0 0.0].

This perfectly matches the "quite poor" descriptor. However, the unchanged first and second sets are closely similar to the membership grades for "quite good" and "quite poor", respectively. Errors occurred are mainly due to fuzzification/defuzzification processes and truncation in computational terms (Gupta et al. 1986) to obtain the inference rules described in the previous section. If one applies different values of α, the resulted membership grades will have different patterns from the previous and the originally assigned ones. This introduces difficulties in recognition of the membership patterns such that appropriate natural words could be assigned to the inferred results. An ambiguous situation may arise in some cases, such as for the time interval 08.00 - 09.00 a.m., if the rule :

IF [surface wind is rather moderate and relative humidity is medium] THEN [visibility is quite poor] is executed, the grades of membership [0.7

0.7 0.8 1.0 1.0 0.8 0.0 0.0] will be obtained. The result does not match or even come close to any predefined patterns.

Due to the difficulties mentioned above, a thorough execution of all predetermined rules was carried out. By sieving out the inferred membership grades of which values lower than 1.0, this yields seven different patterns illustrated in Table 4. These patterns are used as templates for finding suitable natural words to be assigned to the numeric results. Notice that a new descriptor (fair) is introduced to cope with any ambiguity, which may arise.

Table 4. Templates for finding appropriate descriptors

VI Universe	0	1	2	3	4	5	6	7
Quite good	0	0	0	0	1	0	0	0
Quite poor	0	0	0	1	0	0	0	0
Acceptable	1	1	1	0	0	0	0	0
as poor	1	1	1	1	0	0	0	0
Fair	0	0	0	1	1	0	0	0
Acceptable	0	0	0	0	1	0	1	1
as good	0	0	0	0	0	0	1	1

Conclusions

A fuzzy system to assist visibility forecast has been introduced. The system utilizes FORTRAN codes and fuzzy inference mechanisms, which are based upon relevant fuzzy control system theory. Recognition of the patterns of the obtained numeric results is carried out in a simple way by using specific templates suitable for this particular application. The system is useful for a day-to-day prediction as well as for giving advice to inexperienced meteorologists.

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Appendix (i) : *relevant fuzzy logic properties*

X is a fuzzy subset of a universe of discourse U. X is then characterized by a membership function $\mu_x(u)$ that assigns to each element $u \in$ U a number $\mu_x(u)$ in the interval 0 to 1. Such members represent grades of membership in X.

Basic fuzzy logic operators, which are union, intersection and complement, are defined as follows :

- X and Y are fuzzy subsets of the universe of discourse U. "X union Y" can be expressed as "X OR Y" and denoted by "X + Y", whose membership function is defined by

$$\mu_{X+Y}(u) = \max\ [\mu_X(u), \mu_Y(u)].$$

- "X intersect Y" can be expressed as "X and Y" and denoted by "X $\cap$ Y", whose membership function is defined by

$$\mu_{X \cap Y}(u) = \min\ [\mu_X(u), \mu_Y(u)]$$

- "complement X" can be expressed as "NOT X" and denoted by "$\neg$ X", whose membership funcition is defined by

$$\mu_{\neg X}(u) = 1 - \mu_X(u).$$

A linguistic implication or a fuzzy conditional statement has its basic form of : "If X then Y" or "X $\Rightarrow$ Y". The implied relation R is denoted by R = X x Y, where x stands for the Cartesian product of the fuzzy subsets X and Y. The membership function of R is defined by

$$\mu_R(u,v) = \mu_{XxY}(u,v)$$
$$= \min\ [\mu_X(u), \mu_Y(v)],\ u \in U,\ v \in V$$

where X is a fuzzy subset of the universe of discourse U, and
Y is a fuzzy subset of the universe of discourse V.

Appendix (ii) : *notations and collection of forecast rules*

- notations :

RH:	=	relative humidity	SW:	=	surface wind	VI:	=	visibility	G:	=	good
H:	=	high	L:	=	low	M:	=	medium	P:	=	poor
LI:	=	light	MO:	=	moderate	QG:	=	quite good	QP:	=	quite poor
RC:	=	rather calm	RM:	=	rather moderate						

(A,u) (B,v) ⇒ (C,w) stahds for IF [A.is u] and [B is v] THEN [C is w]

- collection of forecast rules:
- forecast rules for 05.00 - 06.00 a.m.

(SW, RC) (RH, L)	⇒	(VI, QG)	:	(SW, LI) (RH, L)	⇒	(VI, QG)
(SW, RC) (RH, M)	⇒	(VI, QP)	:	(SW, LI) (RH, M)	⇒	(VI, QP)
(SW, RC) (RH, H)	⇒	(VI, QP)	:	(SW, LI) (RH, H)	⇒	(VI, QP)

- forecast rules for 06.00 - 08.00 a.m.

(SW, RC) (RH, L)	⇒	(VI, QP)	:	(SW, LI) (RH, L)	⇒	(VI, P)
(SW, MO) (RH, L)	⇒	(VI, QP)	:	(SW, RM) (RH, L)	⇒	(VI, P)
(SW, RC) (RH, M)	⇒	(VI, P)	:	(SW, LI) (RH, M)	⇒	(VI, P)
(SW, MO) (RH, M)	⇒	(VI, QP)	:	(SW, RM) (RH, M)	⇒	(VI, P)
(SW, RC) (RH, H)	⇒	(VI, P)	:	(SW, LI) (RH, H)	⇒	(VI, P)
(SWI, MO) (RH, H)	⇒	(VI, QP)	:	(SW, RM) (RH, H)	⇒	(VI, P)

- forecast rules for 08.00 - 09.00 a.m.

(SW, RC) (RH, L)	⇒	(VI, QP)	:	(SW, LI) (RH, L)	⇒	(VI, QP)
(SW, MO) (RH, L)	⇒	(VI, QG)	:	(SW, RM) (RH, L)	⇒	(VI, QP)
(SW, RC) (RH, M)	⇒	(VI, QP)	:	(SW, LI) (RH, M)	⇒	(VI, QP)
(SW, MO) (RH, M)	⇒	(VI, QG)	:	(SW, RM) (RH, M)	⇒	(VI, QP)
(SW, RC) (RH, H)	⇒	(VI, QP)	:	(SW, LI) (RH, H)	⇒	(VI, P)
(SW, MO) (RH, H)	⇒	(VI, QP)	:	(SW, RM) (RH, H)	⇒	(VI, QP)

- forecast rules for 09.00 - 10.00 a.m.

(SW, LI) (RH, L)	⇒	(VI, QP)	:	(SW, MO) (RH, L)	⇒	(VI, G)
(SW, RM) (RH, L)	⇒	(VI, G)	:	(SW, LI) (RH, M)	⇒	(VI, QP)
(SW, MO) (RH, M)	⇒	(VI, G)	:	(SW, RM) (RH, M)	⇒	(VI, QG)
(SW, LI) (RH, H)	⇒	(VI, QP)	:	(SW, MO) (RH, H)	⇒	(VI, QG)
(SW, RM) (RH, H)	⇒	(VI, QG)				

Appendix (iii) : *relational matrices Q_{11} and Q_{21}*

- 05.00 - 06.00 a.m.

$$Q_{11} = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.6 & 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.6 & 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.6 & 0.6 & 0.8 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 0.6 & 0.4 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 0.8 & 0.6 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 0.6 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 0.6 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

$$Q_{21} = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.7 & 0.7 & 0.7 & 0.7 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.4 & 0.4 & 0.4 & 0.4 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.3 & 0.3 & 0.3 & 0.3 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.2 & 0.2 & 0.2 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

- 08.00 - 09.00 a.m.

$$Q_{11} = \begin{bmatrix} 0.0 & 0.0 & 0.8 & 1.0 & 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 1.0 & 0.8 & 0.0 & 0.0 \\ 0.4 & 0.4 & 0.8 & 0.8 & 0.8 & 0.8 & 0.0 & 0.0 \\ 0.6 & 0.6 & 0.8 & 1.0 & 1.0 & 0.8 & 0.0 & 0.0 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 0.4 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \end{bmatrix}$$

$$Q_{21} = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.8 & 0.8 & 0.8 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 0.8 & 0.8 & 0.8 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 0.7 & 0.7 & 0.8 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 0.4 & 0.4 & 0.8 & 1.0 & 1.0 & 0.8 & 0.0 & 0.0 \\ 0.3 & 0.3 & 0.8 & 1.0 & 1.0 & 0.8 & 0.0 & 0.0 \\ 0.2 & 0.2 & 0.8 & 1.0 & 1.0 & 0.8 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 1.0 & 0.8 & 0.0 & 0.0 \end{bmatrix}$$

- 06.00 - 08.00 a.m.

$$Q_{11} = \begin{bmatrix} 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.6 & 0.3 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.6 & 0.3 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \end{bmatrix}$$

$$Q_{21} = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 0.6 & 0.6 & 0.3 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 0.5 & 0.4 & 0.3 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 0.5 & 04. & 0.3 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 0.5 & 0.4 & 0.3 & 0.0 & 0.0 \\ 1.0 & 1.0 & 1.0 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 0.3 & 0.3 & 0.6 & 1.0 & 0.6 & 0.3 & 0.0 & 0.0 \\ 0.2 & 0.2 & 0.8 & 1.0 & 0.6 & 0.2 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 0.6 & 0.0 & 0.0 & 0.0 \end{bmatrix}$$

- 09.00 - 10.00 a.m.

$$Q_{11} = \begin{bmatrix} 0.0 & .0. & 0.8 & 1.0 & 0.6 & 0.7 & 1.0 & 1.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 0.6 & 0.7 & 1.0 & 1.0 \\ 0.0 & 0.0 & 0.8 & 0.8 & 0.6 & 0.7 & 0.8 & 0.8 \\ 0.0 & 0.0 & 0.8 & 1.0 & 1.0 & 0.8 & 1.0 & 1.0 \\ 0.0 & 0.0 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \\ 0.0 & 0.0 & 0.8 & 1.0 & 1.0 & 0.8 & 0.4 & 0.4 \\ 0.0 & 0.0 & 0.8 & 1.0 & 1.0 & 0.8 & 0.0 & 0.0 \end{bmatrix}$$

$$Q_{21} = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 0.6 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 0.8 & 0.6 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 0.6 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 0.6 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.8 & 1.0 & 0.6 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.7 & 0.7 & 1.0 & 0.8 & 1.0 & 1.0 \\ 0.0 & 0.0 & 0.4 & 0.6 & 1.0 & 0.8 & 1.0 & 1.0 \\ 0.0 & 0.0 & 0.3 & 0.6 & 1.0 & 0.8 & 1.0 & 1.0 \\ 0.0 & 0.0 & 0.3 & 0.6 & 1.0 & 0.8 & 1.0 & 1.0 \\ 0.0 & 0.0 & 0.3 & 0.6 & 1.0 & 0.8 & 1.0 & 1.0 \end{bmatrix}$$