CHAPTER II
LITERATURE REVIEW

2.1 Introduction

Literature review is conducted on experimental studies that investigate the deformation moduli as well as single and multi-stage triaxial compressive strengths. Data sources include journals, technical reports, and conference proceedings. This chapter presents a summary of the findings from the literature review.

2.2 Single stage vs. Multi–stage triaxial compression test

Prayanto, Nurhandoko, and Sunardi (2022) conduct a study the multi-stage triaxial compression test as a cost-effective alternative to the traditional triaxial compression test. This method is particularly useful for determining the reliability of shear strength parameters in rock or soil samples. The study aimed to obtain key material properties such as Young's modulus, Poisson's ratio, cohesive strength, and the internal friction coefficient, all of which are crucial in geotechnical engineering. Shear strength, a fundamental property of rock and soil, plays a critical role in ensuring the stability of various structures such as foundations, retaining walls, slopes, and embankments. Traditionally, shear strength is determined through laboratory tests conducted on either compacted specimens or samples extracted from exploration drilling. However, variability in soil and rock properties often complicates the assessment process, leading to challenges in accurately characterizing shear strength. To address these issues, the multi-stage triaxial compression test was proposed as an effective method for minimizing the effects of soil variability. This approach allows for the extraction of extensive shear strength data from a single specimen by subjecting it to multiple stages of increasing confining stresses. The results demonstrate that this method is both practical and effective for measuring shear strength, providing comparable outcomes to traditional shear tests. Additionally, the study found significant insights into Poisson's ratio behavior during testing. Specifically, Poisson's ratio was observed to increase as the material approached the yield point. Before reaching the yield point, the ratio was relatively low, peaked at the yield point, and then decreased after surpassing it. This variation in Poisson's ratio highlights the material's deformation behavior during loading and offers valuable information for geotechnical applications. Overall, the multi-stage triaxial compression test offers a reliable and economical approach for evaluating shear strength and associated parameters, making it a valuable tool for addressing challenges in geotechnical engineering

Aghababaei, Behnia, and Moradian (2019) investigat the effectiveness of single stage triaxial experiments and two variants of the multi-stage triaxial test method: (Crawford and Wylie, 1987) modifies version and the original approach suggested by the International Society for Rock Mechanics (ISRM) (Kovari, Tisa, Einstein, and Franklin, 1983). Their study focused on carbonate rocks subjected to varying confining pressures to evaluate and compare the yield strength results from these methods. The researchers analyze the confidence and prediction intervals for all three testing approaches based on their fit to the yield strength. Multi-stage triaxial tests are useful and economical for determining shear strength values, but specimen integrity must be carefully considered. Crawford and Wylie (1987) modified approach is clearly more dependable option than the ISRM method, especially when maintaining the specimen's structural integrity is essential for accurate results.

Al-Maamori, El Naggar, and Micic (2019) investigate the strength characteristics of swelling Queenston shale, with specimens soaked in various fluids for 100 days. They find that the multi-stage triaxial compression tests were well-suited for studying this type of shale, as it had minimal influence on the measured strength. However, the test's reliability depends on precise monitoring of deformation curves during loading to accurately determine the maximum strength at each confining pressure. Failure to identify the imminent strength at any stage compromises the test's advantage over the traditional single stage triaxial compression tests.

Tsoi and Homenok (2017) conduct multi-stage triaxial compression tests on sandstone specimens, creating Mohr-Coulomb envelopes for each sample. They determine key parameters such as elasticity modulus, Poisson's ratio, elastic limit, and specific energy intensity relative to the loading stage. The results revealed a consistent increase in elastic limit, elasticity modulus, and Poisson's ratio across successive stages, highlighting the variation of these properties under loading-unloading conditions. This variation should be considered in theoretical stress-strain calculations. However, the study concluded that Mohr-Coulomb envelopes derived from multi-stage tests provide a reliable method for describing sandstone strength, especially in situations with limited core samples. This method serves as an alternative to traditional strength envelope techniques, which rely on separate experimental tests (splitting tensile, uniaxial, and conventional triaxial compression). Furthermore, multi–stage triaxial compression makes it possible to describe the stepwise mechanical behavior of rock from an elastic state to a plastic state.

Yang (2012) investigates stress paths and mechanical behaviors of red sandstone under single-stage and multi-stage triaxial compression tests. The stress path for the single stage triaxial test (Fig. 2.1a) followed the conventional triaxial compression method, while the multi-stage triaxial test (Fig. 2.1b) used a single specimen to evaluate the peak strength under varying confining pressures. This approach allows for efficient confirmation of rock peak strength across different conditions. The study found that the peak strength of red sandstone under multi-stage triaxial compression aligns more closely with the nonlinear Hoek-Brown criterion than the linear Mohr-Coulomb criterion. Additionally, for specimens exhibiting larger post-peak circumferential deformation, the internal friction angle under multi-stage compression was approximately the same as in single stage tests, but the cohesion value was significantly lower. This reduction in cohesion correlates with increased circumferential strain after peak strength, reflecting specimen damage caused by the multi-stage testing process. This damage is interpreted as a loss of cohesion within the sandstone.

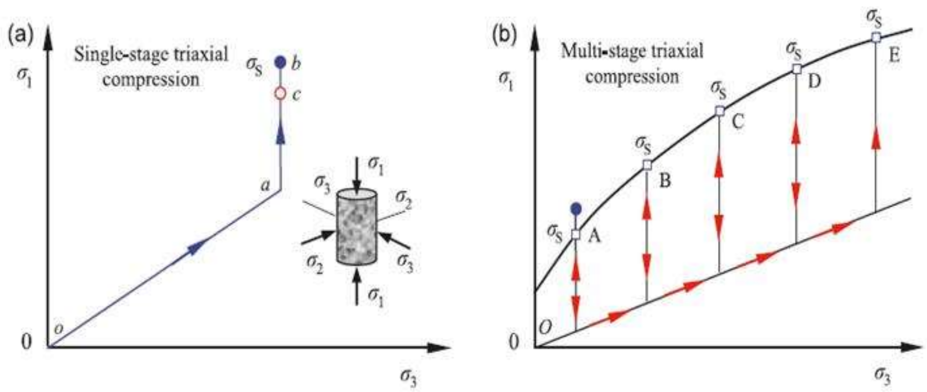


Figure 2.1 Stress path for single stage, (a) and multi-stage triaxial tests (b) (Yang 2012).

Cain, Yuen, Le Bel, Crawford, and Lau (1987) investigate a modified multistage testing procedure provides an accurate determination of the peak strength envelope from a single specimen, with an average discrepancy of approximately +5% compared to single stage tests. This method relies on the volumetric strain/axial strain relationship to predict instability, enabling controlled stage transitions. Unlike the ISRM standard, which maintains the maximum axial stress while increasing confining stress, the modified approach allows partial elastic recovery by relaxing stresses to a hydrostatic level. This adjustment enhances the accuracy of replicating single-stage test conditions. Future improvements, such as servo-controlled confining stress with direct volume change measurement, may further refine the accuracy of this method.

2.3 Factors affecting multi–stage triaxial compression test

2.3.1 Effect of multi–stage triaxial compression test

2.3.1.1 Strength Criteria

Shia et al. (2016) conduct a series of multi-stage triaxial compression tests alongside conventional triaxial tests on sandstone samples to study radial strain variations under different loading paths. The observed variation in radial strain was too minor to effectively compare single and multi-stage tests directly. To address this, they developed a model to correct the strength parameter discrepancies between the two methods. The corrected strength parameters from the multi-stage tests closely matched the conventional triaxial test results, demonstrating the model's effectiveness. Using the Mohr-Coulomb criterion, the data in Figure. 2.2 shows the fitted curve, yielding a high adjusted R-square of 0.932 and an R-square of 0.939, indicating a strong correlation. The fitted parameters included a cohesion value of 22.02 MPa and an internal friction angle of 18.59°, confirming that the Mohr-Coulomb criterion accurately describes the sandstone's strength behaviors.

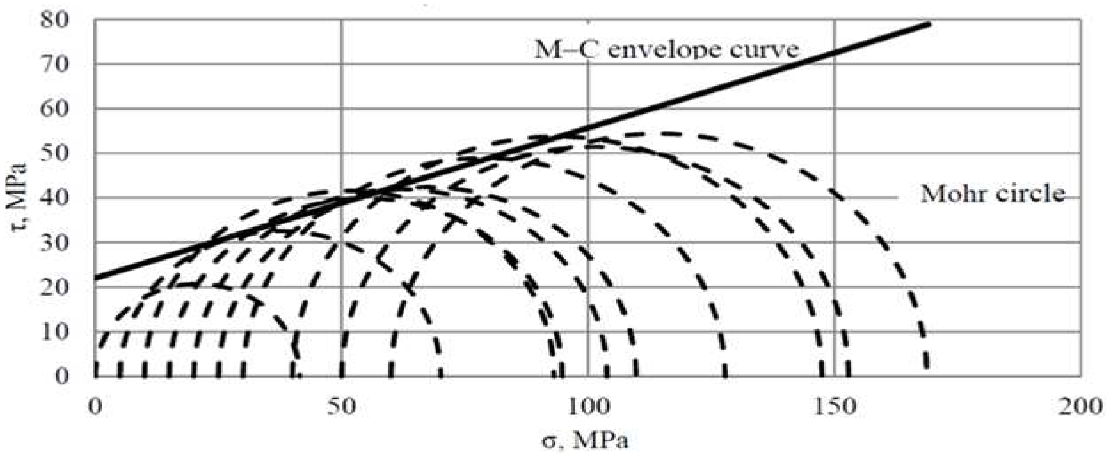


Figure 2.2 Conventional triaxial test data for sandstone (Shia et al., 2016).

Venter, Purvis, and Hamman (2016) conduct a study demonstrates that the intact Hoek–Brown parameters (σ_{ci} and m_i) can be effectively estimated using multistage triaxial testing. This approach optimizes specimen usage, reducing the required number by 80% compared to single-stage testing. Moreover, multistage testing allows for probabilistic analysis due to multiple estimates of σ_{ci} and m_i. However, the study highlights that these parameters are highly sensitive to input variations and should be derived as parameter pairs. Since the research is based on a single homogeneous rock type, further validation across various rock types is necessary.

Yang (2012) investigate the mechanical behavior of red sandstone under multi-stage and single stage triaxial compression, focusing on the relationship between the crack damage threshold and confining pressure (σ_3). The multi-stage triaxial compression findings show that the crack damage threshold increased nonlinearly with higher σ_3 under multi-stage triaxial compression, contrasting with the linear relationship observed in single-stage tests. The difference in crack damage thresholds between the two methods grew as σ_3 increased, indicating that multi-stage loading conditions facilitate easier crack creation and reopening. Importantly, the crack damage threshold in multi-stage triaxial compression was found to be independent of the fractured extent of the specimen's post-peak strength. Single Stage Triaxial Compression Results. The study also presented stress-strain curves for single-stage compression tests (Figure 2.4), showing brittle post-peak behavior even at a high σ_3 of 65 MPa. A transition in failure mode was observed: from class I (stable) at $\sigma_3 = 5$ MPa to class II (unstable) at $\sigma_3 > 5$ MPa. With increasing confining pressure, strainsoftening became more pronounced, while the magnitude of stress drops decreased. This study highlights the distinct differences between multi-stage and single stage triaxial compression tests for red sandstone. Multi-stage loading introduces nonlinear effects on the crack damage threshold and emphasizes the ease of crack initiation and reopening under these conditions. Conversely, single stage tests reveal predictable linear behavior and clear transitions in failure mode and post-peak strain response, providing critical insights into the rock's mechanical behavior.

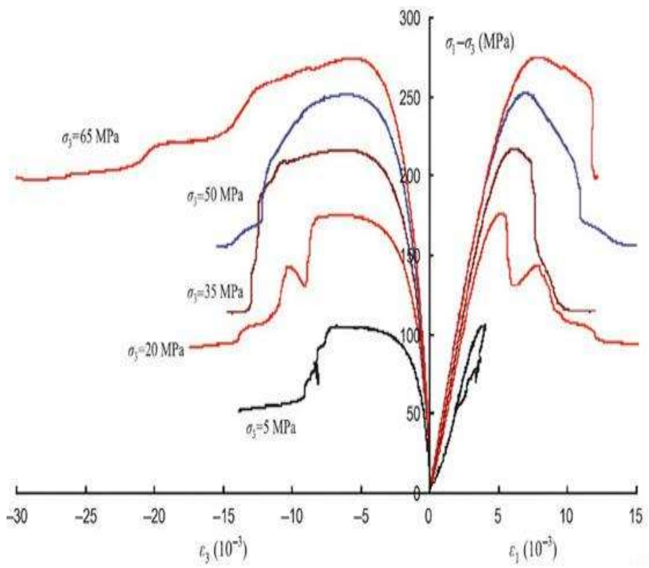


Figure 2.3 Axial deviatoric stress–axial strain and lateral strain curves of red sandstone under triaxial compression test σ_3 = 5 MPa (Yang, 2012).

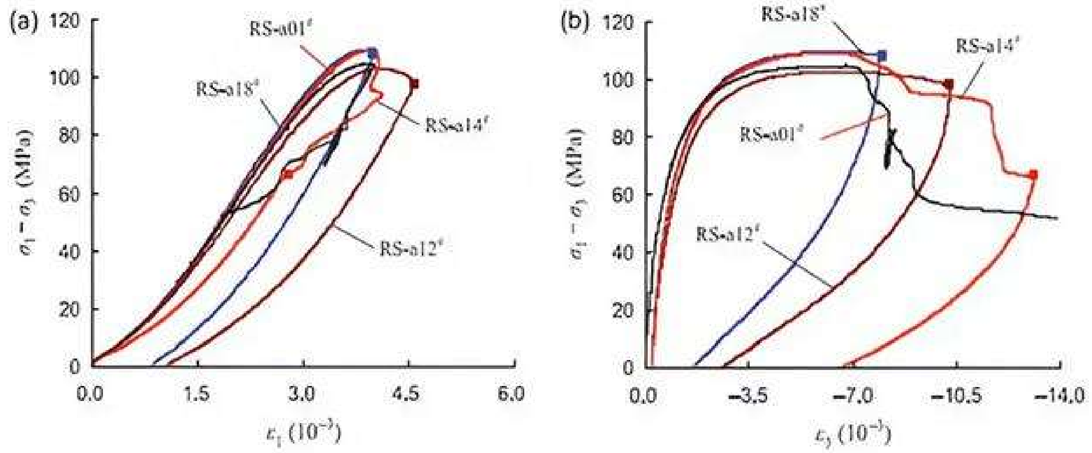


Figure 2.4 Single stage triaxial experimental results of red sandstone. σ_1, axial strain, σ_3, Lateral strain (Yang, 2012).

2.3.1.2 Experimental Techniques

Melati, Wattimena, Kramadibrata, Simangunsong, and Sianturi (2014) investigate how different stress paths impact the results of multi-stage triaxial testing. The study measured stress-strain rates to identify fracture initiation at each stage. The test results were analyzed in terms of the strength envelope and stress-strain curves, as shown in Figure 2.5, and validated using numerical simulations. Multi-stage triaxial tests were conducted on concrete and Tanjung claystone samples using these stress paths. Results showed that both axial load and confining pressure were simultaneously increased. produced higher triaxial compressive strength for both materials. The failure envelopes from this scenario were also closer to those obtained from conventional triaxial tests. Similar findings were noted in studies on Pierre and Raton shale, where constants of failure criteria aligned more closely with single stage triaxial test results. In contrast, tests on Lyons sandstone by (Kim and Ko,1979) yielded different outcomes. This research highlights the significant influence of stress paths on multi-stage triaxial test results, particularly in strength evaluation and failure criteria.

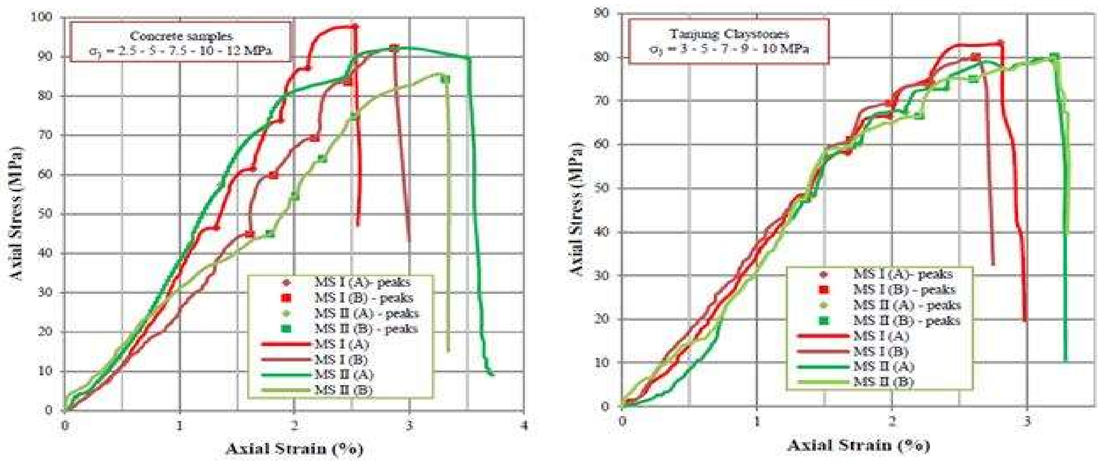


Figure 2.5 Stress-strain curve from multistage triaxial test on: (left) concrete samples, (right) Tanjung claystone (Melati, Wattimena, Kramadibrata, Simangunsong, and Sianturi, 2014).

2.3.1.3 Confining pressure conditions

Hamza, Stace, and Reddish (2005) investigate the impact of strain rate variations on the mechanical properties of silty mudstone, utilizing multi-stage triaxial testing. Experiments were conducted on both intact and fractured rock samples, revealing that changes in strain rate significantly affect the stiffness and strength of fractured rocks more than intact samples. Slower strain rates caused a notable reduction in the elastic modulus and pre-failure axial stress of fractured samples in Figure. 2.6. The result is attributed to partial creeping and relaxation phenomena. However, at lower confining pressures, these effects were not apparent in intact samples, which diverges from previous findings in other rock types and may be due to the narrow strain rate range used in this study. At higher confining pressures, strain rate impacts on intact samples were minimal, whereas fractured samples exhibited changes under all tested conditions, most notably at 10 MPa confinement. The experimental results were modeled using a non-linear logarithmic function, underscoring the potential sensitivity of medium-to-weak rocks, such as coal measures, to strain rate alterations. The study calls for further exploration into elastic-viscoplastic models and broader strain rate applications to refine understanding and predictive tools across different rock types and conditions.

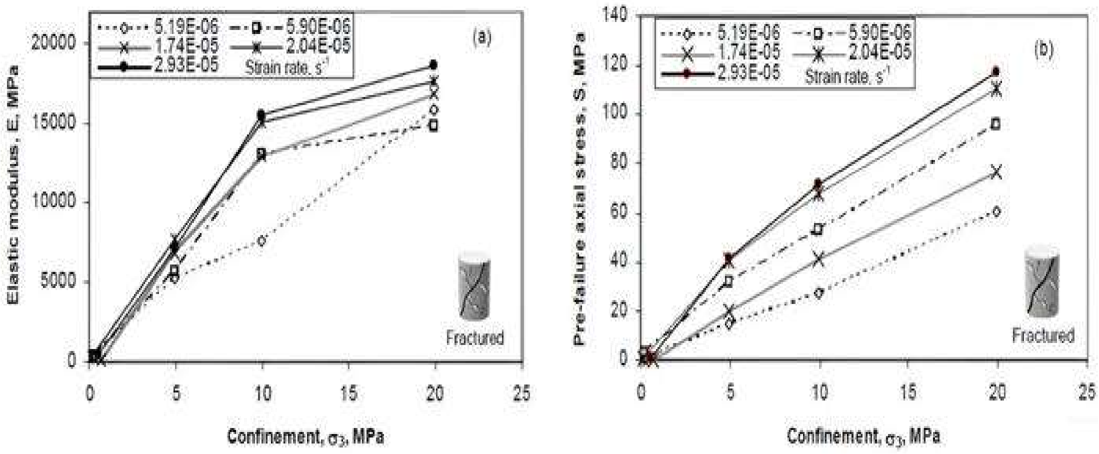


Figure 2.6 Single stage triaxial experimental results of red sandstone. σ_1, axial strain, σ_3, circumferential strain (Hamza, Stace, and Reddish, 2005).

2.3.2 Effect of multi–stage triaxial test on strength and elasticity properties in rock

2.3.2.1 Strength Criteria

Al-Ajmi and Zimmerman (2005) presented the most basic and significant failure criterion in 1776. He proposed that rock failure under compression occurs when the shear stress τ, developed on a particular plane, reaches a value sufficient to overcome both the natural cohesiveness of the rock and the frictional force opposing motion along the failure plane. The criteria might be expressed as

$$\tau = c + \sigma_n \tan\phi \qquad (2.1)$$

where σ_n is the normal stress acting on the failure plane, c is the cohesion of the material, and f is the angle of internal friction. As the sign of t only affects the direction of sliding, Eq. (2.1) should be written in terms of $|\tau|$, but for simplicity we will omit the absolute value sign.

Yang (2012) describes the well-known Mohr-Coulomb and Hoek-Brown criteria as typical strength criteria that have been widely employed in rock mechanics and engineering practice. We study the relationship between multi-stage triaxial strength and the confining pressure using the Mohr–Coulomb and Hoek–Brown criteria. The linear Mohr–Coulomb criterion can be expressed with the following equation:

$$\sigma_s=\sigma_0+q\ \sigma_3=[2C\cos\phi+\sigma_3\cdot(1+\sin\phi)]/(1-\sin\phi) \qquad (2.2)$$

where s_0 is usually regarded as the uniaxial compressive strength (UCS) of rock material. The σ_0 and q are related to the cohesion C and the internal friction angle f of rock material.

However, the nonlinear Hoek–Brown criterion is an empirical strength criterion, which can be expressed as the following equation (Hoek and Brown 1980, 1997):

where σ_c is usually regarded as the UCS of intact rock material, m and s are material constants for a specific rock. The parameter s represents the fragmented extent, which ranges from 0 to 1. The rock remains more complete when the value s approaches 1. By using the linear Mohr–Coulomb Eq. (2.2) and the nonlinear Hoek–Brown Eq. (2.3) criteria, the influence of σ_3 on peak strength σ_s for red sandstone under single-stage and multi-stage triaxial compression is examined. The peak strength values of red sandstone under single and multi-stage triaxial compression can be obtained by these criteria.

2.3.2.2 Deformation modulus

Minaeian, Dewhurst, and Rasouli (2020) investigated the impact of lateral stress anisotropy ($\sigma_2 \neq \sigma_3$) on the mechanical properties and behavior of rock, focusing on compressive strength, elastic modulus, inelastic deformation, dilatancy, and failure modes. They demonstrated that increasing the intermediate principal stress (σ_2) beyond quasi-isotropic conditions ($\sigma_2 = \sigma_3$) led to significant changes in the rock's response. In some cases, the compressive strength more than doubled, and the onset of dilatancy was delayed to over 80% of the peak stress. The measured failure stresses aligned well with the Mogi-type true triaxial failure criterion, indicating that this empirical model effectively captures the rock's behavior under varying stress conditions. To evaluate the effects of σ_2, multi-stage true triaxial tests were performed on synthetic sandstones. In these tests, a single sample was subjected to increasing intermediate principal stress levels while maintaining constant minimum stress. The results were compared with single-stage true triaxial test data, which apply a specific stress configuration to each sample. Distinct differences emerged between the strengths and elastic properties obtained from the two methods, as depicted in Figure 2.7. The study highlighted potential mechanisms underlying these differences, offering insights into the rock's mechanical behavior and supporting the suitability of multistage testing for investigating the influence of intermediate principal stress on rock properties.

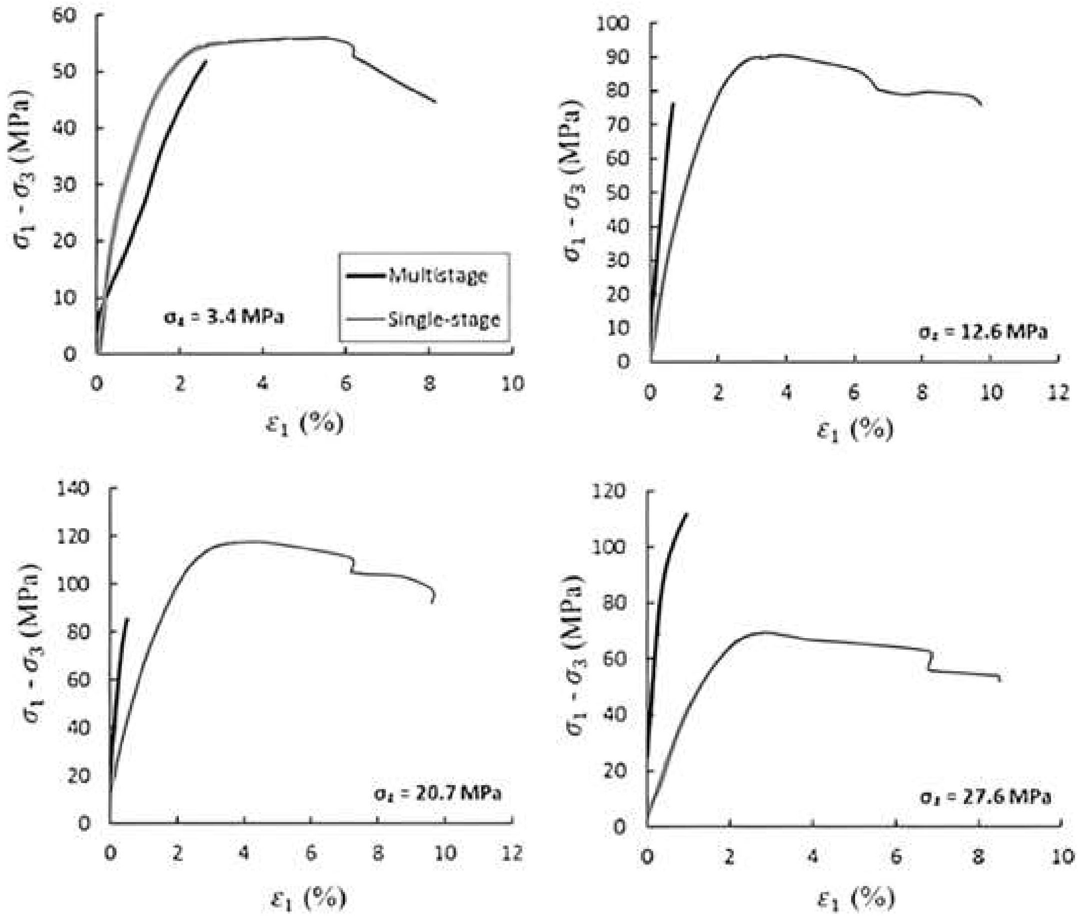


Figure 2.7 Differential stress versus axial strain curves recorded in multi–stage and single stage true triaxial testing of the sandstone samples under σ_3 = 3.4 MPa. (Minaeian, Dewhurst, and Rasouli, 2020).

Yang, Ni, and Wen (2014) examine the spatial evolution of acoustic emission (AE) distribution in red sandstone during multi-stage triaxial deformation under five different confining pressures. This analysis sheds light on the re-fracture deformation mechanisms in pre-damaged specimens. Their findings revealed that both the peak strength and crack damage threshold of red sandstone under multi-stage triaxial compression increase nonlinearly with higher confining pressures (σ_3), as illustrated in Figure 2.8. Furthermore, the disparity between peak strength and crack damage threshold widens as σ_3 increases. These results emphasize the significant influence of confining pressure on the progressive failure characteristics of red sandstone, providing valuable insights into the mechanical behavior of pre-damaged rocks under varying stress conditions.

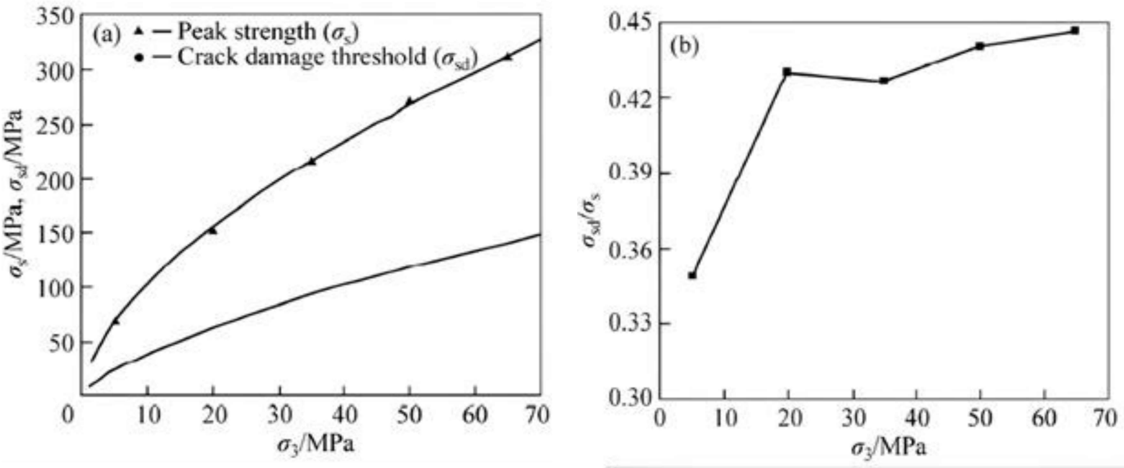


Figure 2.8 Relationship between peak strength, crack damage threshold of red sandstone under multi-stage triaxial compression and confining pressure: (a) σ_s and σ_{sd}; (b) σ_{sd} / σ_s (Yang, Ni, and Wen, 2014).

Yang (2012) states that confining pressure has a significant effect on the deformation parameters of red sandstone under multi-stage triaxial compression, which are lower than those under single stage triaxial compression (Youn and Tonon, 2010). Under both single and multi-stage triaxial compression, the elastic modulus of red sandstone increases with confining pressure, but its increasing values differ depending on the cracked extent of the specimen following peak strength. The elastic modulus is more sensitive to the confining pressure for the specimen with more compressed post-peak circumferential deformation. Red sandstone during multi-stage triaxial compression has a Poisson's ratio larger than that under single stage triaxial compression. Red sandstone's peak axial strain grows nonlinearly under multi-stage triaxial compression; its peak circumferential strain has no known connection with the confining pressure.

Goodman (1970) has introduced a method for assessing the elastic constants of an equivalent continuous material that is representative of a rock mass and is frequently traversed by a single set of joints. This method is based on the concept of joint stiffness. The criterion can be written as:

$$\frac{1}{E_r} = \frac{1}{k_n s} + \frac{1}{E_i} \quad (2.4)$$

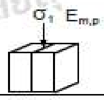
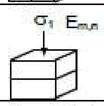
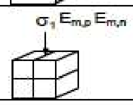
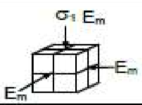
where E_r is the rock deformation modulus, k_n is the joint normal stiffness, s is the average joint spacing and E_i is the equivalent deformation modulus.

Thaweeboon, Dasri, Sartkaew, and Fuenkajorn (2016) modified Goodman's (1970) equation to calculate the deformation modulus along three principal directions. The parameter N is proposed whose value depends on joint set direction. The proposed equation can only predict the deformation modulus in the directions normal and parallel to the joint planes. It is proposed as:

$$\frac{1}{E_m} = \frac{N}{k_n s} + \frac{1}{E_i} \quad (2.5)$$

where E_m is the jointed rock deformation modulus, E_i is the intact deformation modulus, s is the joint spacing, k_n is the joint normal stiffness, and N is a parameter whose value varies with the joint set direction (Table 2.1).

Table 2.1 Parameter N defined for modified Goodman's equation.

Number of joint sets	Orientation of joint set to σ_1	Case	N
1	1 parallel		0.5*
1	1 normal		1.0* (original Goodman)
2	1 parallel, 1 normal		1.5
3	2 parallels, 1normal		2.0*

2.4 Mechanical Behaviour of rock Under single and multi–stage triaxial compression test.

Wang, Feng, Yang, Han, and Kong (2021) investigate the reliability of rapid procedures employed to determine the mechanical parameters of Jinping marble through true triaxial multistage loading experiments. This work investigated the multistage loading strength, deformation features, and failure mode of the marble based on the findings of three types of multistage loading tests conducted on Jinping marble. Test data lead one to the following conclusions. The Jinping marble true triaxial multistage loading test has been validated. The strength obtained from the multi-stage loading test is somewhat lower than that obtained from the single-stage loading test due to unloading under a fixed condition before the peak strength; yet the difference between the two strength parameters obtained by the Mogi–Coulomb criterion (Al-Ajmi and Zimmerman, 2005) fitting is rather small in Figure 2.9. In engineering, these variations are usually minor. Marble with clearly plastic portions would be suited for the real triaxial multi-stage loading test. To minimize the effect of microfracturing on the strength values, the unloading point should not surpass the peak strength of the specimen during the test. By means of the last stage of the multi-stage loading test, someone can assess the potential error of the strength parameters to confirm the dependability of the chosen data set. For reference in engineering practice, the Jinping marble multi-stage loading test shows good reproducibility and can quickly acquire dependable strength criteria. The macro-mechanical characteristics of Jinping marble under various multi-stage loading situations are shown by the test results to be closely correlated with the formation of interior microcracks. By means of the failure mode analysis, we find that internal microcrack development influences the macro failure mode of the specimen under real triaxial multi–stage stress.

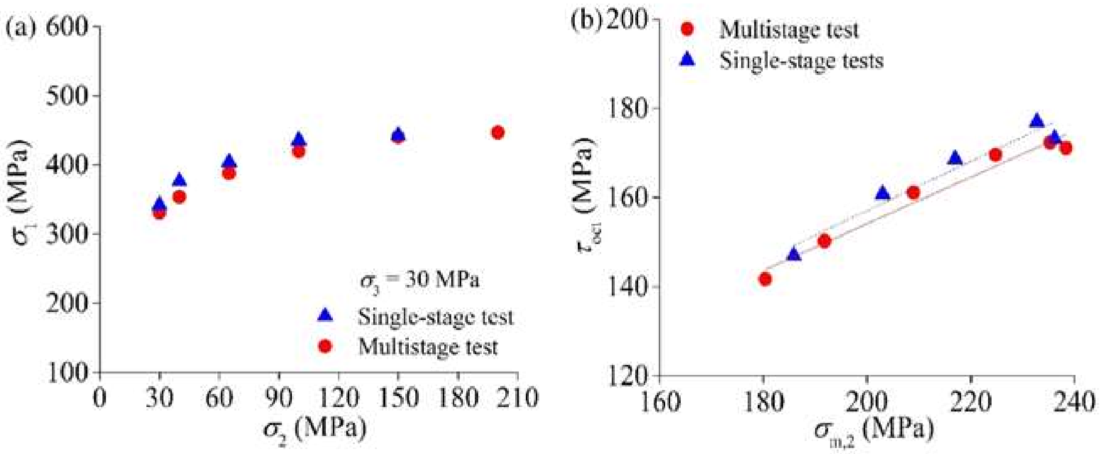


Figure 2.9 Comparative analysis of single and multi-stage tests (a) comparison of strength (b) Mogi–Coulomb strength data fitting comparison. (Wang, Feng, Yang, Han, and Kong, 2021).

Yang et al. (2019) investigate the strength reduction and damage mechanical behavior of mudstone from the Yuncheng coal mine in Shandong province, China, at a depth of 960 m. Using both experimental testing and numerical simulation methods, they examined the mechanical behavior of mudstone under conventional and multi-stage triaxial compression and analyzed their findings, leading to the following conclusions:

1. The Particle Flow Code (PFC) proved effective in simulating the mechanical behavior of mudstone under both conventional and multi-stage triaxial compression at various confining pressures.

2. The peak strength of intact and damaged mudstone specimens increased with confining pressure and adhered well to the linear Mohr–Coulomb criterion. However, the cohesion and internal friction angle of intact specimens were consistently higher than those of damaged specimens, and these strength parameters attenuated with increasing damage extent. The study derived exponential attenuation equations linking cohesion and internal friction angle to the post-stress reduction ratio, with good agreement between experimental and numerical results.

3. A damage variable was defined as the ratio of the micro-crack area to the total specimen area. With increasing post-stress reduction ratio, the damage variable

rose rapidly before stabilizing at a constant level when the ratio exceeded 0.4. Additionally, under reducing confining pressure, the rate of micro-crack formation increased more significantly than under single stage triaxial compression, reflecting a heightened sensitivity of the damaged specimens to stress variations.

These studies highlight the dynamic mechanical behavior and damage mechanisms of mudstone under stress conditions, providing valuable insights for understanding the strength reduction in geological materials subjected to multi-stage deformation.

Vergara, Kudella, and Triantafyllidis (2015) explore the mechanical behavior of bedded and jointed rock to inform the design of slopes, tunnels, and tunnel portals for a planned rail line. Triaxial tests conducted on the material demonstrated ductile behavior, supporting the use of the multi-stage testing technique to evaluate mechanical properties. The study revealed that strength parameters derived from the multi-stage tests were influenced by material composition, failure mode, and confining stress. The observed failure modes highlighted the role of material composition and the presence of discontinuities in determining the failure behavior and resulting strength of the rock specimens. Based on the Mohr–Coulomb failure criterion, the internal friction angle obtained from tests with decreasing confining stress was found to approximate that of intact rock, whereas tests with increasing confining stress showed a contrasting trend. For cohesion, the opposite pattern was observed, with decreasing confining stress tests producing results more divergent from the intact material values. These studies underscore the importance of considering material heterogeneity and discontinuities when assessing the strength and deformation characteristics of bedded and jointed rock for engineering applications.

Amann, Kaiser, and Button (2012) investigate the brittle failure behavior of clay shale under triaxial compression, revealing key insights into its fracture mechanisms. The study found that brittle failure is consistently initiated at a differential stress of 2.1 MPa, indicating negligible friction mobilization at the onset of failure. Volumetric behavior prior to rupture was found to depend on confining stress, as shown in Figures 2.10 and 2.11. At low confining pressures ($\sigma_3 < 0.5$ MPa), volumetric strain at rupture was negative (indicating expansion). As confinement increased, dilation was increasingly suppressed, and for $\sigma_3 \geq 2$ MPa, volumetric strain reversal disappeared entirely, reflecting the suppression of dilatant fracturing and an associated shift in the failure mode. The rupture surface's characteristics also varied with confining stress. At low confinement ($\sigma_3 < 0.5$), failure planes were steeply inclined, consisting of axial cracks interconnected with bedding-parallel shears. As confinement increased, axial cracks became shorter while bedding-parallel shears extended, and eventually, inclined fractures predominated with axial fractures no longer visible. These observations indicate a transition from axial splitting to macroscopic shear failure as confinement increases. Although crack initiation occurred at a consistent differential stress, rupture stress depended on confining pressure, suggesting the mobilization of friction during failure. The study noted that neither the linear Mohr–Coulomb criterion nor the non-linear Hoek–Brown failure envelope adequately captured the stress dependence of failure. Instead, the data supported a bi- (or tri-) linear or S-shaped failure envelope (e.g., Kaiser and Kim, 2008) to account for the evolving fracture processes. This research highlights the importance of understanding stress-dependent fracture mechanisms in brittle materials like clay shale and offers refined models for capturing complex failure behaviors under varying confining pressures.

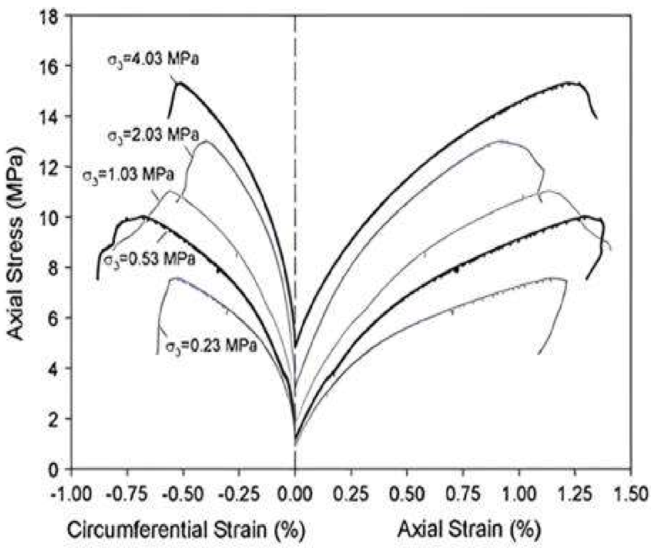


Figure 2.10 Axial and radial strain behavior at different confining stress levels (Amann, Kaiser, and Button, 2012)

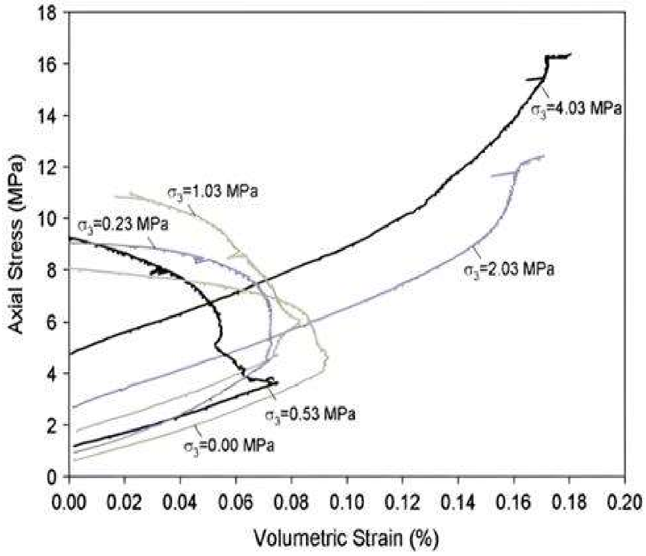


Figure 2. 11 Volumetric strain versus axial stress behavior for different confining stresses in the pre-rupture phase. For confining stress $\sigma_3 \geq \sigma_2$ MPa. (Amann et al., 2012)