APPENDIX
LIST OF PUBLICATION

COMPARISON BETWEEN SINGLE AND MULTI-STAGE TRIAXIAL COMPRESSIVE STRENGTHS OF SOME ROCK TYPES

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Abstract: This study aims at correlating the triaxial compressive strengths obtained from multi-stage testing with those from the conventional single stage test condition. Results from ten rock types are used to determine Hoek-Brown parameters under confining pressures ranging from 0 to 40 MPa. The ratio of Hoek-Brown parameters obtained from multi-stage testing to that of single stage testing (i.e. m_m/m_s) can correlate well with the rock uniaxial compressive strengths (with $R^2 > 0.9$) using an exponential equation. As a result, the triaxial compressive strength of rock can be closely predicted providing that the Hoek-Brown parameters from multi-stage testing and the uniaxial compressive strengths of the rock are known. The correlation of the strength results between the two test conditions however strongly depends on the loading path used during the multi-stage testing. Different loading paths would likely result in a different m_m/m_s–σ_c relation.

Keywords: Hoek-Brown criterion, Strength criteria, Multi-stage triaxial testing,

1. INTRODUCTION

The triaxial compression test (ASTM D7012-14) [1] has been widely used to determine rock strength and deformation under confinement, which are important parameters for the design and stability study of geological structures in civil as well as mining engineering works, including foundations for dams, structures, and bridges, and additional host rocks for underground mining and tunnels. The triaxial compression test is used in the laboratory to simulate these structures. A significant limitation of the traditional triaxial test method is that it is expensive, time-consuming, and requires a large number of standard samples. The multi-stage triaxial compression test ([2], [3] and [4]) is more popular nowadays because it requires fewer samples to determine the triaxial strength. It is found that the multi-stage strength is often lower than the single stage strength. The differences and representativeness of the multi-stage test results and the deformation modulus however require more investigation.

The multi-stage triaxial test concept was first introduced in the mid-1970s [5]. It is necessary to test a variety of rocks to conduct a comparative study of the single stage and multi-stage strengths. Accurate lateral and axial stresses of the specimen are typically difficult to measure due to non-uniform deformations of the specimen and local strain measurement by measuring equipment [6].

The objective of this study is to develop mathematical correlations between single stage and multi-stage triaxial compressive test results in terms of strength and deformability. Hoek-Brown strength criterion and Goodman stiffness relation between intact and fractured rocks are employed. Ten rock types with strength varying from soft to strong rocks are used. A polyaxial load frame is used to load and unload the specimens, which allows determining the compressive strengths and deformation moduli of the rocks under both single and multi-stage testing.

2. SAMPLE PREPARATION

Rock specimens come from various locations in Thailand. Rock specimens are prepared from ten different rock types, ranging from soft to hard rocks, including Tak Fa gypsum, Maha Sarakham salt, Khao Khad bedded limestone, Phu Kradung sandstone, Khao Khad marble, Pha Wihan sandstone, Phu Phan bedded sandstone, Phu Phan sandstone, Rayoung-Bang Lamung granite, and Buriram basalt. Prismatic specimens are prepared with nominal dimensions of $54 \times 54 \times 108$ mm³. The bedding planes for sedimentary rocks are oriented perpendicular to the primary axis

3. TEST METHOD

There are two types of triaxial tests: single stage triaxial compression tests and multi-stage triaxial compression tests. The objective of the triaxial compression tests is to determine the parameters (m and s) of the specimens in accordance with the Hoek-Brown criterion [7] under varying confining pressures. After the confining pressure is increased a desired magnitude, the axial stress increases at rate of 1 MPa/s until failure occurs while the confining pressure is maintained constant. To find the strength envelope, the specimens are tested at different confining pressures in a single-stage behavior under axial stress until it fails, as per the ASTM standard procedure (ASTM D7012-14) [1]. The multi-stage triaxial compression testing aims to measure the maximum failure stress and axial and lateral strains during loading and reloading phases by conducting a multi-stage loading using polyaxial compression frame [8]. These tests are carried out under constant confining pressures between 0 and 40

MPa. The multi-stage stress path is shown in **Fig 1.** Under hydrostatic conditions, confining pressure (σ_3) and axial (σ_1) stress are first simultaneously increased to the required confining pressure. The axial stress is then raised to failure and then released to the original confining pressure. Both axial and confining stresses then increased to the next level of confining pressure. After that, axial stress is once raised to reach the peak strength. This cycle is repeated to allow analysis of rock strength at various confining pressures. Axial stress and strains and lateral strains are plotted continuously from the first to the last loading stages.

4. TEST RESULTS

The results of single stage triaxial compression tests conducted under confining pressures (σ_3) ranging from 0 to 40 MPa. They are performed to compare with the multi-stage triaxial tests. Extension failure is observed in high-loading specimens. The high confining pressures result in multiple fractures. Some post-tested single stage specimens are shown in **Fig 2(a).** The results of multi-stage triaxial compression tests. These tests are performed to compare with the single stage triaxial tests. The specimens exhibit varying failure patterns depending on the applied stress level at each stage. Under confining pressure, deformation was observed to gradually increase as each subsequent step was initiated at a higher confining pressure.

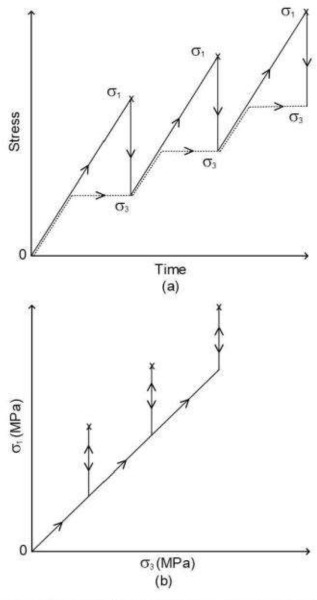

Fig.1. Stress path for multi-stage triaxial compression test used in this study, (a) variations over time and (b) confining pressure (σ_3).

Some post-tested multi-stage specimens are shown in **Fig 2(b).** The test results are presented as stress-strain curves in **Fig 3 and 4.**

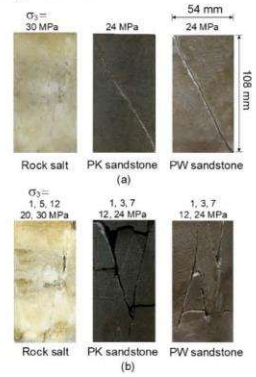

Fig.2. Some post-test specimens for single stage (a) and multi-stage triaxial compression tests (b).

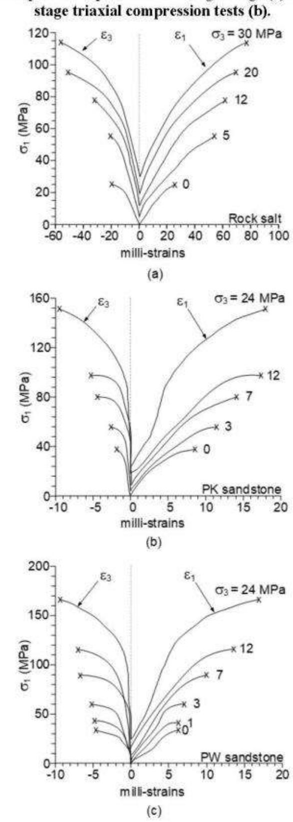

Fig.3. Some stress-strain curves obtained from Rock salt (a), PK sandstone (b), and PW sandstone (c) specimens with single stage triaxial compression tests.

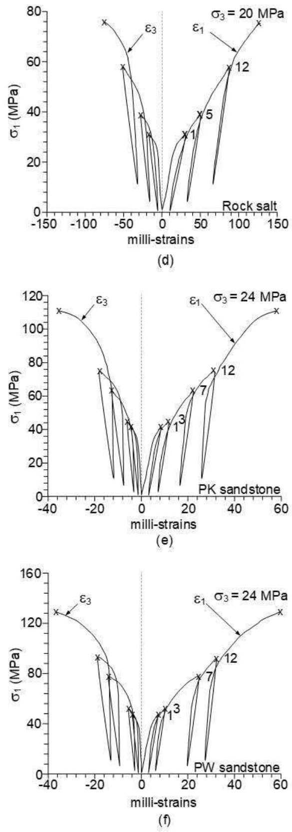

Fig.4. Some stress-strain curves obtained from Rock salt (d), PK sandstone (e), and PW sandstone (f) specimens with multi-stage triaxial compression tests.

4.1. Strength results

Hoek-Brown criterion [7] is applied to the triaxial compression strengths, using data obtained from both single and multi-stage test conditions, based on the compressive strengths of test results in **Table 1.** It represents the principal stresses at failure ($\sigma_{1,f}$):

$$\sigma_{1,f} = \sigma_3 + (m \cdot \sigma_c \cdot \sigma_3 + s \cdot \sigma_c^2)^{1/2} \quad (1)$$

where $\sigma_{1,f}$ is compressive strength at failure, σ_3 and σ_c are confining pressure and uniaxial compression strength of rock. The parameters m and s are empirical constants. Regression analyses are performed on the test results to determine these parameters. The $\sigma_{1,f} - \sigma_3$ curves obtained from the regression are compared with the test data for all rock types in **Fig 5.** Good correlations are obtained ($R^2 > 0.9$).

Table 1: Summary of single and multi-stage triaxial compression test results.

Rock Types	Confining pressure σ_3 (MPa)	$\sigma_{1,f}$ (MPa) Single stage	Multi-stage
Tak Fa gypsum	0	10.8	-
	1	-	11.3
	3	17.9	13.3
	7	28.3	18.1
	12	40.8	24.6
	18	51.3	33.1
Maha Sarakham salt	0	25.1	-
	1	-	27.2
	5	55.3	35.9
	12	77.1	54.1
	20	95.6	71.5
	30	114.2	89.5
Khao Khad bedded limestone	0	32.6	-
	1	-	35.2
	5	64.2	50.3
	8	78.3	61.2
	16	107.1	85.6
	30	149.1	121.5
Phu Kradung sandstone	0	38.9	-
	1	-	40.4
	3	58.0	43.7
	7	77.7	62.4
	12	97.2	73.7
	24	151.3	110.7
Khao Khad marble	0	41.7	-
	1	-	43.5
	3	65.4	50.3
	7	83.2	66.3
	12	105.2	85.3
	20	139.7	109.3
	35	186.9	147.2
Pha Wihan sandstone	0	43.3	-
	1	-	46.8
	3	63.4	51.3
	7	88.9	76.6
	12	106.9	91.3
	24	166.4	129.0
Phu Phan bedded sandstone	0	53.7	-
	1	-	55.3
	3	78.3	63.5
	7	105.4	84.6
	12	130.2	108.4
	24	186.5	144.9
Phu Phansandstone	0	55.0	-
	1	-	57.2
	4	95.7	75.3
	12	138.8	110.5
	20	180.4	147.3
	36	235.9	195.3
Rayong-Bang Lamung granite	0	59.1	-
	1	-	61.0
	5	-	86.2
	10	140.3	115.4
	20	191.3	157.4
	30	225.3	191.9
Buriram basalt	0	100.6	-
	1	-	107.3
	10	192.1	175.3
	20	271.2	240.5
	30	331.9	290.3
	40	376.5	325.4

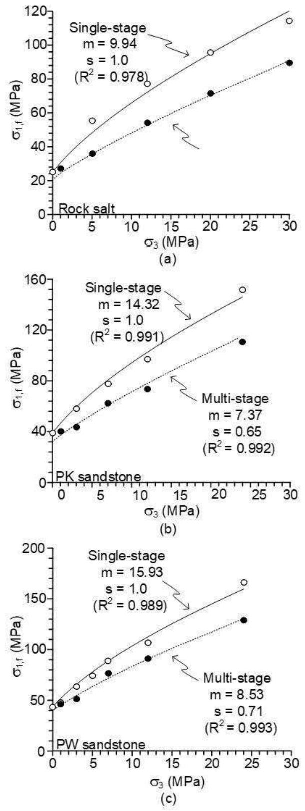

Fig.5. Compressive strengths at failure as a function of confining pressure for Maha Sarakham salt (a), Phu Kradung sandstone (b), Pha Wihan sandstone (c).

An attempt is made to correlate Hoek-Brown parameters obtained from the two test conditions. Since both parameters depend on uniaxial compression strength of rocks. They are plotted as a function of σ_c in **Fig 6.** The parameter m increases with σ_c. where single stage test condition gives higher values than the multi-stage condition. The parameter s for the single stage test is equal to one for all rock types as it is calibrated from the intact rocks. For multi-stage testing however parameter s shows the values of less than one. The weaker rocks give the lower s values than the stronger rocks (higher σ_c).

To correlate the Hoek-Brown parameters between the two test conditions, the subscript m denoting multi-stage is assigned for the multi-stage parameters (m_m and s_m), where subscript s (denoting single stage) is using for the single stage parameters (m_s and s_s). After several trials, ratios of the Hoek-Brown parameters are proposed as: m_m/m_s and s_m/s_s. They are plotted in **Fig 7**. Exponential equations are fitted to these ratios. Good correlation is obtained. They can be presented as:

$$m_m/m_s = 1 - a \cdot \exp(-b \cdot \sigma_c) \quad (2)$$

$$s_m/s_s = 1 - c \cdot \exp(-d \cdot \sigma_c) \quad (3)$$

when a, b, c, d are empirical constants. The m_m/m_s ratio allows predicting then m_s value for single stage testing of the m_m value is known for the multi-stage testing. Extrapolation of the two equations above allows predicting the uniaxial compressive strengths of rocks at which the m_m/m_s and s_m/s_s approach one are given in **Table 2.**

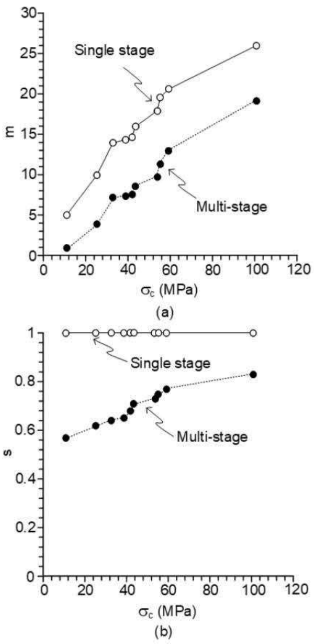

Fig.6. Hoek-Brown parameters, m (a), and s (b) as a function of uniaxial compressive strength (σ_c).

4.2. Strengths from researches obtained elsewhere

Fig 8 compares the m_m/m_s ratio obtained from this study with those published by other researches who conduct both multi-stage and single stage compression testing on various rock types. The solid points in the diagram represent the results obtained from the same loading path are used in this study. The open data points are those from different loading paths of multi-stage testing. The error between the prediction and the results obtained elsewhere are calculated using equations as follows [9]:

$$\text{Error} = 1/n \, |m_m/m_{s\,(i,p)} - m_m/m_{s\,(i,t)} / m_m/m_{s\,(i,p)}| \cdot 100 \quad (4)$$

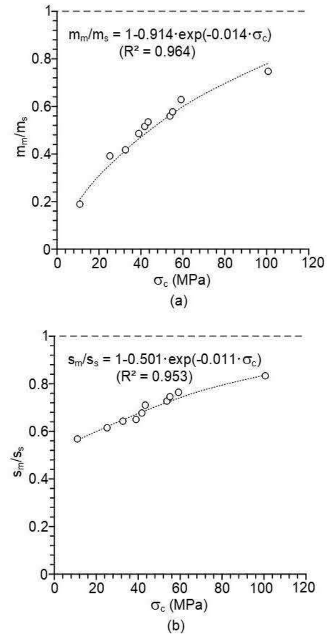

Fig.7. Relationship between m_m/m_s ratios (a), and s_s/s_m ratios (b) as a function of uniaxial compressive strength (σ_c).

Table 2: m_m/m_s and s_m/s_s ratios obtained from single and multi-stage strength results.

Rock Types	m_m/m_s	s_m/s_s
Tak Fa gypsum	0.19	0.57
Maha Sarakham salt	0.39	0.62
Khao Khad bedded limestone	0.42	0.64
Phu Kradung sandstone	0.43	0.65
Khao Khad marble	0.51	0.68
Pha Wihan sandstone	0.54	0.71
Phu Phan bedded sandstone	0.56	0.73
Phu Phan sandstone	0.58	0.75
Rayong-Bang Lamung granite	0.63	0.77
Buriram basalt	0.74	0.83

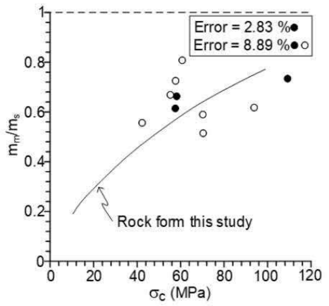

Fig.8. Solid points are results from [3]. Open points are from others: [9], [10], [11], [12].

DISCUSSIONS

Due to the fact that the $\sigma_{1,f} - \sigma_3$ relations for all tested rocks are not linear, as shown in **Fig 5,** the conventional Coulomb strength criterion is not applicable here. In addition, the criterion is only for intact rock specimens, and hence it is not valid for the multi-stage testing specimens where they are failed after the first loading cycle. Hoek-Brown criterion seems more appropriate for this study as it is capable of describing the compressive strengths of the intact specimens for the single stage tests and the failed specimens for multi-stage testing.

Rock types selected for this study have uniaxial compressive strengths ranging from soft rock (gypsum – 10.8 MPa) to strong rock (basalt – 100.6 MPa). This allows deriving an appropriate mathematical relations between m_m/m_s ratios and the strength (i.e., Equation (2)). Missing from this study is the very strong rock specimens (σ_c > 250 MPa), as they are rarely encountered in the geo-engineering projects and mines in Thailand.

Good correlations between m_m/m_s ratio and σ_c and between s_m/s_s ratio and σ_c support that the triaxial compression strengths for the single stage testing that uses several specimens to accomplish can be predicted by the results of the multi-stage testing that uses much fewer specimens. As a minimum the multi-stage testing requires only one uniaxial compression test to obtain σ_c and one multi-stage triaxial testing to obtain m_m and s_m. The three parameters can be used to predict m_s and s_s (always equals to 1) for the single stage testing.

Comparison of the correlation equation obtained from this study with the multi-stage strength results obtained elsewhere by other researchers suggests that the multi-stage strengths are sensitive to the loading path. If a multi-stage testing uses the same loading path that used here the prediction by Equation (2) can give good correlation with the error of about 2.83%, as shown in **Fig 8.** For multi-stage strengths obtained under different loading paths (i.e., not allowing axial stress to drop from the failure stress to the current confining pressure), as performed by [10], [11], and [12], larger error can be obtained, as shown in **Fig 8.**

CONCLUSIONS

Test results and analyses performed in this study can be concluded as follows.

1. Hoek-Brown criterion can be used to correlate the triaxial compressive strengths between single and multi-stage testing.
2. Good correlation between m_m/m_s ratio and uniaxial compressive strength allows predicting the conventional single stage triaxial compressive strengths from the multi-stage compressive strength results.

3. The multi-stage triaxial strengths are sensitive to the loading paths used for each loading cycle. Different sets of compressive strengths would likely be obtained under different loading paths.

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