CHAPTER VI
ANALYSIS OF TEST RESULTS

6.1 Introduction

The primary objective of this chapter is to determine mathematical correlations of the strengths and deformation moduli between the single and multi-stage triaxial compression tests. The findings will allow predicting the single stage test results from the multi-stage results. The correlation relations will also be applied to the multi-stage test results obtained elsewhere from other researchers.

6.2 Compressive strengths

Hoek-Brown criterion (Hoek-Brown, 1980) is applied to the triaxial compression strengths, using data obtained from both single and multi-stage test conditions, based on the compressive strengths of test results in Table 6.1. It represents the principal stresses at failure ($\sigma_{1,f}$) as (Hoek and Brown, 1980):

$$\sigma_{1,f} = \sigma_3 + (m \cdot \sigma_c \cdot \sigma_3 + s \cdot \sigma_c^2)^{1/2} \tag{6.1}$$

where σ_3 and σ_c are confining pressure and uniaxial compressive strength of rock. The parameters m and s are empirical constants. Regression analyses are performed on the test results to determine these parameters. Table 6.2 gives the analysis results for both single and multi-stage test conditions. The $\sigma_{1,f} - \sigma_3$ curves obtained from the regression are compared with the test data for all rock types in Figure 6.1. Good correlations are obtained ($R^2 > 0.9$).

6.3 Hoek-Brown parameters

An attempt is made to correlate Hoek-Brown parameters obtained from the two test conditions. Since both parameters depend on uniaxial compression strength of rocks. They are plotted as a function of σ_c in Figure 6.2.

Table 6.1 Compressive strengths of tested rocks.

Rock Types	Confining pressure σ_3 (MPa)	$\sigma_{1,f}$ (MPa)	
		Single stage	Multi-stage
Tak Fa gypsum	0	10.8	-
	1	-	11.3
	3	17.9	13.3
	7	28.3	18.1
	12	40.8	24.6
	18	51.3	33.1
Maha Sarakham salt	0	25.1	-
	1	-	27.2
	5	55.3	35.9
	12	77.1	54.1
	20	95.6	71.5
	30	114.2	89.5
Khao Khad bedded limestone	0	32.6	-
	1	-	35.2
	5	64.2	50.3
	8	78.3	61.2
	16	107.1	85.6
	30	149.1	121.5
Phu Kradung sandstone	0	38.9	-
	1	-	40.4
	3	58.0	4.37
	7	77.7	62.4
	12	97.2	73.7
	24	151.3	110.7
Khao Khad marble	0	41.7	-
	1	-	43.5
	3	65.4	50.3
	7	83.2	66.3
	12	105.2	85.3
	20	139.7	109.3
	35	186.9	147.2
Pha Wihan sandstone	0	43.3	-
	1	-	46.8
	3	63.4	51.3
	7	88.9	76.6
	12	106.9	91.3
	24	166.4	129.0
Phu Phan bedded sandstone	0	53.7	-
	1	-	55.3
	3	78.3	63.5
	7	105.4	84.6
	12	130.2	108.4
	24	186.5	144.9
Phu Phan sandstone	0	55.0	-
	1	-	57.2
	4	95.7	75.3
	12	138.8	110.5
	20	180.4	147.3
	36	235.9	195.3
Rayong-Bang Lamung granite	0	59.1	-
	1	-	61.0
	5	-	86.2
	10	140.3	115.4
	20	191.3	157.4
	30	225.3	191.9
Buriram basalt	0	100.6	-
	1	-	107.3
	10	192.1	175.3
	20	271.2	240.5
	30	331.9	290.3
	40	376.5	325.4

Table 6.2 Hoek-Brown parameters obtained from single and multi-stage strength results.

Rock Types	Parameters	Single stage	Multi-stage
Tak Fa gypsum	m	5.01	0.88
	s	1.00	0.57
	R^2	0.995	0.998
Maha Sarakham salt	m	9.94	3.89
	s	1.00	0.62
	R^2	0.978	0.995
Khao Khad bedded limestone	m	13.92	7.18
	s	1.00	0.64
	R^2	0.998	0.999
Phu Kradung sandstone	m	14.32	7.37
	s	1.00	0.65
	R^2	0.991	0.992
Khao Khad marble	m	14.66	7.55
	s	1.00	0.68
	R^2	0.999	0.998
Pha Wihan sandstone	m	15.93	8.53
	s	1.00	0.71
	R^2	0.989	0.993
Phu Phan bedded sandstone	m	17.92	9.70
	s	1.00	0.73
	R^2	0.999	0.995
Phu Phan sandstone	m	19.58	11.31
	s	1.00	0.75
	R^2	0.996	0.998
Rayong-Bang Lamung granite	m	20.60	12.96
	s	1.00	0.77
	R^2	0.996	0.999
Buriram basalt	m	26.00	19.14
	s	1.00	0.83
	R^2	0.998	0.996

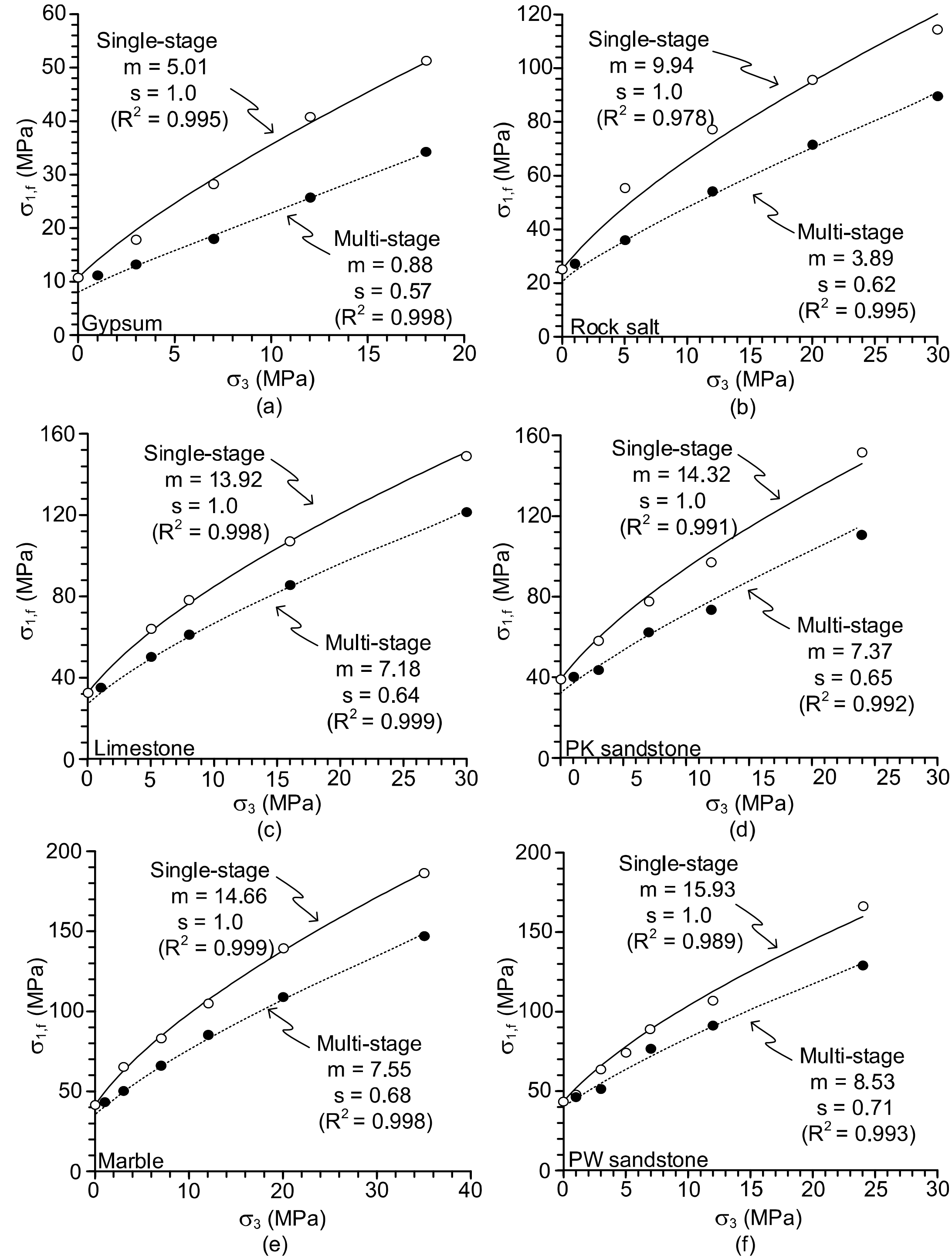

Figure 6.1 Compressive strengths at failure as a function of confining pressure for Tak Fa gypsum (a), Maha Sarakham salt (b), Bedded limestone (c), Phu Kradung sandstone (d), Khao Khad marble (e), Pha Wihan sandstone (f).

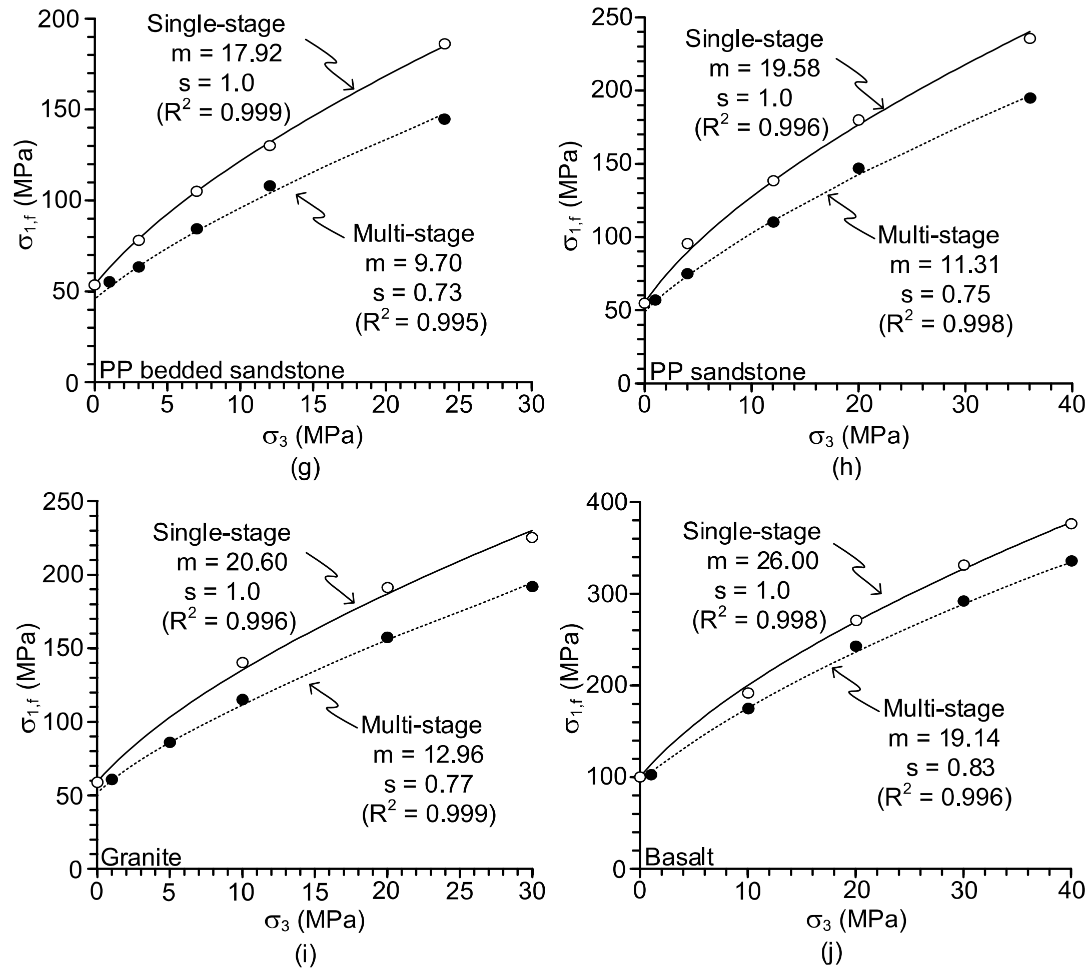

Figure 6.1 Compressive strengths at failure as a function of confining pressure for Phu Phan bedded sandstone (g), Phu Phan sandstone (h), Rayong-Bang Lamung granite (i), Burirum basalt (j). (cont.)

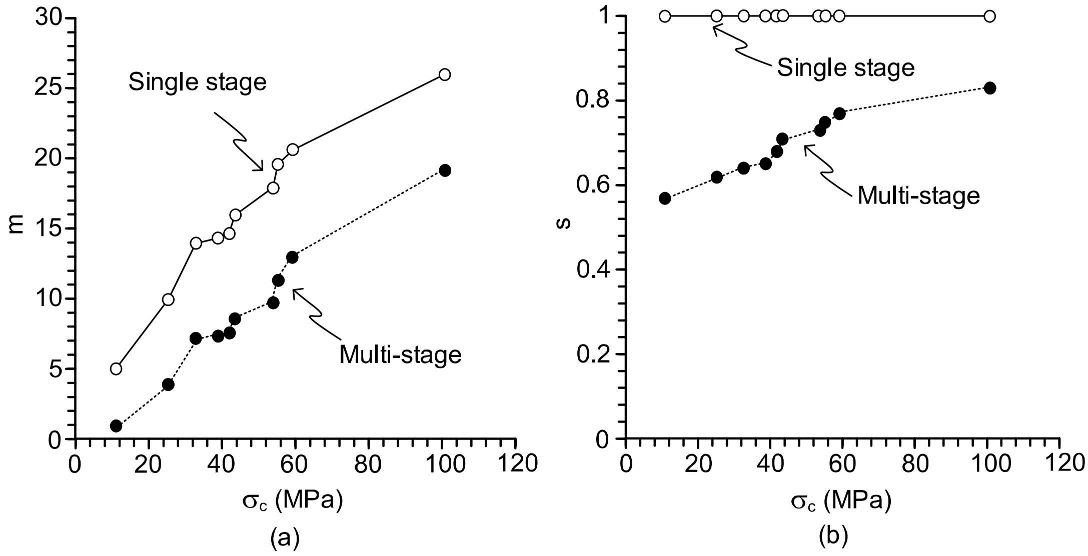

Figure 6.2 Hoek-Brown parameters, m (a), and s (b) as a function of uniaxial compressive strength (σ_c).

The parameter m increases with σ_c. where single stage test condition gives higher values than the multi-stage condition. The parameter s for the single stage test is equal to one for all rock types as it is calibrated from the intact rocks. For multi-stage testing however parameter s shows the values of less than one. The weaker rocks give the lower s values than the stronger rocks (higher σ_c).

To correlate the Hoek-Brown parameters between the two test conditions, the subscript m denoting multi-stage is assigned for the multi-stage parameters (m_m and s_m), where subscript s (denoting single stage) is using for the single stage parameters (m_s and s_s). After several trials, ratios of the Hoek-Brown parameters are proposed as: m_m/m_s and s_m/s_s. They are plotted in Figure 6.3.

Exponential equations are fitted to these ratios. Good correlation is obtained. They can be presented as:

$$m_m/m_s = 1 - a \cdot \exp(-b \cdot \sigma_c) \quad (6.2)$$

$$s_m/s_s = 1 - c \cdot \exp(-d \cdot \sigma_c) \quad (6.3)$$

when a, b, c, d are empirical constants. The m_m/m_s ratio allows predicting then m_s value for single stage testing of the m_m value is known for the multi-stage testing. Extrapolation of the two equations above allows predicting the uniaxial compressive strengths of rocks at which the m_m/m_s and s_m/s_s approach one are given in Table 6.3.

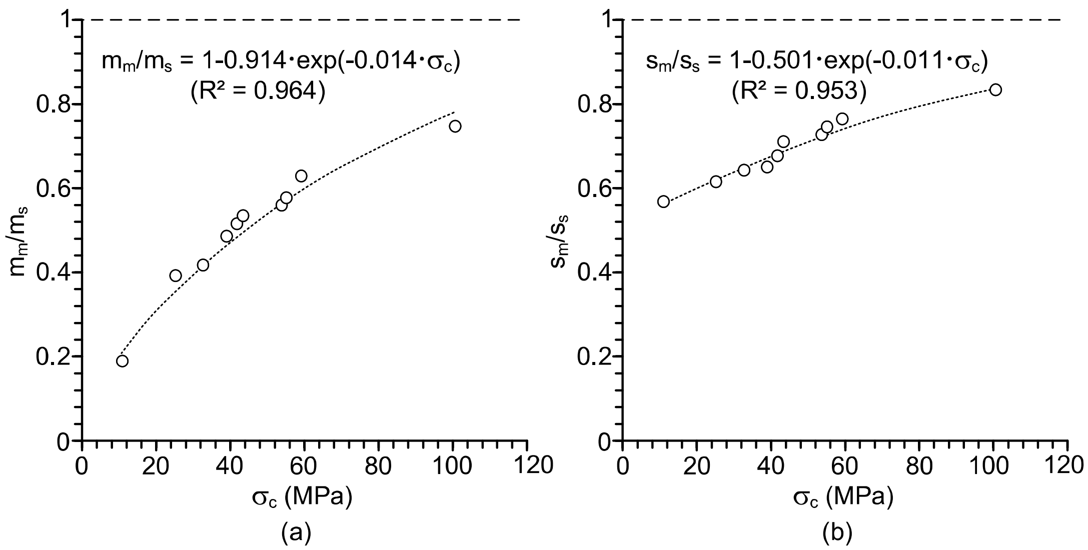

Figure 6.3 Relationships between m_m/m_s ratios (a), and s_m/s_s ratios (b) as a function of uniaxial compressive strength (σ_c).

Table 6.3 m_m/m_s and s_m/s_s ratios obtained from single and multi-stage strength results.

Rock Types	m_m/m_s	s_m/s_s
Tak Fa gypsum	0.19	0.57
Maha Sarakham salt	0.39	0.62
Khao Khad bedded limestone	0.42	0.64
Phu Kradung sandstone	0.43	0.65
Khao Khad marble	0.51	0.68
Pha Wihan sandstone	0.54	0.71
Phu Phan bedded sandstone	0.56	0.73
Phu Phan sandstone	0.58	0.75
Rayong-Bang Lamung granite	0.63	0.77
Buriram basalt	0.74	0.83

6.4 Strengths from researches obtained elsewhere

Figure 6.4 compares the m_m/m_s ratio obtained from this study with those published by other researches who conduct both multi-stage and single stage compression testing on various rock types. The solid points in the diagram represent the results obtained from the same loading path are used in this study. The open data points are those from different loading paths of multi-stage testing. The error between the prediction and the results obtained elsewhere are calculated using equations as follows (Khair, Fahmi, Al Hakim, and Rahim, 2017):

$$\text{Error} = 1/n \sum_{i=1}^{n} \mid m_m/m_{s\,(i,\,p)} - m_m/m_{s\,(i,\,t)} \,/\, m_m/m_{s\,(i,\,p)} \mid \cdot 100 \qquad (6.4)$$

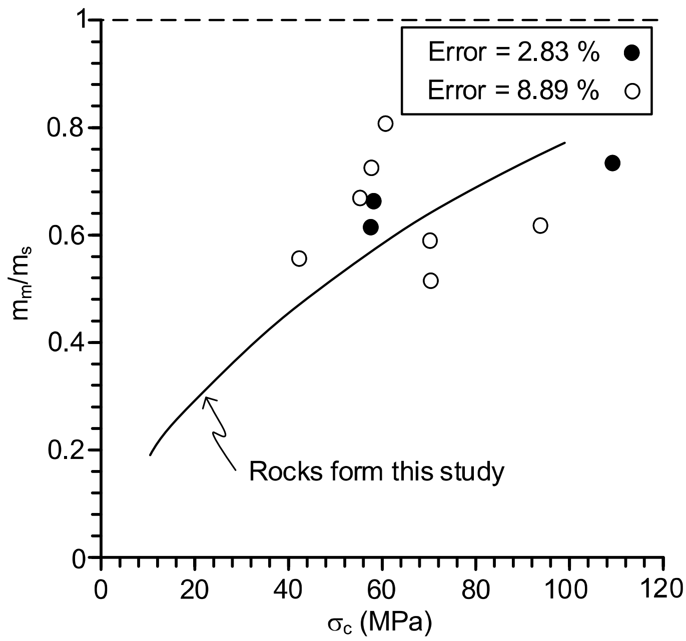

Figure 6.4 Solid points are results from Yang et al. (2012). Open points are from others: Aghababaei et al. (2019), Shi et al. (2016), Venter et al. (2016), Cain et al. (1986).

6.5 Deformation moduli

Figure 6.5 plots the elastic or deformation moduli as a function of confining pressure (σ_3), where E_m and E_s represent the values obtained from multi-stage and single stage testing. Both increase with confining pressure, where the single stage testing gives higher values than those from multi-stage tests. The Poisson's ratios from both test conditions are also presented as a function of confining pressure in Figure 6.6. They are denoted by ν_s for single stage and ν_m for multi-stage. The single stage testing shows the decrease of ν_s as σ_3 increases, while the multi-stage testing shows the increase of ν_m with σ_3. This is probably due to the accumulation of the induced fractures in the specimens as the loading cycles increase.

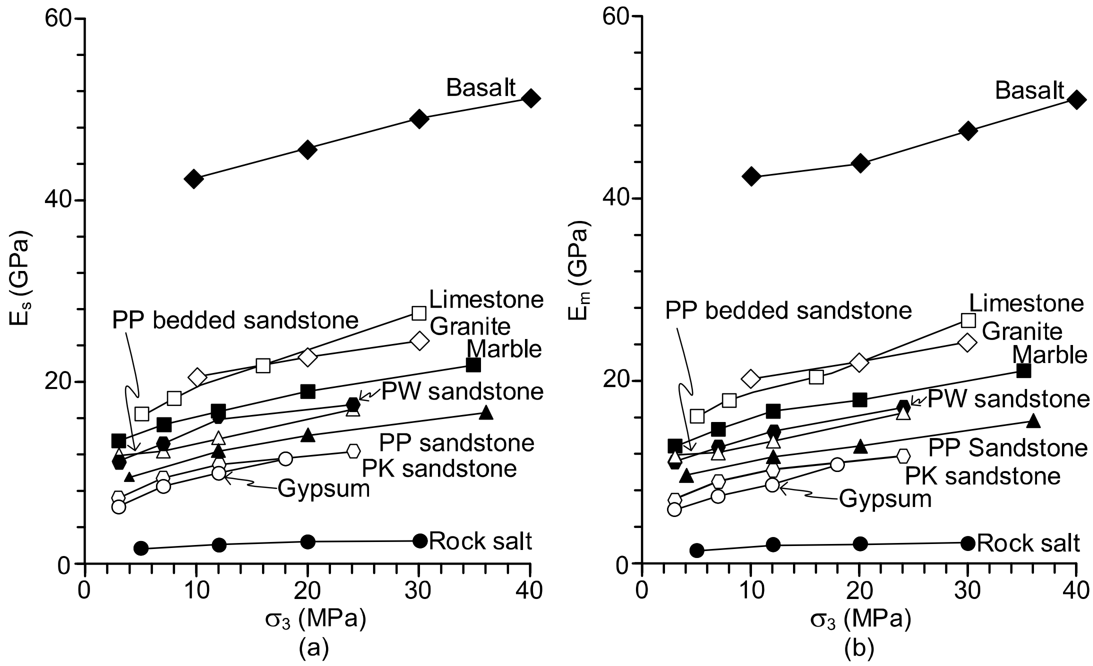

Figure 6.5 Elastic moduli for single stage, E_s (a), and multi-stage, E_m (b), as a function of confining pressure (σ_3).

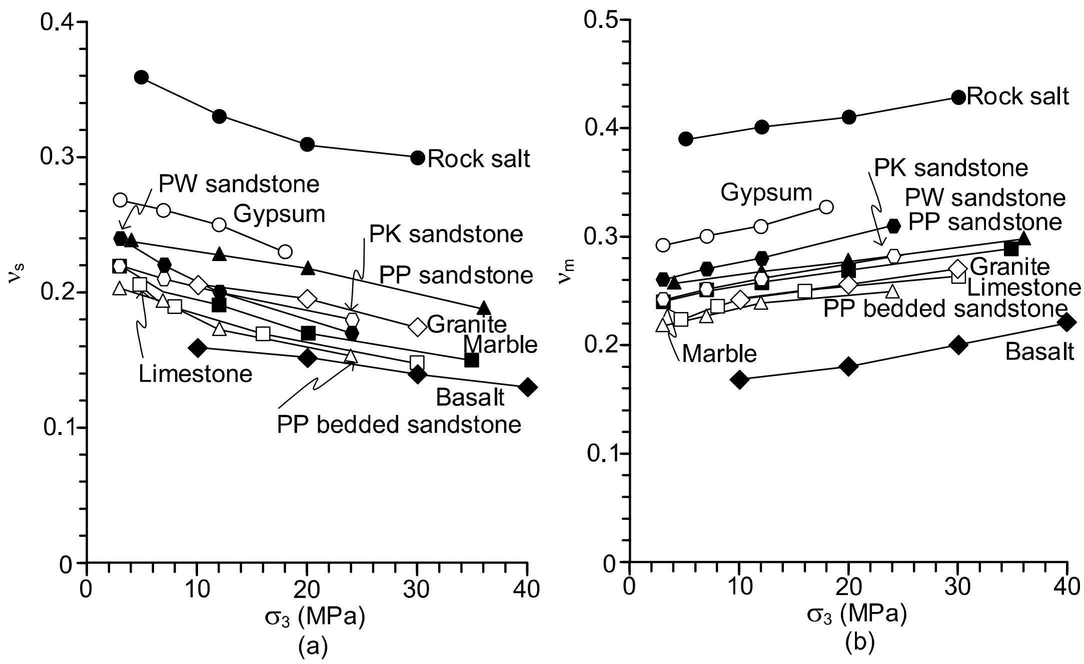

Figure 6.6 Poisson's ratios for single stage, ν_s (a), and multi-stage, ν_m (b), as a function of confining pressure (σ_3).

To correlate the deformation moduli obtained between both test conditions, empirical equation are proposed as:

$$\frac{1}{E_m} = \frac{1}{E_s} + \frac{1}{\beta} \tag{6.6}$$

$$\frac{1}{G_m} = \frac{1}{G_s} + \frac{1}{\alpha} \tag{6.7}$$

where E_m is elastic moduli obtained from multi-stage, E_s is elastic moduli obtained from single stage, G_m and G_s are shear moduli obtained from multi-stage and single stage testing ($G = E/2(1+\nu)$). The parameters β and α are empirical constants correlating the two test conditions. Their numerical values are obtained from regression analysis and are given in Table 6.4.

To allow predicting the multi-stage deformation and shear moduli for different rock types, the constants β and α are correlated with the uniaxial compressive strength of the rocks tested here. Figure 6.7 plots these constants as a function of σ_c. Exponential equation is proposed to represent their variations with the evolution of σ_c, as follows:

$$\beta = 1 + 98.85 \cdot \exp(0.027 \cdot \sigma_c) \tag{6.7}$$

$$\alpha = 1 + 15.39 \cdot \exp(0.029 \cdot \sigma_c) \tag{6.8}$$

The numerical values in the equations are obtained from regression analysis. Good correlations are obtained ($R^2 > 0.8$). If σ_c of any rock is known, constants β and α can be determined, and subsequently E_s and G_s can be predicted from E_m and G_m. They can be substituted in equations (6.7 and 6.8) to predict the deformation and shear moduli of single stage testing from the results of multi-stage testing.

Table 6.4 Empirical constants β and α.

Rock Types	β (GPa)	α (GPa)
Tak Fa gypsum	0.072	0.021
Maha Sarakham salt	0.018	0.004
Khao Khad bedded limestone	0.431	0.080
Phu Kradung sandstone	0.228	0.045
Khao Khad marble	0.431	0.065
Pha Wihan sandstone	0.223	0.050
Phu Phan bedded sandstone	0.402	0.065
Phu Phan sandstone	0.150	0.038
Rayong-Bang Lamung granite	0.854	0.122
Buriram basalt	1.490	0.286

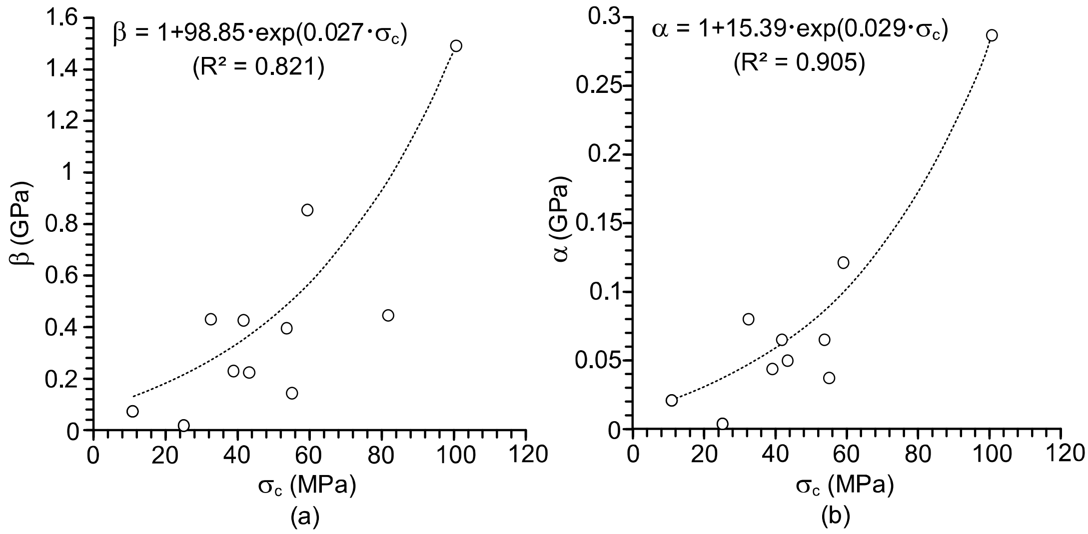

Figure 6.7 Constants β (a), and α (b) as a function of uniaxial compressive strength (σ_c).