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AXIAL FORM FACTOR OF THE NUCLEON IN THE PERTURBATIVE CHIRAL QUARK MODEL

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A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Physics

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Suranaree University of Technology has approved this thesis submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy.

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แบบจำลองควาร์กเชิงไคแรลเพอร์เทอร์เบชันที่หนึ่งวงวนได้ถูกนำมาประยุกต์เพื่อวิเคราะห์ฟอร์มแฟกเตอร์เชิงแกนของนิวคลีออน แบบจำลองนี้มีพื้นฐานอยู่บนลากรานเจียนยังผลโดยอธิบายแบริออนด้วยเวเลนซ์ควาร์กเชิงสัมพัทธภาพและกลุ่มหมอกของโกลด์สโตนโบซอนเชิงเพอร์เทอร์เบชันที่ถูกกำหนดด้วยความสมมาตรเชิงไคแรล ซึ่งได้นำแบบจำลองนี้มาประยุกต์ใช้เพื่อให้ได้สมการเชิงวิเคราะห์สำหรับฟอร์มแฟกเตอร์เชิงแกนของนิวคลีออน ผลลัพธ์ที่ได้ถูกแสดงในพจน์ของพารามิเตอร์หลักมูลทางฟิสิกส์ของพายออน-นิวคลีออนพลังงานต่ำ (ค่าคงตัวการสลายตัวของพายออนอย่างอ่อน, ฟอร์มแฟกเตอร์ของพายออน-นิวคลีออนอย่างแรง) และขึ้นอยู่กับพารามิเตอร์ของแบบจำลองเพียงตัวเดียว (รัศมีแก่นของควาร์กสามตัวเชิงนิวคลีออน) ผลลัพธ์ที่ได้สอดคล้องกับข้อมูลจากการทดลองเป็นอย่างดี

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The perturbative chiral quark model (PCQM) at one loop is used to analyze the axial form factor of the nucleon. The model is based on an effective Lagrangian, where baryons are described by relativistic valence quarks and a perturbative cloud of Goldstone bosons as dictated by chiral symmetry. Obtained in the model are analytical expressions for the axial form factor of the nucleon, which is given in terms of fundamental parameters of low-energy pion-nucleon physics (weak pion decay constant, strong pion-nucleon form factor) and of only one model parameter (radius of the nucleonic three-quark core). The results are in good agreement with experimental data.

School of Physics	Student's Signature____________
Academic Year 2004	Advisor's Signature____________
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Khanchai Khosonthongkee

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Chapter I

Introduction

All the known interactions that occur in nature can be reduced to four interactions between the matter particles. There are the strong (nuclear) interaction, electromagnetism, the weak interaction and gravity. Among these four interactions only the weak interaction does not produce observed bound states but it causes reactions which make the particles ultimately decay into the stable leptons and hadrons, that is, electrons, neutrinos and protons.

The history of the weak interactions could be said to have begun in 1896 when Henri Becquerel discovered radioactivity. We today know that the majority of the rays that were discovered stem from the β-decay of heavy nuclei. The distinction between α and β rays was first made by Rutherford in 1899. It was Niels Bohr who first attributed the origin of β rays to the nucleus soon after Rutherford discovered the atomic nucleus. James Chadwick showed in 1914 that the β rays possess a continuous energy spectrum. To understand this problem, Wolfgang Pauli postulated in 1930 the existence of a new particle, today called the electron-antineutrino.

The next milestone in the history of the weak interaction is the monumental work by Enrico Fermi, in which he developed the first theory of β-decay. At that time, and for a long time afterwards, the proton and neutron were regarded as elementary particles. Then Fermi proposed that the elementary β-decay of the neutron

$$n \rightarrow p + e^- + \bar{\nu}_e \tag{1.1}$$

stems from the interaction

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \left(\bar{\psi}_p \hat{O}_i \psi_n \right) \left(\bar{\psi}_e \hat{O}_i \psi_\nu \right), \tag{1.2}$$

where G_F is Fermi constant. Here ψ_p, ψ_n, ψ_e and ψ_ν denote the wave functions of the four particles and the quantities $\hat{O}_i$ are appropriate operators which characterize the decay process. This theory has been called the four-fermion interaction.

The four-fermion interaction was modified to the so-called V-A theory soon after discovery of the parity violation of the weak interaction in 1956 (Lee and Yang, 1956; Wu, Ambler, Hayward, Hoppes and Hudson, 1957) and the experimental results that there exist only the left-handed neutrino and the right-handed antineutrino (Goldhaber, Grodzins and Sunyar, 1958). In the V-A theory the weak interaction is governed by the interaction

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} J_\mu^+ J^\mu \tag{1.3}$$

with

$$J_\mu = \bar{\psi}_l \gamma_\mu (1 - \gamma^5) \psi_{\nu_l} + \bar{\psi}_u \gamma_\mu (1 - \gamma^5) \psi_d. \tag{1.4}$$

The Lagrangian cannot be applied to the baryon decay directly since the baryons are bound states of quarks (for example, the proton is a bound state of two u and one d quarks while the neutron is a bound state of one u and two d quarks). To apply the V-A theory to baryon decay processes, one may modify the current in Eq. (1.4) as follows

$$\bar{\psi}_p \gamma_\mu (1 - \gamma^5) \psi_n \rightarrow \bar{\psi}_p \gamma_\mu (g_V - g_A \gamma^5) \psi_n \tag{1.5}$$

where $g_A/g_V \neq 1$ indicates that the form factors for the vector and the axial vector current are different. Correspondingly, the β-decay process in Eq. (1.1) is respectively described by the interaction

$$\frac{G_F}{\sqrt{2}} \bar{\psi}_e \gamma_\mu (1 - \gamma^5) \psi_{\nu_e} \cdot \bar{\psi}_p \gamma^\mu (g_V - g_A \gamma^5) \psi_n. \tag{1.6}$$

By comparing the theoretical prediction of the V-A interactions with the experimental data for the β-decay processes, the parameters are determined as

$$g_V \approx 1, \quad g_A \approx 1.25 \tag{1.7}$$

It is clear that one has to introduce form factors in describing the β-decay since baryons are bound states of quarks. This is very much similar to the electron-nucleon elastic and inelastic collisions. The form factors have to be momentum-dependent instead of being constant. The constant g_A in Eq. (1.5) is just the value of the axial form factor of the nucleon at zero momentum transfer, $Q^2 = 0$.

Since the nucleons are bound states of quarks, as already mentioned before, one has to understand the structure and interaction of them, which is one of the major research areas in nuclear and particle physics. One of the main research directions is the search for the manifestation of elementary quarks and gluons in strong interaction processes. There is no doubt that Quantum Chromodynamics (QCD) is the fundamental theory of strong interactions, which at least in the perturbative domain, that is for large momentum transfers Q^2, is confirmed in a rather impressive manner. However, in the so-called confinement regime at low momentum transfers Q^2 the properties of QCD are only understood partially in a somewhat qualitative manner. Now perturbative physics dominates and hence traditional approaches in solving QCD cannot be applied. Based on the lack of exact solutions of QCD in the non-perturbative region the main ansatz consists of the development of effective models. The main recipe in constructing these models consists of reducing the elementary degrees of freedom of QCD and introducing effective interactions, characteristic of the fundamental theory.

Our understanding of the structure of hadrons is based to a large extent on the theoretical concept of the constituent quark model. Thereby, quarks and antiquarks form the relevant degrees of freedom, where for example baryons are made up of three of these constituent quarks put together by confinement while mesons are consist of a quark and an antiquark. In a next step it was realized that chiral symmetry, considered to be one of the best symmetries of the strong interaction, is violated by the quark confinement mechanism.

The problem of chiral symmetry breaking is resolved by introduction of Goldstone boson fields in consistency with chiral symmetry. The Goldstone bosons, like the pion, reflect the presence of the sea quarks, which in addition to the valence quarks, should be present. Modern theories which attempt to describe the structure of baryon should contain both features of low-energy QCD, such as confinement and chiral symmetry.

The nucleon axial form factor is of fundamental significance to weak interaction properties, the pion-nucleon interaction and has been, and still is, an important set of parameters for the investigation of the spin-isospin distribution of the nucleon (since in a non-relativistic language this is nothing but the matrix-element of the Gamov-Teller operators $\vec{\sigma}\vec{\tau}$). Hence it provides an important test for theories that attempt to describe the structure of the nucleon.

The theoretical description of axial form factors was performed in detail within approaches of low-energy hadron physics: bag model, constituent quark model, QCD sum rules, chiral perturbation theory, relativistic and non-relativistic quark models, etc. Since the early eighties chiral quark models (Théberge, Thomas and Miller, 1980; Thomas, Théberge and Miller, 1981; Théberge and Thomas 1983; Thomas, 1984; Oset, Tegen and Weise, 1984; Tegen, 1990; Chin, 1982; Diakonov, Petrov and others, 1984, 1986, 1988, 1989; Gutsche, 1987; Gutsche and Robson, 1989) describing the nucleon as a bound system of valence quarks with a surrounding pion cloud, play an important role in the description of low-energy nucleon physics. These models include the two main features of low-energy hadron structure, confinement and chiral symmetry.

With respect to the treatment of the pion cloud, theses approaches fall essentially into two categories. The first type of chiral quark models assumes that the valence quark content dominates the nucleon, thereby treating pion contributions perturbatively (Théberge, Thomas and Miller, 1980; Thomas, Théberge and Miller, 1981; Théberge and Thomas 1983; Thomas, 1984; Oset, Tegen and Weise, 1984; Tegen, 1990; Chin, 1982; Gutsche, 1987; Gutsche and Robson, 1989). Originally, this idea was formulated in the context of the cloudy bag model (Théberge, Thomas and Miller, 1980, Thomas, Théberge and Miller, 1981; Théberge and Thomas 1983; Thomas, 1984). By imposing chiral symmetry the MIT bag model (Chodos et al., 1974) was extended to include the interaction of the confined quarks with the pion fields on the bag surface. With the pion cloud treated as a perturbation on the basic features of the MIT bag, pionic effects generally improve the description of nucleon observables. Later, similar perturbative chiral models (Oset et al., 1984; Tegen, 1990; Chin, 1982; Gutsche, 1987; Gutsche and Robson, 1989) were developed where the rather unphysical sharp bag boundary is replaced by a finite surface thickness of the quark core. By introducing a static quark potential of general form, these quark models contain a set of free parameters characterizing the confinement (coupling strength) and/or the quark masses. The perturbative technique allows a fully quantized treatment of the pion field up to a given order in accuracy. Although formulated on the quark level, where confinement is put in phenomenologically, perturbative chiral quark models are formally close to chiral perturbation theory which is applied on the hadron level.

Alternatively, when the pion cloud is assumed to dominate the nucleon structure this effect has to be treated non-perturbatively. The non-perturbative approaches are based for example on these by Diakonov, Petrov, and others (1984, 1986, 1988, 1989), where the chiral quark soliton model was derived. This model is based on the concept that the QCD instanton vacuum is responsible for the spontaneous breaking of chiral symmetry, which in turn leads to an effective chiral Lagrangian at low energy as derived from QCD. The classical pion field (the soliton) is described by a trial profile function, which is fixed by minimizing the energy of the nucleon. Further quantization of slow rotations of this soliton field leads to a nucleon state, which is built up from rotational excitations of the classical nucleon. On the phenomenological level the chiral quark soliton model tends to have advantages in the description of the nucleon spin structure, that is for large momentum transfers, but has some problems when compared to the original perturbative chiral quark models in the description of low-energy nucleon properties.

As a further development of chiral quark models with a perturbative treatment of the pion cloud (Théberge, Thomas and Miller, 1980; Thomas, Théberge and Miller, 1981; Théberge and Thomas 1983; Thomas, 1984; Oset, Tegen and Weise, 1984; Tegen, 1990; Chin, 1982; Gutsche, 1987; Gutsche and Robson, 1989), we recently extended the relativistic quark model suggested in Gutsche (1987) and Gutsche and Robson (1989) to describe the low-energy properties of the nucleon (Lyubovitskij, Gutsche and Faessler, 2001; Lyubovitskij, Gutsche, Faessler and Drukarev, 2001; Lyubovitskij, Gutsche, Faessler and Vinh-Mau, 2001, 2002). Lyubovitskij, Gutsche and Faessler (2001), Lyubovitskij, Gutsche, Faessler and Drukarev(2001), Lyubovitskij, Gutsche, Faessler and Vinh-Mau (2001, 2002), Cheedket, Lyubovitskij, Gutsche, Faessler, Pumsa-ard and Yan (2003), Pumsa-ard, Lyubovitskij, Gutsche, Faessler and Cheedket (2003), Inoue, Lyubovitskij, Gutsche and Faessler (2004) developed the so-called perturbative chiral quark model (PCQM) in application to baryon properties such as: sigma-term physics, electromagnetic form factors of the baryon octet, πN scattering and electromagnetic corrections, strange nucleon form factors, electromagnetic nucleon-delta transition, etc. The current approach contains several new features: i) generalization of the phenomenological confining potential; ii) SU(3) extension of chiral symmetry to include the kaon and eta-meson cloud contributions; iii) consistent formulation of perturbation theory both on the quark and baryon level by use of renormalization techniques and by allowing to account for excited quark states in the meson loop diagrams; iv) fulfillment of the constraints imposed by chiral symmetry (low-energy theorems), including the current quark mass expansion of the matrix elements (for details see Lyubovitskij, Gutsche, Faessler and Drukarev (2001)); v) possible consistency with chiral perturbation theory. In this dissertation we follow up on the earlier investigations and employ the same model in order to study the axial form factor of the nucleon.

The PCQM is based on an effective chiral Lagrangian describing quarks as relativistic fermions moving in a self-consistent field (static potential) $V_{\rm eff}(r) = S(r) + \gamma^0 V(r)$ which is described by a sum of a scalar potential $S(r)$ providing confinement and the time component of a vector potential $\gamma^0 V(r)$. It is known from lattice simulations that the scalar potential should be a linearly rising one and the vector potential is thought to be responsible for short-range fluctuations of the gluon field configurations (Takahashi, Matsufuru, Nemoto and Suganuma, 2001). In our study we approximate $V_{\rm eff}(r)$ by a relativistic harmonic oscillator potential with a quadratic radial dependence (Lyubovitskij, Gutsche and Faessler, 2001)

$$S(r) = M_1 + c_1 r^2 \,, \quad V(r) = M_2 + c_2 r^2 \,. \tag{1.8}$$

The model potential defines unperturbed wave functions for the quarks, which are subsequently used to calculate baryon properties. This potential has no direct connection to the underlying physical picture and is thought to serve as an approximation of a realistic potential. The vector part of the potential is also a pure long-ranged potential and is not responsible for the short-range fluctuations of gluon fields. In general, we need a vector potential to distinguish between quark and antiquark solutions of the Dirac equation with an effective potential. Note, that this type of the potential was extensively used in chiral potential models (Gutsche and Robson, 1989; Tegen, Brockmann and Weise, 1982; Tegen and Weise, 1983; Oset, Tegen and Weise, 1984; Tegen, 1986,1989; Abbas, 1990) . A positive feature of this potential is that most of the calculations can be done analytically. As was shown in Gutsche and Robson (1989), Tegen, Brockmann and Weise (1982), Tegen and Weise (1983), Oset, Tegen and Weise (1984), Tegen (1986,1989), Abbas (1990) and later on also checked in the PCQM (Lyubovitskij, Gutsche and Faessler, 2001; Lyubovitskij, Gutsche, Faessler and Drukarev, 2001; Lyubovitskij, Gutsche, Faessler and Vinh-Mau, 2001, 2002; Cheedket, Lyubovitskij, Gutsche, Faessler, Pumsa-ard and Yan, 2003; Pumsa-ard, Lyubovitskij, Gutsche, Faessler and Cheedket, 2003; Inoue, Lyubovitskij, Gutsche and Faessler, 2004), this effective potential gives a reasonable description of low-energy baryon properties and can be treated as a phenomenological approximation of the long-ranged potential dictated by QCD.

Baryons in the PCQM are described as bound states of valence quarks supplemented by a cloud of Goldstone bosons (π, K, η) as required by chiral symmetry. The chiral symmetry constraints will in general introduce a nonlinear meson-quark interaction, but when considering meson as small fluctuations we restrict the interaction Lagrangian up to the quadratic term in the meson fields. With the derived interaction Lagrangian we do our perturbation theory in the expansion parameter $1/F$ (where F is the pion leptonic decay constant in the chiral limit). We also treat the mass term of the current quarks $(\hat{m}, m_s)$ as a perturbation. Dressing the baryonic three-quark core by a cloud of Goldstone mesons corresponds effectively to the inclusion of sea-quark contributions. All calculations are performed at one loop or at order of accuracy $O(1/F^2, \hat{m}, m_s)$.

To be consistent, we use the unified Dirac equation with a fixed static potential both for the ground and for the excited quark states. In the Appendix we give details of the solutions to the Dirac equation for any excited state. Inclusion of excited states should be handled consistently. First of all, one should guarantee conservation of local symmetries (like gauge invariance). Second, excited states should be restricted to energies smaller than the typical application scale $\Lambda \approx 1$ GeV of low-energy approaches. An alternative possibility to suppress the inclusion of higher-order excited states is to introduce a meson-quark vertex form factor (Oset, Tegen and Weise, 1984; Gutsche and Robson, 1989). Solving the Dirac equation with a relativistic harmonic oscillator potential Eq. (1.8) one can show, that the energy shift between the first low-lying $1p$ excited states and the $1s$ ground state is about 200 MeV. The excited states ($1d$ and $2s$) lie about ~ 370 MeV above the ground state. The $2p$ and the $1f$ states are 530 MeV heavier when compared to the ground state. As soon as the typical energy of the ground-state quark is about 540 MeV*one can restrict to the low-lying $1p$, $1d$ and $2s$ excited states with energies smaller than the typical scale $\Lambda = 1$ GeV. The requirement of convergence of physical observables when including excited states is physically not meaningful since it takes states with very large energies where the phenomenological low-energy approaches break down. Gutsche and Robson (1989) showed that the inclusion of excited states to the nucleon and Δ masses can be convergent

*This value can be deduced from a calculation of octet and decouplet baryon spectrum. Similar estimates can be found in other chiral quark calculations.

when using a linearly rising confinement potential, e.g., the use of potential with a quadratic radial dependence leads to a nonsatisfactory convergence. However, this statement is sensitive to the quantity one is testing. On the other hand, our approach has some different features in comparison to previous ones (Gutsche and Robson, 1989; Tegen, Brockmann and Weise, 1982; Tegen and Weise, 1983; Oset, Tegen and Weise, 1984; Tegen, 1986,1989; Abbas, 1990). In particular we perform a consistent renormalization procedure when we include meson cloud effects. It gives additional contributions to physical quantities which were not taken into account before. Pumsa-ard, Lyubovitskij, Gutsche, Faessler and Cheedket (2003) demonstrated that excited quark states ($1p$, $1d$ and $2s$) can increase the contribution of loop diagrams but in comparison to the leading order (three-quark core) diagram this effect was of the order of 10%. This is why we were interested to study these effects for the example for the axial form factor. Again, we truncate the set of excited states to the $1p$, $1d$ and $2s$ states with energies which satisfy the condition $\mathcal{E} < \Lambda = 1$ GeV. We do not pretend that we have a more accurate estimate of the whole tower of excited states. The scheme we use is thought to take the excited states into account in an average fashion. We showed that the zeroth-order value of the axial nucleon charge is not changed much in the presence of meson cloud effects in consistency with chiral perturbation theory. As an extension of previous work, which is dominantly aimed to describe the low-energy static properties of baryons, we consider the model prediction for the axial charge and for completeness also for the axial form factor. No further parameters are adjusted in the present work.

In the present dissertation we proceed as follows. First, we describe the basic features of our approach (the underlying effective Lagrangian, the unperturbed, that is valence quark, result for the nucleon description together with the choice of parameters) and a brief overview of perturbation theory when including the meson fields. The full details of the renormalization technique can be found in the Appendix and in Lyubovitskij, Gutsche and Faessler (2001). In chapter III we concentrate on the detailed analysis of the nucleon axial form factor in our approach. We derive analytical expressions in terms of fundamental parameters of low-energy pion-nucleon physics (weak pion decay constant, strong pion-nucleon form factor) and of only one model parameter (radius of the three-quark core of the nucleon). Finally, chapter IV contains the numerical results in comparison with data which are presented to test the phenomenological implications of the model. A summary of our major conclusions are displayed at the end of this chapter.

Chapter II

The Perturbative Chiral Quark Model

In this chapter we will give details of the perturbative chiral quark model. Here, we describe the basic features of our approach: the underlying effective Lagrangian, the unperturbed, that is valence quark, result for the nucleon description together with the choice of parameters and a brief overview of perturbation theory when including the meson fields. The appropriate propagators for quarks and mesons and the renormalization technique can be found as well.

2.1 Effective Lagrangian

Following considerations lay out the basic notions of the perturbative chiral quark model (PCQM), a relativistic quark model suggested in Gutsche (1987), Gutsche and Robson (1989), and extended in Lyubovitskij, Gutsche and Faessler (2001), Lyubovitskij, Gutsche, Faessler and Drukarev (2001), Lyubovitskij, Gutsche, Faessler and Vinh-Mau (2001, 2002), for the study of low-energy properties of baryons. In this model quarks move in an effective static field, represented by a scalar $S(r)$ and vector $V(r)$ component with $V_{\rm eff}(r) = S(r) + \gamma^0 V(r)$ and $r = |\vec{x}|$, providing phenomenological confinement. The interaction of quarks with Goldstone bosons is introduced on the basis of the nonlinear σ-model (Gell-Mann and Lévy, 1960). The PCQM is then defined by the effective, chirally invariant Lagrangian $\mathcal{L}_{\rm inv}$ (Lyubovitskij, Gutsche, Faessler and Drukarev; Lyubovitskij, Gutsche, Faessler and Vinh-Mau)

$$\begin{aligned}\mathcal{L}_{\rm inv}(x) &= \bar\psi(x)\biggl\{{\rm i}\not\partial-\gamma^0V(r)-S(r)\left[\frac{U+U^\dagger}{2}+\gamma^5\frac{U-U^\dagger}{2}\right]\biggr\}\psi(x)\\ &+\frac{F^2}{4}{\rm Tr}\left[\partial_\mu U\,\partial^\mu U^\dagger\right], \end{aligned}\tag{2.1}$$

with an additional mass term for quarks and mesons

$$\mathcal{L}_{\chi SB}(x)=-\bar\psi(x)\mathcal{M}\psi(x)-\frac{B}{2}{\rm Tr}[\hat\Phi^2\mathcal{M}],\tag{2.2}$$

which explicitly breaks chiral symmetry. Here ψ is the quark field, $U={\rm e}^{{\rm i}\hat\Phi/F}$ the chiral field with $\hat\Phi$ being the matrix of pseudoscalar mesons (in the following we restrict to the SU(2) flavor case, that is $\hat\Phi\to\hat\pi=\vec\pi\cdot\vec\tau$), $F=88$ MeV the pion decay constant in the chiral limit, $\mathcal{M}={\rm diag}\{\hat m,\hat m\}$ the mass matrix of current quarks (we restrict to the isospin symmetry limit with $m_u=m_d=\hat m=7$ MeV) and $B=-\langle 0|\bar uu|0\rangle/F^2=1.4$ GeV is the low-energy constant which measures the vacuum expectation value of the scalar quark densities in the chiral limit. We rely on the standard picture of chiral symmetry breaking, and for the mass of pions we use the leading term in their chiral expansion (i.e. linear in the current quark mass): $M_\pi^2=2\hat mB$.

With the unitary chiral rotation $\psi\to\exp\{-{\rm i}\gamma^5\hat\Phi/(2F)\}\psi$ (Thomas, 1981; Morgan, Miller and Thomas, 1986; Jennings and Maxwell, 1984) the Lagrangian equation in Eq. (2.1) transforms into a Weinberg-type form $\mathcal{L}^W$ containing the axial-vector coupling and the Weinberg-Tomozawa term (Lyubovitskij, Gutsche, Faessler and Vinh-Mau, 2001, 2002)(see details in Appendix B):

$$\begin{aligned}\mathcal{L}^W(x) &= \mathcal{L}_0(x)+\mathcal{L}_I^W(x)+o(\vec\pi^2), \qquad (2.3)\\ \mathcal{L}_0(x) &= \bar\psi(x)\biggl\{{\rm i}\not\partial-S(r)-\gamma^0V(r)\biggr\}\psi(x)-\frac{1}{2}\vec\pi(x)\left(\Box+M_\pi^2\right)\vec\pi(x),\\ \mathcal{L}_I^W(x) &= \frac{1}{2F}\partial_\mu\vec\pi(x)\bar\psi(x)\gamma^\mu\gamma^5\vec\tau\psi(x)-\frac{\varepsilon_{ijk}}{4F^2}\pi_i(x)\partial_\mu\pi_j(x)\bar\psi(x)\gamma^\mu\tau_k\psi(x),\end{aligned}$$

where $\mathcal{L}_I^W(x)$ is the $O(\vec{\pi}^2)$ strong interaction Lagrangian and $\Box = \partial^\mu \partial_\mu$.

In our calculation we do a perturbation calculation in the expansion parameter $1/F$ (where F is the pion leptonic decay constant in the chiral limit). We also treat the mass term of the current quarks as a perturbation. Dressing the baryonic three-quark core by a cloud of Goldstone mesons corresponds effectively to the inclusion of sea-quark contributions. All calculations are performed at one loop or at order of accuracy $O(1/F^2, \hat{m})$.

We expand the quark field ψ in the basis of potential eigenstates as

$$\psi(x) = \sum_\alpha b_\alpha u_\alpha(\vec{x}) \mathrm{e}^{-\mathrm{i}\mathcal{E}_\alpha t} + \sum_\beta d_\beta^\dagger v_\beta(\vec{x}) \mathrm{e}^{\mathrm{i}\mathcal{E}_\beta t}, \tag{2.4}$$

where the expansion coefficients b_α and $d_\beta^\dagger$ are the corresponding single quark annihilation and antiquark creation operators. The set of quark $\{u_\alpha\}$ and antiquark $\{v_\beta\}$ wavefunctions in orbits α and β is solutions of the static Dirac equation:

$$\left[-\mathrm{i}\gamma^0 \vec{\gamma} \cdot \vec{\nabla} + \gamma^0 S(r) + V(r) - \mathcal{E}_\alpha\right] u_\alpha(\vec{x}) = 0, \tag{2.5}$$

where $\mathcal{E}_\alpha$ is the single quark energy.

The unperturbed nucleon state is conventionally set up by the product of spin-flavor and color quark wavefunctions, where the nonrelativistic single quark wavefunction is replaced by the relativistic solution $u_0(\vec{x})$ in the ground state.

2.2 Parameters of Model

For a given form of effective potential $V_{\mathrm{eff}}(r)$ the Dirac equation in Eq. (2.5) can be solved numerically. Here, for the sake of simplicity, we use a variational Gaussian ansatz for the quark wavefunction given by the analytical form

$$u_0(\vec{x}) \; = \; N_0 \, \exp\left(-\frac{\vec{x}^2}{2R^2}\right) \begin{pmatrix} 1 \\ \mathrm{i}\rho \, \dfrac{\vec{\sigma} \cdot \vec{x}}{R} \end{pmatrix} \chi_s \, \chi_f \, \chi_c \, , \tag{2.6}$$

where

$$N_0 = \left[\pi^{3/2} R^3 \left(1 + \frac{3}{2}\rho^2\right)\right]^{-1/2} \tag{2.7}$$

is a constant fixed by the normalization condition

$$\int d^3\vec{x}\, u_0^\dagger(\vec{x})\, u_0(\vec{x}) \equiv 1; \tag{2.8}$$

χ_s, χ_f, χ_c are the spin, flavor and color quark wavefunctions, respectively. Our Gaussian ansatz contains two model parameters: the dimensional parameter R and the dimensionless parameter ρ.

The parameter ρ can be related to the axial coupling constant $g_A^{(0)}$ calculated in zeroth-order (or three-quark core) approximation:

$$g_A^{(0)} = \frac{5}{3}\left(1 - \frac{2\rho^2}{1+\frac{3}{2}\rho^2}\right) = \frac{5}{3}\left(\frac{1+2\gamma}{3}\right), \tag{2.9}$$

where $\gamma = (1-\frac{3}{2}\rho^2)/(1+\frac{3}{2}\rho^2)$ is a relativistic reduction factor. In our calculations we use the value $g_A^{(0)} = 1.25$ as obtained in the chiral limit of chiral perturbation theory (Gasser, Sainio and Švarc, 1988) which gives the parameter $\rho = \sqrt{2/13}$.

The parameter R can be physically understood as the mean radius of the three-quark core and is related to the charge radius of the proton in the leading-order (or zeroth-order) approximation as

$$\left\langle r_E^2 \right\rangle_{LO}^P = \int d^3\vec{x} u_0^\dagger(\vec{x}) \vec{x}^2 u_0(\vec{x}) = \frac{3R^2}{2} \frac{1 + \frac{5}{2}\rho^2}{1 + \frac{3}{2}\rho^2}. \tag{2.10}$$

Therefore we have only one free parameter, R. In the numerical evaluation R is varied in the region from 0.55 fm to 0.65 fm corresponding to a change of $\left\langle r_E^2 \right\rangle_{LO}^P$ in the region from 0.5 fm^2 to 0.7 fm^2.

The use of the Gaussian ansatz Eq. (2.6) in its exact form restricts the scalar and the vector parts of the potential to

$$\begin{aligned} S(r) &= \frac{1-3\rho^2}{2\rho R} + \frac{\rho}{2R^3} r^2, & (2.11)\\ V(r) &= \mathcal{E}_0 - \frac{1+3\rho^2}{2\rho R} + \frac{\rho}{2R^3} r^2, & (2.12) \end{aligned}$$

where the single-quark energy $\mathcal{E}_0$ is a free parameter in the Gaussion ansatz.

2.3 Perturbation Theory in the PCQM

According to Gell-Mann and Low theorem the expectation value of an operator $\hat{O}$ in the PCQM takes the form

$$\langle \hat{O} \rangle = ^N\!\langle \phi_0 | \sum_{n=0}^{\infty} \frac{\mathrm{i}^n}{n!} \int d^4x_1 \ldots \int d^4x_n T[\mathcal{L}_I^W(x_1) \ldots \mathcal{L}_I^W(x_n) \hat{O}] | \phi_0 \rangle_c^N . \tag{2.13}$$

The subscript c refers to contributions from connected graphs only, and $\mathcal{L}_I^W(x)$ is the pion-quark interaction Lagrangian as indicated in Eq. (2.3). The superscript N in Eq. (2.13) indicates that the matrix elements are projected on the respective nucleon states, the explicit form of the nucleon wavefunction $|N\rangle$ is given in Appendix E. The projection of "one-body" diagrams on the nucleon state refers to

$$\chi_{f'}^\dagger \chi_{s'}^\dagger I^{f'f} \, J^{s's} \, \chi_f \chi_s \stackrel{Proj.}{\longrightarrow} \langle N | \sum_{i=1}^{3} (I \, J)^{(i)} | N \rangle, \tag{2.14}$$

where the single-particle matrix element of the operators I and J, acting in flavor and spin space, is replaced by the one embedded in the nucleon state. For "two-body" diagrams with two independent quark indices i and j the projection prescription reads as

$$\chi_{f'}^\dagger \chi_{s'}^\dagger \, I_1^{f'f} \, J_1^{s's} \, \chi_f \chi_s \otimes \chi_{k'}^\dagger \chi_{\sigma'}^\dagger \, I_2^{k'k} \, J_2^{\sigma'\sigma} \, \chi_k \chi_\sigma \stackrel{Proj.}{\longrightarrow} \langle N | \sum_{i \neq j}^{3} (I_1 \, J_1)^{(i)} \otimes (I_2 \, J_2)^{(j)} | N \rangle. \tag{2.15}$$

We evaluate Eq. (2.13) using Wick's theorem and the appropriate propagators. For the quark field we use a Feynman propagator for a fermion in a binding potential. The quark propagator $\mathrm{i}G_\psi(x, y)$ is given by

$$\mathrm{i}G_\psi(x,y) = \big\langle \phi_0 \big| T\{\psi(x)\bar{\psi}(y)\} \big| \phi_0 \big\rangle \to \sum_\alpha u_\alpha(\vec{x})\bar{u}_\alpha(\vec{y}) \exp[-\mathrm{i}\mathcal{E}_\alpha(x_0-y_0)]\theta(x_0-y_0), \tag{2.16}$$

where we restrict to the quark states propagating forward in time. The explicit form of the excited quark wavefunctions is obtained analytically as given in Appendix D. For the meson we use the free Feynman propagator for a boson field with

$$\mathrm{i}\Delta_{ij}(x-y) = \big\langle 0 \big| T\{\Phi_i(x)\Phi_j(y)\} \big| 0 \big\rangle = \delta_{ij} \int \frac{d^4k}{(2\pi)^4\mathrm{i}} \frac{\exp[-\mathrm{i}k(x-y)]}{M_\Phi^2 - k^2 - \mathrm{i}\epsilon}. \tag{2.17}$$

2.4 Renormalization of the PCQM

We follow the formalism set out in Lyubovitskij, Gutsche and Faessler (2001), where in present work intermediate excited quark states are included in the loop diagrams (see details in Appendix G). Here we briefly describe the basic ingredients relevant for the further discussion. In the following, we attach the superscript “ 0 ” to the renormalization constants when we restrict to the contribution of the ground state quark propagator and the superscript “ F ” when the excited states are included.

First we introduce the renormalized quark field $\psi^r(x)$. It can be expanded in a set of potential eigenstates which are solutions of the renormalized Dirac equation with the full renormalized quark mass of

$$\begin{aligned} \hat{m}_F^r &= \hat{m} - \frac{3}{\gamma}\left(\frac{1}{4\pi F}\right)^2 \sum_\alpha \int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\ &\quad \times \Big[F_{I_\alpha}(k)F_{I_\alpha}^\dagger(k) - 2\omega(k^2)F_{I_\alpha}(k)F_{II_\alpha}^\dagger(k) + \omega^2(k^2)F_{II_\alpha}(k)F_{II_\alpha}^\dagger(k)\Big]\,. \end{aligned} \tag{2.18}$$

The expression for the renormalized quark mass includes self-energy corrections of the pion cloud, where

$$F_{I_\alpha}(k) \equiv N_0 N_\alpha k \Bigg[\int_0^\infty dr\, r^2\, (g_0(r) g_\alpha(r) - f_0(r) f_\alpha(r)) \int_\Omega d\Omega\, \mathrm{e}^{\mathrm{i}kr\cos\theta}\, C_\alpha\, Y_{l_\alpha 0}(\theta,\phi)$$
$$-2\,\mathrm{i}\frac{\partial}{\partial k} \int_0^\infty dr\, r\, (f_0(r) f_\alpha(r)) \int_\Omega d\Omega\, \cos\theta\, \mathrm{e}^{\mathrm{i}kr\cos\theta}\, C_\alpha\, Y_{l_\alpha 0}(\theta,\phi) \Bigg], \quad (2.19)$$

$$F_{II_\alpha}(k) \equiv N_0 N_\alpha \frac{\partial}{\partial k} \int_0^\infty dr\, r\, (g_0(r) f_\alpha(r) - f_0(r) g_\alpha(r)) \int_\Omega d\Omega\, \mathrm{e}^{\mathrm{i}kr\cos\theta}\, C_\alpha\, Y_{l_\alpha 0}(\theta,\phi), \quad (2.20)$$

with the pion energy $\omega(k^2) = \sqrt{M_\pi^2 + k^2}$; $k = |\vec{k}|$ is the pion momentum and $\Delta\mathcal{E}_\alpha = \mathcal{E}_\alpha - \mathcal{E}_0$ is the excess of the energy of the quark in state α with respect to the ground state. The label $\alpha = (n l_\alpha j m)$ characterizes the quark state (principal quantum number n, non-relativistic orbital angular momentum l_α, total angular momentum and projection j, m). For the Clebsch-Gordan coefficients we use the notation $C_\alpha \equiv \left\langle l_\alpha 0 \frac{1}{2}\frac{1}{2} | j \frac{1}{2} \right\rangle$ and $Y_{l_\alpha\, 0}(\theta,\phi)$ is the usual spherical harmonic. The explicit form of the radial wave functions $g_\alpha(r)$ and $f_\alpha(r)$, of the normalization constants N_α and of the energy difference $\Delta\mathcal{E}_\alpha$ are given in Appendix D.

When restricting the quark propagator to the ground state the expression above for the renormalized quark mass reduces to

$$\hat{m}_0^r = \hat{m} - \frac{27}{400\gamma}\left(\frac{g_A^{(0)}}{\pi F}\right)^2 \int_0^\infty dk\, k^4\, \frac{F_{\pi NN}^2(k^2)}{\omega^2(k^2)}, \quad (2.21)$$

where $F_{\pi NN}(k^2)$ is the πNN form factor normalized to unity at zero recoil ($k^2 = 0$):

$$F_{\pi NN}(k^2) = \exp\left(-\frac{k^2 R^2}{4}\right)\left[1 + \frac{k^2 R^2}{8}\left(1 - \frac{5}{3 g_A^{(0)}}\right)\right]. \quad (2.22)$$

In the second step we renormalize the effective Lagrangian including a set of counterterms. The renormalized interaction Lagrangian $\mathcal{L}_{I;r}^W = \mathcal{L}_{I;r}^{W;\mathrm{str}} + \mathcal{L}_{I;r}^{W;\mathrm{em}}$ contains a part due to the strong interaction,

$$\mathcal{L}_{I;r}^{W;\mathrm{str}} = \mathcal{L}_I^{W;\mathrm{str}} + \delta\mathcal{L}^{W;\mathrm{str}}, \quad (2.23)$$

and a piece due to the electromagnetic interaction,

$$\mathcal{L}_{I;\,r}^{W;\,\mathrm{em}} = \mathcal{L}_{I}^{W;\,\mathrm{em}} + \delta\mathcal{L}^{W;\,\mathrm{em}}. \tag{2.24}$$

The strong interaction term $\mathcal{L}_{I}^{W;\,\mathrm{str}}$ is given by

$$\mathcal{L}_{I}^{W;\,\mathrm{str}}(x) = \frac{1}{2F}\partial_\mu\vec{\pi}(x)\bar{\psi}^r(x)\gamma^\mu\gamma^5\vec{\tau}\psi^r(x) - \frac{\varepsilon_{ijk}}{4F^2}\pi_i(x)\partial_\mu\pi_j(x)\bar{\psi}^r(x)\gamma^\mu\tau_k\psi^r(x). \tag{2.25}$$

The interaction of pions and quarks with the electromagnetic field is described by (Lyubovitskij, Gutsche, Faessler and Vinh-Mau, 2001, 2002)

$$\begin{aligned}\mathcal{L}_{I}^{W;\,\mathrm{em}}(x) &= -eA_\mu^{\mathrm{em}}\bar{\psi}^r(x)Q\gamma^\mu\psi^r(x) \\ &\quad +\frac{e}{4F^2}A_\mu^{\mathrm{em}}(x)\bar{\psi}^r(x)\gamma^\mu\left[\vec{\pi}^2(x)\tau_3 - \vec{\pi}(x)\vec{\tau}\pi^0(x)\right]\psi^r(x) \\ &\quad -eA_\mu^{\mathrm{em}}(x)\varepsilon_{3ij}\left[\pi_i(x)\partial^\mu\pi_j(x) - \frac{\pi_j(x)}{2F}\bar{\psi}^r(x)\gamma^\mu\gamma^5\tau_i\psi^r(x)\right],\end{aligned} \tag{2.26}$$

which is generated by minimal substitution with

$$\partial_\mu\psi^r \to D_\mu\psi^r = \partial_\mu\psi^r + \mathrm{i}eQA_\mu^{\mathrm{em}}\psi^r, \tag{2.27}$$

$$\partial_\mu\pi_i \to D_\mu\pi_i = \partial_\mu\pi_i + e\varepsilon_{3ij}A_\mu^{\mathrm{em}}\pi_j, \tag{2.28}$$

where Q is the quark charge matrix with $Q = \mathrm{diag}\{2/3, -1/3\}$. The set of counterterms, denoted by $\delta\mathcal{L}^{W;\,\mathrm{str}}$ and $\delta\mathcal{L}^{W;\,\mathrm{em}}$ are explained and given in Appendix G and Lyubovitskij, Gutsche and Faessler (2001).

Now we consider the nucleon charge and prove that the properly introduced counterterms guarantee charge conservation. Using Noether's theorem we first derive from the renormalized Lagrangian the electromagnetic current operator:

$$j_r^\mu = j_{\psi^r}^\mu + j_\pi^\mu + j_{\psi^r\pi}^\mu + \delta j_{\psi^r}^\mu. \tag{2.29}$$

It contains the quark component $j_{\psi^r}^\mu$, the charged pion component j_π^μ, the quark-pion component $j^\mu_{\psi^r\pi}$ and the contribution of the counterterm $\delta j^\mu_{\psi^r}$:

$$
\begin{aligned}
j^\mu_{\psi^r} &= \bar{\psi}^r\gamma^\mu Q\psi^r = \frac{1}{3}\left(2\bar{u}^r\gamma^\mu u^r - \bar{d}^r\gamma^\mu d^r\right), && (2.30)\\
j^\mu_{\pi} &= \varepsilon_{3ij}\left(\pi_i\partial^\mu\pi_j\right) \equiv \pi^-\mathrm{i}\partial^\mu\pi^+ - \pi^+\mathrm{i}\partial^\mu\pi^-, && (2.31)\\
j^\mu_{\psi^r\pi} &= -\frac{1}{4F^2}\bar{\psi}^r\gamma^\mu\left(\vec{\pi}^2\tau_3 - \vec{\pi}\cdot\vec{\tau}\pi^0\right)\psi^r - \varepsilon_{3ij}\frac{\pi_j}{2F}\bar{\psi}^r\gamma^\mu\gamma^5\tau_i\psi^r, && (2.32)\\
\delta j^\mu_{\psi^r} &= \bar{\psi}^r(\hat{Z}-1)\gamma^\mu Q\psi^r. && (2.33)
\end{aligned}
$$

Here $\hat{Z}$ is the renormalization constant determined by the nucleon charge conservation condition (Lyubovitskij, Gutsche and Faessler, 2001). The analytical expression for the full renormalization constant $\hat{Z}^F$ is

$$
\begin{aligned}
\hat{Z}^F &= 1 - \frac{3}{(4\pi F)^2}\sum_\alpha\int_0^\infty dk\,k^2\,\frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)^2}\\
&\quad\times\left[F_{I_\alpha}(k)F^\dagger_{I_\alpha}(k) - 2\,\omega(k^2)F_{I_\alpha}(k)F^\dagger_{II_\alpha}(k) + \omega^2(k^2)F_{II_\alpha}(k)F^\dagger_{II_\alpha}(k)\right]. \quad (2.34)
\end{aligned}
$$

When restricting intermediate quark states to the ground state Eq. (2.34) yields the result

$$
\hat{Z}^0 = 1 - \frac{27}{400}\left(\frac{g_A^{(0)}}{\pi F}\right)^2\int_0^\infty dk\,k^4\,\frac{F^2_{\pi NN}(k^2)}{\omega^3(k^2)}. \quad (2.35)
$$

We obtain a value of $\hat{Z}^0 = 0.9 \pm 0.02$ (Lyubovitskij, Gutsche and Faessler, 2001) for our set of parameters. Inclusion of the excited quark states changes the value of the renormalization constant to a value of $\hat{Z}^F = 0.7 \pm 0.05$.

Chapter III

The Axial Form Factor of the Nucleon

For the present purposes we have to construct the partially conserved axial-vector current A_i^μ (see detail in Appendix H):

$$\begin{aligned} A_i^\mu &= F\partial^\mu\pi_i + \bar{\psi}^r\gamma^\mu\gamma^5\frac{\tau_i}{2}\psi^r - \frac{\varepsilon_{ijk}}{2F}\bar{\psi}^r\gamma^\mu\tau_j\psi^r\pi_k + \frac{1}{4F^2}\bar{\psi}^r\gamma^\mu\gamma^5\left(\vec{\pi}\cdot\vec{\tau}\pi_i - \vec{\pi}^2\tau_i\right)\psi^r \\ &\quad + \bar{\psi}^r(\hat{Z}-1)\gamma^\mu\gamma^5\frac{\tau_i}{2}\psi^r + o(\vec{\pi}^2)\,. \end{aligned} \tag{3.1}$$

The axial form factor $G_A(Q^2)$ of the nucleon is defined by the matrix element of the $i = 3$ isospin component and the spatial part of the axial vector current evaluated for nucleon states. In the Breit frame $G_A(Q^2)$ is set up as (Tegen and Weise, 1983):

$$\left\langle N_{s'}\left(\frac{\vec{q}}{2}\right)\left|\int d^3\vec{x}\, \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}}\vec{A}_3(x)\right|N_s\left(-\frac{\vec{q}}{2}\right)\right\rangle = \chi^\dagger_{N_{s'}}\vec{\sigma}_N\frac{\tau_N^3}{2}\chi_{N_s}G_A(Q^2), \tag{3.2}$$

with the space-like momentum transfer squared given as $Q^2 = -q^2 = \vec{q}^{\,2}$. Here, χ_{N_s} and $\chi_{N_{s'}}$ are the nucleon spin wave functions in the initial and final states; $\vec{\sigma}_N$ is the nucleon spin matrix and τ_N^3 is the third component of the isospin matrix of the nucleon.

At zero recoil ($Q^2 = 0$) the axial form factor satisfies the condition:

$$G_A(0) = g_A, \tag{3.3}$$

where g_A is the axial charge of the nucleon.

For low-momentum transfers, that is $Q^2 \leq 1$ GeV^2, the axial form factor can be represented by a dipole fit

$$G_A(Q^2) = \frac{G_A(0)}{(1+\frac{Q^2}{M_A^2})^2}, \tag{3.4}$$

in terms of one adjustable parameter M_A, the axial mass (or sometimes dipole mass). Therefore, the axial radius can be expressed in terms of the axial mass with:

$$\langle r_A^2 \rangle = -6 \frac{1}{G_A(0)} \frac{dG_A(Q^2)}{dQ^2}\bigg|_{Q^2=0} = \frac{12}{M_A^2}. \tag{3.5}$$

In the PCQM the axial form factor of the nucleon up to one loop corrections is given by

$$\begin{aligned} \chi^\dagger_{N_{s'}} \vec{\sigma}_N \frac{\tau_N^3}{2} \chi_{N_s} G_A(Q^2) \;=\; & {}^N\!\langle \phi_0 | \sum_{n=0}^{2} \frac{\mathrm{i}^n}{n!} \int \delta(t)\, d^4x\, d^4x_1 \,\ldots\, d^4x_n \mathrm{e}^{-\mathrm{i}qx} \\ & \times T[\mathcal{L}_I^{W;\mathrm{str}}(x_1) \,\ldots\, \mathcal{L}_I^{W;\mathrm{str}}(x_n)\, \vec{A}_3(x)]\, |\phi_0\rangle_c^N, \end{aligned} \tag{3.6}$$

with the interaction term

$$\mathcal{L}_I^{W;\mathrm{str}}(x) = \frac{1}{2F} \partial_\mu \vec{\pi}(x) \bar{\psi}^r(x) \gamma^\mu \gamma^5 \vec{\tau} \psi^r(x) - \frac{\varepsilon_{ijk}}{4F^2} \pi_i(x) \partial_\mu \pi_j(x) \bar{\psi}^r(x) \gamma^\mu \tau_k \psi^r(x), \tag{3.7}$$

where the superscript r refers to renormalized quantities.

From the works of Lyubovitskij, Gutsche and Faessler (2001), Lyubovitskij, Gutsche, Faessler and Drukarev (2001), Lyubovitskij, Gutsche, Faessler and Vinh-Mau (2001), Lyubovitskij, Wang, Gutsche and Faessler (2002), Simkovic et al. (2002) and Pumsa-ard, Lyubovitskij, Gutsche, Faessler and Cheedket (2003) they conclude that the use of a truncated quark propagator leads to a reasonable description of the experiment. The excited quark states in propagator of Eq. (2.16) were included for the first time by Pumsa-ard et al.(2003) to analyze their influence on the matrix elements for the $N - \Delta$ transition.

Here we follow the formalism set out in Pumsa-ard, Lyubovitskij, Gutsche, Faessler and Cheedket (2003). In the final calculation we include a set of excited states up to $2\hbar\omega$ in the quark propagator: the first p-states ($1p_{1/2}$ and $1p_{3/2}$ in the non-relativistic spectroscopic notation) and the second excited states ($1d_{3/2}$, $1d_{5/2}$

and $2s_{1/2}$). In other words we include the excited states whose energies satisfy the restriction $\mathcal{E} < \Lambda = 1$ GeV (see discussion in the Chapter I). The diagrams to be evaluated are shown in Fig. (3.1).

Next, we present the analytical expressions for the axial form factor of the nucleon obtained in the PCQM. We start with the simplest case, where the quark propagator is restricted to the ground state contribution. The axial form factor of the nucleon is a sum of terms arising from different diagrams: the three-quark diagram (Fig. 3.1(a)), the counterterm (Fig. 3.1(b)), the self-energy diagram (Figs. 3.1(c) and (d)), the exchange diagram (Fig. 3.1(e)) and the vertex-correction diagram (Fig. 3.1(f)). Other possible diagrams at one loop are compensated by the counterterm. The corresponding analytical expressions for the relevant diagrams are given in the following:

(a) For the three-quark diagram ($3q$) (Fig. 3.1(a)) we obtain:

$$G_A(Q^2)\big|_{3q} = G_A(Q^2)\big|_{3q}^{LO} + G_A(Q^2)\big|_{3q}^{NLO}, \tag{3.8}$$

with

$$\begin{aligned} G_A(Q^2)\big|_{3q}^{LO} &= g_A^{(0)} F_{\pi NN}(Q^2) \qquad (3.9)\\ G_A(Q^2)\big|_{3q}^{NLO} &= \frac{3}{2}\,\hat{m}_0^r \frac{\rho R}{\left(1+\frac{3}{2}\rho^2\right)^2} \Bigg\{ \left(1+\frac{9}{2}\rho^2\right) G_A(Q^2)\big|_{3q}^{LO} - \frac{5}{72}\Big[12\left(2-3\rho^2\right) \\ &\quad -4\left(1+5\rho^2\right) Q^2R^2 + \rho^2 Q^4 R^4\Big] \exp\left(-\frac{Q^2R^2}{4}\right)\Bigg\}, \qquad (3.10) \end{aligned}$$

where $G_A(Q^2)|_{3q}^{LO}$ is the leading-order term (LO) evaluated with the unperturbed quark wavefunction $u_0(\vec{x})$; $G_A(Q^2)|_{3q}^{NLO}$ is a correction due to the renormalization of the quark wavefunction $u_0(\vec{x}) \to u_0^r(\vec{x};\hat{m}^r)$ which is referred to as next-to-leading order (NLO).

The modified quark wavefunction $u_0^r(\vec{x};\hat{m}^r)$ is given by (Lyubovitskij,

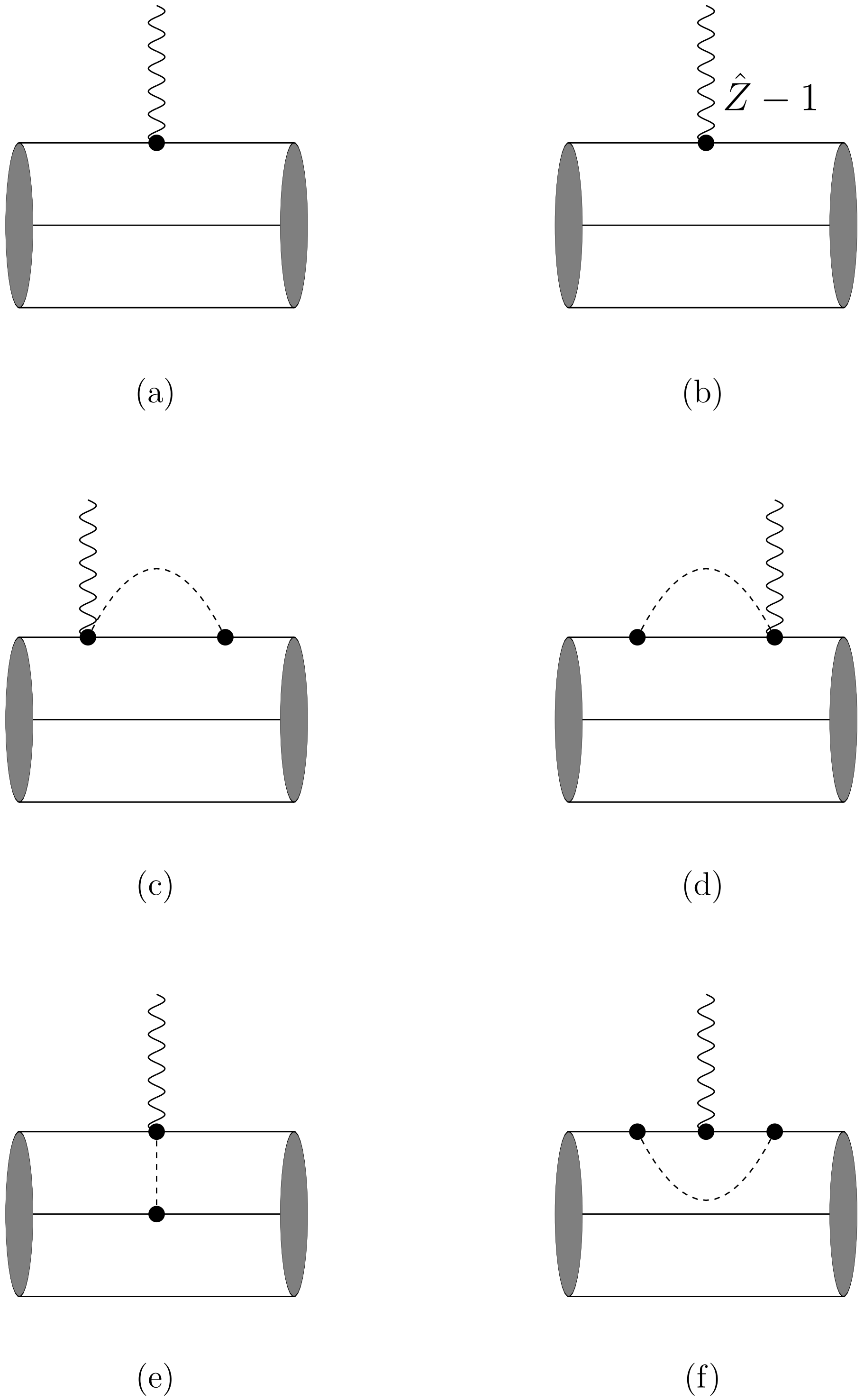

Figure 3.1: Diagrams contributing to the axial form factor of the nucleon: three-quark core (a), counterterm (CT)(b), self-energy (SE)(c and d), exchange (EX)(e) and vertex correction (VC)(f).

Gutsche and Faessler ,2001)

$$u_0^r(\vec{x};\hat{m}^r) = u_0(\vec{x}) + \delta u_0(\vec{x};\hat{m}^r), \tag{3.11}$$

where

$$\delta u_0(\vec{x};\hat{m}^r) = \frac{\hat{m}^r}{2}\frac{\rho R}{1+\frac{3}{2}\rho^2}\left(\frac{\frac{1}{2}+\frac{21}{4}\rho^2}{1+\frac{3}{2}\rho^2} - \frac{\vec{x}^2}{R^2} + \gamma^0\right) u_0(\vec{x}). \tag{3.12}$$

(b) The three-quark counterterm (CT) (Fig. 3.1(b)) results in the expression:

$$G_A(Q^2)\big|_{CT} \equiv (\hat{Z}^0 - 1) G_A(Q^2)\big|_{3q}^{LO}. \tag{3.13}$$

(c) The self-energy diagram I (SE;I) (Fig. 3.1(c)) yields:

$$G_A(Q^2)\big|_{SE;I} = 8\, g_A^{(0)}\, \frac{\rho\, R}{(2+3\rho^2)}\left(\frac{1}{2\pi F}\right)^2 \int_0^\infty dk\, k^4 \frac{F_{\pi NN}(k^2)}{\omega^2(k^2)}\, \mathcal{D}(k,Q^2), \tag{3.14}$$

where

$$\begin{aligned}\mathcal{D}(k,Q^2) \quad &\equiv \quad \exp\left(-\frac{(k+\sqrt{Q^2})^2R^2}{4}\right)\left(\frac{1}{k^2Q^2R^4}\right)^{3/2} \\ &\times \left[2 + k\sqrt{Q^2}R^2 + \exp\left(k\sqrt{Q^2}R^2\right)\left(-2 + k\sqrt{Q^2}R^2\right)\right].\end{aligned} \tag{3.15}$$

(d) For the self-energy diagram II (SE;II) (Fig. 3.1(d)) we also obtain:

$$G_A(Q^2)\big|_{SE;II} = G_A(Q^2)\big|_{SE;I}. \tag{3.16}$$

(e) For the exchange diagram (EX) (Fig. 3.1(e)) we get:

$$G_A(Q^2)\big|_{EX} = \frac{96}{5}\, g_A^{(0)}\, \frac{\rho\, R}{(2+3\rho^2)}\left(\frac{1}{2\pi F}\right)^2 \int_0^\infty dk\, k^4 \frac{F_{\pi NN}(k^2)}{\omega^2(k^2)}\, \mathcal{D}(k,Q^2). \tag{3.17}$$

(f) The vertex-correction diagram (VC) (Fig. 3.1(f)) gives the contribution:

$$G_A(Q^2)\big|_{VC} = \frac{3}{100}\left(g_A^{(0)}\right)^3\left(\frac{1}{2\pi F}\right)^2 F_{\pi NN}(Q^2) \int_0^\infty dk\, k^4\, \frac{F_{\pi NN}^2(k^2)}{\omega^3(k^2)}. \tag{3.18}$$

In the next step we extend the formalism by also including excited states in the quark propagator. The leading-order expression of the three-quark diagram and the exchange term remain the same. In turn the following contributions must be extended.

(a) In the three-quark NLO expression the appropriate renormalized mass has to be inserted with

$$\begin{aligned} G_A(Q^2)\big|_{3q}^{NLO} &= \frac{3}{2}\,\hat{m}_F^r \frac{\rho\, R}{\left(1+\frac{3}{2}\rho^2\right)^2}\left\{\left(1+\frac{9}{2}\rho^2\right) G_A(Q^2)\big|_{3q}^{LO} - \frac{5}{72}\left[12\left(2-3\rho^2\right)\right.\right. \\ &\left.\left. -4\left(1+5\rho^2\right) Q^2R^2 + \rho^2 Q^4 R^4\right] \exp\left(-\frac{Q^2R^2}{4}\right)\right\}. \end{aligned} \tag{3.19}$$

(b) For the three-quark counterterm (CT) the renormalization constant has to be replaced accordingly

$$G_A(Q^2)\big|_{CT} = (\hat{Z}^F - 1) G_A(Q^2)\big|_{3q}^{LO}. \tag{3.20}$$

(c) For the self-energy diagram I (SE;I) we obtain the full expression

$$\begin{aligned} G_A(Q^2)\big|_{SE;I} &= \frac{10}{3}\left(\frac{1}{4\pi F}\right)^2 \sum_\alpha \int_0^\infty dk\, k^2\, \frac{\omega(k^2) F_{II_\alpha}(k) - F_{I_\alpha}(k)}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\ &\times \int_{-1}^{1} dx \frac{2k(1-x^2)F_{III_\alpha}(k_-) + \left(\sqrt{Q^2}x + (1-2x^2)k\right) F_{IV_\alpha}(k_-)}{\sqrt{k_-^2}}, \end{aligned} \tag{3.21}$$

where

$$F_{III_\alpha}(k_-) \equiv N_0 N_\alpha \frac{\partial}{\partial k_-}\int_0^\infty dr\, r\,(g_0(r) f_\alpha(r)) \int_\Omega d\Omega\, \mathrm{e}^{\mathrm{i}k_-\, r\cos\theta} C_\alpha Y_{l_\alpha 0}(\theta,\phi), \tag{3.22}$$

$$\begin{aligned} F_{IV_\alpha}(k_-) &\equiv N_0 N_\alpha \frac{\partial}{\partial k_-}\int_0^\infty dr\, r\,(f_0(r) g_\alpha(r) - g_0(r) f_\alpha(r)) \\ &\times \int_\Omega d\Omega\, \mathrm{e}^{\mathrm{i}k_-\, r\cos\theta} C_\alpha Y_{l_\alpha 0}(\theta,\phi), \end{aligned} \tag{3.23}$$

$$k_\pm^2 \equiv k^2 + Q^2 \pm 2k\sqrt{Q^2}x. \tag{3.24}$$

(d) For the self-energy diagram II (SE;II) we get

$$G_A(Q^2)\big|_{SE;II} = \frac{10}{3}\left(\frac{1}{4\pi F}\right)^2 \sum_{\alpha} \int_0^{\infty} dk\, k^2\, \frac{\omega(k^2)F^{\dagger}_{II_\alpha}(k) - F^{\dagger}_{I_\alpha}(k)}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \times \int_{-1}^{1} dx\, \frac{2k(1-x^2)F_{V_\alpha}(k_+) - (\sqrt{Q^2}x+k)F_{IV_\alpha}(k_+)}{\sqrt{k_+^2}}, \tag{3.25}$$

where

$$F_{V_\alpha}(k_+) \equiv N_0 N_\alpha \frac{\partial}{\partial k_+}\int_0^{\infty} dr\, r\,(f_0(r)g_\alpha(r)) \int_\Omega d\Omega \mathrm{e}^{\mathrm{i}k_+ r\cos\theta} C_\alpha Y_{l_\alpha 0}(\theta,\phi). \tag{3.26}$$

(e) For the vertex-correction diagram (VC) inclusion of excited states results in

$$G_A(Q^2)\big|_{VC} = \sum_{\alpha\,\beta} \frac{5}{9}\frac{\mathcal{F}_{\alpha,\beta}(Q^2)}{(4\pi F)^2}\int_0^{\infty} dk\, k^2 \left[\frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)\,(\omega(k^2)+\Delta\mathcal{E}_\beta)}\right] \times\Big[\omega^2(k^2)\left(F_{II_\alpha}(k)F^{\dagger}_{II_\beta}(k)\right) - \omega(k^2)\left(F_{II_\alpha}(k)F^{\dagger}_{I_\beta}(k) + F_{I_\alpha}(k)F^{\dagger}_{II_\beta}(k)\right) + \left(F_{I_\alpha}(k)F^{\dagger}_{I_\beta}(k)\right)\Big], \tag{3.27}$$

where

$$\mathcal{F}_{\alpha,\beta}(Q^2) \equiv N_\alpha N_\beta \int_0^{\infty} drr^2\Big(\mathcal{A}_{\alpha,\beta}(r) + 2\mathcal{B}_{\alpha,\beta}(r)\Big), \tag{3.28}$$

$$\mathcal{A}_{\alpha,\beta}(r) \equiv (g_\alpha(r)g_\beta(r) - f_\alpha(r)f_\beta(r))\int_\Omega d\Omega\, \exp\left(\mathrm{i}\sqrt{Q^2}r\cos\theta\right)\mathcal{C}_{\alpha\beta;1}(\theta,\phi), \tag{3.29}$$

$$\mathcal{B}_{\alpha,\beta}(r) \equiv f_\alpha(r)f_\beta(r)\int_\Omega d\Omega\, \exp\left(\mathrm{i}\sqrt{Q^2}r\cos\theta\right) \times\Big[\cos^2\theta\, \mathcal{C}_{\alpha\beta;1}(\theta,\phi) + \sin\theta\cos\theta\, \mathcal{C}_{\alpha\beta;2}(\theta,\phi)\Big], \tag{3.30}$$

$$\mathcal{C}_{\alpha\beta;1}(\theta,\phi) \equiv C_\alpha C_\beta Y_{l_\alpha 0}(\theta,\phi)Y_{l_\beta 0}(\theta,\phi) - D_\alpha D_\beta Y^*_{l_\alpha 1}(\theta,\phi)Y_{l_\beta 1}(\theta,\phi), \tag{3.31}$$

$$\mathcal{C}_{\alpha\beta;2}(\theta,\phi) \equiv C_\alpha D_\beta Y_{l_\alpha 0}(\theta,\phi)Y_{l_\beta 1}(\theta,\phi)\mathrm{e}^{-\mathrm{i}\phi} + D_\alpha C_\beta Y^*_{l_\alpha 1}(\theta,\phi)Y_{l_\beta 0}(\theta,\phi)\mathrm{i}\mathrm{e}^{\mathrm{i}\phi}, \tag{3.32}$$

where $D_\alpha = \left\langle l_\alpha 1 \tfrac{1}{2} - \tfrac{1}{2} | j \tfrac{1}{2}\right\rangle$, l_α and l_β are the orbital quantum numbers of the intermediate states α and β, respectively.

Chapter IV

Results and Discussion

4.1 Results

The Q^2-dependence (up to $0.4\,\mathrm{GeV}^2$) of the axial form factor of the nucleon are shown in Figs. (4.1), (4.2) and (4.3), the description of each figure is given below. Due to the lack of covariance, the form factor can be expected to be reasonable up to $Q^2 < \vec{p}^{\,2} = 0.4\,\mathrm{GeV}^2$, where $\vec{p}$ is the typical three-momentum transfer which defines the region where relativistic effect $\leq$ 10% or where the following inequality $\vec{p}^{\,2}/\left(4m_N^2\right) < 0.1$ is fulfilled.

The first result for the Q^2-dependence of the axial form factor $G_A(Q^2)$ of the nucleon is indicated in Fig. (4.1). The numerical values are obtained, when truncating the quark propagator to the ground state or equivalently to the intermediate nucleon and delta baryon states in loop diagrams. Thereby, we also give the individual contributions of the different diagrams of Fig. (3.1), which add up coherently. The leading order three-quark diagram dominates the result for the axial form factor, whereas pion cloud corrections add about 20% of the total result. Here, both the exchange and self-energy terms give the largest, positive contribution.

In the next step we include the intermediate excited quark states with quantum numbers $1p_{1/2}, 1p_{3/2}, 1d_{3/2}, 1d_{5/2}$ and $2s_{1/2}$ in the propagator. The resulting effect on $G_A(Q^2)$ is given in Fig. (4.2). We explicitly indicate the additional terms, which are solely due to the contribution of these excited states. The

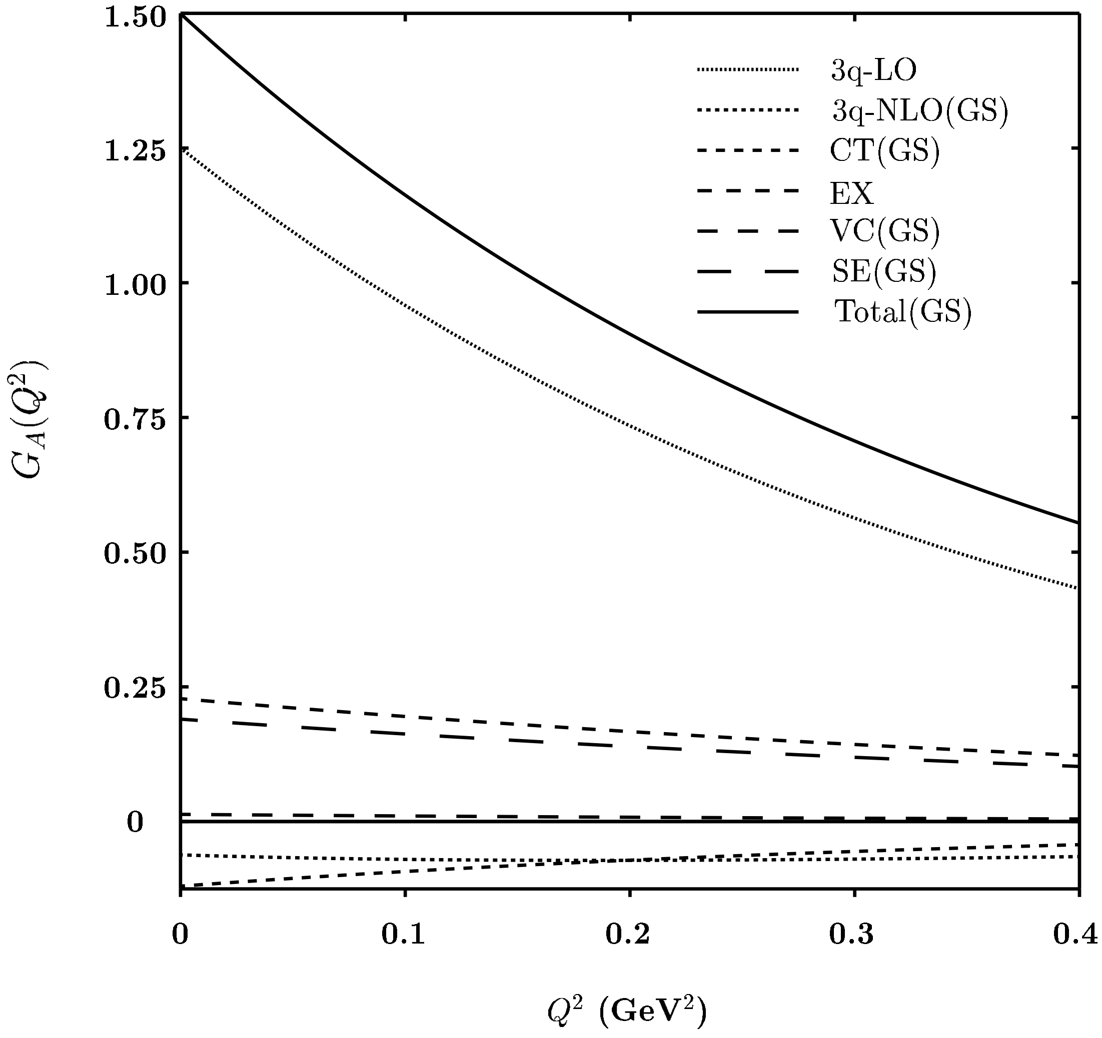

Figure 4.1: Model results for the axial form factor of the nucleon $G_A(Q^2)$. The coherent contributions of the different diagrams of Fig. (3.1) are indicated when restricting to the ground state(GS) quark propagator.

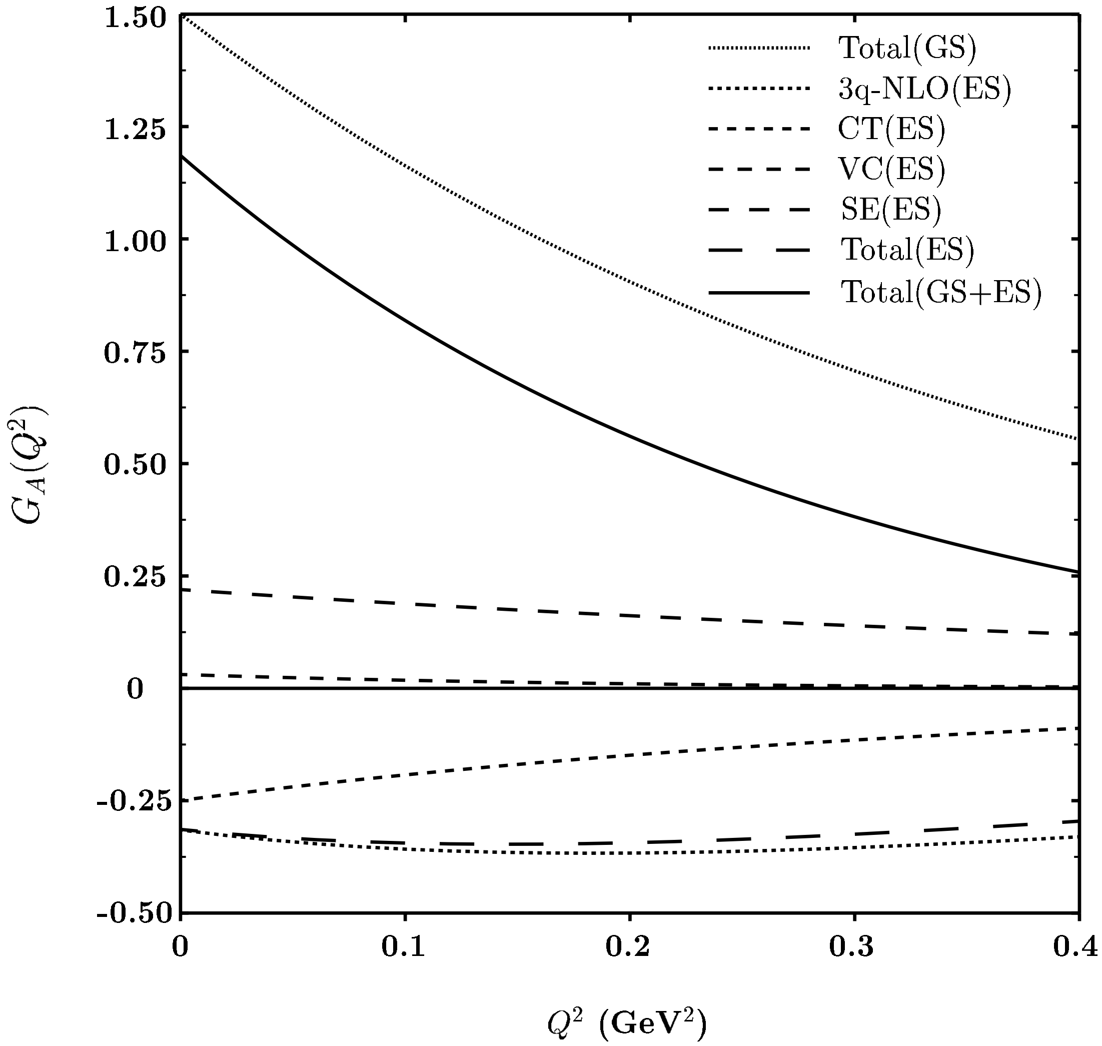

Figure 4.2: Model results for the axial form factor of the nucleon $G_A(Q^2)$ when excited states are included in the quark propagator. The full ground-state result is contained in the curve labelled by Total(GS). Excited state(ES) contributions of the individual diagrams are indicated separately.

previous result, where the quark propagator is restricted to the ground state, is contained in the curve denoted by Total(GS). The inclusion of the intermediate excited states tends to induce a cancellation of the original pion cloud corrections generated for the case of the ground state quark propagator, thereby regaining approximately the tree level result. In Fig. (4.3) we give for completeness the full result for $G_A(Q^2)$ including excited states in comparison with experimental data and with the dipole fit using an axial mass of $M_A = 1.069$ GeV and normalized to $G_A(0) = 1.267$ at zero recoil. The model clearly underestimates the finite Q^2-behavior, but it should be noted that a similar effect occurs in the discussion of the electromagnetic form factors of the nucleon (Lyubovitskij, Gutsche and Faessler, 2001). The stiffness of the form factors can be traced to the Gaussian ansatz of the single quark wave functions and can also be improved when in addition resorting to a fully covariant description of the valence quark content of the nucleon (Ivanov, Locher and Lyubovitskij, 1996). Hence the applicability of the PCQM is mostly for static quantities and low Q^2 observables of baryons.

For the comparison with the data near $Q^2 = 0$ we first turn to the results for the axial charge, g_A. In Table (4.1) we list the numerical values for the complete set of Feynman diagrams (Fig. (3.1)), again indicating separately the contributions of ground and excited states in the quark propagator.

The prediction for the axial charge including loop corrections are relevant for several reasons:

(i) the tree level result for g_A was previously adjusted to fix one of the parameters, ρ. Since loop corrections essentially do not change this results, the previous model predictions remain meaningful;

(ii) the predicted small loop corrections to g_A are consistent with similar results in chiral perturbation theory;

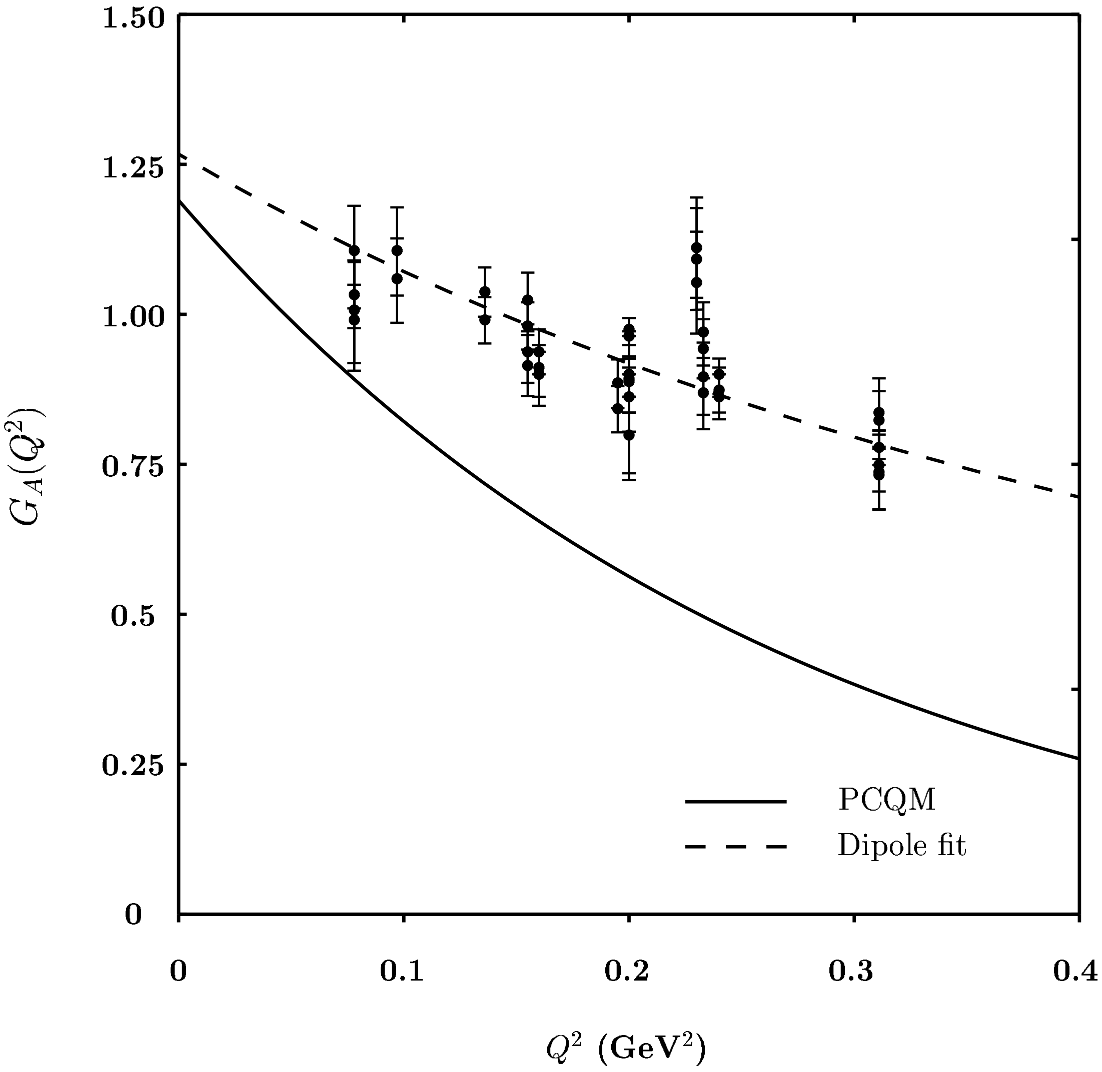

Figure 4.3: The axial form factor of the nucleon in the PCQM in comparison with a dipole fit (axial mass $M_A = 1.069$ GeV) and with experimental data. Data are taken from Refs.(Amaldi et al., 1970, 1972; Bloom et al., 1973; Brauel et al., 1973; Guerra et al., 1975; Esaulov, Pilipenko and Titov, 1978)

Table 4.1: Contributions of the individual diagrams of Fig. (3.1) to the axial charge g_A. Separate results for the inclusion of ground (GS) and excited states (ES) in the quark propagator are indicated.

	g_A
GS quark propagator	
3q-core	
LO	1.25
NLO	-0.062 ± 0.013
Counterterm	-0.120 ± 0.024
Exchange	0.228 ± 0.042
Vertex correction	0.013 ± 0.003
Self-energy	0.190 ± 0.034
GS contribution	1.499 ± 0.042
ES quark propagator	
NLO	-0.315 ± 0.054
Counterterm	-0.249 ± 0.044
Vertex correction	0.031 ± 0.005
Self-energy	0.220 ± 0.038
ES contribution	-0.314 ± 0.055
Total(GS+ES)	1.185 ± 0.013
Experiment	1.267 ± 0.003

Table 4.2: Comparison of the axial mass and the axial radius between experimental values and the result from the PCQM.

	Model	Experiment
M_A (GeV)	0.779 ± 0.050	1.069 ± 0.016
$\langle r_A^2 \rangle^{1/2}$ (fm)	0.881 ± 0.056	0.639 ± 0.010

(iii) the generic role of excited states in loop diagrams are rather relevant in understanding the nucleon properties. This role was already exemplified in the case of $N - \Delta$ transition (Pumsa-ard, Lyubovitskij, Gutsche, Faessler and Cheedket, 2003) and sigma-terms (Inoue, Lyubovitskij, Gutsche and Faessler, 2004) and again is demonstrated in the case of g_A.

A comparison of the experimentally deduced values for the axial mass and the axial radius with our model results is given in Table (4.2).

4.2 Summary

In summary, we have evaluated the axial form factor of the nucleon and, more important, its low Q^2 limits, such as the axial charge and the axial radius using a perturbative chiral quark model as based on an effective chiral Lagrangian. Since the PCQM is a static model, Lorentz covariance cannot be fulfilled. Approximate techniques to account for Galilei invariance and Lorentz boost effects were shown to change the tree level results by about 10%. Higher order, that is loop contributions, are less sensitive to these correction. The derived quantities contain, in consistency with previous works, only one model parameter R, which is related to the radius of the three-quark core, and are otherwise expressed in terms of fundamental parameters of low energy hadron physics: weak pion decay constant, and set of QCD parameters. In addition, another parameter (ρ of Eq. (2.5)), which is related to the amplitude of the small component of the single quark wave function, was originally set up to reproduce the value for the axial charge in the chiral limit with $g_A^{(0)} = 1.25$ (Gasser, Sainio and Švarc, 1988). Predictions are given for the fixed values of model parameters ρ and R in consistency with previous investigations. In particular, our result for the axial charge, $g_A = 1.185 \pm 0.013$, is in reasonable agreement with the central value of data: $g_A = 1.267 \pm 0.003$. Thereby, contributions of excited quark states in loop diagrams play a considerable role in order to generate a small correction to the tree level result, which is required to account for the data point. This result, obtained in the context of the PCQM, is rather encouraging. Minor pion cloud corrections to the tree level result of g_A justify in turn the appropriate choice for ρ or for $g_A^{(0)}$ used in previous works. Also, recent calculations of the axial charge up to order p^4 in chiral perturbation theory (Kambor and Mojžiš, 1999; Schweizer, 2000) imply rather large p^3 corrections leading to rather large uncertainties when going to the next order in the chiral expansion. Our model result can naturally explain the small correction to the one obtained in the chiral limit, but only when going beyond nucleon and delta states in the loop diagrams.

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Appendices

Appendix A

Quantum Chromodynamics and Chiral Perturbation Theory

Chiral perturbation theory (ChPT) provides a systematic framework for investigating strong-interaction processes at low energies, as opposed to a perturbative treatment of quantum chromodynamics (QCD) at high momentum transfers in terms of the "running coupling constant". The basis of ChPT is the global $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ symmetry of the QCD Lagrangian in the limit of massless u, d, and s quarks. This symmetry is assumed to be spontaneously broken down giving rise to eight massless Goldstone bosons. In this appendix we will give an overview of the foundations for ChPT which is the chiral effective Lagrangian. The method for including the external field to the Lagrangian and the lowest order of the ChPT are also indicated at the end.

A.1 The QCD Lagrangian

The gauge principle has proven to be a tremendously successful method in elementary particle physics to generate interactions between matter fields through the exchange of massless gauge bosons. The best-known example is quantum electrodynamics (QED). In QED the interaction between charged particles is mediated by the exchange of neutral gauge bosons, photons. Because of the neutrality of the photon there do not exist vertices where a photon interacts directly with another photon. Therefore in QED only a single vertex is required, the coupling of the photon to a fermion. The coupling constant, e, in QED is related to the fine structure constant via $\alpha = e^2/4\pi \approx 1/137$, and because of the smallness of α, the theory can be successfully treated perturbatively.

The remarkable success of QED leads quite naturally to a nonabelian generalization involving a triplet of color-charges interacting via the exchange of color gauge bosons called gluons. This is the theory of QCD which is the gauge theory of the strong interactions. The matter fields of QCD are the so-called quarks which are spin-1/2 fermions, with six different flavors in addition to their three possible colors.

The QCD Lagrangian obtained from the gauge principle reads

$$\mathcal{L}_{\mathrm{QCD}} = \sum_{f=\substack{u,d,s,\\ c,b,t}} \bar{q}_f(\mathrm{i}\not{D} - m_f)q_f - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a. \tag{A.1}$$

For each quark flavor f the quark field q_f consists of a color triplet (subscripts r, g, and b standing for "red", "green", and "blue"),

$$q_f = \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix}, \tag{A.2}$$

and the covariant derivative of q_f is

$$D_\mu \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} = \partial_\mu \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix} - \mathrm{i}g \sum_{a=1}^{8} \frac{\lambda_a^C}{2} \mathcal{A}_{\mu,a} \begin{pmatrix} q_{f,r} \\ q_{f,g} \\ q_{f,b} \end{pmatrix}. \tag{A.3}$$

We note that the interaction between quarks and gluons is independent of the quark flavors. Equation (A.1) also contains the generalization of the field strength tensor to the nonabelian case,

$$\mathcal{G}_{\mu\nu,a} = \partial_\mu \mathcal{A}_{\nu,a} - \partial_\nu \mathcal{A}_{\mu,a} + g f_{abc} \mathcal{A}_{\mu,b} \mathcal{A}_{\nu,c}, \tag{A.4}$$

where f_{abc} is the SU(3) structure constants and $\mathcal{A}_\mu$ is the gluon field.

In contradistinction to the abelian case of QED, the squared field strength tensor gives rise to gauge-field self interactions involving vertices with three and four gauge fields of strength g and g^2, respectively. Such interaction terms are characteristic of nonabelian gauge theories and make them much more complicated than abelian theories. These difficulties have heretofore prevented a precise confrontation of experiment with rigorous QCD predictions. Nevertheless there are at least two cases in which these problems can be ameliorated and reliable theoretical predictions can be generated from QCD:

- High energy limit: At very high energies, when the momentum transfer q^2 is large, QCD becomes "asymptotically free", i.e. the running coupling constant $g(q^2)$ approaches zero. Hence, in this limit one can utilize perturbative methods. However, this procedure, "perturbative QCD", is not useful except for interactions at the very highest energies.

- Symmetry: The second way to confront QCD with experimental test is to utilize the symmetry of $\mathcal{L}_{\text{QCD}}$. In order to do so, we separate the quark components into two groups. That involving the heavy quarks, c, b and t, we shall not consider further in these dissertation. Indeed the masses of such quarks are much larger than the QCD scale, $\Lambda_{\text{QCD}} \sim 300$ MeV, but can be treated using heavy-quark symmetry methods. On the other hand, the light quarks, u, d and s, have masses much smaller than the QCD scale and their interactions can be analyzed by exploiting the chiral symmetry of the QCD Lagrangian as will be shown further below. As we shall see, this procedure is capable of rigor but is only useful for energies $E \ll 1$ GeV, low energy method.

A.2 Global Symmetries of $\mathcal{L}_{\rm QCD}$

The six quark flavors are commonly divided into the three light quarks u, d, and s and the three heavy flavors c, b, and t,

$$\begin{pmatrix} m_u = 0.005\,\mathrm{GeV} \\ m_d = 0.009\,\mathrm{GeV} \\ m_s = 0.175\,\mathrm{GeV} \end{pmatrix} \ll 1\,\mathrm{GeV} \leq \begin{pmatrix} m_c = (1.15-1.35)\,\mathrm{GeV} \\ m_b = (4.0-4.4)\,\mathrm{GeV} \\ m_t = 174\,\mathrm{GeV} \end{pmatrix}, \tag{A.5}$$

where the scale of 1 GeV is associated with the masses of the lightest hadrons containing light quarks, e.g., m_ρ= 770 MeV, which are not Goldstone bosons resulting from spontaneous symmetry breaking. The scale associated with spontaneous symmetry breaking, $4\pi F_\pi \approx 1170$ MeV, is of the same order of magnitude, where F_π is the pion decay constant.

In the following, we will approximate the full QCD Lagrangian by its light-flavor version. The Lagrangian $\mathcal{L}^0_{\rm QCD}$, containing only the light-flavor quarks in the so-called chiral limit $m_u, m_d, m_s \to 0$, might be a good starting point in the discussion of low-energy QCD:

$$\mathcal{L}^0_{\rm QCD} = \sum_{l=u,d,s} \bar{q}_l \mathrm{i} \not{D}\, q_l - \frac{1}{4}\mathcal{G}_{\mu\nu,a}\mathcal{G}^{\mu\nu}_a. \tag{A.6}$$

We repeat that the covariant derivative $\not{D}\, q_l$ acts on color and Dirac indices only but is independent of flavor.

In order to fully exhibit the global symmetries of Eq.(A.6), we consider the chirality matrix $\gamma_5 = \gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$, $\{\gamma^\mu, \gamma_5\} = 0$, $\gamma_5^2 = 1$, and introduce projection operators

$$P_R = \frac{1}{2}(1+\gamma^5) = P_R^\dagger, \qquad P_L = \frac{1}{2}(1-\gamma^5) = P_L^\dagger, \tag{A.7}$$

where the indices R and L refer to right-handed and left-handed, respectively. Obviously, the 4×4 matrices P_R and P_L satisfy a completeness relation:

$$P_R + P_L = 1, \tag{A.8}$$

are idempotent, i.e.,

$$P_R^2 = P_R, \qquad P_L^2 = P_L, \tag{A.9}$$

and respect the orthogonality relations

$$P_R P_L = P_L P_R = 0. \tag{A.10}$$

The combined properties of Eqs. (A.8)-(A.10) guarantee that P_R and P_L are indeed projection operators which project from the Dirac field variable q its chiral components q_R and q_L,

$$q_R = P_R q, \qquad q_L = P_L q, \tag{A.11}$$

$$\bar{q}_R = \bar{q} P_L, \qquad \bar{q}_L = \bar{q} P_R \tag{A.12}$$

and one can write the quark field into

$$q = (P_R + P_L)q = P_R q + P_L q = q_R + q_L. \tag{A.13}$$

Our goal is to analyze the symmetry of the QCD Lagrangian with respect to independent global transformations of the left- and right-handed fields. In order to decompose the 16 quadratic forms into their respective projections to right- and left-handed fields, one can show that

$$\bar{q}\Gamma_i q = \begin{cases} \bar{q}_R \Gamma_1 q_R + \bar{q}_L \Gamma_1 q_L & \text{for} \quad \Gamma_1 \in \{\gamma^\mu, \gamma^\mu \gamma_5\} \\ \bar{q}_R \Gamma_2 q_L + \bar{q}_L \Gamma_2 q_R & \text{for} \quad \Gamma_2 \in \{1, \gamma_5, \sigma^{\mu\nu}\} \end{cases} . \tag{A.14}$$

We now apply Eq. (A.14) to Eq. (A.6) the QCD Lagrangian in the chiral limit, which can then be written as

$$\mathcal{L}^0_{\text{QCD}} = \sum_{l=u,d,s} (\bar{q}_{R,l} i \not{D}\, q_{R,l} + \bar{q}_{L,l} i \not{D}\, q_{L,l}) - \frac{1}{4} \mathcal{G}_{\mu\nu,a} \mathcal{G}^{\mu\nu}_a . \tag{A.15}$$

Due to the flavor independence of the covariant derivative $\mathcal{L}^0_{\text{QCD}}$ is invariant under

$$\begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} \mapsto U_L \begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix} = \exp\left(-\mathrm{i}\sum_{a=1}^{8}\Theta_a^L \frac{\lambda_a}{2}\right)\begin{pmatrix} u_L \\ d_L \\ s_L \end{pmatrix}, \tag{A.16}$$

$$\begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix} \mapsto U_R \begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix} = \exp\left(-\mathrm{i}\sum_{a=1}^{8}\Theta_a^R \frac{\lambda_a}{2}\right)\begin{pmatrix} u_R \\ d_R \\ s_R \end{pmatrix}, \tag{A.17}$$

where U_L and U_R are independent unitary 3×3 matrices. Note that the Gell-Mann matrices, λ_a, act in flavor space. According to Noether's theorem, which states that an invariance of the Lagrangian density is associated with a conserved quantity, one obtains the currents associated with the transformations of the left-handed or right-handed quarks

$$L^{\mu,a} = \bar{q}_L \gamma^\mu \frac{\lambda^a}{2} q_L, \quad \partial_\mu L^{\mu,a} = 0, \tag{A.18}$$

$$R^{\mu,a} = \bar{q}_R \gamma^\mu \frac{\lambda^a}{2} q_R, \quad \partial_\mu R^{\mu,a} = 0. \tag{A.19}$$

Instead of these chiral currents one often uses linear combinations,

$$V^{\mu,a} = R^{\mu,a} + L^{\mu,a} = \bar{q}\gamma^\mu \frac{\lambda^a}{2} q, \qquad \partial_\mu V^{\mu,a} = 0, \tag{A.20}$$

$$A^{\mu,a} = R^{\mu,a} - L^{\mu,a} = \bar{q}\gamma^\mu \gamma^5 \frac{\lambda^a}{2} q, \quad \partial_\mu A^{\mu,a} = 0, \tag{A.21}$$

which are vector and axial vector current, respectively.

Eqs. (A.18)-(A.21) state that the QCD Lagrangian in the limit of massless u, d, and s quarks (chiral limit) has the global $\mathrm{SU(3)}_L \times \mathrm{SU(3)}_R$ symmetry or $\mathrm{SU(3)}_V \times \mathrm{SU(3)}_A$ which is the best symmetry for strong interaction.

A.2.1 Chiral Symmetry Breaking due to Quark Masses

The preceding discussion concerns the fictitious world where all of the quark masses are set equal to zero. In reality, the Lagrangian of QCD contains a quark mass term. Let us consider the quark mass matrix of the three light quarks,

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}. \tag{A.22}$$

In particular, applying Eq. (A.14) we see that the quark mass term mixes left- and right-handed fields,

$$\mathcal{L}_M = -\bar{q}Mq = -(\bar{q}_R M q_L + \bar{q}_L M q_R). \tag{A.23}$$

From $\mathcal{L}_M$ one obtains as the variation $\delta\mathcal{L}_M$ under the transformations of Eqs. (A.16) and (A.17),

$$\delta\mathcal{L}_M = -\mathrm{i}\left[\sum_{a=1}^{8} \Theta_a^R \left(\bar{q}_R \frac{\lambda_a}{2} M q_L - \bar{q}_L M \frac{\lambda_a}{2} q_R\right) + \sum_{a=1}^{8} \Theta_a^L \left(\bar{q}_L \frac{\lambda_a}{2} M q_R - \bar{q}_R M \frac{\lambda_a}{2} q_L\right)\right], \tag{A.24}$$

which results in the following divergences,

$$\partial_\mu L^{\mu,a} = \frac{\partial \delta\mathcal{L}_M}{\partial \Theta_a^L} = -\mathrm{i}\left(\bar{q}_L \frac{\lambda_a}{2} M q_R - \bar{q}_R M \frac{\lambda_a}{2} q_L\right), \tag{A.25}$$

$$\partial_\mu R^{\mu,a} = \frac{\partial \delta\mathcal{L}_M}{\partial \Theta_a^R} = -\mathrm{i}\left(\bar{q}_R \frac{\lambda_a}{2} M q_L - \bar{q}_L M \frac{\lambda_a}{2} q_R\right). \tag{A.26}$$

Again, using a linear combination as Eq. (A.20) and Eq. (A.21) the corresponding divergences for vector and axial vector read

$$\partial_\mu V^{\mu,a} = \mathrm{i}\bar{q}[M, \frac{\lambda_a}{2}]q, \tag{A.27}$$

$$\partial_\mu A^{\mu,a} = \mathrm{i}\left(\bar{q}_L \{\frac{\lambda_a}{2}, M\} q_R - \bar{q}_R \{\frac{\lambda_a}{2}, M\} q_L\right) = \mathrm{i}\bar{q}\{\frac{\lambda_a}{2}, M\}\gamma^5 q. \tag{A.28}$$

We are now in the position to summarize the various (approximate) symmetries of the strong interactions in combination with the corresponding currents and their divergences.

- In the limit of massless quarks, the sixteen currents $L^{\mu,a}$ and $R^{\mu,a}$ or, alternatively, $V^{\mu,a}$ and $A^{\mu,a}$ are conserved.

- For equal quark masses, $m_u = m_d = m_s$, the eight vector currents $V^{\mu,a}$ are conserved, because $[\lambda_a, 1] = 0$. The eight axial currents $A^{\mu,a}$ are not conserved. The divergences of the octet axial-vector currents of Eq. (A.28) are proportional to pseudoscalar quadratic forms. This can be interpreted as the microscopic origin of the PCAC relation (partially conserved axial-vector current) which states that the divergences of the axial-vector currents are proportional to renormalized field operators representing the lowest lying pseudoscalar octet.

A.3 Linear Sigma-Model

The field theoretical restoration of chiral symmetry in a Lagrangian with a symmetry breaking mass term for the fermions is realized by the σ-model, which had been introduced long before the emergence of QCD. By introducing phenomenological scalar-isoscalar and pseudoscalar-isovector fields σ and $\pi^i (i = 1, 2, 3)$, respectively, the simplest linear sigma-model based on chiral symmetry is defined as

$$\mathcal{L}_{LS} = \bar{\psi} \mathrm{i} \gamma_\mu \partial^\mu \psi - g \bar{\psi} \left(\sigma + \mathrm{i} \vec{\tau} \cdot \vec{\pi} \gamma^5 \right) \psi + \frac{1}{2} \left[(\partial_\mu \sigma)^2 + (\partial_\mu \vec{\pi})^2 \right] - \mathcal{U}(\sigma, \vec{\pi}) \quad \text{(A.29)}$$

where

$$\mathcal{U}(\sigma, \vec{\pi}) = \frac{\lambda^2}{4} \left(\sigma^2 + \vec{\pi}^2 - f_0^2 \right)^2 . \quad \text{(A.30)}$$

Here, the field operator ψ represents the isospin doublet of u and d quarks. The chiral-symmetric Lagrangian density leads to a conserved axial current

$$A_\mu^i = \bar{\psi} \gamma_\mu \gamma^5 \frac{\tau^i}{2} \psi + \sigma \partial_\mu \pi^i - \pi^i \partial_\mu \sigma, \quad \text{(A.31)}$$

with $\partial^\mu A_\mu^i = 0$.

In order for the system to be stable, g and λ are positive constants. When treating the boson fields classically, they are replaced everywhere by their vacuum expectation values and $\mathcal{U}(\sigma, \vec{\pi})$ plays the role of an effective potential. Since a nonzero classical expectation value for $\vec{\pi}$ would violate parity, we set $\langle\vec{\pi}\rangle = 0$.

Now two possibilities emerge: When $f_0^2 < 0$, $\mathcal{U}(\sigma, \vec{\pi} = 0)$ has an absolute minimum at $\sigma_0 = 0$, in which case the quarks stay massless and the σ and $\vec{\pi}$ fields become degenerate in mass. This manifestation of chiral symmetry in which the physical states in the spectrum of $\mathcal{L}_\sigma$ are classified according to the irreducible representations of the chiral symmetry group is referred to as the Wigner-Weyl mode.

In the case $f_0^2 > 0$, $\mathcal{U}(\sigma, \vec{\pi} = 0)$ has two minima at $\sigma_0 = \pm f_0$. When expanding the boson fields about either one of the equivalent minima, i.e. $\sigma \to \sigma + f_0$ and $\vec{\pi} \to \vec{\pi}$, the symmetry of the original Lagrangian density $\mathcal{L}_\sigma$ is hidden. With the classical vacuum expectation value of the σ-filed being nonzero, the fermions acquire a finite mass term $g\sigma_0$. Hence, chiral symmetry is dynamically broken. The Goldstone theorem then requires the existence of a massless pseudoscalar-isovector excitation, which in fact makes the $\vec{\pi}$ field massless. Thus, chiral symmetry, as manifested in the so called Nambu-Goldstone mode, is realized by two massive fermions (quarks) and an isovector triplet of massless Goldstone bosons.

When making contact between the σ model and a quark model with a built-in confinement phenomenology, the quark mass term $g\sigma$ can be identified as a local mass, which is small for short distances from the baryon center and grows towards infinity for large distances between the quarks. Thus, chiral symmetry exhibits itself in the asymptotic freedom region in the Wigner-Weyl mode with massless quarks and no bosons present, and in the confining region in the Nambu-Goldstone mode with the occurrence of massless Goldstone bosons.

While restricting ourselves to flavor SU(2), the massless pseudoscalar-isovector Goldstone bosons, which are generated by spontaneous symmetry breaking, are identified with the only possible candidate, the pion. In principle, it can be shown on very general grounds that in a chiral symmetric SU(2) $\times$ SU(2) world the pion must appear as a Goldstone boson (with $m_\pi = 0$). Since f_0 specifies the mesonic piece of the axial current Eq.(A.31), it is identified as the pion decay constant with $f_0 = F_\pi = 93$ MeV.

A.4 Nonlinear Sigma-Model

One of the disturbing features of the linear sigma-model as in Eq. (A.29) is the existence of the σ- field, because it cannot really be identified with any existing particle. Then we remove the σ-meson by sending its mass to infinity. Formally this can be achieved by assuming an infinitely large coupling λ in the linear-sigma model. As a consequence the potential gets infinitely steep in the sigma-direction. This confines the dynamics to the circle, defined by the minimum of the potential,

$$\sigma^2 + \pi^2 = F_\pi^2. \tag{A.32}$$

Therefore, the fields can be expressed in terms of angles $\vec{\varphi}$,

$$\begin{aligned} \sigma(x) &= F_\pi \cos\left(\frac{\varphi(x)}{F_\pi}\right) = F_\pi + \mathcal{O}(\varphi^2), && \text{(A.33)} \\ \vec{\pi}(x) &= F_\pi \hat{\varphi} \sin\left(\frac{\varphi(x)}{F_\pi}\right) = \vec{\varphi}(x) + \mathcal{O}(\varphi^3). && \text{(A.34)} \end{aligned}$$

Equivalently, one can choose a complex notation for the fields,

$$U(x) = \exp\left\{ \mathrm{i}\frac{\vec{\tau}\cdot\vec{\varphi}(x)}{F_\pi} \right\} = \cos\left(\frac{\varphi(x)}{F_\pi}\right) + \mathrm{i}\vec{\tau}\cdot\hat{\varphi}\sin\left(\frac{\varphi(x)}{F_\pi}\right) = \frac{1}{F_\pi}\left(\sigma + \mathrm{i}\vec{\tau}\cdot\vec{\pi}\right). \tag{A.35}$$

Let us continue by rewriting the Lagrangian of the linear sigma-model, Eq. (A.29), in terms of the new variables U or φ. After a little algebra we find that the kinetic energy term of the mesons is given by

$$\frac{1}{2}\left[(\partial_\mu\sigma)^2+(\partial_\mu\vec{\pi})^2\right]=\frac{F_\pi^2}{4}\mathrm{Tr}[\partial_\mu U\partial^\mu U^\dagger]. \tag{A.36}$$

Next, we realize that quark-meson coupling term can be written as

$$\begin{aligned}-g\bar{\psi}(\sigma+\mathrm{i}\vec{\tau}\cdot\vec{\pi}\gamma^5)\psi &= -g\bar{\psi}\left\{F_\pi\left[\cos\left(\frac{\varphi}{F_\pi}\right)+\mathrm{i}\gamma^5\vec{\tau}\cdot\hat{\varphi}\sin\left(\frac{\varphi}{F_\pi}\right)\right]\right\}\psi \\ &= -g\bar{\psi}\left\{F_\pi\exp\left(\mathrm{i}\gamma^5\frac{\vec{\tau}\cdot\vec{\varphi}}{F_\pi}\right)\right\}\psi \\ &= -gF_\pi\bar{\psi}\Lambda\Lambda\psi, \end{aligned} \tag{A.37}$$

where we have defined

$$\Lambda\equiv\exp\left(\mathrm{i}\gamma^5\frac{\vec{\tau}\cdot\vec{\varphi}}{2F_\pi}\right). \tag{A.38}$$

If we now redefine the quark fields

$$\psi_W=\Lambda\psi,\qquad \bar{\psi}_W=\bar{\psi}\Lambda, \tag{A.39}$$

the interaction term Eq. (A.37) can be simply written as

$$-gF_\pi\bar{\psi}\Lambda\Lambda\psi=-\mathcal{M}\bar{\psi}_W\psi_W. \tag{A.40}$$

We also have to rewrite the quark kinetic energy term in terms of those fields.

$$\bar{\psi}\mathrm{i}\gamma_\mu\partial^\mu\psi=\bar{\psi}_W\Lambda^\dagger\mathrm{i}\gamma_\mu\partial^\mu\Lambda^\dagger\psi_W. \tag{A.41}$$

After some straightforward algebra, one finds

$$\bar{\psi}_W\Lambda^\dagger\mathrm{i}\gamma_\mu\partial^\mu\Lambda^\dagger\psi_W=\bar{\psi}_W(\mathrm{i}\gamma_\mu\partial^\mu+\gamma^\mu\mathcal{V}_\mu+\gamma^\mu\gamma^5\mathcal{A}_\mu)\psi_W \tag{A.42}$$

with

$$\mathcal{V}_\mu \equiv \frac{\mathrm{i}}{2}\left(\xi\partial_\mu\xi^\dagger+\xi^\dagger\partial_\mu\xi\right), \tag{A.43}$$

$$\mathcal{A}_\mu \equiv \frac{\mathrm{i}}{2}\left(\xi\partial_\mu\xi^\dagger-\xi^\dagger\partial_\mu\xi\right), \tag{A.44}$$

$$\xi \equiv \exp\left(\mathrm{i}\frac{\vec{\tau}\cdot\vec{\varphi}}{2F_\pi}\right). \tag{A.45}$$

Putting everything together, the Lagrangian of the nonlinear sigma-model, which is often referred to as the Weinberg-Lagrangian, reads in the above variables,

$$\mathcal{L}_W = \bar{\psi}(\mathrm{i}\gamma_\mu\partial^\mu + \gamma^\mu\mathcal{V}_\mu + \gamma^\mu\gamma^5\mathcal{A}_\mu - \mathcal{M})\psi + \frac{F_\pi^2}{4}\mathrm{Tr}(\partial_\mu U\partial^\mu U^\dagger). \tag{A.46}$$

Here we have dropped the subscript from the quark fields. Clearly, this Lagrangian depends nonlinearly on the field $\vec{\varphi}$.

A.5 Chiral Effective Lagrangian

In this section, we briefly review how to construct the effective chiral Lagrangian of the strong interactions following closely the work of Gasser and Leutwyler (Gasser and Leutwyler, 1985). It is most economical to use the external field technique since it avoids any complication related to the nonlinear transformation properties of the mesons. The basic objects to consider are currents and densities with external fields coupled to them in accordance with the symmetry requirements.

A.5.1 QCD in the Present of External Sources

Following the procedure of Gasser and Leutwyler (Gasser and Leutwyler, 1984, 1985), we introduce into the Lagrangian of QCD the couplings of the nine vector currents and the eight axial-vector currents as well as the scalar and pseudoscalar quark densities to external fields $v^\mu(x)$, $a^\mu(x)$, $s(x)$, and $p(x)$,

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}^0_{\mathrm{QCD}} + \mathcal{L}_{\mathrm{ext}} = \mathcal{L}^0_{\mathrm{QCD}} + \bar{q}\gamma_\mu(v^\mu + \gamma^5 a^\mu)q - \bar{q}(s - \mathrm{i}\gamma^5 p)q. \tag{A.47}$$

The external fields are color-neutral, Hermitian 3×3 matrices, where the matrix character, with respect to the (suppressed) flavor indices u, d, and s of the quark fields, is

$$v^\mu = \sum_{a=1}^{8} \frac{\lambda_a}{2} v_a^\mu, \quad a^\mu = \sum_{a=1}^{8} \frac{\lambda_a}{2} a_a^\mu, \quad s = \sum_{a=0}^{8} \lambda_a s_a, \quad p = \sum_{a=0}^{8} \lambda_a p_a. \tag{A.48}$$

The ordinary three flavor QCD Lagrangian is recovered by setting $v^\mu = a^\mu = p = 0$ and $s = \mathrm{diag}(m_u, m_d, m_s)$ in Eq. (A.47).

If one defines the generating functional

$$\exp(\mathrm{i}Z[v,a,s,p]) = \langle 0|T \exp\left[\mathrm{i}\int d^4x \mathcal{L}_{\mathrm{ext}}(x)\right]|0\rangle, \tag{A.49}$$

then any Green function consisting of the time-ordered product of color-neutral, Hermitian quadratic forms can be obtained from Eq. (A.47) through a functional derivative with respect to the external fields. The generating functional is related to the vacuum-to-vacuum transition amplitude in the presence of external fields (Gasser and Leutwyler, 1984, 1985),

$$\exp(\mathrm{i}Z[v,a,s,p]) = \langle 0_{\mathrm{out}}|0_{\mathrm{in}}\rangle_{v,a,s,p}, \tag{A.50}$$

where the dynamics is determined by the Lagrangian of Eq. (A.47).

Next, we need to discuss the requirements to be met by the external fields under local $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ transformations. In a first step, we write Eq. (A.47) in terms of the left- and right-handed quark fields. Besides the properties of Eqs. (A.8) - (A.10) we make use of the auxiliary formulae

$$\gamma^5 P_R = P_R\gamma^5 = P_R, \quad \gamma^5 P_L = P_L \gamma^5 = -P_L, \tag{A.51}$$

and

$$\gamma^\mu P_R = P_L \gamma^\mu, \quad \gamma^\mu P_L = P_R \gamma^\mu, \tag{A.52}$$

yielding for the Lagrangian of Eq. (A.47)

$$\mathcal{L}_{\mathrm{QCD}} = \mathcal{L}^0_{\mathrm{QCD}} + \bar{q}_L \gamma^\mu l_\mu q_L + \bar{q}_R \gamma^\mu r_\mu q_R - \bar{q}_R(s+\mathrm{i}p)q_L - \bar{q}_L(s-\mathrm{i}p)q_R, \tag{A.53}$$

where

$$v_\mu = \frac{1}{2}(r_\mu + l_\mu), \quad a_\mu = \frac{1}{2}(r_\mu - l_\mu). \tag{A.54}$$

The Lagrangian $\mathcal{L}_{\mathrm{QCD}}$ in Eq. (A.53) remains invariant under local $\mathrm{SU}(3)_{\mathrm{L}} \times \mathrm{SU}(3)_{\mathrm{R}}$ transformations if the quark and external fields transform as follows:

$$\begin{aligned}
q_R &\rightarrow Rq_R, \qquad q_L \rightarrow Lq_L, &(A.55)\\
r_\mu &\rightarrow Rr_\mu R^\dagger + \mathrm{i}R\partial_\mu R^\dagger, &(A.56)\\
l_\mu &\rightarrow Ll_\mu L^\dagger + \mathrm{i}L\partial_\mu L^\dagger, &(A.57)\\
s + ip &\rightarrow R(s + ip)L^\dagger, &(A.58)\\
s - ip &\rightarrow L(s - ip)R^\dagger, &(A.59)
\end{aligned}$$

with L, R are elements of $\mathrm{SU}(3)_{L,R}$.

The effective meson Lagrangian follows from the low energy representation of the generating functional

$$\exp(\mathrm{i}Z[v, a, s, p]) = \int [DU] \mathrm{e}^{\int \mathrm{i}d^4x \mathcal{L}_{\mathrm{eff}}(U; v, a, s, p)}, \tag{A.60}$$

where the matrix U collects the meson fields. The low energy expansion is now obtained from a perturbative expansion of the meson effective theory,

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_2 + \mathcal{L}_4 + \cdots, \tag{A.61}$$

where the subscript $(n = 2, 4, \ldots)$ denotes the low energy dimension (number of derivatives and/or quark mass terms). The lowest order in this expansion will be discussed.

A.5.2 The Lowest-Order Effective Lagrangian

Let us construct the leading term (called $\mathcal{L}_2$) in the low energy expansion Eq. (A.61). The mesons are described by a unitary 3×3 matrix in flavor space,

$$U^\dagger U = 1, \qquad \mathrm{det} U = 1. \tag{A.62}$$

The matrix U transform linearly under chiral symmetry,

$$U \to R U L^\dagger. \tag{A.63}$$

To write down the Lagrangian, it is convenient to work with the quantity u rather than with U itself (Schweizer, 2000). The matrix $u(x)$ is related to the meson field introduced above by

$$u^2 = U. \tag{A.64}$$

The effective Lagrangian consists of two pieces,

$$\mathcal{L}_{\mathrm{eff}} = \mathcal{L}_\phi + \mathcal{L}_q \tag{A.65}$$

and explicitly involves the following quantities

$$\begin{aligned}
D_\mu &= \partial_\mu + \Gamma_\mu, && \text{(A.66)}\\
\Gamma_\mu &= \frac{1}{2}\left[u^\dagger, \partial_\mu u\right] - \frac{\mathrm{i}}{2} u^\dagger r_\mu u - \frac{\mathrm{i}}{2} u l_\mu u^\dagger, && \text{(A.67)}\\
u_\mu &= \mathrm{i}\left\{u^\dagger, \partial_\mu u\right\} + u^\dagger r_\mu u - u l_\mu u^\dagger, && \text{(A.68)}\\
r_\mu &= v_\mu + a_\mu, \quad l_\mu = v_\mu - a_\mu, && \text{(A.69)}\\
\chi_+ &= u^\dagger \chi u^\dagger + u \chi^\dagger u, \quad \chi = 2B(s + \mathrm{i}p). && \text{(A.70)}
\end{aligned}$$

Considering the mesonic sector of the Lagrangian, only the meson field $U(x)$ and an even number of derivatives thereof are involved

$$\mathcal{L}_\phi = \mathcal{L}_\phi^{(2)} + \mathcal{L}_\phi^{(4)} + \cdots . \tag{A.71}$$

The superscript i attached to $\mathcal{L}_\phi^{(i)}$ denotes the low energy dimension of the Lagrangian. At leading order, the effective meson Lagrangian takes the following form

$$\mathcal{L}_\phi^{(2)} = \frac{F^2}{4}\mathrm{Tr}[u_\mu u^\mu + \chi_+]. \tag{A.72}$$

The physical significance of the low energy constants F and B is given below. The quark Lagrangian contains odd as well as even terms,

$$\mathcal{L}_q = \mathcal{L}_q^{(1)} + \mathcal{L}_q^{(2)} + \cdots . \tag{A.73}$$

Again, at leading order, the effective quark Lagrangian yields

$$\mathcal{L}_q^{(1)} = \bar{\psi}(\mathrm{i}\not{D} - \mathcal{M})\psi + \frac{1}{2}\bar{\psi}\not{u}\gamma^5\psi, \tag{A.74}$$

containing the real parameter $\mathcal{M}$. The quantity F is the pion decay constant in the chiral limit, the constant B occurs in the quark mass expansion of the physical pion mass and $\mathcal{M}$ denotes the quark mass matrix.

Therefore, the well known leading order of the effective Lagrangian reads

$$\mathcal{L}_2 = \bar{\psi}(\mathrm{i}\not{D} - \mathcal{M})\psi + \frac{1}{2}\bar{\psi}\not{u}\gamma^5\psi + \frac{F^2}{4}\mathrm{Tr}[u_\mu u^\mu + \chi_+]. \tag{A.75}$$

On the other hand in our model, PCQM, we introduce the phenomenological effective potential, V_{eff}, as already mentioned in Chapter II, to confine quarks into the nucleon. Then the leading order of the effective Lagrangian, Eq. (A.75) yields

$$\mathcal{L}_2 = \bar{\psi}(\mathrm{i}\not{D} - \gamma^0 V(r) - S(r) - \mathcal{M})\psi + \frac{1}{2}\bar{\psi}\not{u}\gamma^5\psi + \frac{F^2}{4}\mathrm{Tr}[u_\mu u^\mu + \chi_+]. \tag{A.76}$$

To calculate the higher orders of the effective Lagrangian, i.e. $\mathcal{L}_4$, a very useful method the so called Weinberg's power counting scheme is entering to the chiral effective Lagrangian or chiral perturbation theory to determine the diagram and the order of meson field to be expanded in the order we are interested in (see detail in Scherer, 2003).

Appendix B

Weinberg-Type Form of Effective Lagrangian

With the effective chirally invariant Lagrangian $\mathcal{L}_{\rm inv}$ of Eq. (2.1) given as

$$\begin{aligned}\mathcal{L}_{\rm inv}(x) &= \bar{\psi}(x)\left\{\mathrm{i}\,\partial\!\!\!/ - \gamma^0 V(r) - S(r)\left[\frac{U+U^\dagger}{2} + \gamma^5\frac{U-U^\dagger}{2}\right]\right\}\psi(x) \\ &\quad + \frac{F^2}{4}\mathrm{Tr}\left[\partial_\mu U\,\partial^\mu U^\dagger\right], \end{aligned} \tag{B.1}$$

where $U = \mathrm{e}^{\mathrm{i}\hat{\Phi}/F}$, and using the gamma matrix property, $\gamma^5\gamma^5 = 1$, one can rewrite the above equation into

$$\begin{aligned}\mathcal{L}_{\rm inv}(x) &= \bar{\psi}(x)\left\{\mathrm{i}\,\partial\!\!\!/ - \gamma^0 V(r) - S(r)\exp\left\{\mathrm{i}\gamma^5\hat{\Phi}/F\right\}\right\}\psi(x) \\ &\quad + \frac{F^2}{4}\mathrm{Tr}\left[\partial_\mu U\,\partial^\mu U^\dagger\right]. \end{aligned} \tag{B.2}$$

With the unitary chiral rotation $\psi \to \exp\{-\mathrm{i}\gamma^5\hat{\Phi}/(2F)\}\psi$ and again using the gamma matrices properties, $\{\gamma^\mu, \gamma^5\} = 0$ and $\{\gamma^0, \gamma^5\} = 0$, the first term on the right hand side of Eq. (B.2) results in

$$\begin{aligned}\mathcal{L}^{\psi}_{\rm inv} &= \bar{\psi}(x)\left\{\mathrm{i}\exp\left\{-\mathrm{i}\gamma^5\hat{\Phi}/(2F)\right\}\partial\!\!\!/\exp\left\{-\mathrm{i}\gamma^5\hat{\Phi}/(2F)\right\} - \gamma^0 V(r) - S(r)\right\}\psi(x) \\ &= \bar{\psi}(x)\left\{\mathrm{i}\,\partial\!\!\!/ - \gamma^0 V(r) - S(r)\right\}\psi(x) + \bar{\psi}(x)\mathrm{i}\Lambda\left\{\partial\!\!\!/\Lambda\right\}\psi(x), \end{aligned} \tag{B.3}$$

where $\Lambda \equiv \exp\{-\mathrm{i}\gamma^5\hat{\Phi}/(2F)\}$.

Now we define $\xi \equiv \mathrm{e}^{\mathrm{i}\hat{\phi}/(2F)}$, $\xi^\dagger = \mathrm{e}^{-\mathrm{i}\hat{\phi}/(2F)}$ with the properties from Eqs. (A.7)-(A.13) and from the definition of Λ, ξ, ψ_R and ψ_L one can show that

$$\Lambda\psi_R = \xi^\dagger\psi_R, \qquad \Lambda\psi_L = \xi\psi_L, \tag{B.4}$$

$$\bar{\psi}_R\Lambda = \bar{\psi}_R\xi, \qquad \bar{\psi}_L\Lambda = \bar{\psi}_L\xi^\dagger \tag{B.5}$$

and with these properties

$$\gamma^5 P_{R/L}\psi = \pm P_{R/L}\psi \tag{B.6}$$

the interaction term between quark and meson field in Eq. (B.3) yields

$$\begin{aligned}
\bar{\psi}\mathrm{i}\Lambda\left(\partial\!\!\!/\Lambda\right)\psi &= \bar{\psi}\mathrm{i}\Lambda\left(\partial\!\!\!/\Lambda\right)\left(\psi_R+\psi_L\right)\\
&= \bar{\psi}_R\mathrm{i}\Lambda\left(\partial\!\!\!/\Lambda\right)\psi_R+\bar{\psi}_L\mathrm{i}\Lambda\left(\partial\!\!\!/\Lambda\right)\psi_L\\
&= \bar{\psi}_R\xi\left(\mathrm{i}\;\partial\!\!\!/\xi^\dagger\right)\psi_R+\bar{\psi}_L\xi^\dagger\left(\mathrm{i}\;\partial\!\!\!/\xi\right)\psi_L\\
&= \bar{\psi}\xi\left(\mathrm{i}\;\partial\!\!\!/\xi^\dagger\right)\frac{1}{2}(1+\gamma^5)\psi+\bar{\psi}\xi^\dagger\left(\mathrm{i}\;\partial\!\!\!/\xi\right)\frac{1}{2}(1-\gamma^5)\psi\\
&= \bar{\psi}\left\{\frac{\mathrm{i}}{2}\left(\xi\partial_\mu\xi^\dagger+\xi^\dagger\partial_\mu\xi\right)\gamma^\mu+\frac{\mathrm{i}}{2}\left(\xi\partial_\mu\xi^\dagger-\xi^\dagger\partial_\mu\xi\right)\gamma^\mu\gamma^5\right\}\psi\\
&\equiv \bar{\psi}\left\{\mathcal{V}_\mu\gamma^\mu+\mathcal{A}_\mu\gamma^\mu\gamma^5\right\}\psi. \qquad \text{(B.7)}
\end{aligned}$$

Eq. (B.7) consist of two parts, vector and axial vector :

$$\mathcal{V}_\mu \equiv \frac{\mathrm{i}}{2}\left(\xi\partial_\mu\xi^\dagger+\xi^\dagger\partial_\mu\xi\right), \tag{B.8}$$

$$\mathcal{A}_\mu \equiv \frac{\mathrm{i}}{2}\left(\xi\partial_\mu\xi^\dagger-\xi^\dagger\partial_\mu\xi\right). \tag{B.9}$$

Finally, the effective chiral invariant Lagrangian in Eq. (B.2), after the unitary chiral rotation of quark field, reads

$$\begin{aligned}
\mathcal{L}_{\mathrm{inv}}(x) &= \bar{\psi}(x)\left\{\mathrm{i}\;\partial\!\!\!/-\gamma^0V(r)-S(r)\right\}\psi(x)+\bar{\psi}\left\{\mathcal{V}_\mu\gamma^\mu+\mathcal{A}_\mu\gamma^\mu\gamma^5\right\}\psi\\
&\quad+\frac{F^2}{4}\mathrm{Tr}\left[\partial_\mu U\,\partial^\mu U^\dagger\right]. \qquad \text{(B.10)}
\end{aligned}$$

When considering mesons as small fluctuations we restrict the interaction Lagrangian up to the quadratic term in the meson field and we also restrict the pseudoscalar meson to the SU(2) flavor case, that is $\hat{\Phi}\to\hat{\pi}=\vec{\pi}\cdot\vec{\tau}$ the Lagrangian in Eq. (B.10) transforms into a Weinberg-type form $\mathcal{L}^W$ containing the axial-vector coupling and the Weinberg-Tomozawa term, that is

$$\mathcal{L}^W(x) = \mathcal{L}_0(x)+\mathcal{L}_I^W(x)+o(\vec{\pi}^2), \tag{B.11}$$

with

$$
\begin{aligned}
\mathcal{L}_0(x) &= \bar{\psi}(x)\Big\{\mathrm{i}\,\partial\!\!\!/ - S(r) - \gamma^0 V(r)\Big\}\psi(x) + \frac{1}{2}\partial_\mu\vec{\pi}(x)\partial^\mu\vec{\pi}(x), \\
\mathcal{L}_I^W(x) &= \frac{1}{2F}\partial_\mu\vec{\pi}(x)\bar{\psi}(x)\gamma^\mu\gamma^5\vec{\tau}\psi(x) - \frac{\varepsilon_{ijk}}{4F^2}\pi_i(x)\partial_\mu\pi_j(x)\bar{\psi}(x)\gamma^\mu\tau_k\psi(x),
\end{aligned}
$$

where $\mathcal{L}_I^W(x)$ is the $O(\vec{\pi}^2)$ strong interaction Lagrangian.

Appendix C

Gell-Mann and Low Theorem

We consider a system described by the Hamiltonian H which might be written as

$$H = H_0 + H_I \tag{C.1}$$

where H_0 and H_I are respectively the free and interaction parts of the Hamiltonian. Let $|\psi_0\rangle$ and $|n\rangle$ be the eigenstates of the free and full Hamiltonian, respectively, one has

$$\begin{aligned} H|n\rangle &= E^{(n)}|n\rangle, \\ H_0|\psi_0\rangle &= E_0|\psi_0\rangle, \end{aligned} \tag{C.2}$$

hence

$$\begin{aligned} \mathrm{e}^{-\mathrm{i}Ht}|\psi_0\rangle &= \sum_n \mathrm{e}^{-\mathrm{i}E^{(n)}t}|n\rangle\langle n|\psi_0\rangle \\ &= \mathrm{e}^{-\mathrm{i}Et}|\psi\rangle\langle\psi|\psi_0\rangle + \sum_{n\neq 0} \mathrm{e}^{-\mathrm{i}E^{(n)}t}|n\rangle\langle n|\psi_0\rangle. \end{aligned} \tag{C.3}$$

Note that we have rewritten $|0\rangle$ and $E^{(0)}$ in the above equation respectively as $|\psi\rangle$ and E, that is

$$H|\psi\rangle = E|\psi\rangle. \tag{C.4}$$

Multiplying the above equation by $\mathrm{e}^{\mathrm{i}E_0 t}$, one derives

$$\mathrm{e}^{\mathrm{i}E_0 t}\mathrm{e}^{-\mathrm{i}Ht}|\psi_0\rangle = \mathrm{e}^{-\mathrm{i}(E-E_0)t}|\psi\rangle\langle\psi|\psi_0\rangle + \sum_{n\neq 0} \mathrm{e}^{-\mathrm{i}(E^{(n)}-E_0)t}|n\rangle\langle n|\psi_0\rangle. \tag{C.5}$$

Since $E^{(n)} > E$ for all $n \neq 0$, we can get rid of all the $n \neq 0$ terms in the series by sending t to ∞ in a slightly imaginary direction $t \to \infty(1 - \mathrm{i}\varepsilon)$. Then the exponential factor $\mathrm{e}^{-\mathrm{i}(E-E_0)t}$ dies slowest and we have

$$\begin{aligned}
|\psi\rangle &= \lim_{t\to\infty(1-\mathrm{i}\varepsilon)} \frac{\mathrm{e}^{\mathrm{i}E_0 t}\mathrm{e}^{-\mathrm{i}Ht}|\psi_0\rangle}{\mathrm{e}^{-\mathrm{i}(E-E_0)t}\langle\psi|\psi_0\rangle} \\
&= \lim_{t\to\infty(1-\mathrm{i}\varepsilon)} \frac{\mathrm{e}^{\mathrm{i}H(-t)}\mathrm{e}^{-\mathrm{i}H_0(-t)}|\psi_0\rangle}{\mathrm{e}^{-\mathrm{i}(E-E_0)t}\langle\psi|\psi_0\rangle} \\
&= \lim_{t\to\infty(1-\mathrm{i}\varepsilon)} \frac{U(0,-t)|\psi_0\rangle}{\mathrm{e}^{-\mathrm{i}(E-E_0)t}\langle\psi|\psi_0\rangle}. \qquad \text{(C.6)}
\end{aligned}$$

Here we have used

$$U(t_0, t) = \mathrm{e}^{\mathrm{i}H(t-t_0)}\mathrm{e}^{-\mathrm{i}H_0(t-t_0)}. \qquad \text{(C.7)}$$

In the same way, we can derive

$$\langle\psi| = \lim_{t\to\infty(1-\mathrm{i}\varepsilon)} \frac{\langle\psi_0|U(t,0)}{\mathrm{e}^{-\mathrm{i}(E-E_0)t}\langle\psi_0|\psi\rangle}. \qquad \text{(C.8)}$$

Now we evaluate the expectation value of the operation $O(x) \equiv O(x^0, \vec{x})$ in the state $|\psi\rangle$

$$\begin{aligned}
\langle\psi|O(x^0,\vec{x})|\psi\rangle &= \lim_{t\to\infty(1-\mathrm{i}\varepsilon)} \frac{\langle\psi_0|U(t,0)U^\dagger(x^0,0)O_I(x)U(x^0,0)U(0,-t)|\psi_0\rangle}{\mathrm{e}^{-\mathrm{i}(E-E_0)t}\langle\psi_0|\psi\rangle\mathrm{e}^{-\mathrm{i}(E-E_0)t}\langle\psi|\psi_0\rangle} \\
&= \lim_{t\to\infty(1-\mathrm{i}\varepsilon)} \frac{\langle\psi_0|U(t,x^0)O_I(x)U(x^0,t)|\psi_0\rangle}{\mathrm{e}^{-\mathrm{i}(E-E_0)(2t)}|\langle\psi_0|\psi\rangle|^2}. \qquad \text{(C.9)}
\end{aligned}$$

To get rid of the denominator in the equation, one may divide it by 1 in the form

$$1 = \langle\psi|\psi\rangle = \lim_{t\to\infty(1-\mathrm{i}\varepsilon)} \frac{\langle\psi_0|U(t,0)U(0,-t)|\psi_0\rangle}{\mathrm{e}^{-\mathrm{i}(E-E_0)(2t)}|\langle\psi_0|\psi\rangle|^2}. \qquad \text{(C.10)}$$

Then finally we derive

$$\langle\psi|O(x^0,\vec{x})|\psi\rangle = \lim_{t\to\infty(1-\mathrm{i}\varepsilon)} \frac{\langle\psi_0|U(t,x^0)O_I(x)U(x^0,t)|\psi_0\rangle}{\langle\psi_0|U(t,-t)|\psi_0\rangle}. \qquad \text{(C.11)}$$

The above equation holds for a product of arbitrarily many operators, for example, for two operators

$$\langle\psi|TO(x)P(y)|\psi\rangle = \lim_{t\to\infty(1-\mathrm{i}\varepsilon)} \frac{\langle\psi_0|T\left\{O_I(x)P_I(y)\exp[-\mathrm{i}\int_{-t}^{t} dzH_I(z)]\right\}|\psi_0\rangle}{\langle\psi_0|T\left\{\exp[-\mathrm{i}\int_{-t}^{t} dzH_I(z)]\right\}|\psi_0\rangle}. \qquad \text{(C.12)}$$

Appendix D

Solutions of the Dirac Equation for the Effective Potential

In this section we indicate the solutions to the Dirac equation with the effective potential $V_{\rm eff}(r) = S(r) + \gamma^0 V(r)$, with $r = |\vec{x}|$. The scalar $S(r)$ and the time-like vector $V(r)$ parts are given by

$$S(r) = M_1 + C_1 r^2, \tag{D.1}$$

$$V(r) = M_2 + C_2 r^2, \tag{D.2}$$

with the particular choice

$$M_1 = \frac{1 - 3\rho^2}{2\rho R}, \tag{D.3}$$

$$M_2 = \mathcal{E}_0 - \frac{1 + 3\rho^2}{2\rho R}, \tag{D.4}$$

$$C_1 = C_2 = \frac{\rho}{2R^3}. \tag{D.5}$$

The quark wave function $u_\alpha(\vec{x})$ in state α and eigenenergy $\mathcal{E}_\alpha$ with the specific choice of $V_{\rm eff}$ satisfies the Dirac equation

$$[-\mathrm{i}\gamma^0\vec{\gamma}\cdot\vec{\nabla} + \gamma^0 S(r) + V(r) - \mathcal{E}_\alpha]u_\alpha(\vec{x}) = 0. \tag{D.6}$$

The solution of the Dirac spinor $u_\alpha(\vec{x})$ to Eq. (D.6) can be written in the analytical form (Tegen and Brockmann, 1982; Pumsa-ard, Lyubovitskij, Gutsche, Faessler and Cheedket, 2003):

$$u_\alpha(\vec{x}) = N_\alpha \begin{pmatrix} g_\alpha(r) \\ \mathrm{i}\vec{\sigma}\cdot\hat{x} f_\alpha(r) \end{pmatrix} \mathcal{Y}_\alpha(\hat{x})\chi_f\,\chi_c\,. \tag{D.7}$$

Where χ_f and χ_c are flavor and color part of the Dirac spinor, respectively. For the particular choice of the potential, Eq. (D.1) and Eq. (D.3), the radial functions $g_\alpha(r)$ and $f_\alpha(r)$ satisfy the form

$$g_\alpha(r) = \left(\frac{r}{R_\alpha}\right)^l L_{n-1}^{l+1/2}\left(\frac{r^2}{R_\alpha^2}\right) \mathrm{e}^{-\frac{r^2}{2R_\alpha^2}}, \tag{D.8}$$

where for the $j = l + \frac{1}{2}$

$$f_\alpha(r) = \rho_\alpha \left(\frac{r}{R_\alpha}\right)^{l+1} \left[L_{n-1}^{l+3/2}\left(\frac{r^2}{R_\alpha^2}\right) + L_{n-2}^{l+3/2}\left(\frac{r^2}{R_\alpha^2}\right)\right] \mathrm{e}^{-\frac{r^2}{2R_\alpha^2}}, \tag{D.9}$$

and for $j = l - \frac{1}{2}$

$$f_\alpha(r) = -\rho_\alpha \left(\frac{r}{R_\alpha}\right)^{l-1} \left[\left(n + l - \frac{1}{2}\right) L_{n-1}^{l-1/2}\left(\frac{r^2}{R_\alpha^2}\right) + n\, L_n^{l-1/2}\left(\frac{r^2}{R_\alpha^2}\right)\right] \mathrm{e}^{-\frac{r^2}{2R_\alpha^2}}. \tag{D.10}$$

The label $\alpha = (nljm_j)$ characterizes the state with principle quantum number $n = 1, 2, 3, ...$, orbital angular momentum l, total angular momentum $j = l \pm \frac{1}{2}$ and projection m_j. Due to the quadratic nature of the potential the radial wave functions contain the associated Laguerre polynomials $L_n^k(x)$ with

$$L_n^k(x) = \sum_{m=0}^{n} (-1)^m \frac{(n+k)!}{(n-m)!(k+m)!m!} x^m. \tag{D.11}$$

The angular dependence, $\mathcal{Y}_\alpha(\hat{x}) \equiv \mathcal{Y}_{ljm_j}(\hat{x})$, is defined by

$$\mathcal{Y}_{ljm_j}(\hat{x}) = \sum_{m_l, m_s} \left\langle l\, m_l \frac{1}{2}\, m_s \middle| j\, m_j \right\rangle Y_{lm_l}(\hat{x}) \chi_{\frac{1}{2}m_s}, \tag{D.12}$$

where $Y_{lm_l}(\hat{x})$ is the usual spherical harmonic.

The two coefficients R_α and ρ_α in the radial function of state α are of the form

$$R_\alpha = R(1 + \Delta\mathcal{E}_\alpha \rho R)^{-1/4}, \tag{D.13}$$

$$\rho_\alpha = \rho \left(\frac{R_\alpha}{R}\right)^3 \tag{D.14}$$

and are related to the Gaussian parameter ρ, R of Eq. (2.6).

The quantity $\Delta\mathcal{E}_\alpha = \mathcal{E}_\alpha - \mathcal{E}_0$ is the excess of the energy of the quark state α with respect to the ground state. $\Delta\mathcal{E}_\alpha$ depends on the quantum number n and l and is related to the parameters ρ and R by

$$\left(\Delta\mathcal{E}_\alpha + \frac{3\rho}{R}\right)^2 \left(\Delta\mathcal{E}_\alpha + \frac{1}{\rho R}\right) = \frac{\rho}{R^3}(4n + 2l - 1)^2. \tag{D.15}$$

The normalization constant, which results in

$$N_\alpha = \left[2^{-2(n+l+1/2)}\pi^{1/2}R_\alpha^3 \frac{(2n+2l)!}{(n+l)!(n-l)!}\left\{1 + \rho_\alpha^2\left(2n + l - \frac{1}{2}\right)\right\}\right]^{-1/2}, \tag{D.16}$$

is obtained from the normalization condition

$$\int d^3\vec{x}\, u_\alpha^\dagger(\vec{x}) u_\alpha(\vec{x}) = 1. \tag{D.17}$$

Appendix E

The Wave Function of the Nucleon

In our calculations we work first on the quark level and finally project the operators from the quark level on the nucleon level. In this Appendix we are going to show the wave function for nucleon, proton and neutron, before we project our calculation from the quark level on the baryon level.

The wave function of the nucleons are formed by using the SU(2) flavor (u,d) combining it with the SU(2) spin (up, down) and the SU(3) color (r,g,b). Therefore, the nucleon wave function are given by (Close, 1979)

$$|N\rangle \equiv \frac{1}{\sqrt{2}}\left(\phi_{\mathrm{M,S}}\chi_{\mathrm{M,S}} + \phi_{\mathrm{M,A}}\chi_{\mathrm{M,A}}\right)\psi_c \tag{E.1}$$

where ϕ, χ and ψ_c refer to the flavor, spin and color wavefunctions, respectively, while the subscript $\mathrm{M,S}$ and $\mathrm{M,A}$ stand for the mixed symmetric and mixed antisymmetric wave functions, respectively. The explicit forms for the flavor part are given by

$$\text{proton}\quad \left\{\begin{array}{lcl} \phi_{\mathrm{M,S}} &=& \frac{1}{\sqrt{6}}(\mathrm{udu}+\mathrm{duu}-2\mathrm{uud}) \\ \phi_{\mathrm{M,A}} &=& \frac{1}{\sqrt{2}}(\mathrm{udu}-\mathrm{duu}) \end{array}\right. \tag{E.2}$$

$$\text{neutron}\quad \left\{\begin{array}{lcl} \phi_{\mathrm{M,S}} &=& -\frac{1}{\sqrt{6}}(\mathrm{udd}+\mathrm{dud}-2\mathrm{ddu}) \\ \phi_{\mathrm{M,A}} &=& \frac{1}{\sqrt{2}}(\mathrm{udd}-\mathrm{dud}) \end{array}\right. \tag{E.3}$$

and for the spin up wave function we have

$$\text{spin up}(\uparrow)\quad \left\{\begin{array}{lcl} \chi_{\mathrm{M,S}} &=& \frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) \\ \chi_{\mathrm{M,A}} &=& \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \end{array}\right. \tag{E.4}$$

The explicit form of the color wavefunction is given by

$$\psi_c = \frac{1}{\sqrt{6}}(\mathrm{rgb} - \mathrm{rbg} - \mathrm{grb} + \mathrm{gbr} - \mathrm{bgr} + \mathrm{brg}). \tag{E.5}$$

The factor $1/\sqrt{2}$ in Eq. (E.1) is the normalization constant from the condition

$$\langle N|N\rangle \equiv 1. \tag{E.6}$$

Appendix F

Nucleon Mass

Before setting out to present the renormalization scheme of the PCQM (Appendix G), we first define and discuss the quantities relevant for mass and wave function renormalization. Following the Gell-Mann and Low theorem (Gell-Man and Low, 1951) we define the mass shift of the nucleonic three-quark core, Δm_N, due to the interaction with Goldstone meson as

$$\Delta m_N \;=\; {}^N\!\langle\phi_0|\sum_{n=1}^{\infty}\frac{\mathrm{i}^n}{n!}\int \mathrm{i}\delta(t_1)d^4x_1\ldots d^4x_n\,T:[\mathcal{L}_I^W(x_1)\ldots\mathcal{L}_I^W(x_n)]|\phi_0\rangle_c^N, \quad \text{(F.1)}$$

where the strong interaction Lagrangian $\mathcal{L}_I^W$ treated as a perturbation is defined by

$$\mathcal{L}_I^W(x) = \frac{1}{2F}\partial_\mu\vec{\pi}(x)\bar{\psi}(x)\gamma^\mu\gamma^5\vec{\tau}\psi(x) - \frac{\varepsilon_{ijk}}{4F^2}\pi_i(x)\partial_\mu\pi_j(x)\bar{\psi}(x)\gamma^\mu\tau_k\psi(x). \quad \text{(F.2)}$$

We evaluate Eq. (F.1) at one loop to order $o(1/F^2)$ using Wick's theorem and the appropriate propagators.

The total nucleon mass is given by $m_N^r = m_N^{\rm core} + \Delta m_N$. The superscript r refers to the renormalization value of the nucleon mass at one loop. The diagrams that contribute to the nucleon mass shift Δm_N at one loop are meson cloud (Fig. F.1(a)) and meson exchange diagrams (Fig. F.1(b)). The explicit expression for the nucleon mass with one loop correction can be written by

$$m_N^r = m_N^{\rm core} + \Delta m_N = 3(\mathcal{E}_0 + \gamma\hat{m}) + \Delta m_N^{MC} + \Delta m_N^{EX}, \quad \text{(F.3)}$$

where Δm_N^{MC} and Δm_N^{EX} are the contributions to nucleon mass shift by the meson cloud and meson exchange diagrams, respectively.

(a) (b)

Figure F.1: Diagrams contributing to the nucleon mass: meson cloud diagram (a) and meson exchange diagram (b).

F.1 Meson Cloud Contribution to the Nucleon Mass Shift

$$
\begin{aligned}
&\Delta m_N^{MC}|_\alpha \\
&= 2\, {}^N\!\langle \phi_0 | \frac{-1}{2!} \int \mathrm{i}\delta(t_1) d^4x_1 d^4x_2 \\
&\times N\left\{ \left[\frac{1}{2F} \partial_\mu \pi_m \bar{\psi} \gamma^\mu \gamma^5 \tau_m \psi \right]_{x_1} \left[\frac{1}{2F} \partial_\nu \pi_n \bar{\psi} \gamma^\nu \gamma^5 \tau_n \psi \right]_{x_2} \right\} |\phi_0\rangle_c^N \\
&= \frac{-\mathrm{i}}{4F^2}\ {}^N\!\langle \phi_0 | \int \delta(t_1) d^4x_1 d^4x_2 \\
&\times \bar{\psi}(x_1) \gamma^\mu \gamma^5 \tau_m \psi(x_1) \bar{\psi}(x_2) \gamma^\nu \gamma^5 \tau_n \psi(x_2) \partial_\mu \pi_m(x_1) \partial_\nu \pi_n(x_2) |\phi_0\rangle_c^N \\
&= \frac{-1}{4F^2(2\pi)^4}\ {}^N\!\langle \phi_0 | b_0^\dagger \int d^3x_1 d^3x_2 d^4k \bar{u}_0(x_1) \gamma^\mu k_\mu \gamma^5 \tau_m u_\alpha(x_1) \bar{u}_\alpha(x_2) \gamma^\nu k_\nu \gamma^5 \tau_m u_0(x_2) \\
&\times \frac{\mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}}{M_\pi^2 - k^2 - \mathrm{i}\epsilon} \int dt_1 dt_2 \delta(t_1) \mathrm{e}^{\mathrm{i}\mathcal{E}_0 t_1} \mathrm{e}^{-\mathrm{i}\mathcal{E}_\alpha(t_1-t_2)} \Theta(t_1 - t_2) \mathrm{e}^{-\mathrm{i}k_0(t_1-t_2)} \mathrm{e}^{-\mathrm{i}\mathcal{E}_0 t_2} b_0 |\phi_0\rangle^N \\
&= \frac{\mathrm{i}}{4F^2(2\pi)^4}\ {}^N\!\langle \phi_0 | b_0^\dagger \int d^3x_1 d^3x_2 d^3k\, \mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}_1-\vec{x}_2)} \\
&\times \int dk_0 \left\{ \left[\frac{1}{M_\pi^2 - k_0^2 + \vec{k}^2 - \mathrm{i}\epsilon} \right] \left[\frac{1}{k_0 + \Delta\mathcal{E}_\alpha - \mathrm{i}\eta} \right] \right. \\
&\times \left. \left[\bar{u}_0(x_1)(\gamma^0 k_0 - \vec{\gamma}\cdot\vec{k}) \gamma^5 \tau_m u_\alpha(x_1) \right] \left[\bar{u}_\alpha(x_2)(\gamma^0 k_0 - \vec{\gamma}\cdot\vec{k}) \gamma^5 \tau_m u_0(x_2) \right] \right\} b_0 |\phi_0\rangle^N
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{-\pi}{4F^2(2\pi)^4}\ {}^N\!\big\langle \phi_0 | b_0^\dagger \int d^3k \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\times\Bigg\{\Bigg[\int d^3x_1 \bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_m u_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\Bigg]\Bigg[\int d^3x_2 \bar{u}_\alpha(x_2)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_m u_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\Bigg] \\
&-\omega(k^2)\Bigg[\int d^3x_1 \bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_m u_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\Bigg]\Bigg[\int d^3x_2 \bar{u}_\alpha(x_2)\gamma^0\gamma^5\tau_m u_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\Bigg] \\
&-\omega(k^2)\Bigg[\int d^3x_1 \bar{u}_0(x_1)\gamma^0\gamma^5\tau_m u_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\Bigg]\Bigg[\int d^3x_2 \bar{u}_\alpha(x_2)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_m u_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\Bigg] \\
&+\omega^2(k^2)\Bigg[\int d^3x_1 \bar{u}_0(x_1)\gamma^0\gamma^5\tau_m u_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\Bigg]\Bigg[\int d^3x_2 \bar{u}_\alpha(x_2)\gamma^0\gamma^5\tau_m u_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\Bigg]\Bigg\} \\
&\times b_0|\phi_0\big\rangle^N
\end{aligned}
\tag{F.4}
$$

with the pion energy $\omega(k^2) = \sqrt{M_\pi^2+k^2}$; $k = |\vec{k}|$ is the pion momentum and $\Delta\mathcal{E}_\alpha = \mathcal{E}_\alpha - \mathcal{E}_0$ is the excess of the energy of the quark in state α with respect to the ground state.

It can be shown that (see Appendix J)

$$
\begin{aligned}
\int d^3x \bar{u}_0(x)\vec{\gamma}\cdot\vec{k}\,\gamma^5\tau_m u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} &\equiv F_{I_\alpha}(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_m\right]_{0,\alpha}, && \text{(F.5)} \\
\int d^3x \bar{u}_\alpha(x)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_m u_0(x)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}} &\equiv F_{I_\alpha}^\dagger(k)\left[\tau_m(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0}, && \text{(F.6)} \\
\int d^3x \bar{u}_0(x)\gamma^0\gamma^5\tau_m u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} &\equiv F_{II_\alpha}(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_m\right]_{0,\alpha}, && \text{(F.7)} \\
\int d^3x \bar{u}_\alpha(x)\gamma^0\gamma^5\tau_m u_0(x)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}} &\equiv F_{II_\alpha}^\dagger(k)\left[\tau_m(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0}. && \text{(F.8)}
\end{aligned}
$$

With the above expressions $\Delta m_N^{MC}|_\alpha$ can be written as

$$
\begin{aligned}
\Delta m_N^{MC}|_\alpha &= \frac{-\pi}{4F^2(2\pi)^4}\ {}^N\!\big\langle \phi_0 | b_0^\dagger \int d^3k \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\times\Bigg\{ F_{I_\alpha}(k)F_{I_\alpha}^\dagger(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_m\right]_{0,\alpha}\left[\tau_m(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0} \\
&-\omega(k^2)F_{I_\alpha}(k)F_{II_\alpha}^\dagger(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_m\right]_{0,\alpha}\left[\tau_m(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0} \\
&-\omega(k^2)F_{II_\alpha}(k)F_{I_\alpha}^\dagger(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_m\right]_{0,\alpha}\left[\tau_m(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0} \\
&+\omega^2(k^2)F_{II_\alpha}(k)F_{II_\alpha}^\dagger(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_m\right]_{0,\alpha}\left[\tau_m(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0}\Bigg\} b_0|\phi_0\big\rangle^N
\end{aligned}
\tag{F.9}
$$

with the identity of the Pauli matrices

$$\sum_{m=1}^{3} \tau_m \tau_m = 3\,\mathbf{I} \tag{F.10}$$

and one can show that

$$F_{II_\alpha}(k) F^\dagger_{I_\alpha}(k) = F_{I_\alpha}(k) F^\dagger_{II_\alpha}(k), \tag{F.11}$$

then we get

$$\begin{aligned} \Delta m_N^{MC}|_\alpha \quad &= \quad \frac{-3\pi}{4F^2(2\pi)^4} \; {}^N\!\langle \phi_0 | b_0^\dagger \left[\chi_c^\dagger \chi_f^\dagger \chi_s^\dagger \right]_0 \int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)(\omega(k^2) + \Delta\mathcal{E}_\alpha)} \\ & \quad \times \left[F_{I_\alpha}(k) F^\dagger_{I_\alpha}(k) - 2\omega(k^2) F_{I_\alpha}(k) F^\dagger_{II_\alpha}(k) + \omega^2(k^2) F_{II_\alpha}(k) F^\dagger_{II_\alpha}(k) \right] \\ & \quad \times \int_\Omega d\Omega (\vec{\sigma} \cdot \hat{k})(\vec{\sigma} \cdot \hat{k}) \left[\chi_s \chi_f \chi_c \right]_0 b_0 | \phi_0 \rangle^N . \end{aligned} \tag{F.12}$$

However, some spin algebra gives that

$$\int_\Omega d\Omega\, (\vec{\sigma} \cdot \hat{k})(\vec{\sigma} \cdot \hat{k}) = 4\pi \mathbf{I} \tag{F.13}$$

and we also use the one body projection operator which gives the result

$$\langle P \uparrow | \sum_{i=1}^{3} \mathbf{I} | P \uparrow \rangle = 3, \tag{F.14}$$

where $\mathbf{I}$ is the identity matrix. Then, the result of $\Delta m_N^{MC}|_\alpha$ yields

$$\begin{aligned} \Delta m_N^{MC}|_\alpha \quad &= \quad -\frac{9}{(4\pi F)^2} \int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)(\omega(k^2) + \Delta\mathcal{E}_\alpha)} \\ & \quad \times \left[F_{I_\alpha}(k) F^\dagger_{I_\alpha}(k) - 2\omega(k^2) F_{I_\alpha}(k) F^\dagger_{II_\alpha}(k) + \omega^2(k^2) F_{II_\alpha}(k) F^\dagger_{II_\alpha}(k) \right] . \end{aligned} \tag{F.15}$$

The expression in Eq. (F.15) is the mass shift of the nucleon due to the self-energy correction of the meson cloud when the intermediate quark propagator is in the state α. The full expressions for $F_{I_\alpha}(k)$ and $F_{II_\alpha}(k)$ are shown explicitly in Appendix J.

The label $\alpha = (n l_\alpha j m)$ characterizes the quark state (principal quantum number n, non-relativistic orbital angular momentum l_α, total angular momentum and projection j, m).

F.2 Meson Exchange Contribution to the Nucleon Mass Shift

$$
\begin{aligned}
&\Delta m_N^{ME} \\
&= {}^N\!\left\langle\phi_0\right| \frac{-1}{2!}\int \mathrm{i}\delta(t_1)d^4x_1d^4x_2 \\
&\quad \times N\left\{\left[\frac{1}{2F}\partial_\mu\pi_m\bar{\psi}\gamma^\mu\gamma^5\tau_m\psi\right]_{x_1}\left[\frac{1}{2F}\partial_\nu\pi_n\bar{\psi}\gamma^\nu\gamma^5\tau_n\psi\right]_{x_2}\right\}\left|\phi_0\right\rangle_c^N \\
&= \frac{-\mathrm{i}}{8F^2}\ {}^N\!\left\langle\phi_0\right|\int\delta(t_1)d^4x_1d^4x_2 \\
&\quad \times\bar{\psi}(x_1)\gamma^\mu\gamma^5\tau_m\psi(x_1)\bar{\psi}(x_2)\gamma^\nu\gamma^5\tau_n\psi(x_2)\partial_\mu\pi_m(x_1)\partial_\nu\pi_n(x_2)\left|\phi_0\right\rangle_c^N \\
&= \frac{-1}{8F^2(2\pi)^4}\ {}^N\!\left\langle\phi_0\right|b_0^\dagger(x_1)b_0^\dagger(x_2)\int d^3x_1d^3x_2d^4k\frac{\mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}_2-\vec{x}_1)}}{M_\pi^2-k^2-\mathrm{i}\epsilon} \\
&\quad \times\bar{u}_0(x_1)\gamma^\mu k_\mu\gamma^5\tau_m u_0(x_1)\bar{u}_0(x_2)\gamma^\nu k_\nu\gamma^5\tau_m u_0(x_2)\int dt_1dt_2\delta(t_1)\mathrm{e}^{-\mathrm{i}k_0(t_2-t_1)} \\
&\quad \times b_0(x_1)b_0(x_2)\left|\phi_0\right\rangle^N \\
&= \frac{-1}{8F^2(2\pi)^3}\ {}^N\!\left\langle\phi_0\right|b_0^\dagger(x_1)b_0^\dagger(x_2)\int d^3x_1d^3x_2d^3k\,\mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}_2-\vec{x}_1)} \\
&\quad \times\int dk_0\left\{\left[\frac{\delta(k_0)}{M_\pi^2-k_0^2+\vec{k}^2-\mathrm{i}\epsilon}\right]\left[\bar{u}_0(x_1)(\gamma^0k_0-\vec{\gamma}\cdot\vec{k})\gamma^5\tau_m u_0(x_1)\right]\right. \\
&\quad \times\left.\left[\bar{u}_0(x_2)(\gamma^0k_0-\vec{\gamma}\cdot\vec{k})\gamma^5\tau_m u_0(x_2)\right]\right\}b_0(x_1)b_0(x_2)\left|\phi_0\right\rangle^N \\
&= \frac{-1}{8F^2(2\pi)^3}\ {}^N\!\left\langle\phi_0\right|b_0^\dagger(x_1)b_0^\dagger(x_2)\int d^3k\frac{1}{\omega^2(k^2)} \\
&\quad \times\left[\int d^3x_1\bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_m u_0(x_1)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_1}\right] \\
&\quad \times\left[\int d^3x_2\bar{u}_0(x_2)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_m u_0(x_2)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_2}\right]b_0(x_1)b_0(x_2)\left|\phi_0\right\rangle^N \\
&= \frac{-1}{8F^2(2\pi)^3}\ {}^N\!\left\langle\phi_0\right|b_0^\dagger(x_1)b_0^\dagger(x_2)\int d^3k\frac{1}{\omega^2(k^2)} \\
&\quad \times\left[\frac{3}{5}g_A^{(0)}k\,F_{\pi NN}(k^2)\left[(\vec{\sigma}\cdot\hat{k})\tau_m\right]_{0,0}\right]_{x_1}\left[\frac{3}{5}g_A^{(0)}k\,F_{\pi NN}(k^2)\left[(\vec{\sigma}\cdot\hat{k})\tau_m\right]_{0,0}\right]_{x_2} \\
&\quad \times b_0(x_1)b_0(x_2)\left|\phi_0\right\rangle^N
\end{aligned}
$$

$$\begin{aligned}
&= \frac{-9}{200F^2(2\pi)^3}\left(g_A^{(0)}\right)^2 \; {}^N\!\big\langle \phi_0 | b_0^\dagger(x_1)\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_{0_{x_1}} b_0^\dagger(x_2)\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_{0_{x_2}} \\
&\quad \times \int_0^\infty dk k^4 \frac{F_{\pi NN}^2(k^2)}{\omega^2(k^2)} \int_\Omega d\Omega \left[(\vec\sigma\cdot\hat k)\tau_m\right]_{x1}\left[(\vec\sigma\cdot\hat k)\tau_m\right]_{x2} \\
&\quad \times \left[\chi_s\chi_f\chi_c\right]_{0_{x_1}} b_0(x_1)\left[\chi_s\chi_f\chi_c\right]_{0_{x_2}} b_0(x_2)|\phi_0\big\rangle^N \\
&= \frac{-3}{100}\left(\frac{g_A^{(0)}}{2F\pi}\right)^2 \; {}^N\!\big\langle \phi_0 | b_0^\dagger(x_1)\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_{0_{x_1}} b_0^\dagger(x_2)\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_{0_{x_2}} \\
&\quad \times \int_0^\infty dk k^4 \frac{F_{\pi NN}^2(k^2)}{\omega^2(k^2)} \left(\vec\sigma_{x_1}\cdot\vec\sigma_{x_2}\right)\tau_{m_{x_1}}\tau_{m_{x_2}} \\
&\quad \times \left[\chi_s\chi_f\chi_c\right]_{0_{x_1}} b_0(x_1)\left[\chi_s\chi_f\chi_c\right]_{0_{x_2}} b_0(x_2)|\phi_0\big\rangle^N \\
&= -\frac{9}{10}\left(\frac{g_A^{(0)}}{2\pi F}\right)^2 \int_0^\infty dk\, k^4 \frac{F_{\pi NN}^2(k^2)}{\omega^2(k^2)}. \qquad \text{(F.16)}
\end{aligned}$$

In order to get the expression in Eq. (F.16) we have used the spin algebra

$$\int_\Omega d\Omega\left[\vec\sigma\cdot\hat k\right]_{x_1}\left[\vec\sigma\cdot\hat k\right]_{x_2} = \frac{4\pi}{3}\left(\vec\sigma_{x_1}\cdot\vec\sigma_{x_2}\right) \qquad \text{(F.17)}$$

and the "two-body" operator results in

$$\big\langle P\uparrow | \sum_{i\neq j}^{3} (\sigma_n\tau_m)^i \otimes (\sigma_n\tau_m)^j | P\uparrow\big\rangle = 30. \qquad \text{(F.18)}$$

The expression in Eq. (F.16) is the mass shift of the nucleon due to correction of the meson exchange.

Appendix G

Renormalization of the PCQM

To redefine our perturbation series up to a given order in terms of renormalized quantities a set of counterterms, $\delta\mathcal{L}$, has to be introduced in the Lagrangian. Thereby, the counterterms play a dual role:

(i) to maintain the proper definition of physical parameters, such as nucleon mass and, in particular, the nucleon charge

(ii) to effectively reduce the number of Feynman diagrams to be evaluated.

G.1 Renormalization of the Quark Field

First, we introduce the renormalized quark field $\psi^r(x)$ with renormalized mass $\mathcal{M}^r$, substituting the original field $\psi(x)$:

$$\psi_i^r(x;m_i^r) = \sum_\alpha b_\alpha u_\alpha^r(\vec{x};m_i^r)\exp[-\mathrm{i}\mathcal{E}_\alpha^r(m_i^r)t], + \sum_\beta d_\beta^\dagger v_\beta^r(\vec{x};m_i^r)\exp[\mathrm{i}\mathcal{E}_\beta^r(m_i^r)t], \tag{G.1}$$

where "i" is the SU(2) flavor index; $\mathcal{E}_\alpha^r(m_i^r)$ is the renormalized energy of the quark field in the state α obtained from the solution of the Dirac equation

$$[-\mathrm{i}\gamma^0\vec{\gamma}\cdot\vec{\nabla} + \gamma^0 m_i^r + \gamma^0 S(r) + V(r) - \mathcal{E}_\alpha^r(m_i^r)]u_\alpha^r(\vec{x};m_i^r) = 0. \tag{G.2}$$

Using the derivations of the previous Appendix, the renormalized mass m_i^r of the quark field is given by

$$m_u^r = m_d^r = \hat{m}^r = \hat{m} - \delta\hat{m} = \hat{m} + \frac{1}{3\gamma}\Delta m_N^{MC}, \tag{G.3}$$

the meson exchange contribution will be included when introducing nucleon mass renormalization. Thereby, the full expression for renormalized quark mass is written as:

$$\begin{aligned}\hat{m}^r &= \hat{m} - \frac{3}{\gamma}\left(\frac{1}{4\pi F}\right)^2 \sum_\alpha \int_0^\infty dk\, k^2 \,\frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\ &\quad\times \left[F_{I_\alpha}(k)F^\dagger_{I_\alpha}(k) - 2\omega(k^2)F_{I_\alpha}(k)F^\dagger_{II_\alpha}(k) + \omega^2(k^2)F_{II_\alpha}(k)F^\dagger_{II_\alpha}(k)\right].\end{aligned} \tag{G.4}$$

The summation on state α appears when we take into account the intermediate excited quark state propagator. When restricting the quark propagator to the ground state the expression above for the renormalized quark mass reduces to

$$\hat{m}^r_0 = \hat{m} - \frac{27}{400\gamma}\left(\frac{g_A^{(0)}}{\pi F}\right)^2 \int_0^\infty dk\, k^4\, \frac{F^2_{\pi NN}(k^2)}{\omega^2(k^2)}. \tag{G.5}$$

For the quark masses we will use in the following the compact notation:

$$\mathcal{M}^r = \mathrm{diag}\{\hat{m}^r, \hat{m}^r\} \quad \text{and} \quad \delta\mathcal{M} = \mathrm{diag}\{\delta\hat{m}, \delta\hat{m}\}. \tag{G.6}$$

The solutions of Eq. (G.2), $\mathcal{E}^r_\alpha(\hat{m}^r)$ and $u^r_\alpha(\vec{x};\hat{m}^r)$, are functions of $\hat{m}^r$. While, the propagator of the renormalized quark field $\psi^r(x)$ is given by

$$\mathrm{i}G_{\psi^r}(x,y) = \langle\phi_0|T\{\psi^r(x)\bar{\psi}^r(y)\}|\phi_0\rangle \to \sum_\alpha u^r_\alpha(\vec{x})\bar{u}^r_\alpha(\vec{y})\exp[-\mathrm{i}\mathcal{E}^r_\alpha(x_0-y_0)]\theta(x_0-y_0). \tag{G.7}$$

Again, we restrict to the quark states propagating forward in time. Eq. (G.7) differs from the unperturbed quark propagator $\mathrm{i}G_\psi(x,y)$ by the term of order $\hat{m}^r$, which in turn only contributes to the two-loop calculations. Thus, to the order of accuracy we are working in (up to one-loop perturbation theory) it is sufficient to use the unperturbed quark propagator $\mathrm{i}G_\psi(x,y)$ instead of the renormalized one. In this present work intermediate excited quark states are included in the loop diagram but we do not have to use the renormalization wave function for excited states, only the ground state is modified.

For the renormalized ground state wave function $u_0^r(\vec{x};\hat{m}^r)$ we again consider the Gaussian ansatz

$$u_0^r(\vec{x};\hat{m}^r) = N_0(\hat{m}^r)\,\exp\left[-c(\hat{m}^r)\frac{\vec{x}^{\,2}}{2R^2}\right]\begin{pmatrix} 1 \\ \mathrm{i}\rho(\hat{m}^r)\,\dfrac{\vec{\sigma}\cdot\vec{x}}{R} \end{pmatrix}\chi_s\,\chi_f\,\chi_c \qquad \text{(G.8)}$$

with normalization

$$\int d^3x\, u_0^{r\dagger}(x;\hat{m}^r)\, u_0^r(x;\hat{m}^r) \equiv 1. \qquad \text{(G.9)}$$

In Eq. (G.8) the functions $N(\hat{m}^r)$, $c(\hat{m}^r)$ and $\rho(\hat{m}^r)$ are normalized at the point $\hat{m}^r = 0$ as follows:

$$N(0) = N, \qquad c(0) = 1, \qquad \rho(0) = \rho. \qquad \text{(G.10)}$$

The product $\rho(\hat{m}^r)\,c(\hat{m}^r)$ can be shown to be $\hat{m}^r$-invariant and we therefore obtain the additional condition

$$\rho(\hat{m}^r)\,c(\hat{m}^r) \equiv \rho. \qquad \text{(G.11)}$$

Treating $\hat{m}^r$ as a small perturbation, Eq. (G.2) can be solved perturbatively, resulting in:

$$\mathcal{E}_0^r(\hat{m}^r) = \mathcal{E}_0 + \delta\mathcal{E}_0(\hat{m}^r) \qquad \text{(G.12)}$$

and

$$u_0^r(\vec{x};\hat{m}^r) \;\; = \;\; u_0(\vec{x}) + \delta u_0(\vec{x};\hat{m}^r) \qquad \text{(G.13)}$$

where

$$\delta\mathcal{E}_0(\hat{m}^r) = \gamma\hat{m}^r \qquad \text{(G.14)}$$

and

$$\delta u_0^r(\vec{x};\hat{m}^r) = \frac{\hat{m}^r}{2}\frac{\rho R}{1+\frac{3}{2}\rho^2}\left(\frac{\frac{1}{2}+\frac{21}{4}\rho^2}{1+\frac{3}{2}\rho^2} - \frac{\vec{x}^{\,2}}{R^2} + \gamma^0\right)u_0(\vec{x}). \qquad \text{(G.15)}$$

For our set of model parameters the ground state quark energy $\mathcal{E}_0$ is about 400 MeV and for the energy corrections $\delta\mathcal{E}_0$ relative to $\mathcal{E}_0$ we obtain

$$\left|\frac{\delta\mathcal{E}_0(\hat{m}^r)}{\mathcal{E}_0}\right| \approx 14.\%. \tag{G.16}$$

Given the small corrections expressed in Eq. (G.16), the perturbative treatment of a finite (renormalized) quark mass is a meaningful procedure.

G.2 Renormalized Effective Lagrangian

Having set up renormalized fields and masses for the quarks we are in the position to rewrite the original Lagrangian. The renormalized effective Lagrangian including the photon field A_μ is now written as

$$\mathcal{L}^r_{\rm full}(x) \;=\; \mathcal{L}^r_\psi(x) \;+\; \mathcal{L}_\pi(x) \;+\; \mathcal{L}_{\rm ph}(x) \;+\; \mathcal{L}^W_{I;\,r}(x). \tag{G.17}$$

The renormalized quark Lagrangian $\mathcal{L}^r_\psi(x)$ defines free nucleon dynamics at one-loop with

$$\begin{aligned}
\mathcal{L}^r_\psi(x) &= \mathcal{L}^r_{\bar\psi\psi}(x) + \mathcal{L}^r_{(\bar\psi\psi)^2}(x), && \text{(G.18)}\\
\mathcal{L}^r_{\bar\psi\psi}(x) &= \bar\psi^r(x)[\mathrm{i}\,\partial\!\!\!/ - \mathcal{M}^r - S(r) - \gamma^0 V(r)]\psi^r(x), && \text{(G.19)}\\
\mathcal{L}^r_{(\bar\psi\psi)^2}(x) &= \left[\frac{1}{2F}\partial_\mu\pi_m(x)\bar\psi^r(x)\gamma^\mu\gamma^5\tau_m\psi^r(x)\right]^2. && \text{(G.20)}
\end{aligned}$$

The parameters $\delta\mathcal{M}$ of Eq. (G.6) guarantee the proper nucleon mass renormalization due to the meson cloud diagrams of Fig. F.1(a). The terms contained in $\mathcal{L}^r_{(\bar\psi\psi)^2}(x)$ are introduced for the purpose of nucleon mass renormalization due to the meson exchange diagram of Fig. F.1(b).

The free meson Lagrangian $\mathcal{L}_\Phi$ is written as

$$\mathcal{L}_\pi = -\frac{1}{2}\vec\pi(x)(\Box + M_\pi^2)\vec\pi(x). \tag{G.21}$$

For the photon field A_μ we have the usual kinetic term

$$\mathcal{L}_{\rm ph} = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) \quad \text{with} \quad F_{\mu\nu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x). \tag{G.22}$$

The renormalized interaction Lagrangian $\mathcal{L}^{W}_{I;\,r}(x) = \mathcal{L}^{W;\,{\rm str}}_{I;\,r}(x) + \mathcal{L}^{W;\,{\rm em}}_{I;\,r}(x)$ contains a part due to the strong interaction,

$$\mathcal{L}^{W;\,{\rm str}}_{I;\,r}(x) = \mathcal{L}^{W;\,{\rm str}}_{I}(x) + \delta\mathcal{L}^{W;\,{\rm str}}(x) \tag{G.23}$$

and a piece due to the electromagnetic interaction,

$$\mathcal{L}^{W;\,{\rm em}}_{I;\,r}(x) = \mathcal{L}^{W;\,{\rm em}}_{I}(x) + \delta\mathcal{L}^{W;\,{\rm em}}(x). \tag{G.24}$$

The strong interaction term $\mathcal{L}^{W;\,{\rm str}}_{I}$ is given by

$$\mathcal{L}^{W;\,{\rm str}}_{I}(x) = \frac{1}{2F} \partial_\mu \vec{\pi}(x) \bar{\psi}^r(x) \gamma^\mu \gamma^5 \vec{\tau} \psi^r(x) - \frac{\varepsilon_{ijk}}{4F^2} \pi_i(x) \partial_\mu \pi_j(x) \bar{\psi}^r(x) \gamma^\mu \tau_k \psi^r(x). \tag{G.25}$$

The interaction of pions and quarks with the electromagnetic field is described by (Lyubovitskij, Gutsche, Faessler and Vinh-Mau, 2001, 2002)

$$\begin{aligned} \mathcal{L}^{W;\,{\rm em}}_{I}(x) &= -eA^{\rm em}_\mu \bar{\psi}^r(x) Q \gamma^\mu \psi^r(x) \\ &\quad + \frac{e}{4F^2} A^{\rm em}_\mu(x) \bar{\psi}^r(x) \gamma^\mu \left[\vec{\pi}^2(x) \tau_3 - \vec{\pi}(x) \vec{\tau} \pi^0(x) \right] \psi^r(x) \\ &\quad - eA^{\rm em}_\mu(x) \varepsilon_{3ij} \left[\pi_i(x) \partial^\mu \pi_j(x) - \frac{\pi_j(x)}{2F} \bar{\psi}^r(x) \gamma^\mu \gamma^5 \tau_i \psi^r(x) \right], \end{aligned} \tag{G.26}$$

which is generated by minimal substitution with

$$\begin{aligned} \partial_\mu \psi^r \to D_\mu \psi^r &= \partial_\mu \psi^r + {\rm i} e Q A^{\rm em}_\mu \psi^r, && \text{(G.27)} \\ \partial_\mu \pi_i \to D_\mu \pi_i &= \partial_\mu \pi_i + e \varepsilon_{3ij} A^{\rm em}_\mu \pi_j, && \text{(G.28)} \end{aligned}$$

where Q is the quark charge matrix with $Q = {\rm diag}\{2/3, -1/3\}$. The set of counterterms, denoted by $\delta\mathcal{L}^{W;\,{\rm str}}(x)$ and $\delta\mathcal{L}^{W;\,{\rm em}}(x)$, is explicitly given by

$$\delta\mathcal{L}^{W;\,{\rm str}}(x) = \delta\mathcal{L}^{W;\,{\rm str}}_1(x) + \delta\mathcal{L}^{W;\,{\rm str}}_2(x) + \delta\mathcal{L}^{W;\,{\rm str}}_3(x), \tag{G.29}$$

with

$$\delta\mathcal{L}_1^{W;\,\rm str}(x) \;=\; \bar{\psi}^r(x)\,(Z-1)\,[\mathrm{i}\,\partial\!\!\!/ - \mathcal{M}^r - S(r) - \gamma^0 V(r)]\psi^r(x), \quad \text{(G.30)}$$

$$\delta\mathcal{L}_2^{W;\,\rm str}(x) \;=\; -\,\bar{\psi}^r(x)\,\delta\mathcal{M}\,\psi^r(x), \quad \text{(G.31)}$$

$$\delta\mathcal{L}_3^{W;\,\rm str}(x) \;=\; -\left[\frac{1}{2F}\partial_\mu\pi_m(x)\bar{\psi}(x)\gamma^\mu\gamma^5\tau_m\psi(x)\right]^2 \quad \text{(G.32)}$$

and

$$\delta\mathcal{L}^{W;\,\rm em}(x) \;=\; -eA_\mu(x)\bar{\psi}^r(x)\,(Z-1)\,\gamma^\mu Q\psi^r(x). \quad \text{(G.33)}$$

Here, $Z = \mathrm{diag}\{\hat{Z}, \hat{Z}\}$ is the diagonal matrix of renormalization constants for u and d quarks. The value of $\hat{Z}$ is determined by the charge conservation condition. The simplest way to fix $\hat{Z}$ is on the quark level. The same set of values for $\hat{Z}$ is also obtained when requiring charge conservation on the baryon level. Results for $\hat{Z}$ will be discussed below.

Now we briefly explain the role of each counterterm and why the constant $\hat{Z}$ is identical in $\delta\mathcal{L}_1^{W;\,\rm str}$ and $\delta\mathcal{L}^{W;\,\rm em}$. The counterterm $\delta\mathcal{L}^{W;\,\rm em}$ is introduced to guarantee charge conservation. The counterterm $\delta\mathcal{L}_1^{W;\,\rm str}$, containing the same renormalization constants $\hat{Z}$ as in $\delta\mathcal{L}^{W;\,\rm em}$, is added to fulfil electromagnetic local gauge invariance on the Lagrangian level. The same term also leads to conservation of the vector current (baryon number conservation). Alternatively, $\delta\mathcal{L}^{W;\,\rm em}$ can also be deduced from $\delta\mathcal{L}_1^{W;\,\rm str}$ by minimal substitution. In covariant theories the equality of the renormalization constants in $\delta\mathcal{L}_1^{W;\,\rm str}$ and $\delta\mathcal{L}^{W;\,\rm em}$ is known as the Ward identity. The counterterms $\delta\mathcal{L}_2^{W;\,\rm str}$ and $\delta\mathcal{L}_3^{W;\,\rm str}$ compensate the contributions of the meson cloud (Fig. F.1(a)) and meson exchange diagrams (Fig. F.1(b)) to the nucleon mass m_N^r (The contribution of meson cloud and exchange diagrams is already taken into account in the renormalized quark Lagrangian $\mathcal{L}_\psi^r$.).

G.3 Renormalization of the Nucleon Mass

Now we illustrate the explicit role of the counterterms when performing the calculation of the nucleon mass. The renormalized nucleon mass m_N^r is defined by the expectation value of the Hamiltonian $\mathcal{H}_\psi^r$ (as derived from the Lagrangian $\mathcal{L}_\psi^r$) averaged over state $|\phi_0\rangle$ and projected on the respective nucleon states:

$$m_N^r \equiv {}^N\langle\phi_0| \int \delta(t)\, d^4x\, \mathcal{H}_\psi^r(x)\, |\phi_0\rangle^N, \tag{G.34}$$

By inclusion of the counterterms the strong interaction Lagrangian $\mathcal{L}_{I;\,r}^{W;\,\mathrm{str}}$ should give a zero contribution to the shift of the renormalized nucleon mass at one loop, that is

$$\begin{aligned}
\Delta m_N^r &= {}^N\langle\phi_0| \sum_{n=1}^{2} \frac{\mathrm{i}^n}{n!} \int \mathrm{i}\delta(t_1) d^4x_1 \ldots d^4x_n T[\mathcal{L}_{I;\,r}^{W;\,\mathrm{str}}(x_1) \ldots \mathcal{L}_{I;\,r}^{W;\,\mathrm{str}}(x_n)]|\phi_0\rangle_c^N \\
&= {}^N\langle\phi_0| -\frac{\mathrm{i}}{2} \int \delta(t_1) d^4x_1 d^4x_2 T[\mathcal{L}_I^{W;\,\mathrm{str}}(x_1)\mathcal{L}_I^{W;\,\mathrm{str}}(x_2)]|\phi_0\rangle_c^N \\
&\quad - {}^N\langle\phi_0| \int \delta(t) d^4x \sum_{i=1}^{3} \delta\mathcal{L}_i^{W;\,\mathrm{str}}(x)\, |\phi_0\rangle^N \\
&\equiv 0.
\end{aligned} \tag{G.35}$$

To prove Eq. (G.35), we first note that the contribution of the counterterm $\delta\mathcal{L}_1^{W;\,\mathrm{str}}$ is equal to zero due to the equation of motion (G.2), that is

$${}^N\langle\phi_0| \int \delta(t) d^4x \delta\mathcal{L}_1^{W;\,\mathrm{str}}(x)\, |\phi_0\rangle^N \equiv 0. \tag{G.36}$$

The counterterms $\delta\mathcal{L}_2^{W;\,\mathrm{str}}$ and $\delta\mathcal{L}_3^{W;\,\mathrm{str}}$ compensate the contribution of the meson cloud (Fig. F.1(a)) and exchange diagrams (Fig. F.1(b)), respectively, with

$$\begin{aligned}
&{}^N\langle\phi_0| -\frac{\mathrm{i}}{2} \int \delta(t_1) d^4x_1 d^4x_2 T[\mathcal{L}_I^{W;\,\mathrm{str}}(x_1)\mathcal{L}_I^{W;\,\mathrm{str}}(x_2)]|\phi_0\rangle_c^N \\
&\qquad - {}^N\langle\phi_0| \int \delta(t) d^4x [\delta\mathcal{L}_2^{W;\,\mathrm{str}}(x) + \delta\mathcal{L}_3^{W;\,\mathrm{str}}(x)]|\phi_0\rangle^N \equiv 0,
\end{aligned} \tag{G.37}$$

hence Eq. (G.35) is fulfilled. The calculation of the nucleon mass m_N^r at one-loop can then either be done with the "unrenormalized" Lagrangian $\mathcal{L}_{\mathrm{eff}}$ or with the "renormalized" version $\mathcal{L}^r_{\rm full}$ (G.17). Both results for m^r_N are identical and are given by Eq. (F.3).

G.4 Renormalization of Nucleon Charge

Now we consider the nucleon charge and prove that the properly introduced counterterms guarantee charge conservation. Using Noether's theorem we first derive from the renormalized Lagrangian the electromagnetic current operator:

$$j^\mu_r = j^\mu_{\psi^r} + j^\mu_\pi + j^\mu_{\psi^r\pi} + \delta j^\mu_{\psi^r}. \tag{G.38}$$

It contains the quark component ($j^\mu_{\psi^r}$), the charged pion component (j^μ_π), the quark-pion component ($j^\mu_{\psi^r\pi}$) and the contribution of the counterterm ($\delta j^\mu_{\psi^r}$):

$$\begin{aligned} j^\mu_{\psi^r} &= \bar\psi^r\gamma^\mu Q\psi^r = \frac{1}{3}\left(2\bar u^r\gamma^\mu u^r - \bar d^r\gamma^\mu d^r\right), && \text{(G.39)}\\ j^\mu_\pi &= \varepsilon_{3ij}\pi_i\partial^\mu\pi_j \equiv \pi^- \mathrm{i}\partial^\mu\pi^+ - \pi^+\mathrm{i}\partial^\mu\pi^-, && \text{(G.40)}\\ j^\mu_{\psi^r\pi} &= -\frac{1}{4F^2}\bar\psi^r\gamma^\mu\left(\vec\pi^2\tau_3 - \vec\pi\cdot\vec\tau\,\pi^0\right)\psi^r - \varepsilon_{3ij}\frac{\pi_j}{2F}\bar\psi^r\gamma^\mu\gamma^5\tau_i\psi^r, && \text{(G.41)}\\ \delta j^\mu_{\psi^r} &= \bar\psi^r(\hat Z-1)\gamma^\mu Q\psi^r = \frac{1}{3}(\hat Z-1)\left(2\bar u^r\gamma^\mu u^r - \bar d^r\gamma^\mu d^r\right), && \text{(G.42)} \end{aligned}$$

where Q is the quark charge matrix.

The renormalized nucleon charge Q^r_N at one loop is defined as

$$Q^r_N = {}^N\!\langle\phi_0|\sum_{n=0}^{2}\frac{\mathrm{i}^n}{n!}\int\delta(t)d^4x d^4x_1\ldots d^4x_n T[\mathcal{L}^{W;\,\rm str}_{I;r}(x_1)\ldots\mathcal{L}^{W;\,\rm str}_{I;r}(x_n)j^0_r(x)]|\phi_0\rangle^N_c. \tag{G.43}$$

Charge conservation requires that the nucleon charge is not changed after renormalization, that is

$$Q^r_N \equiv Q_N = \begin{cases} 1 & \text{for } N = \mathrm{p} \text{ (proton)},\\ 0 & \text{for } N = \mathrm{n} \text{ (neutron)}. \end{cases} \tag{G.44}$$

Thereby, Q_N is the nucleon charge in the three-quark core approximation, which is defined as the expectation value of the quark charge operator $\hat{Q}_\psi = \int d^3x j^0_\psi(x)$ taken between the unperturbed 3q-states $|\phi_0\rangle$:

$$Q_N = {}^N\langle\phi_0| \int \delta(t) d^4x j^0_\psi(x) |\phi_0\rangle^N. \tag{G.45}$$

Eqs.(G.43)-(G.45) completely define the charge conservation condition within our approach.

From nucleon charge conservation we obtain a condition on the renormalization constant $\hat{Z}$. In the one-loop approximation following diagrams contribute to the nucleon charge : the three-quark diagram (Fig. (G.1)) with an insertion of the quark current $j^\mu_{\psi^r}$, the three-quark diagram (Fig. (G.2)) with the counterterm $\delta j^\mu_{\psi^r}$ (three-quark counterterm diagram), the self-energy (Figs. (G.3) and (G.4)), the vertex correction diagram (Fig. (G.8)) with the quark current $j^\mu_{\psi^r}$, and finally the meson-cloud diagram (Fig. (G.9)) generated by the pion current j^μ_π.

G.4.1 Quark Current Component

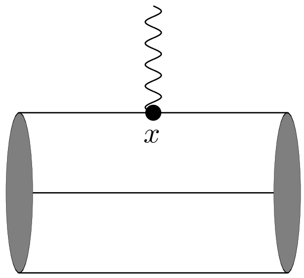

Figure G.1: Three-quark core diagram

The contribution of the three-quark diagram Fig. (G.1), with an insertion of the quark current $j^\mu_{\psi^r}$, to the nucleon charge is trivially given by

$$Q^{r;3q}_N = {}^N\langle\phi_0| \int \delta(t) d^4x j^0_{\psi^r}(x) |\phi_0\rangle^N_c \equiv Q_N. \tag{G.46}$$

G.4.2 Counterterm Current Component

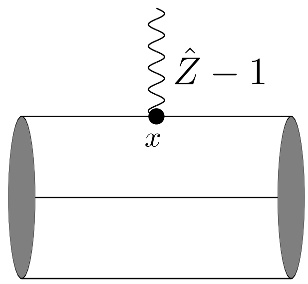

Figure G.2: Three-quark core counterterm diagram

The three-quark counterterm diagram Fig. (G.2), with the counterterm $\delta j^{\mu}_{\psi^r}$, is simply related to the one of Fig. (G.1) with

$$\begin{aligned} Q_N^{r;CT} &= (\hat{Z}-1)Q^{r;3q} \\ &= (\hat{Z}-1)\,{}^{N}\!\left\langle \phi_0 \right| \int \delta(t) d^4x j^0_{\psi^r}(x) \left| \phi_0 \right\rangle_c^N \\ &= (\hat{Z}-1)Q_N. \end{aligned} \tag{G.47}$$

G.4.3 Pion Current Component

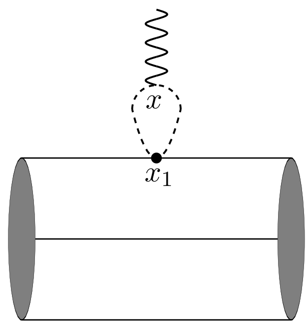

Figure G.3: Pion loop I diagram

The pion loop diagram Fig. (G.3), with the pion current j^{μ}_{π} results in

$$
\begin{aligned}
&Q_N^{r;LoopI} \\
&= \;{}^N\!\big\langle\phi_0|\mathrm{i}\int\delta(t)d^4xd^4x_1T[\mathcal{L}_I^{W;\,\mathrm{str}}(x_1)j_\pi^0(x)]|\phi_0\big\rangle_c^N \\
&= \;2\,{}^N\!\big\langle\phi_0|\mathrm{i}\int\delta(t)d^4xd^4x_1N\bigg\{\bigg[-\frac{\varepsilon_{ijk}}{4F^2}\pi_i\partial_\mu\pi_j\bar{\psi}\gamma^\mu\tau_k\psi\bigg]_{x_1}\bigg[\varepsilon_{3lm}\pi_l\frac{\partial}{\partial t}\pi_m\bigg]_x\bigg\}|\phi_0\big\rangle_c^N \\
&= \;-\frac{2\mathrm{i}\varepsilon_{ijk}\varepsilon_{3lm}}{4F^2}\,{}^N\!\big\langle\phi_0|\int\delta(t)d^4xd^4x_1\bar{\psi}(x_1)\gamma^\mu\tau_k\psi(x_1) \\
&\quad\times\pi_i(x_1)\pi_l(x)\partial_\mu\pi_j(x_1)\frac{\partial}{\partial t}\pi_m(x)|\phi_0\big\rangle_c^N \\
&= \;\frac{2\mathrm{i}\varepsilon_{ijk}\varepsilon_{3ij}}{4F^2(2\pi)^8}\,{}^N\!\big\langle\phi_0|b_0^\dagger\int d^3xd^3x_1d^4k_1d^4k_2k_2^0\bar{u}_0(x_1)\gamma^\mu k_{2\mu}\tau_ku_0(x_1) \\
&\quad\times\bigg[\frac{\mathrm{e}^{\mathrm{i}\vec{k}_1\cdot(\vec{x}-\vec{x}_1)}}{M_\pi^2-k_1^2-\mathrm{i}\epsilon}\bigg]\bigg[\frac{\mathrm{e}^{\mathrm{i}\vec{k}_2\cdot(\vec{x}_1-\vec{x})}}{M_\pi^2-k_2^2-\mathrm{i}\epsilon}\bigg]\int dtdt_1\delta(t)\mathrm{e}^{-\mathrm{i}k_1^0(t-t_1)}\mathrm{e}^{-\mathrm{i}k_2^0(t_1-t)}b_0|\phi_0\big\rangle^N \\
&= \;\frac{2\mathrm{i}\varepsilon_{ijk}\varepsilon_{3ij}}{4F^2(2\pi)^7}\,{}^N\!\big\langle\phi_0|b_0^\dagger\int d^3xd^3x_1d^3k_1d^3k_2\mathrm{e}^{\mathrm{i}\vec{k}_1\cdot(\vec{x}-\vec{x}_1)}\mathrm{e}^{\mathrm{i}\vec{k}_2\cdot(\vec{x}_1-\vec{x})} \\
&\quad\times\int dk_1^0dk_2^0\bigg[\frac{1}{M_\pi^2-(k_1^0)^2+\vec{k}_1^2-\mathrm{i}\epsilon}\bigg]\bigg[\frac{1}{M_\pi^2-(k_2^0)^2+\vec{k}_2^2-\mathrm{i}\epsilon}\bigg]\delta(k_2^0-k_1^0) \\
&\quad\times\Big\{(k_2^0)^2\left[\bar{u}_0(x_1)\gamma^0\tau_ku_0(x_1)\right]-k_2^0\Big[\bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}_2\tau_ku_0(x_1)\Big]\Big\}\,b_0|\phi_0\big\rangle^N \\
&= \;\frac{2\mathrm{i}\varepsilon_{ijk}\varepsilon_{3ij}}{4F^2(2\pi)^7}\,{}^N\!\big\langle\phi_0|b_0^\dagger\int d^3x_1d^3k_1\left[\bar{u}_0(x_1)\gamma^0\tau_ku_0(x_1)\right]\mathrm{e}^{-\mathrm{i}\vec{k}_1\cdot\vec{x}_1} \\
&\quad\int d^3k_2\mathrm{e}^{\mathrm{i}\vec{k}_2\cdot\vec{x}_1}\bigg[-\frac{\pi\mathrm{i}}{\omega(k_1^2)+\omega(k_2^2)}\bigg]\int d^3x\mathrm{e}^{-\mathrm{i}\vec{x}\cdot(\vec{k}_2-\vec{k}_1)}b_0|\phi_0\big\rangle^N \\
&= \;\frac{\varepsilon_{ijk}\varepsilon_{3ij}}{4F^2(2\pi)^3}\,{}^N\!\big\langle\phi_0|b_0^\dagger\int d^3k_1\frac{1}{2\omega(k_1^2)}\bigg[\int d^3x_1\bar{u}_0(x_1)\gamma^0\tau_ku_0(x_1)\bigg]b_0|\phi_0\big\rangle^N \\
&= \;\frac{1}{4F^2(2\pi)^3}\,{}^N\!\big\langle\phi_0|b_0^\dagger\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_0\int d^3k\frac{1}{\omega(k^2)}\tau_3\left[\chi_s\chi_f\chi_c\right]_0b_0|\phi_0\big\rangle^N. \qquad \text{(G.48)}
\end{aligned}
$$

G.4.4 Quark-Pion Current Component

The contribution of the nucleon charge from the quark-pion current $j^\mu_{\psi^r\pi}$,

$$
j^\mu_{\psi^r\pi}=j^{\mu;1}_{\psi^r\pi}+j^{\mu;2}_{\psi^r\pi}+j^{\mu;3}_{\psi^r\pi}, \qquad \text{(G.49)}
$$

where

$$
\begin{aligned}
j^{\mu;1}_{\psi^r\pi} &\equiv -\frac{1}{4F^2}\bar{\psi}^r\gamma^\mu\vec{\pi}^2\tau_3\psi^r, &\text{(G.50)}\\
j^{\mu;2}_{\psi^r\pi} &\equiv \frac{1}{4F^2}\bar{\psi}^r\gamma^\mu\vec{\pi}\cdot\vec{\tau}\,\pi^0\psi^r, &\text{(G.51)}\\
j^{\mu;3}_{\psi^r\pi} &\equiv -\varepsilon_{3ij}\frac{\pi_j}{2F}\bar{\psi}^r\gamma^\mu\gamma^5\tau_i\psi^r, &\text{(G.52)}
\end{aligned}
$$

can be shown separately.

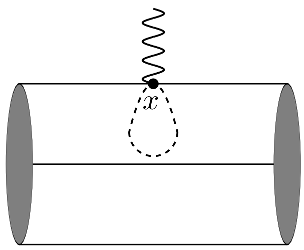

Figure G.4: Pion loop II diagram

Pion Loop II;1

The charge contribution to the nucleon with the insertion of $j^{\mu;1}_{\psi^r\pi}$ yields $Q_N^{r;LoopII;1}$

$$
\begin{aligned}
&= {}^N\!\langle\phi_0|\int\delta(t)d^4x j^{0;1}_{\psi^r\pi}(x)]|\phi_0\rangle^N_c\\
&= {}^N\!\langle\phi_0|\int\delta(t)d^4x\left[-\frac{1}{4F^2}\bar{\psi}(x)\gamma^0\pi_i(x)\pi_i(x)\tau_3\psi(x)\right]|\phi_0\rangle^N_c\\
&= \frac{3\mathrm{i}}{4F^2(2\pi)^4}\,{}^N\!\langle\phi_0|b_0^\dagger\int d^3x\bar{u}_0(x)\gamma^0\tau_3u_0(x)\int d^3kdk_0\frac{1}{M_\pi^2-k_0^2+\vec{k}^2-\mathrm{i}\epsilon}b_0|\phi_0\rangle^N\\
&= -\frac{3\pi}{4F^2(2\pi)^4}\,{}^N\!\langle\phi_0|b_0^\dagger\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_0\int d^3k\frac{1}{\omega(k^2)}\tau_3\left[\chi_s\chi_f\chi_c\right]_0 b_0|\phi_0\rangle^N. \qquad \text{(G.53)}
\end{aligned}
$$

Pion Loop II;2

The charge contribution to the nucleon with the insertion of $j^{\mu;2}_{\psi^r\pi}$ results in

$$
\begin{aligned}
&Q_N^{r;LoopII;2} \\
&= {}^N\big\langle\phi_0\big|\int\delta(t)d^4x j^{0;2}_{\psi^r\pi}(x)]\big|\phi_0\big\rangle_c^N \\
&= {}^N\big\langle\phi_0\big|\int\delta(t)d^4x\left[\frac{1}{4F^2}\bar{\psi}(x)\gamma^0\underbrace{\pi_i(x)\pi_3(x)}\tau_i\psi(x)\right]\big|\phi_0\big\rangle_c^N \\
&= -\frac{\mathrm{i}}{4F^2(2\pi)^4}\,{}^N\big\langle\phi_0\big|b_0^\dagger\int d^3x\bar{u}_0(x)\gamma^0\tau_3u_0(x)\int d^3kdk_0\frac{1}{M_\pi^2-k_0^2+\vec{k}^2-\mathrm{i}\epsilon}b_0\big|\phi_0\big\rangle^N \\
&= \frac{\pi}{4F^2(2\pi)^4}\,{}^N\big\langle\phi_0\big|b_0^\dagger\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_0\int d^3k\frac{1}{\omega(k^2)}\tau_3\left[\chi_s\chi_f\chi_c\right]_0 b_0\big|\phi_0\big\rangle^N. \qquad \text{(G.54)}
\end{aligned}
$$

For the pion loop correction to the nucleon charge, we obtain

$$
\begin{aligned}
Q_N^{r;Loop} &= Q_N^{r;LoopI}+Q_N^{r;LoopII;1}+Q_N^{r;LoopII;2} \\
&= \frac{2\pi-3\pi+\pi}{4F^2(2\pi)^4}\,{}^N\big\langle\phi_0\big|b_0^\dagger\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_0\int d^3k\frac{1}{\omega(k^2)}\tau_3\left[\chi_s\chi_f\chi_c\right]_0 b_0\big|\phi_0\big\rangle_c^N \\
&= 0. \qquad \text{(G.55)}
\end{aligned}
$$

Thereby, the contribution from the pion loop to the nucleon charge vanishes for the one loop correction.

The nucleon charge with the insertion of $j^{\mu;3}_{\psi^r\pi}$ are the self-energy I, II and the exchange term. The analytical expressions for each diagram are shown separately.

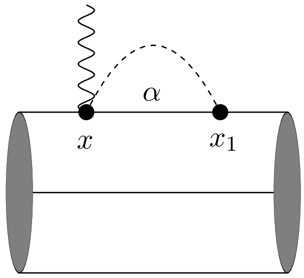

Figure G.5: Self energy I diagram

Self-Energy I

$$
\begin{aligned}
& Q_N^{r;SE;I}\Big|_\alpha \\
&= {}^N\big\langle\phi_0|\mathrm{i}\int\delta(t)d^4xd^4x_1T[\mathcal{L}_I^{W;\,\mathrm{str}}(x_1)j^{0;3}_{\psi^r\pi}(x)]|\phi_0\big\rangle_c^N \\
&= {}^N\big\langle\phi_0|\mathrm{i}\int\delta(t)d^4xd^4x_1N\Bigg\{\bigg[\frac{1}{2F}\partial_\mu\pi_k\bar\psi\gamma^\mu\gamma^5\tau_k\psi\bigg]_{x_1}\bigg[-\frac{\varepsilon_{3ij}}{2F}\pi_j\bar\psi\gamma^0\gamma^5\tau_i\psi\bigg]_x\Bigg\}|\phi_0\big\rangle_c^N \\
&= -\frac{\mathrm{i}\varepsilon_{3ij}}{4F^2}\,{}^N\big\langle\phi_0|\int\delta(t)d^4xd^4x_1\bar\psi(x_1)\gamma^\mu\gamma^5\tau_k\psi(x_1)\bar\psi(x)\gamma^0\gamma^5\tau_i\psi(x) \\
&\quad\times\partial_\mu\pi_k(x_1)\pi_j(x)|\phi_0\big\rangle_c^N \\
&= \frac{\mathrm{i}\varepsilon_{3ij}}{4F^2(2\pi)^4}\,{}^N\big\langle\phi_0|b_0^\dagger\int d^3xd^3x_1d^4k\frac{\mathrm{e}^{\mathrm{i}\vec k\cdot(\vec x_1-\vec x)}}{M_\pi^2-k^2-\mathrm{i}\epsilon} \\
&\quad\times\bar u_0(x_1)\gamma^\mu k_\mu\gamma^5\tau_ju_\alpha(x_1)\bar u_\alpha(x)\gamma^0\gamma^5\tau_iu_0(x) \\
&\quad\times\int dtdt_1\delta(t)\mathrm{e}^{\mathrm{i}\mathcal{E}_0t_1}\mathrm{e}^{-\mathrm{i}\mathcal{E}_\alpha(t_1-t)}\Theta(t_1-t)\mathrm{e}^{-\mathrm{i}\mathcal{E}_0t}\mathrm{e}^{-\mathrm{i}k_0(t_1-t)}b_0|\phi_0\big\rangle^N \\
&= \frac{\varepsilon_{3ij}}{4F^2(2\pi)^4}\,{}^N\big\langle\phi_0|b_0^\dagger\int d^3xd^3x_1d^3k\mathrm{e}^{\mathrm{i}\vec k\cdot(\vec x_1-\vec x)} \\
&\quad\int dk_0\bigg[\frac{1}{M_\pi^2-k_0^2+\vec k^2-\mathrm{i}\epsilon}\bigg]\bigg[\frac{1}{k_0+\Delta\mathcal{E}_\alpha-\mathrm{i}\eta}\bigg] \\
&\quad\times\Big[\bar u_0(x_1)(\gamma^0k_0-\vec\gamma\cdot\vec k)\gamma^5\tau_ju_\alpha(x_1)\bar u_\alpha(x)\gamma^0\gamma^5\tau_iu_0(x)\Big]b_0|\phi_0\big\rangle^N \\
&= \frac{\varepsilon_{3ij}\pi\mathrm{i}}{4F^2(2\pi)^4}\,{}^N\big\langle\phi_0|b_0^\dagger\int d^3k\frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\quad\times\Bigg\{\omega(k^2)\bigg[\int d^3x_1\bar u_0(x_1)\gamma^0\gamma^5\tau_ju_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec k\cdot\vec x_1}\bigg]\bigg[\int d^3x\bar u_\alpha(x)\gamma^0\gamma^5\tau_iu_0(x)\mathrm{e}^{-\mathrm{i}\vec k\cdot\vec x}\bigg] \\
&\quad-\bigg[\int d^3x_1\bar u_0(x_1)\vec\gamma\cdot\vec k\gamma^5\tau_ju_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec k\cdot\vec x_1}\bigg]\bigg[\int d^3x\bar u_\alpha(x)\gamma^0\gamma^5\tau_iu_0(x)\mathrm{e}^{-\mathrm{i}\vec k\cdot\vec x}\bigg]\Bigg\} \\
&\quad\times b_0|\phi_0\big\rangle^N \\
&= \frac{\varepsilon_{3ij}\pi\mathrm{i}}{4F^2(2\pi)^4}\,{}^N\big\langle\phi_0|b_0^\dagger\int d^3k\frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\quad\times\Bigg\{\omega(k^2)F_{II_\alpha}(k)F^\dagger_{II_\alpha}(k)\Big[(\vec\sigma\cdot\hat k)\tau_j\Big]_{0,\alpha}\Big[\tau_i(\vec\sigma\cdot\hat k)\Big]_{\alpha,0} \\
&\quad-F_{I_\alpha}(k)F^\dagger_{II_\alpha}(k)\Big[(\vec\sigma\cdot\hat k)\tau_j\Big]_{0,\alpha}\Big[\tau_i(\vec\sigma\cdot\hat k)\Big]_{\alpha,0}\Bigg\}b_0|\phi_0\big\rangle^N
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{4F^2(2\pi)^3}\, {}^N\!\big\langle \phi_0 | b_0^\dagger \left[\chi_c^\dagger \chi_f^\dagger \chi_s^\dagger\right]_0 \tau_3 \int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\quad \times \left\{ \left[\omega(k^2) F_{II_\alpha}(k) F^\dagger_{II_\alpha}(k) - F_{I_\alpha}(k) F^\dagger_{II_\alpha}(k)\right] \right\} \int_\Omega d\Omega\, (\vec{\sigma}\cdot\hat{k})(\vec{\sigma}\cdot\hat{k}) \\
&\quad \times \left[\chi_s \chi_f \chi_c\right]_0 b_0 | \phi_0 \big\rangle^N \\
&= \frac{q_N^{SE;I}}{2(2\pi F)^2} \int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \left[\omega(k^2) F_{II_\alpha}(k) F^\dagger_{II_\alpha}(k) - F_{I_\alpha}(k) F^\dagger_{II_\alpha}(k)\right].
\end{aligned}
\tag{G.56}
$$

We have used the Pauli matrices algebra

$$\varepsilon_{3ij}\tau_j\tau_i = -2\,\mathrm{i}\,\tau_3. \tag{G.57}$$

After the projection on the nucleon we obtain

$$q_p^{SE;I} \equiv \big\langle p\uparrow | \sum_{i=1}^{3} (\tau_3)^i | p\uparrow \big\rangle = \quad 1, \tag{G.58}$$

$$q_n^{SE;I} \equiv \big\langle n\uparrow | \sum_{i=1}^{3} (\tau_3)^i | n\uparrow \big\rangle = -1. \tag{G.59}$$

Self-Energy II

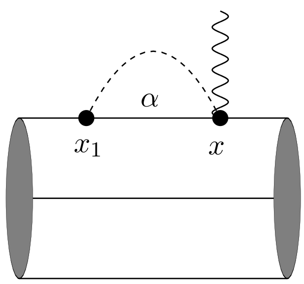

Figure G.6: Self energy II diagram

$$
\begin{aligned}
&Q_N^{r;SE;II}\Big|_\alpha \\
&= {}^N\!\big\langle \phi_0 | \mathrm{i} \int \delta(t) d^4x d^4x_1 T[\mathcal{L}_I^{W;\,\mathrm{str}}(x_1) j^{0;3}_{\psi^r\pi}(x)] | \phi_0 \big\rangle_c^N \\
&= {}^N\!\big\langle \phi_0 | \mathrm{i} \int \delta(t) d^4x d^4x_1 N \left\{ \left[-\frac{\varepsilon_{3ij}}{2F} \pi_j \bar{\psi}\gamma^0\gamma^5\tau_i\psi \right]_x \left[\frac{1}{2F}\partial_\mu\pi_k\bar{\psi}\gamma^\mu\gamma^5\tau_k\psi\right]_{x_1} \right\} | \phi_0 \big\rangle_c^N
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\mathrm{i}\varepsilon_{3ij}}{4F^2}\, {}^N\!\big\langle\phi_0\big| \int \delta(t) d^4x d^4x_1 \bar{\psi}(x)\gamma^0\gamma^5\tau_i \psi(x)\bar{\psi}(x_1)\gamma^\mu\gamma^5\tau_k\psi(x_1) \\
&\quad \times \pi_j(x)\partial_\mu\pi_k(x_1)\big|\phi_0\big\rangle_c^N \\
&= -\frac{\mathrm{i}\varepsilon_{3ij}}{4F^2(2\pi)^4}\, {}^N\!\big\langle\phi_0\big| b_0^\dagger \int d^3x d^3x_1 d^4k \frac{\mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}-\vec{x}_1)}}{M_\pi^2 - k^2 - \mathrm{i}\epsilon} \\
&\quad \times \bar{u}_0(x)\gamma^0\gamma^5\tau_i u_\alpha(x)\bar{u}_\alpha(x_1)\gamma^\mu k_\mu\gamma^5\tau_j u_0(x_1) \\
&\quad \times \int dt dt_1 \delta(t) \mathrm{e}^{\mathrm{i}\mathcal{E}_0 t}\mathrm{e}^{-\mathrm{i}\mathcal{E}_\alpha(t-t_1)}\Theta(t-t_1)\mathrm{e}^{-\mathrm{i}\mathcal{E}_0 t_1}\mathrm{e}^{-\mathrm{i}k_0(t-t_1)} b_0\big|\phi_0\big\rangle^N \\
&= -\frac{\varepsilon_{3ij}}{4F^2(2\pi)^4}\, {}^N\!\big\langle\phi_0\big| b_0^\dagger \int d^3x d^3x_1 d^3k \mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}-\vec{x}_1)} \\
&\quad \int dk_0 \left[\frac{1}{M_\pi^2 - k_0^2 + \vec{k}^2 - \mathrm{i}\epsilon}\right]\left[\frac{1}{k_0 + \Delta\mathcal{E}_\alpha + \mathrm{i}\eta}\right] \\
&\quad \times \left[\bar{u}_0(x)\gamma^0\gamma^5\tau_i u_\alpha(x)\bar{u}_\alpha(x_1)(\gamma^0 k_0 - \vec{\gamma}\cdot\vec{k})\gamma^5\tau_j u_0(x_1)\right] b_0\big|\phi_0\big\rangle^N \\
&= -\frac{\varepsilon_{3ij}\pi\mathrm{i}}{4F^2(2\pi)^4}\, {}^N\!\big\langle\phi_0\big| b_0^\dagger \int d^3k \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\quad \times \Bigg\{ \omega(k^2)\left[\int d^3x \bar{u}_0(x)\gamma^0\gamma^5\tau_i u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}\right]\left[\int d^3x_1 \bar{u}_\alpha(x_1)\gamma^0\gamma^5\tau_j u_0(x_1)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_1}\right] \\
&\quad - \left[\int d^3x \bar{u}_0(x)\gamma^0\gamma^5\tau_i u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}\right]\left[\int d^3x_1 \bar{u}_\alpha(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_0(x_1)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_1}\right]\Bigg\} \\
&\quad \times b_0\big|\phi_0\big\rangle^N \\
&= -\frac{\varepsilon_{3ij}\pi\mathrm{i}}{4F^2(2\pi)^4}\, {}^N\!\big\langle\phi_0\big| b_0^\dagger \int d^3k \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\quad \times \Bigg\{ \omega(k^2) F_{II_\alpha}(k) F_{II_\alpha}^\dagger(k) \left[(\vec{\sigma}\cdot\hat{k})\tau_i\right]_{0,\alpha}\left[\tau_j(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0} \\
&\quad - F_{II,\alpha}(k) F_{I_\alpha}^\dagger(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_i\right]_{0,\alpha}\left[\tau_j(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0}\Bigg\} b_0\big|\phi_0\big\rangle^N \\
&= \frac{1}{4F^2(2\pi)^3}\, {}^N\!\big\langle\phi_0\big| b_0^\dagger \left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_0 \tau_3 \int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\quad \times \Bigg\{ \left[\omega(k^2) F_{II_\alpha}(k) F_{II_\alpha}^\dagger(k) - F_{II_\alpha}(k) F_{I_\alpha}^\dagger(k)\right] \Bigg\} \int_\Omega d\Omega\, (\vec{\sigma}\cdot\hat{k})(\vec{\sigma}\cdot\hat{k}) \\
&\quad \times \left[\chi_s\chi_f\chi_c\right]_0 b_0\big|\phi_0\big\rangle^N \\
&= \frac{q_N^{SE;II}}{2(2\pi F)^2}\int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)}\left[\omega(k^2) F_{II_\alpha}(k) F_{II_\alpha}^\dagger(k) - F_{II_\alpha}(k) F_{I_\alpha}^\dagger(k)\right].
\end{aligned}
\tag{G.60}
$$

Here we also obtain

$$q_p^{SE;II} \equiv \left\langle p\uparrow \right| \sum_{i=1}^{3} (\tau_3)^i \left| p\uparrow \right\rangle = \quad 1, \tag{G.61}$$

$$q_n^{SE;II} \equiv \left\langle n\uparrow \right| \sum_{i=1}^{3} (\tau_3)^i \left| n\uparrow \right\rangle = -1. \tag{G.62}$$

Exchange

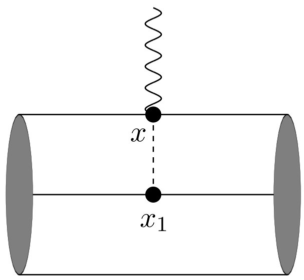

Figure G.7: Exchange diagram

$$
\begin{aligned}
&Q_N^{r;EX} \\
&= {}^N\!\left\langle \phi_0 \right| \mathrm{i} \int \delta(t) d^4x d^4x_1 T[\mathcal{L}_I^{W;\,\mathrm{str}}(x_1) j^{;0;3}_{\psi^r\pi}(x)] \left| \phi_0 \right\rangle_c^N \\
&= {}^N\!\left\langle \phi_0 \right| \mathrm{i} \int \delta(t) d^4x d^4x_1 N\left\{ \left[\frac{1}{2F} \partial_\mu \pi_k \bar{\psi} \gamma^\mu \gamma^5 \tau_k \psi \right]_{x_1} \left[-\frac{\varepsilon_{3ij}}{2F} \pi_j \bar{\psi} \gamma^0 \gamma^5 \tau_i \psi \right]_x \right\} \left| \phi_0 \right\rangle_c^N \\
&= -\frac{\mathrm{i}\varepsilon_{3ij}}{4F^2} {}^N\!\left\langle \phi_0 \right| \int \delta(t) d^4x d^4x_1 \bar{\psi}(x_1) \gamma^\mu \gamma^5 \tau_k \psi(x_1) \bar{\psi}(x) \gamma^0 \gamma^5 \tau_i \psi(x) \\
&\quad \times \partial_\mu \pi_k(x_1) \pi_j(x) \left| \phi_0 \right\rangle_c^N \\
&= \frac{\mathrm{i}\varepsilon_{3ij}}{4F^2(2\pi)^4} {}^N\!\left\langle \phi_0 \right| b_0^\dagger(x_1) b_0^\dagger(x) \int d^3x d^3x_1 d^4k \frac{\mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}_1-\vec{x})}}{M_\pi^2 - k^2 - \mathrm{i}\epsilon} \\
&\quad \times \bar{u}_0(x_1) \gamma^\mu k_\mu \gamma^5 \tau_j u_0(x_1) \bar{u}_0(x) \gamma^0 \gamma^5 \tau_i u_0(x) \\
&\quad \times \int dt dt_1 \delta(t) \mathrm{e}^{-\mathrm{i}k_0(t_1-t)} b_0(x) b_0(x_1) \left| \phi_0 \right\rangle^N \\
&= \frac{\mathrm{i}\varepsilon_{3ij}}{4F^2(2\pi)^3} {}^N\!\left\langle \phi_0 \right| b_0^\dagger(x_1) b_0^\dagger(x) \int d^3x d^3x_1 d^3k \mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}_1-\vec{x})} \\
&\quad \times \int dk_0 \left[\frac{\delta(k_0)}{M_\pi^2 - k_0^2 + \vec{k}^2 - \mathrm{i}\epsilon} \right] \\
&\quad \times \left[\bar{u}_0(x_1) \left(\gamma^0 k_0 - \vec{\gamma}\cdot\vec{k} \right) \gamma^5 \tau_j u_0(x_1) \bar{u}_0(x) \gamma^0 \gamma^5 \tau_i u_0(x) \right] b_0(x) b_0(x_1) \left| \phi_0 \right\rangle^N
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{\mathrm{i}\varepsilon_{3ij}}{4F^2(2\pi)^3}\,{}^N\!\big\langle\phi_0|b_0^\dagger(x_1)b_0^\dagger(x)\int d^3k\frac{1}{M_\pi^2+\vec{k}^2} \\
&\quad\times\left[\int d^3x_1\bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_0(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x_1}}\right]\left[\int d^3x\bar{u}_0(x)\gamma^0\gamma^5\tau_i u_0(x)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}}\right] \\
&\quad\times b_0(x)b_0(x_1)|\phi_0\big\rangle^N \\
&= 0.
\end{aligned}
\tag{G.63}
$$

Because of this condition:

$$
\int d^3x\bar{u}_0(x)\gamma^0\gamma^5\tau_i u_0(x)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}} = 0. \tag{G.64}
$$

G.4.5 Vertex Correction

The insertion of the quark current $j^\mu_{\psi^r}$ with two vertices of the pion-quark interaction results for the nucleon charge in

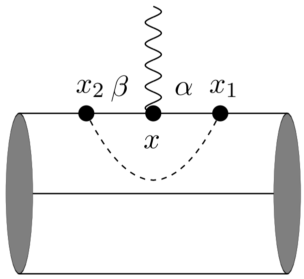

Figure G.8: Vertex correction diagram

$$
\begin{aligned}
&Q_N^{r;VC}\Big|_{\beta,\alpha} \\
&= {}^N\!\big\langle\phi_0|\frac{\mathrm{i}^2}{2!}\int\delta(t)d^4xd^4x_1d^4x_2T[\mathcal{L}_I^{W;\,\mathrm{str}}(x_1)\mathcal{L}_I^{W;\,\mathrm{str}}(x_2)j^0_{\psi^r}(x)]|\phi_0\big\rangle_c^N \\
&= 2\,{}^N\!\big\langle\phi_0|\frac{-1}{2}\int\delta(t)d^4xd^4x_1d^4x_2 \\
&\quad\times N\left\{\left[\frac{1}{2F}\,\partial_\mu\pi_i\bar{\psi}\gamma^\mu\gamma^5\tau_i\psi\right]_{x_1}\left[\frac{1}{2F}\,\partial_\nu\pi_j\bar{\psi}\gamma^\nu\gamma^5\tau_j\psi\right]_{x_2}\left[\bar{\psi}\gamma^0Q\psi\right]_x\right\}|\phi_0\big\rangle_c^N
\end{aligned}
$$

$$
\begin{aligned}
=&\frac{-1}{4F^2}\,{}^N\!\big\langle\phi_0\big|\int \delta(t)d^4xd^4x_1d^4x_2\\
&\times\bar{\psi}(x_1)\gamma^\mu\gamma^5\tau_i\underbrace{\psi(x_1)\bar{\psi}(x)}_{}\gamma^0Q\underbrace{\psi(x)\bar{\psi}}_{}(x_2)\gamma^\nu\gamma^5\tau_j\psi(x_2)\partial_\mu\underbrace{\pi_i(x_1)\partial_\nu\pi_j}_{}(x_2)\big|\phi_0\big\rangle_c^N\\
=&\frac{\mathrm{i}}{4F^2(2\pi)^4}\,{}^N\!\big\langle\phi_0\big|b_0^\dagger\int d^3xd^3x_1d^3x_2d^4k\bar{u}_0(x_1)\gamma^\mu k_\mu\gamma^5\tau_iu_\alpha(x_1)\\
&\times\bar{u}_\alpha(x)\gamma^0Q\,u_\beta(x)\bar{u}_\beta(x_2)\gamma^\nu k_\nu\gamma^5\tau_iu_0(x_2)\frac{\mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}}{M_\pi^2-k^2-\mathrm{i}\epsilon}\\
&\times\int dtdt_1dt_2\delta(t)\mathrm{e}^{\mathrm{i}\varepsilon_0t_1}\mathrm{e}^{-\mathrm{i}\varepsilon_\alpha(t_1-t)}\Theta(t_1-t)\mathrm{e}^{-\mathrm{i}\varepsilon_\beta(t-t_2)}\Theta(t-t_2)\mathrm{e}^{-\mathrm{i}\varepsilon_0t_2}\mathrm{e}^{-\mathrm{i}k_0(t_1-t_2)}\\
&b_0\big|\phi_0\big\rangle^N\\
=&\frac{-\mathrm{i}}{4F^2(2\pi)^4}\,{}^N\!\big\langle\phi_0\big|b_0^\dagger\int d^3xd^3x_1d^3x_2d^3k\mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}\\
&\times\int dk_0\left[\frac{1}{M_\pi^2-k_0^2+\vec{k}^2-\mathrm{i}\epsilon}\right]\left[\frac{1}{k_0+\Delta\mathcal{E}_\alpha-\mathrm{i}\eta}\right]\left[\frac{1}{k_0+\Delta\mathcal{E}_\beta-\mathrm{i}\eta}\right]\\
&\times\bar{u}_0(x_1)(\gamma^0k_0-\vec{\gamma}\cdot\vec{k})\gamma^5\tau_iu_\alpha(x_1)\bar{u}_\alpha(x)\gamma^0Q\,u_\beta(x)\\
&\times\bar{u}_\beta(x_2)(\gamma^0k_0-\vec{\gamma}\cdot\vec{k})\gamma^5\tau_iu_0(x_2)b_0\big|\phi_0\big\rangle^N\\
=&\frac{1}{8F^2(2\pi)^3}\,{}^N\!\big\langle\phi_0\big|b_0^\dagger\int d^3k\frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)^2}\\
&\times\Bigg\{\left[\int d^3x_1\bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_iu_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\right]Q_{\alpha\alpha}\left[\int d^3x_2\bar{u}_\alpha(x_2)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_iu_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\right]\\
&-\omega(k^2)\left[\int d^3x_1\bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_iu_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\right]Q_{\alpha\alpha}\left[\int d^3x_2\bar{u}_\alpha(x_2)\gamma^0\gamma^5\tau_iu_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\right]\\
&-\omega(k^2)\left[\int d^3x_1\bar{u}_0(x_1)\gamma^0\gamma^5\tau_iu_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\right]Q_{\alpha\alpha}\left[\int d^3x_2\bar{u}_\alpha(x_2)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_iu_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\right]\\
&+\omega^2(k^2)\left[\int d^3x_1\bar{u}_0(x_1)\gamma^0\gamma^5\tau_iu_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\right]Q_{\alpha\alpha}\left[\int d^3x_2\bar{u}_\alpha(x_2)\gamma^0\gamma^5\tau_iu_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\right]\Bigg\}\\
&b_0\big|\phi_0\big\rangle^N. \qquad\qquad (G.65)
\end{aligned}
$$

Here we use

$$
\begin{aligned}
\int d^3x\bar{u}_\alpha(x)\gamma^0Q\,u_\beta(x) &= \delta_{\alpha\beta}\,Q_{\alpha\beta}\\
&\equiv \delta_{\alpha\beta}\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_\alpha Q\left[\chi_s\chi_f\chi_c\right]_\beta. \qquad (G.66)
\end{aligned}
$$

We obtain

$$
\begin{aligned}
Q_N^{r;VC} &= \frac{1}{8F^2(2\pi)^3}\,{}^N\!\left\langle \phi_0 | b_0^\dagger \int d^3k \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)^2}\right. \\
&\times\left\{ F_{I_\alpha}(k)F_{I_\alpha}^\dagger(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_i\right]_{0,\alpha} Q_{\alpha\alpha}\left[\tau_i(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0}\right. \\
&-\omega(k^2)F_{I_\alpha}(k)F_{II_\alpha}^\dagger(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_i\right]_{0,\alpha} Q_{\alpha\alpha}\left[\tau_i(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0} \\
&-\omega(k^2)F_{II_\alpha}(k)F_{I_\alpha}^\dagger(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_i\right]_{0,\alpha} Q_{\alpha\alpha}\left[\tau_i(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0} \\
&\left.\left.+\omega^2(k^2)F_{II_\alpha}(k)F_{II_\alpha}^\dagger(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_i\right]_{0,\alpha} Q_{\alpha\alpha}\left[\tau_i(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0}\right\} b_0|\phi_0\right\rangle^N \\
&= \frac{1}{8F^2(2\pi)^3}\,{}^N\!\left\langle \phi_0 | b_0^\dagger \left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_0 \tau_i Q \tau_i \int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)^2}\right. \\
&\times\left[F_{I_\alpha}(k)F_{I_\alpha}^\dagger(k) - 2\omega(k^2)F_{I_\alpha}(k)F_{II_\alpha}^\dagger(k) + \omega^2(k^2)F_{II_\alpha}(k)F_{II_\alpha}^\dagger(k)\right] \\
&\times\left.\int_\Omega d\Omega(\vec{\sigma}\cdot\hat{k})(\vec{\sigma}\cdot\hat{k})\left[\chi_s\chi_f\chi_c\right]_0 b_0|\phi_0\right\rangle^N \\
&= \frac{q_N^{VC}}{4(2\pi F)^2}\int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)^2} \\
&\times\left[F_{I_\alpha}(k)F_{I_\alpha}^\dagger(k) - 2\omega(k^2)F_{I_\alpha}(k)F_{II_\alpha}^\dagger(k) + \omega^2(k^2)F_{II_\alpha}(k)F_{II_\alpha}^\dagger(k)\right].
\end{aligned}
\tag{G.67}
$$

When projecting on the nucleon, we obtain

$$
q_p^{VC} \equiv \sum_{j=1}^{3}\langle p\uparrow|\sum_{i=1}^{3}[\tau_j\, Q\, \tau_j]^{(i)}|p\uparrow\rangle = 1, \tag{G.68}
$$

$$
q_n^{VC} \equiv \sum_{j=1}^{3}\langle n\uparrow|\sum_{i=1}^{3}[\tau_j\, Q\, \tau_j]^{(i)}|n\uparrow\rangle = 2. \tag{G.69}
$$

G.4.6 Meson cloud

The insertion of the pion current j_π^μ with two vertices of the pion-quark interaction gives

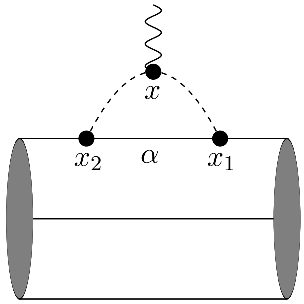

Figure G.9: Meson cloud diagram

$$
\begin{aligned}
& Q_N^{r;MC} \\
&= {}^N\big\langle\phi_0\big|\frac{\mathrm{i}^2}{2!}\int\delta(t)d^4xd^4x_1d^4x_2T[\mathcal{L}_I^{W;Str}(x_1)\mathcal{L}_I^{W;Str}(x_2)j_\pi^0(x)]\big|\phi_0\big\rangle_c^N \\
&= 4\,{}^N\big\langle\phi_0\big|\frac{-1}{2}\int\delta(t)d^4xd^4x_1d^4x_2 \\
&\quad\times N\left\{\left[\frac{1}{2F}\,\partial_\mu\pi_i\bar{\psi}\gamma^\mu\gamma^5\tau_i\psi\right]_{x_1}\left[\frac{1}{2F}\,\partial_\nu\pi_j\bar{\psi}\gamma^\nu\gamma^5\tau_j\psi\right]_{x_2}\left[\varepsilon_{3kl}\pi_k\frac{\partial}{\partial t}\pi_l\right]_x\right\}\big|\phi_0\big\rangle_c^N \\
&= \frac{-\varepsilon_{3kl}}{2F^2}\,{}^N\big\langle\phi_0\big|\int\delta(t)d^4xd^4x_1d^4x_2 \\
&\quad\times\bar{\psi}(x_1)\gamma^\mu\gamma^5\tau_i\psi(x_1)\bar{\psi}(x_2)\gamma^\nu\gamma^5\tau_j\psi(x_2)\partial_\mu\pi_i(x_1)\pi_k(x)\partial_\nu\pi_j(x_2)\frac{\partial}{\partial t}\pi_l(x)\big|\phi_0\big\rangle_c^N \\
&= \frac{-\mathrm{i}\varepsilon_{3ij}}{2F^2(2\pi)^8}\,{}^N\big\langle\phi_0\big|b_0^\dagger\int d^3xd^3x_1d^3x_2d^4k_1d^4k_2\left[\frac{\mathrm{e}^{\mathrm{i}\vec{k}_1\cdot(\vec{x}_1-\vec{x})}}{M_\pi^2-k_1^2-\mathrm{i}\epsilon}\right]\left[\frac{\mathrm{e}^{\mathrm{i}\vec{k}_2\cdot(\vec{x}-\vec{x}_2)}}{M_\pi^2-k_2^2-\mathrm{i}\epsilon}\right] \\
&\quad\times\bar{u}_0(x_1)\gamma^\mu k_{1\mu}\gamma^5\tau_iu_\alpha(x_1)\bar{u}_\alpha(x_2)\gamma^\nu k_{2\nu}\gamma^5k_2^0\tau_ju_0(x_2) \\
&\quad\times\int dtdt_1dt_2\delta(t)\mathrm{e}^{\mathrm{i}\varepsilon_0t_1}\mathrm{e}^{-\mathrm{i}\varepsilon_\alpha(t_1-t_2)}\Theta(t_1-t_2)\mathrm{e}^{-\mathrm{i}\varepsilon_0t_2}\mathrm{e}^{-\mathrm{i}k_1^0(t_1-t)}\mathrm{e}^{-\mathrm{i}k_2^0(t-t_2)}b_0\big|\phi_0\big\rangle^N \\
&= \frac{-\varepsilon_{3ij}}{2F^2(2\pi)^7}\,{}^N\big\langle\phi_0\big|b_0^\dagger\int d^3xd^3x_1d^3x_2d^3k_1d^3k_2\mathrm{e}^{\mathrm{i}\vec{k}_1\cdot\vec{x}_1}\mathrm{e}^{-\mathrm{i}\vec{k}_2\cdot\vec{x}_2}\int d^3x\mathrm{e}^{-\mathrm{i}(\vec{k}_1-\vec{k}_2)\cdot\vec{x}} \\
&\quad\times\int dk_1^0dk_2^0\left[\frac{1}{M_\pi^2-(k_1^0)^2+\vec{k}_1^2-\mathrm{i}\epsilon}\right]\left[\frac{1}{M_\pi^2-(k_2^0)^2+\vec{k}_2^2-\mathrm{i}\epsilon}\right]\left[\frac{\delta(k_1^0-k_2^0)k_2^0}{k_1^0+\Delta\mathcal{E}_\alpha-\mathrm{i}\eta}\right] \\
&\quad\times\left[\bar{u}_0(x_1)(\gamma^0k_1^0-\vec{\gamma}\cdot\vec{k}_1)\gamma^5\tau_iu_\alpha(x_1)\bar{u}_\alpha(x_2)(\gamma^0k_2^0-\vec{\gamma}\cdot\vec{k}_2)\gamma^5\tau_ju_0(x_2)\right]b_0\big|\phi_0\big\rangle^N
\end{aligned}
$$

$$
\begin{aligned}
=&\frac{\mathrm{i}\varepsilon_{3ij}}{4F^2(2\pi)^3}\,{}^N\!\big\langle\phi_0|b_0^\dagger\int d^3x_1d^3x_2d^3k_1d^3k_2\\
&\times\frac{\delta^3(\vec{k}_1-\vec{k}_2)\mathrm{e}^{\mathrm{i}\vec{k}_1\cdot\vec{x}_1}\mathrm{e}^{-\mathrm{i}\vec{k}_2\cdot\vec{x}_2}}{(\omega(k_1^2)+\omega(k_2^2))(\omega(k_1^2)+\Delta\mathcal{E}_\alpha)(\omega(k_2^2)+\Delta\mathcal{E}_\alpha)}\\
&\times\Bigg\{\Big[\omega(k_1^2)\omega(k_2^2)+\Delta\mathcal{E}_\alpha(\omega(k_1^2)+\omega(k_2^2))\Big]\Big[\bar{u}_0(x_1)\gamma^0\gamma^5\tau_iu_\alpha(x_1)\bar{u}_\alpha(x_2)\gamma^0\gamma^5\tau_ju_0(x_2)\Big]\\
&-\Delta\mathcal{E}_\alpha\Big[\bar{u}_0(x_1)\gamma^0\gamma^5\tau_iu_\alpha(x_1)\bar{u}_\alpha(x_2)\vec{\gamma}\cdot\vec{k}_2\gamma^5\tau_ju_0(x_2)\Big]\\
&-\Delta\mathcal{E}_\alpha\Big[\bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}_1\gamma^5\tau_iu_\alpha(x_1)\bar{u}_\alpha(x_2)\gamma^0\gamma^5\tau_ju_0(x_2)\Big]\\
&-\Big[\bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}_1\gamma^5\tau_iu_\alpha(x_1)\bar{u}_\alpha(x_2)\vec{\gamma}\cdot\vec{k}_2\gamma^5\tau_ju_0(x_2)\Big]\Bigg\}b_0|\phi_0\big\rangle^N\\
=&\frac{\mathrm{i}\varepsilon_{3ij}}{8F^2(2\pi)^3}\,{}^N\!\big\langle\phi_0|b_0^\dagger\int d^3k\frac{1}{(\omega(k^2))(\omega(k^2)+\Delta\mathcal{E}_\alpha)^2}\Bigg\{\omega(k^2)\Big[\omega(k^2)+2\Delta\mathcal{E}_\alpha\Big]\\
&\times\Big[\int d^3x_1\bar{u}_0(x_1)\gamma^0\gamma^5\tau_iu_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\Big]\Big[\int d^3x_2\bar{u}_\alpha(x_2)\gamma^0\gamma^5\tau_ju_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\Big]\\
&-\Delta\mathcal{E}_\alpha\Big[\int d^3x_1\bar{u}_0(x_1)\gamma^0\gamma^5\tau_iu_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\Big]\Big[\int d^3x_2\bar{u}_\alpha(x_2)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_ju_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\Big]\\
&-\Delta\mathcal{E}_\alpha\Big[\int d^3x_1\bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_iu_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\Big]\Big[\int d^3x_2\bar{u}_\alpha(x_2)\gamma^0\gamma^5\tau_ju_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\Big]\\
&-\Big[\int d^3x_1\bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_iu_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\Big]\Big[\int d^3x_2\bar{u}_\alpha(x_2)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_ju_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}_1\cdot\vec{x}_2}\Big]\Bigg\}\\
&\times b_0|\phi_0\big\rangle^N\\
=&\frac{\mathrm{i}\varepsilon_{3ij}}{8F^2(2\pi)^3}\,{}^N\!\big\langle\phi_0|b_0^\dagger\int d^3k\frac{1}{(\omega(k^2))(\omega(k^2)+\Delta\mathcal{E}_\alpha)^2}\Bigg\{\omega(k^2)\Big[\omega(k^2)+2\Delta\mathcal{E}_\alpha\Big]\\
&\times F_{II_\alpha}(k)F_{II_\alpha}^\dagger(k)\,\Big[(\vec{\sigma}\cdot\hat{k})\tau_i\Big]_{0,\alpha}\Big[\tau_j(\vec{\sigma}\cdot\hat{k})\Big]_{\alpha,0}\\
&-\Delta\mathcal{E}_\alpha F_{II_\alpha}(k)F_{I_\alpha}^\dagger(k)\,\Big[(\vec{\sigma}\cdot\hat{k})\tau_i\Big]_{0,\alpha}\Big[\tau_j(\vec{\sigma}\cdot\hat{k})\Big]_{\alpha,0}\\
&-\Delta\mathcal{E}_\alpha F_{I_\alpha}(k)F_{II_\alpha}^\dagger(k)\,\Big[(\vec{\sigma}\cdot\hat{k})\tau_i\Big]_{0,\alpha}\Big[\tau_j(\vec{\sigma}\cdot\hat{k})\Big]_{\alpha,0}\\
&-F_{I_\alpha}(k)F_{I_\alpha}^\dagger(k)\,\Big[(\vec{\sigma}\cdot\hat{k})\tau_i\Big]_{0,\alpha}\Big[\tau_j(\vec{\sigma}\cdot\hat{k})\Big]_{\alpha,0}\Bigg\}b_0|\phi_0\big\rangle^N
\end{aligned}
$$

$$
\begin{aligned}
Q_N^{r;MC} = & \frac{-1}{4F^2(2\pi)^3} \, {}^N\langle\phi_0|b_0^\dagger \left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_0 \tau_3 \int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)^2} \\
& \times\Bigg\{\omega(k^2)\Big[\omega(k^2)+2\Delta\mathcal{E}_\alpha\Big] F_{II_\alpha}(k)F^\dagger_{II_\alpha}(k) - 2\Delta\mathcal{E}_\alpha F_{I_\alpha}(k)F^\dagger_{II_\alpha}(k) \\
& -F_{I_\alpha}(k)F^\dagger_{I_\alpha}(k)\Bigg\}\int_\Omega d\Omega(\vec{\sigma}\cdot\hat{k})(\vec{\sigma}\cdot\hat{k})\left[\chi_s\chi_f\chi_c\right]_0 b_0|\phi_0\rangle^N \\
= & -\frac{q_N^{MC}}{2(2\pi F)^2}\int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)^2} \\
& \times\Bigg\{\omega(k^2)\Big[\omega(k^2)+2\Delta\mathcal{E}_\alpha\Big] F_{II_\alpha}(k)F^\dagger_{II_\alpha}(k) - 2\Delta\mathcal{E}_\alpha F_{I_\alpha}(k)F^\dagger_{II_\alpha}(k) \\
& -F_{I_\alpha}(k)F^\dagger_{I_\alpha}(k)\Bigg\}. \qquad (G.70)
\end{aligned}
$$

Projecting on the nucleon, we obtain

$$
q_p^{MC} \equiv \langle p\uparrow|\sum_{i=1}^{3}\tau_3^{(i)}|p\uparrow\rangle = \quad 1, \qquad (G.71)
$$

$$
q_n^{MC} \equiv \langle n\uparrow|\sum_{i=1}^{3}\tau_3^{(i)}|n\uparrow\rangle = -1. \qquad (G.72)
$$

We also obtain a set of diagrams Figs. G.10(c), G.10(d), G.10(g) and G.10(h) generated by the counterterms $\delta\mathcal{L}_2^{W;\,\mathrm{str}}(x)$ and $\delta\mathcal{L}_3^{W;\,\mathrm{str}}(x)$. The contribution of the counterterm $\delta\mathcal{L}_1^{W;\,\mathrm{str}}(x)$ is equal to zero due to the equation of motion (Eq. (G.2)). By the definition of the counterterms $\delta\mathcal{L}_2^{W;\,\mathrm{str}}(x)$ and $\delta\mathcal{L}_3^{W;\,\mathrm{str}}(x)$, the self-energy and the meson exchange current diagrams of Figs. G.10(a), G.10(b), G.10(e) and G.10(f) are compensated by the counterterm diagrams of Figs. G.10(c), G.10(d), G.10(g) and G.10(h), respectively.

To guarantee charge conservation, the sum of three-quark, meson cloud, vertex correction, self-energy I and II, and the counterterm diagram have to equal the nucleon charge of the nucleon before renormalization:

$$
Q_N^r = Q_N^{3q} + Q_N^{3q;CT} + Q_N^{SE;I} + Q_N^{SE;II} + Q_N^{VC} + Q_N^{MC}.
$$

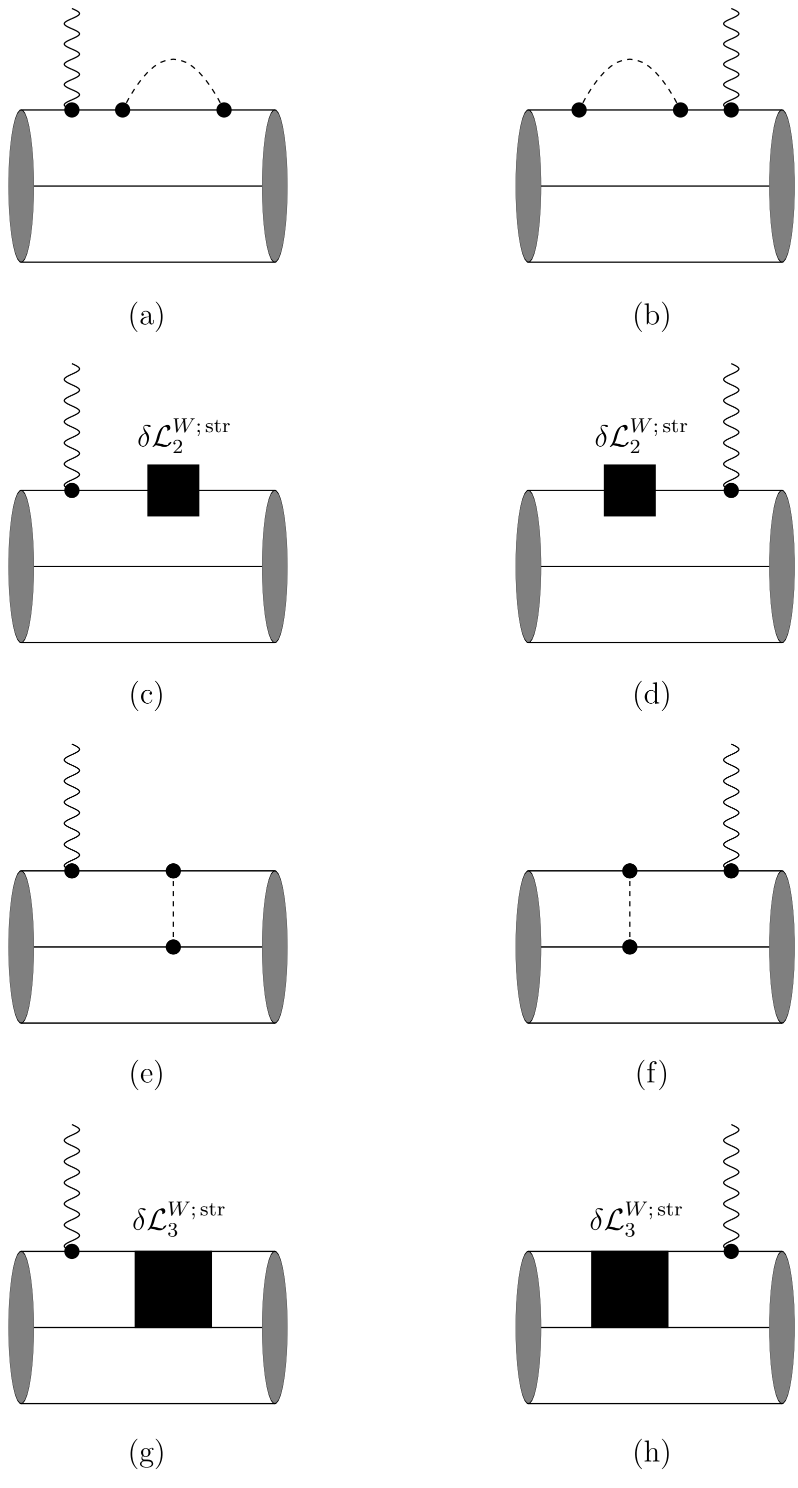

Figure G.10: Diagrams contributing to nucleon charge where their sum equals zero: self-energy diagram (a) and (b), self-energy counterterm diagrams (c) and (d), exchange current diagrams (e) and (f), and exchange current counterterm diagrams (g) and (h).

From the previous calculation we obtain:

$$
\begin{aligned}
Q_N^r &= Q_N + (Z-1)Q_N + \frac{1}{4(2\pi F)^2}\int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)\Big[\omega(k^2)+\Delta\mathcal{E}_\alpha\Big]^2} \\
&\quad \times \Bigg\{ \Big[q_N^{VC} + 2\, q_N^{MC}\Big] F_{I_\alpha}(k^2) F^\dagger_{I_\alpha}(k^2) \\
&\quad +2\left[2\,\Delta\mathcal{E}_\alpha \Big[q_N^{MC} - q_N^{SE}\Big] - \omega(k^2)\Big[2\, q_N^{SE} + q_N^{VC}\Big]\right] F_{I_\alpha}(k^2) F^\dagger_{II_\alpha}(k^2) \\
&\quad +\left[4\,\Delta\mathcal{E}_\alpha \omega(k^2)\Big[q_N^{SE} - q_N^{MC}\Big] + \omega^2(k^2)\Big[4\, q_N^{SE} - 2 q_N^{MC} + q_N^{VC}\Big]\right] \\
&\quad \times F_{II_\alpha}(k^2) F^\dagger_{II_\alpha}(k^2) \Bigg\}. \qquad \text{(G.73)}
\end{aligned}
$$

Here we use

$$
q_N^{SE;I} = q_N^{SE;II} \equiv q_N^{SE}. \qquad \text{(G.74)}
$$

Then, for the neutron charge renormalization results in

$$
Q_n^r = 0 \qquad \text{(G.75)}
$$

as we expected since after renormalization the neutron charge should not be changed.

For the proton, the renormalization charge is calculated by

$$
\begin{aligned}
1 &= 1 + 1(\hat{Z}^F - 1) + \frac{3}{(4\pi F)^2}\sum_\alpha \int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)^2} \\
&\quad \times \Big[F_{I_\alpha}(k) F^\dagger_{I_\alpha}(k) - 2\,\omega(k^2) F_{I_\alpha}(k) F^\dagger_{II_\alpha}(k) + \omega^2(k^2) F_{II_\alpha}(k) F^\dagger_{II_\alpha}(k)\Big],
\end{aligned}
\qquad \text{(G.76)}
$$

and the full renormalization constant, $\hat{Z}^F$, is

$$
\begin{aligned}
\hat{Z}^F &= 1 - \frac{3}{(4\pi F)^2}\sum_\alpha \int_0^\infty dk\, k^2 \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)^2} \\
&\quad \times \Big[F_{I_\alpha}(k) F^\dagger_{I_\alpha}(k) - 2\,\omega(k^2) F_{I_\alpha}(k) F^\dagger_{II_\alpha}(k) + \omega^2(k^2) F_{II_\alpha}(k) F^\dagger_{II_\alpha}(k)\Big].
\end{aligned}
\qquad \text{(G.77)}
$$

The summation over state α appears when we take into account the intermediate excited quark state propagator. When restricting intermediate quark states to the ground state Eq. (G.77) yields the result

$$\hat{Z}^0 = 1 - \frac{27}{400}\left(\frac{g_A^{(0)}}{\pi F}\right)^2 \int_0^\infty dk\, k^4\, \frac{F_{\pi NN}^2(k^2)}{\omega^3(k^2)}. \tag{G.78}$$

Appendix H

Axial Vector Current

In QCD, the axial vector current is given by the operator $A_i^\mu = \bar{\psi}\gamma^\mu\gamma^5\frac{\tau_i}{2}\psi$. The corresponding representation in the framework of the effective theory may be obtained from the effective Lagrangian - it is given by a term linear in the external field $a_i^\mu(x)$. The representation of the axial vector current in terms of effective field consists of an infinite string of terms

$$A_i^\mu = A_i^{\mu^{(0)}} + A_i^{\mu^{(1)}} + \mathcal{O}(p^2). \tag{H.1}$$

According to section A.5, the effective Lagrangian involves the quantities u_μ, D_μ and χ_+. In the absence of the external fields v_μ, p and s, these reduce to

$$\begin{aligned} u_\mu &= \mathrm{i}\left\{u^\dagger, \partial_\mu u\right\} - \left[u^\dagger, [u, a_\mu]\right] + 2a_\mu, && \text{(H.2)}\\ D_\mu &= \partial_\mu + \frac{1}{2}\left[u^\dagger, \partial_\mu u\right] + \frac{1}{2}\left\{u^\dagger, [u, a_\mu]\right\}, && \text{(H.3)}\\ \chi_+ &= M^2(U^\dagger + U), && \text{(H.4)} \end{aligned}$$

while the chiral field is $U = \mathrm{e}^{\mathrm{i}\hat{\Phi}/F}$.

In the following we restrict to the SU(2) flavor case, that is, only pions are considered. The effective Lagrangian reads

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_q^{(1)} + \mathcal{L}_\pi^{(2)}, \tag{H.5}$$

where

$$\begin{aligned} \mathcal{L}_q^{(1)} = {} & \bar{\psi}\Big\{\mathrm{i}\gamma^\mu\partial_\mu - \gamma^0 V(r) - S(r) - M\Big\}\psi + \bar{\psi}\Big\{-\frac{1}{4F^2}\varepsilon_{ijk}\pi_i\partial_\mu\pi_j\gamma^\mu\tau_k \\ & -\frac{1}{2F}\partial_\mu\pi_i\gamma^\mu\gamma^5\tau_i + \frac{1}{12F^3}\left(\pi_i\tau_i\pi_k\partial_\mu\pi_k - \vec{\pi}^2\tau_i\partial_\mu\pi_i\right)\gamma^\mu\gamma^5\Big\}\psi + \mathcal{O}(\pi^4), \end{aligned} \tag{H.6}$$

$$\mathcal{L}_\pi^{(2)} = \frac{1}{2}\partial_\mu\pi_i\partial^\mu\pi_i - \frac{1}{2}M_\pi^2\vec{\pi}^2 + \frac{1}{6F^2}\left\{\pi_i(\partial_\mu\pi_i)\pi_k(\partial^\mu\pi_k) - \vec{\pi}^2(\partial_\mu\pi_i)(\partial^\mu\pi_i)\right\} + \frac{1}{24F^2}M_\pi^2\vec{\pi}^4 + \mathcal{O}(\pi^6). \quad \text{(H.7)}$$

The relevant contributions to the effective field theory representation of the operator A_i^μ read:

$$A_i^\mu = A_i^{\mu^{(0)}} + A_i^{\mu^{(1)}}, \quad \text{(H.8)}$$

where

$$A_i^{\mu^{(0)}} = \bar{\psi}\left\{\gamma^\mu\gamma^5\frac{\tau_i}{2} - \frac{\varepsilon_{ijk}}{2F}\gamma^\mu\tau_j\pi_k + \frac{1}{4F^2}\gamma^\mu\gamma^5\left(\pi_i\pi_j\tau_j - \vec{\pi}^2\tau_i\right)\right\}\psi + \mathcal{O}(\pi^3), \quad \text{(H.9)}$$

$$A_i^{\mu^{(1)}} = F\partial^\mu\pi_i + \frac{2}{3F}\left(\pi_i\pi_j\partial^\mu\pi_j - \vec{\pi}^2\partial^\mu\pi_i\right) + \mathcal{O}(\pi^4). \quad \text{(H.10)}$$

One can show that, using Eqs. (H.5) and (H.8) with the Euler Lagrange equation which states that

$$\frac{\partial\mathcal{L}}{\partial\phi_i} - \partial_\mu\left(\frac{\partial\mathcal{L}}{\partial\partial_\mu\phi_i}\right) = 0, \quad \text{(H.11)}$$

the divergence of axial vector current up to the order we are interested in, $O(1/F^2, \hat{m})$, results in

$$\partial_\mu A_i^\mu = -FM_\pi^2\pi_i. \quad \text{(H.12)}$$

Eq. (H.12) is well known as the partially conserved axial vector current, which states that the divergence of axial vector current is proportional to the mass of the pion squared. In the chiral limit, $M_\pi \to 0$, the conservation of the axial vector current is regained.

In Eq.(3.1), the axial vector current has the term linear in F, that is $F\partial^\mu\pi_i$. To obtain the PCAC in the order we are interested in $O(\hat{m}, 1/F^2)$ we have to expand the effective Lagrangian up to $1/F^3$.

The counterterm axial vector current part in Eq. (3.1), $\bar{\psi}^r(\hat{Z}-1)\gamma^\mu\gamma^5\frac{\tau_i}{2}\psi^r$, results from the renormalization techniques of the PCQM.

Appendix I

Calculation of the Diagrams for the Axial Form Factor

I.1 Three-Quark Core

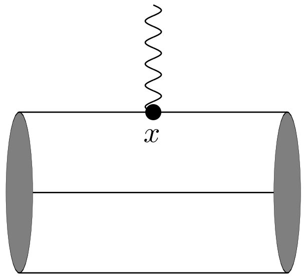

Figure I.1: Three-quark core diagram

$$
\begin{aligned}
&\chi^\dagger_{N_{s'}}\sigma^3_N\tfrac{\tau^3_N}{2}\chi_{N_s}G_A(Q^2)\big|_{3q}\\
&\quad = \; {}^N\!\big\langle\phi_0\big|\int\delta(t)d^4x\mathrm{e}^{-\mathrm{i}qx}\bar{\psi}^r(x)\gamma^3\gamma^5\frac{\tau_3}{2}\psi^r(x)\big|\phi_0\big\rangle^N\\
&\quad = \; {}^N\!\big\langle\phi_0\big|b_0^\dagger\int\delta(t)d^4x\mathrm{e}^{-\mathrm{i}qx}\bar{u}^r_0(x)\gamma^3\gamma^5\frac{\tau_3}{2}u^r_0(x)b_0\big|\phi_0\big\rangle^N\\
&\quad = \; {}^N\!\big\langle\phi_0\big|b_0^\dagger\int\delta(t)d^4x\mathrm{e}^{-\mathrm{i}qx}\left[\bar{u}_0(x)+\delta\bar{u}_0(x;\hat{m}^r)\right]\\
&\qquad\quad \times\gamma^3\gamma^5\frac{\tau_3}{2}\left[u_0(x)+\delta u_0(x;\hat{m}^r)\right]b_0\big|\phi_0\big\rangle^N\\
&\quad = \; {}^N\!\big\langle\phi_0\big|b_0^\dagger\int\delta(t)d^4x\mathrm{e}^{-\mathrm{i}qx}\Big\{\bar{u}_0(x)\gamma^3\gamma^5\frac{\tau_3}{2}u_0(x)+\bar{u}_0(x)\gamma^3\gamma^5\frac{\tau_3}{2}\delta u_0(x;\hat{m}^r)\\
&\qquad\quad +\delta\bar{u}_0(x;\hat{m}^r)\gamma^3\gamma^5\frac{\tau_3}{2}u_0(x)+O((\hat{m}^r)^2)\Big\}b_0\big|\phi_0\big\rangle^N. \qquad\qquad (\mathrm{I.1})
\end{aligned}
$$

Here we separated the contribution of the three-quark core diagram, into the three-quark core leading-order(LO) term and the three quark core next-to-leading order

(NLO) terms, that is,

$$G_A(Q^2)\big|_{3q} = G_A(Q^2)\big|_{3q}^{LO} + G_A(Q^2)\big|_{3q}^{NLO} \tag{I.2}$$

with

$$G_A(Q^2)\big|_{3q}^{LO} \equiv 2\ {}^N\!\big\langle\phi_0|b_0^\dagger \int \delta(t) d^4x e^{-\mathrm{i}qx}\bar{u}_0(x)\gamma^3\gamma^5\frac{\tau_3}{2}u_0(x)b_0|\phi_0\big\rangle^N, \tag{I.3}$$

$$G_A(Q^2)\big|_{3q}^{NLO} \equiv 2\ {}^N\!\big\langle\phi_0|b_0^\dagger \int \delta(t) d^4x \mathrm{e}^{-\mathrm{i}qx}\bar{u}_0(x)\gamma^3\gamma^5\tau_3\delta u_0(x;\hat{m}^r), \tag{I.4}$$

where

$$\delta u_0(x;\hat{m}^r) = \frac{\hat{m}^r}{2}\frac{\rho R}{1+\frac{3}{2}\rho^2}\left(\frac{\frac{1}{2}+\frac{21}{4}\rho^2}{1+\frac{3}{2}\rho^2} - \frac{x^2}{R^2} + \gamma^0\right) u_0(x) \tag{I.5}$$

is the additional term of the quark wave function resulting from renormalization. The factor 2 in the right hand side of Eqs. (I.3) and (I.4) come from the calculation of $\chi^\dagger_{N_{s'}}\sigma^3_N\frac{\tau^3_N}{2}\chi_{N_s}$ on the nucleon level. For a proton with spin up we get

$$\chi^\dagger_{p\uparrow_{s'}}\sigma^3\frac{\tau^3}{2}\chi_{p\uparrow_s} = \frac{1}{2}. \tag{I.6}$$

I.1.1 Three-Quark Core Leading Order (LO)

Now we come to consider the expression of the three quark leading order term

$$\begin{aligned} G_A(Q^2)\big|_{3q}^{LO} &= {}^N\!\big\langle\phi_0|b_0^\dagger(x) \int \delta(t) d^4x e^{-\mathrm{i}qx}\bar{u}_0(x)\gamma^3\gamma^5\tau_3 u_0(x)b_0(x)|\phi_0\big\rangle^N \\ &= {}^N\!\big\langle\phi_0|b_0^\dagger(x) \int d^3x \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}}\bar{u}_0(x)\gamma^3\gamma^5\tau_3 u_0(x)b_0(x)|\phi_0\big\rangle^N \\ &= \left(\frac{2-\rho^2}{2+3\rho^2}\right) F_{\pi NN}(Q^2)\, {}^N\!\big\langle\phi_0|b_0^\dagger(x)\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\sigma_3\tau_3\,\chi_s\chi_f\chi_c b_0(x)|\phi_0\big\rangle^N \\ &= \frac{5}{3}\left(\frac{2-\rho^2}{2+3\rho^2}\right) F_{\pi NN}(Q^2). \end{aligned} \tag{I.7}$$

Details of the integration are indicated in Appendix K and the "one-body" projection operator yields

$$\big\langle P\uparrow|\sum_{i=1}^{3}(\sigma_3\tau_3)^{(i)}|P\uparrow\big\rangle = \frac{5}{3}. \tag{I.8}$$

At zero momentum transfer, $Q^2 = 0$ the leading order axial charge $g_A^{(0)}$ is related to the pion nucleon form factor, $F_{\pi NN}$, which is normalized to one. Then we obtain

$$G_A(0)\big|_{3q}^{LO} = \frac{5}{3}\left(\frac{2-\rho^2}{2+3\rho^2}\right) \equiv g_A^{(0)}, \tag{I.9}$$

and Eq. (I.7) can therefore be written as

$$G_A(Q^2)\big|_{3q}^{LO} = g_A^{(0)} F_{\pi NN}(Q^2). \tag{I.10}$$

I.1.2 Three-Quark Core Next to Leading Order (NLO)

For the next to leading order term we obtain

$$\begin{aligned}
&G_A(Q^2)\big|_{3q}^{NLO} \\
&= 2\ {}^N\!\big\langle\phi_0|b_0^\dagger \int \delta(t)d^4x\mathrm{e}^{-\mathrm{i}qx}\bar{u}_0(x)\gamma^3\gamma^5\tau_3\delta u_0(x;\hat{m}^r)b_0|\phi_0\big\rangle^N. \\
&= 2\ {}^N\!\big\langle\phi_0|b_0^\dagger \int \delta(t)d^4x\mathrm{e}^{-\mathrm{i}qx}u_0^\dagger(x)\,\sigma_3\tau_3\left[\frac{\hat{m}^r}{2}\frac{\rho R}{1+\frac{3}{2}\rho^2}\left(\frac{\frac{1}{2}+\frac{21}{4}\rho^2}{1+\frac{3}{2}\rho^2}-\frac{x^2}{R^2}+\gamma^0\right)\right] \\
&\quad\times u_0(x)b_0|\phi_0\big\rangle^N \\
&= \frac{3}{2}\,\hat{m}_0^r\frac{\rho\,R}{\left(1+\frac{3}{2}\rho^2\right)^2}\Bigg\{\left(1+\frac{9}{2}\rho^2\right)G_A(Q^2)\Big|_{3q}^{LO} \\
&\quad-\frac{5}{72}\left[12\left(2-3\rho^2\right)-4\left(1+5\rho^2\right)Q^2R^2+\rho^2Q^4R^4\right]\exp\left(-\frac{Q^2R^2}{4}\right)\Bigg\}.
\end{aligned} \tag{I.11}$$

I.2 Three-Quark Core Counterterm

$$\begin{aligned}
&\chi_{N_{s'}}^\dagger\sigma_N^3\frac{\tau_N^3}{2}\chi_{N_s}G_A(Q^2)\big|_{CT} \\
&\qquad= {}^N\!\big\langle\phi_0|\int\delta(t)d^4x\mathrm{e}^{-\mathrm{i}qx}(x)(\hat{Z}^0-1)\bar{\psi}^r(x)\gamma^3\gamma^5\frac{\tau_3}{2}\psi^r(x)|\phi_0\big\rangle_c^N \\
&\qquad= (\hat{Z}^0-1)\ {}^N\!\big\langle\phi_0|b_0^\dagger\int\delta(t)d^4x\mathrm{e}^{-\mathrm{i}qx}\bar{u}_0^r(x)\gamma^3\gamma^5\frac{\tau_3}{2}u_0^r(x)b_0|\phi_0\big\rangle_c^N \\
&G_A(Q^2)\Big|_{CT} \equiv (\hat{Z}^0-1)G_A(Q^2)\big|_{3q}^{LO}.
\end{aligned} \tag{I.12}$$

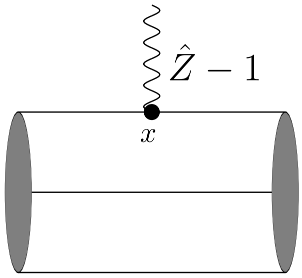

Figure I.2: Three-quark core counterterm diagram

I.3 Exchange Term

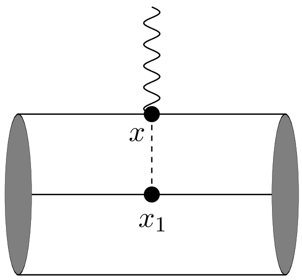

Figure I.3: Exchange diagram

$$
\begin{aligned}
&\chi^\dagger_{N_{s'}}\sigma^3_N \tfrac{\tau^3_N}{2}\chi_{N_s} G_A(Q^2)\big|_{EX} \\
&= \quad {}^N\!\big\langle\phi_0\big|\mathrm{i}\int \delta(t)d^4x d^4x_1 \mathrm{e}^{-\mathrm{i}qx} \\
&\quad \times N\Big\{\Big[\frac{1}{2F}\partial_\mu\pi_m\bar\psi\gamma^\mu\gamma^5\tau_m\psi\Big]_{x_1}\Big[-\frac{\varepsilon_{3ij}}{2F}\bar\psi\gamma^3\tau_i\psi\pi_j\Big]_x\Big\}\big|\phi_0\big\rangle^N_c \\
&= \quad \frac{-\mathrm{i}\varepsilon_{3ij}}{4F^2}\,{}^N\!\big\langle\phi_0\big|b_0^\dagger(x_1)b_0^\dagger(x)\int\delta(t)d^4xd^4x_1\mathrm{e}^{-\mathrm{i}qx} \\
&\quad \times\bar u_0(x_1)\gamma^\mu\gamma^5\tau_m u_0(x_1)\bar u_0(x)\gamma^3\tau_i u_0(x)\partial_\mu\pi_m(x_1)\pi_j(x)b_0(x)b_0(x_1)\big|\phi_0\big\rangle^N_c \\
&= \quad \frac{\mathrm{i}\varepsilon_{3ij}}{4F^2(2\pi)^4}\,{}^N\!\big\langle\phi_0\big|b_0^\dagger(x_1)b_0^\dagger(x)\int d^3xd^3x_1d^4k\frac{\mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x_1}-\vec{x})}}{M_\pi^2-k^2-\mathrm{i}\epsilon}\,\mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}} \\
&\quad \times\bar u_0(x_1)\gamma^\mu k_\mu\gamma^5\tau_j u_0(x_1)\bar u_0(x)\gamma^3\tau_i u_0(x)\int dtdt_1\delta(t)\mathrm{e}^{-\mathrm{i}q_0t}\mathrm{e}^{-\mathrm{i}k_0(t_1-t)} \\
&\quad \times b_0(x)b_0(x_1)\big|\phi_0\big\rangle^N
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{\mathrm{i}\varepsilon_{3ij}}{4F^2(2\pi)^3}\,{}^N\!\Big\langle \phi_0|b_0^\dagger(x_1)b_0^\dagger(x)\int d^3x d^3x_1 d^3k \int dk_0 \frac{\delta(k_0)}{M_\pi^2 - k_0^2 + \vec{k}^2 - \mathrm{i}\epsilon} \\
&\quad \times \left[\bar{u}_0(x_1)(\gamma^0 k_0 - \vec{\gamma}\cdot\vec{k})\gamma^5\tau_j u_0(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\right]\left[\bar{u}_0(x)\gamma^3\tau_i u_0(x)\mathrm{e}^{\mathrm{i}(\vec{q}-\vec{k})\cdot\vec{x}}\right] \\
&\quad \times b_0(x)b_0(x_1)|\phi_0\Big\rangle^N \\
&= \frac{-\mathrm{i}\varepsilon_{3ij}}{4F^2(2\pi)^3}\,{}^N\!\Big\langle \phi_0|b_0^\dagger(x_1)b_0^\dagger(x)\int d^3k \frac{1}{\omega^2(k^2)} \\
&\quad \times \left[\int d^3x_1 \bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_0(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\right]\left[\int d^3x \bar{u}_0(x)\gamma^3\tau_i u_0(x)\mathrm{e}^{\mathrm{i}(\vec{q}-\vec{k})\cdot\vec{x}}\right] \\
&\quad \times b_0(x)b_0(x_1)|\phi_0\Big\rangle^N \\
&= \frac{-\mathrm{i}\varepsilon_{3ij}}{4F^2(2\pi)^3}\,{}^N\!\Big\langle \phi_0|b_0^\dagger(x_1)b_0^\dagger(x)\int d^3k \frac{1}{\omega^2(k^2)} \\
&\quad \times \left[\frac{3}{5}g_A^{(0)}k\,F_{\pi NN}(k^2)\big[(\vec{\sigma}\cdot\hat{k})\tau_j\big]_{0,0;x_1}\right]\left[\frac{F_{III_0}(|\vec{q}-\vec{k}|)}{|\vec{q}-\vec{k}|}\big[\varepsilon_{3mn}k_m\sigma_n\tau_i\big]_{0,0;x}\right] \\
&\quad \times b_0(x)b_0(x_1)|\phi_0\Big\rangle^N \\
&= \frac{-3\mathrm{i}g_A^{(0)}\varepsilon_{3ij}}{20F^2(2\pi)^3}\,{}^N\!\Big\langle \phi_0|b_0^\dagger(x_1)\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_{x_1} b_0^\dagger(x)\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_x \int d^3k \frac{k\,F_{\pi NN}(k^2)}{\omega^2(k^2)} \\
&\quad \times \frac{F_{III_0}(|\vec{q}-\vec{k}|)}{|\vec{q}-\vec{k}|}\big[(\vec{\sigma}\cdot\hat{k})\tau_j\big]_{x_1}\big[\varepsilon_{3mn}k_m\sigma_n\tau_i\big]_x\left[\chi_c\chi_f\chi_s\right]_x b_0(x)\left[\chi_c\chi_f\chi_s\right]_{x_1} \\
&\quad \times b_0(x_1)|\phi_0\Big\rangle^N \\
&= \frac{-3\mathrm{i}g_A^{(0)}\varepsilon_{3ij}}{20F^2(2\pi)^3}\,{}^N\!\Big\langle \phi_0|b_0^\dagger(x_1)\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_{x_1} b_0^\dagger(x)\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_x \int_0^\infty dk\,k^3\frac{F_{\pi NN}(k^2)}{\omega^2(k^2)} \\
&\quad \times \int_\Omega d\Omega\,\frac{F_{III_0}(k,Q^2,\cos\theta)}{\sqrt{k^2+Q^2-2k\sqrt{Q^2}\cos\theta}}\big[(\vec{\sigma}\cdot\hat{k})\tau_j\big]_{x_1}\big[\varepsilon_{3mn}k_m\sigma_n\tau_i\big]_x \\
&\quad \times \left[\chi_c\chi_f\chi_s\right]_x b_0(x)\left[\chi_c\chi_f\chi_s\right]_{x_1} b_0(x_1)|\phi_0\Big\rangle^N \\
&= \frac{-3\mathrm{i}g_A^{(0)}\pi}{20F^2(2\pi)^3}\,{}^N\!\Big\langle \phi_0|b_0^\dagger(x_1)\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_{x_1} b_0^\dagger(x)\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_x \{\varepsilon_{3ij}\varepsilon_{3mn}[\sigma_m\tau_j]_{x_1}[\sigma_n\tau_i]_x\} \\
&\quad \times \int_0^\infty dk\,k^4\frac{F_{\pi NN}(k^2)}{\omega^2(k^2)}\int_{-1}^1 dx\,(1-x^2)\frac{F_{III_0}(k_-)}{\sqrt{k_-^2}} \\
&\quad \times \left[\chi_c\chi_f\chi_s\right]_x b_0(x)\left[\chi_c\chi_f\chi_s\right]_{x_1} b_0(x_1)|\phi_0\Big\rangle^N \\
&= \frac{12}{5}\frac{g_A^{(0)}\rho\,R}{2+3\rho^2}\frac{1}{(2\pi F)^2}\int_0^\infty dk\,k^4\frac{F_{\pi NN}(k^2)}{\omega^2(k^2)}\int_{-1}^1 dx\,(1-x^2)\exp\left(-\frac{k_-^2R^2}{4}\right) \\
&= \frac{96}{5}\frac{g_A^{(0)}\rho\,R}{2+3\rho^2}\frac{1}{(2\pi F)^2}\int_0^\infty dk\,k^4\frac{F_{\pi NN}(k^2)}{\omega^2(k^2)}\mathcal{D}(k,Q^2).
\end{aligned}
\tag{I.13}
$$

Where

$$\begin{aligned}
\mathcal{D}(k,Q^2) &\equiv \exp\left(-\frac{(k+\sqrt{Q^2})^2R^2}{4}\right)\left(\frac{1}{k^2Q^2R^4}\right)^{3/2} \\
&\quad \times\left[2+k\sqrt{Q^2}R^2+\exp\left(k\sqrt{Q^2}R^2\right)\left(-2+k\sqrt{Q^2}R^2\right)\right], \quad \text{(I.14)} \\
k_\pm^2 &\equiv k^2+Q^2\pm 2k\sqrt{Q^2}x. \quad \text{(I.15)}
\end{aligned}$$

We have used

$$\left\langle P\uparrow\right|\sum_{k\neq l}^{3}\varepsilon_{3ij}\varepsilon_{3mn}[\sigma_m\tau_j]^{(k)}[\sigma_n\tau_i]^{(l)}\left|P\uparrow\right\rangle = 8. \quad \text{(I.16)}$$

I.4 Self-Energy Term I

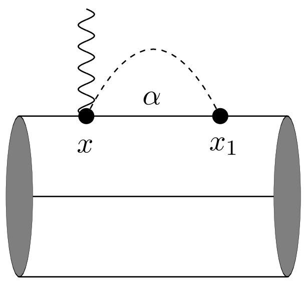

Figure I.4: Self energy I diagram

$$\chi^\dagger_{N_{s'}}\sigma^3_N\tfrac{\tau^3_N}{2}\chi_{N_s}G_A(Q^2)\big|^{\alpha}_{SE;I}$$

$$\begin{aligned}
&= {}^N\!\left\langle\phi_0\right|\mathrm{i}\int\delta(t)d^4xd^4x_1\mathrm{e}^{-\mathrm{i}qx} \\
&\quad\times N\left\{\left[\frac{1}{2F}\partial_\mu\pi_m\bar{\psi}\gamma^\mu\gamma^5\tau_m\psi\right]_{x_1}\left[-\frac{\varepsilon_{3ij}}{2F}\bar{\psi}\gamma^3\tau_i\psi\pi_j\right]_x\right\}\left|\phi_0\right\rangle^N_c \\
&= \frac{-\mathrm{i}\varepsilon_{3ij}}{4F^2}\,{}^N\!\left\langle\phi_0\right|\int\delta(t)d^4xd^4x_1\mathrm{e}^{-\mathrm{i}qx} \\
&\quad\times\bar{\psi}(x_1)\gamma^\mu\gamma^5\tau_m\psi(x_1)\bar{\psi}(x)\gamma^3\tau_i\psi(x)\partial_\mu\pi_m(x_1)\pi_j(x)\left|\phi_0\right\rangle^N_c
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\mathrm{i}\varepsilon_{3ij}}{4F^2(2\pi)^4}\,{}^N\!\langle\phi_0|b_0^\dagger(x_1)\int d^3x d^3x_1 d^4k \frac{\mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}_1-\vec{x})}}{M_\pi^2-k^2-\mathrm{i}\epsilon}\\
&\quad\times\bar{u}_0(x_1)\gamma^\mu k_\mu\gamma^5\tau_j u_\alpha(x_1)\bar{u}_\alpha(x)\gamma^3\tau_i u_0(x)\,\mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}}\\
&\quad\times\int dt dt_1\delta(t)\mathrm{e}^{-\mathrm{i}q_0 t}\mathrm{e}^{\mathrm{i}\mathcal{E}_0 t_1}\mathrm{e}^{-\mathrm{i}\mathcal{E}_\alpha(t_1-t)}\Theta(t_1-t)\mathrm{e}^{-\mathrm{i}\mathcal{E}_0 t}\mathrm{e}^{-\mathrm{i}k_0(t_1-t)}b_0(x)|\phi_0\rangle^N\\
&= \frac{\varepsilon_{3ij}}{4F^2(2\pi)^4}\,{}^N\!\langle\phi_0|b_0^\dagger(x_1)\int d^3x d^3x_1 d^3k\,\mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}_1-\vec{x})}\,\mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}}\\
&\quad\int dk_0\left[\frac{1}{M_\pi^2-k_0^2+\vec{k}^2-\mathrm{i}\epsilon}\right]\left[\frac{1}{k_0+\Delta\mathcal{E}_\alpha-\mathrm{i}\eta}\right]\\
&\quad\times\left[\bar{u}_0(x_1)(\gamma^0 k_0-\vec{\gamma}\cdot\vec{k})\gamma^5\tau_j u_\alpha(x_1)\bar{u}_\alpha(x)\gamma^3\tau_i u_0(x)\right]b_0(x)|\phi_0\rangle^N\\
&= \frac{\varepsilon_{3ij}\pi\mathrm{i}}{4F^2(2\pi)^4}\,{}^N\!\langle\phi_0|b_0^\dagger(x_1)\int d^3k\frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)}\\
&\quad\times\Bigg\{\omega(k^2)\left[\int d^3x_1\bar{u}_0(x_1)\gamma^0\gamma^5\tau_j u_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\right]\left[\int d^3x\bar{u}_\alpha(x)\gamma^3\tau_i u_0(x)\,\mathrm{e}^{\mathrm{i}(\vec{q}-\vec{k})\cdot\vec{x}}\right]\\
&\quad-\left[\int d^3x_1\bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\right]\left[\int d^3x\bar{u}_\alpha(x)\gamma^3\tau_i u_0(x)\,\mathrm{e}^{\mathrm{i}(\vec{q}-\vec{k})\cdot\vec{x}}\right]\Bigg\}\\
&\quad\times b_0(x)|\phi_0\rangle^N\\
&= \frac{\varepsilon_{3ij}\pi\mathrm{i}}{4F^2(2\pi)^4}\,{}^N\!\langle\phi_0|b_0^\dagger(x_1)\int d^3k\frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)}\\
&\quad\times\Bigg\{\omega(k^2)\left[F_{II_\alpha}(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_j\right]_{0,\alpha}\right]\left[\int d^3x\bar{u}_\alpha(x)\gamma^3\tau_i u_0(x)\,\mathrm{e}^{\mathrm{i}(\vec{q}-\vec{k})\cdot\vec{x}}\right]\\
&\quad-\left[F_{I_\alpha}(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_j\right]_{0,\alpha}\right]\left[\int d^3x\bar{u}_\alpha(x)\gamma^3\tau_i u_0(x)\,\mathrm{e}^{\mathrm{i}(\vec{q}-\vec{k})\cdot\vec{x}}\right]\Bigg\}\\
&\quad\times b_0(x)|\phi_0\rangle^N.
\end{aligned}
$$

Using the result in Eq. (K.9)

$$
\begin{aligned}
\int d^3x\bar{u}_\alpha(x)\gamma^3\tau_i u_0(x)\,\mathrm{e}^{\mathrm{i}(\vec{q}-\vec{k})\cdot\vec{x}} &= \frac{F_{III_\alpha}(|\vec{q}-\vec{k}|)}{|\vec{q}-\vec{k}|}\left[\varepsilon_{3mn}k_m\sigma_n\tau_i\right]_{\alpha,0}\\
&\quad+\frac{F_{IV_\alpha}(|\vec{q}-\vec{k}|)}{|\vec{q}-\vec{k}|}\left[\Big((\sqrt{Q^2}-k_3)\mathbf{I}+\mathrm{i}\varepsilon_{3mn}\sigma_m k_n\Big)\tau_i\right]_{\alpha,0},
\end{aligned}
$$

we obtain

$$
\begin{aligned}
&\chi^{\dagger}_{N_{s'}}\sigma^3_N\frac{\tau^3_N}{2}\chi_{N_s}G_A(Q^2)\Big|^{\alpha}_{SE;I}\\
&= \frac{\varepsilon_{3ij}\pi\mathrm{i}}{4F^2(2\pi)^4}\,{}^N\!\Big\langle\phi_0|b_0^{\dagger}(x_1)\Big[\chi_c^{\dagger}\chi_f^{\dagger}\chi_s^{\dagger}\Big]_0\int_0^{\infty}dk\,k^2\,\frac{\omega(k^2)F_{II_\alpha}(k)-F_{I_\alpha}(k)}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)}\\
&\quad\times\int_{\Omega}d\Omega\Bigg\{\frac{F_{III_\alpha}(k,Q^2,\cos\theta)}{\sqrt{k^2+Q^2-2k\sqrt{Q^2}\cos\theta}}\Big[(\vec{\sigma}\cdot\hat{k})\tau_j\Big]_{\alpha}\Big[\varepsilon_{3mn}k_m\sigma_n\tau_i\Big]_{\alpha}\\
&\quad+\frac{F_{IV_\alpha}(k,Q^2,\cos\theta)}{\sqrt{k^2+Q^2-2k\sqrt{Q^2}\cos\theta}}\Big[(\vec{\sigma}\cdot\hat{k})\tau_j\Big]_{\alpha}\Big[\Big((\sqrt{Q^2}-k_3)\mathbf{I}+\mathrm{i}\varepsilon_{3mn}\sigma_m k_n\Big)\tau_i\Big]_{\alpha}\Bigg\}\\
&\quad\times\Big[\chi_s\chi_f\chi_c\Big]_0 b_0(x)|\phi_0\Big\rangle^N\\
&= \frac{1}{(4\pi F)^2}\,{}^N\!\Big\langle\phi_0|b_0^{\dagger}(x_1)\Big[\chi_c^{\dagger}\chi_f^{\dagger}\chi_s^{\dagger}\Big]_0\sigma_3\tau_3\int_0^{\infty}dk\,k^2\,\frac{\omega(k^2)F_{II_\alpha}(k)-F_{I_\alpha}(k)}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)}\\
&\quad\times\int_{-1}^{1}dx\Bigg\{\frac{\mathrm{i}\,k\,(1-x^2)\,F_{III_\alpha}(k_-)}{\sqrt{k_-^2}}+\frac{F_{IV_\alpha}(k_-)}{\sqrt{k_-^2}}\Big[(\sqrt{Q^2}\,x+k\left(1-2x^2\right)\Big]\Bigg\}\\
&\quad\times\Big[\chi_c\chi_f\chi_s\Big]_0 b_0(x)|\phi_0\Big\rangle^N\\
&= \frac{5}{3}\left(\frac{1}{4\pi F}\right)^2\int_0^{\infty}dk\,k^2\,\frac{\omega(k^2)F_{II_\alpha}(k)-F_{I_\alpha}(k)}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)}\\
&\quad\times\int_{-1}^{1}dx\Bigg\{\frac{\mathrm{i}\,k\,(1-x^2)\,F_{III_\alpha}(k_-)}{\sqrt{k_-^2}}+\frac{F_{IV_\alpha}(k_-)}{\sqrt{k_-^2}}\Big[(\sqrt{Q^2}\,x+k\left(1-2x^2\right)\Big]\Bigg\}.
\end{aligned}
$$

Finally we obtain

$$
\begin{aligned}
&G_A(Q^2)\Big|_{SE;I}\\
&= \frac{10}{3}\left(\frac{1}{4\pi F}\right)^2\sum_{\alpha}\int_0^{\infty}dk\,k^2\,\frac{\omega(k^2)F_{II_\alpha}(k)-F_{I_\alpha}(k)}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)}\\
&\quad\times\int_{-1}^{1}dx\Bigg\{\frac{\mathrm{i}\,k\,(1-x^2)\,F_{III_\alpha}(k_-)}{\sqrt{k_-^2}}+\frac{F_{IV_\alpha}(k_-)}{\sqrt{k_-^2}}\Big[(\sqrt{Q^2}\,x+k\left(1-2x^2\right)\Big]\Bigg\}.
\end{aligned}
\tag{I.17}
$$

When we restrict the quark propagator to only the ground state and use these results

$$F_{I_0}(k) = \frac{3}{5}g_A^{(0)}\,k\,F_{\pi NN}(k^2), \tag{I.18}$$

$$F_{II_0}(k) = 0, \tag{I.19}$$

$$F_{III_0}(k_-) \;=\; \frac{2\,\mathrm{i}\,k_-\,R\,\rho}{2+3\rho^2}\exp\left(-\frac{k_-^2R^2}{4}\right), \tag{I.20}$$

$$F_{IV_0}(k_-) \;=\; 0, \tag{I.21}$$

then Eq. (I.17) reduces to

$$\begin{aligned}
& G_A(Q^2)\big|_{SE;I}^{GS} \\
&= \frac{g_A^{(0)}\rho R}{2+3\rho^2}\left(\frac{1}{2\pi F}\right)^2\int_0^\infty dk\,k^4\,\frac{F_{\pi NN}(k)}{\omega^2(k^2)}\int_{-1}^1 dx\,\left(1-x^2\right)\exp\left(-\frac{k_-^2R^2}{4}\right) \\
&= 8\frac{g_A^{(0)}\rho R}{2+3\rho^2}\left(\frac{1}{2\pi F}\right)^2\int_0^\infty dk\,k^4\,\frac{F_{\pi NN}(k)}{\omega^2(k^2)}\mathcal{D}(k,Q^2).
\end{aligned} \tag{I.22}$$

I.5 Self-Energy Term II

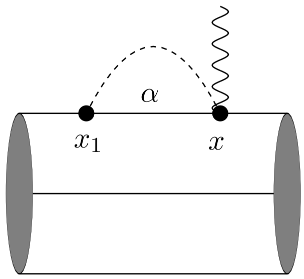

Figure I.5: Self energy II diagram

$$\begin{aligned}
& \chi^\dagger_{N_{s'}}\sigma^3_N\tfrac{\tau^3_N}{2}\chi_{N_s}G_A(Q^2)\big|^{\alpha}_{SE;II} \\
&= {}^N\!\big\langle\phi_0\big|\mathrm{i}\int\delta(t)d^4xd^4x_1\mathrm{e}^{-\mathrm{i}qx} \\
&\quad\times N\Bigg\{\left[-\frac{\varepsilon_{3ij}}{2F}\bar\psi\gamma^3\tau_i\psi\pi_j\right]_x\left[\frac{1}{2F}\partial_\mu\pi_m\bar\psi\gamma^\mu\gamma^5\tau_m\psi\right]_{x_1}\Bigg\}\big|\phi_0\big\rangle^N_c \\
&= \frac{-\mathrm{i}\varepsilon_{3ij}}{4F^2}\,{}^N\!\big\langle\phi_0\big|\int\delta(t)d^4xd^4x_1\mathrm{e}^{-\mathrm{i}qx} \\
&\quad\times\bar\psi(x)\gamma^3\tau_i\psi(x)\bar\psi(x_1)\gamma^\mu\gamma^5\tau_m\psi(x_1)\pi_j(x)\partial_\mu\pi_m(x_1)\big|\phi_0\big\rangle^N_c \\
&= \frac{-\mathrm{i}\varepsilon_{3ij}}{4F^2(2\pi)^4}\,{}^N\!\big\langle\phi_0\big|b_0^\dagger(x)\int d^3xd^3x_1d^4k\frac{\mathrm{e}^{\mathrm{i}\vec k\cdot(\vec x-\vec x_1)}}{M_\pi^2-k^2-\mathrm{i}\epsilon} \\
&\quad\times\bar u_0(x)\gamma^3\tau_iu_\alpha(x)\bar u_\alpha(x_1)\gamma^\mu k_\mu\gamma^5\tau_ju_0(x_1)\,\mathrm{e}^{\mathrm{i}\vec q\cdot\vec x} \\
&\quad\times\int dtdt_1\delta(t)\mathrm{e}^{-\mathrm{i}q_0t}\mathrm{e}^{\mathrm{i}\mathcal{E}_0t}\mathrm{e}^{-\mathrm{i}\mathcal{E}_\alpha(t-t_1)}\Theta(t-t_1)\mathrm{e}^{-\mathrm{i}\mathcal{E}_0t_1}\mathrm{e}^{-\mathrm{i}k_0(t-t_1)}b_0(x_1)\big|\phi_0\big\rangle^N
\end{aligned}$$

$$
\begin{aligned}
&= \frac{-\varepsilon_{3ij}}{4F^2(2\pi)^4}\, {}^N\!\big\langle\phi_0|b_0^\dagger(x)\int d^3x d^3x_1 d^3k\, \mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}-\vec{x}_1)}\, \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}} \\
&\quad \int dk_0 \left[\frac{1}{M_\pi^2 - k_0^2 + \vec{k}^2 - \mathrm{i}\epsilon}\right]\left[\frac{1}{k_0 + \Delta\mathcal{E}_\alpha - \mathrm{i}\eta}\right] \\
&\quad \times\left[\bar{u}_0(x)\gamma^3\tau_i u_\alpha(x)\bar{u}_\alpha(x_1)(\gamma^0 k_0 - \vec{\gamma}\cdot\vec{k})\gamma^5\tau_j u_0(x_1)\right] b_0(x_1)|\phi_0\big\rangle^N \\
&= \frac{-\varepsilon_{3ij}\pi\mathrm{i}}{4F^2(2\pi)^4}\, {}^N\!\big\langle\phi_0|b_0^\dagger(x)\int d^3k \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\quad \times\Bigg\{\omega(k^2)\left[\int d^3x\bar{u}_0(x)\gamma^3\tau_i u_\alpha(x)\, \mathrm{e}^{\mathrm{i}(\vec{q}+\vec{k})\cdot\vec{x}}\right]\left[\int d^3x_1\bar{u}_\alpha(x_1)\gamma^0\gamma^5\tau_j u_0(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\right] \\
&\quad - \left[\int d^3x\bar{u}_0(x)\gamma^3\tau_i u_\alpha(x)\, \mathrm{e}^{\mathrm{i}(\vec{q}+\vec{k})\cdot\vec{x}}\right]\left[\int d^3x_1\bar{u}_\alpha(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_0(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\right]\Bigg\} \\
&\quad \times b_0(x_1)|\phi_0\big\rangle^N \\
&= \frac{-\varepsilon_{3ij}\pi\mathrm{i}}{4F^2(2\pi)^4}\, {}^N\!\big\langle\phi_0|b_0^\dagger(x)\int d^3k \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\quad \times\Bigg\{\left[\int d^3x\bar{u}_0(x)\gamma^3\tau_i u_\alpha(x)\, \mathrm{e}^{\mathrm{i}(\vec{q}+\vec{k})\cdot\vec{x}}\right]\omega(k^2)\left[F^\dagger_{II_\alpha}(k)\Big[\tau_j(\vec{\sigma}\cdot\hat{k})\Big]_{\alpha,0}\right] \\
&\quad - \left[\int d^3x\bar{u}_0(x)\gamma^3\tau_i u_\alpha(x)\, \mathrm{e}^{\mathrm{i}(\vec{q}+\vec{k})\cdot\vec{x}}\right]\left[F^\dagger_{I_\alpha}(k)\Big[\tau_j(\vec{\sigma}\cdot\hat{k})\Big]_{\alpha,0}\right]\Bigg\} \\
&\quad \times b_0(x_1)|\phi_0\big\rangle^N.
\end{aligned}
$$

Using the result in Eq. (K.11)

$$
\begin{aligned}
\int d^3x\bar{u}_0(x)\gamma^3\tau_i u_\alpha(x)\, \mathrm{e}^{\mathrm{i}(\vec{q}+\vec{k})\cdot\vec{x}} &= \frac{F_{V_\alpha}(|\vec{q}+\vec{k}|)}{|\vec{q}+\vec{k}|}\Big[\varepsilon_{3mn}\sigma_m k_n\tau_i\Big]_{0,\alpha} \\
&\quad -\frac{F_{IV_\alpha}(|\vec{q}+\vec{k}|)}{|\vec{q}+\vec{k}|}\Big[\Big((\sqrt{Q^2}+k_3)\mathbf{I} + \mathrm{i}\varepsilon_{3mn}k_m\sigma_n\Big)\tau_i\Big]_{0,\alpha},
\end{aligned}
$$

we obtain

$$\chi^{\dagger}_{N_{s'}}\sigma^3_N\frac{\tau^3_N}{2}\chi_{N_s}G_A(Q^2)\Big|^{\alpha}_{SE;II}$$

$$\begin{aligned}
&= \frac{-\varepsilon_{3ij}\pi\mathrm{i}}{4F^2(2\pi)^4}\,{}^N\!\langle\phi_0|b_0^{\dagger}(x)\Big[\chi_c^{\dagger}\chi_f^{\dagger}\chi_s^{\dagger}\Big]_0\int_0^{\infty}dk\,k^2\,\frac{\omega(k^2)F^{\dagger}_{II_\alpha}(k)-F^{\dagger}_{I_\alpha}(k)}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\quad\times\int_{\Omega}d\Omega\Bigg\{\frac{F_{V_\alpha}(k,Q^2,\cos\theta)}{\sqrt{k^2+Q^2+2k\sqrt{Q^2}\cos\theta}}\Big[\varepsilon_{3mn}\sigma_m k_n\tau_i\Big]_\alpha\Big[\tau_j(\vec{\sigma}\cdot\hat{k})\Big]_\alpha \\
&\quad-\frac{F_{IV_\alpha}(k,Q^2,\cos\theta)}{\sqrt{k^2+Q^2+2k\sqrt{Q^2}\cos\theta}}\Big[\Big((\sqrt{Q^2}+k_3)\mathbf{I}+\mathrm{i}\varepsilon_{3mn}k_m\sigma_n\Big)\tau_i\Big]_\alpha\Big[\tau_j(\vec{\sigma}\cdot\hat{k})\Big]_\alpha\Bigg\} \\
&\quad\times\Big[\chi_s\chi_f\chi_c\Big]_0 b_0(x_1)|\phi_0\rangle^N \\
&= \frac{1}{(4\pi F)^2}\,{}^N\!\langle\phi_0|b_0^{\dagger}(x_1)\Big[\chi_c^{\dagger}\chi_f^{\dagger}\chi_s^{\dagger}\Big]_0\sigma_3\tau_3\int_0^{\infty}dk\,k^2\,\frac{\omega(k^2)F^{\dagger}_{II_\alpha}(k)-F^{\dagger}_{I_\alpha}(k)}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\quad\times\int_{-1}^{1}dx\Bigg\{\frac{\mathrm{i}\,k\;(1-x^2)\;F_{V_\alpha}(k_+)-(\sqrt{Q^2}\,x+k)F_{IV_\alpha}(k_+)}{\sqrt{k_+^2}}\Bigg\} \\
&\quad\times\Big[\chi_s\chi_f\chi_c\Big]_0 b_0(x)|\phi_0\rangle^N \\
&= \frac{5}{3}\left(\frac{1}{4\pi F}\right)^2\int_0^{\infty}dk\,k^2\,\frac{\omega(k^2)F^{\dagger}_{II_\alpha}(k)-F^{\dagger}_{I_\alpha}(k)}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\quad\times\int_{-1}^{1}dx\Bigg\{\frac{\mathrm{i}\,k\;(1-x^2)\;F_{V_\alpha}(k_+)-(\sqrt{Q^2}\,x+k)F_{IV_\alpha}(k_+)}{\sqrt{k_+^2}}\Bigg\}.
\end{aligned}$$

Finally we obtain

$$\begin{aligned}
G_A(Q^2)\Big|_{SE;II} &= \frac{10}{3}\left(\frac{1}{4\pi F}\right)^2\sum_{\alpha}\int_0^{\infty}dk\,k^2\,\frac{\omega(k^2)F^{\dagger}_{II,\alpha}(k)-F^{\dagger}_{I_\alpha}(k)}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)} \\
&\quad\times\int_{-1}^{1}dx\Bigg\{\frac{\mathrm{i}\,k\;(1-x^2)\;F_{V_\alpha}(k_+)-(\sqrt{Q^2}\,x+k)F_{IV_\alpha}(k_+)}{\sqrt{k_+^2}}\Bigg\}.
\end{aligned} \tag{I.23}$$

When we restrict the quark propagator to only the ground state and use these results

$$F_{III_0}(k_+) = \frac{2\,\mathrm{i}\,k_+\,R\,\rho}{2+3\rho^2}\exp\left(-\frac{k_+^2R^2}{4}\right), \tag{I.24}$$

$$F_{IV_0}(k_+) = 0, \tag{I.25}$$

the result in Eq. (I.23) reduces to

$$
\begin{aligned}
& G_A(Q^2)\big|^{GS}_{SE;II} \\
&\quad = \frac{g_A^{(0)}\rho R}{2+3\rho^2}\left(\frac{1}{2\pi F}\right)^2 \int_0^\infty dk\, k^4\, \frac{F_{\pi NN}(k)}{\omega^2(k^2)} \int_{-1}^{1} dx\left(1-x^2\right)\exp\left(-\frac{k_+^2 R^2}{4}\right) \\
&\quad = 8\frac{g_A^{(0)}\rho R}{2+3\rho^2}\left(\frac{1}{2\pi F}\right)^2 \int_0^\infty dk\, k^4\, \frac{F_{\pi NN}(k)}{\omega^2(k^2)}\mathcal{D}(k,Q^2). \qquad \text{(I.26)}
\end{aligned}
$$

I.6 Vertex Correction

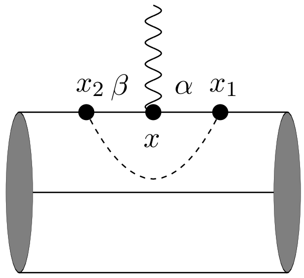

Figure I.6: Vertex correction diagram

$$
\begin{aligned}
& \chi^\dagger_{N_{s'}}\sigma^3_N \tfrac{\tau^3_N}{2}\chi_{N_s} G_A(Q^2)\big|^{\beta,\alpha}_{VC} \\
&\quad = 2\,{}^N\!\big\langle\phi_0\big|\frac{-1}{2}\int \delta(t)d^4x d^4x_1 d^4x_2 \\
&\qquad \times N\left\{\left[\frac{1}{2F}\,\partial_\mu\pi_i\bar{\psi}\gamma^\mu\gamma^5\tau_i\psi\right]_{x_1}\left[\frac{1}{2F}\,\partial_\nu\pi_j\bar{\psi}\gamma^\nu\gamma^5\tau_j\psi\right]_{x_2}\left[\bar{\psi}\gamma^3\gamma^5\frac{\tau_3}{2}\psi\right]_x\right\}\big|\phi_0\big\rangle^N_c \\
&\quad = \frac{-1}{8F^2}\,{}^N\!\big\langle\phi_0\big|\int \delta(t)d^4x d^4x_1 d^4x_2 \\
&\qquad \times\bar{\psi}(x_1)\gamma^\mu\gamma^5\tau_i\psi(x_1)\bar{\psi}(x)\gamma^3\gamma^5\tau_3\psi(x)\bar{\psi}(x_2)\gamma^\nu\gamma^5\tau_j\psi(x_2)\partial_\mu\pi_i(x_1)\partial_\nu\pi_j(x_2)\big|\phi_0\big\rangle^N_c \\
&\quad = \frac{\mathrm{i}}{8F^2(2\pi)^4}\,{}^N\!\big\langle\phi_0\big|b_0^\dagger\int d^3x d^3x_1 d^3x_2 d^4k\,\bar{u}_0(x_1)\gamma^\mu k_\mu\gamma^5\tau_i u_\alpha(x_1) \\
&\qquad \times\bar{u}_\alpha(x)\gamma^3\gamma^5\tau_3\, u_\beta(x)\bar{u}_\beta(x_2)\gamma^\nu k_\nu\gamma^5\tau_i u_0(x_2)\frac{\mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}}{M_\pi^2-k^2-\mathrm{i}\epsilon} \\
&\qquad \times\int dt dt_1 dt_2\Big\{\delta(t)\mathrm{e}^{-\mathrm{i}q_0 t}\mathrm{e}^{\mathrm{i}\varepsilon_0 t_1}\mathrm{e}^{-\mathrm{i}\varepsilon_\alpha(t_1-t)}\mathrm{e}^{-\mathrm{i}\varepsilon_\beta(t-t_2)}\mathrm{e}^{-\mathrm{i}\varepsilon_0 t_2}\mathrm{e}^{-\mathrm{i}k_0(t_1-t_2)} \\
&\qquad \times\Theta(t_1-t)\Theta(t-t_2)\Big\}b_0\big|\phi_0\big\rangle^N
\end{aligned}
$$

$$
\begin{aligned}
=&\frac{-\mathrm{i}}{8F^2(2\pi)^4}\,{}^N\!\big\langle\phi_0|b_0^\dagger\int d^3x d^3x_1 d^3x_2 d^3k \mathrm{e}^{\mathrm{i}\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}\mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}}\\
&\times\int dk_0\left[\frac{1}{M_\pi^2-k_0^2+\vec{k}^2-\mathrm{i}\epsilon}\right]\left[\frac{1}{k_0+\Delta\mathcal{E}_\alpha-\mathrm{i}\eta}\right]\left[\frac{1}{k_0+\Delta\mathcal{E}_\beta-\mathrm{i}\eta}\right]\\
&\times\bar{u}_0(x_1)(\gamma^0k_0-\vec{\gamma}\cdot\vec{k})\gamma^5\tau_i u_\alpha(x_1)\bar{u}_\alpha(x)\gamma^3\gamma^5\tau_3\,u_\beta(x)\\
&\times\bar{u}_\beta(x_2)(\gamma^0k_0-\vec{\gamma}\cdot\vec{k})\gamma^5\tau_i u_0(x_2)b_0|\phi_0\big\rangle^N\\
=&\frac{1}{16F^2(2\pi)^3}\,{}^N\!\big\langle\phi_0|b_0^\dagger\int d^3k\frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)(\omega(k^2)+\Delta\mathcal{E}_\beta)}\\
&\times\Bigg\{\Big[\int d^3x_1\bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_i u_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\Big]\Gamma_{\alpha\beta}\Big[\int d^3x_2\bar{u}_\beta(x_2)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_i u_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\Big]\\
&-\omega(k^2)\Big[\int d^3x_1\bar{u}_0(x_1)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_i u_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\Big]\Gamma_{\alpha\beta}\Big[\int d^3x_2\bar{u}_\beta(x_2)\gamma^0\gamma^5\tau_i u_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\Big]\\
&-\omega(k^2)\Big[\int d^3x_1\bar{u}_0(x_1)\gamma^0\gamma^5\tau_i u_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\Big]\Gamma_{\alpha\beta}\Big[\int d^3x_2\bar{u}_\beta(x_2)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_i u_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\Big]\\
&+\omega^2(k^2)\Big[\int d^3x_1\bar{u}_0(x_1)\gamma^0\gamma^5\tau_i u_\alpha(x_1)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}_1}\Big]\Gamma_{\alpha\beta}\Big[\int d^3x_2\bar{u}_\beta(x_2)\gamma^0\gamma^5\tau_i u_0(x_2)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}_2}\Big]\Bigg\}\\
&\times b_0|\phi_0\big\rangle^N. \qquad (\mathrm{I}.27)
\end{aligned}
$$

Here we have used

$$
\int d^3x\bar{u}_\alpha(x)\gamma^3\gamma^5\tau_3\,u_\beta(x)\mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}}=\Big[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\Big]_\alpha\Gamma(Q^2)\Big[\chi_s\chi_f\chi_c\Big]_\beta\equiv\Gamma_{\alpha\beta} \qquad (\mathrm{I}.28)
$$

then

$$
\begin{aligned}
&\chi^\dagger_{N_{s'}}\sigma_N^3\tfrac{\tau_N^3}{2}\chi_{N_s}G_A(Q^2)\Big|_{VC}^{\beta,\alpha}\\
&\quad=\frac{1}{16F^2(2\pi)^3}\,{}^N\!\big\langle\phi_0|b_0^\dagger\int d^3k\frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)(\omega(k^2)+\Delta\mathcal{E}_\beta)}\\
&\quad\times\Bigg\{F_{I_\alpha}(k)F^\dagger_{I_\beta}(k)\,\Big[(\vec{\sigma}\cdot\hat{k})\tau_i\Big]_{0,\alpha}\Gamma_{\alpha\beta}\,\Big[\tau_i(\vec{\sigma}\cdot\hat{k})\Big]_{\beta,0}\\
&\quad-\omega(k^2)F_{I_\alpha}(k)F^\dagger_{II_\beta}(k)\,\Big[(\vec{\sigma}\cdot\hat{k})\tau_i\Big]_{0,\alpha}\Gamma_{\alpha\beta}\,\Big[\tau_i(\vec{\sigma}\cdot\hat{k})\Big]_{\beta,0}\\
&\quad-\omega(k^2)F_{II_\alpha}(k)F^\dagger_{I_\beta}(k)\,\Big[(\vec{\sigma}\cdot\hat{k})\tau_i\Big]_{0,\alpha}\Gamma_{\alpha\beta}\,\Big[\tau_i(\vec{\sigma}\cdot\hat{k})\Big]_{\beta,0}\\
&\quad+\omega^2(k^2)F_{II_\alpha}(k)F^\dagger_{II_\beta}(k)\,\Big[(\vec{\sigma}\cdot\hat{k})\tau_i\Big]_{0,\alpha}\Gamma_{\alpha\beta}\,\Big[\tau_i(\vec{\sigma}\cdot\hat{k})\Big]_{\beta,0}\Bigg\}b_0|\phi_0\big\rangle^N
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{16F^2(2\pi)^3}\, {}^{N}\!\left\langle \phi_0 | b_0^\dagger \int d^3k \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)(\omega(k^2)+\Delta\mathcal{E}_\beta)} \right. \\
&\quad \times \Big\{ F_{I_\alpha}(k)F^\dagger_{I_\beta}(k) - \omega(k^2)F_{I_\alpha}(k)F^\dagger_{II_\beta}(k) - \omega(k^2)F_{II_\alpha}(k)F^\dagger_{I_\beta}(k) \\
&\quad \left. +\omega^2(k^2)F_{II_\alpha}(k)F^\dagger_{II_\beta}(k) \Big\} \left[(\vec{\sigma}\cdot\hat{k})\tau_i\right]_{0,\alpha} \Gamma_{\alpha\beta} \left[\tau_i(\vec{\sigma}\cdot\hat{k})\right]_{\beta,0} b_0|\phi_0 \right\rangle^N \\
&= \frac{1}{16F^2(2\pi)^3}\, {}^{N}\!\left\langle \phi_0 | b_0^\dagger \left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_0 \int d^3k \frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)(\omega(k^2)+\Delta\mathcal{E}_\beta)} \right. \\
&\quad \times \Big\{ F_{I_\alpha}(k)F^\dagger_{I_\beta}(k) - \omega(k^2)F_{I_\alpha}(k)F^\dagger_{II_\beta}(k) - \omega(k^2)F_{II_\alpha}(k)F^\dagger_{I_\beta}(k) \\
&\quad +\omega^2(k^2)F_{II_\alpha}(k)F^\dagger_{II_\beta}(k) \Big\} \left[(\vec{\sigma}\cdot\hat{k})\tau_i\right] \left[\chi_s\chi_f\chi_c\right]_\alpha \Gamma_{\alpha\beta} \left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_\beta \left[\tau_i(\vec{\sigma}\cdot\hat{k})\right] \\
&\quad \left. \times \left[\chi_s\chi_f\chi_c\right]_0 b_0|\phi_0 \right\rangle^N .
\end{aligned} \tag{I.29}
$$

One can show that

$$
\left[\chi_s\chi_f\chi_c\right]_\alpha \Gamma_{\alpha\beta} \left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_\beta = \sigma_3\tau_3\, \mathcal{F}_{\alpha,\beta}(Q^2) \tag{I.30}
$$

where

$$
\mathcal{F}_{\alpha,\beta}(Q^2) \equiv N_\alpha N_\beta \int_0^\infty dr r^2 \Big(\mathcal{A}_{\alpha,\beta}(r) + 2\mathcal{B}_{\alpha,\beta}(r) \Big), \tag{I.31}
$$

$$
\mathcal{A}_{\alpha,\beta}(r) \equiv (g_\alpha(r)g_\beta(r) - f_\alpha(r)f_\beta(r)) \int_\Omega d\Omega\, \exp\left(\mathrm{i}\sqrt{Q^2}r\cos\theta\right) \mathcal{C}_{\alpha\beta;1}(\theta,\phi), \tag{I.32}
$$

$$
\begin{aligned}
\mathcal{B}_{\alpha,\beta}(r) &\equiv f_\alpha(r)f_\beta(r) \int_\Omega d\Omega\, \exp\left(\mathrm{i}\sqrt{Q^2}r\cos\theta\right) \\
&\quad \times \Big[\cos^2\theta\, \mathcal{C}_{\alpha\beta;1}(\theta,\phi) + \sin\theta\cos\theta\, \mathcal{C}_{\alpha\beta;2}(\theta,\phi) \Big],
\end{aligned} \tag{I.33}
$$

$$
\mathcal{C}_{\alpha\beta;1}(\theta,\phi) \equiv C_\alpha C_\beta Y_{l_\alpha 0}(\theta,\phi)Y_{l_\beta 0}(\theta,\phi) - D_\alpha D_\beta Y^*_{l_\alpha 1}(\theta,\phi)Y_{l_\beta 1}(\theta,\phi), \tag{I.34}
$$

$$
\mathcal{C}_{\alpha\beta;2}(\theta,\phi) \equiv C_\alpha D_\beta Y_{l_\alpha 0}(\theta,\phi)Y_{l_\beta 1}(\theta,\phi)\mathrm{e}^{-\mathrm{i}\phi} + D_\alpha C_\beta Y^*_{l_\alpha 1}(\theta,\phi)Y_{l_\beta 0}(\theta,\phi)\mathrm{e}^{\mathrm{i}\phi}, \tag{I.35}
$$

where $D_\alpha = \left\langle l_\alpha 1 \frac{1}{2} -\frac{1}{2} | j \frac{1}{2} \right\rangle$, l_α and l_β are the orbital quantum numbers of the intermediate states α and β, respectively.

$$
\begin{aligned}
&\chi^{\dagger}_{N_{s'}}\sigma^3_N\frac{\tau^3_N}{2}\chi_{N_s}G_A(Q^2)\Big|^{\beta,\alpha}_{VC} \\
&= \frac{-\mathcal{F}_{\alpha,\beta}(Q^2)}{16F^2(2\pi)^3}\,{}^{N}\Big\langle\phi_0|b^{\dagger}_0\Big[\chi^{\dagger}_c\chi^{\dagger}_f\chi^{\dagger}_s\Big]_0\tau_3\int_0^{\infty}dkk^2\frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)(\omega(k^2)+\Delta\mathcal{E}_\beta)} \\
&\quad\times\Bigg\{F_{I_\alpha}(k)F^{\dagger}_{I_\beta}(k)-\omega(k^2)F_{I_\alpha}(k)F^{\dagger}_{II_\beta}(k)-\omega(k^2)F_{II_\alpha}(k)F^{\dagger}_{I_\beta}(k) \\
&\quad+\omega^2(k^2)F_{II_\alpha}(k)F^{\dagger}_{II_\beta}(k)\Bigg\}\int_\Omega d\Omega(\vec{\sigma}\cdot\hat{k})\sigma_3(\vec{\sigma}\cdot\hat{k})\Big[\chi_s\chi_f\chi_c\Big]_0 b_0|\phi_0\Big\rangle^{N}. \qquad \text{(I.36)}
\end{aligned}
$$

With this algebra

$$
\int_\Omega d\Omega(\vec{\sigma}\cdot\hat{k})\sigma_3(\vec{\sigma}\cdot\hat{k}) = -\frac{4\pi}{3}\,\sigma_3, \qquad \text{(I.37)}
$$

we finally obtain

$$
\begin{aligned}
G_A(Q^2)\Big|^{\beta,\alpha}_{VC} &= \frac{5}{9}\frac{\mathcal{F}_{\alpha,\beta}(Q^2)}{(4\pi F)^2}\int_0^{\infty}dkk^2\frac{1}{\omega(k^2)(\omega(k^2)+\Delta\mathcal{E}_\alpha)(\omega(k^2)+\Delta\mathcal{E}_\beta)} \\
&\quad\times\Bigg\{F_{I_\alpha}(k)F^{\dagger}_{I_\beta}(k)-\omega(k^2)F_{I_\alpha}(k)F^{\dagger}_{II_\beta}(k)-\omega(k^2)F_{II_\alpha}(k)F^{\dagger}_{I_\beta}(k) \\
&\quad+\omega^2(k^2)F_{II_\alpha}(k)F^{\dagger}_{II_\beta}(k)\Bigg\}. \qquad \text{(I.38)}
\end{aligned}
$$

Appendix J

Vertex Function for qqπ System

In this appendix we will show the expression of the integral form for the vertex function for qqπ. There are

$$\int d^3x \bar{u}_\beta(x)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}, \tag{J.1}$$

$$\int d^3x \bar{u}_\alpha(x)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_\beta(x)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}}, \tag{J.2}$$

$$\int d^3x \bar{u}_\beta(x)\gamma^0\gamma^5\tau_j u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}, \tag{J.3}$$

$$\int d^3x \bar{u}_\alpha(x)\gamma^0\gamma^5\tau_j u_\beta(x)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}}. \tag{J.4}$$

Eqs. (J.1) and (J.2) are related by the the hermitian conjugate, as the following

$$\left[\int d^3x \bar{u}_\alpha(x)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_\beta(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}\right]^\dagger = \int d^3x \bar{u}_\beta(x)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_\alpha(x)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}}. \tag{J.5}$$

Again Eqs. (J.3) and (J.4) are also related by the the hermitian conjugate,

$$\left[\int d^3x \bar{u}_\beta(x)\gamma^0\gamma^5\tau_j u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}\right]^\dagger = \int d^3x \bar{u}_\alpha(x)\gamma^0\gamma^5\tau_j u_\beta(x)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}}. \tag{J.6}$$

J.1 $\int d^3x \bar{u}_\beta(\vec{x})\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_\alpha(\vec{x})\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}$

$$
\begin{aligned}
&\int d^3x \bar{u}_\beta(\vec{x})\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_\alpha(\vec{x})\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \\
&= N_\beta N_\alpha \chi_c^\dagger \chi_f^\dagger \int d^3x \mathcal{Y}_\beta^\dagger(\hat{x})\big[g_\beta(x)\; -\mathrm{i}(\vec{\sigma}\cdot\hat{x})\, f_\beta(x)\big]\gamma^0\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j \begin{pmatrix} g_\alpha(x) \\ \mathrm{i}\vec{\sigma}\cdot\hat{x} f_\alpha(x) \end{pmatrix} \\
&\quad \times\mathcal{Y}_\alpha(\hat{x})\chi_{f'}\chi_{c'}\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \\
&= N_\beta N_\alpha \chi_c^\dagger \chi_f^\dagger \int d^3x \mathcal{Y}_\beta^\dagger(\hat{x})\left[g_\beta(x)g_\alpha(x)\,(\vec{\sigma}\cdot\vec{k}) + f_\beta(x)f_\alpha(x)(\vec{\sigma}\cdot\hat{x})(\vec{\sigma}\cdot\vec{k})(\vec{\sigma}\cdot\hat{x})\right] \\
&\quad \times\tau_j\mathcal{Y}_\alpha(\hat{x})\chi_{f'}\chi_{c'}\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \\
&= N_\beta N_\alpha \chi_c^\dagger \chi_f^\dagger \int d^3x \mathcal{Y}_\beta^\dagger(\hat{x}) \\
&\quad \times\left\{\left[g_\beta(x)g_\alpha(x) - f_\beta(x)f_\alpha(x)\right](\vec{\sigma}\cdot\vec{k}) + 2f_\beta(x)f_\alpha(x)(\vec{\sigma}\cdot\hat{x})(\hat{x}\cdot\vec{k})\right\} \\
&\quad \times\tau_j\mathcal{Y}_\alpha(\hat{x})\chi_{f'}\chi_{c'}\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \\
&= N_\beta N_\alpha k\, \chi_c^\dagger \chi_f^\dagger \Bigg\{\Bigg\{\int_0^\infty dx\, x^2\Big[g_\beta(x)g_\alpha(x) - f_\beta(x)f_\alpha(x)\Big] \\
&\quad \times\int_\Omega d\Omega\mathcal{Y}_\beta^\dagger(\hat{x})(\vec{\sigma}\cdot\hat{k})\mathcal{Y}_\alpha(\hat{x})\mathrm{e}^{\mathrm{i}kx\cos\theta}\Bigg\} \\
&\quad -2\,\mathrm{i}\frac{\partial}{\partial k}\Bigg\{\int_0^\infty dx\, x f_\beta(x)f_\alpha(x)\int_\Omega d\Omega\cos\theta\mathcal{Y}_\beta^\dagger(\hat{x})(\vec{\sigma}\cdot\hat{k})\mathcal{Y}_\alpha(\hat{x})\mathrm{e}^{\mathrm{i}kx\cos\theta}\Bigg\}\Bigg\}\tau_j\chi_{f'}\chi_{c'},
\end{aligned}
\tag{J.7}
$$

here we have used

$$
\begin{aligned}
\gamma^0\vec{\gamma}\cdot\vec{k}\gamma^5 &= (\vec{\sigma}\cdot\vec{k})\,\mathbf{I}, && \text{(J.8)} \\
(\vec{\sigma}\cdot\hat{x})(\vec{\sigma}\cdot\vec{k})(\vec{\sigma}\cdot\hat{x}) &= 2(\vec{\sigma}\cdot\hat{x})(\vec{\sigma}\cdot\vec{k}) - (\vec{\sigma}\cdot\vec{k}) && \text{(J.9)}
\end{aligned}
$$

and

$$
(\vec{\sigma}\cdot\vec{x})\,\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} = -\mathrm{i}\left(\vec{\sigma}\cdot\hat{k}\right)\frac{\partial}{\partial k}\,\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}. \tag{J.10}
$$

When $\beta = 1s_{1/2}$ only the contribution from the projection $l_m = 0$ of the orbital angular momentum l that belongs to state α does not vanish, then the result yields

$$
\begin{aligned}
&\int d^3x \bar{u}_0(\vec{x})\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_\alpha(\vec{x})\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \\
&= N_0 N_\alpha k\left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_0(\vec{\sigma}\cdot\hat{k})\Bigg\{\bigg\{\int_0^\infty dx\,x^2\left[g_0(x)g_\alpha(x)-f_0(x)f_\alpha(x)\right] \\
&\quad\times\int_\Omega d\Omega\mathrm{e}^{\mathrm{i}kx\cos\theta}C_\alpha Y_{l_\alpha 0}(\theta,\phi)\bigg\} \\
&\quad -2\,\mathrm{i}\,\frac{\partial}{\partial k}\bigg\{\int_0^\infty dx\,x f_0(x)f_\alpha(x)\int_\Omega d\Omega\cos\theta\mathrm{e}^{\mathrm{i}kx\cos\theta}C_\alpha Y_{l_\alpha 0}(\theta,\phi)\bigg\}\Bigg\}\tau_j\left[\chi_s\chi_f\chi_c\right]_\alpha \\
&\equiv \left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_0(\vec{\sigma}\cdot\hat{k})F_{I_\alpha}(k)\tau_j\left[\chi_s\chi_f\chi_c\right]_\alpha \\
&\equiv F_{I_\alpha}(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_j\right]_{0,\alpha} \qquad \text{(J.11)}
\end{aligned}
$$

where

$$
\begin{aligned}
F_{I_\alpha}(k) \equiv N_0 N_\alpha k&\Bigg\{\bigg\{\int_0^\infty dx\,x^2\left[g_0(x)g_\alpha(x)-f_0(x)f_\alpha(x)\right]\int_\Omega d\Omega\mathrm{e}^{\mathrm{i}kx\cos\theta}C_\alpha Y_{l_\alpha 0}(\theta,\phi)\bigg\} \\
&-2\,\mathrm{i}\,\frac{\partial}{\partial k}\bigg\{\int_0^\infty dx\,x f_0(x)f_\alpha(x)\int_\Omega d\Omega\cos\theta\mathrm{e}^{\mathrm{i}kx\cos\theta}C_\alpha Y_{l_\alpha 0}(\theta,\phi)\bigg\}\Bigg\}. \qquad \text{(J.12)}
\end{aligned}
$$

For the Clebsch-Gordan coefficients we use the notation $C_\alpha \equiv \left\langle l_\alpha 0\frac{1}{2}m_s|jm_j\right\rangle$ and $Y_{l_\alpha 0}(\theta,\phi)$ is the usual spherical harmonic. The explicit form of the radial wave functions $g_\alpha(r)$ and $f_\alpha(r)$, of the normalization constants (N_α) and of the energy difference ($\Delta\mathcal{E}_\alpha$) are already given in Appendix D. With the relation in Eq. (J.5) we get

$$
\begin{aligned}
\int d^3x \bar{u}_\alpha(x)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_0(x)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}} &= \left[\int d^3x \bar{u}_0(x)\vec{\gamma}\cdot\vec{k}\gamma^5\tau_j u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}\right]^\dagger \\
&\equiv \left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_\alpha\tau_j F_{I_\alpha}^\dagger(k)(\vec{\sigma}\cdot\hat{k})\left[\chi_s\chi_f\chi_c\right]_0 \\
&\equiv F_{I_\alpha}^\dagger(k)\left[\tau_j(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0}. \qquad \text{(J.13)}
\end{aligned}
$$

For the special case when state β and α are in the state $1s_{1/2}$ then $F_{I_{1s1/2}}$ or F_{I_0} is

$$
\begin{aligned}
F_{I_0}(k) &= N_0^2 k\Bigg\{\bigg\{\int_0^\infty dx\,x^2\exp\left(-\frac{x^2}{R^2}\right)\left(1-\frac{\rho^2x^2}{R^2}\right)\int_\Omega d\Omega\mathrm{e}^{\mathrm{i}kx\cos\theta}\bigg\} \\
&\quad -2\,\mathrm{i}\,\frac{\partial}{\partial k}\bigg\{\int_0^\infty dx\,x\exp\left(-\frac{x^2}{R^2}\right)\left(\frac{\rho^2x^2}{R^2}\right)\int_\Omega d\Omega\cos\theta\mathrm{e}^{\mathrm{i}kx\cos\theta}\bigg\}\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{4}N_0^2 k\, R^3\pi^{3/2}(-4+(2+k^2R^2)\rho^2)\exp\left(-\frac{k^2R^2}{4}\right) \\
&= \frac{3}{5}g_A^{(0)} k\, F_{\pi NN}(k^2), \qquad \text{(J.14)}
\end{aligned}
$$

where $F_{\pi NN}$ is the πNN form factor normalized to unity at zero recoil ($k^2 = 0$):

$$
F_{\pi NN}(k^2) = \exp\left(-\frac{k^2R^2}{4}\right)\left[1+\frac{k^2R^2}{8}\left(1-\frac{5}{3g_A^{(0)}}\right)\right], \qquad \text{(J.15)}
$$

we also obtain

$$
F_{I_0}^{\dagger}(k) \;=\; \frac{3}{5}g_A^{(0)} k\, F_{\pi NN}(k^2). \qquad \text{(J.16)}
$$

J.2 $\int d^3x \bar{u}_\beta(x)\gamma^0\gamma^5\tau_j u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}$

$$
\begin{aligned}
&\int d^3x \bar{u}_\beta(x)\gamma^0\gamma^5\tau_j u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \\
&= N_\beta N_\alpha \chi_c^\dagger\chi_f^\dagger \int d^3x \mathcal{Y}_\beta^\dagger(\hat{x})\left[g_\beta(x) \;-\mathrm{i}(\vec{\sigma}\cdot\hat{x})\, f_\beta(x)\right]\gamma^5\tau_j \begin{pmatrix} g_\alpha(r) \\ \mathrm{i}\vec{\sigma}\cdot\hat{r} f_\alpha(r)\end{pmatrix} \\
&\quad \times\mathcal{Y}_\alpha(\hat{x})\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}\chi_{f'}\chi_{c'} \\
&= N_\beta N_\alpha \chi_c^\dagger\chi_f^\dagger \int d^3x \mathcal{Y}_\beta^\dagger(\hat{x})\left[g_\beta(x)f_\alpha(x) - f_\beta(x)g_\alpha(x)\right]\mathrm{i}(\vec{\sigma}\cdot\hat{x}) \\
&\quad \times\tau_j\mathcal{Y}_\alpha(\hat{x})\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}\chi_{f'}\chi_{c'} \\
&= N_\beta N_\alpha \chi_c^\dagger\chi_f^\dagger \int_0^\infty dx\, x\left[g_\beta(x)f_\alpha(x) - f_\beta(x)g_\alpha(x)\right] \\
&\quad \times\frac{\partial}{\partial k}\int_\Omega d\Omega \mathcal{Y}_\beta^\dagger(\hat{x})(\vec{\sigma}\cdot\hat{k})\mathcal{Y}_\alpha(\hat{x})\mathrm{e}^{\mathrm{i}kx\cos\theta}\tau_j\chi_{f'}\chi_{c'}. \qquad \text{(J.17)}
\end{aligned}
$$

When $\beta = 1s_{1/2}$ only the contribution from the projection $l_m = 0$ of the orbital angular momentum l of state α does not vanish, then the result yields

$$\begin{aligned}
&\int d^3x \bar{u}_0(x)\gamma^0\gamma^5\tau_j u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \\
&\qquad = N_0 N_\alpha \chi_c^\dagger \chi_f^\dagger \chi_s^\dagger (\vec{\sigma}\cdot\hat{k}) \int_0^\infty dx\, x\, [g_0(x) f_\alpha(x) - f_0(x) g_\alpha(x)] \\
&\qquad\quad \times \frac{\partial}{\partial k} \int_\Omega d\Omega \mathrm{e}^{\mathrm{i}kx\cos\theta} C_\alpha Y_{l_\alpha 0}(\theta,\phi)\tau_j \chi_{s'}\chi_{f'}\chi_{c'} \\
&\qquad \equiv \left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_0 (\vec{\sigma}\cdot\hat{k}) F_{II_\alpha}(k)\tau_j \left[\chi_s\chi_f\chi_c\right]_\alpha \\
&\qquad \equiv F_{II_\alpha}(k)\left[(\vec{\sigma}\cdot\hat{k})\tau_j\right]_{0,\alpha}, \qquad \text{(J.18)}
\end{aligned}$$

where

$$\begin{aligned}
F_{II_\alpha}(k) \equiv\; & N_0 N_\alpha \int_0^\infty dx\, x\, [g_0(x) f_\alpha(x) - f_0(x) g_\alpha(x)] \\
& \times \frac{\partial}{\partial k} \int_\Omega d\Omega \mathrm{e}^{\mathrm{i}kx\cos\theta} C_\alpha Y_{l_\alpha 0}(\theta,\phi). \qquad \text{(J.19)}
\end{aligned}$$

With the relation in Eq. (J.5) we get

$$\begin{aligned}
\int d^3x \bar{u}_\alpha(x)\gamma^0\gamma^5\tau_j u_0(x)\mathrm{e}^{-\mathrm{i}\vec{k}\cdot\vec{x}} &= \left[\int d^3x \bar{u}_0(x)\gamma^0\gamma^5\tau_j u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}\right]^\dagger \\
&\equiv \left[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\right]_\alpha \tau_j F_{II_\alpha}^\dagger(k)(\vec{\sigma}\cdot\hat{k})\left[\chi_s\chi_f\chi_c\right]_0 \\
&\equiv F_{II_\alpha}^\dagger(k)\left[\tau_j(\vec{\sigma}\cdot\hat{k})\right]_{\alpha,0}. \qquad \text{(J.20)}
\end{aligned}$$

Appendix K

Vertex Functions for Quark-Axial Vector Current and Quark-Pion-Axial Vector Current

In this appendix we will show the expression of the integral form for the vertex function for the quark-axial vector current and the quark-pion-axial vector current.

K.1 Vertex Function for Quark-Axial Vector Current

$$
\begin{aligned}
&\int d^3x \bar{u}_\alpha(x)\gamma^3\gamma^5\tau_3 u_\beta(x)\mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}} \\
&= N_\alpha N_\beta \chi_c^\dagger \chi_f^\dagger \int d^3x\, \mathcal{Y}_\alpha^\dagger(\hat{x})\Big(g_\alpha(r) \quad -\mathrm{i}(\vec{\sigma}\cdot\hat{x})f_\alpha(r)\Big)\sigma_3\tau_3\begin{pmatrix} g_\beta(r) \\ \mathrm{i}(\vec{\sigma}\cdot\hat{x})f_\beta(r)\end{pmatrix}\mathcal{Y}_\beta(\hat{x}) \\
&\quad\times \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}}\chi_f\chi_c \\
&= N_\alpha N_\beta \chi_c^\dagger \chi_f^\dagger \int d^3x\, \mathcal{Y}_\alpha^\dagger(\hat{x})\Big(g_\alpha(r)g_\beta(r)\sigma_3 + f_\alpha(r)f_\beta(r)(\vec{\sigma}\cdot\hat{x})\sigma_3(\vec{\sigma}\cdot\hat{x})\Big)\tau_3\mathcal{Y}_\beta(\hat{x}) \\
&\quad\times \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}}\chi_f\chi_c \\
&= N_\alpha N_\beta \chi_c^\dagger \chi_f^\dagger \int d^3x\, \mathcal{Y}_\alpha^\dagger(\hat{x})\Big\{\Big[g_\alpha(r)g_\beta(r) - f_\alpha(r)f_\beta(r)\Big]\sigma_3 \\
&\quad +2f_\alpha(r)f_\beta(r)(\vec{\sigma}\cdot\hat{x})\hat{x}_3\Big)\Big\}\tau_3\mathcal{Y}_\beta(\hat{x})\mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}}\chi_f\chi_c \\
&= N_\alpha N_\beta \chi_c^\dagger \chi_f^\dagger \Big\{\int_0^\infty dx\, x^2\Big[g_\alpha(r)g_\beta(r) - f_\alpha(r)f_\beta(r)\Big]\int_\Omega d\Omega\, \mathcal{Y}_\alpha^\dagger(\hat{x})\,\sigma_3\tau_3\,\mathcal{Y}_\beta(\hat{x})\,\mathrm{e}^{\mathrm{i}qx\cos\theta} \\
&\quad +2\int_0^\infty dx\, x^2 f_\alpha(r)f_\beta(r)\int_\Omega d\Omega(\vec{\sigma}\cdot\hat{x})\cos\theta\,\mathcal{Y}_\alpha^\dagger(\hat{x})\,\tau_3\mathcal{Y}_\beta(\hat{x})\mathrm{e}^{\mathrm{i}qx\cos\theta}\Big\}\chi_f\chi_c. \qquad \text{(K.1)}
\end{aligned}
$$

Here we have used the Pauli spin matrix algebra,

$$(\vec{\sigma}\cdot\hat{x})\sigma_i(\vec{\sigma}\cdot\hat{x}) = 2(\vec{\sigma}\cdot\hat{x})\hat{x}_i - \sigma_i. \qquad \text{(K.2)}$$

When states α and β are in the ground state one obtains

$$
\begin{aligned}
&\int d^3x\bar{u}_0(x)\gamma^3\gamma^5\tau_3 u_0(x)\mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}} \\
&\quad= N_0^2\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\int_0^\infty dx\,x^2\,\exp\left(-\frac{\vec{x}^{\,2}}{R^2}\right) \\
&\qquad\times\left\{\sigma_3\left[1-\left(\frac{\rho\,x}{R}\right)^2\right]\int_\Omega d\Omega\,\mathrm{e}^{\mathrm{i}qx\cos\theta}+2\left(\frac{\rho\,x}{R}\right)^2\int_\Omega d\Omega\,\mathrm{e}^{\mathrm{i}qx\cos\theta}(\vec{\sigma}\cdot\hat{x})\cos\theta\right\} \\
&\qquad\times\tau_3\,\chi_s\chi_f\chi_c,
\end{aligned}
\tag{K.3}
$$

hence

$$
\begin{aligned}
\int d^3x\,\bar{u}_0(x)\gamma^3\gamma^5\tau_3 u_0(x)\mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{x}} &= \chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\mathrm{e}^{-\frac{Q^2R^2}{4}}\left[\frac{4-\rho^2(2+Q^2R^2)}{4+6\rho^2}\right]\sigma_3\tau_3\,\chi_s\chi_f\chi_c \\
&= \left(\frac{2-\rho^2}{2+3\rho^2}\right)F_{\pi NN}(Q^2)\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\sigma_3\tau_3\,\chi_s\chi_f\chi_c.
\end{aligned}
\tag{K.4}
$$

K.2 Vertex Function for Quark-Pion-Axial Vector Current

$$
\begin{aligned}
&\int d^3x\bar{u}_\beta(x)\gamma^3\tau_j u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \\
&= N_\beta N_\alpha\chi_c^\dagger\chi_f^\dagger\int d^3x\mathcal{Y}_\beta^\dagger(\hat{x})\big[g_\beta(x)\;\;-\mathrm{i}(\vec{\sigma}\cdot\hat{x})\,f_\beta(x)\big]\begin{pmatrix}0 & \sigma_3\\ \sigma_3 & 0\end{pmatrix}\begin{pmatrix}g_\alpha(x)\\ \mathrm{i}\vec{\sigma}\cdot\hat{x}f_\alpha(x)\end{pmatrix} \\
&\quad\times\tau_j\mathcal{Y}_\alpha(\hat{x})\chi_{f'}\chi_{c'}\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \\
&= N_\beta N_\alpha\chi_c^\dagger\chi_f^\dagger\int d^3x\mathcal{Y}_\beta^\dagger(\hat{x})\Big[-\mathrm{i}(\vec{\sigma}\cdot\hat{x})(\vec{\sigma}\cdot\hat{q})f_\beta(x)g_\alpha(x)+\mathrm{i}(\vec{\sigma}\cdot\hat{q})(\vec{\sigma}\cdot\hat{x})g_\beta(x)f_\alpha(x)\Big] \\
&\quad\times\tau_j\mathcal{Y}_\alpha(\hat{x})\chi_{f'}\chi_{c'}\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}.
\end{aligned}
\tag{K.5}
$$

Here we choose the direction of momentum transfer in z direction

$$\vec{q}\equiv|\vec{q}|\hat{k} \tag{K.6}$$

and with the Pauli matrix algebra

$$(\vec{\sigma}\cdot\hat{x})(\vec{\sigma}\cdot\hat{q})=2\,\mathrm{i}\,\hat{q}\cdot(\vec{\sigma}\times\hat{x})+(\vec{\sigma}\cdot\hat{q})(\vec{\sigma}\cdot\hat{x}). \tag{K.7}$$

$$
\begin{aligned}
&\int d^3x \bar{u}_\beta(x)\gamma^3\tau_j u_\alpha(x) \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \\
&= N_\beta N_\alpha \chi_c^\dagger \chi_f^\dagger \int d^3x \mathcal{Y}_\beta^\dagger(\hat{x}) \Big\{ 2\,\hat{q}\cdot(\vec{\sigma}\times\hat{x}) f_\beta(x) g_\alpha(x) \\
&\quad + \mathrm{i}(\vec{\sigma}\cdot\hat{q})(\vec{\sigma}\cdot\hat{x})\big[g_\beta(x)f_\alpha(x) - f_\beta(x)g_\alpha(x)\big]\Big\} \tau_j \mathcal{Y}_\alpha(\hat{x}) \chi_{f'}\chi_{c'} \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \\
&= N_\beta N_\alpha \chi_c^\dagger \chi_f^\dagger \Bigg\{ -2\,\mathrm{i}\,\Bigg[\int_0^\infty dx\, x f_\beta(x) g_\alpha(x) \int_\Omega d\Omega \mathcal{Y}_\beta^\dagger(\hat{x}) \big[\hat{q}\cdot(\vec{\sigma}\times\hat{k})\big] \frac{\partial}{\partial k} \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \mathcal{Y}_\alpha(\hat{x}) \Bigg] \\
&\quad + \Bigg[\int_0^\infty dx\, x \big[g_\beta(x)f_\alpha(x) - f_\beta(x)g_\alpha(x)\big] \int_\Omega d\Omega \mathcal{Y}_\beta^\dagger(\hat{x}) (\vec{\sigma}\cdot\hat{q})(\vec{\sigma}\cdot\hat{k}) \frac{\partial}{\partial k} \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \mathcal{Y}_\alpha(\hat{x}) \Bigg] \Bigg\} \\
&\quad \times \tau_j \chi_{f'}\chi_{c'}. \qquad \text{(K.8)}
\end{aligned}
$$

For the case when the state α is in the ground state, we obtain

$$
\begin{aligned}
&\int d^3x \bar{u}_\beta(x)\gamma^3\tau_j u_0(x) \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \\
&= N_\beta N_0 \Big[\chi_c^\dagger \chi_f^\dagger \chi_s^\dagger\Big]_\beta \\
&\quad \times \Bigg\{ -2\,\mathrm{i}\,\Bigg[\int_0^\infty dx\, x f_\beta(x) g_0(x) \int_\Omega d\Omega\, C_\beta Y_{l_\beta 0}(\theta,\phi) \big[\hat{q}\cdot(\vec{\sigma}\times\hat{k})\big] \frac{\partial}{\partial k} \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \Bigg] \\
&\quad + \Bigg[\int_0^\infty dx\, x \big[g_\beta(x)f_0(x) - f_\beta(x)g_0(x)\big] \int_\Omega d\Omega\, C_\beta Y_{l_\beta 0}(\theta,\phi) (\vec{\sigma}\cdot\hat{q})(\vec{\sigma}\cdot\hat{k}) \frac{\partial}{\partial k} \mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \Bigg] \Bigg\} \\
&\quad \times \tau_j \Big[\chi_s \chi_f \chi_c\Big]_0 \\
&= \Big[\chi_c^\dagger \chi_f^\dagger \chi_s^\dagger\Big]_\beta \Big\{ \big[\hat{q}\cdot(\vec{\sigma}\times\hat{k})\big] F_{III_\beta}(k) + (\vec{\sigma}\cdot\hat{q})(\vec{\sigma}\cdot\hat{k}) F_{IV_\beta}(k) \Big\} \tau_j \Big[\chi_s \chi_f \chi_c\Big]_0 \\
&= F_{III_\beta}(k) \big[\hat{q}\cdot(\vec{\sigma}\times\hat{k})\tau_j\big]_{\beta,0} + F_{IV_\beta}(k) \big[(\vec{\sigma}\cdot\hat{q})(\vec{\sigma}\cdot\hat{k})\tau_j\big]_{\beta,0}, \qquad \text{(K.9)}
\end{aligned}
$$

where

$$
\begin{aligned}
F_{III_\beta}(k) &\equiv -2\,\mathrm{i}\, N_\beta N_0 \frac{\partial}{\partial k} \Bigg[\int_0^\infty dx\, x f_\beta(x) g_0(x) \int_\Omega d\Omega\, C_\beta Y_{l_\beta 0}(\theta,\phi)\, \mathrm{e}^{\mathrm{i}kx\cos\theta} \Bigg], \\
F_{IV_\beta}(k) &\equiv N_\beta N_0 \frac{\partial}{\partial k} \Bigg[\int_0^\infty dx\, x \big[g_\beta(x)f_0(x) - f_\beta(x)g_0(x)\big] \\
&\quad \times \int_\Omega d\Omega\, C_\beta Y_{l_\beta 0}(\theta,\phi)\, \mathrm{e}^{\mathrm{i}kx\cos\theta} \Bigg]. \qquad \text{(K.10)}
\end{aligned}
$$

For the case when the state β is in the ground state, we obtain

$$
\begin{aligned}
&\int d^3x\bar{u}_0(x)\gamma^3\tau_j u_\alpha(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} \\
&= N_0N_\alpha\Big[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\Big]_0 \\
&\quad\times\Bigg\{-2\,\mathrm{i}\Bigg[\int_0^\infty dx\,xf_0(x)g_\alpha(x)\int_\Omega d\Omega\big[\hat{q}\cdot(\vec{\sigma}\times\hat{k})\big]\frac{\partial}{\partial k}\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}C_\alpha Y_{l_\alpha 0}(\theta,\phi)\Bigg] \\
&\quad+\Bigg[\int_0^\infty dx\,x\big[g_0(x)f_\alpha(x)-f_0(x)g_\alpha(x)\big]\int_\Omega d\Omega(\vec{\sigma}\cdot\hat{q})(\vec{\sigma}\cdot\hat{k})\frac{\partial}{\partial k}\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}}C_\alpha Y_{l_\alpha 0}(\theta,\phi)\Bigg]\Bigg\} \\
&\quad\times\tau_j\Big[\chi_s\chi_f\chi_c\Big]_\alpha \\
&= \Big[\chi_c^\dagger\chi_f^\dagger\chi_s^\dagger\Big]_0\Big\{\big[\hat{q}\cdot(\vec{\sigma}\times\hat{k})\big]F_{V_\alpha}(k)-(\vec{\sigma}\cdot\hat{q})(\vec{\sigma}\cdot\hat{k})F_{IV_\alpha}(k)\Big\}\tau_j\Big[\chi_s\chi_f\chi_c\Big]_\alpha \\
&= F_{V_\alpha}(k)\big[\hat{q}\cdot(\vec{\sigma}\times\hat{k})\tau_j\big]_{0,\alpha}-F_{IV_\alpha}(k)\big[(\vec{\sigma}\cdot\hat{q})(\vec{\sigma}\cdot\hat{k})\tau_j\big]_{0,\alpha}, \qquad \text{(K.11)}
\end{aligned}
$$

where

$$
F_{V_\alpha}(k) \equiv -2\,\mathrm{i}\,N_0N_\alpha\frac{\partial}{\partial k}\Bigg[\int_0^\infty dx\,xf_0(x)g_\alpha(x)\int_\Omega d\Omega\,C_\alpha Y_{l_\alpha 0}(\theta,\phi)\,\mathrm{e}^{\mathrm{i}kx\cos\theta}\Bigg]. \qquad \text{(K.12)}
$$

For the special case when the states β and α are in the ground state, we obtain

$$
\int d^3x\bar{u}_0(x)\gamma^3\tau_j u_0(x)\mathrm{e}^{\mathrm{i}\vec{k}\cdot\vec{x}} = F_{III_0}(k)[\hat{q}\cdot(\vec{\sigma}\times\hat{k})\tau_j\big]_{0,0} \qquad \text{(K.13)}
$$

where

$$
\begin{aligned}
F_{III_0}(k) &\equiv -2\,\mathrm{i}\,N_0^2\frac{\partial}{\partial k}\Bigg[\int_0^\infty dx\,xf_0(x)g_0(x)\int_\Omega d\Omega\,\mathrm{e}^{\mathrm{i}kx\cos\theta}\Bigg] \\
&= \frac{2\,\mathrm{i}\,k\,R\,\rho}{2+3\rho^2}\exp\left(-\frac{k^2R^2}{4}\right). \qquad \text{(K.14)}
\end{aligned}
$$

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