รหัสโครงการ SUT 1-103-42-12-16

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Mathematical Modeling of Steady Natural Convection in a Two-Layer System

แบบจำลองทางคณิตศาสตร์ของการเคลื่อนที่ของของไหลแบบคงตัว ในชั้นของของไหลสองชั้น

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ผลการวิจัยเป็นความรับผิดชอบของหัวหน้าโครงการวิจัยแต่เพียงผู้เดียว

มกราคม 2543

ACKNOWLEDGEMENT

This is to acknowledge to the Suranaree University of Technology Research Fund Administration for financial support of this projectt.

ABSTRACT

The Navier-Stokes equations in the Oberbeck-Boussinesq approach are used for description convective flows of viscous incompressible fluids in a two-layer systems. A finite-difference method is utilized to developed the numerical algorithm for modeling buoyancy driven flow in cavity vertical or horizontal sides which are differentially heated. The algorithm is based on the method of splitting (O.M. Belotserkovskii, V.A.Gushin, V.V. Shennikov [1]). The approximation is carried out on a staggered grid. Critical comparison with benchmark solution [2] confirms the accuracy of method, and results for the buoyancy-driven flow in square cavity with vertical sides, which are differently heated, are presented for Rayleigh numbers of 10^{6}. The results of two-dimensional (2-D) numerical simulations of thermal convection in two-layer system are compared with the experimental data of N.L. Dobretsov and A.G. Kirdyashkin [3]. The dependence of two-layer convection for wide range of relation of the viscosity, the thermal diffusivity and layer thickness was made.

บทคัดย่อ

แนวคิดของโอเบอร์เบค-บูสสิเนสค ได้ใช้สมการนาเวียร์ – สโตกส์ ศึกษาการไหลแบบพาของของไหลที่มีความหนืดและอัดตัวไม่ได้ในระบบการไหล 2 ชั้น ระเบียบวิธีการผลต่างสืบเนื่องเป็นวิธีการหนึ่งที่ถูกนำมาพัฒนาตามขั้นตอนวิธีการเชิงตัวเลข สำหรับการไหลจำลองของแรงลอยตัวภายในอาณาบริเวณตามแนวตั้งและแนวนอน ด้วยความร้อนที่แตกต่างกัน วิธีการผลต่างสืบเนื่องที่ใช้คือวิธีสปริททิ้ง [O.M. Belotserkovskii, V.A. Gushin และ V.V. Shennikov [1]] การประมาณค่าพิจารณาตรงจุดที่เหลื่อมกัน แสดงการเปรียบเทียบความแม่นตรงของผลเฉลยกับผลเฉลยที่ได้มีผู้วิจัยอื่นๆทำมาแล้ว ได้ผลลัพธ์ที่มีความถูกต้องตรงกัน ในกรณีของการไหลของแรงลอยตัวภายในช่องสี่เหลี่ยมจัตุรัสที่ตั้งตรงกับขนาดการไหลหมุนวน ซึ่งอยู่ในความร้อนที่แตกต่างกัน เมื่อกำหนดจำนวนเรลายด์ (Rayleigh Numbers) เท่ากับ 10^6 งานวิจัยนี้แสดงผลการเปรียบเทียบวิธีการจำลองเชิงตัวเลขของการพาความร้อนในสองมิติในระบบการไหล 2 ชั้น กับผลที่ได้จากการทดลองของ N.L. Dobretsov และ A.G. Kirdyashkin [3] นอกจากนี้การศึกษาการพาความร้อนของของไหลในชั้นของของไหล 2 ชั้น ยังได้พิจารณาถึงตัวแปรต่างๆเช่น ความหนืด การกระจายความร้อนและความหนาของชั้นด้วย

CONTENTS

1. Introduction

1.1 Background

Buoyancy-driven flow in a cavity has a wide variety of practical problems such as nuclear reactor insulation, ventilation of rooms, solar energy collection, solar-chimneys power-generators, and crystal growth in liquids. The two-layer convection problem is an extension of the broadly studied problem of natural convection in a differently heated cavity with a single liquid.

The study of thermal convection in a two-layer system of immiscible liquids is inspired by the development of liquid encapsulated crystal growth techniques.

Thermal convection in two horizontal layers of immiscible liquids are differently heated from the side was studied analytically, numerically and experimentally A. Prakash and J.N.Koster, 1997 [4]. Flow in the two-layer system was numerically simulated using the commercial finite element computer code FIDAP. The interface and the free surface are both considered to be deformable. An early study Villers and Platten 1988, 1990 [5,6] performed a one-dimensional (1-D) analysis of convective flow in a two layer system. They assumed that the temperature gradient across the cavity is constant, and a parallel flow with negligible vertical velocity develops in both layers. In these works the problem of fluid differentially heated from up to down doesn't considerate.

Mantle convection is now a generally accepted principle of geodynamics. There are several models of mantle convection. One of these models implies that convection takes place in two discrete layers, of the upper and the lower mantle and there is no significant mass transfer across the boundary between them. For instance, this problem was studied by F.M.Richer (1979) [7], F.M. Richer and D.P. McKenzie (1981) [8], N.L. Dobretsov and A. G. Kirdyashkin (1993) [3], L. Gserpes and M. Rabinovicz (1985) [9], and L.Gserpes, M. Rabinovicz, and C. Rosemberg-Borot (1988) [10]. However, the results of numerical modeling (experiments) depend on many general assumptions and realizations of boundary conditions. Therefore, there is a need for numerical methods that establish a link between numerical simulation and laboratory experiment. It is the purpose of the present article to study a finite- difference method which matches experimental data. Here, we present a numerical model for the study of thermal convection in two-layers. The results of two-dimensional (2-D) numerical simulations are compared with the experimental data of N.L.Dobretsov and A.G.Kyrdyashkin [3].

1.2 Objectives

The objectives of the project was:

1) To developed the numerical algorithm for modeling buoyancy-driven flow in cavity vertical or horizontal sides, which are differentially heated.

2) To made compare of experimental data (Dobretsov and Kirdyashkin, 1993, [3])) with data of numerical simulation.

3) To investigate the dependence two-layer convection for wide range of relation of the viscosity, the thermal diffusivity and layer thickness.

1.3 Hypothesis

The main hypothesis in such kind of an investigation is that fluid is viscous incompressible and effect of buoyancy account as Bousinesq approximation. We used the Navier-Stokes equations with Bousinesq approximation. Next important hypothesis of our research is that boundary separated two fluids is horizontal line. This assumption is well supported experimentally for two-layer model of mantle convection [3]

1.4 Usefulness and benefits

The numerical model which give a good agreement with experimental data would be useful in numerical simulation of practical problems such as nuclear reactor insulation, ventilation of rooms, solar energy collection, solar-chimneys power-generators, simulation of mantle convection, and crystal growth in liquids.
The usefulness of numerical code for simulation of convection in a two-layer system is to provide the cheap tool for study in detail the problem of two-layer convection which so reach by different phenomena's.

1.5 Scope and limitations.

The present research attempts to describe basis two-dimensional models of two-layer convection in a rectangular box, with constant viscosity fluids in each layer. The viscosity ratio and Prandtl number be taken as the main quantities varied in the models. This research concentrate on the structure of the flow verse of parameters.

2. Methodology

The method of study of the boundary value problem for the Navier-Stokes equations in Bousinesq approach (see APPENDIX-C, pp. 7.3-7.4) is that of numerical simulation. The numerical scheme use to solve the system of incompressible Navier-Stokes equations. It is a finite difference algorithm for primitive variable formulation. The methods for solving the time-dependent Navier-Stokes equations base on discretization in time of fractional-step type and characterized by the orthogonal projection onto the space of the solenoidal vector fields, in order to satisfy the incompressibility condition (see

APPENDIX-B, pp. 6.6-6.7, and APPENDIX-C, pp. 7.5-7.7). We verify numerical algorithm by compared with data of laboratory physical modeling, and benchmark solutions (see APPENDIX-C, pp. 7.7-7.9)

3. Results

The numerical model which give a good agreement with experimental data would be useful in numerical simulation of practical problems such as nuclear reactor insulation, ventilation of rooms, solar energy collection, solar-chimneys power-generators, simulation of mantle convection, and crystal growth in liquids.

The main results of present project are the finite-difference method for simulation of convection flow in two-layer system of immiscible fluids. Considered finite-difference method matches experimental data and can be used to numerical modeling of thermal convection in two-layer system.

The results of project was presented at the Third Annual National Symposium on Computational Science and Engineering - ANSCSE'99. This forum organized by Faculty of Science, Chulalongkorn University, and NSTDA/GREC/CSEP on March 24-26, 1999 (see Appendix B). And in more detail the results was published in the Thailand Journal of Mathematics, Vol.1, No. 1 (1999), pp. 47-60. (see Appendix C)

4. Summary and conclusions

Thermal convection in a two-layer system of immiscible liquids is distinguished from its single layer counterpart by the interface. Across the interface, the two liquids are mechanically and thermally coupled. In addition to buoyancy, along the interface tangent stress provides a driving force for flow. From the interface tangent stress and from viscous and thermal coupling a cell structure develops within the layers. The cell structure depends on the viscosity contrast between the layers, on the layer depths and on the Rayleigh number. The increase of the viscosity contrast change the flow structure from the dominating viscous coupling towards thermal coupling.

If it is assumed that mantle convection occurs in separate layers above and below the 700-km discontinuity, this two-layer model can fit the observation within certain limits.

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APPENDIX -A. (BIODATA)

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Educational Background:

1974 Master's (MS Science Mathematics) Novosibirsk State U. Russia.

1983 Doctoral Ph.D. Fluid Mechan. Institut of Theoretical and Applied Mechanics of RAS, Novosibirsk, Russia

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Over 40 research papers. Some of latest are

1. G.G.Chernykh, A.G.Demenkov, N.P.Moshkin, O.F.Voropaeva, Numerical Models of Turbulent Wakes in Homogeneous and Stratified Fluids. Computational Fluid Dynamics'96, Proceedings of the Third ECCOMAS computational Fluid Dynamics Conference, 9-13 September 1996, Paris, France, John Wiley & Sons, Ltd., pp.160-166.
2. Chernykh G.G., Moshkin N.P., Rychkova E.V., Tuchkov S.A., Comparison of some numerical algorithms for two-dimensional convection of fluid with nonlinear viscosity. International conference on the methods of aerophysical research, September 2-6, 1996, Novosibirsk, Russia, Proceedings, part 1, Novosibirsk 1996, pp. 79-84.
3. N.P. Moshkin, G.G. Chernykh, O.F.Voropaeva .Numerical models of turbulent Wakes in Stratified Fluids. Presented at the Fourth International Conference on Computational Physics, 2-4 June 1997, Singapore.

Research Experience in Thailand:

Completed Research Projects:

"Application of Group Analysis to Kinetic Equations / coinvestigator Project begun in March 1997—Funded for 2 years.

APPENDIX -B

Copy of article from Proceeding of ANSCSE'99
(ISBN 974-7578-49-2)

The Third Annual National Sympos
Computational Science
and Engineering

ANSCSE '99

Organized by

Faculty of Science,
Chulalongkorn University

NSTDA/GREC/CSEP

March 24 - 26, 1999

Faculty of Science, Chulalongkorn University, Bangkok, Thailan

ISBN 974-7578-49-2

Distributed by

Faculty of Science
Chulalongkorn University
254 Phyathai Road
Bangkok 10330,Thailand
Tel : (02) 218-5041
FAX : (02) 253-5036
http://www.sc.chula.ac.th

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Numerical Simulation of Free Convection Flows in a Two-Layer System of Immiscible Fluids.

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Two-dimensional model of convection in superimposed horizontal layers of two immiscible liquids that are heated from below and cooled from above have been solved numerically. The usual Boissinesq approximation of the Navier-Stokes equations was used. In the models to be considered, the two plane horizontal layers have constant viscosity, their viscosity ratio and depths are varied. The upper and lower surfaces are no-slip boundaries with no vertical motion. The boundary conditions at the interface between the layers prescribe the continuity of the horizontal velocity and the tangent stress, together with no fluid motion across the interface surface.

The method for solving the time-dependent Navier-Stokes equations is written by using the primitive variables of velocity and pressure. This method is based on discretization in time of fractional-step type and characterized by the orthogonal projection onto the space of the solenoidal vector fields in order to satisfy the incompressibility condition.

To verify our algorithm we made the comparison with the experimental data of N.L. Dobretsov and A.G. Kirdyashkin [1]. They studied the free convection flows in two horizontal liquid which were heated from beneath and cooled from above. The data of numerical simulation are in satisfactory agreement with these experimental data.

1. Introduction

Numerical convection in enclosures has recently been receiving increased attentions. This attention is due in part to recognition of the importance of this process in many diverse applications such as home heating, solar collection, crystal growth, nuclear reactor design as well as dramatically increasing number of research publications about convection in the earth's mantle. It is now generally accepted that the earth's surface consists of plates in relative motion. The movement of plates is presumably associated with convection in the mantle [2]. There are several models of mantle convection. One of these models implies that convection takes place in two discrete layers of the upper and lower mantle and there is no significant mass transfer across the boundary between them [1].

Numerical studies of convection (particularly mantle convection) have made a great contribution towards understanding the nature of convectiv flows (particularly in the understanding of mantle convection problems). However, the results of numerical modeling (experiments) depend on many general assumption and realization of boundary conditions. Therefore, there is a need for numerical methods that establish a link between numerical

[1]The author is on leave from Institute of Computational Technologies, Russian Academy of Sciences, Novosibirsk, Russia..

simulation and laboratory experiment. It is the intent of this research to study the finite-difference method which has to be successful in comparison with experimental data.

Here,, we present a numerical model for a study of thermal convection in two-layers. The results of two-dimensional (2-D) numerical simulations are compared with the experimental data N.L. Dobretsov and A.G. Kyrdyashkin [1]

2. Mathematical Formulation

We consider a rectangular, two-dimensional cavity of aspect ratio H / L see Fig. 1. The two layered heights are not necessary equal. Each layer consists of constant viscosity fluids. The way in which the two layers may differ is in their viscosity. Mechanically the box's boundaries may be free-slip or no-slip boundaries. The upper and lower horizontal boundaries are isothermal surfaces. The vertical walls are insulated. The interface between the layers is fixed at the depth $z = d$. There is no mass flux across it, but tangent velocities and tangential stresses are continuous. Then, utilizing the primitive variable formulation of the Bousiinesq approximation to the Navier-Stokes equations, the no-dimensional governing equations of fluid flow and equation satisfied by temperature in each liquid layer are:

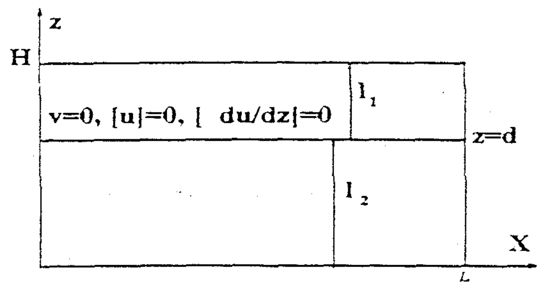

Fig. 1

$$\frac{\partial u_i}{\partial t}+u_k\frac{\partial u_i}{\partial x_k}=-\frac{\partial p}{\partial x}+\frac{\Pr}{c}\frac{\partial \sigma_{ik}}{\partial x_k}+\frac{Ra\cdot\Pr}{c^2}T\cdot g_i,\quad i,k=1,2;\quad (x_1,x_2)=(x,z), \tag{1}$$

$$\frac{\partial u_k}{\partial x_k}=\frac{\partial u_1}{\partial x_1}+\frac{\partial u_2}{\partial x_2},\qquad u=(u,v)=(u_1,u_2), \tag{2}$$

$$\frac{\partial T}{\partial t}+u_k\frac{\partial T}{\partial x_k}=\frac{1}{c}\frac{\partial^2 T}{\partial x_k\,\partial x_k}, \tag{3}$$

$$\sigma_{ik}=\gamma_l\left(\partial u_i/\partial x_k+\partial u_k/\partial x_i\right) \tag{4}$$

where $u=u_1$ or $v=u_2$ are horizontal and vertical velocities, respectively, and t is the time; $\vec{g}=(g_1,g_2)=(0,-g)$ is the acceleration of gravity; T is the fluid temperature. The dimensionless viscosities $\gamma_l, l=1,2$, of equation (4) are defined as $\gamma_l = {}^{\nu_l}\!/_{\nu_1}, l=1,2$, i.e. $\gamma_1 \equiv 1$, $\gamma_2 = {}^{\nu_2}\!/_{\nu_1}$, and ν_1 , ν_2 are the kinematic viscosities of the layers. The governing equations have been scaled using thermophysical properties of layers, the container's height H, and the applied temperature difference $\Delta T = T_{z=0} - T_{z=H}$. The following time, length, velocity, temperature and pressure scales are used:

$$t^* = \frac{H^2}{c \cdot k}, \qquad x^* = H, \quad u^* = \frac{c \cdot k}{H}, \qquad T^* = \Delta T, \quad p^* = \frac{\rho \cdot k^2 c^2}{H^2},$$

Here k is coefficient of the thermal diffusivity, ρ is the density of the fluids, c is non-dimensional multiple which was chosen as $c = 1$ or $c = \Pr$ etc. Non-dimensional parameters appearing in the above mentioned problem are

$$x_d = {}^{L}\!/_{H}, \qquad \Pr = {}^{\nu_1}\!/_{k}, \qquad Ra = \frac{g\alpha\Delta T H^3}{k^2 \nu_1},$$

aspect ratio, the Prandtl number and the Rayleigh number respectively (α is the coefficient of thermal expansion). These equations (1)-(4) must be completed with the boundary conditions. The upper and the lower boundaries are isothermal surfaces

$$T=0 \quad at \quad z=0, \tag{5}$$

$$T=1 \quad at \quad z=1. \tag{6}$$

Mechanically the top and bottom horizontal boundaries may be free-slip or no-slip boundaries with

$$v=0, \quad {}^{\partial u}\!/_{\partial z}=0, \quad at\ z=0\ or\ z=1, \tag{7}$$

for free-slip and

$$v=0, \quad u=0,\ at\ z=0\ or\ z=1, \tag{7'}$$

for no-slip. The vertical sides of the box are insulated

$$ {}^{\partial T}\!/_{\partial x}=0, \quad at \quad x=0\ or\ x=x_d, \tag{8}$$

and mechanically they may be free-slip or no-slip boundaries

$$u=0, \quad {\partial v}/{\partial x}=0, \quad at \quad x=0 \ or \ x=x_d, \tag{9}$$

for free-slip and

$$u=0, \ v=0, \quad at \quad x=0 \ or \ x=x_d, \tag{9'}$$

for no-slip. The boundary conditions at the interface boundary prescribe the continuity of the horizontal velocity and tangential stresses as well as with reqirement ni fluid motion across interface.

$$v=0, \quad at \quad z=d,$$

$$[u]=u(x,d-0)-u(x,d+0)=0, \quad 0<x<x_d, \tag{10}$$

$$\left[\nu \frac{\partial u}{\partial z} \right] = o, \quad at \quad z=d.$$

Numerical simulation

When one use the equation of fluid dynamics in primitive (velocity-pressure) variables, one of the main ideas in the construction of a numerical method is that the pressure in a subsequent time level may be determined by the condition of vanishing of the divergence of the velocity vector (MAC method F.Harlow and J.Welch [3]). The same idea was also realized in the splitting method (O.M.Belotserkovskii, V.A.Gushchin, V.V.Shennikov [4]) in which the computation process is divided into three stage.

Now we describe an algorithm for direct numerical solution of the equations (1)-(4) with boundary conditions (5)-(10). The algorithm is based on the method of splitting (O.M.Belotserkovskii, V.A.Gushchin, V.V.Shennikov [4]). The approximation is carried out on staggered grids, i.e. the pressure is specified at the centers of the cells and the velocity components are specified at the centers of the corresponding cell's surfaces. The solution is divided into four stages. For given u^n, v^n, p^n, T^n these steps are:

1. An intermediate velocity field u_i^*, not satisfying the condition of incompressibility, is calculated as solution of discretized version of the momentum equation

$$\frac{u_i^* - u_i^n}{\tau} + u_k^n \frac{\partial u_i^n}{\partial x_k} = -\frac{\partial p^n}{\partial x_i} + \frac{\Pr}{c} \frac{\partial \sigma_{ij}^n}{\partial x_i} + \frac{Ra \cdot \Pr}{c^2} T^n g_i, \quad i=1,2.$$

Here τ is step in time and equation (1) descretized only with respect to time. An explicit and implicit finite-difference approximation for convective and diffusive terms was used. The boundary, which separate the two fluids, passed through the grid line where the vertical component of velocity vector was defined. Boundary conditions on the interface boundary (10) are realized at this stage.

2. The Poisson equation for the pressure correction is solved

$$\Delta\Phi^{n+1} = \frac{1}{\tau}\frac{\partial u_i^*}{\partial x_i}; \qquad \Phi^{n+1} = p^{n+1} - p^n, \tag{11}$$

The boundary conditions for the pressure correction can be reduced to the finite-difference analogue of the homogeneous conditions of the Neuman type

$$\left[\frac{\partial \Phi^{n+1}}{\partial n}\right]_\gamma = 0, \tag{12}$$

The Neuman problem obtained in this way has a solution only if the total fluid flux across all outer boundaries is equal to zero. The solution of this boundary value problem (11)-(12) was obtained by means of an iterative scheme of stabilizing corrections.

3. Preliminary values of the velocity components are corrected with the allowance for the pressure found so that for each cell responded with a difference analog of discontinuity equation be realized.

$$u_i^{n+1} = u_i^* - \tau\frac{\partial}{\partial x_i}\left[\Phi^{n+1}\right].$$

4. The next approximation for temperature filed T^{n+1} is defined by the implicit finite-difference scheme of stabilizing correction [5].

Comparison with laboratory experiment

In the experimental research of N.L. Dobretsov and A.G. Kyrdyashkin [1] a two-layer model of mantle convection was derived from experimental works on hydrodynamic and heat exchange within a horizontal two-layer medium consisting of two immiscible liquids of different densities and viscosities, the upper one cooled and the lower one heated. The viscosity of the thinner upper layer (model of the asthenosphere) is less than that of the lower layer (model of the lower mantle).

Two immiscible liquids, glycerin and hexadecan , were used in the experiments. Hexadecan has the following properties at $T = 30^\circ C$, density, ρ, is $766.5\, kg/m^3$; thermoconductivity, λ, is 0.147 $W/m\cdot{}^\circ C$; dynamic viscosity, μ is $2.754\cdot 10^{-3}\, N\cdot s\cdot m^{-2}$; kinematic viscosity, ν , is $0.359\cdot 10^{-5} m^2 s^{-1}$; thermal diffusivity, a , is $1.154\cdot 10^{-7} m^2 s^{-1}$; coefficient of thermal expansion, β , is $0.529\cdot 10^{-3}\,{}^\circ C^{-1}$; Prandtl number, $\Pr = \nu/a$ is 31.13.

Glycerin has the following physical properties at $T = 40^{\circ}C$; $\rho = 1250 \, {}^{kg}/_{m^3}$
$\lambda = 0.283 \, W \cdot m^{-1} {}^{\circ}C^{-1}$; $\mu = 330 \cdot 10^{-3} N \cdot s \cdot m^{2}$; $\nu = 2.64 \cdot 10^{-4} m^2 s^{-1}$; $a = 9.18 \cdot 10^{-8} m^2 s^{-1}$;
$\beta = 4.4 \cdot 10^{-4} \, {}^{\circ}C^{-1}$; $\Pr = 2.88 \cdot 10^{3}$.

Experimental investigations were carried out when the layers had a thickness of $l_1 / l_2 << 1$ and $V_2 / V_1 = 73.5$. Fig. 2-4 reproduced from Ref. [1] for comparison with our numerical calculations. Fig. 2 shows the lines of points in the two-layer system of liquids obtained from the videofilm and also cross-sections A-A and B-B, for which velocity profiles are found that are given in Fig. 3. Fig. 3 shows profiles of horizontal and vertical components of velocity in the liquid layers of glycerin ($l_2 = 19mm$) and hexedecan ($l_1 = 7mm$). Fig. 4 shows experimentally measured temperature profiles in different vertical cross-sections parallel to the roll axis.

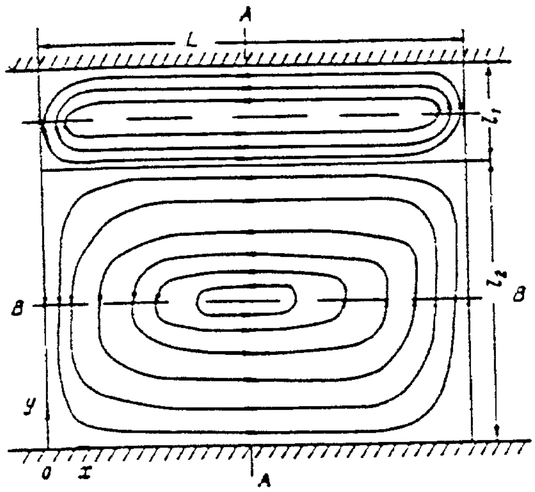

Fig. 2. Experimental lines of flow. The A-A and B-B section corresponds to fig.3. (From Ref.[1])

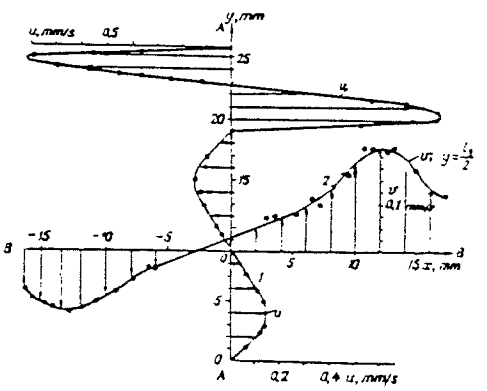

Fig.3. Velocity profiles in the horizontal two-layer model: glycerin (lower layer) and hexadecan (upper layer). (From Ref. [1])

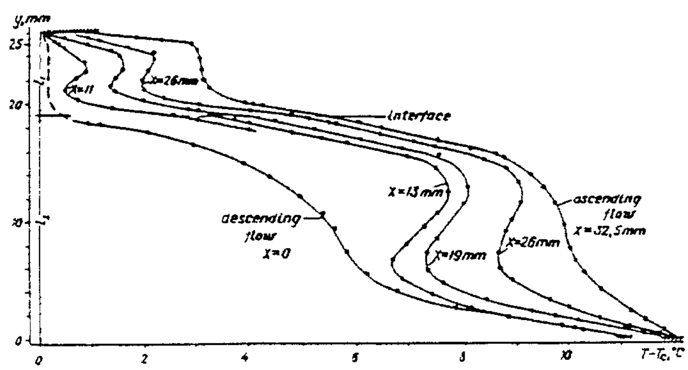

Fig. 4. Temperature profiles in the horizontal layers of glycerin-hexadecan in several vertical sections from descending flow (x=0) up to ascending flow (x=32.5, see Fig.2) (From. Ref.[1]).

In Fig.5-7 we portray the numerical solutions in the steady state. The aspect ratio of the domain $x_d = L/H$ corresponds to the size of the experimental box $100/25$. The non-dimensional parameters was taken according to the conditions of the experiment and physical properties of glycerin and hexadecan.

$$Ra = 384615, \quad \Pr = 31, \quad \nu_2/\nu_1 = 735.$$

The main calculations were carried out on grid which contained 50x200 nodes in z and x directions respectively.

In Fig.5 we portray the steady state solution in terms of the stream lines, and isotherms. It is to be noted that the flow structure in the two-layer system in numerical simulation is very similar to the flow pattern in the laboratory experiment. Correlation between descending flows in the upper and lower layers as well as between ascending flows was occured both in numerical simulation and experimental data. Fig. 6 shows the profile of the horizontal components of velocity. This profile is drawn for $x = 1.7$ section (see Fig. 5). Near the liquid interface boundary a counter current appears. As it pointed out in [1] the reason for the counter current is thermal coupling. In Fig. 7 one draws the temperature profiles in different vertical cross-sections. The profile is marked by a plus symbol (+) this corresponds to the section $x = 1.3$ in the region of ascending flows. The profile mark by cross sign (×) corresponds to the section $x = 1.7$ in the region closer to the section A-A in the experimental study see Fig.3. The profile is marked by diamond symbol (◊) that corresponds to the section $x = 2.14$ which located in the region of descending flows. The counter currents near the interface provide a horizontal temperature gradient.

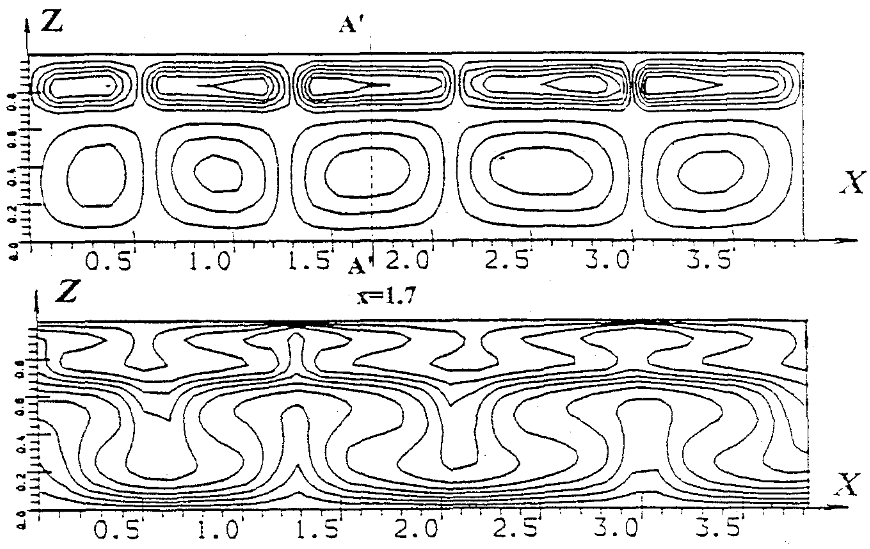

Fig. 5. The stream lines and isotherms. Result of numerical simulation.

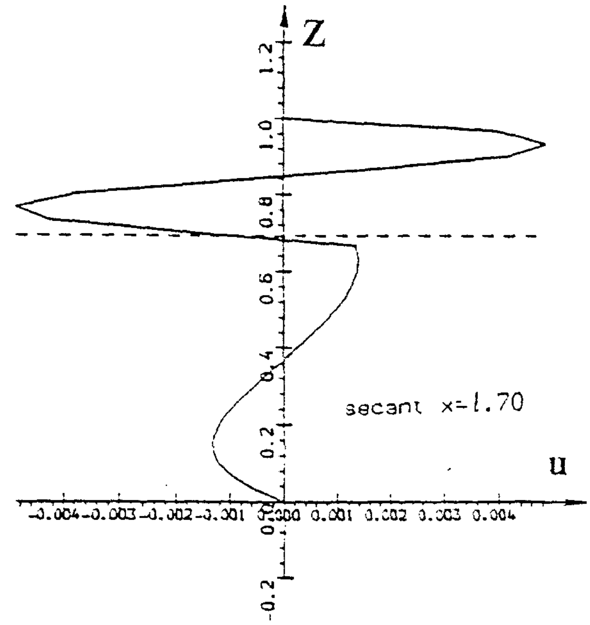

Fig. 6. The profile of horizontal component of velocity at the section x=1.7 (see Fig. 5)

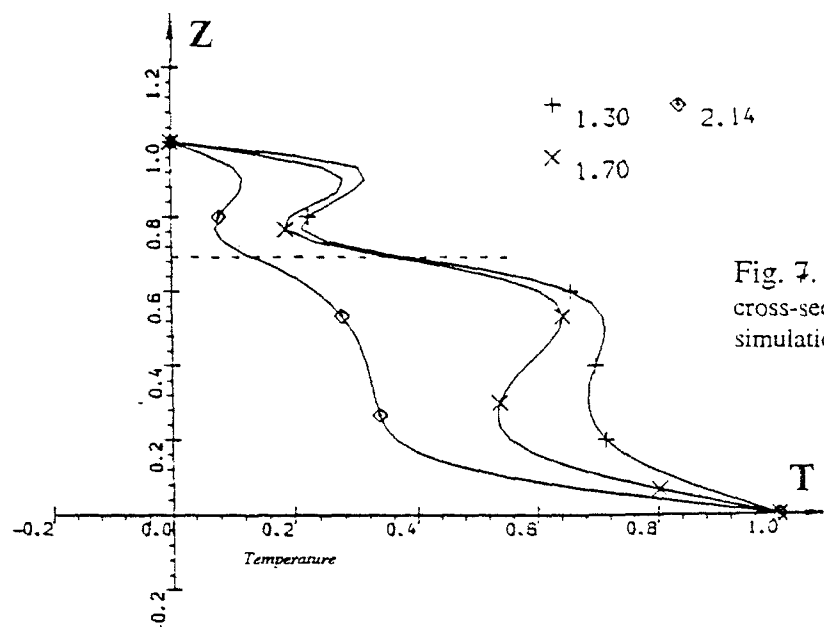

Fig. 7. Temperature profiles in different cross-section. Results of numerical simulation.

In conclusion, the results of numerical simulation reported here indicate only qualitative agreements with experimental data. The developed numerical models can be considered as a good tools to simulate natural convective flows in many layers system of fluids.

Acknowledgment

We thanks Prof. N.L. Dobretsov and Prof. A.G. Kyrdyashkin for their attention and useful discussions while working with this problem. These discussions stimulated our interest in the problem of covective flows in two-layers fluids.

This work was supported by the Suranaree University of Technology grant.

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APPENDIX-C

Copy of article from Thailand Mathematical Journal, vol.1, No.1 (1999), pp. 47-60. (ISSN 0859-5399)

วารสารคณิตศาสตร์ไทย

Thailand Journal of Mathematics

VOLUME 1 NUMBER 1 OCTOBER 1999 ISSN 0859-5399

CONTENTS

Published Under The Support of

The National Research Council, Thailand

Thailand Journal of Mathematics,
Vol. 1, No. 1 (1999), pp.47-60

On Convection Flows in a Two-Layer of Immiscible Fluids

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1 Introduction

One of the areas of interest in fluid dynamics is the study of the behavior of a fluid layer, heated from below, in the presence of a gravitational field. H. Bernard (1900) [1] and Lord Rayleigh (1920) [2] experimentally established the conditions under which the fluid layers first become unstable. S. Chandrasekhar (1961) [3] studied the linear instability of fluid layers of infinite horizontal extent, both with free and no-slip boundary conditions. Several thousand articles as well as a lot of handbooks, have been published in this area of fluid dynamics in resent years. The study of thermal convection in two-layer systems of immiscible liquids was inspired by the development of liquid encapsulated crystal growth techniques. D. Villers and J.K. Platten (1988-1990) [4]-[5] conducted one dimensional analysis of convective flow in a two-layer system. The numerical simulation of flow in a two-layer system with a free surface was achieved by N. Ramachandran (1990) [6], T. Doi and J.N. Koster (1993) [7], and J.P. Fontaine and Sani R.L. (1992) [8].

Mantle convection is now a generally accepted principle of geodynamics. There are several models of mantle convection. One of these models implies that convection takes place in two discrete layers, of the upper and the lower mantle and there is no significant mass transfer across the boundary between them. For instance, this problem was studied by F.M. Richer (1979) [9], F.M. Richer and D.P. McKenzie (1981) [10], N.L. Dobretsov and A. G. Kirdyashkin (1993) [11], L. Gserpes and M. Rabinovicz (1985) [12], and L.Gserpes, M. Rabinovicz, and C. Rosemberg-Borot (1988) [13]. However, the results of numerical modeling (experiments) depend on many general assumptions and realizations of boundary conditions. Therefore, there is a need for numerical methods that establish a link between numerical simulation and laboratory experiment. It is the purpose of the present article to study a finite- difference method which matches experimental data. Here, we present a numerical model for the study of thermal convection in two-layers. The results of two-dimensional (2-D) numerical simulations are compared with the experimental data of N.L. Dobretsov and A.G. Kyrdyashkin [11].

2 Mathematical Formulation

The problem of convection has been thoroughly investigated in a number of places. We here enumerate the basic equations and the boundary conditions. The equations of the Boussinesq approximation are treated in terms of velocity and pressure

$$\frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial x_k} = -c' \frac{\partial p}{\partial x_i} + \frac{\Pr}{c} \frac{\partial \sigma_{ik}}{\partial x_k} + \frac{Ra \cdot \Pr}{c^2} T g_i, \quad (1)$$

$$\sigma_{ik} = \eta(x_1, x_2, x_3, T, p) \left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right),$$

$$\frac{\partial T}{\partial t} + u_k \frac{\partial T}{\partial x_k} = \frac{1}{c} \frac{\partial^2 T}{\partial x_k \cdot \partial x_k}, \quad (2)$$

$$\frac{\partial u_k}{\partial x_k} = 0, \quad i, k = 1, 2; \quad g = (0, g_2), \quad (3)$$

where $x_1 = x,$; $x_2 = z$ are standard cartesian coordinates, $u = u_1$ or $v = u_2$ are horizontal and vertical components of velocity vector, respectively, and t is the time; g_2 is the acceleration of gravity; T is the fluid temperature. η is the coefficient of dynamic viscosity, and c' is the pressure scaling factor. The governing equations have been scaled using thermophysical properties of layers, the container's height , and the applied temperature difference . The following time, length, velocity, viscosity, temperature and pressure scales are used:

$$t' = c\frac{k}{H^2}, \quad u' = c^{-1}\frac{H}{k}u; \; p' = T' = \frac{T - T_0}{T_1}.$$

Here H is scale of length, k is the coefficient of the thermal diffusivity, ρ is the density of the fluids, T_0, T_1 are the temperatures of upper and lower boundaries, and c is a non-dimensional multiple which was chosen as $c = 1$ or $c = Pr$ etc. Non-dimensional parameters appearing in the above mentioned problem are

$$x_l = L/H, \quad Ra = \frac{g\alpha T_1 H^3}{k\nu}, \quad Pr = \frac{\nu}{\kappa},$$

the aspect ratio, the Prandtl number and the Rayleigh number respectively (α is the coefficient of thermal expansion).

We consider a rectangular, two-dimensional cavity of aspect ratio H/L. The two layered heights are not necessary equal. Each layer consists of constant viscosity fluid. The way in which the two layers may differ is in their viscosity. We denoted these viscosities as ν_1 and ν_2. ν_1 and ν_2 are kinematic viscosities of the upper and lower layers correspondingly. Mechanically the box's boundaries may be free-slip or no-slip boundaries. The upper and lower horizontal boundaries are isothermal surfaces. The vertical walls are insulated. The interface between the layers is fixed at the depth $z = d$. There is no mass flux across it, but tangent velocities and tangential stresses are continuous.

The governing equations (1)-(3) must be completed with the boundary conditions. The upper and the lower boundaries are isothermal surfaces.

$$T = 0 \text{ at } z = 0, \quad T = 1 \text{ at } z = 1.$$

Mechanically the top and bottom horizontal boundaries may be free-slip or no-slip boundaries with

$$v = 0, \; \partial u/\partial z = 0, \text{ at } z = 0 \text{ and } z = 1,$$

for free-slip and

$$v = 0, \; u = 0, \text{ at } z = 0 \text{ and } z = 1,$$

for no-slip. The vertical sides of the box are insulated

$$\frac{\partial T}{\partial x} = 0, \text{ at } x = 0 \text{ or } x = x_l$$

and mechanically they may be free-slip or no-slip boundaries

$$u = 0, \; \frac{\partial v}{\partial x} = 0, \text{ at } x = 0 \text{ and } x = x_l,$$

for free-slip and

$$u = 0, \; v = 0, \text{ at } x = 0 \text{ and } x = x_l,$$

for no-slip. The boundary conditions at the interface boundary prescribe the continuity of the horizontal velocity and of the tangential stresses as well as excluding fluid motion across interface

$$v = 0, \; [u] = u(x, d-0) - u(x, d+0) = 0, \; \left[\nu \frac{\partial u}{\partial z}\right] = 0,$$
$$\text{at } z = d, \; 0 < x < x_l.$$

3 Numerical Simulation

When using the equations of fluid dynamics in primitive (velocity-pressure) variables the main idea in the construction of a numerical method is that the pressure in a subsequent time level may be determined by the condition of vanishing of the divergence of the velocity vector (MAC method F.Harlow and J.Welch [14]). The same idea was also realized in the splitting method (O.M.Belotserkovskii, V.A.Gushchin, V.V.Shennikov [15]) in which the computation process is divided into three stages. An implicit method of the same class with pressure correction was studied by A.I. Tolstykh (1987) [16].

Now we describe an algorithm for direct numerical solution of the equations (1)-(3) with boundary conditions presented above. The algorithm is based on the method of splitting (O.M.Belotserkovskii, V.A.Gushchin, V.V.Shennikov [15]). The approximation is carried out on a staggered grid ω_h, i.e. the pressure and temperature is specified at the centers of the cells and the velocity components are specified at the centers of the corresponding cell's surfaces.

The boundary, which separate the two fluids, passed through the grid line where the vertical component of velocity vector was defined. Let j_d be the number on the horizontal grid line corresponding to this boundary. It is easy to see that $v_{i,j_d} = 0$ on this boundary. Let us introduce $u^+ = u(x, d+0)$ and $u^- = u(x, d-0)$ two tangent components of the velocity vector from different sides of the $z = d$ boundary. One can express some approximation of boundary condition $\left[\nu \frac{\partial u}{\partial z}\right] = 0$, $[u] = 0$ in the form

$$\begin{cases} \nu_1 \left[\sum\limits_{m=1}^{N^+} C_m^+ u_{i+1/2,j_d+m} + C^+ u^+\right] = \nu_2 \left[\sum\limits_{m=0}^{N^-} C_m^- u_{i+1/2,j_d-m} + C^- u^-\right], \\ u^+ = u^-, \end{cases}$$

where

$$\left(\frac{\partial u}{\partial z}\right)_{z=d_1-0} = \sum_{m=0}^{N^-} C_m^- u_{i+1/2,j_d-m} + C^- u^- + O(h^\beta),$$

$$\left(\frac{\partial u}{\partial z}\right)_{z=d_1+0} = \sum_{m=1}^{N^+} C_m^+ u_{i+1/2,j_d+m} + C^+ u^+ + O(h^\alpha).$$

Here N^+, N^- are number of grid points used to approximate the partial derivatives by one side difference. These two equations can be solved with respect to $u^+ = u^- = u_\gamma$

$$u_\gamma = \frac{\nu_1 \sum\limits_{m=1}^{N^+} C_m^+ u_{i+1/2,j_d+m} - \nu_2 \sum\limits_{m=0}^{N^-} C_m^- u_{i+1/2,j_d-m}}{\eta_1 C^+ - \eta_2 C^-}.$$

These are reduced boundary conditions on the boundary which separate two fluids and we can consider each domain as independent.

In order to describe the numerical methods it is convenient to present the momentum equation in the vector form

$$\frac{\partial \mathbf{u}}{\partial t} + L\mathbf{u} = -\frac{1}{\rho}\nabla p, \tag{4}$$

$$div\mathbf{u} = 0. \tag{5}$$

Here, the operator L contains convective terms and also diffusive terms in the viscous case. Introducing the difference analogues L_h, div_h, and ∇_h of the operators L, div, and ∇ on the grid ω_h we can write out the following approximation of (4), (5):

(1) Splitting method:

$$\begin{cases} \dfrac{\mathbf{u}^* - \mathbf{u}^n}{\tau} + L_h\mathbf{u}^n = 0 \\ \mathbf{u}^{n+1} = \mathbf{u}^* - \tau\dfrac{1}{\rho}grad_h\, p^{n+1};\ div_h\, \mathbf{u}^{n+1} = 0; \\ \dfrac{1}{\rho}div_h\, grad_h\, p^{n+1} = \dfrac{1}{\tau}div_h\, \mathbf{u}^*. \end{cases}$$

(2) Implicit method with pressure correction of second order $O(\tau^2)$ with respect to time approximation:

$$\begin{cases} \dfrac{\mathbf{u}^* - \mathbf{u}^n}{0.5\tau} + L_h\mathbf{u}^* = -\dfrac{1}{\rho}grad_h\, p^n, \\ \dfrac{\mathbf{u}^{n+1} - \mathbf{u}^n}{\tau} + L_h\mathbf{u}^* = -\dfrac{1}{2\rho}grad_h\, (p^{n+1} + p^n), \\ div_h\, \mathbf{u}^{n+1} = div_h\, \mathbf{u}^n = 0, \end{cases}$$

$$\mathbf{u}^{n+1} + \mathbf{u}^n - 2\,\mathbf{u}^* = \frac{\tau}{2\rho}grad_h\, \delta p:\ \ \delta p = p^{n+1} - p^n,$$

$$\frac{1}{\rho}div_h\, grad_h\, \delta p = \frac{4}{\tau}div_h\, \mathbf{u}^*.$$

(3) Implicit method with pressure correction of first order $O(\tau)$ with respect to time approximation :

$$\begin{cases} \mathbf{u}^* - \mathbf{u}^n + \tau \cdot L_h\mathbf{u}^* = -\tau \cdot \dfrac{1}{\rho}grad_h\, p^n, \\ \mathbf{u}^{n+1} = \mathbf{u}^* - \tau\dfrac{1}{\rho}grad_h\, \delta p,\ div_h\, \mathbf{u}^{n+1} = 0, \\ \dfrac{1}{\rho}div_h\, grad_h\, \delta p = \dfrac{1}{\tau}div_h\, \mathbf{u}^*,\ \ \delta p = p^{n+1} - p^n. \end{cases}$$

The order of spatial approximation of these schemes depends on the choice of operator L_h. We have the Poisson equation for the pressure p^{n+1} in case 1 and for the pressure correction δp in cases 2,3. The boundary conditions for the pressure or for the pressure correction can be reduced to the finite-difference analogue of homogeneous conditions of the Neuman type. The Neuman problem obtained in this way has a solution only if the total fluid flux across all outer boundaries is equal to zero. The solution of the boundary value problem for Poisson equation was obtained by means of an iterative scheme of stabilizing corrections [17]. The temperature field T^{n+1} is defined by the implicit finite difference scheme of stabilizing correction or by the predictor corrector scheme [17]. The method of stabilizing correction can be presented in the following form:

$$\frac{\widetilde{T}_{i,j}-T^n_{i,j}}{\Delta t}=-\frac{(uT)^n_{i+1/2,j}-(uT)^n_{i-1/2,j}}{h_x}-\frac{(vT)^n_{i,j+1/2}-(vT)^n_{i,j-1/2}}{h_y}+$$
$$+\frac{1}{c}\left[\frac{T^n_{i+1,j}-2T^n_{i,j}+T^n_{i-1,j}}{h_x^2}+\frac{\widetilde{T}_{i,j+1}-2\widetilde{T}_{i,j}+\widetilde{T}_{i,j-1}}{h_z^2}\right]$$

$$\frac{T^{n+1}_{i,j}-\widetilde{T}_{i,j}}{\Delta t}=\frac{1}{c}\left\{\frac{T^{n+1}_{i+1,j}-2T^{n+1}_{i,j}+T^{n+1}_{i-1,j}}{h_x^2}-\frac{T^n_{i+1,j}-2T^n_{i,j}+T^n_{i-1,j}}{h_x^2}\right\}.$$

The predictor corrector (Yanenko N.N., 1968 [17]) in our case has the form:

$$\frac{T^{n+1/3}_{i,j}-T^n_{i,j}}{0.5\cdot\Delta t}=-\left(v\frac{\partial T^{n+1/3}}{\partial z}\right)_{i,j}+\frac{1}{c}\frac{T^{n+1/3}_{i,j+1}-2T^{n+1/3}_{i,j}+T^{n+1/3}_{i,j-1}}{h_z^2}=\Lambda_{zz}T^{n+1/3}_{i,j}.$$

$$\frac{T^{n+2/3}_{i,j}-T^{n+1/3}_{i,j}}{0.5\cdot\Delta t}=-\left(u\frac{\partial T^{n+2/3}}{\partial x}\right)_{i,j}+\frac{1}{c}\frac{T^{n+2/3}_{i+1,j}-2T^{n+2/3}_{i,j}+T^{n+2/3}_{i-1,j}}{h_x^2}=\Lambda_{xx}T^{n+2/3}_{i,j},$$

$$\frac{T^{n+1}_{i,j}-T^n_{i,j}}{\Delta t}=(\Lambda_{zz}+\Lambda_{xx})\,T^{n+2/3}_{i,j}.$$

4 Solution of Bench Mark Problem

The mechanism of buoyancy-driven flow in a square cavity can be applied for investigation of processes such as nuclear reactor insulation, cooling of radioactive waste containers, ventilation of rooms, solar energy collection, crystal growth in liquids and processes in the Earth's mantle. A common practice in computational fluid dynamics to compare and test numerical codes is the use of benchmark solutions. In [18], [19] it was proposed that buoyancy-driven flow in a square cavity with vertical sides which are differently heated would be a suitable benchmark solution for testing and validating computer codes. The benchmark comparison for mantle convection codes is presented in [20].

The problem being considered in [18] is that of the two-dimensional flow of a Boussinesq fluid of Prandtl number 0.71 in an upright square cavity of side L. Both velocity components are zero on boundaries. The horizontal walls are insulated, and the vertical sides are at different temperatures. The solution of this problem (velocities, temperature and rates of heat transfer) has been obtained at Rayleigh numbers of $Ra = 10^3, 10^4, 10^5, 10^6$.

The benchmark values and results of our numerical experiments appear in Table 1.

Table 1.

Ra	10^3	10^4	10^5	10^6	
$Nu_{\max}$	1.505	3.528	7.717	17.925	*Benchmark*
$Nu_{\max}$	1.515	3.620	8.920	19.200	21×21
$Nu_{\max}$	1.510		7.530		41×41
$\overline{Nu}$	1.118	2.243	4.519	8.800	*Benchmark*
$\overline{Nu}$	1.111	2.222	5.150	10.900	21×21
$\overline{Nu}$	1.113		4.430		41×41
$u_{\max}$	3.649	16.178	34.73	64.63	*Benchmark*
$u_{\max}$	3.494	17.07	42.59	59.13	21×21
$u_{\max}$	3.650		37.75		41×41

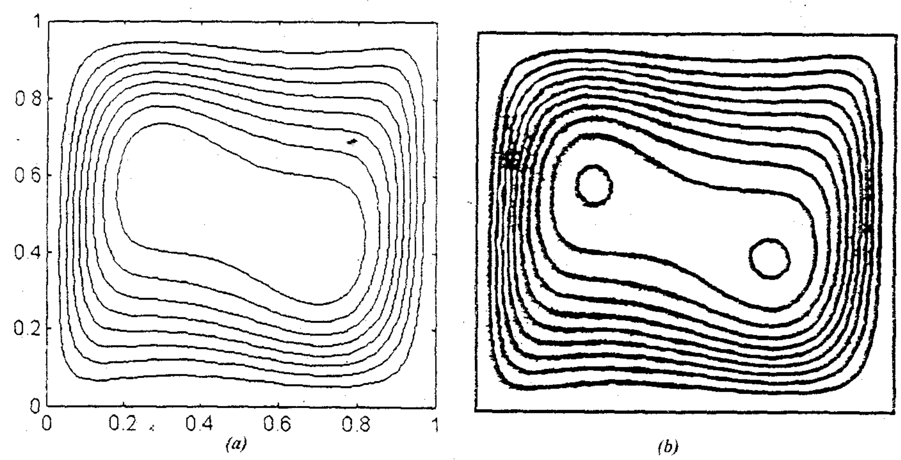

Figure 1. Contour maps of stream function

Where $Nu_{\max}$ is the maximum value of the local Nuselt number on the boundary at $x = 0$, $\overline{Nu} = \int_0^1 \frac{\partial T}{\partial x} dz \mid_{x=0}$ is average Nuselt number, and $u_{\max}$ is the maximum horizontal velocity on the vertical mid-plane of the cavity.

Figures 1-3 show the streamlines, isotherms and vorticity contours for the benchmark solution and the similar data from recent work computed at the

41×41 grid for the $Ra = 10^5$. Figures 1(a),2(a), 3(a) are results of our calculations and figures 1(b),2(b), 3(b) are benchmark solutions.

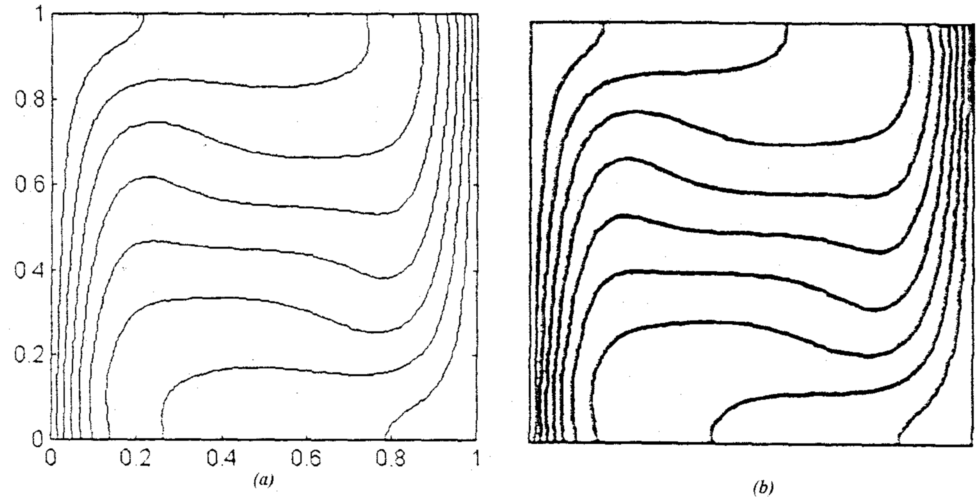

Figure 2. Contour maps of temperature T

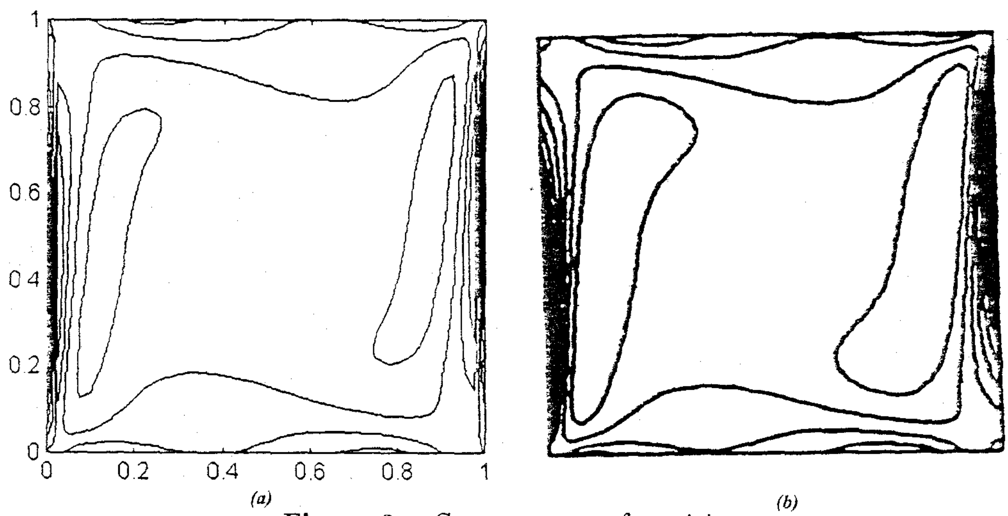

Figure 3. Contour maps of vorticity.

5 Comparison with laboratory experiment

In the experimental research of N.L. Dobretsov and A.G. Kyrdyashkin [11] a two-layer model of mantle convection was derived from experimental work on hydrodynamic and heat exchange within a horizontal two-layer medium consisting of two immiscible liquids of different densities and viscosities. the upper one cooled and the lower one heated. The viscosity of the thinner upper layer (model of the asthenosphere) is less than that of the lower layer (model of the lower mantle). Two immiscible liquids. glycerin and hexadecan , were used in the experiments. Hexadecan has the following properties at $T = 30\,°C$, density, ρ , is $766.5\,kg/m^3$; thermoconductivity, λ , is $0.147\,W/m \cdot° C^{-1}$; dynamic viscosity, μ is $2.754{\times}10^{-3}N \cdot s \cdot m^2$; kinematic viscosity, ν, is $0.359 \times 10^{-5}m^2s^{-1}$; thermal diffusivity, a, is $1.154 \times 10^{-7}m^2s^{-1}$; coefficient of thermal expansion, β, is $0.529{\times}10^{-3}\,°C^{-1}$; Prandtl number, $\Pr = \nu/a$ is 31.13. Glycerin has the following physical properties at $T = 40°C$; $\rho = 1259\,kg/m^3$; $\lambda = 0.283\,W \cdot m^{-1}\,°C^{-1}$; $\mu = 330 \times 10^{-3}N \cdot s \cdot m^2$; $\nu = 2.64 \times 10^{-4}m^2s^{-1}$; $a = 9.18 \times 10^{-8}m^2s^{-1}$; $\beta = 4.4 \times 10^{-4}\,°C^{-1}$; $\Pr = 2.88 \times 10^3$.

Experimental investigations were carried out when the layers had a thickness of $l_1/l_2 \ll 1$ and $\nu_1/\nu_2 = 73.5$. Fig. 4-6 reproduced from Ref. [11] for comparison with our numerical calculations. Fig. 4 shows the lines of points in the two-layer system of liquids obtained from the videofilm and also cross-sections $A - A$ and $B - B$, for which velocity profiles are found that are given in Fig. 5. Fig. 5 shows profiles of horizontal and vertical components of velocity in the liquid layers of glycerin ($l_2 = 19\,mm$) and hexedecan ($l_1 = 7mm$). Fig. 6 shows experimentally measured temperature profiles in different vertical cross-sections parallel to the roll axis.

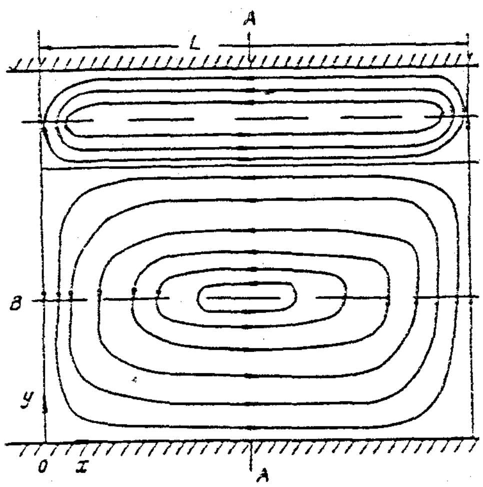

Figure 4. Experimental lines of flow. The $A - A$ and $B - B$ section corresponds to fig. 4. (From Ref.[11])

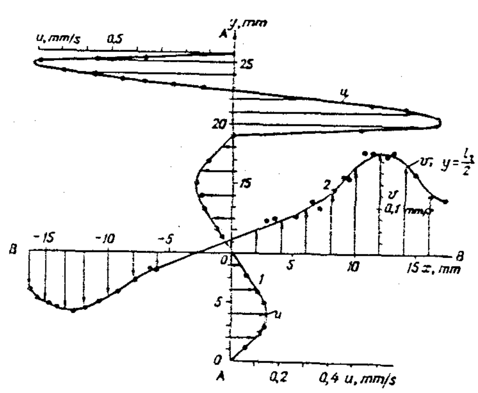

Figure 5. Velocity profiles in the horizontal two- layer model: glycerin (lower layer) and hexadecan (upper layer). (From Ref. [11])

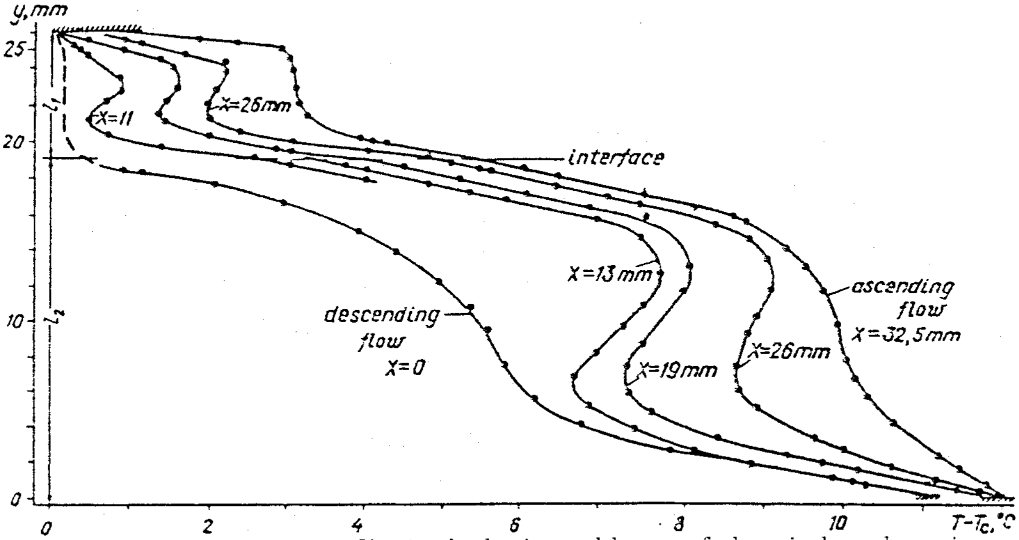

Figure 6. Temperature profiles in the horizontal layers of glycerin-hexadecan in several vertical sections from descending flow ($x = 0$) up to ascending flow ($x = 32.5$. see Fig.4) (From. Ref.[11]).

In Fig.7-9 we portray the numerical solutions in the steady state. The aspect ratio of the domain $x_d = L/H$ corresponds to the size of the experimental box 100/25. The non- dimensional parameters are taken according to the conditions of the experiment and the physical properties of glycerin and hexadecan.

$$Ra = 38461.5,\ \Pr = 31,\ \nu_2/\nu_1 = 73.5\,.$$

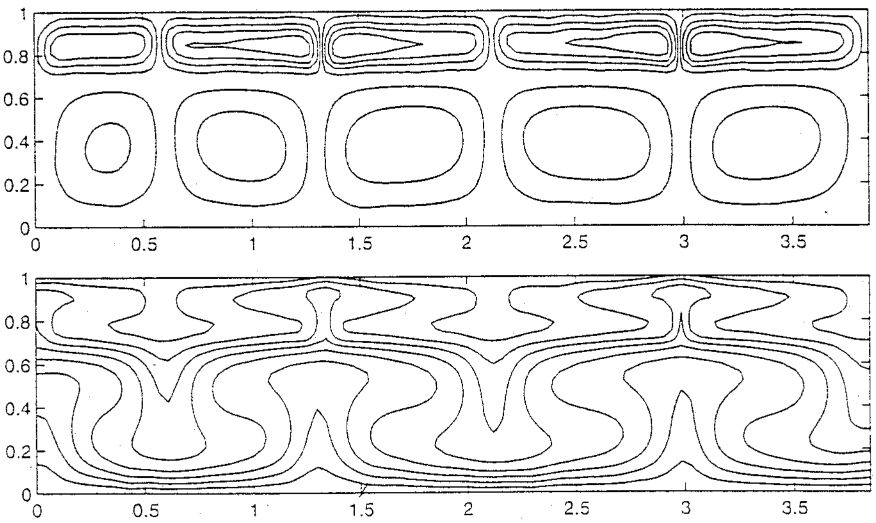

Figure 7. The stream lines and isotherms. Result of numerical simulation.

The main calculations were carried out on a grid which contained 50×200 nodes in z and x directions respectively. In Fig.7 we portray the steady state solution in terms of the stream lines, and isotherms. It is to be noted that the flow structure in the two-layer system in numerical simulation is very similar to the flow pattern in the laboratory experiment. Correlation between descending flows in the upper and lower layers as well as between ascending flows occurred in both numerical simulation and experimental data. Fig. 8 shows the profile of the horizontal components of velocity. This profile is drawn for $x = 1.7$ section (see Fig. 7). Near the liquid interface boundary a counter current appears. As pointed out in [11] the reason for the counter current is thermal coupling. In Fig. 9 the temperature profiles are drawn in different vertical cross-sections. The profile marked by a plus symbol (+) corresponds to the section $x = 1.3$ in the region of ascending flows. The profile marked by a cross ($\times$) corresponds to the section $x = 1.7$ in the region closer to the section $A - A$ in the experimental study (see Fig.4). The profile marked by a diamond ($\diamond$) corresponds to the section located in the region of descending flows. The counter currents near the interface provide a horizontal temperature gradient.

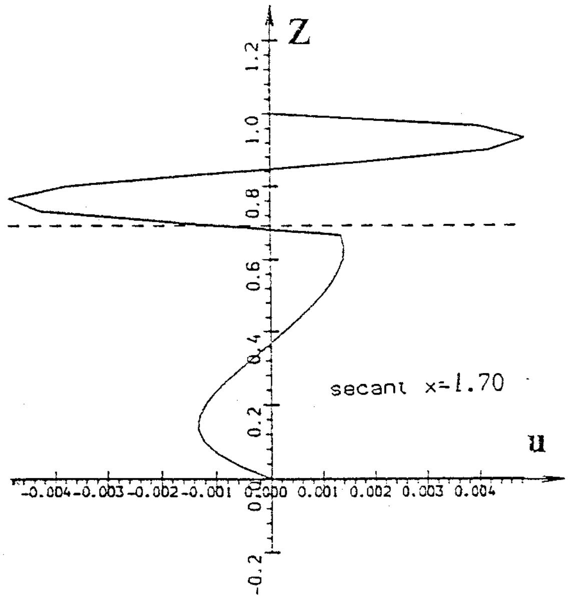

Figure 8. The profile of horizontal component of velocity at the section $x = 1.7$ (see Fig. 7)

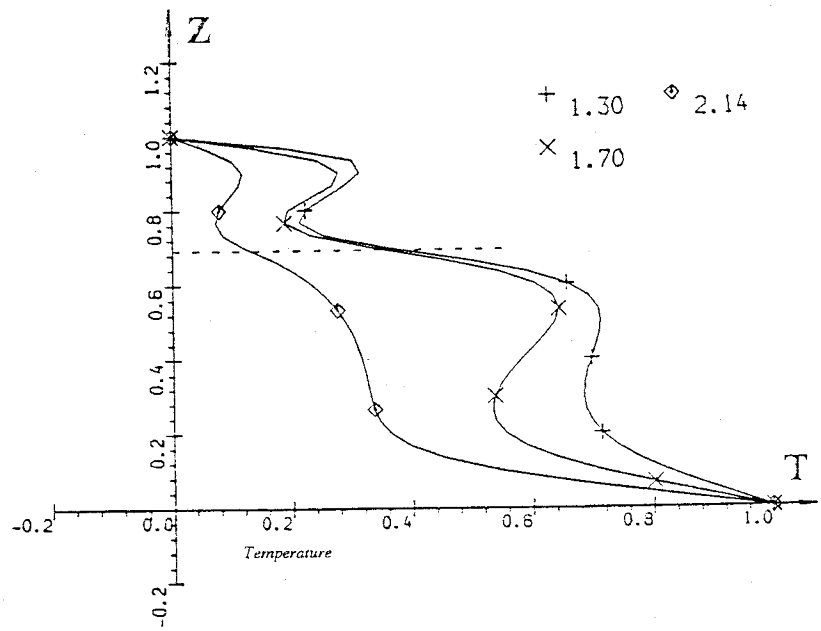

Figure 9. Temperature profiles in different cross-sections. Results of numerical simulation.

In conclusion, the results of the numerical simulation reported here indicate only qualitative agreements with experimental data. The developed numerical models can be considered as a good tools to simulate natural convective flows in a many—layered system of fluids.

6 Acknowledgment

We thank Prof. N.L. Dobretsov and Prof. A.G. Kyrdyashkin for their attention and for the useful discussions while working with this problem. These discussions stimulated our interest in the problem of convective flows in two-layers fluids. This work was supported by a Suranaree University of Technology grant.

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