TORSIONAL RESONANCE SUPPRESSION VIA POLE-ZERO ASSIGNMENT

S. SUJITJORN [1], C. U-THAIWASIN [2], Y. PREMPRANEERAT [3]

[1] School of Electrical Engineering, Suranaree University of Technology, Nakhon Ratchasima, Thailand.
[2] Department of Electrical Engineering, South-East Asia University, Bangkok, Thailand.
[3] Department of Control Engineering, King Mongkut Institute of Technology-Ladkrabang, Thailand.

ABSTRACT

Torsional resonance in a 2-mass rotary system is suppressed via active compensation. To achieve this requires two compensators namely forward or input and feedback compensators. The former provides desired performance of the overall system. The latter suppresses resonant behaviour of and stabilizes the system. The design approach is transfer function synthesis based on pole-zero assignment. The CB-segment method is applied for testing stability. The test result confirms stability robustness.

KEY WORDS

torsional resonance, active compensation, pole-zero assignment, robustness, 2-mass rotary system.

INTRODUCTION

A rotary system having its rotating components coupled by a long and deflective shaft usually experiences torsional resonance. This phenomenon arises because of the non-homogeneous twist of the shaft. The resonance causes a limit-cycle oscillation to angular motion of the system, and shortens the life-span of mechanical components.

The problem of torsional resonance has become an interest among engineers for almost 40 years. This is evident by Cannon [1] who discussed it in terms of modelling of distributed parameter system. Early solutions to the problem of torsional resonance suppression were proposed by Waagen [2] , Tal and Kuo [3], respectively. Their recommendations were quite similar on the basis of modification of mechanical characteristics of the system. The modification could be changing components' dimension, and rearranging the connection of subsystems. Tal and Kuo [3] also suggested the possibility of using a notch filter to reject frequency component that could excite the resonance.

Recently, proposed technology to resolve such problem has become more complex and expensive. Fujikawa et. al. [4] employed a 32-bit DSP to execute state-space control law. In [5], H_∞ approach with an additional adaptive loop was proposed. Estimation of load speed and shaft torque by Kalman filter, and speed governing by an LQ controller were proposed in [6]. All these approaches need a high performance processor to realize controller and/or observer. We believe that the problem of torsional resonance suppression can be resolved by using low-profile analog technology. In this regard, real-time and robusted performance can still be achieved satisfactorily.

This article describes our work to resolve the problem of torsional resonance suppression in a rotary 2-mass system with a long and deflective shaft. The following section describes the system and its model. The active compensation scheme based on 2-degrees-of-freedom configuration is presented. Simulation as well as practical results follow to confirm the attainment of robusted performance and stability.

SYSTEM DESCRIPTION AND MODEL

Mechanical systems with physical structure *rendering torsional resonance are commonly* encountered in industry, for example material transporting systems, manipulators, xy plotters, etc. Figure 1 shows the diagram representing our system

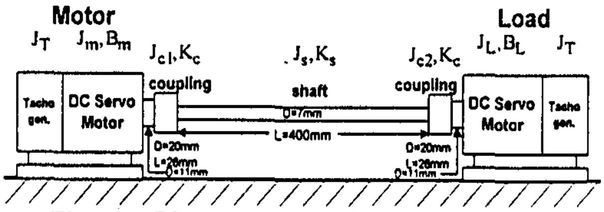


Figure 1 Diagram represents a 2-mass rotary system with a long and deflective shaft.

that consists of a dc motor and a rotating load. The motor and load are coupled together through a shaft of 400 mm long. Oscillation occurring with this system well reflects that of those industrial applications. Torsional resonance causes high frequency oscillation superimposing the speed performance waveform as can be seen quite clearly from figure 2. The waveform

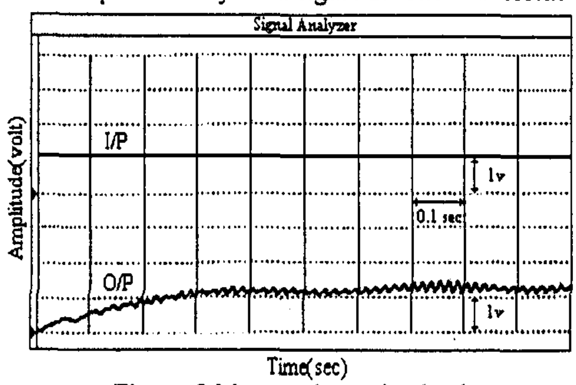


Figure 2 Measured speed at load.

indicates a sluggish response with limited average speed. After the transient stage is over, this average speed is about 143 rpm due to the saturation limit of the drive amplifier.

Through out this work, a linear operating range of the system operation is assumed. Motor and load may assume second-order dynamics, while first-order for shaft with coupling. This leads naturally to a third-order transfer function having a pair of complex poles. However, higher-order models may be assumed according to various applications.

Observed data was obtained from exciting the open-loop system with a unit-step command and passed through MATLAB™ to fit a third-order model. The model assumes no zero and is expressed by

$$G(s)=\frac{1.32\times10^{6}}{s^{3}+13.38s^{2}+1.63\times10^{5}s+7.31\times10^{5}} \quad (1)$$

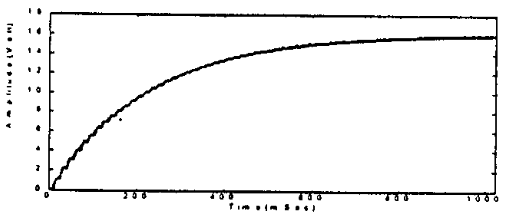


Figure 3 Simulated unit-step response plotted against measured data.

Figure 3 shows plot of the simulated result based on equation (1) against the measured response. The difference is hardly noticeable. The rms value of the squared error is 0.120. Frequency response plot for the model (1) is illustrated in figure 4 that exhibits a resonance peak at 400 rad/sec. It is this resonance that needs to be suppressed. Regarding to this, our approach used is pole-zero assignment method [7,8] to achieve robusted performance. The design and compensator realization are in the next section.

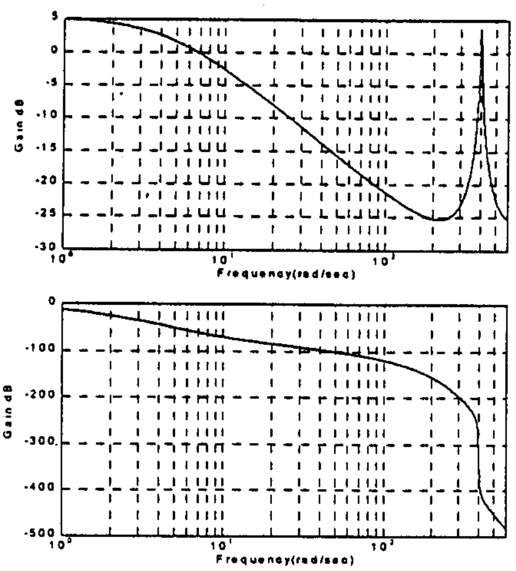


Figure 4 Frequency response plot for the model (1).

COMPENSATION AND REALIZATION

Suppression of torsional resonance can be achieved by transfer function synthesis with pole-zero assignment method. Theoretically, the achieved system consists of a plant and two compensators. Figure 5 depicts the block diagram of our compensated system so called 2-degrees-of-freedom (DOF) configuration.

Referring to figure 5, the feedback compensator, $G_{fb}(s)$, helps to eliminate oscillation due to resonance and stabilize the feedback loop. The forward or input compensator, $G_{ff}(s)$, drives the system to reach desirable performance. Pole-zero assignment technique [7,8] well suits our objective of resonance suppression. We

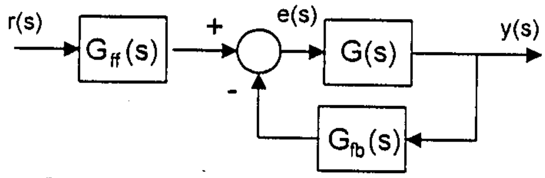


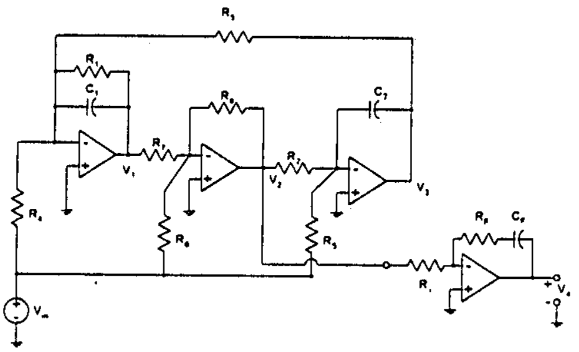
Figure 5 A control system of 2-DOF structure

utilize the same poles for both compensators in our design to ensure the desired characteristic polynomial of the compensated system. Zeros of the compensators are obtained from solving a diophantine equation such that the overall transfer function is achieved. Equations (2) and (3) express the input and the feedback compensators, respectively.

$$G_{ff}(s)=15.093\frac{(s^{2}+4\times10^{3}s+4\times10^{6})}{(s^{2}+7.186\times10^{3}s+19.160\times10^{6})}.10^{-3}\frac{(s+2000)}{s} \quad (2)$$

$$G_{fb}(s)=16.83\frac{(s^{2}+2.714\times10^{2}s+5.037\times10^{4})}{(s^{2}+7.186\times10^{3}s+19.160\times10^{6})}.\frac{(s+142.36)}{s} \quad (3)$$

Realization of the compensators employs op-amp technology. The compensation circuits are of biquad structure with a series connected PI-element. The combination possesses a third-order transfer function. Figure 6 gives the details of circuit configuration and component values. The compensation circuits are built



(a) Compensation circuit

$$G_{ff}(s)=15.093\frac{(s^{2}+4\times10^{3}s+4\times10^{6})}{(s^{2}+7.186\times10^{3}s+19.160\times10^{6})}.10^{-3}\frac{(s+2000)}{s}$$

C=0.01µF, R_1=13.91 kΩ, R_2=R_3=R_7=R_8= 22.84 kΩ
R_4=2.08 kΩ, R_5= 7.25 kΩ, R_6= 1.51 kΩ
C_F= 0.1 µF, R_F = 5 kΩ, R_i = 5 MΩ

(b) Component values for input compensator

$$G_{fb}(s)=16.83\frac{(s^{2}+2.714\times10^{2}s+5.037\times10^{4})}{(s^{2}+7.186\times10^{3}s+19.160\times10^{6})}.\frac{(s+142.36)}{s}$$

C=0.01 µF, R_1=13.91 kΩ, R_2=R_3=R_7=R_8= 22.84 kΩ
R_4=858.8 kΩ, R_5= 516.11 kΩ, R_6= 1.35 kΩ
C_F= 0.1 µF, R_F = R_i = 70.24 kΩ

(c) Component values for feedback compensator

Figure 6 The combined biquad circuit and PI-element with corresponding component values to realize the compensators.

into our test bed as illustrated by figure 7. Experiments were conducted using this equipment. Practical results are presented in subsequent section. Simulation results can be found in the following section.

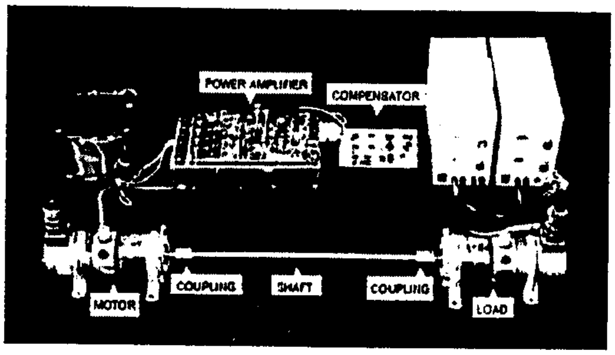


Figure 7 Experimental equipment.

SIMULATION RESULTS

In order to assess the performance of compensated system before attempting experimental works, simulation and stability test are needed. This section presents a comparison of simulation results for resonance suppression using between a notch filter and our compensation scheme. Then, practical results and stability test follow in subsequent sections.

Comparison of performances : With our compensated system, a fast response without any limit-cycle oscillation can be achieved. Figure 8(a) and 8(b) show the simulated responses when the 2-mass system is compensated by a notch filter (see appendix) and our compensation scheme, respectively. The response in figure 8(b) is much faster with less overshoot. Limit-cycle oscillation is eliminated completely. The bandwidth of our compensated system is about 150 rad/sec as can be observed from figure 9.

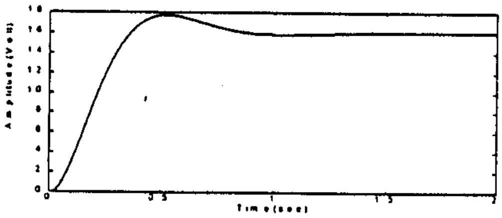


Figure 8(a) Simulation result for a system using notch filter. Response shows P.O. of 10, rise-time 0.3 seconds, and settling time 1.4 seconds.

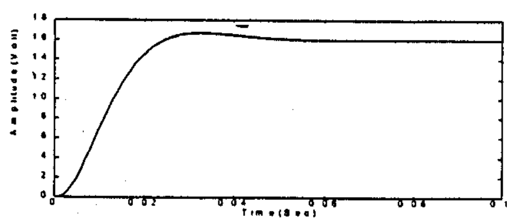


Figure 8(b) Simulation results for a system using pole-zero assignment scheme. Response shows P.O. of 4, rise-time 20 milli-seconds, and settling time 70 milli-seconds.

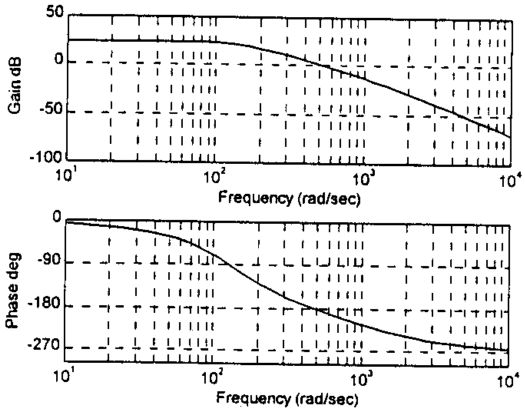


Figure 9 Frequency response plot for compensated system using 2-DOF configuration.

It is necessary to ascertain that the performance is robusted due to uncertainty in the plant model. So far, the design has been based on the nominal plant model of equation (1). In practice, several factors regarded as disturbances may contribute to inaccuracy of the model. This effects the performance of the compensated system. To assess the performance robustness, we assume an interval plant model in which its coefficients can vary within a reasonable range. The defined range for the coefficient variation is $\pm$ 30% except that of the highest order. Within this range, we obtain the uppermost and the lowermost plant models as follows

$$G_p^+(s) = \frac{1.72\times10^6}{s^3+17.39s^2+211.86\times10^3 s+950.30\times10^3} \quad (4)$$

$$G_p^-(s) = \frac{924.0\times10^3}{s^3+9.37s^2+114.08\times10^3 s+511.70\times10^3} \quad (5).$$

Simulation results obtained from using the models (4) and (5) represent the boundary of possible responses. These simulated responses due to unit-step input are illustrated in figure 10 indicating insignificant differences among them. Therefore, it can be concluded that the system performance is robusted.

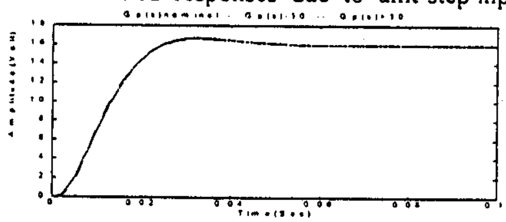


Figure 10 Unit-step responses obtained from simulation based on interval plant models.

PRACTICAL RESULTS

Our compensated system can provide a smooth and fast response due to unit-step input. An experimental result is shown in figure 11 for an input of 1 volt representing 143 rpm command. The lower curve is the measured speed response that shows 4 P.O., rise-time 30 milli-seconds, settling time 70 milli-seconds, and no steady-state error. The steady-state speed is limited to 143 rpm due to the saturation limit of power drive amplifier. This saturation greatly effects the magnitude of drive control signal during transient. Hence, a further investigation of the system when non-linearity present will be necessary.

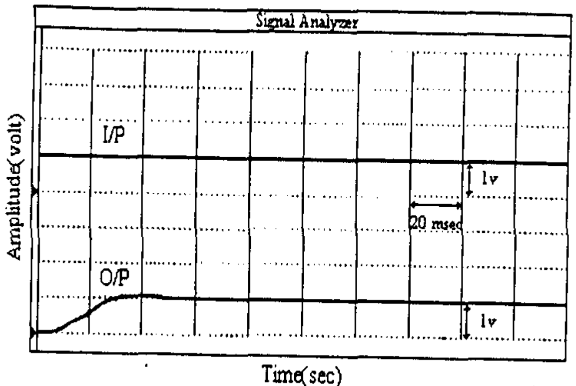


Figure 11 Measured speed of the compensated system.

STABILITY TEST

Since our system is assumed linear, stability test can be done conveniently by checking that all closed-loop poles lie in the left-half s-plane. Subject to the plant model uncertainty, a family of system transfer functions can be derived. On the basis of these transfer functions, the robustness of stability can be guaranteed iff all characteristic polynomials belonging to such family are Hurwitz. In other words, our fixed compensator can stabilize the whole set of interval plants. Regarding this, we apply the family of Kharitonov's polynomials [9] and the CB-segment method [10]. This approach has the following advantages: (i) it can be applied for closed-loop control with a fixed compensator and an interval plant, and (ii) it requires a small number of transfer functions for stability test.

Let $G(s) = N(s)/D(s)$ and $G_{fb}(s) = P_1(s)/P_2(s)$.
The characteristic polynomial of the closed-loop system is given by

$$Q(s) = P_1(s)N(s) + P_2(s)D(s) \tag{6}$$

The problem is to determine the Hurwitz stability of the family Q(s). Only the coefficients of N(s) and D(s) are perturbed independently. Assuming these uncertain polynomials enter the characteristic polynomial linearly, hence the term linear interval system.

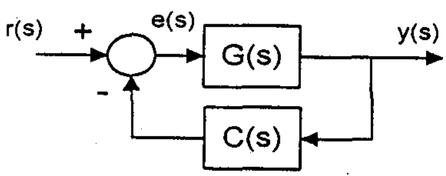


Figure 12 A closed-loop system with plant G(s) and controller C(s).

CB Theorem [10] : The control system of figure 12 is stable for all $G(s) \in \mathbf{G}(s)$ iff it is stable for all $G(s) \in \mathbf{G}_{CB}(s)$.

$\mathbf{G}_{CB}(s)$ is a one parameter family of transfer functions and there are at most 32 distinct elements.

$$G_{CB}(s) := \left\{ \frac{N(s)}{D(s)} : (N(s), D(s)) \in (\mathbf{N}(s) \times \mathbf{D}(s))_{CB} \right\} \tag{7}$$

$$(\mathbf{N}(s) \times \mathbf{D}(s))_{CB} = \{ N(s), D(s)) : N(s) \in \mathbf{K}_N(s), D(s) \in \mathbf{S}_D(s) \ \ or \ \ N(s) \in \mathbf{S}_N(s), D(s) \in \mathbf{K}_D(s) \} \tag{8}$$

$$\mathbf{K}_N(s) : \left\{ K_n^1(s), K_n^2(s), K_n^3(s), K_n^4(s) \right\} \tag{9}$$

$$\mathbf{K}_D(s) := \left\{ K_d^1(s), K_d^2(s), K_d^3(s), K_d^4(s) \right\} \tag{10}$$

$$\mathbf{S}_N(s) := \left[\lambda K_n^i(s) + (1-\lambda) K_n^j(s) : \lambda \in [0,1], (i,j) \in \{(1,2),(1,3),(2,4),(3,4)\} \right] \tag{11}$$

$$\mathbf{S}_D(s) := \left[\mu K_d^i(s) + (1-\mu) K_d^j(s) : \mu \in [0,1], (i,j) \in \{(1,2),(1,3),(2,4),(3,4)\} \right] \tag{12}$$

$\mathbf{K}_N(s)$ and $\mathbf{K}_D(s)$ are sets of Kharitonov's polynomial derived from the numerator and the denominator of the plant model. Using the nominal plant (1) results in linear interval plant model of $\pm 30\%$ variation bounded by (4) and (5). For our case of having 3[rd]-order compensator, the theorem gives 12 distinct characteristic polynomials of order 6. Each polynomial has the parameter λ embedded. Closed-loop poles are computed with the parameter λ varied from 0 to 1 with a step of 0.1. Figure 13 illustrates the map of these closed-loop poles. Since all the poles lie in the left-half s-plane, the stability robustness is assured.

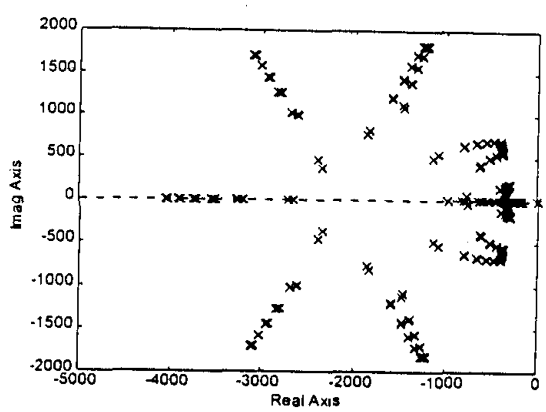


Figure 13 Map of the closed-loop system poles.

CONCLUSION

The work herein demonstrates the effectiveness of active compensation on suppression of torsional resonance in a 2-mass rotary system. The use of input and feedback compensators successfully eliminates limit-cycle oscillation in speed performance of the system. The compensated system responds smoothly and rapidly with an overshoot of about 4 percent. Both simulation and practical results confirm this claim. The system stability is robusted as demonstrated by the results of our investigation based on the CB-segment method. Since our realization and performance analyses assume linearity of the system, our future works will take account of the non-linearity due to power amplifier. Furthermore, performance and stability robustness will be analyzed under the assumption of non-deterministic variation in the coefficients of interval plant model. These studies will give an insightful understanding of the nature and practical limits of an actual system.

REFERENCES

[1] R. H. Cannon Jr., *Dynamics of Physical Systems* (McGraw-Hill, 1967).

[2] H. Waagen, Reduce torsional resonance in incremental motion servo, *Control Engineering* , 16(14), 1969, 85-88.

[3] J. Tal and B. C. Kuo (ed.), *Incremental Motion Control* (S.R.L. Publishing, 1978).

[4] K. Fujikawa, Z. Q. Yang, H. Kobayashi and T. Koga, Robust and fast speed control for torsional system based on state-space method, *Proc. IECON'91*, 1991, 687-692.

[5] H. Hirata, H. Sado, M. Anabuki and P. Teschareon, Speed control of dc motor with torsional oscillation and load fluctuation, *Proc. Sch. Engineering*, Tokai University-Japan, 35(1), 1995, 31-41.

[6] J-K. Ji and S-K. Sul, Kalman filter and LQ based speed controller for torsional vibration suppression in a 2-mass motor drive system, *IEEE Trans. IE*, 42(6), 1995, 564-571.

[7] C. T. Chen and B. Seo, Application of the linear algebraic method for control system design, *IEEE Contr. Syst.*, Jan., 1990, 43-47.

[8] K. J. Astrom, Robustness of a design method based on assignment of poles and zeros, *IEEE Trans. AC*, AC25(3), 1980, 588-591.

[9] P. C. Park and V. Hahn, *Stability Theory* (Prentice Hall, 1993).

[10] S. P. Bhattacharyya and L. H. Keel (ed.), *Control of Uncertain Dynamic System* (CRC Press, 1991).

[11] B. C. Kuo, *Automatic Control Systems* (Prentice Hall, 1995).

APPENDIX

Notch filter : $G_{notch}(s) = \dfrac{s^2 + 8.90s + 1.63 \times 10^5}{s^2 + 36.62 \times 10^3 s + 1.63 \times 10^5}$

The design of this filter follows the procedures described in [11].