การวิเคราะห์สิทธิเลือกจริงของการลงทุนพลังงานทดแทนภายใต้ความไม่แน่นอนในประเทศจีน-กรณีศึกษาในโครงการพลังงานลม

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A REAL OPTIONS ANALYSIS OF RENEWABLE ENERGY INVESTMENT UNDER UNCERTAINTY IN CHINA - A CASE STUDY OF A WIND POWER PROJECT

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A Thesis Submitted in Partial Fulfillment of the Requirements for the

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สาระสำคัญของวิทยานิพนธ์ฉบับนี้คือการกำหนดราคาสิทธิเลือกจริง โดยใช้วิธีการสองวิธี วิธีการแรกจะใช้ต้นไม้ทวินามพร้อมกับโปรแกรมพลวัต ในการกำหนดราคาสิทธิเลือกจริงของโครงการพลังงานลม ในวิธีที่สองจะสมมุติว่าข้อมูลจริงของโครงการพลังงานลมสอดคล้องกับกระบวนการคืนกลับสู่ค่าเฉลี่ยซึ่งเป็นตัวแบบต่อเนื่อง ในส่วนนี้จะกำหนดราคาสิทธิเลือกจริงด้วยการจำลองแบบมอนติคาร์โล วิทยานิพนธ์นี้ได้เสนอกรณีศึกษาพลังงานลม และได้ให้ข้อสรุปว่าสิทธิเลือกจริง ได้ให้ทางเลือกพร้อมกับค่าความเสี่ยงหลายทางสำหรับการตัดสินใจแก่นักลงทุนซึ่งจะช่วยให้นักลงทุนตัดสินใจได้อย่างถูกต้อง นอกจากนี้งานวิจัยนี้ก็ยังช่วยให้เห็นประโยชน์ของการวิเคราะห์สิทธิเลือกจริงด้วย

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REAL OPTIONS / INVESTMENT / BINOMIAL TREE / REVERTING MEAN PROCESS / DYNAMIC PROGRAMMING

The core contents of this thesis are to price the real option by using two methods of real options analysis (ROA). Firstly, we construct a framework by using the binomial tree with dynamic programming to price a wind power project (WPP) value. Secondly, considering that the reality data of the WPP follows a mean reverting process, we consider to model under the continuous time process, and in this part, we price the WPP value by using Monte Carlo simulation. This study shows a case study and concludes that ROA can give flexibility to investors when making decisions, revealing uncertainty and allowing them to make decisions that positively influence the final project value. In addition, this work also contributes a better understanding of the usefulness of ROA.

School of Mathematics
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LIST OF ABBREVIATIONS

AWR	Abandoned Wind Rate
CERs	Certified Emission Reductions
CDM	Clean Development Mechanism
DCF	Discounted Cash Flow
NPV	Net Present Value
OGE	On-grid Electricity
O&M	Operation and Maintenance
O-U	Ornstein-Uhlenbeck
PDD	Project Design Document
PDE	Partial Differential Equations
PMR	Project Monitor Reports
ROT	Real Option Theory
ROA	Real Option Analysis
ROV	Real Option Valuation
WPP	Wind Power Project

CHAPTER I

INTRODUCTION

Project valuation is probably the most important part to be considered in an investment process, since it assigns a dollar value to a project. If the project's net revenues during the production phase are higher than the investment costs, the project is considered worthy of investments. Most investment decisions share three important characteristics at varying degrees. First, the investment is partially or completely irreversible. Second, there is uncertainty over the future rewards from the investment. Third, there is some leeway about the timing of the investment. One may postpone action to get more information about the future.(Dixit and Pindyck, 1994)

Real Options Theory (ROT), also called Real Options Analysis (ROA), is an important new framework in the theory of investment decision. A real option itself is the right， but not the obligation, to undertake certain business initiatives, such as deferring, abandoning, expanding, staging, or contracting a capital investment project. ROA gives flexibility to investors when making decisions about real assets, revealing uncertainty associated with cash-flows and allowing investors to make decisions that positively influence the final project value. (Santos, Soares, Mendes, and Ferreira, 2014)

1.1 Research Background

This research focuses on structuring a real options analysis framework in a wind power investment in China. Most power stations in the world burn fossil fuels such as coal, oil, and natural gas to generate electricity, while some use nuclear power, however with the seriousness of the atmospheric pollution, there is an increasing use of cleaner renewable sources such as solar, wind, wave and hydroelectric. Renewable energy sources have been supported and subsidized by different countries. Among them, wind energy seems to be the most successful in terms of market penetration (Munoz, Contreras, Caamano, and Correia, 2009).

In China, wind power (WP) has entered the large-scale development phase. Figure 1.1 shows the WP capacity in China from 2003 to 2010. China's total wind power installed capacity nearly doubled each year from 2006 to 2009. By the end of 2010, total installed capacity was 41 GW, operational wind power capacity was 31 GW and 50 TWh of electricity had been generated from WP (Agency, 2011).

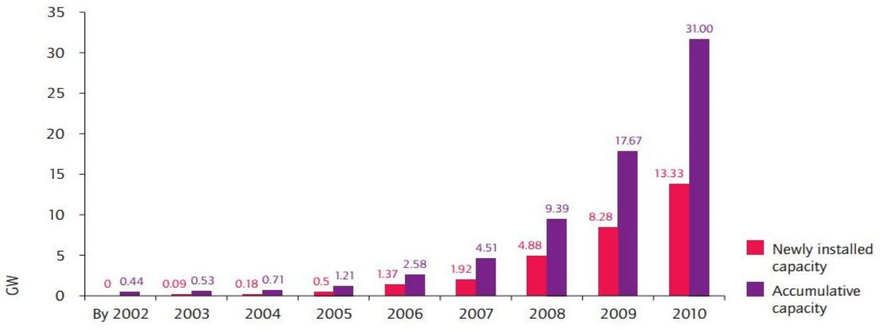

Figure 1.1 Wind power capacity in China from 2002-2010.

From the Global Wind 2015 Report (GWEC, 2016), China had added 30.8 GW of wind installed capacity in 2015, that makes the cumulative wind power installed capacity in China reached 145.4Gw, while global new installed wind capacity was 63.5GW in 2015, and cumulative wind power installed capacity is 439.2GW.

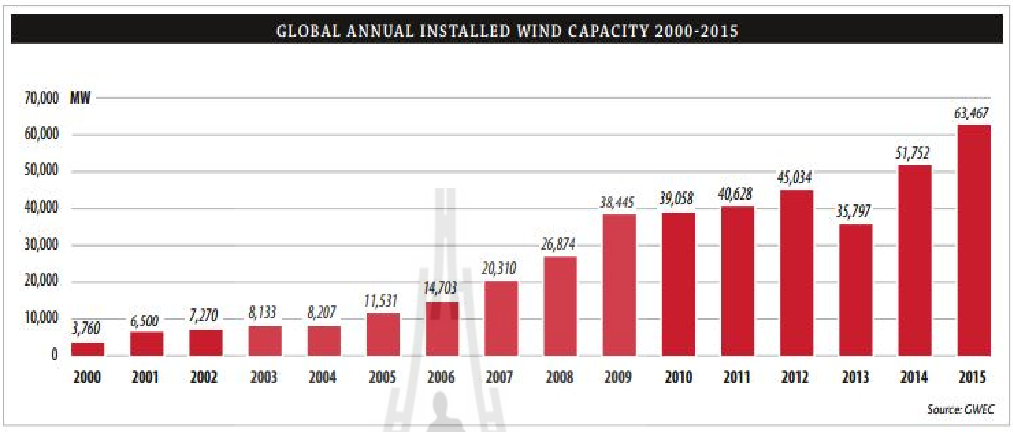

Figure 1.2 Global cumulative installed wind capacity 2000-2015.

China has identified wind power as a key growth component of the country's economy. The Chinese government has proposed a low-carbon development strategy and wind power has become to be one of the main energy technologies used to realize low carbon targets. Some researchers from Harvard and Beijing Tsinghua University have found that China could meet all of its electricity demands with wind power by 2030.

However, investments in wind energy in China as well as in other countries of the world are irreversible, costly, and are subject to numerous uncertainty sources, such characteristics draw complexity into the decision process. Any wind power investment project faces uncertainties including the price of electricity, practical generating volumes, national support policy, conventional power generation costs, CDM (Clean Development Mechanism), etc.

Investment in the power sector has three important characteristics. First, the investment is partially or completely irreversible. Once invested, the capital costs become totally or partially sunk. Second, there are always uncertainties over the future return from the investment. Future energy price and carbon price are unpredictable factors which will make cash inflow of the project return uncertain. Third, the investors have choices to invest at flexible timing. They can invest in a power plant now if they think the return of the investment is high enough to recover all the investment risks, or they can postpone the investment to get better information on the future prices. They will never invest until future major uncertainty is cleared. In other words, investors have the opportunity or option but not the obligation to invest in a project in a period of time. They can also have flexibility to abandon, expand, contract, extend and shorten the operation of the project even after the investment. A good project evaluation methodology or model should incorporate in a quantitative way all the three characteristics: irreversibility, uncertainty and flexibility. (Yang and Blyth, 2007)

1.2 Research Objectives

Because of the advantage of the Real Option Analysis (ROA) method, wind power investment decision makers could count on strategic flexibility that allows them to decide an optimum decision of completion, expansion, contraction, interruption or abandonment according to the market conditions.

In this study, two models are proposed which evaluate the value of an electric power generation project, by considering electricity prices and electric energy production, with empirical analysis illustrating the decision making process through a particular wind power project (WPP). The modeling methodology in this study is developed to help the investment decision of a potential investor in Chinese wind power.

1.3 Research Outline

The thesis is organized as follows: Chapter II gives a literature review on the real option theory (ROT), and in particular, gives a brief summary of literature about the ROA application in the energy area. Because our subject focus is wind power investment, so at the end of chapter II, the thesis gives an introduction into the wind power investment environment in China. Chapter III presents a ROA model by using binomial tree with dynamic programming, the WPP pricing problems in this chapter are divided into two option valuation problems, one is delay option and another one is Research and Development (R&D) option. We also offer a numerical calculation and scenario analysis to show how the binomial tree can be used to model the real option problem. Chapter IV presents an ROA model by using PDE and contingent claim analysis, the pricing problem in this chapter is considered to valuate under a continuous stochastic process. Chapter V summarizes the research of the thesis, puts forward limitations of this study and provides suggestions for future research.

CHAPTER II

LITERATURE REVIEW

2.1 Traditional Valuation Tools and Limitations

The traditional valuation tools depending on a discounted cash flow (DCF) series do not get at some of the intrinsic attributes of the asset or investment opportunity. Traditional methods assume that the investment is an all-or-nothing strategy and do not account for managerial flexibility, for example, management can alter the course of an investment over time when certain aspects of the project's uncertainty become known (Mun, 2002). One of the value-added features of using real option analysis is that it takes into account management's ability to create, execute, and abandon strategic and flexible options.

There are three traditional tools to appraise a project value: DCF analysis, Monte Carlo simulation, and decision trees. Irrespective of the tools, the building blocks for the calculations are provided by the present value of the cash flow stream. Any valuation starts with estimation of development and production phase costs and net revenues (free cash flows) over the project life. (Kodukula and Papudesu, 2006)

However, the orthodox theory of investment has not recognized the important qualitative and quantitative implications of the interaction between irreversibility, uncertainty, and the choice of timing. One of the most commonly used approaches of valuation is the discounted cash flow (DCF) approach. According to Net Present Value (NPV) theory, the DCF approach can neither properly deal with unexpected market developments nor allow for management's flexibility to adapt and revise later decisions in response to them. (Venetsanos, Angelopoulou, and Tsoutsos, 2002) By using this theory, the future cash flows of an investment project are estimated and if there is uncertainty about those cash flows, the expected value is determined. The expected cash flows are discounted at the cost of capital for the corporation and the results summed. If the NPV is positive the project is worthwhile and should be pursued. If it is negative the project should be turned down. If the NPV is zero it does not matter to the corporation whether the project is accepted or rejected. Specifically, traditional approaches underestimate the value of a project by ignoring the value of its flexibility. (Mun, 2002)

2.2 Real Options Theory and Method

The method which allows us to value the company and its investment projects in conditions of uncertainty, taking into consideration the ability of the company to react to changes taking place in the economy, is defined as Real Option Valuation (ROV), also often termed Real Options Analysis (ROA) (Dzyuma, 2012).

Options formulations first appeared in the seminal works of the late Fisher Black, Myron Scholes (Black and Scholes, 1973), and Robert Merton (Merton, 1973). Their works led to the Black-Scholes formula that determines the foundation for options and derivatives pricing, expanding the scope of options by considering equity as an option on the firm. Real options provide an analytical framework to evaluate management flexibility in decision-making concerning whether or how to proceed with business investment while it considers the dynamic uncertainty involved in the future values of the underlying factors. In 1977, Stewart Myers first introduced the term real options in his papers (Myers, 1977). Since then the concept of ROV had gradually been gaining theoretical significance, yet it was not until the 1990s that the first attempts to apply it in practice were made. Trigeorgis was the first to systematize the wealth of dispersed knowledge in the real options area, and to prove the usefulness and possibilities of valuation (Trigeorgis, 1996). He explained how management's flexibility to revise their original operating strategy according to the future conditions of the dynamic market represents an asymmetry or skewness in the probability distribution of the NPV.

Today, ROA is well accepted for enriching the value of projects under uncertainty by modeling the managers' flexibility to make decisions to adjust the projects in response to changes in their environments. Because of this, ROA could be used to deal with current energy and environmental issues by enhancing the value of electricity generation projects, especially renewable energy projects.

Calculation of the real option value of a project basically starts with the computation of the underlying asset value by the traditional DCF method using a risk-adjusted discount rate. Next it incorporates the investment cost (strike price) and the value created by the uncertainty of the asset value and flexibility due to the contingent decision. If there is no uncertainty, management can make a decision today and there is no option value. Uncertainty creates future management decision opportunities that are reflected in the value of the option. The higher the uncertainty is, the higher the option value. (Kodukula and Papudesu, 2006)

There are three commonly used methods to solve real options valuation problems, partial differential equations (PDE), simulation, and trees or lattices. In the first method, a PDE has to be formulated for the assessment of specific RO under fixed assumptions (Dixit and Pindyck, 1994). This approach is highly accurate and can be computationally inexpensive for simple options. However, a new set of PDE has to be formulated whenever the RO or assumptions change. This can be time consuming or even unfeasible for complex options (Trigeorgis, 1996). The most widely used PDE is the Black and Scholes formula. (Black and Scholes, 1973);(Merton, 1973). As for the second method, one simulation can be used to model the evolution of uncertainty. This is a robust approach that can handle many types of RO, however, it tends to be computationally expensive. In the third method one uses binomial trees or lattices trees to simulate the evolution of uncertainty in discrete scenarios (Dixit and Pindyck, 1994). This approach facilitates the modeling of multiple interrelated options. Nonetheless, it is less accurate than the PDE approach and can become computationally expensive or prohibitive for large amounts of scenarios. The most widely used tree approach is the binomial tree (Cox, Ross, and Rubinstein, 1979).

There are several models available to solve real option problems, and each of these approaches has its own advantages and disadvantages in certain situations. The Black-Scholes and binomial tree methods are by far the most commonly used, followed by simulations. The former two methods are able to address most of the issues presented in the preceding discussion on the input variables. The difference lies in how easily one can adjust the model to account for those issues and how effectively one can explain the results. The following presents a brief discussion on the real world applicability of the models to real option problems.

The famous Black-Scholes formula is from the Black-Scholes partial differential equation, (Higham, 2004)

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S^2\frac{\partial^2 V}{\partial S^2}+rS\frac{\partial V}{\partial S}-rV=0.$$

Use $C(S,t)$ to denote the European call option value,

$$C(S,t)=SN(d_1)-Ee^{-r(T-t)}N(d_2),$$

where $N(\cdot)$ is the N(0,1) distribution function, and

$$d_1=\frac{\log(S/E)+(r+1/2\sigma^2)(T-t)}{\sigma\sqrt{T-t}},\ \ d_2=d_1-\sigma\sqrt{T-t}\ .$$

This formula may seem to be the right method for real options analysis (ROA) because it is so widely employed in financial options valuation and easy to use. We just need to identify the input parameters: current asset value is S, E is strike price, σ is volatility of the asset, r is the risk free rate, T is the time to expiration. But its application in real options is limited for many reasons. The primary two reasons are:

first, it is difficult to explain the derivation of the equation because of its mathematical complexity, second, Black and Scholes developed their model for European financial options, which means that the option is exercised only at a fixed date and no dividends are paid during the option life. However, real options can be exercised at any time during their life (Kodukula and Papudesu, 2006). Although these limitations may be overcome by making adjustments to the Black-Scholes approach, the already complex model becomes even more complex.

Simulations are more easily applicable to European options, where there is a fixed exercise date. The computations, however, become tedious when simulating all the possible option exercise dates for an American option. In order to consider any exercise date during the year, one would have to run 1,000 simulations, 365 times each. It becomes an even bigger challenge when dealing with sequential options, because each decision leads to a new path. This can involve millions of simulations, which can be an enormous computational task even with today's fast computers.

The binomial method offers the most flexibility compared to Black-Scholes and the simulation approach. Input parameters such as the strike price and volatility can be changed easily over the option life. Jumps and leakage can also be accommodated without any complex changes. The key advantage which the binomial method offers to a practitioner is that it is transparent in its underlying framework, making the results easy to explain to upper management for buy-in and approval.

While Black-Scholes gives the most accurate option value, the binomial method will be an approximation of the Black-Scholes equation because of the underlying mathematical framework of binomial method. Binomial is our method of choice for solving the options problems in wind power investment project presented in this research, because it is the most effective for communication and illustrative purposes for an ordinary investor. However, for comparison, we also briefly present Black-Scholes solutions wherever appropriate. The binomial method offers the most flexibility compared to Black-Scholes and the simulation approach. Input parameters such as the strike price and volatility can be changed easily over the option life. Jumps and leakage can also be accommodated without any complex changes.

2.3 ROA Application in the Power Sector

ROA is useful in project appraisal when the project revenue streams resulting from the investment are uncertain, and now ROA is widely used as a tool to help decision making in many fields (Trigeorgis, 1996). In the energy investment area, there have been a growing number of publications on real options analysis in energy investment in recent years. (Konstantinos Venetsanosa, 2002; Luna, Assuad, and Dyner, 2003; Davis and Owens, 2003; Fleten and Maribu, 2004; Kumbaroğlu, Madlener, and Demirel, 2006; Zhou et al., 2007; Correia, Carvalho, Ferreira, Guedes, and Sousa, 2008; Munoz et al., 2009; Lamothe, Méndez, and Goyanes, 2009; Cheng, Hou, and Wu, 2010; Martinez-Cesena and Mutale, 2012; Lee and Shih, 2010; Yang, Nguyen, De T'Serclaes, and Buchner, 2010; Zhao, Li, and Xia, 2014; Wesseh and Lin, 2015; Díaz, Moreno, Coto, and Gómez-Aleixandre, 2015)

Konstantinos Venetsanos et al. (2002) present a framework for the appraisal of power projects under uncertainty within a competitive market environment; the study focuses on the electricity from Renewable Energy Sources.

Real options are also being discussed by Luna Assuad, and Dyner (2003), where it is proposed to employ the real options methodology to assess an investment project of wind energy generation in Colombia, backed up by a model with systems dynamics which reproduces the interaction between market values. Graham A. Daviset al. (2003) use "real option" pricing techniques to estimate the value of renewable electric technologies in the face of uncertain fossil fuel prices. Fleten, Stein-Erik Maribu, et al. (2004) present a method by solving a PDE for the evaluation of investments in small-scale wind power under uncertainty.

Some papers combine the wind speed and electricity price distribution to determine the revenues of a wind farm. Kumbaroğlu, Madlener, and Demirel (2006) present a policy planning model which integrates learning curve information on renewable power generation technologies into a dynamic programming formulation by featuring real options analysis. Hui Zhou et al. (2007), for evaluation of wind power generation asset investment, use a model for the mean reversion process with long-term periodic mean to describe the special characteristics of electricity price such as fluctuation, uncertainty and periodicity. Correia, Carvalho, Ferreira, Guedes, and Sousa (2008) focus on establishing the market-based value of a power plant and on determining the best execution of investment when it is done in multiple, modular stages. A comprehensive methodology is developed to establish a process for the plant present value

A similar framework is also applied by Munoz et al. (2009). The paper presents a decision-making tool for investment in a wind energy plant by using RO approach, considering market price sand wind regimes obtained from Geometric Brownian motion with Mean Reversion (GBM-MR) and Weibull models. Compound real option has also been focus by some other. Lamothe, Méndez, and Goyanes (2009) model the wind resource variability using two different models; one for the annual variation and another for variations within a year. Cheng, Hou, and Wu et al. (2010) evaluate wind projects based on their fuel and emission savings. Theoretical valuation of the project is thus given by the solution of a partial differential equation derived by Ito's lemma. Martinez-Cesena and Mutale (2012) present a RO study of wind power projects planning considering the wind resource assessment, Lee and Shih (2010) present a policy benefit evaluation model that integrates cost efficiency curve information on renewable power generation technologies into real options analysis (ROA) methods. The framework based on the uncertainty of CDM is being discussed. Yang et al. (2010) focus on risks from China's uncertain electricity market regulation and an uncertain energy policy framework. Zhao, Li, and Xia (2014) construct a financial model of net present value (NPV) to analyze the cost price of wind power electricity, give a sensitivity analysis to examine the impact of different variables with and without certified emission reductions (CERs) income brought about by the CDM.

Wesseh and Lin (2015) use the real options approach from a policy perspective which could provide insights about the viability of renewable energy programs and robustness of feed-in tariffs. Díaz, Moreno, Coto, and Gómez-Aleixandre (2015), due to prospect valuation of a wind power distributed generation project, argue that the value of a distributed generation wind-based project can be revisited by means of the Longstaff–Schwartz method.

A brief summary of real options literature addressing wind power projects is presented in Table 2.1 as follows:

Table 2.1 Real options literature addressing wind power projects.

References	Uncertainty	Tool	Year
(Konstantinos Venetsanosa, 2002)	energy market	PDE	2002
(Davis and Owens, 2003)	fuel prices	PDE	2003
(Luna, Assuad, and Dyner, 2003)	wind	Binomial Tree	2003
(Fleten and Maribu, 2004)	price	PDE	2004
(Zhou et al., 2007)	Price, wind	Simulation	2007
(Munoz et al., 2009)	Price, wind	Tree, Simulation	2009
(Lamothe, Méndez, and Goyanes, 2009)	Cash flows	Tree, Simulation	2009
(Cheng, Hou, and Wu, 2010)	Price, cost,policy	Tree	2010
(Yang, Nguyen, De T'Serclaes, and Buchner, 2010)	CER Price, policy	Simulation	2010
(Cheng et al., 2010)	fuel price	PDE	2010
(Wesseh and Lin, 2015)	Cost policy	Tree	2015
(Martinez-Cesena and Mutale, 2012)	wind	Tree, Simulation	2012
(Zhao, Li, and Xia, 2014)	CER Price	Sim	2014

2.4 WP Investment Policies and Environment in China

2.4.1 Feed-in Tariffs

According to the Renewable Energy Law of China (MOFCOM, 2009), anyone who wishes to construct a project of electricity generation by using renewable energies (RE) shall obtain an administrative license. The on-grid electricity (OGE) prices for projects of electricity generation by using renewable energy shall be determined by the administrative department of price of the State Council in light of the conditions of different areas and the characteristics of different type of renewable energies. Under this law, a power grid company signs a long-term power purchase agreement with WPP investors and agrees to buy all electricity generated by the WPP within the coverage of their power grid. The bidding competition determines the in-grid tariffs, the agreement, and the duration of the agreement which normally covers the total operational period of a wind project.

Nowadays, competitive tendering for WP development has been adopted by many countries, as well as by China. In addition to competitive tendering, many policy mechanisms have been implemented to facilitate the development of wind power farms, such as feed-in tariffs (FIT) for example. A FIT is a kind of tariff system that is based on the fixed regional primary energy price plus a fixed premium for WP. This type of policy instrument is a mean to palliate uncertainty (Yang et al., 2010).

2.4.2 The Clean Development Mechanism

In addition to tariffs, investors may also benefit from the CDM.

The Clean Development Mechanism (CDM) is one of the Flexible Mechanisms defined in the Kyoto Protocol (Solomon, 2007) that provides for emissions reduction projects which generate CER units which may be traded in emissions trading schemes (Wikipedia). The CDM allows net global greenhouse gas emissions to be reduced at a much lower global cost by financing emissions reduction projects in developing countries where costs are lower than in industrialized countries (called Annex 1 countries). If a firm invests in a CDM project in developing countries it may claim CERs from the project and may trade the CERs in Annex 1 countries to recover part of its investment cost or make a profit.

2.4.3 Abandoned Wind Power Rationing

Apart from the above, we also need to consider the effect of different abandoned wind rate (AWR) in the investment process. China's wind resources are mainly distributed in the "Three Northern Areas", but electrical loads are mainly distributed in coastal regions, and still several problems exist, for example wind power and power grid construction paces are not synchronous, local load levels are low, the number of flexibly adjustable power supplies is limited, and the cross-provincial market is not mature. All of these problems are likely to cause an increase in the AWR. In March 2011, the State Electricity Regulatory Commission issued the "Wind Power and Photovoltaic Power Generation Regulatory Report", which provided statistics regarding non-purchased wind power electricity during January-June 2010. Regionally, the amount of wind electricity curtailed in the north and northeast were the largest, accounting for 57.20% and 38.33% of the total wind electricity curtailed nationwide, respectively. (GWEC, 2012). According to the data of the National Energy Administration of China, in 2015, the national average AWR was 15%; the most serious area was Gansu province, where the AWR reached as high as 39%; an abandoned wind statistics table is shown in Table 2.2.

Table 2.2 Abandoned wind statistics from 2011 to 2015.

	National average AWR	Abandoned WP loss（Gwh）	Electricity revenue loss（million Yuan）
2011	16.23%	12300	6600
2012	17.12%	20800	11200
2013	10.74%	16200	8800
2014	8%	12600	6800
2015	15%	33900	18300
Total	13%	95900	51800

The abandoned rate will affect the electricity output directly, and the AWR can be calculated by formula:

$$\text{AWR}=\frac{\text{Abandoned wind electricity}}{\text{Generation power of wind farm}}\times 100\%,$$

where Abandoned wind electricity = Generation electricity − on-grid electricity.

The above data shows that abandoned wind rate (AWR) is a main factor which may negatively affect the revenue of a wind farm, and have the investment profits.

CHAPTER III

REAL OPTION ANALYSIS BY USING BINOMIAL METHOD

3.1 Methodology

In this chapter, we will model two decision processes regarding the wind power investment.

The first problem simply addresses the question whether to invest, and if so, when to invest. This will be called the stage one problem. The second question which we will call the stage two problem, is whether to apply to register an already started project as a CDM project.

Since both models employ binomial trees and dynamic programming, we will begin by briefly reviewing these two concepts.

3.1.1 Binomial Tree

Figure 3.1 is a graphical representation of a process which can follow a binomial tree in discrete time. It shows the ways to represent such a process of the state variable.

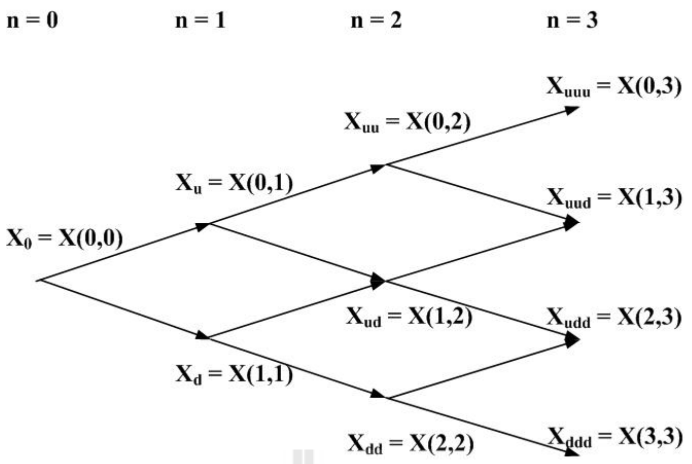

Figure 3.1 Binomial tree of state variable.

At each time step, the state variable can transition in two ways, up or down. For example, the left-most node in Figure 3.1 shows the state variable taking the value X_0 at date 0 and evolving to take one of the two values X_u and X_d at date 1. We will use $X(i,n)$ to denote the various states, where i refers to the number of down moves and n refers to the date n. For example, as will be used in our model, if each up-move modifies the state variable by a factor U, and each down-move by a factor D (D=1/U), then $X(i,n) = X(0,0)U^{n-i}D^{i}$.

3.1.2 Dynamic Programming

Assume the wind power firm's current status is described by state variable x_t, and x_t affects the firm's operation and expansion opportunities. At any date t, x_t is known, but the future values $x_{t+1}, x_{t+2}, ...$ are random variables. At each data t, the firm faces many options, which are represented by a control variable denoted as u_t. For example, in the stage 1 of the ROA considered below, we can denote $u_t = 0$ to describe "wait and see", and $u_t = 1$ to describe "to invest", The value u_t must be chosen using only the information x_t. We will use $\pi(x_t, u_t)$ to denote the immediate profit flow of the firm at date t, and $F_t(x_t)$ to denote the outcome when the firm makes all decisions optimally from date t onwards, that is the expected net present value of all of the firm's current and future cash flows.

Bellman's Principle of Optimality states: an optimal policy has the property that, whatever the initial action, the remaining choices constitute an optimal policy with respect to the sub-problem starting at the state that results from the initial actions. (Dixit and Pindyck, 1994), thus, the Bellman Equation is:

$$F_t(x_t) = \max_{u_t} \left\{ \pi(x_t, u_t) + \frac{1}{R_f} E_t[F_{t+1}(x_{t+1})] \right\}.$$

Here $R_f = 1 + r$, where r denotes the discount rate over one time period and E_t is the expected value at date t of $F_{t+1}(x_{t+1})$. If the many-period problem has a fixed finite time horizon *T,* suppose at the end of the horizon the firm gets a termination payoff $\Omega_T(x_T)$, then at the period before,

$$F_{T-1}(x_{T-1}) = \max_{u_{T-1}} \left\{ \pi_T(x_{T-1}, u_{T-1}) + \frac{1}{R_f} E_{T-1}[\Omega_T(x_T)] \right\}.$$

By using recursion, we can get $F_t(x_t)$ for all t, $0 \le t \le T$.

3.2 Modeling Process

The main objective of this study is to evaluate the market value of a wind power investment, taking into consideration the current Chinese tariff policy, and CDM regulation. A cash flow model is developed to undertake project financial analysis. We model the project benefits by putting different options and using different parameters.

Figure 3.2 is a illustration of the study process of this study:

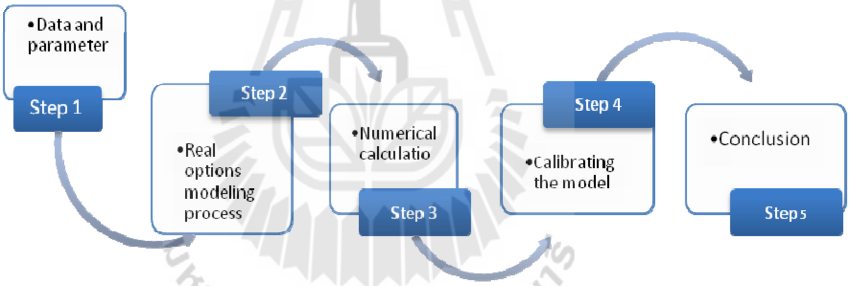

Figure 3.2 Framework of the study.

In order to do this, based on the method from (Guthrie, 2009) we continue the study step by step.

Step 1: Develop a relevant database with sorted primary information including capital cost, Operating and maintenance (O&M) costs, CER prices, and electricity prices (EP).

Step 2: Involve the development of a cash flow spreadsheet.

Because the focus of this study is the application of real options analysis, we will appropriately simplify the cash flow model which will be used to calculate the market value of the WPP, and only the main factors that affect the market value of the project will be considered. These include power gain, CDM gains (if as a CDM project), static investment cost, O&M costs, and loan interest.

Step 3: To identifies RO style.

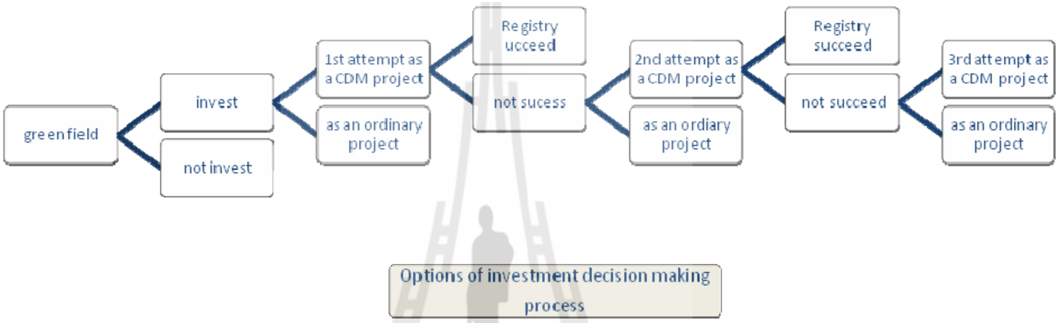

Figure 3.3 Total process of investment.

Figure 3.3 gives an intuitive view of total process of investment.

In the first stage, the investor faces immediately the choice to invest or not invest. Since the owner has the development rights for a limited amount of time (5 years in our model), the question can be considered as a delay option pricing problem.

In the second stage, once the investor has built a wind farm, one assumes that he/she intends to apply for CDM projects. But due to the complexity of the CDM approval procedure, the applicant may face failure and thus the question can be considered as an R&D option problem.

3.3 Methodology Procedures

3.3.1 NPV modeling

By using the formula $PV = FV/(1+r)^n$ where FV is the future value of a project, PV is the present value, r is the discount rate per time period, and n is number of the time period, the NPV of a project can be calculated as follows:

Project NPV =PV of free cash flows in production phase − PV of investment costs.

Figure 3.4 shows the key variables and the overall modeling framework of DCF. From DCF, one can calculate the market value of a WPP by using ROA and binomial tree method.

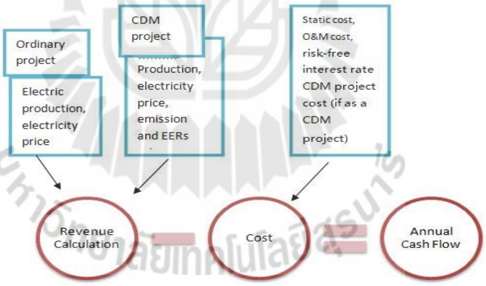

Figure 3.4 WPP NPV modeling.

3.3.2 Real Option Pricing of Stage 1

Next, without loss generality, two stages of the real options model are established respectively.

Stage 1 Investment or wait and see.

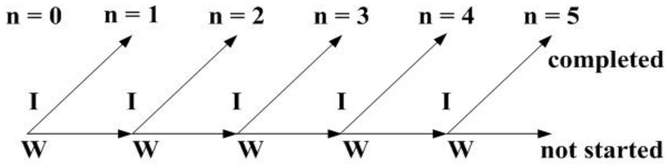

Figure 3.5 Decision tree for option to delay.

At this stage, at any point in time the project will be in one of two possible states— "not started" and "complete"—corresponding to before and after investment respectively. A decision tree of this stage is shown in Figure 3.5.

We will make the following assumptions:

1) The project has a 5-year duration.

2) The investment horizon is split into years. That is, we have dates

n=0,1,2,3,4,5,

at each date n=0,1,2,3,4 a decision to invest or not to invest can be made.

3) There is no residual value from a completed project after 5 years, because government may take ownership of the power plant without compensation.

4) If the decision to invest is made at some time t=n, then the project will begin generating power from time t=n+1onwords.

First we let X denote the annual value of power production of a completed project; we assume that X follows a binomial tree as outlined in Figure 3.1; up and down moves at every date n occur with risk-neutral probabilities π_u and π_d respectively. Thus $X(i,n)$ denotes the annualized power production at date n, if i down moves have taken place.

We define the state variable $Y_1(i,n)$ to be the cash flow of the completed project at date n, and

$$Y_1(i,n)=\begin{cases}0 & \text{if investment not completed}\\ X(i,n)P_f-C_O & \text{if investment completed,}\end{cases} \tag{3.1}$$

where P_f denotes the feed-in tariff, and C_O denotes the cost of operation and maintenance.

Next let $Y'(i,n)$ denote the multi-period (from the date n onwards) cash flow of income at node (i,n); then by using backward induction, we can calculate $Y'(i,n)$ by the following formula:

$$\begin{cases}Y'(i,5)=Y_1(i,5)\\ Y'(i,n)=Y_1(i,n)+\dfrac{\pi_u Y'(i,n+1)+\pi_d Y'(i+1,n+1)}{R_f}\end{cases} \qquad i=0,1,2,3,4,$$

where $R_f=1+r$, r is the risk-free interest rate.

Next let $V_b(i,n)$ denote the market value of the project rights at node (i,n) before investment begins, and $V_a(i,n)$ denote their market value immediately after its completion. Without loss of generality, we assume that the project can be sold as soon as the construction is completed, we thus must have $V_a(i,n)=Y'(i,n)$. That is, immediately after the construction is completed, the project is worth whatever the owner will receive from the imminent sale of the completed project.

This leaves us with the problem of calculating the market value $V_b(i,n)$ of the project rights before investment occurs. Since the investment option expires at date 5, the project rights will be worthless at this date and this lead to the terminal condition

$$V_b(i,5)=0, \qquad i=0,1,2,3,4,5. \tag{3.2}$$

According to the decision tree, we see that the owner can choose between investing and waiting at all earlier dates up to n = 4.

If the owner invests at node (i,n) she pays C_s immediately at date $n+1$ where C_s denotes the average static investment cost. Moreover, she will (briefly) own a completed project and so we have

$$payoff_{invest}(i,n) = -C_s + \frac{\pi_u Y'(i,n+1) + \pi_d Y'(i+1,n+1)}{R_f}. \tag{3.3}$$

If the owner waits at node (i,n) she pays nothing and thus

$$payoff_{wait}(i,n) = \frac{\pi_u V_b(i,n+1) + \pi_d V_b(i+1,n+1)}{R_f}. \tag{3.4}$$

Since the owner seeks to maximize the market value of project rights, we get the recursive equation

$$V_b(i,n) = \max\left\{ -C_s + \frac{\pi_u Y'(i,n+1) + \pi_d Y'(i+1,n+1)}{R_f}, \frac{\pi_u V_b(i,n+1) + \pi_d V_b(i+1,n+1)}{R_f} \right\}. \tag{3.5}$$

Equation (3.5) and terminal condition (3.2) completely determine the market value of the project rights before investment as well as the optimal investment.

It is optimal to invest at date n if and only if the payment payoff from investing exceeds the payoff from waiting, that is

$$payoff_{invest}(i,n) > payoff_{wait}(i,n). \tag{3.6}$$

It then follows that

$$\frac{\pi_u Y'(i,n+1) + \pi_d Y'(i+1,n+1)}{R_f} > payoff_{wait}(i,n) + C_s. \tag{3.7}$$

3.3.3 Real Option Pricing of Stage 2

Now, we move to the stage 2.

Suppose that the wind farm is built immediately, and starts to operate at date n=1. The investor now faces the choice to attempt to register the wind farm as a CDM project. He or she can do so any time over the remaining 4 years of the project. If application for registration is made at date t=n and is approved, then the CDM will generate cash flow from date t=n+1 onwards. If the CDM registration is refused then the owner can reapply the following year. Each registration attempt costs C_c, lasts one period, and succeeds with some constant probability q between 0 and 1. The registration rights to the project will be lost if registration is not successfully completed on or before date 4.

At date 0, the manager of the wind farm faces to choose action "attempt" or action "maintain". These are represented by the labels "A" (for attempt), "M" (for maintain), "As" (for register succeed), and "Af" (for register failed) as appearing in the decision tree shown in Figure 3.6.

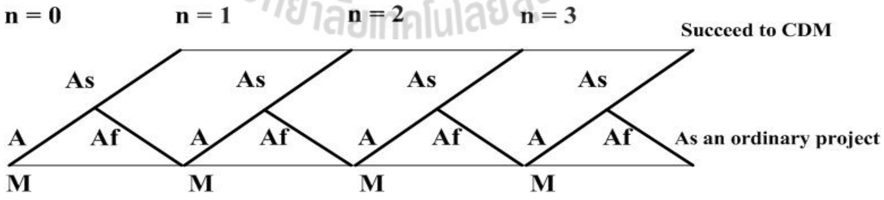

Figure 3.6 Decision tree for R&D option.

We define the state variable $Y_2(i,n)$ to be the cash flow of the completed project at date n, and

$$Y_2(i,n) = X(i,n)P_f - C_O + G_c P_e \qquad n = 0,1,2,3,4,5,$$

where $X(i,n)$ denotes the annual electricity generation of the wind farm at node

(i,n), P_f denote the feed-in tariff, C_O the cost of operation and maintenance, G_c the annual estimation of emission reductions (tCO2 e), P_e the CERs price.

We let $V^*_b(i,n)$ denote the market value of the development rights at node (i, n) before investment begins and $V^*_a(i,n)$ denote their market value immediately after the register succeeds. Without loss of generality, we assume that the project can be sold as soon as construction is completed; we thus must have $V^*_a(i,n) = Y^*(i,n)$ (where $Y^*(i,n)$ denotes the multi-period cash flow at this stage which can be obtained by using backward induction). This leaves us with the problem of calculating the market value, $V^*_b(i,n)$ of the project rights before investment occurs. Since the investment option expires at date 4, the project rights will be worthless at this date, leading to the terminal condition

$$V^*_b(i,4) = 0 \qquad i = 0,1,2,3. \tag{3.8}$$

If the manager does not attempt to register the CDM project, there is no cash flow of cost for CDM project,

$$payoff_{wait}(i,n) = \frac{\pi_u V^*_b(i,n+1) + \pi_d V^*_b(i+1,n+1)}{R_f}. \tag{3.9}$$

Suppose, instead, that the manger attempts registration, which involves an immediate negative cash flow of C_c, if an up move occurs then she owns $V^*_a(i,n+1) = Y^*(i,n+1)$, or it fails and worth $V^*_b(i,n+1)$.

So, by assumption, we get the expected value:

$$qY^*(i,n) + (1-q)V^*_b(i,n+1).$$

Similarly, if a down move occurs, the expected value is:

$$qY^*(i+1,n+1)+(1-q)V^*{}_b(i+1,n+1).$$

Thus, the payoff from attempting registration is:

$$\begin{aligned} payoff_{attempt}(i,n) = -C_c + \frac{1}{R_f}\{&\pi_u[qY^*(i,n)+(1-q)V^*{}_b(i,n+1)] \\ &+\pi_d[qY^*(i+1,n+1)+(1-q)V^*{}_b(i+1,n+1)]\}, \end{aligned} \tag{3.10}$$

and we get the recursive equation

$$\begin{aligned} V^*{}_b(i,n) = \max\Bigg\{ &\frac{\pi_u V^*{}_b(i,n+1)+\pi_d V^*{}_b(i+1,n+1)}{R_f}, \\ &-C_c + q\frac{\pi_u Y^*(i,n+1)+\pi_d Y^*(i+1,n+1)}{R_f} \\ &+(1-q)\frac{\pi_u V^*{}_b(i,n+1)+\pi_d V^*{}_b(i+1,n+1)}{R_f}\Bigg\}, \end{aligned} \tag{3.11}$$

for all $n = 3,2,1,0.$

Equation (3.11) and terminal condition (3.8) completely determine the market value, as well as the optimal investment.

$$payoff_{attempt}(i,n) > payoff_{wait}(i,n)$$

i.e.

$$\begin{aligned} q\frac{\pi_u Y^*(i,n+1)+\pi_d Y^*(i+1,n+1)}{R_f} &+ (1-q)\frac{\pi_u V^*{}_b(i,n+1)+\pi_d V^*{}_b(i+1,n+1)}{R_f} \\ &> \frac{\pi_u V^*{}_b(i,n+1)+\pi_d V^*{}_b(i+1,n+1)}{R_f} + C_c. \end{aligned} \tag{3.12}$$

3.4 Numerical Calculation

In this study, we carry out an empirical analysis with the data of a realistic WPP. The WPP is project No. 0689 in the CDM database, The project is located in Danianzi town, Songshan District, Chifeng City in Inner Mongolia Autonomous Region, and its installed capacity is 49.3MW. Some data we used are collected from the No.0689 project design document (PDD) and project monitor reports (PMR), and some others data are estimated according to government or agency reports.

3.4.1 Valuation with Traditional Method

In order to compare the RO method with the traditional method, we first give a result by using the traditional NPV method. Suppose that the land development right can be held for 5 years; to keep matters as simple as possible, we assume that the wind farm can be built instantly. A table of some parameters is given in the Table 3.1. Suppose the annual average cost including static and O&M cost is 49.78 million Yuan, and the annual output is 101.82GW, the OGE price is 0.545 Yuan/kwh, thus the

$$\text{Annual revenue=}\frac{(100.81\times 10^{6})\times 0.545}{10^{6}}-49.78=5.16 \text{ million Yuan}$$

$$\text{NPV of project=}\sum_{n=1}^{5}\frac{5.16}{(1+0.05)^{n}}=22.35 \text{ million Yuan.}$$

Table 3.1 Parameters.

Project's characteristics	Symbol	Value (unit)	Note
Total static investment cost	C_S	514.95 (Million Yuan)	PDD of project (PDD, 2006)Average annual cost $C_s = 34.33$
Cost of Operation and Maintenance	C_O	15.4485 (Million Yuan)	According IRENA (IRENA, 2015) Suppose $C_O = 3\% C_S$
Cost of CDM project	C_c	0.772425 (Million Yuan)	Suppose $C_c = 5\% C_O$
Feed-in tariff	P_f	0.545 (Yuan/kwh (excl. VAT))	PDD
CERs price	P_e	7 (EUR/tCO2 e)	Suppose 1Eur=9Rmb
Annual average electricity output	X	100.81 (Gwh/year)	To calculate by using the data from the PMR.

Table 3.1 (Continued).

Project's characteristics	Symbol	Value (unit)	Note
Annual estimation of emission reductions (tCO2 e)	G_c	125557 (Ton/year)	PDD
Risk free rate	R_f	5%	Risk-free interest rate
Project life time	T'	20 (year)	
Holding period of land development right	T	5 (year)	

3.4.2 Parameter Estimation

Suppose the electricity generation follows a random walk. Figure 3.7 shows a monthly data from one project of the database in the CDM website.

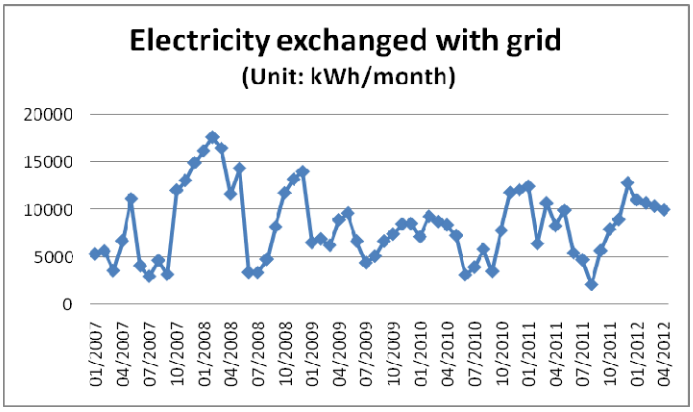

Figure 3.7 Electricity exchanged with grid, monthly data from 2007-2012.

We calculate the normalized drift and normalized volatility by formulas

$$\hat{\mu} = \frac{\sum_{i=1}^{n} ln\left(\frac{x_{i+1}}{x_i}\right)}{n}, \quad \sigma^2 = \frac{\sum_{i=1}^{n}\left(ln\left(\frac{x_{i+1}}{x_i}\right) - \hat{\mu}\right)^2}{n-1},$$

and get estimation $\hat{\mu} = 0.0100$ and $\hat{\sigma} = 1.6097$ from the data. Since $\Delta t = 1/12$, we can obtain:

$$U = e^{\hat{\sigma}\sqrt{\Delta t}} = 1.1436 \quad , \quad D = e^{-\hat{\sigma}\sqrt{\Delta t}} = 0.8745$$

$$\pi_u = \frac{1}{2} + \frac{\hat{\mu}\sqrt{\Delta t}}{2\hat{\sigma}} = 0.5108, \pi_d = 1 - \pi_u = 0.4892.$$

3.4.3 Calculation Results of ROA

We can fill the binomial tree for electricity generation according the previous description.

The calculated result of stage 1 is as follows:

Table 3.2 Binomial tree of delay options (a).

Tree for annual electricity output						
X(i,n)	0	1	2	3	4	5
0	100.81	115.28	131.83	150.75	172.40	197.15
1		88.15	100.81	115.28	131.83	150.75
2			77.09	88.15	100.81	115.28
3				67.41	77.09	88.15
4					58.95	67.41
5						51.55

Table 3.3 Binomial tree of delay options (b).

Multi-period cash flow of income in stage 1						
Y'(i,n)	0	1	2	3	4	5
0	207.44	244.47	230.77	203.09	158.14	92.00
1		168.77	161.93	144.39	113.65	66.71
2			109.28	99.51	79.64	47.38
3				65.19	53.63	32.60
4					33.74	21.29
5						12.64

Table 3.4 Binomial tree of delay options (c).

Market value of projects rights						
$V_b(i,n)$	0	1	2	3	4	5
0	28.18	50.39	63.08	34.15	41.51	0.00
1		7.87	13.61	19.86	29.96	0.00
2			2.68	5.51	11.33	0.00
3				0.00	0.00	0.00
4					0.00	0.00
5						0.00

Table 3.5 Binomial tree of delay options (d).

Policy of stage 1						
policy(i,n)	0	1	2	3	4	5
0	wait	invest	invest	wait	invest	wait
1		wait	invest	wait	invest	wait
2			wait	wait	invest	wait
3				wait	wait	wait
4					wait	wait
5						wait

Tables 3.2 to 3.5 present the evolution of the market value of project rights and the policy of stage 1. In Table 3.2, from the left side to the right side, regarding the annual electricity output, the value presented by the first node of the tree gives the current output of the wind power farm. The output can increase or decrease

depending on coefficients *U* and *D*, respectively. We consider the last column of the binomial tree to represent the possible output of the wind power farm in the 5th year. According to Table 3.3, we can get the multi-period cash flow of income in stage 1 by using the formula of market value of projects rights, and the last sheet shows the optimal policy in different nodes. If the power generation output moves up, the optimal time to invest is in the 3rd year; this may be relevant with the climate, power grid, and AWR. On the other hand, one can see from the results that the project value in the first year is 28.18 million Yuan, which is greater than the result by using the traditional method. And most importantly, the ROA results is dynamic, while the result of the traditional method is static.

Calculated results of Stage 2:

Table 3.6 Binomial tree of R&D options (a).

Multi-period cash flow of income in stage 2						
Y^*(i,n)	0	1	2	3	4	5
0	250.71	276.11	256.08	222.07	170.79	98.32
1		130.52	187.24	163.38	126.31	73.04
2			134.59	118.49	92.29	53.71
3				84.17	66.28	38.92
4					46.39	27.62
5						18.97

Table 3.7 Binomial tree of R&D options (b).

Market value of R&D rights						
$V^*_b(i,n)$	0	1	2	3	4	5
0	82.97	75.76	53.37	27.66	8.34	0.00
1		45.54	31.46	15.11	1.95	0.00
2			15.06	6.51	0.00	0.00
3				0.00	0.00	0.00
4					0.00	0.00
5						0.00

Table 3.8 Binomial tree of R&D options (c).

Policy of stage 2						
policy(i,n)	0	1	2	3	4	5
0	attempt	attempt	attempt	attempt	attempt	wait
1		attempt	attempt	attempt	attempt	wait
2			attempt	attempt	wait	wait
3				wait	wait	wait
4					wait	wait
5						wait

The CDM project profits can increase the profit of the WPP, but because of the complexity of the CDM project application, the profit is uncertain. The trees presented from the Tables 3.6 to 3.8 show the evolution of the market value of projects rights and the policy of stage 2. According to Table 3.6, we can simulate and get the multi-period cash flow of income in stage 2 by using formula market value of projects rights and Table 3.7 gives the market value of R&D options. The results shows that the R&D options value will gradually decrease with the passage of time, although the simulation just calculates for 5 years. The last sheet shows the optimal policy in different nodes. The maximum value is at the node (0,0), that means, the investor should attempt to register as a CDM project as soon as possible and the minimum value is at the node (3,3) for the first time. It means he should give up registering the project as a CDM project if the power generation output continues to move down for three years.

3.5 Scenario Analysis

Considering the future policy uncertainty and the uncertainty of wind power consumption in China, scenario analysis focuses on the effect of feed-in-tariff and AWR to options value. With the gradual establishment of the China carbon market trading system, investors are faced with great opportunities. The price of CERs will also become one of the important factors affecting the profit of WPP, so the scenario analyses also focus on the CER price.

Based on the stage 1, the scenario analysis starts from the current state with a feed-in-tariff of p=0.545 Yuan/mwh and the wind abandoned rate is r. The abandoned rate will affect the electricity output directly. By changing these two factors respectively, we can get a figure of the delay options price at node (0,0).

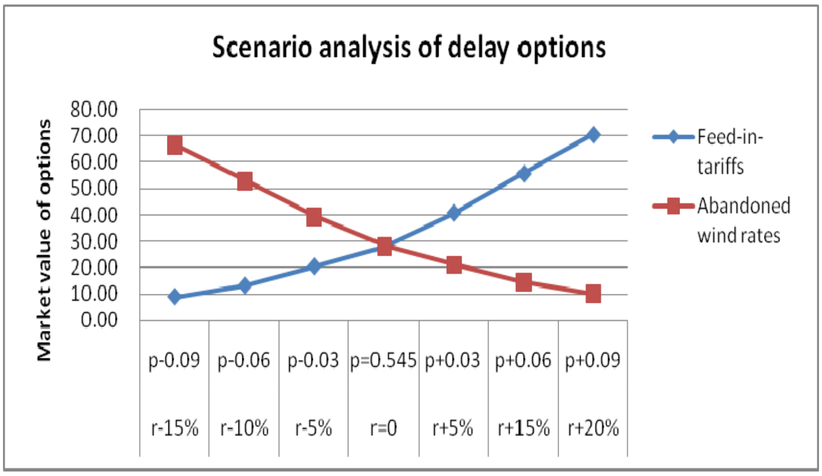

Figure 3.8 Scenario analysis of delay options.

From Figure 3.8, feed-in-tariff and AWR are the main factors of influence to the WPP. Beginning with the current state, the delay options price moves up as the feed-in-tariff moves up, and moves down as the AWR moves up. The delay options value move up to about 70 million Yuan as the feed-in-tariffs move to 0.635 Yuan/mwh. While the AWR increases by 20%, the value of the delay options moves down to about 10 million Yuan.

Now China has claimed that she plans to launch a national emission trading system in 2017. As a first step, carbon-trading pilots have been initiated in seven provinces and cities. Thus the CER price will be another main factor calculating the market value of a wind project in the future. Thus, in the stage 2, scenario analyses focuses on the impact of CER price changes on options value.

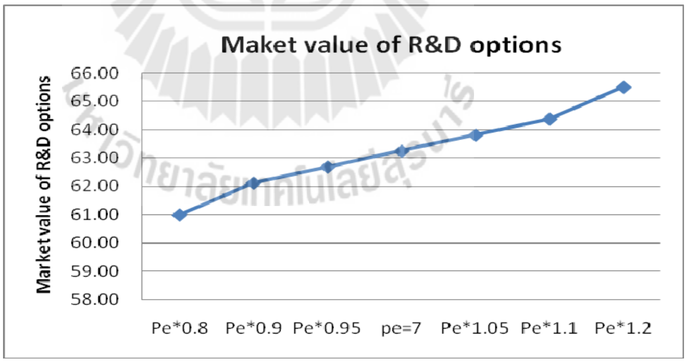

Figure 3.9 Scenario analysis of R&D options.

In Figure 3.9, the horizontal axis represents the various value of CERs price. At the middle of the horizontal axis CERs price "pe" equals 7. It increases to the right hand side and decreases to the left hand side. As the price of CERs goes up, the market value of R&D options goes up from about 61 million Yuan to about 65.5

million. One can see from Figure 3.9 that when the CERs price (carbon gain) changes by about 5% it will make R&D options change by about 0.9%, which is not too much and not contrary to expectation, after all, carbon gain is an additional revenue of a WPP.

CHAPTER IV

ROA UNDER CONTINUOUS TIME PROCESSES

4.1 Modeling under Mean Reverting Process

Many commodity researches rely on the mean-reverting process, for example Gibson and Schwartz (Gibson and Schwartz, 1990) use a mean reverting model for estimating commodity convenience yield. Also, an Ornstein-Uhlenbeck (O-U) process generally produces positive results, which is suitable for modeling construction projects. Therefore, our research will adopt the mean-reverting O-U process for evaluating real options. Mean reverting processes are naturally attractive to model commodity prices since they embody the economic argument that when prices are "too high", demand will reduce and supply will increase, producing a counter-balancing effect. When prices are "too low" the opposite will occur, again pushing prices back towards some kind of long term mean. Mean reverting processes are also useful for modeling other processes, observed or unobserved, such as interest rates or commodity "convenience yield".

We consider the revenue of a completed wind power farm, and we suppose the wind farm is a CDM project. Thus the revenue of the WPP comes from electricity income and carbon emission income. Let V be the revenue of the wind farm, we have

$$V = G_e P_f + G_e F_e P_e, \tag{4.1}$$

where G_e is the electricity output of the wind power farm, F_e is the emission factor of carbon, P_f and P_e are feed-in tariff and Certified Emission Reductions (CERs) prices, respectively.

According to the description in Chapter III, we have the monthly historical data on grid electricity of a wind power farm. Figure 4.1 shows the time series plot of the monthly revenue *V*.

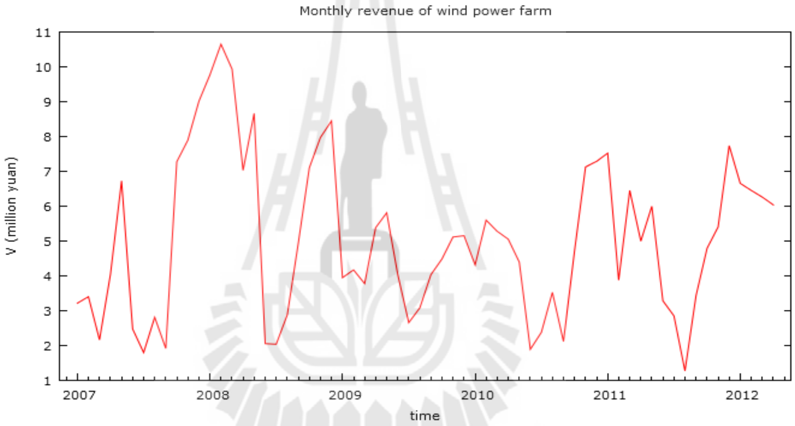

Figure 4.1 Historical monthly data of $V = G_e P_f + G_e F_e P_e$.

Giving a brief analysis on the data, Figure 4.2 shows the autocorrelation function (ACF) and partial autocorrelation function (PACF) of *V*.

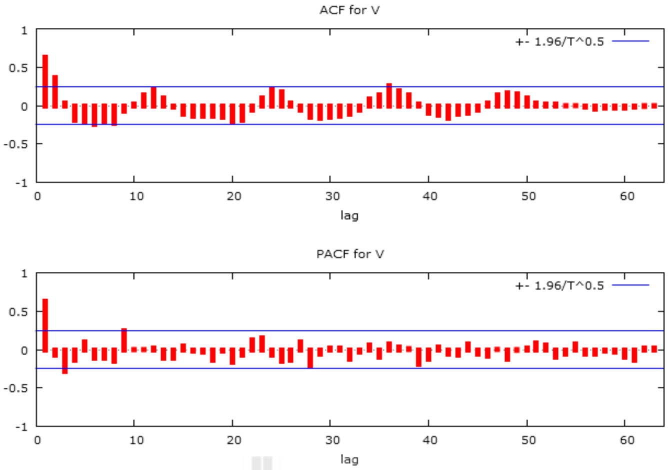

Figure 4.2 ACF and PACF of *V*.

One can see from Figure 4.2 that the series of *V* is a stationary processes. The O-U process (4.2) is a continuous time mean reverting process and can be used to model a stationary series (Arratia, Cabana, and Cabana, 2012). Thus in this study, we suppose the WPP's revenue follows a one-factor mean-reverting process, i.e. O-U process

$$dV_t = q(\frac{p}{q} - V_t)dt + \sigma dz_t, \tag{4.2}$$

where

$dz_t = \varepsilon_t \sqrt{dt}$, $\varepsilon_t \sim N(0,1)$ is a Brownian Motion,

q measures the speed of mean reversion,

$\frac{p}{q}$ is the "long run mean", to which the process tends to revert, and

σ is a measure of the process volatility.

4.2 Real Option Valuation under O-U Process

Now, our starting point is to consider this problem: at what point is it optimal to pay a sunk cost *I* in return worth *V* for a WPP. Note that the WPP investment opportunity is equivalent to a perpetual call option: the right but not the obligation to buy a share of stock at a pre-specified price. Therefore the decision to invest is equivalent to decide when to exercise such an option. Thus the investment decision can be viewed as a problem of option valuation. Alternatively, it can be viewed as a problem in dynamic programming. We will derive the optimal investment rule by using contingent claim methods.

In what follows, we will denote the value of the investment opportunity (that is, the value of the option to invest in the WPP) by *F*, where $F = F(V_t)$ is a function of *V*, and *V* is a function of *t*. Once we hold the option, we want a rule that maximizes its expected present value: .

$$F(V_t) = \max E[e^{-r(T-t)}(V_T - I)^+],\ T \geq t \quad . \tag{4.3}$$

Here T is the unknown time when the decision is made and r is the discount rate. In order to calculate $F(V)$, we assume that *F* is smooth enough to apply Ito's formula, and notice that $(dt)^2 = dtdz = 0, (dz)^2 = dt$. So we can obtain

$$\begin{aligned} dF &= \frac{\partial F}{\partial V_t} dV_t + \frac{1}{2}\frac{\partial^2 F}{\partial V_t^2}(dV_t)^2 \\ &= \left[\frac{\partial F}{\partial V_t}(p - qV_t) + \frac{\sigma^2}{2}\frac{\partial^2 F}{\partial V_t^2}\right]dt + \frac{\partial F}{\partial V_t}\sigma dz_t. \end{aligned} \tag{4.4}$$

Equations (4.2) and (4.4) can be transformed into discrete form as follows:

$$\Delta V_t = q\left(\frac{p}{q} - V_t\right)\Delta t + \sigma \Delta z_t \text{ and} \tag{4.5}$$

$$\Delta F = \left[\frac{\partial F}{\partial V_t}(p - qV_t) + \frac{\sigma^2}{2}\frac{\partial^2 F}{\partial V_t^2}\right]\Delta t + \frac{\partial F}{\partial V_t}\sigma \Delta z_t, \tag{4.6}$$

where ΔV and ΔF are changes of V and F over a short interval length Δt.

To simplify the problem, we will assume that the real project V is tradable and consider the following portfolio: hold $a = a(V)$ units of the project (or equivalently of the asset) and go short one unit of the option to invest which is worth $F(V)$. The value of this portfolio is $\Pi = -F(V) + aV$. Note that this portfolio is dynamic, i.e. as V changes the value of the portfolio Π may change from one short interval of time to the next. So, after a short interval of length Δt, the change of the portfolio will be as follows:

$$\begin{aligned}\Delta\Pi &= -\Delta F + a\Delta V_t \\ &= -\left[\frac{\partial F}{\partial V_t}(p - qV_t) + \frac{\sigma^2}{2}\frac{\partial^2 F}{\partial V_t^2}\right]\Delta t - \frac{\partial F}{\partial V_t}\sigma\Delta z + a\left[q\left(\frac{p}{q} - V_t\right)\Delta t + \sigma\Delta z_t\right] \\ &= -\frac{\sigma^2}{2}\frac{\partial^2 F}{\partial V_t^2}\Delta t - \frac{\partial F}{\partial V_t}(p - qV_t)\Delta t + a(p - qV_t)\Delta t + \left(-\frac{\partial F}{\partial V_t}\sigma + a\sigma\right)\Delta z_t.\end{aligned} \tag{4.7}$$

In order for the diffusion coefficient to vanish, we must let

$-\frac{\partial F}{\partial V_t}\sigma + a\sigma = 0$ and it follows that we have $a = \frac{\partial F}{\partial V_t}$. Thus the portfolio value can be rewritten as follows:

$$\Pi = -F + \frac{\partial F}{\partial V_t}V_t \tag{4.8}$$

and

$$\Delta\Pi = -\frac{\partial F}{\partial t}\Delta t - \frac{\sigma^2}{2}\frac{\partial^2 F}{\partial V_t^2}\Delta t. \tag{4.9}$$

For arbitrage reasons, the wealth process must be

$$\Delta\Pi = r\Pi\Delta t \tag{4.10}$$

Substituting equations (4.8) and (4.9) into (4.10), one gets

$$-\frac{\sigma^2}{2}\frac{\partial^2 F}{\partial V_t^2}\Delta t = r\left(-F+\frac{\partial F}{\partial V_t}V_t\right)\Delta t.$$

After rearranging this differential equation, it follows that

$$\frac{\sigma^2}{2}\frac{\partial^2 F}{\partial V_t^2}+rV_t\frac{\partial F}{\partial V_t}=rF. \tag{4.11}$$

Since we are dealing with a free boundary problem, we need three conditions to determine uniquely the optimal point to invest, i.e

$$(1)F(0)=0, \tag{4.12}$$

$$(2)F(V_t^*)=V_t^*-I, \tag{4.13}$$

$$(3)F'(V_t^*)=1, \tag{4.14}$$

where V^* is the optimal price for investment. The two first conditions are natural. If the project is worthless then a contract on it must be worthless. Second, at the point on the boundary, the contract must have the same value of its payoff. The third is the "smooth pasting" condition. Thus we combine equation (4.11) and the boundary conditions, the option value $F(V_t)$ can be calculated. Since solving this system requires complex mathematical skills, it is not conducive to the practical application of ROA. We will use the Monte Carlo simulation method to solve this problem and all MATLAB code will be shown in the Appendix of thesis.

4.3 Solution and Parameters Estimation of O-U Process

4.3.1 The Explicit Solution of O-U Process

We go back to find the solution of equation (4.2). Let $f(V_t,t)=V_te^{qt}$ and by using Ito's lemma, one gets

$$
\begin{aligned}
df(V_t,t) &= qV_t e^{qt}dt + e^{qt}dV_t \\
&= qV_t e^{qt}dt + e^{qt}[q(\frac{p}{q} - V_t)dt + \sigma dz_t] \\
&= pe^{qt}dt + \sigma e^{qt}dz_t.
\end{aligned}
\tag{4.15}
$$

Integrating on the both sides of equation (4.15) from 0 to t, we have

$$
V_t e^{qt} = V_0 + \int_0^t e^{qs} p dS + \int_0^t e^{qs} \sigma dz_s. \tag{4.16}
$$

Thus we can get the explicit solution of O-U process,

$$
\begin{aligned}
V_t &= V_0 e^{-qt} + \frac{p}{q}(1 - e^{-qt}) + e^{-qt}\int_0^t e^{qs}\sigma dz_s \\
&= V_0 e^{-qt} + \frac{p}{q}(1 - e^{-qt}) + e^{-qt}\sigma\int_0^t e^{qs} dz_s.
\end{aligned}
\tag{4.17}
$$

Additionally, according to the properties of Brownian Motion $\{z_t\}$, we can get the mean and variance of V_t:

$$
E(V_t) = V_0 e^{-qt} + \frac{p}{q}(1 - e^{-qt}), \tag{4.18}
$$

$$
\begin{aligned}
Var(V_t) &= E[V_t - E(V_t)]^2 \\
&= E[V_0 e^{-qt} + \frac{p}{q}(1 - e^{-qt}) + e^{-qt}\sigma\int_0^t e^{qs}dz_s - V_0 e^{-qt} - \frac{p}{q}(1 - e^{-qt})]^2 \\
&= E(e^{-qt}\sigma^2\int_0^t e^{qs}dz_s)^2 \\
&= \sigma^2 e^{-2qt} E(\int_0^t e^{qs}dz_s)^2.
\end{aligned}
\tag{4.19}
$$

By Ito isometry: $E(\int_0^t e^{qs}dz_s)^2 = E\int_0^t e^{2qs}ds$, we have

$$\begin{aligned} Var(V_t) &= \sigma^2 e^{-2qt} \int_0^t e^{2qs} ds \\ &= \sigma^2 e^{-2qt} \left. \frac{e^{2qs}}{2q} \right|_{s=0}^{s=t} \\ &= \frac{\sigma^2}{2q}(1 - e^{-2qt}). \end{aligned} \tag{4.20}$$

Hence, the O-U mean reverting model is a Gaussian model in the sense that, given V_0, the time t value of the process V_t is normally distributed with

$$E(V_t) = V_0 e^{-qt} + \frac{p}{q}(1 - e^{-qt}), \quad Var(V_t) = \frac{\sigma^2}{2q}(1 - e^{-2qt}).$$

As time $t \to \infty$, we can see from expressions above equations that

$$\lim_{t\to\infty} E(V_t) := E(V_\infty) = \frac{p}{q}, \ \lim_{t\to\infty} Var(V_t) := Var(V_\infty) = \frac{\sigma^2}{2q},$$

and the O-U stochastic process converges in distribution to $N(\frac{p}{q}, \frac{\sigma^2}{2q})$ as time $t \to \infty$.

4.3.2 Parameter Estimation Method of O-U Process

Using Euler's method, we can first discretize the O-U process and then use the Maximum Likelihood Estimation method (MLE) to obtain the parameters p, q, and σ. According to Euler's discretization method, we assume that the time-step is Δ, and then we obtain:

$$V_{t+1} - V_t = q(\frac{p}{q} - V_t)\Delta + \sigma\sqrt{\Delta}\varepsilon_t. \tag{4.21}$$

Let $\mu = \frac{p}{q}$, we rewrite the discrete form as

$$V_{t+1} = q\mu\Delta + (1 - q\Delta)V_t + \sigma\sqrt{\Delta}\varepsilon_t. \tag{4.22}$$

According to the section (4.3.1), we have the explicit solution of the O-U process as,

$$V_t = V_0 e^{-qt} + \frac{p}{q}(1 - e^{-qt}) + e^{-qt}\sigma\int_0^t e^{qs} dz_s .$$

The probability density function of standard normal distribution is

$$\varphi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} .$$

Thus the conditional probability density of an observation V_{i+1} condition on previous observation V_i is

$$\begin{aligned} & f(V_i | V_{i-1}, \mu, q, \hat{\sigma}) \\ & = \frac{1}{\sqrt{2\pi\hat{\sigma}^2}} \exp\left[-\frac{(v_i - v_{i-1}e^{-q\Delta} - \mu(1 - e^{-q\Delta}))^2}{2\hat{\sigma}^2} \right], \end{aligned} \tag{4.23}$$

where $\hat{\sigma}^2 = \sigma^2 \frac{1 - e^{-2q\Delta}}{2q}$.

The log-likelihood function of the set of data $v_0, v_1, v_2, \cdots, v_n$ can be obtained from the following function:

$$\begin{aligned} L(\mu, q, \hat{\sigma}) &= \sum_{i=1}^{n} \ln f(V_{i+1} | V_i, \mu, q, \hat{\sigma}) \\ &= \frac{n}{2}\ln(2\pi) - n\ln(\hat{\sigma}) - \frac{1}{2\hat{\sigma}^2} \sum_{i=1}^{n} \left[v_i - v_{i-1}e^{-q\Delta} - \mu(1 - e^{-q\Delta}) \right]^2 . \end{aligned} \tag{4.24}$$

In order to derive the maximum likelihood, we set all the partial derivatives equal to zero:

$$\begin{cases} \dfrac{\partial L(\mu, q, \hat{\sigma})}{\partial \mu} = 0 \\ \dfrac{\partial L(\mu, q, \hat{\sigma})}{\partial q} = 0 \\ \dfrac{\partial L(\mu, q, \hat{\sigma})}{\partial \hat{\sigma}} = 0 \end{cases} \Rightarrow \begin{cases} \dfrac{1}{\hat{\sigma}^2} \sum_{i=1}^{n} \left[v_i - v_{i-1} e^{-q\Delta} - \mu(1 - e^{-q\Delta}) \right] = 0 \\ -\dfrac{\Delta e^{-q\Delta}}{\hat{\sigma}^2} \sum_{i=1}^{n} \left[(v_i - \mu)(v_{i-1} - \mu) - e^{-q\Delta} (v_{i-1} - \mu)^2 \right] = 0 \\ \dfrac{n}{\hat{\sigma}} - \dfrac{1}{\hat{\sigma}^2} \sum_{i=1}^{n} \left[v_i - v_{i-1} e^{-q\Delta} - \mu(1 - e^{-q\Delta}) \right]^2 = 0. \end{cases} \tag{4.25}$$

Solving these equations, we obtain:

$$\begin{cases} \mu = \dfrac{\sum_{i=1}^{n} (v_i - v_{i-1} e^{-q\Delta})}{n(1 - e^{-q\Delta})}, \\ q = \dfrac{1}{\Delta} \ln \dfrac{\sum_{i=1}^{n} \left[(v_i - \mu)(v_{i-1} - \mu) \right]}{\sum_{i=1}^{n} (v_{i-1} - \mu)^2}, \\ \hat{\sigma}^2 = \dfrac{1}{n} \sum_{i=1}^{n} \left[v_i - \mu - e^{-q\Delta} (v_{i-1} - \mu) \right]^2. \end{cases} \tag{4.26}$$

We use symbols as follows:

$$V_x = \sum_{i=1}^{n} v_{i-1}, \; V_y = \sum_{i=1}^{n} v_i, \; V_{xx} = \sum_{i=1}^{n} v_{i-1}^{\,2}, \; V_{xy} = \sum_{i=1}^{n} v_i v_{i-1}, \; V_{yy} = \sum_{i=1}^{n} v_i^{\,2}. \tag{4.27}$$

Thus we can rewrite the estimation of parameters as follows:

$$\mu = \frac{V_y V_{xx} - V_x V_{xy}}{n(V_{xx} - V_{xy}) - (V_x^{\,2} - V_x V_y)}, \tag{4.28}$$

$$q = -\frac{1}{\Delta} \ln \frac{V_{xy} - \mu(V_x + V_y) + n\mu^2}{V_{xx} - 2\mu V_x + n\mu^2}, \; p = \mu q, \tag{4.29}$$

$$\hat{\sigma}^2 = \frac{1}{n} \left[V_{yy} - 2e^{-q\Delta} V_{xy} + e^{-2q\Delta} V_{xx} - 2\mu(1 - e^{-q\Delta})(V_y - e^{-q\Delta} V_x) + n\mu^2 (1 - e^{-q\Delta})^2 \right], \text{and}$$

$$\sigma^2 = \hat{\sigma}^2 \frac{2q}{1 - e^{-2q\Delta}}. \tag{4.30}$$

Using the algorithm on the historical monthly data with MATLAB software and assuming that the initial monthly revenue is $V_0 = 5.031$ then we get the estimation of parameters: $\mu = 5.139,\ q = 5.389,\ \sigma = 7.406$.

4.4 Monte Carlo Simulation Basic Procedure (Pricing Procedure)

For the purpose of pricing the real option, we will solve the problem by simulation. Simulation is not an analytical method, but is meant to imitate a real-life system, especially when other analyses are too mathematically complex or too difficult to reproduce. A simulation calculates numerous scenarios of a model by repeatedly picking values from the probability distribution for the uncertain variables and using those values for the event. As all those scenarios produce associated results, each scenario can have a forecast. Forecasts are events (usually with formulas or functions) that one defines as important outputs of the model. These usually are events such as totals, net profit, or gross expenses (Mun, 2002).

One type of simulation is Monte Carlo simulation which randomly generates values for uncertain variables over and over to simulate a real-life model. In recent years researchers have begun to apply the Monte Carlo simulation method to the pricing of real options. As a reference of decision makers, this method is based on the computer and modern statistical techniques to simulate the possible risks of investment projects and the probability distribution of the project value by using the method of mathematical statistics.

In this study, the following basic steps are involved such calculations; more details will be shown in the next section.

Step 1.Generate a random revenue path $V_i = (V_{i1}, V_{i2}, V_{i3}, \cdots, V_{in})$ which follows the O-U process, where $i = 1,2,3,\cdots,m$ denote the simulation times.

Step 2. Using equation (4.3) and the simulation of revenue paths to calculate the value of option F_i.

Step 3. Repeat the above two steps to get a large number of samples $V_1, V_2, V_3, \cdots, V_m$ and $F_1, F_2, F_3, \cdots, F_m$.

Step 4. Calculating the average of $F_1, F_2, F_3, \cdots, F_m$, we obtain the option value

$$F = \frac{\sum_{i=1}^{m} F_i}{m}.$$

4.5 Numerical Calculation

4.5.1 Monte Carlo Data Simulation

Mean reversion is the theory suggesting that prices and returns eventually move back towards their mean or average. This mean or average can be the historical average of the price, return, or another relevant average. Because the historical annual sample data is unavailable, we need to simulate annual data of on grid electricity. Figure 4.3 shows a histogram of monthly on grid electricity historical data from 2007 to 2011.

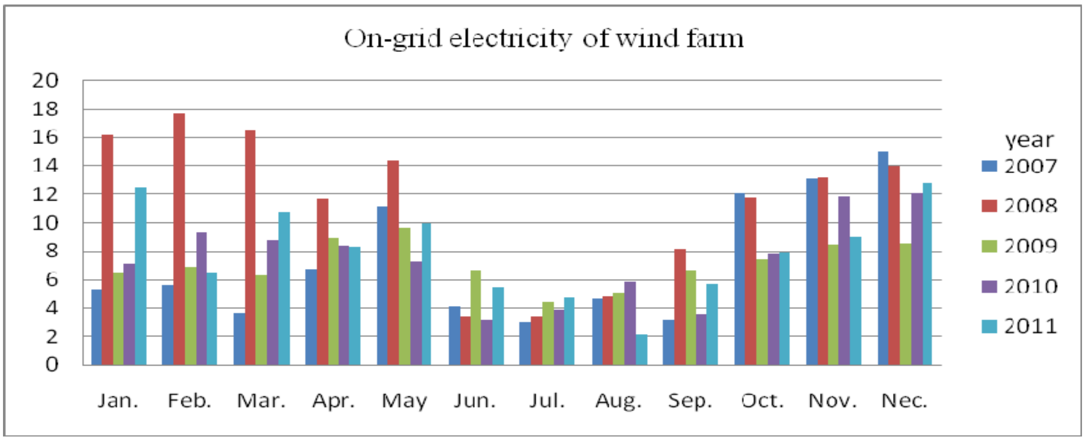

Figure 4.3 OGE historical data from 2007 to 2011.

The data for the histogram in Figure 4.3 is shown in Table 4.1.

Table 4.1 Table monthly OGE from 2007 to 2011.

Month(Gwh)	2007	2008	2009	2010	2011
Jan.	5.31901	16.21047	6.53894	7.14620	12.47605
Feb.	5.62989	17.67216	6.90734	9.28281	6.41836
Mar.	3.57051	16.48416	6.25526	8.74791	10.69763
Apr.	6.71283	11.65560	8.92105	8.37510	8.27833
May	11.15392	14.36952	9.63379	7.28175	9.94163
Jun.	4.09018	3.39240	6.68772	3.12184	5.43568
Jul.	2.96721	3.36336	4.39220	3.93134	4.71288
Aug.	4.64902	4.76256	5.09862	5.83857	2.09069
Sep.	3.16417	8.19446	6.67822	3.49657	5.67635
Oct.	12.06622	11.78033	7.42845	7.81239	7.94024
Nov.	13.09409	13.21852	8.47918	11.81753	8.95553
Dec.	14.95098	14.01494	8.54082	12.08499	12.83414

Studying the OGE historical data of project No.0968, we find that the data presents a seasonal feature. We suppose that the each monthly output follows one normal distribution and then simulate the annual data as follows.

Step 1. Let X_i denote the OGE of the month and assume that $X_i \sim N(\mu_i, \sigma_i^2)$, $i = 1,2,\cdots,12$. We denote x_{ij} $(i = 1,2,\cdots,12,\ j = 1,2,\cdots,n)$ for the sample data from the distribution $N(\mu_i, \sigma_i^2)$. Substituting the historical monthly data from

Table 4.1 into the formula $\hat{\mu}_i = \dfrac{\sum_{j=1}^{5} x_{ij}}{5}$ and $\hat{\sigma}_i^2 = \dfrac{\sum_{j=1}^{5} (x_{ij} - \hat{\mu}_i)^2}{5-1}$, $(i = 1,2,\cdots,12)$ we obtain the parameter estimation of μ_i and $\sigma_i^{\,2}$.

Step 2. Use MATLAB program to generate random number $\hat{x}_i$ from the normal distribution $N(\mu_i, \sigma_i^{\,2})$, $(i = 1,2,\cdots,12)$. Let $\hat{G}_e = \sum_{i=1}^{12} \hat{x}_i$, thus we obtain an annual data of OGE. By simulating 60 times, we can get a simulation data set $\left\{\hat{G}_{e1}, \hat{G}_{e2}, \hat{G}_{e3}, \cdots, \hat{G}_{e60}\right\}$. The simulation of this data set is shown in Figure 4.4.

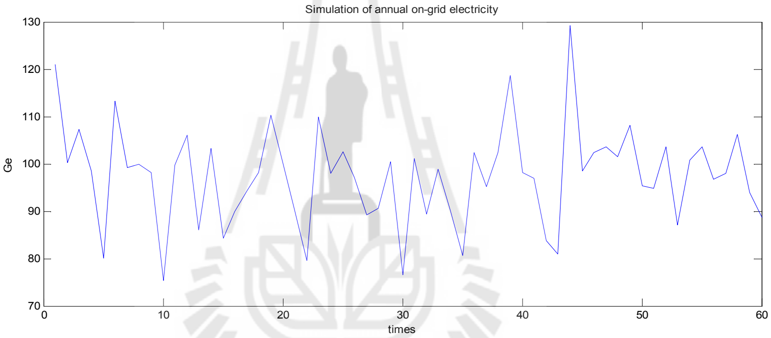

Figure 4.4 Simulation path of annual OGE.

4.5.2 Parameters Estimation

According to the system requirements previously described, suppose we take 100 times simulations to get a data set $\left\{\hat{G}_{e1}, \hat{G}_{e2}, \hat{G}_{e3}, \cdots, \hat{G}_{e100}\right\}$. Calculating revenue by using form $V = G_e P_f + G_e F_e P_e$, it follows an O-U process as the described in section 4.3. We can estimate the parameters by using the simulated data and obtain the O-U process $dV_t = 30.722(58.531 - V_t)dt + 46.539 dz_t$.

A parameter description table which will be used in sections 4.5.2 and 4.5.3 is shown in Table 4.2.

Table 4.2 Parameter description used in sections 4.5.2 and 4.5.3.

Parameters	Representation	Value
P_f	Feed-in tariff	0.545 Yuan/kwh
P_e	CERs price	56 Yuan/tCO2 e
F_e	Baseline emission factor of carbon	1.024 t/mwh
r	Risk-free rate	0.05%
μ	Estimation of long run mean of O-U process,	58.531
q	Estimation of the speed of mean reversion,	30.722
σ	Estimation of the process volatility	46.539
I	Annual average cost	49.7785 Million Yuan
T	Time to invest	1,2,...,5 (year)
V_0	Annual average revenue from 2007 to 2011	59.3234 Million Yuan
nstep	Time period number	60
npath	Simulation times	10000

4.5.3 Real Option Value

We price the real option according to the steps described in section 4.4.

Let $\{z_s\}_{t\geq 0}$ be a standard Brownian Motion and let $W_t = \int_0^t e^{qs} dz_s$. By the properties of Ito's integral, one gets

$$E(W_t) = 0,\ Var(W_t) = \int_0^t e^{2qs} ds = \frac{e^{2qt}-1}{2q}, \text{ i.e. } W_t \sim N(0, \frac{e^{2qt}-1}{2q})$$

So, we can rewrite the explicit solution of the O-U process in a discrete form:

$$V_t = V_{t-1} e^{-q\Delta t} + \frac{p}{q}(1 - e^{-q\Delta t}) + e^{-q\Delta t} \sigma \sqrt{\frac{e^{2q\Delta t}-1}{2q}} \varepsilon_t, \tag{4.31}$$

where $\varepsilon_t \sim N(0,1)$. In order to get the simulation paths of O-U process with the initial value V_0 and the year T to invest, we divide the interval $[0,T]$ into n time periods with the time subinterval is $dt = T/n.$, by using the parameters p, q, σ as in Table 4.2, one can get simulation paths of V_t with different simulated times as shown in Figure 4.5.

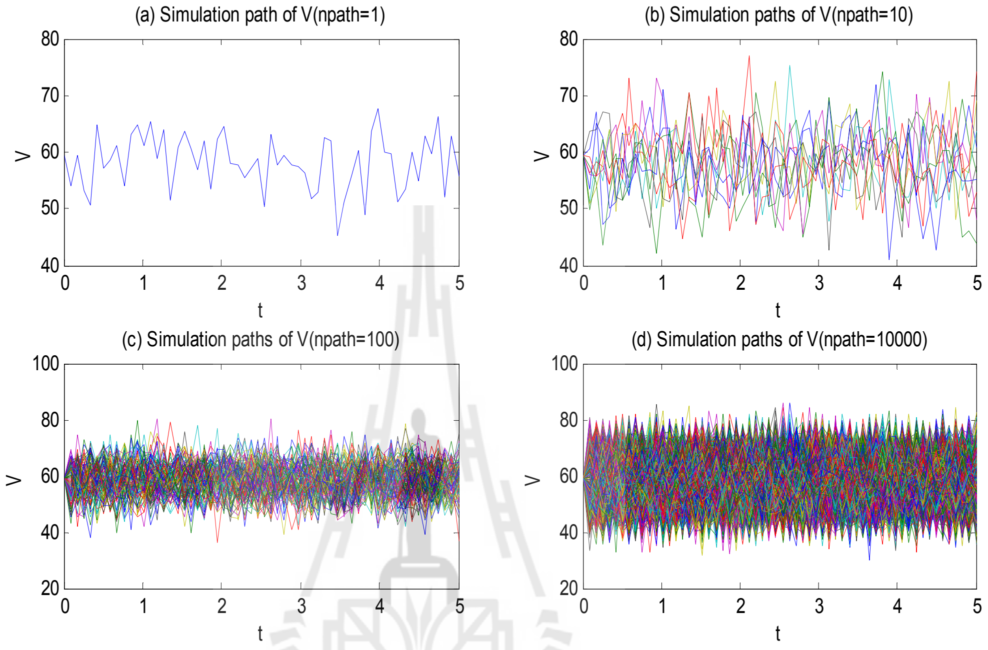

Figure 4.5 Simulation paths of annual revenue.

Figure 4.5 shows four simulation results with different simulated times. Figure 4.5(a), 4.5(b), 4.5(c), and 4.5(d) show the simulation paths which were generated one time, 10 times, 100 times, and 10000 times respectively.

Following with the steps described in section 4.4, using the parameters shown in Table 4.2, and inputting the simulation step number nstep=60 and times $m = 10000$, we call the MATLAB code to obtain the value of option. Table 4.3 shows the values of option F when we change the investment time T.

Table 4.3 Option values of various time.

V0	I	σ	r	q	μ	T	F
59.3234	49.7785	46.539	0.05	30.722	58.531	5	6.8289
59.3234	49.7785	46.539	0.05	30.722	58.531	4	7.1715
59.3234	49.7785	46.539	0.05	30.722	58.531	3	7.5500
59.3234	49.7785	46.539	0.05	30.722	58.531	2	7.9490
59.3234	49.7785	46.539	0.05	30.722	58.531	1	8.3448

One can see from the Table 4.3 that the investment opportunity value (or option value F) will decrease as time goes on. If the investor invests in the WPP in the first year, this investment opportunity value is worth 8.34 million Yuan, but if he (or she) invests in the last year of development right, the opportunity value is worth only 6.83 million Yuan.

4.5.4 Critical Value

As we know, if the real option (or investment opportunity) value F is positive, then it is worthwhile to invest. If it is negative the project should be abandoned. If F equals to zero then the investment opportunity is almost worthless, the investor may not accept the project. Now we need to calculate the critical value to help the investor make a decision. In order to reach this aim, we consider the effect of the crucial parameters on the project value.

Firstly, we consider the effect of cost on the option value. By fixing the other parameters and changing the cost value, we obtain the option values shown in Figure 4.6 and the simulated calculation results will be shown in Appendix B (B-1). From the simulated results, the WPP investment opportunity value F will equal zero when the critical cost is $I^* = 61$ million Yuan. Thus if the annual average cost including static investment cost and O&M cost is higher than 61 million Yuan, the WPP is worthless to invest.

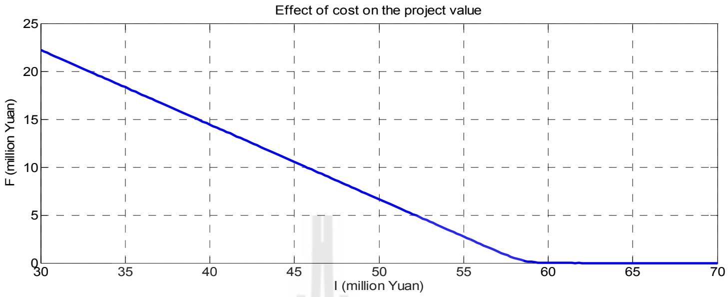

Figure 4.6 Effect of cost on the WPP investment opportunity value F.

Next, we fix the others parameters and change only the parameter μ. We shall consider the effect of long-run mean μ on the option values. After inputting various values of $\mu = \frac{p}{q}$ into (4.2) and (4.3), the simulated path results of F are shown in the Figure 4.7 while the numerical results table will be shown in Appendix B (B-2). According to the calculated results, with the increase of long-run mean of the revenue, the WPP value increases gradually. From the numerical results table in Appendix B (B-2), the WPP value will equal zero when the long-run mean of the revenue goes to 47 million Yuan.

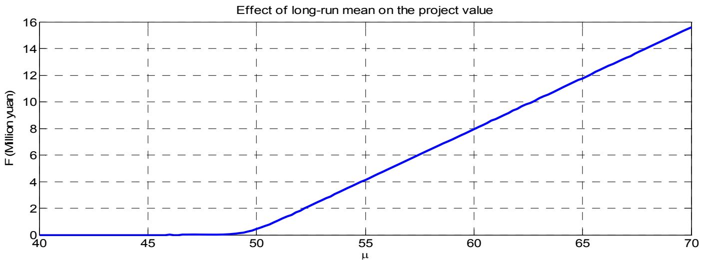

Figure 4.7 Effect of long-run mean on the WPP value.

Now, we move to consider the other two parameters of the O-U process, volatility and speed of reversion.

Similarly as described above, we fix the remaining other parameters and only change volatility σ within the interval [0,70]. We get the simulated path of F shown in Figure 4.8 while the numerical results table will be shown in Appendix B (B-3). Although the volatility σ changes greatly, the WPP value F does not change by much. The variation range of F is between about 6.8 million Yuan and 6.85 million Yuan. This means that the volatility of the O-U process influences the project value but the effect was not very significant.

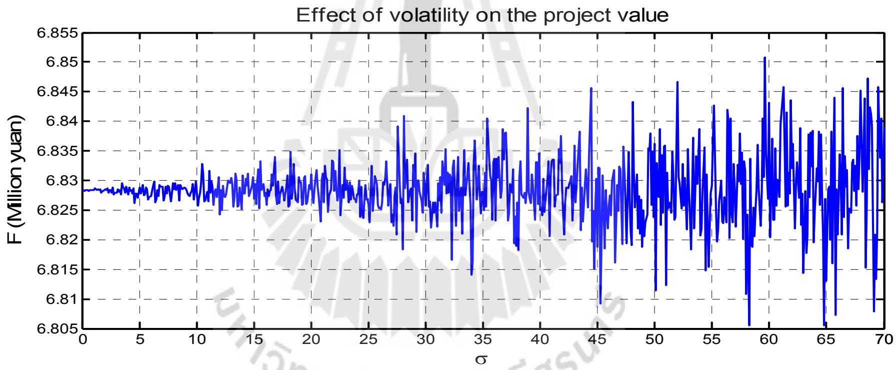

Figure 4.8 Effect of volatility on the WPP value.

Now, we fix the others parameters and only change reversion speed q within the variation range (0, 32]. We can get the simulated path of F shown in Figure 4.9, while the table of numerical results will be shown in Appendix B (B-4). From Figure 4.9, the simulated results indicate that the WPP value F becomes smaller with increasing reverting speed q, and the smaller reverting speed is, the larger the project value will be. Numerical results of the simulation show that the project value seems to be stabilizing after q reaches a certain value. In this simulated calculation, the WPP value will be almost stable at 6.8 million Yuan when $q \geq 5$.

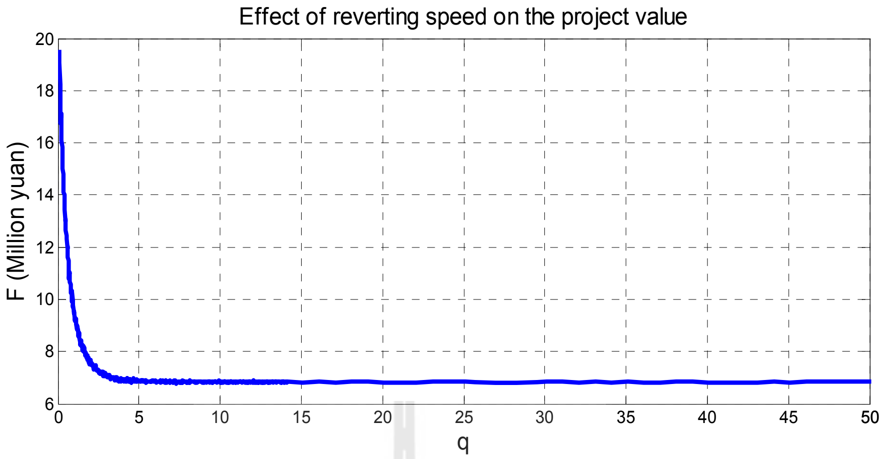

Figure 4.9 Effect of reverting speed on WPP value.

4.6 Scenario Analysis under O-U Process

By considering the uncertainties of the future policy and of wind power consumption in China, the scenario analysis in this study will focus on the effects of feed-in-tariff and abandoned wind rate (AWR) on the option value. On the other hand, with the gradual establishment of the China carbon market trading system, investors are faced with great opportunities. The price of CERs will also become one of the important factors affecting the profit of WPP, so the scenario analyses also focus on the CERs price.

4.6.1 Case 1: Vary P_f

Firstly, we shall consider the case of the OGE price change. Suppose that P_f increases 5% from $P_f = 0.436$ to $P_f = 0.654$ Yuan/Kwh; then the option value will increase from $F = 0.001$ to $F = 14.919$ million Yuan as shown in Table 4.4. This means for a WPP investor that if his (or her) expected return is over 6.827 million Yuan, he(or she) can invest when the OGE price $P_f \geq 0.545$ Yuan/Kwh, otherwise he may give up to invest.

Table 4.4 Real option values under various OGE price.

Pf	V0	I	σ	q	μ	F
0.436	59.323	49.779	38.117	30.722	47.940	0.001
0.463	59.323	49.779	40.223	30.722	50.588	0.780
0.491	59.323	49.779	42.328	30.722	53.236	2.772
0.518	59.323	49.779	44.434	30.722	55.884	4.798
0.545	59.323	49.779	46.539	30.722	58.532	6.827
0.572	59.323	49.779	48.644	30.722	61.180	8.844
0.600	59.323	49.779	50.750	30.722	63.828	10.889
0.627	59.323	49.779	52.855	30.722	66.476	12.913
0.654	59.323	49.779	54.961	30.722	69.124	14.919

4.6.2 Case 2: Vary AWR

Secondly, we shall consider the case of the wind abandoned rate change. Suppose the current AWR is r=0, then we change it to r=10%, that means the output electricity of the wind farm is not all transported onto the grid which is about 10% of the wind farm output will be abandoned. Thus, in order to obverse the effect of AWR, we increase r from 5% to 20%. In this case we shall suppose that the current level is r=0, but in fact, it may be a non-zero level in reality. If the causes of abandoned wind power have been addressed, for example, the transmission network construction has been improved, national policy has been changed, or market demand levels have increased, then the current AWR may decrease. Thus we also consider AWR level improving from -5% to -20% on the other side. Table 4.5 shows the calculated results of WPP value F by using various AWRs.

Table 4.5 Real option values under various AWR levels.

AWR level	σ	μ	F
-20%	55.847	70.238	15.776
-15%	53.520	67.312	13.538
-10%	51.193	64.385	11.295
-5%	48.866	61.459	9.067
Current level	46.539	58.532	6.820
5%	44.212	55.605	4.583
10%	41.885	52.679	2.351
15%	39.558	49.752	0.282
20%	37.231	46.826	0.000

Table 4.5 shows simulated results of project value with different AWR. From the table, the project value will increase with improved AWR. With the current level of AWR, i.e. r=0, the project value is 6.82 million Yuan. In this simulated calculation, the project value will reach to 15.78 million Yuan after AWR has decreased by about 20% from the current level. For a WPP investor, if his (or her) expected return is more than 6.827 million Yuan, he (or she) should pay attention to the changes of AWR according the government report and compare with the current level of AWR. If the level of r decreases by 5% from the current level, the expected value will increase from F=6.82 to F=9.067 million Yuan and it is worth to invest.

4.6.3 Case 3: Various CERs Price

Finally, we consider the effect of CERs price on the project value, by a calculation process similar to the above two cases. The simulated calculation results are shown in Table 4.6. According to the results, When Pe is reduced by 5 to 20 percent on one hand and grows by 5 to 20 percent on the other hand, although the CERs price changes greatly, the project value changes not by much, it just changes from 5.99 million Yuan to 7.68 million Yuan.

Table 4.6 Real option values under various CERs prices.

Yuan/tco2	σ	μ	F
44.8	45.653	57.418	5.992
47.6	45.874	57.696	6.186
50.4	46.096	57.975	6.408
53.2	46.317	58.253	6.610
56.0	46.539	58.532	6.817
58.8	46.760	58.811	7.036
61.6	46.982	59.089	7.251
64.4	47.204	59.368	7.465
67.2	47.425	59.646	7.684

The simulated calculation results of the three factors affecting the project value are visually presented by the graphs in Figure 4.10:

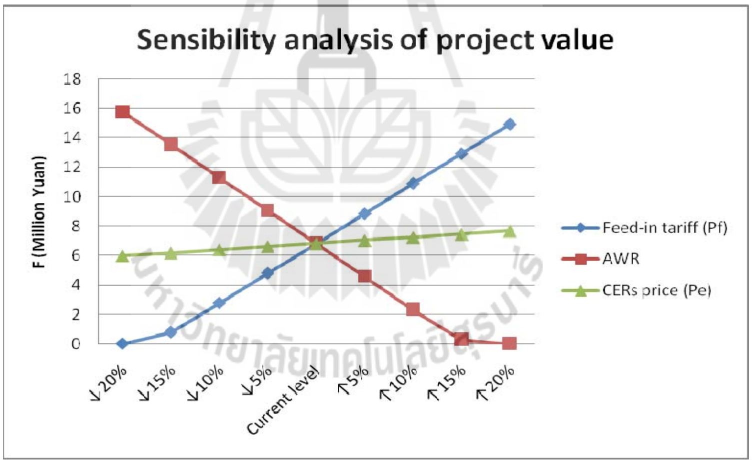

Figure 4.10 Sensibility analysis of project value.

CHAPTER V

CONCLUSION

This thesis focuses on the real options applications in the renewable energy markets, especially on the wind power investment in China's investment environment. The real options method has been used for the valuation of WPP and the decision making in operation and investment. This section provides a summary of the main findings throughout the thesis.

To carry out any real options analysis on a power-related asset or an investment opportunity, the electricity price modeling is the starting point. But in view of the particularity of the investment environment in China, where the electricity price of WP is modified seldomly, we change the starting point to the generation power of the wind farm.

Two ROA frameworks are presented in this study, one is shown in chapter III. In this chapter, we assume that the electricity production of the wind farm follows a GBM process, and use the binomial method with dynamic programming to price the WPP's investment value. It is a ROA underlying discrete time. In chapter IV, considering that the electricity production of the wind farm tends to exhibit mean reversion, we assume that the electricity production of the wind power follows a O-U process, use contingent claim analysis to obtain the project value equation, and obtain the project value through Monte Carlo simulation calculation to get the project value. After the modeling process, we carry out an empirical analysis with the data of a realistic WPP named No.0689 project in CDM database.

One can see from these two frameworks that real options analysis can predict a dynamic series of future decisions, they can let an investor or managing person have a lot of flexibility in acting and can adjust to changes taking place in the economy. On the other hand, comparing the two frameworks shown in the thesis, we find there is little difference in project value between the two frameworks. However if we pay attention to the change of the option value over time, we find that the project value in the first framework shown in chapter III varies greatly, while the project value in the second framework shown in chapter IV varies not so much. The major cause of this is originates from the difference of the stochastic processes used in the two frameworks. At last, from the results of scenario analysis in Chapter III and Chapter IV, the calculated results of those two frameworks are similar, the major influence factors to a WPP value are OGE price and AWR, while the effect of the CERs price on the WPP value is comparatively small.

In the end, we should note, there are also some limitations in this thesis, these limitations as follows:

(i) Because of the non-availability of data, some data in the model are estimated or generated by simulation, and thus effect the accuracy of the model.

(ii) The model only considers the primary factors relevant to the wind energy project and economic evaluation. In the real world, a WPP faces more uncertainties, such as investment cost, tax, policy, technology, etc.

(iii) The options considered in the thesis are simplistic, in the reality, usually investment projects are composed of a set of a large number of related options.

Because of the great uncertainty of the development of renewable energy, the investment projects of renewable energy have increased complexity and uncertainty. So the ROA frameworks employed in this study need a lot of additional works to be improved in the future.

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APPENDICES

APPENDIX A

MATLAB Code

The MATLAB codes used in the Chapter IV is shown below:

```
%%%%%%%%%%
  function [ mu, sigma, q ] = maxlikehood(V, deltat)
  n = length(V)-1;
Vx = sum( V(1:end-1) );
Vy = sum( V(2:end) );
Vxx = sum( V(1:end-1).^2 );
Vxy = sum( V(1:end-1).*V(2:end) );
Vyy = sum( V(2:end).^2 );
  mu = (Vy*Vxx - Vx*Vxy) / ( n*(Vxx - Vxy) - (Vx^2 - Vx*Vy) );
  q = -(1/deltat)*log((Vxy - mu*Vx - mu*Vy + n*mu^2) / (Vxx -2*mu*Vx + n*mu^2));
  a1 = exp(- q*deltat);
  a2 = exp(-2*q*deltat);
  sigmah2 = (1/n)*(Vyy - 2*a1*Vxy + a2*Vxx - ...
  2*mu*(1-a1)*(Vy - a1*Vx) + n*mu^2*(1-a1)^2)
```

```
sigma = sqrt(sigmah2*2*q/(1-a2))

end



%%%%%%%%%%%%%%%%%%%%%%%%

x=xlsread('mlh.xls');

[m,n]=size(x);

mu=zeros(n,1);

s=zeros(n,1);

q=zeros(n,1);

for i=1:n

[mu(i,1),s(i,1),q(i,1)]=maxlikehood(x(:,i),1/12);

end

a=[mu s q];

%dlmwrite('mlh.txt', a, 'newline', 'pc');

xlswrite('output_mlh.xls',a);

%xlswrite('output.xls',a);



%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%% function to calculate option value by using simulation%%



function[a,F,sig]=simov(V0,I,sigma,r,q,mu,T,nstep,npath)
```

```
rm=randn(npath,nstep-1);
dt=T/(nstep-1);
V=[V0*ones(npath,1),zeros(npath,nstep-1)];
dZt=sqrt(1-exp(-2*q*dt))*rm/sqrt(2*q);% Calculate the random term
for i=1:(nstep-1)
V(:,i+1)=V(:,i)*exp(-q*dt)+mu*(1-exp(-q*dt))+sigma*dZt(:,i);
end
a=V;
F1=max(mean(V(:,1:end),2)-I,0);
F=exp(-r*T)*mean(F1);
sig=std(F1)/sqrt(npath);
end
```

```
%%%%%%%%%operate function simov to calculate option value %%%%%
%%%% function[a,F,sig]=simov(V0,I,sigma,r,q,mu,t,nstep,npath)%%%%
%%%%%%%%%%% giving the simulation fig of V %%%%%%%%%%%%
[a,F1,s]=simov(59.3234,49.7785,46.539,0.05,30.722,58.531,5,60,1);
tvals=(0:5/59:5);
subplot(2,2,1);
plot(tvals,a);
%set(gca,'xtick',0:0.5:5)
%grid on
title('Simulation path of V(npath=1)');
xlabel('t');
ylabel('V');


[a,F1,s]=simov(59.3234,49.7785,46.539,0.05,30.722,58.531,5,60,10);
tvals=(0:5/59:5);
subplot(2,2,2);
plot(tvals,a);
%set(gca,'xtick',0:0.5:5)
%grid on
title('Simulation path of V(npath=10)');
xlabel('t');
```

```
ylabel('V');


[a,F1,s]=simov(59.3234,49.7785,46.539,0.05,30.722,58.531,5,60,100);
tvals=(0:5/59:5);
subplot(2,2,3);
plot(tvals,a);
%set(gca,'xtick',0:0.5:5)
%grid on
title('Simulation path of V(npath=100)');
xlabel('t');
ylabel('V');


[a,F1,s]=simov(59.3234,49.7785,46.539,0.05,30.722,58.531,5,60,10000);
tvals=(0:5/59:5);
subplot(2,2,4);
plot(tvals,a);
%set(gca,'xtick',0:0.5:5)
%grid on
title('Simulation path of V(npath=10000)');
xlabel('t');
ylabel('V');
```

```
%%%%%%%%%%%%%%%%%

%%%%%%%%%% creating a option values table by using various parameter%%%

x=xlsread('data.xls');

[m,n]=size(x);

p=zeros(m,1);

for i=1:m

[b,p(i,1),s]=simov(x(i,1),x(i,2),x(i,3),x(i,4),x(i,5),x(i,6),x(i,7),x(i,8),x(i,9));

end

a=[x p];

plot(a(:,3),a(:,10));%%%%%%%%%parameter c3 and option F c10%%%%

plot(a(:,3),a(:,10),'LineWidth',3);

grid on

title('Effect of volatility on the project value');

xlabel('Volatility');

ylabel('F (Million Yuan)');

xlswrite('output.xls',a);


%%%%%%%%%%%%%%%%%%%%%%


%%%%%This code is used to simulate paths of annual on-grid electricity%%%
```

```
a=xlsread('Book1.xlsx');
npath=input('Input simulation times：');%%% Input simulated times
[m,n]=size(a);
p=zeros(m,2);
for i=1:m
    p(i,1)=mean(a(i,:),2);
    p(i,2)=var(a(i,:),0,2);
end
x=zeros(m,npath);
for i=1:m
x(i,:)= p(i,1) + sqrt(p(i,2))* randn(1,npath);
end
b=sum(x);
tvals=(1:npath);
plot(tvals,b);
%grid on
title('Simulation of annual on-grid electricity');
xlabel('times');
ylabel('Ge');
xlswrite('output_ele.xls',b);
```

APPENDIX B

DETAILED RESULTS OF THE CALCULATIONS OF CHAPTER IV

The tables below show some of the results of calculations performed in Chapter IV.

B-1

Effect of cost on option value

V0	I	sigma	r	q	mu	t	nstep	npath	F
59.3234	36.7785	46.539	0.05	30.722	58.531	5	60	10000	16.9461
59.3234	37.7785	46.539	0.05	30.722	58.531	5	60	10000	16.1701
59.3234	38.7785	46.539	0.05	30.722	58.531	5	60	10000	15.3958
59.3234	39.7785	46.539	0.05	30.722	58.531	5	60	10000	14.6062
59.3234	40.7785	46.539	0.05	30.722	58.531	5	60	10000	13.8364
59.3234	41.7785	46.539	0.05	30.722	58.531	5	60	10000	13.0660
59.3234	42.7785	46.539	0.05	30.722	58.531	5	60	10000	12.2691
59.3234	43.7785	46.539	0.05	30.722	58.531	5	60	10000	11.5012
59.3234	44.7785	46.539	0.05	30.722	58.531	5	60	10000	10.7143
59.3234	45.7785	46.539	0.05	30.722	58.531	5	60	10000	9.9432
59.3234	46.7785	46.539	0.05	30.722	58.531	5	60	10000	9.1672
59.3234	47.7785	46.539	0.05	30.722	58.531	5	60	10000	8.3744
59.3234	48.7785	46.539	0.05	30.722	58.531	5	60	10000	7.5994
59.3234	49.7785	46.539	0.05	30.722	58.531	5	60	10000	6.8261
59.3234	50.7785	46.539	0.05	30.722	58.531	5	60	10000	6.0481
59.3234	51.7785	46.539	0.05	30.722	58.531	5	60	10000	5.2674
59.3234	52.7785	46.539	0.05	30.722	58.531	5	60	10000	4.4924
59.3234	53.7785	46.539	0.05	30.722	58.531	5	60	10000	3.7096
59.3234	54.7785	46.539	0.05	30.722	58.531	5	60	10000	2.9315
59.3234	55.7785	46.539	0.05	30.722	58.531	5	60	10000	2.1544
59.3234	56.7785	46.539	0.05	30.722	58.531	5	60	10000	1.3772
59.3234	57.7785	46.539	0.05	30.722	58.531	5	60	10000	0.6635
59.3234	58.7785	46.539	0.05	30.722	58.531	5	60	10000	0.1752
59.3234	59.7785	46.539	0.05	30.722	58.531	5	60	10000	0.0179
59.3234	60.7785	46.539	0.05	30.722	58.531	5	60	10000	0.0004

B-1 (Continued)

59.3234	61.7785	46.539	0.05	30.722	58.531	5	60	10000	0
59.3234	62.7785	46.539	0.05	30.722	58.531	5	60	10000	0
59.3234	63.7785	46.539	0.05	30.722	58.531	5	60	10000	0
59.3234	64.7785	46.539	0.05	30.722	58.531	5	60	10000	0
59.3234	65.7785	46.539	0.05	30.722	58.531	5	60	10000	0
59.3234	66.7785	46.539	0.05	30.722	58.531	5	60	10000	0
59.3234	67.7785	46.539	0.05	30.722	58.531	5	60	10000	0
59.3234	68.7785	46.539	0.05	30.722	58.531	5	60	10000	0
59.3234	69.7785	46.539	0.05	30.722	58.531	5	60	10000	0
59.3234	65.7785	46.539	0.05	30.722	58.531	5	60	10000	0
59.3234	66.7785	46.539	0.05	30.722	58.531	5	60	10000	0
59.3234	67.7785	46.539	0.05	30.722	58.531	5	60	10000	0
59.3234	68.7785	46.539	0.05	30.722	58.531	5	60	10000	0
59.3234	69.7785	46.539	0.05	30.722	58.531	5	60	10000	0

B-2

Effect of long-run mean on option value

V0	I	sigma	r	q	mu	t	nstep	npath	F
59.323	49.779	46.539	0.05	30.723	40	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	40.2	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	40.4	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	40.6	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	40.8	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	41	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	41.2	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	41.4	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	41.6	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	41.8	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	42	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	42.2	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	42.4	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	42.6	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	42.8	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	43	5	60	10000	0

B-2 (Continued)

59.323	49.779	46.539	0.05	30.723	43.2	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	43.4	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	43.6	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	43.8	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	44	5	60	10000	0
59.323	49.779	46.539	0.05	30.723	44.2	5	60	10000	0.0000
59.323	49.779	46.539	0.05	30.723	44.4	5	60	10000	0.0000
59.323	49.779	46.539	0.05	30.723	44.6	5	60	10000	0.0000
59.323	49.779	46.539	0.05	30.723	44.8	5	60	10000	0.0000
59.323	49.779	46.539	0.05	30.723	45	5	60	10000	0.0000
59.323	49.779	46.539	0.05	30.723	45.2	5	60	10000	0.0000
59.323	49.779	46.539	0.05	30.723	45.4	5	60	10000	0.0000
59.323	49.779	46.539	0.05	30.723	45.6	5	60	10000	0.0000
59.323	49.779	46.539	0.05	30.723	45.8	5	60	10000	0.0000
59.323	49.779	46.539	0.05	30.723	46	5	60	10000	0.0000
59.323	49.779	46.539	0.05	30.723	46.2	5	60	10000	0.0000
59.323	49.779	46.539	0.05	30.723	46.4	5	60	10000	0.0000
59.323	49.779	46.539	0.05	30.723	46.6	5	60	10000	0.0000
59.323	49.779	46.539	0.05	30.723	46.8	5	60	10000	0.0001
59.323	49.779	46.539	0.05	30.723	47	5	60	10000	0.0002
59.323	49.779	46.539	0.05	30.723	47.2	5	60	10000	0.0003
59.323	49.779	46.539	0.05	30.723	47.4	5	60	10000	0.0007
59.323	49.779	46.539	0.05	30.723	47.6	5	60	10000	0.0015
59.323	49.779	46.539	0.05	30.723	47.8	5	60	10000	0.0034
59.323	49.779	46.539	0.05	30.723	48	5	60	10000	0.0054
59.323	49.779	46.539	0.05	30.723	48.2	5	60	10000	0.0128
59.323	49.779	46.539	0.05	30.723	48.4	5	60	10000	0.0208
59.323	49.779	46.539	0.05	30.723	48.6	5	60	10000	0.0374
59.323	49.779	46.539	0.05	30.723	48.8	5	60	10000	0.0590
59.323	49.779	46.539	0.05	30.723	49	5	60	10000	0.0885
59.323	49.779	46.539	0.05	30.723	49.2	5	60	10000	0.1288
59.323	49.779	46.539	0.05	30.723	49.4	5	60	10000	0.1829
59.323	49.779	46.539	0.05	30.723	49.6	5	60	10000	0.2545
59.323	49.779	46.539	0.05	30.723	49.8	5	60	10000	0.3279
59.323	49.779	46.539	0.05	30.723	50	5	60	10000	0.4384
59.323	49.779	46.539	0.05	30.723	50.2	5	60	10000	0.5446
59.323	49.779	46.539	0.05	30.723	50.4	5	60	10000	0.6652

B-2 (Continued)

59.323	49.779	46.539	0.05	30.723	50.6	5	60	10000	0.7975
59.323	49.779	46.539	0.05	30.723	50.8	5	60	10000	0.9315
59.323	49.779	46.539	0.05	30.723	51.4	5	60	10000	1.3763
59.323	49.779	46.539	0.05	30.723	51.6	5	60	10000	1.5217
59.323	49.779	46.539	0.05	30.723	51.8	5	60	10000	1.6811
59.323	49.779	46.539	0.05	30.723	52	5	60	10000	1.8221
59.323	49.779	46.539	0.05	30.723	52.2	5	60	10000	1.9864
59.323	49.779	46.539	0.05	30.723	52.4	5	60	10000	2.1360
59.323	49.779	46.539	0.05	30.723	52.6	5	60	10000	2.2975
59.323	49.779	46.539	0.05	30.723	52.8	5	60	10000	2.4475
59.323	49.779	46.539	0.05	30.723	53	5	60	10000	2.6024
59.323	49.779	46.539	0.05	30.723	53.2	5	60	10000	2.7561
59.323	49.779	46.539	0.05	30.723	53.4	5	60	10000	2.8922
59.323	49.779	46.539	0.05	30.723	53.6	5	60	10000	3.0479
59.323	49.779	46.539	0.05	30.723	53.8	5	60	10000	3.2056
59.323	49.779	46.539	0.05	30.723	54	5	60	10000	3.3718
59.323	49.779	46.539	0.05	30.723	54.2	5	60	10000	3.5109
59.323	49.779	46.539	0.05	30.723	54.4	5	60	10000	3.6671
59.323	49.779	46.539	0.05	30.723	54.6	5	60	10000	3.8138
59.323	49.779	46.539	0.05	30.723	54.8	5	60	10000	3.9739
59.323	49.779	46.539	0.05	30.723	55	5	60	10000	4.1358
59.323	49.779	46.539	0.05	30.723	55.2	5	60	10000	4.2873
59.323	49.779	46.539	0.05	30.723	55.4	5	60	10000	4.4377
59.323	49.779	46.539	0.05	30.723	55.6	5	60	10000	4.5886
59.323	49.779	46.539	0.05	30.723	55.8	5	60	10000	4.7387
59.323	49.779	46.539	0.05	30.723	56	5	60	10000	4.8862
59.323	49.779	46.539	0.05	30.723	56.2	5	60	10000	5.0330
59.323	49.779	46.539	0.05	30.723	56.4	5	60	10000	5.1949
59.323	49.779	46.539	0.05	30.723	56.6	5	60	10000	5.3499
59.323	49.779	46.539	0.05	30.723	56.8	5	60	10000	5.5032
59.323	49.779	46.539	0.05	30.723	57	5	60	10000	5.6535
59.323	49.779	46.539	0.05	30.723	57.2	5	60	10000	5.8173
59.323	49.779	46.539	0.05	30.723	57.4	5	60	10000	5.9657
59.323	49.779	46.539	0.05	30.723	57.6	5	60	10000	6.1068
59.323	49.779	46.539	0.05	30.723	57.8	5	60	10000	6.2732
59.323	49.779	46.539	0.05	30.723	58	5	60	10000	6.4259
59.323	49.779	46.539	0.05	30.723	58.2	5	60	10000	6.5720
59.323	49.779	46.539	0.05	30.723	58.4	5	60	10000	6.7302

B-2 (Continued)

59.323	49.779	46.539	0.05	30.723	58.6	5	60	10000	6.8829
59.323	49.779	46.539	0.05	30.723	58.8	5	60	10000	7.0396
59.323	49.779	46.539	0.05	30.723	59	5	60	10000	7.1888
59.323	49.779	46.539	0.05	30.723	59.2	5	60	10000	7.3442
59.323	49.779	46.539	0.05	30.723	59.4	5	60	10000	7.4837
59.323	49.779	46.539	0.05	30.723	59.6	5	60	10000	7.6423
59.323	49.779	46.539	0.05	30.723	59.8	5	60	10000	7.7987
59.323	49.779	46.539	0.05	30.723	60	5	60	10000	7.9541
59.323	49.779	46.539	0.05	30.723	60.2	5	60	10000	8.1025
59.323	49.779	46.539	0.05	30.723	60.4	5	60	10000	8.2608
59.323	49.779	46.539	0.05	30.723	60.6	5	60	10000	8.4099
59.323	49.779	46.539	0.05	30.723	60.8	5	60	10000	8.5734
59.323	49.779	46.539	0.05	30.723	61	5	60	10000	8.7111
59.323	49.779	46.539	0.05	30.723	61.2	5	60	10000	8.8746
59.323	49.779	46.539	0.05	30.723	61.4	5	60	10000	9.0207
59.323	49.779	46.539	0.05	30.723	61.6	5	60	10000	9.1715
59.323	49.779	46.539	0.05	30.723	61.8	5	60	10000	9.3419
59.323	49.779	46.539	0.05	30.723	62	5	60	10000	9.4815
59.323	49.779	46.539	0.05	30.723	62.2	5	60	10000	9.6474
59.323	49.779	46.539	0.05	30.723	62.4	5	60	10000	9.7959
59.323	49.779	46.539	0.05	30.723	62.6	5	60	10000	9.9337
59.323	49.779	46.539	0.05	30.723	62.8	5	60	10000	10.0857
59.323	49.779	46.539	0.05	30.723	63	5	60	10000	10.2485
59.323	49.779	46.539	0.05	30.723	63.2	5	60	10000	10.4027
59.323	49.779	46.539	0.05	30.723	63.4	5	60	10000	10.5492
59.323	49.779	46.539	0.05	30.723	63.6	5	60	10000	10.7016
59.323	49.779	46.539	0.05	30.723	63.8	5	60	10000	10.8608
59.323	49.779	46.539	0.05	30.723	64	5	60	10000	11.0072
59.323	49.779	46.539	0.05	30.723	64.2	5	60	10000	11.1625
59.323	49.779	46.539	0.05	30.723	64.4	5	60	10000	11.3186
59.323	49.779	46.539	0.05	30.723	64.6	5	60	10000	11.4773
59.323	49.779	46.539	0.05	30.723	64.8	5	60	10000	11.6217
59.323	49.779	46.539	0.05	30.723	65	5	60	10000	11.7699
59.323	49.779	46.539	0.05	30.723	65.2	5	60	10000	11.9189
59.323	49.779	46.539	0.05	30.723	65.4	5	60	10000	12.0724
59.323	49.779	46.539	0.05	30.723	65.6	5	60	10000	12.2313
59.323	49.779	46.539	0.05	30.723	65.8	5	60	10000	12.3839
59.323	49.779	46.539	0.05	30.723	66	5	60	10000	12.5393

B-2 (Continued)

59.323	49.779	46.539	0.05	30.723	66.2	5	60	10000	12.6903
59.323	49.779	46.539	0.05	30.723	66.4	5	60	10000	12.8477
59.323	49.779	46.539	0.05	30.723	66.6	5	60	10000	12.9974
59.323	49.779	46.539	0.05	30.723	66.8	5	60	10000	13.1498
59.323	49.779	46.539	0.05	30.723	67	5	60	10000	13.3172
59.323	49.779	46.539	0.05	30.723	67.2	5	60	10000	13.4465
59.323	49.779	46.539	0.05	30.723	67.4	5	60	10000	13.6148
59.323	49.779	46.539	0.05	30.723	67.6	5	60	10000	13.7690
59.323	49.779	46.539	0.05	30.723	67.8	5	60	10000	13.9174
59.323	49.779	46.539	0.05	30.723	68	5	60	10000	14.0687
59.323	49.779	46.539	0.05	30.723	68.2	5	60	10000	14.2219
59.323	49.779	46.539	0.05	30.723	68.4	5	60	10000	14.3730
59.323	49.779	46.539	0.05	30.723	68.6	5	60	10000	14.5378
59.323	49.779	46.539	0.05	30.723	68.8	5	60	10000	14.6909
59.323	49.779	46.539	0.05	30.723	69	5	60	10000	14.8372
59.323	49.779	46.539	0.05	30.723	69.2	5	60	10000	14.9866
59.323	49.779	46.539	0.05	30.723	69.4	5	60	10000	15.1430
59.323	49.779	46.539	0.05	30.723	69.6	5	60	10000	15.2947
59.323	49.779	46.539	0.05	30.723	69.8	5	60	10000	15.4549
59.323	49.779	46.539	0.05	30.723	70	5	60	10000	15.5994

B-3

Effect of the volatility on the options value									
V0	I	sigma	r	q	mu	t	nstep	npath	F
59.3234	49.7785	0	0.05	30.723	58.532	5	60	10000	6.82831
59.3234	49.7785	0.1	0.05	30.723	58.532	5	60	10000	6.82829
59.3234	49.7785	0.2	0.05	30.723	58.532	5	60	10000	6.82834
59.3234	49.7785	0.3	0.05	30.723	58.532	5	60	10000	6.82837
59.3234	49.7785	0.4	0.05	30.723	58.532	5	60	10000	6.82827
59.3234	49.7785	0.5	0.05	30.723	58.532	5	60	10000	6.82830
59.3234	49.7785	0.6	0.05	30.723	58.532	5	60	10000	6.82830
59.3234	49.7785	0.7	0.05	30.723	58.532	5	60	10000	6.82813
59.3234	49.7785	0.8	0.05	30.723	58.532	5	60	10000	6.82841
59.3234	49.7785	0.9	0.05	30.723	58.532	5	60	10000	6.82842
59.3234	49.7785	1	0.05	30.723	58.532	5	60	10000	6.82817

B-3 (Continued)

59.3234	49.7785	1.1	0.05	30.723	58.532	5	60	10000	6.82832
59.3234	49.7785	1.2	0.05	30.723	58.532	5	60	10000	6.82832
59.3234	49.7785	1.3	0.05	30.723	58.532	5	60	10000	6.82815
59.3234	49.7785	1.4	0.05	30.723	58.532	5	60	10000	6.82829
59.3234	49.7785	1.5	0.05	30.723	58.532	5	60	10000	6.82849
59.3234	49.7785	1.6	0.05	30.723	58.532	5	60	10000	6.82861
59.3234	49.7785	1.7	0.05	30.723	58.532	5	60	10000	6.82835
59.3234	49.7785	1.8	0.05	30.723	58.532	5	60	10000	6.82871
59.3234	49.7785	1.9	0.05	30.723	58.532	5	60	10000	6.82814
59.3234	49.7785	2	0.05	30.723	58.532	5	60	10000	6.82878
59.3234	49.7785	2.1	0.05	30.723	58.532	5	60	10000	6.82811
59.3234	49.7785	2.2	0.05	30.723	58.532	5	60	10000	6.82834
59.3234	49.7785	2.3	0.05	30.723	58.532	5	60	10000	6.82862
59.3234	49.7785	2.4	0.05	30.723	58.532	5	60	10000	6.82800
59.3234	49.7785	2.5	0.05	30.723	58.532	5	60	10000	6.82789
59.3234	49.7785	2.6	0.05	30.723	58.532	5	60	10000	6.82836
59.3234	49.7785	2.7	0.05	30.723	58.532	5	60	10000	6.82780
59.3234	49.7785	2.8	0.05	30.723	58.532	5	60	10000	6.82855
59.3234	49.7785	2.9	0.05	30.723	58.532	5	60	10000	6.82857
59.3234	49.7785	3	0.05	30.723	58.532	5	60	10000	6.82809
59.3234	49.7785	3.1	0.05	30.723	58.532	5	60	10000	6.82878
59.3234	49.7785	3.2	0.05	30.723	58.532	5	60	10000	6.82753
59.3234	49.7785	3.3	0.05	30.723	58.532	5	60	10000	6.82780
59.3234	49.7785	3.4	0.05	30.723	58.532	5	60	10000	6.82827
59.3234	49.7785	3.5	0.05	30.723	58.532	5	60	10000	6.82880
59.3234	49.7785	3.6	0.05	30.723	58.532	5	60	10000	6.82832
59.3234	49.7785	3.7	0.05	30.723	58.532	5	60	10000	6.82826
59.3234	49.7785	3.8	0.05	30.723	58.532	5	60	10000	6.82795
59.3234	49.7785	3.9	0.05	30.723	58.532	5	60	10000	6.82928
59.3234	49.7785	4	0.05	30.723	58.532	5	60	10000	6.82874
59.3234	49.7785	4.1	0.05	30.723	58.532	5	60	10000	6.82794
59.3234	49.7785	4.2	0.05	30.723	58.532	5	60	10000	6.82856
59.3234	49.7785	4.3	0.05	30.723	58.532	5	60	10000	6.82810
59.3234	49.7785	4.4	0.05	30.723	58.532	5	60	10000	6.82936
59.3234	49.7785	4.5	0.05	30.723	58.532	5	60	10000	6.82896
59.3234	49.7785	4.6	0.05	30.723	58.532	5	60	10000	6.82786
59.3234	49.7785	4.7	0.05	30.723	58.532	5	60	10000	6.82977
59.3234	49.7785	4.8	0.05	30.723	58.532	5	60	10000	6.82719

B-3 (Continued)

59.3234	49.7785	4.9	0.05	30.723	58.532	5	60	10000	6.82786
59.3234	49.7785	5	0.05	30.723	58.532	5	60	10000	6.82903
59.3234	49.7785	5.1	0.05	30.723	58.532	5	60	10000	6.82828
59.3234	49.7785	5.2	0.05	30.723	58.532	5	60	10000	6.82911
59.3234	49.7785	5.3	0.05	30.723	58.532	5	60	10000	6.82731
59.3234	49.7785	5.4	0.05	30.723	58.532	5	60	10000	6.82742
59.3234	49.7785	5.5	0.05	30.723	58.532	5	60	10000	6.82807
59.3234	49.7785	5.6	0.05	30.723	58.532	5	60	10000	6.82904
59.3234	49.7785	5.7	0.05	30.723	58.532	5	60	10000	6.82826
59.3234	49.7785	5.8	0.05	30.723	58.532	5	60	10000	6.82776
59.3234	49.7785	5.9	0.05	30.723	58.532	5	60	10000	6.82781
59.3234	49.7785	6	0.05	30.723	58.532	5	60	10000	6.82773
59.3234	49.7785	6.1	0.05	30.723	58.532	5	60	10000	6.82810
59.3234	49.7785	6.2	0.05	30.723	58.532	5	60	10000	6.82887
59.3234	49.7785	6.3	0.05	30.723	58.532	5	60	10000	6.82651
59.3234	49.7785	6.4	0.05	30.723	58.532	5	60	10000	6.82901
59.3234	49.7785	6.5	0.05	30.723	58.532	5	60	10000	6.82917
59.3234	49.7785	6.6	0.05	30.723	58.532	5	60	10000	6.82816
59.3234	49.7785	6.7	0.05	30.723	58.532	5	60	10000	6.82888
59.3234	49.7785	6.8	0.05	30.723	58.532	5	60	10000	6.82769
59.3234	49.7785	6.9	0.05	30.723	58.532	5	60	10000	6.82793
59.3234	49.7785	7	0.05	30.723	58.532	5	60	10000	6.82859
59.3234	49.7785	7.1	0.05	30.723	58.532	5	60	10000	6.83001
59.3234	49.7785	7.2	0.05	30.723	58.532	5	60	10000	6.82896
59.3234	49.7785	7.3	0.05	30.723	58.532	5	60	10000	6.82825
59.3234	49.7785	7.4	0.05	30.723	58.532	5	60	10000	6.82846
59.3234	49.7785	7.5	0.05	30.723	58.532	5	60	10000	6.82836
59.3234	49.7785	7.6	0.05	30.723	58.532	5	60	10000	6.82965
59.3234	49.7785	7.7	0.05	30.723	58.532	5	60	10000	6.82957
…	…	…	…	…	…	…	…	…	…
59.3234	49.7785	68	0.05	30.723	58.532	5	60	10000	6.83563
59.3234	49.7785	68.1	0.05	30.723	58.532	5	60	10000	6.83910
59.3234	49.7785	68.2	0.05	30.723	58.532	5	60	10000	6.81725
59.3234	49.7785	68.3	0.05	30.723	58.532	5	60	10000	6.82189
59.3234	49.7785	68.4	0.05	30.723	58.532	5	60	10000	6.82592
59.3234	49.7785	68.5	0.05	30.723	58.532	5	60	10000	6.82905
59.3234	49.7785	68.6	0.05	30.723	58.532	5	60	10000	6.82332
59.3234	49.7785	68.7	0.05	30.723	58.532	5	60	10000	6.84136

B-3 (Continued)

59.3234	49.7785	68.8	0.05	30.723	58.532	5	60	10000	6.81076
59.3234	49.7785	68.9	0.05	30.723	58.532	5	60	10000	6.82978
59.3234	49.7785	69	0.05	30.723	58.532	5	60	10000	6.83294
59.3234	49.7785	69.1	0.05	30.723	58.532	5	60	10000	6.84191
59.3234	49.7785	69.2	0.05	30.723	58.532	5	60	10000	6.82178
59.3234	49.7785	69.3	0.05	30.723	58.532	5	60	10000	6.82057
59.3234	49.7785	69.4	0.05	30.723	58.532	5	60	10000	6.83204
59.3234	49.7785	69.5	0.05	30.723	58.532	5	60	10000	6.83020
59.3234	49.7785	69.6	0.05	30.723	58.532	5	60	10000	6.81972
59.3234	49.7785	69.7	0.05	30.723	58.532	5	60	10000	6.82093
59.3234	49.7785	69.8	0.05	30.723	58.532	5	60	10000	6.82425
59.3234	49.7785	69.9	0.05	30.723	58.532	5	60	10000	6.83028
59.3234	49.7785	70	0.05	30.723	58.532	5	60	10000	6.82481

B-4

Effect of the reverting-speed on the options value									
V0	I	sigma	r	q	mu	t	nstep	npath	F
59.3234	49.7785	46.539	0.05	0.1	58.532	5	60	10000	19.48198
59.3234	49.7785	46.539	0.05	0.12	58.532	5	60	10000	18.79189
59.3234	49.7785	46.539	0.05	0.14	58.532	5	60	10000	18.03071
59.3234	49.7785	46.539	0.05	0.16	58.532	5	60	10000	17.87555
59.3234	49.7785	46.539	0.05	0.18	58.532	5	60	10000	17.88754
59.3234	49.7785	46.539	0.05	0.2	58.532	5	60	10000	17.07454
59.3234	49.7785	46.539	0.05	0.22	58.532	5	60	10000	16.66652
59.3234	49.7785	46.539	0.05	0.24	58.532	5	60	10000	16.46955
59.3234	49.7785	46.539	0.05	0.26	58.532	5	60	10000	16.00562
59.3234	49.7785	46.539	0.05	0.28	58.532	5	60	10000	15.78353
59.3234	49.7785	46.539	0.05	0.3	58.532	5	60	10000	15.44736
59.3234	49.7785	46.539	0.05	0.32	58.532	5	60	10000	14.90446
59.3234	49.7785	46.539	0.05	0.34	58.532	5	60	10000	14.48049
59.3234	49.7785	46.539	0.05	0.36	58.532	5	60	10000	14.61013
59.3234	49.7785	46.539	0.05	0.38	58.532	5	60	10000	14.21139
59.3234	49.7785	46.539	0.05	0.4	58.532	5	60	10000	13.88608
59.3234	49.7785	46.539	0.05	0.42	58.532	5	60	10000	13.81859
59.3234	49.7785	46.539	0.05	0.44	58.532	5	60	10000	13.58868
59.3234	49.7785	46.539	0.05	0.46	58.532	5	60	10000	13.03272
59.3234	49.7785	46.539	0.05	0.48	58.532	5	60	10000	12.93468
59.3234	49.7785	46.539	0.05	0.5	58.532	5	60	10000	12.25982
59.3234	49.7785	46.539	0.05	0.52	58.532	5	60	10000	12.44145
59.3234	49.7785	46.539	0.05	0.54	58.532	5	60	10000	12.41093
59.3234	49.7785	46.539	0.05	0.56	58.532	5	60	10000	12.17556
59.3234	49.7785	46.539	0.05	0.58	58.532	5	60	10000	12.19809
59.3234	49.7785	46.539	0.05	0.6	58.532	5	60	10000	11.95977
59.3234	49.7785	46.539	0.05	0.62	58.532	5	60	10000	11.72636
59.3234	49.7785	46.539	0.05	0.64	58.532	5	60	10000	11.32464
59.3234	49.7785	46.539	0.05	0.66	58.532	5	60	10000	11.38860
59.3234	49.7785	46.539	0.05	0.68	58.532	5	60	10000	11.16018
59.3234	49.7785	46.539	0.05	0.7	58.532	5	60	10000	11.07293
59.3234	49.7785	46.539	0.05	0.72	58.532	5	60	10000	10.88590
59.3234	49.7785	46.539	0.05	0.74	58.532	5	60	10000	10.64603
59.3234	49.7785	46.539	0.05	0.76	58.532	5	60	10000	10.79269
59.3234	49.7785	46.539	0.05	0.78	58.532	5	60	10000	10.62271
59.3234	49.7785	46.539	0.05	0.8	58.532	5	60	10000	10.33649

B-4 (Continued)

59.3234	49.7785	46.539	0.05	0.82	58.532	5	60	10000	10.39183
59.3234	49.7785	46.539	0.05	0.84	58.532	5	60	10000	10.18161
59.3234	49.7785	46.539	0.05	0.86	58.532	5	60	10000	9.98757
59.3234	49.7785	46.539	0.05	0.88	58.532	5	60	10000	10.27677
59.3234	49.7785	46.539	0.05	0.9	58.532	5	60	10000	10.08905
59.3234	49.7785	46.539	0.05	0.92	58.532	5	60	10000	9.95617
59.3234	49.7785	46.539	0.05	0.94	58.532	5	60	10000	9.82930
59.3234	49.7785	46.539	0.05	0.96	58.532	5	60	10000	9.64721
59.3234	49.7785	46.539	0.05	0.98	58.532	5	60	10000	9.63140
59.3234	49.7785	46.539	0.05	1	58.532	5	60	10000	9.49705
59.3234	49.7785	46.539	0.05	1.02	58.532	5	60	10000	9.61301
59.3234	49.7785	46.539	0.05	1.04	58.532	5	60	10000	9.45184
59.3234	49.7785	46.539	0.05	1.06	58.532	5	60	10000	9.21486
59.3234	49.7785	46.539	0.05	1.08	58.532	5	60	10000	9.19199
59.3234	49.7785	46.539	0.05	1.1	58.532	5	60	10000	9.14916
59.3234	49.7785	46.539	0.05	1.12	58.532	5	60	10000	8.97011
59.3234	49.7785	46.539	0.05	1.14	58.532	5	60	10000	8.84706
59.3234	49.7785	46.539	0.05	1.16	58.532	5	60	10000	9.05599
59.3234	49.7785	46.539	0.05	1.18	58.532	5	60	10000	8.85766
59.3234	49.7785	46.539	0.05	1.2	58.532	5	60	10000	8.77501
59.3234	49.7785	46.539	0.05	1.22	58.532	5	60	10000	8.80424
59.3234	49.7785	46.539	0.05	1.24	58.532	5	60	10000	8.79044
59.3234	49.7785	46.539	0.05	1.26	58.532	5	60	10000	8.65616
…	…	…	…	…	…	…	…	…	…
59.3234	49.7785	46.539	0.05	31.1	58.532	5	60	10000	6.82467
59.3234	49.7785	46.539	0.05	32.1	58.532	5	60	10000	6.82911
59.3234	49.7785	46.539	0.05	33.1	58.532	5	60	10000	6.83599
59.3234	49.7785	46.539	0.05	34.1	58.532	5	60	10000	6.82556
59.3234	49.7785	46.539	0.05	35.1	58.532	5	60	10000	6.83323
59.3234	49.7785	46.539	0.05	36.1	58.532	5	60	10000	6.82772
59.3234	49.7785	46.539	0.05	37.1	58.532	5	60	10000	6.82788
59.3234	49.7785	46.539	0.05	38.1	58.532	5	60	10000	6.83182
59.3234	49.7785	46.539	0.05	39.1	58.532	5	60	10000	6.82198
59.3234	49.7785	46.539	0.05	40.1	58.532	5	60	10000	6.83184
59.3234	49.7785	46.539	0.05	41.1	58.532	5	60	10000	6.84221
59.3234	49.7785	46.539	0.05	42.1	58.532	5	60	10000	6.82830
59.3234	49.7785	46.539	0.05	43.1	58.532	5	60	10000	6.83741
59.3234	49.7785	46.539	0.05	44.1	58.532	5	60	10000	6.83444

B-4 (Continued)

59.3234	49.7785	46.539	0.05	45.1	58.532	5	60	10000	6.82957
59.3234	49.7785	46.539	0.05	46.1	58.532	5	60	10000	6.82866
59.3234	49.7785	46.539	0.05	47.1	58.532	5	60	10000	6.82477
59.3234	49.7785	46.539	0.05	48.1	58.532	5	60	10000	6.82666
59.3234	49.7785	46.539	0.05	49.1	58.532	5	60	10000	6.82758
59.3234	49.7785	46.539	0.05	50.1	58.532	5	60	10000	6.82751

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