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Strong interactions in pionium*

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Pionium is investigated in various pion-pion strong interactions which reproduce well the pion-pion scattering data. It is found that the ground-state pionium wave functions in those realistic pion-pion strong interactions are considerably different from the hydrogen-like one at small distance. One may suggest that some pion-pion interactions may need to be largely improved before applied to the pion-pion atomic system.

1. INTRODUCTION

Pionium is the $\pi^+\pi^-$ atomic state, bound mainly by the Coulomb force and effected by the strong interaction between the two pions. Pionium decays predominantly into $\pi^0\pi^0$ via strong interaction, which probes the low energy interactions of the pions, in particular, at zero energy. It has been believed that pionium might be employed to test more accurately the predictions of chiral perturbation theory. The investigation of pionium has recently become of particular interest due to the pionium DIRAC experiment. The preliminary result [1] of the pionium lifetime, based on part of the collected data, has been published as $\tau_{1s} = \left[2.91\,^{+0.49}_{-0.62}\right] \times 10^{-15}$ seconds.

The nonrelativistic formula of the pionium lifetime in the lowest order of electromagnetic interactions reads [2]

$$\Gamma_0 = \frac{2}{9}\,\frac{64\pi\, p}{M^3}\,|\psi(0)|^2\,|a_0 - a_2|^2 \tag{1}$$

where M is the mass of the $\pi\pi$ system, p is the center-of-mass momentum of the π^0 in the pionium system, $\psi(0)$ is the $1s$ pionium function at the origin ($r = 0$), and a_0 and a_2 are the S-wave $\pi\pi$ scattering lengths for isospin $I = 0$ and 2, respectively.

Any reasonable prediction of the pionium lifetime in the potential model (or say, in the quantum mechanics regime) must be based on the accurate knowledge of the wave function of the pionium state. The evaluation of the pionium wave function has been a challenge to numerical methods. Required is an approach, which is able to overcome the longstanding problem, that is, accounting for both the strong short-range interaction and the long-range Coulomb force. In this work we apply the numerical approach, which has been successfully applied to the protonium problem [3], to study the pionium problem here. The aim of this work is to reveal whether the pion-pion interactions, which reproduce well the pion-pion scattering data, are applicable to pionium.

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2. STRONG INTERACTIONS IN PIONIUM

Since Pionium has a small $\pi^0\pi^0$ component, the coupling of the $\pi^+\pi^-$ and $\pi^0\pi^0$ configurations must be properly treated. The dynamical equations of the ($\pi^+\pi^-$, $\pi^0\pi^0$) system may take the general form

$$E\Psi = (H_0 + \mathbf{V_c} + \mathbf{V_s})\mathbf{\Psi} \tag{2}$$

with

$$\Psi = \begin{pmatrix} \psi_{\pi^+\pi^-} \\ \psi_{\pi^0\pi^0} \end{pmatrix},\ H_0 = \begin{pmatrix} H^0_{\pi^+\pi^-} & 0 \\ 0 & H^0_{\pi^0\pi^0} \end{pmatrix},\ \mathbf{V_c} = \begin{pmatrix} V_c & 0 \\ 0 & 0 \end{pmatrix} \tag{3}$$

where V_c is the coulomb interaction between extended charges of pions. The charge distribution of π^+ and π^- is described by the form factor $F(q) = 1/\left(1 + q^2/a^2\right)$, with $a = 0.77$ GeV. The strong interaction matrix $\mathbf{V_s}$ takes, for example, for S-wave pionium the form

$$\mathbf{V_s} = \begin{pmatrix} \frac{2}{3}V^0 + \frac{1}{3}V^2 & \frac{\sqrt{2}}{3}\left(V^2 - V^0\right) \\ \frac{\sqrt{2}}{3}\left(V^2 - V^0\right) & \frac{1}{3}V^0 + \frac{2}{3}V^2 \end{pmatrix} \tag{4}$$

where V^0 and V^2 are respectively the isospin 0 and 2 strong interactions of the $\pi\pi$ system. The binding energy of pionium is derived as $E_b = E - 2m_\pi$ with m_π the mass of $\pi^\pm$.

The pionium problem is more difficult than other exotic problems, for example, the protonium problem in term of evaluating their wave functions since the Bohr radius of pionium is much larger than the one of protonium. Here we solve the pionium dynamical equation in eq. (1) by expanding the pionium wave function Ψ in the complete basis of Sturmian functions [4].

Table 1
Energy shift of the $1s$ pionium compared to the pure Coulomb interaction level.

	Model B	Model C	Model D	Model E	Data
$\Delta E(eV)$	-1.36	-2.97	-3.93	-2.87	–
$\tau(10^{-15}s)$	1.10	2.68	2.59	2.24	$2.91^{+0.49}_{-0.62}$

Studied first in the work is the pion-pion interactions in the work [5], which are worked out in the meson-exchange model and reproduce well the pion-pion phase shift data. The work considers the contributions of the ρ-exchange in the t-channel and the exchanges of ρ, f_2 and ε (a scalar meson) in the s-channel for the very low energy pion-pion scattering. For the ε-exchange both the scalar coupling and the gradient coupling are studied. For our convenience, we may call the interaction with the ε scaler coupling Model A and the one with the ε gradient coupling Model B. It is found that the pion-pion potential in Model A supports a number of pion-pion deep bound states which have never been observed. The deep bound states stem mainly from the large contribution of the ε scaler coupling at zero energy. The predictions for the energy shift and the pionium lifetime in

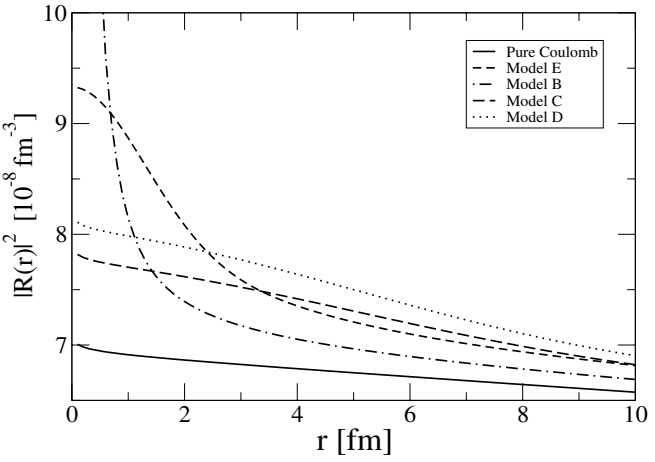

Figure 1. Squared 1s radial wave functions for the $\pi^+\pi^-$ component of pionium. For comparison the pure Coulomb interaction wave function is also plotted. All the wave functions have be multiplied by a factor 10^8.

Model B are shown in Table 1 while the 1s radial wave function for the $\pi^+\pi^-$ component of pionium is plotted in Fig. 1 as the dash-dotted curve. The lifetime in Model B and also in other models below is evaluated using eq. (1) with the scattering length a_0 and a_2 taken from [6]. Although Model B does not support any deep bound state, it is obvious that the interaction in the model is also too strong at zero energy. The main contributor to the pion-pion interaction at zero energy in Model B is the t-channel ρ-exchange. The pion-pion potentials in both Model A and B reproduce very well the pion-pion scattering data, but both of them fail to give reasonable predictions for the pionium properties.

The pion-pion interaction has been studied intensively in the chiral perturbation theory (ChPT) and considerable successes have been achieved in the regime. However, the ChPT success in reproducing pion-pion experimental data does not necessarily guarantee a practical potential which is applicable to multi-pion systems where the off-shell effects play important roles. In this work we study the pionium system in the pion-pion potentials derived in the chiral Lagrangian [7]. In analogy to the works [8], where meson-meson potentials provided by the lowest order chiral Lagrangian combined with Lippmann-Schwinger equations are applied to study the reactions of $\gamma\gamma$ to two mesons and two mesons to two mesons, and the derivation of the nucleon-nucleon interaction in chiral perturbation theory [9], we impose a cutoff of the momentum on the pion-pion potentials derived in the chiral perturbation theory. Devoted to Model C is the potential derived from the tree diagram of the leading order Lagrangian $\mathcal{L}_2$ in [7] with the cutoff $\Lambda = 0.1$ GeV for all momenta and to Model D is the potential derived from the tree diagrams of the chiral Lagrangian $\mathcal{L}_{eff} = \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6$ in [7] with the same cutoff. Shown in Table 1 are the predictions of Model C and D for the energy shifts and lifetimes of pionium, and the long dashed and dotted curves in Fig. 1 are the 1s radial wave functions for the $\pi^+\pi^-$

component of pionium. All the parameters of the potentials are taken from the works [7]. It is found that the predictions of Model C and D are fair good with a reasonable cutoff $\Lambda = 0.1$ GeV, hence it is possible to construct a pion-pion potential in the framework of the chiral perturbation theory. Of course, to get a practical pion-pion potential one needs to reproduce not only the pionium data but also the pion-pion scattering data by solving Lippmann-Schwinger equations for both bound and scattering problems.

The last model interaction we study here is a simple, local potential which has been widely employed for studying the influence of the hadronic interaction on pionium wave functions [10]. The potential is independent of both the energy of the pionium system and pion masses, and reproduce very well the phase shifts given by two-loop chiral perturbation theory [7]. For convenience, we may call the pion-pion interaction here Model E. In consistence with the works [10], we solve for the $(\pi^+\pi^-, \pi^0\pi^0)$ system here the coupled Schrödinger equations employed in the works [10].

The predictions of Model E for the energy shift and lifetime of the $1s$ pionium state are listed in Table 1 while the evaluated $1s$ radial wave functions for the $\pi^+\pi^-$ component of pionium is plotted in Fig. 1 as the dashed curve. It is clear that the ground state pionium wave function in Model E is considerably different from the hydrogen-like one at small distances, and the $1s$ pionium lifetime is much shorter than the experimental value.

3. SUMMARY AND CONCLUSIONS

The pionium system has been studied in various strong interactions, which may lead us to some points. The interaction in the meson-exchange model with the scaler coupling for the ε-exchange is unreasonably strong for the pionium system. The pionium system strongly favors the gradient coupling for the ε-exchange, and demands a much weaker coupling for the t-channel ρ-exchange. A practical pion-pion potential may be derived from the chiral perturbation theory, which can reproduce both the pionium and pion-pion scattering data and is applicable to other multi-pion systems, for example, the pion gas probably produced in high-energy heavy-ion collisions. The local pion-pion potential, which has been widely applied to the pionium system, is indeed too strong at zero energy though it reproduces well the pion-pion phase shift data.

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