ความน่าจะเป็นรูอินและเงินทุนเริ่มต้นน้อยสุดในกระบวนการส่วนเกินเวลาจำกัดสำหรับการประกันวินาศภัย

นายสุนทร บุญตา

มหาวิทยาลัยเทคโนโลยีสุรนารี

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรดุษฎีบัณฑิต
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RUIN PROBABILITY AND MINIMUM INITIAL CAPITAL IN FINITE TIME SURPLUS PROCESS FOR NON-LIFE INSURANCE

Soontorn Boonta

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy in Applied Mathematics

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RUIN PROBABILITY AND MINIMUM INITIAL CAPITAL IN FINITE TIME SURPLUS PROCESS FOR NON-LIFE INSURANCE

Suranaree University of Technology has approved this thesis submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy.

Thesis Examining Committee

(Assoc. Prof. Dr. Prapasri Asawakun)

Chairperson

(Prof. Dr. Pairote Sattayatham)

Member (Thesis Advisor)

(Prof. Dr. Samruam Chongcharoen)

Member

(Asst. Prof. Dr. Eckart Schulz)

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สุนทร บุญตา : ความน่าจะเป็นรูอินและเงินทุนเริ่มต้นน้อยสุดในกระบวนการส่วนเกินเวลาจำกัดสำหรับการประกันวินาศภัย (RUIN PROBABILITY AND MINIMUM INITIAL CAPITAL IN FINITE TIME SURPLUS PROCESS FOR NON-LIFE INSURANCE) อาจารย์ที่ปรึกษา : ศาสตราจารย์ ดร.ไพโรจน์ สัตยธรรม, 156 หน้า.

การวิจัยครั้งนี้มีวัตถุประสงค์เพื่อหาความสัมพันธ์ระหว่างเงินทุนเริ่มต้นและความน่าจะเป็นรูอิน และคำนวณเงินทุนเริ่มต้นน้อยสุดที่บริษัทประกันภัยต้องสำรองเพื่อรับประกันภายใต้ความน่าจะเป็นรูอินที่กำหนด ของกระบวนการส่วนเกินเวลาจำกัดซึ่งคิดอัตราดอกเบี้ยคงที่

ในการศึกษาครั้งนี้เราจำแนกค่าสินไหมทดแทนออกเป็นค่าสินไหมทดแทนมาตรฐานหรือค่าสินไหมทดแทนมูลค่าสูงและพิจารณาการขาดสภาพคล่องของบริษัทประกันภัยด้วยความน่าจะเป็นรูอินซึ่งถูกประมาณค่าโดยวิธีจำลองสถานการณ์ การประมาณค่าพารามิเตอร์การแจกแจงค่าสินไหมทดแทนใช้วิธีค้นหาจากย่านใกล้เคียงแบบสุ่ม และการคำนวณเงินทุนเริ่มต้นน้อยสุดใช้วิธีการวิเคราะห์การถดถอย

มหาวิทยาลัยเทคโนโลยีสุรนารี

สาขาวิชาคณิตศาสตร์ ลายมือชื่อนักศึกษา_______________

ปีการศึกษา 2557 ลายมือชื่ออาจารย์ที่ปรึกษา_______________

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SURPLUS PROCESS / RUIN PROBABILITY / MINIMUM INITIAL CAPITAL/ RANDOM NEIGHBORHOON SEARCH/

The objective of this study was to find the relationship between initial capital and ruin probability, and what minimum initial capital an insurance company had to hold for ensuring that the ruin probability was not greater than the given quantity in finite time surplus processes that include a constant interest rate from investment.

In this study, we divided claim severities into standard or large claim severities and considered the insolvency of the insurance company with the ruin probability which was approximated by a simulation approach. The random neighborhood search was used for parameter estimation of the claim severities and the minimum initial capital was computed by regression analysis.

School of Mathematics | Student's Signature S. boonta

Academic Year 2014 | Advisor's Signature P. Sattayatham

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CHAPTER I

INTRODUCTION

1.1 Introduction and Motivation

We start by introducing a surplus of non-life insurance. A surplus can be described as

$$\text{Surplus} = \text{Initial capital} + \text{Income} - \text{Outflow}.$$

In the following, we assume that all of the processes are defined in a probability space $(\Omega, \mathbb{F}, P)$. Lundberg (1903) was the first actuary who considered the surplus process of non-life insurance under three assumptions in the model:

1. Claims happen at times T_i satisfying $0 \le T_1 \le T_2 \le \dots$. We call them *claim arrivals* or *claim times*.

2. The *i*-th claim arriving at time T_i causes the *claim size* or *claim severity* Y_i. The sequence of claim sizes $\{Y_i, i \in \mathbb{N}\}$ constitutes an independent and identically distributed (i.i.d.) sequence of non-negative random variables.

3. The claim severity process $\{Y_i, i \in \mathbb{N}\}$ and the claim arrival process $\{T_i, i \in \mathbb{N}\}$ are mutually independent.

Next, we define the *claim number process*

$$N(t) = \#\{i \ge 1 : T_i \le t\},\ t \ge 0.$$

Thus, $N(t)$ is the number of claims in $[0,t]$.

Cramer (1930) proposed the Cramer-Lundberg model by considering $N(t)$ as a Poisson process. The premium income was computed in the time interval $[0,t]$ as ct where $c>0$ is a constant and is called *the premium rate*. Thus for each $t \geq 0$, the surplus process at time t, denoted by $U(t)$, by this model is

$$U(t) = u + ct - \sum_{k=1}^{N(t)} Y_k, \tag{1.1}$$

where $u \geq 0$ is *the initial capital*, $Y_1, Y_2, \ldots$ are i.i.d. and non-negative random variables.

Next, we will give the definition of ruin, ruin time and ruin probability. The event that U falls below zero is called *ruin*:

$$\text{ruin} = \{\, U(t) < 0 \text{ for some } t > 0 \,\}.$$

The time T when the process falls below zero for the first time is called *ruin time*, so that

$$T = \inf\{t > 0 \,:\, U(t) < 0 \mid U(0) = u\}.$$

The function

$$\Phi(u) = P\left(\{U(t) < 0 \text{ for some } t > 0\} \mid U(0) = u\right) = P(T < \infty) \tag{1.2}$$

is called the *ruin probability*. In the definition of (1.2) we made use of the fact that

$$\text{ruin} = \bigcup_{t \geq 0} \{U(t) < 0\} = \left\{\inf_{t \geq 0} U(t) < 0\right\} = \{T < \infty\}.$$

Let $\tilde{T} \in \mathbb{N}$, the function

$$\Phi(u, \tilde{T}) = P(T \leq \tilde{T} \mid U(0) = u)$$

is called the *finite-time ruin probability* in the period $[0, \tilde{T}]$.

Chan and Zhang (2007) considered the discrete time surplus process

$$U(n) = u + cn - \sum_{k=1}^{n} Y_k, \quad U(0) = u; \; n \in \mathbb{N}, \tag{1.3}$$

where $\{Y_n, n \in \mathbb{N}\}$ is a sequence of i.i.d. random variables. They investigated two cases of $\{Y_n,\, n \in \mathbb{N}\}$. For the first case, $\{Y_n,\, n \in \mathbb{N}\}$ is assumed to be a sequence of exponential random variables with exponential density function given by

$$f(y) = \alpha e^{-\alpha y}.$$

They proposed the recursive and explicit formulas of the ruin probabilities as follows: For each $n \in \mathbb{N}$,

$$\begin{aligned}\Phi_n(u) &= \Phi_{n-1}(u) + \frac{(\alpha c_n(u))^{n-1}}{(n-1)!} e^{-\alpha c_n(u)} \frac{c_1(u)}{c_n(u)} \\ &= \sum_{k=1}^{n} \frac{(\alpha c_k(u))^{k-1}}{(k-1)!} e^{-\alpha c_k(u)} \frac{c_1(u)}{c_k(u)}\end{aligned}$$

where

$$c_n(u) = u + nc.$$

For the second case, $\{Y_n,\, n \in \mathbb{N}\}$ is assumed to be a sequence of geometric random variables. The geometric mass function is given by

$$p_k = P(Y = k) = pq^k,\ k \geq 0,$$

where $p + q = 1,\ 0 < p < 1.$ For simplicity, they set premium rate c to 1 in the surplus process (1.3). They expressed the recursive formula of ruin probabilities as follows.

$$\begin{aligned}&\Phi_1(u) = q^{u+2} \\ &\Phi_{n+1}(u) = \Phi_n(u) + h_{n+1}(u),\end{aligned}$$

for $n \geq 2$, where $h_{n+1}(u)$ is a term defined by

$$h_{n+1}(u) = p^n q^{d(u)+n} \frac{d(u)[d(u)+(n+1)][d(u)+(n+2)]\ldots[d(u)+(2n-1)]}{n!},$$

with $d(u) = u + 2.$

Sattayatham et al. (2013) generalized the results of Chan and Zhang by considering the discrete time surplus process (1.3)

$$U(n) = u + cn - \sum_{k=1}^{n} Y_k,\ U(0) = u;\ n \in \mathbb{N}, \tag{1.4}$$

where $\{Y_n,\ n \in \mathbb{N}\}$ is any sequence of i.i.d. random variables, i.e., they did not assume that $\{Y_n, n \in \mathbb{N}\}$ is a sequence of exponential or geometric random variables. Since the formula of ruin probability was difficult to find explicitly, they proposed the ruin probability in the recursive form:

$$\Phi_N(u) = \Phi_1(u) + \int_{-\infty}^{u+c} \Phi_N(u + c - y) dF_{Y_1}(y),$$

where

$$\Phi_N(u) = P\left(\{\ U(i) < 0 \text{ for some } i \in \{1, 2, ..., N\}\} \mid\ U(0) = u\right).$$

Moreover, they expressed a condition for the bound of the ruin probability, i.e.,

for each $h > 0$, if

$$E[e^{hY_1}] \le e^{hc},$$

then

$$\Phi_N(u) \le e^{-hu}$$

for all $u \ge 0,\ N \in \mathbb{N}.$ They approximated the minimum initial capital under the condition that ruin probability was not greater than a given quantity. This minimum initial capital is approximated by applying the bisection technique which is a numerical method.

On substituting $t = T_n$ to the surplus process (1.1), the surplus process at time T_n has the form

$$U(T_n) = u + cT_n - \sum_{k=1}^{n} Y_k. \tag{1.5}$$

For convenience, we set $U_n := U(T_n)$ for all $n = 0, 1, 2, 3, \ldots$ and $U_0 = u$.

Note that we can rewrite (1.5) in the form:

$$\begin{aligned} U_n &= u + c\sum_{k=1}^{n}(T_k - T_{k-1}) - \sum_{k=1}^{n} Y_k \\ &= u + c\sum_{k=1}^{n} Z_k - \sum_{k=1}^{n} Y_k \\ &= u + c(\sum_{k=1}^{n-1} Z_k + Z_n) - (\sum_{k=1}^{n-1} Y_k + Y_n) \\ &= u + c\sum_{k=1}^{n-1} Z_k - \sum_{k=1}^{n-1} Y_k + cZ_n - Y_n \\ &= U_{n-1} + cZ_n - Y_n, \quad U_0 = u, \end{aligned} \tag{1.6}$$

where the inter arrival time $Z_k = T_k - T_{k-1}$ for $k \in \mathbb{N}$.

In this thesis, we study (1.6) by considering the following.

1) We assume that an insurance company is allowed to invest in a risk-free asset with a constant interest rate r for one unit time, i.e., we will include a constant interest rate r from investment in the surplus process (1.6).

2) We will divide the occurrence of the claim severities $Y_n, n \in \mathbb{N}$ into two classes as follows.

Class A: the claim severities Y_n happen exactly once per day, in this class the inter

arrival time $Z_n = 1$ for all $n \in \mathbb{N}$.

Class B: the claim severities Y_n do not happen every day, in this class the inter arrival time Z_n, $n \in \mathbb{N}$ are assumed to be i.i.d. Poisson random variables with parameter λ.

3) For each class, we will consider two cases of claim severities Y_n. For the first case, we will consider the claim severities Y_n of arbitrary size. For the second case, we will consider claim severities Y_n in the form of standard claims and large claims.

Indeed, consider a set of i.i.d. random variables on $(\Omega, \mathbb{F}, P)$, say $\{Y_1, Y_2, \ldots, Y_n\}$. Now, let $k \in \{1,2,\ldots,n\}$. If

$$Y_k(\omega) \leq \frac{1}{n}(Y_1(\omega) + Y_2(\omega) + \cdots + Y_k(\omega) + \cdots + Y_n(\omega)) \tag{1.7}$$

for all $\omega \in \Omega$, then Y_k is called *a standard claim severity*. In order to avoid confusion, a standard claim severity is denoted by V_k. If Y_k does not satisfy (1.7), Y_k is called *a large claim severity*. In order to avoid confusion, a large claim severity is denoted by W_k.

Next, we will give more explanation about Class A and Class B.

Class A : the claim severities happen once per day (i.e., $Z_n = 1$ for all $n \in \mathbb{N}$)

Case 1. Claim severities Y_n of arbitrary size

The surplus process is of the form

$$\begin{aligned} U_0 &= u, \\ U_n &= U_{n-1}(1+r_0) + c - Y_n, \quad n \in \mathbb{N} \end{aligned} \tag{1.8}$$

where r_0 is a daily interest rate which is given by $r_0 = (1+r)^{\frac{1}{365}} - 1$, we consider r as compound interest rate in the range $r = 2\%$ to $r = 8\%$ per annum and

$$c = (1+\theta)\frac{EY_1}{EZ_1} = (1+\theta)EY_1.$$

Case 2. Claim severities Y_n in the form of standard claims V_n or large claims W_n

Let $\{T_n^L, n \in \mathbb{N}\}$ be an arrival time process of large claims. The inter arrival time process $\{Z_n^L, n \in \mathbb{N}\}$ of the arrival time process $\{T_n^L, n \in \mathbb{N}\}$ is assumed to be i.i.d. such that $Z_1^L \sim \text{Poisson}(\lambda^L)$. Let premium rate $c = (1+\theta)(\frac{EW_1}{EZ_1^L} + EV_1)$, the surplus process is given by

$U_0 = u,$

$$U_n = \begin{cases} U_{n-1}(1+r_0) + \ c - W_n, & n = T_k^L, \ \text{for some } k = 1,2,3, \dots, \\ U_{n-1}(1+r_0) + \ c - V_n, & n \neq T_k^L, \ \text{for all } k = 1,2,3, \dots, \end{cases} \tag{1.9}$$

$n \in \mathbb{N},$

where $\{V_n,\ n \in \mathbb{N}\}$ and $\{W_n,\ n \in \mathbb{N}\}$ are assumed to be sequences of i.i.d. random variables, r_0 is mentioned in the previous case.

Class B : the claim severities do not happen every day

Case 1. Claim severities Y_n of arbitrary size

The surplus process is of the form

$$U_0 = u,$$
$$U_n = U_{n-1}(1+r_0)^{Z_n} + c\sum_{k=0}^{Z_n-1}(1+r_0)^k - Y_n,\ \ n \in \mathbb{N}, \tag{1.10}$$

where r_0 is the daily interest rate which is given by $r_0 = (1+r)^{\frac{1}{365}} - 1$, we consider r as compound interest rate in the range $r = 2\%$ to $r = 8\%$ per annum and

$$c = (1+\theta)\frac{EY_1}{EZ_1}.$$

Case 2. Claim severities Y_n in the form of standard claims V_n or large claims W_n

Let $\{T_n^L, n \in \mathbb{N}\}$ be an arrival time process of large claims. The inter arrival time process $\{Z_n^L,\ n \in \mathbb{N}\}$ of the arrival time process $\{T_n^L, n \in \mathbb{N}\}$ is assumed to be i.i.d. such that $Z_1^L \sim \text{Poisson}(\lambda^L)$. Let $\{T_n^l, n \in \mathbb{N}\}$ be an arrival time process of standard claims. The inter arrival time process $\{Z_n^l,\ n \in \mathbb{N}\}$ of the arrival time process $\{T_n^l, n \in \mathbb{N}\}$ is assumed to be i.i.d. such that $Z_1^l \sim \text{Poisson}(\lambda^l)$. Let

$$c = (1+\theta)\left(\frac{EW_1}{EZ_1^L} + \frac{EV_1}{EZ_1^l}\right),$$

the surplus process is given by

$$U_0 = u,$$

$$U_n = \begin{cases} U_{n-1}(1+r_0)^{Z_n} + c\sum_{k=0}^{Z_n-1}(1+r_0)^k - W_n, n = T_k^L, \text{ for some} \\ \quad k = 1,2,3,\ldots, \\ U_{n-1}(1+r_0)^{Z_n} + c\sum_{k=0}^{Z_n-1}(1+r_0)^k - V_n, n \neq T_k^L, \text{ for all k} \\ \quad k = 1,2,3,\ldots, \end{cases} \tag{1.11}$$

for all $n \in \mathbb{N}$, where $\{V_n,\ n \in \mathbb{N}\}$ and $\{W_n,\ n \in \mathbb{N}\}$ are assumed to be sequences of i.i.d. random variables, r_0 is mentioned in the previous case.

All of these settings motivate us to study the relationship between initial capital and ruin probability, and what minimum initial capital an insurance company has to hold for ensuring that the ruin probability is not greater than the given quantity in surplus processes (1.8), (1.9), (1.10) and (1.11).

To compute the ruin probabilities in (1.8), (1.9), (1.10) and (1.11), we found the distribution and parameters of claim severities. In this research, we applied randomized neighborhood search (RNS) to estimate parameters for the claim severities.

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1.2 Outline of the Thesis

This thesis is organized as the follows.

In Chapter II, we introduced the terminology and some mathematical and statistical background used in this thesis.

In Chapter III, we proposed the RNS for the estimation of the Weibull parameters for the claim severity of fire accidents. The RNS is also used for the parameter estimation of motor claim severities in Chapter IV.

In Chapter IV, we computed what the minimum initial capital an insurance company had to hold for ensuring that the ruin probability was not greater than the given quantity in surplus processes (1.8), (1.9), (1.10) and (1.11), respectively.

The conclusion of the thesis was presented in the last chapter.

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CHAPTER II

PRELIMINARIES

In this chapter, we introduce the definitions, and some of the mathematical and statistical concepts that are used for estimating parameters and approximating the minimum initial capital in chapter III and chapter IV.

2.1 Probability Theory

We recall some definitions of probability theory. Most of these results can be found in Brzezniak and Zastawniak (1999).

Definition 2.1 Let Ω be a non-empty set. A σ*-field* $\mathcal{F}$ on Ω is a family of subsets of Ω such that

1. the empty set $\emptyset \in \mathcal{F}$;
2. if A belongs to $\mathcal{F}$, then so does the complement $\Omega \backslash A$;
3. if $A_1, A_2, \ldots$ is a sequence of sets in $\mathcal{F}$, then their union $A_1 \cup A_2 \cup \ldots$ also belongs to $\mathcal{F}$.

Definition 2.2 Let $\mathcal{F}$ be a σ-field on Ω. A *probability measure* P is a function

$$P : \mathcal{F} \rightarrow [0,1]$$

such that

1. $P(\Omega) = 1$;
2. if $A_1, A_2, \ldots$ are pairwise disjoint sets (that is, $A_i \cap A_j = \emptyset$ for $i \neq j$) belonging to $\mathcal{F}$, then

$$P(A_1 \cup A_2 \cup \dots) = P(A_1) + P(A_2) + \cdots.$$

The triple $(\Omega, \mathcal{F}, P)$ is called a *probability space*. A set belonging to $\mathcal{F}$ is called *an event.*

Definition 2.3 If $\mathcal{F}$ is a σ-field on Ω, then a function $X: \Omega \to \mathbb{R}$ is said to be $\mathcal{F}$-*measurable* if

$$(X \in B) \coloneqq \{\omega \in \Omega: X(\omega) \in B\} = X^{-1}(B) \in \mathcal{F}$$

for every Borel set $B \in \mathcal{B}(\mathbb{R})$. If $(\Omega, \mathcal{F}, P)$ is a probability space, then such a function X is called a *random variable.*

Definition 2.4 The σ-field $\sigma(X)$ *generated by a random variable* $X: \Omega \to \mathbb{R}$ consists of all sets of the form $(X \in B)$, where B is a Borel set in $\mathbb{R}$.

Theorem 2.5 Let X be a random variable on the probability space $(\Omega, \mathcal{F}, P)$. If

$$P_X: \mathcal{B}(\mathbb{R}) \to [0,1]$$

is given by

$$P_X(B) = P(X \in B),$$

then P_X is *a probability measure on* $\mathbb{R}$.

Definition 2.6 The probability P_X in Theorem 2.5 is called the *distribution of* X.

Definition 2.7 Let X be a random variable on the probability space $(\Omega, \mathcal{F}, P)$. We define $F_X: \mathbb{R} \to [0,1]$ by

$$F_X(x) = P(X \leq x).$$

The function F_X is called *the distribution function of* X.

Definition 2.8 A random variable X is called *discrete* if it takes values in some countable subset $\{x_1, x_2, \dots\}$ of $\mathbb{R}$. The discrete random variable X has *probability mass function* $f: \mathbb{R} \to [0,1]$ given by

$$f(x) = P(X = x).$$

Definition 2.9 A random variable X is called *continuous* if its distribution function can be expressed as

$$F(x) = \int_{-\infty}^{x} f(t)dt; \;\; x \in \mathbb{R},$$

for some integrable function $f: \mathbb{R} \to [0,1]$ called the *probability density function* (pdf) of X.

Definition 2.10 A random variable $X: \Omega \to \mathbb{R}$ is said to be integrable if

$$\int_{\Omega} |X| \, dP < \infty.$$

Then

$$E[X] := \int_{\Omega} X \, dP$$

exists and is called the *expectation of* X.

Definition 2.11 Two events $A, B \in \mathcal{F}$ are called *independent* if

$$P(A \cap B) = P(A)P(B).$$

In general, we say that n events $A_1, A_2, \ldots, A_n \in \mathcal{F}$ are independent if

$$P\left(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_k}\right) = P\left(A_{i_1}\right)P\left(A_{i_2}\right) \ldots P\left(A_{i_k}\right)$$

for any indices $1 \leq i_1 < i_2 < \cdots < i_k \leq n$.

Definition 2.12 Two random variable X and Y are called *independent* if for any Borel sets $A, B \in \mathcal{B}(\mathbb{R})$ the two events $(X \in A)$ and $(Y \in B)$ are independent. We say that n random variable; $X_1, X_2, \ldots, X_n$ are *independent* if for any Borel sets $B_1, B_2, \ldots, B_n \in \mathcal{B}(\mathbb{R})$ the events

$$(X_1 \in B_1), (X_2 \in B_2), \ldots, (X_n \in B_n)$$

are independent.

In the following, we recall the distributions of random variables which can be found in Hoel, G., Port, C., and Stone, J. (1971) and Suhov, Y. and Kekbert, M. (2005).

Definition 2.13 A random variable X is said to be *Poisson distributed with parameter* λ denoted by $X \sim Poi(\lambda)$, if its probability mass function is

$$f(x;\lambda) = \frac{\lambda^x e^{-\lambda}}{x!},\ x = 0, 1, 2, \ldots.$$

Definition 2.14 A random variable X is said to be *normally distributed with parameters* $\mu,\ \sigma^2,$ denoted by $X \sim N(\mu,\ \sigma^2)$, if its probability density function is

$$f(x;(\mu,\sigma^2)) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2};\ -\infty < x < \infty.$$

The cumulative distribution function is

$$F(x;(\mu,\sigma^2)) = \Phi\left(\frac{x-\mu}{\sigma}\right),$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{1}{2}t^2}\, dt.$$

Definition 2.15 A random variable X is said to be *log normally distributed with parameters* $\mu,\ \sigma^2,$ denoted by $X \sim LN(\mu,\ \sigma^2)$, if its probability density function is

$$f(x;(\mu,\sigma^2)) = \frac{1}{\sigma x\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2\right);\ 0 < x < \infty.$$

The cumulative distribution function is

$$F(x;(\mu,\sigma^2)) = \Phi\left(\frac{\ln x-\mu}{\sigma}\right),$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{1}{2}t^2} dt.$$

Definition 2.16 A random variable X *is Weibull with 2 parameters* α, β, denoted by $X \sim Weibull(\alpha, \beta)$, if it has probability density function

$$f(x;(\alpha,\beta)) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right), \; x \geq 0.$$

The cumulative distribution function is

$$F(x;(\alpha,\beta)) = 1 - \exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right).$$

Definition 2.17 (See Singh, V. (1998)) A random variable X is said to be *log logistically distributed with parameters* a, b, c denoted by $X \sim LLG(\mu, \sigma^2)$, if its probability density function is

$$f(x;\,(a,b,c)) = \frac{b}{a}\left(\frac{x-c}{a}\right)^{b-1}\left(1+\left(\frac{x-c}{a}\right)^{b}\right)^{-2}, \; c < x < \infty.$$

The cumulative distribution function is

$$F(x\,;\,(a,b,c)) = \left(1+\left(\frac{a}{x-c}\right)^{b}\right)^{-1}, \; c < x < \infty.$$

2.2 Maximum Likelihood Estimation (MLE)

The basic idea behind MLE is to obtain the most likely values of the parameters, for a given distribution. Under MLE, one determines the likely value(s) for the parameter(s) of the assumed distribution. It is mathematically formulated as follows.

Let $X_1, \ldots, X_n$ be i.i.d. random variables with pdf $f(.;\boldsymbol{\theta}), \boldsymbol{\theta} \in \Theta$ such that Θ is a parameter space. We consider the joint p.d.f. of X's $f(x_1;\boldsymbol{\theta}) \ldots f(x_n;\boldsymbol{\theta})$. Treating the x's as if they were constants and looking at this joint p.d.f. as a function of $\boldsymbol{\theta}$, we denote it by $L(\boldsymbol{\theta}|x_1, \dots., x_n)$ and call it *the likelihood function.*

The estimate $\widehat{\boldsymbol{\theta}} = \widehat{\boldsymbol{\theta}}(x_1, \dots, x_n)$ is called *a maximum likelihood estimate* of $\boldsymbol{\theta}$ if

$$L(\widehat{\boldsymbol{\theta}}|x_1, \dots, x_n) = \max[L(\boldsymbol{\theta}|x_1, \dots., x_n); \boldsymbol{\theta} \in \Theta];$$

$\widehat{\boldsymbol{\theta}} = \widehat{\boldsymbol{\theta}}(X_1, \dots, X_n)$ is called *a maximum likelihood estimator of* $\boldsymbol{\theta}$*.*

Let $\boldsymbol{\theta} = (\theta_1, \theta_2, \dots, \theta_k)$. Now we have k unknown parameters which need to be estimated, with n independent observations $x_1, \dots, x_n$. The likelihood function is given by

$$L \coloneqq L((\theta_1, \theta_2, \dots, \theta_k)|x_1, x_2, \dots, x_n) = \prod_{i=1}^{n} f(x_i; (\theta_1, \theta_2, \dots, \theta_k)).$$

The logarithmic likelihood function is given by

$$\Lambda := \ln L((\theta_1, \theta_2, \dots, \theta_k)|x_1, x_2, \dots, x_n) = \sum_{i=1}^{n} \ln f(x_i; (\theta_1, \theta_2, \dots, \theta_k)).$$

The maximum likelihood estimators of $\theta_1, \theta_2, \dots, \theta_k$ are obtained by maximizing L or Λ. By maximizing Λ, which is much easier to work with than L, the maximum likelihood estimators of $\theta_1, \theta_2, \dots, \theta_k$ are the simultaneous solutions of k equations such that

$$\frac{\partial \Lambda}{\partial \theta_j} = 0, j = 1, 2, \dots, k.$$

In this part, the maximum likelihood estimation is used for parameter estimation of some distributions as explained below.

Maximum likelihood normal distribution

Suppose that $X_1, \dots, X_n$ are i.i.d. random variables with normal density:

$$f(x; (\mu, \sigma^2)) = \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right); \; -\infty < x < \infty.$$

We have two unknown parameters μ and σ^2, i.e., $\boldsymbol{\theta} = (\mu, \sigma^2)$. Thus the likelihood function is

$$L(\boldsymbol{\theta}|x_1,\dots,x_n) = \prod_{i=1}^{n} f(x_i;\boldsymbol{\theta}) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\left(-\frac{1}{2}\left(\frac{x_i-\mu}{\sigma}\right)^2\right).$$

The log likelihood function is

$$\ln L(\boldsymbol{\theta}|x_1,\dots,x_n) = \sum_{i=1}^{n} \ln f(x_i;\boldsymbol{\theta})$$

$$= \sum_{i=1}^{n} \left[-\ln\sigma - \frac{1}{2}\ln 2\pi - \frac{1}{2\sigma^2}(x_i-\mu)^2\right]$$

$$= -n\ln\sigma - \frac{n}{2}\ln 2\pi - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2.$$

Setting the partial derivatives $\frac{\partial \ln L}{\partial \mu}$ and $\frac{\partial \ln L}{\partial \sigma}$ to be 0, we have

$$\frac{\partial \ln L}{\partial \mu} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu) = 0$$

$$\frac{\partial \ln L}{\partial \sigma} = -\frac{n}{\sigma} + \sigma^{-3}\sum_{i=1}^{n}(x_i-\mu)^2 = 0.$$

Solving these equations will give us the MLE for the parameters μ and σ in the following.

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad (2.1)$$

and

$$\hat{\sigma} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i-\hat{\mu}\right)^2}\,. \qquad (2.2)$$

Maximum likelihood log-normal distribution

For determining the maximum likelihood estimators of the log normal distribution parameters μ and σ we can use the same procedure as for the normal distribution.

Suppose that $X_1,\dots,X_n$ are i.i.d. random variables with log normal density:

$$f(x;(\mu,\ \sigma^2)) = \frac{1}{\sigma x\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2\right);\ 0 < x < \infty.$$

Thus the log likelihood function as

$$\ln L((\mu,\sigma^2)|x_1,\ldots,x_n) = \sum_{i=1}^{n}\ln f(x_i;(\mu,\sigma^2))$$

$$= \sum_{i=1}^{n}\ln\frac{1}{\sigma x_i\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{\ln x_i-\mu}{\sigma}\right)^2\right)$$

$$= \sum_{i=1}^{n}\left[-\ln x_i - \ln\sigma - \frac{1}{2}\ln 2\pi - \frac{1}{2\sigma^2}(\ln x_i-\mu)^2\right]$$

$$= -\sum_{i=1}^{n}\ln x_i - n\ln\sigma - \frac{n}{2}\ln 2\pi - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(\ln x_i-\mu)^2.$$

Setting the partial derivatives $\frac{\partial\ln L}{\partial\mu}$ and $\frac{\partial\ln L}{\partial\sigma}$ to be 0, we have

$$\frac{\partial\ln L}{\partial\mu} = \frac{1}{\sigma^2}\sum_{i=1}^{n}(\ln x_i-\mu) = 0$$

$$\frac{\partial\ln L}{\partial\sigma} = -\frac{n}{\sigma} + \sigma^{-3}\sum_{i=1}^{n}(\ln x_i-\mu)^2 = 0.$$

Solving these equations will give us the MLE for the parameters μ and σ as follows

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n}\ln x_i \tag{2.3}$$

and

$$\hat{\sigma} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\ln x_i-\hat{\mu})^2}. \tag{2.4}$$

Maximum likelihood Weibull distribution

Suppose that $X_1,\ldots,X_n$ are i.i.d. random variables with Weibull density

$$f(x;(\alpha,\beta)) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}\exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right),\ \ x \geq 0.$$

Thus the log likelihood function is

$$\ln L((\alpha,\beta)|x_1,\dots,x_n) = \sum_{i=1}^{n} \ln f(x_i;(\alpha,\beta))$$

$$= \sum_{i=1}^{n} \ln\left[\frac{\alpha}{\beta}\left(\frac{x_i}{\beta}\right)^{\alpha-1} \exp\left(-\left(\frac{x_i}{\beta}\right)^{\alpha}\right)\right].$$

On differentiating with respect to β and α, and equal to zero, one gets

$$\frac{\partial \ln L}{\partial \beta} = -\frac{n}{\beta} + \frac{1}{\beta^{\alpha+1}} \sum_{i=1}^{n} x_i^{\alpha} = 0,$$

$$\frac{\partial \ln L}{\partial \alpha} = \frac{n}{\alpha} - n\ln\beta + \sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} \left(\frac{x_i}{\beta}\right)^{\alpha} \ln\left(\frac{x_i}{\beta}\right) = 0.$$

After solving the above two equations, we obtain

$$\hat{\beta} = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^{\alpha}\right)^{\frac{1}{\alpha}}, \tag{2.5}$$

$$\alpha = \left[\frac{\sum_{i=1}^{n} x_i^{\alpha} \ln x_i}{\sum_{i=1}^{n} x_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n} \ln x_i\right]^{-1}. \tag{2.6}$$

We can solve α from equation (2.6) by the Newton-Raphson method. By inserting α into (2.5), we obtain β.

Maximum likelihood log-logistic distribution

Suppose that $X_1,\dots,X_n$ are i.i.d. random variables with log logistic density

$$f(x;(a,b,c)) = \frac{b}{a}\left(\frac{x-c}{a}\right)^{b-1}\left(1+\left(\frac{x-c}{a}\right)^{b}\right)^{-2}, c < x < \infty.$$

Singh and Guo (1992) expressed the estimation equations as

$$2\sum_{i=1}^{n}\left[\frac{\left(\frac{x_i-c}{a}\right)^{b}}{1+\left(\frac{x_i-c}{a}\right)^{b}}\right] = n, \tag{2.7}$$

and

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$$2b\sum_{i=1}^{n}\left[\frac{\left(\ln\left(\frac{x_i-c}{a}\right)\right)\left(\frac{x_i-c}{a}\right)^b}{1+\left(\frac{x_i-c}{a}\right)^b}\right]-b\sum_{i=1}^{n}\ln\left(\frac{x_i-c}{a}\right)-n=0 \tag{2.8}$$

$$2b\sum_{i=1}^{n}\left[\frac{\left(\frac{x_i-c}{a}\right)^b}{1+\left(\frac{x_i-c}{a}\right)^b}\right]-a(b-1)\sum_{i=1}^{n}\frac{1}{x_i-c}=0 \tag{2.9}$$

where n is the sample size. We estimate parameters a, b and c by using an iterative scheme. First, with an assumed value of b and c, equation (2.7) is solved for a. With this value of a and the initial guess of c, equation (2.8) is solved to give a new value of b. Then, a new value of c is calculated from equation (2.9).

Maximum likelihood Poisson distribution

Suppose that $X_1, \ldots, X_n$ are i.i.d. random variables with Poisson distribution, parameter λ :

$$f(x;\lambda)=\frac{\lambda^x e^{-\lambda}}{x!},\ x=0,1,2,\ldots$$

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Then the log likelihood function as

$$\ln L(\lambda; x_1,\ldots,x_n)=\sum_{i=1}^{n}\ln f(x_i,\lambda)=\sum_{i=1}^{n}\ln\frac{\lambda^{x_i}e^{-\lambda}}{x_i!}=\sum_{i=1}^{n}\left(-\lambda+x_i\ln\lambda-\ln\left(x_i!\right)\right).$$

Setting the partial derivative to be 0, we have

$$\frac{\partial \ln L}{\partial\lambda}=\sum_{i=1}^{n}\left(-1+\frac{x_i}{\lambda}\right)=0.$$

After solving the equation, we obtain MLE for the parameter λ as follows.

$$\hat{\lambda}=\frac{\sum_{i=1}^{n}x_i}{n}. \tag{2.10}$$

2.3 Least Squares Linear Regression

In this topic we follow Miller, S. J. (n.d.).

Prediction

Given data $\{(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\}$, we may define the error associated to $y = \beta_0 + \beta_1 x$ by

$$\varepsilon(\beta_0, \beta_1) = \sum_{i=1}^{n} (y_i - (\beta_0 + \beta_1 x_i))^2. \tag{2.11}$$

The goal is to find values of β_0 and β_1 that minimize the error. In multivariable calculus we learn that this requires us to find the values of (β_0, β_1) such that

$$\frac{\partial \varepsilon}{\partial \beta_0} = 0, \ \frac{\partial \varepsilon}{\partial \beta_1} = 0.$$

Differentiating $\varepsilon(\beta_0, \beta_1)$ yields

$$\frac{\partial \varepsilon}{\partial \beta_1} = \sum_{i=1}^{n} 2(y_i - (\beta_0 + \beta_1 x_i)) \cdot (-x_i)$$
$$\frac{\partial \varepsilon}{\partial \beta_0} = \sum_{i=1}^{n} 2(y_i - (\beta_0 + \beta_1 x_i)) \cdot 1.$$

Setting $\frac{\partial \varepsilon}{\partial \beta_0} = \frac{\partial \varepsilon}{\partial \beta_1} = 0$ yields

$$\sum_{i=1}^{n} (y_i - (\beta_0 + \beta_1 x_i)) \cdot x_i = 0$$
$$\sum_{i=1}^{n} (y_i - (\beta_0 + \beta_1 x_i)) = 0.$$

We may rewrite these equations as

$$\left(\sum_{i=1}^{n} x_i^2\right)\beta_1 + \left(\sum_{i=1}^{n} x_i\right)\beta_0 = \sum_{i=1}^{n} x_i y_i$$
$$\left(\sum_{i=1}^{n} x_i\right)\beta_1 + \left(\sum_{i=1}^{n} 1\right)\beta_0 = \sum_{i=1}^{n} y_i.$$

We have obtained that the values of β_0 and β_1 which minimize the error (defined in (2.11)) satisfy the following matrix equation

$$\begin{pmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} 1 \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_0 \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} y_i \end{pmatrix}.$$

We will show the matrix is invertible, which implies

$$\begin{pmatrix} \beta_1 \\ \beta_0 \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} 1 \end{pmatrix}^{-1} \begin{pmatrix} \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} y_i \end{pmatrix}$$

Denote the matrix by M. The determinant of M is

$$\det M = \sum_{i=1}^{n} x_i^2 \cdot \sum_{i=1}^{n} 1 - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i$$

As

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i,$$

we find that

$$\begin{aligned} \det M &= n\sum_{i=1}^{n} x_i^2 - (n\bar{x})^2, \\ &= n^2\left(\frac{1}{n}\sum_{i=1}^{n} x_i^2 - \bar{x}^2\right) \\ &= n^2 \cdot \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 \end{aligned}$$

where the last equality follows from algebra. Thus, as long as all the x_i are not equal, $\det M$ will be non-zero and M will be invertible. Thus

$$\begin{pmatrix}\beta_1\\ \beta_0\end{pmatrix}=\begin{pmatrix}\dfrac{n}{n\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2} & \dfrac{-\sum_{i=1}^{n}x_i}{n\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}\\[2ex] \dfrac{-\sum_{i=1}^{n}x_i}{n\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2} & \dfrac{\sum_{i=1}^{n}x_i^2}{n\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}\end{pmatrix}\begin{pmatrix}\sum_{i=1}^{n}x_iy_i\\[2ex] \sum_{i=1}^{n}y_i\end{pmatrix}$$

$$\begin{pmatrix}\beta_1\\ \beta_0\end{pmatrix}=\begin{pmatrix}\dfrac{n\sum_{i=1}^{n}x_iy_i-\left(\sum_{i=1}^{n}x_i\right)\left(\sum_{i=1}^{n}y_i\right)}{n\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}\\[3ex] \dfrac{\left(\sum_{i=1}^{n}x_i^2\right)\left(\sum_{i=1}^{n}y_i\right)-\left(\sum_{i=1}^{n}x_i\right)\left(\sum_{i=1}^{n}x_iy_i\right)}{n\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2}\end{pmatrix}$$

Since

$$n\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2=n^2\left(\frac{1}{n}\sum_{i=1}^{n}\left(x_i-\bar{x}\right)^2\right)=n^2\left(\frac{\sum_{i=1}^{n}x_i^2}{n}-\left(\bar{x}\right)^2\right)=n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2,$$

we have

$$\begin{pmatrix}\beta_1\\ \beta_0\end{pmatrix}=\begin{pmatrix}\dfrac{n\sum_{i=1}^{n}x_iy_i-\left(\sum_{i=1}^{n}x_i\right)\left(\sum_{i=1}^{n}y_i\right)}{n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2}\\[3ex] \dfrac{\left(\sum_{i=1}^{n}x_i^2\right)\left(\sum_{i=1}^{n}y_i\right)-\left(\sum_{i=1}^{n}x_i\right)\left(\sum_{i=1}^{n}x_iy_i\right)}{n\sum_{i=1}^{n}x_i^2-\left(\sum_{i=1}^{n}x_i\right)^2}\end{pmatrix} \tag{2.12}$$

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2.4 Chi-Squared Goodness of Fit Test

A goodness-of-fit test is an inferential procedure used to determine whether a frequency distribution follows a claimed distribution. It is a test of the agreement or conformity between the observed frequencies (O_i) and the expected frequencies (E_i) for several classes or categories. The steps of the Chi-squared goodness of fit test are shown in the following.

Step 1 H_0: data is assumed of distribution F with parameters (α, β).

H_1: data is not assumed of distribution F with parameters (α, β).

Step 2 The test statistic is $\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$.

Step 3 Test statistic distribution: Chi-squared; DF $= k - 1 -$ Number of estimated parameter.

Step 4 Establish test significance level 0.05 and obtain Chi-squared critical value.

Step 5 When Chi-squared critical value $>$ Test statistics, the data is assumed from distribution F with parameters (α, β).

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CHAPTER III

RANDOMIZED NEIGHBORHOOD SEARCH FOR THE SEVERITY OF FIRE ACCIDENTS

The problem of estimating parameters in actuarial science is an important issue. Choosing an appropriate estimator is very important. In practice, constructive methods for parameter estimation are needed. The maximum likelihood estimation (MLE), the method of moments (MOM), the least squares method (LSM) and the weighted least squares method (WLSM) are frequently used for parameter estimation. Here, we consider the problem of the estimation of Weibull parameters. Many authors have investigated various aspects of this problem. Seyit and Ali (2009) presented power density method for Weibull parameters estimation. El-Mezouar (2010) proposed the Coefficient of Variation (CV) estimator comparing with Cran (1988) of the estimation of Weibull parameters. Yeliz et al. (2011) compared the method based on quantiles, maximum spacing method, MLE, MOM, LSM and WLSM for Weibull parameters estimation.

In this chapter, we propose the randomized neighborhood search technique (RNS) for the estimation of the Weibull parameters for the claim severity of fire accidents; the data were provided by the Thai Reinsurance Public Co., Ltd. Five estimation methods (MLE, MOM, LSM, WLSM and RNS) were used to estimate the parameters of Weibull distribution.

3.1 The Severity of Fire Accidents

We consider a data set of fire insurance in Thailand from 2000 to 2004. These data were provided by the Thai Reinsurance Public Co., Ltd. They are composed of the claim times and the claim severities. In this study, the claim severities (y_i) are considered as excess of 20 million baht. The quantity of y_i is shown in Table 3.1.

Table 3.1 Claim times and claim severities y_i (million Baht).

2000					
6-Mar	12-Mar	13-Mar	25-Mar	13-Jul	26-Aug
15.5	6.4	44.9	107.3	37.7	1.8
3-Sep	24-Oct				
47.3	28.5				
2001					
16-Jan	28-Jan	17-Feb	22-Feb	9-Mar	19-Jun
3.6	2.3	64.6	1.4	31.5	0.7
20-Jun	5-Jul	6-Aug	24-Aug	18-Sep	23-Oct
20.1	9.3	6.7	12.4	56.5	13.2
29-Nov	1-Dec				
5.7	40.2				
2002					
27-Jan	2-Mar	10-Apr	13-Apr	2-Jun	23-Aug
112.2	0.9	45.8	35.3	13	2.1
26-Oct	29-Oct				
4.2	24.4				
2003					
9-Jan	5-Feb	8-Apr	14-Apr	7-May	23-Nov
0.4	10.8	49.9	102.7	138.9	13.1
2004					
2-Jan	4-Jan	6-Jan	7-Feb	28-Feb	5-Mar
40	84.3	9.2	43.1	70	7.2
14-Mar	22-Apr	8-Jul	1-Nov	24-Dec	
2.4	7.5	37.2	14.2	33.2	

3.2 Weibull Distribution

Catastrophe insurance covers large insurance losses that happen infrequently, but involve high-cost claims. Examples include large-scale fire, windstorm or flood insurance. In the case of catastrophes, claim severity has heavy tails. The Weibull distribution with a shape parameter of less than one and a scale parameter greater than zero is a clear example of a heavy-tailed distribution. The probability density and cumulative distribution function for a two parameter Weibull random variable Y, are given by (3.1) and (3.2). For $y \geq 0$

$$f(y;\alpha,\beta) = \frac{\alpha}{\beta}\left(\frac{y}{\beta}\right)^{\alpha-1} \exp\left(-\left(\frac{y}{\beta}\right)^{\alpha}\right), \tag{3.1}$$

$$F(y;\alpha,\beta) = 1 - \exp\left(-\left(\frac{y}{\beta}\right)^{\alpha}\right). \tag{3.2}$$

where $\alpha > 0$ and $\beta > 0$ are the shape and scale parameters respectively.

3.3 Estimation of the Weibull Parameters

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3.3.1 Maximum Likelihood Estimation

Let $y_1, y_2, \ldots, y_n$ be samples for Weibull distribution, then the log likelihood function is defined by,

$$\ln L((\alpha,\beta) \mid y_1, y_2, ..., y_n) = \sum_{i=1}^{n} \ln \frac{\alpha}{\beta}\left(\frac{y_i}{\beta}\right)^{\alpha-1} \exp\left(-\left(\frac{y_i}{\beta}\right)^{\alpha}\right).$$

By (2.5) and (2.6), we obtain maximum likelihood estimation for the parameters α and β,

$$\beta = \left(\frac{1}{n} \sum_{i=1}^{n} y_i^{\alpha} \right)^{\frac{1}{\alpha}}, \tag{3.3}$$

$$\alpha = \left[\frac{\sum_{i=1}^{n} y_i^{\alpha} \ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n} \sum_{i=1}^{n} \ln y_i \right]^{-1}. \tag{3.4}$$

We can solve for α from equation (3.4) by Newton-Raphson method. By inserting α into (3.3), we obtain β.

The Newton-Raphson method

This method is a standard iterative approach to finding roots of nonlinear equations. Let $g:\mathbb{R}\to\mathbb{R}$ be a continuous function on $[a,b]$. The Newton-Raphson method is used for solving an equation of the form $g(x)=0$. We make an initial guess for the root that we are trying to find, and we call this initial guess x_0. The sequence $x_0, x_1, x_2, \ldots$ generated in the manner described below should converge to the exact root. To implement it analytically, we need a formula for each approximation in terms of the previous one, i.e., we need x_{k+1} in terms of x_k. The equation of the tangent line to the graph $z = g(x)$ at the point $(x_0, g(x_0))$ is

$$z - g(x_0) = g^{/}(x_0)(x - x_0).$$

Let the tangent line intersect the x-axis when $x = x_1$(at the point $x = x_1$, value of $z = 0$), so

$$-g(x_0) = g^{/}(x_0)(x_1 - x_0).$$

Solving this for x_1 gives

$$x_1 = x_0 - \frac{g(x_0)}{g^{/}(x_0)}$$

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and, more generally

$$x_{k+1} = x_k - \frac{g(x_k)}{g^{/}(x_k)}. \tag{3.5}$$

In this topic, we want to find α such that equation (3.4) holds. To use the Newton-Raphson method, we set

$$g(\alpha) := \alpha - \left[\frac{\sum_{i=1}^{n} y_i^{\alpha} \ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n} \ln y_i \right]^{-1} = 0.$$

Note that g is a continuous function on $[0.1, 1.1]$. By (C.1) and (C.2) in Appendix C, we obtain

$$g^{/}(\alpha) = 1 + \left[\frac{\sum_{i=1}^{n} y_i^{\alpha} \ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n} \ln y_i \right]^{-2} \cdot \left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha} (\ln y_i)^2\right) - \left(\sum_{i=1}^{n} y_i^{\alpha} \ln y_i\right)^2}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^2} \right] \tag{3.6}$$

and

$g^{//}(\alpha)$

$$=\left[\frac{\sum_{i=1}^{n} y_i^{\alpha}\ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}}-\frac{1}{n}\sum_{i=1}^{n}\ln y_i\right]^{-2}\left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha}\left(\ln y_i\right)^{3}\right)-\left(\sum_{i=1}^{n} y_i^{\alpha}\left(\ln y_i\right)^{2}\right)\left(\sum_{i=1}^{n} y_i^{\alpha}\ln y_i\right)}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^{2}}-\right.$$

$$\left.2\left(\frac{\sum_{i=1}^{n} y_i^{\alpha}\ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}}\right)\left(\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha}(\ln y_i)^{2}\right)-\left(\sum_{i=1}^{n} y_i^{\alpha}\ln y_i\right)\left(\sum_{i=1}^{n} y_i^{\alpha}\ln y_i\right)}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^{2}}\right)\right]$$

$$-2\left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha}\left(\ln y_i\right)^{2}\right)-\left(\sum_{i=1}^{n} y_i^{\alpha}\ln y_i\right)^{2}}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^{2}}\right]\left[\frac{\sum_{i=1}^{n} y_i^{\alpha}\ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}}-\frac{1}{n}\sum_{i=1}^{n}\ln y_i\right]^{-3}\times$$

$$\left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha}(\ln y_i)^{2}\right)-\left(\sum_{i=1}^{n} y_i^{\alpha}\ln y_i\right)\left(\sum_{i=1}^{n} y_i^{\alpha}\ln y_i\right)}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^{2}}\right].$$

Since $g^{/}(\alpha)$ and $g^{//}(\alpha)$ exist and are continuous for all $\alpha\in(0.1,1.1)$, we have that the Newton-Raphson method iteration will converge to the exact root α^{*} if the initial guess α_0 is close enough to α^{*}. By (3.5) and (3.6), we obtain the Newton-Raphson formula:

$$\alpha_{k+1}=\alpha_k-\frac{g(\alpha_k)}{g^{/}(\alpha_k)}$$

$$=\alpha_k-\frac{\alpha_k-\left[\frac{\sum_{i=1}^{n} y_i^{\alpha_k}\ln y_i}{\sum_{i=1}^{n} y_i^{\alpha_k}}-\frac{1}{n}\sum_{i=1}^{n}\ln y_i\right]^{-1}}{1+\left[\frac{\sum_{i=1}^{n} y_i^{\alpha_k}\ln y_i}{\sum_{i=1}^{n} y_i^{\alpha_k}}-\frac{1}{n}\sum_{i=1}^{n}\ln y_i\right]^{-2}\cdot\left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha_k}\right)\left(\sum_{i=1}^{n} y_i^{\alpha_k}\left(\ln y_i\right)^{2}\right)-\left(\sum_{i=1}^{n} y_i^{\alpha_k}\ln y_i\right)^{2}}{\left(\sum_{i=1}^{n} y_i^{\alpha_k}\right)^{2}}\right]}. \qquad (3.7)$$

Setting $k=0$ and using an initial guess $\alpha_0=1$ in formula (3.7), we obtain

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$$\alpha_1 = 1 - \frac{1 - \left[\dfrac{\sum_{i=1}^{n} y_i \ln y_i}{\sum_{i=1}^{n} y_i} - \dfrac{1}{n} \sum_{i=1}^{n} \ln y_i \right]^{-1}}{1 + \left[\dfrac{\sum_{i=1}^{n} y_i \ln y_i}{\sum_{i=1}^{n} y_i} - \dfrac{1}{n} \sum_{i=1}^{n} \ln y_i \right]^{-2} \cdot \left[\dfrac{\left(\sum_{i=1}^{n} y_i\right)\left(\sum_{i=1}^{n} y_i (\ln y_i)^2\right) - \left(\sum_{i=1}^{n} y_i \ln y_i\right)^2}{\left(\sum_{i=1}^{n} y_i\right)^2} \right]}.$$

$$\alpha_1 = 1 - \frac{1 - \left[\dfrac{5758.565}{1459.6} - \dfrac{1}{47}(126.35437) \right]^{-1}}{1 + \left[\dfrac{5758.565}{1459.6} - \dfrac{1}{47}(126.35437) \right]^{-2} \cdot \left[\dfrac{(1459.6)(23696.27673) - (5758.565)^2}{(1459.6)^2} \right]}$$

$$\approx 0.85643.$$

Again repeat the procedure until the 5th decimal place remains unchanged.

Table 3.2 Newton-Raphson for estimation α.

k	α_k
0	1
1	0.85643
2	0.86327
3	0.86329
4	0.86329

Thus we estimate the parameter $\alpha = 0.86329$. By inserting $\alpha = 0.86329$ into (3.3), we obtain $\beta = 28.86685$.

3.3.2 Methods of moments (MOM)

We know that the k th moment μ_k for the Weibull distribution is given by

$$\mu_k = \beta^k \Gamma(1 + \frac{k}{\alpha}),$$

where $\Gamma(t)$ denotes the gamma function as

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$$\Gamma(t) = \int_0^\infty e^{-x} x^{t-1} dx,\ t > 0 .$$

In particular, the mean μ (the first moment) and the variance σ^2 are (3.8) and (3.9), respectively.

$$\mu = \beta\Gamma(1+\frac{1}{\alpha}), \tag{3.8}$$

$$\sigma^2 = \mu_2 - (\mu)^2 = \beta^2[\Gamma(1+\frac{2}{\alpha}) - \Gamma^2(1+\frac{1}{\alpha})]. \tag{3.9}$$

The coefficient of variation CV for the Weibull distribution can be determined by the following equation (3.10),

$$CV = \frac{\sigma}{\mu} = \frac{\sqrt{\Gamma(1+\frac{2}{\alpha}) - \Gamma^2(1+\frac{1}{\alpha})}}{\Gamma(1+\frac{1}{\alpha})}. \tag{3.10}$$

The shape parameter α as appearing in (3.10) was determined by bisection and the scale β was calculated from (3.8). In the following, we give the details of bisection method.

Let $g:\mathbb{R}\to\mathbb{R}$ be a continuous function on $[a,b]$ which also satisfies $g(a)g(b)<0$. In other words, the value of the function g at one endpoint of the interval is supposed to be positive, and at the other endpoint negative. This guarantees that there exists at least one root $\hat{x}\in(a,b)$ such that $g(\hat{x})=0.$ Denote $a_0 = a,\ b_0 = b,$ and take the initial guess x_0 at the midpoint of the original interval (a,b), i.e., $x_0 = \frac{a_0+b_0}{2}$. Then, for all $k = 0,1,2,\ldots$, define

$$(a_{k+1}, b_{k+1}) = \begin{cases} (a_k, x_k), & if\ \ g(a_k)\cdot g(x_k) < 0, \\ (x_k, b_k), & if\ \ g(x_k)\cdot g(b_k) < 0, \end{cases}$$

and take the new iterate x_{k+1}, again at the midpoint

$$x_{k+1} = \frac{a_{k+1} + b_{k+1}}{2}.$$

In this part, we want to find α such that equation (3.10) holds. Firstly, from Table 3.1, we compute $CV = \frac{\sigma}{\mu} = \frac{33.47145}{31.05532} = 1.07780$ and then we set

$$g(\alpha) = 1.07780 - \frac{\sqrt{\Gamma(1+\frac{2}{\alpha}) - \Gamma^2(1+\frac{1}{\alpha})}}{\Gamma(1+\frac{1}{\alpha})}.$$

We set $a_0 = 0.92,\ b_0 = 0.93$. Note that g is continuous on $[a_0, b_0]$, $g(a_0) = -0.01031$ and $g(b_0) = 0.00167$. By the bisection method, the initial guess yields

$$\alpha_0 = \frac{a_0 + b_0}{2} = 0.92500.$$

Since $g(\alpha_0) = -0.00428$ which is negative, so we set $a_1 = \alpha_0 = 0.92500$ and $b_1 = b_0 = 0.93$. It follows that

$$\alpha_1 = \frac{a_1 + b_1}{2} = 0.9275.$$

The next steps are shown in the following Table 3.3.

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Table 3.3 Bisection method $a_0 = 0.92859,\ b_0 = 0.92860$.

k	α_k	$g(\alpha_k)$
1	0.92750	- 0.00130
2	0.92875	0.00019
3	0.92813	- 0.00055
4	0.92844	- 0.00018
5	0.92860	0.00001
6	0.92852	- 0.00008
7	0.92856	- 0.00004
8	0.92858	- 0.00001
9	0.92859	- 0.00002
10	0.92860	0.00001
11	0.92860	0.00001

Thus we estimate the parameter $\alpha = 0.92860$. Inserting $\alpha = 0.92860$ in (3.8), we get

$$\beta = \frac{\mu}{\Gamma(1+\frac{1}{\alpha})} = \frac{31.05532}{\Gamma(1+\frac{1}{0.92860})} \approx 30.0055.$$

Another method of moments has been proposed by Cran (1988). Let $y_{(1)} \le y_{(2)} \le \ldots \le y_{(n)}$ be an ordered random sample of the cumulative distribution function $F(y)$ as given by (3.2). Then $F(y)$ can be estimated by $S_n(y)$ where

$$S_n(y) = \begin{cases} 0, & y < y_{(1)}, \\ \frac{r}{n}, & y_{(r)} \le y < y_{(r+1)},\ r = 1, \ldots, n-1, \\ 1, & y_{(n)} \le y. \end{cases}$$

The population moment μ_k is estimated by

$$m_k = \int_0^\infty [1 - S_n(y)]^k dy$$
$$= \sum_{r=0}^{n-1} \left(1 - \frac{r}{n}\right)^k (y_{(r+1)} - y_{(r)}), \quad y_{(0)} = 0.$$

Cran expresses the parameters in terms of lower order moments as follows:

$$\alpha = (\ln 2)\left(\ln(\mu_1 - \mu_2) - \ln(\mu_2 - \mu_4)\right)^{-1}, \tag{3.11}$$

$$\beta = \mu_1 \left(\Gamma(1 + \frac{1}{\alpha})\right)^{-1}. \tag{3.12}$$

Therefore, α and β can be obtained by substituting m_1, m_2 and m_4 for μ_1, μ_2 and μ_4 respectively. To compute m_1, m_2 and m_4, we arrange the data from Table 3.1 in order from the smallest to the largest, that is,

$$0.4,\ 0.7,\ 0.9, \ldots, 112.2,\ 138.9 \tag{3.13}$$

Thus $y_{(1)} = 0.4,\ y_{(2)} = 0.7,\ y_{(3)} = 0.9, \ldots,\ y_{(46)} = 112.2,\ y_{(47)} = 138.9.$

We compute

$$m_1 = \sum_{r=0}^{n-1} \left(1 - \frac{r}{n}\right)(y_{(r+1)} - y_{(r)}), \quad y_{(0)} = 0$$
$$= \left(1 - \frac{0}{47}\right)(y_{(1)} - y_{(0)}) + \left(1 - \frac{1}{47}\right)(y_{(2)} - y_{(1)}) + \ldots + \left(1 - \frac{46}{47}\right)(y_{(47)} - y_{(46)})$$

$$= 0.4 + \left(1 - \frac{1}{47}\right)(0.7 - 0.4) + \ldots + \left(1 - \frac{46}{47}\right)(138.9 - 112.2)$$
$$= 31.05532,$$

$$m_2 = \sum_{r=0}^{n-1}\left(1-\frac{r}{n}\right)^2 (y_{(r+1)} - y_{(r)}), \quad y_{(0)} = 0$$
$$= \left(1-\frac{0}{47}\right)^2 (y_{(1)} - y_{(0)}) + \left(1-\frac{1}{47}\right)^2 (y_{(2)} - y_{(1)}) + \ldots + \left(1-\frac{46}{47}\right)^2 (y_{(47)} - y_{(46)})$$
$$= 0.4 + \left(1-\frac{1}{47}\right)^2 (0.7 - 0.4) + \ldots + \left(1-\frac{46}{47}\right)^2 (138.9 - 112.2)$$
$$= 14.08818, \text{ and}$$

$$m_4 = \sum_{r=0}^{n-1}\left(1-\frac{r}{n}\right)^4 (y_{(r+1)} - y_{(r)}), \quad y_{(0)} = 0$$
$$= \left(1-\frac{0}{47}\right)^4 (y_{(1)} - y_{(0)}) + \left(1-\frac{1}{47}\right)^4 (y_{(2)} - y_{(1)}) + \ldots + \left(1-\frac{46}{47}\right)^4 (y_{(47)} - y_{(46)})$$
$$= 0.4 + \left(1-\frac{1}{47}\right)^4 (0.7 - 0.4) + \ldots + \left(1-\frac{46}{47}\right)^4 (138.9 - 112.2)$$
$$= 5.87609.$$

By substituting m_1, m_2 and m_4 for μ_1, μ_2 and μ_4 in (3.11), we obtain

$$\alpha = (\ln 2)\big(\ln(31.05532 - 14.08818) - \ln(14.08818 - 5.87609)\big)^{-1}$$
$$= 0.95518.$$

Inserting $\alpha = 0.95518$ in (3.12), we obtain

$$\beta = \mu_1\left(\Gamma(1+\frac{1}{\alpha})\right)^{-1} = 31.05532\left(\Gamma(1+\frac{1}{0.95518})\right)^{-1} = 30.42391.$$

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3.3.3 Least squares method (LSM)

We note from (3.2) that a probability F_i is assigned to each y_i. Since the true value of F_i is unknown, a prescribed estimator must be used. The following four expressions are often used to define the probability estimator equation (3.14a)-(3.14d) (See Wu et al. (2006)):

$$F_i = \frac{i-0.5}{n}, \tag{3.14a}$$

$$F_i = \frac{i}{n+1}, \tag{3.14b}$$

$$F_i = \frac{i-0.3}{n+0.4}, \tag{3.14c}$$

$$F_i = \frac{i-3/8}{n+1/4}, \tag{3.14d}$$

where F_i is the probability for the i th ranked y_i and n is the sample size.

By applying the logarithm to (3.2), we get a linear form

$$\ln\ln\left[\frac{1}{1-F}\right] = \alpha \ln y - \alpha \ln \beta. \tag{3.15}$$

The shape parameter α can be obtained from the slope term in (3.15) and the scale parameter β can be solved from the intercept term.

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3.3.4 Weighted least squares method (WLSM)

For this method, we follow the technique given by Wu et al. (2006). Equation (3.15) can be rewritten in the form $Y = mS + b$, where

$$Y = \ln\ln\left[\frac{1}{1-F}\right],\ m = \alpha,\ S = \ln y \text{ and } b = -\alpha \ln \beta.$$

The WLSM is based on the hypothesis that a straight line fit must minimize the weighted sum of the squares of deviations for the data Y_i from the fitting function $Y(S_i)$, so the equation

$$l^2 = \sum_{i=1}^{n} W_i (Y_i - b - mS_i)^2$$

gives the minimum value. By solving $\frac{\partial l^2}{\partial m} = \frac{\partial l^2}{\partial b} = 0$, we compute

$$m = \alpha = \frac{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i Y_i W_i - \sum_{i=1}^{n} S_i W_i \sum_{i=1}^{n} Y_i W_i}{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i^2 W_i - (\sum_{i=1}^{n} S_i W_i)^2}, \qquad (3.16)$$

and

$$b = \frac{\sum_{i=1}^{n} Y_i W_i - \alpha \sum_{i=1}^{n} S_i W_i}{\sum_{i=1}^{n} W_i} \qquad (3.17)$$

where W_i is the weight factor for the i th datum point. The parameter β can be calculated from

$$\beta = \exp\left(-\frac{b}{m}\right). \qquad (3.18)$$

It is clear that LSM is a special case of WLSM where $W_i = 1$.

Bergman (1986) derived the weight factor based on the theory of error propagation, as equations (3.19a) and (3.19b),

$$W_i = \left[(1 - F_i)\ln(1 - F_i)\right]^2, \qquad (3.19a)$$

$$W_i = 3.3F_i - 27.5\left[1 - (1 - F_i)^{0.025}\right]. \qquad (3.19b)$$

Similar to LSM, the probability F for each datum ranked in ascending order is also approximated by F_i as shown from (3.14a) to (3.14d).

Next, we estimated parameters α and β by least squares method (LSM) with F_i given by (3.14a). Firstly, we set $W_i = 1$ in (3.16) and (3.17), thus

$$m = \alpha = \frac{n\sum_{i=1}^{n} S_i Y_i - \sum_{i=1}^{n} S_i \sum_{i=1}^{n} Y_i}{n\sum_{i=1}^{n} S_i^2 - (\sum_{i=1}^{n} S_i)^2}, \tag{3.20}$$

$$b = \frac{\sum_{i=1}^{n} Y_i - \alpha \sum_{i=1}^{n} S_i}{n}. \tag{3.21}$$

Note that $Y_i = \ln\ln\left[\frac{1}{1-F_i}\right]$ where F_i is the probability estimator for the i th ranked y_i as mentioned in (3.14a), i.e., $F_i(y_i) = \frac{i-0.5}{n}$. Note also that we must arrange the data y_i from Table 3.1 in order from the smallest to the largest as mentioned in (3.13), i.e.,

$$y_{(1)} := y_1 = 0.4,\ y_{(2)} := y_2 = 0.7, \ldots,\ y_{(46)} := y_{46} = 112.2,\ y_{(47)} := y_{47} = 138.9.$$

Thus

$$Y_1(y_1) = \ln\ln\left[\frac{1}{1-F_1(y_1)}\right] = \ln\ln\left[\frac{1}{1-\frac{1-0.5}{47}}\right] \approx -4.53795,$$

$$Y_2(y_2) = \ln\ln\left[\frac{1}{1-F_2(y_2)}\right] = \ln\ln\left[\frac{1}{1-\frac{2-0.5}{47}}\right] \approx -3.42851.$$

$Y_3(y_3), \ldots, Y_{47}(y_{47})$ are shown in Table 3.4.

Table 3.4 Values of $Y_i(y_i)$ by using estimator (3.14a).

i	$Y_i(y_i)$	i	$Y_i(y_i)$	i	$Y_i(y_i)$
3	-2.90665	18	-0.76409	33	0.16212
4	-2.55894	19	-0.69266	34	0.22111
5	-2.29617	20	-0.62369	35	0.28097
6	-2.08382	21	-0.55686	36	0.34203
7	-1.90485	22	-0.49189	37	0.40465
8	-1.74958	23	-0.42852	38	0.46929
9	-1.61199	24	-0.36651	39	0.53654
10	-1.48808	25	-0.30567	40	0.60718
11	-1.37502	26	-0.24578	41	0.68226
12	-1.27077	27	-0.18666	42	0.76333
13	-1.17380	28	-0.12814	43	0.85274
14	-1.08293	29	-0.07002	44	0.95451
15	-0.99721	30	-0.01213	45	1.07632
16	-0.91589	31	0.04573	46	1.23683
17	-0.83836	32	0.10374	47	1.51365

We compute $\sum_{i=1}^{47} Y_i(y_i) \approx -26.84016$, this summation was used in (3.20) and (3.21).

Table 3.5 shows $S_i = \ln y_i$, $S_i^2 = (\ln y_i)^2$, and $S_i Y_i(y_i)$ for all $i = 1, 2, ..., 47$.

Table 3.5 Values of $S_i = \ln y_i$, $S_i^2 = (\ln y_i)^2$, and $S_i Y_i(y_i)$ for all $i = 1, ..., 47$.

i	S_i	S_i^2	$S_i Y_i(y_i)$	i	S_i	S_i^2	$S_i Y_i(y_i)$
1	-0.91629	0.83959	4.15808	25	3.00072	9.00432	-0.91722
2	-0.35667	0.12722	1.22286	26	3.19458	10.20536	-0.78516
3	-0.10536	0.01110	0.30625	27	3.34990	11.22186	-0.62531
4	0.33647	0.11321	-0.86101	28	3.44999	11.90241	-0.44207
5	0.58779	0.34549	-1.34966	29	3.50255	12.26786	-0.24524
6	0.74194	0.55047	-1.54606	30	3.56388	12.70126	-0.04322
7	0.83291	0.69374	-1.58656	31	3.61631	13.07769	0.16536
8	0.87547	0.76645	-1.53170	32	3.62966	13.17443	0.37653
9	1.28093	1.64079	-2.06486	33	3.68888	13.60783	0.59803
10	1.43508	2.05947	-2.13551	34	3.69387	13.64465	0.81674
11	1.74047	3.02922	-2.39317	35	3.76352	14.16411	1.05745
12	1.85630	3.44584	-2.35893	36	3.80444	14.47375	1.30123
13	1.90211	3.61801	-2.23270	37	3.82428	14.62515	1.54748
14	1.97408	3.89700	-2.13779	38	3.85651	14.87267	1.80981
15	2.01490	4.05983	-2.00928	39	3.91002	15.28826	2.09789
16	2.21920	4.92486	-2.03255	40	4.03424	16.27510	2.44950
17	2.23001	4.97296	-1.86955	41	4.16821	17.37401	2.84381
18	2.37955	5.66224	-1.81819	42	4.24850	18.04971	3.24299
19	2.51770	6.33880	-1.74391	43	4.43438	19.66374	3.78138
20	2.56495	6.57897	-1.59974	44	4.63181	21.45368	4.42109
21	2.57261	6.61833	-1.43260	45	4.67563	21.86150	5.03246
22	2.58022	6.65752	-1.26918	46	4.72028	22.28107	5.83820
23	2.65324	7.03969	-1.13696	47	4.93375	24.34193	7.46799
24	2.74084	7.51220	-1.00455	Sum	126.35437	437.03538	11.36244

We calculate $\sum_{i=1}^{47} S_i \approx 126.35437$, $\sum_{i=1}^{47} S_i^2 \approx 437.03538$, and $\sum_{i=1}^{47} S_i Y_i(y_i) \approx 11.36244$. These summations were used in (3.20) and (3.21). By (3.20) and setting $n = 47$, we obtain

$$m = \alpha = \frac{n\sum_{i=1}^{n} S_i Y_i - (\sum_{i=1}^{n} S_i)(\sum_{i=1}^{n} Y_i)}{n\sum_{i=1}^{n} S_i^2 - (\sum_{i=1}^{n} S_i)^2}$$

$$\approx \frac{(47)(11.36244) - (126.35437)(-26.84016)}{(47)(437.03538) - (126.35437)^2}$$

$$\approx 0.85797.$$

Substituting α in (3.21) and setting $n = 47$, we get

$$b = \frac{\sum_{i=1}^{n} Y_i - \alpha \sum_{i=1}^{n} S_i}{n} \approx \frac{-26.84016 - (0.85797)(126.35437)}{47} \approx -2.87762.$$

By inserting b and m in (3.18), we have

$$\beta = \exp\left(-\frac{b}{m}\right) \approx \exp\left(-\frac{-2.87762}{0.85797}\right) \approx 28.61683.$$

In the following, we estimated parameters α and β by LSM with F_i given by (3.14b), (3.14c) and (3.14d), respectively. Firstly, we must calculate $\sum_{i=1}^{47} Y_i(y_i)$ and $\sum_{i=1}^{47} S_i Y_i(y_i)$ (See Appendix B for details). These summations were used in (3.20) and (3.21).

LSM with F_i as given by (3.14b)

Note that $Y_i = \ln\ln\left[\frac{1}{1-F_i}\right]$ where F_i is the probability estimator for the i th ranked y_i as mentioned in (3.14b), i.e., $F_i(y_i) = \frac{i}{n+1}$. By (B.1) in Appendix B, we have $\sum_{i=1}^{47} Y_i(y_i) \approx -25.72047$ and $\sum_{i=1}^{47} S_i Y_i(y_i) \approx 8.57754$. From Table 3.5, we know $\sum_{i=1}^{47} S_i \approx 126.35437$, $\sum_{i=1}^{47} S_i^2 \approx 437.03538$. By (3.20) and setting $n = 47$, we estimate

$$m=\alpha=\frac{n\sum_{i=1}^{n}S_iY_i-(\sum_{i=1}^{n}S_i)(\sum_{i=1}^{n}Y_i)}{n\sum_{i=1}^{n}S_i^2-(\sum_{i=1}^{n}S_i)^2}$$
$$\approx\frac{(47)(8.57754)-(126.35437)(-25.72047)}{(47)(437.03538)-(126.35437)^2}$$
$$\approx 0.79844.$$

Inserting α in (3.21) and setting $n=47,$ we get

$$b=\frac{\sum_{i=1}^{n}Y_i-\alpha\sum_{i=1}^{n}S_i}{n}\approx\frac{-25.7205-(0.79844)(126.35437)}{47}\approx-2.69376.$$

By (3.18), we obtain

$$\beta=\exp\left(-\frac{b}{m}\right)\approx\exp\left(-\frac{-2.69376}{0.79844}\right)\approx 29.18879.$$

LSM with F_i as given by (3.14c)

Note that $Y_i=\ln\ln\left[\frac{1}{1-F_i}\right]$ where F_i is the probability estimator for the i th ranked y_i as mentioned in (3.14c), i.e., $F_i=\frac{i-0.3}{n+0.4}$. By (B.2) in Appendix B, we have $\sum_{i=1}^{47}Y_i(y_i)\approx-26.32864$ and $\sum_{i=1}^{47}S_iY_i(y_i)\approx 10.11662.$ From Table 3.5, we know $\sum_{i=1}^{47}S_i\approx 126.35437,$ $\sum_{i=1}^{47}S_i^2\approx 437.03538.$ By (3.20) and setting $n=47,$ we estimate

$$m=\alpha=\frac{n\sum_{i=1}^{n}S_iY_i-(\sum_{i=1}^{n}S_i)(\sum_{i=1}^{n}Y_i)}{n\sum_{i=1}^{n}S_i^2-(\sum_{i=1}^{n}S_i)^2}$$
$$\approx\frac{(47)(10.11662)-(126.35437)(-26.32864)}{(47)(437.03538)-(126.35437)^2}$$
$$\approx 0.83104.$$

Substituting α in (3.21) and setting $n=47,$ we get

$$b = \frac{\sum_{i=1}^{n} Y_i - \alpha \sum_{i=1}^{n} S_i}{n} \approx \frac{-26.32864 - (0.83104)(126.35437)}{47} \approx -2.79435.$$

Parameter β can be calculated from (3.18), i.e.,

$$\beta = \exp\left(-\frac{b}{m}\right) \approx \exp\left(-\frac{-2.79435}{0.83104}\right) \approx 28.86020.$$

LSM with F_i as given by (3.14d)

Note that $Y_i = \ln\ln\left[\frac{1}{1-F_i}\right]$ where F_i is the probability estimator for the i th ranked y_i as mentioned in (3.14d), i.e., $F_i = \frac{i - 3/8}{n + 1/4}$. By (B.3) in Appendix B, we have $\sum_{i=1}^{47} Y_i(y_i) \approx -26.50724$ and $\sum_{i=1}^{47} S_i Y_i(y_i) \approx 10.55655$. From Table 3.5, we know $\sum_{i=1}^{47} S_i \approx 126.35437$, $\sum_{i=1}^{47} S_i^2 \approx 437.03538$. By (3.20) and setting $n = 47$, we estimate

$$\begin{aligned} m = \alpha &= \frac{n\sum_{i=1}^{n} S_i Y_i - (\sum_{i} S_i)(\sum_{i=1}^{n} Y_i)}{n\sum_{i=1}^{n} S_i^2 - (\sum_{i=1}^{n} S_i)^2} \\ &\approx \frac{(47)(10.55655) - (126.35437)(-26.50724)}{(47)(437.03538) - (126.35437)^2} \\ &\approx 0.84050. \end{aligned}$$

Inserting α in (3.21) and setting $n = 47$, we obtain

$$b = \frac{\sum_{i=1}^{n} Y_i - \alpha \sum_{i=1}^{n} S_i}{n} \approx \frac{-26.50724 - (0.84050)(126.35437)}{47} \approx -2.82356.$$

Parameter β can be calculated from (3.18), that is,

$$\beta = \exp\left(-\frac{b}{m}\right) \approx \exp\left(-\frac{-2.82356}{0.84050}\right) \approx 28.77206.$$

Next, we will estimate parameters α and β by WLSM with equation (3.19a) and choosing $F_i = \dfrac{i-0.5}{n}$ as mentioned in equation (3.14a). Firstly, we computed $\sum_{i=1}^{47} W_i(y_i)$, $\sum_{i=1}^{47} S_i Y_i W_i(y_i)$, $\sum_{i=1}^{47} S_i W_i(y_i)$, $\sum_{i=1}^{47} Y_i W_i(y_i)$, $\sum_{i=1}^{47} S_i^2 W_i(y_i)$. These summations were used in (3.16) and (3.17). Consider

$$W_1(y_1) = \left[(1-F_1(y_1))\ln(1-F_1(y_1))\right]^2 = \left[(1-\frac{1-0.5}{47})\ln(1-(1-\frac{1-0.5}{47}))\right]^2 \approx 0.00011197,$$

$$W_2(y_2) = \left[(1-F_2(y_2))\ln(1-F_2(y_2))\right]^2 = \left[(1-\frac{2-0.5}{47})\ln(1-(1-\frac{1-0.5}{47}))\right]^2 \approx 0.00098597.$$

For values of $W_3(y_3)$, $W_4(y_4)$, ..., and $W_{47}(y_{47})$ are shown in the Table 3.6.

Table 3.6 Values of $W_i(y_i)$ by using estimator (3.19a) and (3.14a).

i	$W_i(y_i)$	i	$W_i(y_i)$	i	$W_i(y_i)$
3	0.00267817	18	0.08546095	33	0.13162978
4	0.00512997	19	0.09201464	34	0.12838776
5	0.00828235	20	0.09834154	35	0.12407237
6	0.01207590	21	0.10437816	36	0.11865396
7	0.01645078	22	0.11006107	37	0.11211255
8	0.02134679	23	0.11532707	38	0.10444078
9	0.02670332	24	0.12011325	39	0.09564796
10	0.03245939	25	0.12435717	40	0.08576591
11	0.03855366	26	0.12799702	41	0.07485749
12	0.04492442	27	0.13097185	42	0.06302981
13	0.05150964	28	0.13322183	43	0.05045583
14	0.05824697	29	0.13468856	44	0.03741217
15	0.06507378	30	0.13531546	45	0.02435354
16	0.07192715	31	0.13504826	46	0.01208607
17	0.07874398	32	0.13383560	47	0.00233607

We compute $\sum_{i=1}^{47} W_i(y_i) \approx 3.481578707$. Next, we calculate

$$S_1Y_1W_1(y_1) = (\ln y_1)\left[\ln\ln\left(\frac{1}{1-F_1(y_1)}\right)\right]\left[(1-F_1(y_1))\ln(1-F_1(y_1))\right]^2$$
$$\approx (\ln 0.4)(-4.53795)(0.00011197)$$
$$\approx 0.00046557,$$

$$S_2Y_2W_2(y_2) = (\ln y_2)\left[\ln\ln\left(\frac{1}{1-F_2(y_2)}\right)\right]\left[(1-F_2(y_2))\ln(1-F_2(y_2))\right]^2$$
$$\approx (\ln 0.7)(-3.42851)(0.00098597)$$
$$\approx 0.00120570.$$

The values of $S_3Y_3W_3(y_3)$, $S_4Y_4W_4(y_4)$, ..., $S_{47}Y_{47}W_{47}(y_{47})$ are shown in the Table 3.7.

Table 3.7 Values of $S_iY_iW_i(y_i)$ for $i = 3, 4, ..., 47$.

i	$S_iY_iW_i(y_i)$	i	$S_iY_iW_i(y_i)$	i	$S_iY_iW_i(y_i)$
3	0.00082018	18	-0.15538413	33	0.07871903
4	-0.00441697	19	-0.16046510	34	0.10485979
5	-0.01117835	20	-0.15732094	35	0.13120052
6	-0.01867009	21	-0.14953179	36	0.15439568
7	-0.02610020	22	-0.13968785	37	0.17349221
8	-0.03269693	23	-0.13112220	38	0.18901848
9	-0.05513856	24	-0.12066016	39	0.20065857
10	-0.06931751	25	-0.11406280	40	0.21008370
11	-0.09226546	26	-0.10049852	41	0.21288043
12	-0.10597363	27	-0.08189742	42	0.20440480
13	-0.11500573	28	-0.05889337	43	0.19079276
14	-0.12451982	29	-0.03303132	44	0.16540250
15	-0.13075144	30	-0.00584799	45	0.12255830
16	-0.14619537	31	0.02233134	46	0.07056086
17	-0.14721545	32	0.05039265	47	0.01744576

We compute $\sum_{i=1}^{47} S_iY_iW_i(y_i) \approx -0.18616028$. Next, consider

$$S_1W_1(y_1) = (\ln y_1)\left[(1-F_1(y_1))\ln(1-F_1(y_1))\right]^2$$
$$\approx (\ln 0.4)(0.00011197)$$
$$\approx -0.00010260,$$

$$S_2W_2(y_2) = (\ln y_2)\left[(1-F_2(y_2))\ln(1-F_2(y_2))\right]^2$$
$$\approx (\ln 0.7)(0.00098597)$$
$$\approx -0.00035167.$$

Table 3.8 shows the values of $S_iW_i(y_i)$.

Table 3.8 Values of $S_iW_i(y_i)$ for $i = 3, 4, ..., 47$.

i	$S_iW_i(y_i)$	i	$S_iW_i(y_i)$	i	$S_iW_i(y_i)$
3	-0.00028217	18	0.20335828	33	0.48556639
4	0.00172609	19	0.23166494	34	0.47424732
5	0.00486826	20	0.25224108	35	0.46694923
6	0.00895956	21	0.26852452	36	0.45141161
7	0.01370200	22	0.28398144	37	0.42875023
8	0.01868845	23	0.30599063	38	0.40277693
9	0.03420518	24	0.32921121	39	0.37398554
10	0.04658197	25	0.37316102	40	0.34600033
11	0.06710133	26	0.40889712	41	0.31202206
12	0.08339311	27	0.43874314	42	0.26778186
13	0.09797687	28	0.45961367	43	0.22374040
14	0.11498424	29	0.47175341	44	0.17328615
15	0.13111735	30	0.48224848	45	0.11386812
16	0.15962099	31	0.48837621	46	0.05704968
17	0.17560021	32	0.48577772	47	0.01152560

We calculate $\sum_{i=1}^{47} S_iW_i(y_i) \approx 11.03029348.$ Next, we consider

$$Y_1W_1(y_1) = \left[\ln\ln\left(\frac{1}{1-F_1(y_1)}\right)\right]\left[(1-F_1(y_1))\ln(1-F_1(y_1))\right]^2$$
$$\approx (-4.53795)(0.00011197)$$
$$\approx -0.00050811,$$

$$Y_2W_2(y_2) = \left[\ln\ln\left(\frac{1}{1-F_2(y_2)}\right)\right]\left[(1-F_2(y_2))\ln(1-F_2(y_2))\right]^2$$
$$\approx (-3.42851)(0.00098597)$$
$$\approx -0.00338040.$$

Table 3.9 shows the values of $Y_i W_i(y_i)$.

Table 3.9 Values of $Y_i W_i(y_i)$ for $i = 3, 4, ..., 47$.

i	$Y_i W_i(y_i)$	i	$Y_i W_i(y_i)$	i	$Y_i W_i(y_i)$
3	-0.00778451	18	-0.06529990	33	0.02133955
4	-0.01312730	19	-0.06373489	34	0.02838754
5	-0.01901770	20	-0.06133491	35	0.03486109
6	-0.02516397	21	-0.05812450	36	0.04058305
7	-0.03133620	22	-0.05413803	37	0.04536593
8	-0.03734791	23	-0.04941962	38	0.04901283
9	-0.04304560	24	-0.04402306	39	0.05131905
10	-0.04830204	25	-0.03801181	40	0.05207515
11	-0.05301193	26	-0.03145904	41	0.05107233
12	-0.05708869	27	-0.02444769	42	0.04811228
13	-0.06046227	28	-0.01707060	43	0.04302579
14	-0.06307736	29	-0.00943065	44	0.03571010
15	-0.06489217	30	-0.00164090	45	0.02621215
16	-0.06587741	31	0.00617517	46	0.01494844
17	-0.06601547	32	0.01388357	47	0.00353600

We compute $\sum_{i=1}^{47} Y_i W_i(y_i) \approx -0.61195459$. Finally, we calculate

$$
\begin{aligned}
S_1{}^2 W_1(y_1) &= (\ln y_1)^2 \left[(1-F_1(y_1))\ln(1-F_1(y_1))\right]^2 \\
&\approx (\ln 0.4)^2 (0.00011197) \\
&\approx 9.40074 \times 10^{-5},
\end{aligned}
$$

$$
\begin{aligned}
S_2{}^2 W_2(y_2) &= (\ln y_2)^2 \left[(1-F_2(y_2))\ln(1-F_2(y_2))\right]^2 \\
&\approx (\ln 0.7)^2 (0.00098597) \\
&\approx 0.00012543.
\end{aligned}
$$

The values of $S_3{}^2 W_3(y_3), ..., S_{47}{}^2 W_{47}(y_{47})$ are shown in the Table 3.10.

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Table 3.10 Values of $S_i^2 W_i(y_i)$ for $i = 3, 4, ..., 47.$

i	$S_i^2 W_i(y_i)$	i	$S_i^2 W_i(y_i)$	i	$S_i^2 W_i(y_i)$
3	0.00002973	18	0.48390041	33	1.79119589
4	0.00058078	19	0.58326200	34	1.75180651
5	0.00286150	20	0.64698559	35	1.75737416
6	0.00664743	21	0.69080947	36	1.71736739
7	0.01141252	22	0.73273368	37	1.63966270
8	0.01636115	23	0.81186719	38	1.55331337
9	0.04381458	24	0.90231527	39	1.46229132
10	0.06684906	25	1.11975167	40	1.39584859
11	0.11678760	26	1.30625583	41	1.30057483
12	0.15480246	27	1.46974744	42	1.13766996
13	0.18636255	28	1.58566143	43	0.99215037
14	0.22698821	29	1.65233986	44	0.80262887
15	0.26418874	30	1.71867713	45	0.53240505
16	0.35423145	31	1.76611917	46	0.26929065
17	0.39159099	32	1.76320802	47	0.05686449

We compute $\sum_{i=1}^{47} S_i^2 W_i(y_i) \approx 37.23780646.$

By (3.16) and setting $n = 47,$ parameter α can be calculated as follows

$$m = \alpha = \frac{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i Y_i W_i - \sum_{i=1}^{n} S_i W_i \sum_{i=1}^{n} Y_i W_i}{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i^2 W_i - (\sum_{i=1}^{n} S_i W_i)^2}$$

$$\approx \frac{(3.48157871)(-0.186160277) - (11.03029348)(-0.611954586)}{(3.48157871)(37.23780646) - (11.03029348)^2}$$

$$\approx 0.76474777.$$

By inserting α in (3.17) and setting $n = 47,$ we obtain

$$b = \frac{\sum_{i=1}^{n} Y_i W_i - \alpha \sum_{i=1}^{n} S_i W_i}{\sum_{i=1}^{n} W_i} \approx \frac{0.00353600 - (0.76474777)(11.03029348)}{3.48157871} \approx -2.59863350.$$

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The parameter β can be calculated by substituting α and b into (3.18), thus

$$\beta \approx \exp\left(-\frac{-2.59863350}{0.76474777}\right) \approx 29.90503490.$$

In the following, we will estimate the parameters α and β by choosing F_i as given by (3.14b), (3.14c) and (3.14d), respectively. Firstly, we must compute $\sum_{i=1}^{47} W_i(y_i)$, $\sum_{i=1}^{47} S_i Y_i W_i(y_i)$, $\sum_{i=1}^{47} S_i W_i(y_i)$, $\sum_{i=1}^{47} Y_i W_i(y_i)$, and $\sum_{i=1}^{47} S_i^2 W_i(y_i)$ (See Appendix B for details). These summations were used in (3.16) and (3.17).

WLSM with equation (3.19a) and F_i as given by (3.14b)

Note that $W_i = \left[(1-F_i)\ln(1-F_i)\right]^2$, $Y_i = \ln\ln\left[\frac{1}{1-F_i}\right]$, and $F_i = \frac{i}{n+1}$. By (B.4) in Appendix B, we have

$\sum_{i=1}^{47} W_i(y_i) \approx 3.55542462$, $\sum_{i=1}^{47} S_i Y_i W_i(y_i) \approx -0.14562181$, $\sum_{i=1}^{47} S_i W_i(y_i) \approx 11.27649724$,

$\sum_{i=1}^{47} Y_i W_i(y_i) \approx -0.62542647$, and $\sum_{i=1}^{47} S_i^2 W_i(y_i) \approx 38.23033561.$ By (3.16) and setting $n = 47$, we estimate

$$\begin{aligned} m = \alpha &= \frac{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i Y_i W_i - \sum_{i=1}^{n} S_i W_i \sum_{i=1}^{n} Y_i W_i}{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i^2 W_i - (\sum_{i=1}^{n} S_i W_i)^2} \\ &\approx \frac{(3.55542462)(-0.14562181) - (11.27649724)(-0.62542647)}{(3.55542462)(38.23033561) - (11.27649724)^2} \\ &\approx 0.74550610. \end{aligned}$$

By (3.17) and setting $n = 47$, we obtain

$$b = \frac{\sum_{i=1}^{n} Y_i W_i - \alpha \sum_{i=1}^{n} S_i W_i}{\sum_{i=1}^{n} W_i} \approx \frac{-0.62542647 - (0.74550610)(11.27649724)}{3.55542462} \approx -2.5403784.$$

Inserting b and α in (3.18), we get

$$\beta = \exp\left(-\frac{b}{m}\right) \approx \exp\left(-\frac{-2.5403784}{0.74550610}\right) \approx 30.1923639.$$

WLSM with equation (3.19a) and F_i as given by (3.14c)

Note that $W_i = \left[(1-F_i)\ln(1-F_i)\right]^2$, $Y_i = \ln\ln\left[\frac{1}{1-F_i}\right]$, and $F_i = \frac{i-0.3}{n+0.4}$. By (B.5) in Appendix B, we have

$\sum_{i=1}^{47} W_i(y_i) \approx 3.51128404$, $\sum_{i=1}^{47} S_i Y_i W_i(y_i) \approx -0.16904739$, $\sum_{i=1}^{47} S_i W_i(y_i) \approx 11.12948575$, $\sum_{i=1}^{47} Y_i W_i(y_i) \approx -0.61712472$, and $\sum_{i=1}^{47} S_i^2 W_i(y_i) \approx 37.63681236$. By (3.16) and setting $n = 47$, we estimate

$$\begin{aligned} m = \alpha &= \frac{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i Y_i W_i - \sum_{i=1}^{n} S_i W_i \sum_{i=1}^{n} Y_i W_i}{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i^2 W_i - (\sum_{i=1}^{n} S_i W_i)^2} \\ &\approx \frac{(3.51128404)(-0.16904739) - (11.12948575)(-0.61712472)}{(3.51128404)(37.63681236) - (11.12948575)^2} \\ &\approx 0.75707560. \end{aligned}$$

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By (3.17) and setting $n = 47$, we obtain

$$b = \frac{\sum_{i=1}^{n} Y_i W_i - \alpha \sum_{i=1}^{n} S_i W_i}{\sum_{i=1}^{n} W_i} \approx \frac{-0.61712472 - (0.75707560)(11.12948575)}{3.51128404} \approx -2.57540737.$$

Inserting b and α in (3.18), we get

$$\beta = \exp\left(-\frac{b}{m}\right) \approx \exp\left(-\frac{-2.57540737}{0.75707560}\right) \approx 30.0175923.$$

WLSM with equation (3.19a) and F_i as given by (3.14d)

Note that $W_i = \left[(1-F_i)\ln(1-F_i)\right]^2$, $Y_i = \ln\ln\left[\dfrac{1}{1-F_i}\right]$, and $F_i = \dfrac{i-3/8}{n+1/4}$.

By (B.6) in Appendix B, we have

$\sum_{i=1}^{47} W_i(y_i) \approx 3.50016495$, $\sum_{i=1}^{47} S_i Y_i W_i(y_i) \approx -0.17535197$, $\sum_{i=1}^{47} S_i W_i(y_i) \approx 11.09237394$, $\sum_{i=1}^{47} Y_i W_i(y_i) \approx -0.61515518$, and $\sum_{i=1}^{47} S_i^2 W_i(y_i) \approx 37.48736716$. By (3.16) and setting $n = 47$, we estimate

$$\begin{aligned}
m = \alpha &= \frac{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i Y_i W_i - \sum_{i=1}^{n} S_i W_i \sum_{i=1}^{n} Y_i W_i}{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i^2 W_i - (\sum_{i=1}^{n} S_i W_i)^2} \\
&\approx \frac{(3.50016495)(-0.17535197) - (11.09237394)(-0.61515518)}{(3.50016495)(37.48736716) - (11.09237394)^2} \\
&\approx 0.75995737.
\end{aligned}$$

By (3.17) and setting $n = 47$, we obtain

$$b = \frac{\sum_{i=1}^{n} Y_i W_i - \alpha \sum_{i=1}^{n} S_i W_i}{\sum_{i=1}^{n} W_i} \approx \frac{-0.61515518 - (0.75995737)(11.09237394)}{3.500164949} \approx -2.58413149.$$

Inserting b and α in (3.18), we get

$$\beta = \exp\left(-\frac{b}{m}\right) \approx \exp\left(-\frac{-2.58413149}{0.75995737}\right) \approx 29.97500184.$$

Next, we will estimate parameters α and β by WLSM with equations (3.19b) and F_i given by equations (3.14a), (3.14b), (3.14c), and (3.14d), respectively. Firstly, we must compute $\sum_{i=1}^{47} W_i(y_i)$, $\sum_{i=1}^{47} S_i Y_i W_i(y_i)$, $\sum_{i=1}^{47} S_i W_i(y_i)$, $\sum_{i=1}^{47} Y_i W_i(y_i)$, and

$\sum_{i=1}^{47} S_i^2 W_i(y_i)$ (See Appendix B for details). These summations were used in (3.16) and (3.17).

WLSM with equation (3.19b) and F_i as given by (3.14a)

Note that $W_i = 3.3F_i - 27.5\left[1-(1-F_i)^{0.025}\right]$, $Y_i = \ln\ln\left[\dfrac{1}{1-F_i}\right]$, and $F_i = \dfrac{i-0.5}{n}$. By (B.7) in Appendix B, we have

$\sum_{i=1}^{47} W_i(y_i) \approx 46.22994532$, $\sum_{i=1}^{47} S_i Y_i W_i(y_i) \approx 24.01573359$, $\sum_{i=1}^{47} S_i W_i(y_i) \approx 151.51192964$,

$\sum_{i=1}^{47} Y_i W_i(y_i) \approx -3.57043308$, and $\sum_{i=1}^{47} S_i^2 W_i(y_i) \approx 541.38822118$. By (3.16) and setting $n = 47$, we estimate

$$m = \alpha = \frac{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i Y_i W_i - \sum_{i=1}^{n} S_i W_i \sum_{i=1}^{n} Y_i W_i}{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i^2 W_i - (\sum_{i=1}^{n} S_i W_i)^2}$$
$$\approx \frac{(46.22994532)(24.01573359) - (151.51192964)(-3.57043308)}{(46.22994532)(541.3882212) - (151.51192964)^2}$$
$$\approx 0.79672993.$$

By (3.17) and setting $n = 47$, we obtain

$$b = \frac{\sum_{i=1}^{n} Y_i W_i - \alpha \sum_{i=1}^{n} S_i W_i}{\sum_{i=1}^{n} W_i} \approx \frac{-3.570433081 - (0.79672993)(151.5119296)}{46.22994532} \approx -2.6883986.$$

Inserting b and α in (3.18), we get

$$\beta = \exp\left(-\frac{b}{m}\right) \approx \exp\left(-\frac{-2.6883986}{0.79672993}\right) \approx 29.20357146.$$

WLSM with equation (3.19b) and F_i as given by (3.14b)

Note that $W_i = 3.3F_i - 27.5\left[1-(1-F_i)^{0.025}\right]$, $Y_i = \ln\ln\left[\frac{1}{1-F_i}\right]$, and $F_i = \frac{i}{n+1}$. By (B.8) in Appendix B, we have

$\sum_{i=1}^{47} W_i(y_i) \approx 47.18265341$, $\sum_{i=1}^{47} S_i Y_i W_i(y_i) \approx 25.32254715$, $\sum_{i=1}^{47} S_i W_i(y_i) \approx 154.95191040$, $\sum_{i=1}^{47} Y_i W_i(y_i) \approx -3.66731344$, and $\sum_{i=1}^{47} S_i^2 W_i(y_i) \approx 557.34074630$. By (3.16) and setting $n = 47$, we estimate

$$
\begin{aligned}
m = \alpha &= \frac{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i Y_i W_i - \sum_{i=1}^{n} S_i W_i \sum_{i=1}^{n} Y_i W_i}{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i^2 W_i - (\sum_{i=1}^{n} S_i W_i)^2} \\
&\approx \frac{(47.18265341)(25.32254715) - (154.95191040)(-3.66731344)}{(47.18265341)(557.34074630) - (154.95191040)^2} \\
&\approx 0.77099148.
\end{aligned}
$$

By equation (3.17) and setting $n = 47$, we obtain

$$
b = \frac{\sum_{i=1}^{n} Y_i W_i - \alpha \sum_{i=1}^{n} S_i W_i}{\sum_{i=1}^{n} W_i} \approx \frac{-3.66731344 - (0.77099148)(154.95191040)}{47.18265341} \approx -2.60972851.
$$

We obtain parameter β by substituting b and α in equation (3.18), i.e.,

$$
\beta = \exp\left(-\frac{b}{m}\right) \approx \exp\left(-\frac{-2.60972851}{0.77099148}\right) \approx 29.51502032.
$$

WLSM with equation (3.19b) and F_i as given by (3.14c)

Note that $W_i = 3.3F_i - 27.5\left[1-(1-F_i)^{0.025}\right]$, $Y_i = \ln\ln\left[\frac{1}{1-F_i}\right]$, and $F_i = \frac{i-0.3}{n+0.4}$. By (B.9) in Appendix B, we have

$\sum_{i=1}^{47} W_i(y_i) \approx 46.65946112$, $\sum_{i=1}^{47} S_i Y_i W_i(y_i) \approx 24.88565987$, $\sum_{i=1}^{47} S_i W_i(y_i) \approx 153.11698387$, $\sum_{i=1}^{47} Y_i W_i(y_i) \approx -3.54541039$, and $\sum_{i=1}^{47} S_i^{\,2} W_i(y_i) \approx 548.87999728$. By (3.16) and setting $n = 47$, we estimate

$$
\begin{aligned}
m = \alpha &= \frac{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i Y_i W_i - \sum_{i=1}^{n} S_i W_i \sum_{i=1}^{n} Y_i W_i}{\sum_{i=1}^{n} W_i \sum_{i=1}^{n} S_i^2 W_i - (\sum_{i=1}^{n} S_i W_i)^2} \\
&\approx \frac{(46.65946112)(24.88565987) - (153.11698387)(-3.54541039)}{(46.65946112)(548.87999728) - (153.11698387)^2} \\
&\approx 0.78684298.
\end{aligned}
$$

By (3.17) and setting $n = 47$, we obtain

$$
b = \frac{\sum_{i=1}^{n} Y_i W_i - \alpha \sum_{i=1}^{n} S_i W_i}{\sum_{i=1}^{n} W_i} \approx \frac{-3.54541039 - (0.78684298)(153.11698387)}{46.65946112} \approx -2.65807687.
$$

Inserting b and α in (3.17), we get

$$
\beta = \exp\left(-\frac{b}{m}\right) \approx \exp\left(-\frac{-2.65807687}{0.78684298}\right) \approx 29.31660668.
$$

WLSM with equation (3.19b) and F_i as given by (3.14d)

Note that $W_i = 3.3F_i - 27.5\left[1 - (1 - F_i)^{0.025}\right]$, $Y_i = \ln\ln\left[\frac{1}{1 - F_i}\right]$, and $F_i = \frac{i - 3/8}{n + 1/4}$. By (B.10) in Appendix B, we have

$\sum_{i=1}^{47} W_i(y_i) \approx 46.50831852$, $\sum_{i=1}^{47} S_i Y_i W_i(y_i) \approx 24.63536317$, $\sum_{i=1}^{47} S_i W_i(y_i) \approx 152.56244221$,

$\sum_{i=1}^{47} Y_i W_i(y_i) \approx -3.54054183$, and $\sum_{i=1}^{47} S_i^{\,2} W_i(y_i) \approx 546.30092085$. By (3.16) and setting $n = 47$, we estimate

$$m=\alpha=\frac{\sum_{i=1}^{n}W_i\sum_{i=1}^{n}S_iY_iW_i-\sum_{i=1}^{n}S_iW_i\sum_{i=1}^{n}Y_iW_i}{\sum_{i=1}^{n}W_i\sum_{i=1}^{n}S_i^2W_i-(\sum_{i=1}^{n}S_iW_i)^2}$$
$$\approx\frac{(46.50831852)(24.63536317)-(152.56244221)(-3.54054183)}{(46.50831852)(546.30092085)-(152.56244221)^2}$$
$$\approx 0.79067283.$$

By (3.17) and setting $n=47,$ we calculate

$$b=\frac{\sum_{i=1}^{n}Y_iW_i-\alpha\sum_{i=1}^{n}S_iW_i}{\sum_{i=1}^{n}W_i}\approx\frac{-3.54054183-(0.79067283)(152.56244221)}{46.50831852}\approx -2.66979165.$$

We obtain parameter β by substituting b and α in equation (3.18), i.e.,

$$\beta=\exp\left(-\frac{b}{m}\right)\approx\exp\left(-\frac{-2.66979165}{0.79067283}\right)\approx 29.27129321.$$

The summary results of parameters estimation α and β using different estimation methods are shown in Table 3.11.

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Table 3.11 Shape α and scale β parameters using various estimation methods.

Method	Type	W_i	F_i	α	β
1	MLE	-	-	0.8633	28.8668
2	MOM (CV)	-	-	0.9286	30.0055
3	MOM (Cran)	-	-	0.9552	30.4239
4	LSM_1	-	3.14a	0.8580	28.6168
5	LSM_2	-	3.14b	0.7984	29.1888
6	LSM_3	-	3.14c	0.8310	28.8602
7	LSM_4	-	3.14d	0.8405	28.7721
8	WLSM_1	3.19a	3.14a	0.7647	29.9050
9	WLSM_2	3.19a	3.14b	0.7455	30.1924
10	WLSM_3	3.19a	3.14c	0.7571	30.0176
11	WLSM_4	3.19a	3.14d	0.7600	29.9750
12	WLSM_5	3.19b	3.14a	0.7967	29.2036
13	WLSM_6	3.19b	3.14b	0.7710	29.5150
14	WLSM_7	3.19b	3.14c	0.7868	29.3166
15	WLSM_8	3.19b	3.14d	0.7907	29.2713

3.4 Chi-Squared Test

The chi-squared test statistic is defined as $\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$ where k is the total number of intervals, O_i is the observed frequency for interval i, E_i is the expected frequency for interval i, and

$$E_i = n[F(y_i) - F(y_{i-1})],\ i = 1,\ 2, \ldots, F(y_0) = 0.$$

Here n is the sample size, F is the cumulative distribution function as in (3.2), and y_i, y_{i-1} are the endpoints of the interval.

We performed the chi-squared goodness of fit for all methods in Table 3.11. The null hypothesis H_0: data is assumed Weibull(α, β). We found that the chi-squared value is less than the chi-squared critical value for degree of freedom 4 at a significance level of 0.05.

For example, H_0: data is assumed Weibull ($\alpha = 0.9286, \beta = 30.0055$). The chi-squared critical value for degree of freedom 4 at a significance level of 0.05 is 9.49, whereas the chi-squared value is 4.0569 (see Table 3.12). Thus we can assume that the distribution of the data (in Table 3.1) is Weibull at a 5% degree of significance.

Table 3.12 Chi-squared statistic, $\alpha = 0.9286$ and $\beta = 30.0055$.

Row i	y_i	$F(y_i) - F(y_{i-1})$	E_i	O_i	$(O_i - E_i)^2 / E_i$
1	6	0.20094	9.4441	11	0.2563
2	12	0.14658	6.8892	7	0.0018
3	18	0.11571	5.4384	6	0.0580
4	30	0.16883	7.9350	3	3.0692
5	42	0.11295	5.3088	7	0.5388
6	66	0.12996	6.1082	7	0.1302
7	∞	0.12503	5.8763	6	0.0026
	Totals	1	47	47	4.0569

3.5 Randomized Neighborhood Search (RNS)

Randomized neighborhood search is a numerical optimization method whose objective functions may be discontinuous and non-differentiable. This optimization is also known as a direct-search or derivative-free method. Randomized neighborhood search operates by iterative random moving from the initial solution to a better solution. In this research, RNS is applied for minimizing the chi-squared value.

RNS Algorithm

The RNS algorithm is as follows:

Step 1 Start from the initial parameters α and β. Compute the chi-squared value.

Step 2 Randomly change the value α to α' and β to β'. We can do this by choosing a uniform variate u from the interval [0,1] and let

$$\alpha' = \alpha + 2(0.5 - u)(0.1998),$$

$$\beta' = \beta + 2(0.5 - u)(4.9950).$$

Step 3 Compute chi-squared value with α' and β'.

Step 4 Compare the chi-squared values which were obtained from steps 1 and 3.

If the chi-squared value of step 3 is greater than or equal to that of step 1, then repeat step 2.

If not, we set $\alpha = \alpha'$ and $\beta = \beta'$ then go on to step 2.

Step 5 Repeat until a termination criterion is met (adequate fitness reached).

From Table 3.1, we compute the mean (μ) and variance (σ^2):

when we replace μ and σ in (3.10) and then approximate α by bisection, we get α =0.9286. The approximate value of β = 30.0055 can be obtained from (3.8). These two parameters α and β were used as the initial parameters for the RNS algorithm. We iterated RNS 10,000 times and obtained the results shown in Table 3.13. Table 3.14 shows the shape parameter α, scale parameter β and chi-squared value using different estimation methods.

Table 3.13 Parameters α, β and chi-squared value by RNS.

Times	α	β	χ^2
1	0.9286000000	30.0055000000	4.0569000000
2	0.8381502696	33.0173413017	3.2266108642
3	0.8381502696	33.0173413017	3.2266108642
4	0.8381502696	33.0173413017	3.2266108642
5	0.8381502696	33.0173413017	3.2266108642
6	0.7076414583	28.7762666642	2.6481293827
7	0.7076414583	28.7762666642	2.6481293827
8	0.7076414583	28.7762666642	2.6481293827
9	0.7076414583	28.7762666642	2.6481293827
10	0.7076414583	28.7762666642	2.6481293827
20	0.7095244694	29.5717808657	2.6140305005
30	0.7148708496	26.9714010325	2.5856511398
40	0.7148708496	26.9714010325	2.5856511398
50	0.7148708496	26.9714010325	2.5856511398
60	0.7148708496	26.9714010325	2.5856511398
70	0.7148708496	26.9714010325	2.5856511398
80	0.7131632905	30.1643026571	2.5559654901
90	0.7131632905	30.1643026571	2.5559654901
100	0.7131632905	30.1643026571	2.5559654901
200	0.7160097628	28.1949938030	2.4788283052
300	0.7160097628	28.1949938030	2.4788283052
400	0.7160097628	28.1949938030	2.4788283052
500	0.7160097628	28.1949938030	2.4788283052
600	0.7160097628	28.1949938030	2.4788283052
700	0.7160097628	28.1949938030	2.4788283052
800	0.7157118026	28.4146840369	2.4783086176
900	0.7157118026	28.4146840369	2.4783086176
1,000	0.7158868924	28.4191078089	2.4745204778
2,000	0.7157825238	28.7701321511	2.4707423531
3,000	0.7158410970	28.7428309644	2.4697671190
4,000	0.7158324217	28.7679580993	2.4697088719
5,000	0.7158182813	28.8071827518	2.4696879774
6,000	0.7158147162	28.8246778976	2.4696432384
7,000	0.7158161544	28.8206039617	2.4696395832
8,000	0.7158168650	28.8183923908	2.4696393961
9,000	0.7158170080	28.8179670891	2.4696392513
10,000	0.7158169062	28.8182970081	2.4696391693

Table 3.14 Chi-squared value for various estimation methods.

Method	Type	α	β	χ^2
1	MLE	0.8633	28.8668	5.9412
2	MOM (CV)	0.9286	30.0055	4.0569
3	MOM (Cran)	0.9552	30.4239	4.4097
4	LSM_1	0.8580	28.6168	5.9758
5	LSM_2	0.7984	29.1888	3.4099
6	LSM_3	0.8310	28.8602	6.0731
7	LSM_4	0.8405	28.7721	6.0239
8	WLSM_1	0.7647	29.9050	3.7284
9	WLSM_2	0.7455	30.1924	3.4214
10	WLSM_3	0.7571	30.0176	3.8360
11	WLSM_4	0.7600	29.9750	3.7936
12	WLSM_5	0.7967	29.2036	3.4216
13	WLSM_6	0.7710	29.5150	3.6609
14	WLSM_7	0.7868	29.3166	3.4988
15	WLSM_8	0.7907	29.2713	3.4662
16	RNS	0.7158	28.8183	2.4696

In this study, we have used RNS to estimate the Weibull parameters for the claim severities which considered as excess of 20 million baht. Table 3.14 shows RNS has the smallest chi-squared value (i.e., chi-squared value = 2.4696). Therefore RNS gives a more accurate estimation of parameters than do MLE, MOM, LSM or WLSM.

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CHAPTER IV

MINIMUM INITIAL CAPITAL PROBLEM

In this chapter, we computed the minimum initial capital an insurance company has to hold for ensuring that the ruin probability is not greater than a given quantity. The claims are considered from motor insurance under two scenarios: where the claims happen every day and where the claims do not happen every day. We estimate parameters for some selected distributions by applying the randomized neighborhood search (RNS) technique introduced in Chapter III. In this Chapter, we apply the simulation method and regression analysis to approximate the minimum initial capital.

4.1 Model Description

We assume that the insurance company can be allowed to invest in non-risky investments with a constant rate of return (interest rate) r and we are interested in two types of the insurance companies: where the claim severities happen every day, say Class A, and do not happen every day, say Class B. Moreover, for each class we will consider two types of claim severities Y_n. For the first type, we considered claim severities Y_n of arbitrary size. For the second type, we considered claim severities Y_n in the form of standard claims V_n or large claims W_n.

In what follows, we first recall the surplus processes in both Class A and Class B.

Class A : the claim severities happen every day

Case 1. Claim severities Y_n of arbitrary size

The surplus process is of the form

$$\begin{aligned} &U_0 = u, \\ &U_n = U_{n-1}(1 + r_0) + c - Y_n, 1n = 1,2,3, \dots, \end{aligned} \tag{4.1}$$

where

- $\{Y_n,\ n \in \mathbb{N}\}$ is an i.i.d. process,
- u is the initial capital,
- r_0 is the daily interest rate which is defined by $r_0 = (1 + r)^{\frac{1}{365}} - 1$, $r = 2\%$ to $r = 8\%$ per annum,
- c is the premium rate for one unit time which is computed by

$$c = (1 + \theta)\frac{EY_1}{EZ_1} = (1 + \theta)EY_1.$$

Case 2. Claim severities Y_n in the form of standard claims V_n or large claims W_n

Let $\{T_n^L, n \in \mathbb{N}\}$ be the arrival time process of large claims. The inter arrival time process $\{Z_n^L,\ n \in \mathbb{N}\}$ of the arrival time process $\{T_n^L, n \in \mathbb{N}\}$ is assumed to be i.i.d. such that $Z_1^L \sim \text{Poisson}(\lambda^L)$. Therefore, the surplus process is given by

$$\begin{aligned} &U_0 = u, \\ &U_n = \begin{cases} U_{n-1}(1 + r_0) + \ c - W_n, & n = T_k^L, \text{afor some } k = 1,2,3, \dots, \\ U_{n-1}(1 + r_0) + \ c - V_n, & n \neq T_k^L, \text{afor all } k = 1,2,3, \dots, \end{cases} \end{aligned} \tag{4.2}$$

for all $n = 1,2,3, \dots$ where

- the two processes, $\{V_n,\ n \in \mathbb{N}\}$ and $\{W_n,\ n \in \mathbb{N}\}$ are i.i.d. random variables and mutually independent,
- u is the initial capital,

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- r_0 is as mentioned in previous case
- c is the premium rate for one unit time which is computed by

$$c = (1+\theta)\left(\frac{EW_1}{EZ_1^L} + EV_1\right).$$

Class B : the claim severities do not happen every day

Case 1. Claim severities $\boldsymbol{Y_n}$ of arbitrary size

The surplus process is of the form

$$U_0 = u,$$
$$U_n = U_{n-1}(1+r_0)^{Z_n} + c\sum_{k=0}^{Z_n-1}(1+r_0)^k - Y_n, an = 1,2,3,..., \quad (4.3)$$

where

- $\{Y_n,\ n \in \mathbb{N}\}$ is an i.i.d. process,
- u is the initial capital,
- r_0 is the daily interest rate which is defined by $r_0 = (1+r)^{\frac{1}{365}} - 1$, $r = 2\%$ to $r = 8\%$ per annum,
- c is the premium rate for one unit time which is computed by

$$c = (1+\theta)\frac{EY_1}{EZ_1}.$$

Case 2. Claim severities $\boldsymbol{Y_n}$ in the form of standard claims $\boldsymbol{V_n}$ or large claims $\boldsymbol{W_n}$

Let $\{T_n^L, n \in \mathbb{N}\}$ be the arrival time process of large claims. The inter arrival time process $\{Z_n^L,\ n \in \mathbb{N}\}$ of the arrival time process $\{T_n^L, n \in \mathbb{N}\}$ is assumed to be i.i.d. such that $Z_1^L \sim \text{Poisson}(\lambda^L)$. Let $\{T_n^l, n \in \mathbb{N}\}$ be the arrival time process of standard claims. The inter arrival time process $\{Z_n^l,\ n \in \mathbb{N}\}$ of the arrival time process

$\{T_n^l, n \in \mathbb{N}\}$ is assumed to be i.i.d. such that $Z_1^l \sim \text{Poisson}(\lambda^l)$. Therefore, the surplus process is given by

$$U_0 = u,$$

$$U_n = \begin{cases} U_{n-1}(1+r_0)^{Z_n} + c\sum_{k=0}^{Z_n-1}(1+r_0)^k - W_n, n = T_k^L, \text{afor some} \\ \quad k = 1,2,3,\ldots, \\ U_{n-1}(1+r_0)^{Z_n} + c\sum_{k=0}^{Z_n-1}(1+r_0)^k - V_n, n \neq T_k^L, a\text{for all k} \\ \quad k = 1,2,3,\ldots, \end{cases} \tag{4.4}$$

for all $n = 1,2,3,\ldots$, where

- the two processes, $\{V_n,\ n \in \mathbb{N}\}$ and $\{W_n,\ n \in \mathbb{N}\}$ are i.i.d. random variables and mutually independent,
- u is the initial capital,
- r_0 is as mentioned in the previous case,
- c is the premium rate for one unit time which is computed by

$$c = (1+\theta)\left(\frac{EW_1}{EZ_1^L} + \frac{EV_1}{EZ_1^l}\right).$$

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4.2 Parameter Estimation of Motor Insurance

Now we apply the RNS to estimate parameters of claim severities in Class A and Class B for some selected distributions.

4.2.1 Class A: Claims happen every day

Case 1. Claim severities Y_n of arbitrary size

The data of motor insurance claims are shown in Figure 4.1.

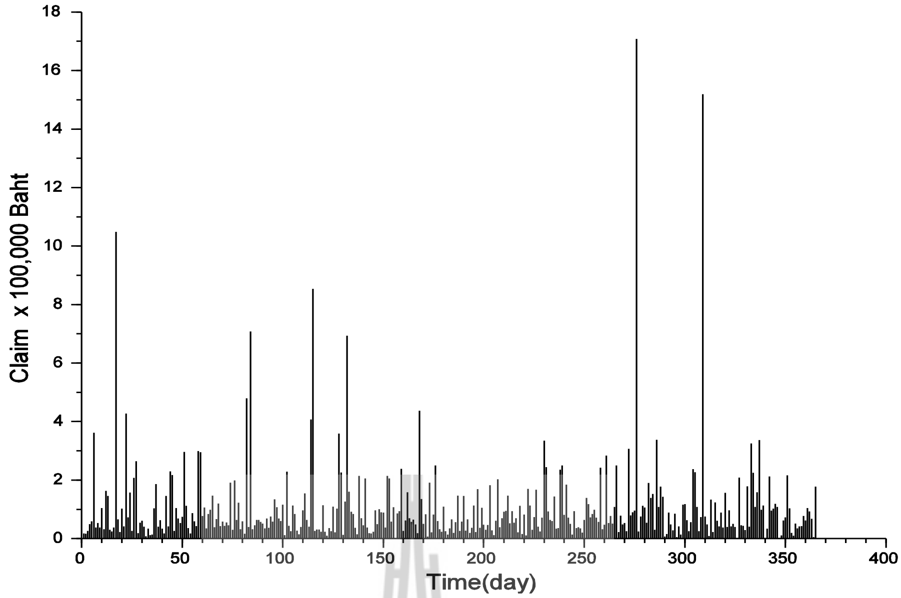

Figure 4.1 Claim severities (Class A).

Next, we estimated the parameters of the claim severities Y_n in Figure 4.1.

Parameter Estimation for Log normal Distribution

Step 1. Suppose that $Y_1, Y_2, \ldots, Y_n$ are i.i.d. random variables with log normal distribution as mentioned in Chapter II, i.e.,

$$f(y;(\mu,\ \sigma^2)) = \frac{1}{\sigma y\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{\ln y - \mu}{\sigma}\right)^2\right);\ 0 < y < \infty.$$

By equation (2.3) and (2.4), we obtain

$$\hat{\mu} = \frac{1}{n}\sum_{k=1}^{n} \ln y_k \ \text{ and } \ \hat{\sigma} = \sqrt{\frac{1}{n}\sum_{k=1}^{n}(\ln y_k - \hat{\mu})^2}. \tag{4.5}$$

By equation (4.5) and using data from Figure 4.1, we have μ =11.06300 and σ =0.99392. These two parameters were used as the initial parameters for the RNS algorithm.

Step 2. In the second step of RNS, we set

$$\mu' = \mu + 2(0.5 - u)(0.1998),$$

$$\sigma' = \sigma + 2(0.5 - u)(9.9900),$$

where u is a uniform variate which is chosen from the interval $[0,1]$. We iterate RNS 10,000 times and obtain the results shown in Table 4.1.

Table 4.1 Parameter $\mu = 11.06300$, $\sigma = 0.99392$ and chi-squared value.

Number of interations	μ	σ	Chi-squared value
1	11.06300	0.99392	5.84276
10	11.24731	0.79862	3.31241
100	11.24119	0.87107	2.96807
1,000	11.08944	0.93456	1.23767
10,000	11.08142	0.93882	1.23460

Case 2. Claim severities Y_n in the form of standard claims V_n or large claims W_n

Standard claim severities

Note that the average of claim severities in Figure 4.1 is 105,718.31507 Baht. Figure 4.2 shows the standard claim severities, i.e., the cost of the claims is less than or equal 105,718.31507 Baht.

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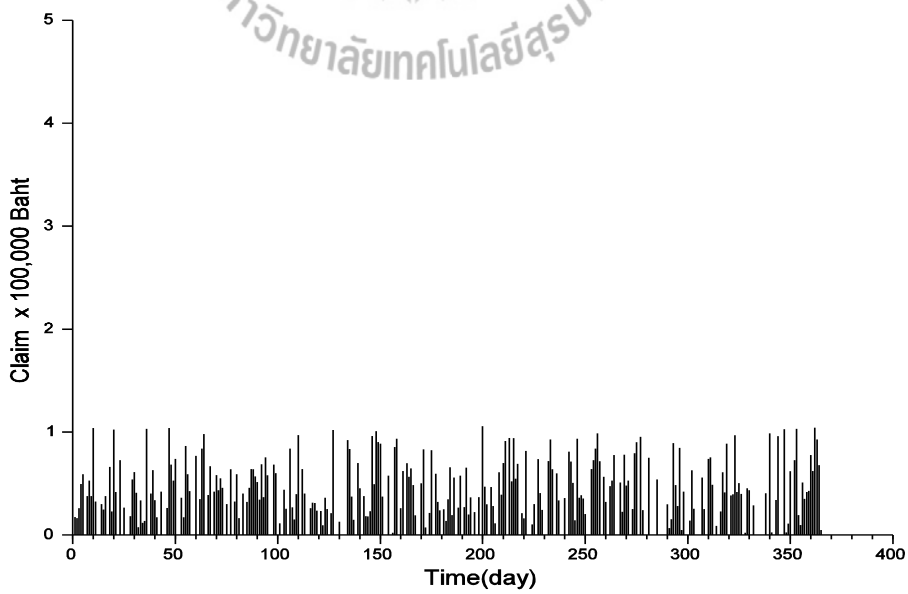

Figure 4.2 Standard claim severities (Class A).

Next, we will estimate the parameters of the standard claim severities.

Parameter Estimation for Weibull Distribution

Step 1. Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. variables with Weibull distribution as mentioned in Chapter II, i.e.,

$$f(x;(\alpha,\beta)) = \frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left(-\left(\frac{x}{\beta}\right)^{\alpha}\right), \; x \geq 0.$$

By equation (2.5) and (2.6), we obtain

$$\hat{\beta} = \left(\frac{1}{n}\sum_{i=1}^{n} x_i^{\alpha}\right)^{\frac{1}{\alpha}}, \tag{4.6}$$

and

$$\alpha = \left[\frac{\sum_{i=1}^{n} x_i^{\alpha} \ln x_i}{\sum_{i=1}^{n} x_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n} \ln x_i\right]^{-1}. \tag{4.7}$$

We solve α in equation (4.7) by the Newton-Raphson method with data in the Figure 4.2 and get $\alpha = 0.30772$. By inserting α into (4.6), we obtain $\beta = 13{,}986.33219$. These two parameters α and β will be used as the initial parameters for the RNS algorithm.

Step 2. In the second step of RNS, we set

$$\alpha' = \alpha + 2(0.5-u)(0.1998),$$
$$\beta' = \beta + 2(0.5-u)(9{,}990),$$

where u is a uniform variate which is chosen from the interval [0,1]. We iterate RNS 10,000 times and obtain the results that shown in Table 4.2.

Table 4.2 Parameter $\alpha = 0.30772$, $\beta = 13{,}986.33219$ and chi-squared value.

Number of interations	α	β	Chi-squared value
1	0.30772	13,986.33291	847.25120
10	1.67086	57,486.73701	8.82050
100	1.81108	56,219.75446	5.90283
1,000	1.81367	55,983.49513	5.89124
10,000	1.81280	56,012.19774	5.89089

Large Claim severities

The average of claim severities in Figure 4.1 is 105,718.31507 Baht. In Figure 4.3 shows the large claim severities, i.e., the cost of the claims is greater than 105,718.31507 Baht.

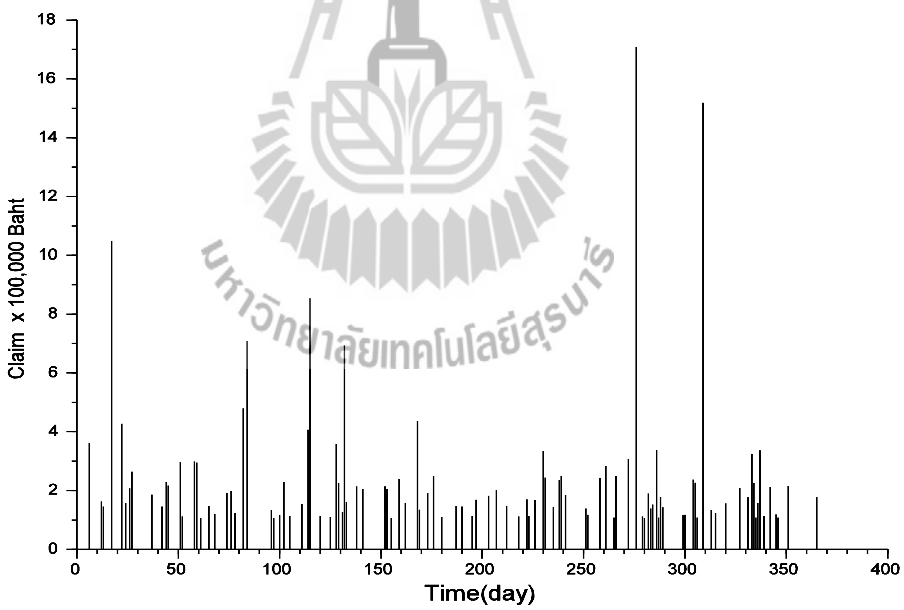

Figure 4.3 Large claim severities w_i (Class A).

The claim severities s_i as shown in Figure 4.4 represent the portion of the claims exceeding the average claim severity, i.e., $s_i = w_i - 105{,}718.31507$ where w_i is the claim severity as shown in Figure 4.3. For convenience, we still call the amount s_i a large claim severity.

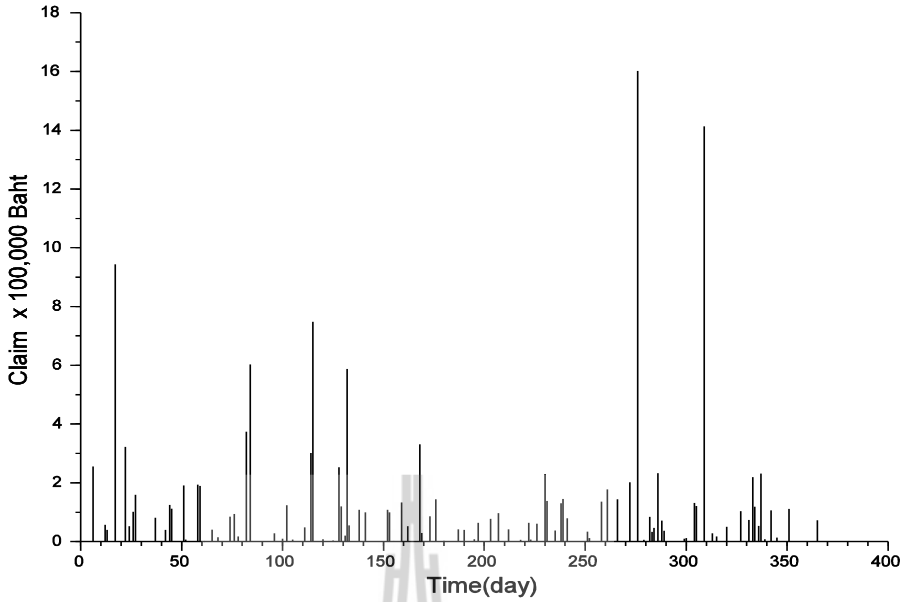

Figure 4.4 Large claim severities s_i (Class A).

Parameter Estimation for Weibull Distribution

Step 1. Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. variables with Weibull distribution as stated in Chapter II. By equation (2.5) and (2.6), we have

$$\hat{\beta} = \left(\frac{1}{n} \sum_{i=1}^{n} x_i^{\alpha} \right)^{\frac{1}{\alpha}}, \tag{4.8}$$

and

$$\alpha = \left[\frac{\sum_{i=1}^{n} x_i^{\alpha} \ln x_i}{\sum_{i=1}^{n} x_i^{\alpha}} - \frac{1}{n} \sum_{i=1}^{n} \ln x_i \right]^{-1}. \tag{4.9}$$

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We solve for α in equation (4.9) by The Newton-Raphson method with the data of Figure 4.4 and get $\alpha = 0.71562$. By inserting α into (4.8), we obtain $\beta =$ 107,096.08173. These two parameters α and β were used as the initial parameters for RNS algorithm.

Step 2. In the second step of RNS, we set

$$\alpha^{/} = \alpha + 2(0.5 - u)(0.8910),$$
$$\beta^{/} = \beta + 2(0.5 - u)(9,900),$$

where u is a uniform variate which is chosen from the interval [0,1]. Table 4.3 shows the results after 10,000 RNS iterations.

Table 4.3 Parameter $\alpha = 0.71562$, $\beta = 107{,}096.08173$ and chi-squared value.

Number of interations	α	β	Chi-squared value
1	0.71562	107,096.08173	13.20821
10	1.05933	109,933.04886	6.71933
100	0.96780	101,715.89982	5.32939
1,000	0.95217	100,025.86779	5.28664
10,000	0.95217	100,025.86779	5.28664

4.2.2 Class B claims do not happen every day

Case 1. Claim severities Y_n of arbitrary size

The data of motor insurance claims are shown in Figure 4.5.

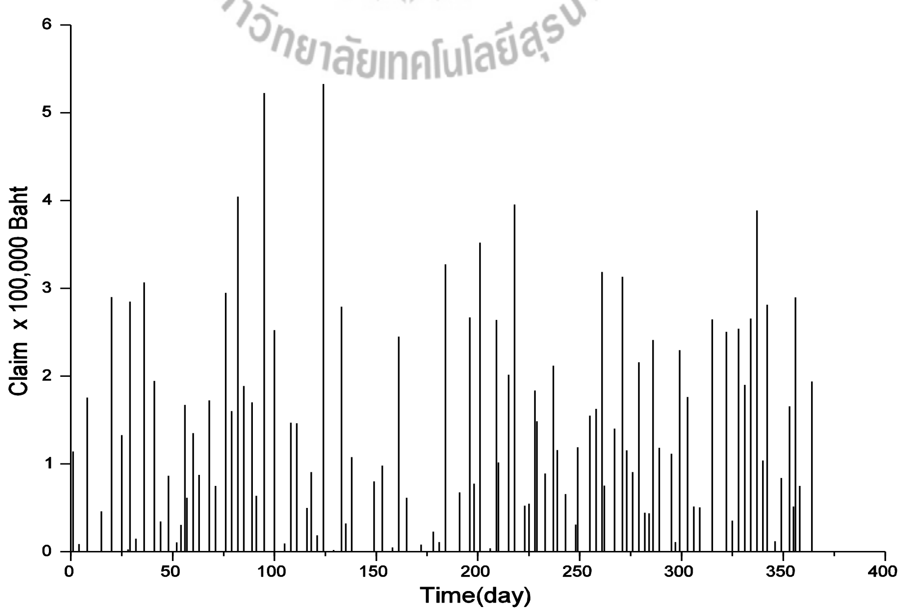

Figure 4.5 Claim severities (Class B).

Next, we estimated parameters of claim severities Y_n.

Parameter Estimation for the Weibull Distribution

Step 1. Suppose that $Y_1, Y_2, \ldots, Y_n$ are i.i.d. variables with Weibull density as stated in Chapter II, i.e.,

$$f(y;(\alpha,\beta)) = \frac{\alpha}{\beta}\left(\frac{y}{\beta}\right)^{\alpha-1} \exp\left(-\left(\frac{y}{\beta}\right)^{\alpha}\right), \; y \geq 0.$$

By equation (2.5) and (2.6), we obtain

$$\hat{\beta} = \left(\frac{1}{n}\sum_{i=1}^{n} y_i^{\alpha}\right)^{\frac{1}{\alpha}}, \tag{4.10}$$

and

$$\alpha = \left[\frac{\sum_{i=1}^{n} y_i^{\alpha} \ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n} \ln y_i\right]^{-1}. \tag{4.11}$$

We can solve for α in (4.11) by the Newton-Raphson method and obtain $\alpha =$ 1.16437. By inserting the α into (4.10), we get $\beta = 154{,}408.43936$. These two parameters α and β were used as the initial parameters for RNS algorithm.

Step 2. In the second step of RNS, we set

$$\alpha' = \alpha + 2(0.5 - u)(0.3960),$$

$$\beta' = \beta + 2(0.5 - u)(99),$$

where u is a uniform variate which is chosen from the interval [0,1]. Table 4.4 shows the results after 10,000 RNS iterations.

Table 4.4 Parameter $\alpha = 1.16437$, $\beta = 154{,}408.43936$ and chi-squared value.

Number of iterations	α	β	Chi-squared value
1	1.16437	154,408.43936	3.79761
10	1.22294	154,385.35203	3.49774
100	1.22753	154,544.49528	3.48842
1,000	1.22748	155,025.62699	3.46303
10,000	1.22747	155,025.98864	3.46301

Case 2. Claim severities Y_n in the form of standard claims V_n or large claims W_n

Standard claim severities

The average of claim severities in Figure 4.5 is 146,953.19417 Baht. Figure 4.6 shows the standard claim severities, i.e., the cost of claims is less than or equal 146,953.19417 Baht.

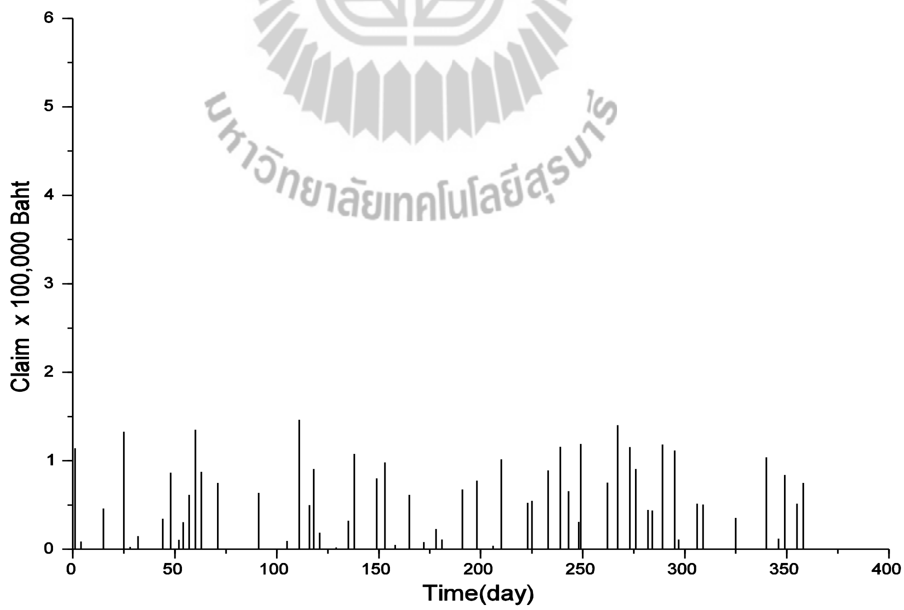

Figure 4.6 Standard claim severities (Class B).

Parameter Estimation for Log logistic Distribution

Step 1. Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. variables with log logistic density as mentioned in Chapter II, i.e.,

$$f(x;(a,b,c)) = \frac{b}{a}\left(\frac{x-c}{a}\right)^{b-1}\left(1+\left(\frac{x-c}{a}\right)^{b}\right)^{-2}, \; c < x < \infty.$$

Singh and Guo expressed the estimation equations

$$2\sum_{i=1}^{n}\left[\frac{\left(\frac{x_i-c}{a}\right)^{b}}{1+\left(\frac{x_i-c}{a}\right)^{b}}\right] = n, \tag{4.12}$$

and

$$2b\sum_{i=1}^{n}\left[\frac{\left(\ln\left(\frac{x_i-c}{a}\right)\right)\left(\frac{x_i-c}{a}\right)^{b}}{1+\left(\frac{x_i-c}{a}\right)^{b}}\right] - b\sum_{i=1}^{n}\ln\left(\frac{x_i-c}{a}\right) - n = 0 \tag{4.13}$$

$$2b\sum_{i=1}^{n}\left[\frac{\left(\frac{x_i-c}{a}\right)^{b}}{1+\left(\frac{x_i-c}{a}\right)^{b}}\right] - a(b-1)\sum_{i=1}^{n}\frac{1}{x_i-c} = 0 \tag{4.14}$$

where n is the sample size. We estimate parameters a, b and c by using an iterative scheme. First, with an assumed value of b and c, equation (4.12) is solved for a. With this value of a and the initial guess of c, equation (4.13) is solved to give a new value of b. Then, a new value of c is calculated from equation (4.14). By the above process, we get $a =$ 209,629.29688, $b =$ 8.530133 and $c = -$144,611.60680. These three parameters $a,$ b and c were used as the initial parameters for RNS algorithm.

Step 2. In the second step of RNS, we set

$$a' = a + 2(0.5 - u)(9.9900),$$

$$b' = b + 2(0.5 - u)(0.0999),$$

$$c' = c + 2(0.5 - u)(9.9900),$$

where u is a uniform variate which is chosen from the interval $[0,1]$. Table 4.5 shows the results after 10,000 RNS iterations.

Table 4.5 Parameter $a = 209{,}629.29688$, $b = 8.530133$, $c = -144{,}611.60680$ and chi-squared value.

Number of iterations	a	b	c	Chi-squared value
1	209,629.29688	8.53013	−144,611.60680	5.88220
10	209,628.32501	7.89717	−144,620.02978	3.81220
100	209,357.29532	6.70409	−144,917.26798	2.07037
1,000	207,495.59645	6.86245	−146,908.09926	1.68737
10,000	208,015.54932	6.90185	−147,922.34491	1.67697

Large Claim severities

The average of the claim severities in Figure 4.5 is 146,953.19417 Baht. Figure 4.7 shows the large claim severities, i.e., the cost of claims is greater than 146,953.19417 Baht.

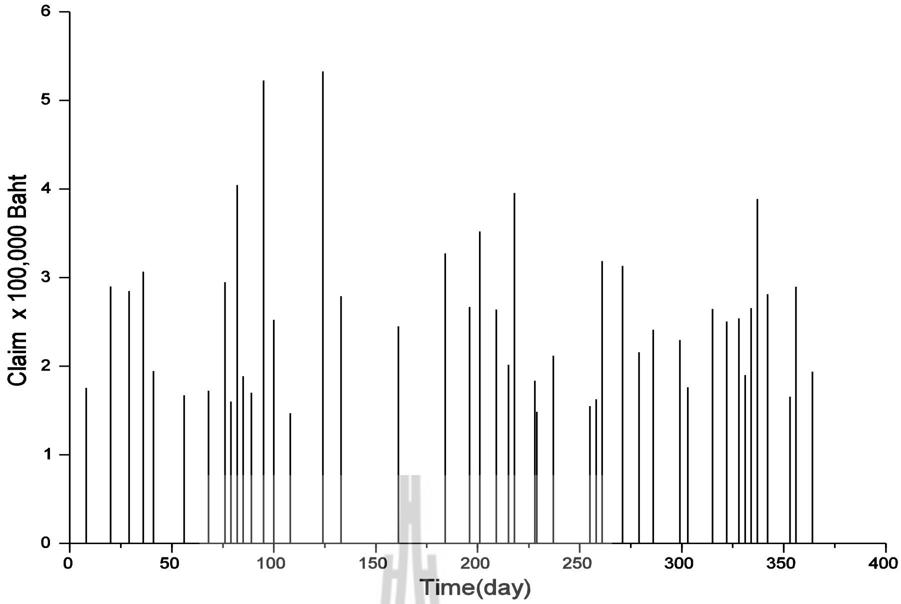

Figure 4.7 Large claim severities w_i (Class B).

The claim severities s_i as shown in Figure 4.8 represent amount in excess of average the claim, i.e., $s_i = w_i - 146{,}953.19417$ where w_i are the claim severities shown in Figure 4.7. For convenience, we still call the amount s_i large claim severities.

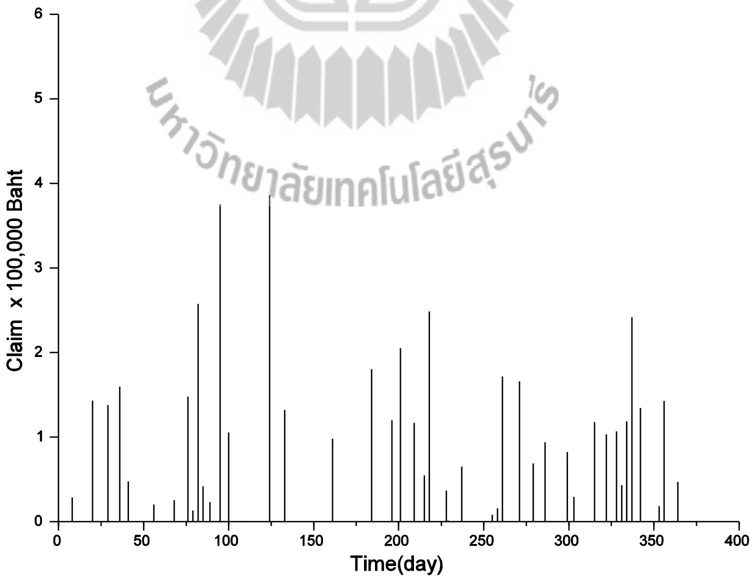

Figure 4.8 Large claim severities s_i (Class B).

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Parameter Estimation for Normal Distribution

Step 1. Suppose that $X_1, X_2, \ldots, X_n$ are i.i.d. variables with normal density (μ, σ^2) as indicated in Chapter II, i.e.,

$$f(x;\mu,\sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; -\infty < x < \infty.$$

By equation (2.1) and (2.2), we obtain

$$\hat{\mu} = \left(\frac{1}{n}\sum_{i=1}^{n} x_i\right), \tag{4.15}$$

and

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \mu\right)^2. \tag{4.16}$$

By equation (4.15) and (4.16) with the data of Figure 4.8, we estimate $\mu =$ 108,502.9836 and $\sigma^2 = 8{,}021{,}344{,}204.8573$. It follows $\sigma = 89{,}561.95735$. These two parameters μ and σ were used as the initial parameters for RNS algorithm.

Step 2. In the second step of RNS, we set

$$\mu' = \mu + 2(0.5 - u)(0.0999),$$

$$\sigma' = \sigma + 2(0.5 - u)(0.0999),$$

where u is a uniform variate which is chosen from the interval [0,1]. We iterate RNS 10,000 times and obtain the results that shown in Table 4.6.

Table 4.6 Parameter $\mu = 108{,}502.9836$, $\sigma = 89{,}561.95735$ and chi-squared value.

Number of iterations	μ	σ	Chi-squared value
1	108,502.98360	89,561.95735	3.66728
10	108,502.43336	89,562.52805	3.66711
100	108,496.90933	89,567.28180	3.66553
1,000	108,461.24015	89,598.29757	3.65530
10,000	108,436.30790	89,619.11735	3.64829

Distribution of the data in Figure 4.1, Figure 4.6, and Figure 4.8 are log normal, log logistic, and normal at 0.05 degree of significance, respectively. Distribution of data in Figure 4.2, Figure 4.4, and Figure 4.5 are Weibull at 0.05 degree of significance. The summary results of parameter estimation in each case are shown in Table 4.7 and Table 4.8.

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Table 4.7 Class A, claim severities happen every day.

Distributions	PDF	MLE	Chi-squared value	RNS	Chi-squared value
Claims of any size: log-normal distribution.	$\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln x-\mu}{\sigma}\right)^2}$	$\mu = 11.0630$ $\sigma = 0.99392$	5.84276	$\mu = 11.08142$ $\sigma = 0.93882$	1.23460
Standard claims: Weibull distribution.	$\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$	$\alpha = 0.30772$ $\beta = 13,986.33219$	847.25120	$\alpha = 1.81280$ $\beta = 56,012.19774$	5.89089
Large claims: Weibull distribution.	$\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$	$\alpha = 0.71562$ $\beta = 107,096.08173$	13.20812	$\alpha = 0.95217$ $\beta = 100,025.86779$	5.28664

Table 4.8 Class B, claim severities do not happen every day.

Distributions	PDF	MLE	Chi-squared value	RNS	Chi-squared value
Claims of any size: Weibull distribution.	$\frac{\alpha}{\beta}\left(\frac{x}{\beta}\right)^{\alpha-1}e^{-\left(\frac{x}{\beta}\right)^{\alpha}}$	$\mu = 1.16437$ $\sigma = 154,408.43936$	3.79761	$\mu = 1.22747$ $\sigma = 155,025.98864$	3.46301
Standard claims: log logistic distribution .	$\frac{b}{a}\left(\frac{x-c}{a}\right)^{b-1}\left(1+\left(\frac{x-c}{a}\right)^{b}\right)^{-2}$	$a = 209,629.29688$ $b = 8.530133$ $c = -144,611.60680$	5.88220	$a = 208,015.54932$ $b = 6.90185$ $c = -147,922.34491$	1.67697
Large claims: Normal distribution.	$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$	$\mu = 108,502.9836$ $\sigma = 89,561.95735$	3.66728	$\mu = 108,436.30790$ $\sigma = 89,619.11735$	3.64829

4.3 Ruin Probability Simulation

In this part, we consider (4.1)-(4.4) under the condition that $0 \le T_1 \le \ldots \le T_n \le \tilde{T}$ when $\tilde{T} = 1$ year (365 days). A simulation method is used to compute the finite time ruin probability, $P\left(\{U_n < 0 \;\; \exists n \in \mathbb{N}\}, \sum_{i=1}^{n} Z_i \le 365 \mid U_0 = u \right)$. The flowcharts for computing the ruin probabilities of equations (4.1)-(4.4) are listed in the Figures 4.9-4.12, respectively.

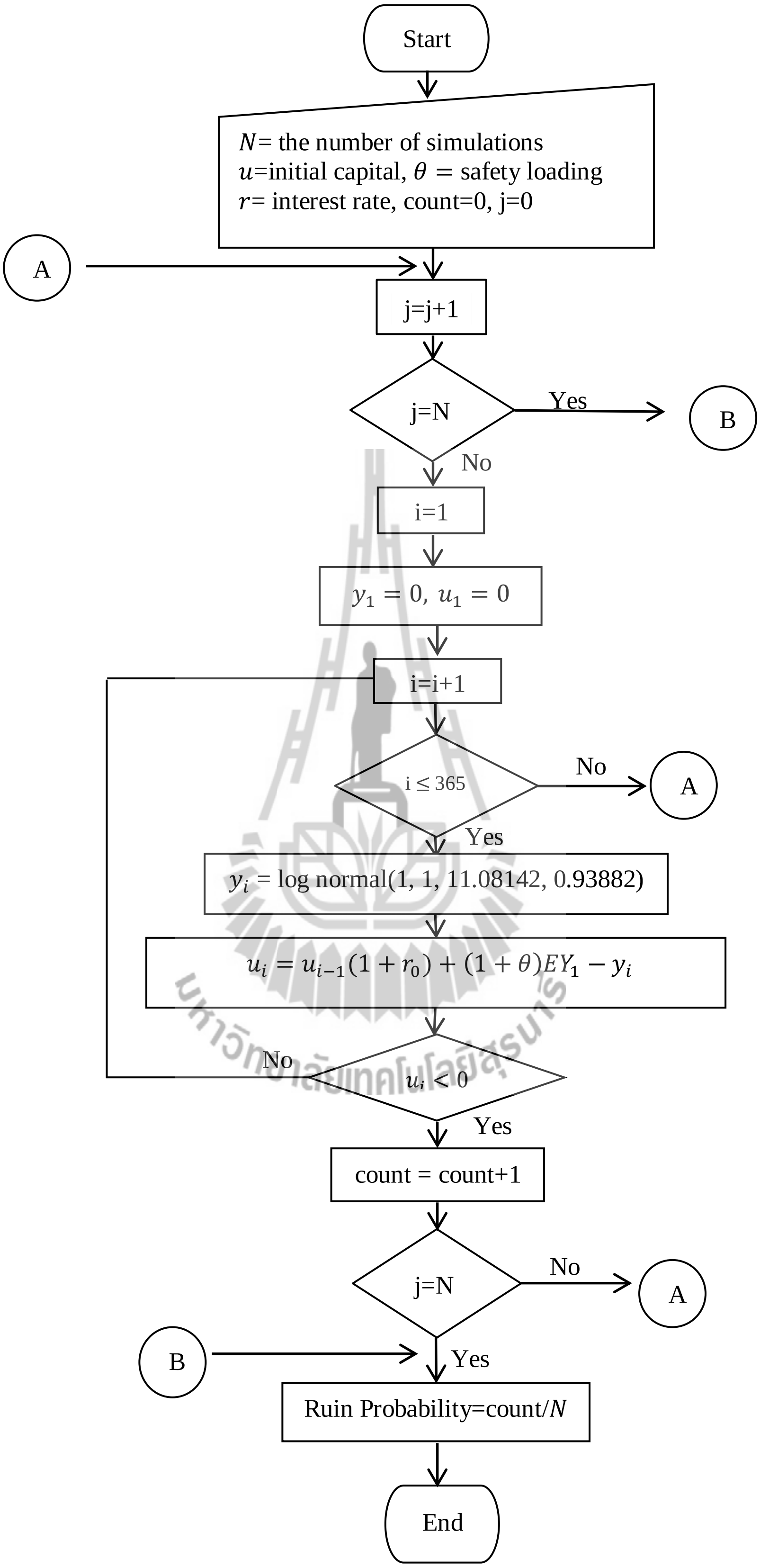

Figure 4.9 Flowchart for computing the ruin probability of equation (4.1).

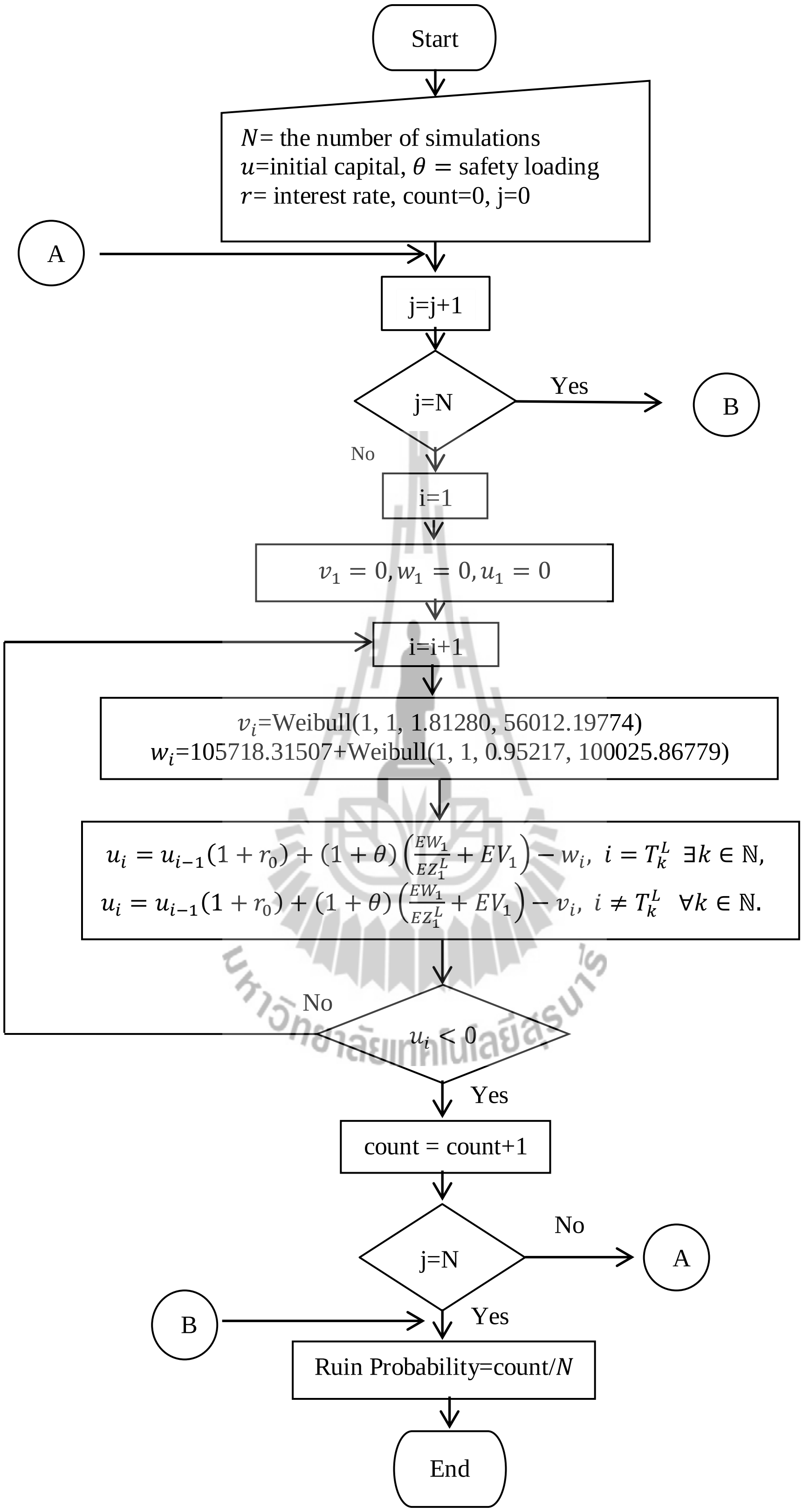

Figure 4.10 Flowchart for computing the ruin probability of equation (4.2).

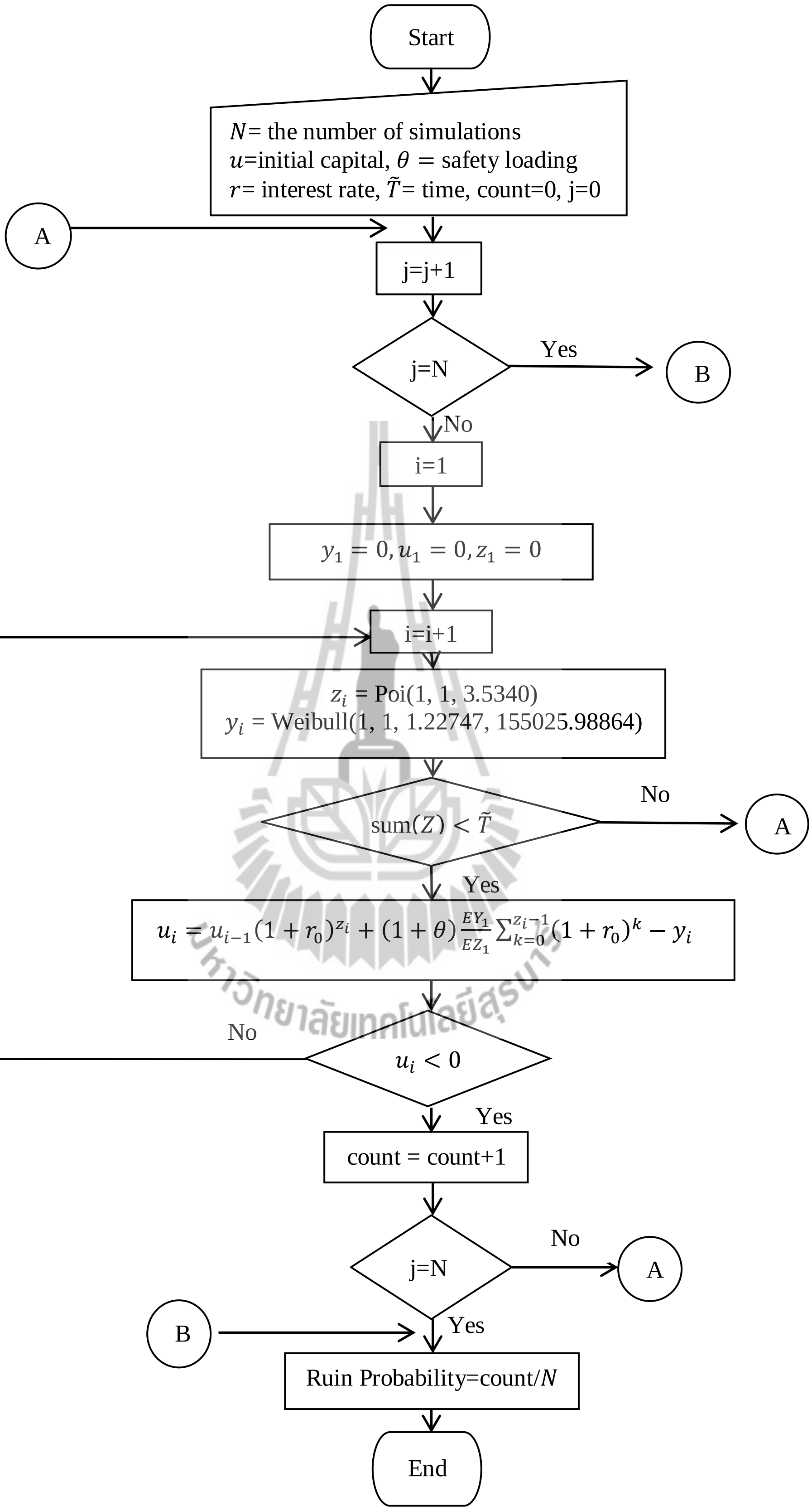

Figure 4.11 Flowchart for computing the ruin probability of equation (4.3).

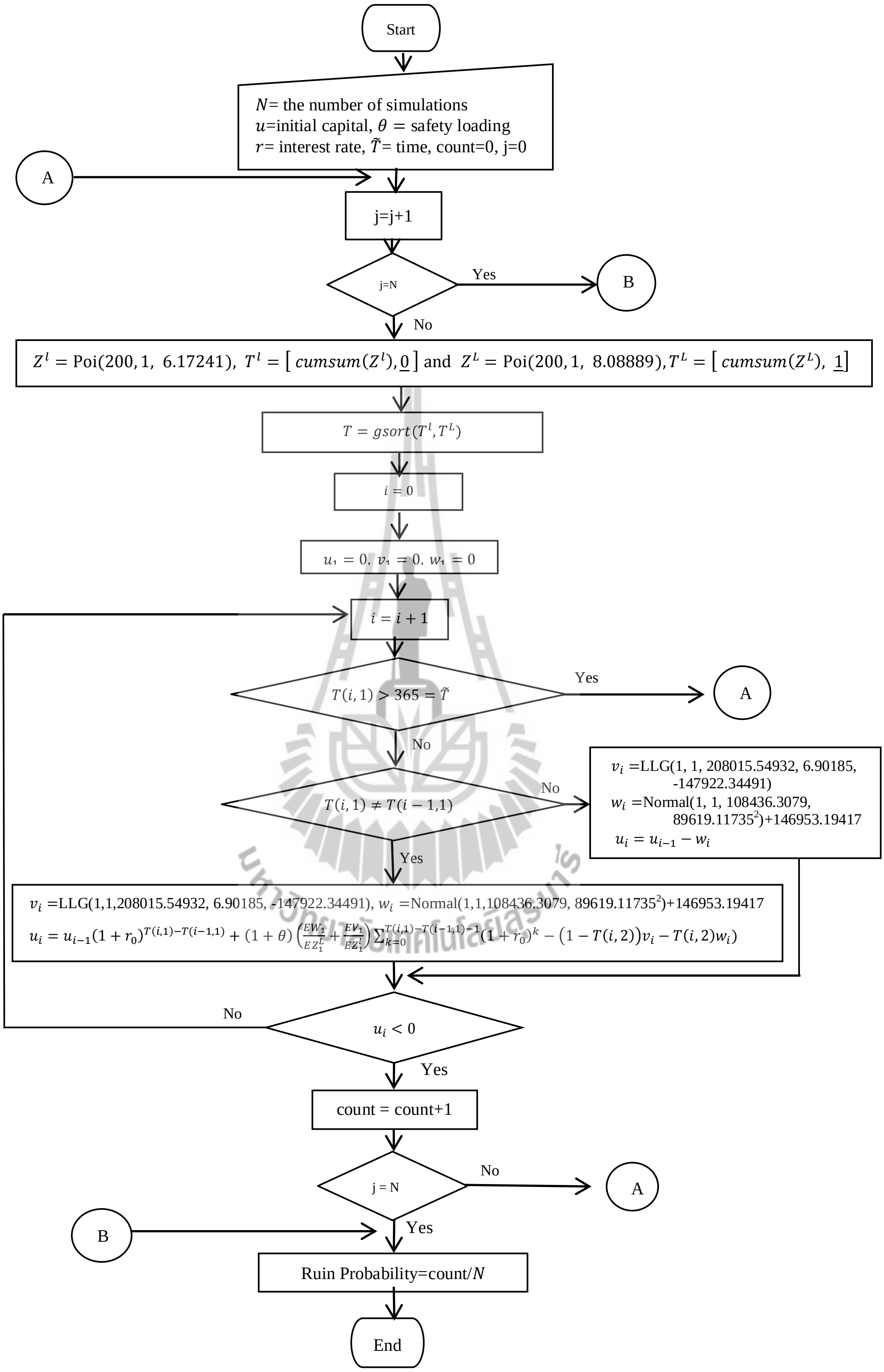

Figure 4.12 Flowchart for computing the ruin probability of equation (4.4).

4.4 Simulation Results

4.4.1 Class A : Claim severities happen every day

Case 1. Claim severities Y_n of arbitrary size

To obtain the ruin probability, we set the initial capital

$$u = 0,\ 20{,}000,\ 40{,}000, \ldots, 1{,}080{,}000$$

Baht and compute 10,000 simulations (see flowchart Figure 4.9). The simulation results are shown in Figure 4.13.

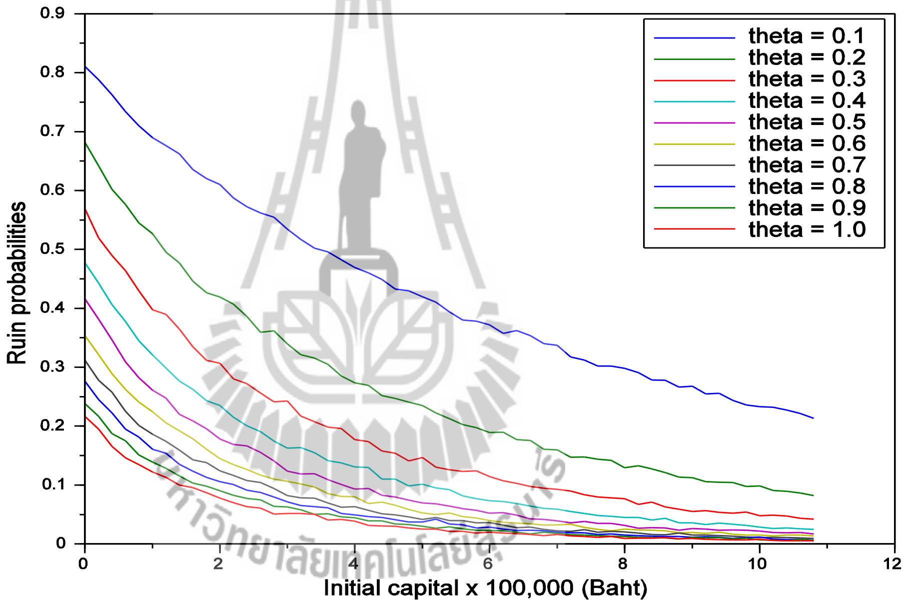

Figure 4.13 The relation between the initial capital and ruin probability in case $r = 2\%$ (Class A: arbitrary size of Y_n).

Case 2. Claim severities Y_n in the form of standard claims V_n or large claims W_n

In this case, we set the initial capital

$$u = 0,\ 20{,}000,\ 40{,}000, \ldots, 1{,}320{,}000$$

Baht and compute 10,000 simulations (see flowchart Figure 4.10). The simulation results as shown in Figure 4.14.

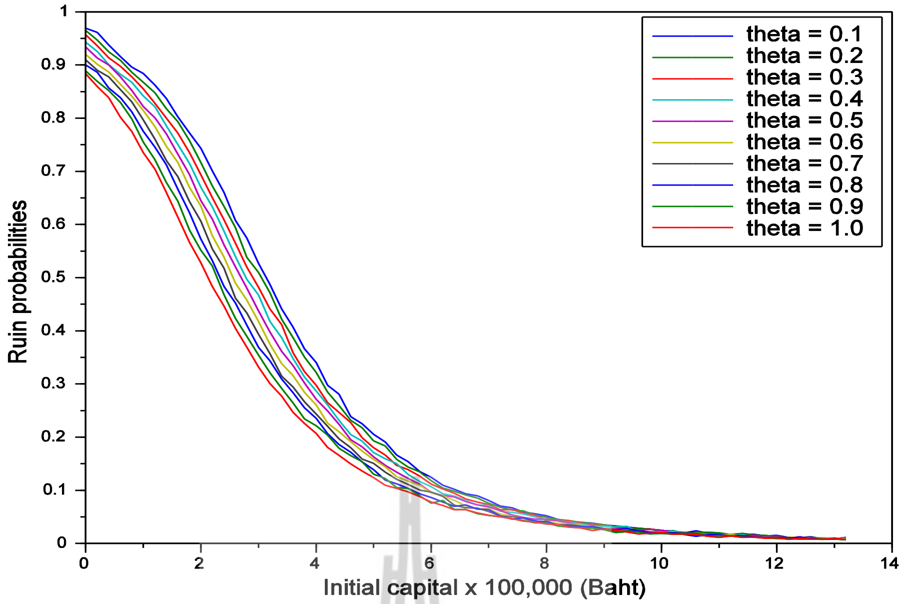

Figure 4.14 The relation between the initial capital and ruin probability in case $r = 2\%$ (Class A: Y_n in the form of V_n or W_n).

The relation between the ruin probability and initial capital are illustrated in Figure 4.13 and Figure 4.14: top curves are plotted in case $\theta = 0.1$, the next curves are plotted in case $\theta = 0.2$, and so on to a bottom curves are plotted in case $\theta = 1.0$, respectively.

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4.4.2 Class B : Claim severities do not happen every day

Case 1. Claim severities Y_n of arbitrary size

To obtain the ruin probability, we set the initial capital $u = 0$, 20,000, 40,000, …, 1,080,000 Baht. The inter arrival time process $\{Z_n,\ n \in \mathbb{N}\}$ is assumed to be i.i.d. such that $Z_1 \sim \text{Poisson}(\lambda)$ and we obtain the estimated parameter $\lambda = 3.5340$ by the MLE. We simulate 10,000 times (see Figure 4.11) such that the simulation results as shown in Figure 4.15.

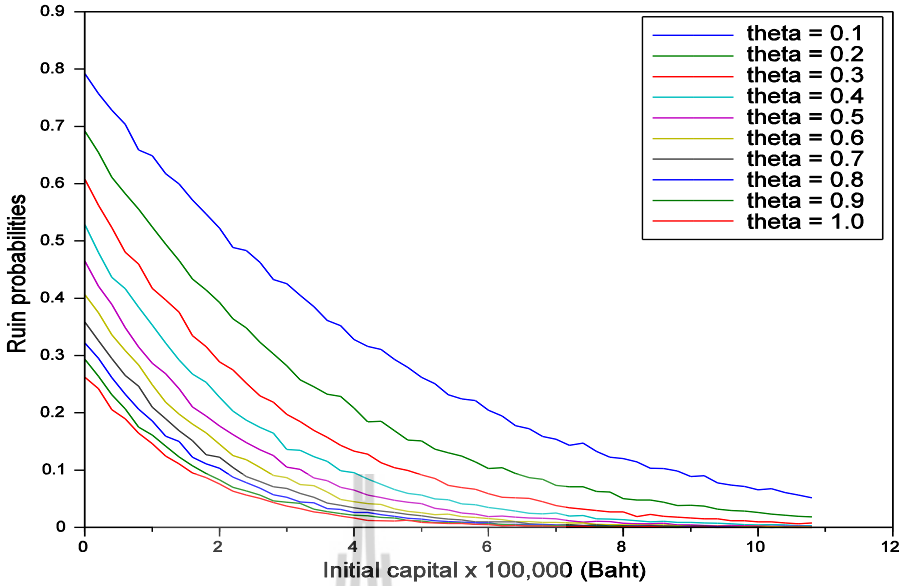

Figure 4.15 The relation between the initial capital and ruin probability in case $r = 2\%$ (Class B: arbitrary size of Y_n).

Case 2. Claim severities Y_n in the form of standard claims V_n or large claims W_n

We set the initial capital $u = 0$, 20,000, 40,000, …, 380,000 baht. The $\{Z_n^L,\ n \in \mathbb{N}\}$ is assumed to be i.i.d. such that $Z_1^L \sim \text{Poisson}(\lambda^L)$ and $\{Z_n^l,\ n \in \mathbb{N}\}$ is assumed to be i.i.d. such that $Z_1^l \sim \text{Poisson}(\lambda^l)$. We obtain $\lambda^L = 8.08889$ and $\lambda^l = 6.17241$ by the MLE. We simulate 10,000 times (see Figure 4.12) and the simulation results are shown in Figure 4.16.

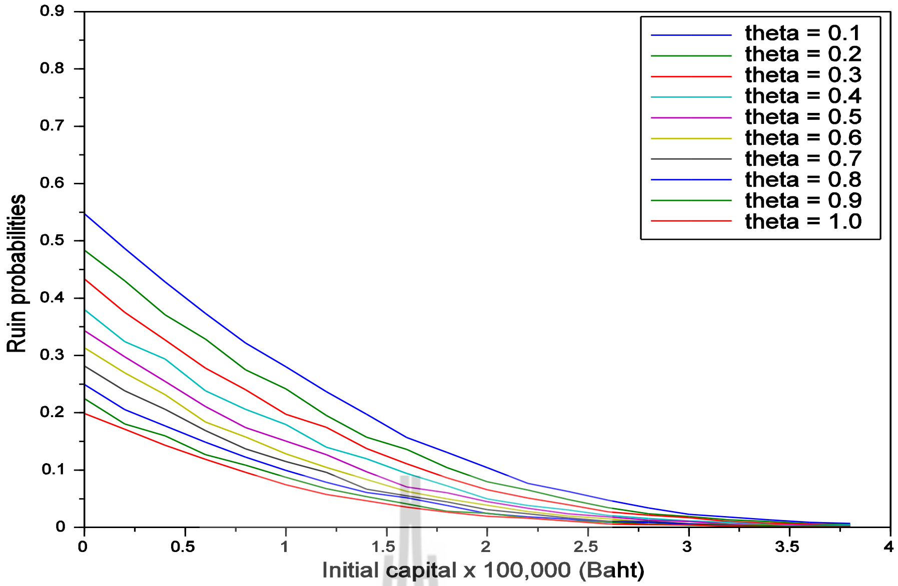

Figure 4.16 The relation between the initial capital and ruin probability in case $r = 2\%$ (Class B: Y_n in the form of V_n or W_n).

The relation between the ruin probability and initial capital are illustrated in Figure 4.15 and Figure 4.16 : top curves are plotted in case θ =0.1, the next curves are plotted in case θ =0.2, and so on to a bottom curves are plotted in case θ =1.0, respectively.

4.5 Minimum initial capital

4.5.1 Regression analysis and minimum initial capital for Class A

By the Figure 4.13 and Figure 4.14, we may consider the relationship between ruin probability $\Phi(u, 365) \coloneqq y$ and initial capital u as an exponential function,

$$y = \gamma \exp(-\delta u). \tag{4.17}$$

By taking the natural logarithmic function in (4.17) and applying the least squares linear regression method as mentioned in (2.12) (setting $\beta_0 = \ln \gamma$ and $\beta_1 = -\delta$), we obtain the approximated parameters as the following:

$$\gamma = \exp\left(\frac{\sum_{i=1}^{n} u_i^2 \sum_{i=1}^{n} \ln y_i - \sum_{i=1}^{n} u_i \sum_{i=1}^{n} u_i \ln y_i}{n\sum_{i=1}^{n} u_i^2 - (\sum_{i=1}^{n} u_i)^2} \right) \tag{4.18}$$

and

$$\delta = -\frac{n\sum_{i=1}^{n} u_i \ln y_i - \sum_{i=1}^{n} u_i \sum_{i=1}^{n} \ln y_i}{n\sum_{i=1}^{n} u_i^2 - (\sum_{i=1}^{n} u_i)^2} \tag{4.19}$$

where u_i is initial capital and y_i is the ruin probability at initial capital u_i. Finally, we consider that if the ruin probability has to be not greater than α, i.e., $\Phi(u, 365) \leq \alpha$, then $u \geq -\frac{1}{\delta}\ln\left(\frac{\alpha}{\gamma}\right)$.

In case of a non-risky portfolio or where the premium rate is high enough, u may be negative. This means that this portfolio need not have an initial capital. Therefore, the minimum initial capital required is

$$u = \max\left\{0, \frac{\ln\gamma - \ln\alpha}{\delta}\right\}. \tag{4.20}$$

4.5.2 Computation of the minimum initial capital for Class A

From equations (4.18), (4.19) and (4.20), we obtain the minimum initial capital (MIC) as shown in Table 4.9. In the case of claim severities of arbitrary size Y_n, the premium rate c can be computed by

$$c = (1+\theta)E[Y_1],$$

whereas in the case of claim severities Y_n in the form of standard and large claims, the premium rate c can be computed by

$$c = (1+\theta)\left(\frac{EW_1}{EZ_1^L} + EV_1\right)$$

where $E[Y_1] = 100{,}922.84$ and $\dfrac{EW_1}{EZ_1^L} + EV_1 = 110{,}761.88.$

Table 4.9 MIC (Baht) in case $r = 2\%, \alpha = 0.01$ (Class A).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	111,015.12	3,607,331	121,838.07	1,253,546
0.2	121,107.41	2,168,619	132,914.26	1,251,740
0.3	131,199.69	1,649,863	143,990.44	1,233,715
0.4	141,291.98	1,371,444	155,066.63	1,221,931
0.5	151,384.26	1,213,677	166,142.82	1,217,123
0.6	161,476.54	1,109,123	177,219.01	1,211,131
0.7	171,568.83	1,014,795	188,295.20	1,193,571
0.8	181,661.11	936,616	199,371.38	1,193,372
0.9	191,753.40	877,268	210,447.57	1,167,637
1.0	201,845.68	827,785	221,523.76	1,165,608

From Table 4.9, we perform linear regression analysis between the premium rate c and the MIC. The results are shown in the Figure 4.17.

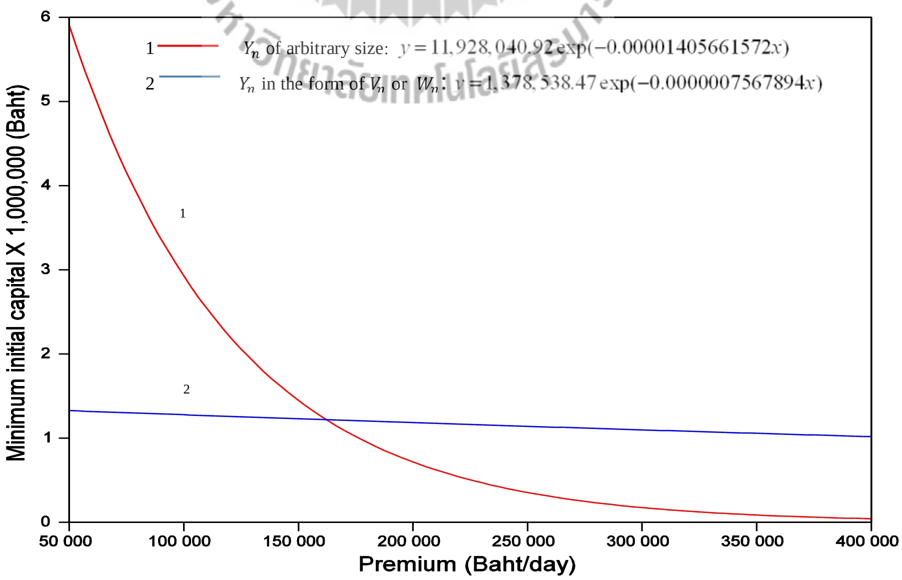

Figure 4.17 MIC under a ruin probability not greater than $\alpha = 0.01$ and interest rate $r = 2\%$ (Class A).

From Figure 4.17, we can conclude that considering claims in the form of standard and large claims is better than considering claims of arbitrary size in the view of the sensitivity analysis. Nevertheless, in the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as the following:

$$\mathrm{MIC} = \begin{cases} 11,928,040.92\exp(-0.00001405661572x), & \text{if } x \le 162,248 \\ 1,378,538.47\exp(-0.0000007567894x), & \text{if } x > 162,248 \end{cases} \tag{4.21}$$

where $x = 162,248$ is the point of intersection of the two curves.

4.5.3 Regression analysis and minimum initial capital for Class B

By the Figure 4.15 and Figure 4.16, we may consider the relationship between ruin probability $\Phi(u, 365) \coloneqq y$ and initial capital u as an exponential function,

$$y = \gamma \exp(-\delta u). \tag{4.22}$$

By taking the natural logarithmic function in (4.22) and applying the least squares linear regression method as mentioned in (2.12) (setting $\beta_0 = \ln \gamma$ and $\beta_1 = -\delta_1$), we obtain the approximated parameters as the following:

$$\gamma = \exp\left(\frac{\sum_{i=1}^{n} u_i^2 \sum_{i=1}^{n} \ln y_i - \sum_{i=1}^{n} u_i \sum_{i=1}^{n} u_i \ln y_i}{n\sum_{i=1}^{n} u_i^2 - (\sum_{i=1}^{n} u_i)^2} \right) \tag{4.23}$$

and

$$\delta = -\frac{n\sum_{i=1}^{n} u_i \ln y_i - \sum_{i=1}^{n} u_i \sum_{i=1}^{n} \ln y_i}{n\sum_{i=1}^{n} u_i^2 - (\sum_{i=1}^{n} u_i)^2} \tag{4.24}$$

where u_i is initial capital and y_i is the ruin probability at initial capital u_i. Finally, we consider that if the ruin probability has to be not greater than α, i.e., $\Phi(u, 365) \leq \alpha$, then $u \geq -\frac{1}{\delta}\ln\left(\frac{\alpha}{\gamma}\right)$.

In case of non-risky portfolio or where the premium rate is hight enough, u may be negative. This means that this portfolio need not have an initial capital. Therefore, the minimum initial capital required is

$$u = \max\left\{0, \frac{\ln\gamma - \ln\alpha}{\delta}\right\}. \tag{4.25}$$

4.5.4 Computation of the minimum initial capital for Class B

From equations (4.23), (4.24) and (4.25), we obtain the MIC as shown in Table 4.10. In the case of claim severities of arbitrary size Y_n, the premium rate c can be computed by

$$c = (1+\theta)\frac{E[Y_1]}{E[Z_1]},$$

whereas in the case of claim severities Y_n in the form of standard and large claims, the premium rate c can be computed by

$$c = (1+\theta)\left(\frac{EW_1}{EZ_1^L} + \frac{EV_1}{EZ_1^l}\right)$$

where $\frac{E[Y_1]}{E[Z_1]} = 41,031.60$ and $\left(\frac{EW_1}{EZ_1^L} + \frac{EV_1}{EZ_1^l}\right) = 24,333.87.$

Table 4.10 MIC (Baht) in case $r = 2\%, \alpha = 0.01$ (Class B).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	45,134.76	1,785,265	26,767.26	376,945
0.2	49,237.92	1,298,091	29,200.64	350,979
0.3	53,341.08	1,018,725	31,634.03	333,964
0.4	57,444.24	859,854	34,067.42	310,735
0.5	61,547.40	749,394	36,500.81	292,543
0.6	65,650.56	667,704	38,934.19	275,678
0.7	69,753.72	588,863	41,367.58	265,542
0.8	73,856.88	548,840	43,800.97	249,637
0.9	77,960.04	501,839	46,234.35	242,256
1.0	82,063.20	469,560	48,667.74	225,168

From Table 4.10, we perform linear regression analysis between the premium rate c and the MIC. The results are shown in the Figure 4.18.

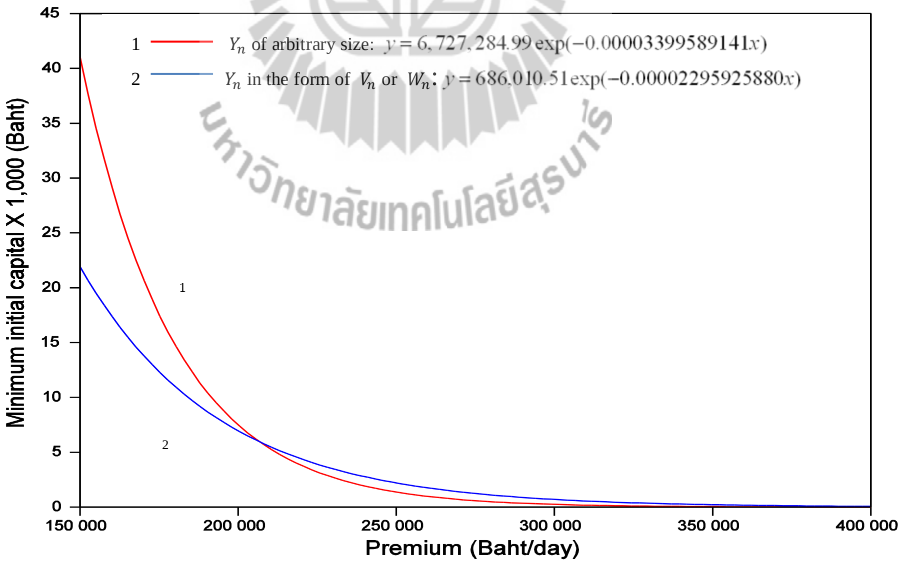

Figure 4.18 MIC under a ruin probability not greater than $\alpha = 0.01$ and interest rate 2% (Class B).

From Figure 4.18, we can conclude that considering claims in the form of standard and large claims is better than considering claims of arbitrary size in the view of the sensitivity analysis. Nevertheless, in the view of the fair decision for insured (customers), if the ruin probability is not greater than $\alpha = 0.01,$ then the insurer ought to choose the minimum initial capital as the following:

$$\text{MIC} = \begin{cases} 6,727,284.99\exp(-0.00003399589141x), & \text{if } x \le 206,860 \\ 686,010.51\exp(-0.00002295925880x), & \text{if } x > 206,860 \end{cases} \tag{4.26}$$

where $x = 206,860$ is the point of intersection of the two curves.

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CHAPTER V

CONCLUSIONS

This thesis is divided into two parts which are the estimation of the Weibull parameters for the claim severity of fire accidents, and the minimum initial capital which a motor insurance company has to hold for ensuring that the ruin probability is not greater than the given quantity.

In the first part, we proposed the Randomized Neighborhood Search technique (RNS) for the estimation of the Weibull parameters for the claim severity. Five estimation methods (MLE, MOM, LSM, WLSM and RNS) were used to estimate the Weibull parameters. By Table 3.14 in Chapter III, RNS has the smallest chi-squared value. Thus RNS gives a more accurate estimation of parameters than do MLE, MOM, LSM or WLSM.

In the second part, we computed what minimum initial capital a motor insurance company has to hold for ensuring that the ruin probability is not greater than a given quantity. The claim severities were considered in two cases, where the claims happen every day and the claims do not happen every day. Moreover, we consider the claim severities in the form of standard claims and large claims. RNS was applied to parameters estimation for distribution of clam severities. Simulation approach and regression analysis were used for approximating the minimum initial capital (MIC) in equation (4.1) to (4.4).

For Class A from Figure 4.17, we conclude that considering claims in the form of standard and large claims is better than considering claims of arbitrary size in the view of the sensitivity analysis. Nevertheless, if the ruin probability is not greater than $\alpha = 0.01$, then in the view of the fair decision for insured (customers), the insurer ought to hold the MIC (baht) as the following:

$$\text{MIC} = \begin{cases} 11,928,040.92\exp(-0.00001405661572x), & \text{if } x \le 162,248 \\ 1,378,538.47\exp(-0.0000007567894x), & \text{if } x > 162,248 \end{cases}$$

where $x = 162,248$ (baht) is the point of intersection of the two curves.

For Class B from Figure 4.18, we conclude that considering claims in the form of standard and large claims is better than considering claims of arbitrary size in the view of the sensitivity analysis. Nevertheless, if the ruin probability is not greater than $\alpha = 0.01$, then in the view of the fair decision for insured, the insurer ought to hold the MIC (baht) as the following:

$$\text{MIC} = \begin{cases} 6,727,284.99\exp(-0.00003399589141x), & \text{if } x \le 206,860 \\ 686,010.51\exp(-0.00002295925880x), & \text{if } x > 206,860 \end{cases}$$

where $x = 206,860$ (baht) is the point of intersection of the two curves.

In the other cases, the interest rate $r = 3\%$ to $r = 8\%$, we obtained the MIC which ought to hold listed in Appendix A.

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APPENDICES

APPENDIX A

MINIMUM INITIAL CAPITAL PROBLEM (Continued)

In Chapter IV, we computed what the minimum initial capital an insurance company has to hold for ensuring that the ruin probability is not greater than the given quantity in case the interest rate $r = 2\%$. In this chapter, we will compute the minimum initial capital for interest rates $r = 3\%$ to $r = 8\%$ for class A and class B.

Class A : claim severities happen every day

A.1. Claim severities Y_n of arbitrary size

We set the initial capital $u = 0, 20{,}000, 40{,}000, \ldots, 1{,}080{,}000$ Baht and simulate 10,000 times. The simulation results as shown in Figure A.1 to Figure A.6 for the interest rate $r = 3\%$ to $r = 8\%$, respectively.

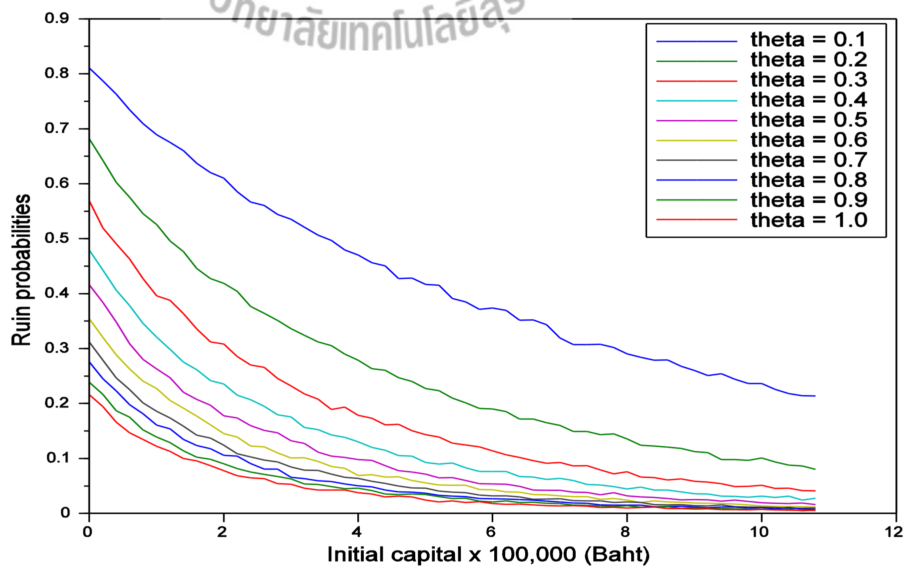

Figure A.1 The relation between the initial capital and ruin probability in case $r = 3\%$ (Class A: arbitrary size of Y_n).

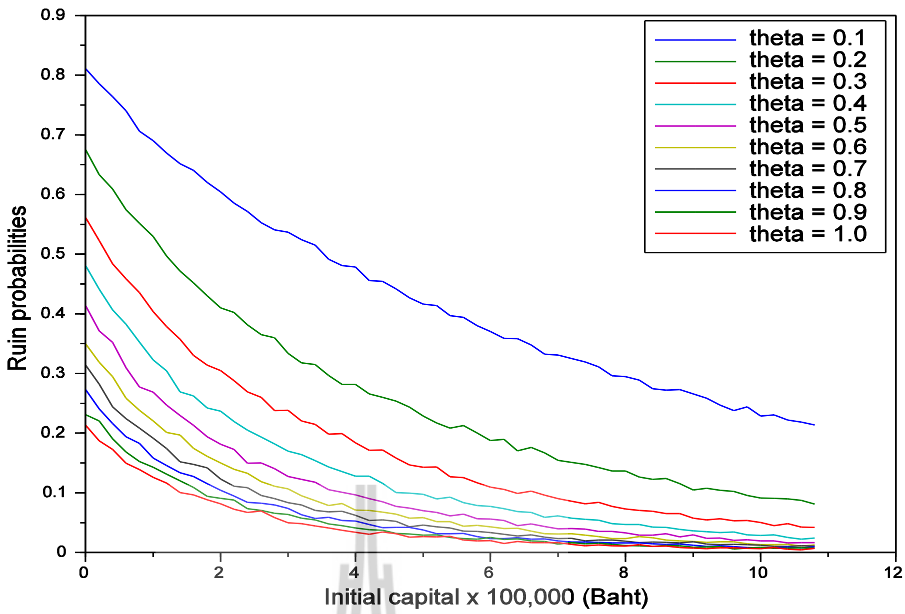

Figure A.2 The relation between the initial capital and ruin probability in case r = 4% (Class A: arbitrary size of Y_n).

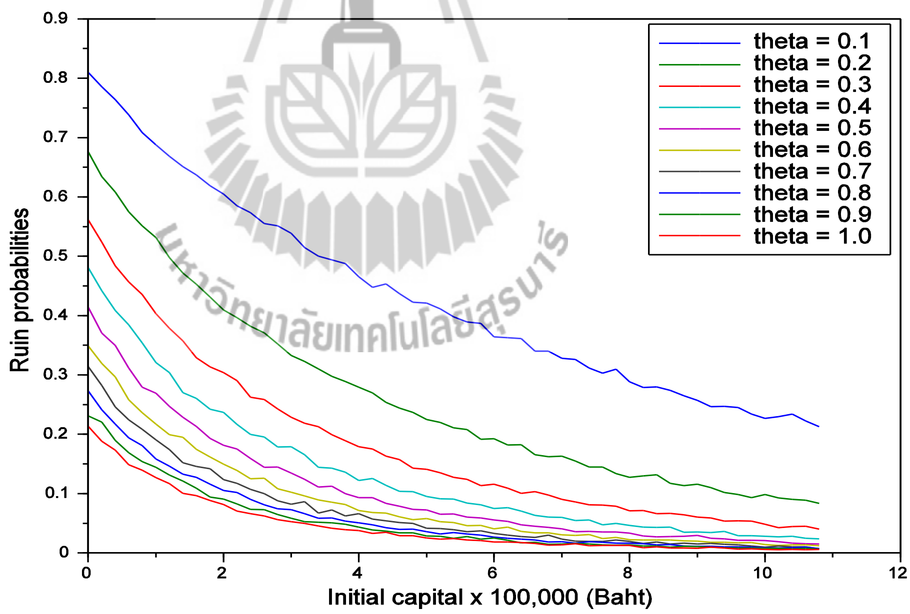

Figure A.3 The relation between the initial capital and ruin probability in case $r = 5\%$ (Class A: arbitrary size of Y_n).

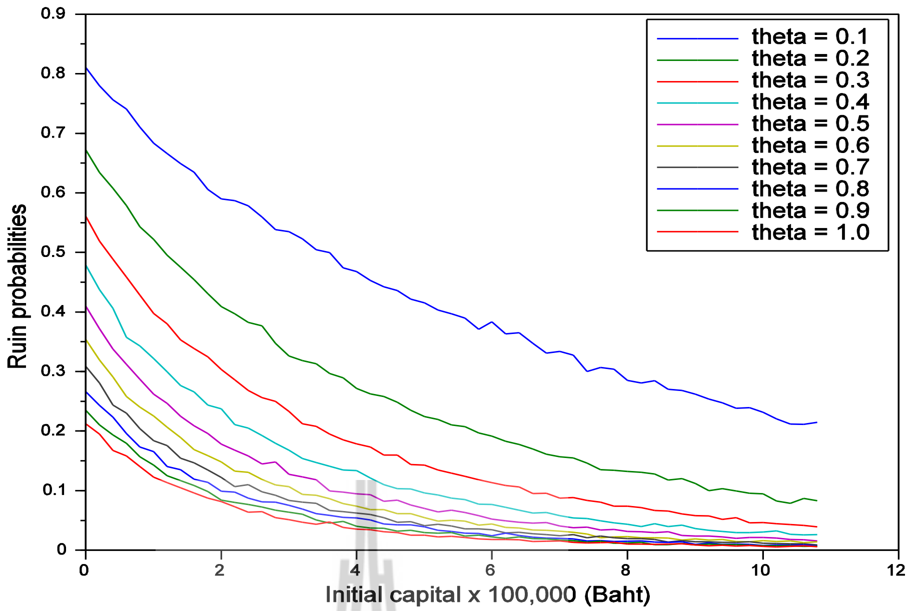

Figure A.4 The relation between the initial capital and ruin probability in case $r = 6\%$ (Class A: arbitrary size of Y_n).

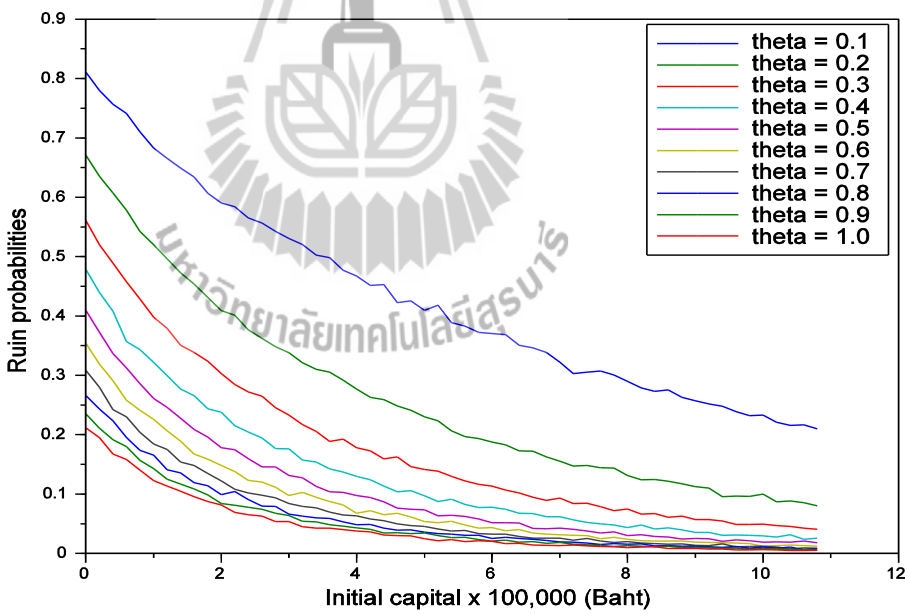

Figure A.5 The relation between the initial capital and ruin probability in case $r = 7\%$ (Class A: arbitrary size of Y_n).

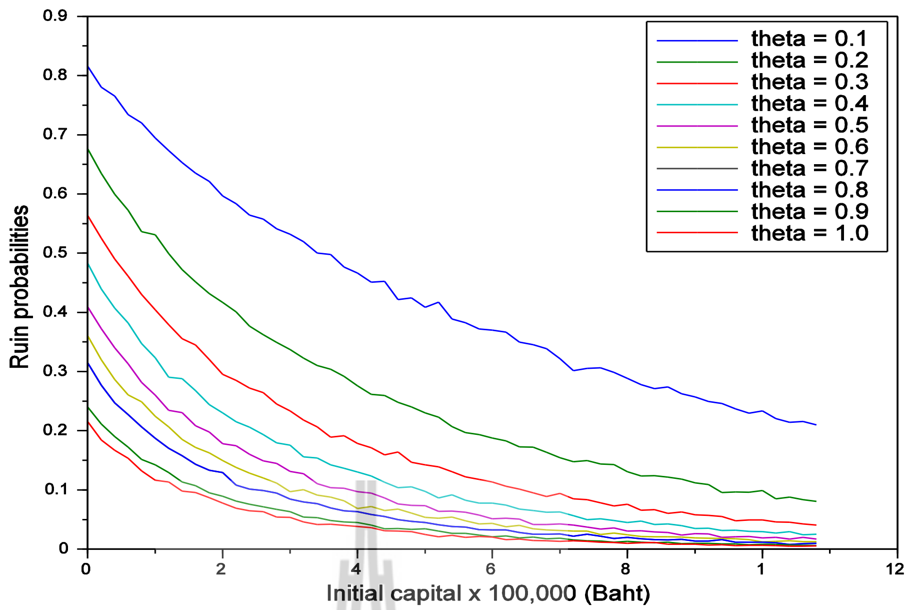

Figure A.6 The relation between the initial capital and ruin probability in case $r = 8\%$ (Class A: arbitrary size of Y_n).

The relations between the ruin probability and initial capital are illustrated in Figure A.1 to Figure A.6 : top curves are plotted in case $\theta = 0.1$, the next curves are plotted in case $\theta = 0.2$, and so on to a bottom curves are plotted in case $\theta = 1.0$, respectively.

A.2. Claim severities Y_n in the form of standard claims V_n or large claims W_n

We set the initial capital

$$u = 0,\ 20{,}000,\ 40{,}000,\ ...,\ 1{,}320{,}000$$

Baht and simulate 10,000 times. Figure A.7 to Figure A.12 show the simulation results in case of on interest rate of $r = 3\%$ to $r = 8\%$, respectively.

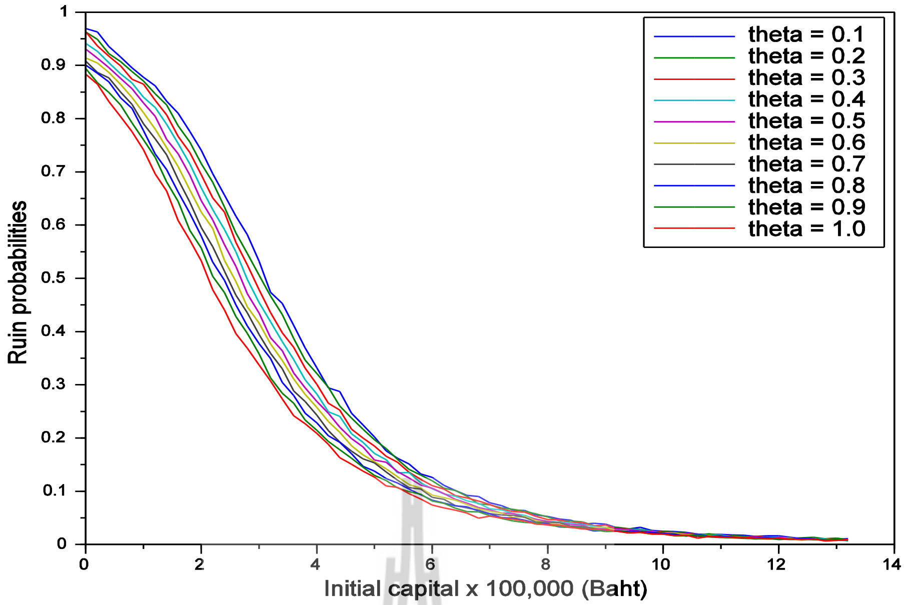

Figure A.7 The relation between the initial capital and ruin probability in case $r = 3\%$ (Class A: Y_n in the form of V_n or W_n).

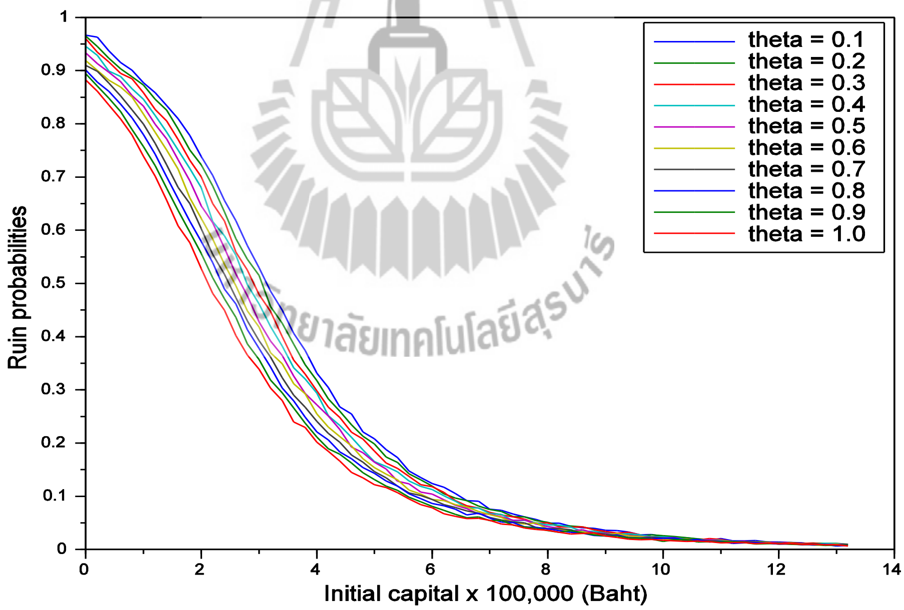

Figure A.8 The relation between the initial capital and ruin probability in case $r = 4\%$ (Class A: Y_n in the form of V_n or W_n).

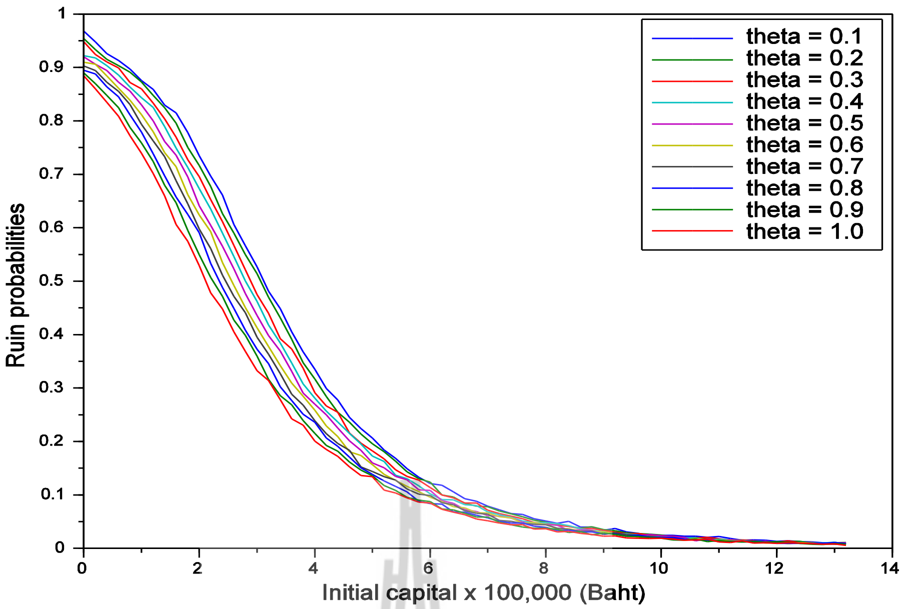

Figure A.9 The relation between the initial capital and ruin probability in case $r = 5\%$ (Class A: Y_n in the form of V_n or W_n).

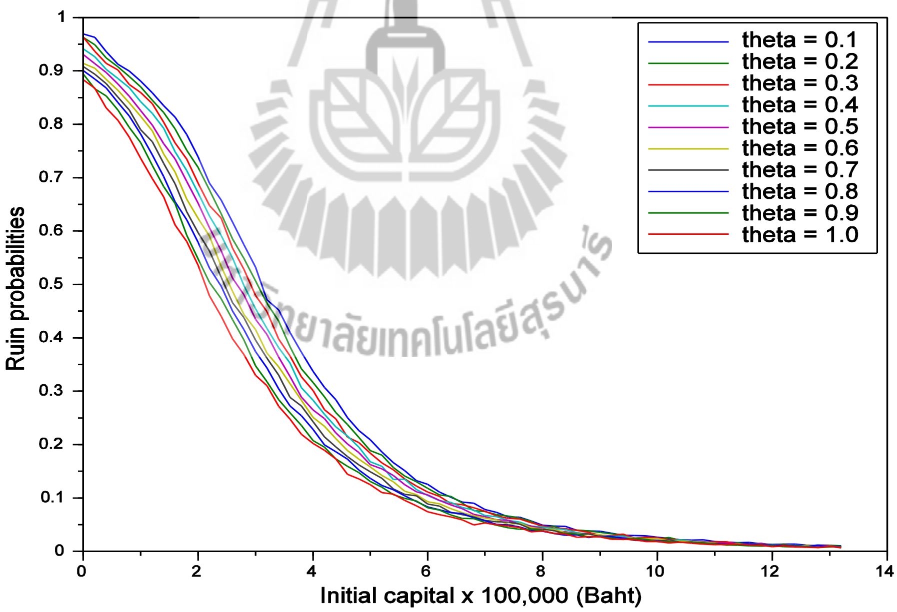

Figure A.10 The relation between the initial capital and ruin probability in case $r = 6\%$ (Class A: Y_n in the form of V_n or W_n).

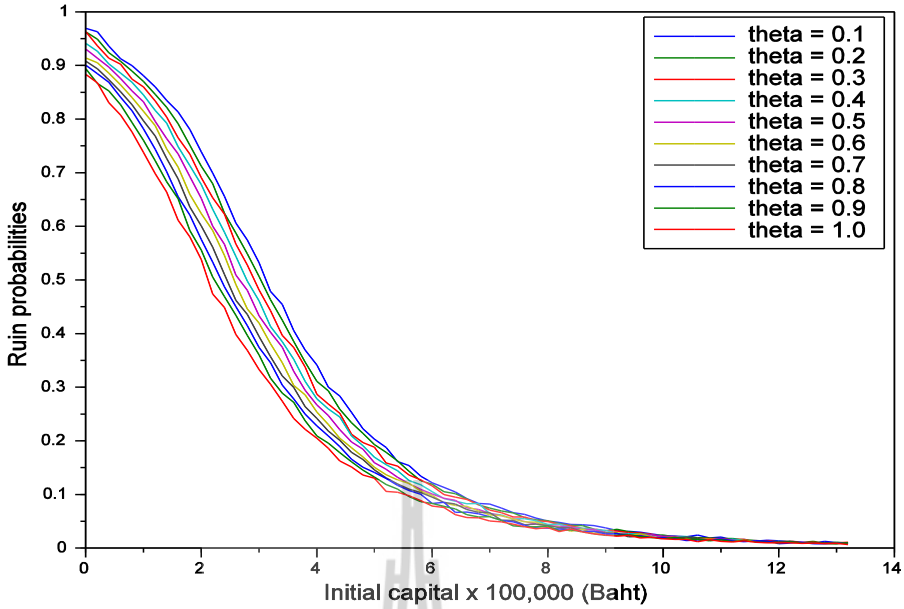

Figure A.11 The relation between the initial capital and ruin probability in case $r = 7\%$ (Class A: Y_n in the form of V_n or W_n).

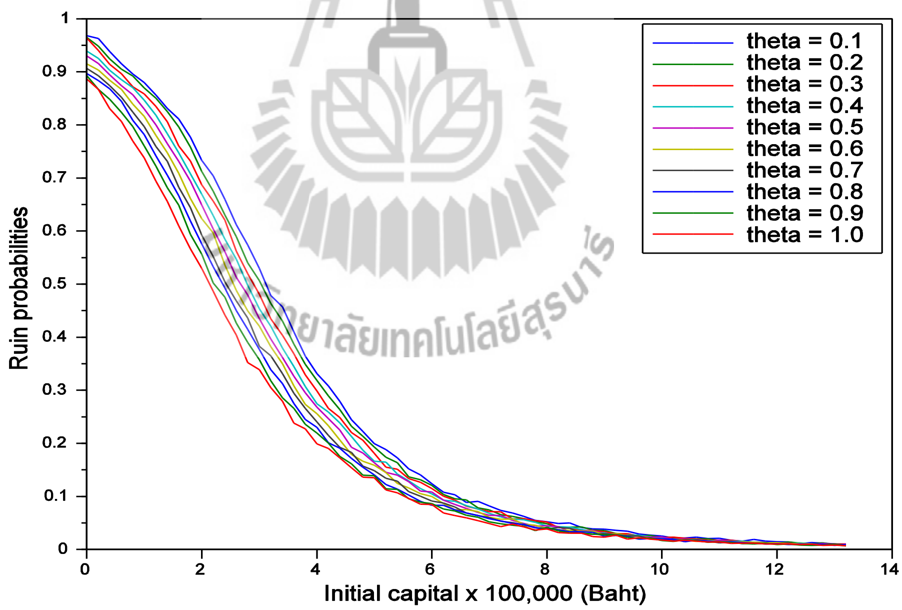

Figure A.12 The relation between the initial capital and ruin probability in case $r = 8\%$ (Class A: Y_n in the form of V_n or W_n).

The relations between the ruin probability and initial capital are illustrated in Figure A.7 to Figure A.12: top curves are plotted in case $\theta = 0.1$, the next curves are plotted in case $\theta = 0.2$, and so on to a bottom curves are plotted in case $\theta = 1.0$, respectively.

Minimum initial capital for Class A

From the Figure A.1 to Figure A.12, we may consider the relationship between ruin probability $\Phi(u, 365) \coloneqq y$ and initial capital u to be an exponential function

$$y = \gamma \exp(-\delta u).$$

By applying (4.18), (4.19), (4.20) and setting $\alpha = 0.01$, we obtain the MIC in case of the interest rate $r = 3\%$ to $r = 8\%$ that are shown in Table A.1 to Table A.6. In the case of considering the claim severities Y_n of arbitrary size, the premium rate c can be computed by

$$c = (1+\theta)E[Y_1],$$

whereas in the case of considering claim severities Y_n in the form of standard and large claims, the premium rate c can be computed by

$$c = (1+\theta)\left(\frac{EW_1}{EZ_1^L} + EV_1\right)$$

where $E[Y_1] = 100{,}922.84$ and $\frac{EW_1}{EZ_1^L} + EV_1 = 110{,}761.88$.

Table A.1 MIC (Baht) in case $r = 3\%, \alpha = 0.01$ (Class A).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	111,015.12	3,570,676	121,838.07	1,261,170
0.2	121,107.41	2,162,086	132,914.26	1,250,189
0.3	131,199.69	1,635,269	143,990.44	1,230,931
0.4	141,291.98	1,388,738	155,066.63	1,220,679
0.5	151,384.26	1,209,122	166,142.82	1,213,976
0.6	161,476.54	1,093,415	177,219.01	1,199,157
0.7	171,568.83	1,009,399	188,295.20	1,188,588
0.8	181,661.11	944,035	199,371.38	1,178,214
0.9	191,753.40	874,909	210,447.57	1,174,565
1.0	201,845.68	825,059	221,523.76	1,163,126

Table A.2 MIC (Baht) in case $r = 4\%, \alpha = 0.01$ (Class A).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	111,015.12	3,593,686	121,838.07	1,253,082
0.2	121,107.41	2,156,912	132,914.26	1,246,846
0.3	131,199.69	1,655,214	143,990.44	1,240,168
0.4	141,291.98	1,374,227	155,066.63	1,227,345
0.5	151,384.26	1,210,324	166,142.82	1,213,071
0.6	161,476.54	1,094,578	177,219.01	1,202,918
0.7	171,568.83	1,018,570	188,295.20	1,202,918
0.8	181,661.11	936,369	199,371.38	1,182,907
0.9	191,753.40	879,066	210,447.57	1,175,218
1.0	201,845.68	835,518	221,523.76	1,170,480

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Table A.3 MIC (Baht) in case $r = 5\%, \alpha = 0.01$ (Class A).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	111,015.12	3,583,733	121,838.07	1,262,195
0.2	121,107.41	2,167,388	132,914.26	1,238,411
0.3	131,199.69	1,650,647	143,990.44	1,232,973
0.4	141,291.98	1,367,891	155,066.63	1,221,244
0.5	151,384.26	1,203,259	166,142.82	1,208,628
0.6	161,476.54	1,100,850	177,219.01	1,197,249
0.7	171,568.83	1,002,929	188,295.20	1,187,625
0.8	181,661.11	937,495	199,371.38	1,178,335
0.9	191,753.40	881,667	210,447.57	1,174,294
1.0	201,845.68	829,227	221,523.76	1,159,241

Table A.4 MIC (Baht) in case $r = 6\%, \alpha = 0.01$ (Class A).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	111,015.12	3,563,904	121,838.07	1,258,356
0.2	121,107.41	2,139,191	132,914.26	1,241,892
0.3	131,199.69	1,631,129	143,990.44	1,233,375
0.4	141,291.98	1,376,157	155,066.63	1,222,956
0.5	151,384.26	1,207,248	166,142.82	1,216,184
0.6	161,476.54	1,089,907	177,219.01	1,201,673
0.7	171,568.83	1,015,854	188,295.20	1,193,984
0.8	181,661.11	933,230	199,371.38	1,189,633
0.9	191,753.40	879,777	210,447.57	1,174,343
1.0	201,845.68	837,705	221,523.76	1,165,044

Table A.5 MIC (Baht) in case $r = 7\%, \alpha = 0.01$ (Class A).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	111,015.12	3,548,480	121,838.07	1,255,179
0.2	121,107.41	2,150,474	132,914.26	1,243,793
0.3	131,199.69	1,634,109	143,990.44	1,235,384
0.4	141,291.98	1,378,759	155,066.63	1,217,830
0.5	151,384.26	1,213,776	166,142.82	1,209,772
0.6	161,476.54	1,098,346	177,219.01	1,202,989
0.7	171,568.83	1,000,259	188,295.20	1,200,557
0.8	181,661.11	942,595	199,371.38	1,193,931
0.9	191,753.40	878,248	210,447.57	1,178,815
1.0	201,845.68	814,526	221,523.76	1,161,427

Table A.6 MIC (Baht) in case $r = 8\%, \alpha = 0.01$ (Class A).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	111,015.12	3,519,739	121,838.07	1,268,103
0.2	121,107.41	2,145,019	132,914.26	1,239,946
0.3	131,199.69	1,632,098	143,990.44	1,234,405
0.4	141,291.98	1,375,105	155,066.63	1,221,475
0.5	151,384.26	1,208,156	166,142.82	1,214,304
0.6	161,476.54	1,096,964	177,219.01	1,205,586
0.7	171,568.83	1,001,775	188,295.20	1,192,079
0.8	181,661.11	942,165	199,371.38	1,187,282
0.9	191,753.40	878,799	210,447.57	1,171,511
1.0	201,845.68	815,222	221,523.76	1,167,113

From Table A.1 to Table A.6, we perform regression analysis between the premium rate c and the MIC. The results are plotted in the Figure A.13 to Figure A.18, respectively.

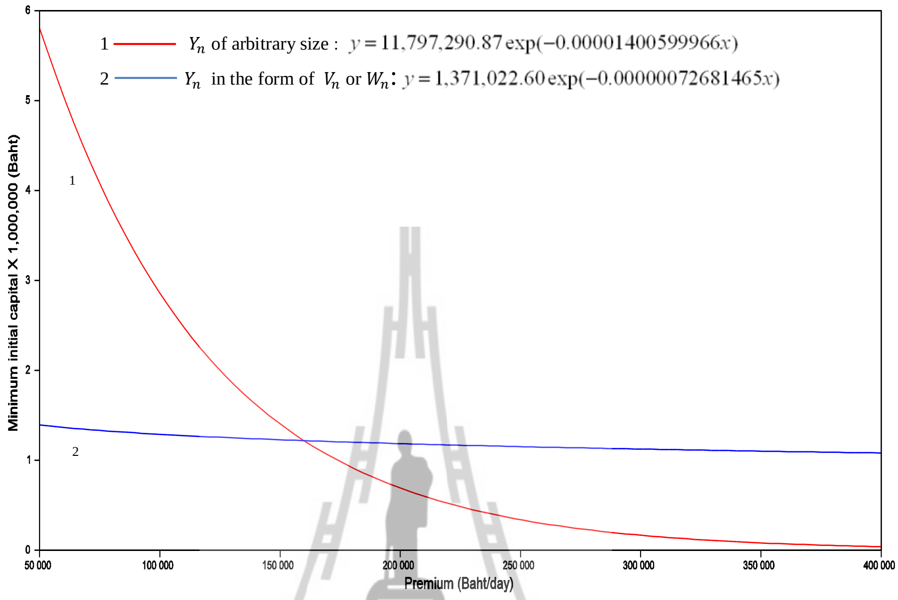

Figure A.13 MIC under a ruin probability not greater than $\alpha = 0.01$ and interest rate $r = 3\%$ (Class A).

In the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as follows:

$$\text{MIC} = \begin{cases} 11,797,290.87\exp(-0.00001400599966x), & \text{if } x \le 162,082 \\ 1,371,022.60\exp(-0.00000072681465x), & \text{if } x > 162,082 \end{cases}$$

where x =162,082 is the point of intersection of the two curves.

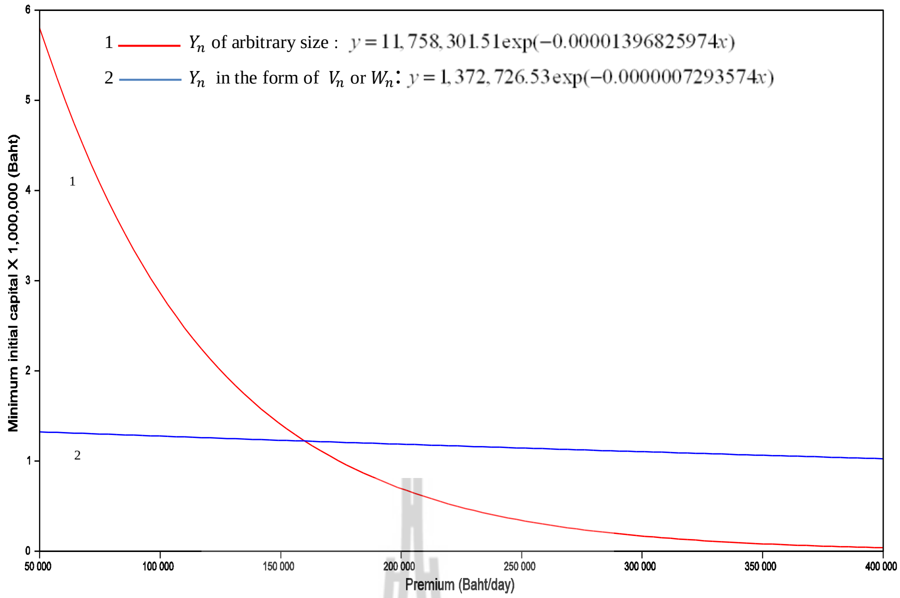

Figure A.14 MIC under a ruin probability not greater than $\alpha = 0.01$ and interest rate $r = 4\%$ (Class A).

In the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as follows:

$$\text{MIC} = \begin{cases} 11,758,301.51\exp(-0.00001396825974x), & \text{if } x \le 162,231 \\ 1,372,726.53\exp(-0.0000007293574x), & \text{if } x > 162,231 \end{cases}$$

where x =162,231 is the point of intersection of the two curves.

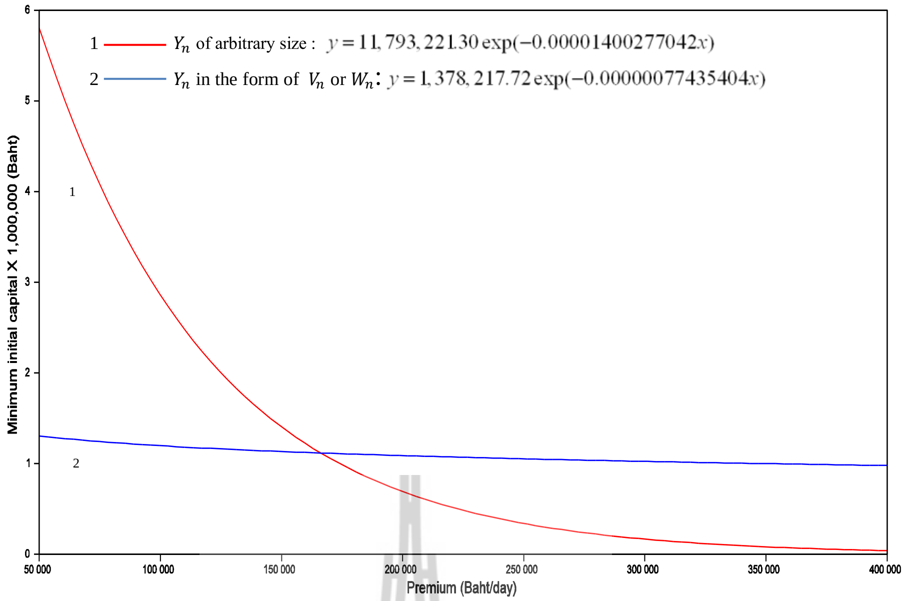

Figure A.15 MIC under a ruin probability not greater than $\alpha = 0.01$ and interest rate $r = 5\%$ (Class A).

In the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as follows:

$$\text{MIC} = \begin{cases} 11,793,221.30\exp(-0.00001400277042x), & \text{if } x \le 162,282 \\ 1,378,217.72\exp(-0.00000077435404x), & \text{if } x > 162,282 \end{cases}$$

where x =162,282 is the point of intersection of the two curves.

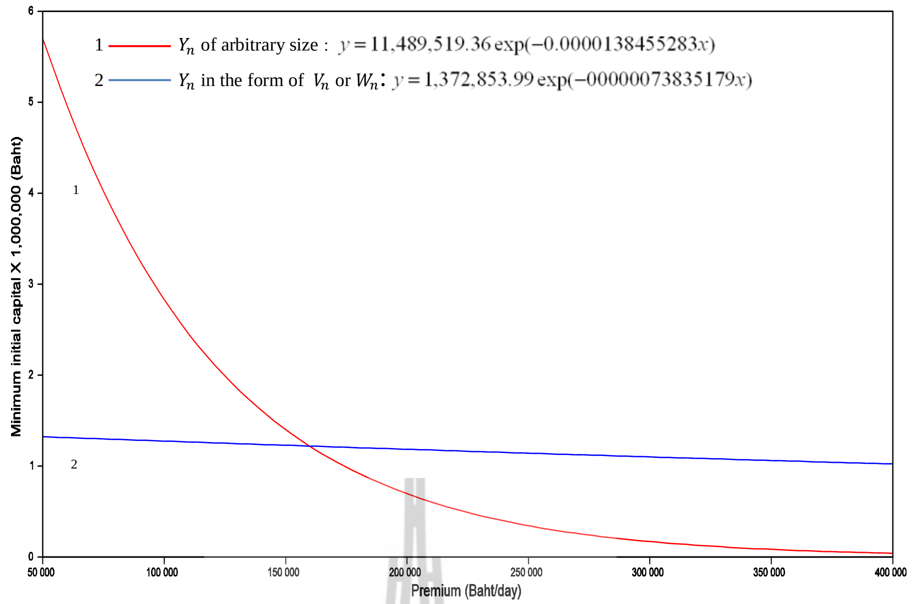

Figure A.16 MIC under a ruin probability not greater than $\alpha = 0.01$ and interest rate $r = 6\%$ (Class A).

In the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as follows:

$$\text{MIC} = \begin{cases} 11{,}489{,}519.36\exp(-0.0000138455283x), & \text{if } x \le 162{,}090 \\ 1{,}372{,}853.99\exp(-00000073835179x), & \text{if } x > 162{,}090 \end{cases}$$

where x =162,090 is the point of intersection of the two curves.

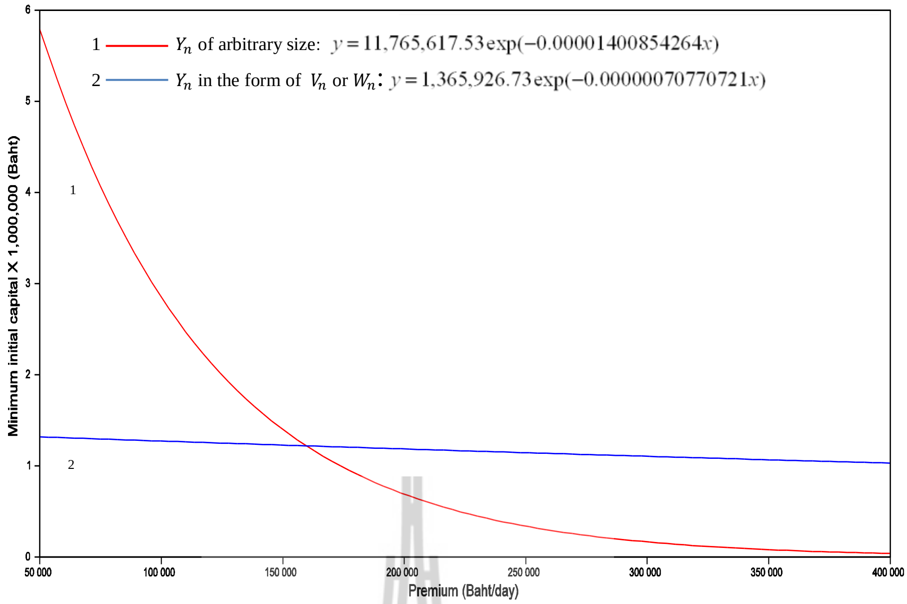

Figure A.17 MIC under a ruin probability not greater than $\alpha = 0.01$ and interest rate $r = 7\%$ (Class A).

In the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as follows:

$$\text{MIC} = \begin{cases} 11,765,617.53\exp(-0.00001400854264x), & \text{if } x \le 161,896 \\ 1,365,926.73\exp(-0.00000070770721x), & \text{if } x > 161,896 \end{cases}$$

where x =161,896 is the point of intersection of the two curves.

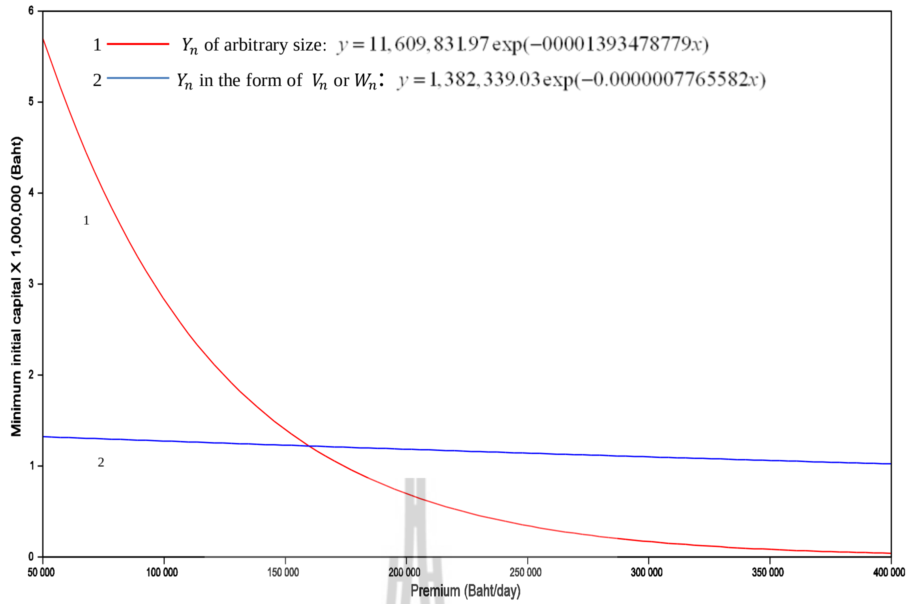

Figure A.18 MIC under a ruin probability not greater than $\alpha = 0.01$ and interest rate $r = 8\%$ (Class A).

In the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as follows:

$$\text{MIC} = \begin{cases} 11,609,831.97\exp(-00001393478779x), & \text{if } x \le 161,730 \\ 1,382,339.03\exp(-0.0000007765582x), & \text{if } x > 161,730 \end{cases}$$

where x =161,730 is the point of intersection of the two curves.

Class B : claim severities do not happen every day

B.1. Claim severities Y_n of arbitrary size

We set the initial capital $u = 0,\ 20,000,\ 40,000,\ ...,\ 1,080,000$ Baht and simulate 10,000 times. The simulation results as shown in Figure A.19 to Figure A.24 for the interest rate r =3% to r =8%, respectively.

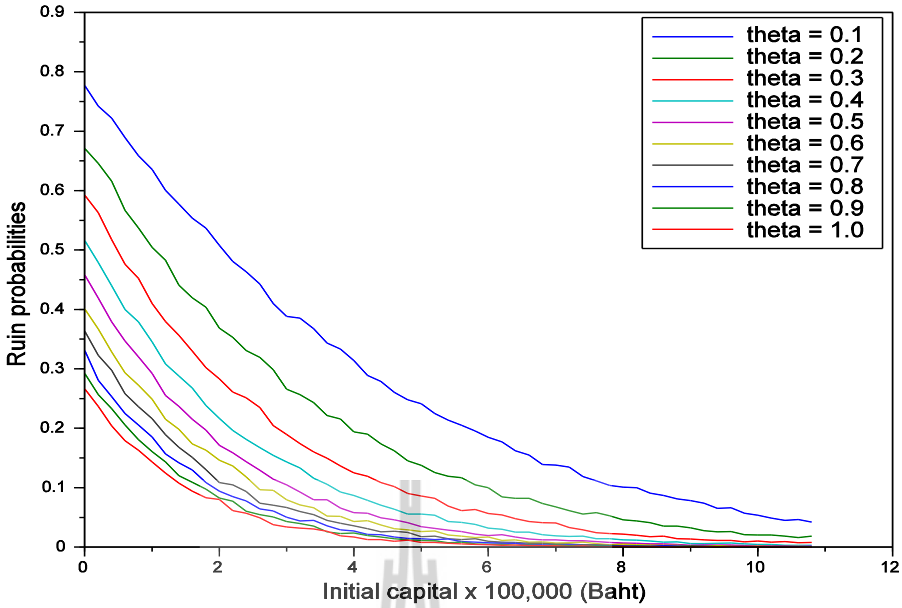

Figure A.19 The relation between the initial capital and ruin probability in case $r = 3\%$ (Class B: arbitrary size of Y_n).

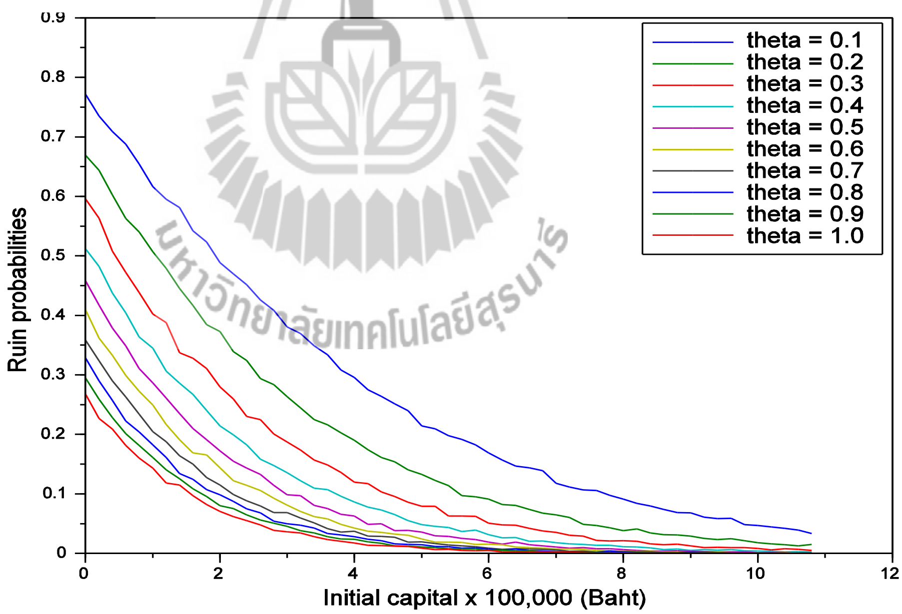

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Figure A.20 The relation between the initial capital and ruin probability in case $r = 4\%$ (Class B: arbitrary size of Y_n).

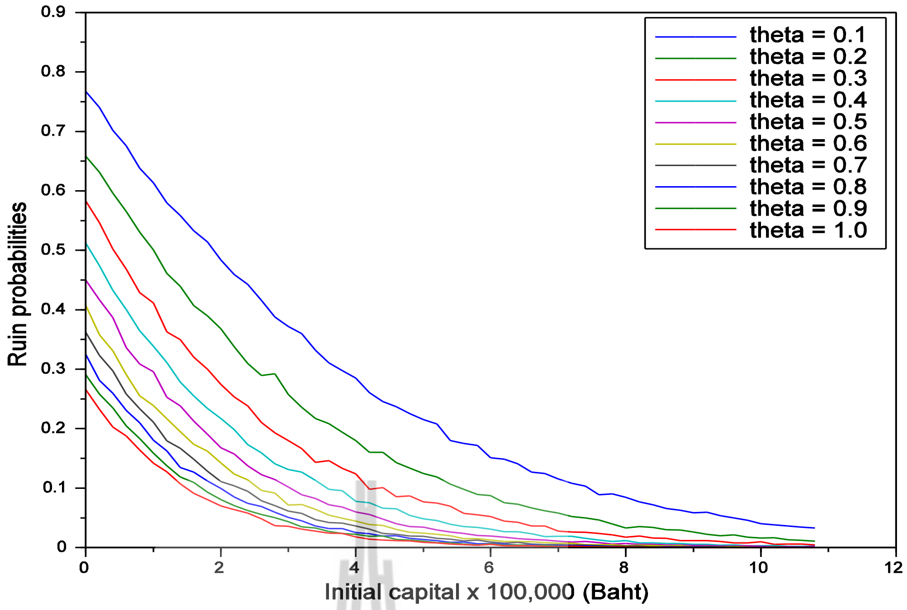

Figure A.21 The relation between the initial capital and ruin probability in case $r = 5\%$ (Class B: arbitrary size of Y_n).

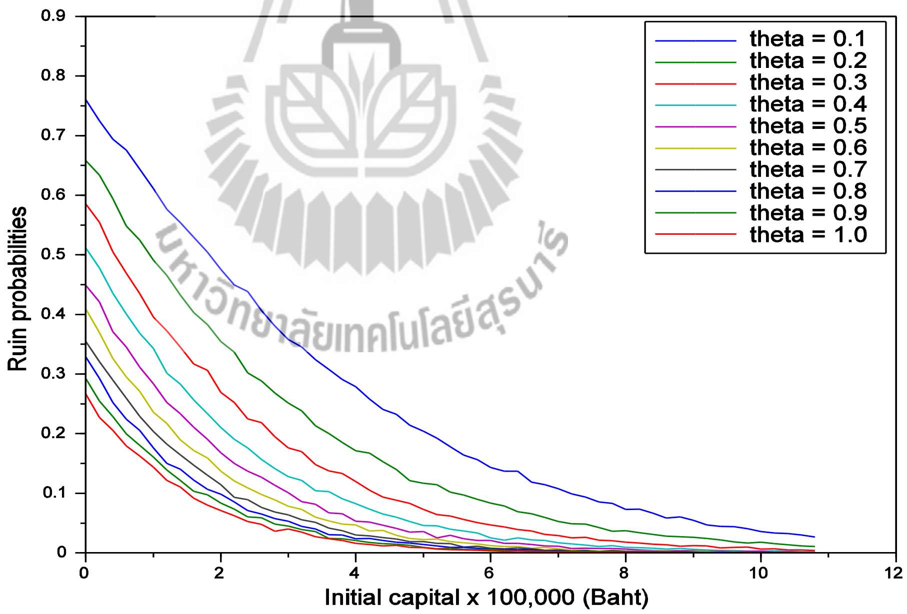

Figure A.22 The relation between the initial capital and ruin probability in case $r = 6\%$ (Class B: arbitrary size of Y_n).

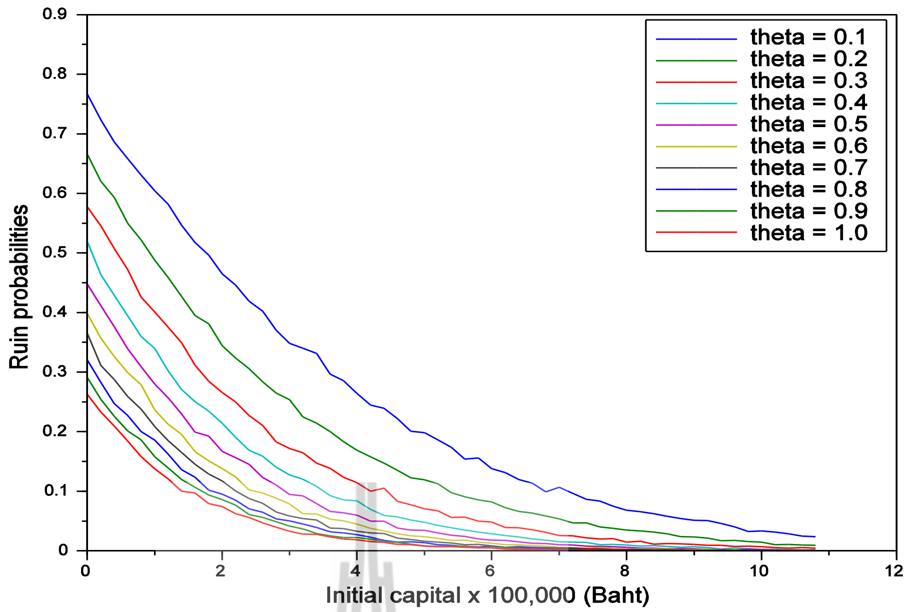

Figure A.23 The relation between the initial capital and ruin probability in case $r = 7\%$ (Class B: arbitrary size of Y_n).

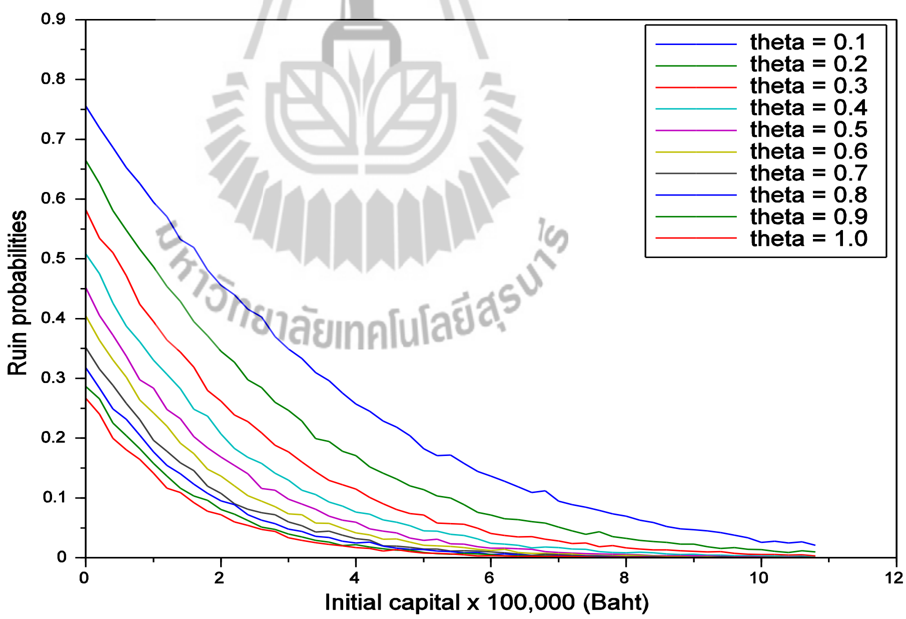

Figure A.24 The relation between the initial capital and ruin probability in case $r = 8\%$ (Class B: arbitrary size of Y_n).

The relations between the ruin probability and initial capital are illustrated in Figure A.19 to Figure A.24 : top curves are plotted in case $\theta = 0.1$, the next curves are plotted in case $\theta = 0.2$, and so on to a bottom curves are plotted in case $\theta = 1.0$, respectively.

B.2 Claim severities Y_n in the form of standard claims V_n or large claims W_n

We set the initial capital

$$u = 0,\ 20,000,\ 40,000,\ ...,\ 380,000$$

Baht and simulate 10,000 times. Figure A.25 to Figure A.30 show the simulation results in case of an interest rate of r =3% to r =8%, respectively.

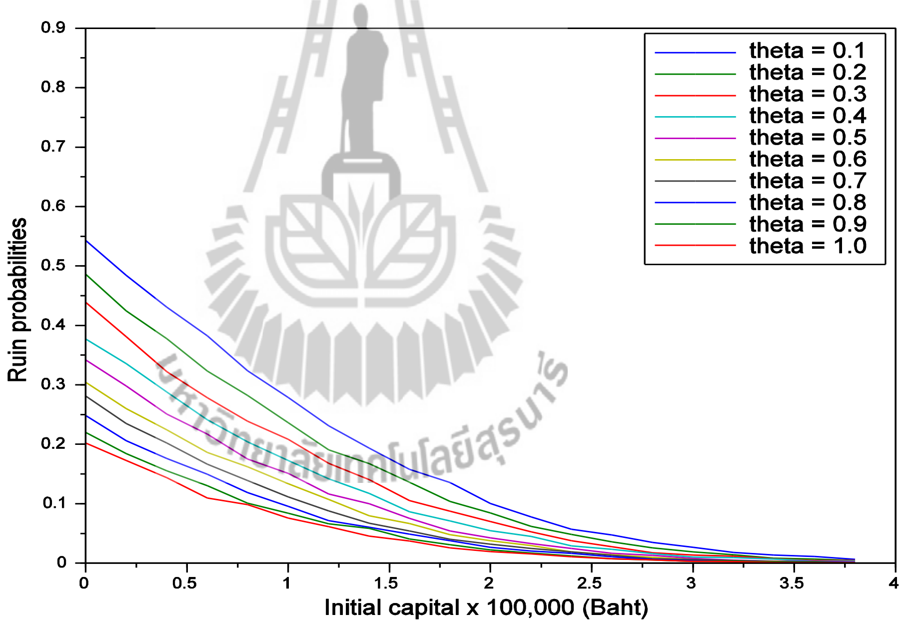

Figure A.25 The relation between the initial capital and ruin probability in case r = 3% (Class B: Y_n in the form of V_n or W_n).

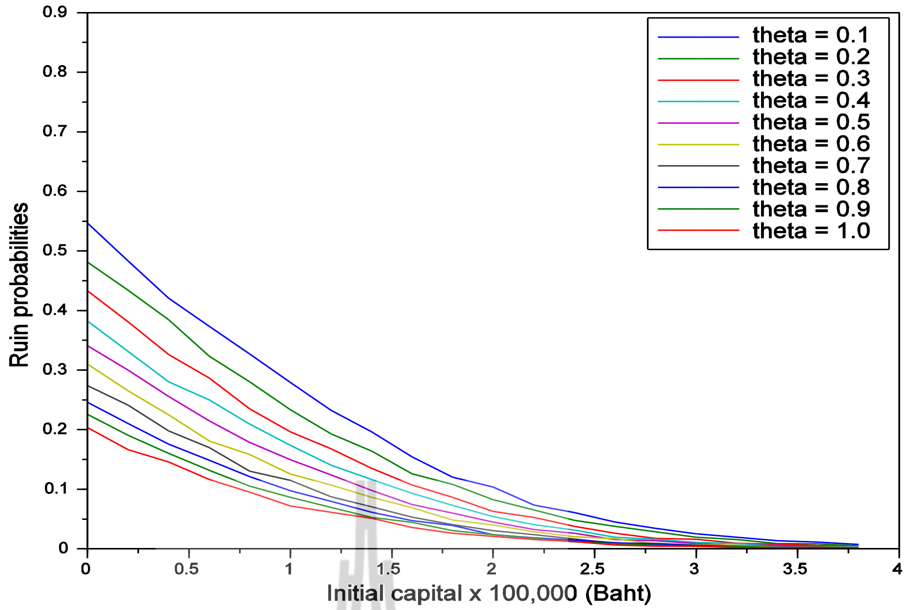

Figure A.26 The relation between the initial capital and ruin probability in case $r = 4\%$ (Class B: Y_n in the form of V_n or W_n).

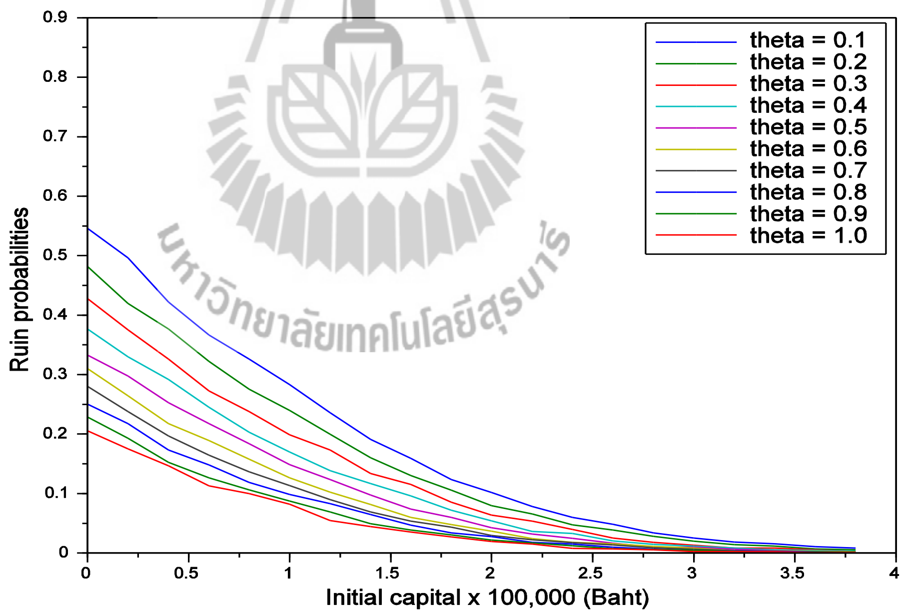

Figure A.27 The relation between the initial capital and ruin probability in case $r = 5\%$ (Class B: Y_n in the form of V_n or W_n).

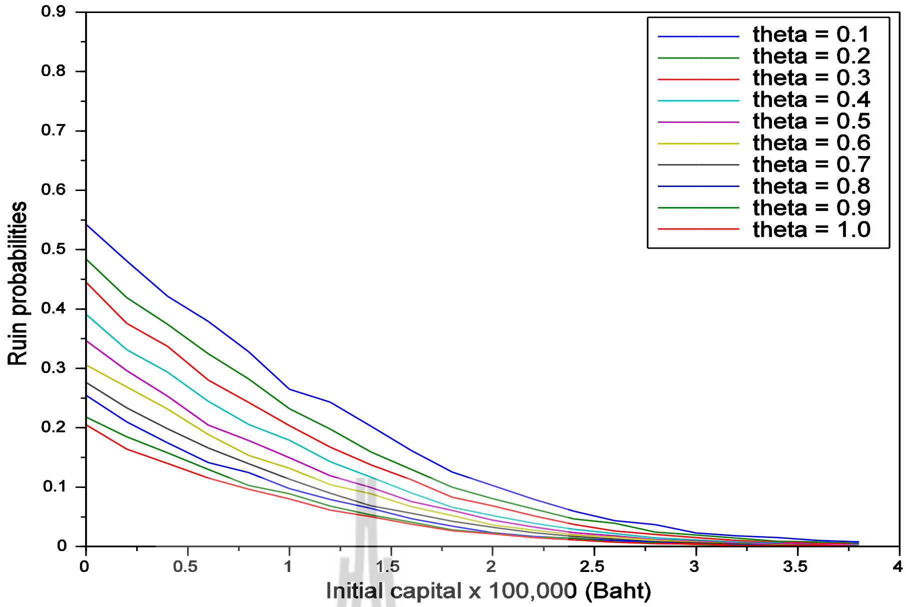

Figure A.28 The relation between the initial capital and ruin probability in case $r = 6\%$ (Class B: Y_n in the form of V_n or W_n).

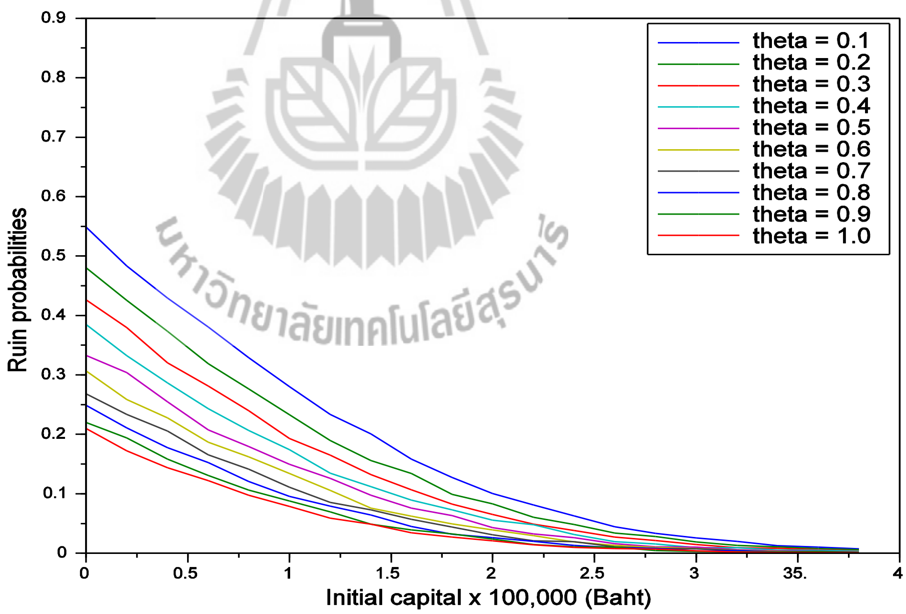

Figure A.29 The relation between the initial capital and ruin probability in case $r = 7\%$ (Class B: Y_n in the form of V_n or W_n).

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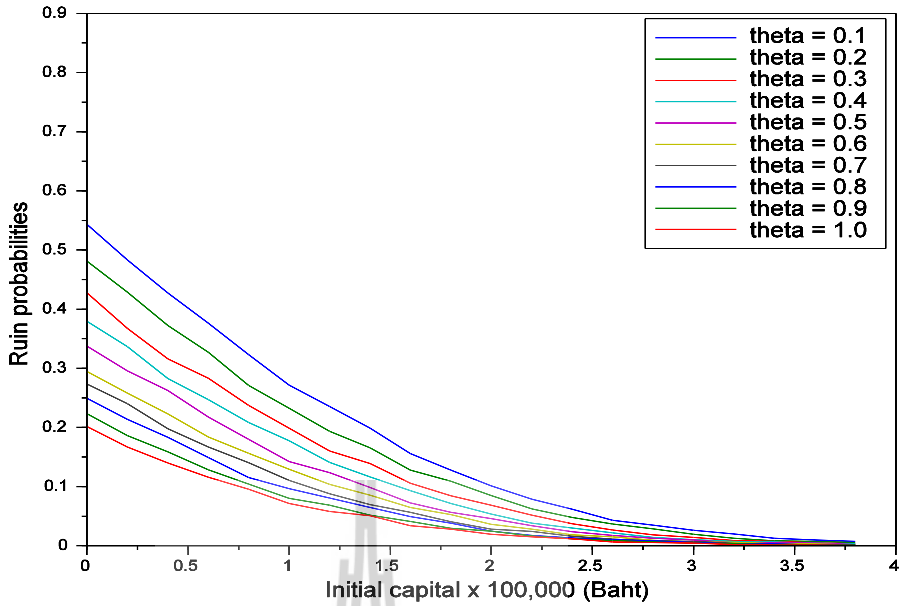

Figure A.30 The relation between the initial capital and ruin probability in case $r = 8\%$ (Class B: Y_n in the form of V_n or W_n).

The relation between the ruin probability and initial capital are illustrated in Figure A.25 to Figure A.30 : top curves are plotted in case $\theta = 0.1$, the next curves are plotted in case $\theta = 0.2$, and so on to a bottom curves are plotted in case $\theta = 1.0$, respectively.

Minimum initial capital for Class B

From the Figure A.19 to Figure A.30, we may consider the relationship between ruin probability $\Phi(u, 365) \coloneqq y$ and initial capital u to be an exponential function

$$y = \gamma \exp(-\delta u).$$

By applying (4.18), (4.19), (4.20) and setting $\alpha = 0.01$, we obtain the MIC in case of the interest rate $r = 3\%$ to $r = 8\%$ that are shown in Table A.7 to Table A.12. In the case of considering the claim severities Y_n of arbitrary size, the premium rate c can be computed by

$$c = (1+\theta)\frac{E[Y_1]}{E[Z_1]}$$

whereas in the case of considering claim severities Y_n in the form of standard and large claims, the premium rate c can be computed by

$$c = (1+\theta)(\frac{EW_1}{EZ_1^L} + \frac{EV_1}{EZ_1^l})$$

where $\frac{E[Y_1]}{E[Z_1]} =$ 41,031.60 and $\frac{EW_1}{EZ_1^L} + \frac{EV_1}{EZ_1^l} = 24,333.87$.

Table A.7 MIC (Baht) in case $r = 3\%, \alpha = 0.01$ (Class B).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	45,134.76	1,650,902	26,767.26	380,802
0.2	49,237.92	1,234,754	29,200.64	349,028
0.3	53,341.08	993,694	31,634.03	330,236
0.4	57,444.24	841,327	34,067.42	314,158
0.5	61,547.40	729,310	36,500.81	285,090
0.6	65,650.56	657,620	38,934.19	269,056
0.7	69,753.72	597,799	41,367.58	259,189
0.8	73,856.88	540,543	43,800.97	249,639
0.9	77,960.04	495,794	46,234.35	238,834
1.0	82,063.20	466,556	48,667.74	227,605

Table A.8 MIC (Baht) in case $r = 4\%, \alpha = 0.01$ (Class B).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	45,134.76	1,564,578	26,767.26	382,780
0.2	49,237.92	1,183,700	29,200.64	357,592
0.3	53,341.08	964,020	31,634.03	331,125
0.4	57,444.24	817,236	34,067.42	309,925
0.5	61,547.40	705,646	36,500.81	290,040
0.6	65,650.56	645,195	38,934.19	280,091
0.7	69,753.72	587,708	41,367.58	261,590
0.8	73,856.88	539,613	43,800.97	250,907
0.9	77,960.04	499,857	46,234.35	240,674
1.0	82,063.20	463,611	48,667.74	227,445

Table A.9 MIC (Baht) in case $r = 5\%, \alpha = 0.01$ (Class B).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	45,134.76	1,508,501	26,767.26	386,608
0.2	49,237.92	1,147,098	29,200.64	357,814
0.3	53,341.08	942,236	31,634.03	322,973
0.4	57,444.24	796,086	34,067.42	310,920
0.5	61,547.40	715,240	36,500.81	288,712
0.6	65,650.56	630,524	38,934.19	278,039
0.7	69,753.72	569,019	41,367.58	261,867
0.8	73,856.88	539,175	43,800.97	249,961
0.9	77,960.04	496,020	46,234.35	239,617
1.0	82,063.20	463,716	48,667.74	227,567

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Table A.10 MIC (Baht) in case $r = 6\%, \alpha = 0.01$ (Class B).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	45,134.76	1,444,173	26,767.26	383,250
0.2	49,237.92	1,140,768	29,200.64	354,359
0.3	53,341.08	929,768	31,634.03	330,882
0.4	57,444.24	786,737	34,067.42	307,816
0.5	61,547.40	704,278	36,500.81	295,642
0.6	65,650.56	622,604	38,934.19	279,344
0.7	69,753.72	572,469	41,367.58	258,709
0.8	73,856.88	530,053	43,800.97	248,477
0.9	77,960.04	496,024	46,234.35	237,039
1.0	82,063.20	465,034	48,667.74	229,054

Table A.11 MIC (Baht) in case $r = 7\%, \alpha = 0.01$ (Class B).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	45,134.76	1,404,190	26,767.26	381,000
0.2	49,237.92	1,105,155	29,200.64	357,452
0.3	53,341.08	912,801	31,634.03	331,884
0.4	57,444.24	780,262	34,067.42	312,815
0.5	61,547.40	696,703	36,500.81	289,252
0.6	65,650.56	628,954	38,934.19	275,669
0.7	69,753.72	570,640	41,367.58	265,770
0.8	73,856.88	531,067	43,800.97	247,322
0.9	77,960.04	495,577	46,234.35	235,987
1.0	82,063.20	470,417	48,667.74	226,171

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Table A.12 MIC (Baht) in case $r = 8\%, \alpha = 0.01$ (Class B).

Safety loading θ	Y_n of arbitrary size		Y_n in the form of V_n or W_n	
	Premium rate c (Baht)	MIC (Baht)	Premium rate c (Baht)	MIC (Baht)
0.1	45,134.76	1,408,705	26,767.26	380,771
0.2	49,237.92	1,102,626	29,200.64	353,706
0.3	53,341.08	916,390	31,634.03	324,661
0.4	57,444.24	786,095	34,067.42	310,935
0.5	61,547.40	692,935	36,500.81	295,027
0.6	65,650.56	619,333	38,934.19	276,722
0.7	69,753.72	566,899	41,367.58	262,295
0.8	73,856.88	523,190	43,800.97	252,053
0.9	77,960.04	494,418	46,234.35	241,342
1.0	82,063.20	467,588	48,667.74	226,782

From Table A.7 to Table A.12, we perform regression analysis between the premium rate c and the MIC. The results are shown in the Figure A.31 to Figure A.36, respectively.

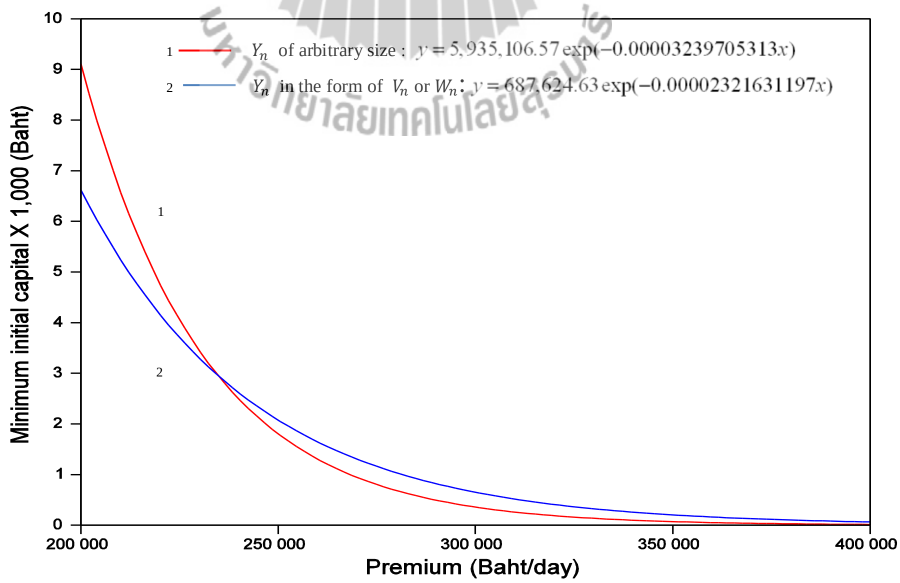

Figure A.31 MIC under a ruin probability not greater than $\alpha = 0.01$ and interest rate $r = 3\%$ (Class B).

In the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as follows:

$$\text{MIC} = \begin{cases} 5{,}935{,}106.57\exp(-0.00003239705313x), & \text{if } x \le 234{,}774 \\ 687{,}624.63\exp(-0.00002321631197x), & \text{if } x > 234{,}774 \end{cases}$$

where $x = 234{,}774$ is the point of intersection of the two curves.

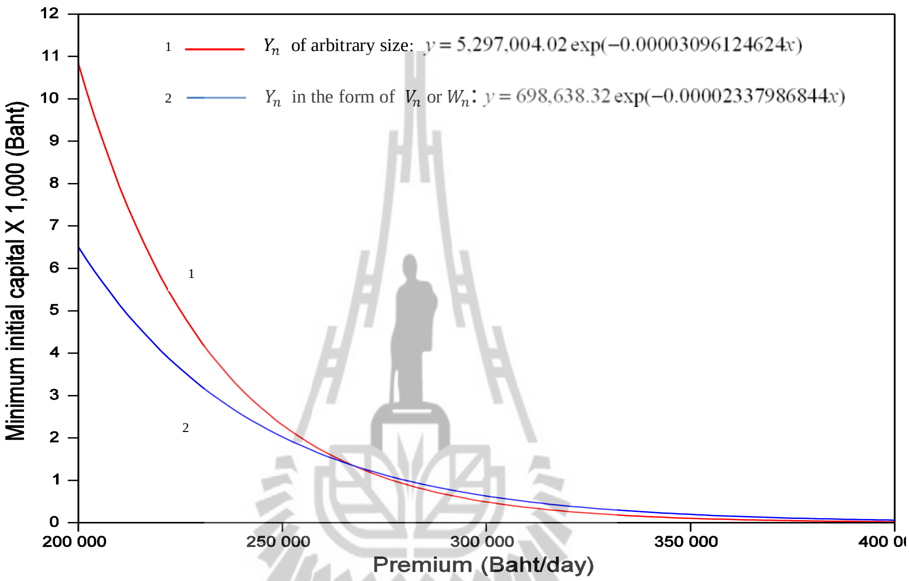

Figure A.32 MIC under a ruin probability not greater than $\alpha = 0.01$ and interest rate $r = 4\%$ (Class B).

In the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as follows:

$$\text{MIC} = \begin{cases} 5{,}297{,}004.02\exp(-0.00003096124624x), & \text{if } x \le 267{,}203 \\ 698{,}638.32\exp(-0.00002337986844x), & \text{if } x > 267{,}203 \end{cases}$$

where $x = 267{,}203$ is the point of intersection of the two curves.

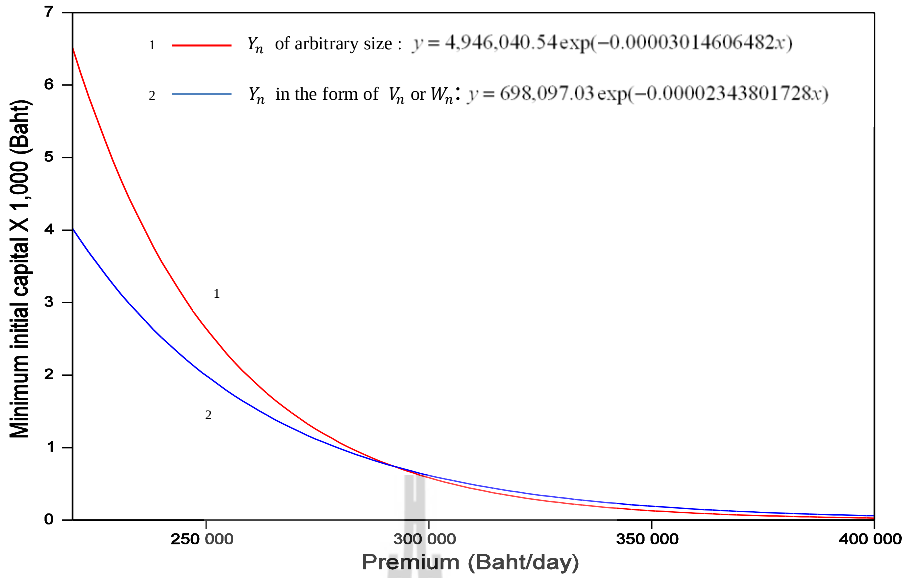

Figure A.33 MIC under a ruin probability not greater than $\alpha = 0.01$ and interest rate $r = 5\%$ (Class B).

In the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as follows:

$$\text{MIC} = \begin{cases} 4,946,040.54\exp(-0.00003014606482x), & \text{if } x \le 291,886 \\ 698,097.03\exp(-0.00002343801728x), & \text{if } x > 291,886. \end{cases}$$

where $x = 291,886$ is the point of intersection of the two curves.

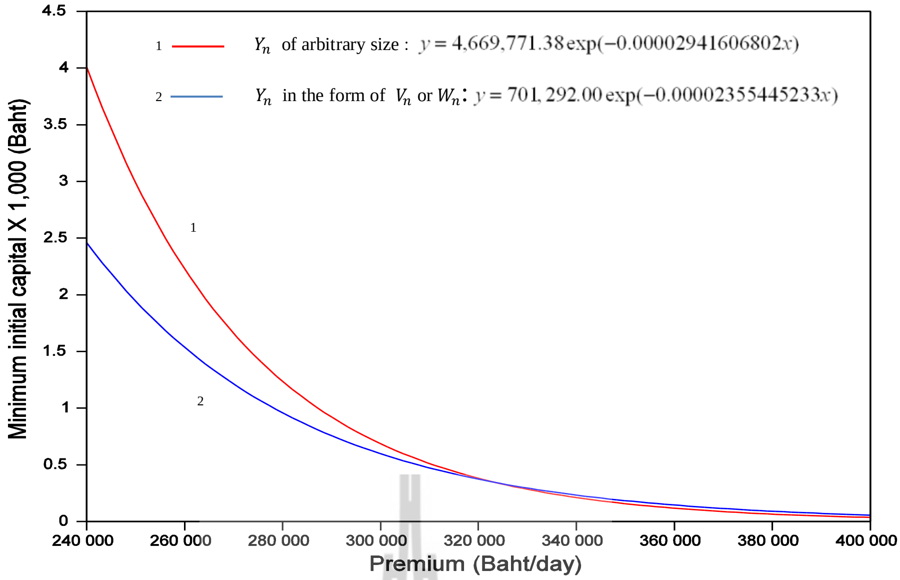

Figure A.34 MIC under a ruin probability not greater than $\alpha = 0.01$ and interest rate $r = 6\%$ (Class B).

In the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as follows:

$$\text{MIC} = \begin{cases} 4,669,771.38\exp(-0.00002941606802x), & \text{if } x \le 323,450 \\ 701,292.00\exp(-0.00002355445233x), & \text{if } x > 323,450 \end{cases}$$

where $x = 323,450$ is the point of intersection of the two curves.

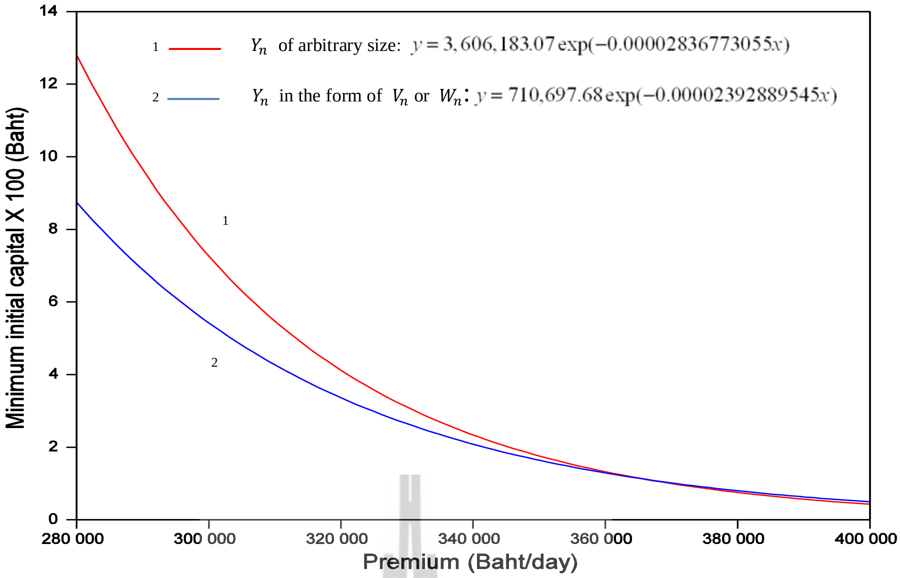

Figure A.35 MIC under the ruin probability is not greater than $\alpha = 0.01$ and interest rate $r = 7\%$ (Class B).

In the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as follows:

$$\text{MIC} = \begin{cases} 3,606,183.07\exp(-0.00002836773055x), & \text{if } x \le 365,897 \\ 710,697.68\exp(-0.00002392889545x), & \text{if } x > 365,897 \end{cases}$$

where $x = 365,897$ is the point of intersection of the two curves.

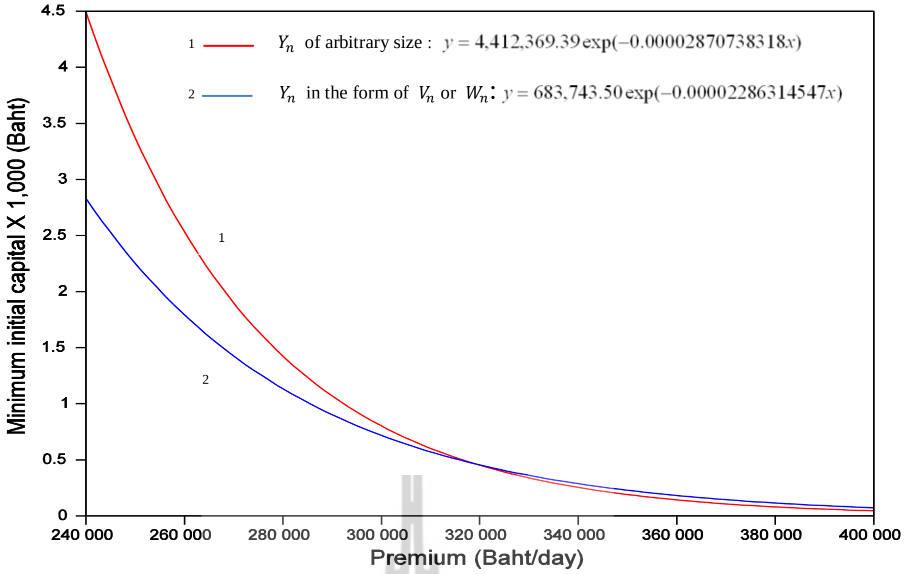

Figure A.36 MIC under a ruin probability not greater than $\alpha = 0.01$ and interest rate $r = 8\%$ (Class B).

In the view of the fair decision for insured (customers), the insurer ought to choose the minimum initial capital as follows:

$$\text{MIC} = \begin{cases} 4{,}412{,}369.39\exp(-0.00002870738318x), & \text{if } x \leq 319{,}047 \\ 683{,}743.50\exp(-0.00002286314547x), & \text{if } x > 319{,}047 \end{cases}$$

where $x = 319{,}047$ is the point of intersection of the two curves.

APPENDIX B

COMPUTATION TERMS OF LINEAR LEAST SQUARES AND WEIGHTED LINEAR LEAST SQUARES METHODS

We start this part by ordering the data y_i from Table 3.1 from the smallest to the largest as mentioned in (3.13). Table B.1 shows the *i* th ranked y_i.

Table B.1 The data y_i sorted from the smallest to the largest.

i	$y_{(i)} \coloneqq y_i$	i	$y_{(i)} \coloneqq y_i$	i	$y_{(i)} \coloneqq y_i$
1	0.4	17	9.3	33	40.0
2	0.7	18	10.8	34	40.2
3	0.9	19	12.4	35	43.1
4	1.4	20	13.0	36	44.9
5	1.8	21	13.1	37	45.8
6	2.1	22	13.2	38	47.3
7	2.3	23	14.2	39	49.9
8	2.4	24	15.5	40	56.5
9	3.6	25	20.1	41	64.6
10	4.2	26	24.4	42	70.0
11	5.7	27	28.5	43	84.3
12	6.4	28	31.5	44	102.7
13	6.7	29	33.2	45	107.3
14	7.2	30	35.3	46	112.2
15	7.5	31	37.2	47	138.9
16	9.2	32	37.7		

To estimate parameter α and β by LSM, we must calculate $\sum_{i=1}^{n} S_i Y_i$ and $\sum_{i=1}^{n} Y_i$ in (3.20) and (3.21). In the following, we set $n = 47$ and compute $\sum_{i=1}^{47} S_i Y_i(y_i)$ and $\sum_{i=1}^{47} Y_i(y_i)$. The probability estimator F_i is given by (3.14b), (3.14c) and (3.14d), respectively.

LSM with F_i is given by (3.14b)

From (3.14b), we have $F_i = \dfrac{i}{n+1}$. We compute

$$Y_1(y_1) = \ln\ln\left[\frac{1}{1-F_1(y_1)}\right] = \ln\ln\left[\frac{1}{1-\frac{1}{47+1}}\right] = -3.86069277, \text{ and}$$

$$S_1Y_1(y_1) = (\ln y_1)(-3.86069277)=(\ln 0.4)(-3.86069277)=3.53751701.$$

For the other terms, $Y_2(y_2),\ldots,Y_{47}(y_{47})$ and $S_2Y_2(y_2),\ldots,S_{47}Y_{47}(y_{47})$ are shown in Table B.2.

Table B.2 Values of $Y_i(y_i)$ and S_iY_i by using equation (3.14b).

i	$Y_i(y_i)$	$S_iY_i(y_i)$	i	$Y_i(y_i)$	$S_iY_i(y_i)$
2	-0.36651292	1.12596912	25	1.16903218	-0.92099178
3	0.09404783	0.28873976	26	1.18114314	-0.79308114
4	0.32663426	-0.82156978	27	1.19266012	-0.63761848
5	0.47588500	-1.29740198	28	1.20363410	-0.45883398
6	0.58319808	-1.49383051	29	1.21411000	-0.26639853
7	0.66572981	-1.53880915	30	1.22412754	-0.06898569
8	0.73209937	-1.49003322	31	1.23372204	0.13483011
9	0.78719501	-2.01356720	32	1.24292499	0.34136165
10	0.83403245	-2.08672979	33	1.25176463	0.55750972
11	0.87459138	-2.34264206	34	1.26026633	0.77111499
12	0.91023509	-2.31276041	35	1.26845297	1.00546921
13	0.94193873	-2.19213807	36	1.27634526	1.24265972
14	0.97042178	-2.10175141	37	1.28396201	1.48194327
15	0.99622889	-1.97792362	38	1.29132032	1.73617644
16	1.01978144	-2.00332038	39	1.29843580	2.01445023
17	1.04141152	-1.84496438	40	1.30532274	2.35276140
18	1.06138513	-1.79659270	41	1.31199423	2.73050161
19	1.07991830	-1.72554649	42	1.31846232	3.11032068
20	1.09718870	-1.58525720	43	1.32473809	3.61909700
21	1.11334405	-1.42201692	44	1.33083176	4.21603793
22	1.12850840	-1.26229349	45	1.33675282	4.76811932
23	1.14278681	-1.13349704	46	1.34251001	5.45791693
24	1.15626901	-1.00455328	47	1.34811149	6.67815607

Therefore,

$$\sum_{i=1}^{47} Y_i(y_i) = -25.72047107 \text{ and } \sum_{i=1}^{47} S_iY_i(y_i) = 8.57754347. \qquad \text{(B.1)}$$

LSM with F_i is given by (3.14c)

By (3.14c), we have $F_i = \dfrac{i-0.3}{n+0.4}$. We calculate

$$Y_1(y_1) = \ln\ln\left[\frac{1}{1-F_1(y_1)}\right] = \ln\ln\left[\frac{1}{1-\frac{1-0.3}{47+0.4}}\right] = -4.20786736,$$

$S_1 = \ln y_1 = \ln 0.4 = -0.91629073$ and $S_1Y_1(y_1) = 3.85562987.$

The other terms, $Y_2(y_2),...,Y_{47}(y_{47})$ and $S_2Y_2(y_2),...,S_{47}Y_{47}(y_{47})$ are shown in Table B.3.

Table B.3 Values of $Y_i(y_i)$ and S_iY_i by using equation (3.14c).

i	$Y_i(y_i)$	$S_iY_i(y_i)$	i	$Y_i(y_i)$	$S_iY_i(y_i)$
2	-3.30978759	1.18051830	25	-0.30617562	-0.91874726
3	-2.83618941	0.29882238	26	-0.24678332	-0.78836984
4	-2.50992756	-0.84452094	27	-0.18815229	-0.63029212
5	-2.25930237	-1.32798780	28	-0.13010459	-0.44885923
6	-2.05477937	-1.52451755	29	-0.07246456	-0.25381075
7	-1.88128511	-1.56693953	30	-0.01505517	-0.05365488
8	-1.73007541	-1.51462694	31	0.04230586	0.15299106
9	-1.59562067	-2.04388452	32	0.09981051	0.36227822
10	-1.47419864	-2.11559966	33	0.15766570	0.58160975
11	-1.36318164	-2.37257153	34	0.21609992	0.79824436
12	-1.26064355	-2.34013009	35	0.27537179	1.03636805
13	-1.16512863	-2.21619994	36	0.33578156	1.27746006
14	-1.07550808	-2.12314010	37	0.39768726	1.52086908
15	-0.99088726	-1.99654174	38	0.46152803	1.77988759
16	-0.91054341	-2.02068111	39	0.52785925	2.06394074
17	-0.83388258	-1.85957015	40	0.59740795	2.41008744
18	-0.76040902	-1.80942834	41	0.67116468	2.79755828
19	-0.68970291	-1.73646258	42	0.75054592	3.18869079
20	-0.62140377	-1.59386921	43	0.83770617	3.71470906
21	-0.55519790	-1.42830891	44	0.93620685	4.33633422
22	-0.49080858	-1.26639256	45	1.05269765	4.92202327
23	-0.42798839	-1.13555675	46	1.20236971	5.67552531
24	-0.36651292	-1.00455328	47	1.43872009	7.09829137

We get

$$\sum_{i=1}^{47} Y_i(y_i) = -26.32863880 \text{ and } \sum_{i=1}^{47} S_i Y_i(y_i) = 10.11662191. \tag{B.2}$$

LSM with F_i is given by (3.14d)

By (3.14d), we have $F_i = \dfrac{i - 3/8}{n + 1/4}$. Next, we compute

$$Y_1(y_1) = \ln\ln\left[\frac{1}{1-F_1(y_1)}\right] = \ln\ln\left[\frac{1}{1-\dfrac{1-0.3}{47+0.4}}\right] = -4.31880578,$$

$S_1 = \ln y_1 = \ln 0.4 = -0.91629073$ and $S_1Y_1(y_1) = 3.95728171.$

The other terms, $Y_2(y_2),\ldots,Y_{47}(y_{47})$ and $S_2Y_2(y_2),\ldots,S_{47}Y_{47}(y_{47})$ are shown in Table B.4.

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Table B.4 Values of $Y_i(y_i)$ and S_iY_i by using equation (3.14d).

i	$Y_i(y_i)$	$S_iY_i(y_i)$	i	$Y_i(y_i)$	$S_iY_i(y_i)$
2	-3.35249745	1.19575184	25	-0.30598568	-0.91817729
3	-2.86192868	0.30153428	26	-0.24640885	-0.78717356
4	-2.52795270	-0.85058590	27	-0.18759698	-0.62843189
5	-2.27291253	-1.33598767	28	-0.12937047	-0.44632652
6	-2.06552518	-1.53249027	29	-0.07155198	-0.25061436
7	-1.89001850	-1.57421366	30	-0.01396272	-0.04976151
8	-1.73731434	-1.52096439	31	0.04358145	0.15760397
9	-1.60170331	-2.05167598	32	0.10127459	0.36759235
10	-1.47935777	-2.12300344	33	0.15932606	0.58773463
11	-1.36758415	-2.38023395	34	0.21796723	0.80514195
12	-1.26441301	-2.34712734	35	0.27746027	1.04422809
13	-1.16835894	-2.22234434	36	0.33810997	1.28631834
14	-1.07827211	-2.12859652	37	0.40028032	1.53078565
15	-0.99324254	-2.00128740	38	0.46441855	1.79103490
16	-0.91253602	-2.02510312	39	0.53109149	2.07657889
17	-0.83554984	-1.86328818	40	0.60104304	2.42475225
18	-0.76178143	-1.81269405	41	0.67528982	2.81475278
19	-0.69080554	-1.73923867	42	0.75529145	3.20885213
20	-0.62225732	-1.59605851	43	0.84327963	3.73942391
21	-0.55581945	-1.42990792	44	0.94297097	4.36766438
22	-0.49121220	-1.26743399	45	1.06138513	4.96264272
23	-0.42818557	-1.13607993	46	1.21489638	5.73465470
24	-0.36651292	-1.00455328	47	1.46451763	7.22557010

We calculate

$$\sum_{i=1}^{47} Y_i(y_i) = -26.50724001 \text{ and } \sum_{i=1}^{47} S_iY_i(y_i) = 10.55654592. \tag{B.3}$$

To estimate the parameters α and β by WLSM, we must calculate $\sum_{i=1}^{n} W_i(y_i)$, $\sum_{i=1}^{n} S_iY_iW_i(y_i)$, $\sum_{i=1}^{n} S_iW_i(y_i)$, $\sum_{i=1}^{n} Y_iW_i(y_i)$, and $\sum_{i=1}^{n} S_i^2W_i(y_i)$ in (3.16) and (3.17). In the following, we set $n = 47$ and compute $\sum_{i=1}^{47} W_i(y_i)$, $\sum_{i=1}^{47} S_i Y_i W_i(y_i)$, $\sum_{i=1}^{47} S_i W_i(y_i)$, $\sum_{i=1}^{47} Y_i W_i(y_i)$, and $\sum_{i=1}^{47} S_i^2 W_i(y_i)$. The probability estimator F_i is given by (3.14b), (3.14c) and (3.14d), respectively.

WLSM with equation (3.19a) and F_i is given by (3.14b)

By (3.19a), we have $W_i = \left[(1-F_i)\ln(1-F_i)\right]^2$. Note that F_i from (3.14b) is given by $F_i = \dfrac{i}{n+1}$. We compute

$$W_1(y_1) = \left[(1-F_1(y_1))\ln(1-F_1(y_1))\right]^2 = \left[(1-\frac{1}{47+1})\ln(1-(\frac{1}{47+1}))\right]^2 = 0.00042497,$$

$$S_1 Y_1 W_1(y_1) = (-0.91629073)(-3.86069278)(0.00042497) = 0.00150334,$$

$$S_1 W_1(y_1) = (-0.91629073)(0.00042497) = -0.00038940,$$

$$Y_1 W_1(y_1) = (-3.86069278)(0.00042497) = -0.00164068,$$

and $S_1^2 W_1(y_1) = (-0.91629073)^2(0.00042497) = 0.00035680$. The other terms,

$W_2(y_2)$, ..., $W_{47}(y_{47})$, $S_2 Y_2 W_2(y_2)$, ..., $S_{47} Y_{47} W_{47}(y_{47})$, $S_2 W_2(y_2)$, …, $S_{47} W_{47}(y_{47})$, $Y_2 W_2(y_2)$, …, $Y_{47} W_{47}(y_{47})$, and $S_2^2 W_2(y_2)$, …, $S_{47}^2 W_{47}(y_{47})$ are shown in Table B.5.

Table B.5 Values of W_i, $S_iY_iW_i$, S_iW_i, Y_iW_i, and $S_i^2W_i$ by using equation (3.19a) and (3.14b).

i	$W_i(y_i)$	$S_iY_iW_i(y_i)$	$S_iW_i(y_i)$	$Y_iW_i(y_i)$	$S_i^2W_i(y_i)$
2	0.00166352	0.00187307	-0.00059334	-0.00525149	0.00021163
3	0.00366084	0.00105703	-0.00038571	-0.01003250	0.00004064
4	0.00636173	-0.00522660	0.00214054	-0.01553353	0.00072023
5	0.00971062	-0.01259858	0.00570777	-0.02143392	0.00335495
6	0.01365158	-0.02039314	0.01012862	-0.02748634	0.00751480
7	0.01812830	-0.02789600	0.01509923	-0.03349225	0.01257629
8	0.02308413	-0.03439612	0.02020944	-0.03928881	0.01769273
9	0.02846205	-0.05731024	0.03645800	-0.04474099	0.04670028
10	0.03420469	-0.07137594	0.04908662	-0.04973640	0.07044344
11	0.04025436	-0.09430155	0.07006135	-0.05418178	0.12193941
12	0.04655305	-0.10766605	0.08641633	-0.05800041	0.16041446
13	0.05304244	-0.11627635	0.10089242	-0.06113027	0.19190824
14	0.05966393	-0.12539875	0.11778143	-0.06352259	0.23251009
15	0.06635866	-0.13125236	0.13370626	-0.06514078	0.26940516
16	0.07306754	-0.14637768	0.16215173	-0.06595956	0.35984768
17	0.07973126	-0.14710134	0.17780187	-0.06596430	0.39650072
18	0.08629040	-0.15502869	0.20533198	-0.06515053	0.48859691
19	0.09268537	-0.15993291	0.23335362	-0.06352351	0.58751358
20	0.09885656	-0.15671307	0.25356206	-0.06109792	0.65037385
21	0.10474436	-0.14894825	0.26946662	-0.05789767	0.69323312
22	0.11028925	-0.13921740	0.28457018	-0.05395570	0.73425278
23	0.11543190	-0.13084172	0.30626876	-0.04931390	0.81260511
24	0.12011325	-0.12066016	0.32921121	-0.04402306	0.90231527
25	0.12427470	-0.11445598	0.37291356	-0.03814284	1.11900911
26	0.12785822	-0.10140194	0.40845370	-0.03174184	1.30483930
27	0.13080655	-0.08340467	0.43818938	-0.02489763	1.46789240
28	0.13306346	-0.06105404	0.45906727	-0.01769689	1.58377636
29	0.13457401	-0.03585032	0.47135218	-0.01023549	1.65093452
30	0.13528491	-0.00933272	0.48213958	-0.00261869	1.71828905
31	0.13514493	0.01822161	0.48872579	0.00503873	1.76738336
32	0.13410544	0.04577845	0.48675716	0.01261233	1.76676306
33	0.13212107	0.07365878	0.48737872	0.01996779	1.79788134
34	0.12915056	0.09958994	0.47706500	0.02696089	1.76221467
35	0.12515780	0.12584232	0.47103428	0.03343737	1.77274833
36	0.12011325	0.14925990	0.45696340	0.03923310	1.73848883
37	0.11399575	0.16893523	0.43595213	0.04417434	1.66720479
38	0.10679496	0.18541490	0.41185587	0.04807841	1.58832639
39	0.09851474	0.19845304	0.38519471	0.05075498	1.50611940
40	0.08917783	0.20981416	0.35976484	0.05200834	1.45137793
41	0.07883268	0.21525277	0.32859152	0.05164148	1.36963992
42	0.06756371	0.21014479	0.28704408	0.04946335	1.21950541
43	0.05550751	0.20088707	0.24614150	0.04530216	1.09148543
44	0.04288029	0.18078491	0.19861342	0.03903114	0.91994006
45	0.03002831	0.14317858	0.14040124	0.03062232	0.65646407
46	0.01753477	0.09570330	0.08276907	0.02027491	0.39069341
47	0.00650443	0.04343757	0.03209124	0.00880416	0.15833029

We calculate

$$\sum_{i=1}^{47} W_i(y_i) = 3.55542462, \sum_{i=1}^{47} S_i Y_i W_i(y_i) = -0.14562181,$$
$$\sum_{i=1}^{47} S_i W_i(y_i) = 11.27649724, \sum_{i=1}^{47} Y_i W_i(y_i) = -0.62542647, \text{ and} \quad \text{(B.4)}$$
$$\sum_{i=1}^{47} S_i^2 W_i(y_i) = 38.23033561.$$

WLSM with equation (3.19a) and F_i is given by (3.14c)

By (3.19a), we $W_i = \left[(1-F_i)\ln(1-F_i)\right]^2$. Note that F_i from (3.14c) is given by

$F_i = \dfrac{i-0.3}{n+0.4}$. We calculate

$$W_1(y_1) = \left[(1-F_1(y_1))\ln(1-F_1(y_1))\right]^2 = \left[(1-\frac{1-0.3}{47+0.4})\ln(1-(\frac{1-0.3}{47+0.4}))\right]^2 = 0.00021487,$$

$S_1 Y_1 W_1(y_1) = (-0.91629073)(-4.20786736)(0.00021487) = 0.000828448,$

$S_1 W_1(y_1) = (-0.91629073)(0.00021487) = -0.00019688,$

$Y_1 W_1(y_1) = (-4.20786736)(0.00021487) = -0.00090413,$

and $S_1^2 W_1(y_1) = (-0.91629073)^2(0.00021487) = 0.00018040.$

The other terms, $W_2(y_2),\ ...,\ W_{47}(y_{47}), S_2 Y_2 W_2(y_2),\ ...,\ S_{47} Y_{47} W_{47}(y_{47}),$

$S_2 W_2,\ \ldots,\ S_{47} W_{47}$ $Y_2 W_2,\ \ldots,\ Y_{47} W_{47}$, and $S_2^2 W_2,\ \ldots,\ S_{47}^2 W_{47}$ are shown in Table B.6.

Table B.6 Values of W_i, $S_iY_iW_i$, S_iW_i, Y_iW_i, and $S_i^2W_i$ by using equation (3.19a) and (3.14c).

i	$W_i(y_i)$	$S_iY_iW_i(y_i)$	$S_iW_i(y_i)$	$Y_iW_i(y_i)$	$S_i^2W_i(y_i)$
2	0.00124003	0.00146387	-0.00044229	-0.00410422	0.00015775
3	0.00305897	0.00091409	-0.00032229	-0.00867582	0.00003396
4	0.00561450	-0.00474156	0.00188912	-0.01409198	0.00063564
5	0.00884900	-0.01175136	0.00520132	-0.01999256	0.00305727
6	0.01270448	-0.01936820	0.00942592	-0.02610489	0.00699345
7	0.01712256	-0.02683001	0.01426153	-0.03221241	0.01187856
8	0.02204448	-0.03338917	0.01929926	-0.03813862	0.01689590
9	0.02741112	-0.05602516	0.03511183	-0.04373775	0.04497593
10	0.03316296	-0.07015956	0.04759166	-0.04888880	0.06829805
11	0.03924017	-0.09310012	0.06829620	-0.05349148	0.11886722
12	0.04558256	-0.10666911	0.08461481	-0.05746336	0.15707030
13	0.05212960	-0.11552962	0.09915611	-0.06073769	0.18860557
14	0.05882049	-0.12488415	0.11611642	-0.06326192	0.22922322
15	0.06559415	-0.13096145	0.13216584	-0.06499640	0.26630136
16	0.07238922	-0.14627552	0.16064640	-0.06591353	0.35650706
17	0.07914416	-0.14717411	0.17649261	-0.06599693	0.39358107
18	0.08579724	-0.15524396	0.20415850	-0.06524100	0.48580457
19	0.09228663	-0.16025228	0.23234972	-0.06365036	0.58498606
20	0.09855040	-0.15707645	0.25277679	-0.06123959	0.64835966
21	0.10452666	-0.14929636	0.26890656	-0.05803298	0.69179231
22	0.11015359	-0.13949768	0.28422014	-0.05406433	0.73334959
23	0.11536956	-0.13100868	0.30610336	-0.04937683	0.81216627
24	0.12011325	-0.12066016	0.32921121	-0.04402306	0.90231527
25	0.12432380	-0.11422215	0.37306088	-0.03806492	1.11945117
26	0.12794093	-0.10086477	0.40871795	-0.03157369	1.30568346
27	0.13090523	-0.08250854	0.43851997	-0.02463012	1.46899985
28	0.13315833	-0.05976935	0.45939460	-0.01732451	1.58490564
29	0.13464326	-0.03417391	0.47159472	-0.00975686	1.65178404
30	0.13530477	-0.00725976	0.48221035	-0.00203704	1.71854124
31	0.13508983	0.02066754	0.48852655	0.00571509	1.76666283
32	0.13394821	0.04852652	0.48618649	0.01336944	1.76469169
33	0.13183316	0.07667545	0.48631664	0.02078557	1.79396348
34	0.12870234	0.10273592	0.47540934	0.02781257	1.75609887
35	0.12451901	0.12904752	0.46863015	0.03428902	1.76370035
36	0.11925352	0.15234161	0.45369259	0.04004313	1.72604523
37	0.11288536	0.17168386	0.43170570	0.04489307	1.65096526
38	0.10540590	0.18761066	0.40649895	0.04864778	1.56766740
39	0.09682211	0.19983509	0.37857648	0.05110845	1.48024198
40	0.08716191	0.21006782	0.35163211	0.05207122	1.41856853
41	0.07648199	0.21396281	0.31879331	0.05133201	1.32879889
42	0.06487972	0.20688135	0.27564116	0.04869521	1.17106016
43	0.05251234	0.19506806	0.23285977	0.04398991	1.03258913
44	0.03963015	0.17184957	0.18355940	0.03710202	0.85021266
45	0.02663989	0.13112215	0.12455823	0.02804375	0.58238801
46	0.01424644	0.08085601	0.06724721	0.01712948	0.31742585
47	0.00387521	0.02750740	0.01911936	0.00557535	0.09433021

We calculate

$$\sum_{i=1}^{47} W_i(y_i) = 3.3.51128404, \sum_{i=1}^{47} S_i Y_i W_i(y_i) = -0.16904739,$$
$$\sum_{i=1}^{47} S_i W_i(y_i) = 11.12948575, \sum_{i=1}^{47} Y_i W_i(y_i) = -0.61712472, \text{ and} \qquad \text{(B.5)}$$
$$\sum_{i=1}^{47} S_i^2 W_i(y_i) = 37.63681236.$$

WLSM with equation (3.19a) and F_i is given by (3.14d)

By (3.19a), we have $W_i = \left[(1-F_i)\ln(1-F_i)\right]^2$. Note that F_i from (3.14d) is given by

$F_i = \dfrac{i - 3/8}{n + 1/4}$. We compute

$$W_1(y_1) = \left[(1-F_1(y_1))\ln(1-F_1(y_1))\right]^2$$
$$= \left[(1-\frac{1-3/8}{47+1/4})\ln(1-(\frac{1-3/8}{47+1/4}))\right]^2$$
$$= 0.00017265,$$

$S_1Y_1W_1(y_1) = (-0.91629073)(-4.31880578)(0.00017265) = 0.00068323,$

$Y_1W_1(y_1) = (-4.31880578)(0.00017265) = -0.00074564,$

$S_1W_1(y_1) = (-0.91629073)(0.00017265) = -0.00015820,$ and

$S_1^2W_1(y_1) = (-0.91629073)^2(0.00017265) = 0.00014496.$

The other terms,

$W_2(y_2),\ ...,\ W_{47}(y_{47}),\ S_2Y_2W_2(y_2),\ ...,\ S_{47}Y_{47}W_{47}(y_{47}),\ S_2W_2(y_2),\ \ldots,\ S_{47}W_{47}(y_{47})$

$Y_2W_2(y_2),\ \ldots,\ Y_{47}W_{47}(y_{47}),$ and $S_2{}^2W_2(y_2),\ \ldots,\ S_{47}{}^2W_{47}(y_{47})$ are shown in Table B.7.

Table B.7 Values of W_i, $S_iY_iW_i$, S_iW_i, Y_iW_i, and $S_i^2W_i$ by using equation (3.19a) and (3.14d).

i	$W_i(y_i)$	$S_iY_iW_i(y_i)$	$S_iW_i(y_i)$	$Y_iW_i(y_i)$	$S_i^2W_i(y_i)$
2	0.00114198	0.00136553	-0.00040732	-0.00382850	0.00014528
3	0.00291416	0.00087872	-0.00030704	-0.00834011	0.00003235
4	0.00543145	-0.00461991	0.00182753	-0.01373044	0.00061491
5	0.00863572	-0.01153721	0.00507596	-0.01962823	0.00298358
6	0.01246845	-0.01910778	0.00925081	-0.02575390	0.00686352
7	0.01687073	-0.02655814	0.01405179	-0.03188599	0.01170386
8	0.02178326	-0.03313156	0.01907056	-0.03784437	0.01669568
9	0.02714636	-0.05569553	0.03477269	-0.04348041	0.04454151
10	0.03289997	-0.06984676	0.04721424	-0.04867083	0.06775643
11	0.03898371	-0.09279034	0.06784982	-0.05331350	0.11809033
12	0.04533681	-0.10641126	0.08415862	-0.05732445	0.15622348
13	0.05189820	-0.11533566	0.09871595	-0.06063572	0.18776835
14	0.05860649	-0.12474957	0.11569396	-0.06319374	0.22838925
15	0.06540002	-0.13088424	0.13177470	-0.06495809	0.26551325
16	0.07221688	-0.14624663	0.16026395	-0.06590050	0.35565832
17	0.07899493	-0.14719031	0.17615982	-0.06600420	0.39283894
18	0.08567185	-0.15529686	0.20386012	-0.06526323	0.48509457
19	0.09218522	-0.16033210	0.23209440	-0.06368206	0.58434326
20	0.09847253	-0.15716792	0.25257706	-0.06127525	0.64784736
21	0.10447129	-0.14938433	0.26876412	-0.05806718	0.69142587
22	0.11011909	-0.13956868	0.28413114	-0.05409184	0.73311994
23	0.11535372	-0.13105104	0.30606132	-0.04939280	0.81205474
24	0.12011325	-0.12066016	0.32921121	-0.04402306	0.90231527
25	0.12433625	-0.11416272	0.37309825	-0.03804511	1.11956331
26	0.12796188	-0.10072821	0.40878485	-0.03153094	1.30589719
27	0.13093014	-0.08228067	0.43860340	-0.02456210	1.46927932
28	0.13318213	-0.05944271	0.45947667	-0.01722983	1.58518880
29	0.13466033	-0.03374781	0.47165453	-0.00963521	1.65199351
30	0.13530902	-0.00673318	0.48222552	-0.00188928	1.71859533
31	0.13507471	0.02128831	0.48847186	0.00588675	1.76646507
32	0.13390673	0.04922309	0.48603593	0.01356135	1.76414522
33	0.13175800	0.07743874	0.48603936	0.02099248	1.79294062
34	0.12858589	0.10352990	0.47497919	0.02802751	1.75450994
35	0.12435352	0.12985344	0.46800732	0.03450316	1.76135632
36	0.11903122	0.15311204	0.45284687	0.04024564	1.72282775
37	0.11259868	0.17236445	0.43060936	0.04507104	1.64677252
38	0.10504773	0.18814415	0.40511765	0.04878611	1.56234040
39	0.09638618	0.20015352	0.37687201	0.05118988	1.47357746
40	0.08664339	0.21008875	0.34954027	0.05207640	1.41012957
41	0.07587827	0.21357857	0.31627690	0.05123982	1.31830995
42	0.06419175	0.20598185	0.27271836	0.04848348	1.15864267
43	0.05174677	0.19350310	0.22946493	0.04363700	1.01753513
44	0.03880311	0.16947898	0.17972873	0.03659021	0.83246972
45	0.02578472	0.12796035	0.12055977	0.02736752	0.56369271
46	0.01343225	0.07702930	0.06340401	0.01631879	0.29928487
47	0.00327356	0.02365333	0.01615094	0.00479419	0.07968476

We calculate

$$\sum_{i=1}^{47} W_i(y_i) = 3.50016495, \sum_{i=1}^{47} S_i Y_i W_i(y_i) = -0.17535197,$$
$$\sum_{i=1}^{47} S_i W_i(y_i) = 11.09237394, \sum_{i=1}^{47} Y_i W_i(y_i) = -0.61515518 \text{ and} \tag{B.6}$$
$$\sum_{i=1}^{47} S_i^2 W_i(y_i) = 37.48736716.$$

WLSM with equation (3.19b) and F_i is given by (3.14a)

By (3.19b), we have $W_i = 3.3F_i - 27.5\left[1-(1-F_i)^{0.025}\right]$. Note that F_i from (3.14a) is given by $F_i = \dfrac{i-0.5}{n}$. We compute

$$\begin{aligned} W_1(y_1) &= 3.3F_1(y_1) - 27.5\left[1-(1-F_1(y_1))^{0.025}\right] \\ &= 3.3\left(\frac{1-0.5}{47}\right) - 27.5\left[1-(1-\frac{1-0.5}{47})^{0.025}\right] \\ &= 0.02775435, \end{aligned}$$

$S_1 Y_1 W_1(y_1) = (-0.91629073)(-4.53795190)(0.02775435) = 0.00068323$=0.11540492,

$S_1 W_1(y_1) = (-0.91629073)(0.02775435) = -0.02543106,$

$Y_1 W_1(y_1) = (-4.53795190)(0.02775435) = -0.12594793$, and

$S_1^2 W_1(y_1) = (-0.91629073)^2(0.02775435) = 0.02330224.$

The other terms $W_2(y_2)$, ..., $W_{47}(y_{47})$, $S_2Y_2W_2(y_2)$, ..., $S_{47}Y_{47}W_{47}(y_{47})$,

$S_2W_2(y_2),\ \ldots,\ S_{47}W_{47}(y_{47})\ \ Y_2W_2(y_2),\ \ldots,\ Y_{47}W_{47}(y_{47})$, and

$S_2^2W_2(y_2),\ \ldots,\ S_{47}^2W_{47}(y_2)$ are shown in Table B.8.

Table B.8 Values of W_i, $S_iY_iW_i$, S_iW_i, Y_iW_i, and $S_i^2W_i$ by using equation (3.19b) and (3.14a).

i	$W_i(y_i)$	$S_iY_iW_i(y_i)$	$S_iW_i(y_i)$	$Y_iW_i(y_i)$	$S_i^2W_i(y_i)$
2	0.08302894	0.10153303	-0.02961434	-0.28466543	0.01056269
3	0.13797992	0.04225585	-0.01453764	-0.40105963	0.00153169
4	0.19259278	-0.16582480	0.06480212	-0.49283353	0.02180412
5	0.24685200	-0.33316586	0.14509631	-0.56681426	0.08528568
6	0.30074094	-0.46496419	0.22313093	-0.62668929	0.16554917
7	0.35424177	-0.56202697	0.29505120	-0.67477586	0.24575084
8	0.40733533	-0.62391661	0.35660935	-0.71266578	0.31220034
9	0.46000100	-0.94983682	0.58923086	-0.74151903	0.75476575
10	0.51221655	-1.09384612	0.73507405	-0.76221721	1.05489339
11	0.56395795	-1.34964741	0.98154974	-0.77545167	1.70835412
12	0.61519919	-1.45121276	1.14199303	-0.78177791	2.11987936
13	0.66591206	-1.48678391	1.26663634	-0.78165082	2.40927852
14	0.71606586	-1.53079875	1.41357204	-0.77544879	2.79050574
15	0.76562715	-1.53835936	1.54266446	-0.76349052	3.10831928
16	0.81455936	-1.65563081	1.80767296	-0.74604732	4.01159414
17	0.86282241	-1.61308579	1.92410640	-0.72335219	4.29078497
18	0.91037226	-1.65522849	2.16627279	-0.69560681	5.15474605
19	0.95716035	-1.66919989	2.40983923	-0.66298695	6.06724373
20	1.00313295	-1.60475226	2.57298520	-0.62564676	6.59957675
21	1.04823041	-1.50169129	2.69669037	-0.58372236	6.93753864
22	1.09238627	-1.38644012	2.81859345	-0.53733473	7.27258225
23	1.13552616	-1.29104715	3.01282565	-0.48659232	7.99375544
24	1.17756646	-1.18292825	3.22752128	-0.43159332	8.84611951
25	1.21841277	-1.11755175	3.65611535	-0.37242789	10.97097778
26	1.25795791	-0.98770202	4.01865113	-0.30918025	12.83791511
27	1.29607951	-0.81044569	4.34174206	-0.24193101	14.54441948
28	1.33263701	-0.58911874	4.59758107	-0.17075967	15.86159743
29	1.36746780	-0.33536076	4.78962417	-0.09574761	16.77589753
30	1.40038243	-0.06052097	4.99079907	-0.01698175	17.78662379
31	1.43115826	0.23665374	5.17551014	0.06544069	18.71624265
32	1.45953124	0.54955216	5.29760228	0.15140596	19.22849561
33	1.48518492	0.88819049	5.47866814	0.24077515	20.21014635
34	1.50773557	1.23143226	5.56937467	0.33337212	20.57252929
35	1.52671156	1.61442345	5.74581407	0.42896601	21.62450341
36	1.54152421	2.00587223	5.86463296	0.52724537	22.31163129
37	1.55142541	2.40080374	5.93309150	0.62777861	22.68982743
38	1.55544410	2.81506602	5.99858619	0.72995164	23.13360941
39	1.55228753	3.25652315	6.06947686	0.83286590	23.73178201
40	1.54018077	3.77267463	6.21345984	0.93516351	25.06659221
41	1.51659148	4.31289727	6.32147845	1.03471099	26.34927756
42	1.47772462	4.79224022	6.27810602	1.12798531	26.67250356
43	1.41750833	5.36014079	6.28577323	1.20876843	27.87351881
44	1.32529553	5.85924808	6.13851987	1.26500124	28.43247072
45	1.17963651	5.93647684	5.51554227	1.26966389	25.78862745
46	0.92556845	5.40364984	4.36894501	1.14477243	20.62265684
47	0.31223698	2.33178244	1.54050053	0.47261828	7.60045105

We calculate

$$\sum_{i=1}^{47} W_i(y_i) = 46.22994532,\ \sum_{i=1}^{47} S_i Y_i W_i(y_i) = 24.01573359,$$
$$\sum_{i=1}^{47} S_i W_i(y_i) = 151.51192964, \sum_{i=1}^{47} Y_i W_i(y_i) = -3.57043308 \text{ and} \qquad \text{(B.7)}$$
$$\sum_{i=1}^{47} S_i^2 W_i(y_i) = 541.38822118.$$

WLSM with equation (3.19b) and F_i is given by (3.14b)

By (3.19b), we have $W_i = 3.3F_i - 27.5\left[1-(1-F_i)^{0.025}\right]$. Note that F_i from (3.14b) is given by $F_i = \dfrac{i}{n+1}$. We compute

$$\begin{aligned} W_1(y_1) &= 3.3F_1(y_1) - 27.5\left[1-(1-F_1(y_1))^{0.025}\right] \\ &= 3.3\left(\frac{1}{47+1}\right) - 27.5\left[1-(1-\frac{1}{47+1})^{0.025}\right] \\ &= 0.05427959, \end{aligned}$$

$S_1Y_1W_1(y_1) = (-0.91629073)(-3.86069277)(0.05427959) = 0.19201497,$

$S_1W_1(y_1) = (-0.91629073)(0.05427959) = -0.04973588,$

$Y_1W_1(y_1) = (-3.86069277)(0.05427959) = -0.20955682$, and

$S_1^2W_1(y_1) = (-0.91629073)^2(0.05427959) = 0.04557253.$ The other terms,

$W_2(y_2),\ ...,\ W_{47}(y_{47}),\ S_2Y_2W_2(y_2),\ ...,\ S_{47}Y_{47}W_{47}(y_{47}),\quad S_2W_2(y_2),\ \ldots,\ S_{47}W_{47}(y_{47})$

$Y_2W_2(y_2),\ \ldots,\ Y_{47}W_{47}(y_{47}),$ and $S_2^2W_2(y_2),\ \ldots,\ S_{47}^2W_{47}(y_{47})$ are shown in Table B.9.

Table B.9 Values of W_i, $S_iY_iW_i$, S_iW_i, Y_iW_i, and $S_i^2W_i$ by using equation (3.19b) and (3.14b).

i	$W_i(y_i)$	$S_iY_iW_i(y_i)$	$S_iW_i(y_i)$	$Y_iW_i(y_i)$	$S_i^2W_i(y_i)$
2	0.10825583	0.12189272	-0.03861214	-0.34174735	0.01377198
3	0.16191554	0.04675145	-0.01705951	-0.44372841	0.00179740
4	0.21524469	-0.17683854	0.07242386	-0.52556650	0.02436862
5	0.26822828	-0.34799989	0.15766100	-0.59205136	0.09267104
6	0.32085023	-0.47929586	0.23805077	-0.64600584	0.17661875
7	0.37309335	-0.57411946	0.31075285	-0.68929424	0.25882889
8	0.42493916	-0.63317347	0.37202095	-0.72323938	0.32569271
9	0.47636781	-0.95919859	0.61019565	-0.74882758	0.78162026
10	0.52735789	-1.10045342	0.75680315	-0.76682133	1.08607648
11	0.57788631	-1.35378078	1.00579158	-0.77782654	1.75054622
12	0.62792810	-1.45224725	1.16562167	-0.78233520	2.16374116
13	0.67745618	-1.48507749	1.28859451	-0.78075370	2.45104531
14	0.72644116	-1.52679874	1.43405371	-0.77342253	2.83093822
15	0.77485102	-1.53259612	1.56124965	-0.76063022	3.14576664
16	0.82265081	-1.64803313	1.82562953	-0.74262371	4.05144342
17	0.86980229	-1.60475423	1.93967162	-0.71961609	4.32549565
18	0.91626348	-1.64615228	2.18029122	-0.69179255	5.18810355
19	0.96198818	-1.65995533	2.42199425	-0.65931511	6.09784639
20	1.00692536	-1.59623568	2.58271256	-0.62232639	6.62452692
21	1.05101847	-1.49456605	2.70386296	-0.58095271	6.95599093
22	1.09420460	-1.38120735	2.82328512	-0.53530670	7.28468778
23	1.13641352	-1.28812136	3.01518004	-0.48548959	8.00000221
24	1.17756646	-1.18292825	3.22752128	-0.43159332	8.84611951
25	1.21757468	-1.12137627	3.65360048	-0.37370242	10.96343135
26	1.25633771	-0.99637775	4.01347527	-0.31189602	12.82138040
27	1.29374118	-0.82491329	4.33390886	-0.24624982	14.51817900
28	1.32965409	-0.61009048	4.58729006	-0.17683846	15.82609358
29	1.36392550	-0.36334775	4.77721707	-0.10373806	16.73244107
30	1.39638014	-0.09633024	4.97653538	-0.02702957	17.73578967
31	1.42681297	0.19237734	5.15979625	0.05319716	18.65941637
32	1.45498196	0.49667504	5.28108995	0.13683789	19.16856146
33	1.48059859	0.82544810	5.46174972	0.22376662	20.14773633
34	1.50331512	1.15922882	5.55304612	0.31382527	20.51221379
35	1.52270712	1.53103513	5.73074328	0.40680903	21.56778412
36	1.53824905	1.91152013	5.85217281	0.50244484	22.26422741
37	1.54927922	2.29594392	5.92488389	0.60035914	22.65843921
38	1.55494824	2.69966451	5.99667389	0.70002782	23.12623461
39	1.55414014	3.13073795	6.07672058	0.80069594	23.76010510
40	1.54534738	3.63583367	6.23430321	0.90124363	25.15067937
41	1.52646238	4.16800799	6.36262249	0.99995048	26.52077477
42	1.49440833	4.64808912	6.34898666	1.09405539	26.97363963
43	1.44443281	5.22754243	6.40516664	1.17886609	28.40295478
44	1.36860905	5.77010769	6.33914000	1.24575599	29.36170546
45	1.25215727	5.97043526	5.85462239	1.27692674	27.37404019
46	1.06213132	5.79702450	5.01356040	1.22810952	23.66542390
47	0.69453086	4.63818545	3.42664456	0.94009252	16.90622216

We calculate

$$\sum_{i=1}^{47} W_i(y_i) = 47.18265341, \sum_{i=1}^{47} S_i Y_i W_i(y_i) = 25.32254715,$$
$$\sum_{i=1}^{47} S_i W_i(y_i) = 154.95191040, \sum_{i=1}^{47} Y_i W_i(y_i) = -3.66731344 \text{ and} \quad \text{(B.8)}$$
$$\sum_{i=1}^{47} S_i^2 W_i(y_i) = 557.34074630.$$

WLSM with equation (3.19b) and F_i is given by (3.14c)

By (3.19b), we have $W_i = 3.3F_i - 27.5\left[1-(1-F_i)^{0.025}\right]$. Note that F_i from (3.14c) is given by $F_i = \dfrac{i-0.3}{n+0.4}$. We calculate

$$\begin{aligned} W_1(y_1) &= 3.3F_1(y_1) - 27.5\left[1-(1-F_1(y_1))^{0.025}\right] \\ &= 3.3\left(\frac{1-0.3}{47+0.4}\right) - 27.5\left[1-(1-\frac{1-0.3}{47+0.4})^{0.025}\right] \\ &= 0.03850741, \end{aligned}$$

$S_1Y_1W_1(y_1) = (-0.91629073)(-4.20786736)(0.03850741) = 0.14847032,$

$S_1W_1(y_1) = (-0.91629073)(0.03850741) = -0.03528398,$

$Y_1W_1(y_1) = (-4.20786736)(0.03850741) = -0.16203407,$ and

$S_1^2W_1(y_1) = (-0.91629073)^2(0.03850741) = 0.03233039.$ The other terms, $W_2(y_2), ..., W_{47}(y_{47}), S_2Y_2W_2(y_2), ..., S_{47}Y_{47}W_{47}(y_{47}),$ $S_2W_2(y_2), \ldots, S_{47}W_{47}(y_{47})$ $Y_2W_2(y_2), \ldots, Y_{47}W_{47}(y_{47}),$ and $S_2^2W_2(y_2), \ldots, S_{47}^2W_{47}(y_{47})$ are shown in Table B.10.

Table B.10 Values of W_i, $S_iY_iW_i$, S_iW_i, Y_iW_i, and $S_i^2W_i$ by using equation (3.19b) and (3.14c).

i	$W_i(y_i)$	$S_iY_iW_i(y_i)$	$S_iW_i(y_i)$	$Y_iW_i(y_i)$	$S_i^2W_i(y_i)$
2	0.09325569	0.11009005	-0.03326197	-0.30865652	0.01186371
3	0.14768323	0.04413105	-0.01555998	-0.41885761	0.00163941
4	0.20177570	-0.17040381	0.06789192	-0.50644240	0.02284375
5	0.25551779	-0.33932451	0.15018995	-0.57729195	0.08827965
6	0.30889310	-0.47091296	0.22917933	-0.63470718	0.17003670
7	0.36188406	-0.56705043	0.30141653	-0.68080709	0.25105258
8	0.41447177	-0.62777011	0.36285708	-0.71706742	0.31767003
9	0.46663593	-0.95374995	0.59772976	-0.74457394	0.76565228
10	0.51835465	-1.09663091	0.74388273	-0.76415771	1.06753460
11	0.56960427	-1.35142687	0.99137696	-0.77647408	1.72545807
12	0.62035921	-1.45172125	1.15157156	-0.78205184	2.13765996
13	0.67059172	-1.48616533	1.27553756	-0.78132561	2.42620959
14	0.72027163	-1.52923759	1.42187456	-0.77465796	2.80689559
15	0.76936608	-1.53607149	1.55019804	-0.76235505	3.12349870
16	0.81783916	-1.65259213	1.81495150	-0.74467805	4.02774670
17	0.86565154	-1.60973976	1.93041539	-0.72185173	4.30485412
18	0.91276002	-1.65157384	2.17195457	-0.69407095	5.16826610
19	0.95911700	-1.66547077	2.41476548	-0.66150578	6.07964652
20	1.00466985	-1.60131234	2.57692728	-0.62430563	6.60968797
21	1.04936019	-1.49881051	2.69959687	-0.58260258	6.94501592
22	1.09312302	-1.38432286	2.82049442	-0.53651416	7.27748717
23	1.13588565	-1.28986261	3.01377948	-0.48614587	7.99628618
24	1.17756646	-1.18292825	3.22752128	-0.43159332	8.84611951
25	1.21807336	-1.11910156	3.65509687	-0.37294437	10.96792161
26	1.25730193	-0.99121891	4.01655553	-0.31028115	12.83122054
27	1.29513307	-0.81631217	4.33857158	-0.24368225	14.53379865
28	1.33143017	-0.59762472	4.59341752	-0.17322518	15.84723323
29	1.36603543	-0.34671448	4.78460723	-0.09898916	16.75832544
30	1.39876524	-0.07505058	4.98503561	-0.02105865	17.76608348
31	1.42940424	0.21868607	5.16916706	0.06047218	18.69330414
32	1.45769748	0.52809204	5.29094639	0.14549353	19.20433697
33	1.48334013	0.86272508	5.47186292	0.23387185	20.18504269
34	1.50596339	1.20212679	5.56282847	0.32543857	20.54834851
35	1.52511534	1.58058081	5.73980664	0.41997374	21.60189428
36	1.54023368	1.96758701	5.85972323	0.51718207	22.29295254
37	1.55060662	2.35826965	5.92996022	0.61665650	22.67785253
38	1.55531437	2.76828474	5.99808588	0.71782117	23.13167994
39	1.55313911	3.20558707	6.07280652	0.81983884	23.74480105
40	1.54241992	3.71736688	6.22249312	0.92145393	25.10303463
41	1.52080703	4.25454630	6.33904978	1.02071196	26.42251866
42	1.48481720	4.73462293	6.30823882	1.11442350	26.80052260
43	1.42895910	5.30816733	6.33655033	1.19704786	28.09868388
44	1.34380546	5.82718960	6.22425442	1.25807987	28.82957703
45	1.21098577	5.96050013	5.66211975	1.27480187	26.47396932
46	0.98624450	5.59745564	4.65535315	1.18583052	21.97458432
47	0.50072347	3.55428106	2.47044653	0.72040091	12.18857605

We calculate

$$\sum_{i=1}^{47} W_i(y_i) = 46.65946112, \sum_{i=1}^{47} S_i Y_i W_i(y_i) = 24.88565987,$$
$$\sum_{i=1}^{47} S_i W_i(y_i) = 153.11698387, \sum_{i=1}^{47} Y_i W_i(y_i) = -3.54541039 \text{ and} \tag{B.9}$$
$$\sum_{i=1}^{47} S_i^2 W_i(y_i) = 548.87999728.$$

WLSM with equation (3.19b) and F_i is given by (3.14d)

By (3.19b), we have $W_i = 3.3F_i - 27.5\left[1-(1-F_i)^{0.025}\right]$. Note that F_i from (3.14d) is given by $F_i = \dfrac{i-3/8}{n+1/4}$. We compute

$$\begin{aligned} W_1(y_1) &= 3.3F_1(y_1) - 27.5\left[1-(1-F_1(y_1))^{0.025}\right] \\ &= 3.3\left(\frac{1-3/8}{47+1/4}\right) - 27.5\left[1-(1-\frac{1-3/8}{47+1/4})^{0.025}\right] \\ &= 0.03449772, \end{aligned}$$

$S_1Y_1W_1(y_1) = (-0.91629073)(-4.31880578)(0.03449772) = 0.13651720,$

$S_1W_1(y_1) = (-0.91629073)(0.03449772) = -0.03160994,$

$Y_1W_1(y_1) = (-4.31880578)(0.03449772) = -0.14898896,$ and

$S_1^2W_1(y_1) = (-0.91629073)^2(0.03449772) = 0.02896390.$ The other terms,

$W_2(y_2),\ ...,\ W_{47}(y_{47}),\ S_2Y_2W_2(y_2),\ ...,\ S_{47}Y_{47}W_{47}(y_{47}),\ S_2W_2(y_2),\ \ldots,\ S_{47}W_{47}(y_{47})$

$Y_2W_2(y_2),\ \ldots,\ Y_{47}W_{47}(y_{47}),$ and $S_2^2W_2(y_2),\ \ldots,\ S_{47}^2W_{47}(y_{47})$ are shown in Table B.11.

Table B.11 Values of W_i , $S_iY_iW_i$ **,** S_iW_i **,** Y_iW_i **,** and $S_i^2W_i$ by using equation (3.19b) and (3.14d).

i	$W_i(y_i)$	$S_iY_iW_i(y_i)$	$S_iW_i(y_i)$	$Y_iW_i(y_i)$	$S_i^2W_i(y_i)$
2	0.08944226	0.10695074	-0.03190181	-0.29985494	0.01137858
3	0.14406499	0.04344053	-0.01517876	-0.41230372	0.00159924
4	0.19835151	-0.16871500	0.06673978	-0.50142323	0.02245608
5	0.25228643	-0.33705156	0.14829060	-0.57342499	0.08716324
6	0.30585327	-0.46871715	0.22692396	-0.63174762	0.16836336
7	0.35903434	-0.56519677	0.29904298	-0.67858156	0.24907563
8	0.41181068	-0.62634938	0.36052738	-0.71544460	0.31563045
9	0.46416184	-0.95230971	0.59456062	-0.74344956	0.76159282
10	0.51606582	-1.09560950	0.74059807	-0.76344597	1.06282082
11	0.56749881	-1.35077993	0.98771248	-0.77610237	1.71908016
12	0.61843508	-1.45154588	1.14799979	-0.78195736	2.13102971
13	0.66884670	-1.48640767	1.27221834	-0.78145302	2.41989607
14	0.71870331	-1.52982936	1.41877856	-0.77495773	2.80078384
15	0.76797182	-1.53693234	1.54738875	-0.76278229	3.11783826
16	0.81661610	-1.65373180	1.81223729	-0.74519160	4.02172330
17	0.86459652	-1.61099247	1.92806269	-0.72241348	4.29960755
18	0.91186957	-1.65294055	2.16983572	-0.69464531	5.16322420
19	0.95838730	-1.66686426	2.41292834	-0.66205926	6.07502116
20	1.00409668	-1.60259705	2.57545712	-0.62480651	6.60591709
21	1.04893884	-1.49988595	2.69851288	-0.58302061	6.94222724
22	1.09284824	-1.38511301	2.81978541	-0.53682039	7.27565778
23	1.13575156	-1.29030456	3.01342371	-0.48631243	7.99534224
24	1.17756646	-1.18292825	3.22752128	-0.43159332	8.84611951
25	1.21819998	-1.11852356	3.65547681	-0.37275175	10.96906170
26	1.25754667	-0.98990748	4.01733737	-0.30987063	12.83371821
27	1.29548623	-0.81412486	4.33975461	-0.24302931	14.53776170
28	1.33188057	-0.59445362	4.59497138	-0.17230602	15.85259404
29	1.36657012	-0.34248210	4.78648002	-0.09778079	16.76488498
30	1.39936911	-0.06963471	4.98718775	-0.01953900	17.77375345
31	1.43005949	0.22538306	5.17153666	0.06232406	18.70187332
32	1.45838294	0.53609041	5.29343437	0.14769714	19.21336749
33	1.48403034	0.87221602	5.47440903	0.23644471	20.19443501
34	1.50662739	1.21304892	5.56528120	0.32839540	20.55740856
35	1.52571490	1.59319436	5.74206312	0.42332526	21.61038661
36	1.54072088	1.98185753	5.86157676	0.52093309	22.30000414
37	1.55092019	2.37412636	5.93115941	0.62080282	22.68243856
38	1.55537426	2.78572958	5.99831683	0.72234465	23.13257062
39	1.55283785	3.22459029	6.07162861	0.82469897	23.74019538
40	1.54160849	3.73801864	6.21921960	0.92657305	25.08982846
41	1.51927006	4.27636962	6.33264336	1.02594761	26.39581531
42	1.48222659	4.75624594	6.29723261	1.11951307	26.75376276
43	1.42477816	5.32784952	6.31801043	1.20148640	28.01647087
44	1.33706333	5.83984387	6.19302612	1.26081191	28.68493342
45	1.19963193	5.95334468	5.60903344	1.27327150	26.22575743
46	0.96458204	5.53154492	4.55310019	1.17186722	21.49192141
47	0.43774116	3.16292945	2.15970732	0.64107965	10.65546516

We calculate

$$\sum_{i=1}^{47} W_i(y_i) = 46.50831852, \sum_{i=1}^{47} S_i Y_i W_i(y_i) = 24.63536317,$$
$$\sum_{i=1}^{47} S_i W_i(y_i) = 152.56244221, \sum_{i=1}^{47} Y_i W_i(y_i) = -3.54054183 \text{ and} \tag{B.10}$$
$$\sum_{i=1}^{47} S_i^2 W_i(y_i) = 546.30092085.$$

มหาวิทยาลัยเทคโนโลยีสุรนารี

APPENDIX C

COMPUTATION DERIVATIVE OF FUNCTION USING IN THE NEWTON-RAPHSON METHOD

Let $g:\mathbb{R}\to\mathbb{R}$ be a continuous function on $[a,b]$. From (3.5), we obtain the Newton-Raphson formula

$$x_{k+1} = x_k - \frac{g(x_k)}{g^{/}(x_k)}.$$

We set

$$g(\alpha) := \alpha - \left[\frac{\sum_{i=1}^{n} y_i^{\alpha}\ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n}\ln y_i\right]^{-1} = 0.$$

In this chapter, we will compute $g^{/}(\alpha)$ and $g^{//}(\alpha)$.

$g^{/}(\alpha)$

$$= 1 + \left[\frac{\sum_{i=1}^{n} y_i^{\alpha}\ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n}\ln y_i\right]^{-2} \cdot \frac{d}{d\alpha}\left[\frac{\sum_{i=1}^{n} y_i^{\alpha}\ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n}\ln y_i\right]$$

$$= 1 + \left[\frac{\sum_{i=1}^{n} y_i^{\alpha}\ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n}\ln y_i\right]^{-2} \cdot \left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha}(\ln y_i)^2\right) - \left(\sum_{i=1}^{n} y_i^{\alpha}\ln y_i\right)\left(\sum_{i=1}^{n} y_i^{\alpha}\ln y_i\right)}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^2}\right] \quad \text{(C.1)}$$

$$= 1 + \left[\frac{\sum_{i=1}^{n} y_i^{\alpha}\ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n}\ln y_i\right]^{-2} \cdot \left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha}(\ln y_i)^2\right) - \left(\sum_{i=1}^{n} y_i^{\alpha}\ln y_i\right)^2}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^2}\right].$$

$$
\begin{aligned}
&g^{//}(\alpha) \\
&= \left[\frac{\sum_{i=1}^{n} y_i^{\alpha} \ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n} \ln y_i \right]^{-2} \frac{d}{d\alpha} \left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha} (\ln y_i)^2\right) - \left(\sum_{i=1}^{n} y_i^{\alpha} \ln y_i\right)^2}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^2} \right] \\
&\quad + \left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha} (\ln y_i)^2\right) - \left(\sum_{i=1}^{n} y_i^{\alpha} \ln y_i\right)^2}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^2} \right] \frac{d}{d\alpha} \left[\frac{\sum_{i=1}^{n} y_i^{\alpha} \ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n} \ln y_i \right]^{-2} \\
&= \left[\frac{\sum_{i=1}^{n} y_i^{\alpha} \ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n} \ln y_i \right]^{-2} \frac{d}{d\alpha} \left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha} (\ln y_i)^2\right)}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)} - \left(\frac{\sum_{i=1}^{n} y_i^{\alpha} \ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} \right)^2 \right] \\
&\quad + \left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha} (\ln y_i)^2\right) - \left(\sum_{i=1}^{n} y_i^{\alpha} \ln y_i\right)^2}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^2} \right] \frac{d}{d\alpha} \left[\frac{\sum_{i=1}^{n} y_i^{\alpha} \ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n} \ln y_i \right]^{-2} \\
&= \left[\frac{\sum_{i=1}^{n} y_i^{\alpha} \ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n} \ln y_i \right]^{-2} \left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha} (\ln y_i)^3\right) - \left(\sum_{i=1}^{n} y_i^{\alpha} (\ln y_i)^2\right)\left(\sum_{i=1}^{n} y_i^{\alpha} \ln y_i\right)}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^2} - \right. \\
&\qquad \left. 2\left(\frac{\sum_{i=1}^{n} y_i^{\alpha} \ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} \right) \left(\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha} (\ln y_i)^2\right) - \left(\sum_{i=1}^{n} y_i^{\alpha} \ln y_i\right)\left(\sum_{i=1}^{n} y_i^{\alpha} \ln y_i\right)}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^2} \right) \right] \\
&\quad - 2\left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha} (\ln y_i)^2\right) - \left(\sum_{i=1}^{n} y_i^{\alpha} \ln y_i\right)^2}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^2} \right] \left[\frac{\sum_{i=1}^{n} y_i^{\alpha} \ln y_i}{\sum_{i=1}^{n} y_i^{\alpha}} - \frac{1}{n}\sum_{i=1}^{n} \ln y_i \right]^{-3} \times \\
&\qquad \left[\frac{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)\left(\sum_{i=1}^{n} y_i^{\alpha} (\ln y_i)^2\right) - \left(\sum_{i=1}^{n} y_i^{\alpha} \ln y_i\right)\left(\sum_{i=1}^{n} y_i^{\alpha} \ln y_i\right)}{\left(\sum_{i=1}^{n} y_i^{\alpha}\right)^2} \right] \qquad \text{(C.2)}
\end{aligned}
$$

CURRICULUM VITAE

Name: Mr. Soontorn Boonta

Education Background:

- 2003 B.Sc. Second Class Honors in Mathematics, Khonkaen University, Khonkaen, Thailand.
- 2006 M.Sc. in Mathematics, Chulalongkorn University, Bangkok, Thailand.

Work Experience:

- 2006 – 2009 Lecturer in Mathematics at Faculty of Science and Engineering, Kasetsart University Chalermphrakiat Sakon Nakhon Province Campus, Thailand.

Publication:

- Soontorn Boonta, Anchalee Sattayatham and Pairote Sattayatham. (2013). **Estimation of Weibull parameters using a randomized neighborhood search for the severity of fire accidents.** Journal of Mathematics and Statistics. 9(1): 12-17.

Scholarship:

- 2011 The Center of Excellence in Mathematics (CEM), the Commission of Higher Education (CHE), Sriayudthaya Road, Bangkok 10140, Thailand.