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CONSTRUCTION OF PENTAQUARK WAVE FUNCTIONS

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เราสร้างฟังก์ชันคลื่นของเพนตะควาร์กจากทฤษฎีกลุ่ม การจัดเรียงโครงสร้างของเพนตะควาร์กประกอบไปด้วยการจัดเรียงของสปิน รส และสีของควาร์กแบบ $\mathrm{SU_{SF}(6)} \otimes \mathrm{SU_C(3)}$ โดย $\mathrm{SU_{SF}(6)} = \mathrm{SU_F(3)} \otimes \mathrm{SU_S(2)}$ สมมาตรการสับเปลี่ยนของการจัดเรียงของควาร์กทั้งสี่ตัวในเพนตะควาร์กถูกอธิบายโดยยังทาบลอยด์ [4] [31] [211] [22] และ [1111] ของกลุ่มการสับเปลี่ยน S_4 เนื่องจากฟังก์ชันคลื่นเชิงสีของแอนติควาร์กคือ [11] หรือแอนติทริปเปต ดังนั้นฟังก์ชันคลื่นเชิงสีของควาร์กสี่ตัวที่เหลือจึงเป็น [211] หรือทริปเปต และเนื่องจากฟังก์ชันคลื่นรวมของเพนตะควาร์กต้องเป็นอสมมาตรจากกฎการกีดกันของเพาลี ดังนั้นฟังก์ชันคลื่นเชิงรัศมี-สปิน-รส จึงต้องเป็น [31] ซึ่งเป็นตัวแทนสังยุคของฟังก์ชันคลื่นเชิงสีของควาร์กสี่ตัว [211] เราสามารถคำนวณการจัดเรียงที่เป็นไปได้ของฟังก์ชันคลื่นเชิงรัศมี สปิน รสและสี โดยใช้เมตริกตัวแทนของกลุ่ม S_4 ฟังก์ชันคลื่นส่วนของสปิน รส และสีสามารถคำนวณได้จากการกระทำของเมตริกตัวแทนบนสถานะตั้งต้น ส่วนฟังก์ชันคลื่นเชิงรัศมี เราให้สมมุติฐานว่าอันตรกริยาระหว่างควาร์กเป็นฮามอนิกอย่างง่าย สมการชโรดิงเจอร์เชิงรัศมีจะถูกคำนวณและได้ผลลัพธ์เป็นสถานะไอเกนและค่าไอเกน ซึ่งสถานะไอเกนจะถูกควบคุมสมมาตรโดยเมตริกตัวแทน ในท้ายที่สุดมวลของเพนตะควาร์กที่สถานะพื้นจะถูกคำนวณโดยฟังก์ชันคลื่นรวมของเพนตะควาร์กซึ่งให้สมมุติฐานว่ามีอันตรกริยาแบบหนึ่งกลูออนแลกเปลี่ยนและโกลสโตนโบซอนแลกเปลี่ยน เพื่อคำนวณมวลที่เปลี่ยนไป

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PENTAQUARK/GROUP THEORY/YOUNG TABLOIDS

We construct the wave function of pentaquarks in group theory approach. The algebraic structure of pentaquark states consists of the usual spin-flavor and color algebras $\mathrm{SU_{SF}}(6) \otimes \mathrm{SU_C}(3)$ with $\mathrm{SU_{SF}}(6) = \mathrm{SU_F}(3) \otimes \mathrm{SU_S}(2)$. The permutation symmetry of the four-quark configurations of pentaquark states is characterized by the Young tabloids [4], [31], [211], [22] and [1111] of the permutation group S_4. Since the color part of the antiquark states is a [11] antitriplet, the color part of the four-quark configuration must be a [211] triplet to make it possible to construct a fully antisymmetric total color wave function. The total wave function of the four-quark configuration is antisymmetric, which implies that its spatial-spin-flavor part must be a [31] state, the conjugate representation of [211], the color state of four quarks.

All the possible configurations of spatial, spin, flavor and color wave functions are determined by applying the matrix representations of S_4 while the explicit forms of the spin, flavor and color wave functions are derived by applying projection operators.

We assume that the dominant interaction between quarks is the harmonic oscillator potential, and the spatial wave functions of various symmetries are derived by applying the matrix representations of S_4.

Finally the mass of ground state pentaquarks is calculated by the total pentaquark wave functions and one-gluon exchange and Goldstone-boson exchange interactions.

School of Physics Student's Signature ____________

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Narongrit Ritjoho

CONTENTS

CONTENTS (Continued)

LIST OF TABLES

CHAPTER I

INTRODUCTION

The hadronic structure and the mechanism of the hadron-hadron interaction are the fundamental and significant research subjects in low and medium energy physics. In the energy scale of GeV, the nonperturbative effect of Quantum Chromodynamics (QCD), which is still far from being strictly solved, becomes more serious and cannot be ignored. An efficient way to investigate hadronic structure and hadron-hadron interaction is to employ a phenomenological model or a QCD inspired model. Amongst various models, the quark model illustrates its success in describing the major characteristics of quarkonia, heavy flavor hadronic systems, the nucleon and its lower lying resonances (De Rujula et al., 1975; Isgur and Karl, 1979a; Isgur and Karl, 1979b; Lucha et al., 1991; Mukherjee et al., 1993; Oka and Yazaki, 1980; Oka and Yazaki, 1987; Faessler et al., 1982; Faessler et al., 1983) and the process of nucleon-antinucleon annihilations (Green and Niskanen, 1984; Maruyama and Ueda, 1981; Maruyama et al., 1987; Gutsche et al., 1989; Dover et al., 1992; Thierauf et al., 1985; Bauer et al., 1996; Yan et al., 1997) and meson-baryon reaction (Yan et al., 1995). In recent years, the constituent quark potential model has been extended to the chiral quark model and successfully applied to theoretical estimations in low- and medium-energy physics, such as the descriptions of the properties of baryons and their resonances (Pumsa-ard et al., 2003; Cheedket et al., 2004; Glozman and Riska, 1996), to the investigations of the baryon-baryon interaction (Fernandez et al., 1993; Zhang et al., 1997) and to the predictions of exotic hadronic state candidates (Yuan et al., 1999; Shen et al., 1999; Zhang et al., 2000; Li et al., 2001).

In the light-quark sector, quark-model calculations have made a considerable success in explaining the low-mass excitations spectrum but a number of challenging problems are still being faced, for instance, the mass ordering problem of low-lying baryons and the missing resonance problem at high masses. Theoretical works in the three-quark picture always predict a larger mass for the Ropper resonance $N^+_{1/2}(1440)$ than for $N^-_{1/2}(1520)$ and $N^-_{1/2}(1535)$. A similar situation happens to the Δ resonances.

The Roper resonance $N^+_{1/2}(1440)$ is the lowest-mass nucleon resonance and has the quantum numbers of the nucleon. Its most natural explanation as the first radial excitation is incompatible with quark models in which the radial excitation requires two harmonic- oscillator quanta while the negative parity states like $N^-_{1/2}(1535)$ require one quantum only. Even including anharmonicity, the mass of the first radial excitation should always be above the first orbital-angular-momentum excitation. For instance, within the constituent quark model with one-gluon-exchange (Capstick and Isgur, 1986) or instanton induced forces (Loring et al., 2001) the Roper resonance $N^+_{1/2}(1440)$ should have a mass 80 MeV above the $N^-_{1/2}(1535)$ mass. All quark models encounter the same problem.

At high masses, above 1.8 GeV, conventional quark models predict a large number of baryon resonances which have not been observed. The lowest mass example of this type of resonances is the not well established quartet of nucleon resonances consisting of $N^+_{1/2}(1880)$, $N^+_{3/2}(1900)$, $N^+_{5/2}(1890)$, and $N^+_{7/2}(1990)$. Conventionally, baryons are treated as bound states of three constituent quarks. However, recent investigations on baryon strange magnetic moment and strangeness spin (Zou and Riska, 2005; An et al., 2006), on baryon decays (Li and Riska, 2006a; Li and Riska, 2006b; An and Zou, 2009; An et al., 2010) and the annihilation reaction $p\overline{p} \rightarrow \phi X$ (Srisuphaphon et al., 2011) reveal that low-lying baryons may possess a considerable $q^4\overline{q}$ component.

The baryon spectrum will be studied, assuming that baryons are composed of the q^3 element as well as $q^4\overline{q}$ pentaquark element. The building of pentaquark wave functions is the first and also essential step for investigating baryons and baryons resonances in the three-quark plus pentaquark picture. However, it is not straightforward to calculate the wave function of pentaquark states, especially the spatial part of higher excitations. Theoretical work of systemically constructing spatial wave functions of higher excitation pentaquark systems has never been seen before.

To construct the spatial wave functions of pentaquark states to any order we develop an approach relying on previous work (Yan, 2006; Yan and Srisuphaphon, 2012).

CHAPTER II

PENTAQUARK WAVE FUNCTIONS

This chapter shows how to construct pentaquark wave functions. In constituent quark model, we treat quarks as a substructure of nucleon. Phenomenologically quark has one important property that is color confinement. This property requires that color wave functions of bound states of quarks cluster are colorless or singlet. In other words, quark never appears individually. Group of quarks can form a particle only if their color wave function is singlet or antisymmetric. As a consequence, the possible multiquark states are, for example, one quark and one antiquark ($q\bar{q}$) the so-called meson, three-quarks (q^3) the so-called baryon and four-quarks cluster with one antiquark ($q^4\bar{q}$) the so-called pentaquark, etc. Since the pentaquark consists of four identical particles, its wave functions must follow Pauli exclusion principle that is their total wave functions have to be antisymmetric. Both color confinement property and Pauli exclusion principle manifestly show an importance of symmetry of wave functions. One possible way to be concern with symmetries of wave functions is group theory approach (Appendix I). In the following section we use the group theory approach to figure out possible pentaquark configurations or to illustrate that how each parts of wave functions are combined and formed the total wave functions in the required symmetries.

2.1 Pentaquark configurations

Quark wave functions contain contributions connected to spatial degrees of freedom and the initial degree of freedom of color, flavor and spin. The internal degree of freedom are taken to be the three light flavor u, d and s with spin $s = \frac{1}{2}$ and three possible colors R, G and B. In term of group theory, quark transforms under fundamental representation of $SU(n)$, whereas antiquark transforms under conjugate representation of $SU(n)$ with $n = 2, 3, 3, 6$ for spin, flavor, color and spin-flavor degree of freedom, respectively. The corresponding algebraic structure consists of usual spin-flavor and color algebras

$$SU_{SF}(6) \otimes SU_C(3) \tag{2.1}$$

with

$$SU_{SF}(6) = SU_F(3) \otimes SU_S(2) \tag{2.2}$$

Since one significant property of physical particles or multiquark states is color confinement property which indicates that their color wave functions must be colorless or antisymmetric. Therefore, pentaquark color requires [222] configuration.

$$\Psi^c_{[222]}(q^4\overline{q}) = \yng(2,2,2) \tag{2.3}$$

From group theory antiquark is [11] antitriplet consequently remaining four quarks must be [211] triplet. Here four-quark wave functions are important because they are identical particles. Their symmetry need to be fixed by Pauli exclusion principle.

$$\Psi^c_{[211]}(q^4) = \yng(2,1,1)\,, \Psi^c_{[11]}(\overline{q}) = \yng(1,1) \tag{2.4}$$

Pauli exclusion principle requires total wave functions of group of identical particles be antisymmetric. Hence spatial-spin-flavor wave functions of the four-quark core must be [31], the conjugate configuration of color [211].

$$\Psi^{osf}_{[31]}(q^4) = \yng(3,1) \tag{2.5}$$

Consequently, the total wave functions of the q^4 configurations can be written in a general form as

$$\Psi^A = \sum_{i,j=\lambda,\rho,\eta} a_{ij}\ \Psi^c_{[211]_i}\Psi^{\rm osf}_{[31]_j} \qquad (2.6)$$

Applying the representation matrix of permutation (12) (Appendix A). (12) is a permutation operator exchanging between the first particle and the second particle. When we substitute one quantum number of the first with one of the second particle and vise versa in antisymmetric wave functions, we should get -1 as the eigenvalue of the permutation operator (12). So that we can write

$$(12)\Psi^A = -\Psi^A \qquad (2.7)$$

$$\begin{aligned} (12)\Psi^A &= +a_{\lambda\lambda}\Psi^c_{[211]_\lambda}\Psi^{osf}_{[31]_\lambda} - a_{\lambda\rho}\Psi^c_{[211]_\lambda}\Psi^{osf}_{[31]_\rho} + a_{\lambda\eta}\Psi^c_{[211]_\lambda}\Psi^{osf}_{[31]_\eta} \\ &\quad -a_{\rho\lambda}\Psi^c_{[211]_\rho}\Psi^{osf}_{[31]_\lambda} + a_{\rho\rho}\Psi^c_{[211]_\rho}\Psi^{osf}_{[31]_\rho} - a_{\rho\eta}\Psi^c_{[211]_\rho}\Psi^{osf}_{[31]_\eta} \qquad (2.8) \\ &\quad -a_{\eta\lambda}\Psi^c_{[211]_\eta}\Psi^{osf}_{[31]_\lambda} + a_{\eta\rho}\Psi^c_{[211]_\eta}\Psi^{osf}_{[31]_\rho} - a_{\eta\eta}\Psi^c_{[211]_\eta}\Psi^{osf}_{[31]_\eta} \end{aligned}$$

For this method we can nail down some coefficients by comparing the operated wave function with minus wave function or solving Eq.(2.7). So that we have $a_{\lambda\lambda} = a_{\lambda\eta} = a_{\rho\rho} = a_{\eta\rho} = 0$

$$\begin{aligned} \Psi^A &= a_{\lambda\rho}\Psi^c_{[211]_\lambda}\Psi^{osf}_{[31]_\rho} + a_{\rho\lambda}\Psi^c_{[211]_\rho}\Psi^{osf}_{[31]_\lambda} + a_{\rho\eta}\Psi^c_{[211]_\rho}\Psi^{osf}_{[31]_\eta} \\ &\quad +a_{\eta\lambda}\Psi^c_{[211]_\eta}\Psi^{osf}_{[31]_\lambda} + a_{\eta\eta}\Psi^c_{[211]_\eta}\Psi^{osf}_{[31]_\eta} \qquad (2.9) \end{aligned}$$

Since now we consider the cluster of four identical particles, we cannot consider only (12) permutation operator but we need to determine all possible permutation operators in S_4 permutation group (Appendix A), for example, $(13), (14), (23)$, etc. Similar to (12), applying (13) representation matrix we should get -1 as a eigenvalue of (13) operator or we can write

$$(13)\Psi^A = -\Psi^A \tag{2.10}$$

$$\begin{aligned}
(13)\Psi^A &= a_{\lambda\rho}(\frac{-1}{2}\Psi^c_{[211]_\lambda} - \frac{\sqrt{3}}{2}\Psi^c_{[211]_\rho})(\frac{-\sqrt{3}}{2}\Psi^{osf}_{[31]_\lambda} + \frac{1}{2}\Psi^{osf}_{[31]_\rho}) \\
&\quad + a_{\rho\lambda}(\frac{-\sqrt{3}}{2}\Psi^c_{[211]_\lambda} + \frac{1}{2}\Psi^c_{[211]_\rho})(\frac{-1}{2}\Psi^{osf}_{[31]_\lambda} - \frac{\sqrt{3}}{2}\Psi^{osf}_{[31]_\rho}) \\
&\quad + a_{\rho\eta}(\frac{-\sqrt{3}}{2}\Psi^c_{[211]_\lambda} + \frac{1}{2}\Psi^c_{[211]_\rho})\Psi^{osf}_{[31]_\eta} \\
&\quad - a_{\eta\lambda}\Psi^c_{[211]_\eta}(\frac{-\sqrt{3}}{2}\Psi^{osf}_{[31]_\lambda} + \frac{1}{2}\Psi^{osf}_{[31]_\rho}) \\
&\quad - a_{\eta\eta}\Psi^c_{[211]_\eta}\Psi^{osf}_{[31]_\eta}
\end{aligned} \tag{2.11}$$

So that $a_{\eta\lambda} = a_{\rho\eta} = 0$ and $a_{\lambda\rho} = -a_{\rho\lambda}$

$$\Psi^A = a_{\lambda\rho}[\Psi^c_{[211]_\lambda}\Psi^{osf}_{[31]_\rho} - \Psi^c_{[211]_\rho}\Psi^{osf}_{[31]_\lambda}] + a_{\eta\eta}\Psi^c_{[211]_\eta}\Psi^{osf}_{[31]_\eta} \tag{2.12}$$

Similarly applying the permutation (34) we obtain $a_{\lambda\rho} = a_{\eta\eta}$. As a result, normalized total wave function of pentaquark is

$$\Psi^A = \frac{1}{\sqrt{3}}[\Psi^c_{[211]_\lambda}\Psi^{osf}_{[31]_\rho} - \Psi^c_{[211]_\rho}\Psi^{osf}_{[31]_\lambda} + \Psi^c_{[211]_\eta}\Psi^{osf}_{[31]_\eta}] \tag{2.13}$$

Eq.(2.13) shows that the total wave functions of the four-quark core which are antisymmetric (A) consist of color wave functions (Ψ^C) in [211] configuration mixing with orbital-spin-flavor coupling wave functions in [31] configuration. From now on, we use this method to determine all possible configurations of orbital-spin-flavor wave functions $\psi^{osf}_{[31]_{\lambda,\rho,\eta}}$ in Eq.(2.13).

For the orbital-spin-flavor and spin-flavor wave functions we can write in the

Table 2.1 Possible configurations of spatial-spin-flavor wave functions.

$[31]_{OSF}$	
$[4]_O$	$[31]_{SF}$
$[1111]_O$	$[211]_{SF}$
$[22]_O$	$[31]_{SF}, [211]_{SF}$
$[211]_O$	$[31]_{SF}, [211]_{SF}, [22]_{SF}$
$[31]_O$	$[4]_{SF}, [31]_{SF}, [211]_{SF}, [22]_{SF}$

general forms,

$$\Psi^{osf}_{[31]} = \sum_{i,j=S,A,\lambda,\rho,\eta} a_{ij} \Psi^{o}_{[X]_i} \Psi^{sf}_{[Y]_j} \tag{2.14}$$

$$\Psi^{sf}_{[Z]} = \sum_{i,j=S,A,\lambda,\rho,\eta} a_{ij} \Psi^{s}_{[X]_i} \Psi^{f}_{[Y]_j} \tag{2.15}$$

The possible spatial, spin and flavor configurations are determined by applying S_4 representation matrices in Yanamouchi basis and are shown in the Table 2.1 and Table 2.2

We have already known all the possible configurations of the spatial, spin, flavor and color parts of pentaquark wave functions. The following procedure is to work out the explicit wave functions of each part by applying group theory approaches.

2.2 Color wave functions

We have already derived all the possible symmetries of the color wave functions of the four-quark core, which are $[211]_{\lambda,\rho,\eta}$. In term of group theory, the color wave functions in each symmetry can be obtained by projection operator process. A projection operator is derived, as detailed in Appendix C, using representation

Table 2.2 Possible configurations of spin-flavor wave functions.

$[FS]$	$[F][S]$
$[4]_{FS}$	$[22]_F[22]_S, [31]_F[31]_S, [4]_F[4]_S$
$[31]_{FS}$	$[31]_F[22]_S, [31]_F[31]_S, [31]_F[4]_S, [211]_F[22]_S$
	$[211]_F[31]_S, [22]_F[31]_S, [4]_F[31]_S$
$[22]_{FS}$	$[22]_F[22]_S, [22]_F[4]_S, [4]_F[22]_S, [211]_F[31]_S$
	$[31]_F[31]_S$
$[211]_{FS}$	$[211]_F[22]_S, [211]_F[31]_S, [211]_F[4]_S, [22]_F[31]_S$
	$[31]_F[22]_S, [31]_F[31]_S$

matrices in Yamanouchi basis of all permutations in S_4 permutation group. After applying the projection operator on the corresponding principle term, we will gain the wave function in the required symmetry. A principle term is the product state derived by reading out the single particle states from the Weyl tableau according to Young tableau. The advantage of applying projection operators onto principle term is fixing normalization coefficients of wave functions consistently. For the definition of principle term, please see Appendix B.

Since the color configuration of pentaquark is [211], then there should be three quarks with color R, G, B in the first column because Young tabloids do not allow same color quantum number in the same column. Remaining box can be any color. Thus possible color configurations are $RRGB, RGGB$ and $RGBB$. The following equations are the color wave functions of four-quark core in λ, ρ and η mixed symmetries.

(1) For $RRGB$ configuration

$$\psi^c_{[211]_\lambda}(R) = \left| \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline R & R \\ \hline G \\ \cline{1-1} B \\ \cline{1-1}\end{array} \right\rangle = P_{[211]_\lambda}|RRGB\rangle$$

$$= \frac{1}{\sqrt{16}}(2|RRGB\rangle - |RGRB\rangle - |GRRB\rangle - 2|RRBG\rangle$$

$$+|RBRG\rangle + |BRRG\rangle + |RGBR\rangle + |GRBR\rangle$$

$$-|RBGR\rangle - |BRGR\rangle) \qquad (2.16)$$

$$\psi^c_{[211]_\rho}(R) = \left| \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline R & R \\ \hline G \\ \cline{1-1} B \\ \cline{1-1}\end{array} \right\rangle = P_{[211]_\rho}|RGRB\rangle$$

$$= \frac{1}{\sqrt{48}}(3|RGRB\rangle - 3|GRRB\rangle - 3|RBRG\rangle + 3|BRRG\rangle$$

$$-|RGBR\rangle + |GRBR\rangle + |RBGR\rangle - |BRGR\rangle$$

$$+2|GBRR\rangle - 2|BGRR\rangle) \qquad (2.17)$$

$$\psi^c_{[211]_\eta}(R) = \left| \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 \\ \cline{1-1} 3 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline R & R \\ \hline G \\ \cline{1-1} B \\ \cline{1-1}\end{array} \right\rangle = P_{[211]_\eta}|RGBR\rangle$$

$$= \frac{1}{\sqrt{6}}(|BRGR\rangle + |RGBR\rangle + |GBRR\rangle - |RBGR\rangle$$

$$-|GRBR\rangle - |BGRR\rangle) \qquad (2.18)$$

(2) For $RGGB$ configuration

$$\psi^c_{[211]_\lambda}(G) = \left| \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline R & G \\ \hline G \\ \cline{1-1} B \\ \cline{1-1}\end{array} \right\rangle = P_{[211]_\lambda}|RGGB\rangle$$

$$= \frac{1}{\sqrt{16}}(|RGGB\rangle + |GRGB\rangle - 2|GGRB\rangle - |RGBG\rangle$$

$$-|GRBG\rangle + |GBRG\rangle + |BGRG\rangle + 2|GGBR\rangle$$

$$-|GBGR\rangle - |BGGR\rangle) \qquad (2.19)$$

$$\psi^c_{[211]_\rho}(G) = \left| \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline R & G \\ \hline G \\ \cline{1-1} B \\ \cline{1-1}\end{array} \right\rangle = P_{[211]_\rho}|RGGB\rangle$$

$$= \frac{1}{\sqrt{48}}(3|RGGB\rangle - 3|GRGB\rangle - |RGBG\rangle + |GRBG\rangle$$

$$-2|RBGG\rangle + 2|BRGG\rangle - |GBRG\rangle + |BGRG\rangle$$

$$+3|GBGR\rangle - 3|BGGR\rangle) \qquad (2.20)$$

$$\psi^c_{[211]_\eta}(G) = \left| \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 \\ \cline{1-1} 3 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline R & G \\ \hline G \\ \cline{1-1} B \\ \cline{1-1}\end{array} \right\rangle = P_{[211]_\eta}|RGBG\rangle$$

$$= \frac{1}{\sqrt{6}}(|RGBG\rangle - |GRBG\rangle - |RBGG\rangle + |BRGG\rangle$$

$$+|GBRG\rangle - |BGRG\rangle) \qquad (2.21)$$

(3) For $RGBB$ configuration

$$\psi^c_{[211]_\lambda}(B) = \left| \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline R & B \\ \hline G \\ \cline{1-1} B \\ \cline{1-1}\end{array} \right\rangle = P_{[211]_\lambda}|RGGB\rangle$$

$$= \frac{1}{\sqrt{16}}(|RBGB\rangle + |BRGB\rangle - |GBRB\rangle - |BGRB\rangle$$

$$-|RBBG\rangle - |BRBG\rangle + 2|BBRG\rangle + |GBBR\rangle$$

$$+|BGBR\rangle - 2|BBGR\rangle) \qquad (2.22)$$

$$\psi^c_{[211]_\rho}(B) = \left| \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} 4 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|}\hline R & B \\ \hline G \\ \cline{1-1} B \\ \cline{1-1}\end{array} \right\rangle = P_{[211]_\rho}|RGGB\rangle$$

$$= \frac{1}{\sqrt{48}}(2|RGBB\rangle - 2|GRBB\rangle + |RBGB\rangle - |BRGB\rangle$$

$$-|GBRB\rangle + |BGRB\rangle - 3|RBBG\rangle + 3|BRBG\rangle$$

$$+3|GBBR\rangle - 3|BGBR\rangle) \quad (2.23)$$

$$\psi^c_{[211]_\eta}(B) = \left| \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 \\ \cline{1-1} 3 \\ \cline{1-1} \end{array}, \begin{array}{|c|c|}\hline R & B \\ \hline G \\ \cline{1-1} B \\ \cline{1-1} \end{array} \right\rangle = P_{[211]_\eta}|RGBG\rangle$$

$$= \frac{1}{\sqrt{6}}(|RGBB\rangle - |GRBB\rangle - |RBGB\rangle + |BRGB\rangle$$

$$+|GBRB\rangle - |BGRB\rangle) \quad (2.24)$$

The pentaquark color wave functions $\Psi^c_{[211]}$ could be derived by coalescing the color wave functions of four-quark core $\psi^c_{[211]}$ with antiquark color states. We have

$$\Psi^c_{[211]_j} = \frac{1}{\sqrt{3}}\left[\psi^c_{[211]_j}(R)\,\bar{R} + \psi^c_{[211]_j}(G)\,\bar{G} + \psi^c_{[211]_j}(B)\,\bar{B}\right] \quad (2.25)$$

where $j = \lambda, \rho, \eta$

2.3 Spin wave functions

Similarly spin wave functions of the four-quark core are derived by operating the projection operators on the principal terms of all spin-state configurations. In each of configurations we can derive total spin quantum number S and total spin projection quantum number S_z :

(1) Spin wave functions with the symmetry [4]

(1.1) $S = 2, S_z = 2$

$$\chi_{[4]_{S=2}} = \left| \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 4 \\ \hline \end{array}, \begin{array}{|c|c|c|c|}\hline \uparrow & \uparrow & \uparrow & \uparrow \\ \hline \end{array} \right\rangle = P_{[4]}(\uparrow\uparrow\uparrow\uparrow)$$

$$= \ |\uparrow\uparrow\uparrow\uparrow\rangle \tag{2.26}$$

(1.2) $S = 2, S_z = 1$

$$\begin{aligned}\chi_{[4]_{S=1}} &= \left|\begin{array}{|c|c|c|c|}\hline 1&2&3&4\\\hline\end{array}, \begin{array}{|c|c|c|c|}\hline \uparrow&\uparrow&\uparrow&\downarrow\\\hline\end{array}\right\rangle = P_{[4]}(\uparrow\uparrow\uparrow\downarrow)\\ &= \frac{1}{\sqrt{4}}(|\uparrow\uparrow\uparrow\downarrow\rangle + |\uparrow\uparrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\uparrow\rangle + |\downarrow\uparrow\uparrow\uparrow\rangle)\end{aligned} \tag{2.27}$$

(1.3) $S = 2, S_z = 0$

$$\begin{aligned}\chi_{[4]_{S=0}} &= \left|\begin{array}{|c|c|c|c|}\hline 1&2&3&4\\\hline\end{array}, \begin{array}{|c|c|c|c|}\hline \uparrow&\uparrow&\downarrow&\downarrow\\\hline\end{array}\right\rangle = P_{[4]}(\uparrow\uparrow\downarrow\downarrow)\\ &= \frac{1}{\sqrt{6}}(|\uparrow\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle\\ &\quad + |\downarrow\uparrow\downarrow\uparrow\rangle + |\downarrow\downarrow\uparrow\uparrow\rangle)\end{aligned} \tag{2.28}$$

(2) Spin wave functions with the symmetry [31]

(2.1) $S = 1, S_z = 1$

$$\begin{aligned}\chi_{[31]\lambda} &= \left|\begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3\\\cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \uparrow&\uparrow&\uparrow\\\hline \downarrow\\\cline{1-1}\end{array}\right\rangle = P_{[31]\lambda}(\uparrow\uparrow\downarrow\uparrow)\\ &= \frac{1}{\sqrt{6}}(2|\uparrow\uparrow\downarrow\uparrow\rangle - |\uparrow\downarrow\uparrow\uparrow\rangle - |\downarrow\uparrow\uparrow\uparrow\rangle)\end{aligned} \tag{2.29}$$

$$\begin{aligned}\chi_{[31]\rho} &= \left|\begin{array}{|c|c|c|}\hline 1&3&4\\\hline 2\\\cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \uparrow&\uparrow&\uparrow\\\hline \downarrow\\\cline{1-1}\end{array}\right\rangle = P_{[31]\rho}(\uparrow\downarrow\uparrow\uparrow)\\ &= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\uparrow\uparrow\rangle - |\downarrow\uparrow\uparrow\uparrow\rangle)\end{aligned} \tag{2.30}$$

$$\begin{aligned}\chi_{[31]\eta} &= \left|\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4\\\cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \uparrow&\uparrow&\uparrow\\\hline \downarrow\\\cline{1-1}\end{array}\right\rangle = P_{[31]\eta}(\uparrow\uparrow\uparrow\downarrow)\\ &= \frac{1}{2\sqrt{3}}(3|\uparrow\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\uparrow\rangle - |\uparrow\downarrow\uparrow\uparrow\rangle - |\uparrow\uparrow\downarrow\uparrow\rangle)\end{aligned} \tag{2.31}$$

(2.2) $S=1,,S_z=0$

$$\chi_{[31]\lambda} = \left|\begin{array}{|c|c|c|}\hline 1 & 2 & 4\\\hline 3 \\\cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \uparrow & \uparrow & \downarrow\\\hline \downarrow \\\cline{1-1}\end{array}\right\rangle = P_{[31]\lambda}(\uparrow\uparrow\downarrow\downarrow)$$

$$= \frac{1}{\sqrt{12}}(2|\uparrow\uparrow\downarrow\downarrow\rangle - |\uparrow\downarrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle$$

$$+|\downarrow\uparrow\downarrow\uparrow\rangle - 2|\downarrow\downarrow\uparrow\uparrow\rangle) \tag{2.32}$$

$$\chi_{[31]\rho} = \left|\begin{array}{|c|c|c|}\hline 1 & 3 & 4\\\hline 2 \\\cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \uparrow & \uparrow & \downarrow\\\hline \downarrow \\\cline{1-1}\end{array}\right\rangle = P_{[31]\rho}(\uparrow\downarrow\uparrow\downarrow)$$

$$= \frac{1}{\sqrt{4}}(|\uparrow\downarrow\uparrow\downarrow\rangle + |\uparrow\downarrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle - |\downarrow\uparrow\downarrow\uparrow\rangle) \tag{2.33}$$

$$\chi_{[31]\eta} = \left|\begin{array}{|c|c|c|}\hline 1 & 2 & 3\\\hline 4 \\\cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \uparrow & \uparrow & \downarrow\\\hline \downarrow \\\cline{1-1}\end{array}\right\rangle = P_{[31]\eta}(\uparrow\uparrow\downarrow\downarrow)$$

$$= \frac{1}{\sqrt{6}}(|\uparrow\uparrow\downarrow\downarrow\rangle + |\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle$$

$$-|\downarrow\uparrow\downarrow\uparrow\rangle - |\downarrow\downarrow\uparrow\uparrow\rangle) \tag{2.34}$$

(3) Spin wave functions with the symmetry [22]

(3.1) $S=0,S_z=0$

$$\chi_{[22]\lambda} = \left|\begin{array}{|c|c|}\hline 1 & 2\\\hline 3 & 4\\\hline\end{array}, \begin{array}{|c|c|}\hline \uparrow & \uparrow\\\hline \downarrow & \downarrow\\\hline\end{array}\right\rangle = P_{[22]\lambda}(\uparrow\uparrow\downarrow\downarrow)$$

$$= \frac{1}{\sqrt{12}}(2|\uparrow\uparrow\downarrow\downarrow\rangle - |\uparrow\downarrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle$$

$$-|\downarrow\uparrow\downarrow\uparrow\rangle + 2|\downarrow\downarrow\uparrow\uparrow\rangle) \tag{2.35}$$

$$\chi_{[22]\rho} = \left|\begin{array}{|c|c|}\hline 1 & 3\\\hline 2 & 4\\\hline\end{array}, \begin{array}{|c|c|}\hline \uparrow & \uparrow\\\hline \downarrow & \downarrow\\\hline\end{array}\right\rangle = P_{[22]\rho}(\uparrow\downarrow\uparrow\downarrow)$$

$$= \frac{1}{\sqrt{4}}(|\uparrow\downarrow\uparrow\downarrow\rangle - |\downarrow\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\downarrow\uparrow\rangle + |\downarrow\uparrow\downarrow\uparrow\rangle) \tag{2.36}$$

Combination of the spin wave functions of the four-quark core with ones of anti-quark yields the total spin wave function of the pentaquark state. Clebsch-Gordan coeffients are taken into account for coupling spins of four-quark core and anti-quark. For example,

$$\begin{aligned}\chi(q^4\bar{s})_{[31]_\alpha} = & \sqrt{\frac{2}{3}}\,\chi_{[31]_\alpha}(s_{q^4}=1, m_{q^4}=1)\chi_{\bar{s}}(-1/2) \\ & -\sqrt{\frac{1}{3}}\,\chi_{[31]_\alpha}(s_{q^4}=1, m_{q^4}=0)\chi_{\bar{s}}(1/2) \end{aligned} \tag{2.37}$$

2.4 Flavor wave functions

The same approaches have been used for getting the flavor wave functions. To make it easy, we define flavor states in order of α, β, γ. We list the flavor wave functions of the four-quark core for various flavor-state configurations:

(1) For $\alpha\alpha\alpha\alpha$ configuration

$$\begin{aligned}\Psi^f_{[4]} &= \left|\begin{array}{|c|c|c|c|}\hline 1&2&3&4\\\hline\end{array}, \begin{array}{|c|c|c|c|}\hline \alpha&\alpha&\alpha&\alpha\\\hline\end{array}\right\rangle = P_{[4]}(\alpha\alpha\alpha\alpha) \\ &= |\alpha\alpha\alpha\alpha\rangle \end{aligned} \tag{2.38}$$

(2) For $\alpha\alpha\alpha\beta$ configuration

$$\begin{aligned}\Psi^f_{[4]} &= \left|\begin{array}{|c|c|c|c|}\hline 1&2&3&4\\\hline\end{array}, \begin{array}{|c|c|c|c|}\hline \alpha&\alpha&\alpha&\beta\\\hline\end{array}\right\rangle = P_{[4]}(\alpha\alpha\alpha\beta) \\ &= \frac{1}{\sqrt{4}}(|\alpha\alpha\alpha\beta\rangle + |\alpha\alpha\beta\alpha\rangle + |\alpha\beta\alpha\alpha\rangle + |\beta\alpha\alpha\alpha\rangle) \end{aligned} \tag{2.39}$$

$$\begin{aligned}\Psi^f_{[31]\lambda} &= \left|\begin{array}{|c|c|c|}\hline 1&2&4\\\hline 3 \\\cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \alpha&\alpha&\alpha\\\hline \beta \\\cline{1-1}\end{array}\right\rangle = P_{[4]}(\alpha\alpha\beta\alpha) \\ &= \frac{1}{\sqrt{6}}(2|\alpha\alpha\beta\alpha\rangle - |\alpha\beta\alpha\alpha\rangle - |\beta\alpha\alpha\alpha\rangle) \end{aligned} \tag{2.40}$$

$$\Psi^{f}_{[31]\rho} = \left| \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \alpha & \alpha & \alpha \\ \hline \beta \\ \cline{1-1}\end{array} \right\rangle = P_{[4]}(\alpha\beta\alpha\alpha)$$

$$= \frac{1}{\sqrt{2}}(|\alpha\beta\alpha\alpha\rangle - |\beta\alpha\alpha\alpha\rangle) \qquad (2.41)$$

$$\Psi^{f}_{[31]\eta} = \left| \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \alpha & \alpha & \alpha \\ \hline \beta \\ \cline{1-1}\end{array} \right\rangle = P_{[4]}(\alpha\alpha\alpha\beta)$$

$$= \frac{1}{\sqrt{12}}(3|\alpha\alpha\alpha\beta\rangle - |\beta\alpha\alpha\alpha\rangle - |\alpha\beta\alpha\alpha\rangle - |\alpha\alpha\beta\alpha\rangle) \qquad (2.42)$$

(3) For $\alpha\alpha\beta\beta$ configuration

$$\Psi^{f}_{[4]} = \left| \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 4 \\ \hline \end{array}, \begin{array}{|c|c|c|c|}\hline \alpha & \alpha & \beta & \beta \\ \hline \end{array} \right\rangle = P_{[4]}(\alpha\alpha\beta\beta)$$

$$= \frac{1}{\sqrt{6}}(|\alpha\alpha\beta\beta\rangle + |\beta\alpha\alpha\beta\rangle + |\beta\alpha\beta\alpha\rangle + |\beta\beta\alpha\alpha\rangle$$

$$+|\alpha\beta\beta\alpha\rangle + |\alpha\beta\alpha\beta\rangle) \qquad (2.43)$$

$$\Psi^{f}_{[31]\lambda} = \left| \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \alpha & \alpha & \beta \\ \hline \beta \\ \cline{1-1}\end{array} \right\rangle = P_{[4]}(\alpha\alpha\beta\beta)$$

$$= \frac{1}{\sqrt{12}}(2|\alpha\alpha\beta\beta\rangle - |\alpha\beta\alpha\beta\rangle - |\beta\alpha\alpha\beta\rangle + |\alpha\beta\beta\alpha\rangle$$

$$+|\beta\alpha\beta\alpha\rangle - 2|\beta\beta\alpha\alpha\rangle) \qquad (2.44)$$

$$\Psi^{f}_{[31]\rho} = \left| \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \alpha & \alpha & \beta \\ \hline \beta \\ \cline{1-1}\end{array} \right\rangle = P_{[4]}(\alpha\beta\alpha\beta)$$

$$= \frac{1}{\sqrt{4}}(|\alpha\beta\alpha\beta\rangle + |\alpha\beta\beta\alpha\rangle - |\beta\alpha\alpha\beta\rangle - |\beta\alpha\beta\alpha\rangle) \qquad (2.45)$$

$$\Psi^{f}_{[31]\eta} = \left| \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1}\end{array}, \begin{array}{|c|c|c|}\hline \alpha & \alpha & \beta \\ \hline \beta \\ \cline{1-1}\end{array} \right\rangle = P_{[4]}(\alpha\alpha\beta\beta)$$

$$= \frac{1}{\sqrt{6}}(3|\alpha\alpha\beta\beta\rangle + |\alpha\beta\alpha\beta\rangle + |\beta\alpha\alpha\beta\rangle - |\alpha\beta\beta\alpha\rangle$$

$$-|\beta\alpha\beta\alpha\rangle - |\beta\beta\alpha\alpha\rangle) \tag{2.46}$$

$$\begin{aligned}
\Psi^{f}_{[22]\lambda} &= \left| \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array}, \begin{array}{|c|c|}\hline \alpha & \alpha \\ \hline \beta & \beta \\ \hline \end{array} \right\rangle = P_{[22]}(\alpha\alpha\beta\beta) \\
&= \frac{1}{\sqrt{12}}(2|\alpha\alpha\beta\beta\rangle - |\alpha\beta\alpha\beta\rangle - |\beta\alpha\alpha\beta\rangle + |\alpha\beta\beta\alpha\rangle \\
&\quad -|\beta\alpha\beta\alpha\rangle + 2|\beta\beta\alpha\alpha\rangle)
\end{aligned} \tag{2.47}$$

$$\begin{aligned}
\Psi^{f}_{[22]\rho} &= \left| \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array}, \begin{array}{|c|c|}\hline \alpha & \alpha \\ \hline \beta & \beta \\ \hline \end{array} \right\rangle = P_{[22]}(\alpha\beta\alpha\beta) \\
&= \frac{1}{\sqrt{4}}(|\alpha\beta\alpha\beta\rangle - |\alpha\beta\beta\alpha\rangle - |\beta\alpha\alpha\beta\rangle + |\beta\alpha\beta\alpha\rangle)
\end{aligned} \tag{2.48}$$

2.5 Spatial wave functions

Previously we have already calculated the color, spin and flavor wave functions of four-quark core and pentaquark. The remaining and important part is spatial wave functions. In this study we make an assumption that in pentaquark mode, four-quark core and one antiquark give the same contribution to spatial part. To the first order approximation, we assume that the internal interaction between quarks is harmonic oscillation. The explicit form of the spatial wave functions are derived from non-relativistic Schrodinger equation. Consequently, Hamiltonian can be written in term of kinetic energy and harmonic oscillation potential :

$$H = \sum_{i=1}^{5} \frac{p_i^2}{2m_i} + \sum_{i<j}^{5} C(\vec{r_i} - \vec{r_j})^2 \tag{2.49}$$

After introducing the Jacobi coordinates,

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r_1} - \vec{r_2})$$

$$
\begin{aligned}
\vec{\lambda} &= \frac{1}{\sqrt{6}}(\vec{r_1} + \vec{r_2} - 2\vec{r_3}) \\
\vec{\eta} &= \frac{1}{\sqrt{12}}(\vec{r_1} + \vec{r_2} + \vec{r_3} - 3\vec{r_4}) \\
\vec{\xi} &= \frac{1}{2\sqrt{5}}(\vec{r_1} + \vec{r_2} + \vec{r_3} + \vec{r_4} - 4\vec{r_5}) \\
\vec{R}_{cm} &= \frac{\vec{r_1} + \vec{r_2} + \vec{r_3} + \vec{r_4} + \vec{r_5}}{5}
\end{aligned} \tag{2.50}
$$

$$
\begin{aligned}
\vec{p}_\rho &= m\frac{d\vec{\rho}}{dt} = \frac{1}{\sqrt{2}}(\vec{p_1} - \vec{p_2}) \\
\vec{p}_\lambda &= \frac{1}{\sqrt{6}}(\vec{p_1} + \vec{p_2} - 2\vec{p_3}) \\
\vec{p}_\eta &= \frac{1}{\sqrt{12}}(\vec{p_1} + \vec{p_2} + \vec{p_3} - 3\vec{p_4}) \\
\vec{p}_\xi &= \frac{1}{2\sqrt{5}}(\vec{p_1} + \vec{p_2} + \vec{p_3} + \vec{p_4} - 4\vec{p_5}) \\
\vec{P}_{cm} &= \vec{p_1} + \vec{p_2} + \vec{p_3} + \vec{p_4} + \vec{p_5}
\end{aligned} \tag{2.51}
$$

where $\vec{r_i}\,(i = 1, 2, 3, 4)$ is the coordinate of the i^{th} quark, and $\vec{r_5}$ is the coordinate of antiquark and assuming that five quarks have the same mass, the Hamiltonian is rewritten as,

$$
H = \frac{P_{CM}^2}{2m} + \frac{p_\lambda^2}{2m} + \frac{p_\rho^2}{2m} + \frac{p_\eta^2}{2m} + \frac{p_\xi^2}{2m} + 5C[\lambda^2 + \rho^2 + \eta^2 + \xi^2] \tag{2.52}
$$

where P_{CM} is the momentum at center of mass. We solve Schrodinger equation by Hamiltonian in Eq.(2.52) using separation of variables method. Eigenstates of the Hamiltonian take the general form in terms of the Jacobi coordinates in the center-of-mass system,

$$
\Psi^o = \Psi_{n_\lambda l_\lambda m_\lambda}(\lambda)\Psi_{n_\rho l_\rho m_\rho}(\rho)\Psi_{n_\eta l_\eta m_\eta}(\eta)\Psi_{n_\xi l_\xi m_\xi}(\xi) \tag{2.53}
$$

where $\Psi_{n_r l_r m_r}(r) = R_{n_r l_r}(r) Y_{l_r m_r}(\hat{r})$, $R_{nl}(r) = L_n^{2l+1}(r) e^{-\alpha^2 r^2}$. Principle quantum number N of each state which is by-product after solving the Schrodinger equation is defined as

$$N = 2(n_\lambda + n_\rho + n_\eta + n_\xi) + l_\lambda + l_\rho + l_\eta + l_\xi \tag{2.54}$$

After that we do linear combination of different pentaquark states to construct the building blogs of spatial wave functions in the required symmetry ; symmetric (S), antisymmetric (A), mixed symmetry (λ, ρ, η). Therefore we write a combination of spatial wave functions in the form of

$$\begin{aligned} \Psi^o_{NLM} \quad = \quad & \sum_{n_\lambda, n_\rho, n_\eta, n_\xi, l_\lambda, l_\rho, l_\eta, l_\xi} A(n_\lambda, n_\rho, n_\eta, n_{xi}, l_\lambda, l_\rho, l_\eta, l_\xi) \\ & \cdot \Psi_{n_\lambda l_\lambda m_\lambda} \Psi_{n_\rho l_\rho m_\rho} \Psi_{n_\eta l_\eta m_\eta} \Psi_{n_\xi l_\xi m_\xi} \\ & \cdot C(l_\lambda, l_\rho, m_\lambda, m_\rho, l_{\lambda\rho}, m_{\lambda\rho}) \\ & \cdot C(l_\eta, l_\xi, m_\eta, m_\xi, l_{\eta\xi}, m_{\eta\xi}) \\ & \cdot C(l_{\lambda\rho}, l_{\eta\xi}, m_{\lambda\rho}, m_{\eta\xi}, LM) \end{aligned} \tag{2.55}$$

Where $C(l_i l_j, m_i m_j, LM)$ is the Clebsch-Gordon coefficient which is appeared when we couple with two wave functions that carry angular momentum quantum number.

We can determine the coefficients A by applying the representation matrices of the S_4 permutation group to get the spatial wave functions with the [4], [31], [22], [211] and [1111] symmetries. Results of the four-quark (q^4) spatial wave functions are shown in Table 2.3 and results of the pentaquark $(q^4\overline{q})$ spatial wave functions are shown in Table 2.4

Table 2.3 Spatial wave functions of four-quark (q^4).

NL	Symmetry	Wave function
11	[4]	-
11	[1111]	-
11	[211]	-
11	[31]	$\Psi^{\lambda[31]} = \Psi_{01M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)$ $\Psi^{\rho[31]} = \Psi_{000}(\lambda)\Psi_{01M}(\rho)\Psi_{000}(\eta)$ $\Psi^{\eta[31]} = \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{01M}(\eta)$
11	[22]	-
20	[4]	$\Psi^{S} = \frac{1}{\sqrt{3}}\{\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta) + \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)$ $+\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\}$
20	[1111]	-
20	[211]	-
20	[31]	$\Psi^{\lambda[31]} = \frac{1}{2}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho) - \sqrt{\frac{3}{8}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)$ $+\sqrt{\frac{3}{8}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\rho[31]} = \sqrt{\frac{2}{3}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta) + \frac{1}{\sqrt{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}$ $\Psi^{\eta[31]} = \frac{1}{\sqrt{6}}\{\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta) + \Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)$ $-2\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\}$
20	[22]	$\Psi^{\lambda[22]} = -\sqrt{\frac{4}{7}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho) - \sqrt{\frac{3}{14}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)$ $+\sqrt{\frac{3}{14}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)$ $\Psi^{\rho[22]} = \frac{1}{\sqrt{3}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta) - \sqrt{\frac{2}{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}$
21	[4]	-
21	[1111]	-
21	[211]	$\Psi^{\lambda[211]} = [\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)$ $\Psi^{\rho[211]} = [\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\lambda)$ $\Psi^{\eta[211]} = [\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
21	[31]	-
21	[22]	-

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
22	[4]	$\Psi^{S} = \frac{1}{\sqrt{3}}\{\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta) + \Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)$ $+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{02M}(\eta)\}$
22	[1111]	-
22	[211]	-
22	[31]	$\Psi^{\lambda[31]} = \sqrt{\frac{5}{11}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho) - \sqrt{\frac{3}{11}}[\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)$ $+\sqrt{\frac{3}{11}}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)$ $\Psi^{\rho[31]} = \sqrt{\frac{2}{3}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta) + \sqrt{\frac{1}{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}$ $\Psi^{\eta[31]} = \frac{1}{\sqrt{6}}[\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta) + \Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)$ $-2\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{02M}(\eta)]$
22	[22]	$\Psi^{\lambda[22]} = -\sqrt{\frac{10}{13}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta) - \sqrt{\frac{3}{26}}\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)$ $+\sqrt{\frac{3}{26}}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)$ $\Psi^{\rho[22]} = -\sqrt{\frac{2}{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM} + \sqrt{\frac{1}{3}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
30	[4]	-

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
30	[1111]	$\Psi^{A} = [\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho) \otimes \Psi_{01m_3}(\eta)]_{LM}$
30	[211]	-
30	[31]	-
30	[22]	-
31	[4]	$\Psi^{S} = \sqrt{\frac{3}{31}}\{-\frac{2\sqrt{2}}{3}[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta) - \frac{2}{3}[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)$ $-\frac{2}{3}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM} + \Psi_{11M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)$ $-\frac{5}{3}[\Psi_{01M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta) + \sqrt{2}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{11M}(\eta)$ $-\frac{5}{\sqrt{18}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{01M}(\eta) - \frac{5}{\sqrt{18}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{01M}(\eta)\}$
31	[1111]	-
31	[211]	-
31	[31]	$\Psi^{\lambda[31]_1} = \frac{1}{\sqrt{3}}\{\Psi_{11M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta) + \Psi_{01M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)$ $+\Psi_{01M}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\}$ $\Psi^{\rho[31]_1} = \frac{1}{\sqrt{3}}\{\Psi_{100}(\lambda)\Psi_{01M}(\rho)\Psi_{000}(\eta) + \Psi_{000}(\lambda)\Psi_{11M}(\rho)\Psi_{000}(\eta)$ $+\Psi_{000}(\lambda)\Psi_{01M}(\rho)\Psi_{100}(\eta)\}$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
31	[31]	$\Psi^{\eta[31]_1} = \frac{1}{\sqrt{3}}\{\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{01M}(\eta) + \Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{01M}(\eta)$ $+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{11M}(\eta)\}$ $\Psi^{\lambda[31]_2} = \frac{1}{2}\{[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta) + [\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)$ $+\sqrt{2}\Psi_{11M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\}$ $\Psi^{\rho[31]_2} = \frac{1}{2}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta) + \Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}$ $+\sqrt{2}\Psi_{000}(\lambda)\Psi_{11M}(\rho)\Psi_{000}(\eta)\}$ $\Psi^{\eta[31]_2} = \frac{1}{2}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho) + \Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}$ $+\sqrt{2}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{11M}(\eta)\}$
31	[22]	-
32	[4]	-
32	[1111]	-
32	[211]	-
32	[31]	$\Psi^{\lambda[31]} = \frac{1}{\sqrt{2}}\{[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta) + [\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\}$ $\Psi^{\rho[31]} = \frac{1}{\sqrt{2}}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta) - \Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{02m_2}(\eta)_{LM}\}$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\eta[31]} = \frac{1}{\sqrt{2}}\{[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho) + \Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\}$
32	[22]	-
33	[4]	$\Psi^{S} = -\sqrt{\frac{7}{17}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta) - \sqrt{\frac{7}{34}}[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)$ $-\sqrt{\frac{7}{34}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM} + \frac{1}{\sqrt{17}}\Psi_{03M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)$ $+\sqrt{\frac{2}{17}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{03M}(\eta)$
33	[1111]	-
33	[211]	$\Psi^{\lambda[211]} = \sqrt{\frac{7}{156}}\{\frac{3}{\sqrt{7}}\Psi_{03M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta) + [\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho) \otimes \Psi_{01m_3}(\eta)]_{LM}$ $+\sqrt{2}[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho) - \sqrt{2}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}$ $-4[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\}$ $\Psi^{\rho[211]} = \sqrt{\frac{21}{524}}\{[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta) + \frac{3}{\sqrt{7}}\Psi_{000}(\lambda)\Psi_{03M}(\rho)\Psi_{000}(\eta)$ $-2\sqrt{\frac{5}{3}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho) \otimes \Psi_{01m_3}(\eta)]_{LM} - 4\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{02m_2}(\eta)]_{LM}\}$ $\Psi^{\eta[211]} = \sqrt{\frac{7}{144}}\{-3\sqrt{2}[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta) + 3\sqrt{\frac{2}{7}}\Psi_{000}(\lambda)\Psi_{03M}(\rho)\Psi_{000}(\eta)\}$
33	[31]	$\Psi^{\lambda[31]_1} = \sqrt{\frac{7}{23}}\{[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta) + [\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)$ $+\frac{3}{\sqrt{7}}\Psi_{03M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\}$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
33	[31]	$\Psi^{\rho[31]_1} = \sqrt{\frac{7}{23}}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}$ $+\frac{3}{\sqrt{7}}\Psi_{000}(\lambda)\Psi_{03M}(\rho)\Psi_{000}(\eta)\}$ $\Psi^{\eta[31]_1} = \sqrt{\frac{7}{23}}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)+\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}$ $+\frac{3}{\sqrt{7}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{03M}(\eta)\}$ $\Psi^{\lambda[31]_2} = \sqrt{\frac{2}{5}}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)-\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}$ $+\frac{1}{\sqrt{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\}$ $\Psi^{\rho[31]_2} = \sqrt{\frac{6}{23}}\{-\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}+\frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\}$ $\Psi^{\eta[31]_2} = \sqrt{\frac{14}{31}}\{\frac{1}{\sqrt{7}}\Psi_{03M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)-[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)$ $+\frac{1}{\sqrt{2}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)+\frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}$ $-\frac{1}{\sqrt{14}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{03M}(\eta)\}$
33	[22]	-
40	[4]	$\Psi^{S_1} = \sqrt{\frac{5}{18}}\{\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)+\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)$ $+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{200}(\eta)+\frac{1}{\sqrt{5}}([\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)$ $+[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)+\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM})\}$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
40	[4]	$\Psi^{S_2} = \sqrt{\frac{5}{18}}\{\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta) + \Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)$ $+\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta) - \frac{1}{\sqrt{5}}([\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)$ $+[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho) + \Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{02m_2}(\eta)]_{LM})\}$
40	[4]	$\Psi^{S_3} = \sqrt{\frac{120}{917}}\{-\frac{5}{2\sqrt{6}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta) - \frac{\sqrt{3}}{2\sqrt{2}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{200}(\eta)$ $+\frac{\sqrt{5}}{2\sqrt{6}}[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta) + \frac{3\sqrt{3}}{2\sqrt{10}}[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)$ $+\frac{3\sqrt{3}}{2\sqrt{10}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{02m_2}(\eta)]_{LM} + [\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)$ $-\frac{5}{3}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_1}(\eta)]_{LM}\Psi_{100}(\rho) - \frac{2\sqrt{2}}{3}[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho) \otimes \Psi_{01m_3}(\eta)]_{LM}\}$
40	[1111]	–
40	[211]	$\Psi^{\lambda[211]} = \sqrt{\frac{5}{1171}}\{-\frac{5\sqrt{3}}{2}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta) - \frac{5\sqrt{3}}{2}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)$ $+\frac{5\sqrt{3}}{2}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta) - \frac{2\sqrt{3}}{\sqrt{5}}[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)$ $-\frac{2\sqrt{3}}{\sqrt{5}}[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho) + \frac{2\sqrt{3}}{\sqrt{5}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{02m_2}(\eta)]_{LM}$ $-3\sqrt{2}[\Psi_{01m_1}(\lambda) \otimes \Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho) + \frac{9}{2}\frac{1}{\sqrt{2}}[\Psi_{11m_1}(\lambda)\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)$ $+\frac{5}{2\sqrt{2}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta) + 3\sqrt{3}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)$ $+[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho) \otimes \Psi_{01m_3}(\eta)]_{LM}\}$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\rho[211]} = \sqrt{\frac{4}{351}}\{\frac{3}{2}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta) - 3\sqrt{2}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}$
		$-\frac{9}{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta) + \frac{9}{2}\frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}$
		$-\frac{5}{2\sqrt{2}}\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM} + 5[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)$
		$+[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM} + 2\sqrt{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\eta)]_{LM}\}$
		$\Psi^{\eta[211]} = \sqrt{\frac{2}{189}}\{\frac{15}{2}\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM} - \frac{9}{2}\Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}$
		$+3\sqrt{2}[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\}$
40	[31]	$\Psi^{\lambda[31]_1} = \sqrt{\frac{1}{18}}\{[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho) + [\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)$
		$-\sqrt{6}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta) + \sqrt{6}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)$
		$-\frac{\sqrt{3}}{\sqrt{2}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta) + \frac{\sqrt{3}}{\sqrt{2}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\}$
		$\Psi^{\rho[31]_1} = \frac{1}{3}\{\sqrt{2}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta) + \sqrt{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$+\sqrt{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta) + \Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}$
		$+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM} + \Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\}$
		$\Psi^{\eta[31]_1} = \sqrt{\frac{2}{45}}\{\sqrt{3}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta) + \sqrt{3}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
		$-2\sqrt{3}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{200}(\eta)+\sqrt{3}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)$
		$-\frac{\sqrt{3}}{2}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)-\frac{\sqrt{3}}{2}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\}$
40	[31]	$\Psi^{\lambda[31]_2}=\sqrt{\frac{16}{1083}}\{\frac{5\sqrt{3}}{2}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)+\frac{5\sqrt{3}}{2}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)$
		$-\frac{5\sqrt{3}}{4}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)-\frac{\sqrt{3}}{2}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)$
		$-\frac{5\sqrt{3}}{2}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)+\Psi_{01m_l}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}$
		$+\frac{2\sqrt{3}}{\sqrt{5}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)+\sqrt{2}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)$
		$-\frac{11}{2\sqrt{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)\}$
		$\Psi^{\rho[31]_2}=\sqrt{\frac{80}{6777}}\{-4[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)+2[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$+\sqrt{2}\Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}-\frac{11}{2\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}$
		$-\frac{5}{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)+[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_l}(\rho)\otimes\Psi_{01m_l}(\eta)]_{LM}\}$
		$\Psi^{\eta[31]_2}=\sqrt{\frac{80}{6777}}\{-\frac{5\sqrt{3}}{2\sqrt{2}}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)-\frac{5\sqrt{3}}{2\sqrt{2}}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)$
		$-\sqrt{\frac{3}{2}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{200}(\eta)-\frac{5\sqrt{3}}{2\sqrt{2}}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)$
		$+\frac{15\sqrt{3}}{4\sqrt{2}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)+\frac{15\sqrt{3}}{4\sqrt{2}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)$
		$-\frac{3}{2}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)+\frac{5}{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
		$+\sqrt{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}+\sqrt{\frac{6}{5}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)$
		$+\sqrt{\frac{6}{5}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\}$
40	[31]	$\Psi^{\lambda[31]_3}=\sqrt{\frac{4}{849}}\{\frac{5\sqrt{5}}{2}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)+\frac{5\sqrt{5}}{4}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)$
		$-\frac{5\sqrt{5}}{4}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)+2[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$+[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)-\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}$
		$+\frac{3\sqrt{5}}{2}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)-\frac{9\sqrt{5}}{2}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)$
		$-\frac{5\sqrt{15}}{2\sqrt{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)\}$
		$\Psi^{\rho[31]_3}=\sqrt{\frac{8}{965}}\{-2\sqrt{15}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)-\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\eta)]_{LM}$
		$-\frac{5\sqrt{15}}{2\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}-\frac{5\sqrt{5}}{2\sqrt{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\rho)]_{LM}\Psi_{100}(\eta)\}$
		$\Psi^{\eta[31]_3}=\sqrt{\frac{16}{3711}}\{-\frac{9\sqrt{5}}{2\sqrt{2}}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)-\frac{9\sqrt{5}}{2\sqrt{2}}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)$
		$+\frac{3\sqrt{5}}{\sqrt{2}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{200}(\eta)-\frac{5\sqrt{5}}{2\sqrt{2}}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)$
		$-\frac{15\sqrt{5}}{4\sqrt{2}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)+\frac{15\sqrt{5}}{4\sqrt{2}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)$
		$-\sqrt{2}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)+\frac{3}{\sqrt{2}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)$
		$+\frac{3}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}-\frac{\sqrt{15}}{2}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
		$+\frac{5\sqrt{5}}{2\sqrt{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)+\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\}$
40	[22]	$\Psi^{\lambda[22]_1}=\sqrt{\frac{4}{31}}\{\sqrt{\frac{3}{2}}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)-\sqrt{\frac{3}{2}}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)$
		$+\frac{\sqrt{3}}{2\sqrt{2}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)-\frac{\sqrt{3}}{2\sqrt{2}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)+[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)\}$
		$\Psi^{\rho[22]_1}=\sqrt{\frac{2}{9}}\{-\frac{1}{\sqrt{2}}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)-\frac{1}{\sqrt{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$-\frac{1}{\sqrt{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)+\Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}$
		$+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}+\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\}$
40	[22]	$\Psi^{\lambda[22]_2}=\sqrt{\frac{320}{10243}}\{\frac{11\sqrt{3}}{8}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)+\frac{5\sqrt{3}}{8}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)$
		$-\frac{5\sqrt{3}}{8}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)+\frac{5\sqrt{3}}{8}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)$
		$-\frac{17\sqrt{3}}{8}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)+\frac{\sqrt{3}}{2\sqrt{5}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$+\sqrt{2}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)+\frac{7}{2\sqrt{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\}$
		$\Psi^{\rho[22]_2}=\sqrt{\frac{16}{287}}\{-\frac{5}{2}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)-[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta)$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
		$+\sqrt{2}\Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}+\frac{7}{2\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}$
		$-\frac{5}{4}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)+[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\}$
40	[22]	$\Psi^{\lambda[22]_3}=\sqrt{\frac{1}{136}}\{-\frac{3\sqrt{5}}{4}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)-\frac{5\sqrt{5}}{4}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)$
		$+\frac{5\sqrt{5}}{2}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)+\frac{5\sqrt{5}}{4}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)$
		$-\frac{5\sqrt{5}}{4}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)-[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$+[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)-\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}$
		$-\sqrt{\frac{15}{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)\}$
		$\Psi^{\rho[22]_3}=\sqrt{\frac{12}{435}}\{\sqrt{15}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)-\sqrt{\frac{15}{2}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}$
		$-\frac{5\sqrt{5}}{2\sqrt{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)-\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\eta)]_{LM}\}$
41	[4]	–
41	[1111]	–
41	[211]	$\Psi^{\lambda[211]}=\sqrt{\frac{1}{3}}\{[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)+[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)\}$
		$\Psi^{\rho[211]}=\sqrt{\frac{1}{3}}\{\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
		$+\Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\}$
		$\Psi^{\eta[211]}=\sqrt{\frac{1}{3}}\{-[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)-[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$-[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)\}$
41	[31]	–
41	[22]	–
42	[4]	$\Psi^{S_1}=\frac{1}{3}\{\Psi_{12M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)+\Psi_{000}(\lambda)\Psi_{12M}(\rho)\Psi_{000}(\eta)$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{12M}(\eta)+\Psi_{02M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)$
		$+\Psi_{100}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)+\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)$
		$+\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{100}(\eta)+\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{02M}(\eta)$
		$+\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{02M}(\eta)\}$
42	[4]	$\Psi^{S_2}=\sqrt{\frac{7}{36}}\{-\sqrt{\frac{5}{14}}\Psi_{02M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)-\sqrt{\frac{5}{14}}\Psi_{100}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)$
		$-\sqrt{\frac{5}{14}}\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)-\sqrt{\frac{5}{14}}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{100}(\eta)$
		$-\sqrt{\frac{5}{14}}\Psi_{100}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)-\sqrt{\frac{5}{14}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{02M}(\eta)$
		$+[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)+[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
		$+\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\}$
42	[1111]	–
42	[211]	–
42	[31]	$\Psi^{\lambda[31]}=\sqrt{\frac{5}{28}}\{-\sqrt{\frac{3}{5}}\Psi_{12M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)+\sqrt{\frac{3}{5}}\Psi_{000}(\lambda)\Psi_{12M}(\rho)\Psi_{000}(\eta)$
		$-\sqrt{\frac{3}{5}}\Psi_{02M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)+\sqrt{\frac{3}{5}}\Psi_{100}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)$
		$-\sqrt{\frac{3}{5}}\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)+\sqrt{\frac{3}{5}}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{100}(\eta)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)+[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)\}$
		$\Psi^{\rho[31]}=\frac{1}{3}\{\sqrt{2}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)+\sqrt{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$+\sqrt{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)+\Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}$
		$+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}+\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\}$
		$\Psi^{\eta[31]}=\sqrt{\frac{10}{27}}\{\sqrt{\frac{3}{10}}\Psi_{12M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)+\sqrt{\frac{3}{10}}\Psi_{000}(\lambda)\Psi_{12M}(\rho)\Psi_{000}(\eta)$
		$-\sqrt{\frac{3}{10}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{12M}(\eta)+\sqrt{\frac{3}{10}}\Psi_{02M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)$
		$+\sqrt{\frac{3}{10}}\Psi_{100}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)+\sqrt{\frac{3}{10}}\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)$
		$+\sqrt{\frac{3}{10}}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{100}(\eta)-\sqrt{\frac{3}{10}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{02M}(\eta)$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
		$-\sqrt{\frac{3}{10}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{02M}(\eta)\}$
42	[22]	$\Psi^{\lambda[22]} = \sqrt{\frac{10}{39}}\{\frac{\sqrt{3}}{2\sqrt{5}}\Psi_{12M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta) - \frac{\sqrt{3}}{2\sqrt{5}}\Psi_{000}(\lambda)\Psi_{12M}(\rho)\Psi_{000}(\eta)$
		$+\frac{\sqrt{3}}{2\sqrt{5}}\Psi_{02M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta) - \frac{\sqrt{3}}{2\sqrt{5}}\Psi_{100}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)$
		$+\frac{\sqrt{3}}{2\sqrt{5}}\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta) - \frac{\sqrt{3}}{2\sqrt{5}}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{100}(\eta)$
		$+[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho) + [\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)\}$
		$\Psi^{\rho[22]} = \sqrt{\frac{2}{9}}\{-\frac{1}{\sqrt{2}}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta) - \frac{1}{\sqrt{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$-\frac{1}{\sqrt{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta) + \Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}$
		$+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM} + \Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\}$
43	[4]	–
43	[1111]	–
43	[211]	–
43	[31]	–

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
43	[22]	–
44	[4]	$\Psi^{S_1} = \sqrt{\frac{5}{57}}\{\Psi_{04M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta) + \Psi_{000}(\lambda)\Psi_{04M}(\rho)\Psi_{000}(\eta)$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{04M}(\eta) + \sqrt{\frac{14}{5}}[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$+\sqrt{\frac{14}{5}}[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho) + \sqrt{\frac{14}{5}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{02m_2}(\eta)]_{LM}\}$
		$\Psi^{S_2} = \sqrt{\frac{120}{1217}}\{\frac{\sqrt{21}}{2\sqrt{5}}[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho) + \frac{\sqrt{21}}{2\sqrt{5}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{02m_2}(\eta)]_{LM}$
		$-\frac{1}{2\sqrt{6}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{04M}(\eta) + [\Psi_{03m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)$
		$-\sqrt{7}[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho) \otimes \Psi_{01m_3}(\eta)]_{LM}\}$
44	[1111]	–
44	[211]	$\Psi^{\lambda[211]} = \sqrt{\frac{7}{1276}}\{\frac{4}{\sqrt{3(14)}}\Psi_{04M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta) + \frac{3}{\sqrt{7}}[\Psi_{03m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)$
		$-\frac{4}{\sqrt{7}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\rho) - 2\sqrt{\frac{3}{5}}[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$-2\sqrt{\frac{3}{5}}[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho) + 2\sqrt{\frac{3}{5}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{02m_2}(\eta)]_{LM}$
		$+[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho) \otimes \Psi_{01m_3}(\eta)]_{LM}\}$
		$\Psi^{\rho[211]} = \sqrt{\frac{7}{108}}\{\sqrt{\frac{2}{7}}[\Psi_{03m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta) - 3\sqrt{\frac{2}{7}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{03m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$+\frac{3}{\sqrt{7}}\Psi_{000}(\lambda)[\Psi_{03m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM} - \frac{4}{\sqrt{7}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{03m_2}(\eta)]_{LM}$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
		$+[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}+2\sqrt{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\eta)]_{LM}\}$
		$\Psi^{\eta[211]}=\frac{\sqrt{7}}{12}\{-3\sqrt{\frac{2}{7}}\Psi_{000}(\lambda)[\Psi_{03m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}+3\sqrt{2}[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\}$
44	[31]	$\Psi^{\lambda[31]_1}=\sqrt{\frac{105}{487}}\{-\frac{4}{\sqrt{3(14)}}\Psi_{000}(\lambda)\Psi_{04M}(\rho)\Psi_{000}(\eta)+\frac{3}{\sqrt{7}}[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)$
		$-\frac{2}{\sqrt{7}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\rho)+2\sqrt{\frac{3}{5}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\}$
		$\Psi^{\rho[31]_1}=\sqrt{\frac{7}{40}}\{-\sqrt{\frac{2}{7}}[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)+3\sqrt{\frac{2}{7}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{03m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$+\frac{3}{\sqrt{7}}\Psi_{000}(\lambda)[\Psi_{03m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}-\frac{2}{\sqrt{7}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{03m_2}(\eta)]_{LM}$
		$+[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\}$
		$\Psi^{\eta[31]_1}=\sqrt{\frac{105}{572}}\{-\frac{4}{\sqrt{21}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{04M}(\eta)-\sqrt{\frac{2}{7}}[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)$
		$+\sqrt{\frac{6}{5}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)+\sqrt{\frac{6}{5}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}$
		$+\sqrt{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\}$
44	[31]	$\Psi^{\lambda[31]_2}=\sqrt{\frac{21}{286}}\{\frac{2}{3}\sqrt{\frac{5}{14}}\Psi_{04M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)-2\sqrt{\frac{5}{14}}\Psi_{000}(\lambda)\Psi_{04M}(\rho)\Psi_{000}(\eta)$
		$-5\sqrt{\frac{5}{21}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\rho)+2[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$+[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)-\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\}$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\rho[31]_2} = \sqrt{\frac{21}{355}}\{-\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho) \otimes \Psi_{02m_3}(\eta)]_{LM} - 5\sqrt{\frac{5}{21}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{03m_2}(\eta)]_{LM}$
		$-8\sqrt{\frac{5}{42}}[\Psi_{03m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\}$
		$\Psi^{\eta[31]_2} = \sqrt{\frac{9}{149}}\{-\sqrt{\frac{5}{7}}\Psi_{04M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta) - \sqrt{\frac{5}{7}}\Psi_{000}(\lambda)\Psi_{04M}(\rho)\Psi_{000}(\eta)$
		$+\frac{2}{3}\sqrt{\frac{5}{7}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{04M}(\eta) - \sqrt{2}[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)$
		$+\frac{3}{\sqrt{2}}[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho) + \frac{3}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{02m_2}(\eta)]_{LM}$
		$-\sqrt{\frac{10}{21}}[\Psi_{03m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho) + \sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho) \otimes \Psi_{01m_3}(\eta)]_{LM}\}$
44	[22]	$\Psi^{\lambda[22]_1} = \sqrt{\frac{7}{274}}\{[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho) - \Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{02m_2}(\eta)]_{LM}$
		$-[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta) - \frac{1}{3}\sqrt{\frac{5}{14}}\Psi_{04M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)$
		$-2\sqrt{\frac{5}{21}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\rho) - \sqrt{\frac{5}{14}}\Psi_{000}(\lambda)\Psi_{04M}(\rho)\Psi_{000}(\eta)\}$
		$\Psi^{\rho[22]_1} = \sqrt{\frac{21}{130}}\{-\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho) \otimes \Psi_{02m_3}(\eta)]_{LM} - 2\sqrt{\frac{5}{21}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{03m_2}(\eta)]_{LM}$
		$+4\sqrt{\frac{5}{42}}[\Psi_{03m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\}$
44	[22]	$\Psi^{\lambda[22]_2} = \sqrt{\frac{105}{437}}\{\frac{3}{\sqrt{7}}[\Psi_{03m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho) + [\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho) \otimes \Psi_{01m_3}(\eta)]_{LM}$
		$+\frac{1}{2}\sqrt{\frac{3}{5}}[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta) + \frac{1}{2}\sqrt{\frac{7}{6}}\Psi_{04M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)$
		$\frac{4}{\sqrt{7}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{03m_2}(\rho)]_{LM}\Psi_{000}(\eta) + \frac{5}{2}\frac{1}{\sqrt{42}}\Psi_{000}(\lambda)\Psi_{04M}(\rho)\Psi_{000}(\eta)\}$

Table 2.3 Spatial wave functions of four-quark (q^4) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\rho[22]_2} = \sqrt{\frac{7}{47}}\{[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\rho) \otimes \Psi_{01m_3}(\eta)]_{LM} + \frac{2}{\sqrt{7}}\Psi_{000}(\lambda)[\Psi_{03m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}$ $+\frac{4}{\sqrt{7}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{03m_2}(\eta)]_{LM} - \frac{3}{\sqrt{14}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{03m_2}(\rho)]_{LM}\Psi_{000}(\eta)$ $-\frac{5}{\sqrt{14}}[\Psi_{03m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$).

NL	Symmetry	Wave function
11	[4]	$\Psi^{S} = \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{01M}(\xi)$
11	[1111]	–
11	[211]	–
11	[31]	$\Psi^{\lambda[31]} = \Psi_{01M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$ $\Psi^{\rho[31]} = \Psi_{000}(\lambda)\Psi_{01M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$ $\Psi^{\eta[31]} = \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{01M}(\eta)\Psi_{000}(\xi)$
11	[22]	–
20	[4]	$\Psi^{S_1} = \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi)$ $\Psi^{S_2} = \frac{1}{\sqrt{3}}\{\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + \Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$ $+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)\}$
20	[1111]	–
20	[211]	–
20	[31]	$\Psi^{\lambda[31]_1} = \sqrt{\frac{3}{8}}\{\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
		$-\sqrt{\frac{2}{3}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
		$\Psi^{\rho[31]_1} = \frac{1}{\sqrt{2}}\{-\frac{2}{\sqrt{3}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$-\sqrt{\frac{2}{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
		$\Psi^{\eta[31]_1} = \frac{1}{\sqrt{3}}\{-\frac{1}{\sqrt{2}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \frac{1}{\sqrt{2}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\sqrt{2}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)\}$
20	[31]	$\Psi^{\lambda[31]_2} = [\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\xi)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\eta)$
		$\Psi^{\rho[31]_2} = [\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\xi)]_{LM}\Psi_{000}(\lambda)\Psi_{000}(\eta)$
		$\Psi^{\eta[31]_2} = [\Psi_{01m_1}(\eta) \otimes \Psi_{01m_2}(\xi)]_{LM}\Psi_{000}(\lambda)\Psi_{000}(\rho)$
20	[22]	$\Psi^{\lambda[22]} = \sqrt{\frac{4}{7}}\{[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi) - \frac{1}{2}\sqrt{\frac{3}{2}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\frac{1}{2}\sqrt{\frac{3}{2}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[22]} = \sqrt{\frac{2}{3}}\{-\frac{1}{\sqrt{2}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
21	[4]	–

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
21	[1111]	–
21	[211]	$\Psi^{\lambda[211]} = -[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$ $\Psi^{\rho[211]} = -\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$ $\Psi^{\eta[211]} = [\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
21	[31]	$\Psi^{\lambda[31]} = [\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\xi)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\eta)$ $\Psi^{\rho[31]} = \Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\xi)]_{LM}\Psi_{000}(\eta)$ $\Psi^{\eta[31]} = \Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta) \otimes \Psi_{01m_2}(\xi)]_{LM}$
21	[22]	–
22	[4]	$\Psi^{S} = \frac{1}{2}\{\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{02M}(\xi) + \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{02M}(\eta)\Psi_{000}(\xi)$ $+\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + \Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
22	[1111]	–
22	[211]	–
22	[31]	$\Psi^{\lambda[31]} = \sqrt{\frac{3}{14}}\{\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$ $-\sqrt{\frac{5}{3}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi) + [\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\xi)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\eta)\}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\rho[31]} = \frac{1}{\sqrt{6}}\{-\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$-\sqrt{\frac{5}{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
		$\Psi^{\eta[31]} = \frac{1}{2}\{-\frac{1}{\sqrt{2}}\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \frac{1}{\sqrt{2}}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\frac{2}{\sqrt{2}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{02M}(\eta)\Psi_{000}(\xi) + \Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
22	[22]	$\Psi^{\lambda[22]} = \sqrt{\frac{10}{11}}\{\frac{1}{2\sqrt{5}}\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \frac{1}{2\sqrt{5}}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[22]} = \sqrt{\frac{2}{3}}\{-\frac{1}{\sqrt{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
30	[4]	$-$
30	[1111]	$\Psi^{A} = [\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
30	[211]	$\Psi^{\lambda[211]} = [\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}\Psi_{000}(\rho)$
		$\Psi^{\rho[211]} = \Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\eta[211]} = -[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho) \otimes \Psi_{01m_3}(\xi)]_{LM}\Psi_{000}(\eta)$
30	[31]	–
30	[22]	–
31	[4]	$\Psi^{S_1} = \frac{1}{\sqrt{3}}\{\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{01M}(\xi) + \Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{01M}(\xi)$ $+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{01M}(\xi)$ $\Psi^{S_2} = \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{11M}(\xi)\}$ $\Psi^{S_3} = \frac{1}{\sqrt{3}}\{[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\xi)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\eta) + \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho) \otimes \Psi_{01m_2}(\xi)]_{LM}$ $+\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{02m_1}(\eta) \otimes \Psi_{01m_2}(\xi)]_{LM}\}$ $\Psi^{S_4} = \sqrt{\frac{25}{93}}\{\frac{1}{\sqrt{2}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{01M}(\eta)\Psi_{000}(\xi) - \frac{3}{5}\Psi_{11M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$ $+\Psi_{01M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + \frac{1}{\sqrt{2}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{01M}(\eta)\Psi_{000}(\xi)$ $+\frac{2\sqrt{2}}{5}[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi) + \frac{2}{5}[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$ $+\frac{2}{5}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi) - \frac{3\sqrt{2}}{5}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{11M}(\eta)\Psi_{000}(\xi)\}$
31	[1111]	–

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
31	[211]	$-$
31	[31]	$\Psi^{\lambda[31]_1} = \Psi_{01M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi)$
		$\Psi^{\rho[31]_1} = \Psi_{000}(\lambda)\Psi_{01M}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi)$
		$\Psi^{\eta[31]_1} = \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{01M}(\eta)\Psi_{100}(\xi)$
31	[31]	$\Psi^{\lambda[31]_2} = [\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\xi)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\eta)$
		$\Psi^{\rho[31]_2} = \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{01m_1}(\rho) \otimes \Psi_{02m_2}(\xi)]_{LM}$
		$\Psi^{\eta[31]_2} = \Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta) \otimes \Psi_{02m_2}(\xi)]_{LM}$
31	[31]	$\Psi^{\lambda[31]_3} = \frac{1}{\sqrt{3}}\{\Psi_{11M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + \Psi_{01M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\Psi_{01M}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[31]_3} = \frac{1}{\sqrt{3}}\{\Psi_{000}(\lambda)\Psi_{11M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + \Psi_{100}(\lambda)\Psi_{01M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\Psi_{000}(\lambda)\Psi_{01M}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\eta[31]_3} = \frac{1}{\sqrt{3}}\{\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{11M}(\eta)\Psi_{000}(\xi) + \Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{01M}(\eta)\Psi_{000}(\xi)$
		$+\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{01M}(\eta)\Psi_{000}(\xi)\}$
31	[31]	$\Psi^{\lambda[31]_4} = \frac{1}{\sqrt{6}}\{-\sqrt{2}\Psi_{01M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \sqrt{2}\Psi_{01M}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)+[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[31]_4}=\frac{1}{\sqrt{6}}\{-\sqrt{2}\Psi_{100}(\lambda)\Psi_{01M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)-\sqrt{2}\Psi_{000}(\lambda)\Psi_{01M}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$+[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
		$\Psi^{\eta[31]_4}=\frac{1}{\sqrt{6}}\{-\sqrt{2}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{01M}(\eta)\Psi_{000}(\xi)-\sqrt{2}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{01M}(\eta)\Psi_{000}(\xi)$
		$+[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)+\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
31	[22]	–
32	[4]	$\Psi^{S}=\frac{1}{\sqrt{3}}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\xi)]_{LM}\Psi_{000}(\eta)+\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{02m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
32	[1111]	–
32	[211]	–
32	[31]	$\Psi^{\lambda[31]_1}=\frac{1}{\sqrt{2}}\{-[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)-[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[31]_1}=\frac{1}{\sqrt{2}}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)-\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
		$\Psi^{\eta[31]_1}=\frac{1}{\sqrt{2}}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)+\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
32	[31]	$\Psi^{\lambda[31]_2}=[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\xi)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\eta)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\rho[31]_2} = \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{01m_1}(\rho) \otimes \Psi_{02m_2}(\xi)]_{LM}$
		$\Psi^{\eta[31]_2} = \Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta) \otimes \Psi_{02m_2}(\xi)]_{LM}$
32	[22]	$-$
33	[4]	$\Psi^{S_1} = \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{03M}(\xi)$
		$\Psi^{S_2} = \frac{1}{\sqrt{3}}\{\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\xi)]_{LM} + \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho) \otimes \Psi_{01m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{02m_1}(\eta) \otimes \Psi_{01m_2}(\xi)]_{LM}\}$
		$\Psi^{S_3} = \sqrt{\frac{7}{17}}\{-\frac{1}{\sqrt{7}}\Psi_{03M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + [\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\frac{1}{\sqrt{2}}[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi) + \frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\sqrt{\frac{2}{7}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{03M}(\eta)\Psi_{000}(\xi)\}$
33	[1111]	$-$
33	[211]	$\Psi^{\lambda[211]} = \sqrt{\frac{7}{156}}\{\frac{3}{\sqrt{7}}\Psi_{03M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + [\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\sqrt{2}[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi) - \sqrt{2}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-4[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[211]} = \sqrt{\frac{21}{524}}\{[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi) + \frac{3}{\sqrt{7}}\Psi_{000}(\lambda)\Psi_{03M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
		$-2\sqrt{\frac{5}{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}-4\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
		$\Psi^{\eta[211]}=\sqrt{\frac{7}{144}}\{-3\sqrt{2}[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+3\sqrt{\frac{2}{7}}\Psi_{000}(\lambda)\Psi_{03M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
33	[31]	$\Psi^{\lambda[31]_1}=\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\xi)]_{LM}$
		$\Psi^{\rho[31]_1}=\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{01m_1}(\rho)\otimes\Psi_{02m_2}(\xi)]_{LM}$
		$\Psi^{\eta[31]_1}=\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta)\otimes\Psi_{02m_2}(\xi)]_{LM}$
33	[31]	$\Psi^{\lambda[31]_2}=\sqrt{\frac{7}{23}}\{[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)+[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$+\frac{3}{\sqrt{7}}\Psi_{03M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[31]_2}=\sqrt{\frac{7}{23}}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\frac{3}{\sqrt{7}}\Psi_{000}(\lambda)\Psi_{03M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\eta[31]_2}=\sqrt{\frac{7}{23}}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)+\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\frac{3}{\sqrt{7}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{03M}(\eta)\Psi_{000}(\xi)\}$
33	[31]	$\Psi^{\lambda[31]_3}=\sqrt{\frac{3}{11}}\{-\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\xi)]_{LM}+\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM}$
		$+\sqrt{\frac{5}{3}}\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}\}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\rho[31]_3} = \frac{1}{\sqrt{5}}\{\sqrt{\frac{10}{3}}\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\xi)]_{LM}$
		$+\sqrt{\frac{5}{3}}\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}\}$
		$\Psi^{\eta[31]_3} = \sqrt{\frac{2}{3}}\{\frac{1}{\sqrt{2}}\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\xi)]_{LM} + \frac{1}{\sqrt{2}}\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM}$
		$-\frac{1}{\sqrt{2}}\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{02m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
33	[31]	$\Psi^{\lambda[31]_4} = \sqrt{\frac{2}{5}}\{-\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi) + \Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\frac{1}{\sqrt{2}}\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
		$\Psi^{\rho[31]_4} = \sqrt{\frac{6}{23}}\{\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
		$\Psi^{\eta[31]_4} = \sqrt{\frac{14}{31}}\{-\frac{1}{\sqrt{7}}\Psi_{03M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + [\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$-\frac{1}{\sqrt{2}}\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi) - \frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\frac{1}{\sqrt{14}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{03M}(\eta)\Psi_{000}(\xi)\}$
33	[22]	$\Psi^{\lambda[22]} = \sqrt{\frac{10}{13}}\{\sqrt{\frac{3}{20}}\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\xi)]_{LM}$
		$-\sqrt{\frac{3}{20}}\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}\}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\rho[22]} = \sqrt{\frac{2}{3}}\{-\sqrt{\frac{1}{2}}\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho) \otimes \Psi_{01m_3}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\eta) \otimes \Psi_{01m_3}(\xi)]_{LM}\}$
40	[4]	$\Psi^{S_1} = \sqrt{\frac{1}{3}}\{\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\xi)]_{LM} + \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho) \otimes \Psi_{02m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{02m_1}(\eta) \otimes \Psi_{02m_2}(\xi)]_{LM}\}$
40	[4]	$\Psi^{S_2} = \sqrt{\frac{1}{18}}\{[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi) + \Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi) - \sqrt{5}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$-\sqrt{5}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi) - \sqrt{5}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
40	[4]	$\Psi^{S_3} = \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{200}(\xi)$
40	[4]	$\Psi^{S_4} = \sqrt{\frac{25}{93}}\{-\frac{3}{5}\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{11m_1}(\lambda) \otimes \Psi_{01m_2}(\xi)]_{LM} + \Psi_{100}(\rho)\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\xi)]_{LM}$
		$+\frac{2\sqrt{2}}{5}\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho) \otimes \Psi_{01m_3}(\xi)]_{LM} + \frac{1}{\sqrt{2}}\Psi_{100}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta) \otimes \Psi_{01m_2}(\xi)]_{LM}$
		$+\frac{2}{5}\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\eta) \otimes \Psi_{01m_3}(\xi)]_{LM} + \frac{1}{\sqrt{2}}\Psi_{000}(\lambda)\Psi_{100}(\rho)[\Psi_{01m_1}(\eta) \otimes \Psi_{01m_2}(\xi)]_{LM}$
		$+\frac{2}{5}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{01m_2}(\eta) \otimes \Psi_{01m_3}(\xi)]_{LM} - \frac{3\sqrt{2}}{5}\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{11m_1}(\eta) \otimes \Psi_{01m_2}(\xi)]_{LM}\}$
40	[4]	$\Psi^{S_5} = \sqrt{\frac{125}{486}}\{-\frac{3}{5}[\Psi_{11m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi) + [\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)\Psi_{000}(\xi)$
		$+\frac{2\sqrt{2}}{5}[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\rho) \otimes \Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi) - \frac{9}{10}\sqrt{\frac{3}{2}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
		$-\frac{9}{10}\sqrt{\frac{3}{2}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)+\frac{3}{10}\sqrt{\frac{3}{2}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{200}(\eta)\Psi_{000}(\xi)$
		$-\frac{\sqrt{5}}{6}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)+\frac{1}{5}\sqrt{\frac{6}{5}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
40	[4]	$\Psi^{S_6}=\sqrt{\frac{1}{3}}\{\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi)+\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi)$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{100}(\xi)\}$
40	[4]	$\Psi^{S_7}=\sqrt{\frac{1}{6}}\{\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)+\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{200}(\eta)\Psi_{000}(\xi)\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$+\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)+\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
40	[1111]	–
40	[211]	$\Psi^{\lambda[211]}=\sqrt{\frac{8}{409}}\{-\frac{3\sqrt{5}}{2}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)+\frac{5\sqrt{5}}{4}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)-\frac{3}{4}\sqrt{\frac{15}{2}}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$-\frac{1}{2}\sqrt{\frac{5}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)+\frac{5\sqrt{5}}{4}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$-\frac{5\sqrt{5}}{4}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)+[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$-\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)+\sqrt{\frac{15}{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$-\frac{5}{4}\sqrt{\frac{5}{6}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)\Psi_{000}(\xi)\}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\rho[211]} = \sqrt{\frac{16}{585}}\{-\frac{\sqrt{15}}{4}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\frac{3\sqrt{15}}{4}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$-\frac{1}{2}\sqrt{\frac{5}{3}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)-\frac{3}{4}\sqrt{\frac{15}{2}}\Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\frac{5}{2}\sqrt{\frac{5}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)\Psi_{000}(\xi)-\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\sqrt{\frac{15}{2}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\xi)-\frac{5}{4}\sqrt{\frac{5}{6}}\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
		$\Psi^{\eta[211]} = \sqrt{\frac{8}{315}}\{-\frac{5\sqrt{15}}{4}\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\sqrt{\frac{15}{2}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\frac{3\sqrt{15}}{4}\Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
40	[31]	$\Psi^{\lambda[31]_1} = \Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\xi)]_{LM}$
		$\Psi^{\rho[31]_1} = \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\xi)]_{LM}$
		$\Psi^{\eta[31]_1} = \Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta)\otimes\Psi_{11m_2}(\xi)]_{LM}$
40	[31]	$\Psi^{\lambda[31]_2} = \sqrt{\frac{6}{11}}\{-\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\xi)]_{LM}+\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\xi)]_{LM}$
		$+\sqrt{\frac{5}{3}}\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{02m_3}(\xi)]_{LM}\}$
		$\Psi^{\rho[31]_2} = \sqrt{\frac{2}{3}}\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\xi)]_{LM}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
		$+\sqrt{\frac{1}{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{02m_3}(\xi)]_{LM}$
		$\Psi^{\eta[31]_2}=\sqrt{\frac{2}{7}}\{\frac{1}{\sqrt{2}}\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\xi)]_{LM}+\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\xi)]_{LM}$
		$-\sqrt{2}\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{02m_1}(\eta)\otimes\Psi_{02m_2}(\xi)]_{LM}\}$
40	[31]	$\Psi^{\lambda[31]_3}=\sqrt{\frac{3}{61}}\{-[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)+\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\sqrt{\frac{5}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)-\sqrt{\frac{20}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)\Psi_{000}(\xi)$
		$+\sqrt{5}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)-\sqrt{5}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[31]_3}=\sqrt{\frac{1}{15}}\{\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\sqrt{\frac{5}{3}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\sqrt{\frac{10}{3}}\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)-\sqrt{\frac{20}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)\Psi_{000}(\xi)\}$
		$\Psi^{\eta[31]_3}=\sqrt{\frac{1}{18}}\{-\frac{1}{\sqrt{2}}\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\sqrt{\frac{5}{2}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)+\sqrt{\frac{5}{2}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$-\sqrt{10}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)-\sqrt{2}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
40	[31]	$\Psi^{\lambda[31]_4}=\sqrt{\frac{6}{221}}\{\frac{5\sqrt{5}}{4}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)+[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$+\sqrt{\frac{5}{12}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)-\frac{5}{4}\sqrt{\frac{15}{2}}\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\frac{1}{2}\sqrt{\frac{5}{3}}\Psi_{100}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)+\frac{9\sqrt{5}}{8}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$-\frac{9\sqrt{5}}{8}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)+\frac{3\sqrt{5}}{4}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$-\frac{9\sqrt{5}}{4}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[31]_4}=\sqrt{\frac{32}{1165}}\{\frac{1}{2}\sqrt{\frac{5}{3}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\frac{5}{4}\sqrt{\frac{15}{2}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\sqrt{\frac{5}{6}}\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)-\frac{9}{4}\sqrt{\frac{5}{3}}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$-\sqrt{15}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\eta[31]_4}=\sqrt{\frac{32}{8667}}\{-\frac{\sqrt{15}}{4}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$+\frac{5}{4}\sqrt{\frac{5}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)\Psi_{000}(\xi)$
		$+\sqrt{\frac{5}{6}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)+\frac{1}{\sqrt{2}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$+\frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)+\frac{3}{2}\sqrt{\frac{5}{4}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{200}(\eta)\Psi_{000}(\xi)$
		$+\frac{19}{8}\sqrt{\frac{5}{2}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)+\frac{19}{8}\sqrt{\frac{5}{2}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$-\frac{9}{4}\sqrt{\frac{5}{2}}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)-\frac{9}{4}\sqrt{\frac{5}{2}}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$-\frac{9}{4}\sqrt{\frac{5}{2}}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
40	[31]	$\Psi^{\lambda[31]_5} = \sqrt{\frac{3}{8}}\{-\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi) + \Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi)$
		$+\sqrt{\frac{2}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{100}(\xi)\}$
		$\Psi^{\rho[31]_5} = \sqrt{\frac{2}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)\Psi_{000}(\xi) + \sqrt{\frac{1}{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\xi)$
		$\Psi^{\eta[31]_5} = \sqrt{\frac{1}{3}}\{\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi) + \Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi)$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{100}(\xi)\}$
40	[31]	$\Psi^{\lambda[31]_6} = \sqrt{\frac{1}{6}}\{[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\xi)]_{LM}\Psi_{000}(\eta)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}\Psi_{000}(\rho)$
		$-\sqrt{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\xi)]_{LM}\Psi_{100}(\rho)\Psi_{000}(\eta) - \sqrt{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\xi)]_{LM}\Psi_{000}(\rho)\Psi_{100}(\eta)\}$
		$\Psi^{\rho[31]_6} = \sqrt{\frac{1}{2}}\{\Psi_{02m_l}(\lambda)\Psi_{01m_l}(\rho)\Psi_{000}(\eta)\Psi_{01m_l}(\xi) + \Psi_{000}(\lambda)\Psi_{01m_l}(\rho)\Psi_{02m_l}(\eta)\Psi_{01m_l}(\xi)\}$
		$\Psi^{\eta[31]_6} = \sqrt{\frac{1}{6}}\{\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}$
		$-\sqrt{2}\Psi_{100}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM} - \sqrt{2}\Psi_{000}(\lambda)\Psi_{100}(\rho)[\Psi_{01m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
40	[31]	$\Psi^{\lambda[31]_7} = \sqrt{\frac{1}{3}}\{\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\xi)]_{LM} + \Psi_{100}(\rho)\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\xi)]_{LM}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
		$+\Psi_{000}(\rho)\Psi_{100}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
		$\Psi^{\rho[31]_7}=\sqrt{\frac{1}{3}}\{\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM}+\Psi_{100}(\lambda)\Psi_{000}(\eta)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)\Psi_{100}(\eta)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
		$\Psi^{\eta[31]_7}=\sqrt{\frac{1}{3}}\{\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{11m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM}+\Psi_{100}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)\Psi_{100}(\rho)[\Psi_{01m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
40	[31]	$\Psi^{\lambda[31]_8}=\sqrt{\frac{1}{18}}\{\sqrt{\frac{3}{2}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)-\sqrt{\frac{3}{2}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$+\sqrt{6}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)-\sqrt{6}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)+[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[31]_8}=\frac{1}{3}\{\Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)+\sqrt{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$+\sqrt{2}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)+\sqrt{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\eta[31]_8}=\sqrt{\frac{2}{45}}\{-2\sqrt{3}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{200}(\eta)\Psi_{000}(\xi)+\sqrt{3}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\sqrt{3}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)-\frac{\sqrt{3}}{2}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$-\frac{\sqrt{3}}{2}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)+\sqrt{3}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
40	[22]	$\Psi^{\lambda[22]_1}=\sqrt{\frac{1}{32}}\{-[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)+\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\sqrt{\frac{20}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)+\sqrt{\frac{40}{3}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\sqrt{5}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)-\sqrt{5}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[22]_1}=\sqrt{\frac{1}{30}}\{\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\sqrt{\frac{20}{3}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\sqrt{\frac{40}{3}}\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)-\sqrt{\frac{20}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)\Psi_{000}(\xi)\}$
40	[22]	$\Psi^{\lambda[22]_2}=\sqrt{\frac{16}{1841}}\{\frac{5\sqrt{5}}{4}\Psi_{100}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+2\sqrt{\frac{5}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)+\sqrt{\frac{15}{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$-\sqrt{\frac{40}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)\Psi_{000}(\xi)-\frac{9\sqrt{5}}{4}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$-\frac{9\sqrt{5}}{4}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)+\frac{9\sqrt{5}}{4}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$+\frac{3\sqrt{5}}{4}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[22]_2}=\sqrt{\frac{4}{305}}\{2\sqrt{\frac{5}{3}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$+\sqrt{\frac{15}{2}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\sqrt{\frac{40}{3}}\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)+\frac{9}{2}\sqrt{\frac{5}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$-\sqrt{15}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
40	[22]	$\Psi^{\lambda[22]_3}=\sqrt{\frac{10}{33}}\{\sqrt{\frac{3}{20}}\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\xi)]_{LM}$
		$-\sqrt{\frac{3}{20}}\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{02m_3}(\xi)]_{LM}\}$
		$\Psi^{\rho[22]_3}=-\frac{1}{\sqrt{3}}\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\xi)]_{LM}$
		$+\sqrt{\frac{2}{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{02m_3}(\xi)]_{LM}$
40	[22]	$\Psi^{\lambda[22]_4}=\sqrt{\frac{4}{7}}\{\sqrt{\frac{3}{8}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi)-\sqrt{\frac{3}{8}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{100}(\xi)\}$
		$\Psi^{\rho[22]_4}=-\frac{1}{\sqrt{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{100}(\xi)+\sqrt{\frac{2}{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\xi)$
40	[22]	$\Psi^{\lambda[22]_5}=\sqrt{\frac{4}{27}}\{[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)+[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)\Psi_{000}(\xi)-\sqrt{\frac{3}{2}}\Psi_{000}(\lambda)\Psi_{200}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\sqrt{\frac{3}{8}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)-\sqrt{\frac{3}{8}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$+\sqrt{\frac{3}{2}}\Psi_{200}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[22]_5} = \sqrt{\frac{2}{9}}\{\Psi_{000}(\lambda)[\Psi_{11m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi) + \Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi) - \frac{1}{\sqrt{2}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$-\frac{1}{\sqrt{2}}[\Psi_{11m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi) - \frac{1}{\sqrt{2}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
41	[4]	$\Psi^{S} = \sqrt{\frac{1}{3}}\{\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\xi)]_{LM} + \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho) \otimes \Psi_{02m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{02m_1}(\eta) \otimes \Psi_{02m_2}(\xi)]_{LM}\}$
41	[1111]	–
41	[211]	$\Psi^{\lambda[211]_1} = [\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{100}(\xi)$
		$\Psi^{\rho[211]_1} = [\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\lambda)\Psi_{100}(\xi)$
		$\Psi^{\eta[211]_1} = [\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{100}(\xi)$
41	[211]	$\Psi^{\lambda[211]_2} = \sqrt{\frac{1}{3}}\{[\Psi_{11m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi) + [\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)\Psi_{000}(\xi)$
		$+[\Psi_{01m_1}(\lambda) \otimes \Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[211]_2} = \sqrt{\frac{1}{3}}\{\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi) + \Psi_{000}(\lambda)[\Psi_{11m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\eta[211]_2} = \sqrt{\frac{1}{3}}\{-[\Psi_{11m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi) - [\Psi_{01m_1}(\lambda) \otimes \Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$ $-[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)\Psi_{000}(\xi)\}$
41	[31]	$\Psi^{\lambda[31]_1} = \Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda) \otimes \Psi_{11m_2}(\xi)]_{LM}$ $\Psi^{\rho[31]_1} = \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{01m_1}(\rho) \otimes \Psi_{11m_2}(\xi)]_{LM}$ $\Psi^{\eta[31]_1} = \Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta) \otimes \Psi_{11m_2}(\xi)]_{LM}$
41	[31]	$\Psi^{\lambda[31]_2} = \sqrt{\frac{1}{3}}\{[\Psi_{11m_1}(\lambda) \otimes \Psi_{01m_2}(\xi)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\eta) + [\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\xi)]_{LM}\Psi_{100}(\rho)\Psi_{000}(\eta)$ $+[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\xi)]_{LM}\Psi_{000}(\rho)\Psi_{100}(\eta)\}$ $\Psi^{\rho[31]_2} = \sqrt{\frac{1}{3}}\{\Psi_{100}(\lambda)\Psi_{000}(\eta)[\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\xi)]_{LM} + \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{11m_1}(\rho) \otimes \Psi_{01m_2}(\xi)]_{LM}$ $+\Psi_{000}(\lambda)\Psi_{100}(\eta)[\Psi_{01m_1}(\rho) \otimes \Psi_{01m_2}(\xi)]_{LM}\}$ $\Psi^{\eta[31]_2} = \sqrt{\frac{1}{3}}\{\Psi_{100}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta) \otimes \Psi_{01m_2}(\xi)]_{LM} + \Psi_{000}(\lambda)\Psi_{100}(\rho)[\Psi_{01m_1}(\eta) \otimes \Psi_{01m_2}(\xi)]_{LM}$ $+\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{11m_1}(\eta) \otimes \Psi_{01m_2}(\xi)]_{LM}\}$
41	[22]	$-$
42	[4]	$\Psi^{S_1} = \sqrt{\frac{1}{3}}\{\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda) \otimes \Psi_{02m_2}(\xi)]_{LM} + \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho) \otimes \Psi_{02m_2}(\xi)]_{LM}$ $+\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{02m_1}(\eta) \otimes \Psi_{02m_2}(\xi)]_{LM}\}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
42	[4]	$\Psi^{S_2} = \sqrt{\frac{7}{36}}\{\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi) + \Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\sqrt{\frac{5}{14}}\Psi_{100}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \sqrt{\frac{5}{14}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{02M}(\eta)\Psi_{000}(\xi)$
		$-\sqrt{\frac{5}{14}}\Psi_{02M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \sqrt{\frac{5}{14}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{02M}(\eta)\Psi_{000}(\xi)$
		$-\sqrt{\frac{5}{14}}\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi) - \sqrt{\frac{5}{14}}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$+[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
42	[4]	$\Psi^{S_3} = \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{12M}(\xi)$
42	[4]	$\Psi^{S_4} = \sqrt{\frac{1}{3}}\{\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi) + \Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi)$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{02M}(\eta)\Psi_{100}(\xi)\}$
42	[4]	$\Psi^{S_5} = \sqrt{\frac{1}{3}}\{\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{02M}(\xi) + \Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{02M}(\xi)$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{02M}(\xi)\}$
42	[4]	$\Psi^{S_6} = \frac{1}{3}\{\Psi_{12M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + \Psi_{000}(\lambda)\Psi_{12M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{12M}(\eta)\Psi_{000}(\xi) + \Psi_{100}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{02M}(\eta)\Psi_{000}(\xi) + \Psi_{02M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{02M}(\eta)\Psi_{000}(\xi) + \Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$+\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)\}$
42	[1111]	$-$
42	[211]	$-$
42	[31]	$\Psi^{\lambda[31]_1} = \sqrt{\frac{1}{3}}\{\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\xi)]_{LM} + \Psi_{100}(\rho)\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\rho)\Psi_{100}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
		$\Psi^{\rho[31]_1} = \sqrt{\frac{1}{3}}\{\Psi_{100}(\lambda)\Psi_{000}(\eta)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM} + \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)\Psi_{100}(\eta)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
		$\Psi^{\eta[31]_1} = \sqrt{\frac{1}{3}}\{\Psi_{100}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)\Psi_{100}(\rho)[\Psi_{01m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{11m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
42	[31]	$\Psi^{\lambda[31]_2} = \sqrt{\frac{10}{33}}\{\sqrt{\frac{3}{10}}\Psi_{100}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \sqrt{\frac{3}{10}}\Psi_{12M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\frac{1}{\sqrt{2}}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi) - \sqrt{\frac{3}{10}}\Psi_{02M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\frac{1}{\sqrt{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)\Psi_{000}(\xi) + \sqrt{\frac{3}{10}}\Psi_{000}(\lambda)\Psi_{12M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$-\sqrt{\frac{3}{10}}\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi) + \sqrt{\frac{3}{10}}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$+\frac{1}{\sqrt{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[31]_2}=\sqrt{\frac{2}{9}}\{\frac{1}{\sqrt{2}}\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)+\frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)\Psi_{000}(\xi)+\frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
		$\Psi^{\eta[31]_2}=\sqrt{\frac{20}{87}}\{\sqrt{\frac{3}{20}}\Psi_{100}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)+\sqrt{\frac{3}{5}}\Psi_{100}(\lambda)\Psi_{000}(\rho)\Psi_{02M}(\eta)\Psi_{000}(\xi)$
		$+\sqrt{\frac{3}{20}}\Psi_{12M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)+\sqrt{\frac{3}{20}}\Psi_{02M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$-\frac{3}{\sqrt{2}}\Psi_{000}(\lambda)\Psi_{100}(\rho)\Psi_{02M}(\eta)\Psi_{000}(\xi)+\sqrt{\frac{3}{20}}\Psi_{000}(\lambda)\Psi_{12M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\sqrt{\frac{3}{20}}\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)+\sqrt{\frac{3}{20}}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$
		$-\sqrt{\frac{3}{5}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{12M}(\eta)\Psi_{000}(\xi)\}$
42	[31]	$\Psi^{\lambda[31]_3}=\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{03m_2}(\xi)]_{LM}$
		$\Psi^{\rho[31]_3}=\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{01m_1}(\rho)\otimes\Psi_{03m_2}(\xi)]_{LM}$
		$\Psi^{\eta[31]_3}=\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta)\otimes\Psi_{03m_2}(\xi)]_{LM}$
42	[31]	$\Psi^{\lambda[31]_4}=\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\xi)]_{LM}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\rho[31]_4} = \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\xi)]_{LM}$
		$\Psi^{\eta[31]_4} = \Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta)\otimes\Psi_{11m_2}(\xi)]_{LM}$
42	[31]	$\Psi^{\lambda[31]_5} = \sqrt{\frac{3}{22}}\{-\sqrt{2}\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi) + \sqrt{2}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi)$
		$+\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{100}(\xi)\}$
		$\Psi^{\rho[31]_5} = \sqrt{\frac{1}{2}}\{[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{100}(\xi) + \Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\xi)\}$
		$\Psi^{\eta[31]_5} = \sqrt{\frac{1}{6}}\{\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi) + \Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\xi)$
		$-2\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{02M}(\eta)\Psi_{100}(\xi)\}$
42	[22]	$\Psi^{\lambda[22]_1} = \sqrt{\frac{10}{13}}\{\sqrt{\frac{3}{20}}\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi) - \sqrt{\frac{3}{20}}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)\Psi_{100}(\xi)$
		$+\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\xi)\}$
		$\Psi^{\rho[22]_1} = -\frac{1}{\sqrt{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{100}(\xi)$
		$+\sqrt{\frac{2}{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\xi)$
42	[22]	$\Psi^{\lambda[22]_2} = \sqrt{\frac{5}{39}}\{\sqrt{\frac{3}{10}}\Psi_{100}(\lambda)\Psi_{02M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) - \sqrt{\frac{3}{10}}\Psi_{12M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$-\sqrt{2}[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi) - \sqrt{\frac{3}{10}}\Psi_{02M}(\lambda)\Psi_{100}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$-\sqrt{2}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{100}(\rho)\Psi_{000}(\xi) + \sqrt{\frac{3}{10}}\Psi_{000}(\lambda)\Psi_{12M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
		$-\sqrt{\frac{3}{10}}\Psi_{02M}(\lambda)\Psi_{000}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)+\sqrt{\frac{3}{10}}\Psi_{000}(\lambda)\Psi_{02M}(\rho)\Psi_{100}(\eta)\Psi_{000}(\xi)$ $-\sqrt{2}\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
		$\Psi^{\rho[22]_2}=\frac{1}{3}\{-\sqrt{2}\Psi_{100}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)+[\Psi_{11m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$ $+[\Psi_{01m_1}(\lambda)\otimes\Psi_{11m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)-\sqrt{2}\Psi_{000}(\lambda)[\Psi_{11m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$ $+[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{100}(\eta)\Psi_{000}(\xi)-\sqrt{2}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{11m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
43	[4]	$\Psi^{S}=\sqrt{\frac{1}{3}}\{\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\xi)]_{LM}+\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\xi)]_{LM}$ $+\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{02m_1}(\eta)\otimes\Psi_{02m_2}(\xi)]_{LM}\}$
43	[1111]	–
43	[211]	–
43	[31]	$\Psi^{\lambda[31]}=\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{03m_2}(\xi)]_{LM}$ $\Psi^{\rho[31]}=\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{01m_1}(\rho)\otimes\Psi_{03m_2}(\xi)]_{LM}$ $\Psi^{\eta[31]}=\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta)\otimes\Psi_{03m_2}(\xi)]_{LM}$
43	[22]	–

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
44	[4]	$\Psi^{S_1} = \Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{04M}(\xi)$
44	[4]	$\Psi^{S_2} = \sqrt{\frac{1}{3}}\{\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\xi)]_{LM} + \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{02m_1}(\eta)\otimes\Psi_{02m_2}(\xi)]_{LM}\}$
44	[4]	$\Psi^{S_3} = \sqrt{\frac{7}{17}}\{\frac{1}{\sqrt{7}}\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_1}(\xi)]_{LM} + \Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\xi)]_{LM}$
		$+\frac{1}{\sqrt{2}}\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM} + \frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}$
		$-\sqrt{\frac{2}{7}}\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{03m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
44	[4]	$\Psi^{S_4} = \sqrt{\frac{840}{1217}}\{-\frac{1}{\sqrt{7}}[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_1}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\sqrt{\frac{3}{20}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi) - \sqrt{\frac{3}{20}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\frac{1}{2\sqrt{42}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{04M}(\eta)\Psi_{000}(\xi)\}$
44	[4]	$\Psi^{S_5} = \sqrt{\frac{5}{57}}\{\Psi_{04M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi) + \Psi_{000}(\lambda)\Psi_{04M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{04M}(\eta)\Psi_{000}(\xi) + \sqrt{\frac{14}{5}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\sqrt{\frac{14}{5}}\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi) + \sqrt{\frac{14}{5}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
44	[1111]	–
44	[211]	$\Psi^{\lambda[211]_1} = \sqrt{\frac{7}{78}}\{-\frac{3}{\sqrt{14}}\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\xi)]_{LM}$ $-\frac{1}{\sqrt{2}}\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\xi)]_{LM}$ $-\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}+\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}$ $+2\sqrt{2}\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)\otimes\Psi_{01m_l3}(\xi)]_{LM}\}$ $\Psi^{\rho[211]_1} = \sqrt{\frac{27}{761}}\{-\frac{1}{\sqrt{2}}\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\xi)]_{LM}$ $-\frac{3}{\sqrt{14}}\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{03m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM}$ $+\frac{\sqrt{30}}{9}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)\otimes\Psi_{01m_4}(\xi)]_{LM}+\frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{02m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}\}$ $\Psi^{\eta[211]_1} = \sqrt{\frac{7}{72}}\{3\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\xi)]_{LM}$ $-\frac{3}{\sqrt{7}}\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{03m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
44	[211]	$\Psi^{\lambda[211]_2} = \sqrt{\frac{105}{1276}}\{\sqrt{\frac{8}{21}}\Psi_{04M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)-\frac{6}{\sqrt{15}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\xi)]_{LM}\Psi_{000}(\eta)$ $+\frac{3}{\sqrt{7}}[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)+[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$ $-\frac{6}{\sqrt{15}}\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)+\frac{6}{\sqrt{15}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$ $-\frac{4}{\sqrt{7}}\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
		$\Psi^{\rho[211]_2} = \sqrt{\frac{7}{108}}\{\sqrt{\frac{2}{7}}[\Psi_{03m_1}(\lambda) \otimes \Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi) - 3\sqrt{\frac{2}{7}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{03m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\rho) \otimes \Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi) + \frac{3}{\sqrt{7}}\Psi_{000}(\lambda)[\Psi_{03m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+2\sqrt{2}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho) \otimes \Psi_{02m_3}(\eta)]_{LM}\Psi_{000}(\xi) - \frac{4}{\sqrt{7}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
		$\Psi^{\eta[211]_2} = \sqrt{\frac{7}{144}}\{3\sqrt{2}[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\rho) \otimes \Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-3\sqrt{\frac{2}{7}}\Psi_{000}(\lambda)[\Psi_{03m_1}(\rho) \otimes \Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
44	[31]	$\Psi^{\lambda[31]_1} = \Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda) \otimes \Psi_{03m_2}(\xi)]_{LM}$
		$\Psi^{\rho[31]_1} = \Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{01m_1}(\rho) \otimes \Psi_{03m_2}(\xi)]_{LM}$
		$\Psi^{\eta[31]_1} = \Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{01m_1}(\eta) \otimes \Psi_{03m_2}(\xi)]_{LM}$
44	[31]	$\Psi^{\lambda[31]_2} = \sqrt{\frac{2}{5}}\{-\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda) \otimes \Psi_{01m_2}(\eta) \otimes \Psi_{01m_3}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho) \otimes \Psi_{01m_2}(\eta) \otimes \Psi_{01m_3}(\xi)]_{LM}$
		$-\frac{1}{\sqrt{2}}\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda) \otimes \Psi_{02m_2}(\eta) \otimes \Psi_{01m_3}(\xi)]_{LM}\}$
		$\Psi^{\rho[31]_2} = \sqrt{\frac{6}{23}}\{\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda) \otimes \Psi_{01m_2}(\rho) \otimes \Psi_{01m_3}(\eta) \otimes \Psi_{01m_4}(\xi)]_{LM}$
		$-\frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho) \otimes \Psi_{02m_2}(\eta) \otimes \Psi_{01m_3}(\xi)]_{LM}\}$
		$\Psi^{\eta[31]_2} = \sqrt{\frac{14}{31}}\{-\frac{1}{\sqrt{7}}\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{03m_1}(\lambda) \otimes \Psi_{01m_2}(\xi)]_{LM}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$+\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\xi)]_{LM}\}$
		$-\frac{1}{\sqrt{2}}\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}-\frac{1}{\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}$
		$+\frac{1}{\sqrt{14}}\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{03m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
44	[31]	$\Psi^{\lambda[31]_3}=\sqrt{\frac{252}{853}}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)+\frac{1}{2}\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\frac{1}{2}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)+\frac{1}{3}\sqrt{\frac{5}{14}}\Psi_{04M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$-\sqrt{\frac{5}{14}}\Psi_{000}(\lambda)\Psi_{04M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)-\frac{5}{2}\sqrt{\frac{5}{21}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[31]_3}=\sqrt{\frac{84}{355}}\{-\frac{1}{2}\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\frac{5}{2}\sqrt{\frac{5}{21}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-4\sqrt{\frac{5}{42}}[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\eta[31]_3}=\sqrt{\frac{252}{1133}}\{-\frac{1}{2}\sqrt{\frac{10}{21}}[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$+\sqrt{\frac{5}{6}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\frac{3}{2\sqrt{2}}\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)+\frac{3}{2\sqrt{2}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\frac{1}{3}\sqrt{\frac{5}{7}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{04M}(\eta)\Psi_{000}(\xi)-\frac{1}{2}\sqrt{\frac{5}{7}}\Psi_{04M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$-\frac{1}{\sqrt{2}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)-\frac{1}{2}\sqrt{\frac{5}{7}}\Psi_{000}(\lambda)\Psi_{04M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
44	[31]	$\Psi^{\lambda[31]_4} = \sqrt{\frac{5}{11}}\{-\frac{3}{\sqrt{15}}\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\xi)]_{LM}$
		$+\frac{3}{\sqrt{15}}\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{02m_3}(\xi)]_{LM}\}$
		$\Psi^{\rho[31]_4} = \sqrt{\frac{2}{3}}\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\xi)]_{LM}$
		$+\sqrt{\frac{1}{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{02m_3}(\xi)]_{LM}$
		$\Psi^{\eta[31]_4} = \sqrt{\frac{5}{9}}\{\sqrt{\frac{3}{10}}\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\xi)]_{LM}$
		$+\sqrt{\frac{3}{10}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{02m_3}(\xi)]_{LM}$
		$-2\sqrt{\frac{3}{10}}\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{02m_1}(\eta)\otimes\Psi_{02m_2}(\xi)]_{LM}\}$
44	[31]	$\Psi^{\lambda[31]_5} = \sqrt{\frac{23}{7}}\{\frac{3}{\sqrt{7}}\Psi_{000}(\rho)\Psi_{000}(\eta)[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\xi)]_{LM} + \Psi_{01m_l}(\lambda)\Psi_{02m_l}(\rho)\Psi_{000}(\eta)\Psi_{01m_l}(\xi)$
		$+\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}\}$
		$\Psi^{\rho[31]_5} = \sqrt{\frac{23}{7}}\{\Psi_{000}(\eta)[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\xi)]_{LM}$
		$+\frac{3}{\sqrt{7}}\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{03m_1}(\rho)\otimes\Psi_{01m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{02m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}\}$
		$\Psi^{\eta[31]_5} = \sqrt{\frac{23}{7}}\{\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
		$+\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{01m_3}(\xi)]_{LM}$
		$+\frac{3}{\sqrt{7}}\Psi_{000}(\lambda)\Psi_{000}(\rho)[\Psi_{03m_1}(\eta)\otimes\Psi_{01m_2}(\xi)]_{LM}\}$
44	[31]	$\Psi^{\lambda[31]_6}=\sqrt{\frac{35}{146}}\{\frac{3}{\sqrt{7}}[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\sqrt{\frac{3}{5}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)+\sqrt{\frac{3}{5}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\sqrt{\frac{6}{7}}\Psi_{04M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)+\sqrt{\frac{6}{7}}\Psi_{000}(\lambda)\Psi_{04M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\frac{3}{\sqrt{7}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)\}$
		$\Psi^{\rho[31]_6}=\sqrt{\frac{35}{206}}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\frac{3}{\sqrt{7}}\Psi_{000}(\lambda)[\Psi_{03m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\sqrt{\frac{3}{5}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\eta)]_{LM}\Psi_{000}(\xi)+\frac{3}{\sqrt{7}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\sqrt{\frac{6}{7}}[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)+\sqrt{\frac{6}{7}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{03m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
		$\Psi^{\eta[31]_6}=\sqrt{\frac{5}{18}}\{-\sqrt{\frac{3}{10}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$-\sqrt{\frac{3}{10}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-2\sqrt{\frac{3}{7}}\Psi_{000}(\lambda)\Psi_{000}(\rho)\Psi_{04M}(\eta)\Psi_{000}(\xi)+\sqrt{\frac{3}{7}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\bar{q}$) (Continued).

NL	Symmetry	Wave function
		$+\sqrt{\frac{3}{7}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)+\sqrt{\frac{3}{7}}\Psi_{000}(\lambda)\Psi_{04M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
44	[22]	$\Psi^{\lambda[22]_1}=\sqrt{\frac{63}{274}}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)-\sqrt{\frac{5}{14}}\Psi_{000}(\lambda)\Psi_{04M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$
		$+\frac{1}{3}\sqrt{\frac{5}{14}}\Psi_{04M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)-\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)+2\sqrt{\frac{5}{21}}\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
		$\Psi^{\rho[22]_1}=\sqrt{\frac{1}{10}}\{\sqrt{\frac{10}{3}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+2\sqrt{\frac{5}{21}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-4\sqrt{\frac{5}{14}}[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)\}$
44	[22]	$\Psi^{\lambda[22]_2}=\sqrt{\frac{10}{13}}\{\sqrt{\frac{3}{20}}[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\xi)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\eta)-\sqrt{\frac{3}{20}}\Psi_{000}(\lambda)\Psi_{000}(\eta)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\xi)]_{LM}$
		$+\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{02m_3}(\xi)]_{LM}\}$
		$\Psi^{\rho[22]_2}=-\frac{1}{\sqrt{3}}\Psi_{000}(\eta)[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\xi)]_{LM}$
		$+\sqrt{\frac{2}{3}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{01m_2}(\eta)\otimes\Psi_{02m_3}(\xi)]_{LM}$
44	[22]	$\Psi^{\lambda[22]_3}=\sqrt{\frac{70}{211}}\{\frac{3}{\sqrt{7}}[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\rho)\Psi_{000}(\xi)$
		$+[\Psi_{01m_1}(\lambda)\otimes\Psi_{02m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\sqrt{\frac{3}{14}}\Psi_{000}(\lambda)\Psi_{04M}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)+\sqrt{\frac{3}{14}}\Psi_{04M}(\lambda)\Psi_{000}(\rho)\Psi_{000}(\eta)\Psi_{000}(\xi)$

Table 2.4 Spatial wave functions of pentaquark ($q^4\overline{q}$) (Continued).

NL	Symmetry	Wave function
		$+\frac{1}{2}\sqrt{\frac{3}{5}}\Psi_{000}(\rho)[\Psi_{02m_1}(\lambda)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)-\frac{1}{2}\sqrt{\frac{3}{5}}\Psi_{000}(\lambda)[\Psi_{02m_1}(\rho)\otimes\Psi_{02m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\frac{3}{\sqrt{7}}\Psi_{000}(\rho)[\Psi_{01m_1}(\lambda)\otimes\Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\xi)\}$
		$\Psi^{\rho[22]_3}=\sqrt{\frac{14}{75}}\{[\Psi_{02m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{01m_3}(\eta)]_{LM}\Psi_{000}(\xi)$
		$+\frac{3}{\sqrt{7}}\Psi_{000}(\lambda)[\Psi_{03m_1}(\rho)\otimes\Psi_{01m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\frac{1}{\sqrt{2}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)\otimes\Psi_{02m_3}(\eta)]_{LM}\Psi_{000}(\xi)+\frac{3}{\sqrt{7}}\Psi_{000}(\lambda)[\Psi_{01m_1}(\rho)\otimes\Psi_{03m_2}(\eta)]_{LM}\Psi_{000}(\xi)$
		$-\frac{3}{\sqrt{14}}[\Psi_{03m_1}(\lambda)\otimes\Psi_{01m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)-\frac{3}{\sqrt{14}}[\Psi_{01m_1}(\lambda)\otimes\Psi_{03m_2}(\rho)]_{LM}\Psi_{000}(\eta)\Psi_{000}(\xi)\}$

CHAPTER III

GROUND STATE PENTAQUARK MASSES

In this work, we calculate ground state pentaquark masses using the total wave functions of pentaquarks which we have already constructed in chapter II. We expect that derived pentaquark masses could help us to understand more or less some problems in hadronic physics, for example, missing resonances of low-lying baryons, and the properties of the Ropper resonance $N_{1/2+}(1440)$ and the negative parity resonances $N_{1/2-}(1530)$ and $N_{3/2-}(1520)$ which are believed containing some pentaquark constituents.

To the lowest-order approximation, the interaction between quarks is usually assumed the type of simple harmonic oscillators. In non-relativistic constituent quark model, one may consider the contributions from one gluon exchange (OGE) (Vijande et al., 2004) and one Goldstone boson exchange (GBE) (Glozman, 1998), which are treated as perturbations to the harmonic-oscillator-type Hamiltonian. The OGE interaction is spin, color, flavor and spatial dependent while the GBE interaction is independent of color. The Hamiltonian of N-quark system takes the form,

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m_i} + C \sum_{i<j}^{N} ([\vec{r_i} - \vec{r_j}]^2 + V_0) + \sum_{i=1}^{N} m_i + H_{OGE} + H_{GBE} \tag{3.1}$$

with

$$H_{OGE} = -C_{OGE} \sum_{i<j} \frac{\lambda_i^C \cdot \lambda_j^C}{m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j \tag{3.2}$$

$$H_{GBE} = -C_{GBE} \sum_{i<j} \frac{\lambda_i^F \cdot \lambda_j^F}{m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j \tag{3.3}$$

where λ_i^C is the Gell-Mann matrices in color space, λ_i^F is the Gell-Mann matrices in flavor space and $\vec{\sigma_i}$ is Pauli matrices in the spin space and m_i is the invarient mass of the i^{th} quark.

We assume that all five quarks have the same mass m and apply the Jacobi coordinate from Eq.(2.50) and Eq.(2.51) in Chepter II. Non-perturbed Hamiltonian is rewritten as

$$H = \frac{P_{cm}^2}{2m} + \frac{p_\lambda^2}{2m} + \frac{p_\rho^2}{2m} + \frac{p_\eta^2}{2m} + \frac{p_\xi^2}{2m} + 5C[\lambda^2 + \rho^2 + \eta^2 + \xi^2] + 5(m + V_0) \quad (3.4)$$

At cm frame ($P_{cm} = 0$) the Hamiltonian of meson ($q\bar{q}$), baryon (q^3) and pentaquark ($q^4\bar{q}$) could be written as

$$H_{q\bar{q}} = 2(m + V_2^0) + \frac{p_\rho^2}{2m} + 2C\rho^2 \quad (3.5)$$

$$H_{q^3} = 3(m + V_3^0) + \frac{p_\lambda^2}{2m} + \frac{p_\rho^2}{2m} + 3C[\lambda^2 + \rho^2] \quad (3.6)$$

$$H_{q^4\bar{q}} = 5(m + V_5^0) + \frac{p_\lambda^2}{2m} + \frac{p_\rho^2}{2m} + \frac{p_\eta^2}{2m} + \frac{p_\xi^2}{2m} + 5C[\lambda^2 + \rho^2 + \eta^2 + \xi^2] \quad (3.7)$$

Define $2C \equiv m\omega_2^2/2$, $3C \equiv m\omega_3^2/2$ and $5C \equiv m\omega_5^2/2$, we obtain

$$H_{q\bar{q}} = 2(m + V_2^0) + \frac{p_\rho^2}{2m} + \frac{m\omega_2^2\rho^2}{2} \quad (3.8)$$

$$H_{q^3} = 3(m + V_3^0) + \frac{p_\lambda^2}{2m} + \frac{p_\rho^2}{2m} + \frac{m\omega_3^2}{2}[\lambda^2 + \rho^2] \quad (3.9)$$

$$H_{q^4\bar{q}} = 5(m + V_5^0) + \frac{p_\lambda^2}{2m} + \frac{p_\rho^2}{2m} + \frac{p_\eta^2}{2m} + \frac{p_\xi^2}{2m} + \frac{m\omega_5^2}{2}[\lambda^2 + \rho^2 + \eta^2 + \xi^2] \quad (3.10)$$

Corresponding eigenvectors are

$$\Psi_{NLM}^o(q\bar{q}) = \sum_{n_\rho, l_\rho} A(n_\rho, l_\rho)\Psi_{n_\rho l_\rho m_\rho} \quad (3.11)$$

$$\Psi_{NLM}^o(q^3) = \sum_{n_\lambda, n_\rho, l_\lambda, l_\rho} A(n_\lambda, n_\rho, l_\lambda, l_\rho) \cdot\Psi_{n_\lambda l_\lambda m_\lambda}\Psi_{n_\rho l_\rho m_\rho} \cdot C(l_\lambda, l_\rho, m_\lambda, m_\rho, LM) \quad (3.12)$$

$$\Psi^{o}_{NLM}(q^4\bar{q}) = \sum_{n_\lambda,n_\rho,n_\eta,n_\xi,l_\lambda,l_\rho,l_\eta,l_\xi} A(n_\lambda, n_\rho, n_\eta, n_\xi, l_\lambda, l_\rho, l_\eta, l_\xi)$$
$$\cdot\Psi_{n_\lambda l_\lambda m_\lambda}\Psi_{n_\rho l_\rho m_\rho}\Psi_{n_\eta l_\eta m_\eta}\Psi_{n_\xi l_\xi m_\xi}$$
$$\cdot C(l_\lambda, l_\rho, m_\lambda, m_\rho, l_{\lambda\rho}, m_{\lambda\rho})$$
$$\cdot C(l_\eta, l_\xi, m_\eta, m_\xi, l_{\eta\xi}, m_{\eta\xi})$$
$$\cdot C(l_{\lambda\rho}, l_{\eta\xi}, m_{\lambda\rho}, m_{\eta\xi}, LM) \quad (3.13)$$

Corresponding eigenvalues are

$$m_{q\bar{q}} = 2m^{'} + N_{q\bar{q}}\omega_2 + \Delta m_{OGE} + \Delta m_{GBE} \quad (3.14)$$

$$m_{q^3} = 3m^{'} + N_{q^3}\omega_3 + \Delta m_{OGE} + \Delta m_{GBE} \quad (3.15)$$

$$m_{q^4\bar{q}} = 5m^{'} + N_{q^4\bar{q}}\omega_5 + \Delta m_{OGE} + \Delta m_{GBE} \quad (3.16)$$

$$\Delta m_{OGE} = \langle\Psi|H_{OGE}|\Psi\rangle \quad (3.17)$$

$$\Delta m_{GBE} = \langle\Psi|H_{GBE}|\Psi\rangle \quad (3.18)$$

where we define $m^{'} = m+V_2^0+\frac{3\omega_2}{4} = m+V_3^0+\omega_3 = m+V_5^0+\frac{6\omega_5}{5}$ to decrease number of parameters as less as possible. Δm_{OGE} and Δm_{GBE} are mass shift when we consider the first order approximation of one-gluon exchange and Goldstone boson exchange interaction respectively. Moreover, the principle quantum numbers are

$$N_{q\bar{q}} = 2n_\rho + l_\rho \quad (3.19)$$

$$N_{q^3} = 2(n_\lambda + n_\rho) + l_\lambda + l_\rho \quad (3.20)$$

$$N_{q^4\bar{q}} = 2(n_\lambda + n_\rho + n_\eta + n_\xi) + l_\lambda + l_\rho + l_\eta + l_\xi \quad (3.21)$$

Table 3.1 Properties of baryons and mesons.

Particle	Quark content	Mass(MeV)	I	J^P
N	uud	938	$\frac{1}{2}$	$\frac{1}{2}^+$
Δ^{++}	uuu	1232	$\frac{3}{2}$	$\frac{3}{2}^+$
Λ	uds	1115	0	$\frac{1}{2}^+$
Σ^0	uus	1193	1	$\frac{1}{2}^+$
Σ^*	uus	1385	1	$\frac{3}{2}^+$
Ξ	uss	1314	$\frac{1}{2}$	$\frac{1}{2}^+$
Ξ^*	uss	1532	$\frac{1}{2}$	$\frac{3}{2}^+$
Ω	sss	1672	0	$\frac{3}{2}^+$
ρ^0	$\frac{u\bar{u}-d\bar{d}}{\sqrt{2}}$	776	1	1^-
ϕ	$s\bar{s}$	1019	0	1^-
K	$u\bar{s}$	892	$\frac{1}{2}$	1^-
w	$\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}$	783	0	1^-

3.1 Parameters estimation

We use well known baryons and mesons to investigate the coefficients of color-magnetic interaction and the Goldston-boson exchange interaction. Properties of selected baryons and mesons employed in this process are shown in Table 3.1

Considering baryons, color confinement property requires their color wave function singlet or antisymmetric, [111]. Spatial wave function of baryons in ground state has the principle quantum number $N = 0$ therefore the spatial wave function of ground state is $\Psi_S^O = \Psi_{NLM} = \Psi_{000}$, symmetric wave function,[3]. The spatial wave functions are a constant giving no contribution in factor λ, ρ. The remaining part, spin-flavor wave functions have to be [3]

$$\Psi^{A}(q^3) = \boxed{}\boxed{}\boxed{}_{spatial} \otimes \boxed{}\boxed{}\boxed{}_{spin-flavor} \otimes \begin{array}{|c|}\hline \\ \hline \\ \hline \\ \hline \end{array}_{color} \tag{3.22}$$

For spin-flavor wave functions of baryons can be written in general form as

$$\Psi^{SF}(q^3) = \sum_{i,j} a_{ij}\phi_i\chi_j \tag{3.23}$$

where $i, j = S, A, \lambda, \rho$. To calculate the spin-flavor wave functions in symmetric, the representation matrices of permutation operators in S_3 group are applied to Eq.(3.23) and coefficients can be nailed down. The result is

$$\Psi^{SF}(q^3) = \phi^S\chi^S, \frac{1}{\sqrt{2}}(\phi^\lambda\chi^\lambda + \phi^\rho\chi^\rho) \tag{3.24}$$

(1) Color wave functions

$$\begin{aligned}\Psi^{C}(q^3) &= \left| \begin{array}{|c|}\hline 1 \\ \hline 1 \\ \hline 1 \\ \hline \end{array}, \begin{array}{|c|}\hline R \\ \hline G \\ \hline B \\ \hline \end{array} \right\rangle \\ &= \frac{1}{\sqrt{6}}|RGB - RBG - GRB + GBR \\ &\quad +BRG - BGR\rangle \end{aligned} \tag{3.25}$$

$$\Psi^{C}(q\bar{q}) = \frac{1}{\sqrt{3}}|R\bar{R} + G\bar{G} + B\bar{B}\rangle \tag{3.26}$$

(2) Spatial wave function

$$\Psi^{O}_{S} = \Psi_{NLM} = \Psi_{000} \tag{3.27}$$

(3) Spin wave functions

(3.1) $S = \frac{3}{2}, S_z = \frac{3}{2}$

$$\chi^S = |\,\boxed{1\,|\,2\,|\,3}\,,\boxed{\uparrow|\uparrow|\uparrow}\,\rangle = |\uparrow\uparrow\uparrow\rangle \tag{3.28}$$

(3.2) $S = \frac{3}{2}, S_z = \frac{1}{2}$

$$\chi^S = |\,\boxed{1\,|\,2\,|\,3}\,,\boxed{\uparrow|\uparrow|\downarrow}\,\rangle = \frac{1}{\sqrt{3}}|\uparrow\uparrow\downarrow + \downarrow\uparrow\uparrow + \uparrow\downarrow\uparrow\rangle \tag{3.29}$$

(3.3) $S = \frac{1}{2}, S_z = \frac{1}{2}$

$$\chi^\lambda = |\,\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1}\end{array}\,,\begin{array}{|c|c|}\hline \uparrow & \uparrow \\ \hline \downarrow \\ \cline{1-1}\end{array}\,\rangle = \frac{1}{\sqrt{6}}|2\uparrow\uparrow\downarrow - \downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow\rangle \tag{3.30}$$

(3.4) $S = \frac{1}{2}, S_z = \frac{1}{2}$

$$\chi^\rho = |\,\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1}\end{array}\,,\begin{array}{|c|c|}\hline \uparrow & \uparrow \\ \hline \downarrow \\ \cline{1-1}\end{array}\,\rangle = \frac{1}{\sqrt{2}}|\downarrow\uparrow\uparrow - \uparrow\downarrow\uparrow\rangle \tag{3.31}$$

(3.5) For meson spin $1, S = 1, S_z = 1$

$$\chi = |\uparrow\bar{\uparrow}\rangle \tag{3.32}$$

(4) Flavor wave functions : We describe flavor wave functions in term of $|n, S, I, I_z\rangle$ where n is supermultiplet, S is strangeness, I is isospin and I_z is isospin projection.

(4.1) $N\ |8, 0, \frac{1}{2}, +\frac{1}{2}\rangle$

$$\phi^\lambda = \frac{1}{\sqrt{6}}|2uud - udu - duu\rangle$$

$$\phi^\rho = \frac{1}{\sqrt{2}}|duu - udu\rangle \tag{3.33}$$

(4.2) $\Delta^{++}\ |10, 0, \frac{3}{2}, +\frac{3}{2}\rangle$

$$\phi^S = |uuu\rangle \tag{3.34}$$

(4.3) Λ $|8,-1,1,0\rangle$

$$\phi^{\lambda} = \frac{1}{\sqrt{12}}|2uds + 2dus - dsu - usd - sud - sdu\rangle$$

$$\phi^{\rho} = \frac{1}{2}|sud + sdu - dsu - usd\rangle \tag{3.35}$$

(4.4) Σ^0 $|8,-1,1,+1\rangle$

$$\phi^{\lambda} = \frac{1}{\sqrt{6}}|2uus - usu - suu\rangle$$

$$\phi^{\rho} = \frac{1}{\sqrt{2}}|suu - usu\rangle \tag{3.36}$$

(4.5) Σ^* $|10,-1,1,+1\rangle$

$$\phi^{S} = \frac{1}{\sqrt{3}}|uus + usu + suu\rangle \tag{3.37}$$

(4.6) Ξ^0 $|8,-2,\frac{1}{2},+\frac{1}{2}\rangle$

$$\phi^{\lambda} = \frac{1}{\sqrt{6}}|2ssu - sus - uss\rangle$$

$$\phi^{\rho} = \frac{1}{\sqrt{2}}|uss - sus\rangle \tag{3.38}$$

(4.7) Ξ^* $|10,-2,\frac{1}{2},+\frac{1}{2}\rangle$

$$\phi^{S} = \frac{1}{\sqrt{3}}|ssu + uss + sus\rangle \tag{3.39}$$

(4.8) Ω $|10,-3,0,0\rangle$

$$\phi^{S} = |sss\rangle \tag{3.40}$$

However, the flavor wave functions of ρ^0, ϕ, K and ω mesons are shown in Table 3.1

To estimate parameters we use the total wave functions calculate the mass shift Δm_{OGE} and Δm_{GBE} from Eq.(3.2), (3.3), (3.17) and (3.18). For mass shift calculation we need to consider Pauli matrices (σ) :

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and Gell-Mann matrices (λ) :

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Spin basis functions may be written in the matrix form as follows :

$$\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \bar{\uparrow} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \bar{\downarrow} = -\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

Color basis functions may be written in the matrix form as follows :

$$R = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad G = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\bar{R} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \bar{G} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \bar{B} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

We show one example to calculate mass of Δ^{++} meson by calculating Δm_{OGE} and Δm_{GBE}. From Eq.(3.15) we have

$$M_{\Delta^{++}} = 3m_u + \Delta m_{OGE} + \Delta m_{GBE} \tag{3.41}$$

$$\begin{aligned}
\Delta m_{OGE} &= -\frac{C_{OGE}}{m_u^2}\sum_{i<j}\langle\Psi^C|\lambda_i^C\cdot\lambda_j^C|\Psi^C\rangle\langle\phi^S\chi^S|\sigma_i\cdot\sigma_j|\phi^S\chi^S\rangle\langle\Psi^O|\Psi^O\rangle \\
&= -\frac{C_{OGE}}{6m_u^2}\sum_{i<j}\langle RGB - RBG - GRB + GBR + BRG - BGR| \\
&\quad \lambda_i^C\cdot\lambda_j^C|RGB - RBG - GRB + GBR + BRG - BGR\rangle \\
&\quad \langle\uparrow\uparrow\uparrow|\sigma_i\cdot\sigma_j|\uparrow\uparrow\uparrow\rangle\langle uuu|uuu\rangle \\
&= -\frac{C_{OGE}}{6m_u^2}[\sum_{i=1}^{8}(\langle RGB|\lambda_1^i\cdot\lambda_2^i|RGB\rangle - \langle RGB|\lambda_1^i\cdot\lambda_2^i|GRB\rangle \\
&\quad +\langle RBG|\lambda_1^i\cdot\lambda_2^i|RBG\rangle - \langle RBG|\lambda_1^i\cdot\lambda_2^i|BGR\rangle \\
&\quad +\langle GRB|\lambda_1^i\cdot\lambda_2^i|GRB\rangle - \langle GRB|\lambda_1^i\cdot\lambda_2^i|RGB\rangle \\
&\quad +\langle GBR|\lambda_1^i\cdot\lambda_2^i|GBR\rangle - \langle GBR|\lambda_1^i\cdot\lambda_2^i|BGR\rangle \\
&\quad +\langle BRG|\lambda_1^i\cdot\lambda_2^i|BRG\rangle - \langle BRG|\lambda_1^i\cdot\lambda_2^i|RBG\rangle
\end{aligned}$$

$$+\langle BGR|\lambda_1^i \cdot \lambda_2^i|BGR\rangle - \langle BGR|\lambda_1^i \cdot \lambda_2^i|GBR\rangle$$

$$+\langle RGB|\lambda_1^i \cdot \lambda_3^i|RGB\rangle - \langle RGB|\lambda_1^i \cdot \lambda_3^i|BGR\rangle$$

$$+\langle RBG|\lambda_1^i \cdot \lambda_3^i|RBG\rangle - \langle RBG|\lambda_1^i \cdot \lambda_3^i|GBR\rangle$$

$$+\langle GRB|\lambda_1^i \cdot \lambda_3^i|GRB\rangle - \langle GRB|\lambda_1^i \cdot \lambda_3^i|BRG\rangle$$

$$+\langle GBR|\lambda_1^i \cdot \lambda_3^i|GBR\rangle - \langle GBR|\lambda_1^i \cdot \lambda_3^i|RBG\rangle$$

$$+\langle BRG|\lambda_1^i \cdot \lambda_3^i|BRG\rangle - \langle BRG|\lambda_1^i \cdot \lambda_3^i|GRB\rangle$$

$$+\langle BGR|\lambda_1^i \cdot \lambda_3^i|BGR\rangle - \langle BGR|\lambda_1^i \cdot \lambda_3^i|RGB\rangle$$

$$+\langle RGB|\lambda_2^i \cdot \lambda_3^i|RGB\rangle - \langle RGB|\lambda_2^i \cdot \lambda_3^i|RBG\rangle$$

$$+\langle RBG|\lambda_2^i \cdot \lambda_3^i|RBG\rangle - \langle RBG|\lambda_2^i \cdot \lambda_3^i|RGB\rangle$$

$$+\langle GRB|\lambda_2^i \cdot \lambda_3^i|GRB\rangle - \langle GRB|\lambda_2^i \cdot \lambda_3^i|GBR\rangle$$

$$+\langle GBR|\lambda_2^i \cdot \lambda_3^i|GBR\rangle - \langle GBR|\lambda_2^i \cdot \lambda_3^i|GRB\rangle$$

$$+\langle BRG|\lambda_2^i \cdot \lambda_3^i|BRG\rangle - \langle BRG|\lambda_2^i \cdot \lambda_3^i|BGR\rangle$$

$$+\langle BGR|\lambda_2^i \cdot \lambda_3^i|BGR\rangle - \langle BGR|\lambda_2^i \cdot \lambda_3^i|BRG\rangle]$$

$$\sum_{i=1}^{3}\langle\uparrow\uparrow\uparrow|\sigma_1^i \cdot \sigma_2^i|\uparrow\uparrow\uparrow\rangle$$

$$= -\frac{C_{OGE}}{m_u^2}\sum_{i=1}^{8}(\lambda_{11}^i \cdot \lambda_{22}^i - \lambda_{12}^i \cdot \lambda_{21}^i + \lambda_{11}^i \cdot \lambda_{33}^i - \lambda_{13}^i \cdot \lambda_{31}^i + \lambda_{22}^i \cdot \lambda_{33}^i$$

$$-\lambda_{23}^i \cdot \lambda_{32}^i)\sum_{i=1}^{3}\sigma_{11}^i\sigma_{11}^i$$

$$= \frac{8C_{OGE}}{m_u^2} \tag{3.42}$$

$$\begin{aligned}
\Delta m_{GBE} &= -\frac{C_{GBE}}{m_u^2}\sum_{i<j}\langle\Psi^C|\Psi^C\rangle\langle\phi^S|\lambda_i^F\cdot\lambda_j^F|\phi^S\rangle\langle\chi^S|\sigma_i\cdot\sigma_j|\chi^S\rangle\langle\Psi^O|\Psi^O\rangle \\
&= -\frac{C_{GBE}}{m_u^2}\sum_{i<j}[\langle uuu|\lambda_i^F\cdot\lambda_j^F|uuu\rangle\langle\uparrow\uparrow\uparrow|\sigma_i\cdot\sigma_j|\uparrow\uparrow\uparrow\rangle] \\
&= -\frac{C_{GBE}}{m_u^2}\sum_{i=1}^{8}[\langle uuu|\lambda_1^i\cdot\lambda_2^i|uuu\rangle\langle\uparrow\uparrow\uparrow|\sigma_1^i\cdot\sigma_2^i|\uparrow\uparrow\uparrow\rangle \\
&\quad +\langle uuu|\lambda_1^i\cdot\lambda_3^i|uuu\rangle\langle\uparrow\uparrow\uparrow|\sigma_1^i\cdot\sigma_3^i|\uparrow\uparrow\uparrow\rangle \\
&\quad +\langle uuu|\lambda_2^i\cdot\lambda_3^i|uuu\rangle\langle\uparrow\uparrow\uparrow|\sigma_2^i\cdot\sigma_3^i|\uparrow\uparrow\uparrow\rangle] \\
&= -\frac{3C_{GBE}}{m_u^2}\sum_{i=1}^{8}\lambda_{11}^i\cdot\lambda_{11}^i\sum_{i=1}^{3}\sigma_{11}^i\cdot\sigma_{11}^i \\
&= -\frac{4C_{GBE}}{m_u^2}
\end{aligned} \tag{3.43}$$

Input results from Eq.(3.42) and Eq.(3.43) into Eq.(3.41),

$$M_{\Delta^{++}} = 3m_u + \frac{8C_{OGE}}{m_u^2} - \frac{4C_{GBE}}{m_u^2} \tag{3.44}$$

We use the same process as Δ^{++} for calculating mass of 8 baryons and 4 mesons. The results show that

$$M_N = 3m_u - \frac{8}{m_u^2}C_{OGE} - \frac{14}{m_u^2}C_{GBE} \tag{3.45}$$

$$M_{\Delta^{++}} = 3m_u + \frac{8}{m_u^2}C_{OGE} - \frac{4}{m_u^2}C_{GBE} \tag{3.46}$$

$$M_\Lambda = 2m_u + m_s - \frac{8}{3}(\frac{2}{m_u^2} + \frac{1}{m_u m_s})C_{OGE} - 2(\frac{4}{m_u^2} + \frac{3}{m_u m_s})C_{GBE} \tag{3.47}$$

$$M_{\Sigma^0} = 2m_u + m_s - \frac{8}{m_u m_s}C_{OGE} - \frac{2}{3}(\frac{2}{m_u^2} + \frac{19}{m_u m_s})C_{GBE} \tag{3.48}$$

$$M_{\Sigma^*} = 2m_u + m_s + \frac{4}{3}(\frac{2}{m_u^2} + \frac{4}{m_u m_s})C_{OGE} - \frac{1}{2}(\frac{8}{3m_u^2} + \frac{16}{3m_u m_s})C_{GBE} \tag{3.49}$$

$$M_{\Xi} = m_u + 2m_s - \frac{8}{m_u m_s}C_{OGE} - \frac{2}{3}(\frac{19}{m_u m_s} + \frac{2}{m_s^2})C_{GBE} \tag{3.50}$$

$$M_{\Xi^*} = m_u + 2m_s + \frac{8}{3}(\frac{1}{m_s^2} + \frac{2}{m_u m_s})C_{OGE} - (\frac{8}{3m_u m_s} + \frac{4}{3m_s^2})C_{GBE} \tag{3.51}$$

$$M_{\Omega} = 3m_s + \frac{8}{m_s^2}C_{OGE} - \frac{4}{m_s^2}C_{GBE} \tag{3.52}$$

$$M_{\rho^0} = 2m_u + \frac{16}{3m_u^2}C_{OGE} - \frac{2}{3m_u^2}C_{GBE} \tag{3.53}$$

$$M_{\phi} = 2m_s + \frac{16}{3m_s^2}C_{OGE} + \frac{4}{3m_s^2}C_{GBE} \tag{3.54}$$

$$M_K = m_u + m_s + \frac{16}{m_u m_s}C_{OGE} - \frac{2}{3m_u m_s}C_{GBE} \tag{3.55}$$

$$M_w = 2m_u + \frac{16}{3m_u^2}C_{OGE} + \frac{10}{3m_u^2}C_{GBE} \tag{3.56}$$

From calculations of magnetic moments of baryons with the same model (Yan, 2006) we found that up quark mass is approximately 333 to 370 MeV. We use this range of quark mass to solve the parameters $m_u, m_s, C_{OGE}, C_{GBE}$ by the following process : first, consider only 8 baryons with and without OGE and GBE interaction, after that consider both 8 baryons and 4 mesons.

(1) 8 baryons

(1.1) C_{OGE}

$$m_u = 364, m_s = 535, C_{OGE} = 19 \tag{3.57}$$

(1.2) C_{GBE}

$$m_u = 370, m_s = 567, C_{GBE} = 12 \tag{3.58}$$

(1.3) $C_{OGE} + C_{GBE}$

$$m_u = 367, m_s = 538, C_{OGE} = 18, C_{GBE} = 2 \tag{3.59}$$

(2) 8 baryons and 4 mesons

(2.1) C_{OGE}

$$m_u = 360, m_s = 527, C_{OGE} = 17 \tag{3.60}$$

(2.2) C_{GBE}

$$m_u = 370, m_s = 553, C_{GBE} = 11 \tag{3.61}$$

(2.3) $C_{OGE} + C_{GBE}$

$$m_u = 360, m_s = 527, C_{OGE} = 16, C_{GBE} \approx 0 \tag{3.62}$$

Where m_u and m_s are in Mev.

We found that when we consider both the OGE and GBE interactions the GBE interaction gives a small contribution to the mass ($C_{GBE} = 1.81$ for 8 baryons and $C_{GBE} \approx 0$ for 8 baryons with 4 mesons) compared with OGE. The appropriate parameters should come from general case, that is, where 8 baryons and 4 mesons are considered. Therefore we obtain the parameters

$$m_u \approx 360, C_{OGE} \approx 16.4, C_{GBE} \approx 0 \tag{3.63}$$

3.2 Non-strange pentaquark mass in ground state

For ground states, the spatial wave function is symmetric, then the total wave function of pentaquarks can be written as

$$\Psi^A = \frac{1}{\sqrt{3}}[\Psi^o_{[5]}(\Psi^c_{[211]_\lambda}\Psi^{sf}_{[31]_\rho} - \Psi^c_{[211]_\rho}\Psi^{sf}_{[31]_\lambda} + \Psi^c_{[211]_\eta}\Psi^{sf}_{[31]_\eta})] \tag{3.64}$$

The mass shift for one-gluon exchange is

$$\begin{aligned}\Delta m_{OGE}(q^4\overline{q}) &= \langle\Psi(q^4\overline{q})|H_{OGE}(q^4\overline{q})|\Psi(q^4\overline{q})\rangle \\ &= -\frac{1}{3}\frac{C_{OGE}}{m_i m_j}\{\langle\Psi^c_{[211]_\lambda}|\lambda^C_i\cdot\lambda^C_j|\Psi^c_{[211]_\lambda}\rangle\langle\Psi^{sf}_{[31]_\rho}|\vec{\sigma}_i\cdot\vec{\sigma}_j|\Psi^{sf}_{[31]_\rho}\rangle\end{aligned}$$

$$-2\langle\Psi^c_{[211]_\lambda}|\lambda^C_i\cdot\lambda^C_j|\Psi^c_{[211]_\rho}\rangle\langle\Psi^{sf}_{[31]_\lambda}|\vec{\sigma}_i\cdot\vec{\sigma}_j|\Psi^{sf}_{[31]_\rho}\rangle$$

$$+2\langle\Psi^c_{[211]_\lambda}|\lambda^C_i\cdot\lambda^C_j|\Psi^c_{[211]_\eta}\rangle\langle\Psi^{sf}_{[31]_\rho}|\vec{\sigma}_i\cdot\vec{\sigma}_j|\Psi^{sf}_{[31]_\eta}\rangle$$

$$+\langle\Psi^c_{[211]_\rho}|\lambda^C_i\cdot\lambda^C_j|\Psi^c_{[211]_\rho}\rangle\langle\Psi^{sf}_{[31]_\lambda}|\vec{\sigma}_i\cdot\vec{\sigma}_j|\Psi^{sf}_{[31]_\lambda}\rangle$$

$$-2\langle\Psi^c_{[211]_\rho}|\lambda^C_i\cdot\lambda^C_j|\Psi^c_{[211]_\eta}\rangle\langle\Psi^{sf}_{[31]_\lambda}|\vec{\sigma}_i\cdot\vec{\sigma}_j|\Psi^{sf}_{[31]_\eta}\rangle$$

$$+\langle\Psi^c_{[211]_\eta}|\lambda^C_i\cdot\lambda^C_j|\Psi^c_{[211]_\eta}\rangle\langle\Psi^{sf}_{[31]_\eta}|\vec{\sigma}_i\cdot\vec{\sigma}_j|\Psi^{sf}_{[31]_\eta}\rangle\} \tag{3.65}$$

The mass shift for Goldstone-boson exchange is

$$\begin{aligned}\Delta m_{GBE}(q^4\overline{q}) &= \langle\Psi(q^4\overline{q})|H_{GBE}(q^4\overline{q})|\Psi(q^4\overline{q})\rangle \\ &= -\frac{1}{3}\frac{C_{GBE}}{m_i m_j}\{\langle\Psi^{sf}_{[31]_\lambda}|\lambda^F_i\cdot\lambda^F_j\vec{\sigma}_i\cdot\vec{\sigma}_j|\Psi^{sf}_{[31]_\lambda}\rangle \\ &\quad +\langle\Psi^{sf}_{[31]_\rho}|\lambda^F_i\cdot\lambda^F_j\vec{\sigma}_i\cdot\vec{\sigma}_j|\Psi^{sf}_{[31]_\rho}\rangle \\ &\quad +\langle\Psi^{sf}_{[31]_\eta}|\lambda^F_i\cdot\lambda^F_j\vec{\sigma}_i\cdot\vec{\sigma}_j|\Psi^{sf}_{[31]_\eta}\rangle\}\end{aligned} \tag{3.66}$$

There exist seven possible spin-flavor configurations of $[31]_{FS}$ as shown in Table 2.2. They are $[31]_F[22]_S, [31]_F[31]_S, [31]_F[4]_S, [211]_F[22]_S, [211]_F[31]_S, [22]_F[31]_S, [4]_F[31]_S$. The explicit forms of the wave functions for all configurations are expressed in Table 3.2. However, for non-strange pentaquark state, $[211]_F$ configuration is impossible in language of group theory because we have only u and d flavor which we can not fill Young tabloid [211]

Table 3.2 Spin-flavor wave functions of [31].

Configuration	Wave function
$[4]_F[31]_S$	$\Psi^{sf}_{[31]_\lambda} = \Psi^{f}_{[4]_s}\Psi^{s}_{[31]_\lambda}$
	$\Psi^{sf}_{[31]_\rho} = \Psi^{f}_{[4]_s}\Psi^{s}_{[31]_\rho}$
	$\Psi^{sf}_{[31]_\eta} = \Psi^{f}_{[4]_s}\Psi^{s}_{[31]_\eta}$
$[31]_F[4]_S$	$\Psi^{f}_{[31]_\lambda} = \Psi^{f}_{[31]_\lambda}\Psi^{s}_{[4]_s}$
	$\Psi^{sf}_{[31]_\rho} = \Psi^{f}_{[31]_\rho}\Psi^{s}_{[4]_s}$
	$\Psi^{sf}_{[31]_\eta} = \Psi^{f}_{[31]_\eta}\Psi^{s}_{[4]_s}$
$[31]_F[31]_S$	$\Psi^{sf}_{[31]_\lambda} = -\frac{1}{\sqrt{3}}[\Psi^{f}_{[31]_\lambda}\Psi^{s}_{[31]_\lambda} - \Psi^{f}_{[31]_\rho}\Psi^{s}_{[31]_\rho} - \frac{1}{\sqrt{2}}\Psi^{f}_{[31]_\eta}\Psi^{s}_{[31]_\lambda} - \frac{1}{\sqrt{2}}\Psi^{f}_{[31]_\lambda}\Psi^{s}_{[31]_\eta}]$
	$\Psi^{sf}_{[31]_\rho} = \frac{1}{\sqrt{3}}[\Psi^{f}_{[31]_\rho}\Psi^{s}_{[31]_\lambda} + \Psi^{f}_{[31]_\lambda}\Psi^{s}_{[31]_\rho} + \frac{1}{\sqrt{2}}\Psi^{f}_{[31]_\eta}\Psi^{s}_{[31]_\rho} + \frac{1}{\sqrt{2}}\Psi^{f}_{[31]_\rho}\Psi^{s}_{[31]_\eta}]$
	$\Psi^{sf}_{[31]_\eta} = \frac{1}{\sqrt{6}}[\Psi^{f}_{[31]_\lambda}\Psi^{s}_{[31]_\lambda} + \Psi^{f}_{[31]_\rho}\Psi^{s}_{[31]_\rho} - 2\Psi^{f}_{[31]_\eta}\Psi^{s}_{[31]_\eta}]$
$[22]_F[31]_S$	$\Psi^{sf}_{[31]_\lambda} = -\frac{1}{2}\Psi^{f}_{[22]_\rho}\Psi^{s}_{[31]_\rho} + \frac{1}{2}\Psi^{f}_{[22]_\lambda}\Psi^{s}_{[31]_\lambda} + \frac{1}{\sqrt{2}}\Psi^{f}_{[22]_\lambda}\Psi^{s}_{[31]_\eta}$
	$\Psi^{sf}_{[31]_\rho} = -\frac{1}{2}\Psi^{f}_{[22]_\rho}\Psi^{s}_{[31]_\lambda} - \frac{1}{2}\Psi^{f}_{[22]_\lambda}\Psi^{s}_{[31]_\rho} + \frac{1}{\sqrt{2}}\Psi^{f}_{[22]_\rho}\Psi^{s}_{[31]_\eta}$
	$\Psi^{sf}_{[31]_\eta} = \frac{1}{\sqrt{2}}\Psi^{f}_{[22]_\rho}\Psi^{s}_{[31]_\rho} + \frac{1}{\sqrt{2}}\Psi^{f}_{[22]_\lambda}\Psi^{s}_{[31]_\lambda}$
$[31]_F[22]_S$	$\Psi^{sf}_{[31]_\lambda} = -\frac{1}{2}\Psi^{f}_{[31]_\rho}\Psi^{s}_{[22]_\rho} + \frac{1}{2}\Psi^{f}_{[31]_\lambda}\Psi^{s}_{[22]_\lambda} + \frac{1}{\sqrt{2}}\Psi^{f}_{[31]_\eta}\Psi^{s}_{[22]_\lambda}$

Table 3.2 Spin-flavor wave functions of [31] (Continued).

Configuration	Wave function
	$\Psi^{sf}_{[31]_\rho} = -\frac{1}{2}\Psi^{f}_{[31]_\rho}\Psi^{s}_{[22]_\lambda} - \frac{1}{2}\Psi^{f}_{[31]_\lambda}\Psi^{s}_{[22]_\rho} + \frac{1}{\sqrt{2}}\Psi^{f}_{[31]_\eta}\Psi^{s}_{[22]_\rho}$
	$\Psi^{sf}_{[31]_\eta} = \frac{1}{\sqrt{2}}\Psi^{f}_{[31]_\rho}\Psi^{s}_{[22]_\rho} + \frac{1}{\sqrt{2}}\Psi^{f}_{[31]_\lambda}\Psi^{s}_{[22]_\lambda}$

We now show one example of how to evaluate the mass of pentaquark without strange quark in ground state. Each parts of wave functions can be taken into account by following :

(1) Spatial wave functions

For ground state the spatial wave functions are in $NLM = 000$ state which is just a constant.

(2) Flavor wave functions

Since $C_{GBE} \approx 0$ thereby mass of pentaquark ignores the Goldstone-boson exchange interaction term. Flavor wave functions can be ignore because one-gluon exchange gives no contribution to flavor wave functions.

(3) Color wave functions

The color wave functions of four-quark core from Eq.(2.16) to (2.24) in Chapter 2 are combined with anticolor of antiquark in Eq.(2.25). We get

$$\begin{aligned}
\Psi^c_{[31]_\lambda}(q^4\overline{q}) &= \frac{1}{\sqrt{3}}[\psi^c_{[211]_\lambda}(R)\overline{R} + \psi^c_{[211]_\lambda}(G)\overline{G}\psi^c_{[211]_\lambda}(B)\overline{B}] \\
&= \frac{1}{\sqrt{3}}[\frac{1}{\sqrt{16}}(2|RRGB\rangle - |RGRB\rangle - |GRRB\rangle - 2|RRBG\rangle \\
&\quad +|RBRG\rangle + |BRRG\rangle + |RGBR\rangle + |GRBR\rangle \\
&\quad -|RBGR\rangle - |BRGR\rangle)\overline{R} \\
&\quad +\frac{1}{\sqrt{16}}(|RGGB\rangle + |GRGB\rangle - 2|GGRB\rangle - |RGBG\rangle \\
&\quad -|GRBG\rangle + |GBRG\rangle + |BGRG\rangle + 2|GGBR\rangle \\
&\quad -|GBGR\rangle - |BGGR\rangle)\overline{G} \\
&\quad +\frac{1}{\sqrt{16}}(|RBGB\rangle + |BRGB\rangle - |GBRB\rangle - |BGRB\rangle
\end{aligned}$$

$$-|RBBG\rangle - |BRBG\rangle + 2|BBRG\rangle + |GBBR\rangle$$

$$+|BGBR\rangle - 2|BBGR\rangle)\overline{B}] \quad (3.67)$$

$$\begin{aligned}
\Psi^c_{[31]_\rho}(q^4\overline{q}) &= \frac{1}{\sqrt{3}}[\psi^c_{[211]_\rho}(R)\overline{R} + \psi^c_{[211]_\rho}(G)\overline{G}\psi^c_{[211]_\rho}(B)\overline{B}] \\
&= \frac{1}{\sqrt{3}}[\frac{1}{\sqrt{48}}(3|RGRB\rangle - 3|GRRB\rangle - 3|RBRG\rangle + 3|BRRG\rangle \\
&\quad -|RGBR\rangle + |GRBR\rangle + |RBGR\rangle - |BRGR\rangle \\
&\quad +2|GBRR\rangle - 2|BGRR\rangle)\overline{R} \\
&\quad \frac{1}{\sqrt{48}}(3|RGGB\rangle - 3|GRGB\rangle - |RGBG\rangle + |GRBG\rangle \\
&\quad -2|RBGG\rangle + 2|BRGG\rangle - |GBRG\rangle + |BGRG\rangle \\
&\quad +3|GBGR\rangle - 3|BGGR\rangle)\overline{G} \\
&\quad +\frac{1}{\sqrt{48}}(2|RGBB\rangle - 2|GRBB\rangle + |RBGB\rangle - |BRGB\rangle \\
&\quad -|GBRB\rangle + |BGRB\rangle - 3|RBBG\rangle + 3|BRBG\rangle \\
&\quad +3|GBBR\rangle - 3|BGBR\rangle)\overline{B}]
\end{aligned} \quad (3.68)$$

$$\begin{aligned}
\Psi^c_{[31]_\eta}(q^4\overline{q}) &= \frac{1}{\sqrt{3}}[\psi^c_{[211]_\eta}(R)\overline{R} + \psi^c_{[211]_\eta}(G)\overline{G}\psi^c_{[211]_\eta}(B)\overline{B}] \\
&= \frac{1}{\sqrt{3}}[\frac{1}{\sqrt{6}}(|BRGR\rangle + |RGBR\rangle + |GBRR\rangle - |RBGR\rangle \\
&\quad -|GRBR\rangle - |BGRR\rangle)\overline{R} \\
&\quad \frac{1}{\sqrt{6}}(|RGBG\rangle - |GRBG\rangle - |RBGG\rangle + |BRGG\rangle
\end{aligned}$$

$$+|GBRG\rangle - |BGRG\rangle)\overline{G}$$

$$\frac{1}{\sqrt{6}}(|RGBB\rangle - |GRBB\rangle - |RBGB\rangle + |BRGB\rangle$$

$$+|GBRB\rangle - |BGRB\rangle)\overline{B}] \tag{3.69}$$

(4) Spin wave functions

Taking the advantage of the four-quark wave functions Eq.(2.26) to (2.36) in Chapter 2 coupling with spin of antiquark we can get the spin wave functions of pentaquark. Clebsch-Gordan coefficients are determined in this process.

$$\Psi^s(q^4\overline{q})_{[4]_s}(\frac{5}{2},\frac{5}{2}) = |\uparrow\uparrow\uparrow\uparrow\bar{\uparrow}\rangle \tag{3.70}$$

$$\Psi^s(q^4\overline{q})_{[4]_s}(\frac{5}{2},\frac{3}{2}) = \sqrt{\frac{1}{5}}(|\uparrow\uparrow\uparrow\uparrow\bar{\downarrow}\rangle + |\uparrow\uparrow\uparrow\downarrow\bar{\uparrow}\rangle + |\uparrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle + |\uparrow\downarrow\uparrow\uparrow\bar{\uparrow}\rangle$$

$$+|\downarrow\uparrow\uparrow\uparrow\bar{\uparrow}\rangle) \tag{3.71}$$

$$\Psi^s(q^4\overline{q})_{[4]_s}(\frac{5}{2},\frac{1}{2}) = \sqrt{\frac{1}{10}}(|\uparrow\uparrow\uparrow\downarrow\bar{\downarrow}\rangle + |\uparrow\uparrow\downarrow\uparrow\bar{\downarrow}\rangle + |\uparrow\downarrow\uparrow\uparrow\bar{\downarrow}\rangle + |\downarrow\uparrow\uparrow\uparrow\bar{\downarrow}\rangle$$

$$|\uparrow\uparrow\downarrow\downarrow\bar{\uparrow}\rangle + |\uparrow\downarrow\uparrow\downarrow\bar{\uparrow}\rangle + |\downarrow\uparrow\uparrow\downarrow\bar{\uparrow}\rangle + |\uparrow\downarrow\downarrow\uparrow\bar{\uparrow}\rangle$$

$$|\downarrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle + |\downarrow\downarrow\uparrow\uparrow\bar{\uparrow}\rangle) \tag{3.72}$$

$$\Psi^s(q^4\overline{q})_{[4]_s}(\frac{3}{2},\frac{3}{2}) = \sqrt{\frac{4}{5}}|\uparrow\uparrow\uparrow\uparrow\bar{\downarrow}\rangle - \frac{1}{20}(+|\uparrow\uparrow\uparrow\downarrow\bar{\uparrow}\rangle + |\uparrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle + |\uparrow\downarrow\uparrow\uparrow\bar{\uparrow}\rangle$$

$$+|\downarrow\uparrow\uparrow\uparrow\bar{\uparrow}\rangle) \tag{3.73}$$

$$\Psi^s(q^4\overline{q})_{[4]_s}(\frac{3}{2},\frac{1}{2}) = \sqrt{\frac{3}{20}}(|\uparrow\uparrow\uparrow\downarrow\bar{\downarrow}\rangle + |\uparrow\uparrow\downarrow\uparrow\bar{\downarrow}\rangle + |\uparrow\downarrow\uparrow\uparrow\bar{\downarrow}\rangle + |\downarrow\uparrow\uparrow\uparrow\bar{\downarrow}\rangle)$$

$$-\sqrt{\frac{1}{15}}(|\uparrow\uparrow\downarrow\downarrow\bar{\uparrow}\rangle+|\uparrow\downarrow\uparrow\downarrow\bar{\uparrow}\rangle+|\uparrow\downarrow\downarrow\uparrow\bar{\uparrow}\rangle+|\uparrow\downarrow\downarrow\uparrow\bar{\uparrow}\rangle$$
$$+|\downarrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle+|\downarrow\downarrow\uparrow\uparrow\bar{\uparrow}\rangle) \quad (3.74)$$

$$\Psi^{\lambda}(q^4\overline{q})_{[22]_\lambda}(\frac{1}{2},\frac{1}{2}) = 2|\uparrow\uparrow\downarrow\downarrow\bar{\uparrow}\rangle-|\uparrow\downarrow\uparrow\downarrow\bar{\uparrow}\rangle-|\downarrow\uparrow\uparrow\downarrow\bar{\uparrow}\rangle-|\uparrow\downarrow\downarrow\uparrow\bar{\uparrow}\rangle$$
$$-|\downarrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle+2|\downarrow\downarrow\uparrow\uparrow\bar{\uparrow}\rangle \quad (3.75)$$

$$\Psi^{\rho}(q^4\overline{q})_{[22]_\rho}(\frac{1}{2},\frac{1}{2}) = |\uparrow\downarrow\uparrow\downarrow\bar{\uparrow}\rangle-|\downarrow\uparrow\uparrow\downarrow\bar{\uparrow}\rangle-|\uparrow\downarrow\downarrow\uparrow\bar{\uparrow}\rangle+|\downarrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle \quad (3.76)$$

$$\Psi^{\lambda}(q^4\overline{q})_{[31]_\lambda}(\frac{3}{2},\frac{3}{2}) = \sqrt{\frac{1}{6}}(2|\uparrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle-|\uparrow\downarrow\uparrow\uparrow\bar{\uparrow}\rangle-|\downarrow\uparrow\uparrow\uparrow\bar{\uparrow}\rangle) \quad (3.77)$$

$$\Psi^{\rho}(q^4\overline{q})_{[31]_\rho}(\frac{3}{2},\frac{3}{2}) = \sqrt{\frac{1}{2}}(|\uparrow\downarrow\uparrow\uparrow\bar{\uparrow}\rangle-|\downarrow\uparrow\uparrow\uparrow\bar{\uparrow}\rangle) \quad (3.78)$$

$$\Psi^{\eta}(q^4\overline{q})_{[31]_\eta}(\frac{3}{2},\frac{3}{2}) = \sqrt{\frac{1}{12}}(3|\uparrow\uparrow\uparrow\downarrow\bar{\uparrow}\rangle-|\uparrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle-|\uparrow\downarrow\uparrow\uparrow\bar{\uparrow}\rangle$$
$$-|\downarrow\uparrow\uparrow\uparrow\bar{\uparrow}\rangle) \quad (3.79)$$

$$\Psi^{\lambda}(q^4\overline{q})_{[31]_\lambda}(\frac{3}{2},\frac{1}{2}) = \sqrt{\frac{1}{18}}(2|\uparrow\uparrow\downarrow\uparrow\bar{\downarrow}\rangle-|\uparrow\downarrow\uparrow\uparrow\bar{\downarrow}\rangle-|\downarrow\uparrow\uparrow\uparrow\bar{\downarrow}\rangle)$$
$$+\sqrt{\frac{1}{18}}(2|\uparrow\uparrow\downarrow\downarrow\bar{\uparrow}\rangle-|\uparrow\downarrow\uparrow\downarrow\bar{\uparrow}\rangle-|\downarrow\uparrow\uparrow\downarrow\bar{\uparrow}\rangle$$
$$+|\uparrow\downarrow\downarrow\uparrow\bar{\uparrow}\rangle+|\downarrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle-2|\downarrow\downarrow\uparrow\uparrow\bar{\uparrow}\rangle) \quad (3.80)$$

$$\Psi^{\rho}(q^4\overline{q})_{[31]_\rho}(\frac{3}{2},\frac{1}{2}) = \sqrt{\frac{1}{6}}(|\uparrow\downarrow\uparrow\uparrow\bar{\downarrow}\rangle-|\downarrow\uparrow\uparrow\uparrow\bar{\downarrow}\rangle)+\sqrt{\frac{1}{6}}(|\uparrow\downarrow\uparrow\downarrow\bar{\uparrow}\rangle$$
$$+|\uparrow\downarrow\downarrow\uparrow\bar{\uparrow}\rangle-|\downarrow\uparrow\uparrow\downarrow\bar{\uparrow}\rangle-|\downarrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle) \quad (3.81)$$

$$\begin{aligned}\Psi^{\eta}(q^4\overline{q})_{[31]_\eta}(\frac{3}{2},\frac{1}{2}) &= \sqrt{\frac{1}{36}}(3|\uparrow\downarrow\uparrow\uparrow\bar{\downarrow}\rangle - |\downarrow\uparrow\uparrow\uparrow\bar{\downarrow}\rangle - |\uparrow\downarrow\uparrow\uparrow\bar{\downarrow}\rangle - |\downarrow\uparrow\uparrow\uparrow\bar{\downarrow}\rangle)\\ &+\sqrt{\frac{1}{9}}(|\uparrow\uparrow\downarrow\downarrow\bar{\uparrow}\rangle + |\uparrow\downarrow\uparrow\downarrow\bar{\uparrow}\rangle + |\downarrow\uparrow\uparrow\downarrow\bar{\uparrow}\rangle - |\uparrow\downarrow\downarrow\uparrow\bar{\uparrow}\rangle\\ &-|\downarrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle - |\downarrow\downarrow\uparrow\uparrow\bar{\uparrow}\rangle) \end{aligned} \tag{3.82}$$

$$\begin{aligned}\Psi^{\lambda}(q^4\overline{q})_{[31]_\lambda}(\frac{3}{2},\frac{1}{2}) &= \sqrt{\frac{1}{9}}(2|\uparrow\uparrow\downarrow\uparrow\bar{\downarrow}\rangle - |\uparrow\downarrow\uparrow\uparrow\bar{\downarrow}\rangle - |\downarrow\uparrow\uparrow\uparrow\bar{\downarrow}\rangle)\\ &-\sqrt{\frac{1}{36}}(2|\uparrow\uparrow\downarrow\downarrow\bar{\uparrow}\rangle - |\uparrow\downarrow\uparrow\downarrow\bar{\uparrow}\rangle - |\downarrow\uparrow\uparrow\downarrow\bar{\uparrow}\rangle\\ &+|\uparrow\downarrow\downarrow\uparrow\bar{\uparrow}\rangle + |\downarrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle - 2|\downarrow\downarrow\uparrow\uparrow\bar{\uparrow}\rangle) \end{aligned} \tag{3.83}$$

$$\begin{aligned}\Psi^{\rho}(q^4\overline{q})_{[31]_\rho}(\frac{3}{2},\frac{1}{2}) &= \sqrt{\frac{1}{3}}(|\uparrow\downarrow\uparrow\uparrow\bar{\downarrow}\rangle - |\downarrow\uparrow\uparrow\uparrow\bar{\downarrow}\rangle) - \sqrt{\frac{1}{12}}(|\uparrow\downarrow\uparrow\downarrow\bar{\uparrow}\rangle\\ &+|\uparrow\downarrow\downarrow\uparrow\bar{\uparrow}\rangle - |\downarrow\uparrow\uparrow\downarrow\bar{\uparrow}\rangle - |\downarrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle) \end{aligned} \tag{3.84}$$

$$\begin{aligned}\Psi^{\eta}(q^4\overline{q})_{[31]_\eta}(\frac{3}{2},\frac{1}{2}) &= \sqrt{\frac{1}{18}}(3|\uparrow\downarrow\uparrow\uparrow\bar{\downarrow}\rangle - |\downarrow\uparrow\uparrow\uparrow\bar{\downarrow}\rangle - |\uparrow\downarrow\uparrow\uparrow\bar{\downarrow}\rangle - |\downarrow\uparrow\uparrow\uparrow\bar{\downarrow}\rangle)\\ &-\sqrt{\frac{1}{18}}(|\uparrow\uparrow\downarrow\downarrow\bar{\uparrow}\rangle + |\uparrow\downarrow\uparrow\downarrow\bar{\uparrow}\rangle + |\downarrow\uparrow\uparrow\downarrow\bar{\uparrow}\rangle - |\uparrow\downarrow\downarrow\uparrow\bar{\uparrow}\rangle\\ &-|\downarrow\uparrow\downarrow\uparrow\bar{\uparrow}\rangle - |\downarrow\downarrow\uparrow\uparrow\bar{\uparrow}\rangle) \end{aligned} \tag{3.85}$$

Now we calculate, as an example, the mass of the pentaquark ($M_{q^4\bar{q}}$) in the $[4]_F[31]_S$ configuration with the total angular momentum (J) equal to $\frac{3}{2}$ in the ground state ($N = 0$) by taking the advantage of Eq.(3.16)

$$M_{q^4\bar{q}}([4]_F[31]_S, J = \frac{3}{2}) = 5m_u + \Delta m_{OGE} \tag{3.86}$$

where Δm_{OGE} is as shown in Eq.(3.65) and calculated by evaluating the components,

$$\langle \Psi^C_{[211]_\lambda} | \lambda^C_1 \cdot \lambda^C_2 | \Psi^C_{[211]_\lambda} \rangle$$

$$\begin{aligned}
= \; & \frac{1}{48} \sum_{i<j} \langle 2RRGB\bar{R} - RGRB\bar{R} - GRRB\bar{R} - 2RRBG\bar{R} + RBRG\bar{R} \\
& + BRRG\bar{R} + RGBR\bar{R} + GRBR\bar{R} - RBGR\bar{R} - BRGR\bar{R} \\
& + RGGB\bar{G} + GRGB\bar{G} - 2GGRB\bar{G} - RGBG\bar{G} - GRBG\bar{G} \\
& + GBRG\bar{G} + BGRG\bar{G} + 2GGBR\bar{G} - GBGR\bar{G} - BGGR\bar{G} \\
& + RBGB\bar{B} + BRGB\bar{B} - GBRB\bar{B} - BGRB\bar{B} - RBBG\bar{B} \\
& - BRBG\bar{B} + 2BBRG\bar{B} + GBBR\bar{B} + BGBR\bar{B} - 2BBGR\bar{B} | \\
& \lambda^C_1 \cdot \lambda^C_2 | 2RRGB\bar{R} - RGRB\bar{R} - GRRB\bar{R} - 2RRBG\bar{R} + RBRG\bar{R} \\
& + BRRG\bar{R} + RGBR\bar{R} + GRBR\bar{R} - RBGR\bar{R} - BRGR\bar{R} \\
& + RGGB\bar{G} + GRGB\bar{G} - 2GGRB\bar{G} - RGBG\bar{G} - GRBG\bar{G} \\
& + GBRG\bar{G} + BGRG\bar{G} + 2GGBR\bar{G} - GBGR\bar{G} - BGGR\bar{G} \\
& + RBGB\bar{B} + BRGB\bar{B} - GBRB\bar{B} - BGRB\bar{B} - RBBG\bar{B} \\
& - BRBG\bar{B} + 2BBRG\bar{B} + GBBR\bar{B} + BGBR\bar{B} - 2BBGR\bar{B} \rangle \\
= \; & 8[\lambda_{11} \cdot \lambda_{11} + \lambda_{22} \cdot \lambda_{22} + \lambda_{33} \cdot \lambda_{33} + \lambda_{11} \cdot \lambda_{22} + \lambda_{12} \cdot \lambda_{21} + \lambda_{11} \cdot \lambda_{33} \\
& + \lambda_{13} \cdot \lambda_{31} + \lambda_{22} \cdot \lambda_{33} + \lambda_{23} \cdot \lambda_{32} + \lambda_{11} \cdot \lambda_{33} + \lambda_{13} \cdot \lambda_{31}] \\
= \; & \frac{4}{3}
\end{aligned} \tag{3.87}$$

Table 3.3 Pentaquark masses: The fourth column shows our estimations while the fifth column gives the results from (Helminen and Riska, 2002).

Configuration	J	Δm_{OGE}	$M(q^4\bar{q})$	$M(q^4\bar{q})$
$[4]_F[31]_S$	$\frac{1}{2},\frac{3}{2}$	$481, 219$	$2281, 2019$	$1781, -$
$[31]_F[4]_S$	$\frac{3}{2},\frac{5}{2}$	$0, 219$	$1800, 2019$	$1725, -$
$[31]_F[31]_S$	$\frac{1}{2},\frac{3}{2}$	$87, 22$	$1887, 1822$	$1613, -$
$[22]_F[31]_S$	$\frac{1}{2},\frac{3}{2}$	$-306, 22$	$1494, 1822$	$1529, -$
$[31]_F[22]_S$	$\frac{1}{2}$	0	1800	1557

$$\begin{aligned}
\langle\Psi^{sf}_{[31]_\lambda}|\vec{\sigma_i}\cdot\vec{\sigma_i}|\Psi^{sf}_{[31]_\lambda}\rangle &= \langle\Psi^{sf}_{[31]_\lambda}|\vec{\sigma_1}\cdot\vec{\sigma_2}|\Psi^{sf}_{[31]_\lambda}\rangle \\
&= \langle\Psi^{f}_{[4]_s}\Psi^{s}_{[31]_\lambda}|\vec{\sigma_1}\cdot\vec{\sigma_2}|\Psi^{f}_{[4]_s}\Psi^{s}_{[31]_\lambda}\rangle \\
&= \langle\Psi^{f}_{[4]_s}|\Psi^{f}_{[4]_s}\rangle\langle\Psi^{s}_{[31]_\lambda}|\vec{\sigma_1}\cdot\vec{\sigma_2}|\Psi^{s}_{[31]_\lambda}\rangle \\
&= \frac{1}{6}\langle 2\uparrow\uparrow\downarrow\uparrow\bar{\uparrow}-\uparrow\downarrow\uparrow\uparrow\bar{\uparrow}-\downarrow\uparrow\uparrow\uparrow\bar{\uparrow}|\vec{\sigma_1}\cdot\vec{\sigma_2}|2\uparrow\uparrow\downarrow\uparrow\bar{\uparrow}-\uparrow\downarrow\uparrow\uparrow\bar{\uparrow}-\downarrow\uparrow\uparrow\uparrow\bar{\uparrow}\rangle \\
&= \frac{1}{6}[4\vec{\sigma_{11}}\cdot\vec{\sigma_{11}}+\vec{\sigma_{11}}\cdot\vec{\sigma_{22}}+\vec{\sigma_{12}}\cdot\vec{\sigma_{21}}+\vec{\sigma_{22}}\cdot\vec{\sigma_{11}}+\vec{\sigma_{21}}\cdot\vec{\sigma_{12}}] \\
&= -3 \qquad (3.88)
\end{aligned}$$

Calculating every cell elements in Eq.(3.65) and inputting the parameters $m_u = 360$ and $C_{OGE} = 16.4$ we obtain the pentaquark mass,

$$\Delta m_{OGE} = \frac{40}{3}\frac{C_{OGE}}{m_u^2} = 219 \qquad (3.89)$$

$$M_{q^4\bar{q}}\left([4]_F[31]_S, J = \frac{3}{2}\right) = 2019 \tag{3.90}$$

The same process is applied to other spin-flavor configurations: $[31]_F[4]_S, [31]_F[31]_S, [22]_F[31]_S, [31]_F[22]_S$. The theoretical results of the mass shifts and pentaquark masses are shown in Table 3.3. The calculated non-strange pentaquark masses are compared with some other pentaquark models.

CHAPTER IV

CONCLUSIONS AND DISCUSSIONS

We try to pave the way for studying the baryon spectrum in the assumption that baryons may consist of sizable pentaquark parts in addition to the three quark components. The construction of pentaquark wave functions, especially for higher excited states has been a challenge in hadron physics. In this work we have systematically constructed pentaquark wave functions in the framework of Yamanouchi basis. It is expected that the easy approach developed in the work for constructing multiquark systems would be very helpful to the whole physics society.

The method may be outlined as follows:

1. Write the total or subtotal wave function of $q^4\bar{q}$ systems into a linear combination of the products of color, spin, flavor and spatial wave functions;
2. Determine all possible configurations by applying the representations of the permutation group S_4 to the general form above;
3. Nail down the expansion coefficients for a certain configuration by applying the representations of S_4;
4. Work out the explicit form of individual color, spin, flavor wave functions by operating the corresponding projection operators to the principle terms of state configurations;
5. Combine the q^4 wave function with the one of antiquark to form the total wave function of pentaquarks.

We have worked out all the possible spin-flavor-spatial configurations of pentaquarks and all the possible spin-flavor configurations of ground state pentaquarks, and the corresponding wave functions.

The construction of spatial wave functions is much more complicated subject to the dynamics. In this work we show how to construct the pentaquark spatial wave functions with various symmetries to higher excited states, where the interaction between quarks takes the form of harmonic oscillators. In our knowledge, this is the first work to systematically construct multiquark spatial wave functions to higher excited states. The spatial wave functions constructed in the work in the harmonic oscillator interaction may serve as a basis to study baryons for more realistic interactions.

We have estimated the model parameters by calculating the mass of the ground state baryons and the lightest vector mesons, resulting in the one-gluon exchange interaction coefficient $C_{OGE} \approx 16.4$, the Goldstone boson exchange interaction coefficient $C_{GBE} \approx 0$ and the mass of $u(d)$ quark $m_{u(d)} \approx 360$ MeV. It is interesting to note that the Goldstone boson exchange interaction gives a very small contribution and can be ignored. One may conclude that baryon and meson masses take a contribution from color wave functions because GBE interaction is independent of but OGE depend strongly on color. We compare our parameters with other theoretical works. One study (Donald, 2000) shows that $m_{u(d)} = 363$ MeV and $K = 50$ or in our form $C_{OGE} = \frac{3K}{8} = 18.75$ which is a bit higher than our prediction. Another theoretical work (Helminen and Riska, 2002) studies merely GBE interaction with OGE abandoned, showing $C_{GBE} = 21$. We have turned off the OGE interaction and refit our parameters and found that $C_{GBE} = 11$. The discrepancy of C_{GBE} between out result and the parameter in (Helminen and Riska, 2002) should be investigated further.

The masses of ground state pentaquarks are evaluated with all the five possible spin-flavor configurations $[4]_F[31]_S$,$[31]_F[4]_S$,$[31]_F[31]_S$,$[22]_F[31]_S$, and $[31]_F[22]_S$. The lowest ground state pentaquark mass is 1494 Mev in the $[22]_F[31]_S$ configuration with the total angular momentum $J = \frac{1}{2}$ whereas the largest ground state pentaquark mass is 2281 MeV in the $[4]_F[31]_S$ with $J = \frac{1}{2}$. As the estimated pentaquark masses range from about 1.5 to 2.3 GeV, covering a large region of baryon masses, baryons may consist of three-quark parts as well as pentaquark components.

There are some theoretical studies estimating the mass of non-strange pentaquarks. We may compare our results with three studies. The spectroscopy of pentaquark states is studied (Bijker et al., 2004) by using mass operator $M = M_0 + M_{orb} + M_{sf}$, where M_0 is a constant, M_{orb} describes the contribution of the spatial degree of freedom, and M_{sf} contains the spin-flavor dependance assumed to have a generalized Gursey-Radicati form which has a similar structure as the GBE interaction. The way to construct the pentaquark wave functions in that work is similar to ours. We found that our prediction is consistent with the results in (Bijker et al., 2004). While our pentaquark masses range from 1.5 to 2.3 GeV, the results in the work (Bijker et al., 2004) shows that the mass of the spin-$\frac{1}{2}$ pentaquark is 2.2 GeV and the mass of the spin-$\frac{3}{2}$ one is 2.3 GeV.

We compare our results with the pentaquark spectrum in the string dynamics model (Narodetskii et al., 2003), where the pentaquark is assumed to be a state of 2 diquarks and 1 antiquark connected by 7 strings. In other words, pentaquark is treated as three-body system i.e. diquark-diquark-antiquark. Therefore, the wave functions are expressed in terms of λ and ρ Jacobi coordinates whereas our work uses λ, ρ, η and ξ to describe five-body systems. For the mass estimation, the Hamiltonian in (Narodetskii et al., 2003) includes kinetic energy, perturbative one-gluon exchange potential and string potential which is proportional to the total length of the string. The physical mass of pentaquark is given by $M = M^0 + \sum C$ where M^0 takes a contribution from the constituent mass and the mass shift from OGE interaction. The constant C has the meaning of constituent self energy expressed in terms of string tension and spin interaction with the vacuum background fields. The work (Narodetskii et al., 2003) predicts the non-strange pentaquark mass in the $uudd\bar{d}$ configuration with spin-$\frac{1}{2}$ equal to 2.4 GeV which is higher than our estimation.

We may also compare our study with the work (Helminen and Riska, 2002), where only the GBE interaction is taken into account. While our work defines that $m' = m + V_0 + \omega$, the work (Helminen and Riska, 2002) calculates m, V_0 and ω separately. The parameters are estimated as $m_{u(d)} = 340$ MeV, $V_0 = -269$ MeV, $C_{GBE} = 21$ and $\omega = 228$ MeV (Helminen and Riska, 2002), and the mass of the spin-$\frac{1}{2}$ pentaquarks are calculated . It is found that the pentaquark masses range from 1.5 to 1.8 GeV as shown in the fifth column in Table 3.3. Almost all the masses in our work are higher than the ones in (Helminen and Riska, 2002) except for the $[22]_F[31]_S$ configuration. This alteration may stem from the difference in the interaction and mass formula.

One may conclude that the prediction of the ground state pentaquark masses in the present work is consistent with other theoretical results where the pentaquark masses range from about 1.5 GeV to 2.4 GeV. In our knowledge, however, this is the first work to determine the parameters of the OGE and GBE interactions together in the naive quark model and then apply the predetermined interactions to pentaquark calculations.

For future works, the ground state wave functions may be applied to calculate other physical quantities, for example, pentaquark magnetic moment, transition amplitude and branching ratio of annihilation processes. We may also predict the mass of strange-pentaquark states and excited pentaquark states. Furthermore, the excited pentaquark wave functions constructed in the work may serve as a powerful basis for studying more realistic potentials.

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APPENDICES

APPENDIX A

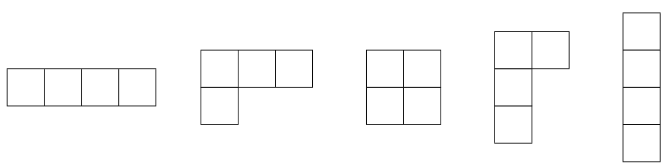
PERMUTATION GROUP

The group theory employed in this thesis is the primary and exquisite approach for determining symmetries of wave functions. Our model assumes that pentaquark $q^4\overline{q}$ consists of one antiquark $\overline{q}$ and four quarks q^4. Since the four quarks are identical particles then their symmetries are significant. S_4 permutation group is engaged in determination of four identical particle symmetries. S_4 permutation group has 4! or 24 members : $e, (12), (13), (14), (23), (24), (34), (123), (132), (124), (142), (134), (143), (234), (243)$ $, (1234), (1243), (1324), (1342), (1423), (1432), (12)(34), (13)(24)$ and $(14)(23)$. The conjugacy class of S_4 which represents the same alpha patterns of any members can be represented by Young tabloids. Young tabloids can be systemically constructed by the following rules :

(1) Number of the box equal to number of n in S_n

(2) Number of the top box and right box always larger than or equal to the below and left box

In S_4 Young tabloids are [4], [31], [22], [211] and [1111].



Each Young tabloids represents a conjugacy class of S_4 and each conjugacy class represents the corresponding irreducible representation of S_4. The number of Young tableaux, filled Young tabloid by number, could represent dimensions of irreducible representation. Young tableaux can be constructed by the following rules :

(1) the number in a box differs from any number in other boxes.

(2) the numbers in a row must increase from left to right.

(3) the numbers in a column must increase from top to bottom.

Therefore,

$$[4] \qquad \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 4 \\ \hline \end{array} \qquad r = 1$$

$$[31] \qquad \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1} \end{array} \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1} \end{array} \quad \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1} \end{array} \qquad r = 3$$

$$[211] \qquad \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 \\ \cline{1-1} 4 \\ \cline{1-1} \end{array} \quad \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 \\ \cline{1-1} 4 \\ \cline{1-1} \end{array} \quad \begin{array}{|c|c|}\hline 1 & 4 \\ \hline 2 \\ \cline{1-1} 3 \\ \cline{1-1} \end{array} \qquad r = 3$$

$$[22] \qquad \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & 4 \\ \hline \end{array} \quad \begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & 4 \\ \hline \end{array} \qquad r = 2$$

$$[1111] \qquad \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 3 \\ \hline 4 \\ \hline \end{array} \qquad r = 1$$

when r is dimension of irreducible representation.

The Yamanouchi basis utilized in this thesis is written in the form

$$\phi^{[\lambda]}_{(r)} = |[\lambda](r_n, r_{n-1}, ..., r_2, r_1)\rangle \tag{A.1}$$

where $[\lambda]$ is young tabloid, r_i stands for the row from which a box is removed in the order of large number to small number.

For example, one Yamanouchi basis of [211] of S_4 is

$$\phi_1 = \begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & \multicolumn{1}{c}{} \\ \cline{1-1} 4 & \multicolumn{1}{c}{} \\ \cline{1-1}\end{array} = |[211](3211)\rangle \tag{A.2}$$

With the aid of group theory, the operation of the element $(n-1, n)$ on the Yamanouchi basis satisfies the followings :

$$(n-1,n)|[\lambda](r,r,r_{n-2},...,r_2,1)\rangle = +|[\lambda](r,r,r_{n-2},...,r_2,1)\rangle \tag{A.3}$$

$$(n-1,n)|[\lambda](r,r-1,r_{n-2},...,r_2,1)\rangle = -|[\lambda](r,r-1,r_{n-2},...,r_2,1)\rangle \tag{A.4}$$

when $|[\lambda](r-1,r,r_{n-2},...,r_2,1)\rangle$ not exists, and

$$\begin{aligned}(n-1,n)|[\lambda](r,s,r_{n-2},...,r_2,r_1)\rangle &= \sqrt{1-\sigma_{rs}^2}|[\lambda](s,r,r_{n-2},...,r_2,r_1)\rangle \\ &\quad +\sigma_{rs}|[\lambda](r,s,r_{n-2},...,r_2,r_1)\rangle \end{aligned}\tag{A.5}$$

when $|[\lambda](r,s,r_{n-2},...,r_2,r_1)\rangle$ and $|[\lambda](s,r,r_{n-2},...,r_2,r_1)\rangle$ all exist and $r \neq s$. For $[\lambda] = [\lambda_1, \lambda_2, ..., \lambda_r ... \lambda_s ... \lambda_n]$ we have

$$\sigma_{rs} = \frac{1}{(\lambda_r - r) - (\lambda_s - s)} \tag{A.6}$$

For other elements there is additional formula from group theory

$$(i,n) = (n-1,n)(i,n-1)(n-1,n) \tag{A.7}$$

Finally we can derive matrix representation of all permutation element. The results are

(1) Matrix representation of $S_4[211]$

$$D^{[211]}(12) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \qquad D^{[211]}(13) = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$D^{[211]}(34) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -\frac{1}{3} & \frac{2\sqrt{2}}{3} \\ 0 & \frac{2\sqrt{2}}{3} & \frac{1}{3} \end{pmatrix} \quad D^{[211]}(23) = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

(2) Matrix representation of $S_4[31]$

$$D^{[31]}(12) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad D^{[31]}(13) = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$D^{[31]}(34) = \begin{pmatrix} \frac{1}{3} & 0 & \frac{2\sqrt{2}}{3} \\ 0 & 1 & 0 \\ \frac{2\sqrt{2}}{3} & 0 & -\frac{1}{3} \end{pmatrix} \quad D^{[31]}(23) = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(3) Matrix representation of $S_4[22]$

$$D^{[22]}(12) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad D^{[22]}(13) = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

$$D^{[22]}(34) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad D^{[22]}(23) = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

At this stage, group theory provides enough material to investigate the symmetries of wave functions.

APPENDIX B

WEYL TABLEAUX

In this thesis we use Weyl tableaux as the means to investigate the principal term which is the most crucial step for calculating wave functions. Resembling to Young tableaux, Weyl tableaux is the young tabloid filled with the possible state instead of number. The following rules are the method to obtain the principal term :

(1) State in the same column can not be the same

(2) State in the same row can be the same and be in the order, for example, u, d, s or r, g, b or $\uparrow, \downarrow$

(3) Read the state in order of the number in Young tableaux.

For instance, the principal terms of uud configuration are

$$\Psi^{f}_{[21]_\lambda} = |\begin{array}{|c|c|}\hline 1 & 2 \\ \hline 3 & \\ \cline{1-1} \end{array}, \begin{array}{|c|c|}\hline u & u \\ \hline d & \\ \cline{1-1} \end{array}\rangle = |uud\rangle \tag{B.1}$$

$$\Psi^{f}_{[21]_\rho} = |\begin{array}{|c|c|}\hline 1 & 3 \\ \hline 2 & \\ \cline{1-1} \end{array}, \begin{array}{|c|c|}\hline u & u \\ \hline d & \\ \cline{1-1} \end{array}\rangle = |udu\rangle \tag{B.2}$$

The main reason that we use Weyl tableaux is to give the normalization constants consistently or to take the principle term positive sign.

APPENDIX C

PROJECTION OPERATORS

In the frame work of group theory, we introduce the projection operator as

$$P^{[\lambda]}_{(r)} = \sum_i \langle [\lambda](r)|\, R_i \,|[\lambda](r)\rangle \, R_i \tag{C.1}$$

when $P^{[\lambda]}_{(r)}$ is the projection operator, $|[\lambda](r)\rangle$ is the Yamanouchi basis function and R_i is the permutation element of S_n. In Yamanouchi technique the operation of $P_{[X]}$ on any principal term results in the wave function of the irreducible representation $[X]$. For S_4 the projection operators could be written by

(1) Projection operator of [4] configuration.

$$\begin{aligned}
P_{[4]_s} &= \sum_i^{24} \langle [4](1111)|R_i|[4](1111)\rangle \, R_i \\
&= 1 + (12) + (13) + (14) + (23) + (24) + (34) \\
&\quad +(12)(34) + (14)(23) + (13)(24) \\
&\quad +(123) + (124) + (132) + (134) + (142) + (143) + (234) + (243) \\
&\quad +(1234) + (1243) + (1324) + (1342) + (1423) + (1432)
\end{aligned}$$

(2) Projection operator of [1111] configuration.

$$\begin{aligned}
P_{[1111]_A} &= \sum_i^{24} \langle [1111](4321)|R_i|[1111](4321)\rangle \, R_i \\
&= 1 - (12) - (13) - (14) - (23) - (24) - (34)
\end{aligned}$$

$$+(12)(34)+(14)(23)+(13)(24)$$

$$+(123)+(124)+(132)+(134)+(142)+(143)+(234)+(243)$$

$$-(1234)-(1243)-(1324)-(1342)-(1423)-(1432)$$

(3) Projection operators of [31] configuration.

$$\begin{aligned} P_{[31]_\lambda} &= \sum_i^{24} \langle [31](1211)|R_i|[31](1211)\rangle\, R_i \\ &= 6+6(12)-3(13)+5(14)-3(23)+5(24)+2(34) \\ &\quad +2(12)(34)-4(14)(23)-4(13)(24) \\ &\quad -3(123)+5(124)-3(132)-(134)+5(142)-(143)-(234)-(243) \\ &\quad -(1234)-(1243)-4(1324)-(1342)-4(1423)-(1432) \end{aligned}$$

$$\begin{aligned} P_{[31]_\rho} &= \sum_i^{24} \langle [31](1121)|R_i|[31](1121)\rangle\, R_i \\ &= 2-2(12)+(13)+(14)+(23)+(24)+2(34) \\ &\quad +2(12)(34) \\ &\quad -(123)-(124)-(132)+(134)-(142)+(143)+(234)+(243) \\ &\quad -(1234)-(1243)-(1342)-(1432) \end{aligned}$$

$$\begin{aligned} P_{[31]_\eta} &= \sum_i^{24} \langle [31](2111)|R_i|[31](2111)\rangle\, R_i \\ &= 3+3(12)+3(13)-(14)+3(23)-(24)-(34) \end{aligned}$$

$$-(12)(34)-(14)(23)-(13)(24)$$

$$+3(123)-(124)+3(132)-(134)-(142)-(143)-(234)-(243)$$

$$-(1234)-(1243)-(1324)-(1342)-(1423)-(1432)$$

(4) Projection operators of [211] configuration.

$$
\begin{aligned}
P_{[211]_\lambda} &= \sum_i^{24} \langle [211](3211)|R_i|[211](3211)\rangle\, R_i \\
&= 2+2(12)-(13)-(14)-(23)-(24)-2(34) \\
&\quad -2(12)(34) \\
&\quad -(123)-(124)-(132)+(134)-(142)+(143)+(234)+(243) \\
&\quad +(1234)+(1243)+(1342)+(1432)
\end{aligned}
$$

$$
\begin{aligned}
P_{[211]_\rho} &= \sum_i^{24} \langle [211](3121)|R_i|[211](3121)\rangle\, R_i \\
&= 6-6(12)+3(13)-5(14)+3(23)-5(24)-2(34) \\
&\quad -4(14)(23)-4(13)(24)+2(12)(34) \\
&\quad -3(123)+5(124)-3(132)-(134)+5(142)-(143)-(234)-(243) \\
&\quad +(1234)+(1243)+4(1324)+(1342)+4(1423)+(1432)
\end{aligned}
$$

$$
\begin{aligned}
P_{[211]_\eta} &= \sum_i^{24} \langle [211](1321)|R_i|[211](1321)\rangle\, R_i \\
&= 3-3(12)-3(13)+(14)-3(23)+(24)+(34)
\end{aligned}
$$

$$-(12)(34)-(14)(23)-(13)(24)$$

$$+3(123)-(124)+3(132)-(134)-(142)-(143)-(234)-(243)$$

$$+(1234)+(1243)+(1324)+(1342)+(1423)+(1432)$$

(5) Projection operators of [22] configuration.

$$\begin{aligned}
P_{[22]_\lambda} &= \sum_i^{24} \langle [22](2211)|R_i|[22](2211)\rangle\, R_i \\
&= 2+2(12)-(13)-(14)-(23)-(24)+2(34) \\
&\quad +2(12)(34)+2(14)(23)+2(13)(24) \\
&\quad -(123)-(124)-(132)-(134)-(142)-(143)-(234)-(243) \\
&\quad -(1234)-(1243)+2(1324)-(1342)+2(1423)-(1432)
\end{aligned}$$

$$\begin{aligned}
P_{[22]_\rho} &= \sum_i^{24} \langle [22](2121)|R_i|[22](2121)\rangle\, R_i \\
&= 2-2(12)+(13)+(14)+(23)+(24)-2(34) \\
&\quad +2(14)(23)+2(13)(24)+2(12)(34) \\
&\quad -(123)-(124)-(132)-(134)-(142)-(143)-(234)-(243) \\
&\quad +(1234)+(1243)-2(1324)+(1342)-2(1423)+(1432)
\end{aligned}$$

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