On Discrete Hyperbolic Tension Splines

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A hyperbolic tension spline is defined as the solution of a differential multipoint boundary value problem. A discrete hyperbolic tension spline is obtained using the difference analogous of differential operators; its computation does not require exponential functions, even if its continuous extension is still a spline of hyperbolic type. We consider the basic computational aspects and show the main features of this approach.

Keywords: Hyperbolic tension splines, multipoint boundary value problem, discrete hyperbolic tension splines and B-splines, shape preserving interpolation.

1. Introduction

Spline theory is mainly grounded on two approaches: the algebraic one (where splines are understood as smooth piecewise functions, see e.g. [29,31]) and the variational one (where splines are obtained via minimization of quadratic functionals with equality and/or inequality constraints, see e.g. [15]). Although less common, a third approach where splines are defined as the solutions of differential multipoint boundary value problems (DMBVP for short), has been considered, [9]. Even though some of the important classes of splines can be obtained from all three schemes, specific features make sometimes the last one an important tool in practical settings. We want to illustrate this fact by the example of hyperbolic tension splines.

Introduced by Schweikert in 1966, [30], hyperbolic tension splines are solutions of DMBVP where the differential operators depend on *tension* parameters. Their tension properties (that is the possibility of pulling the curve toward a piecewise linear function) have kept hyperbolic splines popular (see for example [11,24,25,27] and references quoted therein) in shape-preserving interpolation and/or approximation. Unfortunately, it is difficult to work with hyperbolic splines for small or large values of the tension parameters. For this