EFFECTS OF THE INTERMEDIATE PRINCIPAL STRESS ON TENSILE STRENGTH OF INTACT ROCKS

Pawinee Masingboon

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Engineering in Geotechnology

Suranaree University of Technology

Academic Year 2012

ผลกระทบของความเค้นหลักกลางต่อค่ากำลังดึงสูงสุดของตัวอย่างหิน

นางสาวภาวิณี มาสิงบุญ

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิศวกรรมศาสตรมหาบัณฑิต

สาขาวิชาเทคโนโลยีธรณี

มหาวิทยาลัยเทคโนโลยีสุรนารี

ปีการศึกษา 2555

EFFECTS OF THE INTERMEDIATE PRINCIPAL STRESS ON TENSILE STRENGTH OF INTACT ROCKS

Suranaree University of Technology has approved this thesis submitted in partial fulfillment of the requirements for a Master's Degree.

Thesis Examining Committee

____

(Assoc. Prof. Kriangkrai Trisarn)

Chairperson

____

(Assoc. Prof. Dr. Kittitep Fuenkajorn)

Member (Thesis Advisor)

____

(Dr. Decho Phueakphum)

Member

____

(Prof. Dr. Sukit Limpijumnong)

Vice Rector for Academic Affairs

____

(Assoc. Prof. Flt. Lt. Dr. Kontorn Chamniprasart)

Dean of Institute of Engineering

ภาวิณี มาสิงบุญ : ผลกระทบของความเค้นหลักกลางต่อค่ากำลังดึงสูงสุดของตัวอย่างหิน (EFFECTS OF THE INTERMEDIATE PRINCIPAL STRESS ON TENSILE STRENGTH OF INTACT ROCKS) อาจารย์ที่ปรึกษา : รองศาสตราจารย์ ดร. กิตติเทพ เฟื่องขจร, 55 หน้า.

วัตถุประสงค์เพื่อหาผลกระทบของความเค้นหลักกลางต่อค่ากำลังดึงของหินทรายชุดภูพาน ชุดพระวิหาร ชุดภูกระดึง และหินอ่อนสระบุรี ซึ่งผลที่ได้นำมาประเมินความสามารถในการคาดคะเนของเกณฑ์การแตกของคูลอมบ์ในกรณีที่มีความเค้นหลักทิศทางเดียวหรือมากกว่าในการรับแรงดึงของหิน การทดสอบในห้องปฏิบัติการจะรวมไปถึงการทดสอบการดัดงอแบบ 4 จุด การทดสอบแรงดึงแบบบราซิลประยุกต์ และการทดสอบการดัดงอในแผ่นกลม สำหรับการทดสอบกำลังรับแรงกดในแกนเดียว กำลังรับแรงกดในสองแกนหรือสามแกนแบบขยาย และการทดสอบกำลังรับแรงกดในสามแกนของตัวอย่างหินทั้งสี่ชนิดจะนำมาสัมพันธ์กับผลการทดสอบกำลังดึงที่ได้ดำเนินการในการศึกษานี้ ผลที่ได้ระบุว่าความเค้นหลักกลางมีผลกระทบกับกำลังรับแรงดึงสูงสุดของตัวอย่างหินซึ่งเป็นจริงกับหินทั้งสี่ชนิด จากเกณฑ์การแตกของคูลอมบ์ส่งผลให้ผลจากการทดสอบการดัดงอแบบ 4 จุด และการทดสอบแรงดึงแบบบราซิลประยุกต์สามารถต่อกันเป็นเส้นตรงแนวเดียวกันกับการทดสอบกำลังกดในสามแกนแบบขยายได้ทั้งนี้เนื่องจากการทดสอบทั้งสามแบบอยู่ภายใต้สภาวะความเค้นเดียวกัน ($\sigma_1 = \sigma_2 \neq \sigma_3$) และการทดสอบการดัดงอในแผ่นกลมจะเข้ากันได้ดีกับการทดสอบกำลังรับแรงกดในแกนเดียวและกำลังรับแรงกดในสามแกน ($\sigma_1 \neq \sigma_2 = \sigma_3$)

ค่ากำลังกดและกำลังดึง และค่าความเค้นยึดติดที่ได้จากการทดสอบในสามแกนแบบขยายจะสูงกว่าผลการทดสอบในสามแกนแบบกด ผลที่ได้จากสภาวะทั้งสองให้ค่ามุมเสียดทานภายในที่ใกล้เคียงกัน ข้อสรุปอีกประการหนึ่งคือค่าความเค้นหลักกลางจะทำให้หินแข็งขึ้นทั้งที่อยู่ในสภาวะกดและสภาวะดึง และที่สำคัญกว่านั้นคือผลการทดสอบกำลังดึงแบบบราซิลเลียนไม่สามารถนำมาสัมพันธ์กับผลการทดสอบที่ได้จากสภาวะความเค้นทั้งสองรูปแบบ การนำกฎการแตกของคูลอมบ์มาประยุกต์ใช้เมื่อหินอยู่ภายใต้สภาวะความเค้นดึงควรใช้ผลที่ได้จากการทดสอบการดัดงอแบบแผ่นกลมแทนที่จะใช้ผลการทดสอบกำลังดึงแบบบราซิลเลียนดังที่นิยมใช้กันในปัจจุบัน

สาขาวิชา เทคโนโลยีธรณี ลายมือชื่อนักศึกษา ____________________

ปีการศึกษา 2555 ลายมือชื่ออาจารย์ที่ปรึกษา______________

PAWINEE MASINGBOON : EFFECTS OF THE INTERMEDIATE PRINCIPAL STRESS ON TENSILE STRENGTH OF INTACT ROCKS. THESIS ADVISOR : ASSOC. PROF. KITTITEP FUENKAJORN, Ph.D., P.E., 55 PP.

TENSION/STRESS/ROCK/TESTING/FAILURE

The objective of this study is to experimentally determine the effects of intermediate principal stresses on the tensile strength of PhuPhan, PhraWihan and PhuKradung sandstones and Saraburi marble. The results are used to assess the predictive capability of the Coulomb criterion when one or more principal stresses are in tension. The laboratory testing involves four-point bending test, Brazilian tests with axial compression, circular plate bending test. Uniaxial, biaxial and triaxial compressive strengths of the four rock types are also determined to correlate their results with those of the tensile testing. The results indicate that the intermediate principal stresses do affect the rock tensile strengths. This holds true for all tested rock types. The four point bending and Brazilian tensile strengths under compression provide a linear transition with the triaxial extension test. This is because they are all under the condition where $\sigma_1 = \sigma_2 \neq \sigma_3$. Based on the Coulomb criterion, the circular plate bending tensile strength can well correlate with the conventional uniaxial and triaxial compressive strengths of the rocks ($\sigma_1 \neq \sigma_2 = \sigma_3$). For all rock types the compressive and tensile strengths and cohesion obtained from the triaxial extension tests are greater than those obtained from the triaxial compression tests. Both stress conditions give similar internal friction angle. This

suggests that σ_2 can strengthen the rock for both compressive and tensile regions. More important the results indicate that the Brazilian tensile strength cannot be correlated with the two stress conditions. It is recommended that an extension of the Coulomb criterion into the tensile region should be correlated with the tensile strength obtained from the circular plate bending test rather than the Brazilian tension test.

School of Geotechnology	Student's Signature________________

Academic Year 2012	Advisor's Signature________________

ACKNOWLEDGMENTS

I wish to acknowledge the funding support of Suranaree University of Technology (SUT).

I would like to express my sincere thanks to Assoc. Prof. Dr. Kittitep Fuenkajorn, thesis advisor, who gave a critical review and constant encouragement throughout the course of this research. Further appreciation is extended to Asst. Prof. Kriangkrai Trisarn : chairman, school of Geotechnology and Dr. Decho Phueakphum, School of Geotechnology, Suranaree University of Technology who are member of my examination committee. Grateful thanks are given to all staffs of Geomechanics Research Unit, Institute of Engineering who supported my work.

Finally, I most gratefully acknowledge my parents and friends for all their supported throughout the period of this research.

Pawinee Masingboon

TABLE OF CONTENTS

Page

TABLE OF CONTENTS (Continued)

Page

TABLE OF CONTENTS (Continued)

Page

LIST OF TABLES

Table **Page**

LIST OF FIGURES

Figure **Page**

LIST OF FIGURES (Continued)

Figure **Page**

SYMBOLS AND ABBREVIATIONS

ϕ	=	Friction angle
$2 \cdot c \cdot \tan(\alpha)$	=	Uniaxial compressive strength of rock
a, c	=	Outside and inside diameters
B	=	Width of the sample
c	=	Cohesion
D	=	Disk diameter
F	=	Maximum values of applied load and moment
J_1	=	The first order of stress invariant
$J_2^{1/2}$	=	The second order of stress invariant
L	=	Disk thickness
L	=	Disk thickness
L/D	=	Length-to-Diameter ratio
M	=	Maximum values of applied load and moment
P	=	Failure load
P_f	=	Failure load
t	=	Thickness of circular rock sample
t	=	Thickness of the sample,
ν	=	Poisson's ratio
σ_1	=	Maximum principal stress
σ_2	=	Intermediate principal stress
σ_3	=	Minimum principal stress

SYMBOLS AND ABBREVIATIONS (Continued)

σ_B	=	Brazilian tensile stress
σ_c	=	Uniaxial compressive strength
σ_n	=	Normal stress
σ_r	=	Radial/tensile stress
σ_x	=	Horizontal stress
σ_y	=	Vertical stress
σ_z	=	Axial stress
τ	=	Shear stress
φ	=	Dipping angle of the foliations
σ_{bend}	=	Tensile stress

CHAPTER I

INTRODUCTION

1.1 Background and rationale

The tensile strength of rock is one of the important parameters for the design of maximum roof span for underground mines and tunnels. The roof beam tensile strength, in many cases, dictates the extraction ratio of the ore and the size of the haulage equipments, and hence affects the economic values of the mine. The Brazilian tension test has long been used to determine the tensile strength from circular disks of rock by applying a line load along the disk diameter until failure. Even though this test method is widely applied and accepted, and results are incorporated into several strength criteria, a question remains on whether they can truly represent the rock tensile strength under in-situ stress states. The sample center, where the tensile crack is initiated, is subjected to biaxial plane stress condition, with the compressive stress in vertical, tensile stress in horizontal, and no stress along the sample axis. The circular thin plates have been performed to improve the tensile region from Coulomb criterion. It includes the effect of intermediate principal stress on the results. Under in-situ condition however, the roof beam is normally subjected to triaxial stress states. The influence of the stress parallel to the incipient crack plane, as normally occurred in the mine roof, has never been studied or quantitatively assessed. This is primarily because a special loading device is required to apply a constant normal stress parallel to the sample axis while the line load is applied.

1.2 Research objectives

The objectives of this study are to determine how the compressive strength criteria can be extended to the tensile strength for intact rocks, and to determine the effect of the intermediate principal stress on rock tensile strength. The effort involves the measurement of the rock tensile strengths under various configurations of the applied principal stresses. Extrapolation of the Coulomb criteria from the compressive to the tensile regions is made from the uniaxial and triaxial compressive strengths of four rock types.

1.3 Research methodology

The research methodology (Figure 1) comprises 5 steps; including literature review, sample collection and preparation, laboratory testing, development of mathematical relations and tensile strength criterion, and discussions and conclusions.

1.3.1 Literature review

Literature review will be carried out to study the previous research on tensile strength test, tensile elastic modulus and tensile strength of rock. The sources of information are from text books, journals, technical reports and conference papers. A summary of the literature review will be given in the thesis.

1.3.2 Sample collection and preparation

Sandstone samples will be prepared for testing. A minimum of 3 sandstone types will be collected and 2 more of marble and limestone. Five samples will be prepared for each type and tested. Sample preparation will be carried out in the laboratory at the Suranaree University of Technology.

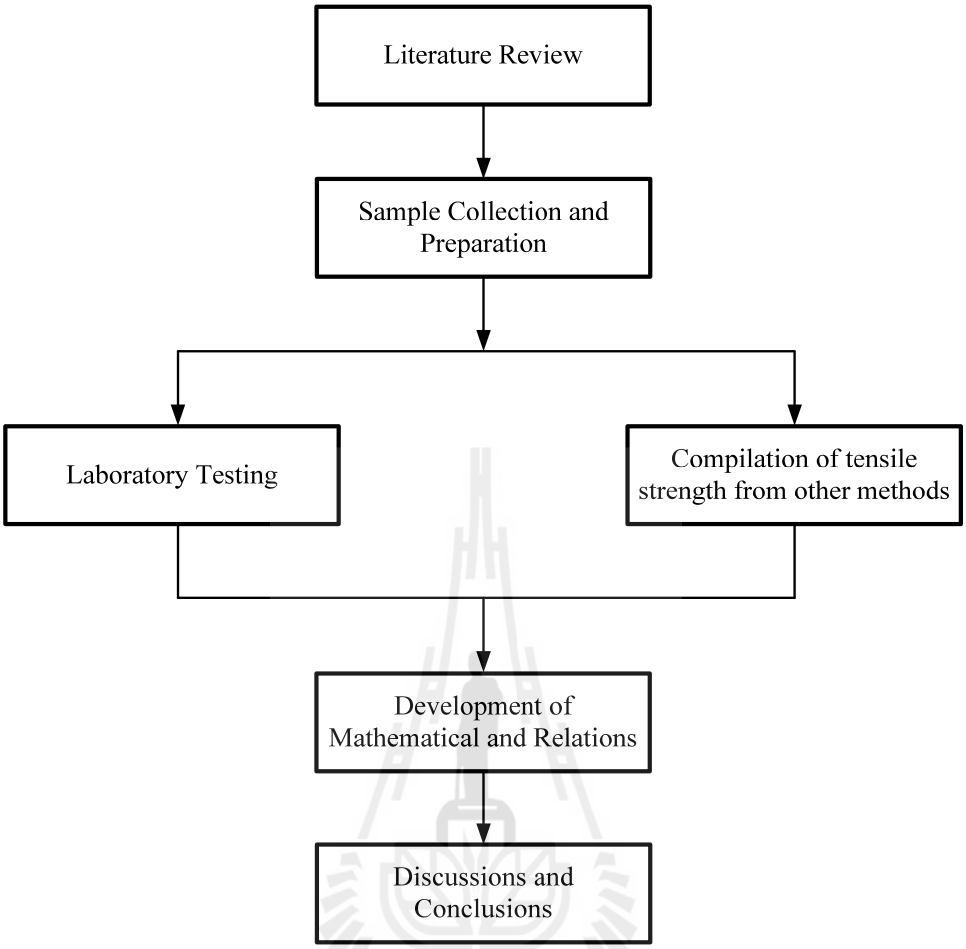


Figure 1 Research methodology

1.3.3 Laboratory testing

Basic characterization tests (included direct tension test) are performed to obtain the tensile strength and the uniaxial and triaxial compressive strengths of the five rock types.

Brazilian tension test under various axial stresses are tested. The specimen is placed in polyaxial load frame which is used to apply axial stresses to the rock. The confining pressures range from 1.4, 3.0, 5.0 to 7.0 MPa. The failure stresses are recorded and mode of failure examined.

Direct tensile strengths are determined from dog-bone shaped specimens of intact rock using a compression-to-tension load converter (CTC). The CTC device will be placed in a compression load frame to apply uniaxial tensile stress at the mid-section of the specimen. The specimens from each rock type will be loaded at a constant rate of 1.0 MPa/s until tensile failure occurred.

Circular plates will be tested with the line load distribution on the top and point load on the bottom with thickness from 0.5 to 1.3 cm, and compared with the basic characterization tests on tensile strength with thin shell theory and to improve the understanding of the transition from compressive to tensile strengths of rock samples.

1.3.3.1 Compilation of tensile strength from other methods.

The results of tensile strength of another method will be compiled to compare with the results of circular plate testing.

1.3.4 Development of mathematical relations

Results from laboratory measurements in terms of intermediate principal stresses and the tensile strength of rock will be used to formulate mathematical relations. Intermediate principal stresses and the applied stresses can be incorporated to the equation, and derive a new failure criterion (modified from Coulomb failure criterion) for rocks under tension.

1.3.5 Conclusions and thesis writing

All research activities, methods, and results will be documented and complied in the thesis. The research or findings will be published in the conference proceedings or journals.

1.4 Scope and limitations

The scope and limitations of the research include as follows.

1. Laboratory experiments will be conducted on five rock types: Phra Wihan (PW) and Phu Phan (PP) sandstones, Phu Kradung (PK) siltstone and Saraburi (SB) marble.

2. Basic characterization tests are performed to obtain the Brazilian tensile strength and the uniaxial and triaxial compressive strengths of the rocks.

3. The tension tests include Brazilian tension test under axial confinement, direct tension test and thin plate under point loading.

4. Up to 20 samples will be tested for each rock type, with a nominal diameter of 10 cm and thickness from 0.5 to 1.3 cm.

5. All tests will be conducted under ambient temperature.

6. Testing will be made under dry condition.

7. No field testing will be performed. scope and limitations of the research include as follows.

1.5 Thesis contents

This research thesis is divided into six chapters. The first chapter includes background and rationale, research objectives, research methodology, and scope and limitations. Chapter II presents results of the literature review to the previous research on tensile strength test, tensile elastic modulus and tensile strength of rock. Chapter III describes sample preparation. Chapter IV basic characterization tests are performed to obtain the tensile strength. Chapter V presents mathematical relations. Chapter VI is the discussions, conclusions and recommendations for future studies.

CHAPTER II

LITERATURE REVIEW

Relevant topics and previous research results will be reviewed to improve an understanding of tensile strength criterion. These include all indirect and direct methods of testing rock tensile strength.

Brazilian tension test is a method for determination of the tensile strength of intact rock. Specifications for the Brazilian tensile strength test have been established by American Society for Testing and Materials (ASTM D3967) and a suggested approach is provided by ISRM (Brown, 1981). The Brazilian tension test (σ_B) can be calculated using the equation (Jaeger and Cook, 1979)

$$\sigma_B = \frac{2P}{\pi DL} \qquad (2.1)$$

where P is the failure load, D is the disk diameter, and L is the disk thickness.

Fuenkajorn and Daemen (1986) conducted ring tension tests on 229 mm (9 in) diameter disks of Grande basaltic andesire and Pomona basalt with various center hole sizes in order to study the relationship between ring tensile strength and relative hole radius (hole radius/disk radius). The tensile strength, σ_R, decreases as the relative hole radius, (mathematical euqation) (available in full paper), increases. A power equation, (mathematical equation) (available in full paper) represent the, coefficients of strength and shape, respectively, adequately represents the ring tensile strength as a function of relative hole radius over the range investigated. This equation can be used to distinguish the effect of the hole size from the strength results, to predict the tensile strength of a ring sample containing arbitrary hole sizes, and to approximate the critical relative hole radius of the material tested.

Claesson and Bohloli (2002) state that the tensile strength of rock is among the most important parameters influencing rock deformability, rock crushing and blasting results. To calculate the tensile strength from the indirect tensile (Brazilian) test, one must know the principal tensile stress, in particular at the rock disc center, where a crack initiates. This stress can be assessed by an analytical solution. A study of this solution for anisotropic (transversely isotropic) rock is presented.

Haimson (2006) studies the effect of the intermediate principal stress (σ_2) on brittle fracture of rocks, and on their compressive strength criteria. Testing equipment emulating Mogi's but considerably more compact was developed at the University of Wisconsin and used for true triaxial testing of some very strong crystalline rocks. Test results revealed three distinct compressive failure mechanisms, depending on loading mode and rock type: shear faulting resulting from extensile microcrack localization, multiple splitting along the σ_1 axis, and nondilatant shear failure. The true triaxial strength criterion for the KTB amphibolite derived from such tests was used in conjunction with logged breakout dimensions to estimate the maximum horizontal in situ stress in the KTB ultra deep scientific hole.

Liao et al. (1997) have studied the tensile behavior of a transversely isotropic rock by a series of direct tensile tests on cylindrical argillite specimens. To study the deformability of argillite under tension, two components of an electrically resistant type of strain gage with a parallel arrangement, or a semiconductor strain gage, are adopted for measuring the small transverse strain observed on specimens during testing. The curves of axial stress and axial strain and average volumetric strain are presented for argillite specimens with differently inclined angles of foliation. Experimental results indicate that the stress-strain behavior depends on the foliation inclination of specimens with respect to the loading direction. The five elastic constants of argillite are calculated by measuring two cylindrical specimens. Based on theoretical analysis results, the range of the foliation inclination of the specimens tested is investigated for feasibility obtaining the five elastic moduli. A dipping angle of the foliations (φ) of 30-60° with respect to the plane normal to the loading direction is recommended. The final failure modes of the specimens are investigated in detail. A sawtoothed failure plane occurs for the specimens with a high inclination of foliation with respect to the plane perpendicular to the loading direction. On the other hand, a smooth plane occurs along the foliation for specimens with low inclination of foliation with respect to the plane normal to the loading direction. A conceptual failure criterion of tensile strength is proposed for specimens with a high inclination of foliation.

Tepnarong (2001) proposes that a modified point load (MPL) testing technique can be correlated the results with the uniaxial compressive strength (UCS) and tensile strength of intact rock. The primary objective is to develop an inexpensive and reliable rock testing method for use in the field and in the laboratory. The test apparatus is similar to that of the conventional point load (CPL), except that the loading points are cut flat to have a circular cross-section area instead of using a half-spherical shape. To derive a new solution, finite element analyses and laboratory experiments have been carried out. The simulation results suggest that the applied stress required failing the MPL specimen increases logarithmically as the specimen thickness or diameter increases. The maximum tensile stress occurs directly below the loading area with a distance approximately equal to the loading diameter. The MPL test, CPL test, UCS test, and Brazilian tension test have been performed on Saraburi mable under variety of sizes and shapes. The UCS test results indicate that the strengths decrease with increasing length-to-diameter ratio. The test results can be postulated that the MPL strength can be correlated with the compressive strength when the MPL specimens are relatively thin, and should de indicator of the tensile strength when the specimens are significantly larger than the diameter of the loading points. Predictive capability of the MPL and CPL techniques has been assessed and compared. Extrapolation of the test results suggest that the MPL results predict the UCS of the rock specimens better than does the CPL testing. The tensile strength predicted by MPL also agrees reasonably well with the Brazilian tensile strength of the rock.

Fuenkajorn and Klanphumeesri (2010) study the direct tensile strength and deformability from dog-bone shaped specimens of intact sandstone, limestone and marble using a compression-to-tension load converter. The device allows a measurement of the rock elastic modulus and Poisson's ratio under uniaxial tensile and compressive stresses on the same specimen. A series of finite difference analyses is performed to obtain a suitable specimen configuration that provides unidirectional tensile stresses at the mid-section. Results indicate that the direct tensile strengths are clearly lower than the Brazilian and ring tensile strengths. The elastic moduli and Poisson's ratios under uniaxial tension are lower than those under uniaxial compression. The discrepancy probably relates to the amount and distribution of the pore spaces and micro-fissures, and the bond strength of cementing materials. The porous and relatively poor-bonding sandstone shows a greater difference between the tensile and compressive elastic moduli and Poisson's ratios compared to those of the dense and well bonding marble and limestone.

Walsri et al. (2009) use a polyaxial load frame to determine the compressive and tensile strengths of three types of sandstone under true triaxial stresses. Results from the polyaxial compression tests on rectangular specimens of sandstones suggest that the rocks are transversely isotropic. The measured elastic modulus in the direction parallel to the bedding planes is slightly greater than that normal to the bedding. Poisson's ratio on the plane normal to the bedding planes is lower than those on the parallel ones. Under the same σ_3, σ_1 at failure increases with σ_2. Results from the Brazilian tension tests under axial compression reveal the effects of the intermediate principal stress on the rock tensile strength. The Coulomb and modified Wiebols and Cook failure criteria derived from the characterization test results predict the sandstone strengths in term of $J_2^{1/2}$ as a function of J_1 under true triaxial stresses. The modified Wiebols and Cook criterion describes the failure stresses better than does the Coulomb criterion when all principal stresses are in compressions. When the minimum principal stresses are in tension, the Coulomb criterion over-estimate the second order of the stress invariant at failure by about 20% while the modified Wiebols and Cook criterion fails to describe the rock tensile strengths.

CHAPTER III

SAMPLE PREPARATION

3.1 Introduction

This chapter describes the rock salt sample preparation procedure to be used in the Brazilian tension tests, circular plate bending tests, biaxial and triaxial extension tests, uniaxial and triaxial compression tests and four-point bending tests.

3.2 Sample preparation

Phra Wihan and Phu Phan sandstones (PWSS, PPSS), Phu Kradung siltstone (PKSS), and Saraburi marble (SBMB) have been selected for testing, primarily because of their highly uniform texture, density and strength. They are commonly found in the north and northeast of Thailand. Their mechanical properties and responses play a significant role in the stability of tunnels, slope embankments and dam foundations in the region.

The sample preparation follows the ASTM (D4543) standard practice, as much as practical. The rock blocks are drilled to obtain cores with nominal diameters of 54 and 100 mm. The cylindrical samples with 54-mm diameter are cut and ground for Brazilian tension testing (L/D = 0.5) as shown in Figure 3.1. The cylindrical samples with 100-mm diameter are cut and ground to obtain 10-mm thick for circular plate bending tests (Figure 3.2). The cubic specimens with a nominal dimension of 55×55×55 mm^3 are prepared for biaxial and triaxial extension test and uniaxial and triaxial compression test (Figure 3.3). The rectangular plates with nominal size of $150\times300\times20$ mm^3 are prepared for four-point bending test (Figure 3.4). All specimens are oven-dried before testing.

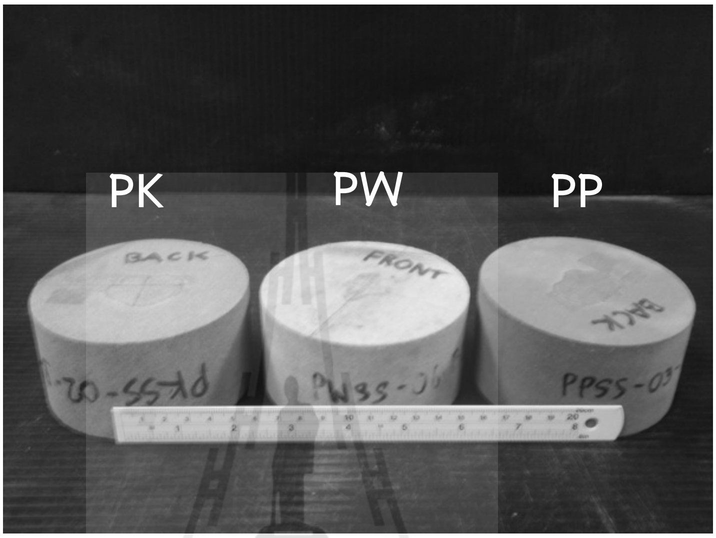


Figure 3.1 Some rock specimens prepared for Brazilian tension testing (L/D = 0.5) with 54-mm in diameter.

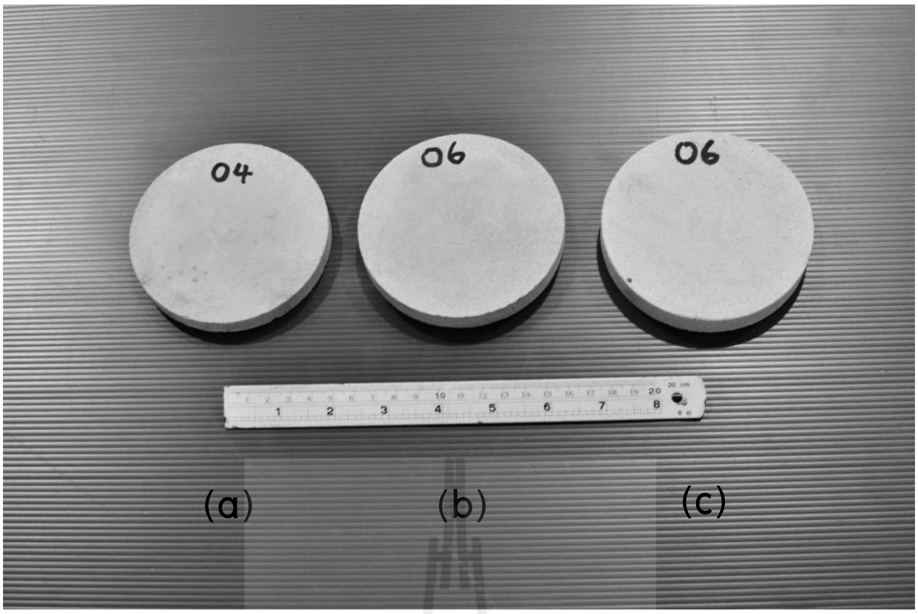


Figure 3.2 Some rock specimens prepared for circular plate bending tests with 100-mm in diameter are cut and ground to obtain 10-mm in thick.

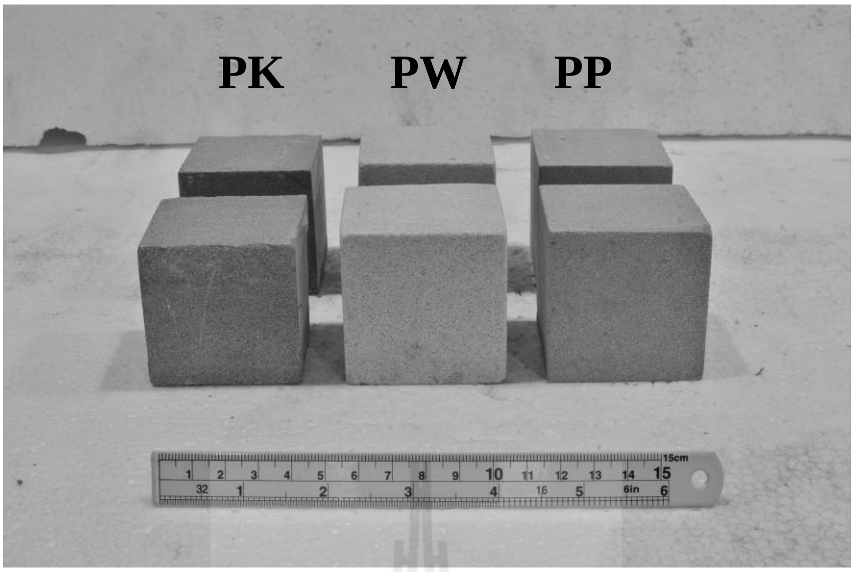
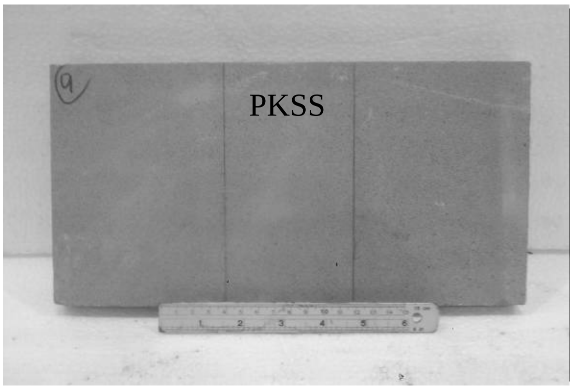
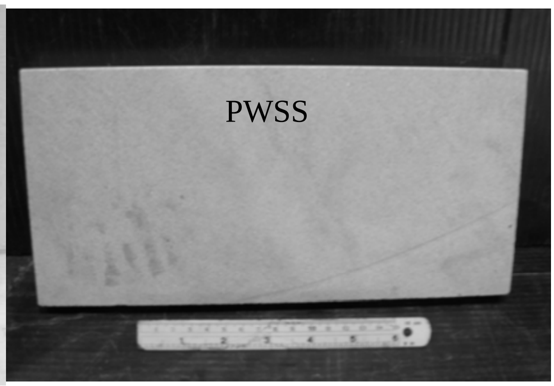


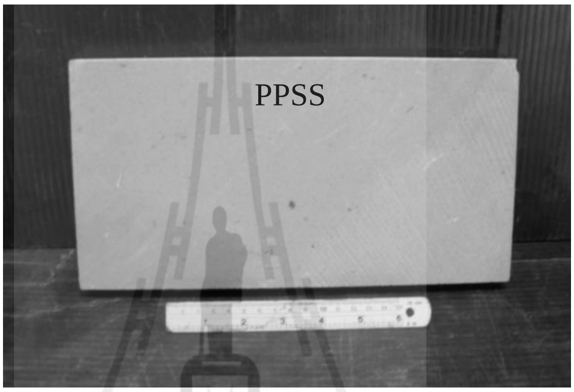
Figure 3.3 Some rock specimens prepared for biaxial and triaxial extension test and uniaxial and triaxial compression test with nominal dimensions of 55×55×55 mm^3.



(a)



(b)



(c)

Figure 3.4 Some rock specimens prepared for four-point bending test with nominal sizes of 150×300×20 mm^3.

CHAPTER IV

LABORATORY TESTING

4.1 Introduction

The objective of this study is to determine the effects of intermediate principal stresses on rock tensile strengths, and to verify whether the indirect (Brazilian) tensile strength can be correlated with the uniaxial and triaxial compressive strengths via the Coulomb criterion. The effort involves the measurement of the rock tensile strengths under various configurations (e.g. Brazilian tension tests, circular plate bending tests and four-point bending tests) of the applied principal stresses. Extrapolation of the Coulomb criteria from the compressive to the tensile regions is made from the uniaxial and triaxial compressive strengths of four rock types. The findings can improve an understanding of the tensile failure of intact rocks under a variety of stress states.

4.2 Basic characterization tests

Basic characterization tests are performed to obtain the Brazilian tensile strength the uniaxial and triaxial compressive strengths of the rocks. The test procedure follows the ASTM (D7012-10; D3967-95) standard practices. The confining pressures for the triaxial compressive strength testing range from 1.4, 3.0, 5.0 to 7.0 MPa. The failure stresses are recorded and mode of failure examined. Table 4.1 shows the results obtained from uniaxial and triaxial compressive strength tests and Table 4.2 summarizes the strength results for the four rock types in terms of uniaxial compressive strength (σ_c), Brazilian tensile strength (σ_B), cohesion (c) and friction angles (ϕ). The Mohr's circles representing the failure stresses are plotted in Figure 4.1. The results show that the Brazilian tensile strength cannot be correlated with the uniaxial and triaxial compression test results.

Table 4.1 Summarizes the strength results obtained from uniaxial and triaxial compressive strength tests.

Rock types	Laboratory tests	Major principal stress, σ_1 (MPa)	Intermediate principal stress , σ_2 (MPa)	Minor principal stress, σ_3 (MPa)
PPSS	Triaxial	80.0	7.0	7.0
	Triaxial	70.0	5.0	5.0
	Triaxial	64.0	3.0	3.0
	Triaxial	56.0	1.4	1.4
	UCS	47.0	0.0	0.0
PWSS	Triaxial	84.0	7.0	7.0
	Triaxial	75.0	5.0	5.0
	Triaxial	61.0	3.0	3.0
	Triaxial	47.5	1.4	1.4
	UCS	38.0	0.0	0.0
PKSS	Triaxial	78.0	7.0	7.0
	Triaxial	69.0	5.0	5.0
	Triaxial	53.0	3.0	3.0
	Triaxial	41.0	1.4	1.4
	UCS	30.3	0.0	0.0
SBMB	Triaxial	69.8	7.0	7.0
	Triaxial	57.1	5.0	5.0
	Triaxial	45.5	3.0	3.0
	Triaxial	34.7	1.4	1.4
	UCS	26.3	0.0	0.0

Table 4.2 Summarizes the strength results.

Rock types	Uniaxial compressive strength, σ_c (MPa)	Brazilian tensile strength, σ_B (MPa)	Friction angles, ϕ (degrees)	Cohesion, c (MPa)
PWSS	38.0 ± 7.5	6.7 ± 0.2	47	8.0
PKSS	30.3 ± 12.7	9.7 ± 0.1	44	7.7
PPSS	47.0 ± 11.1	10.7 ± 0.7	44	9.4
SBMB	22.0 ± 6.9	8.0 ± 0.3	46	5.8

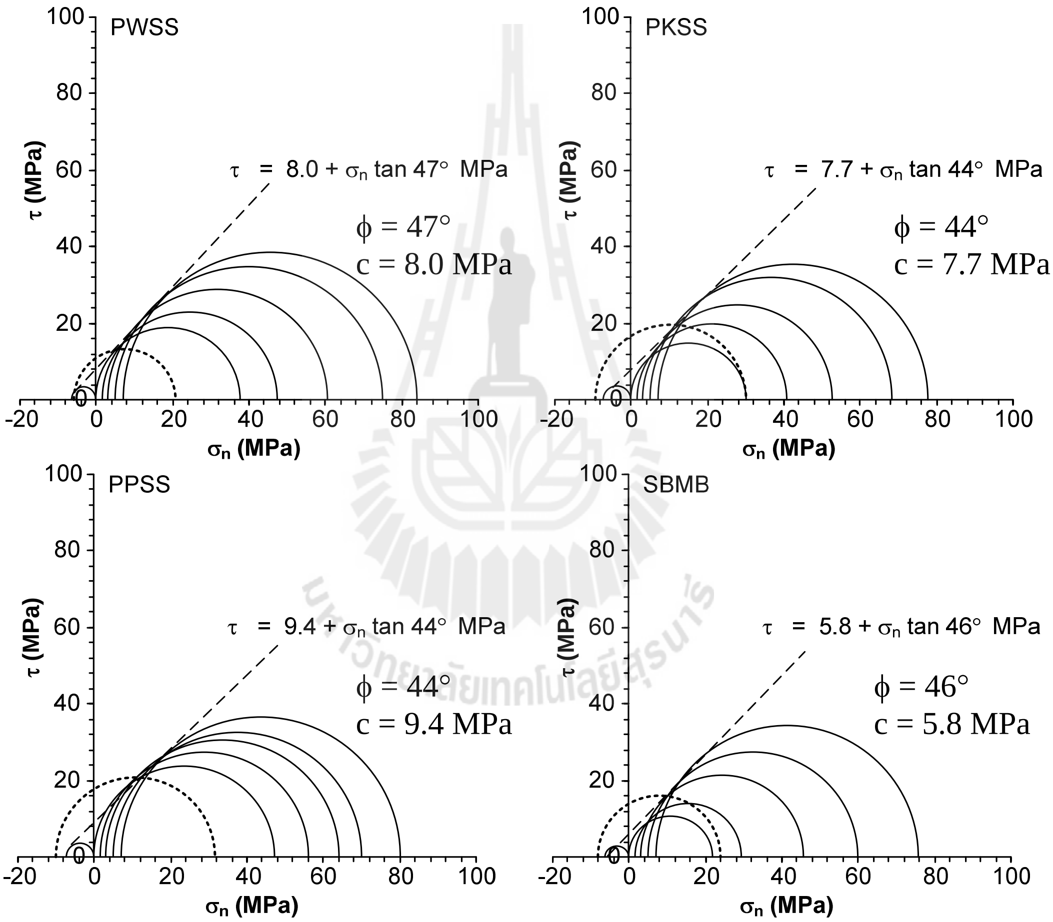


Figure 4.1 Mohr's circles representing Brazilian tensile strength (dash line) and uniaxial and triaxial compressive strengths (solid lines).

4.3 Triaxial extension tests

Triaxial extension tests (Tiwari et. al, 2006; Samsri et. al, 2011; Sriapai et. al, 2011; Pobwandee and Fuenkajorn, 2011) differs from a triaxial compression tests because for triaxial extension the lateral stress is increased while the axial stress is held constant. The results from this test method provide the rock strength for the condition where $\sigma_1 = \sigma_2 > 0$ and $\sigma_3 > 0$. The minimum principal stresses (axial stress) range from 0, 2, 4 to 6 MPa while the intermediate and major principal stresses are equal and increased until failure occurs. Table 4.3 summarizes the strength results obtained from the test.

4.4 Brazilian tension tests with axial confinement

The Brazilian tension test with axial confinement uses disk specimens with a nominal diameter of 54 mm with a thickness-to-diameter ratio of 0.5 (ASTM D3967-95). The polyaxial load frame (Walsri et al., 2009) is used to apply a constant axial stress on the disk specimen while the diametral line load is increased until failure (Figure 4.2). The constant axial stress is varied from zero (Brazilian test) to as high as the rock compressive strength (diametral line load is zero - uniaxial test). Neoprene sheets are used to minimize the friction between the rock surface and loading platen in the axial direction.

The results indicate that the tensile (horizontal) stresses (σ_x) and compressive (vertical) stresses (σ_y) induced at the crack initiation point in the middle of the specimen decrease with the increase of axial stress (σ_z), as shown in Figure 4.3. Assuming that the rocks are linearly elastic and the law of superposition is valid, these stresses can be calculated by (Jaeger and Cook 1979):

Table 4.3 Results obtained from triaxial extension test.

σ_3 (MPa)	$\sigma_1 = \sigma_2$ (MPa)			
	PWSS	PKSS	PPSS	SBMB
0.0	15.0	51.0	71.1	36.0
2.0	56.3	69.4	77.8	55.8
4.0	68.5	79.8	94.4	66.9
6.0	77.8	94.4	102.7	83.5

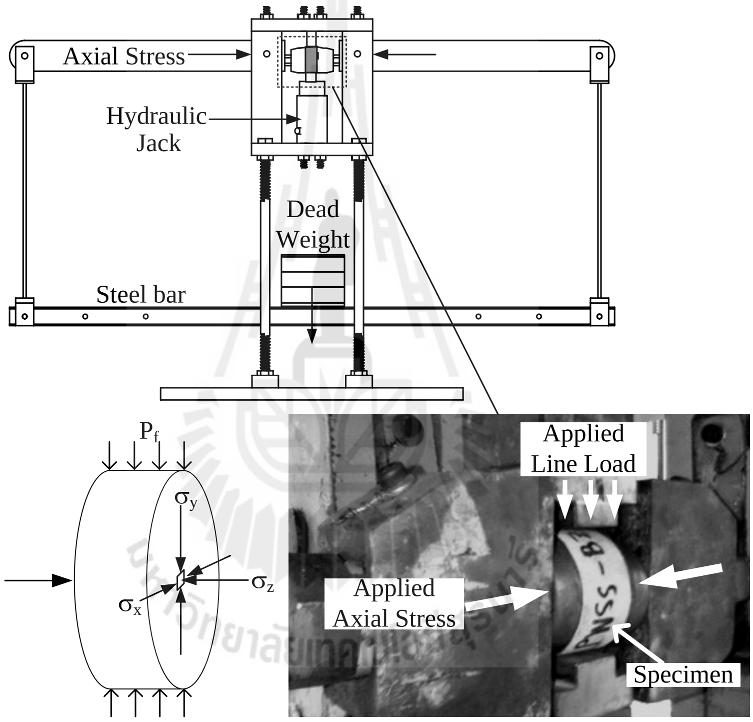


Figure 4.2 Polyaxial load frame used in Brazilian tension test with axial confinement (Walsri et al., 2009).

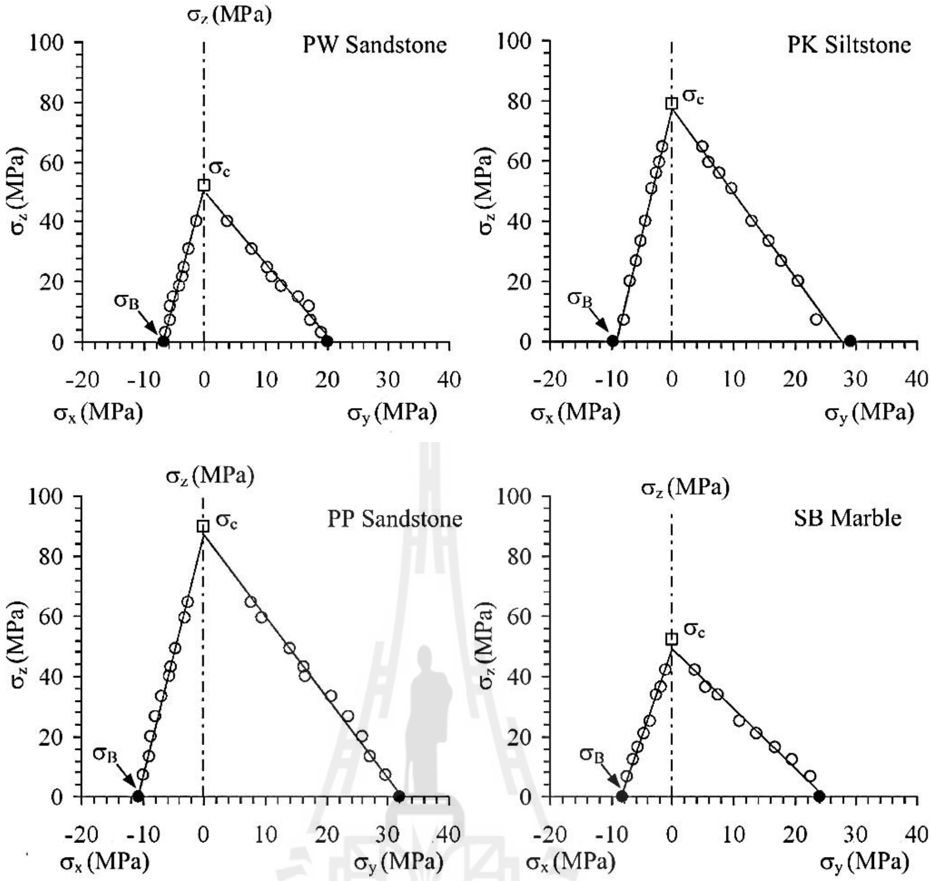


Figure 4.3 Induced compressive (σ_y) and tensile (σ_x) stresses at failure as a function of applied axial stress (σ_z) (Walsri et al., 2009).

$$\sigma_x = -2P/\pi DL \tag{4.1}$$

$$\sigma_y = 6P/\pi DL = -3\sigma_x \tag{4.2}$$

$$\sigma_z = \text{Applied axial stresses} \tag{4.3}$$

where P_f is the failure load, D is the disk diameter and L is the disk thickness. The applied axial stresses are varying from 0 to the uniaxial compressive strength (σ_c).

At the crack initiation point σ_y, σ_x and σ_z represent the principal stresses. The induced tensile stress σ_x is always the minimum principal stress. The magnitudes of the applied axial stress determine whether σ_y or σ_z is the maximum principal stress. Under low applied σ_z, σ_y is the maximum principal stress and σ_z is the intermediate principal stress. Under high σ_z, σ_y becomes the intermediate principal stress and σ_z is the maximum principal stress. The results indicate that the state of stresses where $\sigma_1 = \sigma_2$ are positive while σ_3 is negative. Table 4.4 summarizes the test results obtained from Brazilian tension test under axial compression.

Post-failure observations show that under low σ_z a single splitting extension crack along the loading diameter is normally induced in the disk specimen. Multiple extension cracks are developed as σ_z increases. When σ_z reaches the uniaxial compressive strength of the rocks, the specimens fail without applying the diametral line load. At this point the specimens are crushed, resulting in multiple shear fractures and extension cracks (Figure 4.4). It is postulated that the axial stress produces tensile strains perpendicular to its direction due to the effect of the Poisson's ratio. The line load at failure therefore decreases with increasing σ_z.

Table 4.4 Results obtained from Brazilian tension test under axial compression.

PWSS		
σ_x (MPa)	σ_y (MPa)	σ_z (MPa)
0.0	0.0	55.0
-1.3	3.8	40.3
-2.5	7.6	31.0
-3.4	10.1	25.0
-3.7	11.0	21.7
-4.2	12.6	18.3
-5.1	15.4	15.1
-5.7	17.1	11.9
-5.7	17.2	7.2
-6.3	19.0	3.0
-6.7	20.1	0.0

PKSS		
σ_x (MPa)	σ_y (MPa)	σ_z (MPa)
0.0	0.0	78.0
-1.6	4.8	64.6
-2.0	6.1	59.4
-2.6	7.7	55.9
-3.3	9.7	50.7
-4.3	13.0	40.3
-5.3	15.8	33.3
-5.9	17.7	26.7
-6.8	20.5	20.0
-7.9	23.7	7.2
-9.7	29.1	0.0

PPSS		
σ_x (MPa)	σ_y (MPa)	σ_z (MPa)
0.0	0.0	90.0
-2.6	7.7	64.6
-3.1	9.4	59.4
-4.7	14.1	49.0
-5.4	16.3	43.0
-5.5	16.6	40.3
-7.0	20.8	33.3
-7.8	23.5	26.7
-8.6	25.8	20.0
-9.0	27.1	13.4
-9.9	29.7	7.2
-10.7	32.0	0.0

SBMB		
σ_x (MPa)	σ_y (MPa)	σ_z (MPa)
0.0	0.0	52.0
-1.2	3.6	42.2
-1.8	5.4	36.3
-2.5	7.5	33.8
-3.7	11.1	25.2
-4.6	13.8	21.1
-5.6	16.8	16.3
-6.5	19.5	12.5
-7.5	22.5	6.6
-8.0	24.1	0.0
-	-	-
-	-	-

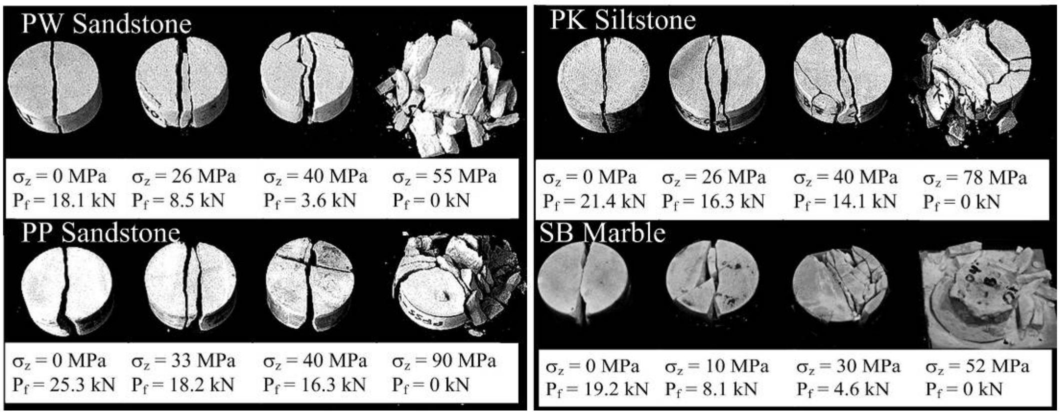


Figure 4.4 Some post-test specimens from Brazilian test with axial confinement (Walsri et al., 2009).

4.5 Four-point bending tests

The test arrangement for the four-point bending is shown in Figure 4.5. The results from this test method provide the rock tensile strengths for the condition where $\sigma_1 = \sigma_2 = 0$ and $\sigma_3 = \sigma_{bend}$ is negative. The 150×300×20 mm^3 rock samples bear on the steel rollers with 8 mm in diameter, which are located at the base L = 100 mm. A press load is applied to the identical rollers, placed at the distance g = 40 mm, through the spherical bearing. The tensile stresses (σ_{bend}) in the bottom surface of the bar can be calculated as (Efimov, 2011):

$$\sigma_{bend} = \frac{6M}{Bt^2} = \frac{3P_f\left(L-g\right)}{2Bt^2} \tag{4.4}$$

where F, M are the maximum values of applied load and moment, respectively, t is the thickness of the sample, B is the width of the sample, and P_f is maximum load as failure. Table 4.5 shows the results from four-point bending tests of all rock types.

4.6 Circular plate bending tests

The circular disks with diameter of 100 mm are cut and ground to obtain 10-mm thick. The plate bending tests on circular disk determines the tensile strength where $\sigma_1 = 0$ and $\sigma_2 = \sigma_3 = \sigma_t$ are negative. The circular disk is simply-supported around its outer circumference. The line load (P_f) is applied concentrically to the circular disk through a tube of diameter c as shown in Figure 4.6.

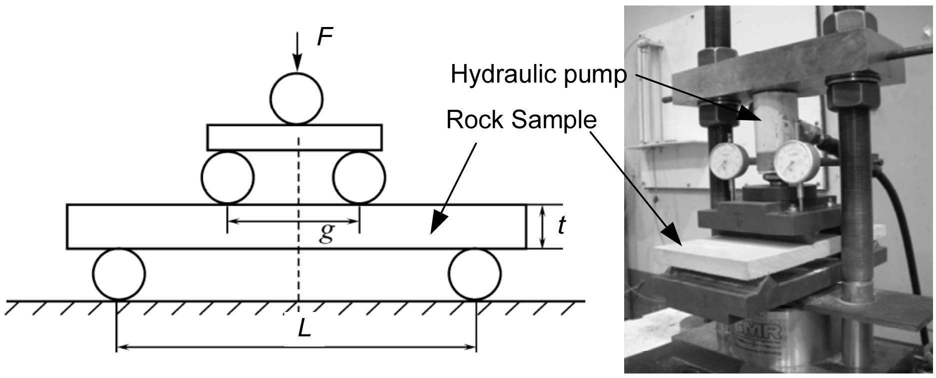


Figure 4.5 Four-point bending tests arrangement.

Table 4.5 Results obtained from four-point bending tests.

Rock Types	Tensile Stresses (σ_{bend}, MPa)
PPSS	13.6±0.35
PKSS	9.4±0.33
PWSS	8.6±0.53
SBMB	8.2±0.11

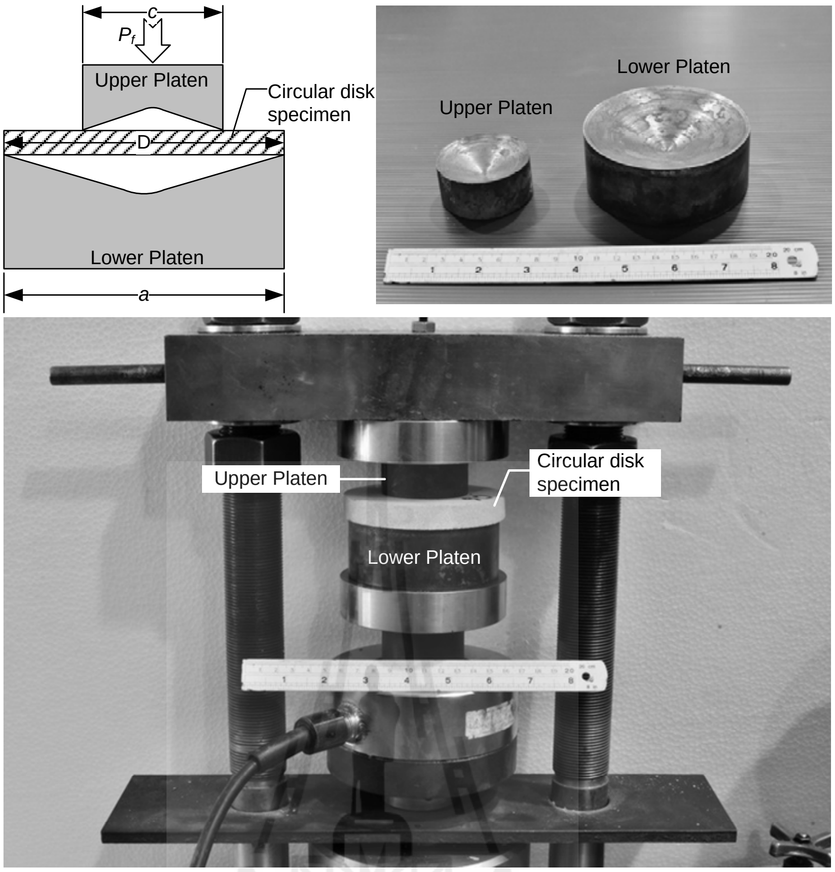


Figure 4.6 Circular plate bending test arrangement.

The governing equation for bending behaviour of a symmetric orthotropic thin plate is expressed below (Ugural, 1999) where the tangential and radial tensile strengths (σ_r, σ_t) are equal.

$$\sigma_r = \sigma_t = \frac{3P_f}{4\pi t^2}\left[(1-\nu)\left(1-\frac{c^2}{a^2}\right)+2(1+\nu)\ln\left(\frac{a}{c}\right)\right] \tag{4.5}$$

where P_f is maximum load as failure, t is the thickness of circular rock sample, v is Poisson's ratio, and a, c are the outside and inside diameters. Table 4.6 shows the results from circular plate bending tests. Figure 4.7 shows the pre- and post-test specimens from circular plate bending tests.

Table 4.6 Results obtained from circular plate bending tests.

Rock Types	No. of Samples	Thickness (mm)	Load (N)	Tensile Stresses, σ_r (MPa)
PKSS	PK_SS-01	7.8	600	4.49
	PK_SS-02	6.0	700	8.94
	PK_SS-03	6.7	790	7.93
	PK_SS-04	6.3	650	7.39
	PK_SS-05	5.5	550	8.27
AVERAGE±SD				7.4±1.72
PPSS	PP_SS-01	11.85	1840	5.92
	PP_SS-02	12.27	1750	5.25
	PP_SS-03	13.25	1845	4.75
	PP_SS-04	12.6	1445	4.11
	PP_SS-05	12.73	1595	4.45
AVERAGE±SD				4.89±0.71
PWSS	PW_SS-01	12	1700	5.33
	PW_SS-02	12	1840	5.77
	PW_SS-03	12	1860	5.83
	PW_SS-04	13	2640	7.06
	PW_SS-05	9.59	1790	8.79
AVERAGE±SD				6.56±1.4
SBMB	SB_MS-01	13	2240	6.03
	SB_MS-02	13	2190	5.89
	SB_MS-03	13	2640	7.11
	SB_MS-04	12.65	2100	5.97
	SB_MS-05	11.8	1840	6.01
AVERAGE±SD				6.2±0.5

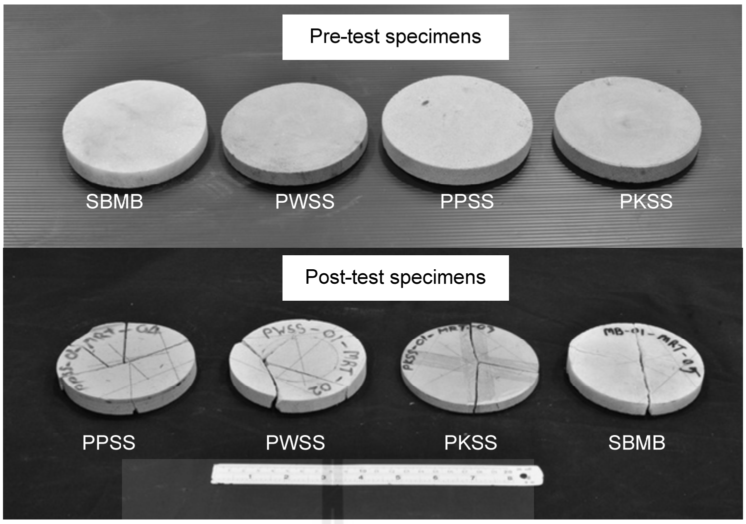


Figure 4.7 Pre-and post-test specimens from circular plate bending test.

CHAPTER V

EXTENSION OF COULOMB STRENGTH CRITERION

5.1 Introduction

The purpose of this chapter is to describe the extrapolation of the Coulomb criterion from the compressive to the tensile regions that are developed from the uniaxial and triaxial compressive strengths, Brazilian tensile strengths, circular plate bending strengths, biaxial and triaxial extension strengths and four-point bending strengths of the four rock types. The findings can improve an understanding of the tensile failure of intact rocks under a variety of stress states.

5.2 Strength criterion

The Coulomb failure criterion is used to describe the rock strengths in compressive and tensile regions. First the σ_x, σ_y, and σ_z from the Brazilian tension test under confinement are defined. The condition where σ_1 equals to σ_2 is selected to develop the strength criterion. Table 5.1 gives the rock strengths from various tests in terms of the principal stresses.

The Coulomb criterion may be written in the form of the principal stresses as (Jaeger and Cook, 1979):

$$\sigma_1 = 2 \cdot c \cdot \tan(\alpha) + \sigma_3 \tan^2(\alpha) \quad (5.1)$$

$$\alpha = \pi/4 + ½ \phi \quad (5.2)$$

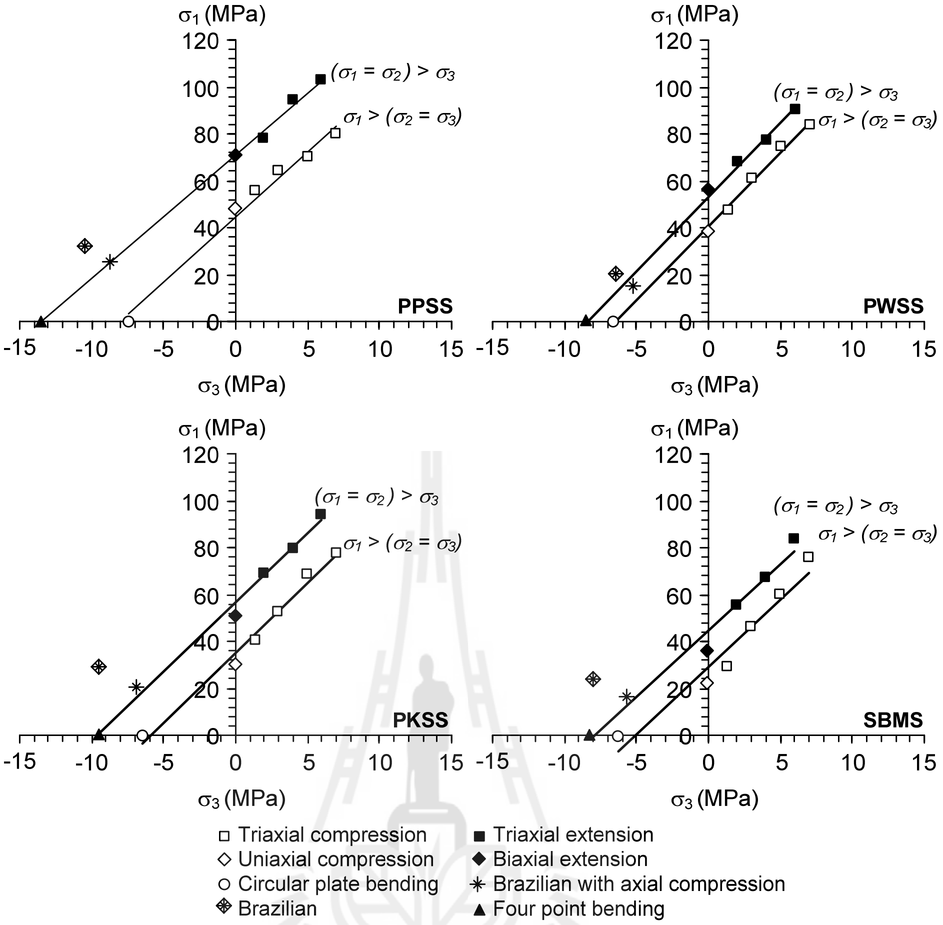
where c is the cohesion, ϕ is internal friction angle, and 2·c·tan(α) is the uniaxial compressive strength of rock. This failure criterion assumes that the intermediate principal stress has no influence on the failure.

Table 5.1. Principal stresses obtained from different test methods

Test Methods		PWSS			PKSS		
		σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)	σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)
$\sigma_1 > (\sigma_2 = \sigma_3)$	TCS	84.0	7.0	7.0	78.0	7.0	7.0
	TCS	75.0	5.0	5.0	69.0	5.0	5.0
	TCS	61.0	3.0	3.0	53.0	3.0	3.0
	TCS	47.5	1.4	1.4	41.0	1.4	1.4
	UCS	38.0	0.0	0.0	30.3	0.0	0.0
	CPB	0.0	-6.6	-6.6	0.0	-7.4	-7.4
$(\sigma_2 = \sigma_2) > \sigma_3$	FPB	0.0	0.0	-8.6	0.0	0.0	-9.4
	MBZ	15.0	15.0	-5.2	20.5	20.5	-6.8
	TXC	56.3	56.3	0.0	51.0	51.0	0.0
	TXC	68.5	68.5	2.0	69.4	69.4	2.0
	TXC	77.8	77.8	4.0	79.8	79.8	4.0
	TXC	90.5	90.5	6.0	94.4	94.4	6.0
Test Methods		**PPSS**			**SBMB**		
		σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)	σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)
$\sigma_1 > (\sigma_2 = \sigma_3)$	TCS	80.0	7.0	7.0	75.5	7.0	7.0
	TCS	70.0	5.0	5.0	59.7	5.0	5.0
	TCS	64.0	3.0	3.0	46.1	3.0	3.0
	TCS	56.0	1.4	1.4	29.5	1.4	1.4
	UCS	47.0	0.0	0.0	22.0	0.0	0.0
	CPB	0.0	-7.5	-7.5	0.0	-6.2	-6.2
$(\sigma_1 = \sigma_2) > \sigma_3$	FPB	0.0	0.0	-13.6	0.0	0.0	-8.2
	MBZ	25.8	25.8	-8.6	16.3	16.3	-5.6
	TXC	71.1	71.1	0.0	36.0	36.0	0.0
	TXC	77.8	77.8	2.0	55.8	55.8	2.0
	TXC	94.4	94.4	4.0	66.9	66.9	4.0
	TXC	102.7	102.7	6.0	83.5	83.5	6.0

TCS – Triaxial compression test
FPB – Four point bending test
UCS – Uniaxial compression test
MBZ – Brazilian tension test under axial compression
CPB – Circular plate bending test TXC – Triaxial extension test

Figure 5.1 presents the test results in the form of the Coulomb criterion. The stresses in compressive region are separated into two conditions; the triaxial extension ($\sigma_1 = \sigma_2$) and the triaxial compression ($\sigma_2 = \sigma_3$). The uniaxial and triaxial compression test results are correlated with those obtained from circular plate bending test. The results from biaxial and triaxial extension test are correlated with those of the Brazilian tension test under axial confinement and four-point bending test. The linear correlation is extended from the compressive to the tensile region. The results indicate that the Coulomb criterion can well correlate with the compressive and tensile strengths obtained from each stress condition. The Brazilian tensile strength (where $\sigma_1 = 3\sigma_B$ is positive, $\sigma_2 = 0$, and $\sigma_3 = \sigma_B$ is negative) cannot be correlated with the failure criterion developed from the two stress conditions.



Coulomb strength criterion:	$\sigma_1 = 2c\tan\alpha + \sigma_3\tan^2\alpha$, $\alpha = \pi/4 + \phi/2$	$(\sigma_1 = \sigma_2) > \sigma_3$		$\sigma_1 > (\sigma_2 = \sigma_3)$	
		c (MPa)	ϕ (degrees)	c (MPa)	ϕ (degrees)
PPSS		15.5	43	9.4	44
PWSS		10.5	47	8.0	47
PKSS		11.7	45	7.7	44
SBMB		9.3	45	5.8	46

Figure 5.1 Major principal stresses (σ_1) as a function of minor principal stresses (σ_3) for various tests.

CHAPTER VI

DISCUSSIONS, CONCLUSIONS AND RECOMMENDATIONS

6.1 Discussions and conclusions

The objective of this study is to determine the effects of intermediate principal stresses on the rock tensile strengths, and to verify whether the indirect (Brazilian) tensile strength can be correlated with the Coulomb criterion. The effort involves the measurement of the tensile strengths of four rock types under various configurations of the applied principal stresses.

Compressive and tensile strength tests are performed on intact rocks prepared from Phu Phan, Phra Wihan and Phu Kradung sandstones and Saraburi marble to determine the effect of the intermediate principal stress on the tensile strength results. The tests are separated into two groups based on the stress state at failure; triaxial extension ($\sigma_1 = \sigma_2 \neq \sigma_3$) and triaxial compression ($\sigma_1 \neq \sigma_2 = \sigma_3$). The first group includes biaxial and triaxial extension tests, four point bending tests and Brazilian tension tests with axial compression. The second group includes uniaxial and triaxial compression tests and circular plate bending tests. The results indicate that the Coulomb criterion can well correlate with the compressive and tensile strengths obtained from each stress condition. For all rock types the compressive and tensile strengths and cohesion obtained from the triaxial extension tests are greater than those obtained from the triaxial compression tests. Both stress conditions give similar internal friction angle. This suggests that σ_2 can strengthen the rock for both compressive and tensile regions. More important the results indicate that the Brazilian tensile strength cannot be correlated with the two stress conditions. It is recommended that an extension of the Coulomb criterion into the tensile region should be correlated with the tensile strength obtained from the circular plate bending test rather than the Brazilian tension test.

6.2 Recommendations

The uncertainties and adequacies of the research investigation and results discussed above lead to the recommendations for further studies testing is required to assess the effect of the intermediate principal stress. More rock types are needed to confirm the applicability and limitations of the proposed concept. The criterion used in this study may not be adequately to describe the rock behavior. Other criteria, such as Mogi may be needed to describe the rock behavior, particularly where the relationship between the shear stress and normal stress at failure is not linear.

REFERENCES

ASTM Standard D3967-95a. 1995. Standard Test Met Splitting Tensile Strength of Intact Rock Core Specimens. **Annual Book of ASTM Standards** (Vol. 04.08). Philadelphia: American Society for Testing and Materials.

ASTM Standard D4543-08. 2008. Standard Practice for Preparing Rock Core Specimens and Determining Dimensional and Shape Tolerances. **Annual Book of ASTM Standards**, American Society for Testing and Materials, West Conshohocken, PA.

ASTM Standard D7012-10. 2010. Standard Test Method for Compressive Strength and Elastic Moduli of Intact Rock Core Specimens under Varying States of Stress and Temperatures. **Annual Book of ASTM Standards**, American Society for Testing and Materials, West Conshohocken, PA.

Bieniawski, Z.T. 1974. Estimate strength of rock materials. **The Journal of the Southern African Institute of Mining and Metallurgy**. 74(8): 312-320.

Claesson, J. and Bohloli, B. (2002). Brazilian test: stress field and tensile strength of anisotropic rocks using an analytical solution. **International Journal of Rock Mechanics & Mining Sciences**.

Efimov, V.P. 2011. Determination of tensile strength by the measured rock bending strength. **Journal of Mining Science**. 7(5): 580-586.

Franklin, J.A. 1971. Triaxial strength of rock materials. **Rock Mechanics and Rock Engineering**. 3(2): 86-98.

Fuenkajorn, K. &Kenkhunthod, N. 2010. Influence of loading rate on deformability and compressive strength of three Thai sandstones. **Geotechical and Geological Engineering**. 28(5): 707-715.

Fuenkajorn, K. &Klanphumeesri, S. 2010. Laboratory determination of direct tensile strength and deformability of intact rocks. **Geotechnical Testing Journal**. 34(1): 1-6.

Fuenkajorn, K. and Daemen, J.J.K. (1986). Shape Effect on Ring Test Tensile Strength. Key to Energy Production: **Proceedings of the 27th U.S. Symposium on Rock Mechanics**, June 23-25, University of Alabama, Tuscaloosa. (pp. 155-163).

Fuenkajorn, K. and Klanphumeesri, S. 2010. Laboratory determination of direct tensile strength and deformability of intact rocks. **Geotechnical Testing Journal,** 34 (1): 1-6.

Haimson, B. (2006). True Triaxial Stresses and the Brittle Fracture of Rock. **Pure and Applied Geophysics.** 163: 1101–1113.

Hoek, E. & Brown, E.T. 1980.Empirical strength criterion for rock mass. **Journal of Geotechnical Engineering Division ASCE**. 106(GT9): 1013-1035.

Hoek, E. & Brown, E.T. 1997. Practical estimates of rock mass strength. **International Journal of Rock Mechanics and Mining Sciences**. 24(8): 1165-1186.

ISRM (1981). **Suggested Method for Rock Characterization**, Testing and Monitoring. Oxford: Pergamon.

Jaeger, J. C. and Cook, N. W. (1979). **Fundamentals of Rock Mechanic**. London: Chapman and Hall. pp. 169-173.

Johnston, I.W. 1985. The strength of intact geomechanical materials. **Journal of Geotechnical Engineering Division ASCE**. 111(6): 730-749.

Kim, M.K. & Lade, P.V. 1984. Modeling rock strength in three dimensions. **International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts**. 21(1): 21-23.

Liao, J. J., Yang, M. T. and Hsieh, H. Y. (1997). Direct tensile behavior of a transversely isotropic rock, **International Journal of Rock Mechanics & Mining Sciences**. Vol. 34. No. 5, pp. 831-849.

Samsri, P., Sriapai, T., C. Walsri & Fuenkajorn, K. (2011). Polyaxial creep testing of rock salt. **Proceedings of the third Thailand Symposium on Rock Mechanics**. March, 10-11, Springfield @ Sea Resort & Spa, Cha-Am, Petchaburi, Suranaree University of Technology, pp. 125-132.

Sheorey, P.R., Biswas, A.K. &Choubey, V.D. 1989. An empirical failure criterion for rocks and jointed rock masses. **Engineering Geology**. 26(2): 141-159.

Sriapai, T., Samsri, P., & Fuenkajorn, K. (2011). Polyaxial strength of Maha Sarakham salt. **Proceedings of the third Thailand Symposium on Rock Mechanics**. March, 10-11, Springfield @ Sea Resort & Spa, Cha-Am, Petchaburi, Suranaree University of Technology, pp. 79-87.

T. Pobwandee & Fuenkajorn, K. (2011). Effect of intermediate principal stresses on compressive strength of Phra Wihan sandstone. **Proceedings of the third Thailand Symposium on Rock Mechanics**. March, 10-11, Springfield @ Sea Resort & Spa, Cha-Am, Petchaburi, Suranaree University of Technology, pp. 55-62.

Tepnarong, P. (2001). Theoretical and Experimental Studies to Determine Compressive and Tensile Strength of Rock, Using Modified Point Load Testing. **M.S. Thesis**, Suranaree University of Technology, Thailand.

Tiwari, R.P. & Rao, K.S., 2006. Post failure behaviour of a rock mass under the influence of triaxial and true triaxial confinement. **Engineering Geology**. 84: 112-129.

Ugural, A.C. 1999. **Stresses in Plates and Shells**, 2nd Edition. WCB/McGraw Hill, Boston, 502 p.

Walsri, C., Poonprakon, P. Thosuwan, R. and Fuenkajorn, K. (2009). Compressive and tensile strengths of sandstones under true triaxial stresses. **Proceedings of the Second Thailand Rock Mechanics Symposium**, Nakhon Ratchasima: Suranaree University of Technology.

Wiebols, G.A. & Cook, N.G.W. 1968.An energy criterion for the strength of rock in polyaxial compression. **International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts**. 5(6): 529-549.

Yudbir, R.K., Lemanza, W. &Prinzl, F. 1983.An empirical failure criterion for rock masses. **Proceedings of the 5th International Congress Society of Rock Mechanics**, Melbourne, Vol. 1, pp. B1-B8.

Zi, G., Oh, H. and Park S.K. (2008). A novel indirect test method to measure the biaxial tensile strength of concretes and other quasibrittle materials. **Cement and Concrete Research.** Vol. 38 : 751 – 756.

APPENDIX A

PUBLICATION

Publication

Masingboon, P., Phueakphum, D. and Fuenkajorn, K., 2013. **Effects of Intermediate Principal Stresses on Rock Tensile Strengths**. In Proceedings of The fourth Thailand Symposium on Rock Mechanics. Nakhon Ratchasima, Thailand, 24-25 January 2013.

Rock Mechanics, Fuenkajorn &Phien-wej (eds) © 2013

Effects of intermediate principal stresses on rock tensile strengths

P. Masingboon,D. Phueakphum& K. Fuenkajorn
Geomechanics Research Unit, Suranaree University of Technology, Thailand

Keywords: Tensile strength,principal stresses,cohesion, internal friction angle

ABSTRACTS: The objective of this study is to experimentally determine the effects of intermediate principal stresses on the tensile strength of rocks. The results are used to assess the predictive capability of the Coulomb criterion when one or more principal stresses are in tension. Four rock types have been used, including two types of sandstone, siltstone and marble. The laboratory testing involves four-point bending test, Brazilian tests with axial compression, circular plate bending test. Uniaxial, biaxial and triaxial compressive strengths of the four rock types are also determined to correlate their results with those of the tensile testing. The results indicate that the intermediate principal stresses do affect the rock tensile strengths. This holds true for all tested rock types. The four point bending and Brazilian tensile strengths under compression provide a linear transition with the triaxial extension test. This is because they are all under the condition where $\sigma_1 = \sigma_2 > \sigma_3$. Based on the Coulomb criterion, the circular plate bending tensile strength can well correlate with the conventional uniaxial and triaxial compressive strengths of the rocks. The intermediate principal stresses do affect the rock tensile strengths. For all rock types the compressive and tensile strengths and cohesion obtained from the triaxial extension tests are greater than those obtained from the triaxial compression tests. Both stress conditions give similar internal friction angle. More important the results indicate that the Brazilian tensile strength can not be correlated with the two stress conditions. It is recommended that an extension of the Coulomb criterion into the tensile region should be correlated with the tensile strength obtained from the circular plate bending test rather than the Brazilian tension test.

1 INTRODUCTION

Various strength criteria have been developed to describe the failure of geological materials, e.g. Mohr-Coulomb (Jaeger & Cook, 1979), Bieniawski (1974), Hoek and Brown (1980), Yudbir et al. (1983), Kim & Lade (1984), Johnston (1985), Sheorey et al. (1989), Mogi (1971), Wiebols& Cook (1968), and Franklin (1971). Some criteria can incorporate the effect of the intermediate principal stress on the rock strength, i.e., multi-axial strength formulation. These failure criteria can be derived from the uniaxial and triaxial compressive strength test data. Extrapolation of these criteria to the tensile region is normally assumed by a continuous transition from the compressive strength to the Brazilian (indirect) tensile strength of the rock (i.e. the Coulomb and Hoek-Brown criteria). Confirmation of the transition between the compressive and tensile strengths has never been attempted.

The objective of this study is to determine how the effects of intermediate principal stresses on rock tensile strengths, and to verify whether the indirect (Brazilian) tensile strength can be correlated with the Coulomb criterion. The effort involves the measurement of the rock tensile strengths under various configurations of the applied principal stresses. Extrapolation of the Coulomb and Mogi criteria from the compressive to the tensile regions are made from the uniaxial and triaxial compressive strengths of four rock types. The findings can improve an understanding of the tensile failure of intact rocks under a variety of stress states.

2 ROCK SPECIMENS

PhraWihan and PhuPhan sandstones (PWSS, PPSS), PhuKradung siltstone (PKSS), and Saraburi marble (SBMB) have been selected for testing, primarily because of their highly uniform texture, density and strength. They are commonly found in the north and northeast of Thailand. Their mechanical properties and responses play a significant role in the stability of tunnels, slope embankments and dam foundations in the region.

The sample preparation follows the ASTM (D4543-08) standard practice, as much as practical. The rock blocks are drilled to obtain cores with nominal diameters of 54 and 100 mm. The cylindrical samples with 54-mm diameter are cut and ground for Brazilian tension testing (L/D = 0.5). The cylindrical samples with 100-mm diameter are cut and ground to obtain 10-mm thick for circular plate bending tests. The cubic specimens with a nominal dimension of 55×55×55 mm^3 are prepared for biaxial and triaxial extension test and uniaxial and triaxial compression test. The rectangular plates with nominals size of 150×300×20 mm^3 are prepared for four-point bending test. All specimens are oven-dried before testing. Figure 1 shows some of rock specimens prepared for laboratory testing.

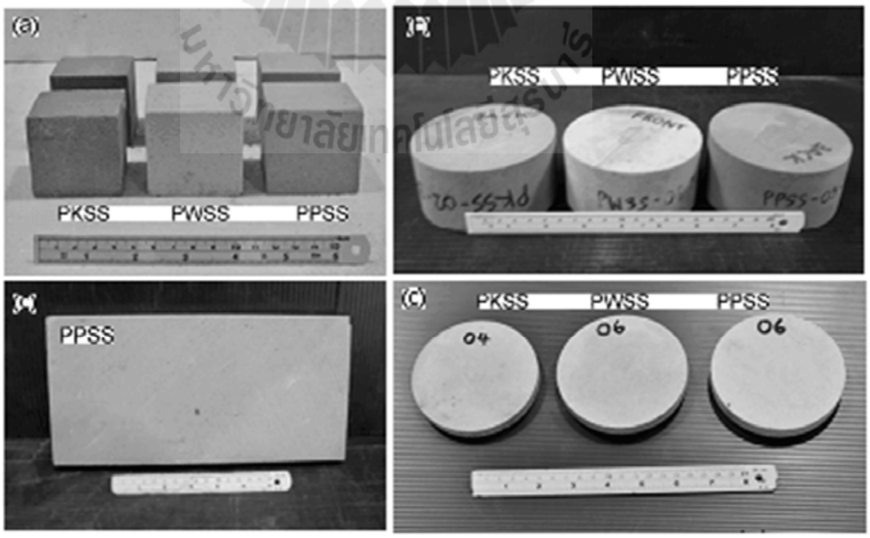


Figure 1. Some rock specimens prepared for laboratory test: a) biaxial and triaxial extension test and uniaxial and triaxial compression test, b) brazillian tension test, c) four-point bending test and d) circular plate bending test.

3 LABORATORY TESTS

3.1 Basic Characterization Tests

Basic characterization tests are performed to obtain the Brazilian tensile strength the uniaxial and triaxial compressive strengths of the rocks. The test procedure follows the ASTM (D7012-10; D3967-95) standard practices. The confining pressures range from 1.7, 3.4, 6.9 to 10.3 MPa. The failure stresses are recorded and mode of failure examined. Table 1 summarizes the strength results for the four rock types. The Mohr's circles representing the failure stresses are plotted in Figure 2. The results shows that the Brazilian tensile strength can not be correlated with the uniaxial and triaxial compression test results.

Table 1.Basic mechanical properties.

Rock Type	σ_c (MPa)	σ_B (MPa)	ϕ (degrees)	c (MPa)
PWSS	38.0 ± 7.5	6.7 ± 0.2	47	8.0
PKSS	30.3 ± 12.7	9.7 ± 0.1	44	7.7
PPSS	47.0 ± 11.1	10.7 ± 0.7	44	9.4
SBMB	22.0 ± 6.9	8.0 ± 0.3	46	5.8

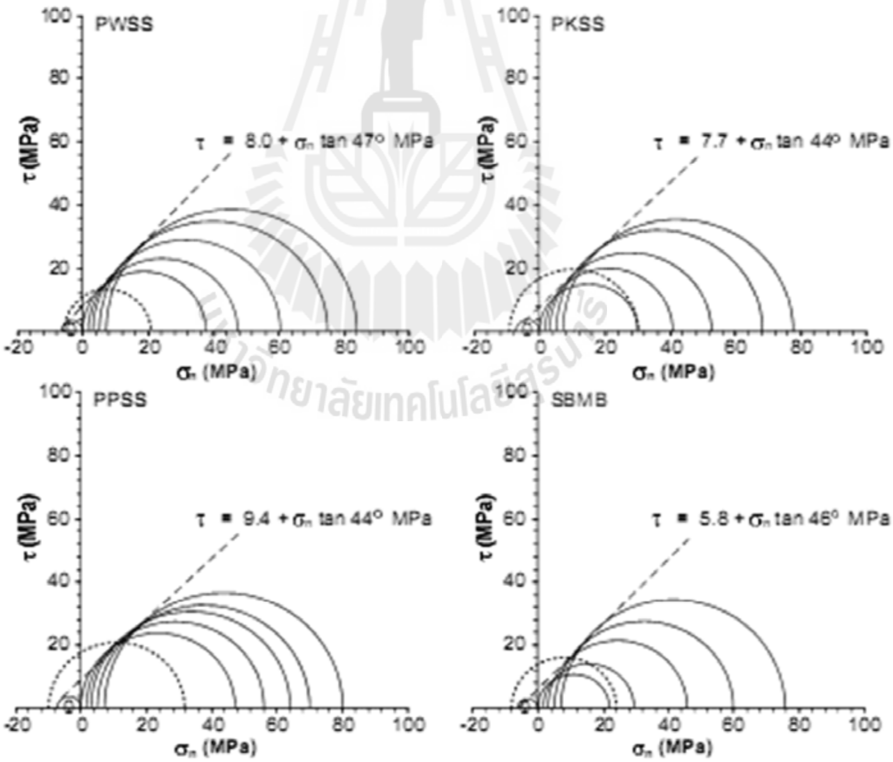


Figure 2. Mohr's circles representing Brazilian tensile strength (dash line) and uniaxial and triaxial compressive strength (solid lines).

3.2 Triaxial Extension Tests

Triaxial extension tests differs from a triaxial compression tests because for triaxial extension the lateral stress is increased while the axial stress is held constant. The results from this test method provide the rock strength for the condition where $\sigma_1 = \sigma_2 > 0$ and $\sigma_3 > 0$.The minimum principal stresses (axial stress) range from 0, 2, 4 to 6 MPa while the intermediate and major principal stresses are increased until failure occurs. Table 2 summarizes the strength results obtained from the test.

3.3 Brazilian Tension Tests with Axial Confinement

The Brazilian tension test with axial confinement uses disk specimens with a nominal diameter of 54 mm with a thickness-to-diameter ratio of 0.5 (ASTM D3967-95). The polyaxial load frame (Walsri et al., 2009) is used to apply a constant axial stress on the disk specimen while the diametral line load is increased until failure (Figure 3). The constant axial stress is varied from zero (Brazilian test) to as high as the rock compressive strength (diametral line load is zero - uniaxial test). Neoprene sheets are used to minimize the friction between the rock surface and loading platen in the axial direction.

Table 2. Results obtained from Brazilian tension test under axial compression.

σ_3 (MPa)	$\sigma_1 = \sigma_2$ (MPa)			
	PWSS	PKSS	PPSS	SBMB
0.0	15.0	51.0	71.1	36.0
2.0	56.3	69.4	77.8	55.8
4.0	68.5	79.8	94.4	66.9
6.0	77.8	94.4	102.7	83.5

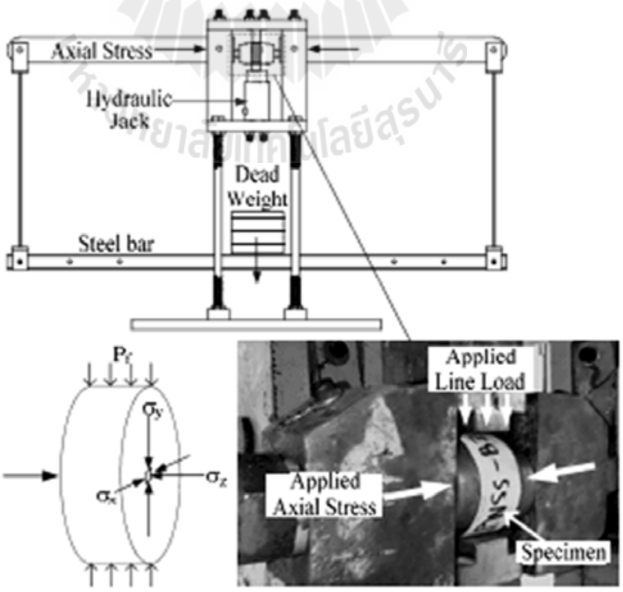


Figure 3. Polyaxial load frame used in Brazilian tension test with axial confinement.

The results indicate that the tensile stresses (σ_x) and compressive stresses (σ_y) induced at the crack initiation point in the middle of the specimen decrease with the increase of axial stress (σ_z), as shown in Figure 4. Assuming that the rocks are linearly elastic and the law of superposition is valid, these stresses can be calculated by (Jaeger & Cook 1979):

$$\sigma_x = -2P/\pi DL \quad (1)$$

$$\sigma_y = 6P/\pi DL = -3\sigma_x \quad (2)$$

$$\sigma_z = \text{Applied axial stresses} \quad (3)$$

where P_f is the failure load, D is the disk diameter and L is the disk thickness. The applied axial stresses are varying from 0 to the uniaxial compressive strength (σ_c).

At the crack initiation point σ_y, σ_x and σ_z represent the principal stresses. The induced tensile stress σ_x is always the minimum principal stress. The magnitudes of the applied axial stress determine whether σ_y or σ_z is the maximum principal stress. Under low applied σ_z, σ_y is the maximum principal stress and σ_z is the intermediate principal stress. Under high σ_z, σ_y becomes the intermediate principal stress and σ_z is the maximum principal stress. The results indicate that the state of stresses where $\sigma_1 = \sigma_2$are positive while σ_3 is negative. Table 3 summarizes the test results obtained from Brazilian tension test under axial compression.

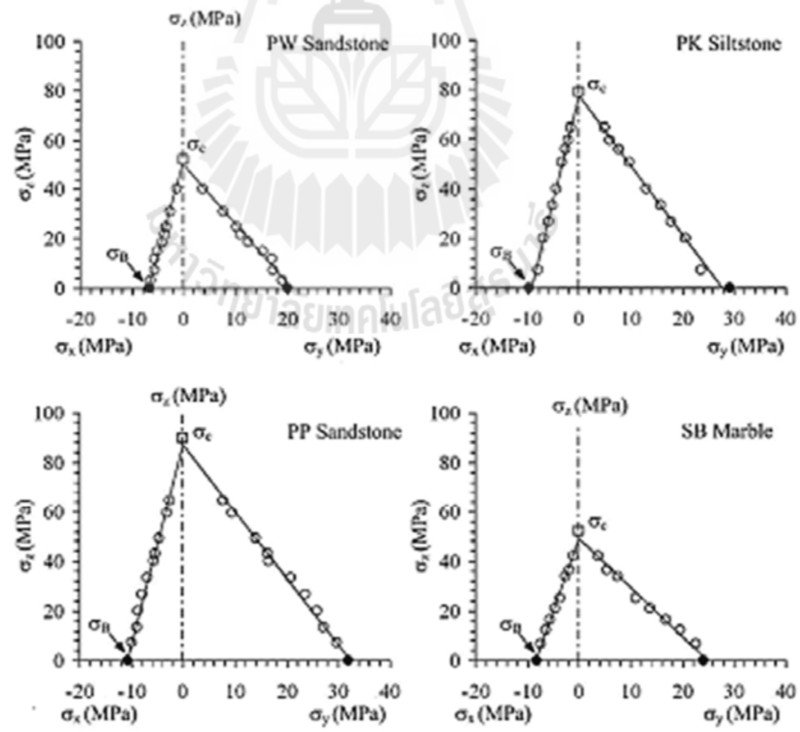


Figure 4.Induced compressive (σ_y) and tensile (σ_x) stresses at failure as a function of applied axial stress (σ_z).

Table 3. Results obtained from Brazilian tension test under axial compression.

PWSS		
σ_x (MPa)	σ_y (MPa)	σ_z (MPa)
0.0	0.0	55.0
-1.3	3.8	40.3
-2.5	7.6	31.0
-3.4	10.1	25.0
-3.7	11.0	21.7
-4.2	12.6	18.3
-5.1	15.4	15.1
-5.7	17.1	11.9
-5.7	17.2	7.2
-6.3	19.0	3.0
-6.7	20.1	0.0

PKSS		
σ_x (MPa)	σ_y (MPa)	σ_z (MPa)
0.0	0.0	78.0
-1.6	4.8	64.6
-2.0	6.1	59.4
-2.6	7.7	55.9
-3.3	9.7	50.7
-4.3	13.0	40.3
-5.3	15.8	33.3
-5.9	17.7	26.7
-6.8	20.5	20.0
-7.9	23.7	7.2
-9.7	29.1	0.0

PPSS		
σ_x (MPa)	σ_y (MPa)	σ_z (MPa)
0.0	0.0	90.0
-2.6	7.7	64.6
-3.1	9.4	59.4
-4.7	14.1	49.0
-5.4	16.3	43.0
-5.5	16.6	40.3
-7.0	20.8	33.3
-7.8	23.5	26.7
-8.6	25.8	20.0
-9.0	27.1	13.4
-9.9	29.7	7.2
-10.7	32.0	0.0

SBMB		
σ_x (MPa)	σ_y (MPa)	σ_z (MPa)
0.0	0.0	52.0
-1.2	3.6	42.2
-1.8	5.4	36.3
-2.5	7.5	33.8
-3.7	11.1	25.2
-4.6	13.8	21.1
-5.6	16.8	16.3
-6.5	19.5	12.5
-7.5	22.5	6.6
-8.0	24.1	0.0
-	-	-
-	-	-

Post-failure observations show that under low σ_z a single splitting extension crack along the loading diameter is normally induced in the disk specimen. Multiple extension cracks are developed as σ_z increases. When σ_z reaches the uniaxial compressive strength of the rocks, the specimens fail without applying the diametral line load. At this point the specimens are crushed, resulting in multiple shear fractures and extension cracks (Figure 5). It is postulated that the axial stress produces tensile strains perpendicular to its direction due to the effect of the Poisson's ratio. The line load at failure therefore decreases with increasing σ_z.

3.4 Four-point Bending Tests

The test arrangement for the four-point bending is shown in Figure 6. The results from this test method provide the rock tensile strength for the condition where $\sigma_1 = \sigma_2 = 0$ and $\sigma_3 = \sigma_{bend}$ is negative. The 150×300×20 mm^3 rock samples bear on the steel rollers with 8 mm in diameter, which are located at the base L = 100 mm. A press load is applied to the identical rollers, placed at the distance g = 40 mm, through the spherical bearing. The tensile stresses (σ_{bend}) in the bottom surface of the bar can be calculated as (Efimov, 2011):

$$\sigma_{bend} = 6M/Bt^2 = 3P_f\,(L\text{-}g)/2Bt^2 \qquad (4)$$

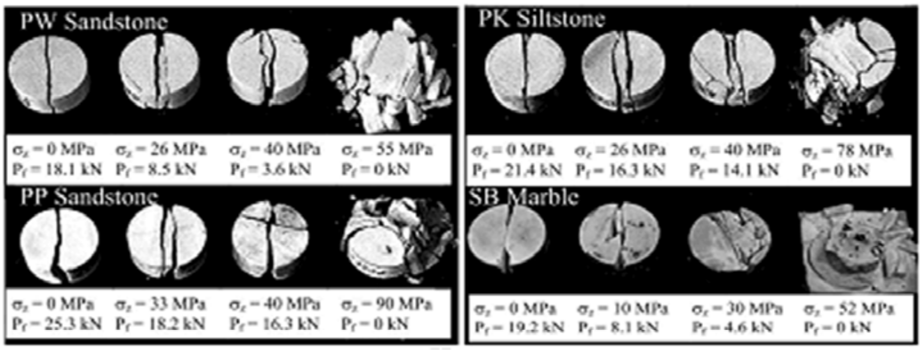


Figure 5. Some post-test specimens from Brazilian test with axial confinement.

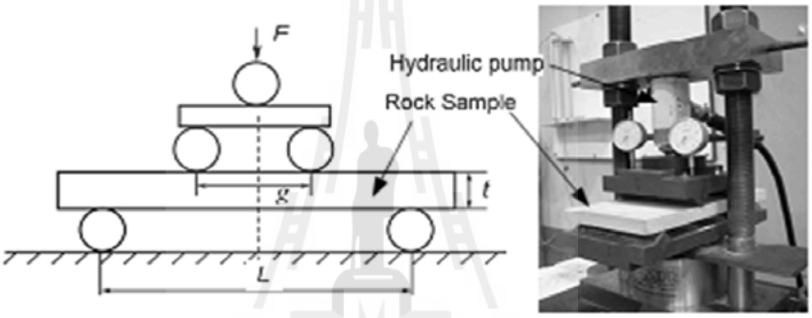


Figure 6.Four-point bending test arrangement.

Where F, M are the maximum values of applied load and moment, respectively, t is the thickness of the sample, B is the width of the sample, and P_f is maximum load as failure.

3.5 *Circular Plate Bending Tests*

The circular disks with diameter of 100 mm are cut and ground to obtain 10-mm thick. The plate bending tests on circular disk determines the tensile strength where $\sigma_1 = 0$ and $\sigma_2 = \sigma_3 = \sigma_t$ are negative. The circular disk is simply-supported around its outer circumference. The line load (P_f) is applied concentrically to the circular disk through a tube of diameter cas shown in Figure 7.

The governing equation for bending behaviour of a symmetric orthotropic thin plate is expressed below (Ugural, 1999) where the tangential and radial tensile strengths (σ_t, σ_r) are equal.

$$\sigma_r = \sigma_t = 3P_f/4\pi t^2[(1-\nu)(1- c^2/a^2) + 2(1 + \nu) \ln(a/c)] \qquad (5)$$

where P_f is maximum load as failure, t is the thickness of circular rock sample, ν is Poisson's ratio, and a, c are the outside and inside diameters. Figure 8 shows the pre- and post-test specimens from circular plate bending tests.

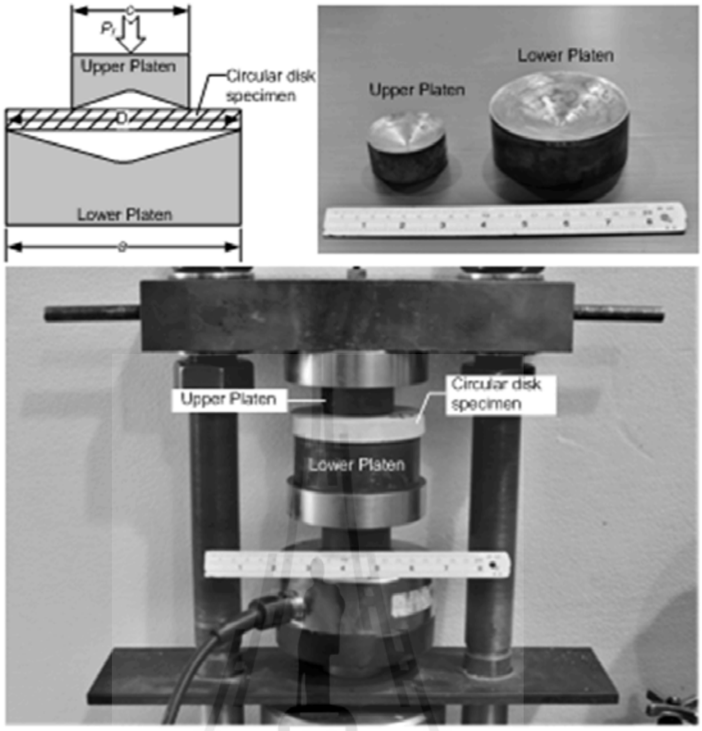


Figure 7.Circular plate bending test arrangement.

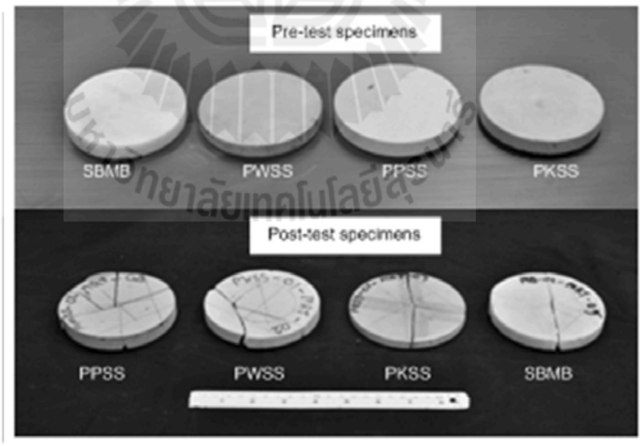


Figure 8. Pre-and post-test specimens from circular plate bending test.

4 STRENGTH CRITERIA

The Coulomb failure criterion is used to describe the rock strength in compressive and tensile regions. First the σ_x, σ_y, and σ_z from the Brazilian tension test under confinement are defined. The condition where $\sigma_1$equal to σ_2 is selected to develop the strength criterion. Table 4 gives the rock strengths from various tests in terms of the principal stresses.

Table 4. Principal stresses obtained from different test methods.

Test Methods		PWSS			PKSS		
		σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)	σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)
$\sigma_1 > (\sigma_2 = \sigma_3)$	TCS	84.0	7.0	7.0	78.0	7.0	7.0
	TCS	75.0	5.0	5.0	69.0	5.0	5.0
	TCS	61.0	3.0	3.0	53.0	3.0	3.0
	TCS	47.5	1.4	1.4	41.0	1.4	1.4
	UCS	38.0	0.0	0.0	30.3	0.0	0.0
	CPB	0.0	-6.6	-6.6	0.0	-7.4	-7.4
$(\sigma_1 = \sigma_2) > \sigma_3$	FPB	0.0	0.0	-8.6	0.0	0.0	-9.4
	MBZ	15.0	15.0	-5.2	20.5	20.5	-6.8
	TXC	56.3	56.3	0.0	51.0	51.0	0.0
	TXC	68.5	68.5	2.0	69.4	69.4	2.0
	TXC	77.8	77.8	4.0	79.8	79.8	4.0
	TXC	90.5	90.5	6.0	94.4	94.4	6.0
Test Methods		PPSS			SBMB		
		σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)	σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)
$\sigma_1 > (\sigma_2 = \sigma_3)$	TCS	80.0	7.0	7.0	75.5	7.0	7.0
	TCS	70.0	5.0	5.0	59.7	5.0	5.0
	TCS	64.0	3.0	3.0	46.1	3.0	3.0
	TCS	56.0	1.4	1.4	29.5	1.4	1.4
	UCS	47.0	0.0	0.0	22.0	0.0	0.0
	CPB	0.0	-7.5	-7.5	0.0	-6.2	-6.2
$(\sigma_1 = \sigma_2) > \sigma_3$	FPB	0.0	0.0	-13.6	0.0	0.0	-8.2
	MBZ	25.8	25.8	-8.6	16.3	16.3	-5.6
	TXC	71.1	71.1	0.0	36.0	36.0	0.0
	TXC	77.8	77.8	2.0	55.8	55.8	2.0
	TXC	94.4	94.4	4.0	66.9	66.9	4.0
	TXC	102.7	102.7	6.0	83.5	83.5	6.0

TCS – Triaxial compression test; FPB – Four point bending test
UCS – Uniaxial compression test; MBZ – Brazilian tension test under axial compression
CPB – Circular plate bending test; TXC – Triaxial extension test

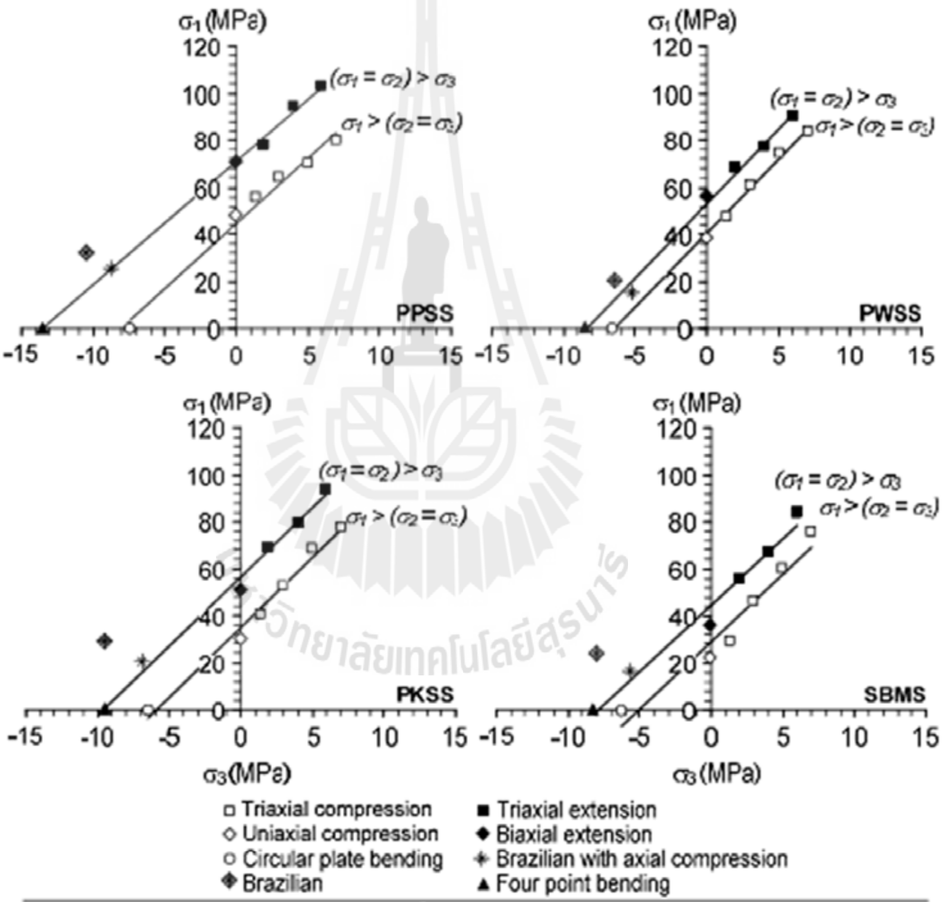
The Coulomb criterion may be written in form of the principal stresses as (Jaeger & Cook, 1979):

$$\sigma_1 = 2 \cdot c \cdot \tan(\alpha) + \sigma_3 \tan^2(\alpha) \qquad (6)$$

$$\alpha = \pi/4 + ½ \phi \qquad (7)$$

where c is the cohesion, ϕ is internal friction angle, and $2 \cdot c \cdot \tan(\alpha)$ is the uniaxial compressive strength of rock. This failure criterion assumes that the intermediate principal stress has no influence on the failure.

Figure 9 presents the test results in the forms of the Coulomb criterion. The stresses in compressive region are separated into two conditions; the triaxial extension ($\sigma_1 = \sigma_2$) and the triaxial compression ($\sigma_2 = \sigma_3$). The uniaxial and triaxial compression test results are correlated with those obtained from circular plate bending test. The results from biaxial and triaxial extension test are correlated with those of the Brazilian tension test under axial confinement and four-point bending test. The linear correlation is extended from the compressive to the tensile region. The results indicate that the Coulomb criterion can well correlate with the compressive and tensile strengths obtained from each stress condition.The Brazilian tensile strength (where $\sigma_1 = 3\sigma_B$ is positive, $\sigma_2 = 0$, and $\sigma_3 = \sigma_B$ is negative) can not be correlated with the failure criterion developed by the two stress conditions.



Coulomb strength criterion:	$\sigma_1 = 2c\tan\alpha + \sigma_3\tan^2\alpha$, $\alpha = \pi/4 + \phi/2$	$(\sigma_1 = \sigma_2) > \sigma_3$		$\sigma_1 > (\sigma_2 = \sigma_3)$	
		c	ϕ	c	ϕ
PPSS		15.5	43	9.4	44
PWSS		10.5	47	8.0	47
PKSS		11.7	45	7.7	44
SBMB		9.3	45	5.8	46

Figure 9. Major principal stress (σ_1) as a function of minor principal stress (σ_3) for various tests.

Hoek, E. & Brown, E.T. 1997. Practical estimates of rock mass strength. *International Journal of Rock Mechanics and Mining Sciences*. 24(8): 1165-1186.

Jaeger, J.C. & Cook, N.G.W. 1979.*Fundamentals of Rock Mechanics*. Chapman and Hall, London, 593 p.

Johnston, I.W. 1985. The strength of intact geomechanical materials.*Journal of Geotechnical EngineeringDivision ASCE*. 111(6): 730-749.

Kim, M.K. & Lade, P.V. 1984. Modeling rock strength in three dimensions.*International Journal of Rock Mechanics and Mining Sciences &Geomechanics Abstracts*. 21(1): 21-23.

Mogi, K. 1971. Fracture and flow of rocks under high triaxial compression.*J. Geophys. Res.* 76: 1255–69.

Sheorey, P.R., Biswas, A.K. &Choubey, V.D. 1989.An empirical failure criterion for rocks and jointed rock masses.*Engineering Geology*. 26(2): 141-159.

Ugural, A.C. 1999. *Stresses in Plates and Shells*, 2nd Edition. WCB/McGraw Hill, Boston, 502 p.

Walsri, C., Poonprakon, P., Thosuwan R. & Fuenkajorn, K. 2009. Compressive and tensile strengths of sandstones under true triaxial stresses.In *Proceeding of the Second Thailand Symposium on Rock Mechanics*.March, 12-13, Jomtien Palme Beach Hotel and Resort, Pattaya, Chonburi, Thailand. pp. 199-218.

Wiebols, G.A. & Cook, N.G.W. 1968.An energy criterion for the strength of rock in polyaxial compression.*International Journal of Rock Mechanics and Mining Sciences &Geomechanics Abstracts*. 5(6): 529-549.

Yudbir, R.K., Lemanza, W. &Prinzl, F. 1983.An empirical failure criterion for rock masses.In *Proceedings of the 5th International Congress Society of Rock Mechanics*, Melbourne, Vol. 1, pp. B1-B8.

BIOGRAPHY

Miss. Pawinee Masingboon was born on October 22, 1986 in Sakon Nakhon province, Thailand. She received her Bachelor's Degree in Engineering (Geotechnology) from Suranaree University of Technology in 2010. For her post-graduate, she continued to study with a Master's degree in the Geological Engineering Program, Institute of Engineering, Suranaree university of Technology. During graduation, 2010-2012, She has published one technical papers related to rock mechanics, titled **"Effects of Intermediate Principal Stresses on Rock Tensile Strengths"** published in the Proceeding of the Fourth Thailand Symposium on Rock Mechanics, Nakhon Ratchasima, Thailand. For her work, she is a good knowledge in geomechanics theory and practice.