TIME-COST TRADE-OFF SCHEDULING UNDER CONSTRUCTION LABOR RESOURCE CONSTRAINTS

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Abstract

Time and cost are the main goals of the construction project management. Planners are searching for the optimal schedules which give both early completion time and small total cost. The scheduling problem named Time-Cost Trade-off (TCT) is attended by previous research. From field surveys conducted in Thailand, contractors are also concerned about the fluctuation of labor resources supply. Most construction labor is seasonal workers who work in agriculture as well. This research, therefore, formulates the TCT model with labor resource constraints. Multi-objective goal programming and binary integer programming are used in the formulation. The mathematical expressions of this model are presented in the paper. Genetic Algorithms is used to search for optimal solutions. This new model is tested with the real data collected. The result shows that the scheduling model with labor resource constraints gives earlier project finish time and less project cost than the one without.

Keywords: Construction Scheduling, labor resource constraints, time-cost trade-off, multi-objective optimization

Introduction

A construction project contains many uncertainties. It requires a number of resources and a large amount of investment. Time and cost are main management goals. Contractors want to get the highest profit so they must plan to complete the job in early time with a minimum cost. Much previous research attempted to formulate and solve the Time-Cost Trade-off scheduling problem for construction projects. The optimal schedule should provide the early completion time with minimum cost.

Various Time-Cost Trade-off (TCT) scheduling models have been proposed in the literature. Some were formulated by using linear programming or integer programming (Perera, 1980; Liu *et al.*, 1995; Moussorakis and Haksever, 2004). These problem models were solved by either the exact method (Perera, 1980) or the approximate method, Genetic Algorithms (Goldberg, 1989). Some models

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characterized the effect of generalized precedence relationships with project network (Elmaghraby and Kamburowski, 1992).

Various types of relationships between activities, such as Start-to-Finish (SF), Finish-to-Start (FS), Start-to-Start (SS) and Finish-to-Finish (FF), can give a direct effect to the project duration. For example, a project network consists of three construction activities, namely; A, B, and C, each of which has the same duration of 4 days. These activities are tied together by FS type of relationship so the project duration of this network is 12 days. If the relationship between A and B is changed to SS type, the project duration will change to 8 days. The research (Chassiakos and Sakellaropoulos, 2005) proposed the TCT model which includes a comprehensive set of activity relationships and time constraints. All four types of activity relationships were formulated as constraints. Also, other activity time constraints were start/finish not earlier than a specified date (SNET/FNET), start/finish not later than a specified date (SNLT/FNLT), and must start/finish on a specified date (MSO/MFO), for example. Their model used a weighted multi-objective function and binary-integer programming. However, the model did not include labor resource constraints which are another main concern of the construction management.

Construction work is labor-intensive. The availability of labor resource can directly affect the project duration and cost. That could result in the project late completion (later than a contracted date) and/or over-budget. Particularly in Thailand, construction labor is seasonal workers who also work in agriculture. The supply of labor resource normally fluctuates seasonally. It can be short at the new crop cycle and at the end. This research proposes a new model which is improved from the recent TCT model. The new model includes various activity relationship types and activity time constraints as found in Chassiakos and Sakellaropoulos (2005). Also, the labor resources are considered as constraints with a limited number of availability.

This research aims to develop the new TCT model by considering the limited labor resources. This new model is formulated using the multi-objective goal programming and the binary integer programming techniques. The approximate optimal solutions of the model are searched using the Genetic Algorithms (GAs). This model can result good schedules in terms of both of project time and cost.

Model Formulation

The new model proposed in this research is detailed into three parts namely the objective function, the decision variables and the constraints. All these parts are explained with mathematical expressions below.

Objective Function

The objective function is used to measure how much a schedule reaches the project goals. We then separate the project goals into four vital aspects i.e. project time, project cost, activity time constraints and labor resources. Therefore, the objective function of this model is multi-objective. For the goal programming method, the term deviation from the setting goal is used as a performance measurement. The objective function is set as the minimization of the summation of the weighted deviation from the managing goals. Each deviation term is assigned to measure one particular aspect and it is then weighted according to contractors' preferences. The deviation terms are values obtained from any feasible schedule that differs from its setting goals. Equation (1) shows the mathematical expression of the weighted multi-objective function.

$$Minimize \quad [W_t d_t + W_c d_c + W_E d_E + W_R d_R] \qquad (1)$$

where W_t, W_c, W_E and W_R = weighted numbers assigned to the four aspects of the multi-objective i.e. project time, project cost, activity time constraints, and labor resources, respectively; d_t, d_c, d_E, and d_R = deviation values of the four aspects of the multi-objective i.e. project time, project cost, activity time constraints, and labor resources,

respectively. Each of these deviation terms is detailed below.

$$d_t = G_t - \max f_i \qquad (2)$$

where d_t = deviation of time aspect; G_t = predefined project duration goal; f_i = finish time of activity i.

$$d_c = G_c - \sum_{i=1}^{H} \sum_{k=1}^{J} c_{ik} y_{ik} \qquad (3)$$

where d_t = deviation of cost aspect; G_c = predefined project direct cost goal; c_{ik} = direct cost of activity i using the execution option k.

$$d_E = G_E - (\sum_{i=1}^{Q} ds_i + \sum_{i=1}^{T} df_i) \qquad (4)$$

where d_E = deviation of activity-time constraint aspect; G_E = predefined activity-time constraint goal; Q = group of activities assigned with start-time constraints (i.e. SNET, SNLT, MSO); T = group of activities assigned with finish-time constraints (i.e. FNET, FNLT, MFO).

$$ds_i = |D_i - s_i| \qquad (5)$$

$$df_i = |D_i - f_i| \qquad (6)$$

where ds_i = deviation of start-time constraints of activity i; D_i = time-constraint assigned to activity i; S_i = star time of activity i; f_i = finish time activity i.

$$d_R = G_R - \max(\sum_{i=1}^{H} R_i)_a \qquad (7)$$

where d_R = deviation of labor resource aspect;

G_R = predefined daily labor goal; $(\sum_{i=1}^{H} R_i)_a$ = total number of labor required (by all activities) on day a.

Decision Variables

In the TCT scheduling model, each construction activity has its own different execution options such as normal and crash options. The activity can be executed in a normal option with predefined duration and cost. The same activity can be executed in a crash option which results in predefined shorter duration and higher cost. Planners need to select execution options of the activities and to trade off between total project time and cost.

The binary-integer programming method is used to formulate this model. A y_{ik} is set as a decision variable and it is a binary-integer value (i.e. 1 or 0). $y_{ik} = 1$ if execution option k is selected for activity i and it equals to 0 if not selected. Where $i = [1, 2, 3, …, H]$ and H is the number of activities in the project network. $k = [1, 2, 3, …, J]$ and J is the number of available execution options (time-cost combinations) of each activity.

Constraint Functions

Constraint functions provide a boundary of the feasible solutions space of the problem model. They define the searching space for the solving algorithms. In this model, some constraint functions are set to relate to the four managing goals. Their mathematical expressions are given in Equation (8) through Equation (16).

$$y_{ik} = 0 \; or \; 1 \qquad (8)$$

$$(\sum_{k=1}^{J} y_k)_i = 1 \quad ; \text{for } i = [1, 2, 3, …, H] \qquad (9)$$

Equation (8) is used to assign all decision variables with a binary value. Equation (9) is used to ensure that each activity can take only one execution option from all available alternatives.

$$D_{\min} \leq \max f_i \leq D_{\max} \qquad (10)$$

$$C_{\min} \leq \sum_{i=1}^{H} \sum_{k=1}^{J} c_{ik} \leq C_{\max} \qquad (11)$$

$$(\sum_{i=1}^{H} R_i)_a \leq G_R \qquad (12)$$

Equations (10) and (11) restrict results of the project finish time and the total direct cost

to their own allowable ranges, respectively. These ranges are derived from the contract documents. Equation (12) sets the upper limit of the number of labor resources required each day. This limitation can be specified according to the availability of labor resources during those seasons.

$$f_i = \max s_p + l_i \qquad (13)$$

$$s_i = \max f_p + l_i \qquad (14)$$

$$s_i = \max s_p + l_i \qquad (15)$$

$$f_i = \max f_p + l_i \qquad (16)$$

Equations (13) to (16) provide the constraints of the activity precedent relationships, SF, FS, SS and FF, respectively. These equations are valid with or without any lag or lead time assigned to an activity relationship. Where l_i = lag or lead time of activity i; s_p = start time of predecessors of activity i; s_i = start time of activity i; f_p = finish time of predecessors of activity i; f_i = finish time of activity i.

$$f_i = s_i + t_{ik} \qquad (17)$$

$$s_i = f_i - t_{ik} \qquad (18)$$

The finish time (f_i) of an activity with FS or SS relationship type is calculated using Equation (17). Equation (18) is used to calculate the start time (s_i) of an activity with SF or FF relationship type, where t_{ik} = duration of activity i using the execution option k.

Genetic Algorithms (GAs) is used as a solution searching tool. In this research, we use a software program called Evolver. This program has been developed by Palisade Corporation. It is an add-ins of Microsoft Excel which is used as a platform of this proposed model. Suitable values of GAs' parameters are initially determined. Previous research reported that the different of both mutation rate and crossover rate does not significantly affect to result of testing (Shen *et al.*, 2009). This research ran a pretest to determine the suitable values of the crossover

rate as 0.50. The mutation rate is set as the automatic number that lets the program choose the right value according to the crossover rate. The 50 of population size and a maximum of 100 generations were assigned. The developed model on Microsoft Excel together with Evolver and these GAs' parameters are then put to the tests.

Testing and Results

A factory building is used as a test case. The construction project includes both structural and architectural work. Project must be completed within 203 days as shown in the project contract. This project uses its own construction labor. The project network consists of 23 activities. Their names and their associated precedence relationships are shown in Table 1. Their execution options including duration, direct cost, and labor required are shown in Table 2. The network diagram of the project is shown in Figure 1.

The model testing was organized into three tests: The number of run-times, the weighting strategy for the multi-objective function, and the effect of labor resource constraint.

The Number of Run-times

GAs is a stochastic searching tool which gives different results for each run time with the same settings. To validate the test results, each test setting was repeatedly run for a number of times. The dispersion of these results was graphed and analyzed. This first test was aimed to determine the suitable number of run-times that can give a steady average result, therefore, this number of run-times would be suggested and used for the other two tests.

Figure 2 shows the graphs of the test results which are separated into three aspects: (a) the average project finish time, (b) the average project direct cost, and (c) the average number of daily labor resources required. Each graph was plotted against the number of run-times. Each sub-figure includes five different lines representing five different test

settings. They are the results from different weightings of the multi-objective function: all equal weighting by assigning 1 on each term, preferable weighting on the term of project finish time, preferable weighting on the terms of project direct cost, activity time constraints, and number of labor resources were assigned by 100 as preferable value.

All graphs tend to be flatter when the number of run-times increases, especially more than 50. The more number of run-times,

the steadier the average result values. This points out that the model can give steady average results from many run-times or more than 50. The average results from repetitive 50 run-times can give the reliability for the analysis and this suitable number of run-times would be used for the other tests.

The Weighting Strategy for the Multi-Objective Function

This model uses the multi-objective

Table 1.   Project activities and their precedence relationships

Activity no.	Activity name	Type of relationship	Activity time constraints
1	Temporary Work , Mobilization		
2	Piling Work (Pre-Bore 14.00 m)	1FS	
3	Footing	2FS-7	
4	Sinking Pit	2SS	
5	Machine Base	4FS	
6	Slab	5FS-7	
7	Steel Wall Frame And Ceiling Support	6FS+54	
8	Gypsum Wall and Ceiling	7FS	
9	M&E Work	8SS+7	
10	Ground Beam And Water Tank (65 m^3)	3FS-7	
11	Ground Floor Slab	10SS+21	
12	1st Column	11SS	
13	2nd Floor Beam ,Slab & PC.Slab	12FS-7	
14	2nd Column And Roof Beam	13FS	
15	Middle Beam & Roof Beam (PC.)	10SS	
16	Steel Structure For Roof & Siding Frame	15FS	
17	Metal Sheet Roofing Work And Siding	16SS+21	
18	Concrete Block, Plastering Work	15FS	
19	Doors & Windows Installation	18FS	
20	Ceiling Work	18FS	
21	Ceramic Tiles Work	18SS+30	
22	Floor Finishing (PVC, Carpet)	21FS	SNET, 1 July'10
23	Painting Work	21SS	

function which combines four different aspects as a summation of four weighted terms. The second test was prepared into five scenario cases which were "all equally weighted" (using weight = 1 for all terms), "preferable weighted on time aspect (using weight = 100), "preferable weighted on direct cost aspect", "preferable weighted on activity time constraint aspect", and "preferable weighted on labor resources". Each of these five scenario cases were repeatedly run for 50 times. Their results were collected and analyzed.

Table 3 shows the best three run-times results of each five scenario cases. Each result shows the three performance measurement values of project duration, project cost, and labor resources required. Since there were

Table 2. Activity's time-cost options

Activity no	Option 1			Option 2		
	Dir. Cost × 10^4 THB	Duration days	Labor man/ days	Dir. Cost × 10^4 THB	Duration days	Labor man/ days
1	205	14	6	209	10	10
2	104	30	5	105	25	8
3	19	30	5	19	25	7
4	87	50	5	91	40	7
5	37	35	5	37	30	7
6	39	14	8	42	12	12
7	24	21	5	24	18	7
8	19	21	4	20	17	6
9	82	30	8	83	26	12
10	29	56	8	30	50	10
11	8	50	8	9	40	10
12	36	21	5	37	18	7
13	120	21	5	122	18	7
14	36	14	5	37	12	7
15	28	56	5	28	50	7
16	130	63	6	132	58	9
17	118	21	6	120	18	9
18	100	50	3	102	45	5
19	178	21	3	180	16	5
20	24	21	4	25	16	7
21	15	29	5	16	22	7
22	16	30	4	17	25	6
23	38	72	3	38	65	6

three performance measurements, it was hard to compare among these results. The total score was established and calculated for the evaluation purpose. First the three performance measurement values were normalized. Each of them was divided by the average value to be the normalized value. Then the total score was the sum of the squares of the three normalized

Figure 1. Factory building project network

values. These total scores were compared.

The result number 15, 3, 12, 6 were slightly different. They were a group of the least total scores and therefore considered as the overall best results. However, these overall best results may not provide the least value of the individual performance measurement. The result number 7 gave the least project duration and it was quite lower than the average results. The result number 14 gave the least project direct cost while four results (number 15, 3, 12, 6) gave the least labor resources. It was noted that the results with the least labor resources were also the overall best results. The value of the labor resources had a strong influence on the overall performance.
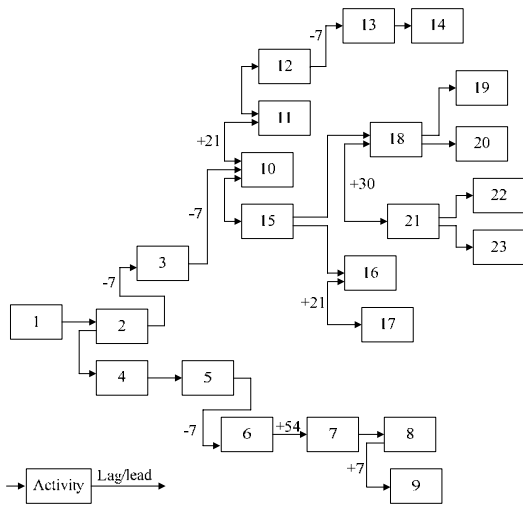
The Effect of Labor Resource Constraint

In this third test, it was divided into two cases which prepared the including and excluding the labor resource constraints (refer to Equation (12)). It is aimed to determine the effective limited labor resource on trade off between time and cost. In the first case, the multi-objective function was set as the preferable weighted on direct cost aspect with number of 75 while the number of 10 and 5
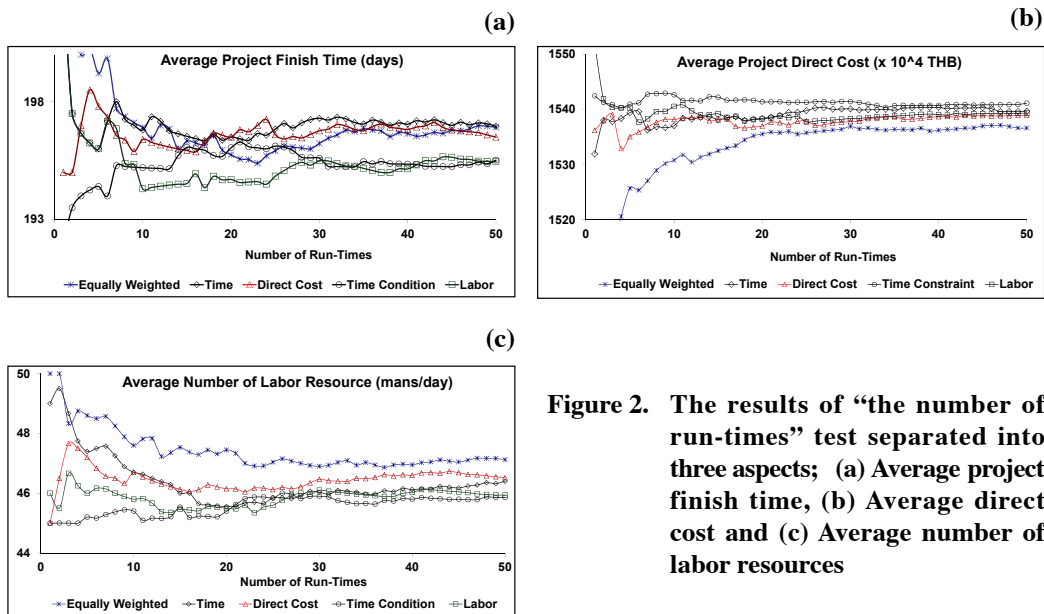

Figure 2. The results of "the number of run-times" test separated into three aspects; (a) Average project finish time, (b) Average direct cost and (c) Average number of labor resources

were respectively set as the weighted on both of activity time constraints and labor resource aspect and project time aspect. In the second case, the weighted on direct cost and project time were set as same as the first case but the time condition was set as different number, 20, in the weighted. The both of cases were also run 50 times. The results from these two cases were then compared to reveal the effect of labor resource constraints on the model. The result of both cases is shown in Figure 3 and the best three of those Pareto optimal solutions were chosen which are shown in Figure 4 (Zheng *et al.*, 2005).

Figure 3 shows that the group of result points of the first case (the model with Equation 12) is on the lower-right position of the second case (the one without). This indicates that the first case, the model including labor resource constraints, provides

better results than the second case. Many schedule results of the first case gives earlier project finish time and lower project direct cost. The best results of the 50 results of the two cases were selected and compared in Table 4.

Figure 4 shows that the model including labor resource constraints (the first case) gives the minimum point which provides project duration 203 days with 1015 × 10^4 THB of project direct cost and 48 workers. The model excluding labor resource constraints (the second case) gives the minimum point which provides project duration 201 days with 1543 × 10^4 THB of project direct cost and 59 workers. It shows that the first case can give a less project duration cost and less number of workers than the second one. However both cases give slightly the same project duration.

Table 3. The result of the weighting strategy for the multi-objective function

Result no.	Weighted on				Resulting			Total score
	Time	Direct cost	Activity time	Resource	Time	Direct cost	Resource	
1	1	1	1	1	185	1,537	49	3.13
2					203	1,516	48	3.24
3					195	1,534	40	2.83
4	100	1	1	1	192	1,515	48	3.12
5					192	1,515	48	3.12
6					195	1,542	40	2.84
7	1	100	1	1	181	1,540	45	2.91
8					203	1,515	47	3.19
9					195	1,535	44	3.00
10	1	1	100	1	185	1,536	45	2.94
11					195	1,528	44	2.99
12					195	1,534	40	2.83
13	1	1	1	100	185	1,542	45	2.95
14					203	1,514	47	3.19
15					195	1,532	40	2.82
				Average	193.3	1529.0	44.7	

The results from the two cases were also compared using ANOVA. The null hypothesis of ANOVA (H_0) was assigned as "The number of labor resource constraint does not affect the results so the two cases should give similar results". The result of ANOVA is shown in Table 5. It shows that the value of all results is higher than F_{crit} value. This fact can reject the assumption of H_0 and indicates that the results from two cases are indeed different. It hence can be deduced that the labor resource constraint has an effect on the results of the model.

The results from the multi-objective goal programming cannot be the best values in all aspects simultaneously. If planners prefer one particular aspect to the others, they must accept to sacrifice the others. Finally, it depends on planners to decide which solution on the Pareto front they like the most.

Conclusions

The new TCT model is formulated using the multi-objective goal programming and binary integer programming by considering labor resource constraints. GAs is used to search for the optimal schedules regarding project time, project cost, activity time constraints and labor resources aspects. The multi-objective function which is formulated accordingly can compact the size of the model and reduce the number of equations required. Also, GAs is an efficient solution searching tool which can give many good results within reasonable run time.

The new model developed in this research is tested. The testing is organized into three parts such as the number of run-times, the weighting strategy for the multi-objective
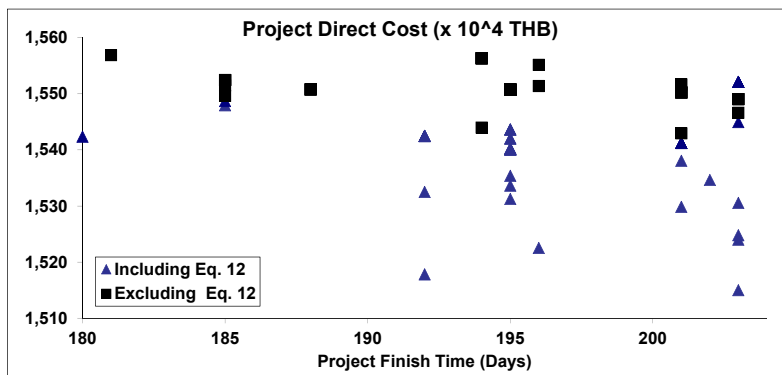

Figure 3. The scattering results of project direct cost against project finish time in case of including and excluding the labor resource constraints

Table 4. The selected best results of two cases

Target	Labor resource constraints	
	Including equation (12)	Excluding equation (12)
Time	203	201
Direct Cost	1,515	1,543
Labor	48	59
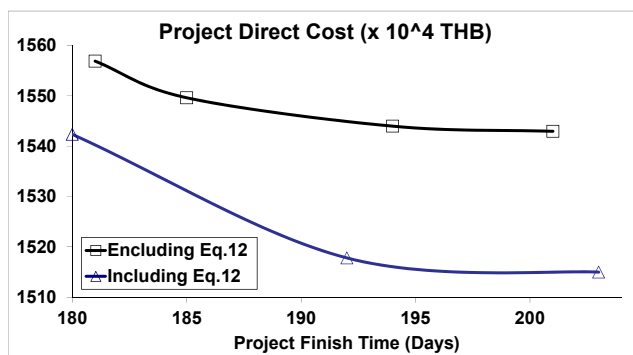
Figure 4. The optimal Pareto frontier was obtained by TCT model

Table 5.   ANOVA results

Result	The result of ANOVA	
	F value	**F_{crit} value**
Time	4.83	3.94
Indirect Cost	83.14	3.94
Labor	369.67	3.94

function, and the effect of labor resource constraint. It is found that the average results from different run-times will be steady after 50 runs. The higher weighted on the project cost term of the multi-objective function will give the shorter project duration. The ANOVA results also proved that the model including labor resource constraints gives better schedule results which have earlier project finish time and less project direct cost.

This model is a helpful tool for contractors who rely on seasonal construction workers and occasionally have a labor resource problem. This model can provide many good (near optimal) schedule regarding project duration and direct cost aspects under labor resource constraints. Contractors can adjust the number of labor resource available and let the model result with good schedules according to the current labor availability. Therefore, they are able to plan their construction projects and then successfully manage them within the allowable duration and budget.

The model in this research was developed on Microsoft Excel so the limitation of this model is the number of the Columns available in an Excel Sheet. This can restrict the number of project time units. Also, this model using the stochastic solving method (GAs) must rely on a number of running-time results for generalization. This requires a few hours to get the final result.

For the future research, this model could be extended to include other managerial aspects such as project cash flow and resource leveling.

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