DESIGN OF POWER SPECTRUM DENSITY MONITORING SYSTEM USING OPTIMAL SLIDING EXPONENTIAL WINDOW TECHNIQUE

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Abstract

Under the restriction of the frequency usage license, there are limited radio frequency ranges that can be used. However, the cost and equipments for monitoring and detecting the violation of licensed frequency bands can be expensive and complicated. Therefore, this paper proposes the design of Power Spectral Monitoring System for monitoring and detecting the signal whose frequency exceeds the allowable range. The main focus for the design is to introduce the efficient power spectrum density computation. To achieve that, we apply sliding DFT computation method which is faster than calculating entire FFT or DFT. Also, we included exponential window, and compared the results between the two sliding DFT methods. The proposed Power Spectral Monitoring System has many advantages over the traditional system based on the usage of spectrum analyzer. This includes a lower cost in determining and detecting excess frequency ranges. In addition, the system uses software implementation which is relatively cheaper and easier to reconfigure than hardware counterpart. The proposed system is also faster in term of processing speed because it uses modified and enhanced algorithms to shorten the detection process.

Keywords: Discrete Fourier Transform (DFT), Power Spectral Density (PSD), exponential window, sliding DFT

Introduction

Power Spectrum Monitoring System plays an important role in detection of public frequency violation. Traditionally, in order to control radio frequency ranges, one need high quality instruments such as spectrum analyzer and wideband receiver to monitor and detect frequency bands which are used to transmit signal. At the same time, the instruments should also be able to envelop the frequency bands. The problem in monitoring and detecting the illegal frequency bands is the lack of equipments used in searching from those frequencies. In addition, the cost of the importing spectral analyzer is quite high, which impedes the ability to monitor the radio frequency. Therefore the ability to scan and estimate the radio frequency is limited to a small

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area.

The Power Spectrum Monitoring System is designed for detecting the frequency range higher than permitted range under the law. The design system will consist of four main parts which are wideband antenna, radio frequency front-end, A/D converter, and LabVIEW program in computer as shown in Figure1.

The function of wideband antenna will receive the signal and then transfer the signal to the RF front-end part for IF downconversion before being analog-to-digital conversion which will be more suitable for processing by LabVIEW program. To verify whether the frequencies of the received signal are out of the certain specified range, the appropriate method must be first verified via MATLAB simulation, and then the verified algorithm will be transferred onto LabVIEW program. Finally, the LabVIEW software will be connected to A/D converter and the remaining two parts for processing.

Hence, the most preferable algorithms must initially be determined. Two approaches are considered, namely parametric approach, and nonparametric approach. In the parametric approach, the signal is assumed to be contaminated by White noise. To do such estimation requires evaluating the coefficient of the filter (system transfer function) correctly. While nonparametric methods such as the periodogram and its improvements, they are based on the idea of estimating the autocorrelation sequence of a random process from measured data, and then taking the FFT to obtain an estimate of the power spectrum. (Aboy *et al.*, 2004)

So, we aim at improving computationally efficient algorithms for evaluating DFT. Since 1965, the Cooley-Tukey FFT and its various form, such as the algorithms of Singleton, have had tremendous influence on the use of DFT in convolution, correlation, and spectrum analysis. In 1990s, split-radix FFT (SRFT) algorithms and mirror FFT and phase FFT are proposed by Duhamel and Price, respectively. Also, the exploitation of symmetry properties in the data to reduce the computation time by Swarztrauber

The recognition that the DFT can be arranged and computed as a linear convolution is also highly significant. On one hand, Goertzel indicated that the DFT can be computed via linear filtering, although the computational savings of this approach is rather modest. On the other hand, Bluestein demonstrated that the computation of the DFT can be formulated as a chirp linear filtering operation. (Proakis and Manolakis, 2007).

In Jacobsen and Lyons (2003), a sliding DFT process, whose spectral bin output rate is equal to the input data rate, is described. This method does not make any assumption, unlike the parametric methods. Also, this approach will provide better frequency solution than nonparametric method as no estimation required.

The sliding technique uses collect set of data for observation. The article prove that the computational complexity of each successive N-point output is O(N) for the sliding DFT compared to $O(N^2)$ for the DFT and $O[N\log_2(N)]$ for the FFT. Hence, sliding DFT requires fewer computations than the Goertzel algorithm for real-time spectral analysis and the traditional radix-2 FFT. Low complexity in DFT computation will lead to low runtime for detecting process, which is our main expectation.

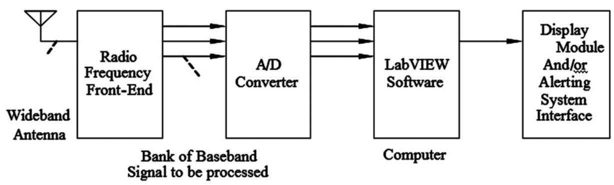

Figure 1. Overall power spectral monitoring system block diagram

Additionally, Jacobsen and Lyons (2003), it shows the effect of time-domain windowing on spectral leakage reduction by introducing the DFT of Hanning window. Therefore, this paper extends the idea of sliding DFT method in Jacobsen and Lyons (2003), but replaces rectangular and Hanning window with exponential window.

The sliding DFT technique is very attractive in many applications. In Brookes *et al*. (2006), the recursive algorithms is used to calculate group delay of voice signal. In Hayashi and Sakai (2004), New per-tone equalization methods using sliding DFT for single carrier block transmission with cyclic pre-fix (SC-CP) systems is proposed. Also Sozanski (2004, 2005), they reveal that the technique is well suited for other applications in power electronics where spectrum analysis is necessary, for example: power spectrum analyzers, energy meters etc.

The organization of this paper is as follows. First, the main result regarding the modified nonparametric PSD estimation is proposed and introduces several efficient computations of the DFT. It also shows the derivation of the update algorithm via optimal exponential window. Next, the experiment result is shown before the conclusion.

Main Results

General Power Density Spectrum Estimation

The nonparametric PSD estimation method makes no assumption about how the data were generated. This method emphasis on obtaining a consistent estimate of the power spectrum through some averaging or smoothing operations performed directly on the periodogram or on autocorrelation, which may reduce the frequency solution further, while the variance of the estimate is decreased. Some examples are the Bartlett method, the Welch method, and the Blackman and Turkey method. These nonparametric methods are relatively simple, well understood, and easy to compute using the FFT algorithm.

However, these methods require the availability of long data records in order to obtain the necessary frequency resolution required in many applications. There are also spectral leakage effects, due to windowing, that are inherent in finite-length data records, which will masks weak signals that are present in the data. The basic limitation of the nonparametric methods is the inherent assumption that the autocorrelation estimate is zero for $|m| \geq N$, where m is number of data points and N is window size. This will severely limits the frequency resolution and the quality of the power spectrum estimate that is achieved.

The second method for determining power density spectrum is the parametric method which does not require assumptions as it extrapolate the values of the autocorrelation for lags $|m| \geq N$. A model for the signal generation can be constructed with a number of parameters that can be estimated from the observed data in which will be able to compute the power density spectrum implied by the model. The modeling approach eliminates the need for window functions and the assumption that the autocorrelation sequence is zero for $|m| \geq N$. As a consequence, parametric power spectrum estimation methods avoid the problem of leakage and provide better frequency resolution than does the FFT based. This is especially true in applications where short data records are available due to time-variance. Some examples are Yule-Walker method and Burg Method for AR Model parameters, (Proakis and Manolakis, 2007).

The modified method propose by this system is the sliding DFT method. This method does not make any assumption, unlike the parametric methods. It uses the sliding technique to collect set of data for observation. The initial group of data will sliding to the next set of data for calculation, and the process will carry on until it obtains the number of set of data its required. This approach will provide better frequency solution than nonparametric method as no estimation required.

Modified Nonparametric PSD Estimator: Sliding DFT

In a receiver, the sampled input data, $x(n)$ may or may not contain a man-made signal.

A common way to window the input data, e.g. by rectangular window or by exponential window, are from $x(0)$ to $x\,(N - 1)$, then from $x\,(N)$ to $x\,(2N - 1)$, and so on. This non-overlapped windowing saves processing time, but a signal may be divided into two data groups; thus, great variation of power spectrum may occur; the sensitivity and frequency resolution of the receiver are also low. One method of improving this approach is to process the data with some degree of overlapping that will double the required processing, but with better the receiver performance and smoother power spectrum.

Proposed by Springer (1988), the most extreme approach (largest overlap region) is to slide the DFT by one point at a time, (Tsui, 2001). So, when a sliding DFT processes, spectral bin output rate is equal to the input data rate, (Jacobsen and Lyons, 2003). For example, the first group of input data contains to, the second group contains to, and the third group to , as shown in Figure 2, where $x_j(n) = j$ -th group of N-points sample data.

$$x_j(n) = [x\,(j)\ \ x(j+1) \ldots\ x\,(j+N-1)]. \quad (1)$$

Sliding DFT with Rectangular Window

In this section, the derivation of sliding DFT is explained. Initially, the definition of $x\,(n)$ must be expressed as followed.

$$x\,(n) = \begin{cases} x_a\,(nT) & \text{where } -N \le n \le N \text{ and } T = \text{Sample Period} \\ 0 & \text{where } n \text{ is otherwise} \end{cases} \quad (2)$$

Let $x\,(n)$ = sampled input data windowed by rectangular window size N

$x_a\,(t)$ = input data before sampling

Next, let us consider the Equation of DFT with L -frequency points

$$X_0(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{k}{L} n} = x(0) + x(1) e^{-j2\pi \frac{k}{L}(1)} + x(2) e^{-j2\pi \frac{k}{L}(2)} + \ldots + x(N-1) e^{-j2\pi \frac{k}{L}(N-1)}, \quad (3)$$

where $X_0\,(k)$ represents the DFT of the first set of data at the k - th frequency point.

By inspection, the second set of data, i.e. $X_1\,(k)$, will generate a DFT which is exactly the same equation except in slightly different term

$$X_1(k) = \sum_{n=0}^{N-1} x(n+1) e^{-j2\pi \frac{k}{L} n} = x(1) + x(2) e^{-j2\pi \frac{k}{L}(1)} + x(3) e^{-j2\pi \frac{k}{L}(2)} + \ldots + x(N) e^{-j2\pi \frac{k}{L}(N-1)} \quad (4)$$

The only difference between (3) and (4) is the set of data points. By arranging the Equations (3) and (4) in series format (Tsui, 2001), the simplification between (3) and (4) can be written as

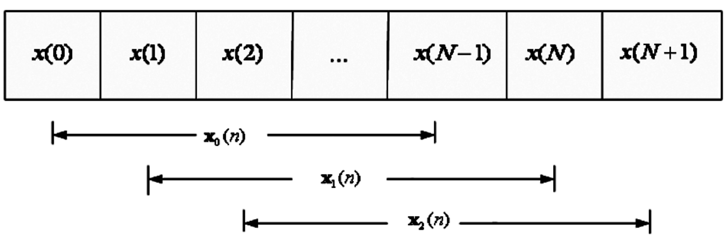

Figure 2. Process of grouping sample for sliding DFT

$$X_1(k) = [X_0(k) - x(0) + x(N)]e^{+j2\pi\frac{k}{L}} . \quad (5)$$

From Proakis and Manolakis (2007), the fact that

$$P_{xx}(k) = \frac{1}{L}\left|\sum_{n=0}^{N-1} x(n)e^{-j2\pi\frac{k}{L}n}\right|^2 = \frac{1}{L}|X(k)|^2, \quad (6)$$

where $X(k)$ is DFT of the sample sequence $x(n)$.

$$P_{xx}(k) = \frac{|X(k)|^2}{L} . \quad (7)$$

So,

$$P_{xx}(k) = \frac{X(k)X^*(k)}{L} , \quad (8)$$

wher $X^*(k)$ is the conjugate pair of $X(k)$.

$$P_{x_1x_1}(k) = \frac{X_1(k)X_1^*(k)}{L} \quad (9)$$

$$P_{x_1x_1}(k) = \frac{1}{L}\{(X_0(k) + x(N) -$$

$$x(0))(X_0(k) + x(N) - x(0))^*\} , \quad (10)$$

$$P_{x_1x_1}(k) = \frac{1}{L}\{(X_0(k) + x(N) -$$

$$x(0))(X_0^*(k) + x(N) - x(0))\} \quad (11)$$

Let $A = x(N) - x(0)$, which is real number.

$$P_{x_1x_1}(k) = \frac{X_0(k)X_0^*(k)}{L} +$$

$$\frac{1}{L}\{A(X_0(k) + X_0^*(k)) + A^2\}, \quad (12)$$

$$P_{x_1x_1}(k) = P_{x_0x_0}(k) + \frac{A}{N}[\mathrm{Re}(X_0(k)) + A]$$

$$= P_{x_0x_0}(k) + \frac{(x(N) - x(0))}{N}$$

$$[\mathrm{Re}(X_0(k)) + (x(N) - x(0))] . \quad (13)$$

Clearly, when the new arrival sample comes, the new power density spectrum is varied by the term $\frac{(x(N) - x(0))}{N}$ [Re $(X_0(k))$ + $(x(N) - x(0))$] that is simpler than re-calculating the entire Discrete Fourier Transform. Despite its simplicity, the sliding DFT still suffer from the Gibbs phenomenon where the over- and undershoot around the discontinuous frequency points may occurs. To avoid, this type of distortion, the proposed of using window technique is proposed. However, to employ the efficiency of sliding technique, the exponential window is used and optimal window parameter is determined.

Sliding DFT with Exponential Window

By inspection, the method sliding DFT can be achieved if DFT can be expressed in term of power series. Likewise, the DFT of the sampled signal which is windowed by exponential window can be possible.

The exponential window is used for the transient signals measured with impact testing and burst random excitation. Used properly, the exponential can minimize leakage errors on lightly damped signals and can also improve the signal-to-noise of heavily damped signals, (Fladung and Rost, 1997).

In this paper, exponential window is defined by the Equation (14)

$$w_{EXP}(n) = \begin{cases} b^{-|n|} & \text{where } -N \le n \le N \text{ and } b > 1 \\ 0 & \text{where } n \text{ is otherwise} \end{cases} \quad (14)$$

In general, the idea to select the window is the main lobe width should be narrow whereas the side lobe height should be as low as possible. So based on Table 1 showing the relationship between main lobe width and side lobe height for different values of b, if b^{-1} is decreased, even though the side lobe height is lower, the main lobe width is also wider. So, the group of b^{-1}s producing narrowest main lobe width are selected ($b^{-1} = 1$ is not included, since it will be rectangular function), then the b^{-1} that makes lowest possible side lobe height chosen. That is why it is 'optimized'

According to Table 1, the value $b^{-1} = 0.9985$ or $b = 1.0015$ is then selected since main lobe width is very close to rectangular window (best) whereas side lobe height is lower than rectangular type for almost 10 dB. Figures 3 and 4 show the time-domain plot and the plot of frequency response between rectangular and exponential window. Clearly, even though exponential window possesses wider main lobe width than rectangular window does, its side lobe height is lower which is beneficial for leakage.

Let $y(n)$ = the signal segmented by exponential window $w_{EXP}(n)$

So, $$y(n) = w_{EXP}(n)\, x(n) \tag{15}$$

Let $$Y(k) = DFT\{y(n)\} \tag{16}$$

Table 1. The variation of main lobe width (sample) and side lobe gain (dB) while changing exponential window parameter *b*

b^{-1}	b	One-sided main lobe width (sample)	Side lobe height (dB)
1.0000	1.0000	4	-13.47
0.9990	1.0010	5	-17.63
0.9987	1.0013	5	-19.49
0.9985	1.0015	5	-21.02
0.9980	1.0020	8	-20.19
0.9975	1.0025	8	-20.27
0.9970	1.0030	8	-20.12
0.9960	1.0040	14	-27.21
0.9950	1.0050	15	-26.16

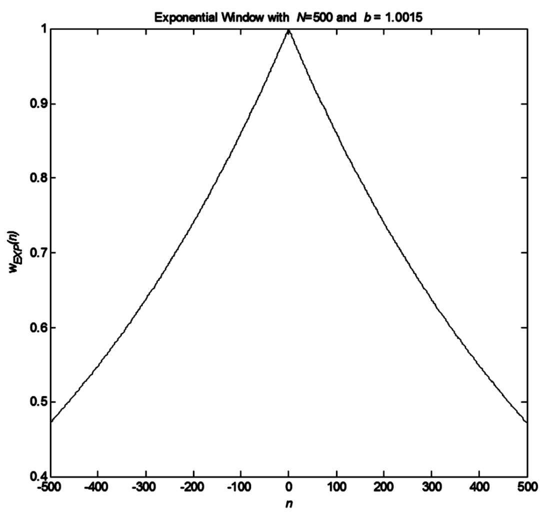

Figure 3. Exponential window in time domain with optimal value *b*

Hence, $Y(k)$

$$= \sum_{n=-N}^{N} y(n)\, e^{-j\frac{2\pi k}{L}n} \cdot e^{-j\frac{2\pi Nk}{L}}$$

$$= \Big(\sum_{-N}^{-1} x(n)\, b^{n}\, e^{-j\frac{2\pi k}{L}n} \cdot e^{-j\frac{2\pi Nk}{L}}\Big) + \Big(\sum_{0}^{N} x(n)\, b^{-n}\, e^{-j\frac{2\pi k}{L}n} \cdot e^{-j\frac{2\pi Nk}{L}}\Big)$$

$$= \Big[e^{-j\frac{2\pi Nk}{L}}\Big]\Big[\Big(\sum_{-N}^{-1} x(n)\big(b e^{-j\frac{2\pi k}{L}}\big)^{n}\Big) + \Big(\sum_{0}^{N} x(n)\big(b^{-1} e^{-j\frac{2\pi k}{L}}\big)^{n}\Big)\Big] \quad (17)$$

Then, let $\lambda = e^{-j\frac{2\pi Nk}{L}}$, (18)

$$u = b e^{-j\frac{2\pi k}{L}} , \quad (19)$$

and $v = b^{-1} e^{-j\frac{2\pi k}{L}}$ (20)

Consequently, $Y(k)\lambda^{-N}$

$$= \Big[\Big(\sum_{-N}^{-1} x(n)u^{n}\Big) + \Big(\sum_{0}^{N} x(n)v^{n}\Big)\Big] \quad (21)$$

Notice that there will be two parts in each DFT computation, i.e. summation from $-N$ to -1 and that from 0 to N. In other words, there are negative side and positive side summation.

For simplicity, let

$$Y^{-}(k) = \sum_{-N}^{-1} x(n)u^{n} \quad (22)$$

$$Y^{+}(k) = \sum_{0}^{N} x(n)v^{n} \quad (23)$$

$Y_0(k)$ = DFT of the former vector $y(n)$

$Y_1(k)$ = DFT of the vector $y(n)$ after arrival of the new sample

So,

$$Y_0(k)\lambda^{-N} = Y_0^{-}(k) + Y_0^{+}(k)$$

$$= \Big[\Big(\sum_{-N}^{-1} x(n)u^{n}\Big) + \Big(\sum_{0}^{N} x(n)v^{n}\Big)\Big] \quad (24)$$

$$Y_1(k)\lambda^{-N} = Y_1^{-}(k) + Y_1^{+}(k)$$

$$= \Big[\Big(\sum_{-N+1}^{0} x(n)u^{n}\Big) + \Big(\sum_{1}^{N+1} x(n)v^{n}\Big)\Big] \quad (25)$$

Base on the same method of sliding DFT in rectangular window, the relationship between $Y_0^{-}(k)$ and $Y_1^{-}(k)$ can be shown below.

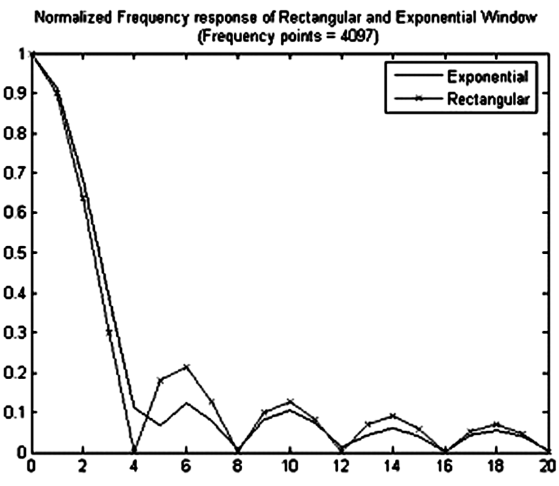

Figure 4. Rectangular window and exponential window plotted in frequency domain

$$Y_1^-(k) = u^{-1}[Y_0^-(k) + x(0) - x(-N)u^{-N+1}] \quad (26)$$

$$Y_1^+(k) = v^{-1}[Y_0^+(k) - x(0) + x(-N + 1)v^{-N+1}] \quad (27)$$

Finally, power spectral density of the later vector $y(n)$ is

$$P_{y_1y_1}(k) = \frac{|Y_1(k)|^2}{L} = \frac{|\lambda^N(Y_1^-(k)+Y_1^+(k))|^2}{L}$$

$$= \frac{|Y_1^-(k)+Y_1^+(k)|^2}{L} \quad (28)$$

Obviously, the proposed equation, called sliding DFT with exponential window equation, reduces number of computation greatly compared to the direct DFT with exponential window. Despite consuming more computations than sliding DFT with rectangular window, exponential window returns more accurate PSD in strong noise situation. And the experiment result is discussed in the next section.

Results and Discussion

In this section, the test signal before sampling is defined by following Equation

$$x(t) = [e^{dt}(\sum_{j=1}^{K} A_j \sin(\omega_j t^c + \phi_j))] + (B \cdot \eta(0,\sigma)),$$

$$\text{where} \quad c \neq 0 \quad \text{and} \quad d \leq 0 \quad (29)$$

where η (0, σ) is AWGN with zero mean and σ is the standard deviation

Also, the method to measure the efficiency of the proposed method is expressed. Through the mean-square error (*MSE*) compared with the PSD of the noise-free initial signal, the sliding DFT according to both rectangular window and exponential window are investigated.

$$MSE = \frac{\sum_{j=0}^{L}(PSD_{s,j} - PSD_{f,j})^2}{L} \quad (30)$$

where $PSD_{f,j}$ = j-th power spectral density of noise-free signal

$PSD_{s,j}$ = j-th power spectral density of windowed signal

To compare the errors between the two approaches, the ratio in Equation (31) will be plotted for each SNR.

$$\kappa = \frac{MSE_{RECT}}{MSE_{EXPO}} \quad (31)$$

Thus, if κ is more than 1, error in rectangular is larger than in exponential window.

Firstly, the most specific case is observed, i.e $c = 1$ and $d = 0$. In this case, the tested signal will be a normal sinusoidal signal contaminated

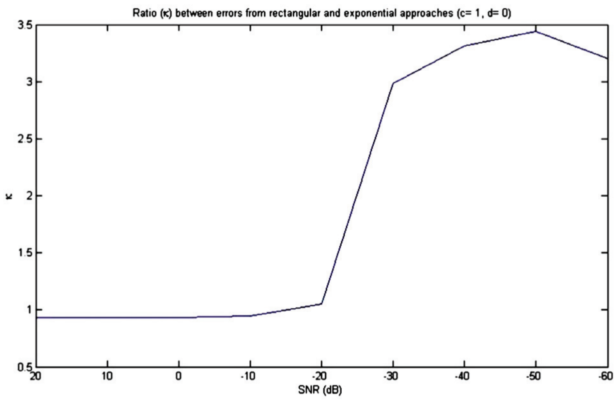

Figure 5. The ratio of Mean-square errors of the two approaches when $c = 1$ and $d = 0$ (specific case)

with AWGN. After that, the error of PSD are analyzed while the factors andare varied. In this section, two results for c = 1. and d = 0 and (specific case) and c = 3 and d = -300 cases are displayed.

From the results in Figures 5 and 6, it is apparent that rectangular window causes marginally lower error when SNR is high, whereas it makes error far greater than exponential approach when SNR is low.

Conclusions

In this paper, the update algorithms for power spectral density (PSD) estimation to approach real-time operation by applying the method of modified sliding DFT, for both rectangular window and exponential window are derived. The corresponding experiment result shows that the sliding exponential window approach with optimal window parameter b carefully chosen can outperform the conventional sliding DFT (rectangular window) especially in low SNR environment. Furthermore, even though sliding DFT with rectangular window requires fewer computations than sliding DFT with exponential window, the approach accumulates errors more than the latter approach under strong-noise situation.

Despite consuming lower operation run-time, leading to detection process shortened, compared to direct DFT method, sliding DFT will accumulate small error leading to error in output PSD. Therefore in the future improvement, the pre-defined condition for recalculating the initial value DFT or FFT may be required. In practical implementation, our work can be extended by feeding the real signal from signal generator through A/D converter and transfer this proposed algorithms to DSP board via LabVIEW or FPGA board via VHDL.

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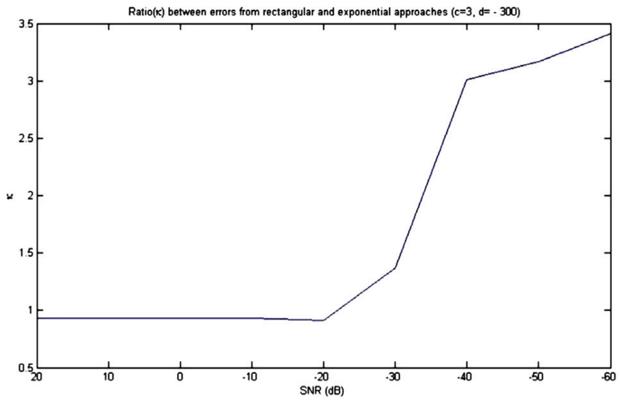

Figure 6. The ratio of Mean-square errors of the two approaches when $c = 3$ and $d = -300$ (an example of general case)

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