QUARK CONFINEMENT STUDY FROM SU(2) LATTICE GAUGE THEORY†

Songvudhi Chimchinda

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Abstract

Nowadays, we know that the constituent particles within any nucleon are the bound states of quark and gluon. The confinement scenarios of this process are still mysterious. However, most physicists believe it can be explained by quantum chromodynamics(QCD) at low energy sector(Infrared). Lattice gauge theory is one of the tools to study this low energy QCD by practically doing simulation. One scenario of confinement is study through Coulomb gauge lattice in which confinement information is known by studying gluon propagator, ghost form factor and Coulomb potential. Those three objects are measured by Monte Carlo simulation and implementing Coulomb gauge fixing by relaxation method. Now, we get the results of gluon propagator, ghost form factor, and Coulomb potential.

Keywords: Lattice QCD, coulomb gauge, Yang-Mills theory

Introduction

Quantum chromodynamics (QCD) is a Yang-Mills theory with gauge group *SU(N)*. Physicists believe that it can describe the strong interaction of quark and gluon within nucleon. Since free quarks cannot be observed in nature as separate entities, this research called them confinement of quarks. Lattice QCD is now a successful simulation method of QCD. It can be mapped with *Yang-Mills* theory, and has been studied to understand the mechanism of the confinement and QCD at low energy sector (infrared regime). The two previous mechanisms are the dual Meissner effect coming from the magnetic monopole and vortex condensation Voigt *et al.* (2008).

In the realistic case of baryon consideration is preformed using *SU(3)* gauge theory. In this work we use *SU(2)* for considering the two color QCD because the properties of *SU(2)* and *SU(3)* are equivalence at low-energy Cucchieri and Mendes (2007), and one advantage of considering *SU(2)* is the ability to reduce the numerical burden. For each group, is the gauge transformation which introduces the additional degrees of freedom, the *gluons*, which are responsible in QCD for the transmission of the strong interaction between quarks. The study of Green function (gluon function, or correlation function) in Coulomb gauge provides the potential of two static colored charges which is

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School of Physics, Institute of Science, Suranaree University of Technology, 111 University Avenue, Nakhon Ratchasima 30000, Thailand. Tel.: 0-4422-4319, Fax: 0-4422-4185. E-mail: songvudhichim@hotmail.com
* *Corresponding author*

Gribov's confinement scenario Cucchieri and Mendes (2007). Gribov pointed out that there exist many configurations of satisfied Coulomb gauge condition, this is called "gauge copies" or *Gribov* copies.

Nowadays, only little information Cucchieri and Zwanziger (2001); Langfeld and Moyaerts (2004); Quandt *et al.* (2007); Voigt *et al.* (2008) we know about this gauge. Our research is to simulate lattice QCD in Coulomb gauge at zero and finite temperature. We found some behavior of gluon propagator and ghost form factor in infrared regime, and hope to find out the feature of Coulomb potential in the future soon. In the present paper of this conference about the using of grid computing, in Section 2, we mention about the method to simulate our system reaching into thermal equilibrium according to classical Monte Carlo simulation. In section 3, we describe how to implement Coulomb gauge fixing into lattice. In section 4, we mention about the measurable observable, gluon propagator. Any measured value must be renormalized for being continuum value, this is used by virtue of renormalisation which mention in Section 5. Another observable quantity, ghost form factor which gives us color charge density, is in Section 6. Later, we talk about Gribov copies influence in Section 7. We can also investigate at finite temperature as mentioned in Section 8. We will talk about the recent result of Coulomb potential in Section 9.

Heatbath Algorithm

Kenneth Wilson, the first person who proposed the formulation of gauge theory by quantizing space-time into lattice in 1974 (Wilson, 1974), gave us the opportunity to study gauge field from lattice both in principle and numerical. Then, in 1980, Michael Creutz could firstly bring Monte Carlo method to simulate gauge field by study on group of transformation in space-time. His procedure is now very standard for studying gauge theory through lattice. It begins from sampling randomly the lattice of *SU*(*2*) and try to simulate system reach on thermalized state and then it can extract every information by measurement. The lattice consists of each link variable which is associated at each space-time x, which is $U_\mu(x)$ linking the site $[x]$ to $[x+\mu]$ where μ is a direction in space-time, as shown in Figure 1.

By the property of *SU(2)* the gauge potential A_μ is hidden in each direction of each space-time, $U_\mu(x) = \exp(ie_0 A_\mu(x))$, where e_0 is coupling constant.

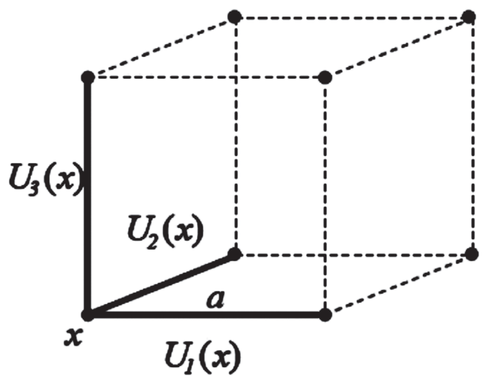

Figure 1. At each point x in space-time, there is any link $U_\mu(x)$ associating with the other

Creutz first successfully used computer to simulate lattice QCD by Monte Calro (MC) simulation method Creutz (1980). His model can be mapped to the classical *SU(2)* Yang-Mills theory from the Lagrangian density Yang and Mills (1954)

$$L = \frac{1}{4} F^{\alpha}_{\mu\nu} F^{\alpha}_{\mu\nu} \quad (1)$$

where α is the internal-symmetry index running from 1 to 3, and $F^{\alpha}_{\mu\nu}$ is defined in terms of potentials

$$F^{\alpha}_{\mu\nu} = \partial_{\mu} A^{\alpha}_{\nu} - \partial_{\nu} A^{\alpha}_{\mu} - e_0 \varepsilon^{\alpha\beta\gamma} A^{\beta}_{\mu} A^{\gamma}_{\nu}. \quad (2)$$

It is the so-called Heat bath algorithm, because it borrows the idea of statistical mechanics to do MC step by Markov process of sampling each configuration $U_i\ (x)$ with probability density proportional to the Boltzmann factor,

$$dP\,(U') \sim \exp\,[-\,\beta S\,(U')]\; dU' \quad (3)$$

where $U_{\mu}(x)$ each was quantized along to space-time to have structure of lattice, $U \rightarrow U\mu\,(x_0, x_1, x_2, x_3)$, β is $4/e_0^2$. According the principle of statistical mechanics, the β value controls the temperature of this system. With this probability density, gauge group $SU(N)$, in simplest case $SU(2)$, U_i is simulated by computer.

$$U_1 \rightarrow U_2 \rightarrow U_3 \rightarrow \ldots \rightarrow U_N. \quad (4)$$

Any observable is associated with each U_i, one kind of them is average plaquette, $< S_{\square} >$, defining internal energy of gauge system, where each $S_{\square} = 1 - \frac{1}{2} Tr(P_{\mu\nu})$, and $P_{\mu\nu}$ is product of each link variable along plaquette, as shown in the Figure 2.

Our Monte Carlo method can simulate an ensemble of configurations $U_{\mu}\,(x)$ and lead them to reach into thermalized state, even we start randomly from any initial configuration. The evidence can be seen from Figure 3 that whether we start system from order state or random (hot) start, after some heatbath steps, the system can be in equilibrium of its temperature. We see the convergence of simulating system. After this convergence, we ensure that our system reach into equilibrium state. After configuration $U_{\mu}\,(x)$ is in equilibrium, any observable O can be measured from N_{MC} step of Monte Calro iterations.

$$P_{\mu\nu}(x) = U^{\dagger}_{\nu}(x) U^{\dagger}_{\mu}(x+\nu) U_{\nu}(x+\mu) U_{\mu}(x).$$

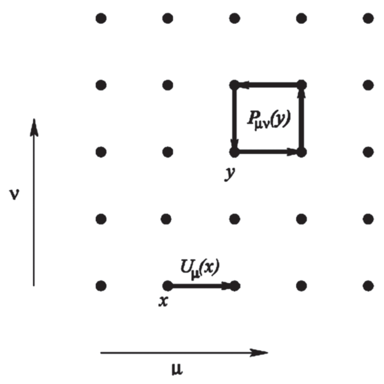

Figure 2. Any plaquettes in the lattice as discretized space

$$\mathrm{O} \quad \frac{1}{N_{MC}}\sum_{i=1}^{N_{MC}} \mathrm{O}(U_i). \qquad (5)$$

Figure 4 shows average plaquette as a function of in which is the characteristic Yang-Mills theory. And it shows in range [2.0 - 3.0], there is the phase transition in this range. Average plaquette in other two ranges is compatible according to asymptotic behavior proving from analytic calculations.

Coulomb Gauge Fixing

Gauge fixing is choosing the gauge. Having gauge fixing in Yang-Mills theory is analogous to choosing a constraint into mechanical problems Christ and Lee (1980). Even we have the freedom to choose any gauge, but in this research we choose Coulomb gauge Ñ . A = 0, because from Gribov's quark confinement scenario Gribov (1978), a study of gauge condition. Finally we should extract potential between quarks within nucleon (combining by gluon exchanges). Numerically, Coulomb gauge fixing on lattice can be provided by satisfying the condition $\sum_{k=1}^{3}\left[U_k\left(x+k\right)-U_k(x)\right]=0$. In order to lead system to this condition, we must make the gauge transform, by finding lattice group W(x) and attributing the lattice by

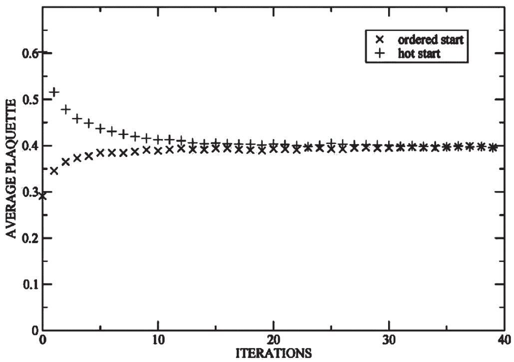

Figure 3. Shows the values of average plaquette, $< S_{\square} >$, v.s. iteration of Monte Calro steps. One initial state starts from ordered state and the other starts from hot state

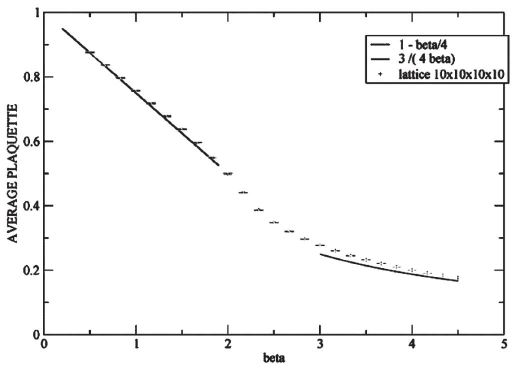

Figure 4. Average plaquette as a function of β for lattice 10^4

$$U_\mu^\Omega(x) = \Omega(x+\hat{\mu})U_\mu(x)\Omega^\dagger(x). \qquad (6)$$

This process can be done arbitrarily, because the system still has the invariance of internal energy ($< S_\square >$ value). The method to get this condition is to minimize this functional

$$F_t\left[U(\mathrm{x},t)\right] = \sum_x \sum_{\mu=1}^{3} \frac{1}{2}\mathrm{Tr}\left[1 - U_\mu^\Omega(\mathrm{x})\right] \to 0. \qquad (7)$$

To get this condition into configuration U_m (x_0, x_1, x_2, x_3), we need to do relaxation method computing by computer, that is, W(x) are computed and attributed to lattice until $F_t\,[U(\mathrm{x},\, t)]<10^{-13}$. That takes very long time to achieve Quandt *et al.* (2007).

There are many configurations $U_m\,(x)$ that satisfied Coulomb gauge condition Ñ . A = 0, whatever by numerical or theoretical point of view, the so-called *Gribov gauge copies* Gribov (1978). In his original work, Gribov predicted the behavior of some measurable quality, the gluon propagator and ghost form factor, and lead us finally to static potential between quarks. This scenario is responsible for the quark confinement.

Gluon Propagator

Gluon propagator is an observable from lattice simulation. It describes correlation function in momentum space from lattice gauge field. The computers do it by using (discrete) Fast Fourier transform,

$$A_\mu^a(x) \to \tilde{A}_\mu^a(p),$$

by

$$\tilde{A}_\mu^a(p) \sim \sum_{x_1=0}^{N_1-1}\sum_{x_2=0}^{N_2-1}\sum_{x_3=0}^{N_3-1} e^{-i\frac{2\pi}{N_1}x_1} e^{-i\frac{2\pi}{N_2}x_2} e^{-i\frac{2\pi}{N_3}x_3} A_\mu^a(x) \qquad (8)$$

and getting the gluon propagator of the lattice momentum from the definition of correlation function, that is, $\left\langle \tilde{A}_\mu^a(p)\tilde{A}_\nu^b(p')\right\rangle$ and we can take advantagethat let $p' = -p$ then our computation is

$$\left\langle \tilde{A}_\mu^a(p)\tilde{A}_\nu^b(p')\right\rangle = \left\langle \tilde{A}_\mu^a(p)\tilde{A}_\nu^b(-p)\right\rangle = \left\langle \left|\tilde{A}_\mu^a(p)\right|^2 \right\rangle.$$

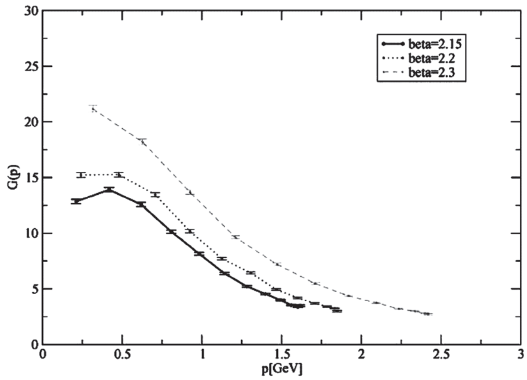

Figure 5. Gluon propagator as a function of physical momentum, *p*, for lattice 24^4 . Data from our program has variety of β

Therefore, gluon propagator is a function of Matsubara frequency l, from the formula,

$$G(l)=\left\langle\sum_{a=1}^{3}\sum_{\mu=0}^{4}\frac{1}{3}(|\tilde{A}_\mu^a(l,0,0)|^2+|\tilde{A}_\mu^a(0,l,0)|^2 +|\tilde{A}_\mu^a(0,0,l)|^2)\right\rangle \quad (9)$$

where for each integer $l_i \in [-\frac{N}{2},\frac{N}{2}]$ corresponds for each $p_i=\frac{2\pi l}{N_i}$ (Matsubara frequency).

Figure 5 shows one example from our computations, gluon propagator as a function of physical momentum, p, for lattice 24^4 at zero temperature, from many values of β (2.15, 2.2, 2.3).

In practice, we want to compute many jobs with several values of β and lattice size, then we need parallel computing or grid computing to compute for each set of parameters. For each curve in Figure 5, we can run one program for one curve, or if it is necessary, we must run many programs for only one curve in the case it may take very long time to measure each one measurement.

Renormalization

In Yang-Mills theories in 4-dimensions, there is the principle of renormalization, in brief, the coupling constant β acquires to depends on the large β value (ultraviolet regime). it allows us to include all any simulated data of lower β (infrared regime) combining them into one set of *renormalized* data by finding some constant to multiply each measurable observable, that is

$$O_{ren}(p) = const \cdot O_{sim}(p). \quad (10)$$

For example the data gluon propagator in Figure 5 can be renormalized into one curve by multiplicative re-curving as in Figure 6. With this procedure, the larger lattice and lower b can extract information at lower energy sector QCD. The renormalized data should be continuous from lower to high energy (momentum). The high energy QCD (ultraviolet regime) have been studied theoretically by Gross, Wilczek, and Politzer (Gross and Wilczek, 1973; David Politzer, 1973) and experiment of deep inelastic scattering.

Ghost form Factor

It begins from defining the ghost propagator by the expectation value of the inverse Faddeev-Popov operator M = -∇·D, where D is the gauge covariant derivative). On lattice it is computed from

$$D^{0ab}(x-y)=\left\langle M^{-1}[A]|^{ab}_{(x,y)}\right\rangle \quad (11)$$

then transform into momentum space to be D^{0ab} (p). We are interested to know behavior of dimensionless quantity, the so-called *ghost form factor*, D (p)

$$D(p)=\frac{D^{0ab}}{|p|^2} \quad (12)$$

The Figure 7 shows the ghost form factor of lattice 24^4 after renormalzation. Its behaviour is predicted to be diverge at lower energy.

Gribov Copies Influence

Both consideration from mathematical and theoretical point of view, Gribov (1978) pointed out that there exist many configurations $U_m(x)$ that satisfied Coulomb gauge condition. We can find the best copies (bc) from n copies by choosing the configuration which gives maximum of functional (7). This was confirmed by our computation. Figure 8 is one evidence which shows how many copies influence the gluon propagator. Qualitatively, it shows the *suppression* of gluon propagator at lowest momentum. Then, we must compute gluon propagator by choosing the best copies from many copies. We show the recent result at zero temperature, at large lattice in Figure 9.

We also found the similar suppression

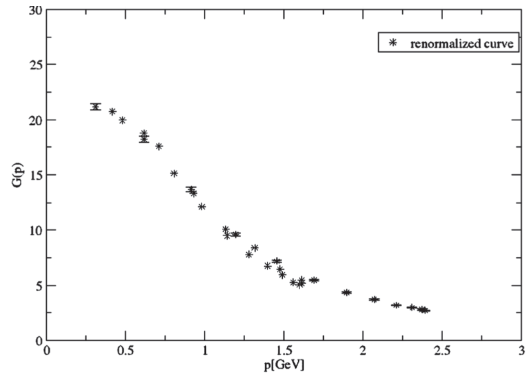

Figure 6. Gluon propagator after doing renormalzation. It can include all set of simulated data into one curve which allows us to see what happen at lower energy sector QCD

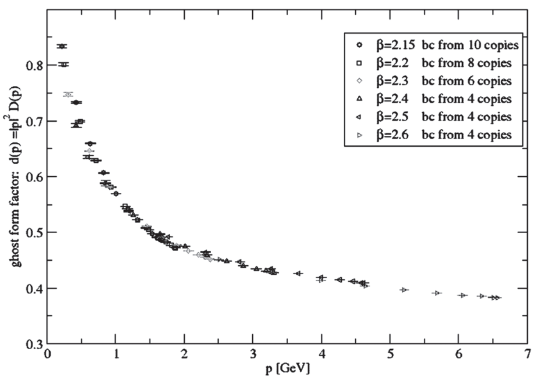

Figure 7. Ghost form factor in Coulomb gauge of lattice 24^4

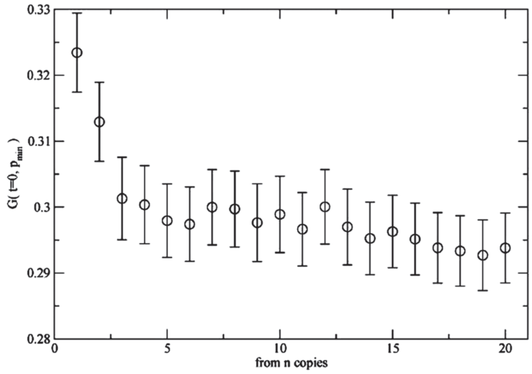

Figure 8. Gribov copies influence to gluon propagator at minimum momentum, p_{min}, for lattice 24^4 at $\beta = 2.15$

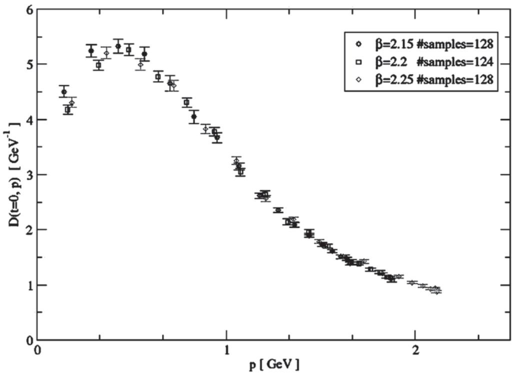

Figure 9. Gluon propagator in Coulomb gauge at lattice 36^4 at β = 2, 15, 2.2, 2.3

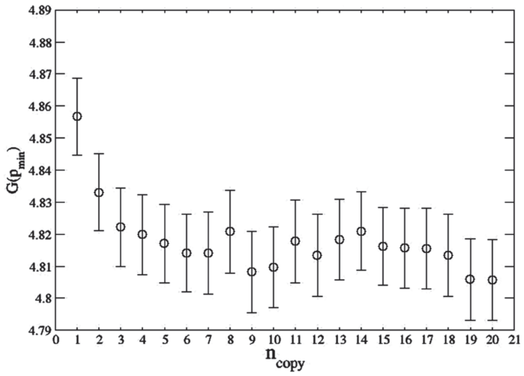

Figure 10. The suppression of Ghost propagator at lowest momentum

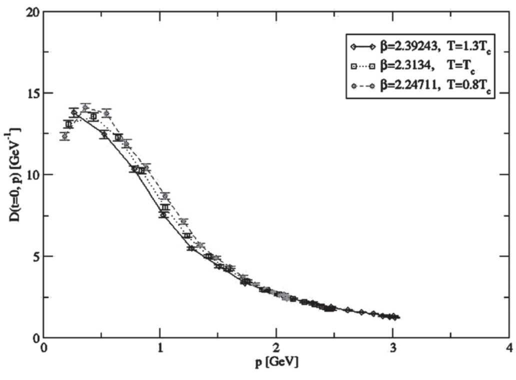

Figure 11. Gluon propagator in Coulomb gauge at lattice 4×36^3 at three different temperatures, $0.8T_c$, T_c, and $1.3T_c$, where T_c is critical temperature

of ghost form factor by Gribov gauge copies influence as in Figure 10. The result of measuring ghost form factor from the best copy has been shown already in Figure 7.

At Finite Temperature

After we have to simulated observables, gluon propagator and ghost form factor, and normalized them from many parameters, we can find their behaviour at finite temperature by tuning up the time-slice lattice size N_0 or coupling constant β

$$T = \frac{1}{N_0 a(\beta)} \tag{13}$$

where a (β) is lattice space relating with coupling constant. Figures 11 and 12 show gluon propagators and ghost form factor at finite temperatures from several lattice volumes, 4×24^3, 4×32^3, 4×36^3. We are interested to know at temperature $T = 0.8T_c$, T_c, $1.3T_c$ where T_c is critical temperature between lower temperature of confinement phase and higher tempera-

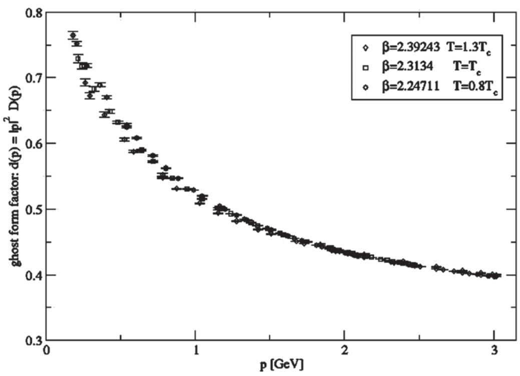

Figure 12. Ghost form factor in Coulomb gauge at lattice 4×36^3 at three different temperatures, $0.8T_c$, T_c, and $1.3T_c$, where T_c is critical temperature

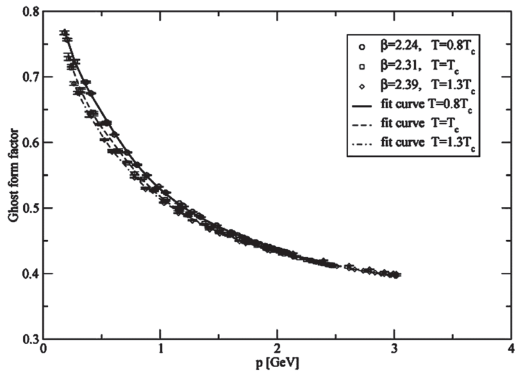

Figure 13. Ghost form factor at finite temperature, measured from the best gauge copy lattice 4×24^3, 4×32^3, 4×36^3

Table 1. Parameters from curve fitting of the ghost form factor for all range at finite temperature

	A	Λ	γ_0	α	κ
$T=0.8T_c$	0.800508	0.148362	0.627315	0.0513664	0.0846121
$T=T_c$	0.814486	0.125575	0.619598	0.01991	0.0968555
$T=1.3T_c$	0.709427	0.222751	0.564559	0.116498	0.0847839

ture of de-confinementphase.

Especially, for the ghost form factor at finite temperature, we made the curve fitting to them by using this Ansatz formula

$$\frac{A}{\left(\left|\log\frac{|p|}{\Lambda}\right|^{\frac{\gamma_0}{\alpha}}+\left(\frac{|p|^2}{\Lambda^2}\right)^{\frac{\kappa}{\alpha}}\right)^{\alpha}} \qquad (14)$$

The curve-fitting is shown in Figure 13, and these numbers of parameters are shown in Table 1.

Coulomb Potential

In the framework of lattice gauge simulation in Coulomb gauge Cucchieri and Zwanziger (2001), we can simulate the (Coulomb) potential between quark and anti-quark by

$$V_{Coul}(\mathrm{x}-\mathrm{y}) \propto -g_0^2\left\langle\left[M^{-1}[A](-\Delta)M^{-1}[A]\right]\right\rangle \qquad (15)$$

and at fixed time-slice t_0, in momentum space, the Coulomb potential is computed from

$$\hat{V}_{Coul}(\mathrm{p}) = -\frac{1}{4N_d}g_0^2 \sum_{a=1}^{3}\sum_{x,y=0}^{N-1}\left\langle M^{-1}[A](-\Delta)M^{-1}[A]\right\rangle e^{-i\frac{2\pi}{N_d}\cdot(x-y)} \qquad (16)$$

This potential was proved that it is the upper bound value of the true potential Zwanziger (2003). This is why we are interested to simulate it. However, its computing is very slow using conjugate-gradient method. We must use the preconditioning technique to speed up the conjugate-gradient process. We use Sternbeck preconditioning procedure Sternbeck *et al.* (2005). In Figure 14, it shows recent data of Coulomb potential measured in the form of dimensional as a function of momentum. The whole results are now being investigated.

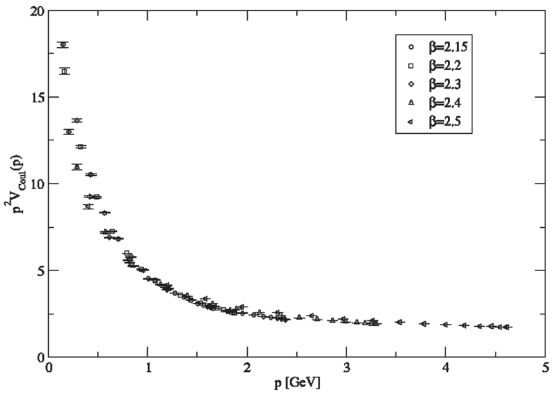

Figure 14. The dimensionless $p^2V(p)$ measured from the lattice 36^4

Conclusions

We are now able to compute to the gluon propagator and ghost form factor in Coulomb gauge at zero temperature and finite temperature. It offers us the opportunity to further compute the static quark-antiquark potential for confirming confinement mechanisms. Now, we can analyze the suppression of gluon propagator and ghost form factor by fitting curve data. The Coulomb potential is being obtained and under investigation. The result will be published in the future.

Acknowledgement

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References

Christ, N.H. and Lee, T.D. (1980), Operator ordering and Feynman rules in gauge theories. Physical Review D., 22(4):939-958.

Creutz, M. (1980) Monte Carlo study of quantized SU(2) gauge theory, Physical Review D., 21(8):2,308-2,315.

Cucchieri, A. and Mendes, T. (2007), Just how different are the SU(2) and SU(3) Landau-gauge propagators in the infrared regime? Physical Review D., 76(11):114507.

Cucchieri, A. and Zwanziger, D. (2001). Numerical study of the gluon propagator and confinement scenario in the minimal Coulomb gauge. Physical Review D., 65(1):014001.

Cucchieri, A. and Zwanziger, D. (2001). Renormalization-group calculation of the color-Coulomb potential. Physical Review D., 65(1):014002.

David Politzer, H. (1973). Reliable Perturbative Results for Strong Interactions? Physical Review Letters, 30: 1346.1349.

Greensite, J. (2003). The Confinement Problem in Lattice Gauge Theory. arXiv:hep-lat/0301023.

Gribov, V.N. (1978). Quantization of non-abelian gauge theories. Nuclear Physics B., 139(1,2):1-19.

Gross, J. D. , Wilczek, F. Asymptotically Free Gauge Theories.I. Physical Review D., 8(10):3,633-3,652.

Langfeld, K. and Moyaerts, L. (2004). Propagators in Coulomb gauge from SU(2) lattice gauge theory. Physical Review D., 70(7):074507.

Quandt, M., Burgio, G., Chimchinda, S., and Reinhardt, H. (2007). Coulomb gauge Green functions and Gribov copies in SU(2) lattice gauge theory. arXiv:0710.0549v1 [hep-lat].

Sternbeck, A., Ilgenfritz, E.-M., and Mueller-Preussker, M. (2005). Towards the infrared limit in SU(3) Landau gauge lattice gluodynamics. Physical Review D., 72(1):014507.

Voigt, A., Ilgenfritz, E.-M. and MØ ller-Presussker, M. (2008) Effective Coulomb potential in SU(3) lattice Yang-Mills theory. Physical Review D., 78(1):014501.

Wilson, K.G. (1974), Confinement of quarks Physical Review D., 10 (8):2,445-2,459.

Yang, C.N. and Mills, R. (1954). Conservation of isotopic spin and isotopic gauge invariance. Physical Review., 96(1):191-195.

Zwanziger, D. (2003). No confinement without coulomb confinement. Physical Review Letter, 90(10):102001.