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วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรมหาบัณฑิต
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STUDY OF $\overline{K}N$ INTERACTIONS

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Suranaree University of Technology has approved this thesis submitted in partial fulfillment of the requirements for the Degree of Master of Science.

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วิทยานิพนธ์นี้ศึกษาอันตรกิริยาที่คู่ควบแบบหลายช่องของแอนติเคออนและนิวคลีออนที่พลังงานต่ำ ในปริภูมิพิกัดซึ่งแตกต่างจากอันตรกิริยาในปริภูมิโมเมนตัมที่ศึกษาด้วยทฤษฎีสนามควอนตัม จากผลการทดลองพบว่า อนุภาคแลมบ์ดา(1405) ($\Lambda(1405)$) เป็นเรโซแนนซ์ที่เกิดขึ้นจากการรวมกันของสถานะสองสถานะ ระหว่างไพออนและซิกมาไฮเปอร์รอน ($\pi\Sigma$) และแอนติเคออนและนิวคลีออน ($\overline{K}N$) สมบัติของอนุภาค $\Lambda(1405)$ เช่น มวล และช่วงเวลาของการสลายตัวขึ้นกับอันตรกิริยาระหว่าง $\pi\Sigma$ และ $\overline{K}N$ ในกรณีการกระเจิงของ $\overline{K}N$ ที่ไอโซสปินเท่ากับศูนย์ ($I=0$) $\Lambda(1405)$ จะปรากฏขึ้นให้เห็นอย่างเด่นชัดที่มวลมีค่าต่ำกว่าขีดเริ่มเปลี่ยนของ $\overline{K}N$ อยู่ประมาณ 28 MeV และยังพบอีกว่า $\Lambda(1405)$ เป็นสถานะกักกันที่ประกอบด้วยอนุภาคมูลฐานในสถานะควาร์กสาม ทำให้สามารถสร้างแบบจำลองที่ประกอบด้วย $\overline{K}$ N π และ Σ ซึ่งอันตรกิริยาระหว่างกันอยู่ในรูปของศักย์หรือการแลกเปลี่ยนอนุภาคเมซอน

งานวิจัยนี้เป็นส่วนหนึ่งของโครงการที่ต้องการหาศักย์ที่สอดคล้องกับผลการทดลองของการกระเจิงของ $\overline{K}N$ ที่พลังงานต่ำและผลการทดลองของอะตอมไฮโดรเจนแบบเคออนิกรวมไปถึงแอมพลิจูดการเปลี่ยนที่ทำนายโดยทฤษฎีอื่น การคำนวณภาคตัดขวางของ $\overline{K}N$ ในวิทยานิพนธ์นี้ใช้สมการของลิฟฟ์มานน์-ชวิงเงอร์ ซึ่งผลที่ได้ แสดงให้เห็นว่ามีความสอดคล้องกับผลการทดลองเฉพาะช่วงโมเมนตัมสี่มิติระหว่าง 200-350 MeV/c ในกรอบห้องปฏิบัติการ

สาขาวิชาฟิสิกส์ ลายมือชื่อนักศึกษา_____

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WANCHALOEM POONSAWAT : STUDY OF $\overline{K}N$ INTERACTIONS.
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82 PP.

$\overline{K}N$ INTERACTIONS/ COUPLED-CHANNEL

In this thesis the coupled-channel interactions of $\overline{K}N$ is directly studied at low energy in coordinate space, unlike interactions in momentum space derived in the framework of quantum field theory. The experiments confirm that the appearance of $\Lambda(1405)$ resonance is a superposition of the two states, $\pi\Sigma$ and $\overline{K}N$. The properties of $\Lambda(1405)$ such as its mass and the decay width depend on the interaction between $\pi\Sigma$ and $\overline{K}N$. Particularly, the $\overline{K}N$ scattering in the isospin equal zero ($I = 0$) channels is dominated by the presence of the $\Lambda(1405)$ located only 28 MeV below the $\overline{K}N$ threshold. Finding that the $\Lambda(1405)$ consists of bound state with only a small admixture of elementary three-quark state suggests a reasonable model is possible with the $\overline{K}$, N, Σ and π as elementary particles interacting via potentials or meson-exchange.

This work is a part of the project which aims to work out potentials which would reproduce the low energy $\overline{K}N$ scattering data, kaonic hydrogen atom data, and transition amplitudes predicted by other theoretical approaches. We have calculated the $\overline{K}N$ cross sections using the Lippmann-Schwinger equation. Our results show some consistencies with experimental data in the range between 200-350 MeV/c of the four-momentum in the laboratory frame.

School of Physics
Academic Year 2009

Student's Signature _____
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B	= Baryon
I	= Isospin
I_3	= Third component of isospin
J	= Total angular momentum
K	= Kaon
N	= Nucleon
$\mathcal{L}$	= Lagrangian density
S	= Strangeness quantum number
$SU(3)$	= Special unitary groups of transformation of order 3
u	= Up quark flavour
d	= Down quark flavour
s	= Strange quark flavour
Y	= Hypercharge quantum number
Λ	= Lambda baryons
π	= Pion
Σ	= Sigma baryons
ψ	= Quantum-mechanical wave function
CPT	= Charged conjugation parity and time reversal

CHAPTER I

INTRODUCTION

The study of quasi-bound $\overline{K}$-nuclear states has become a hot subject in nuclear physics. Recently, definite information about the strong interaction level shifts of the kaonic hydrogen atom was obtained from the experiment KpX at KEK (Iwasaki et al., 1997; Ito et al., 1998) which indicates a repulsive-type for the 1s orbit. For heavier nuclei (Friedman et al., 1993; Batty et al., 1997), reanalyzed all of the existing data of $\overline{K}$ atoms, including a density-dependent term for the $\overline{K}N$ scattering length, and deduced optical potential with a strongly attractive real part and also a strongly absorptive imaginary part. The reason for such a highly attractive potential, despite the fact that the strong interaction shifts appear to be negative, comes from the assertion that the $\Lambda(1405)$ state is not an elementary particle, but the bound state of $\overline{K} + N$. From such a potential one expects deeply bound nuclear states in heavier nuclei, but their widths are estimated to be on the order of 100 MeV or if their potential parameters are strictly applied, and thus such nuclear states may not be identified as discrete state.

Calculations of strong binding of $\overline{K}$ in a nuclear medium based on chiral SU(3) dynamics have a long history (Brown et al., 1994; Waas et al., 1996; Waas and Weise, 1997; Waas et al., 1997; Lutz and Korpa, 2002; Lutz et al., 2008). The recent interest in this topic was prompted by Akaishi and Yamazaki (Brown et al., 1994; Lutz et al., 2008; Akaishi and Yamazaki, 2002) who used a simple potential model to calculate bound states of few-body systems such as K^-pp, K^-pn, and K^-pnn. However, it is noted that the predictive power of all

such investigations is limited because the interactions are constrained just by the scattering processes. The energy range of the $\overline{K}N$ interaction relevant for deeply bound kaonic nuclei lies far below the $\overline{K}N$ threshold. For variational calculations of few-body systems involving anti-kaons, one must use a realistic effective $\overline{K}N$ interaction, particularly in the form of a potential. This potential is in general complex and energy dependent. It must be constrained to reproduce the scattering amplitude in vacuum. Also it must encode the full coupled-channel dynamics. A number of attempts in this direction, using a schematic effective interaction, have been reported in (Dote et al., 2004; Weise, 2007).

However, we have found that all versions of the $\overline{K}N$ interactions give unreasonably large decay widths for the kaonic hydrogen atom. We would like to derive an effective interaction which reproduce not only the $\overline{K}N$ scattering amplitudes but also the kaonic hydrogen atom data. This work is just the first step of the whole project with which such a potential is expected to be worked out.

This chapter is devoted to the observation of $\Lambda(1405)$ and the $\overline{K}N$ states. The state is mainly the bound state of a K^- and a proton (p) related to their quark configurations. The $\overline{K}N$ systems couples to a large number of other channels due to the strong interaction at small distances. However, for processes at low energies close to the $\overline{K}N$ mass threshold, one may consider only the $\pi\Sigma$ and $\pi\Lambda$ channels. In our study, we neglect the $\eta\Sigma$ and $\eta\Lambda$ channels since the couplings are believed weak.

1.1 The observation of $\Lambda(1405)$ resonances

The case of the $\Lambda(1405)$ is one of the examples of dynamically generated resonances which was already described within scattering theory with coupled channels (Jones et al., 1977). Recently the advent of nonperturbative methods

with input from chiral Lagrangian has set of original idea on firmer grounds (Kaiser et al., 1995; Kaiser et al., 1997; Oller and Meissner, 2001; Oset and Ramos, 1998; Jido et al., 2002; Garcia-Recio et al., 2003; Garcia-Recio et al., 2004). The $\Lambda(1405)$ resonance appearing about 30 MeV below the $\overline{K}N$ threshold plays an important key role in the $\overline{K}N$ interaction and related processes, and is a subject to find out its nature, whether it is a genuine three quarks system (Isgur and Karl, 1978; Kimura et al., 2000), or a molecular-like meson-baryon bound state where chiral dynamics plays an important role. The recent discovery of the pentaquark (Nakano et al., 2003), should stimulate again on the nature of the $\Lambda(1405)$ since the existence of that exotic state forces an interpretation of that baryon with at least five quarks (Hosaka, 2003; Jaffe and Wilczek, 2003), although molecular structures with $K\pi N$ (heptaquark) have also been investigated (Bicudo and Marques, 2004; Llanes-Estrada et al., 2004). Within the chiral approach (Kaiser et al., 1995; Kaiser et al., 1997; Oller and Meissner, 2001; Oset and Ramos, 1998; Jido et al., 2002; Garcia-Recio et al., 2003; Garcia-Recio et al., 2004), the $\Lambda(1405)$ stands as a quasi-bound state of meson baryon, mostly $\overline{K}N$ and $\pi\Sigma$, which is also equivalent to five quarks in the quark picture. The existence of the pentaquark makes more easily acceptable the idea of other pentaquark non-exotic state and vice versa. Explorations on the nature of the $\Lambda(1405)$ will provide more clues to understand the nonperturbative nature of the QCD dynamics.

With this chiral unitary approach (Nachera et al., 1999), the photoproduction of the $\Lambda(1405)$ on the proton and nuclei was calculated and found different shapes of $\pi\Sigma$ invariant mass distributions in different $\pi\Sigma$ charge channels, which was lately experimentally confirmed (Ahn and collaboration, 2003), and gave support to the assumption that the $\Lambda(1405)$ is a meson baryon loosely bound state. Additionally, it was found (Jido et al., 2003) that the SU(3) symmetry breaking

leads to two poles of $\overline{K}N$ scattering matrix that might be responsible for the nominal $\Lambda(1405)$, one dominantly coupling to $\pi\Sigma$ and the other to $\overline{K}N$, and these poles are the mixing of the SU(3) singlet and octet. It was concluded that there are two $\Lambda(1405)$ resonances and the experimentally observed one is a superposition of the two states. However, whether the two poles really exist in the $\Lambda(1405)$ region are still unsolved experimentally.

1.2 Coupled-channel of $\overline{K}N$ systems

The appearance of $\Lambda(1405)$ resonances, related to the experimental, confirm that these resonances are superposition of the two states. These two resonances couple differently to $\pi\Sigma$ and $\overline{K}N$ states and, as a consequence, the properties of the $\Lambda(1405)$ (mass and width) will depend on the particular reaction employed to produce it. $\overline{K}N$ scattering in the $I = 0$ channel is dominated by the presence of the $\Lambda(1405)$, located only 28 MeV below the $\overline{K}N$ threshold.

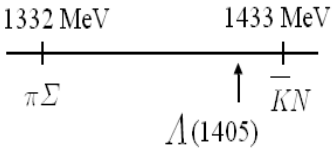

Figure 1.1 Threshold energy spectrum of $\overline{K}N$ scattering at isospin equal to zero.

In 1960s (Dalitz et al., 1967), the $\Lambda(1405)$ was obtained as a $\overline{K}N$ quasi-bound state in a potential model (Schrödinger equation). The study of $\overline{K}N$ scattering has been revisited more recently from the ideas of chiral Lagrangian. However, the presence of a resonance makes charged conjugation parity and time reversal (CPT) not applicable, i.e., non-perturbative techniques implementing unitarization in coupled channels are mandatory.

The dynamically resonances can be generated by coupled-channel of $\overline{K}N$ systems. To understand the coupled-channel interaction of $\overline{K}N$ systems, we will consider the particle properties of kaon and baryon.

Considering to the quark configurations of anti-kaon with $J^P = 0^-,\ I = \frac{1}{2}$, and $S = -1$ and nucleon with $J^P = \frac{1}{2}^+,\ I = \frac{1}{2},$ and $S = 0$

$$\overline{K} = \begin{pmatrix} \bar{K}^0(\bar{d}s) \\ K^-(\bar{u}s) \end{pmatrix}$$

$$N = \begin{pmatrix} p(uud) \\ n(udd) \end{pmatrix}.$$

The $\overline{K}N$ systems can generate $\Lambda(1405)$ with the following diagram.

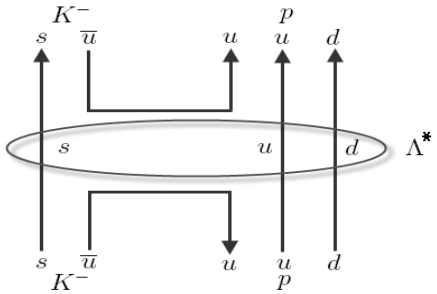

Figure 1.2 Quark diagram for $K^-p \to K^-p$ process.

The strong interaction at lowest energies can be studied in a direct way in simple hadronic atoms like kaonic hydrogen or kaonic helium atoms. In these atoms the strong interaction between anti-kaon and nucleon ($\overline{K}N$) leads to an energy shift and an increased width due to the reduced lifetime and nuclear absorption.

The present theory anticipates that the $\overline{K}N$ interaction is strongly attractive below threshold due to an s-wave resonance with the mass of $\Lambda(1405)$ about 28 MeV below threshold. This resonance is an importance for the existence of

kaonic nuclear clusters.

This work is a part of the whole project which try to calculate a potential which would reproduce the low energy $\overline{K}N$ scattering data, kaonic hydrogen atom data, and transition amplitudes predicted by other theoretical approaches. This potential is needed when we expand our calculations to multiple particles system.

The thesis is organized as follows. In Chapter II we derive the dynamical Lippmann-Schwinger equation for radial outgoing scattered wave function. In Chapter III we construct our numerical procedure. In Chapter IV we describe the coupled-channel interaction. Finally, in Chapter V contains results for the total cross section of $\overline{K}N$ channels, discussions, and conclusions.

CHAPTER II

LIPPMANN-SCHWINGER EQUATION FOR $\overline{K}N$ SYSTEMS

This chapter we describe the analytical procedure to solve our dynamical equations. We start from the scattering process of free particle with the Lippmann-Schwinger equation. In this work we study the s-wave low energy $\overline{K}N$ interactions. A reliable and realistic starting point for a theory of low energy $\overline{K}N$ interactions is the coupled-channel approach based on the chiral SU(3) meson-baryon effective Lagrangian (Kaiser et al., 1995), and subsequently expanded by several groups (Oset and Ramos, 1998; Oller and Meissner, 2001; Lutz and Kolomeitsev, 2002). Unitarization of the chiral interaction correctly reproduces the $\overline{K}N$ scattering observables and provides a framework for generating the $\Lambda(1405)$ resonance dynamically as a $\overline{K}N$ quasi-bound state embedded in the strongly interacting $\pi\Sigma$ continuum. This approach is successful over a wide range of energies and a variety of channels. But the detail of what chiral SU(3) dynamics tells us about the $\overline{K}N$ interactions below threshold would be investigated are needed. Because this interaction can then be used in K^--nuclear few-body calculations (Dote and Weise, 2007). Finally, the derived Lippmann-Schwinger equation must be reduced into only the radial part of the dynamical equations.

2.1 Derivative of Lippmann-Schwinger equation

There are several methods can be used to study the $\overline{K}N$ systems. Since, the $\Lambda(1405)$ is generated from $\overline{K}N$ subthreshold, this enables us to consider the scattering method. Thus, we have to solve the scattering equation by using the integral form of the Schrödinger's equation, known as Lippmann-Schwinger equation.

Starting from the basic Schrödinger equation, we have

$$(E - H_0)\psi_\alpha = V_{\alpha\beta}\psi_\beta, \tag{2.1}$$

where H_0 is the kinetic energy operator of free particles ($\vec{p}^2/2m$) consisting of a momentum $\vec{p}$ and a mass m that can be solved exactly, E is an energy eigenvalue of H_0, $V_{\alpha\beta}$ is the interaction between the initial state (β) and the final state (α), ψ_α is an eigenfunction of the final state, and ψ_β is an eigenfunction of the initial state.

From a time independent formulation of a scattering process, assuming that the harmiltonian can be written as

$$H = H_0 + V, \tag{2.2}$$

where V is the perturbation operator.

The transformation from a differential equation to an integral equation is

$$|\psi_\alpha\rangle = |\phi_\alpha\rangle + \frac{1}{E - H_0 + i\varepsilon} V_{\alpha\beta} |\psi_\beta\rangle , \tag{2.3}$$

where $|\phi_\alpha\rangle$ is the energy eigenket of H_0. However, the operator $E - H_0$ is a singularity, so we can circumvent by adding a small complex term ($i\varepsilon$) to the

denominator.

In order to solve this equation explicitly, let us translate it into a ket equation independent of particular representations. By taking the inner product with the bra $\langle\vec{r}|$, we obtain

$$\langle\vec{r}\,|\psi_\alpha\rangle = \langle\vec{r}\,|\phi_\alpha\rangle + \langle\vec{r}|\,\frac{1}{E-H_0+i\varepsilon}V_{\alpha\beta}\,|\psi_\beta\rangle\,. \tag{2.4}$$

Considering the second term in the right hand side of the equation Eq. (2.4), one can rewrite as the following

$$\begin{aligned}\langle\vec{r}|\,\frac{1}{E-H_0+i\varepsilon}V_{\alpha\beta}\,|\psi_\beta\rangle &= \langle\vec{r}|\,\frac{1}{E-H_0+i\varepsilon}\,|\vec{r}_1\rangle\,\langle\vec{r}_1|\,V_{\alpha\beta}\,|\vec{r}_2\rangle\,\langle\vec{r}_2|\,\psi_\beta\rangle \\ &= \langle\vec{r}|\,\frac{1}{E-H_0+i\varepsilon}\,|\vec{r}_1\rangle\,V_{\alpha\beta}(\hat{r}_1)\delta(\vec{r}_1-\vec{r}_2)\,\langle\vec{r}_2|\,\psi_\beta\rangle \\ &= \langle\vec{r}|\,\frac{1}{E-H_0+i\varepsilon}\,|\vec{r}\,'\rangle\,V_{\alpha\beta}(\vec{r}\,')\psi_\beta(\vec{r}\,'),\end{aligned} \tag{2.5}$$

and then write an operator $\frac{1}{E-H_0+i\varepsilon}$ of the bra-ket notation in term of inverted Green's function. Thus, the formal solution of the Lippmann-Schwinger equation for outgoing scattered wave can takes the form

$$\psi_\alpha(\vec{r}) = \phi_\alpha(\vec{r}) + \int d^3\vec{r}\,'G(\vec{r},\vec{r}\,')V_{\alpha\beta}(\vec{r}\,')\psi_\beta(\vec{r}\,'), \tag{2.6}$$

where $G(\vec{r},\vec{r}\,')$ is the Green's function.

In our system the potential is a spherically symmetric. The scattering state, of course, can be expanded in terms of spherical harmonics and a radial part. To do this, we first expand the scattering wave functions in the spherical harmonics

as

$$\psi(\vec{r}) = (2/\pi)^{1/2} \sum_{ll'ss'JM} R^J_{l's',ls}(k,r) Y^{JM*}_{ls}(\hat{k}) Y^{JM}_{l's'}(\hat{r}) i^{l'}. \tag{2.7}$$

For the outgoing scattered (final) state, the wave function becomes

$$\begin{aligned}\psi_\alpha(\vec{r}) &= (2/\pi)^{1/2} \sum_{ll'ss'JM} R^{\alpha,J}_{l's',ls}(k_\alpha,r) Y^{JM*}_{ls}(\hat{k}) Y^{JM}_{l's'}(\hat{r}) i^{l'} \\ &= (2/\pi)^{1/2} \sum_{ll'ss'JM} Y^{JM*}_{ls}(\hat{k}) Y^{JM}_{l's'}(\hat{r}) i^{l'} R^{\alpha,J}_{l's',ls}(k_\alpha,r).\end{aligned} \tag{2.8}$$

For the incoming particle (initial) state, the wave function becomes

$$\begin{aligned}\psi_\beta(\vec{r}') &= (2/\pi)^{1/2} \sum_{ll'ss'JM} R^{\beta,J}_{l's',ls}(k_\beta,r') Y^{JM*}_{ls}(\hat{k}) Y^{JM}_{l's'}(\hat{r}') i^{l'} \\ &= (2/\pi)^{1/2} \sum_{ll'ss'JM} Y^{JM*}_{ls}(\hat{k}) Y^{JM}_{l's'}(\hat{r}') i^{l'} R^{\beta,J}_{l's',ls}(k_\beta,r'),\end{aligned} \tag{2.9}$$

where $R^J_{l'S',lS}(k,r)$ is the radial functions which generalize the functions $R_l(k,r)$. Furthermore we write the Green's function as (See in Appendix A)

$$\begin{aligned}G(k,\vec{r},\vec{r}') &= \frac{2m}{\hbar^2}\left(-\frac{1}{4\pi}\frac{e^{i\bar{k}|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\right) \\ &= \frac{2m}{\hbar^2}\sum_{JMls} g_l(k,\vec{r},\vec{r}') Y^{JM^*}_{ls}(\hat{r}') Y^{JM}_{ls}(\hat{r}),\end{aligned} \tag{2.10}$$

with

$$g_l(k,\vec{r},\vec{r}') = -ikj_l(kr_<)h^{(1)}_l(kr_>), \tag{2.11}$$

where j_l is the spherical Bessel functions, and h_l is the Hankel functions.

We may then decompose the initial free wave by

$$\phi_\alpha(\vec{k}, s, m_s, \vec{r}) = (2/\pi)^{1/2} \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} i^l j_l(k_\alpha r) Y^*_{lm}(\hat{k}) \chi_{s,m_s} Y_{lm}(\hat{r}) \chi_{s,m_s}, \tag{2.12}$$

where χ_{s,m_s} is the spinor and

$$Y_{lm}(\hat{r})\chi_{s,m_s} = \sum_{JM} C(lm_l, sm_s, JM) Y^{JM}_{ls}(\hat{r}). \tag{2.13}$$

Substituting Eq. (2.13) into Eq. (2.12), we get

$$\begin{aligned}\phi_\alpha(\vec{k}, s, m_s, \vec{r}) = (2/\pi)^{1/2} &\sum_{l=0}^{\infty} \sum_{m=-l}^{+l} i^l j_l(k_\alpha r) Y^*_{lm}(\hat{k}) \chi_{s,m_s} \\ &\times \sum_{JM} C(lm_l, sm_s, JM) Y^{JM}_{ls}(\hat{r}),\end{aligned} \tag{2.14}$$

where

$$Y^{JM}_{ls}(\hat{k}) = \sum_{m=-l}^{+l} C(lm_l, sm_s, JM) Y_{lm}(\hat{k}) \chi_{s,m_s}, \tag{2.15}$$

thus

$$Y^{JM*}_{ls}(\hat{k}) = \sum_{m=-l}^{+l} C(lm_l, sm_s, JM) Y^*_{lm}(\hat{k}) \chi_{s,m_s}. \tag{2.16}$$

Substituting Eq. (2.16) into Eq. (2.14), we get

$$\begin{aligned}\phi_\alpha(\vec{k}, \vec{r}) &= (2/\pi)^{1/2} \sum_{ls} \sum_{JM} i^l j_l(k_\alpha r) Y^{JM}_{ls}(\hat{r}) Y^{JM*}_{ls}(\hat{k}) \\ &= (2/\pi)^{1/2} \sum_{ll'ss'} \sum_{JM} i^{l'} j_l(k_\alpha r) Y^{JM}_{l's'}(\hat{r}) Y^{JM*}_{ls}(\hat{k}) \delta_{ll'} \delta_{ss'}.\end{aligned} \tag{2.17}$$

Consider the second term in the right hand side of the Eq. (2.6), we get

$$
\begin{aligned}
&\int d^3\vec{r}\,' G(\vec{r},\vec{r}\,')V_{\alpha\beta}(\vec{r}\,')\psi_\beta(\vec{r}\,') \\
&\qquad = \int d^3\vec{r}\,' \sum_{JMls} g_l(k_\alpha, r, r') Y_{ls}^{JM*}(\hat{r}') Y_{ls}^{JM}(\hat{r}) V_{\alpha\beta}(\vec{r}\,')(2/\pi)^{1/2} \\
&\qquad\quad \times \sum_{ll'ss'JM} R^{\beta,J}_{l's',ls}(k_\beta r') Y_{ls}^{JM*}(\hat{k}) Y_{l's'}^{JM}(\hat{r}') i^{l'} \\
&\qquad = \int d^3\vec{r}\,' Y_{ls}^{JM*}(\hat{r}') Y_{l's'}^{JM}(\hat{r}') g_l(k_\alpha, r, r') V_{\alpha\beta}(\vec{r}\,')(2/\pi)^{1/2} \\
&\qquad\quad \times \sum_{ll'ss'JM} R^{\beta,J}_{l's',ls}(k_\beta r') Y_{ls}^{JM}(\hat{r}) Y_{ls}^{JM*}(\hat{k}) i^{l'} \\
&\qquad = \int d^3\vec{r}\,' |Y_{ls}^{JM}(\hat{r}')|^2 g_l(k_\alpha, r, r') V_{\alpha\beta}(\vec{r}\,')(2/\pi)^{1/2} \\
&\qquad\quad \times \sum_{ll'ss'JM} R^{\beta,J}_{l's',ls}(k_\beta r') Y_{ls}^{JM}(\hat{r}) Y_{ls}^{JM*}(\hat{k}) i^{l'} \\
&\qquad = \int_0^\infty \int_0^\pi \int_0^{2\pi} r'^2 \sin\theta\, g_l(k_\alpha, r, r') V^{\alpha\beta}_{ls,l's'}(\vec{r}\,')(2/\pi)^{1/2} \\
&\qquad\quad \times \sum_{ll'ss'JM} R^{\beta,J}_{l's',ls}(k_\beta r') Y_{ls}^{JM}(\hat{r}) Y_{ls}^{JM*}(\hat{k}) i^{l'}\, d\phi d\theta dr' \\
&\qquad = \int_0^\infty r'^2 dr' g_l(k_\alpha, r, r') V^{\alpha\beta}_{ls,l's'}(\vec{r}\,')(2/\pi)^{1/2} \\
&\qquad\quad \times \sum_{ll'ss'JM} R^{\beta,J}_{l's',ls}(k_\beta r') \delta_{ll'}\delta_{ss'} Y_{l's'}^{JM}(\hat{r}) Y_{ls}^{JM*}(\hat{k}) i^{l'} \\
&\qquad = \int_0^\infty r'^2 dr' g_l(k_\alpha, r, r') \sum_{l''s''} V^{\alpha\beta}_{l's',l''s''}(r')(2/\pi)^{1/2} \\
&\qquad\quad \times \sum_{ll'ss'JM} \delta_{l'l''}\delta_{s's''} R^{\beta,J}_{l's',ls}(k_\beta r') Y_{l's'}^{JM}(\hat{r}) Y_{ls}^{JM*}(\hat{k}) i^{l'} \\
&\qquad = \int_0^\infty r'^2 dr' g_l(k_\alpha, r, r') \sum_{l''s''} V^{\alpha\beta}_{l's',l''s''}(r')(2/\pi)^{1/2} \\
&\qquad\quad \times \sum_{ll'ss'JM} R^{\beta,J}_{l''s'',ls}(k_\beta r') Y_{l's'}^{JM}(\hat{r}) Y_{ls}^{JM*}(\hat{k}) i^{l'}
\end{aligned}
$$

$$
\begin{aligned}
=&(2/\pi)^{1/2} \sum_{ll'ss'JM} Y_{l's'}^{JM}(\hat{r}) Y_{ls}^{JM*}(\hat{k}) i^{l'} \\
&\times \sum_{l''s''} \int_0^\infty r'^2 dr' g_l(k_\alpha, r, r') V_{l's',l''s''}^{\alpha\beta}(r') R_{l''s'',ls}^{\beta,J}(k_\beta r'). \qquad (2.18)
\end{aligned}
$$

We now substituting Eq. (2.8), (2.17), and (2.18) in the Lippmann-Schwinger equation Eq. (2.6) to obtain the set of coupled integral equations which become the equation of radial scattered wave functions of coupled-channel as follows

$$
\begin{aligned}
R_{l's',ls}^{\alpha,J} \quad (k_\alpha, r) &= j_l(k_\alpha r)\delta_{\alpha\alpha_0}\delta_{ll'}\delta_{ss'} \\
&+ \sum_{l''s''} \int_0^\infty dr' r'^2 g_l(k_\alpha, r, r') V_{l's',l''s''}^{\alpha\beta,J}(r') R_{l''s'',ls}^{\beta,J}(k_\beta, r'), \qquad (2.19)
\end{aligned}
$$

where α and α_0 are final and initial state of the system, $l(l')$ and $s(s')$ are total orbital angular momentum and spin of initial (final) states, J is the total angular momentum, k_α and k_β are the momenta of final state and initial state respectively, g_l is the radial component of the Green's function $G(k, \vec{r}, \vec{r}')$, and $V_{l's',l''s''}^{\alpha\beta,J}$ is the interaction term between the initial state (β) and the final state (α).

In the next chapter, the radial Lippmann-Schwinger of the outgoing scattered wave function will be calculated by the numerical method.

CHAPTER III

NUMERICAL METHOD OF RADIAL PART LIPPMANN-SCHWINGER EQUATION

This chapter we describe the numerical procedure to solve our dynamical equations based on the chiral SU(3) symmetry. The s-wave meson-baryon interaction is studied at $S = -1$ sector by means of a coupled-channel Lippmann-Schwinger equation. This method can describe the $\Lambda(1405)$ resonance successfully.

3.1 The numerical methods

The calculations starting from the radial part of the out scattered wave equation Eq. (2.19). We consider the case of s-wave with $l = l' = 0$ and, $s = s' = 0$. For the total orbital angular momentum (l) equal to zero, the coupling state of the radial part s-wave function can be simply solved as a hard sphere scattering. Suppose that $V(r) \to 0$ at $r \geq R$, so the radial part can be rewritten as

$$R_\alpha(k_\alpha, r) = j_0(k_\alpha r)\delta_{\alpha\alpha_0} + \int r'^2 dr' g(k_\alpha, r, r') V_{\alpha\beta}(r') R_\beta(k_\beta, r'), \qquad (3.1)$$

where $\delta_{\alpha\alpha_0}$ is Dirac delta function.

To find the solution of Eq. (3.1) we substitute kr' with x', so we get $r' = \frac{x'}{k}$. Then, Eq. (3.1) becomes

$$R_\alpha(x) = j_0(x)\delta_{\alpha\alpha_0} + \int_0^\infty \frac{x'^2}{k^3} dx' g(x, x') V_{\alpha\beta}(x') R_\beta(x'). \qquad (3.2)$$

According to the central potential, $V_{\alpha\beta}(x') \rightarrow \frac{2m}{\hbar^2}V_{\alpha\beta}(x')$ where m is the reduce mass of the outgoing particles, the Eq. (3.2) becomes

$$R_\alpha(x) = j_0(x)\delta_{\alpha\alpha_0} + \int_0^\infty \frac{x'^2}{k^3}dx' g(x,x')\frac{2m}{\hbar^2}V_{\alpha\beta}(x')R_\beta(x'). \tag{3.3}$$

We use the numerically Gaussian-Legendre method to solve this above equation by dividing an area under the function of integral and later do the summation over all interval. The size of each interval correspond to the weight function $w(x)$ which is generated by Gaussian-Legendre method.

Next, the integral term of the radial part will be written in the language of Gaussian-Legendre as follows

$$R_\alpha(x) = j_0(x)\delta_{\alpha\alpha_0} + \frac{2m}{\hbar^2 k^3}\int_0^\infty dx' V_{\alpha\beta}(x')f(x')R_\beta(x'), \tag{3.4}$$

where $f(x') = x'^2 g_0(x,x')$. Now let $x' = x_j$, and $x = x_i$, where the indices i and j represent the final and initial state respectively. Therefore, we can use the summation to approximate the value of integration. Then the Eq. (3.4) becomes

$$\begin{aligned} R_\alpha(x_i) &= j(x_i)\delta_{\alpha\alpha_0} + \frac{2m}{\hbar^2 k^3}[\sum_{j=1}^{N} f(x_j)w(x_j)V_{\alpha\beta}(x_j)R_\beta(x_j)] \\ &= j(x_i)\delta_{\alpha\alpha_0} + \sum_{j=1}^{N} F_{ij}^{\alpha\beta}(x_j)R_\beta(x_j), \end{aligned} \tag{3.5}$$

where $\sum_{j=1}^{N} F_{ij}^{\alpha\beta}(x_j) = \frac{2m}{\hbar^2 k^3}[\sum_{j=1}^{N} f(x_j)w(x_j)V_{\alpha\beta}(x_j)]$. To evaluate Eq. (3.5) we use

$$
\begin{aligned}
R_\alpha(x_i) &= j_0(x_i)\delta_{\alpha\alpha_0} + \sum_{j=1}^{N} F_{ij}^{\alpha\beta}(x_j)R_\beta(x_j) \\
R_\alpha(x_i) - \sum_{j=1}^{N} F_{ij}^{\alpha\beta}(x_j)R_\beta(x_j) &= j_0(x_i)\delta_{\alpha\alpha_0} \\
\sum_{j=1}^{N} [\delta_{ij}R_\alpha(x_j) - F_{ij}^{\alpha\beta}(x_j)R_\beta(x_j)] &= j_0(x_i)\delta_{\alpha\alpha_0} \\
\sum_{j=1}^{N} [\delta_{ij}\delta_{\alpha\beta}R_\beta(x_j) - F_{ij}^{\alpha\beta}(x_j)R_\beta(x_j)] &= j_0(x_i)\delta_{\alpha\alpha_0} \\
\sum_{j=1}^{N} [(\delta_{ij}\delta_{\alpha\beta} - F_{ij}^{\alpha\beta}(x_j))R_\beta(x_j)] &= j_0(x_i)\delta_{\alpha\alpha_0}. \qquad (3.6)
\end{aligned}
$$

Note that in the above equation we have to sum over β too. To calculate the approximate solution of a set of real linear equations with multiple right-hand sides, we must rewrite into the matrices form, where the index i runs from one to N. Let we rewrite each term in Eq. (3.6) as follows

$$\sum_{j=1}^{N} (\delta_{ij}\delta\alpha\beta - F_{ij}^{\alpha\beta}(x_j)) = [A_{ij}], \qquad (3.7)$$

$$\sum_{j=1}^{N} R_\beta(x_j) = [R_{j1}], \qquad (3.8)$$

and

$$j_0(x_i)\delta_{\alpha\alpha_0} = [J_{i1}\delta_{\alpha\beta}], \qquad (3.9)$$

where $[A_{ij}]$, $[R_{j1}]$, and $[J_{i1}\delta_{\alpha\beta}]$ are matrices. Thus, the radial part of the out

scattering wave in Eq. (3.6) becomes

$$[A]_{i\times j}[R]_{j\times 1} = [J]_{i\times 1}$$
$$[R]_{j\times 1} = [A]^{-1}_{i\times j}[J]_{i\times 1}. \tag{3.10}$$

Later, we use this equation for our coupled-channel calculation.

Considering the radial Green's function, we can divide it into two cases depending on the range of scattering. First, for $r > r'$ or $x_i > x_j$, the radial Green's function can be written as

$$g_0(x_i, x_j) = kj_0(x_i)n_0(x_j). \tag{3.11}$$

Second, for $r < r'$ or $x_i < x_j$, the radial Green's function can be written as

$$g_0(x_i, x_j) = kj_0(x_j)n_0(x_i), \tag{3.12}$$

where j_0 and n_0 are the spherical Bessel function of the 1st and the 2nd kind for rank $l = 0$ respectively.

Substituting Eq. (3.11), and (3.12) into Eq. (3.1), we then calculate with the numerical program (See in Appendix B). Now, we can use the set of linear equations to evaluate the radial part by using the NAG Fortran Library. The details of our Fortran code are shown in Appendix B.

Finally, we obtain the approximated solutions of the radial part of scattered wave function in the form of matrix elements depend on the interaction (input potential).

3.2 The total cross section

The results from the radial part of the out scattered wave function can be used to get other properties of the scattering process such as, the cross sections, phase shifts, scattering lengths, scattering amplitudes, and transition amplitudes. Those properties can also be obtained from the experiments and compared with the theoretical calculations.

In this work, we succeed in computing the cross section of $\overline{K}N$. The details of analytic solution are given in this section.

3.2.1 Direct process

In case of direct process ($\alpha \to \alpha$), we start with the radial part that satisfies the Schrödinger equation in the spherical coordinates, that is

$$\left[\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} - \frac{l(l+1)}{r^2} - \frac{2m}{\hbar^2}V(r)\right]R_l(r) = -k^2R_l(r). \tag{3.13}$$

We are interested in the far-field solution where the distance $r > R$ and $V(r) \approx 0$, thus the Eq. (3.13) becomes

$$\left[\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} - \frac{l(l+1)}{r^2}\right]R_l(r) = -k^2R_l(r). \tag{3.14}$$

The general solution of Eq. (3.14) according to the far-field condition is

$$R_l(r) = a_l j_l(kr) - b_l n_l(kr), \tag{3.15}$$

where a_l and b_l are constant coefficients. Therefore, the asymptotically general solution of this equation above can be written as

$$\begin{aligned} R_l(r) &= \frac{A_l \sin(kr + \delta_l - l\pi/2)}{kr} \\ &= \frac{A_l \cos\delta_l \sin(kr - l\pi/2)}{kr} + \frac{A_l \sin\delta_l \cos(kr - l\pi/2)}{kr} \\ &= A_l j_l(kr)\cos\delta_l - A_l n_l(kr)\sin\delta_l, \end{aligned} \tag{3.16}$$

where A_l is the amplitude of the asymptotic solution, and δ_l is a phase shift due to the potential scattering.

The wave function across the boundary at $r = R$ should be continuous and differentially continuous. Thus, we have

$$\frac{r}{R_l(r)}\frac{dR_l(r)}{dr}\bigg|_{r=R^-} = kR\frac{j_l'(kR)\cos\delta_l - n_l'(kR)\sin\delta_l}{j_l(kR)\cos\delta_l - n_l(kR)\sin\delta_l} = \beta_l. \tag{3.17}$$

Let $\beta_l = kR\frac{j_l'(kR)\cos\delta_l - n_l'(kR)\sin\delta_l}{j_l(kR)\cos\delta_l - n_l(kR)\sin\delta_l}$, then the phase shift becomes

$$\delta_l = \arctan\left(\frac{\beta_l j_l(kR) - kRj_l'(kR)}{\beta_l n_l(kR) - kRn_l'(kR)}\right). \tag{3.18}$$

We have seen that the phase shift depends on the energy of the incoming particle as well as on the total orbital angular momentum l. We shall see that very often, only the states of small angular momentum contribute to the scattering state significantly. Thus we are motivated to study the scattering of the states of different angular momentum. Therefore, the total cross section σ_{tot} according to the optical theorem is given in

$$\sigma_{tot} = \frac{4\pi}{k^2}\sum_{l=0}^{l=kR}(2l+1)\sin^2\delta_l. \tag{3.19}$$

Therefore the total cross section in case of the direct process are given by the Eq. (3.19).

3.2.2 Cross process

Similarly, the cross process ($\alpha_0 \to \alpha$) can be obtained in the same way but different in the first term of the radial equation on the right hand side of the Eq. (3.1). According to this equation, if the potential equal to zero, then the incoming state for the radial part of the scattered wave can not occur. Thus, the radial part of the out scattering wave is reduced to

$$R_l^{\alpha}(r) = \int r'^2 dr' g_l(r, r') V^{\alpha\beta}(r') R_l^{\beta}(r'), \tag{3.20}$$

where $\alpha \neq \alpha_0$. Considering for the large-distance behavior, the wave function $\left\langle \vec{x} \mid \varphi^{(+)} \right\rangle$ in the presence of the scatterer is given by

$$\left\langle \vec{x} \mid \varphi^{(+)} \right\rangle = \frac{1}{(2\pi)^{\frac{3}{2}}} \left[f(\theta)_{\beta} \frac{e^{ikr}}{r} \right], \tag{3.21}$$

where $f(\theta)$ is the scattering amplitude.

As we will see explicitly later, at sufficiently large distances, the spatial dependence provided that the potential is of finite range. Using the partial method to expand the plane wave in term of the spherical wave, we get

$$\left\langle \vec{x} \mid \varphi^{(+)} \right\rangle = \frac{1}{(2\pi)^{\frac{3}{2}}} \sum_l (2l+1) f_l(k) P_l(\cos\theta) \frac{e^{ikr}}{r}, \tag{3.22}$$

where $f_l(k)$ is the partial-wave amplitude, and P_l is Legendre function of rank l. Next, the full-wave function at any r can be written as (for $r > R$, where R being

the range of the potential)

$$\left\langle \vec{x} \mid \varphi^{(+)} \right\rangle = \frac{1}{(2\pi)^{\frac{3}{2}}} \sum_l i^l (2l+1) R_l(r) P_l(\cos\theta). \tag{3.23}$$

The equivalent between Eq. (3.22) and Eq. (3.23) shows that the definition of the partial-wave amplitude is as follows

$$\begin{aligned} f_l(k)\frac{e^{ikr}}{r} &= i^l R_l(r) \\ f_l(k) &= i^l e^{-ikr} R_l(r) r. \end{aligned} \tag{3.24}$$

In this chapter we have shown already the numerical method that can be used to solve the dynamical equations of the radial outgoing scattered wave function. The next chapter we will consider the interaction term as the input potential appeared in the integral term of the radial part wave function. This step is the most important step in our calculation. The details of calculation will be shown in the next chapter.

CHAPTER IV

STRONG INTERACTION OF $\overline{K}N$

This chapter shows the calculations of the coupling interaction. For variational calculations of few-body systems involving $\overline{K}N$, one must use a realistic effective $\overline{K}N$ interactions, preferentially in the form of a potential. This potential is generally complex and energy dependent. It must be constrained to reproduce the scattering amplitudes. It also must encode the full coupled-channel dynamics. First attempts in this direction, using a schematic effective interaction (Weise, 2007; Dote and Weise, 2007). Here we would like to explicitly derive such an effective interaction in the single $\overline{K}N$ channel and construct an equivalent, energy dependent local potential, starting from chiral SU(3) coupled-channel scattering. The derivation of the radial out scattered wave function in the language of the Lippmann-Schwinger equation shown that the other quantities can be further calculated with. Basically, the most important term in radial Lippmann-Schwinger equation of the out scattering particle is the potentials ($V^{\alpha\beta}$), that is the interaction between the particle β and α.

The s-wave resonance in $\overline{K}N$ coupled-channel systems clearly tell us that the interactions is strong. In this part, we describe the various two-body interactions ($\overline{K}N$). Here, we intend to construct two-body coupled-channel $\overline{K}N$ interactions. The coupled-channel involved $\overline{K}N$ and πY ($Y = \Lambda, \Sigma$) with different total charge states for the mesons and baryons (in the particle basis), or in different total isospin states (in the isospin basis). The physical masses used in the particle basis and average masses used in the isospin basis may be found in Table 4.1.

Table 4.1 Particle masses (in MeV). The fourth column gives the average mass for each isospin multiplet, and the last column specifies the phase convention used for the isospin states.

		Mass	Avg. Mass	States
$\overline{K}$	K^-	493.7	495.7	$\left\vert\frac{1}{2}\ -\frac{1}{2}\right\rangle$
	$\bar{K}^0$	497.7		$\left\vert\frac{1}{2}\ \frac{1}{2}\right\rangle$
N	p	938.3	938.9	$\left\vert\frac{1}{2}\ \frac{1}{2}\right\rangle$
	n	939.6		$\left\vert\frac{1}{2}\ -\frac{1}{2}\right\rangle$
π	π^-	139.6	138.0	$\vert 1\ -1\rangle$
	π^+	139.6		$\vert 1\ 1\rangle$
	π^0	134.9		$\vert 1\ 0\rangle$
Σ	Σ^-	1197.4	1193.1	$\vert 1\ -1\rangle$
	Σ^+	1189.4		$\vert 1\ 1\rangle$
	Σ^0	1192.6		$\vert 1\ 0\rangle$
Λ		1115.7		$\vert 0\ 0\rangle$

It is important to emphasize here that we follow the observation by Oset and Ramos (Oset and Ramos, 1998) and retain the channels in our fit although this channel has a considerably higher threshold as compared with that for $\overline{K}N$. As a result we have two coupled channels: $\overline{K}N$, $\pi\Sigma$ to deal with for $I = 0$, and three coupled channels: $\overline{K}N$, $\pi\Sigma$, $\pi\Lambda$ for $I = 1$. In terms of physical (or particle) channels the following two groups are separately coupled by this direction

$$K^-p \to K^-p,\ \bar{K}^0n,\ \pi^+\Sigma^-,\ \ \pi^0\Sigma^0,\ \pi^-\Sigma^+,\ \pi^0\Lambda.$$

4.1 The formalism appropriate to complete charge-independence

Particles are proposed to possess a strangeness quantum number (S) which is conserved in strong interaction (but not weak). As a definition we can

write the complete charge as

$$Q = e\left[I_z + \frac{1}{2}(N+S)\right] = e\left[I_z + \frac{1}{2}Y\right], \tag{4.1}$$

where Q is total charge, I_z is isospin projection, N is baryon number, and e is the magnitude of electron charge. If we average over all a family of which have the same N and S, where $Y = N + S$, then we have $\sum I_z = 0$.

For N-baryon, the strangeness is $S = 2\frac{\bar{Q}}{e} - N$, and $S = 2\frac{\bar{Q}}{e}$ for mesons. The strangeness quantum number shown in Table 4.2, and Table 4.3 makes a big difference to which strong reactions are available for the various particles. For example, K^+ has only elastic scattering (with $S = +1$) while K^- has many more final states (all having $S = -1$).

Table 4.2 The strangeness of baryons with the corresponding average charged.

	Family	$\overline{Q}/e$	S
N	$\begin{pmatrix} p \\ n \end{pmatrix}$	$\frac{1}{2}$	0
Σ	$\begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix}$	0	-1
Λ	Λ	0	-1

Table 4.3 The strangeness of mesons with the corresponding average charged.

	Family	$\overline{Q}/e$	S
π	$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$	0	0
$\overline{K}$	$\begin{pmatrix} \bar{K}^0 \\ K^- \end{pmatrix}$	$-\frac{1}{2}$	-1

In the framework of SU(3) symmetry, the strong interaction is invariant under the unitary transformation of u, d, and s quarks, the interactions among the channels ($S = -1$) $\overline{K}N$, $\pi\Sigma$, and $\pi\Lambda$ are related with each other under the conservation of isospin. The relations are usefully express in the isospin basis.

In this situation, it is appropriate to consider basic states of definite total isotopic spin I. For the $\overline{K}N$ states of total charge zero with $I = 0$ and $I = 1$, the states are

$$\psi_0 = \frac{1}{\sqrt{2}}\left(\left|K^- p\right\rangle - \left|\bar{K}^0 n\right\rangle\right), \tag{4.2}$$

$$\psi_1 = \frac{1}{\sqrt{2}}\left(\left|K^- p\right\rangle + \left|\bar{K}^0 n\right\rangle\right). \tag{4.3}$$

For the $Y\pi$ states, these take the forms

$$\phi_0 = \frac{1}{\sqrt{3}}\left(\left|\pi^+\Sigma^-\right\rangle - \left|\pi^0\Sigma^0\right\rangle + \left|\pi^-\Sigma^+\right\rangle\right), \tag{4.4}$$

$$\phi_1 = \frac{1}{\sqrt{2}}\left(\left|\pi^+\Sigma^-\right\rangle - \left|\pi^-\Sigma^+\right\rangle\right), \tag{4.5}$$

$$\chi_1 = \left|\pi^0\Lambda\right\rangle. \tag{4.6}$$

In these isospin amplitudes we must use average mass for the $\overline{K}(K^-, \bar{K}^0)$, $N(p, n)$, $\pi(\pi^-, \pi^0, \pi^+)$, $\Sigma(\Sigma^-, \Sigma^0, \Sigma^+)$, and Λ states.

The strong interaction terms, may be interpreted in terms of interactions in isospin basis since the proton, neutron, neutral anti-kaon, negatively charge kaon, neutral pion, negatively charge pion, positively charge pion, lambda , sigma, negatively charge sigma, and positively charge sigma are all isospin eigenstates.

We use the phase convention for nucleon, that are $p = \left|\frac{1}{2}, \frac{1}{2}\right\rangle$ and $n = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$.

The phase convention for anti-kaon are $K^- = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$ and $\bar{K}^0 = \left|\frac{1}{2}, \frac{1}{2}\right\rangle$.

The phase convention for pion are $\pi^- = |1,-1\rangle$, $\pi^0 = |1,0\rangle$, and $\pi^+ = |1,1\rangle$.

The phase convention for sigma are $\Sigma^- = |1,-1\rangle$, $\Sigma^0 = |1,0\rangle$, and $\Sigma^+ = |1,1\rangle$.

The phase convention for lambda are $\Lambda = |0,0\rangle$.

Because the strong interaction is isospin conserve, so we can express the particle basis interactions in the form of isospin basis.

Starting from the spin state of spin $-\frac{1}{2}$ particles $\overline{K}$ and N, the total spin and its projection onto the defined axis can be computed by using the rules for adding angular momentum in quantum mechanics and the Clebsch-Gordan (C_{ij}) coefficients (See in Appendix C). In general,

$$|I, I_z\rangle = \sum_{I_{z1}+I_{z2}=I_z} C^{I_1 I_2 I}_{I_{z1} I_{z2} I_z} |I_1 I_{z1}\rangle |I_2 I_{z2}\rangle . \tag{4.7}$$

For $I = 0$, the isospin basis in terms of the particle basis can be written as

$$\begin{aligned}
\chi(0,0) &= C_1 \chi_1(\tfrac{1}{2},\tfrac{1}{2})\chi_2(\tfrac{1}{2},-\tfrac{1}{2}) - C_2 \chi_1(\tfrac{1}{2},-\tfrac{1}{2})\chi_2(\tfrac{1}{2},\tfrac{1}{2}) \\
&= \frac{1}{\sqrt{2}}\bar{K}^0 n - \frac{1}{\sqrt{2}} K^- p \\
&= \frac{1}{\sqrt{2}}(\bar{K}^0 n - K^- p) \to I(0,0).
\end{aligned} \tag{4.8}$$

For $I = 1$, the isospin basis in term of the particle basis can be written as

$$\begin{aligned}
\chi(1,0) &= C_1 \chi_1(\tfrac{1}{2},\tfrac{1}{2})\chi_2(\tfrac{1}{2},-\tfrac{1}{2}) + C_2 \chi_1(\tfrac{1}{2},-\tfrac{1}{2})\chi_2(\tfrac{1}{2},\tfrac{1}{2}) \\
&= \frac{1}{\sqrt{2}}\bar{K}^0 n + \frac{1}{\sqrt{2}} K^- p \\
&= \frac{1}{\sqrt{2}}(\bar{K}^0 n + K^- p) \to I(1,0).
\end{aligned} \tag{4.9}$$

On the other hands, the particles basis can be written in terms of the linear

combination of isospin basis as follows

$$\bar{K}^0 n = \frac{1}{\sqrt{2}} \left(\left| \overline{K} N(1) \right\rangle + \left| \overline{K} N(0) \right\rangle \right), \tag{4.10}$$

$$K^- p = \frac{1}{\sqrt{2}} \left(\left| \overline{K} N(1) \right\rangle - \left| \overline{K} N(0) \right\rangle \right). \tag{4.11}$$

Similarly to the other particles, for $\pi\Sigma$ with $I = 2$, the isospin basis in term of the particle basis can be written as

$$\begin{aligned} \chi(2,2) &= C_1 \chi_1(1,1)\chi_2(1,1) \\ &= \pi^+\Sigma^+ \rightarrow I(2,2), \end{aligned} \tag{4.12}$$

$$\begin{aligned} \chi(2,1) &= C_1 \chi_1(1,1)\chi_2(1,0) + C_2 \chi_1(1,0)\chi_2(1,1) \\ &= \frac{1}{\sqrt{2}}\pi^+\Sigma^0 + \frac{1}{\sqrt{2}}\pi^0\Sigma^+ \\ &= \frac{1}{\sqrt{2}}(\pi^+\Sigma^0 + \pi^0\Sigma^+) \rightarrow I(2,1), \end{aligned} \tag{4.13}$$

$$\begin{aligned} \chi(2,0) &= C_1 \chi_1(1,1)\chi_2(1,-1) + C_2 \chi_1(1,-1)\chi_2(1,1) + C_3 \chi_1(1,0)\chi_2(1,0) \\ &= \frac{1}{\sqrt{6}}\pi^+\Sigma^- + \frac{1}{\sqrt{6}}\pi^-\Sigma^+ + \sqrt{\frac{2}{3}}\pi^0\Sigma^0 \\ &= \frac{1}{\sqrt{6}}(\pi^+\Sigma^- + \pi^-\Sigma^+ + 2\pi^0\Sigma^0) \rightarrow I(2,0). \end{aligned} \tag{4.14}$$

For $I = 1$, the isospin basis in term of the particle basis can be written as

$$\begin{aligned} \chi(1,1) &= C_1 \chi_1(1,1)\chi_2(1,0) + C_2 \chi_1(1,0)\chi_2(1,1) \\ &= \frac{1}{\sqrt{2}}\pi^+\Sigma^0 - \frac{1}{\sqrt{2}}\pi^0\Sigma^+ \\ &= \frac{1}{\sqrt{2}}(\pi^+\Sigma^0 - \pi^+\Sigma^0) \rightarrow I(1,1), \end{aligned} \tag{4.15}$$

$$\begin{aligned}\chi(1,0) &= C_1\chi_1(1,1)\chi_2(1,-1) + C_2\chi_1(1,0)\chi_2(1,0) + C_3\chi_1(1,-1)\chi_2(1,1)\\ &= \frac{1}{\sqrt{2}}\pi^+\Sigma^- - \frac{1}{\sqrt{2}}\pi^-\Sigma^+\\ &= \frac{1}{\sqrt{2}}(\pi^+\Sigma^- - \pi^-\Sigma^+) \to I(1,0). \end{aligned} \tag{4.16}$$

For $I = 0$, the isospin basis in term of the particle basis can be written as

$$\begin{aligned}\chi(0,0) &= C_1\chi_1(1,1)\chi_2(1,-1) + C_2\chi_1(1,0)\chi_2(1,0) + C_3\chi_1(1,-1)\chi_2(1,1)\\ &= \frac{1}{\sqrt{3}}\pi^+\Sigma^- - \frac{1}{\sqrt{3}}\pi^0\Sigma^0 + \frac{1}{\sqrt{3}}\pi^-\Sigma^+\\ &= \frac{1}{\sqrt{3}}(\pi^+\Sigma^- - \pi^0\Sigma^0 + \pi^-\Sigma^+) \to I(0,0). \end{aligned} \tag{4.17}$$

The particles bases in terms of linear combination of isospin bases can be written as

$$\pi^0\Sigma^0 = \sqrt{\frac{2}{3}}\,|\pi\Sigma(2)\rangle - \frac{1}{\sqrt{3}}\,|\pi\Sigma(0)\rangle\,, \tag{4.18}$$

$$\pi^+\Sigma^- = \frac{1}{\sqrt{6}}\,|\pi\Sigma(2)\rangle + \frac{1}{\sqrt{2}}\,|\pi\Sigma(1)\rangle + \frac{1}{\sqrt{3}}\,|\pi\Sigma(0)\rangle\,, \tag{4.19}$$

$$\pi^-\Sigma^+ = \frac{1}{\sqrt{6}}\,|\pi\Sigma(2)\rangle - \frac{1}{\sqrt{2}}\,|\pi\Sigma(1)\rangle + \frac{1}{\sqrt{3}}\,|\pi\Sigma(0)\rangle\,. \tag{4.20}$$

For $\pi\Lambda$ with $I = 1$, the isospin basis in term of the particle basis can be written as

$$\begin{aligned}\chi(1,0) &= C_1\chi_1(1,0)\chi_2(0,0)\\ &= \pi^0\Lambda^0 \to I(1,0). \end{aligned} \tag{4.21}$$

Thus, the particles bases in terms of linear combination of isospin bases can be

written as

$$\pi^0\Lambda^0 = |\pi\Lambda(1)\rangle\,. \tag{4.22}$$

Finally, we consider the rates of the strong interaction (elastic scattering and charge exchange scattering) where everything else is unchanged. Historically it was assumed that these were interactions occurring at a single vertex. In this case we have

$$\text{Rate} \propto |\langle\psi_\text{f}|\,\text{V}\,|\psi_\text{i}\rangle|^2\,, \tag{4.23}$$

and we define $V = \hat{V}_2 + \hat{V}_1 + \hat{V}_0$ where $\hat{V}_2$ is an operator for the isospin 2 states with eigenvalue V^2_{if}, $\hat{V}_1$ is an operator for the isospin 1 states with eigenvalue V^1_{if}, and $\hat{V}_0$ is an operator for the isospin 0 states with eigenvalue V^0_{if}.

The amplitude for various scattering process can be written in isospin basis as follows:

$$\begin{aligned}
\left\langle K^-p\right|V\left|K^-p\right\rangle =& \left(\frac{1}{\sqrt{2}}\left\langle\overline{K}N(1)\right| - \frac{1}{\sqrt{2}}\left\langle\overline{K}N(0)\right|\right) \\
&\times V\left(\frac{1}{\sqrt{2}}\left|\overline{K}N(1)\right\rangle - \frac{1}{\sqrt{2}}\left|\overline{K}N(0)\right\rangle\right) \\
=&\frac{1}{2}\left\langle\overline{K}N(1)\right|\hat{V}_1\left|\overline{K}N(1)\right\rangle + \frac{1}{2}\left\langle\overline{K}N(0)\right|\hat{V}_0\left|\overline{K}N(0)\right\rangle \\
=&\frac{1}{2}V^1_{11} + \frac{1}{2}V^0_{11}
\end{aligned} \tag{4.24}$$

where $V^1_{11} \equiv V^{I=1}_{\overline{K}N\to\overline{K}N}$ and $V^0_{11} \equiv V^{I=0}_{\overline{K}N\to\overline{K}N}$

$$\begin{aligned}\left\langle K^- p\right| V \left|\bar{K}^0 n\right\rangle =& \left(\frac{1}{\sqrt{2}}\left\langle \overline{K}N(1)\right| - \frac{1}{\sqrt{2}}\left\langle \overline{K}N(0)\right|\right) \\ & \times V\left(\frac{1}{\sqrt{2}}\left|\overline{K}N(1)\right\rangle + \frac{1}{\sqrt{2}}\left|\overline{K}N(0)\right\rangle\right) \\ =& \frac{1}{2}\left\langle \overline{K}N(1)\right| \hat{V}_1 \left|\overline{K}N(1)\right\rangle - \frac{1}{2}\left\langle \overline{K}N(0)\right| \hat{V}_0 \left|\overline{K}N(0)\right\rangle \\ =& \frac{1}{2}V_{12}^1 - \frac{1}{2}V_{12}^0 \end{aligned} \tag{4.25}$$

where $V_{12}^1 \equiv V_{\overline{K}N\to\overline{K}N}^{I=1}$ and $V_{12}^0 \equiv V_{\overline{K}N\to\overline{K}N}^{I=0}$

$$\begin{aligned}\left\langle K^- p\right| V \left|\pi^0\Sigma^0\right\rangle =& \left(\frac{1}{\sqrt{2}}\left\langle \overline{K}N(1)\right| - \frac{1}{\sqrt{2}}\left\langle \overline{K}N(0)\right|\right) \\ & \times V\left(\sqrt{\frac{2}{3}}\left|\pi\Sigma(2)\right\rangle - \frac{1}{\sqrt{3}}\left|\pi\Sigma(0)\right\rangle\right) \\ =& \frac{1}{\sqrt{6}}\left\langle \overline{K}N(0)\right| \hat{V}_0 \left|\pi\Sigma(0)\right\rangle \\ =& \frac{1}{\sqrt{6}}V_{13}^0 \end{aligned} \tag{4.26}$$

where $V_{13}^0 \equiv V_{\overline{K}N\to\pi\Sigma}^{I=0}$

$$\begin{aligned}\left\langle K^- p\right| V \left|\pi^+\Sigma^-\right\rangle =& \left(\frac{1}{\sqrt{2}}\left\langle \overline{K}N(1)\right| - \frac{1}{\sqrt{2}}\left\langle \overline{K}N(0)\right|\right) \\ & \times V\left(\frac{1}{\sqrt{6}}\left|\pi\Sigma(2)\right\rangle + \frac{1}{\sqrt{2}}\left|\pi\Sigma(1)\right\rangle + \frac{1}{\sqrt{3}}\left|\pi\Sigma(0)\right\rangle\right) \\ =& \frac{1}{2}\left\langle \overline{K}N(1)\right| \hat{V}_1 \left|\pi\Sigma(1)\right\rangle - \frac{1}{\sqrt{6}}\left\langle \overline{K}N(0)\right| \hat{V}_0 \left|\pi\Sigma(0)\right\rangle \\ =& \frac{1}{2}V_{14}^1 - \frac{1}{\sqrt{6}}V_{14}^0 \end{aligned} \tag{4.27}$$

where $V_{14}^1 \equiv V_{\overline{K}N\to\pi\Sigma}^{I=1}$ and $V_{14}^0 \equiv V_{\overline{K}N\to\pi\Sigma}^{I=0}$

$$\begin{aligned}\left\langle K^- p\right| V \left|\pi^- \Sigma^+\right\rangle &= \left(\frac{1}{\sqrt{2}}\left\langle \overline{K}N(1)\right| - \frac{1}{\sqrt{2}}\left\langle \overline{K}N(0)\right|\right) \\ &\quad \times V\left(\frac{1}{\sqrt{6}}\left|\pi\Sigma(2)\right\rangle - \frac{1}{\sqrt{2}}\left|\pi\Sigma(1)\right\rangle + \frac{1}{\sqrt{3}}\left|\pi\Sigma(0)\right\rangle\right) \\ &= -\frac{1}{2}\left\langle \overline{K}N(1)\right| \hat{V}_1 \left|\pi\Sigma(1)\right\rangle - \frac{1}{\sqrt{6}}\left\langle \overline{K}N(0)\right| \hat{V}_0 \left|\pi\Sigma(0)\right\rangle \\ &= -\frac{1}{2}V_{15}^1 - \frac{1}{\sqrt{6}}V_{15}^0 \end{aligned} \tag{4.28}$$

where $V_{15}^1 \equiv V_{\overline{K}N\to\pi\Sigma}^{I=1}$ and $V_{15}^0 \equiv V_{\overline{K}N\to\pi\Sigma}^{I=0}$

$$\begin{aligned}\left\langle K^- p\right| V \left|\pi^0 \Lambda^0\right\rangle &= \left(\frac{1}{\sqrt{2}}\left\langle \overline{K}N(1)\right| - \frac{1}{\sqrt{2}}\left\langle \overline{K}N(0)\right|\right) V \left|\pi\Lambda(1)\right\rangle \\ &= \frac{1}{\sqrt{2}}\left\langle \overline{K}N(1)\right| \hat{V}_1 \left|\pi\Lambda(1)\right\rangle \\ &= \frac{1}{\sqrt{2}}V_{16}^1 \end{aligned} \tag{4.29}$$

where $V_{16}^1 \equiv V_{\overline{K}N\to\pi\Lambda}^{I=1}$

$$\begin{aligned}\left\langle \bar{K}^0 n\right| V \left|K^- p\right\rangle &= \left(\frac{1}{\sqrt{2}}\left\langle \overline{K}N(1)\right| + \frac{1}{\sqrt{2}}\left\langle \overline{K}N(0)\right|\right) \\ &\quad \times V\left(\frac{1}{\sqrt{2}}\left|\overline{K}N(1)\right\rangle - \frac{1}{\sqrt{2}}\left|\overline{K}N(0)\right\rangle\right) \\ &= \frac{1}{2}\left\langle \overline{K}N(1)\right| \hat{V}_1 \left|\overline{K}N(1)\right\rangle - \frac{1}{2}\left\langle \overline{K}N(0)\right| \hat{V}_0 \left|\overline{K}N(0)\right\rangle \\ &= \frac{1}{2}V_{21}^1 - \frac{1}{2}V_{21}^0 \end{aligned} \tag{4.30}$$

where $V_{21}^1 \equiv V_{\overline{K}N\to\overline{K}N}^{I=1}$ and $V_{21}^0 \equiv V_{\overline{K}N\to\overline{K}N}^{I=0}$

$$\begin{aligned}\left\langle \bar{K}^0 n\right| V \left|\bar{K}^0 n\right\rangle &= \left(\frac{1}{\sqrt{2}}\left\langle \overline{K}N(1)\right| + \frac{1}{\sqrt{2}}\left\langle \overline{K}N(0)\right|\right) \\ &\quad \times V\left(\frac{1}{\sqrt{2}}\left|\overline{K}N(1)\right\rangle + \frac{1}{\sqrt{2}}\left|\overline{K}N(0)\right\rangle\right) \\ &= \frac{1}{2}\left\langle \overline{K}N(1)\right| \hat{V}_1 \left|\overline{K}N(1)\right\rangle + \frac{1}{2}\left\langle \overline{K}N(0)\right| \hat{V}_0 \left|\overline{K}N(0)\right\rangle \\ &= \frac{1}{2}V_{22}^1 + \frac{1}{2}V_{22}^0 \end{aligned} \tag{4.31}$$

where $V_{22}^1 \equiv V_{\overline{K}N\to\overline{K}N}^{I=1}$ and $V_{22}^0 \equiv V_{\overline{K}N\to\overline{K}N}^{I=0}$

$$\begin{aligned}\left\langle \bar{K}^0 n\right| V \left|\pi^0\Sigma^0\right\rangle =& \left(\frac{1}{\sqrt{2}}\left\langle \overline{K}N(1)\right| + \frac{1}{\sqrt{2}}\left\langle \overline{K}N(0)\right|\right) \\ & \times V\left(\sqrt{\frac{2}{3}}\left|\pi\Sigma(2)\right\rangle - \frac{1}{\sqrt{3}}\left|\pi\Sigma(0)\right\rangle\right) \\ =& -\frac{1}{\sqrt{6}}\left\langle \overline{K}N(0)\right| \hat{V}_0 \left|\pi\Sigma(0)\right\rangle \\ =& -\frac{1}{\sqrt{6}} V_{23}^0 \end{aligned} \tag{4.32}$$

where $V_{23}^0 \equiv V_{\overline{K}N\to\pi\Lambda}^{I=0}$

$$\begin{aligned}\left\langle \bar{K}^0 n\right| V \left|\pi^+\Sigma^-\right\rangle =& \left(\frac{1}{\sqrt{2}}\left\langle \overline{K}N(1)\right| + \frac{1}{\sqrt{2}}\left\langle \overline{K}N(0)\right|\right) \\ & \times V\left(\frac{1}{\sqrt{6}}\left|\pi\Sigma(2)\right\rangle + \frac{1}{\sqrt{2}}\left|\pi\Sigma(1)\right\rangle + \frac{1}{\sqrt{3}}\left|\pi\Sigma(0)\right\rangle\right) \\ =& \frac{1}{2}\left\langle \overline{K}N(1)\right| \hat{V}_1 \left|\pi\Sigma(1)\right\rangle + \frac{1}{\sqrt{6}}\left\langle \overline{K}N(0)\right| \hat{V}_0 \left|\pi\Sigma(0)\right\rangle \\ =& \frac{1}{2} V_{24}^1 + \frac{1}{\sqrt{6}} V_{24}^0 \end{aligned} \tag{4.33}$$

where $V_{24}^1 \equiv V_{\overline{K}N\to\pi\Sigma}^{I=1}$ and $V_{24}^0 \equiv V_{\overline{K}N\to\pi\Sigma}^{I=0}$

$$\begin{aligned}\left\langle \bar{K}^0 n\right| V \left|\pi^-\Sigma^+\right\rangle =& \left(\frac{1}{\sqrt{2}}\left\langle \overline{K}N(1)\right| + \frac{1}{\sqrt{2}}\left\langle \overline{K}N(0)\right|\right) \\ & \times V\left(\frac{1}{\sqrt{6}}\left|\pi\Sigma(2)\right\rangle - \frac{1}{\sqrt{2}}\left|\pi\Sigma(1)\right\rangle + \frac{1}{\sqrt{3}}\left|\pi\Sigma(0)\right\rangle\right) \\ =& -\frac{1}{2}\left\langle \overline{K}N(1)\right| \hat{V}_1 \left|\pi\Sigma(1)\right\rangle + \frac{1}{\sqrt{6}}\left\langle \overline{K}N(0)\right| \hat{V}_0 \left|\pi\Sigma(0)\right\rangle \\ =& -\frac{1}{2} V_{25}^1 + \frac{1}{\sqrt{6}} V_{25}^0 \end{aligned} \tag{4.34}$$

where $V_{25}^1 \equiv V_{\overline{K}N\to\pi\Sigma}^{I=1}$ and $V_{25}^0 \equiv V_{\overline{K}N\to\pi\Sigma}^{I=0}$

$$\begin{aligned}\left\langle \bar{K}^0 n\right| V \left|\pi^0\Lambda^0\right\rangle &= \left(\frac{1}{\sqrt{2}}\left\langle \overline{K}N(1)\right| + \frac{1}{\sqrt{2}}\left\langle \overline{K}N(0)\right|\right) V \left|\pi\Lambda(1)\right\rangle \\ &= \frac{1}{\sqrt{2}}\left\langle \overline{K}N(1)\right| \hat{V}_1 \left|\pi\Lambda(1)\right\rangle \\ &= \frac{1}{\sqrt{2}} V^1_{26} \end{aligned} \tag{4.35}$$

where $V^1_{26} \equiv V^{I=1}_{\overline{K}N\to\pi\Lambda}$

$$\begin{aligned}\left\langle \pi^0\Sigma^0\right| V \left|K^- p\right\rangle &= \left(\sqrt{\frac{2}{3}}\left\langle \pi\Sigma(2)\right| - \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) \\ &\quad \times V\left(\frac{1}{\sqrt{2}}\left|\overline{K}N(1)\right\rangle - \frac{1}{\sqrt{2}}\left|\overline{K}N(0)\right\rangle\right) \\ &= \frac{1}{\sqrt{6}}\left\langle \pi\Sigma(0)\right| \hat{V}_0 \left|\overline{K}N(0)\right\rangle \\ &= \frac{1}{\sqrt{6}} V^0_{31} \end{aligned} \tag{4.36}$$

where $V^0_{31} \equiv V^{I=0}_{\pi\Sigma\to\overline{K}N}$

$$\begin{aligned}\left\langle \pi^0\Sigma^0\right| V \left|\bar{K}^0 n\right\rangle &= \left(\sqrt{\frac{2}{3}}\left\langle \pi\Sigma(2)\right| - \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) \\ &\quad \times V\left(\frac{1}{\sqrt{2}}\left|\overline{K}N(1)\right\rangle + \frac{1}{\sqrt{2}}\left|\overline{K}N(0)\right\rangle\right) \\ &= -\frac{1}{\sqrt{6}}\left\langle \pi\Sigma(0)\right| \hat{V}_0 \left|\overline{K}N(0)\right\rangle \\ &= -\frac{1}{\sqrt{6}} V^0_{32} \end{aligned} \tag{4.37}$$

where $V^0_{32} \equiv V^{I=0}_{\pi\Sigma\to\overline{K}N}$

$$\begin{aligned}\left\langle \pi^0\Sigma^0\right| V \left|\pi^0\Sigma^0\right\rangle &= \left(\sqrt{\frac{2}{3}}\left\langle \pi\Sigma(2)\right| - \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) V \left(\sqrt{\frac{2}{3}}\left|\pi\Sigma(2)\right\rangle - \frac{1}{\sqrt{3}}\left|\pi\Sigma(0)\right\rangle\right) \\ &= \frac{2}{3}\left\langle \pi\Sigma(2)\right| \hat{V}_2 \left|\pi\Sigma(2)\right\rangle + \frac{1}{3}\left\langle \pi\Sigma(0)\right| \hat{V}_0 \left|\pi\Sigma(0)\right\rangle \\ &= \frac{2}{3} V^2_{33} + \frac{1}{3} V^0_{33} \end{aligned} \tag{4.38}$$

where $V_{33}^2 \equiv V_{\pi\Sigma\to\pi\Sigma}^{I=2}$ and $V_{33}^0 \equiv V_{\pi\Sigma\to\pi\Sigma}^{I=0}$

$$\begin{aligned}\left\langle \pi^0\Sigma^0\right| V \left|\pi^+\Sigma^-\right\rangle =& \left(\sqrt{\frac{2}{3}}\left\langle \pi\Sigma(2)\right| - \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) \\ & \times V\left(\frac{1}{\sqrt{6}}\left|\pi\Sigma(2)\right\rangle + \frac{1}{\sqrt{2}}\left|\pi\Sigma(1)\right\rangle + \frac{1}{\sqrt{3}}\left|\pi\Sigma(0)\right\rangle\right) \\ =& \frac{1}{3}\left\langle \pi\Sigma(2)\right| \hat{V}_2 \left|\pi\Sigma(2)\right\rangle - \frac{1}{3}\left\langle \pi\Sigma(0)\right| \hat{V}_0 \left|\pi\Sigma(0)\right\rangle \\ =& \frac{1}{3}V_{34}^2 - \frac{1}{3}V_{34}^0 \end{aligned} \tag{4.39}$$

where $V_{34}^2 \equiv V_{\pi\Sigma\to\pi\Sigma}^{I=2}$ and $V_{34}^0 \equiv V_{\pi\Sigma\to\pi\Sigma}^{I=0}$

$$\begin{aligned}\left\langle \pi^0\Sigma^0\right| V \left|\pi^-\Sigma^+\right\rangle =& \left(\sqrt{\frac{2}{3}}\left\langle \pi\Sigma(2)\right| - \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) \\ & \times V\left(\frac{1}{\sqrt{6}}\left|\pi\Sigma(2)\right\rangle - \frac{1}{\sqrt{2}}\left|\pi\Sigma(1)\right\rangle + \frac{1}{\sqrt{3}}\left|\pi\Sigma(0)\right\rangle\right) \\ =& \frac{1}{3}\left\langle \pi\Sigma(2)\right| \hat{V}_2 \left|\pi\Sigma(2)\right\rangle - \frac{1}{3}\left\langle \pi\Sigma(0)\right| \hat{V}_0 \left|\pi\Sigma(0)\right\rangle \\ =& \frac{1}{3}V_{35}^2 - \frac{1}{3}V_{35}^0 \end{aligned} \tag{4.40}$$

where $V_{35}^2 \equiv V_{\pi\Sigma\to\pi\Sigma}^{I=2}$ and $V_{35}^0 \equiv V_{\pi\Sigma\to\pi\Sigma}^{I=0}$

$$\begin{aligned}\left\langle \pi^0\Sigma^0\right| V \left|\pi^0\Lambda^0\right\rangle =& \left(\sqrt{\frac{2}{3}}\left\langle \pi\Sigma(2)\right| - \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) V \left|\pi\Lambda(1)\right\rangle \\ =& 0 \end{aligned} \tag{4.41}$$

$$\begin{aligned}\left\langle \pi^+\Sigma^-\right| V \left|K^- p\right\rangle =& \left(\frac{1}{\sqrt{6}}\left\langle \pi\Sigma(2)\right| + \frac{1}{\sqrt{2}}\left\langle \pi\Sigma(1)\right| + \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) \\ & \times V\left(\frac{1}{\sqrt{2}}\left|\overline{K}N(1)\right\rangle - \frac{1}{\sqrt{2}}\left|\overline{K}N(0)\right\rangle\right) \\ =& \frac{1}{2}\left\langle \pi\Sigma(1)\right| \hat{V}_1 \left|\overline{K}N(1)\right\rangle + \frac{1}{\sqrt{6}}\left\langle \pi\Sigma(0)\right| \hat{V}_0 \left|\overline{K}N(0)\right\rangle \\ =& \frac{1}{2}V_{41}^1 + \frac{1}{\sqrt{6}}V_{41}^0 \end{aligned} \tag{4.42}$$

where $V_{41}^{1} \equiv V_{\pi\Sigma\to\overline{K}N}^{I=1}$ and $V_{41}^{0} \equiv V_{\pi\Sigma\to\overline{K}N}^{I=0}$

$$\begin{aligned}\left\langle \pi^+\Sigma^- \right| V \left| \bar{K}^0 n \right\rangle =& \left(\frac{1}{\sqrt{6}} \left\langle \pi\Sigma(2) \right| + \frac{1}{\sqrt{2}} \left\langle \pi\Sigma(1) \right| + \frac{1}{\sqrt{3}} \left\langle \pi\Sigma(0) \right| \right) \\ & \times V \left(\frac{1}{\sqrt{2}} \left| \overline{K}N(1) \right\rangle + \frac{1}{\sqrt{2}} \left| \overline{K}N(0) \right\rangle \right) \\ =& \frac{1}{2} \left\langle \pi\Sigma(1) \right| \hat{V}_1 \left| \overline{K}N(1) \right\rangle + \frac{1}{\sqrt{6}} \left\langle \pi\Sigma(0) \right| \hat{V}_0 \left| \overline{K}N(0) \right\rangle \\ =& \frac{1}{2} V_{42}^{1} + \frac{1}{\sqrt{6}} V_{42}^{0} \end{aligned} \tag{4.43}$$

where $V_{42}^{1} \equiv V_{\pi\Sigma\to\overline{K}N}^{I=1}$ and $V_{42}^{0} \equiv V_{\pi\Sigma\to\overline{K}N}^{I=0}$

$$\begin{aligned}\left\langle \pi^+\Sigma^- \right| V \left| \pi^0\Sigma^0 \right\rangle =& \left(\frac{1}{\sqrt{6}} \left\langle \pi\Sigma(2) \right| + \frac{1}{\sqrt{2}} \left\langle \pi\Sigma(1) \right| + \frac{1}{\sqrt{3}} \left\langle \pi\Sigma(0) \right| \right) \\ & \times V \left(\sqrt{\frac{2}{3}} \left| \pi\Sigma(2) \right\rangle - \frac{1}{\sqrt{3}} \left| \pi\Sigma(0) \right\rangle \right) \\ =& \frac{1}{3} \left\langle \pi\Sigma(2) \right| \hat{V}_2 \left| \pi\Sigma(2) \right\rangle - \frac{1}{3} \left\langle \pi\Sigma(0) \right| \hat{V}_0 \left| \pi\Sigma(0) \right\rangle \\ =& \frac{1}{3} V_{43}^{2} - \frac{1}{3} V_{43}^{0} \end{aligned} \tag{4.44}$$

where $V_{43}^{2} \equiv V_{\pi\Sigma\to\pi\Sigma}^{I=2}$ and $V_{43}^{0} \equiv V_{\pi\Sigma\to\pi\Sigma}^{I=0}$

$$\begin{aligned}\left\langle \pi^+\Sigma^- \right| V \left| \pi^+\Sigma^- \right\rangle =& \left(\frac{1}{\sqrt{6}} \left\langle \pi\Sigma(2) \right| + \frac{1}{\sqrt{2}} \left\langle \pi\Sigma(1) \right| + \frac{1}{\sqrt{3}} \left\langle \pi\Sigma(0) \right| \right) \\ & \times V \left(\frac{1}{\sqrt{6}} \left| \pi\Sigma(2) \right\rangle + \frac{1}{\sqrt{2}} \left| \pi\Sigma(1) \right\rangle + \frac{1}{\sqrt{3}} \left| \pi\Sigma(0) \right\rangle \right) \\ =& \frac{1}{6} \left\langle \pi\Sigma(2) \right| \hat{V}_2 \left| \pi\Sigma(2) \right\rangle + \frac{1}{2} \left\langle \pi\Sigma(1) \right| \hat{V}_1 \left| \pi\Sigma(1) \right\rangle \\ & + \frac{1}{3} \left\langle \pi\Sigma(0) \right| \hat{V}_0 \left| \pi\Sigma(0) \right\rangle \\ =& \frac{1}{6} V_{44}^{2} + \frac{1}{2} V_{44}^{1} + \frac{1}{3} V_{44}^{0} \end{aligned} \tag{4.45}$$

where $V_{44}^{2} \equiv V_{\pi\Sigma\to\pi\Sigma}^{I=2}$, $V_{44}^{1} \equiv V_{\pi\Sigma\to\pi\Sigma}^{I=1}$, and $V_{44}^{0} \equiv V_{\pi\Sigma\to\pi\Sigma}^{I=0}$

$$\begin{aligned}\left\langle \pi^+\Sigma^- \right| V \left|\pi^-\Sigma^+\right\rangle =& \left(\frac{1}{\sqrt{6}}\left\langle \pi\Sigma(2)\right| + \frac{1}{\sqrt{2}}\left\langle \pi\Sigma(1)\right| + \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) \\ & \times V\left(\frac{1}{\sqrt{6}}\left|\pi\Sigma(2)\right\rangle - \frac{1}{\sqrt{2}}\left|\pi\Sigma(1)\right\rangle + \frac{1}{\sqrt{3}}\left|\pi\Sigma(0)\right\rangle\right) \\ =& \frac{1}{6}\left\langle \pi\Sigma(2)\right| \hat{V}_2 \left|\pi\Sigma(2)\right\rangle - \frac{1}{2}\left\langle \pi\Sigma(1)\right| \hat{V}_1 \left|\pi\Sigma(1)\right\rangle \\ & + \frac{1}{3}\left\langle \pi\Sigma(0)\right| \hat{V}_0 \left|\pi\Sigma(0)\right\rangle \\ =& \frac{1}{6}V^2_{45} - \frac{1}{2}V^1_{45} + \frac{1}{3}V^0_{45} \end{aligned} \tag{4.46}$$

where $V^2_{45} \equiv V^{I=2}_{\pi\Sigma\to\pi\Sigma}$, $V^1_{45} \equiv V^{I=1}_{\pi\Sigma\to\pi\Sigma}$ and $V^0_{45} \equiv V^{I=0}_{\pi\Sigma\to\pi\Sigma}$

$$\begin{aligned}\left\langle \pi^+\Sigma^- \right| V \left|\pi^0\Lambda^0\right\rangle =& \left(\frac{1}{\sqrt{6}}\left\langle \pi\Sigma(2)\right| + \frac{1}{\sqrt{2}}\left\langle \pi\Sigma(1)\right| + \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) V \left|\pi\Lambda(1)\right\rangle \\ =& \frac{1}{\sqrt{2}}\left\langle \pi\Sigma(1)\right| \hat{V}_1 \left|\pi\Lambda(1)\right\rangle \\ =& \frac{1}{\sqrt{2}}V^1_{46} \end{aligned} \tag{4.47}$$

where $V^1_{46} \equiv V^{I=1}_{\pi\Sigma\to\pi\Lambda}$

$$\begin{aligned}\left\langle \pi^-\Sigma^+ \right| V \left|K^-p\right\rangle =& \left(\frac{1}{\sqrt{6}}\left\langle \pi\Sigma(2)\right| - \frac{1}{\sqrt{2}}\left\langle \pi\Sigma(1)\right| + \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) \\ & \times V\left(\frac{1}{\sqrt{2}}\left|\overline{K}N(1)\right\rangle - \frac{1}{\sqrt{2}}\left|\overline{K}N(0)\right\rangle\right) \\ =& -\frac{1}{2}\left\langle \pi\Sigma(1)\right| \hat{V}_1 \left|\overline{K}N(1)\right\rangle - \frac{1}{\sqrt{6}}\left\langle \pi\Sigma(0)\right| \hat{V}_0 \left|\overline{K}N(0)\right\rangle \\ =& -\frac{1}{2}V^1_{51} - \frac{1}{\sqrt{6}}V^0_{51} \end{aligned} \tag{4.48}$$

where $V^1_{51} \equiv V^{I=1}_{\pi\Sigma\to\overline{K}N}$ and $V^0_{51} \equiv V^{I=0}_{\pi\Sigma\to\overline{K}N}$

$$\begin{aligned}\left\langle \pi^-\Sigma^+\right| V \left|\bar{K}^0 n\right\rangle &= \left(\frac{1}{\sqrt{6}}\left\langle \pi\Sigma(2)\right| - \frac{1}{\sqrt{2}}\left\langle \pi\Sigma(1)\right| + \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) \\ &\quad \times V\left(\frac{1}{\sqrt{2}}\left|\overline{K}N(1)\right\rangle + \frac{1}{\sqrt{2}}\left|\overline{K}N(0)\right\rangle\right) \\ &= -\frac{1}{2}\left\langle \pi\Sigma(1)\right|\hat{V}_1\left|\overline{K}N(1)\right\rangle + \frac{1}{\sqrt{6}}\left\langle \pi\Sigma(0)\right|\hat{V}_0\left|\overline{K}N(0)\right\rangle \\ &= -\frac{1}{2}V^1_{52} + \frac{1}{\sqrt{6}}V^0_{52} \end{aligned} \tag{4.49}$$

where $V^1_{52} \equiv V^{I=1}_{\pi\Sigma\to\overline{K}N}$ and $V^0_{52} \equiv V^{I=0}_{\pi\Sigma\to\overline{K}N}$

$$\begin{aligned}\left\langle \pi^-\Sigma^+\right| V \left|\pi^0\Sigma^0\right\rangle &= \left(\frac{1}{\sqrt{6}}\left\langle \pi\Sigma(2)\right| - \frac{1}{\sqrt{2}}\left\langle \pi\Sigma(1)\right| + \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) \\ &\quad \times V\left(\sqrt{\frac{2}{3}}\left|\pi\Sigma(2)\right\rangle - \frac{1}{\sqrt{3}}\left|\pi\Sigma(0)\right\rangle\right) \\ &= \frac{1}{3}\left\langle \pi\Sigma(2)\right|\hat{V}_2\left|\pi\Sigma(2)\right\rangle - \frac{1}{3}\left\langle \pi\Sigma(0)\right|\hat{V}_0\left|\pi\Sigma(0)\right\rangle \\ &= \frac{1}{3}V^2_{53} - \frac{1}{3}V^0_{53} \end{aligned} \tag{4.50}$$

where $V^2_{53} \equiv V^{I=2}_{\pi\Sigma\to\pi\Sigma}$ and $V^0_{53} \equiv V^{I=0}_{\pi\Sigma\to\pi\Sigma}$

$$\begin{aligned}\left\langle \pi^-\Sigma^+\right| V \left|\pi^+\Sigma^-\right\rangle &= \left(\frac{1}{\sqrt{6}}\left\langle \pi\Sigma(2)\right| - \frac{1}{\sqrt{2}}\left\langle \pi\Sigma(1)\right| + \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) \\ &\quad \times V\left(\frac{1}{\sqrt{6}}\left|\pi\Sigma(2)\right\rangle + \frac{1}{\sqrt{2}}\left|\pi\Sigma(1)\right\rangle + \frac{1}{\sqrt{3}}\left|\pi\Sigma(0)\right\rangle\right) \\ &= \frac{1}{6}\left\langle \pi\Sigma(2)\right|\hat{V}_2\left|\pi\Sigma(2)\right\rangle - \frac{1}{2}\left\langle \pi\Sigma(1)\right|\hat{V}_1\left|\pi\Sigma(1)\right\rangle \\ &\quad + \frac{1}{3}\left\langle \pi\Sigma(0)\right|\hat{V}_0\left|\pi\Sigma(0)\right\rangle \\ &= \frac{1}{6}V^2_{54} - \frac{1}{2}V^1_{54} + \frac{1}{3}V^0_{54} \end{aligned} \tag{4.51}$$

where $V^2_{54} \equiv V^{I=2}_{\pi\Sigma\to\pi\Sigma}$, $V^1_{54} \equiv V^{I=1}_{\pi\Sigma\to\pi\Sigma}$ and $V^0_{54} \equiv V^{I=0}_{\pi\Sigma\to\pi\Sigma}$

$$\begin{aligned}\left\langle \pi^-\Sigma^+\right| V \left|\pi^-\Sigma^+\right\rangle &= \left(\frac{1}{\sqrt{6}}\left\langle \pi\Sigma(2)\right| - \frac{1}{\sqrt{2}}\left\langle \pi\Sigma(1)\right| + \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) \\ &\times V\left(\frac{1}{\sqrt{6}}\left|\pi\Sigma(2)\right\rangle - \frac{1}{\sqrt{2}}\left|\pi\Sigma(1)\right\rangle + \frac{1}{\sqrt{3}}\left|\pi\Sigma(0)\right\rangle\right) \\ &= \frac{1}{6}\left\langle \pi\Sigma(2)\right| \hat{V}_2 \left|\pi\Sigma(2)\right\rangle + \frac{1}{2}\left\langle \pi\Sigma(1)\right| \hat{V}_1 \left|\pi\Sigma(1)\right\rangle \\ &+ \frac{1}{3}\left\langle \pi\Sigma(0)\right| \hat{V}_0 \left|\pi\Sigma(0)\right\rangle \\ &= \frac{1}{6}V^2_{55} + \frac{1}{2}V^1_{55} + \frac{1}{3}V^0_{55} \end{aligned} \tag{4.52}$$

where $V^2_{55} \equiv V^{I=2}_{\pi\Sigma\to\pi\Sigma}$, $V^1_{55} \equiv V^{I=1}_{\pi\Sigma\to\pi\Sigma}$ and $V^0_{55} \equiv V^{I=0}_{\pi\Sigma\to\pi\Sigma}$

$$\begin{aligned}\left\langle \pi^-\Sigma^+\right| V \left|\pi^0\Lambda^0\right\rangle &= \left(\frac{1}{\sqrt{6}}\left\langle \pi\Sigma(2)\right| - \frac{1}{\sqrt{2}}\left\langle \pi\Sigma(1)\right| + \frac{1}{\sqrt{3}}\left\langle \pi\Sigma(0)\right|\right) V \left|\pi\Lambda(1)\right\rangle \\ &= -\frac{1}{\sqrt{2}}\left\langle \pi\Sigma(1)\right| \hat{V}_1 \left|\pi\Lambda(1)\right\rangle \\ &= -\frac{1}{\sqrt{2}}V^1_{56} \end{aligned} \tag{4.53}$$

where $V^1_{56} \equiv V^{I=1}_{\pi\Sigma\to\pi\Lambda}$

$$\begin{aligned}\left\langle \pi^0\Lambda^0\right| V \left|K^- p\right\rangle &= \left\langle \pi\Lambda(1)\right| V \left(\frac{1}{\sqrt{2}}\left|\overline{K}N(1)\right\rangle - \frac{1}{\sqrt{2}}\left|\overline{K}N(0)\right\rangle\right) \\ &= \frac{1}{\sqrt{2}}\left\langle \pi\Lambda(1)\right| \hat{V}_1 \left|\overline{K}N(1)\right\rangle \\ &= \frac{1}{\sqrt{2}}V^1_{61} \end{aligned} \tag{4.54}$$

where $V^1_{61} \equiv V^{I=1}_{\pi\Lambda\to\overline{K}N}$

$$\begin{aligned}\left\langle \pi^0\Lambda^0\right| V \left|\bar{K}^0 n\right\rangle &= \left\langle \pi\Lambda(1)\right| V \left(\frac{1}{\sqrt{2}}\left|\overline{K}N(1)\right\rangle + \frac{1}{\sqrt{2}}\left|\overline{K}N(0)\right\rangle\right) \\ &= \frac{1}{\sqrt{2}}\left\langle \pi\Lambda(1)\right| \hat{V}_1 \left|\overline{K}N(1)\right\rangle \\ &= \frac{1}{\sqrt{2}}V^1_{62} \end{aligned} \tag{4.55}$$

where $V^1_{62} \equiv V^{I=1}_{\pi\Lambda\to\overline{K}N}$

$$\begin{aligned}\left\langle \pi^0\Lambda^0\right| V \left|\pi^0\Sigma^0\right\rangle &= \langle \pi\Lambda(1)| V \left(\sqrt{\frac{2}{3}}\,|\pi\Sigma(2)\rangle - \frac{1}{\sqrt{3}}\,|\pi\Sigma(0)\rangle\right) \\ &= 0 \end{aligned} \tag{4.56}$$

$$\begin{aligned}\left\langle \pi^0\Lambda^0\right| V \left|\pi^+\Sigma^-\right\rangle &= \langle \pi\Lambda(1)| V \left(\frac{1}{\sqrt{6}}\,|\pi\Sigma(2)\rangle + \frac{1}{\sqrt{2}}\,|\pi\Sigma(1)\rangle + \frac{1}{\sqrt{3}}\,|\pi\Sigma(0)\rangle\right) \\ &= \frac{1}{\sqrt{2}}\,\langle \pi\Lambda(1)|\,\hat{V}_1\,|\pi\Sigma(1)\rangle \\ &= \frac{1}{\sqrt{2}} V^1_{64} \end{aligned} \tag{4.57}$$

where $V^1_{64} \equiv V^{I=1}_{\pi\Lambda\to\pi\Sigma}$

$$\begin{aligned}\left\langle \pi^0\Lambda^0\right| V \left|\pi^-\Sigma^+\right\rangle &= \langle \pi\Lambda(1)| V \left(\frac{1}{\sqrt{6}}\,|\pi\Sigma(2)\rangle - \frac{1}{\sqrt{2}}\,|\pi\Sigma(1)\rangle + \frac{1}{\sqrt{3}}\,|\pi\Sigma(0)\rangle\right) \\ &= -\frac{1}{\sqrt{2}}\,\langle \pi\Lambda(1)|\,\hat{V}_1\,|\pi\Sigma(1)\rangle \\ &= -\frac{1}{\sqrt{2}} V^1_{65} \end{aligned} \tag{4.58}$$

where $V^1_{65} \equiv V^{I=1}_{\pi\Lambda\to\pi\Sigma}$

$$\begin{aligned}\left\langle \pi^0\Lambda^0\right| V \left|\pi^0\Lambda^0\right\rangle &= \langle \pi\Lambda(1)| V\,|\pi\Lambda(1)\rangle \\ &= 1\,\langle \pi\Lambda(1)|\,\hat{V}_1\,|\pi\Lambda(1)\rangle \\ &= V^1_{66} \end{aligned} \tag{4.59}$$

where $V^1_{66} \equiv V^{I=1}_{\pi\Lambda\to\pi\Lambda}$.

The interactions of various channels in isospin basis, now are given in the collected Table 4.4, and Table 4.5.

Table 4.4 Isospin basis interaction terms for the first three coupled channels of physical states.

	K^-p	$\bar{K}^0n$	$\pi^0\Sigma^0$
K^-p	$\frac{1}{2}V_{11}^1 + \frac{1}{2}V_{11}^0$	$\frac{1}{2}V_{12}^1 - \frac{1}{2}V_{12}^0$	$\frac{1}{\sqrt{6}}V_{13}^0$
$\bar{K}^0n$		$\frac{1}{2}V_{22}^1 + \frac{1}{2}V_{22}^0$	$-\frac{1}{\sqrt{6}}V_{23}^0$
$\pi^0\Sigma^0$			$\frac{2}{3}V_{33}^2 + \frac{1}{\sqrt{3}}V_{33}^0$
$\pi^-\Sigma^+$			
$\pi^+\Sigma^-$			
$\pi^0\Lambda^0$			

Table 4.5 Isospin basis interaction terms for the last three coupled channels of physical states.

	$\pi^-\Sigma^+$	$\pi^+\Sigma^-$	$\pi^0\Lambda^0$
K^-p	$-\frac{1}{2}V_{14}^1 - \frac{1}{\sqrt{6}}V_{14}^0$	$\frac{1}{2}V_{15}^1 - \frac{1}{\sqrt{6}}V_{15}^0$	$\frac{1}{\sqrt{2}}V_{16}^1$
$\bar{K}^0n$	$-\frac{1}{2}V_{24}^1 + \frac{1}{\sqrt{6}}V_{24}^0$	$\frac{1}{2}V_{25}^1 + \frac{1}{\sqrt{6}}V_{25}^0$	$\frac{1}{\sqrt{26}}V_{2}^1$
$\pi^0\Sigma^0$	$\frac{1}{3}V_{34}^2 - \frac{1}{3}V_{34}^0$	$\frac{1}{3}V_{35}^2 - \frac{1}{3}V_{35}^0$	0
$\pi^-\Sigma^+$	$\frac{1}{6}V_{44}^2 + \frac{1}{2}V_{44}^1 + \frac{1}{3}V_{44}^0$	$\frac{1}{6}V_{45}^2 - \frac{1}{2}V_{45}^1 + \frac{1}{3}V_{45}^0$	$-\frac{1}{\sqrt{2}}V_{46}^1$
$\pi^+\Sigma^-$		$\frac{1}{6}V_{55}^2 + \frac{1}{2}V_{55}^1 + \frac{1}{3}V_{55}^0$	$\frac{1}{\sqrt{2}}V_{56}^1$
$\pi^0\Lambda^0$			V_{66}^1

4.2 Low energy chiral SU(3) symmetry

In this framework, the vector meson exchange is widely use to study $\overline{K}N$ systems. The s-wave interaction of Goldstone boson (meson) with any hadron (H) predicted by chiral SU(3) symmetry are so-called Weinberg-Tomozawa interaction.

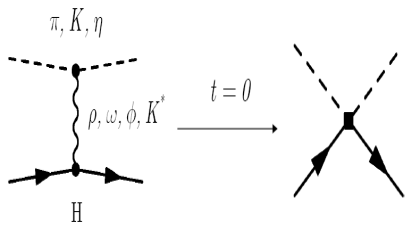

Figure 4.1 T-channel diagram of meson-hadron interaction.

The exchanging process introduces us the coupling constants between the

channels that become the coupling strength. The exact coupling strength can be worked out by starting from the SU(3) matrices of the octet pseudoscalar meson and octet baryon in this following

$$\Phi = \begin{bmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{bmatrix}.$$

Considering the trace of Weinberg-Tomozawa Lagrangian with the lowest order in momentum, we find that the interaction (kernel) becomes

$$\mathcal{L}_{\rm WT} = Tr\left(\bar{B} i\gamma^\mu \frac{1}{4f^2}[(\Phi\partial_\mu\Phi - \partial_\mu\Phi\Phi)B - B(\Phi\partial_\mu\Phi - \partial_\mu\Phi\Phi)]\right), \tag{4.60}$$

where f is the form factor and δ_μ is the covariant derivative. In this example we show only $\overline{K}N$ sector where the term $\gamma^\mu\partial_\mu$ is included in the kernel (K). Since we are interest in the coefficient between the various channels, thus the Lagrangian in the particle basis takes the form

$$\begin{aligned} \mathcal{L}_{\rm WT_{particle}} &= \sum_{i,j} C_{ij}\bar{B}_i\bar{\Phi}_i K_P\,\Phi_j B_j \\ &= 2\bar{p}K^- K_P K^- p + 2\bar{n}\bar{K}^0 K_P \bar{K}^0 n \\ &+ \bar{n}\bar{K}^0 K_P K^- p + \bar{p}K^- K_P \bar{K}^0 n + ...\ , \end{aligned} \tag{4.61}$$

where K_p is the interaction kernel in particles basis.

The coefficients in Eq. (4.61) are the coupling strength (C_{ij}) of particle

from channel $i \to j$ in particle basis. Some example of these coupling strength (C_{ij}) are given in Table 4.6.

Table 4.6 C_{ij} coefficients for $\overline{K}N$.

	K^-p	$\bar{K}^0n$
K^-p	2	1
$\bar{K}^0n$	1	2

The details of full Lagrangian calculation for six channels coupling strength are shown in Appendix D.

For the $\overline{K}N$ systems, the six physical states defined in Eq. (4.10), (4.11), (4.18), (4.19), (4.20), and (4.22) are expressed as linear combinations of the $I = 0$, 1, and 2 states according to Table 4.7.

Table 4.7 The particle basis and isospin basis related for $\overline{K}N$, $\pi\Sigma$ and $\pi\Lambda$.

Index	Particle Bases	Isospin Bases
1	$\|K^-p\rangle$	$= \frac{1}{\sqrt{2}} \left\|\overline{K}N(1) - \overline{K}N(0)\right\rangle$
2	$\left\|\bar{K}^0n\right\rangle$	$= \frac{1}{\sqrt{2}} \left\|\overline{K}N(1) + \overline{K}N(0)\right\rangle$
3	$\|\pi^0\Sigma^0\rangle$	$= \sqrt{\frac{2}{3}} \|\pi\Sigma(2)\rangle - \frac{1}{\sqrt{3}} \|\pi\Sigma(0)\rangle$
4	$\|\pi^-\Sigma^+\rangle$	$= \frac{1}{\sqrt{6}} \|\pi\Sigma(2)\rangle - \frac{1}{\sqrt{2}} \|\pi\Sigma(1)\rangle + \frac{1}{\sqrt{3}} \|\pi\Sigma(0)\rangle$
5	$\|\pi^+\Sigma^-\rangle$	$= \frac{1}{\sqrt{6}} \|\pi\Sigma(2)\rangle + \frac{1}{\sqrt{2}} \|\pi\Sigma(1)\rangle + \frac{1}{\sqrt{3}} \|\pi\Sigma(0)\rangle$
6	$\|\pi^0\Lambda\rangle$	$= \|\pi\Lambda(1)\rangle$

Substituting these physical states back into the Lagrangian (only in $\overline{K}N$ sector) Eq. (4.61), we get

$$\mathcal{L}_{\mathrm{WT_{isospin}}} = 3\overline{K}N(0)K_I\overline{K}N(0) + 1\overline{K}N(1)K_I\overline{K}N(1) + ... \ , \tag{4.62}$$

where K_I is the interaction kernel in isospin basis. Similar to the physical state, the coupled interactions must be related to the coupling strength (C_{ij}) coefficient.

For example, $K^-p \to K^-p$ channel, the interaction can be written as

$$\begin{aligned}\left\langle K^-p\right| K \left|K^-p\right\rangle &= C_{ij}^{I=0}\left\langle \overline{K}N(0)\right| \hat{K}_0 \left|\overline{K}N(0)\right\rangle + C_{ij}^{I=1}\left\langle \overline{K}N(1)\right| \hat{K}_1 \left|\overline{K}N(1)\right\rangle \\ &= (3)\frac{1}{2}K_I^0 + (1)\frac{1}{2}K_I^1. \end{aligned} \tag{4.63}$$

We define $K = \hat{K}_1 + \hat{K}_0$ where $\hat{K}_1$ is an operator for the isospin 1 states with eigenvalue K_I^1, $\hat{K}_0$ is an operator for the isospin 0 states with eigenvalue K_I^0. The coefficients obtained in Eq. (4.63) are called the coupling strength.

The coupling strength C_{ij} for $I = 0$ and $I = 1$ states are collected in the matrices below

$$C_{ij}^{I=0} = \begin{pmatrix} 3 & -\sqrt{\frac{3}{2}} \\ -\sqrt{\frac{3}{2}} & 4 \end{pmatrix}, \quad C_{ij}^{I=1} = \begin{pmatrix} 1 & -1 & -\sqrt{\frac{3}{2}} \\ -1 & 2 & 0 \\ -\sqrt{\frac{3}{2}} & 0 & 0 \end{pmatrix}, \tag{4.64}$$

where channel 1 is $\overline{K}N$, channel 2 is $\pi\Sigma$, and channel 3 is $\pi\Lambda$.

Thus we can rewrite the interactions for each channels related to the coupling strength in term of linear combination (decomposition) of the potential in isospin basis by using i and j as the channel indices.

The interaction obtained in this chapter will be added into the numerical method that be calculated by NAG Fortran program. The results of the scattering properties are shown in the next chapter.

CHAPTER V

RESULTS, DISCUSSIONS AND CONCLUSIONS

This chapter is devoted to report our numerical results of the kaonic hydrogen atoms with realistic interactions and compare our theoretical results with the experimental data. In this study, only real potentials is considered. Finally, the discussions and conclusions are also given.

5.1 Numerical Results with phenomenological $\overline{K}N$ potential

In this section we calculate the cross section of the reaction $\overline{K}N \to \overline{K}N$ with the famous potential taken from Akaishi and Yamazaki (Akaishi and Yamazaki, 2002). The potential takes the general form as follow

$$V^I_{\alpha,\beta}(r) = V^I_{\alpha,\beta} \exp[-\left(\frac{r}{b}\right)^2], \tag{5.1}$$

where α and β stand for a various channel, and I are isospins taking values 0 and 1. The range parameter $b = 0.6$ fm. The potential $(V^I_{\alpha,\beta})$ on the right side of the above equation take the values as shown in Table 5.1

The cross sections of the reactions $\overline{K}N \to \overline{K}N,\ \pi\Lambda,\ \pi\Sigma$ can be derived by solving the radial Lippmann-Schwinger equation. In Fig. 5.1 shown that the cross sections of the reactions $K^-p \to K^-p,\ \bar{K}^0n$ derived for the phenomenological

Table 5.1 The reanalyzed of the existing data of K^- atoms, deduced an optical potential with a strongly attractive.

	$V^{I=0}$	$V^{I=1}$
$\overline{K}N \to \overline{K}N$	-436 MeV	-62 MeV
$\overline{K}N \to \pi\Sigma$	-412 MeV	-285 MeV
$\overline{K}N \to \pi\Lambda$	0 MeV	-62 MeV

potentials (Akaishi and Yamazaki, 2002). The solid line is the theoretical results obtained from our calculations of coupled-channel Lippmann-Schwinger equation. The square represent the experimental data (Ciborowski et al., 1982). It is found that the theoretical results can fit to the experimental data in the range between 200-350 MeV/c in laboratory frame momentum. For other phenomenological potentials, the theoretical results are similar to the potential employed here. We then conclude that better potentials, which could reproduce the low-energy $\overline{K}N$ scattering data and kaonic hydrogen atom data, should be developed.

5.2 Discussions and conclusions

In this work we have derived the dynamical Lippmann-Schwinger equation for the coupled-channel $\overline{K}N$ systems. In the s-wave low energy strong interaction, it easily to transform into the isospin basis because of the conservation law. The radial part of the out scattered wave function consists of the interaction term, related to the initial and final states of the scattering process. All of the possible channels that can produce $\Lambda(1405)$ are evaluated in isospin states $I = 0$. The interactions between different channels are related to each others by the coupling strength. In Figures 5.1 we show the cross sections of the reactions $\overline{K}N \to \overline{K}N$, that are evaluated with the phenomenological $\overline{K}N$ potentials of Akaishi and Yamazaki . The comparison of the theoretical results and the experimental data

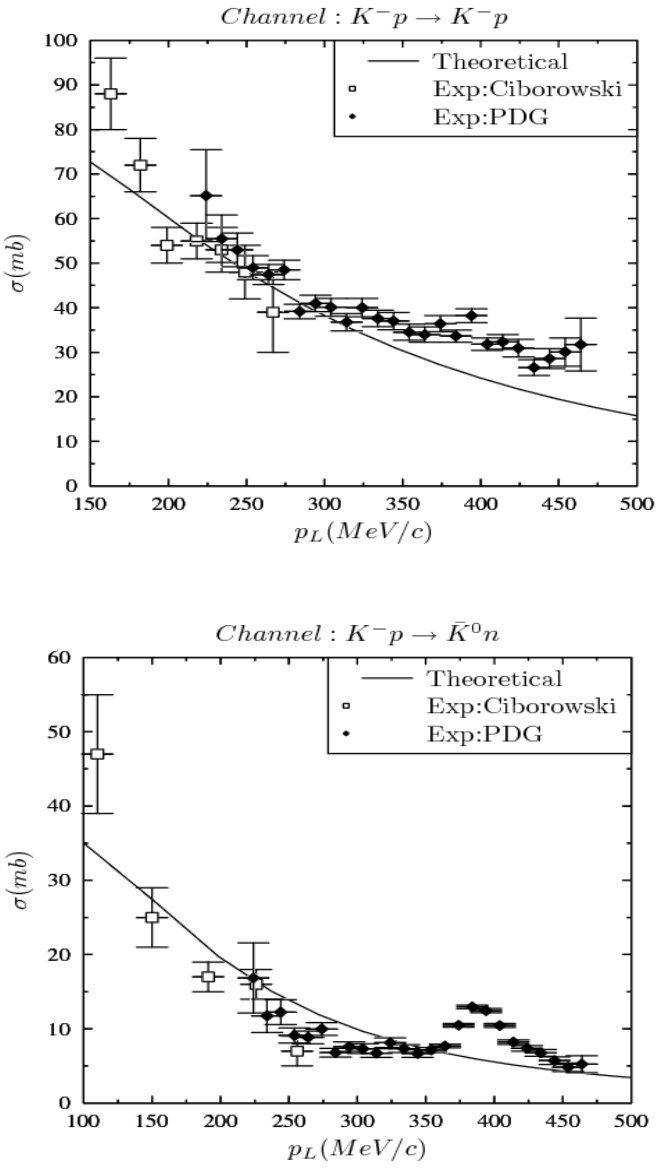

Figure 5.1 Total cross sections of the reaction $K^- p \to K^- p$, $\bar{K}^0 n$ derived with the famous potentials (Akaishi and Yamazaki, 2002). The experimental data are taken from Ciborowski (Ciborowski et al., 1982; Sakitt et al., 1965).

show some differences in the range outside 200-350 MeV/c. This suggests that one could develop a better version of $\overline{K}N$ potentials to fit the entire experimental data.

The final goal of our work is to derive, in the framework of the SU(3) chiral symmetry, a version of interactions for coupled $\overline{K}N$ system in coordinate space.

The potentials are expected to reproduce both low-energy $\overline{K}N$ scattering data and $\overline{K}N$ exotic atom observable data.

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APPENDICES

APPENDIX A

THE SATISFICATION GREEN'S FUNCTION

The formal solution of the Lippmann-Schwinger equation for outgoing scattered waves takes the form

$$\psi_\alpha(\overrightarrow{r}) = \phi_\alpha(\overrightarrow{r}) + \int d^3\overrightarrow{r''} G(\overrightarrow{r}, \overrightarrow{r'}) V_{\alpha\beta}(\overrightarrow{r'}) \psi_\beta(\overrightarrow{r''}). \tag{A.1}$$

This equation is of Helmholtz type.

Let us consider specifically the position basis and work with ket-equation of Lippmann-Schwinger equation, which is the independent of particular representations.

To make any progress we must first evaluate the kernel of the integral equation defined by

$$G_\pm = \frac{\hbar^2}{2m} \langle \vec{r} | \frac{1}{E - H_0 \pm i\varepsilon} | \vec{r'} \rangle . \tag{A.2}$$

Provided that $(\nabla^2 + k^2)G(\vec{r}, \vec{r'}) = \delta^3(\vec{r} - \vec{r'})$, using the Fourier representation of the delta function, we get

$$\delta(x) = \frac{1}{2\pi} \int dk e^{ikx} \tag{A.3}$$

$$\delta^n(x) = \frac{1}{(2\pi)^n} \int d^n k e^{-i\vec{k}.\vec{x}} \tag{A.4}$$

$$\delta^3(\vec{r}) = \frac{1}{(2\pi)^3} \int d^3 k e^{-i\vec{k}.\vec{r}}. \tag{A.5}$$

We see that

$$G(\vec{r},\vec{r'}) = \frac{1}{(2\pi)^3}\int d^3\vec{k'}\frac{1}{k^2-k'^2}e^{-i\vec{k}.(\vec{r}-\vec{r'})}. \tag{A.6}$$

However, we see again that $\nabla^2 + k^2$ is a singularity, as is evidenced by the fact that the denominator in the integrand is zero whenever $k'^2 = k^2$. Therefore we include a term $i\varepsilon$ and write $\vec{k'}\cdot r = k'r\cos\theta$ as follow

$$\begin{aligned}
G^{\pm}(\vec{r}) &= \frac{1}{(2\pi)^3}\int d^3\vec{k'}\frac{1}{k^2-k'^2\pm i\varepsilon}e^{-i\vec{k'}.\vec{r}} \\
&= \int_0^{\infty}\int_0^{\pi}\int_0^{2\pi} k'^2\sin\theta\frac{1}{k^2-k'^2\pm i\varepsilon}e^{-ik'r\cos\theta}d\phi d\theta dk' \\
&= -\frac{1}{(2\pi)^2}\int_0^{\infty}k'^2dk'\int_1^{-1}d\cos\theta\frac{1}{k^2-k'^2\pm i\varepsilon}e^{-ik'r\cos\theta} \\
&= \frac{1}{(2\pi)^2}\int_0^{\infty}k'^2dk'\int_{-1}^{1}dx\frac{1}{k^2-k'^2\pm i\varepsilon}e^{-ik'rx} \\
&= \frac{2}{(2\pi)^2}\int_0^{\infty}k'dk'\frac{1}{ir}\frac{1}{k^2-k'^2\pm i\varepsilon}(e^{-ik'r}-e^{ik'r}) \\
&= -\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}k'dk'\frac{1}{ir}\frac{1}{k'^2-k^2\mp i\varepsilon}e^{ik'r}.
\end{aligned} \tag{A.7}$$

The purpose now of $i\varepsilon$ is to move the poles off of the real axis. Thus,

$$G^{\pm}(\vec{r}) = -\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}k'dk'\frac{1}{ir}\frac{1}{[k'-(k\pm\frac{i\varepsilon}{2k})][k'-(-k\mp\frac{i\varepsilon}{2k})]+O(\varepsilon^2)}e^{ik'r}. \tag{A.8}$$

Since $r \geq 0$ we need to close the contour above. Using the residue theorem, we see that the result will contain either e^{ikr} (outgoing waves) or e^{-ikr} (incoming waves), where we recall a time dependence of e^{-iE_kt}. Incoming waves represent a problem for this particular setup, since they source disturbances in the future,

so we demand that $G(r) \approx e^{ikr}$. The pole that contributes is $k + i\varepsilon$, so the appropriate Green's function is

$$\begin{aligned} G^{+}(\vec{r}) &= -\frac{1}{(2\pi)^2}\frac{1}{ir}2\pi ik\frac{1}{2k}e^{ikr} \\ &= -\frac{1}{4\pi r}e^{ikr}. \end{aligned} \tag{A.9}$$

Thus, we get

$$G^{\pm}(\vec{r}-\vec{r}') = -\frac{1}{4\pi|\vec{r}-\vec{r}'|}e^{\pm ik(\vec{r}-\vec{r}')}. \tag{A.10}$$

Recalling that $H_0 = \frac{P^2}{2m}$, therefore we can also write Eq. (A.2) as

$$\langle\vec{r}'|\,\frac{1}{E - H_0 \pm i\varepsilon}\,|\vec{r}\rangle = -\frac{2m}{\hbar^2}\frac{1}{4\pi|\vec{r}-\vec{r}'|}e^{\pm ik(\vec{r}-\vec{r}')}. \tag{A.11}$$

APPENDIX B

MANUSCRIPTs PROGRAM FORTRAN

The radial part Lippmann-Schwinger equation for the out scattered wave can be evaluated by solving the integral term described in Chapter III. We first classify the equation into a kind of the integral equations. In this case it is a linear non-singular Fredholm integral equations of the second kind with smooth kernel. We set up all dynamical functions as a kernel and use Gaussian-Legendre method to solve the integral term and evaluate all terms in the equation (include the term of incoming wave) with the iteration method. Then the given results are in form of matrix depend on the executable input values. The obtained eigenvalues and eigenstates with the diagonalize matrix codes are shown in the section B.1.

For the interaction (kernel) term inside the integral, they consist of the spherical Bessel function that depend on the range of the scattering. We write the code of subroutine and respective functions to support the main program (see in the section B.2).

B.1 The radial parts evaluation Fortran codes

```
                                        *Written by Mr.Wanchaloem Poonsawat
      Program MAIN
* .. Global Parameter Declaration ..
      INTEGER N, I, J, D, NMAX, IFAIL, M,kr
      INTEGER IA, IB, IC
      INTEGER IAA, IBB
      PARAMETER (NMAX=2000,IA=NMAX,IB=NMAX,IC=NMAX,IAA=NMAX,IBB=NMAX)
      INTEGER NIN, NOUT
      PARAMETER (NIN=5,NOUT=8)
```

```
      REAL jj, yj, ji, yi, jip, yip, jjp, yjp
* .. Local Scalars ..
      DOUBLE PRECISION LOW, UP, Wj(NMAX), Xj(NMAX), Xi(NMAX), J0(IB,1)
      DOUBLE PRECISION A(IA,NMAX),j1(IB,1),besj(nmax),besy(nmax),besjj(nmax),besyy(nmax)
      DOUBLE PRECISION vlow, vhigh , R(IC,1), WKSPCE(NMAX),AA(IAA,NMAX),BB(IBB,NMAX)
      REAL v, k,u,mk,mn,mm
* .. External Functions ..
      EXTERNAL GAULEG, SPHBES, F04AAF, FUNCJ, FUNC
* .. Output File ..
      OPEN(UNIT=74,FILE='RESULT1',STATUS ='NEW')
      OPEN(UNIT=73,FILE='RESULT',STATUS ='NEW')
* .. Executable Statements ..
      low=0
      up=50
      N=1800
      mn=0.938272
      mk=0.493677
      mm=(mn*mk)/(mn+mk)
      kr=10
      k=0.2
      vlow=0
      vhigh=10
* .. Compute Gaussian Points ..
      CALL GAULEG(LOW,UP,Xj,Wj,N)
      do 200 i=1,n
      CALL SPHBES(0,Xj(I),ji,yi,jip,yip)
* .. Compute Spherical Bessel Function ..
* call spherical bessel by sending Xj(I) to get
* ji bessel function for I
* yi neumann function for I
* jip and yip are using in this subroutine
      besj(i)=ji
      besy(i)=yi
200 continue
* .. running the first index I ..
      DO 300 I = 1, N
* .. setting J0 as a spherical bessel function of order zero ..
      J0(I,1) = FUNCJ(Xj(I))
* .. running the second index J ..
```

```
      DO 400 J =1, N
* .. get the values for index J ..
* .. set delta function ..
      IF (I.EQ.J) THEN
      D = 1
      ELSE
      D = 0
      END IF
* .. set potential ..
      IF (Xj(J).LT.kr) then
      u=(2*mm*vhigh)
      else
      u=(2*mm*vlow)
      end if
* .. set matrix elements ..
      IF (Xj(J).LT.Xj(I)) A(I,J) = D-(wj(j)*besj(j)*besy(i)*u*xj(j)*xj(j)/(k*k))
      IF (Xj(J).GT.Xj(I)) A(I,J) = D-(wj(j)*besj(i)*besy(j)*u*Xj(J)*Xj(j)/(k*k))
      IF (Xj(J).EQ.Xj(I)) A(I,J) = D-(wj(j)*besj(i)*besy(i)*u*Xj(J)*Xj(j)/(k*k))
400 CONTINUE
300 CONTINUE
* .. CALCULATE WAVE FUNCTION using NAG Fortran Library F04AAF (RESULT) ..
      IF (N.GE.0 .AND. N.LE.NMAX) THEN
      M = 1
      IFAIL = 0
      CALL F04AEF(A,IA,J0,IB,N,M,R,IC,WKSPCE,AA,IAA,BB,IBB,IFAIL)
      WRITE (NOUT,*) ' Solution'
      DO 700 I=1,N
      WRITE (73,99998) xj(I),R(I,1)*xj(I)
      WRITE (*,99998) xj(I),R(I,1)*xj(I)
700 CONTINUE
      ELSE WRITE (NOUT,99999) 'N is out of range: N = ', N
      END IF
      STOP
99999 FORMAT (1X,A,I5)
99998 FORMAT (1X,e15.4,e15.4)

      END

* .. END OF MAIN PROGRAM ..
```

B.2 The subroutines of Fortran codes

```
      SUBROUTINE GAULEG(x1,x2,x,w,n)
      INTEGER n
      DOUBLE PRECISION x1,x2,x(n),w(n)
      DOUBLE PRECISION EPS
      PARAMETER (EPS=3.d-14)
      INTEGER i,j,m
      DOUBLE PRECISION p1,p2,p3,pp,xl,xm,z,z1
      m=(n+1)/2 xm=0.5d0*(x2+x1)
      xl=0.5d0*(x2-x1)
      do 112 i=1,m
      z=cos(3.141592654d0*(i-.25d0)/(n+.5d0))
113 continue
      p1=1.d0
      p2=0.d0
      do 111 j=1,n
      p3=p2
      p2=p1
      p1=((2.d0*j-1.d0)*z*p2-(j-1.d0)*p3)/j
111 enddo
      pp=n*(z*p1-p2)/(z*z-1.d0)
      z1=z
      z=z1-p1/pp
      if(abs(z-z1).gt.EPS)goto 113
      x(i)=xm-xl*z
      x(n+1-i)=xm+xl*z
      w(i)=2.d0*xl/((1.d0-z*z)*pp*pp)
      w(n+1-i)=w(i)
112 continue
      return
      END
* .....
      SUBROUTINE POINT(x1,x2,x,n)
      INTEGER n,i
      DOUBLE PRECISION x1,x2,x(n)
      DO 222 i=1,n
```

```
      x(i)= (i*(x2-x1)/n)+x1
222 CONTINUE
      RETURN
      END
* .....................................................................
      SUBROUTINE SPHBES(n,x,sj,sy,sjp,syp)
      INTEGER n
      DOUBLE PRECISION x
      REAL sj,sjp,sy,syp
      REAL factor,order,rj,rjp,ry,ryp,RTPIO2
      PARAMETER (RTPIO2=1.2533141)
      if(n.lt.0.or.x.le.0.)pause
      order=n+0.5
      call bessjy(x,order,rj,ry,rjp,ryp)
      factor=RTPIO2/sqrt(x)
      sj=factor*rj
      sy=factor*ry
      sjp=factor*rjp-sj/(2.*x)
      syp=factor*ryp-sy/(2.*x)
      return
      END
* .....................................................................
      SUBROUTINE bessjy(x,xnu,rj,ry,rjp,ryp)
      INTEGER MAXIT
      REAL rj,rjp,ry,ryp,xnu,XMIN
      DOUBLE PRECISION EPS,FPMIN,PI
      PARAMETER (EPS=1.e-10,FPMIN=1.e-30,MAXIT=10000,XMIN=2.)
      PARAMETER (PI=3.141592653589793d0)
      INTEGER i,isign,l,nl
      DOUBLE PRECISION a,b,br,bi,c,cr,ci,d,del,del1,den,di,dlr,dli
      DOUBLE PRECISION dr,e,f,fact,fact2,fact3,ff,gam,gam1,gam2,gammi
      DOUBLE PRECISION gampl,h,p,pimu,pimu2,q,r,rjl,rjl1,rjmu,rjp1,rjpl
      DOUBLE PRECISION rjtemp,ry1,rymu,rymup,rytemp,sum,sum1,temp,w,x2
      DOUBLE PRECISION xi,xi2,xmu,xmu2,x
      if(x.le.0..or.xnu.lt.0.) pause 'first condition'
      if(x.lt.XMIN)then
      nl=int(xnu+.5d0)
      else
      nl=max(0,int(xnu-x+1.5d0))
      endif
```

```
      xmu=xnu-nl
      xmu2=xmu*xmu
      xi=1.d0/x
      xi2=2.d0*xi
      w=xi2/PI
      isign=1
      h=xnu*xi
      if(h.lt.FPMIN)h=FPMIN
      b=xi2*xnu
      d=0.d0
      c=h
      do 11 i=1,MAXIT
      b=b+xi2
      d=b-d
      if(abs(d).lt.FPMIN)d=FPMIN
      c=b-1.d0/c
      if(abs(c).lt.FPMIN)c=FPMIN
      d=1.d0/d
      del=c*d
      h=del*h
      if(d.lt.0.d0)isign=-isign
      if(abs(del-1.d0).lt.EPS) goto 1
11 enddo
      pause
1 continue
      rjl=isign*FPMIN
      rjpl=h*rjl
      rjl1=rjl
      rjp1=rjpl
      fact=xnu*xi
      do 12 l=nl,1,-1
      rjtemp=fact*rjl+rjpl
      fact=fact-xi
      rjpl=fact*rjtemp-rjl
      rjl=rjtemp
12 enddo
      if(rjl.eq.0.d0)rjl=EPS
      f=rjpl/rjl
      if(x.lt.XMIN) then
      x2=.5d0*x
```

```
      pimu=PI*xmu
      if(abs(pimu).lt.EPS)then
      fact=1.d0
      else
      fact=pimu/sin(pimu)
      endif
      d=-log(x2)
      e=xmu*d
      if(abs(e).lt.EPS)then
      fact2=1.d0
      else
      fact2=sinh(e)/e
      endif
      call beschb(xmu,gam1,gam2,gampl,gammi)
      ff=2.d0/PI*fact*(gam1*cosh(e)+gam2*fact2*d)
      e=exp(e)
      p=e/(gampl*PI)
      q=1.d0/(e*PI*gammi)
      pimu2=0.5d0*pimu
      if(abs(pimu2).lt.EPS)then
      fact3=1.d0
      else
      fact3=sin(pimu2)/pimu2
      endif
      r=PI*pimu2*fact3*fact3
      c=1.d0
      d=-x2*x2
      sum=ff+r*q
      sum1=p
      do 13 i=1,MAXIT
      ff=(i*ff+p+q)/(i*i-xmu2)
      c=c*d/i
      p=p/(i-xmu)
      q=q/(i+xmu)
      del=c*(ff+r*q)
      sum=sum+del
      del1=c*p-i*del
      sum1=sum1+del1
      if(abs(del).lt.(1.d0+abs(sum))*EPS) goto 2
13    enddo
```

```
            pause
2 continue
            rymu=-sum
            ry1=-sum1*xi2
            rymup=xmu*xi*rymu-ry1
            rjmu=w/(rymup-f*rymu)
            else
            a=.25d0-xmu2
            p=-.5d0*xi
            q=1.d0
            br=2.d0*x
            bi=2.d0
            fact=a*xi/(p*p+q*q)
            cr=br+q*fact
            ci=bi+p*fact
            den=br*br+bi*bi
            dr=br/den
            di=-bi/den
            dlr=cr*dr-ci*di
            dli=cr*di+ci*dr
            temp=p*dlr-q*dli
            q=p*dli+q*dlr
            p=temp
            do 14 i=2,MAXIT
            a=a+2*(i-1)
            bi=bi+2.d0
            dr=a*dr+br
            di=a*di+bi
            if(abs(dr)+abs(di).lt.FPMIN)dr=FPMIN
            fact=a/(cr*cr+ci*ci)
            cr=br+cr*fact
            ci=bi-ci*fact
            if(abs(cr)+abs(ci).lt.FPMIN)cr=FPMIN
            den=dr*dr+di*di
            dr=dr/den
            di=-di/den
            dlr=cr*dr-ci*di
            dli=cr*di+ci*dr
            temp=p*dlr-q*dli
            q=p*dli+q*dlr
```

```
      p=temp
      if(abs(dlr-1.d0)+abs(dli).lt.EPS)goto 3
14 enddo
      pause
3 continue
      gam=(p-f)/q
      rjmu=sqrt(w/((p-f)*gam+q))
      rjmu=sign(rjmu,rjl)
      rymu=rjmu*gam
      rymup=rymu*(p+q/gam)
      ry1=xmu*xi*rymu-rymup
      endif
      fact=rjmu/rjl
      rj=rjl1*fact
      rjp=rjp1*fact
      do 15 i=1,nl
      rytemp=(xmu+i)*xi2*ry1-rymu
      rymu=ry1
      ry1=rytemp
15 enddo
      ry=rymu
      ryp=xnu*xi*rymu-ry1
      return
      END
* .....
      SUBROUTINE beschb(x,gam1,gam2,gampl,gammi)
      INTEGER NUSE1,NUSE2
      DOUBLE PRECISION gam1,gam2,gammi,gampl,x
      PARAMETER (NUSE1=5,NUSE2=5)
      REAL xx,c1(7),c2(8),chebev
      SAVE c1,c2
      DATA c1/-1.142022680371168d0,6.5165112670737d-3,3.087090173086d-4,
     * -3.4706269649d-6,6.9437664d-9,3.67795d-11,-1.356d-13/
      DATA c2/1.843740587300905d0,-7.68528408447867d-2,
     * 1.2719271366546d-3,-4.9717367042d-6,-3.31261198d-8,
     * 2.423096d-10,-1.702d-13,-1.49d-15/
      xx=8.d0*x*x-1.d0
      gam1=chebev(-1.,1.,c1,NUSE1,xx)
      gam2=chebev(-1.,1.,c2,NUSE2,xx)
      gampl=gam2-x*gam1
```

```
      gammi=gam2+x*gam1
      return
      END
* .....................................................
* .. FUNCTIONS ..
      FUNCTION chebev(a,b,c,m,x)
      INTEGER m
      REAL chebev,a,b,c(m),x
      INTEGER j
      REAL d,dd,sv,y,y2
      if ((x-a)*(x-b).gt.0.) pause
      d=0.
      dd=0.
      y=(2.*x-a-b)/(b-a)
      y2=2.*y
      do 16 j=m,2,-1
      sv=d
      d=y2*d-dd+c(j)
      dd=sv
16 enddo
      chebev=y*d-dd+0.5*c(1)
      return
      END
* .....................................................
      DOUBLE PRECISION FUNCTION FUNCJ(x)
      DOUBLE PRECISION x
      FUNCJ = SIN(x)/x
      RETURN
      END
* .....................................................
      DOUBLE PRECISION FUNCTION FUNC(x,v,k)
      DOUBLE PRECISION x
      REAL v,k
      FUNC = (v*x**2)/k**2
      RETURN

      END

* .....................................................
```

APPENDIX C

CLEBSCH-GORDAN COEFFICIENTS

As an example of the use of the Clebsch-Gordan coefficients table, we choose the case of combining two angular momenta $j_1 = 1$, $m_1 = 1$ and $j_2 = 1$, $m_2 = -1$. By looking at the entry for combining angular momenta 1×1, we get

$$\phi_1(1,1)\phi_2(1,-1) = \sqrt{\frac{1}{6}}\psi(2,0) + \sqrt{\frac{1}{2}}\psi(1,0) + \sqrt{\frac{1}{3}}\psi(0,0).$$

This tells us how two particles of angular momentum (or isospin) equal to one combined and form the states of angular momentum $j = 0, 1$ or 2. Alternatively, a state of particular j, m can be decomposed into constituents. Thus $j = 2, m = 0$ can be decomposed into products of state with $j = j_1 + j_2 = 2$ and $m = m_1 + m_2 = 0$. The fourth column of the 1×1 Clebsch-Gordan's table gives

$$\psi(2,0) = \sqrt{\frac{1}{6}}\phi_1(1,1)\phi_2(1,-1) + \sqrt{\frac{2}{3}}\phi_1(1,0)\phi_2(1,0) + \sqrt{\frac{1}{6}}\phi_1(1,-1)\phi_2(1,1).$$

The sign convention used in the table follows Condon and Shortley (1951).

Table C.1 Clebsch-Gordan coefficients for the addition of $j_1 = \frac{1}{2}$ and $j_2 = \frac{1}{2}$.

		$j =$	1	1	0	1
m_1	m_2	$m =$	+1	0	0	-1
$+\frac{1}{2}$	$+\frac{1}{2}$		1			
$+\frac{1}{2}$	$-\frac{1}{2}$			$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
$-\frac{1}{2}$	$+\frac{1}{2}$			$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	
$-\frac{1}{2}$	$-\frac{1}{2}$					1

Table C.2 Clebsch-Gordan coefficients for the addition of $j_1 = 1$ and $j_2 = \frac{1}{2}$.

		$j =$	$\frac{3}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{3}{2}$
m_1	m_1	$m =$	$+\frac{3}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{3}{2}$
+1	$+\frac{1}{2}$		1					
+1	$-\frac{1}{2}$			$\sqrt{\frac{1}{3}}$	$\sqrt{\frac{2}{3}}$			
0	$+\frac{1}{2}$			$\sqrt{\frac{2}{3}}$	$-\sqrt{\frac{1}{3}}$			
0	$-\frac{1}{2}$					$\sqrt{\frac{2}{3}}$	$\sqrt{\frac{1}{3}}$	
-1	$+\frac{1}{2}$					$\sqrt{\frac{1}{3}}$	$-\sqrt{\frac{2}{3}}$	
-1	$-\frac{1}{2}$							1

Table C.3 Clebsch-Gordan coefficients for the addition of $j_1 = 1$ and $j_2 = 1$.

		$j =$	2	2	1	2	1	0	2	1	2
m_1	m_2	$m =$	+2	+1	+1	0	0	0	-1	-1	-2
+1	+1		1								
+1	0			$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$						
0	+1			$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$						
+1	-1					$\sqrt{\frac{1}{6}}$	$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$			
0	0					$\sqrt{\frac{2}{3}}$	0	$-\sqrt{\frac{1}{3}}$			
-1	+1					$\sqrt{\frac{1}{6}}$	$-\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{3}}$			
0	-1								$\sqrt{\frac{1}{2}}$	$\sqrt{\frac{1}{2}}$	
-1	0								$\sqrt{\frac{1}{2}}$	$-\sqrt{\frac{1}{2}}$	
-1	-1										1

APPENDIX D

COUPLING STRENGTH COEFFICIENTS

The Weinberg-Tomozawa Lagrangian is given by

$$\mathcal{L}_{\rm WT} = Tr\left(\bar{B} i\gamma^{\mu} \frac{1}{4f^2}[(\Phi\partial_{\mu}\Phi - \partial_{\mu}\Phi\Phi)B - B(\Phi\partial_{\mu}\Phi - \partial_{\mu}\Phi\Phi)]\right). \tag{D.1}$$

The octet pseudoscalar meson is defined by

$$\Phi = \begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi_0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{6}} - \frac{\pi_0}{\sqrt{2}} & K_0 \\ K^- & \bar{K}_0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}, \tag{D.2}$$

and the octet Baryon is defined by

$$B = \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma_0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma_0}{\sqrt{2}} & n \\ \Xi^- & \Xi_0 & -\sqrt{\frac{2}{3}}\Lambda \end{pmatrix}. \tag{D.3}$$

Considering the Lagrangian in the particle basis, we get

$$\mathcal{L}_{\rm WT\_particle} = \sum_{i,j} C_{ij} \bar{B}_i \bar{\Phi}_i K_P \, \Phi_j B_j, \tag{D.4}$$

where i and j are the outgoing and incoming particles, C_{ij} is the coupling strength of the Weinberg-Tomozawa interaction and V_{ij}^P is the interaction kernel in particle basis.

Putting Eq. (D.2) and (D.3) into Eq. (D.4), we get the Lagrangian in the particle

basis as

$$
\begin{aligned}
L_{\mathrm{WT\_particle}} \;=\; & -2\bar{n}K^0\,K_P\,K^0n \;-\; \bar{p}K^0\,K_P\,K^0p \;+\; \underline{2\bar{\Xi}^0K^0\,K_P\,K^0\Xi^0} \;+ \\
& \bar{\Xi}^-K^0\,K_P\,K^0\Xi^- \;-\; \bar{\Sigma}^-K^0\,K_P\,K^0\Sigma^- \;+\; \bar{\Sigma}^+K^0\,K_P\,K^0\Sigma^+ \;-\; \tfrac{3}{2}\bar{n}K^0\,K_P\,\eta\Lambda \;- \\
& \tfrac{3}{2}\eta\bar{\Lambda}\,K_P\,K^0n \;+\; \tfrac{\sqrt{3}}{2}\bar{n}K^0\,K_P\,\pi^0\Lambda \;+\; \tfrac{\sqrt{3}}{2}\bar{\Lambda}\pi^0\,K_P\,K^0n \;+\; \underline{\tfrac{3}{2}\bar{\Lambda}\eta\,K_P\,K^0\Xi^0} \;- \\
& \underline{\tfrac{\sqrt{3}}{2}\bar{\Lambda}\pi^0\,K_P\,K^0\Xi^0} \;+\; \tfrac{\sqrt{3}}{2}\bar{n}K^0\,K_P\,\eta\Sigma^0 \;-\; \tfrac{1}{2}\bar{n}K^0\,K_P\,\pi^0\Sigma^0 \;+\; \underline{\tfrac{3}{2}\bar{\Xi}^0K^0\,K_P\,\eta\Lambda} \;- \\
& \underline{\tfrac{\sqrt{3}}{2}\bar{\Xi}^0K^0\,K_P\,\pi^0\Lambda} \;-\; \underline{\tfrac{\sqrt{3}}{2}\bar{\Xi}^0K^0\,K_P\,\eta\Sigma^0} \;+\; \underline{\tfrac{1}{2}\bar{\Xi}^0K^0\,K_P\,\pi^0\Sigma^0} \;+\; \tfrac{\sqrt{3}}{2}\bar{\Sigma}^0\eta\,K_P\,K^0n \;- \\
& \tfrac{1}{2}\bar{\Sigma}^0\pi^0\,K_P\,K^0n \;-\; \underline{\tfrac{\sqrt{3}}{2}\bar{\Sigma}^0\eta\,K_P\,K^0\Xi^0} \;+\; \underline{\tfrac{1}{2}\bar{\Sigma}^0\pi^0\,K_P\,K^0\Xi^0} \;-\; \sqrt{\tfrac{3}{2}}\bar{p}\pi^-\,K_P\,K^0\Lambda \;- \\
& \sqrt{\tfrac{3}{2}}\bar{\Lambda}K^0\,K_P\,\pi^-p \;-\; \tfrac{1}{\sqrt{2}}\bar{p}\pi^-\,K_P\,K^0\Sigma^0 \;-\; \tfrac{1}{\sqrt{2}}\bar{\Sigma}^0K^0\,K_P\,\pi^-p \;+\; \sqrt{\tfrac{3}{2}}\bar{\Lambda}\pi^-\,K_P\,K^0\Xi^- \;+ \\
& \tfrac{1}{\sqrt{2}}\bar{\Sigma}^0\pi^-\,K_P\,K^0\Xi^- \;-\; \bar{n}\pi^-\,K_P\,K^0\Sigma^- \;+\; \sqrt{\tfrac{3}{2}}\bar{\Xi}^-K^0\,K_P\,\pi^-\Lambda \;+\; \tfrac{1}{\sqrt{2}}\bar{\Xi}^-K^0\,K_P\,\pi^-\Sigma^0 \;+ \\
& \sqrt{\tfrac{3}{2}}\bar{\Xi}^-K^0\,K_P\,\eta\Sigma^- \;-\; \tfrac{1}{\sqrt{2}}\bar{\Xi}^-K^0\,K_P\,\pi^0\Sigma^- \;-\; \bar{\Sigma}^-K^0\,K_P\,\pi^-n \;+\; \sqrt{\tfrac{3}{2}}\bar{\Sigma}^-K^0\,K_P\,\eta\Xi^- \;- \\
& \tfrac{1}{\sqrt{2}}\bar{\Sigma}^-K^0\,K_P\,\pi^0\Xi^- \;-\; \bar{n}K^+\,K_P\,K^0p \;-\; \bar{p}K^0\,K_P\,K^+n \;+\; \underline{\bar{\Xi}^0K^0\,K_P\,K^+\Xi^-} \;- \\
& \sqrt{2}\bar{\Sigma}^0K^0\,K_P\,K^+\Sigma^- + \underline{\bar{\Xi}^-K^+\,K_P\,K^0\Xi^0} - \sqrt{2}\bar{\Sigma}^-K^+\,K_P\,K^0\Sigma^0 - \sqrt{\tfrac{3}{2}}\bar{p}\eta\,K_P\,K^0\Sigma^+ + \\
& \tfrac{1}{\sqrt{2}}\bar{p}\pi^0\,K_P\,K^0\Sigma^+ \;+\; \underline{\bar{\Xi}^0K^0\,K_P\,\pi^-\Sigma^+} \;+\; \sqrt{2}\bar{\Sigma}^0K^+\,K_P\,K^0\Sigma^+ \;-\; \sqrt{\tfrac{3}{2}}\bar{\Sigma}^+\eta\,K_P\,K^0p \;+ \\
& \tfrac{1}{\sqrt{2}}\bar{\Sigma}^+K^0\,K_P\,p\pi^0 \;+\; \underline{\bar{\Sigma}^+\pi^-\,K_P\,K^0\Xi^0} \;+\; \sqrt{2}\bar{\Sigma}^+K^0\,K_P\,K^+\Sigma^0 \;+\; \underline{2\bar{n}\bar{K}^0\,K_P\,\bar{K}^0n} \;+ \\
& \bar{p}\bar{K}^0\,K_P\,\bar{K}^0p + \bar{n}K^-\,K_P\,K^-n + \underline{2\bar{p}K^-\,K_P\,K^-p} - \bar{\Xi}^0K^-\,K_P\,K^-\Xi^0 - \bar{n}\pi^-\,K_P\,\pi^-n + \\
& \bar{p}\pi^-\,K_P\,\pi^-p + \bar{\Xi}^0\pi^-\,K_P\,\pi^-\Xi^0 - \bar{n}K^+\,K_P\,K^+n - 2\bar{p}K^+\,K_P\,K^+p + \bar{\Xi}^0K^+\,K_P\,K^+\Xi^0 + \\
& \underline{2\bar{\Xi}^-K^+\,K_P\,K^+\Xi^-} \;+\; \bar{\Sigma}^-K^+\,K_P\,K^+\Sigma^- \;+\; \bar{n}\pi^+\,K_P\,\pi^+n \;-\; \bar{p}\pi^+\,K_P\,\pi^+p \;- \\
& \bar{\Xi}^0\pi^+\,K_P\,\pi^+\Xi^0 + \bar{\Xi}^-\pi^+\,K_P\,\pi^+\Xi^- + \underline{2\bar{\Sigma}^-\pi^+\,K_P\,\pi^+\Sigma^-} + \underline{\tfrac{3}{2}\bar{n}\bar{K}^0\,K_P\,\eta\Lambda} + \underline{\tfrac{3}{2}\bar{\Lambda}\eta\,K_P\,\bar{K}^0n} - \\
& \underline{\tfrac{\sqrt{3}}{2}\bar{n}\bar{K}^0\,K_P\,\pi^0\Lambda} \;-\; \underline{\tfrac{\sqrt{3}}{2}\bar{\Lambda}\pi^0\,K_P\,\bar{K}^0n} \;-\; \tfrac{3}{2}\bar{\Lambda}\eta\,K_P\,\Xi^0\bar{K}^0 \;+\; \tfrac{\sqrt{3}}{2}\bar{\Lambda}\pi^0\,K_P\,\Xi^0\bar{K}^0 \;- \\
& \underline{\tfrac{\sqrt{3}}{2}\bar{n}\bar{K}^0\,K_P\,\eta\Sigma^0} \;+\; \underline{\tfrac{1}{2}\bar{n}\bar{K}^0\,K_P\,\pi^0\Sigma^0} \;-\; 2\bar{\Xi}^0\bar{K}^0\,K_P\,\bar{K}^0\Xi^0 \;-\; \tfrac{3}{2}\bar{\Xi}^0\bar{K}^0\,K_P\,\eta\Lambda \;+ \\
& \tfrac{\sqrt{3}}{2}\bar{\Xi}^0\bar{K}^0\,K_P\,\pi^0\Lambda \;+\; \tfrac{\sqrt{3}}{2}\bar{\Xi}^0\bar{K}^0\,K_P\,\eta\Sigma^0 \;-\; \tfrac{1}{2}\bar{\Xi}^0\bar{K}^0\,K_P\,\pi^0\Sigma^0 \;-\; \underline{\tfrac{\sqrt{3}}{2}\bar{\Sigma}^0\eta\,K_P\,\bar{K}^0n} \;+ \\
& \underline{\tfrac{1}{2}\bar{\Sigma}^0\pi^0\,K_P\,\bar{K}^0n} + \tfrac{\sqrt{3}}{2}\bar{\Sigma}^0\eta\,K_P\,\bar{K}^0\Xi^0 - \tfrac{1}{2}\bar{\Sigma}^0\pi^0\,K_P\,\bar{K}^0\Xi^0 + \underline{\tfrac{3}{2}\bar{p}K^-\,K_P\,\eta\Lambda} + \underline{\tfrac{3}{2}\bar{\Lambda}\eta\,K_P\,K^-p} + \\
& \underline{\tfrac{\sqrt{3}}{2}\bar{p}K^-\,K_P\,\pi^0\Lambda} \;+\; \underline{\tfrac{\sqrt{3}}{2}\bar{\Lambda}\pi^0\,K_P\,K^-p} \;+\; \underline{\tfrac{\sqrt{3}}{2}\bar{p}K^-\,K_P\,\eta\Sigma^0} \;+\; \underline{\tfrac{1}{2}\bar{p}K^-\,K_P\,\pi^0\Sigma^0} \;+ \\
& \underline{\bar{n}\bar{K}^0\,K_P\,K^-p} + \underline{\bar{p}K^-\,K_P\,\bar{K}^0n} + \underline{\tfrac{\sqrt{3}}{2}\bar{\Sigma}^0\eta\,K_P\,K^-p} + \underline{\tfrac{1}{2}\bar{\Sigma}^0\pi^0\,K_P\,K^-p} - \sqrt{2}\bar{n}\pi^0\,K_P\,\pi^-p - \\
& \sqrt{2}\bar{p}\pi^-\,K_P\,\pi^0n \;+\; \sqrt{\tfrac{3}{2}}\bar{n}K^-\,K_P\,\pi^-\Lambda \;+\; \sqrt{\tfrac{3}{2}}\bar{\Lambda}K^-\,K_P\,\pi^-n \;-\; \sqrt{\tfrac{3}{2}}\bar{\Lambda}\pi^-\,K_P\,K^-\Xi^0 \;- \\
& \tfrac{1}{\sqrt{2}}\bar{n}K^-\,K_P\,\pi^-\Sigma^0 - \sqrt{\tfrac{3}{2}}\bar{\Xi}^0K^-\,K_P\,\pi^-\Lambda + \tfrac{1}{\sqrt{2}}\bar{\Xi}^0K^-\,K_P\,\pi^-\Sigma^0 - \tfrac{1}{\sqrt{2}}\bar{\Sigma}^0\pi^-\,K_P\,K^-n +
\end{aligned}
$$

$$\tfrac{1}{\sqrt{2}}\bar{\Sigma}^0\pi^- K_P K^-\Xi^0 - \tfrac{3}{2}\bar{\Lambda}K^- K_P \eta\Xi^- - \tfrac{\sqrt{3}}{2}\bar{\Lambda}K^- K_P \pi^0\Xi^- - \bar{\Xi}^0 K^- K_P \bar{K}^0\Xi^- -$$
$$\tfrac{\sqrt{3}}{2}\bar{\Sigma}^0 K^- K_P \eta\Xi^- - \tfrac{1}{2}\bar{\Sigma}^0 K^- K_P \pi^0\Xi^- + \sqrt{2}\bar{\Xi}^0\pi^- K_P \pi^0\Xi^- + \sqrt{\tfrac{3}{2}}\bar{n}K^- K_P \eta\Sigma^- +$$
$$\tfrac{1}{\sqrt{2}}\bar{n}K^- K_P \pi^0\Sigma^- + \sqrt{2}\bar{\Sigma}^0 K^- K_P \bar{K}^0\Sigma^- - 2\bar{\Sigma}^0\pi^- K_P \pi^0\Sigma^- - \tfrac{3}{2}\bar{\Xi}^-\eta K_P K^-\Lambda -$$
$$\tfrac{\sqrt{3}}{2}\bar{\Xi}^-\pi^0 K_P K^-\Lambda - \tfrac{\sqrt{3}}{2}\bar{\Xi}^-\eta K_P K^-\Sigma^0 - \tfrac{1}{2}\bar{\Xi}^-\pi^0 K_P K^-\Sigma^0 - \bar{\Xi}^-\bar{K}^0 K_P K^-\Xi^0 +$$
$$\sqrt{2}\bar{\Xi}^-\pi^0 K_P \pi^-\Xi^0 - \bar{\Xi}^-\bar{K}^0 K_P \bar{K}^0\Xi^- - 2\bar{\Xi}^- K^- K_P K^-\Xi^- - \bar{\Xi}^-\pi^- K_P \pi^-\Xi^- -$$
$$\sqrt{\tfrac{3}{2}}\bar{\Xi}^-\bar{K}^0 K_P \eta\Sigma^- + \tfrac{1}{\sqrt{2}}\bar{\Xi}^-\bar{K}^0 K_P \pi^0\Sigma^- - \bar{\Xi}^- K^- K_P \pi^-\Sigma^- + \sqrt{\tfrac{3}{2}}\bar{\Sigma}^-\eta K_P K^- n +$$
$$\tfrac{1}{\sqrt{2}}\bar{\Sigma}^-\pi^0 K_P K^- n + \sqrt{2}\bar{\Sigma}^-\bar{K}^0 K_P K^-\Sigma^0 - 2\bar{\Sigma}^-\pi^0 K_P \pi^-\Sigma^0 - \sqrt{\tfrac{3}{2}}\bar{\Sigma}^-\eta K_P \bar{K}^0\Xi^- +$$
$$\tfrac{1}{\sqrt{2}}\bar{\Sigma}^-\pi^0 K_P \bar{K}^0\Xi^- - \bar{\Sigma}^-\pi^- K_P K^-\Xi^- + \bar{\Sigma}^-\bar{K}^0 K_P \bar{K}^0\Sigma^- - \bar{\Sigma}^- K^- K_P K^-\Sigma^- -$$
$$2\bar{\Sigma}^-\pi^- K_P \pi^-\Sigma^- - \tfrac{3}{2}\bar{p}\eta K_P K^+\Lambda - \tfrac{3}{2}\bar{\Lambda}K^+ K_P \eta p - \tfrac{\sqrt{3}}{2}\bar{p}\pi^0 K_P K^+\Lambda -$$
$$\tfrac{\sqrt{3}}{2}\bar{\Lambda}K^+ K_P \pi^0 p - \tfrac{\sqrt{3}}{2}\bar{p}\eta K_P K^+\Sigma^0 - \tfrac{1}{2}\bar{p}\pi^0 K_P K^+\Sigma^0 - \tfrac{\sqrt{3}}{2}\bar{\Sigma}^0 K^+ K_P \eta p -$$
$$\tfrac{1}{2}\bar{\Sigma}^0 K^+ K_P \pi^0 p + \underline{\tfrac{3}{2}\bar{\Lambda}\eta K_P K^+\Xi^-} + \underline{\tfrac{\sqrt{3}}{2}\bar{\Lambda}\pi^0 K_P K^+\Xi^-} + \underline{\tfrac{\sqrt{3}}{2}\bar{\Sigma}^0\eta K_P K^+\Xi^-} +$$
$$\underline{\tfrac{1}{2}\bar{\Sigma}^0\pi^0 K_P K^+\Xi^-} - \sqrt{\tfrac{3}{2}}\bar{n}\eta K_P K^+\Sigma^- - \tfrac{1}{\sqrt{2}}\bar{n}\pi^0 K_P \Sigma^- K^+ + \underline{\tfrac{3}{2}\bar{\Xi}^- K^+ K_P \eta\Lambda} +$$
$$\underline{\tfrac{\sqrt{3}}{2}\bar{\Xi}^- K^+ K_P \pi^0\Lambda} + \underline{\tfrac{\sqrt{3}}{2}\bar{\Xi}^- K^+ K_P \eta\Sigma^0} + \underline{\tfrac{1}{2}\bar{\Xi}^- K^+ K_P \pi^0\Sigma^0} - \sqrt{\tfrac{3}{2}}\bar{\Sigma}^- K^+ K_P \eta n -$$
$$\tfrac{1}{\sqrt{2}}\bar{\Sigma}^- K^+ K_P \pi^0 n + \sqrt{2}\bar{n}\pi^+ K_P \pi^0 p + \sqrt{2}\bar{p}\pi^0 K_P \pi^+ n + \sqrt{\tfrac{3}{2}}\bar{p}\bar{K}^0 K_P \pi^+\Lambda +$$
$$\sqrt{\tfrac{3}{2}}\bar{\Lambda}\pi^+ K_P \bar{K}^0 p + \tfrac{1}{\sqrt{2}}\bar{p}\bar{K}^0 K_P \pi^+\Sigma^0 + \tfrac{1}{\sqrt{2}}\bar{\Sigma}^0\pi^+ K_P \bar{K}^0 p - \sqrt{\tfrac{3}{2}}\bar{\Lambda}\bar{K}^0 K_P \pi^+\Xi^- -$$
$$\sqrt{2}\bar{\Xi}^0\pi^0 K_P \pi^+\Xi^- - \tfrac{1}{\sqrt{2}}\bar{\Sigma}^0\bar{K}^0 K_P \pi^+\Xi^- + \underline{\bar{n}\bar{K}^0 K_P \pi^+\Sigma^-} + \underline{2\bar{\Sigma}^0\pi^0 K_P \pi^+\Sigma^-} -$$
$$\sqrt{2}\bar{\Xi}^-\pi^+ K_P \pi^0\Xi^0 - \sqrt{\tfrac{3}{2}}\bar{\Xi}^-\pi^+ K_P \bar{K}^0\Lambda - \tfrac{1}{\sqrt{2}}\bar{\Xi}^-\pi^+ K_P \bar{K}^0\Sigma^0 + \underline{2\bar{\Sigma}^-\pi^+ K_P \pi^0\Sigma^0} +$$
$$\underline{\bar{\Sigma}^-\pi^+ K_P \bar{K}^0 n} - \sqrt{\tfrac{3}{2}}\bar{n}K^+ K_P \pi^+\Lambda - \sqrt{\tfrac{3}{2}}\bar{\Lambda}K^+ K_P \pi^+ n + \sqrt{\tfrac{3}{2}}\bar{\Lambda}\pi^+ K_P K^+\Xi^0 +$$
$$\tfrac{1}{\sqrt{2}}\bar{n}K^+ K_P \pi^+\Sigma^0 + \sqrt{\tfrac{3}{2}}\bar{\Xi}^0 K^+ K_P \pi^+\Lambda - \tfrac{1}{\sqrt{2}}\bar{\Xi}^0 K^+ K_P \pi^+\Sigma^0 + \tfrac{1}{\sqrt{2}}\bar{\Sigma}^0 K^+ K_P \pi^+ n -$$
$$\tfrac{1}{\sqrt{2}}\bar{\Sigma}^0\pi^+ K_P K^+\Xi^0 + \underline{\bar{\Xi}^- K^+ K_P \pi^+\Sigma^-} + \underline{\bar{\Sigma}^-\pi^+ K_P K^+\Xi^-} + \sqrt{\tfrac{3}{2}}\bar{p}\bar{K}^0 K_P \eta\Sigma^+ -$$
$$\tfrac{1}{\sqrt{2}}\bar{p}\pi^0 K_P \bar{K}^0\Sigma^+ - \sqrt{\tfrac{3}{2}}\bar{\Xi}^0\eta K_P K^-\Sigma^+ - \tfrac{1}{\sqrt{2}}\bar{\Xi}^0\pi^0 K_P K^-\Sigma^+ - \sqrt{2}\bar{\Sigma}^0\bar{K}^0 K_P K^-\Sigma^+ +$$
$$\underline{2\bar{\Sigma}^0\pi^0 K_P \pi^-\Sigma^+} + \underline{\bar{p}K^- K_P \pi^-\Sigma^+} + \sqrt{\tfrac{3}{2}}\bar{\Xi}^0 K^+ K_P \eta\Sigma^+ + \tfrac{1}{\sqrt{2}}\bar{\Xi}^0 K^+ K_P \pi^0\Sigma^+ -$$
$$\bar{\Xi}^0\pi^+ K_P \bar{K}^0\Sigma^+ - 2\bar{\Sigma}^0\pi^+ K_P \pi^0\Sigma^+ - \bar{p}K^+ K_P \pi^+\Sigma^+ + \sqrt{\tfrac{3}{2}}\bar{\Sigma}^+\eta K_P \bar{K}^0 p -$$
$$\tfrac{1}{\sqrt{2}}\bar{\Sigma}^+\pi^0 K_P \bar{K}^0 p - \sqrt{\tfrac{3}{2}}\bar{\Sigma}^+ K^- K_P \eta\Xi^0 - \tfrac{1}{\sqrt{2}}\bar{\Sigma}^+ K^- K_P \pi^0\Xi^0 - \sqrt{2}\bar{\Sigma}^+ K^- K_P \bar{K}^0\Sigma^0 +$$
$$\underline{2\bar{\Sigma}^+\pi^- K_P \pi^0\Sigma^0} + \underline{\bar{\Sigma}^+\pi^- K_P K^- p} + \sqrt{\tfrac{3}{2}}\bar{\Sigma}^+\eta K_P K^+\Xi^0 + \tfrac{1}{\sqrt{2}}\bar{\Sigma}^+\pi^0 K_P K^+\Xi^0 -$$
$$2\bar{\Sigma}^+\pi^+ K_P \pi^0\Sigma^0 - \bar{\Sigma}^+\bar{K}^0 K_P \pi^+\Xi^0 - \bar{\Sigma}^+\pi^+ K_P K^+ p - \bar{\Sigma}^+\bar{K}^0 K_P \bar{K}^0\Sigma^+ +$$

$\bar{\Sigma}^+K^-\,K_P\,K^-\Sigma^+ + \underline{2\bar{\Sigma}^+\pi^-\,K_P\,\pi^-\Sigma^+} - \bar{\Sigma}^+K^+\,K_P\,K^+\Sigma^+ - 2\bar{\Sigma}^+\pi^+\,K_P\,\pi^+\Sigma^+,$

where the underline show the six coupled-channel terms considered in this thesis. We substitute the term of particle basis with the isospin basis in the Lagrangian by using these transformations

$$
\begin{aligned}
K^-p &= \frac{1}{\sqrt{2}}|\bar{K}N(1)\rangle - \frac{1}{\sqrt{2}}|\bar{K}N(0)\rangle, && \text{(D.5)}\\
\bar{K}^0n &= \frac{1}{\sqrt{2}}|\bar{K}N(1)\rangle + \frac{1}{\sqrt{2}}|\bar{K}N(0)\rangle, && \text{(D.6)}\\
\pi^0\Sigma^0 &= \sqrt{\frac{2}{3}}|\pi\Sigma(2)\rangle - \frac{1}{\sqrt{3}}|\pi\Sigma(0)\rangle, && \text{(D.7)}\\
\pi^-\Sigma^+ &= \frac{1}{\sqrt{6}}|\pi\Sigma(2)\rangle - \frac{1}{\sqrt{2}}|\pi\Sigma(1)\rangle + \frac{1}{\sqrt{3}}|\pi\Sigma(0)\rangle, && \text{(D.8)}\\
\pi^+\Sigma^- &= \frac{1}{\sqrt{6}}|\pi\Sigma(2)\rangle + \frac{1}{\sqrt{2}}|\pi\Sigma(1)\rangle + \frac{1}{\sqrt{3}}|\pi\Sigma(0)\rangle, && \text{(D.9)}\\
\pi^0\Lambda &= \pi\Lambda(1)\rangle, && \text{(D.10)}\\
K^0\Xi^0 &= \frac{1}{\sqrt{2}}|\bar{K}\Xi(1)\rangle - \frac{1}{\sqrt{2}}|\bar{K}\Xi(0)\rangle, && \text{(D.11)}\\
K^+\Xi^- &= -\frac{1}{\sqrt{2}}|\bar{K}\Xi(1)\rangle - \frac{1}{\sqrt{2}}|\bar{K}\Xi(0)\rangle. && \text{(D.12)}
\end{aligned}
$$

The Lagrangian in the isospin basis then takes the form

$L_{\text{WT\_isospin}} =$

$3\bar{K}N(0)K_I\bar{K}N(0) - \sqrt{\frac{3}{2}}\bar{K}N(0)K_I\pi\Sigma(0) + \frac{3}{\sqrt{2}}\bar{K}N(0)K_I\eta\Lambda(0) + 4\pi\Sigma(0)K_I\pi\Sigma(0) -$
$\sqrt{\frac{3}{2}}\pi\Sigma(0)K_I\bar{K}N(0) + \sqrt{\frac{3}{2}}\pi\Sigma(0)K_IK\Xi(0) + \frac{3}{\sqrt{2}}\eta\Lambda(0)K_I\bar{K}N(0) -$
$\frac{3}{\sqrt{2}}\eta\Lambda(0)K_IK\Xi(0) + \sqrt{\frac{3}{2}}K\Xi(0)K_I\pi\Sigma(0) - \frac{3}{\sqrt{2}}K\Xi(0)K_I\eta\Lambda(0) + 3K\Xi(0)K_IK\Xi(0) +$
$\bar{K}N(1)K_I\bar{K}N(1) - \bar{K}N(1)K_I\pi\Sigma(1) - \sqrt{\frac{3}{2}}\bar{K}N(1)K_I\pi\Lambda(1) - \sqrt{\frac{3}{2}}\bar{K}N(1)K_I\eta\Sigma(1) -$
$\pi\Sigma(1)K_I\bar{K}N(1) \quad 2\pi\Sigma(1)K_I\pi\Sigma(1) + \pi\Sigma(1)K_I\Xi(1) - \sqrt{\frac{3}{2}}\pi\Lambda(1)K_I\bar{K}N(1) -$
$\sqrt{\frac{3}{2}}\pi\Lambda(1)K_I\Xi(1) - \sqrt{\frac{3}{2}}\eta\Sigma(1)K_I\bar{K}_IKN(1) - \sqrt{\frac{3}{2}}\eta\Sigma(1)K_I\Xi(1) + \Xi(1)K_I\pi\Sigma(1) -$
$\sqrt{\frac{3}{2}}\Xi(1)K_I\pi\Lambda(1) - \sqrt{\frac{3}{2}}Xi(1)K_I\eta\Sigma(1) + \Xi(1)K_I\Xi(1) + \ldots\ .$

Note that we consider the Lagrangian in the $S = -1$ sector only.

Therefore, the coupling strength (C_{ij} coefficients) of the Weinberg-

Tomozawa interaction in particle basis and isospin basis can be written in the Table D.1, D.2, and D.3.

Table D.1 C_{ij} coefficients of Weinberg-Tomozawa interaction in particle basis ($C_{ji} = C_{ij}$).

	K^-p	$\bar{K}^0n$	$\pi^0\Lambda$	$\pi^0\Sigma^0$	$\eta\Lambda$	$\eta\Sigma^0$	$\pi^+\Sigma^-$	$\pi^-\Sigma^+$
K^-p	2	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{\sqrt{3}}{2}$	0	1
$\bar{K}^0n$		2	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$-\frac{\sqrt{3}}{2}$	1	0
$\pi^0\Lambda$			0	0	0	0	0	0
$\pi^0\Sigma^0$				0	0	0	2	2
$\eta\Lambda$					0	0	0	0
$\eta\Sigma^0$						0	0	0
$\pi^+\Sigma^-$							2	0
$\pi^-\Sigma^+$								2

Table D.2 $C_{ji}^{I=0}$ coefficients in isospin basis for $I = 0$ ($C_{ji}^{I=0} = C_{ij}^{I=0}$).

	$\bar{K}N$	$\pi\Sigma$	$\eta\Lambda$
$\bar{K}N$	3	$-\sqrt{\frac{3}{2}}$	$\frac{3}{\sqrt{2}}$
$\pi\Sigma$		4	0
$\eta\Lambda$			0

Table D.3 $C_{ji}^{I=1}$ coefficients in isospin basis for $I = 1$ ($C_{ji}^{I=1} = C_{ij}^{I=1}$).

	$\bar{K}N$	$\pi\Sigma$	$\pi\Lambda$	$\eta\Sigma$
$\bar{K}N$	1	-1	$-\sqrt{\frac{3}{2}}$	$-\sqrt{\frac{3}{2}}$
$\pi\Sigma$		2	0	0
$\pi\Lambda$			0	0
$\eta\Sigma$				0

APPENDIX E

CONFERENCE PROCEEDINGS

การประชุมเสนอผลงานวิจัย ระดับบัณฑิตศึกษาแห่งชาติ ครั้งที่ ๑๔
มหาวิทยาลัยเทคโนโลยีพระจอมเกล้าพระนครเหนือ
วันที่ ๑๐-๑๑ กันยายน ๒๕๕๒

การศึกษาอันตรกิริยาของแอนติเคอนและนิวคลิออน

Study of $\overline{K}N$ Interactions

Wanchaloem Poonsawat[1], Khanchai Khosonthongkee[2], Chinorat Kobdaj[3] and Yupeng Yan[4]

บทคัดย่อ

ผลของการศึกษาอนุภาค $\Lambda(1405)$ ที่เกิดเป็นสภาพพักกันของ $\overline{K}N$ ซึ่งเป็นการผสมกันเพียงชั่วขณะของอนุภาคมูลฐานในสถานะสาม-ควาร์ก ได้ชี้ให้เห็นถึงแบบจำลองที่สมเหตุสมผลและเป็นไปได้ว่า อนุภาคมูลฐาน $\overline{K}, N, \Sigma$ และ π สามารถมีอันตรกิริยาต่อกันได้โดยผ่านทางศักย์หรือการแลกเปลี่ยนอนุภาคเมซอน ในงานวิจัยนี้ซึ่งเป็นเพียงส่วนหนึ่งของงานวิจัยหลัก ซึ่งจะคำนวณหาศักย์ที่ให้ผลสอดคล้องกับทั้ง ข้อมูลการชนกันของ $\overline{K}N$ ที่พลังงานต่ำ, ข้อมูลของอะตอมไฮโดรเจนแบบเคออนิก และทรานสิชั่นแอมพลิจูดที่ได้ถูกทำนายไว้โดยวิธีการในเชิงทฤษฎีซึ่งแตกต่างกับอันตรกิริยาในปริภูมิของโมเมนตัม ที่ต้องอาศัยบริบททางทฤษฎีสนามควอนตัม โดยศักย์ดังกล่าว จะช่วยให้เราสามารถศึกษาระบบของหลายอนุภาคได้สะดวกยิ่งขึ้น

Abstract

Finding that the $\Lambda(1405)$ consist of $\overline{K}N$ bound state with only a small admixture of elementary three-quark state suggests a reasonable model is possible with the $\overline{K}$, N, Σ and π as elementary particles interacting via potentials or meson-exchange. The work is part of a project which is to work out potentials which would reproduce the low-energy $\overline{K}N$ scattering data, kaonic hydrogen atom data, and transition amplitudes predicted by other theoretical approaches. Unlike interactions in momentum space derived in the framework of quantum field theory, such a potential would be conveniently applied to multi-particle systems.

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Introduction

The exploration for quasibound antikaon-nuclear states has become a hot subject in nuclear physics. Recently, definite information about the strong- interaction level shifts of the kaonic hydrogen atom was obtained from the experiment KpX at KEK [1,2], which indicates a repulsive-type for the 1s orbit. For heavier nuclei, Batty et al. [3,4], reanalyzed all of the existing data of $\overline{K}$ atoms, including a density-dependent term for the $\overline{K}N$ scattering length, and deduced optical potential with a strongly attractive real part and also a strongly absorptive imaginary part. The reason for such a highly attractive potential, despite the fact that the strong-interaction shifts appear to be negative, comes from the assertion that the $\Lambda(1405)$ state is not an elementary particle, but the bound state of $\overline{K}+N$. From such a potential one expects deeply bound nuclear states in heavier nuclei, but their widths are estimated to be on the order of 100 MeV or if their potential parameters are strictly applied, and thus such nuclear states may not be identified as discrete state.

Calculations of strong binding of antikaons in a nuclear medium based on chiral SU(3) dynamics have a long history [5,6,7,8,9,10]. The recent revival of this theme was prompted by Akaishi and Yamazaki [10,11], who used a simple potential model to calculate bound states of few-body systems such as K^-pp, K^-pn, and K^-pnn. However, it is noted that the predictive power of all such investigations is limited because the interactions are constrained just by the scattering processes but the energy range of the $\overline{K}N$ interaction relevant for deeply bound kaonic nuclei lies far below the $\overline{K}N$ threshold. For variational calculations of few-body systems involving anti-kaons, one must use a realistic effective $\overline{K}N$ interaction, preferentially in the form of a potential. This potential is in general complex and energy dependent. It must be constrained to reproduce the scattering amplitude in vacuum, and it must encode the full coupled-channel dynamics. A number of attempts in this direction, using a schematic effective interaction, have been reported in [12,13,14].

However, we have found that all versions of the $\overline{K}N$ interactions give unreasonably large decay widths for the kaonic hydrogen atom. Here we would like to derive an effective interaction which reproduce not only the $\overline{K}N$ scattering amplitudes but also the kaonic hydrogen atom data. This work is just the first step of the whole project with which such a potential is expected to be worked out.

Dynamical Equations for Coupled $\overline{K}N$ Systems

We start from the Lippmann-Schwinger equation

$$|\psi_\alpha\rangle = |\phi_\alpha\rangle + \frac{1}{E - H_0 + i\varepsilon} V_{\alpha\beta} |\psi_\beta\rangle \qquad (1)$$

where $V_{\alpha\beta}$ is the interaction between the α and β channels, and $|\phi_\alpha\rangle$ satisfies the homogeneous equation.

$$(E - H_0)|\phi_\alpha\rangle = 0 \qquad (2)$$

The formal solution of the Lippmann-Schwinger equation for outgoing scattered waves takes the form

$$\psi_\alpha(\vec{r}) = \phi_\alpha(\vec{r}) + \int d^3\vec{r}'' G(\vec{r},\vec{r}') V_{\alpha\beta}(\vec{r}') \psi_\beta(\vec{r}'') \qquad (3)$$

where $G(\vec{r},\vec{r}')$ is the Green's function, satisfying the equation,

$$(\nabla^2 + k^2)G(\vec{r},\vec{r}') = \delta^{(3)}(\vec{r} - \vec{r}'). \qquad (4)$$

By solving the above equation, one derives the appropriate Green's function for scattering processes

$$G(\vec{r},\vec{r}') = -\frac{1}{4\pi \mid \vec{r} - \vec{r}' \mid} e^{ik|\vec{r}-\vec{r}'|}$$

In the $\left|JMLS\right\rangle$ basis, we derive the Lippmann–Schwinger equation for the radial part of the scattered wave,

$$R^{\alpha,J}_{L'S',LS}(k_\alpha,r) = j_l(kr)\delta_{\alpha,\beta}\delta_{LL'}\delta_{SS'} + \sum_{L''S''}\int r_1^2 dr_1\; g_l(k_\alpha,r,r_1)V^{\alpha\beta,J}_{L'S',L''S''}(r_1)R^{\beta,J}_{L''S'',LS}(k_\beta,r_1) \qquad (5)$$

where α and β are channels of the system, $L(L')$ and $S(S')$ are total orbital angular momentum and spin of initial(final) states, and J is the total angular momentum. k_α and k_β are the momenta of channels α and β, respectively. g_l is the radial component of the Green's function $G(\vec{r},\vec{r}_1)$ and $V^{\alpha\beta,J}_{L'S',L''S''}$ is the radial part of the interaction from the channel β to α.

Numerical Results with phenomenological $\overline{K}N$ potential

In this section we show the quality of the most popular phenomenological $\overline{K}N$ potentials. As an example, we calculate the cross section of the reaction $\overline{K}N \to \overline{K}N$ with the potential taken from Akaishi and Yamazaki [11]. The potential takes the general form

$$V^I_{\alpha\beta}(r) = V^I_{\alpha,\beta}\, exp[-\left(\frac{r}{b}\right)^2] \qquad (6)$$

where α and β stand for channels, I are isospins taking values 0 and 1. The length parameter b = 0.6 fm. The $V^I_{\alpha,\beta}$ on the right side of the above equation take the values as follows:

$$V^{I=0}_{\bar{K}N,\bar{K}N} = -436\ MeV, V^{I=1}_{\bar{K}N,\bar{K}N} = -62\ MeV$$

for the $\overline{K}N \to \overline{K}N$ channel,

$$V^{I=0}_{\bar{K}N,\pi\Sigma} = -412\ MeV, V^{I=1}_{\bar{K}N,\pi\Sigma} = -285\ MeV$$

for the $\overline{K}N \to \pi\Sigma$ channel, and

$$V^{I=0}_{\bar{K}N,\pi\Lambda} = 0\ MeV, V^{I=1}_{\bar{K}N,\pi\Lambda} = -285\ MeV$$

for the $\overline{K}N \to \pi\Lambda$ channel.

The cross sections of the reactions $\bar{K}N \to \bar{K}N, \pi\Lambda, \pi\Sigma$ can be derived by solving the radial Lippmann–Schwinger equation (5). Shown in Figure below are the cross sections of the reactions $K^-p \to K^-p, \bar{K}^0n$, derived for the phenomenological potentials [11]. It is found that the theoretical results have quite large discrepancies from the experimental data. For other phenomenological potentials the theoretical

results are similar to the potential employed here. One may conclude that better potentials, which would reproduce the low-energy $\overline{K}N$ scattering data and kaonic hydrogen atom data, should be developed.

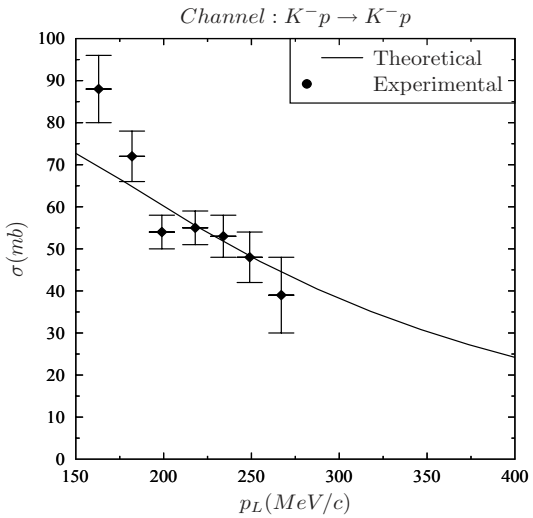

FIG. 1

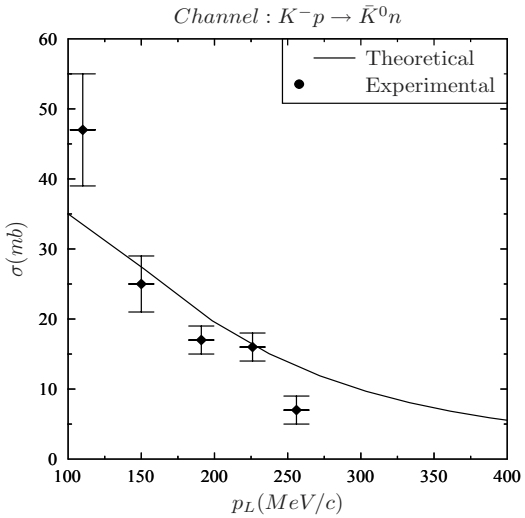

FIG. 2

Figure 1-2 Cross sections of the reaction $K^- p \to K^- p, \bar{K}^0 n$ derived with the potentials [11]. The experimental data taken from J. Ciborowski [16].

Interaction with Chiral Symmetry

By considering in the framework of SU(3) flavor symmetry where the strong interaction is invariant under the unitary transformation of u, d and s quarks, the interactions among the channels $\overline{K}N$, $\pi\Sigma$ and $\pi\Lambda$ are related with each other. One may express the relations in the isospin basis with the following matrices,

$$C_{ij}^{I=0} = \begin{pmatrix} 3 & -\sqrt{\frac{3}{2}} \\ -\sqrt{\frac{3}{2}} & 4 \end{pmatrix} \qquad C_{ij}^{I=1} = \begin{pmatrix} 1 & -1 & -\sqrt{\frac{3}{2}} \\ -1 & 2 & 0 \\ -\sqrt{\frac{3}{2}} & 0 & 0 \end{pmatrix} \qquad (7)$$

where C_{ij}^{I} is the couple strengths for $I=0$ and $I=1$, for $\overline{K}N$ (channel 1), $\pi\Sigma$ (channels 2) and $\pi\Lambda$ (channel 3).

The interactions for the various channels in the particle basis can be derived from the isospin-based interactions as follows:

Table 1

	K^-p	$\bar{K}^0n$	$\pi^-\Sigma^+$
K^-p	$\frac{1}{2}C_{11}^{I=0}V^0(r)+\frac{1}{2}C_{11}^{I=1}V^1(r)$	$-\frac{1}{2}C_{11}^{I=0}V^0(r)+\frac{1}{2}C_{11}^{I=1}V^1(r)$	$-\frac{1}{\sqrt{6}}C_{12}^{I=0}V^0(r)-\frac{1}{2}C_{12}^{I=1}V^1(r)$
$\bar{K}^0n$		$\frac{1}{2}C_{11}^{I=0}V^0(r)+\frac{1}{2}C_{11}^{I=1}V^1(r)$	$\frac{1}{\sqrt{6}}C_{12}^{I=0}V^0(r)-\frac{1}{2}C_{12}^{I=1}V^1(r)$
$\pi^-\Sigma^+$			$\frac{1}{3}C_{22}^{I=0}V^0(r)+\frac{1}{2}C_{22}^{I=1}V^1(r)$
$\pi^+\Sigma^-$			
$\pi^0\Sigma^0$			
$\pi^0\Lambda$			

Table 2

	$\pi^+\Sigma^-$	$\pi^0\Sigma^0$	$\pi^0\Lambda$
K^-p	$-\frac{1}{\sqrt{6}}C_{12}^{I=0}V^0(r)+\frac{1}{2}C_{12}^{I=1}V^1(r)$	$\frac{1}{\sqrt{6}}C_{12}^{I=0}V^0(r)$	$\frac{1}{\sqrt{2}}C_{13}^{I=1}V^1(r)$
$\bar{K}^0n$	$\frac{1}{\sqrt{6}}C_{12}^{I=0}V^0(r)+\frac{1}{2}C_{12}^{I=1}V^1(r)$	$-\frac{1}{\sqrt{6}}C_{12}^{I=0}V^0(r)$	0
$\pi^-\Sigma^+$	$\frac{1}{3}C_{22}^{I=0}V^0(r)-\frac{1}{2}C_{22}^{I=1}V^1(r)$	$-\frac{1}{3}C_{22}^{I=0}V^0(r)+\frac{1}{3}C_{22}^{I=2}V^2(r)$	0
$\pi^+\Sigma^-$	$\frac{1}{3}C_{22}^{I=0}V^0(r)+\frac{1}{2}C_{22}^{I=1}V^1(r)$	$-\frac{1}{3}C_{22}^{I=0}V^0(r)+\frac{1}{3}C_{22}^{I=2}V^2(r)$	0
$\pi^0\Sigma^0$		$\frac{1}{3}C_{22}^{I=0}V^0(r)$	0
$\pi^0\Lambda$			0

Table 1-2. Particle based potentials in terms of isospin based ones.

The isospin based potentials $V^0(r)$ and $V^1(r)$ may be determined by adjusting to experimental data of the reactions $\bar{K}N \to \bar{K}N, \pi\Lambda\pi\Sigma$ and $\bar{K}N$ exotic atoms.

Discussion

In this work we have derived the dynamical equations for the coupled-channel $\overline{K}N$ system. The cross sections of the reactions $\overline{K}N \to \bar{K}N$ are evaluated with one of the most popular phenomenological $\overline{K}N$ potentials and the theoretical results indicate that it is necessary to develop better versions of $\overline{K}N$ potentials.

The final goal of our work is to derive, in the framework of the SU(3) chiral symmetry, a version of interactions for coupled $\overline{K}N$ system in coordinate space. The potentials are expected to reproduce both the low-energy $\overline{K}N$ scattering data and $\overline{K}N$ exotic atom observables.

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