



รายงานการวิจัย

Quantum Field Theory and Theoretical Physics:  
Applications to Cerenkov Radiation  
and  
Multi-Electron Systems

คณะผู้วิจัย

หัวหน้าโครงการ

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ผลงานวิจัยเป็นความรับผิดชอบของหัวหน้าโครงการวิจัยแต่เพียงผู้เดียว

November 2541

#### ACKNOWLEDGMENT

This is to acknowledge with many thanks the Suranaree University of Technology Research Fund Administration for financial support which made this project possible and led to its successful completion.

ABSTRACT

Using quantum field theory methods, an explicit derivation was provided for the photon Green function in a dielectric slab immersed in another dielectric medium. From the vacuum-to-vacuum transition amplitude of field theory, the photon number density emitted by a charged particle per unit length of its path was derived. Unlike the conventional Cerenkov radiation in an infinite medium, the photon number density was shown to be proportional to  $\sin \vartheta_c$  rather than  $\sin^2 \vartheta_c$ , where  $\vartheta_c$  is the Cerenkov-cone half angle. This and other related results may be of interest experimentally. Finally, the quantum correction pioneered by Schwinger and others was derived without any modeling after a harmonic oscillator interaction and without going through the particle density.

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## 1. INTRODUCTION

Quantum Field Theory and Renormalization [1] have become the method and criterion, respectively, for constructing the fundamental interactions in nature [2,3,4]. Undoubtedly, the most important of these fundamental interactions in our world is the electromagnetic one - more precisely referred to as Quantum Electrodynamics. For example, the Legendary Nobel Laureate R. P. Feynman states [5,p.77,p.152]: "Most of the phenomena you are familiar with involve the interaction of light and electrons - all of chemistry and biology, for example". "Quantum Electrodynamics is our best example of a good theory". As far as the accuracy that Quantum Electrodynamics provides when compared with experiments, Feynman [5,p.7] states: "If you were to measure the distance from Los Angeles to New York [this] accuracy would be exact to the thickness of a human hair". Renormalization, on the other hand, is a statement of the high-energy behaviour of a theory and, in a fundamental way, interprets consistently in defining the masses and the coupling parameters in a theory. Quantum Electrodynamics just meets the criterion of renormalizability as an acceptable theory to describe nature. It is a simple matter that no theory may be acceptable to describe nature unless it is renormalizable. Cerenkov radiation is of electromagnetic origin. It is the emission of photons (radiation) by a charged particle and an electron in particular, when the speed of the particle exceeds the speed of light in the medium. There has been much interest in

this form of radiation in recent years both theoretically [6-19] and experimentally [cf.20-22]. It has been applied not only to particles, but also for the first time to strings [17]. Cerenkov radiation was discovered in 1936 [23] and was first explained theoretically in 1937 [24].

One of the objectives of the project was to step out of this form of radiation as conventionally studied in an infinite medium to a medium where an electron may be trapped within a slab of permittivity different from the surrounding medium even for short periods of time and see how the photon number density changes with such a discontinuity in a medium [6]. The differences, as derived in the bulk of the project, will be discussed later in the relevant sections on the analysis of the results and conclusions. As an underlying hypothesis in any such an investigation is that the medium may be described by a permittivity which is well supported experimentally [25] for photons in the visible region, and that the energy quantum of a photon is much less than that of the electron. The latter is also justified for high energy relativistic electrons. The so-called statistical model of the multi-electron atom was invented 70 years ago by Thomas [26] and Fermi [27] when quantum mechanics was still in its infancy. Ever since, it has captivated the heart of physicists, and will continue to do so to the twenty first century, due to its extreme simplicity and remarkable success. The ground state energy of atoms has been vigorously investigated over the years [cf.28-30] as a function of  $Z$  - the atomic number. The quantum correction to the ground-state pioneered by Schwinger [31] and others

has now been derived [32] without recourse to a harmonic oscillator potential [31] and without going through the particle density [28] - the details of which are spelled out in our analysis of results and conclusions. The usefulness of the modified Cerenkov radiation we derived is that any deviation of the form of radiation observed over the orthodox one would reflect the presence of such a cloud of a medium surrounding the electron. The expression obtained is general enough to give an estimate of the size of such a foreign medium. This may be interesting experimentally. The usefulness of a quantum correction to the ground-state is to provide the path to a more accurate expression for the exact ground state energy of all atoms. The scope of research is limited as far as the need of a permittivity to describe the property of matter and of high energy electrons where radiative corrections are unimportant.

## 2. METHODOLOGY

The method of study of the entire project is that of Quantum Field Theory. Modern field theory, as we know it, is due to two giants: Schwinger [33,34] and Feynman [35]. The so-called vacuum-to-vacuum transition amplitude provides all the information one needs to extract from the dynamics. The phase of the vacuum-to-vacuum amplitude is a geometrical phase in spacetime with a proportionality expression corresponding to the ground-state energy of the system. If the absolute value of the vacuum-to-vacuum amplitude is less than one, or more precisely, strictly less than one, then the vacuum (ground state) is unstable and the electron has done something to the system. In our case, the electron has emitted radiation. The ground-state now includes radiated photons and is different from the initial ground-state in the absence of the electron and the radiation. Over the years, we have found this way of study as the clearest and the easiest way for investigating problems not only in high energy particle physics but condensed matter physics and gravitation. The method of the vacuum-to-vacuum amplitude relies heavily on the Green functions of the theory. These were derived for the photon propagator [6] for studying Cerenkov radiation and in the quantum correction [32] to multi-electron systems. As far as using Quantum Field Theory methods in investigating all sorts of problems in all branches of physics, it is worth quoting Feynman [36, p.1] in his famous Cornell lectures on Quantum Field Theory Methods in Fundamental Processes: "These lectures will cover all of physics".



### 3. RESULTS OBTAINED

An exact expression for the photon propagator corresponding to an electron moving in an immediately surrounding medium of permittivity different from the rest of the medium was derived for the first time [6]. The latter was used in the vacuum-to-vacuum transition amplitude of field theory to obtain the exact expression for the number density of Cerenkov photons emitted by the electron as it moves in its immediately surrounding medium with a speed greater than the speed of light in the medium. The most remarkable result obtained was that the photon number density is proportional to  $\sin \vartheta_c$ , near threshold of the electron's speed, rather than to the familiar expression  $\sin^2 \vartheta_c$ , where  $\vartheta_c$  is the Cerenkov-cone half angle. This provides an overall enhancing factor for radiation near threshold, that is, for small  $\sin \vartheta_c$ . This may be of interest experimentally. The second remarkable property of the photon number density derived was that, near threshold, the density is seen to damp out at higher frequencies unlike the conventional expression for an infinite medium. This may be also of interest experimentally. The permittivity, throughout the analysis is assumed to be frequency dependent. As an application, the immediately surrounding medium was considered to be a hydrogen gas trapped in air even for short periods of times. The expression for the number density is explicitly dependent on the thickness of such a surrounding medium. Emission of visible light by the electron was studied. The analysis introduces naturally a scale factor of the order of  $\sim 0.01$  cm into the physics of the

problem. At such a scale, we have derived that, on the average as the electron moves one centimeter, near threshold, it would emit  $(3 \times \sin \vartheta_c)$  photons. For much less distances than .001 cm, this average number derived was  $(7 \times \sin \vartheta_c)$  photons/cm. In practice, with a large number of electrons moving in the medium within such limited regions may lead to a readily detectable amount of radiation. The reduction of the photon number by one half as we move from very small distances to .001 cm may be a significant factor of detectability. As this distance becomes arbitrarily large in comparison to .001 cm one recovers the conventional Cerenkov radiation expression. Finally, by using the method of Green function, we have derived [32] the quantum correction which contributes precisely an energy  $- .04907 Z^{5/3}$  (in units of  $me^4/\hbar^2$ ) to the conventional statistical atom, where Z is the atomic number.

#### 4. SUMMARY AND CONCLUSIONS

The exact photon propagator and the exact number density of Cerenkov photons emitted by the electron, was shown to behave like the sine of the Cerenkov angle, near threshold, rather than the square of the latter, and that it damps out at high frequencies which may be of interest experimentally. The significant dependence of the number density on the thickness of the immediately surrounding medium of the electron immersed in a larger medium is a critical factor on the photon number emitted by the electron and may be a key factor in detecting the presence of such a foreign substance in the medium. Radiative corrections, where, for example, an emitted photon by the electron is reabsorbed by the latter, are not important for photon energies much less than the electron's energy. The high Z quantum correction to multi-electron atoms was derived without any modeling and contributes precisely the amount  $-.04907 Z^{5/3}$  (in units of  $m_e^4/\hbar^2$ ) to the conventional ground state of the statistical atom.

REFERENCES

1. Manoukian, E. B. 1983. Renormalization, Academic Press, New York.
2. Weinberg, S. 1980. Nobel Lectures in Physics 1979, Rev. Mod. Phys. 52, 515.
3. Salam, A. 1980. Nobel Lectures in Physics 1979, Rev. Mod. Phys. 52, 525.
4. Glashow, S. 1980. Nobel Lectures in Physics 1979, Rev. Mod. Phys. 52, 539.
5. Feynman, R. P. 1985. QED. The Strange Theory of Light and Matter, Princeton University Press, New Jersey.
6. Manoukian, E. B. and Bantitadawit, P. 1999. Physica B 269, 115.
7. Manoukian, E. B., Eab, C.-H. and Ungkitchanukit, A. 1997. Physica B 240, 68.
8. Manoukian, E. B. , Ungkitchanukit, A. and Eab, C.-H. 1997. Progr. Theor. Phys. 98, 1.
9. Soln, J. 1997. Journ. Phys. D 30, 2936.
10. Zhevago, N. K. and Glebov, V. I. 1997. Sov. Phys. JETP 84, 466.
11. Orisa, B. D. 1995. Int. Journ. Theor. Phys. 34, 545.
12. Manoukian, E. B. 1994. Il Nuovo Cimento 107A, 503.
13. Manoukian, E. B. 1993. Phys. Rev. E 48, 543.
14. Pratap, R., Sasidharan, K. and Krishan, V. 1993. Phys. Rev. E 47, 640.
15. Fulop, L. 1993. Int. Journ. Theor. Phys. 32, 2041.
16. Ginzburg, M. S., Zotora, I. V., Kovalev, N. F. and Sergeev, A. S. 1993. Sov. Phys. JETP 77, 893.
17. Manoukian, E. B. 1991. Il Nuovo Cimento 104 A, 1459.

18. Pardy, M. 1989. Phys. Lett. A 134, 357.
19. Schwinger, J., Tsai, W.-Y. and Erber, T. 1976. Ann. Phys. 96, 303.
20. Wang, G., Yu, S., Xun, P. et. al. 1991. Appl. Phys. Lett. 59, 2378.
21. Garate, E. P., Kosai, H., Evans, K. and Fisher, A, 1990. Appl. Phys. Lett. 56, 1092.
22. Efimov, N. N. and Sokurov, V. F. 1979. Proc. 16th Int. Conf. on Cosmic Rays, Kyoto 8, 152.
23. Cerenkov, P. A. 1936. C. R. Acad. Sci. (USSR) 3, 413.
24. Tamm, I. E. and Frank, I. B. 1937. Dokl. Acad. Nauk. SSR 14, 109.
25. Born, M. and Wolf, E. 1989. Principles of Optics, Pergamon Press, 6th Edition, Oxford.
26. Thomas, L. H. 1927. Proc. Camb. Phil. Soc. 23, 542.
27. Fermi, E. 1927. Rend. Acad. Naz. Linzei 6, 602.
28. Dreizler, R. M. and Gross, E. K. U. 1990. Density Functional Theory, Springer, Berlin.
29. Morgan, J. D. 1996. In Atomic and Molecular Handbook, G. W. Drake, ed., American Institute of Physics, New York.
30. Lieb, E. H. 1991. The Stability of Matter: From Atoms to Stars, Springer, Berlin.
31. Schwinger, J. 1981. Phys. Rev. A 24, 2353.
32. Manoukian, E. B. and Bantitadawit, P. 1999. Int. Journ. Theor. Phys. 38, 897.
33. Schwinger, J. ed. 1958. Quantum Electrodynamics, Dover, New York.

34. Schwinger, J. 1970, Vol.1, 1973, Vol.2, 1989, Vol.3.  
Particles, Sources and Fields, Addison-Wesley, Reading, MA.
35. Feynman, R. P. and Hibbs, A. R. 1965. Quantum Mechanics and Path Integrals , McGraw-Hill, New York.
36. Feynman, R. P. 1982. The Theory of Fundamental Processes, Benjamin, Menlo Park, California.

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