

## System Identification via Adaptive Tabu Search

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### Abstract

This article presents a novel Adaptive Tabu Search (ATS) method. Our additional back-tracking and adaptive search radius mechanisms enhance the conventional Tabu Search (TS) method to achieve faster and more efficient search. The main application of the ATS method discussed in this article is identification of system models. Examples cover hot-air tube, system with torsional resonance, static nonlinearity and inverted pendulum.

**Keywords:** System Identification, Adaptive Tabu Search, Back-Tracking, Adaptive Radius, Conventional Identification, ARX Model

### 1. Introduction

Regression analysis forms the core of model identification. This conventional approach has been used widely to identify linear [1, 2] and nonlinear system [3] models. Although their closed-form formulae provide an advantage leading to fast computation, its major drawback is the restriction of the class of models to be difference equations and the like. Consequently, it is not possible to apply the method to identify models of other forms that do not confine to the same class, such as static nonlinearity.

To provide more alternatives for identification problems, we turn our interest to artificial intelligent (AI) searching techniques. Three methods are candidates: Genetic Algorithm, Evolutionary Programming, and Tabu Search. We have chosen the TS method since recent researches in optimization problems such as those reported in [4-6] confirm the efficiency of the method. Yet, the TS method has not been widely used for identification problems.

This article gives a brief description of the conventional TS method. The proposed ATS method is then described in details. Since our ATS method provides more efficient search than the conventional TS method does, we have applied the ATS method to identify the models of various systems. Examples of case studies given in this article include die-swell steady-state model for polymer melts, hot-air tube, system with torsional resonance, static nonlinearity, and inverted pendulum. Conclusions are drawn at the final section of this article.

### 2. Tabu and Adaptive Tabu Search Mechanisms

The TS method [7, 8] is an iterative process that searches for the best solution by moving from a current solution to find a better solution repeatedly. One of the most important elements that make the TS method different from other searching methods is its tabu list. It keeps the history of paths. This list is used as information to find directions of a new move. This new move should lead the search to the better local optimum solution and ultimately to the global optimum one. Details of a tabu list are varied to suit each different problem.

#### 2.1 Fundamentals of Tabu Search (TS)

This section briefly details the TS mechanism. Given *count* as a number of iteration and *MAX\_ITERATION* as a predefined maximum number of iteration. The simple TS procedures can be outlined as follows:

- 1) Randomly select an initial solution  $S_0$  within a search space of radius  $R$ .  $S_0$  is currently a local optimum solution and  $S_0 = best\_neighbor$  (see Fig.1).
- 2) Randomly select  $N$  new solutions by moving the current solution around  $S_0$  within the search space. Let  $S_1(r)$  be a set that contains all  $N$  solutions (see Fig. 2)
- 3) Compute a cost value of each member in  $S_1(r)$  using the objective function. Then choose the best solution and assign it as *best\_neighbor1* (see Fig.2).
- 4) If *best\_neighbor1* is better than *best\_neighbor*, then  $best\_neighbor = best\_neighbor1$  (see Fig.3). If there is no better solution, go to step 6.
- 5) Assign  $S_0 = best\_neighbor$  (see Fig.4).
- 6) If *best\_neighbor1* is not in *neighbor\_list*, then store *best\_neighbor1* in *neighbor\_list*.
- 7) If  $count \geq MAX\_ITERATION$ , stop the process. The current best solution is the overall best solution. Otherwise, go back to step 2 and start the process again until all criteria is satisfied (see Fig.5).

#### 2.2 Adaptive Tabu Search (ATS)

In order to improve the performance of the TS method, we have proposed the extended version of the TS method called adaptive tabu search (ATS). This novel ATS consists of two additional steps, namely back-

tracking and adaptive radius, different from the conventional TS.

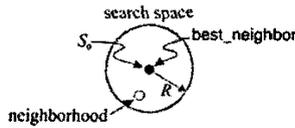


Fig.1 Random  $S_0$  in search space

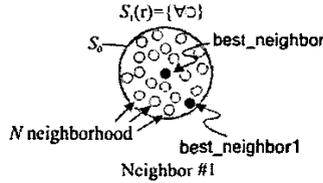


Fig.2 Neighbor hood around  $S_0$

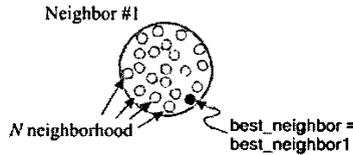


Fig.3 Assign new best neighbor

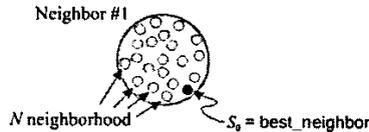


Fig.4 Assign new  $S_0$

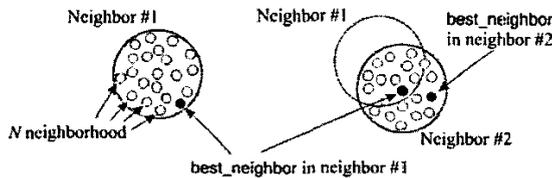


Fig.5 Searching process in the next iteration

The back-tracking process allows the system to go back and look up the previous solutions that have been already searched. The better solution is then chosen among the current and the previous solutions. Fig. 6 demonstrates details of the back-tracking process. If another new solution is assigned within the current search space, the new search space is then introduced to the current search. Given this new search space to explore, the search process is likely to have more chances of escaping from the local optimum. Note that the new solution chosen here is obviously not necessary to be the best solution within the current search space.

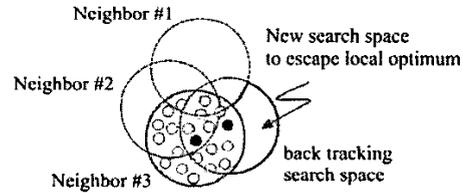


Fig.6 Back-tracking in ATS

The adaptive radius process decreases the search area during the searching process. This added feature continues until the near global solution is found. For a regular searching process, the most important factor is the search radius which should be appropriately chosen. The larger size of the radius, the coarser resolution of the searching step. In this case, the searching process could possibly overlook the desire solution. In contrast, the finer size of the radius, the longer computational time. With this too small step of moving the solution, however, the searching process may not efficiently cover the search area that could contain the desire solution. Consequently, the adaptive radius mechanism has been developed to suitably adjust the search radius during the searching process. By using the cost of the solution as a criterion for adapting the search radius, once the better solution has been found, the search radius will be decreased using the following relationship:

$$radius_{new} = \frac{radius_{old}}{DF} \quad (1)$$

where DF is a decreasing factor (DF = 10 for this work). A simple procedure of the adaptive radius is as follows:

```

If best_cost < a
    radius = radius/DF
End
If best_cost < b
    radius = radius/DF
End
If best_cost < c
    radius = radius/DF
End

```

At the earlier stage of the searching process, the search radius should be efficiently large to speed up the move to the vicinity of the global solution. Once the searching process approaches the near global solution, the finer step of searching is then employed in order to move to the solution in a high resolution step. The criteria of the cost value ( $a$ ,  $b$  and  $c$ ) can be varied for different problems.

The back-tracking process can be added to the conventional TS process during the step 6 and 7 while the adaptive radius process is normally performed at the

end of step 7. The following section presents successful applications of the ATS method in identification domain.

### 3. Applications

Identification of model parameters is very important issue in control context. Many applications rely on the conventional identification methods such as ARX, ARMAX, Box-Jenkin, etc. Such methods, however, may not be well-suited for all cases. This section presents some of the applications that are difficult to solve by the conventional approach. The proposed ATS method has been applied to demonstrate the more desirable performance of solving such problems. During our identification processes, the convergence of solutions and errors, as well as the minimum identification errors were monitored numerically. The detail of each problem, however, is not discussed herein. The problems are presented as follows.

Case 1: Die-swell model for polymer melts

The model for die-swell ratio has been proposed by [9] as

$$B = (a + bS_R^x)^y \quad (2)$$

where B is die-swell ratio,  $S_R$  is recoverable shear strain, a, b, x, and y are model parameters, respectively. Table 1 displays the observed data of  $S_R$  and B from the actual process. The search using ATS method results in  $a=2.6811$ ,  $b=0.0793$ ,  $x=2.7946$ , and  $y=0.5775$ , respectively. Fig.7 shows the agreement between the model and the observed data.

Table 1 Observed data of polymer melts.

$S_R$	1.287	1.374	1.432	1.492	1.565
B	1.827	1.835	1.852	1.860	1.870
$S_R$	1.655	1.722	1.779	1.819	
B	1.882	1.901	1.913	1.924	

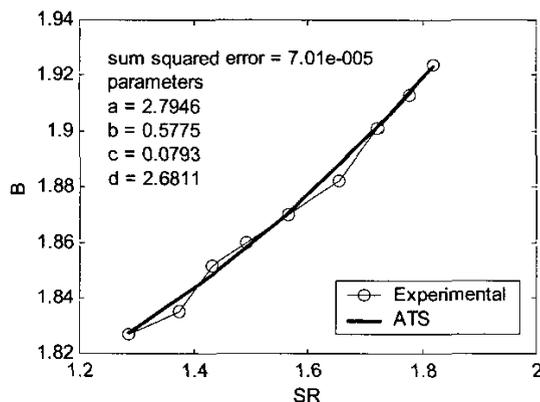


Fig.7 Plot of the die-swell model against observed data

Case 2: Model for hot-air tube

The hot-air tube is an experimental kit made by Feedback Instruments Ltd. [10] to demonstrate transport-lag dynamic. Its dynamic is commonly represented by first-order plus delay model as

$$G(s) = \frac{Ke^{-\tau_d s}}{\tau s + 1} \quad (3)$$

where  $K$ =system gain,  $\tau$ =time constant (sec), and  $\tau_d$ =delay time (sec), respectively. In order to aid the identification of model parameters, the system delay is represented by the first-order Pade's approximation. The ATS results in  $K=0.8645$ ,  $\tau=0.3337$  sec, and  $\tau_d=0.2042$  sec, respectively. Fig. 8 illustrates the model plotted against the empirical data. The undershoot in the plot occurs due to the approximation term that contains a zero in the right-half of the s-plane. The proposed method has shown very satisfactory results.

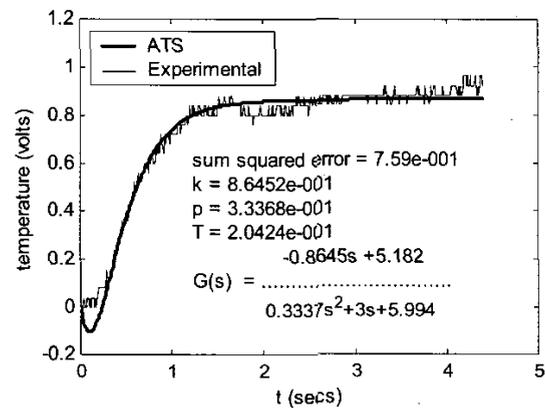


Fig.8 Plot of the hot-air model against observed data

Case 3: Torsional resonance and nonlinear saturation models

A rotational system exhibiting torsional resonance is represented by the diagram shown in Fig.9. Torsional resonance affects speed performance in terms of imposed oscillation with a small amplitude. The dynamic of torsional resonance has been studied for many years such as those appeared in [11,12]. The work [13] provides a reduced order model expressed by the transfer function

$$G(s) = \frac{K}{a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \quad (4)$$

At the first step, the system is considered linear and described by the fifth-order model in equation (4). Table 2 summarizes the coefficients obtained from the ATS.

Fig. 10 depicts the response of the model plotted against the actual response. The amplitude (y-axis) in Fig.10 represents speed measured at load. The agreement between the experimental and the theoretical results can be noticed.

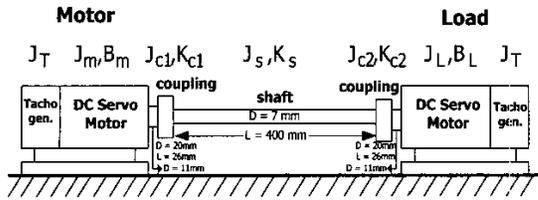


Fig.9 A system exhibiting torsional resonance

Table 2 Model parameters resulted from the ATS method for the system with torsional resonance.

K		a <sub>0</sub>		a <sub>1</sub>	
8.60×10 <sup>11</sup>		8.42×10 <sup>11</sup>		5.38×10 <sup>10</sup>	
a <sub>2</sub>	a <sub>3</sub>	a <sub>4</sub>	a <sub>5</sub>		
1.24×10 <sup>8</sup>	9.44×10 <sup>5</sup>	1590	1		

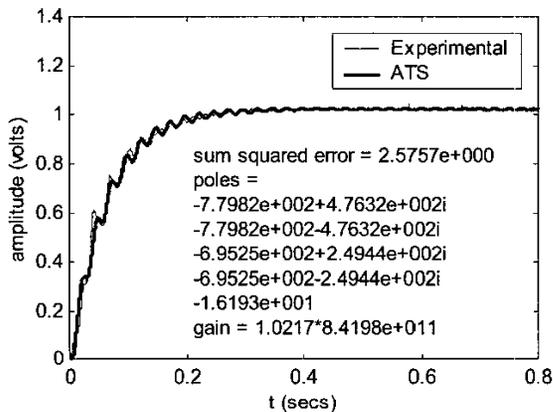


Fig.10 Plot of the linear model for the two-mass rotary

The above transfer function is accurate for a specific operating point. Based on this transfer function, some feedback and feed-forward designs were made for the system to improve its dynamic performance. In order to extend the useful range of the operation, we may economically drive the system beyond the operating point. In this regard, the system becomes nonlinear because of the protective mechanisms of drive amplifier and compensators for eliminating the jiggling effect caused by torsional resonance. The extended system is dominated by a saturation characteristic as shown in Fig. 11 in which  $m$ =slope,  $x_u$ =positive saturation starting point, and  $x_l$ =negative saturation starting point.

This saturation characteristic appears in the feedback loop of the compensated system with an operating point

extension. Obviously, it is impossible to obtain the parameters using any conventional identification method. The ATS method, however, is capable of finding such solutions and results in  $m=1.1447$ ,  $x_u=2.7590$ , and  $x_l=1.9159$ . Fig. 12 depicts the extended nonlinear model plotted against the observed data.

In this case, the ATS method yields very satisfactory results for both linear and nonlinear models.

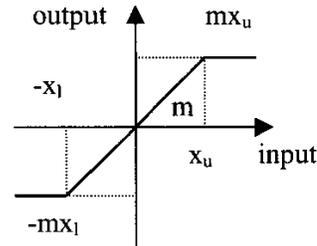


Fig.11 Saturation characteristic

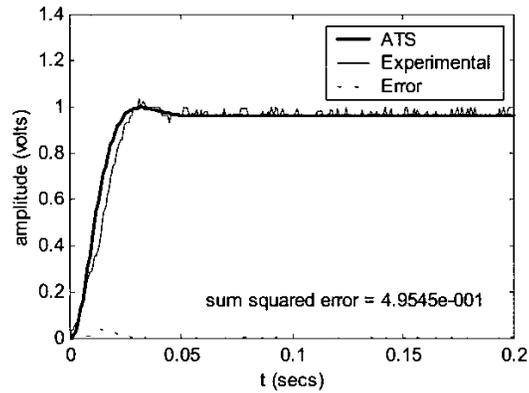


Fig.12 Plot of the nonlinear model of the system with torsional resonance (compensated) against observed speed

#### Case 4: Inverted pendulum on cart

The diagram in Fig. 13 represents an inverted pendulum on a cart [14]. With small angles of oscillation, the system can be considered linear and represented by the following 10<sup>th</sup>-order transfer function.

$$G(s) = \frac{N(s)}{D(s)}$$

$$N(s) = z_{10}s^{10} + z_9s^9 + z_8s^8 + z_7s^7 + z_6s^6 + z_5s^5 + z_4s^4 + z_3s^3 + z_2s^2 + z_1s + z_0$$

$$D(s) = p_{10}s^{10} + p_9s^9 + p_8s^8 + p_7s^7 + p_6s^6 + p_5s^5 + p_4s^4 + p_3s^3 + p_2s^2 + p_1s + p_0$$
(5)

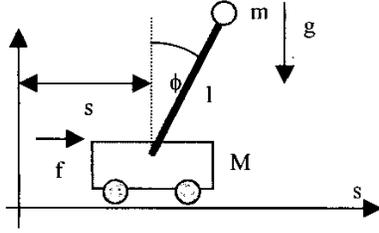


Fig.13 Inverted pendulum on cart

We have applied both the classical ARX method and the ATS method for comparison of their identification performance. The ARX model composes of

$$\begin{aligned}
 N(s) &= 0.003737s^{10} + 49.04s^9 - 594.3s^8 \\
 &\quad + 1.64 \times 10^4 s^7 - 4.79 \times 10^5 s^6 \\
 &\quad + 2.62 \times 10^6 s^5 - 4.24 \times 10^7 s^4 \\
 &\quad + 5.56 \times 10^8 s^3 + 3.72 \times 10^8 s^2 \\
 &\quad + 1.99 \times 10^{10} s - 1.07 \times 10^{10} \\
 D(s) &= s^{10} + 171.5s^9 + 1.12 \times 10^4 s^8 \\
 &\quad + 3.92 \times 10^5 s^7 + 8.34 \times 10^6 s^6 \\
 &\quad + 1.14 \times 10^8 s^5 + 1.02 \times 10^9 s^4 \\
 &\quad + 5.96 \times 10^9 s^3 + 2.29 \times 10^{10} s^2 \\
 &\quad + 5.87 \times 10^{10} s + 8.72 \times 10^{10}.
 \end{aligned} \tag{6}$$

The ATS method results in

$$\begin{aligned}
 N(s) &= -0.004152s^{10} + 35.06s^9 - 3719s^8 \\
 &\quad + 3.14 \times 10^4 s^7 - 2.53 \times 10^6 s^6 \\
 &\quad + 1.48 \times 10^6 s^5 - 2.90 \times 10^8 s^4 \\
 &\quad + 1.17 \times 10^9 s^3 + 4.90 \times 10^8 s^2 \\
 &\quad + 1.61 \times 10^{10} s - 2.99 \times 10^{10} \\
 D(s) &= 0.0851s^{10} + 152.5s^9 + 12145s^8 \\
 &\quad + 971670s^7 + 1.69 \times 10^7 s^6 \\
 &\quad + 3.22 \times 10^8 s^5 + 3.61 \times 10^9 s^4 \\
 &\quad + 1.31 \times 10^{10} s^3 + 6.26 \times 10^{10} s^2 \\
 &\quad + 9.16 \times 10^{10} s + 1.52 \times 10^{11}.
 \end{aligned} \tag{7}$$

The observed angles of oscillation, and the plots of the 10<sup>th</sup>-order models of both methods are depicted in Fig. 14. In the figure, the vertical axis indicates the angles (in radians) of oscillation while the horizontal axis indicates the n<sup>th</sup>-datum point of a specific block of observed data for model validation. The sum squared errors of the ARX model and the ATS method are 7.7966 and 6.2859, respectively.

Because the 10<sup>th</sup>-order model is unlikely practical, we have applied both the classical ARX and the ATS methods again on the basis of a 5<sup>th</sup>-order transfer function and then compare their identification

performance. The 5<sup>th</sup>-order ARX and ATS models are shown as follows:

The ARX 5<sup>th</sup>-order model is

$$\begin{aligned}
 N(s) &= 0.06051s^5 + 0.1136s^4 - 46.39s^3 \\
 &\quad - 45.34s^2 + 6335s + 4470 \\
 D(s) &= s^5 + 14.24s^4 + 1534s^3 + 5360s^2 \\
 &\quad + 2.58 \times 10^4 s + 6.02 \times 10^4.
 \end{aligned} \tag{8}$$

The ATS 5<sup>th</sup>-order model is

$$\begin{aligned}
 N(s) &= 0.00257s^5 + 23.36s^4 - 22.86s^3 \\
 &\quad - 45.27s^2 + 1.25 \times 10^4 s + 1053 \\
 D(s) &= 5.91s^5 + 35.06s^4 + 2579s^3 \\
 &\quad + 1.05 \times 10^4 s^2 + 3.78 \times 10^4 s \\
 &\quad + 8.53 \times 10^4.
 \end{aligned} \tag{9}$$

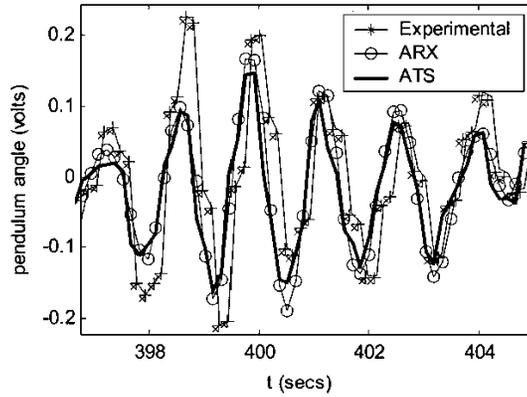


Fig.14 Angles of oscillation of the pendulum (observed data and the 10<sup>th</sup>-order models plotted)

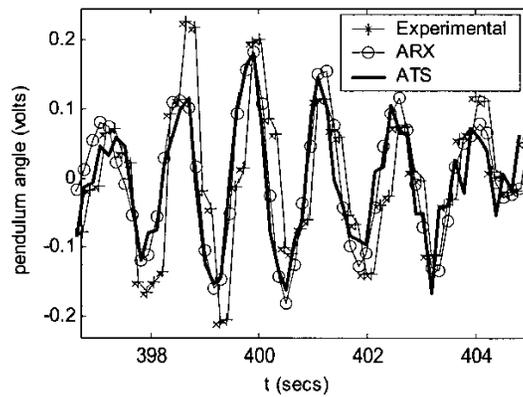


Fig.15 Angles of oscillation of the pendulum (observed data and the 5<sup>th</sup>-order models plotted)

Fig. 15 illustrates comparison plots of the observed data, and the reduced order models. The explanation of the axes in Fig. 14 is still applied. The results show that the ARX model yields the sum squared error of 10.1412, while the ATS-based model gives 7.7107. It is clear that our ATS method provides superior results.

#### 4. Conclusions

We have presented the novel adaptive tabu search (ATS) method and its application to system identification in this article so far. While the conventional TS is reviewed, the adaptive mechanisms, namely back-tracking and adaptive search radius, are highlighted. The article also presents some case studies including die-swell model, hot-air tube, system with torsional resonance, static nonlinear characteristic, and inverted pendulum on cart. Besides its less complicated structure than many other conventional methods, the ATS method has shown very superior results to all cases even when it is compared with the ARX models of the inverted pendulum system. Moreover, to apply the method is so simple and requires no special arrangement of the model forms. The only thing that it consumes is computing time. The ATS method is then proved to be one of the most efficient tools for the system identification.

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