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Optics Communications 240 (2004) 253–260

OPTICS COMMUNICATIONS

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Improved holographic particle sizing by using absolute values of the wavelet transform

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Received 24 January 2004; received in revised form 15 June 2004; accepted 16 June 2004

Abstract

A new method for sizing particle from in-line particle holograms by using absolute values of the wavelet transform is proposed in order to improve accuracy in measurements. The proposed method provides direct calculation of the particle size by using spatial frequency information of a chirp signal at minima position of an envelope function. Simulation and experimental results are presented.

PACS: 42.40; 42.30
Keywords: Particle in-line holography; Fraunhofer holography; Particle sizing; Wavelet transform; Digital analysis

1. Introduction

In-line Fraunhofer holography has been found to be very useful for sizing small particles [1,2]. In in-line particle holography, opaque or semi-transparent particles are illuminated by a collimated coherent light. By recording an interference pattern produced between light waves diffracted from the particles and the light wave transmitted directly on a photographic film, a hologram of the particles is generated. The interference pattern in the hologram contains information about both the three-dimensional (3-D) spatial position and the size of the particles which are encoded as a chirp signal and an envelope function, respectively. In a conventional analyzing method, this information is extracted by illuminating the developed hologram with the coherent light. The light transmitted through the hologram reconstructs images of the particles at the positions with the same distances as the recording distances. Since, in general, these distances are not known in advance, the image planes with the best focus for the particles must

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doi:10.1016/j.optcom.2004.06.036

be investigated by scanning the overall depth along an optical axis with fine steps. Although, this method allows us to freeze moving particles and to analyze them later, in real applications we may deal with a huge number of particles. As a consequence, the conventional reconstruction process is very tedious and time consuming. Therefore, it is not suitable for the human-operator based analysis which is non-repeatable and inaccurate.

To solve this problem, automatic all-optical analysis of particle holograms by using a wavelet transform (WT)-based correlator was proposed by Widjaja [3]. In the method, the WT is used to enhance edge features of both the images of particles reconstructed from the hologram and the image of a reference particle. By correlating these two edge-enhanced images, the position and the size of particles are determined. Although the method is indeed useful for analyzing irregularly shaped particles, the problem in the method is that the optical system becomes complicated.

On the other hand, instead of using the optical reconstruction, alternative methods for particle sizing and tracking by direct analysis of the interference pattern in the hologram were reported [4–7]. By establishing a mathematical relationship between the far-field number given by $N=\lambda z/(2a)^2$ (refer to the definition of λ, z and a in Eq. (1)) [4] and the density and diameter of interference fringes, Murakami reported direct measurements of the diameter and position of particles from the developed in-line holograms. However, this method is applicable for a small value of the far-field number which corresponds to a very big diameter of particles or a very short recording distance. As for a large far-field number, the density of fringes does not vary significantly. This leads to an inaccuracy of the method. Use of the WT for extracting the position of particles from the in-line holograms was proposed by Buraga-Lefebvre et al. [5] and Soontaranon et al. [7]. In the method of Buraga-Lefebvre et al., a diffraction process is regarded as a wavelet transformation with a spherical wave for the wavelet and an axial distance of the wave propagation for its dilation (scale change). To determine the position of particles, the digital holograms are wavelet transformed by using a spherical wave-based analyzing wavelet. The position of particles is obtained if the resultant WT gives a maximum value. In fact, this approach is equivalent to searching the in-focus image plane of particles reconstructed from the hologram. However, since the dilation factor is determined by the axial recording distance, this method is useful only for the short axial distance. For the longer distance, the dilation increases. As a result, the admissibility condition of the wavelet is so violated that this method becomes invalid. An algorithm for extracting 3-D location and image reconstruction of object obtained by optical scanning holography was proposed by Kim et al. [6]. In optical scanning holography, the hologram is generated by scanning optically the object with a time-dependent Fresnel zone plate produced by a superposition of a plane wave and a temporal-frequency-shifted spherical wave. The optical scanning is done by using a 2-D scanning mirror, while the temporal frequency shift is achieved by using an acousto-optic modulator. The total intensity reflected from the object which is proportional to the correlation between the time-dependent Fresnel zone plate is detected by an area-integrating photodetector. After preprocessing electrically an output current of the photodetector, a Wigner distribution function is used for extracting the depth location of the object. The image of the object is then reconstructed by convolving the hologram with a free-space impulse response at the measured depth location. However, unlike a wavelet transform, the Wigner distribution function is not a multiresolution signal representation, because the window function is the analyzed signal itself. Since the resolution is fixed, the accuracy in measurement of the depth location may not be optimized.

Our previous work [7] is based on a signal processing approach applied directly to digitally-recorded in-line holograms. Our proposed method excels in the point that it obviates the need for searching all depth planes and has a larger far-field number. The WT is employed to determine a space-varying frequency of the transmittance of the hologram. Since this spatial frequency corresponds to the inverse of the recording distance, the position of the particle with respect to the recording plane can be measured. In the computation process, a Morlet wavelet is used as an analyzing wavelet with the dilation factor which is not determined by the axial distance. Furthermore, the size of particles can be calculated by using a combination of the resultant recording distance and the position of the minima of the envelope function which are determined in its reconstruction process through determination of maximum and minimum amplitudes of the transmittance of the holograms.

However, the results in our study show that errors in measurements of the particle size are mainly determined by the accuracy in detecting the positions of the minima from the envelope. This is due to the fact that the amplitude of the chirp signal around the minima is very small. The measured maximum and minimum amplitudes cannot be used for determining the exact position of the minima.

In order to improve the accuracy in measurements of the size of the particles, a novel method for sizing the particles by using the absolute value of the WT is proposed. This interest stems from the fact that the absolute value of the WT extracts the positions of the minima of the envelope function. In the WT domain, these minima are functions of the dilation and the spatial translation. The frequencies of the fringes at the minima are directly obtained by using values of the dilation of the wavelet at the corresponding minima. Since the particle size is inversely proportional to these frequencies, the particle size can be accurately measured.

2. In-line particle hologram

An amplitude transmittance of the in-line Fraunhofer hologram of a small spherical particle with a radius of a can be mathematically expressed as [2]

$$I(r) = 1 - \frac{2\pi a^2}{\lambda z}\sin\left(\frac{\pi r^2}{\lambda z}\right)\left[\frac{2J_1\left(\frac{2\pi ar}{\lambda z}\right)}{\frac{2\pi ar}{\lambda z}}\right] + \frac{\pi^2 a^4}{\lambda^2 z^2}\left[\frac{2J_1\left(\frac{2\pi ar}{\lambda z}\right)}{\frac{2\pi ar}{\lambda z}}\right]^2, \quad (1)$$

where λ and z are the wavelength of the illuminating light and the distance between the particle and the recording plane, respectively. J_1 denotes the first-order Bessel function while r represents the radius coordinate in the hologram plane. The first term of Eq. (1) corresponds to the light transmitted directly. The second term corresponding to a modulation of the chirp signal by an Airy function encodes the recording distance z into the frequency of the chirp signal, while the particle size a is encoded in the Airy function. The third term is a square of the Airy function whose amplitude is much smaller than the other terms [8].

The second term of Eq. (1) shows that the frequency of the chirp signal is $r/\lambda z$, while the minimum positions of the interference pattern corresponding to the zero-crossing positions of the Bessel function appear at $2\pi ar/\lambda z = 3.83$, 7.02, 10.17,... [9]. Thus, the frequencies of the chirp signal at the positions of the first three minima are found to be $3.83/2\pi a$, $7.02/2\pi a$, and $10.17/2\pi a$, respectively. By taking this into consideration, the size of particles can be calculated, provided that the spatial frequencies of the chirp signal at the minimum positions are known. In this work, the frequencies of the chirp signal at the minima are determined by using the absolute values of the WT.

3. Method

In this work, the interference pattern of a hologram of the particles being studied is captured by a CCD sensor and stored into a frame memory of the computer. The captured pattern or its 1-D cross-sectional profile is digitally wavelet transformed. By taking the absolute values of the resultant WT output, the frequencies of the chirp signal at the minima of the envelope function are determined. The size of the particles is finally calculated from the relationship of these frequencies with the positions of the minima. All digital computations were conducted by using the Matlab 6.1.

3.1. Wavelet transform

The WT is a useful mathematical method for representing simultaneously time-frequency information of signals. This signal representation has been introduced in signal analysis to overcome the inability of Fourier analysis in providing local frequency spectra. The WT of a signal pattern $s(r)$ is defined as [10,11]

$$W(t,d) = \frac{1}{\sqrt{d}} \int_{-\infty}^{\infty} g^*\left(\frac{r-t}{d}\right) s(r)\,\mathrm{d}r, \tag{2}$$

where d and t are the dilation and the translation (shift) parameters, respectively. Eq. (2) can be considered as a cross correlation between the signal $s(r)$ and the dilated (scaled) wavelet $g(r/d)$. The WT is computed by dilating and translating the analyzing wavelet $g(r)$ into a set of functions having different frequency responses. The frequency is inversely proportional to the dilation. By varying the dilation factor, the WT provides a multi-resolution decomposition of the signal in such a way that it gives a good spatial resolution at the high frequency and a good frequency resolution at the low frequency. When the frequency content f of the signal $s(r)$ in the region subtended by $g^*[(r-t)/d]$ matches the center frequency of the dilated wavelet $g(r/d)$, a correlation peak is generated along the dilation d at the position t where the frequency f occurs. This peak is found to be proportional to the amplitude of the signal $s(r)$.

3.2. Absolute value of wavelet transform

In the general case of signals that $s(r)$ is an amplitude-modulated signal, the amplitudes of the resultant correlation peaks are determined by the amplitude of the modulating function. Therefore, besides its excellent ability of providing the time-frequency information simultaneously, the WT extracts the modulating information of the signals being analyzed. A further insight into this property may be gained by analyzing mathematically the modulated signal with the use of the WT.

For the sake of simplicity, we consider that a modulated signal is given by

$$s(x) = \cos(2\pi f_1 x)\cos(2\pi f_0 x), \tag{3}$$

where the first cosine term corresponds to the carrier signal while the second one is the modulating signal. Here, the carrier frequency f_1 is greater than the modulating frequency f_0. By using a Morlet wavelet given by [11]

$$g(x) = \exp\left(\mathrm{i}2\pi f_g x\right)\exp\left(-x^2/2\right), \tag{4}$$

as an analyzing wavelet, the WT of the signal $s(x)$ is found to be

$$\begin{aligned} W(t,d) = \sqrt{\frac{\pi d}{8}}\Big\{ &\exp\left[-\mathrm{i}2\pi(f_0+f_1)t - 2\pi^2 d^2\left(f_g/d+f_0+f_1\right)^2\right] \\ &+\exp\left[-\mathrm{i}2\pi(-f_0+f_1)t - 2\pi^2 d^2\left(f_g/d-f_0+f_1\right)^2\right] \\ &+\exp\left[-\mathrm{i}2\pi(f_0-f_1)t - 2\pi^2 d^2\left(f_g/d+f_0-f_1\right)^2\right] \\ &+\exp\left[-\mathrm{i}2\pi(-f_0-f_1)t - 2\pi^2 d^2\left(f_g/d-f_0-f_1\right)^2\right]\Big\}, \end{aligned} \tag{5}$$

where f_g denotes the frequency of the wavelet. Since the first and second terms of Eq. (5) are much smaller than the other terms, the first two terms can be neglected. Therefore, Eq. (5) may be approximated as

$$\begin{aligned} W(t,d) = \sqrt{\frac{\pi d}{8}}\Big\{ &\exp\left[\mathrm{i}2\pi(f_1-f_0)t - 2\pi^2 d^2\left(f_g/d+f_0-f_1\right)^2\right] \\ &+\exp\left[\mathrm{i}2\pi(f_1+f_0)t - 2\pi^2 d^2\left(f_g/d-f_0-f_1\right)^2\right]\Big\}. \end{aligned} \tag{6}$$

The absolute value of the WT given by Eq. (6) results in

$$\begin{aligned} |W(t,d)| = \sqrt{\frac{\pi d}{8}}\Bigg\{ &\exp\left\{-4\pi^2(f_0-f_1)^2\left[d-\frac{f_g}{f_1-f_0}\right]^2\right\} \\ &+\exp\left\{-4\pi^2(f_0+f_1)^2\left[d-\frac{f_g}{f_1+f_0}\right]^2\right\} \\ &+2\cos(4\pi f_0 t)\exp\Bigg\{-2\pi^2(f_0-f_1)^2 \\ &\times\left[d-\frac{f_g}{f_1-f_0}\right]^2 - 2\pi^2(f_0+f_1)^2\left[d-\frac{f_g}{f_1+f_0}\right]^2\Bigg\}\Bigg\}^{\frac{1}{2}}. \end{aligned} \tag{7}$$

Eq. (7) shows that the first two terms are a summation of two Gaussian functions, while the third term is a multiplication of the cosine function corresponding to the modulating function with a product of two other Gaussian functions. The first and third Gaussian functions have the same mean values, while the means of the second and fourth Gaussians are the same. Since $f_1 \gg f_0$ the difference between the values of the two means is not significant as well as the variances. As a result, either the summation or the multiplication of the two Gaussian functions produces a Gaussian-like function whose maximum peak appears at the same position of the average value of the two means, $d=f_g/f_1$. This can be mathematically verified by substituting this average value into the first and second derivatives of the summation and the multiplication of the Gaussian functions. The first derivative test yields zero for the critical value $d=f_g/f_1$, while the second derivative test gives a negative result. These tests verify that the peak position of the Gaussian-like function is at $d=f_g/f_1$. This position is regarded as the one where the frequency of the dilated wavelet f_g/d matches the frequency of the modulating signal f_1 Therefore, the cosine function in the third term of Eq. (7) is confined by the Gaussian-like function centered at the dilation $d=f_g/f_1$.

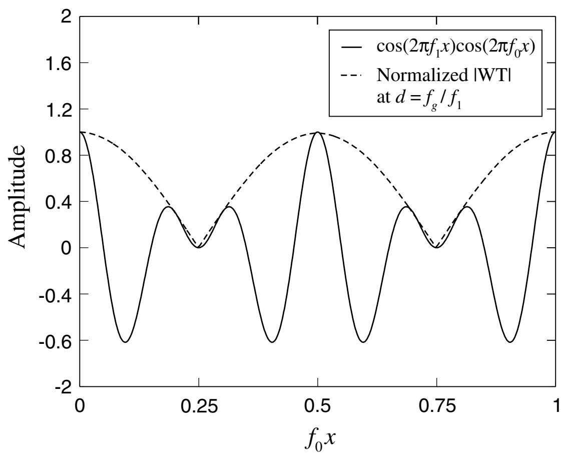

Fig. 1. Modulated signal $s(x)$ of Eq. (3) and the normalized absolute value of its WT given by Eq. (8).

When the dilation $d=f_g/f_1$ is achieved, Eq. (7) reduces to

$$|W(t,d)|_{d=\frac{f_g}{f_1}} = \sqrt{\frac{\pi f_g}{2f_1}} \exp\left[\frac{-2\pi^2 f_g^2 f_0^2}{f_1^2}\right] |\cos(2\pi f_0 t)|. \tag{8}$$

Eq. (8) confirms that the absolute value of the WT gives the information about the modulating signal with its absolute value $|\cos(2\pi f_0 t)|$. Fig. 1 shows the modulated signal $s(x)$ of Eq. (3) and the normalized absolute value of its WT given by Eq. (8), which are represented by the solid and broken lines, respectively. Here, the carrier frequency f_1 is five times higher than the modulating frequency f_0. It is clear from the figure that the minima of the absolute value of the resultant WT output coincide with the zero-crossing points of the carrier signal of Eq. (3).

4. Results and discussions

In a preliminary verification, the in-line hologram of an optical fiber with the diameter of 125 μm was simulated under illumination of the coherent light operating at the wavelength of 543.5 nm. For the line object, the envelope function of the interference pattern is a sinc function with the same argument as the Airy function of Eq. (1) [2]. The frequency of the interference fringe at the minimum position becomes $n/2a$, where n represents the order of the minima. Fig. 2(a) shows the simulated in-line hologram of the optical fiber and the absolute value of its WT which is obtained by retrieving the amplitude of the resultant absolute value of the WT along the dilation $d=f_g/f_1$. In the case of in-line holograms, f_1 stands for the frequency of the chirp signal determined by $r/\lambda z$. As a function of dilation $2a/d$, Fig. 2(b) illustrates a 3-D plot for the absolute value of the WT of the fringe which is cut along the dilation $d=f_g/f_1$ represented by the dash line. In the WT domain, the path of the dilation is nonlinear, because it is inversely proportional to the space-varying frequency f_1 of the chirp signal. Evidently, the minima of the absolute value of the resultant WT output appear at the correct zero-crossing points of the chirp

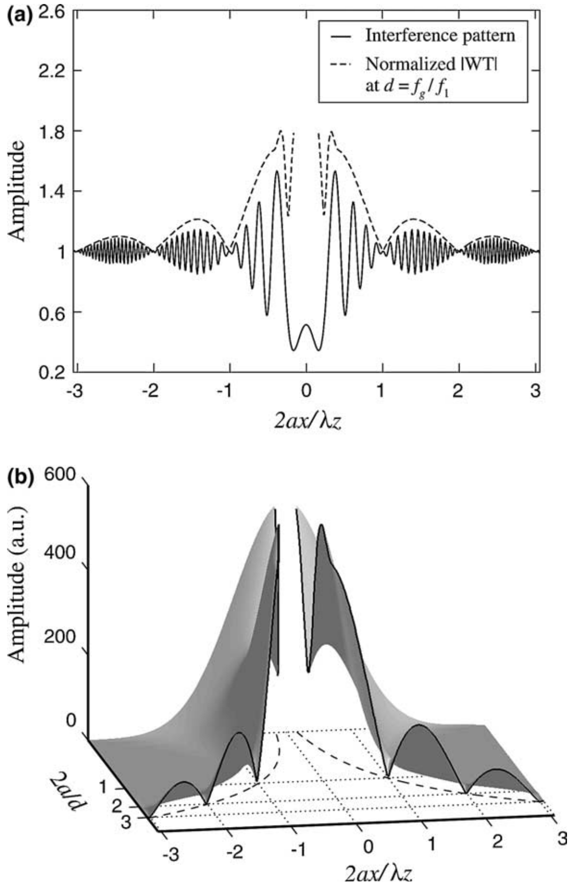

Fig. 2. (a) Simulated in-line hologram of the optical fiber and the normalized absolute value of its WT which is retrieved along the dilation $d=f_g/f_1$ and (b) the 3-D plot of the absolute value of the WT which is cut along the dilation $d=f_g/f_1$.

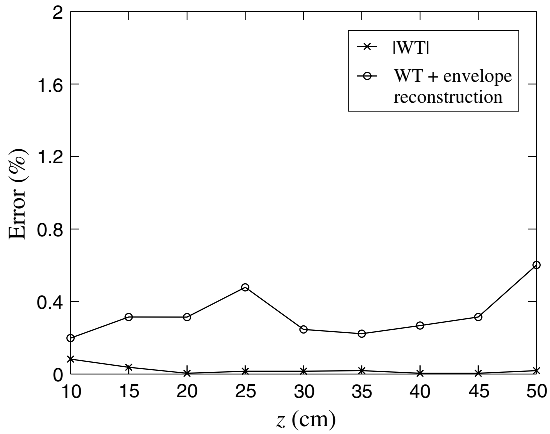

Fig. 3. Errors in measurements of the fiber size from the simulated holograms by using the envelope reconstruction method and the absolute value of WT.

signal. By determining the frequencies of the chirp function at these minima, the fiber size can then be calculated.

Fig. 3 shows the errors in measurements of the diameter by using our previous and proposed methods for given values of the recording distance z from 10 to 50 cm. The errors in the envelope reconstruction method used previously is represented by the circle sign, while the errors in the proposed current method is shown as the cross sign. It is obvious from the figure that, in comparison with the previous method, the errors in measurements using the proposed method are significantly reduced. The reason of this reduction is that the proposed method employs only the frequency of the chirp signal at the minimum position of the envelope function for evaluating the object size. Since the information of the recording distance is not required, its measurement errors do not affect the measurements in the size of the object.

Next, feasibility of the proposed method was experimentally verified by generating optically the in-line holograms of the optical fiber. The col-

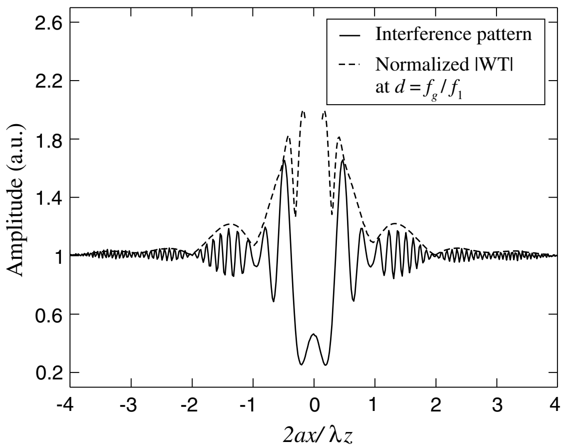

Fig. 4. Experimentally generated in-line hologram of the optical fiber and the normalized absolute of its WT which is retrieved along the dilation $d=f_g/f_1$.

limated coherent light was obtained from a He–Ne laser with the wavelength of 543.5 nm. The generated hologram was recorded by using a CCD camera HAMAMATSU C5948 with the resolution of 640×480 pixels in the area of 8.3×6.3 mm. Fig. 4 shows the digitized in-line hologram of the optical fiber recorded at the distance $z = 15$ cm and the normalized absolute value of its WT. It is obvious from the figure that, since the interference fringes have not been faithfully sampled, the resultant absolute value of its WT does not appear exactly at the zero-crossing points of the chirp signal. This may be caused by the limited spatial-resolution of the CCD combined with the speckle noise. In this figure, the information about the absolute value of the envelope function is also obtained by retrieving the amplitude value of the resultant output along the dilation $d=f_g/f_1$. Note that, in our previous work [7], the dependencies of the measurable recording distance and the object size on the aperture and the resolution of the CCD were discussed.

Fig. 5 shows a comparison of the errors in measurements of the diameter of the optical fiber, where the circle and cross signs correspond to the envelope reconstruction method and the proposed absolute value of the WT, respectively. Although the errors in experimental measurements using the proposed method are higher than the simulation results in Fig. 3, the errors of measurement have been reduced to less than 1% which are smaller than those of the envelope reconstruction method. Therefore, the accuracy in measurements of the object size has been improved.

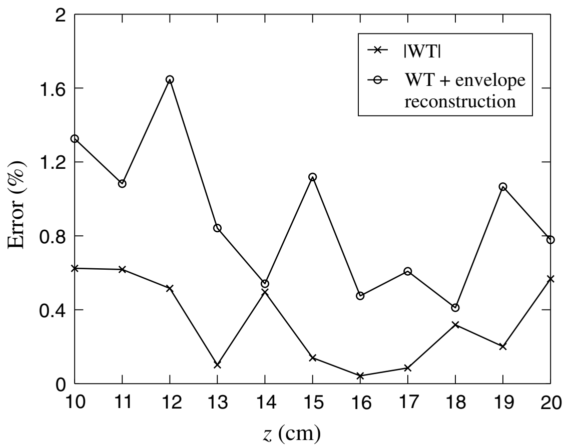

Fig. 5. Errors in measurements of the fiber size from the experimentally generated holograms by using the envelope reconstruction method and the absolute value of the WT.

5. Conclusions

We have proposed and verified experimentally a novel method for sizing the particles from the in-line holograms by using the absolute value of the WT. The proposed method takes an advantage of the property of the WT whose absolute value could extract the positions of the minima of the envelope function. Since the resultant extracted minima are functions of the dilation of the wavelet, the frequencies of the fringes at the minima of the envelope function are measured. The size of the particles can be finally calculated from the relationship between the frequencies with the positions of the minima. Unlike the previous method, the information of the recording distance is not used for determining the object size and, therefore, the errors in measurements do not affect the accuracy of the proposed method. The experimental results agree well with the theory.

Acknowledgement

The financial support from the Thailand Research Fund through the Royal Golden Jubilee Ph.D. Program (Grant No. PHD/0188/2542) is acknowledged.

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