

# THE NEUTRINO IN ELEMENTARY PARTICLE PHYSICS - WHAT'S NEW ?

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## Abstract

We discuss the role of the neutrino in nature and how it is described within the generally accepted theory of the strong, weak and electromagnetic forces, the Standard Model. The Standard Model in its present form regards the neutrino as a massless particle with half-integer spin and no charge. This view is challenged by recent evidence for the phenomenon of neutrino oscillations, both in vacuum and in matter. If confirmed by independent experiments this would require that at least one of the neutrinos has a non-vanishing (albeit very small) mass. The different oscillation models presently discussed could shed new light on the observed solar neutrino deficit and on preliminary findings of a non-vanishing neutrino mass at recent accelerator based and underground neutrino experiments. While challenging the present form of the Standard Model a neutrino mass can be implemented in a mildly revised form of this model.

## Introduction

Stable matter consists of electrons, protons, bound neutrons, and photons. The presence of the *weak force* in nature gives rise to nuclear  $\beta$ -decay. If the neutron is "liberated" from the nucleus through a fission process (as one example) and becomes a free neutron ( $n$ ) it decays within 15 minutes into an electron ( $e^-$ ), a proton ( $p$ ), and an anti-electron neutrino ( $\bar{\nu}_e$ ), or symbolically. Hence the weak force adds another particle to our world, the  $\nu_e$ . All forces in nature (except for gravity) can be explained by the generally accepted Standard Model (SM). The SM has six fermions as fundamental particles, see Fig.1, and force-mediating "gauge" bosons, the photon ( $\gamma$ ), the  $W^\pm$ ,  $Z^0$  bosons, and the unobservable eight gluons ( $G_a$  with  $a=1,2, \dots 8$ ).

Where do the observed masses come from? The mass giving mechanism is known as the *Higgs effect* and postulates the existence of the *Higgs boson*. There has been recent evidence for the Higgs boson with rest mass around  $114 \text{ GeV}/c^2$  [1]. While one of the corner stones of the SM seems to have been established, there appears to have been the first glimpse at *Physics beyond the SM* in the form of a tiny, but crucial, deviation from the expected value of the muon's anomalous magnetic moment [2].

Neutrinos were postulated by Pauli in 1930 [3] to properly explain the  $\beta$ -decay of the free neutron<sup>3</sup>  $n \rightarrow p + e^- + \bar{\nu}_e$  without violating energy-momentum conservation. This 3-body decay allows the emerging electron to have a variety

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<sup>3</sup> The neutron is heavier than the proton (see rest mass in  $\text{MeV}/c^2$  in brackets, data taken from ref.[4]).

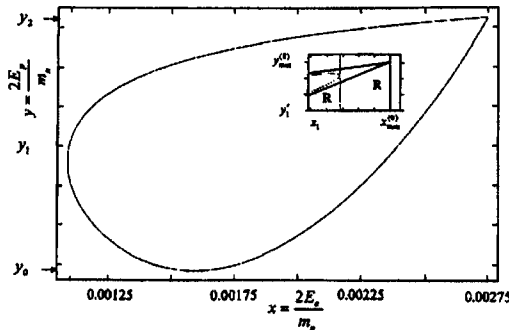
$Q/ e $			
0	$\begin{pmatrix} \nu_e' \\ e^- \end{pmatrix}$	$\begin{pmatrix} \nu_\mu' \\ \mu^- \end{pmatrix}$	$\begin{pmatrix} \nu_\tau' \\ \tau^- \end{pmatrix}$
-1	$\begin{pmatrix} u \\ d' \end{pmatrix}$	$\begin{pmatrix} c \\ s' \end{pmatrix}$	$\begin{pmatrix} t \\ b' \end{pmatrix}$
$+\frac{2}{3}$			
$-\frac{1}{3}$			

**Figure 1.** The three Fermion generations in the Standard Model.  $d', s', b'$  are mixed (GIM mechanism).  $\nu_e', \nu_\mu', \nu_\tau'$  are mixed ( $\nu$ -oscillations).

of energies, ranging from its rest mass (i.e. kinetic energy = 0) to a maximal energy  $E_e^{\max} = 1.29$  MeV. Soon after its experimental verification it was realized that  $n \rightarrow p + e^- + \bar{\nu}_e$  is the fundamental process behind the  $\beta$ -decay of nuclei<sup>4</sup>  ${}_Z^A X \rightarrow {}_{Z+1}^A X' + e^- + \bar{\nu}_e$ . Since about 1964 we know that the fundamental process behind both is the weak decay of the slightly heavier down quark  $d \rightarrow u + e^- + \bar{\nu}_e$ , via the exchange of a virtual  $W^-$  (80419), [5-7]. The  $\nu_e$  is a member of the first of three Fermion generations in the Standard Model (SM), (Fig.1)

Consequently the first source of neutrinos was identified after the Second World War in the form of the newly built nuclear reactors with their high flux of neutrons. Reines and Cowan [8] demonstrated in 1953 that indeed there are (anti) neutrinos "behind a reactor". Man-made neutrinos had been discovered soon after naturally occurring neutrinos were discovered in 1965 by Reines and Sellschop [9] in the deepest mine then (near Johannesburg, South Africa) with the biggest detector at that time [5, 9].

Naturally occurring neutrinos come from two principal sources: (I) atmospheric neutrinos, and (II) solar neutrinos. The former (I) are produced by cosmic ray inducing cascades  $\pi(K) \rightarrow \mu \rightarrow e$  in the upper atmosphere, while the latter (II) are by-products of nuclear burning in the sun ("p-p" chain and "CNO" cycle, see below). Of course the experimental detection of neutrinos depends crucially on their expected rest mass. The  $\beta$ -decay is a 3-body decay and as such has a continuous energy spectrum for the charged lepton in the final state (initially only decays involving the electron  $e(0.51)$  were considered, later also the  $\mu(105.6)$  and the  $\tau(1777)$  were produced as



**Figure 2.** Dalitz plot boundary for  $\beta$ -decay,  $n \rightarrow p + e^- + \bar{\nu}_e$ . The point R, at which the (massive) neutrino is at rest, is at the "intersection" of the upperboundary curve with the straight line  $y = 2(1 - r_p) - x$ ; note that the straight line is tangential to the upper boundary curve in R (see the almost vertical solid and dashed curves in theinset). The 3 points  $y_0, y_1, y_2$  are numerically very close together due to the near-degeneracy of the proton and neutron masses:  $y_0 = 1.997246960, y_1 = 1.997247760, y_2 = 1.997248560$ . The coordinates in the inset are  $x_1 = 0.002751430, x_{\max}^{(0)} = 0.002751440, y_1 = 1.997248559980, y_{\max}^{(0)} = 1.9972485600106$ . Solid curves are for a massless neutrino, while the dashed curves apply for a 3 eV neutrino. The maximal electron energies for the two cases 0 eV and 3 eV are almost exactly 3 eV apart. The inset is at the upper right-hand tip of the Dalitz plot and only then displays a difference between a massive (3 eV) and a massless neutrino.

<sup>4</sup>for example  $(X', X) = (Co, Ni), (B, Be), etc.$

final state leptons in  $\tau$  decays). Near the endpoint energy of the final state lepton ( $E_1^{\max} - m_e c^2 = 18.6 \text{ keV}$  for tritium  $\beta$ -decay,  $E_1^{\max} = 1.29 \text{ MeV}$  for  $\beta$ -decay of the free neutron) effects linear in the neutrino mass are observable, because the antineutrino/neutrino leaves the interaction region with minimal total energy (almost non-relativistic (Fig. 2) for an illustration). Quite generally, if one considers the Dalitz plot for a three-body decay  $M \rightarrow m_1 + m_2 + m_3$  in the  $E_1 - E_2$  plane ( $E_1$  is the energy of the final-state lepton of rest mass  $m_1$  and  $E_2$  and is the corresponding energy of the non-leptonic final-state particle of rest mass  $m_2$ )

$$y(x) = \frac{2(2-x)(a-x)}{4(1-x) + x \frac{\min}{\max}} \pm \sqrt{x_{\max} - x}$$

$$\sqrt{x - x_{\min}} \frac{\sqrt{(x + x_{\min})(b-x)}}{4(1-x) + x \frac{\min}{\max}}$$

with  $y = 2E_2/M$ ,  $x = 2E_1/M$ ,  $a = 1 - r_3^2 + r_1^2 + r_2^2$ ,  $b = 1 + r_1^2 - (r_2 - r_3)^2$ ,  $x_{\max} = 1 + r_1^2 - (r_2 + r_3)^2$ ,  $y_{\max} = 1 + r_2^2 - (r_1 + r_3)^2$ ,  $x_{\min} = 2r_1$ ,  $y_{\min} = 2r_2$ ,  $r_i = m_i/M$ , one notices that the upper right-hand corner (i.e.  $E_{1,2}$  both are near their maximal values) of the Dalitz plot boundary depends *linearly* on the antineutrino/neutrino mass ( $m_3$  here). This technique has been used to continuously lower the upper bound on the neutrino's rest mass. The upper limit  $m_{\nu_e} < 500 \text{ eV}/c^2$  set in 1949 by Hanna and Pontecorvo (from tritium  $\beta$ -decay) has come down considerably, to presently [4]  $m_{\nu_e} < 3 \text{ eV}/c^2$ . Other 3-body decays have been used over the years to lower the upper limits on  $m_{\nu_\mu}$  and  $m_{\nu_\tau}$ :  $\pi^- \rightarrow \mu^- \bar{\nu}_\mu \gamma$ ,  $\tau^- \rightarrow \pi^- \omega \nu_\tau$ ,  $\tau^- \rightarrow 5\pi \nu_\tau$  with the best present limits [4,10,11]:  $m_{\nu_\mu} < 0.19 \text{ MeV}/c^2$ ,  $m_{\nu_\tau} < 18.2 \text{ MeV}/c^2$ . The present limit for  $m_{\nu_e}$  of a few  $\text{eV}/c^2$  seems to be a natural barrier for such *kinematical* determinations of the neutrino's rest mass. Although it is highly unlikely that such experiments will lower  $m_{\nu_e}$  to, say  $10^{-3} \text{ eV}/c^2$ , more surprises can not be excluded. Recently two experiments (Mainz in Germany [12] and Moscow in Russia [13]) have reached

unprecedented levels of sensitivity in determining the number of electrons versus electron energy measured from  $\beta$ -decay of tritium. Both experiments see a decay anomaly (anomalous peak) between 5 and 15 eV below the maximum  $\beta$ -energy. While probably not directly related to  $m_\nu$ , this unexpected effect has been interpreted as pointing to neutrino relics [14].

All above mass limits refer to Dirac neutrinos, which are distinct from their anti-particles (but have the same Dirac-mass). Some electrically neutral particles like photons ( $\gamma$ ) and the  $\pi^0(140)$  are known to be identical with their own anti-particle<sup>5</sup>. Is such a phenomenon restricted to bosons? Neutral fermions comprise neutral leptons (neutrinos) and neutral baryons (neutron,  $\Lambda$ ,  $\Sigma^0 \dots$ ). All neutral baryons are different from their anti-particles. If an (uncharged) neutrino is, however, identical to its own (uncharged) anti-neutrino, different experimental techniques have to be used to detect it, for example, the neutrinoless double-beta decay ( $0\nu\beta\beta$ ) where a neutrino is emitted as a neutrino from one nucleon in a nucleus and re-absorbed as an anti-neutrino by another nucleon in the *same* nucleus  ${}^A_Z X \rightarrow {}^A_Z X' - 2e^-$

Present upper limits are [4, 15]  $m_{\nu_\mu} \leq$  several eV which may be lowered to [15]  $m_{\nu_\mu} = 0.04 \text{ eV}$  with the next generation of  $0\nu\beta\beta$  experiments. It is apparent that a completely new type of experiment is called for should neutrinos turn out to have rest masses even lower than  $10^{-2} \text{ eV}$ . There is growing evidence that neutrinos have indeed tiny masses and that *dynamical* neutrino mass effects are the signal to look for. According to Bruno Pontecorvo tiny neutrino mass *differences* may lead to neutrino oscillations which might be easier to detect than the rest-masses themselves, by detecting simultaneously a deficit of one neutrino flavour and an excess of the other neutrino flavour. Such experiments fall into two categories: (i) accelerator experiments, and (ii) underground cosmic ray experiments with directional sensitivity. In the next section 2 we give the basic principle of

<sup>5</sup> The neutral and flavoured mesons  $K^0, D^0, \dots$  are, however, different from their anti-particles, up to small CP-violation effects.

vacuum oscillations leading to disappearance and re-appearance probabilities as functions of neutrino energy, distance from source, and neutrino mass information. In section 3 we present the basic principle of matter oscillations and show how that could solve the solar neutrino deficit.

Before we turn to the oscillation phenomenon we mention that the choice of detector depends crucially on the anticipated energy of the antineutrinos/neutrinos. The first generation of chlorine detectors had a threshold of  $E_\nu = 0.814 \text{ MeV}$  (using the reaction  $^{37}_{17}\text{Cl} + \nu_e \rightarrow ^{37}_{18}\text{Ar} + e^-$  termed *inverse*  $\beta$ -decay, which excluded most of the neutrinos from the dominant p-p chain, in the sun). The next generation of Gallium detectors is capable of reaching down to 0.2332 MeV and will be able to have a fresh look at the solar neutrino deficit. Finally, detectors using *Cerenkov* effect in water have a threshold of  $E_\nu \geq 7 \text{ MeV}$  and will be mostly looking at the blue light cone emitted by high-energetic leptons, emerging from the reaction  $\nu_l + A \rightarrow A' + l$  ( $l = \mu^-$  mostly). The difficulty to identify  $\nu_\tau$  (as being distinct from  $\nu_\mu, \nu_e$ ) may be demonstrated by the fact that only in the year 2000 has the *DONUT* collaboration at *Fermi Lab* [16] experimentally verified that  $\nu_\tau \neq \nu_{\mu,e}$  by showing that  $\nu_\tau + A \rightarrow A' + \tau^-$  with no  $\mu^-$  and  $e^-$  in the final state<sup>6</sup>.

### Vacuum oscillations

The idea of neutrino oscillations goes back to Bruno Pontecorvo (1957, [17] for a review). This phenomenon occurs if the mass matrix is not diagonal in lepton flavour space ( $e^-, \mu^-, \tau^-$  neutrino flavours). In that case the mass eigenstates are different from the flavour eigenstates.

We consider here only two-flavour oscillations in order to bring the basic message across,

$$|\nu_e\rangle = \sum_{i=1}^2 U_{ei} |\nu_i\rangle \quad (2.1)$$

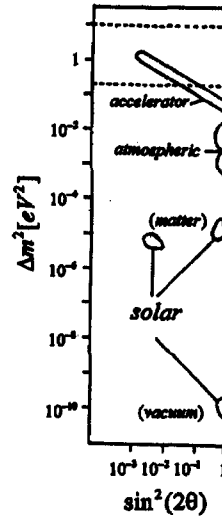


Figure 3. Schematic view of present preliminary experimental evidence for neutrino mass differences in the double-logarithmic  $\Delta m^2 (\text{eV}^2)$ -  $\sin^2(2\theta)$  plane. The dashed lines give limits from cosmological, kinematical considerations (*non-luminous matter in galaxies*) whereas "accelerator neutrinos" refers to the LSND collaboration based at the LANSCE facility in Los Alamos, "atmospheric neutrinos" refers to Kamiokande and Super-K experiments and "solar neutrinos" finally refers to the solar neutrino deficit, which hints at matter oscillations and vacuum oscillations as indicated, see text.

with  $|\nu_i\rangle$  different mass components (one could be zero or very close to zero), and

$$U = \begin{pmatrix} \cos\theta & \sin\theta e^{i\psi} \\ -\sin\theta e^{-i\psi} & \cos\theta \end{pmatrix} \quad (2.2)$$

As in ref. [13] we consider a  $\nu_e$  source at the origin which emits  $\nu_e$ 's of total energy  $E$ . One assumes  $m_i \ll E$  (for  $i = 1, 2$ ) and easily derives (see ref. [15]) the probability of finding a  $\nu_e$  at a distance  $r$  from the source (re-appearance mode)

$$P_{\nu_e \rightarrow \nu_e}(r) = 1 - \sin^2(2\theta) \sin^2\left(\frac{\pi r}{L}\right) \quad (2.3)$$

<sup>6</sup> Note that if oscillation lengths turn out to be  $L > 100 \text{ km}$ , the *DONUT* experiment is not sensitive to neutrino oscillations.

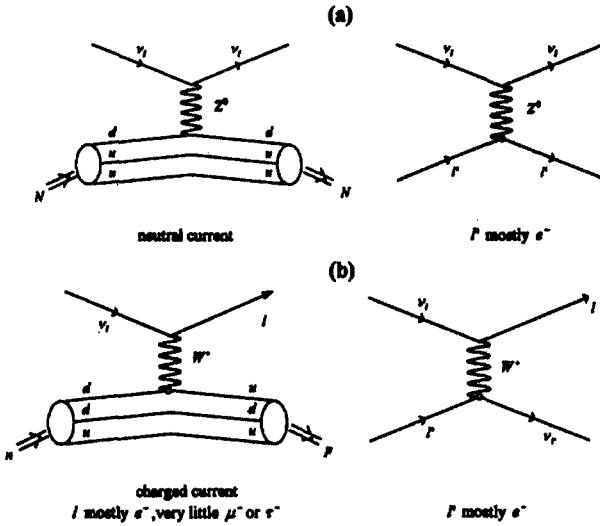


Figure 4. Feynman diagrams for (a) hadronic and leptonic neutral current scattering, (b) hadronic and leptonic charged current scattering processes, see text.

where  $L = \frac{4\pi E}{|m_1^2 - m_2^2|}$  is the oscillation length and  $p_{\nu_e \rightarrow \nu_\tau}(r) \equiv 1 - p_{\nu_\tau \rightarrow \nu_e}(r)$  describes the "disappearance mode". A more practical and widely used formula is

$$p_{\nu_e \rightarrow \nu_\tau}(r) = \sin^2(2\theta) \sin^2\left(1.27 \frac{\Delta m^2 (eV^2) L(m)}{E(MeV)}\right) \quad (2.4)$$

with  $\Delta m^2 \equiv |m_1^2 - m_2^2|$  The all-important oscillation length  $L$  depends on the neutrino energy  $E$  and on  $\Delta m_2$ ,

$$L = 2.5m \frac{E(MeV)}{\Delta m^2 (eV^2)} \quad (2.5)$$

The American Los Alamos group *LSND* (Liquid Scintillator Neutrino Detector, [18]) have reported evidence for  $\nu_e \leftrightarrow \nu_\mu$  neutrino oscillations and seem to favour  $\Delta m^2 \approx 1eV^2$  relatively large, but a small oscillation angle, (Fig. 3). *LSND* detects neutrinos emitted by the *LANSCE* accelerator with two detectors  $\approx 30m$  apart. According to eq. (2.5)  $L = \frac{60m}{2n+1}$  ( $n = 0,1,\dots$ ) should produce the largest  $\nu_\mu$  deficit for selected neutrino energies  $E_\nu \approx 24MeV, 8MeV, 5MeV, \dots$

More recently the Japanese *Super-Kamiokande* collaboration [19] have reported evidence for  $\nu_\mu \leftrightarrow \nu_\tau$  neutrino oscillations favouring a maximal mixing angle and much

smaller  $\Delta m^2 \approx (10^{-3} - 10^{-2})eV^2$ , (Fig. 2). This experiment compares neutrino fluxes for neutrinos which have traveled different distances  $r_1 \approx 30km$  and  $r_2 \approx 12000km$ . This time  $L \approx \frac{24000}{2n+1} km$  should produce the largest  $\nu_\mu$  deficit for neutrinos of energy  $E_\nu \approx 10GeV, 3GeV, \dots$  (For  $\Delta m^2 \approx 10^{-3} eV^2$  assumed). This simple exercise explains why long-baseline neutrino experiments have been proposed. The *K2K Long Baseline Neutrino Oscillation* experiment connecting the Japanese *KEK* Laboratory and its "near detector" with the detectors in the *Kamioka* mine 250 km way, has reported its first neutrino interactions. The *CERN-Gran Sasso* and *Fermilab-Soudan* projects will involve detectors  $\approx 800 km$  apart. While *vacuum* oscillations might have been seen for the first time only relatively recently, the first evidence for *matter* oscillations goes back nearly forty years when the solar neutrino deficit was announced by Ray Davies [20]. We turn to matter oscillations in the next chapter.

### Matter oscillations

The fusion of light nuclei to heavier, more strongly bound nuclei produces the energy radiated from the sun's surface. The fusion chain begins with hydrogen nuclei which, after several

steps, fuse into  ${}^4\text{He}$  (this "p-p" chain reaction is the dominant source of energy for the sun). For stars heavier than the sun the CNO (carbon-nitrogen-oxygen) cycle of nuclear fusion is the dominant source of energy generation. The CNO cycle results in the fusion of four protons into a  ${}^4\text{He}$  nucleus ( $4{}^1\text{H} \rightarrow {}^4\text{He} + 2e^+ + 2\nu_e + \text{energy}$ ), see ref. [20,21] for details. The extreme dense shell of the sun absorbs all of the charged leptons and all nucleons generated in this chain, and only the weakly interacting antineutrinos/neutrinos can penetrate the sun's shell. Thus, on earth we are able to detect the neutrinos that are produced in the fusion chain, but not the other particles. However, the reactions that create the most neutrinos,  $p + p \rightarrow {}^2\text{He} + e^+ + \nu_e$  and  ${}^7\text{Be} + e^- \rightarrow {}^7\text{Li} + \nu_e$ , have neutrino energies below 1 MeV and can not be detected with chlorine detectors, which since 1965 has seen less electron-neutrinos than predicted by the standard solar model [20,21]. All solar neutrino detectors (*Super-Kamiokande*, *SNO*, *Borexino* etc) have confirmed the Davis result [20]. Detectors using gallium ( $E_\nu \geq 0.2\text{MeV}$ ), chlorine ( $E_\nu \geq 0.8\text{MeV}$ ) and *Cerenkov* effect in water ( $E_\nu \geq 7\text{MeV}$ ) measure significantly lower neutrino rates than calculated by Bahcall in the standard solar model. The deficit in the solar neutrino flux compared with solar model calculations seems to hint at neutrino *matter* oscillations.

The solar neutrino deficit can not be explained by the vacuum oscillation phenomenon of the previous section. Here the *MSW* mechanism [22] comes into play. The surprising result will be that in matter  $\Delta m^2$  is much smaller than in vacuum ( $< 10^{-5} eV^2$ ), because the high density of the sun affects the neutrino flavour eigenstates as they emerge from the core. The sun is made up primarily of  $e$ ,  $p$  and  $n$  (no  $\mu$ 's or  $\tau$ 's) so that the electron neutrino flavour eigenstate  $|\nu_e\rangle$  is affected differently from the other neutrino flavour eigenstates  $|\nu_{\mu,\tau}\rangle$ . This additional phase shift to  $|\nu_e\rangle$  changes the quantum mechanical probability of flavour mixing. In order to quantify the effect on a neutrino by a large number of surrounding interacting particles, a refractive index is introduced, in complete analogy to light travelling through matter.

All neutrino flavour  $|\nu_{e,\mu,\tau}\rangle$  interact with the matter in the sun ( $e$ ,  $n$ ,  $p$ ) by means of exchange of  $Z^0$  bosons (a "neutral current" interaction (Fig. 4a) while only  $|\nu_e\rangle$  participate in the charged current interaction (Fig. 4b). Mikheyev and Smirnov [22] have derived a neutral current refractive index

$$n_{nc} = 1 + 2\pi\rho_e A_{nc}/\bar{p}^2 \quad (3.1)$$

and a charged current refractive index

$$n_{cc} = 2\pi\rho_e A_{cc}/\bar{p}^2 \quad (3.2)$$

where  $A_{nc}/A_{cc}$  is the forward scattering amplitude of the neutral/charged current, is the electron density ( $\approx 10^{23}/\text{cm}^3$  in the sun's core) and  $|\bar{p}|$  is the neutrino's momentum. The Standard Model gives

$$A_{cc} = \frac{G_F |\bar{p}|}{\sqrt{2}\pi}$$

and a similar relation for  $A_{nc}$ , so that for very high-energetic neutrinos,  $|\bar{p}| \rightarrow \infty$ ,  $n_{nc} \rightarrow 1$  and  $n_{cc} \rightarrow 0$ , as expected. The expressions  $(n_{nc} - 1)$  and  $n_{cc}$  are proportional to  $\rho_e$  and, hence, vanish in the vacuum.

In analogy to birefringent materials (different refractive indices for independent polarizations of light) the matter in the sun gives rise to different refractive indices for  $\nu_e$  compared to  $\nu_{\mu,\tau}$ . It is well known that when polarised light travel through such a birefringent material, a phase shift develops between the polarization states. Similarly, a phase shift develops between  $\nu_e$ 's and other neutrino flavours after travelling a distance  $r$ . Each flavour eigenstate develops a phase  $e^{i|\bar{p}|r(n-1)}$ , with  $n = n_{nc} + n_{cc}$ , that is a result of the weak interaction refraction. Then the total phase experienced by the different neutrino flavours is

$$\delta_e = e^{i|\bar{p}|r(n_{nc} + n_{cc} - 1)} = e^{i\rho_e r \left( \frac{2\pi A_{nc}}{|\bar{p}|} + \sqrt{2}G_F \right)} \quad (3.3a)$$

$$\delta_{\mu,\tau} = e^{i|\bar{p}|r(n_{nc} - 1)} = e^{i\rho_e 2\pi r \frac{A_{nc}}{|\bar{p}|}} \quad (3.3b)$$

The extra term ( $\sqrt{2}G_F\rho_e$ ) present in  $\delta_e$  is the matter oscillation term. From this one obtains

$$P_{\nu_e \rightarrow \nu_\mu}^{MSW}(r) = R(\theta) \sin^2 \left( \frac{\pi r K}{L_0} \right) \quad (3.4)$$

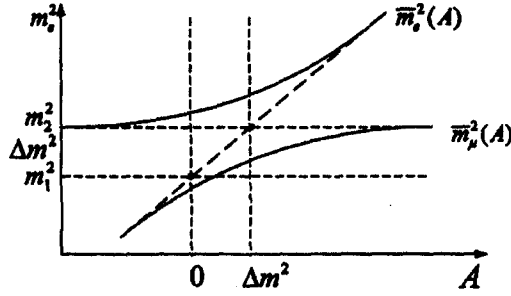


Figure 5.  $\Delta m^2$  as a function of  $A$ . The sun's surface corresponds to the point  $A = 0$ , see text  $\theta = 0 : m_e$ , sloping curve;  $m_\mu$ , constant.  $\theta \neq 0$  : same association but no level crossing (solid curve).

with  $L_0$  as in (2.3),

$$K^2 = \sin^2(2\theta) + (C - \cos(2\theta))^2,$$

$$C = \sqrt{2} G_F \rho_e \frac{2E_\nu}{\Delta m^2}, R(\theta) = \sin^2(2\theta) / K^2$$

is maximal for  $C = \cos(2\theta)$  or, equivalently,

$$\sqrt{2} G_F \rho_e = \frac{2}{L_0} \cos 2\theta \quad (3.5)$$

which is known as the matter oscillation resonance equation. Under the condition (3.5)  $R(\theta) \rightarrow 1$  and (3.4) is practically insensitive to the mixing angle  $\theta$ . Due to the dependence on  $\rho_e$  a neutrino created in the core and travelling outwards will encounter a decreasing  $\rho_e$  until, at some distance from the center, condition (3.5) is fulfilled. Following the MSW model we now take a look at the mass squared matrix for a  $2\nu$  scenario. Standard procedures [22, 23] obtain the eigenvalues as a function of a parameter  $A$  (which contains the weak interaction strength and the neutrino energy) and of the mixing angle  $\theta$ ,

$$m_e^2 = \frac{1}{2}(m_1^2 + m_2^2 + A) \pm \frac{1}{2} \sqrt{(\Delta m^2 \cos 2\theta - A)^2 + (\Delta m^2)^2 \sin^2 2\theta} \quad (3.6)$$

Without mixing, i.e. for  $\theta = 0$ , (3.6) simplifies to

$$m_e^2 = \frac{1}{2}(m_1^2 + m_2^2 + A) \pm \frac{1}{2}|A - \Delta m^2| \quad (3.7)$$

which gives the two dashed lines crossing at  $A = \Delta m^2$  (Fig.5). For small  $A$  and  $\theta = 0$  one expects the muon-neutrino  $m_\mu$  to be heavier than the electron-neutrino  $m_e$ ,  $m_\mu$  has then (for  $\theta = 0$ ) no  $A$ -dependence while  $m_e$  exhibits a linear  $A$ -dependence

$$m_e^2 = m_1^2 + A \quad (3.8)$$

which leads to level crossing at  $A = \Delta m^2$ . For

$\theta \neq 0$  no level crossing occurs and one obtains the solid curves in Fig.5. For  $A \gg \Delta m^2$ ,  $m_e^2(A)$  (for decreasing) will start off on the upper dashed curve and following the solid curve end up on the horizontal dashed curve for small  $A$ , hence will emerge as a muon-neutrino with mass  $\geq m_2^2$ . For  $\theta = 0$  in (3.6) one obtains a minimal distance between the two mass squared eigenvalues for

$$A^{(0)} = \Delta m^2 \cos 2\theta \quad (3.9)$$

(Note via  $A = \rho_e E_\nu$ , one obtains resonant neutrino energy  $E_\nu^{(0)}$  compliant with (3.9)) which is exactly the resonant condition (3.5) which maximizes (3.4).

Let us return to the interpretation of Fig.5. Recall that  $A \sim E_\nu \rho_e$ , and, hence, the same avoidance of level crossing occurs if  $m_e^2$  is plotted versus  $\rho_e$  with a closest approach for  $E_\nu = E_\nu^{(0)}$ . An electron-neutrino created in the core with energy  $E_\nu > E_\nu^{(0)}$  will have to move on the upper solid curve from right to left until it leaves the sun's surface (note a neutrino in the core starts with a certain high value of  $A$  and -upon travelling to the surface of the sun - continually lowers the value of  $A$  thereby moving from the right to the left in Fig. 5) as a muon-neutrino. A detector on earth designed to detect electron-neutrinos from the sun will miss this transmuted neutrino and will signal a solar neutrino deficit. The electron neutrinos from the sun that are detected as such must have energies  $E_\nu < E_\nu^{(0)}$ , otherwise they would appear as muon-neutrinos, if the MSW model is correct. This implies that  $E_\nu^{(0)}$  must be greater than the threshold of the detectors, which find a deficit of  $\nu_e$ 's. We conclude that  $E_\nu^{(0)} > 5 \text{ MeV}$ . The observation of a high-energy  $\nu_e$ -deficit of  $^8\text{B}$  neutrinos (with a

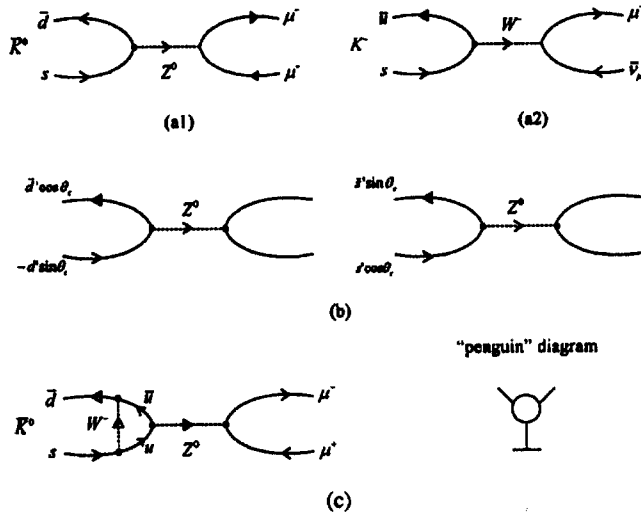


Figure 6. Feynman diagrams for flavour-changing neutral currents; (a1) contribution to  $K_L^0 \rightarrow \mu^+ \mu^-$  without mixing; note the  $\bar{d}sZ^0$  coupling is  $\frac{g}{\sqrt{2}} \cos \theta_c \sin \theta_c$ ; (a2) the corresponding (allowed) charged current decay of the K; (b) the GIM mechanism leads to exact cancellation. Now the  $\bar{q}qZ^0$  coupling is  $\frac{g}{\cos \theta_w}$ , the same for d' and s'. (c) second order effect leads to observed  $>10^{-8}$  suppression of this *SI mode* [3]. Diagram (c) is also known as a "penguin" diagram.

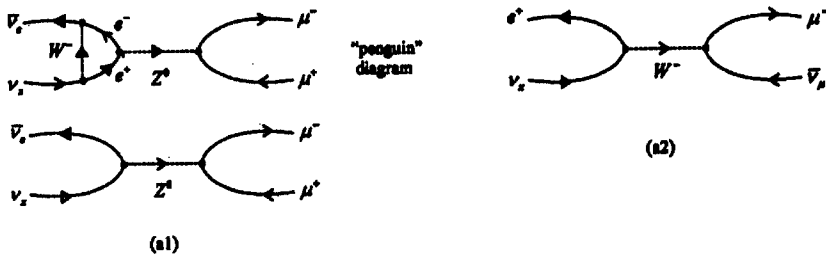


Figure 7. The top (penguin) diagram is the (supposedly allowed) lepton number changing neutral current decay of a high-energetic neutrino. The  $\nu_e \nu_x$  ( $x \neq e$ ) coupling has, however, to date not been observed. (a1) The analogous diagram to Fig.6 (a1), with  $x = \mu$  or  $\tau$ . (a2) The analogous diagram to Fig.6 (a2), with  $x = \mu$  or  $\tau$ . Both (a1) and (a2) are forbidden by lepton family number conservation.

maximum energy of 14 MeV) puts  $E_{\nu}^{(0)} < 14 \text{ MeV}$ . ( $m_1 \approx m_{\nu_e}$ )

Thus a rough estimate is

$$5 \text{ MeV} < E_{\nu}^{(0)} < 14 \text{ MeV}$$

The resonance condition (3.5, 3.9) allows to estimate  $\Delta m^2$ . If  $\theta$  is small (3.5) reads

$$\Delta m^2 = 2\sqrt{2} E_{\nu}^{(0)} G_F \rho_e$$

and  $\rho_e = 4 \times 10^{25} \text{ cm}^{-3}$  in the region where the  ${}^8\text{B}$  process occurs [23]. Using  $G_F = 1.17 \times 10^{-5} \text{ GeV}^2$  and  $E_{\nu}^{(0)} = 6 \text{ MeV}$  one gets  $\Delta m^2 \approx 6 \times 10^{-5} \text{ eV}^2$ . Assuming a normal mass hierarchy for all leptons one obtains for the mass eigenvalues

$$m_{\nu_\mu} \approx 8 \text{ meV for } m_{\nu_\mu} \gg m_{\nu_e}$$

Of course the chosen parameters can vary. But it is relatively safe to say that values for  $\Delta m^2 \approx (10^{-4} - 10^{-6}) \text{ eV}^2$  along with a mixing angle given by  $\sin^2(2\theta) = 10^{-3} - 10^{-2}$  allows electron-neutrinos created in the  $p-p$  process to remain electron-neutrinos and those created in the  ${}^7\text{Be}$  process to oscillate into muon-neutrinos - thus supporting results from the chlorine, water, and gallium detectors. Another parameter set seems to be also



compatible with observations within the rather large uncertainties:  $\Delta m^2 \approx (10^{-4} - 10^{-5}) eV^2$  and  $\sin^2(2\theta)$  close to 1 (Fig. 3). Hence, with the right parameters, the MSW mechanism seems to be capable of explaining the solar neutrino deficit. A question remains, of course. Is neutrino mixing (if confirmed) unnatural within the SM? It has frequently been pointed out [24] that a neutrino mass would be easy to implement in the SM without changing its structure. Let us take a look at the three Fermion generations in the SM as depicted in our Fig.1. The mixing of (d, s, b)-quarks is a natural consequence of the non-observation of flavour-changing neutral currents (named *S1 mode*), (Fig. 6b) for a typical example. The observed (suppressed) *S1 mode* is due to loop contributions ("penguin" diagram in Fig. 6c). In Fig. 7 we have tentatively drawn analogous diagrams in the mixed neutrino sector. While there are indeed neutral mesonic bound states of those mixed (d, s, b) quarks and anti-quarks (Fig. 6(a1)) which do not decay (at the dominant tree-diagram level) into  $\mu^+ \mu^-$  pairs, there are no such  $\nu_e \bar{\nu}_x$  ( $x = \mu, \tau$ ) bound states because neutrinos are not participating in the strong interactions. The weak interaction is too weak and too short-ranged (due to the high mass of the exchanged W-boson at about  $90 \text{ GeV}/c^2$ ) to give rise to neutrino-antineutrino bound states. The processes shown in diagrams in Fig. 7(a1) and Fig. 7(a2) are forbidden by well-tested charged-lepton family number conservation,  $L_\mu \neq L_e \neq L_\tau$ . Such exotic  $\nu_e \bar{\nu}_x$  ( $x = \mu, \tau$ ) bound states, if they existed as weakly bound states, would have to decay into  $\mu^+ \mu^-$  pairs via a second-order "penguin" diagram, (Fig. 7). That would determine its lifetime and spatial extent (intrinsic radius). Our calculations show, however, that such states do not exist due to the strength and range of the weak interactions.

We leave here the still speculative interpretation of (possibly existing) neutrino oscillations and await its experimental verification.

## Summary and Outlook

The three known neutrino species are important members of the Standard Model. The all-

important question of their rest-mass is crucial for the dark matter problem in Astrophysics as well as for the understanding of the solar neutrino deficit (why are less electron-neutrinos from the sun observed by terrestrial neutrino detectors than expected to be produced in the sun? The solar neutrino flux on earth is almost a factor 2 smaller than expected). Neutrinos and anti-neutrinos can therefore be considered to be the link/messenger between the Standard Model of Elementary Particle Physics and Astrophysics. There is growing evidence that neutrinos have tiny masses and mix, giving rise to vacuum and matter oscillation phenomena. Those mixing parameters, once confirmed by independent experiments, are additional parameters of the Standard Model and (like any other SM parameter) are *not* explained by the model. Once understood they will shed new light on both the mass-giving mechanism of the Standard Model and on the interior of stars.

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