รหัสโครงการ SUT1-105-44-12-05

รายงานการวิจัย

ทฤษฎีสนามควอนตัม, ศึกษาด้านฟิสิกส์ทฤษฎี

(Quantum Field Theory and Theoretical Physics : Theory and Applications)

คณะผู้วิจัย

หัวหน้าโครงการ

Professor Dr. Edouard Berge Manoukian

School of Physics

Institute of Science

Suranaree University of Technology

ผู้ร่วมวิจัย

1. นางสาวงามจิตต์ เจียรกุลประเสริฐ
2. นายเสกสรร สุขะเสนา
3. นายนัฏพงษ์ ยงรัมย์

ได้รับทุนอุดหนุนการวิจัยจากมหาวิทยาลัยเทคโนโลยีสุรนารี ปีงบประมาณ พ.ศ. 2544

ผลงานวิจัยเป็นความรับผิดชอบของหัวหน้าโครงการวิจัยแต่เพียงผู้เดียว

กันยายน 2544

ACKNOWLEDGMENT

This is to acknowledge with many thanks the Suranaree University of Technology Research Fund Administration for financial support which made this project possible and led to its successful completion.

บทคัดย่อ

เมื่อเร็วๆ มานี้เราได้หารูปแบบนิพจน์ที่ชัดแจ้งสำหรับค่าเฉลี่ยของจำนวนโฟตอนที่ถูกปล่อยออกมาต่อ 1 รอบของการโคจรในการแผ่รังสีซิงโครตอน ซึ่งได้นำเราไปสู่การวิเคราะห์ระบบฟิสิกส์พลังงานสูงสำหรับอนุภาคพลังงานสูงและสิ่งที่ได้คือ ค่าเฉลี่ยที่มีความผิดพลาดสัมพัทธ์ 2.2%, .64%, .017% ที่ 0.8, 0.9, 0.99 ของความเร็วแสงตามลำดับ ส่วนค่าเฉลี่ยที่ได้จากสูตรการคำนวณแบบเดิมที่รู้จักกันดี จะมีความผิดพลาด 160%, 82%, 17% ที่เงื่อนไขความเร็วแสงเดียวกันตามลำดับ การกระจายความแปรปรวนแบบดิแรค(Dirac picture perturbation expansion) ถูกพัฒนาขึ้นมาอย่างมากสำหรับฟังก์ชันต่างๆ ที่ขึ้นกับเวลาในระบบพิกัดทั่วไปและระบบโมเมนตัมทั่วไป ภายในการกระจายแบบ Schwinger-Feymann-Dyson ในทฤษฎีสนาม และกฎที่มีรายละเอียดปลีกย่อยได้ถูกสร้างขึ้นมาเพื่อใช้ในการคำนวณ นอกจากนี้ระเบียบวิธีที่ได้มายังผลให้เราได้นิพจน์ของ path integral ที่มีจำนวนอตรรกยะที่นับไม่ได้เข้ามาเกี่ยวข้อง ในรูปแบบของ resolution of the identity และในอันดับสุดท้ายขอบเขตล่างที่เป็นจำนวนบวกที่กำหนดอย่างชัดเจนถูกหามาสำหรับภาวะปกติของ eigenstates ของโมเมนตัมเชิงมุมกำลังสองในกฎเกณฑ์แบบควันตัม ซึ่งการกำหนดนี้ได้นำไปสู่การกำจัดค่ากึ่งจำนวนเต็มของโมเมนตัมเชิงมุม

ABSTRACT

Our recent derivation of an explicit expression for the mean number of photons emitted per revolution in synchrotron radiation has led us to a systematic high-energy analysis for high-energetic particles and a novel expression was obtained for the mean number with relative errors of 2.2%, .64%, .017% for speeds of 0.8, 0.9, 0.99, in units of the speed of light, in comparison to the well known formula tabulated in the literature of 160%, 82%, 17%, respectively. A Dirac picture perturbation expansion is developed for the time evolution of arbitrary functions of generalized coordinates and generalized momenta much in the spirit of the Schwinger-Feynman-Dyson expansion in field theory and detailed rules are derived for computations. The formalism also allows us to obtain a path integral expression, as a resolution of the identity, involving an uncountable infinite number of Lagrange multipliers. Finally a rigorous positive definite lower bound was derived for the norm of the eigenstates of the square of the orbital angular momentum in the quantum regime which leads systematically the elimination of half-odd integral values for the orbital angular momentum.

CONTENTS

1. INTRODUCTION

Background 1

Objectives 3

Hypotheses 3

Usefulness and benefits 3

Scope and limitations 3

2. METHODOLOGY

Method of research 4

3. RESULTS OBTAINED

Discussion of the results 6

4. SUMMARY AND CONCLUSIONS

Summary of results, conclusions and recommendations 10

5. REFERENCES 11

6. BIODATA 13

I. INTRODUCTION

Synchrotron radiation provides a fascinating theoretical laboratory for quantum field theorists. Much theoretical work has been carried out over the years and recent work on the subject is summarized in a treatise published recently [1] which is useful for practitioners interested in the theoretical aspects of this form of radiation as a very particular form of radiation that may be handled and described by the sophisticated techniques of field theory. Earlier work both of theoretical and experimental nature was discussed in a remarkable review [2] on the subject. The pioneering rigorous theoretical study was due to Julian Schwinger [3, see also 4-6] who has laid the foundation for much of the theoretical work carried out over the years. It is surprizing that although the main features of synchrotron radiation have been well known for a long time an explicit and exact expression for the mean number of photons per revolution was obtained only recently [7]. Our recent result [7] provided us the opportunity to carry out a systematic analysis for high-energy particles and an explicit expression for the latter was derived. Our novel expression involves relative errors of 2.2%, .64%, .017% for speeds of the charged particle, relative to that of the speed of light, of 0.8, 0.9, 0.99, respectively. These are to be compared with the crude and well known formula tabulated in the literature [8,9] with corresponding relative errors of 160%, 82%, 17% !. These latter very large relative errors has urged us to carry out the investigation reported here.

With the very rapid progress of field theory in describing the basic interactions occurring in nature, it has become more and more urgent to extend the powerful techniques of field theory in describing physics, in general, in a more unified language. Such a unification has always been the goal of physics research over the years. It was our aim to extend these methods in describing classical dynamics in such a spirit. Some earlier work on the subject are given in the references [10-13], which are, however, only tangentially related to our present investigations. We have developed a Dirac picture in the spirit of the Schwinger-Feynman-Dyson [14-24] perturbation theory to all orders in a coupling parameter and detailed rules were derived for computations starting from the time evolution of arbitrary functions of the generalized coordinates and generalized momenta. These methods are, in particular, useful for the complexification of the time operation and for phase space analyses and study such properties as the so-called geometrical Berry phase [25,26]. Since the classical limit of the path integral [27 and, e.g., 28], starting from the quantum regime, reduces to just a phase factor involving the classical action, such a limit [11,12] is not of very practical value, as it stands, for actual computations. Instead, we develop a path integral expression by direct integration of Hamilton's equations, as a resolution of the identity, that may be applied to any function of the generalized coordinates and momenta, in the same spirit of developing the resolution of the identity of a self-adjoint operator, in quantum physics, that may be applied to any vector in the underlying Hilbert space.

A rigorous and explicit positive lower bound is derived for the norm of eigenstates of orbital angular momentum in quantum physics. The latter bound is restricted to a sub-interval in the azimuthal angle near the origin which is shown to be non-normalizable for $j = 3/2, 5/2, \ldots$ in contradiction with the very normalizability condition of the corresponding eigenstates. This fact was used to dismiss with half-odd integral values of the orbital angular momentum. For a partial list of contributions to orbital angular momentum and half-odd integral values, a problem considered first by Pauli [29], by completely different approaches, see, e.g., [30-34].

From above, we may summarize, in turn, by stating that the objectives of this project were to obtain, for the first time, an explicit expression, with negligible relative errors, of the mean number of photons emitted per revolution in the very celebrated form of radiation known as synchrotron radiation and correct the crude and well known tabulated result with relative errors as high as 160%. Our aim was also to extend field theory methods to a relatively unexplored area of physics by such techniques. Such unifications have always been the goal of physics research over the years. The underlying hypotheses involved as of the validity of field theory which has been tested repeatedly in the history of physics as a framework to describe most if not all of the phenomenae in physics. The usefulness of our results are that they are of practical value for many computations that may be carried out by our newly established results. The scope of the research is unlimitted as our results are obtained from well established theories which have been tested over the years.

2. METHODOLOGY

We have used the theoretical foundation laid by Julian Schwinger, to carry out a systematic theoretical analysis of the mean number of photons emitted per revolution in synchrotron radiation. The basic idea for obtaining the explicit expression for the latter, for the first time, was to incorporate at the outset the correct boundary condition for vanishing speeds of the charged particle. This has led us to the investigation for radiation off high-energy charged particles. This investigation turned out to be far from trivial for the following reasons. At high energies, the integrand obtained develops a singularity near the origin. By rewriting the integrand in a form suitable for studying its behaviour near the origin, made the investigation at infinity or the upper limit of integration rather difficult. These points, in turn, has led us to a systematic analysis of some given functions at both ends. These properties of the integrand are reflected quite clearly in the expression obtained for the mean number. The so-called Dirac picture obtained was an extension of the field theory one which, in our formalism, describes the time evolution of non-commuting operators, and specific rules were then possible to derive for such methods with no constraints used in the order of the perturbation expansion obtained. These methods are reminiscent of Feynman diagrammatic techniques well known in particle and condensed matter physics problems. In developing our path integral expression, we have realized that the well known Feynman path integral is not useful in our limit as it reduces just to a phase factor involving the classical action. Instead, we had to use and develop a resolution of the identity operation in the spirit of developing the resolution of the identity of a self-adjoint operator. To dismiss with half-odd integral values for the otbital angular momentum, a rigorous lower bound was derived for the norm of the eigenstates of this operator which is positive definite. This bound was shown not to be integrable near the origin in the azimuthal angular part which was a sufficient condition for establishing this important result of physics. The periodic time table of elements would have looked quite different if half-odd integral values were allowed for the orbital angular momentum !

3. RESULTS OBTAINED

The new results obtained in the project were numerous. The exact expression for the mean number of photons emitted per revolution in synchrotron radiation we obtained earlier was used to do a systematic high-energy analysis for radiating charged particles. This analysis has led to the following expression for the mean number of photons emitted per revolution:

$$\langle N \rangle \simeq 5\pi\alpha / \sqrt{3(1-\beta^2)} + a_0\alpha + \pi\alpha\sqrt{(1-\beta^2)}/10\sqrt{3}$$

where α is the fine-structure constant, a_0 is a numerical constant: $a_0 = -9.55797$, and β is the speed of the charged particle as measured relative to the speed of light. The relative errors are 2.2%, .64%, .017% in comparison to the well known formula tabulated in the literature with relative errors of 160%, 82%, 17% for $\beta = 0.8, 0.9, 0.99$, respectively.

A systematic perturbation theory was developed for any function of generalized coordinates and generalized momenta $f[q(t), p(t)]$, in a standard notation, leading to the expansion

$$f[q(t),p(t)] = \sum_{n=0}^{\infty} (\lambda)^n \int_0^t dt_1 \int_0^{t_1} dt_2 \ldots \int_0^{t_{n-1}} dt_n B(t_n) \ldots B(t_1)\, e^{tA}\, f[q,p]$$

where λ denotes so-called a coupling parameter, defined by writing the Hamiltonian in question as

$$H = H_1 + \lambda H_2$$

with

$$H_1 = p^2/2m,\ H_2 = V(q)$$

Here

$$B(t) = e^{tA} B e^{-tA}$$

$$A = \left[\frac{\partial H_1}{\partial p} \frac{\partial}{\partial q} - \frac{\partial H_1}{\partial q} \frac{\partial}{\partial p} \right]$$

$$B = \left[\frac{\partial H_2}{\partial p} \frac{\partial}{\partial q} - \frac{\partial H_2}{\partial q} \frac{\partial}{\partial p} \right]$$

Upon defining

$$q + \frac{t_i p}{m} = u_i \qquad\qquad F[q] = -V'(q)$$

the rule for evaluating the integrand in the perturbation expansion is as follows:

$$B(t_n)\ldots B(t_1)[q + \frac{tp}{m}]$$

$$= \left(\frac{t-t_1}{m}\right) F[u_n] \times \sum \left(\frac{t_1 - t_2}{m}\right)^{\delta(k_1,2)} \cdots \left(\frac{t_1 - t_n}{m}\right)^{\delta(k_1,n)} F^{(k_1)}[u_1]$$

$$\times \left(\frac{t_2 - t_3}{m}\right)^{\delta(k_2,3)} \cdots \left(\frac{t_2 - t_n}{m}\right)^{\delta(k_2,n)} F^{(k_2)}[u_2]$$

$$\vdots$$

$$\times \left(\frac{t_{n-1} - t_n}{m}\right)^{\delta(k_{n-1},n)} F^{(k_{n-1})}[u_{n-1}]$$

$$F^{(a)}[u] = \left(\frac{d}{du}\right)^a F[u]$$

and the sum is over all k's and δ's such that

$$\left.\begin{array}{l} k_1 + k_2 + \ldots + k_{n-1} = n-1 \\ k_1 = 1,\ldots,n-1 \\ k_2 = 0,1,\ldots,n-2 \\ \vdots \\ k_{n-1} = 0,1 \end{array}\right\}$$

$$\delta(k_i, j) = 0, \text{ if } k_i = 0 \qquad\qquad \delta(k_i, j) = 0, \text{ if } 1 \le j \le i$$

and for $j > i$, the $\delta(k_i, j)$ are zero or one <u>such</u> <u>that</u>

$$\sum_{j=i+1}^{n} \delta(k_i, j) = k_i\,, \quad i = 1,\ldots,n-1$$

$$\sum_{i=1}^{n-1} \delta(k_i, j) = 1\,, \quad (\,\delta(k_i, j) = 0, \quad j = 1,2,\ldots,i\,)$$

i.e., for a fixed j, t_j appears only <u>once</u> in the product:

$$\prod_{j=2}^{n} (t_1 - t_j)^{\delta(k_1,j)} \prod_{j=3}^{n} (t_2 - t_j)^{\delta(k_2,j)} \ldots$$

The path integral expression obtained in the formalism as a <u>resolution</u> <u>of</u> <u>the</u> <u>identity</u> is given by:

$$f[q(t),p(t)] = \int [dq][dp][d\lambda][d\eta] \exp i\int_0^t dt' \left[\lambda(t')\left(\frac{\partial H(t')}{\partial p(t')} - \dot{q}(t') \right) + \eta(t')\left(\frac{\partial H(t')}{\partial q(t')} + \dot{p}(t') \right) \right] \times f[q(t),p(t)]$$

in terms of an uncountable infinite number of Lagrange multipliers $\lambda(\cdot), \eta(\cdot)$.

Finally upon considering the familiar orbital angular momentum eigenstate

$$\langle \theta,\phi | j,\ j \rangle = A_j e^{ij\phi} (\sin\theta)^j \ ;\ A_j = \left(\Gamma\left(j + 3/2\right) / 2\pi\Gamma\left(j + 1\right)\Gamma(1/2) \right)^{1/2}$$

with

$$\langle \theta,\phi | j,\ -j \rangle = (\hbar)^{-2j} \left((2j)! \right)^{-1} \left(L_- \right)^{2j} \langle \theta,\phi | j,\ j \rangle \ ;\ L_- = \hbar e^{-i\phi} \left(i \cot\theta \frac{\partial}{\partial\phi} - \frac{\partial}{\partial\theta} \right)$$

a rigorous <u>non</u>-vanishing lower bound was derived for the norm:

$$\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta \left| \langle \theta,\ \phi | j,\ -j \rangle \right|^2 \geq 2\pi \int_0^{\alpha_K} \sin\theta d\theta \left| \langle \theta,\ \phi | j,\ -j \rangle \right|^2$$

$$\geq \left| A_j \right|^2 \left((2j)! \right)^{-2} \pi C_K^2 \int_0^{\alpha_K} (\sin\theta)^{-2K} d\theta$$

where A_j and C_K, α_K are strictly positive constants. Here $K = (2j-1)/2$. The lower bound diverges at the origin for $K = 1, 2, 3, \ldots$ and this fact was used to give a clear account of the non-existence of half-odd integral values for the orbital angular momentum in quantum physics.

4. SUMMARY AND CONCLUSIONS

The exact and explicit expression for the mean number of photons emitted per revolution in synchrotron radiation we derived recently was used to carry out a systematic analysis for high-energy particles with our result summarized in Sect.3. This, in turn, improved tremendously the crude and well known formula tabulated for this physical quantity in the literature with very large relative errors as high as 160% !. The quantum field theory techniques were used to develop a perturbation theory for the time development of arbitrary functions of generalized coordinates and generalized momenta and explicit rules were derived and spelled out for computations with no restrictions on the order of the perturbation expansion. Finally, a rigorous positive definite lower bound was derived for the norm of orbital angular momentum states which automatically allows one to dismiss half-odd integral values for the orbital angular momentum. The properties of elements in the periodic table would have been different if half-odd integral values were allowed for the orbital angular momentum.

5. REFERENCES

1. Bordovitsyn, V. A. (Ed.). 1999. Synchrotron Radiation and its Development, World Scientific, Singapore.
2. Pollock, H. C. 1983. Am. J. Phys. 51, 278.
3. Schwinger, J. 1949. Phys. Rev. 75, 1912.
4. Schwinger, J. 1946. Phys. Rev. 70, 798.
5. Schwinger, J. and Tsai, W.-T. 1978. Ann. Phys. (NY) 110, 63.
6. Schwinger, J., Tsai, W.-T. and Erber, T. 1976. Ann. Phys. (NY), 303.
7. Manoukian, E. B. and Jearnkulprasert, N. 2000. Phys. Lett. A 268, 35.
8. Review of Particle Physics. 1998. Europ. Phys. J. 3, 79.
9. Review of Particle Physics. 1996. Phys. Rev. D 54 (I), 75.
10. Schwartz. 1976. J. Math. Phys. 18, 110.
11. Gozzi, E., Reuter, M. and Thacker, W. D. 1989. Phys. Rev. D 40, 3363.
12. Abrikosov, A. A. 1993. Phys. Lett. A 182, 172.
13. Wetterich, C. 1997. Phys. Lett. B 339, 123.
14. Schwinger, J. (Ed.). 1958. Quantum Electrodynamics, Dover Publications, Inc., New York.
15. Manoukian, E. B. 1986. Phys. Rev. D 34, 3739.
16. Manoukian, E. B. 1989. Int. J. Theor. Phys. 28, 495.
17. Manoukian, E. B. 1988. Int. J. Theor. Phys. 27, 787.
18. Manoukian, E. B. 1987. Nuovo Cimento A 98, 459.
19. Manoukian, E. B. 1987. J. Phys. G.: Nucl. Phys. 13, 1103.
20. Manoukian, E. B. 1985. Nuovo Cimento 90 A, 295.
21. Manoukian, E. B. 1985. J. Math. Phys. 26, 1065.

22. Manoukian, E. B. 1982. J. Phys. G.: Nucl. Phys. 8, 599.

23. Manoukian, E. B. 1981. J. Math. Phys. 22, 2258.

24. Manoukian, E. B. and Charuchittapan, D. 2000. Int. J. Theor. Phys. 39, 2197.

25. Berry, M. V. 1984. Proc. Roy. Soc. Lond. A 392, 45.

26. Shapere, A. and Wilczek, F. (Eds.). 1989. Geometric Phases in Physics, World Scientific, Singapore.

27. Feynman, R. P. and Hibbs, A. R. 1965. Quantum Mechanics and Path Integrals, McGraw-Hill, New York.

28. Swanson, M. S. 1992. Path Integrals and Quantum Processes, Academic, New York.

29. Pauli, W. 1958. Encyclopedia of Phys. (Ed. S. Flügge), Vol. 5. Part 1, pp. 45, 46. Springer, Berlin.

30. Biedenharn, L. C. and Louck, J. D. 1981. Angular Momentum in Quantum Mechanics, Addison-Wesley, Reading, MA.

31. DeLange, O. L. and Raab, R. E. 1991. Operator Methods in Quantum Mechanics, Oxford, Oxford.

32. Levy-Leblond, J. M. 1963. J. Math. Phys. 4, 776.

33. Panders, D. 1965. J. Math. Phys. 5, 1098.

34. Schwinger, J. 1965. in: Quantum Theory of Angular Momentum, (Biendeharn, L. C. and Van Dam, H. Eds.), Academic, New York.

BIODATA

Edouard Berge Manoukian, Professor Dr.
School of Physics, Institute of Science
Suranaree University of Technology
111 University Avenue, Muang District
Nakhon Ratchasima 30000
e-mail: edouard@ccs.sut.ac.th

Degrees:

B.Sc., Honours Physics and Mathematics, 1967. Sir George Williams University, Canada

M.Sc., Physics, 1968. McGill University, Canada

Ph.D., Physics, 1971. University of Toronto, Canada.

Special Honours:

- Was asked by the Nobel Prize Committee to recommend a candidate for the Nobel Prize in Physics.
- Honorary Professor title granted by the Institute for Basic Research - IBR, Molise, Italy.
- Chief and Development Award by the Department of National Defence for research for the years 1978 - 1995 , Canada.

Publications:

Over 150 refereed research papers and four books published by: Academic Press, New York; Hermann, Paris; Gordon & Breach, New York; Springer-Verlag, Berlin and New York.

Research Interests and Publications in:

Quantum Field Theory, Quantum Electrodynamics and Gauge Theories,

Renormalization, High-Energy and Particle Physics,

Relativity and Gravitation, Condensed Matter Physics.