A STUDY OF TWO QUBITS SYSTEM WITH QUANTUM OPERATOR FORMALISM

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วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรมหาบัณฑิต

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A STUDY OF TWO QUBITS SYSTEM WITH QUANTUM OPERATOR FORMALISM

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ศิริรัชนี ธรรมสุวรรณ : การศึกษาระบบสองคิวบิทด้วยตัวดำเนินการทางควอนตัม (A STUDY OF TWO QUBITS SYSTEM WITH QUANTUM OPERATOR FORMALISM) อาจารย์ที่ปรึกษา : ผู้ช่วยศาสตราจารย์ ดร.อายุทธ ลิ้มพิรัตน์, 46 หน้า

วิทยานิพนธ์นี้ได้ทำการศึกษาระบบเปิดที่ประกอบด้วยอะตอมสองระดับชั้นสองอะตอมภายในโพรงเดียวกัน โดยที่อะตอมทั้งสองมีคุณสมบัติเหมือนกันและเกิดอันตรกิริยากับโพรงผ่านการแลกเปลี่ยนโฟตอน ซึ่งสามารถศึกษาด้วยตัวดำเนินการทางควอนตัมที่เรียกว่า "ตัวดำเนินการเคร้าส์" ในวิทยานิพนธ์ ได้ทำการสร้างตัวดำเนินการเคร้าส์ที่จะนำมาประยุกต์ใช้กับเมทริกซ์ความหนาแน่นที่มีการวิวัฒน์ของเวลาเพื่อวิเคราะห์เสถียรภาพของการพัวพันของระบบสองคิวบิทได้

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TWO QUBITS SYSTEM/KRAUS OPERATOR/ENTANGLEMENT/DENSITY MATRIX

An open system consisting of two two-level atoms inside a cavity, where the atoms are identical and allowed to interact via photon exchanges with the cavity, is studied in the quantum operator formalism, the approach of Kraus operators. We have constructed in the work the Kraus operators which can be applied to the time evolution of the density matrix for analyzing the stability of the entanglement of two qubits systems.

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Siriratchanee Thammasuwan

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LIST OF ABBREVIATIONS

Qubit	Quantum bit
EPR Paradox	Einstein-Podolsky-Rosen Paradox

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CHAPTER I

INTRODUCTION

In quantum mechanics, two interesting and beneficial phenomena are quantum superposition and quantum entanglement, which have been recognized as critical resources of quantum computing and quantum information processing (Nielsen and Chuang, 2010; Marinescu and Marinescug, 2011; Preskill, 2004; Barnett, 2010). The simplest entanglement is the Bell state (Sych and Leuchs, 2009) in which a pair of quantum bits (or qubits) is formed. A quantum bit is a two-state quantum mechanical system, such as the polarization of a single photon and the spin of electron. An essential feature of the multiple qubits is that it can have entanglement. Quantum entanglement is a physical phenomenon that occurs when pairs or groups of particles are generated or interact in ways such that the quantum state of each particle cannot be described independently. The Bell system has been studied intensively. For instance, the Bell singlet state is explored in the suppression of disentanglement (Liu and Chen, 2006). The system consists of two atoms inside the same cavity, where the interaction of these atoms is allowed by exchanging photons inside a lossless cavity (environment). It is shown in their work that, with the success of suppressing decoherence, the entanglement can be stabilized and even enhanced.

The innovative work of Ting Yu and J. H. Eberly (Yu and Eberly, 2004) discussed two initially entangled qubits and examined the dynamics of their disentanglement due to spontaneous emission. The decoherence decay of a single qubit under the master equation and the Kraus representation are constructed in the Ting Yu and J. H. Eberly work. In conclusion, the dynamical evolution of the initially atoms in the entangled state will disentangle in a finite-time which is not caused by the spontaneous emission.

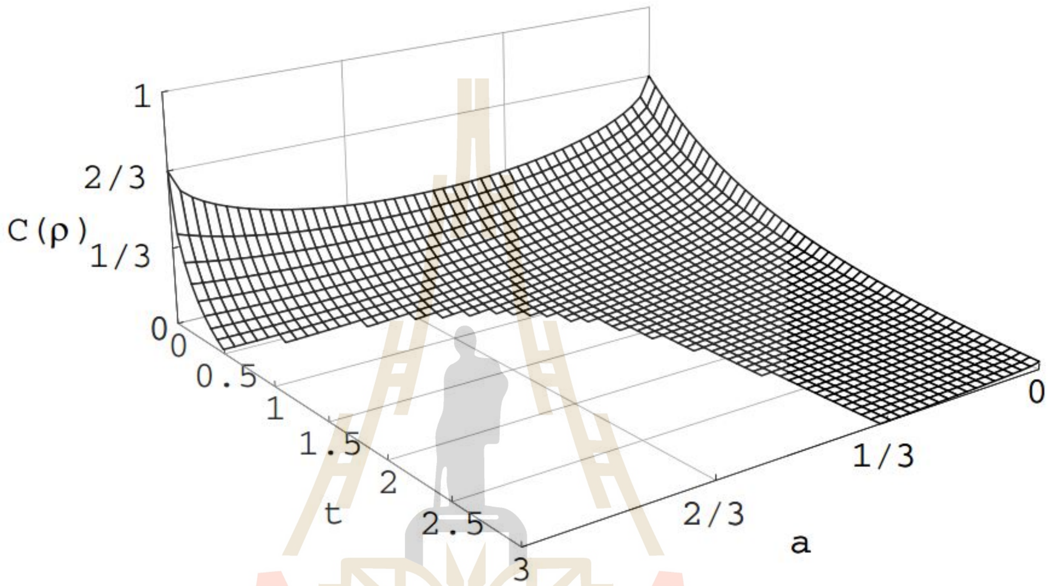

Figure 1.1 The entanglement decay of the atoms initially in the entangled states by spontaneous emission with between 0 and 1 (Yu and Eberly, 2004).

The study of R. Tanas and Z. Ficek (Tanas and Ficek, 2004) discussed about the creation of entanglement in a system of two two-level atoms through the spontaneous emission. In this system, the two two-level atoms are separated by an arbitrary distance and interact with each other through the dipole-dipole interaction. It is shown that the spontaneous emission can lead to a transient entanglement between the atoms even if the atoms are prepared initially in an unentangled state, and for sufficiently large dipole-dipole interactions the entanglement exhibits oscillatory behavior with considerable entanglement.

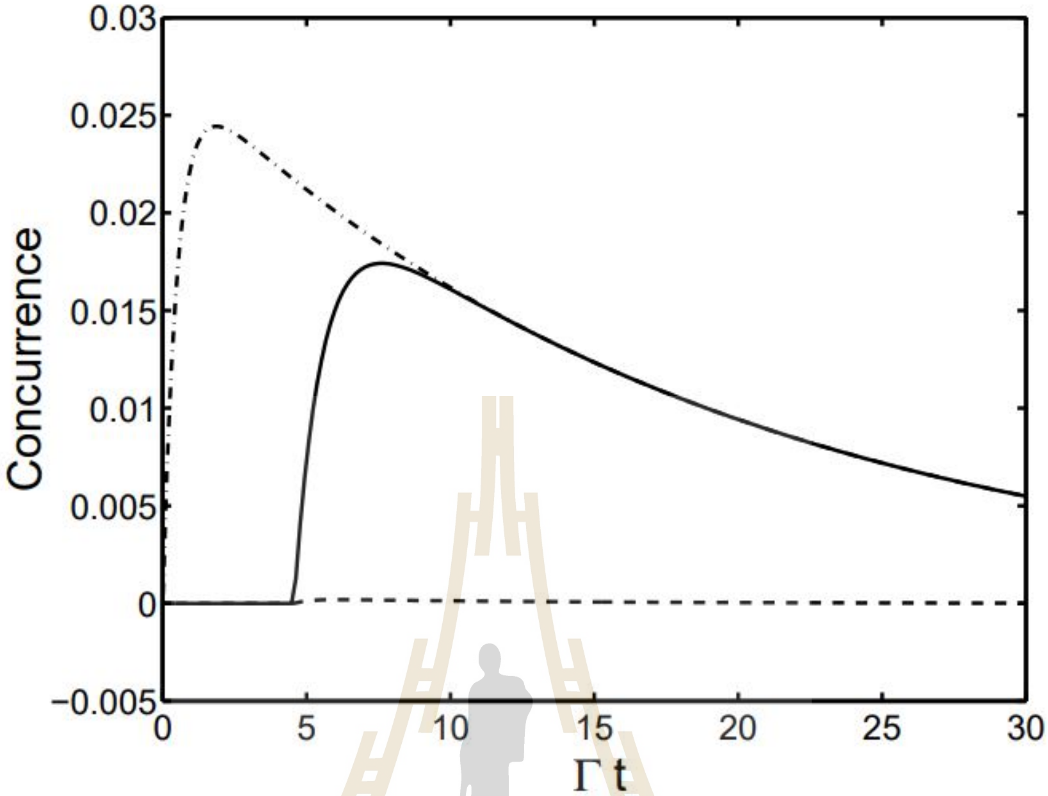

Figure 1.2 The time evolution of the concurrence $\zeta(t)$ (solid line) for two initially excited atoms (Tanas and Ficek, 2004).

Our research follows the work of Ru-Fen Liu and Chia-Chu Chen, in which the master equation of the density matrix is solved directly (Liu and Chen, 2006). In general, it is difficult (or say, impossible) to directly solve the master equation of a system with interactions. We study the master equation in the approach of Kraus operators in the interaction picture. It is believed that the Kraus operator representation may provide another approach for solving the master equation of interacting systems.

Open quantum system is a quantum mechanical system which interacts with the external system called "environment". Normally, these interactions importantly change the dynamics of the system and result in quantum distribution, where information limited in the system is lost to its environment. In general, consider bipartite system AB. The system go through a unitary evolutions and describing the evolution of subsystem A alone. Initially, assume that the two subsystems are not entangled and can be described by the density matrix (ρ). The evolution of the total system is control by the unitary time-evolution operator $U(t)$, which induces to the new density matrix ($\rho^{'}$) is known as the Kruas representation or the operator-sum representation. The Kraus operators (Kraus, 1983) in one-qubit system are derived in Ref. (M. ArsenijeviÄĞ and DugiÄĞ, 2017), but it is still unclear to apply the Kraus operator representation of two-qubit with photon bath. (Breuer and Petruccione, 2002; Dodd and Halliwell, 2004; Rivas and Huelga, 2012)

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CHAPTER II

THEORY

2.1 The Fundamental concepts of quantum mechanics

This chapter describes the mathematical description of quantum physics and concepts (Peres, 1993) that are applied to study quantum information (Nielsen and Chuang, 2010). Probability interpretation of quantum theory has the probability amplitudes more than probabilities as the fundamental quantities. We will see that the rule for the dynamic evolution and quantum measurements, together with the existence of entangled states, are significant for quantum information.

2.1.1 Pure quantum state

The state of a physical system is a pure quantum state if it is entirely identified by a state vector $|\Psi\rangle$ that resides in the Hilbert space $\mathcal{H}$. If the basis $|\psi_i\rangle$ is selected for the Hilbert space, the state vector $|\Psi\rangle$ can be expanded by linear combinations of the basis vectors as,

$$|\Psi\rangle = \sum_i \alpha_i |\psi_i\rangle, \tag{2.1}$$

where α_i are complex, and the general state of $|\Psi\rangle$ is normalized by

$$\langle\Psi|\Psi\rangle = 1$$

$$\sum_i |\alpha_i|^2 = 1. \tag{2.2}$$

2.1.2 Mixed quantum state

The state of a physical system is called a mixed state when the quantum mechanical system can not be described by wave functions. As mentioned above, the states described by wave functions are called pure states. It is necessary to have a new formalism instead of the wave function method to study mixed states. It is found that the density operator may be a convenient mean. We define the density operator to be a combination of pure states,

$$\rho = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|, \tag{2.3}$$

with conditions $0 \leq p_i \leq 1$ and $\sum_i p_i = 1$. As an special instance, if the state is a pure state then the density matrix reduces to

$$\rho = |\Psi\rangle\langle\Psi|. \tag{2.4}$$

It is convenient to introduce a measurement of the purity of a state, $P = Tr\,\rho^2$ with the properties,

$$\begin{aligned} \rho^2 &= \rho \qquad \text{for pure state,} \\ \rho^2 &\neq \rho \qquad \text{for mixed state.} \end{aligned} \tag{2.5}$$

To ensure that ρ describes the physical state, it has to satisfy the following conditions:

- $\rho = \rho^\dagger$ is Hermitian.
- Eigenvalues of ρ are positive ($\lambda_i = 0$).
- $Tr\,\rho = 1$, normalization.

We will use the density matrix formalism, because it can be used for both pure

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and mixed states.

2.2 Single Qubit

Qubit is a quantum system which can be identified as two states system. We name the vector states $|0\rangle$ and $|1\rangle$. The qubit can hold only one-bit of information under its ability to prepare the data in these states. Then the qubit is given by,

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle, \tag{2.6}$$

where $|\alpha|^2 + |\beta|^2 = 1$. Using this criterion, we can reparameterize the qubit state vector to,

$$|\Psi\rangle = \cos\frac{\theta}{2}|0\rangle + \sin\frac{\theta}{2}e^{i\psi}|1\rangle, \tag{2.7}$$

where the spherical polar coordinates. θ and ψ, define a point on the sphere in 3-dimensional space. A sphere where any pure qubit state is located is called the Bloch sphere.

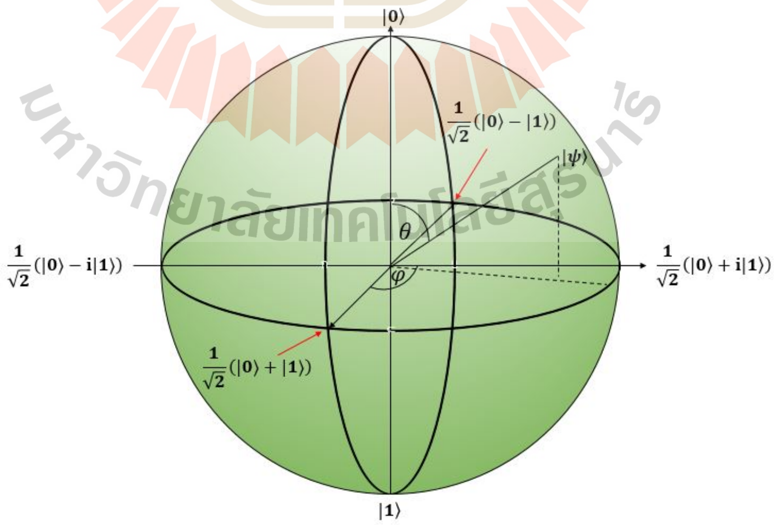

Figure 2.1 The pure state on the surface of the Bloch sphere.

By including mixed states, one can decompose the density matrix of any

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arbitrary state with the well known Pauli matrices.

$$\rho = \frac{1}{2}(\mathbb{1} + \hat{n} \cdot \vec{\sigma}) \tag{2.8}$$

where $\hat{n} = (n_1, n_2, n_3)$ is called the Bloch vector. If the Bloch vector $\hat{n}$ satisfies $|\hat{n}|^2 = 1$, then it is a pure state and located on the surface of the 3-dimensional ball (see Figure 2.2, Bloch sphere). If $|\hat{n}|^2 < 1$, ρ is the mixed state and the vector lies inside the sphere (Bloch ball).

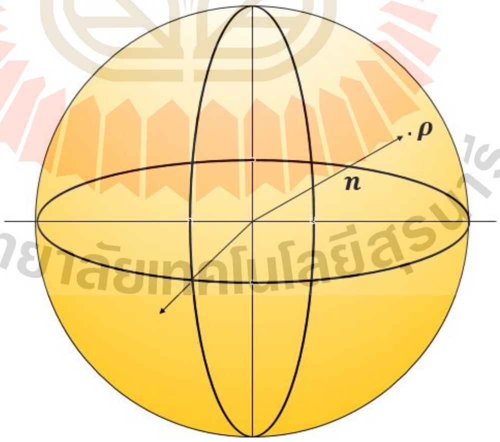

Figure 2.2 Bloch ball.

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2.3 Two Qubits

Since the density matrix of two qubits are living in Hilbert space $\mathcal{H} \equiv \mathbb{C}^2 \otimes \mathbb{C}^2$, one needs to create a basis by using the 4x4 hermitian matrices with 16 elements. These basis operators can be tensor products of the basis operators of the single qubit, such as, $\mathbb{1}_2 \otimes \mathbb{1}_2, \mathbb{1}_2 \otimes \sigma_i, \sigma_i \otimes \mathbb{1}_2, \sigma_i \otimes \sigma_j$. For any two qubits, the density matrix may given by,

$$\rho = \frac{1}{4}(\mathbb{1}_1 \otimes \mathbb{1}_2 + \sum_i a_i \mathbb{1}_2 \otimes \sigma_i + \sum_i b_i \sigma_i \otimes \mathbb{1}_2 + \sum_{i,j=1}^{3} c_i \sigma_i \otimes \sigma_j), \tag{2.9}$$

where the coefficients a_i, b_i and c_i are real and responsible for the non-local correlations between subsystems.

2.4 Quantum entanglement

The entanglement occurs in a quantum mechanical system consisting of two or more parties. This property makes the Einstein, Podolsky, Rosen (EPR Paradox) (A. Einstein and Rosen, 1935) and other physicists dislike the quantum mechanics since it predicts highly counterintuitive processes.

For a system of two qubits, which are in the product space of two Hilbert spaces of the two subsystems $\mathcal{H} = \mathcal{H}^A \otimes \mathcal{H}^B$, the density operators usually cannot be written in the form of the product of ρ_A and ρ_B. If the density operators can be written as a product $\rho = \rho_A \otimes \rho_B$, we say the state is separable. If a state is not separable then it is called entangled state. For instance, the four Bell vectors are the typical entangled states,

$$\left|\Phi^+\right\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1 \otimes |0\rangle_2 + |1\rangle_1 \otimes |1\rangle_2),$$

$$\left|\Phi^-\right\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1 \otimes |0\rangle_2 - |1\rangle_1 \otimes |1\rangle_2),$$

$$\left|\Psi^{+}\right\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1 \otimes |1\rangle_2 + |1\rangle_1 \otimes |0\rangle_2),$$

$$\left|\Psi^{-}\right\rangle = \frac{1}{\sqrt{2}}(|0\rangle_1 \otimes |1\rangle_2 - |1\rangle_1 \otimes |0\rangle_2), \tag{2.10}$$

where $|00\rangle$, $|01\rangle$, $|10\rangle$, and $|11\rangle$ are the well-known form of the computational basis.

2.5 Closed and open systems

As we know, the time evolution of a closed quantum system is described by a unitary operator. For an open quantum system, however, the time evolution is not unitary. The time evolution of open systems can be described by the so-called Kraus representation, which is constructed from a larger closed system which is the system together with the environment.

The time evolution of closed systems is given by the Schrödinger equation,

$$i\hbar\frac{d}{dt}|\psi(t)\rangle = H(t)|\psi(t)\rangle, \tag{2.11}$$

where H is the Hamiltonian of the system. Since we are describing the closed system, the evolution of the state can be expressed by the time-evolution unitary operator $U(t)$, which will change the initial state $|\psi(0)\rangle$ to the state $|\psi(t)\rangle$,

$$|\psi(t)\rangle = U(t)|\psi(0)\rangle. \tag{2.12}$$

We consider the isolated system, the Hamiltonian is time-independent. So the unitary operator is,

$$U(t) = \exp[-iHt]. \tag{2.13}$$

If the system has been considered as a quantum statistical ensemble then the state of the vector cannot explain this ensemble. Moreover, the system can be

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described by $\rho(t)$,

$$\rho(0) = \sum_i a_i |\psi_i(0)\rangle\langle\psi_i(0)|, \tag{2.14}$$

since the state vectors depend on time, the density matrix of the time can be written by,

$$\begin{aligned} \rho(t) &= \sum_i U(t)|\psi_i(t)\rangle\langle\psi_i(t)|U^\dagger(t), \\ &= U(t)\rho(0)U^\dagger(t) \end{aligned} \tag{2.15}$$

By differentiate the equation with t,

$$\frac{d}{dt}\rho(t) = -i[H(t), \rho(t)]. \tag{2.16}$$

This equation is the time-evolution equation for the closed system, which is called Liouville-von Neumann equation.

Figure 2.3 The schematic picture of an open quantum system.

Now we turn to the open system. Shown in Figure 2.3 is a system with external interactions. The combined system is closed and the time evolution of the dynamical system is given by a unitary operator $U(t)$. The interactions between the system and environment are related to the correlation between them, which will have an impact on the characteristics of the system. Therefore, the

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reduced system must be described by the master equation of, which described in the section on Born-Markovian dynamics.

A reservoir or heat bath can represent the environment comprises with an infinite number of degrees of freedom. That typically leads to behavior that is not reversible. Next, we need to determine the density matrix that shows the states of the system. The density matrix without any indices should be the state of the total system, while S and E stand for specific subsystem. Consider the quantum mechanical system that is the state of the tensor product $S \otimes E$ of Hilbert spaces. The mixed state is described by the density ρ matrix, that is the non-negative trace class operator on the tensor product. Partial trace of ρ with respect to the environmental system E is indicated by ρ_S, known as the reduced state of ρ in system A.

$$\rho_S = Tr_E \rho, \tag{2.17}$$

from the equation (2.17), we can reduce the environment term then, the density matrix of the system can be written as,

$$\rho_S(t) = Tr_E[U(t)\rho(0)U^\dagger(t)], \tag{2.18}$$

where $U(t)$ is the time evolution operator of the total system. To obtain the equations of motion of systems, we can take the partial trace over the environment on both sides of the equation (2.24) for the total system,

$$\frac{d}{dt}Tr_E \rho(t) = \frac{d}{dt}\rho_S(t) = -i[H(t), \rho(t)] + D[\rho(t)], \tag{2.19}$$

where $D[\rho(t)]$ is the decoherence term for an open system, the equation (2.21) called **"the master equation"**.

2.6 Two-level model

An atom with two energy eigenvalues is described by two-dimensional state space $|0\rangle$ and $|1\rangle$. These states are the complete orthogonal basis. The eigenvalue of the energy are E_0 and E_1.

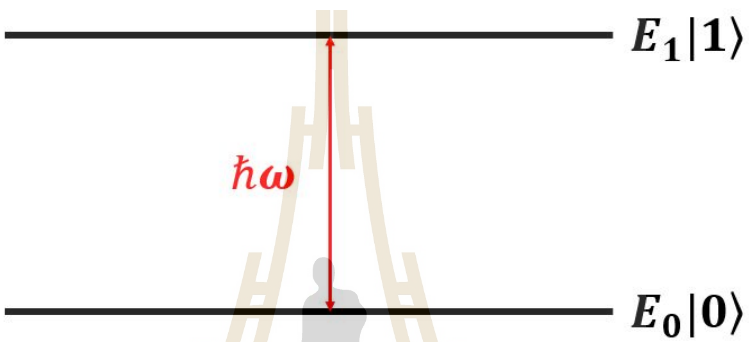

Figure 2.4 Two-level atom model.

The Hamiltonian in the energy representation,

$$H_{atom} = E_0|0\rangle\langle 0| + E_1|1\rangle\langle 1|. \tag{2.20}$$

A possible alternative for the operator in this space is,

$$\begin{aligned} |0\rangle\langle 0| + |1\rangle\langle 1| &= \mathbb{1}, \\ |0\rangle\langle 0| - |1\rangle\langle 1| &= \sigma_z, \\ |0\rangle\langle 1| &= \sigma_-, \\ |1\rangle\langle 0| &= \sigma_+, \end{aligned} \tag{2.21}$$

where the operator σ_+ generates the transition from the ground state to excited

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state by absorbing energy and σ_- reduces the transition from excited state to ground state while releasing energy. On the contrary, with σ_z is the Hermitian operator.

If we exclude from the Hamiltonian in the term $\frac{1}{2}(E_0 + E_1) \cdot \mathbb{1}$ where the $\mathbb{1}$ refers to the unit matrix, we will calculate the corresponding energy and find the Hamiltonian of two-level systems as,

$$H_{atom} = \frac{1}{2}\hbar\omega\sigma_z, \tag{2.22}$$

where ω is the transition frequency $\omega = \frac{1}{\hbar}(E_1 - E_0)$.

2.7 Creation and annihilation operators

We start with the Hamiltonian of the harmonic oscillator,

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2, \tag{2.23}$$

where all symbols have the meaning generally inspired by unique identification.

$$H = \hbar\omega(x\sqrt{\frac{m\omega}{2\hbar}} - ip\frac{1}{\sqrt{2m\omega}})(x\sqrt{\frac{m\omega}{2\hbar}} + ip\frac{1}{\sqrt{2m\omega}}), \tag{2.24}$$

we will calculate the quantum mechanical model on the right-hand side of the equation. It differs from a classic for two reasons. First, there is an operator that operates with the state instead of a function. Secondly, the position and momentum operators do not commute. We get,

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + \frac{i\omega}{2}[x,p] = H - \frac{1}{2}\hbar\omega, \tag{2.25}$$

since $[x,p] = i\hbar$. Now we introduce the notation,

$$a = x\sqrt{\frac{m\omega}{2\hbar}} + ip\frac{1}{\sqrt{2\hbar m\omega}}$$

$$a^\dagger = x\sqrt{\frac{m\omega}{2\hbar}} - ip\frac{1}{\sqrt{2\hbar m\omega}}, \tag{2.26}$$

so we can write the Hamiltonian as,

$$H = \hbar\omega(a^\dagger a + \frac{1}{2}). \tag{2.27}$$

Let's consider some features of a and $a^\dagger$. That gives $(a)^\dagger \neq a$, which looks slightly, because we have chosen the name wisely from the start. Please note that both the operators are Hermitian. Next, we will look at the commutation properties of the two operators. First, because x commutes with itself, and similarly for p, we have ($[x, p] = i\hbar$),

$$[a, a^\dagger] = \frac{1}{2\hbar}[x, -ip] + \frac{1}{2\hbar}[ip, x] = 1, \tag{2.28}$$

with the commutator, and the equation (2.27), we can find the commutation coefficients of H with a and $a^\dagger$,

$$\begin{aligned}[H, a] &= [\hbar\omega a^\dagger a, a] = (\hbar\omega a^\dagger aa - a\hbar\omega a^\dagger a) \\ &= \hbar\omega a(a^\dagger a - aa^\dagger) = -\hbar\omega a(aa^\dagger - a^\dagger a).\end{aligned} \tag{2.29}$$

We can calculate the similarity with $a^\dagger$ as well, or we can use conjugation equation Eq (2.28),

$$[H, a^\dagger] = \hbar\omega a^\dagger. \tag{2.30}$$

These new operators work on the energy-eigenstate $|E\rangle$, with energy E. The formalities are more common than it already is, and a note that the operator works in a quantum state, and are not the representative of this state-specific. With one of the above commutators, we have,

$$[H, a]|E\rangle = (Ha - aH)|E\rangle = (E - \hbar\omega)a|E\rangle. \tag{2.31}$$

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This equation means that $|E\rangle$ is an eigenvector H with energy $E - \hbar\omega$. The annihilation operator term used for a is, therefore, appropriate. We can repeat the calculation with the $a^\dagger$ instead of a, and we will get that $a^\dagger|E\rangle$ is the state of energy with energy $E + \hbar\omega$. Hence, $a^\dagger$ is the raising operators.

Now, we will find the energy of the lowest state. First, we realize that the energy is always positive, with the definition of the Hamiltonian we have used, because it consists of two non-negative terms, proportional to x^2 and p^2. This means that there is the lowest energy state which we will denote $|E_0\rangle$. The application of a in this state gives a state with the lower energy. The solution is that the state must vanish. Concerning the wave functions, this must mean that it is identical zero,

$$a|E_0\rangle = 0. \tag{2.32}$$

If we use the Hamiltonian on this state, we have,

$$H|E_0\rangle = \hbar\omega(a^\dagger a + \frac{1}{2})|E_0\rangle = \frac{1}{2}\hbar\omega|E_0\rangle \tag{2.33}$$

The lowest energy for the state is $\frac{\hbar\omega}{2}$. If the raising operator applied to this states, it would increase energy with $\hbar\omega$. This energy was precisely the amount that reduced by using a lower operator. We can repeat the arguments for the excited states. We, therefore, conclude that the operators are working on raising and lowering the same ladder of eigenstates. It is also seen clearly from the lowering operator decreases the energy and convert it to the lower energy eigenstates, and the raising operator will return the energy eigenstate to the state.

As a result, the final factor we can find that there is a change of state,

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which normalized by application of raising and lowering operators. We know from previous studies of the Hamiltonian and the actions of the a and $a^\dagger$, that, $a^\dagger a|E_n\rangle = n|E_n\rangle$. The formula in the set notation of the same equations read bra-ket as,

$$a^\dagger a|n\rangle = n|n\rangle, \tag{2.34}$$

we will choose a phase factor, and the normalized factor $n = \sqrt{n}$. Therefore, conclude that,

$$a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$
$$a|n\rangle = \sqrt{n}|n-1\rangle. \tag{2.35}$$

2.8 Dynamical map

We introduce the concept of dynamic maps and relationships with the theory of open systems. We assume that the initial state of the total system is the product states,

$$\rho(0) = \rho_S(0) \otimes \rho_E(0), \tag{2.36}$$

where ρ_S and ρ_E are the density matrices of system and environment, which have no relation at $t = 0$. The evolution of the state system can be expressed with the help of quantum dynamically map $K(t)$ in such a way,

$$\rho_S(0) \mapsto \rho_S(t) = K(t)\rho_S(0). \tag{2.37}$$

The dynamical map $K(t)$ is one of the one-parameters series which are trace-preserving. The map is a general description of the time evolution, where the initial state $\rho(0)$ is mapped onto the complete physical states $\rho(t)$, as shown in the commutative diagram in Figure 2.5.

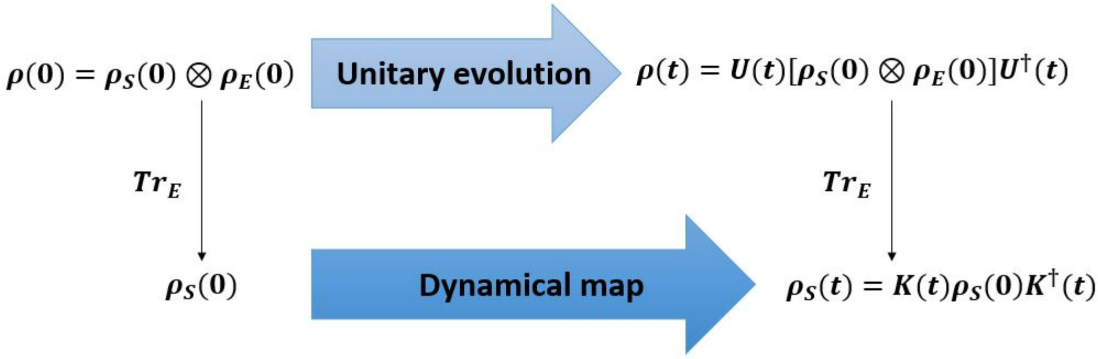

Figure 2.5 Diagram of the commutative dynamical map.

Now we have defined a dynamic map, K_t with all the features to guarantee one to get the physical state. We will discuss how to create such a map in the next chapter, which turns out to be the Kraus representation.

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CHAPTER III

CALCULATION METHOD: KRAUS REPRESENTATION

The time evolution of a closed quantum system can be described by an unitary operator $U(t)$. For open quantum systems, however, the time evolution is not unitary. The evolution of an open system may be described by the Kraus representation, which is constructed by considering a larger (closed) system. Usually, the general solutions for the master equation can be derived from the method of the Kraus representation.

3.1 Kraus representation for one qubit

The simplest system for the Kraus representation is one qubit system consist of a two-level atom inside a cavity. In this section we show the derivation of the Klaus operators for this simple qubit system, where the interaction between the atom and the cavity is simulated by the coupling between the atom and photons. The dynamical system is given by the total Hamiltonian ($\hbar = 1$):

$$\begin{aligned} H_{total} &= H_{atom} + H_\gamma + H_i \\ &= \frac{1}{2}\omega_0\sigma_z + \omega(a^\dagger a + \frac{1}{2}) + (\lambda a\sigma_+ + \lambda^* a^\dagger \sigma_-) \end{aligned} \quad (3.1)$$

with, $H_0 = H_{atom} + H_\gamma$, $[H_0, H_i] = 0$,

$\sigma_- = \sigma_x - i\sigma_y$

$\sigma_+ = \sigma_x + i\sigma_y$

σ_x, σ_y and σ_z are the Pauli matrices.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{3.2}$$

a, $a^\dagger$ are annihilation and creation operators for a photon.

ω is angular frequency.

and λ^*, λ are the photon-atom coupling constants.

The solution for the master equation of the density matrix $\rho_s(t)$ can be written in terms of the Kraus operators. The evaluation of the Kraus operators is to perform a partial trace over the environment to obtain the reduced state of the system, $\rho_S(t)$:

$$\begin{aligned}
\rho(t) &= U(t)\rho(0)U^\dagger(t) \\
\rho_S(t) &= \mathrm{Tr}_E(U(t)(\rho_S(0) \otimes \rho_E)U^\dagger(t)) \\
&= \sum_{n=0}^{\infty} \langle n| (U(t)\rho_S(0) \otimes \rho_E U^\dagger(t)) |n\rangle \\
&= \sum_{n=0}^{\infty} \langle n| (U(t)\rho_S(0) \otimes |0\rangle \langle 0| U^\dagger(t)) |n\rangle \\
&= \sum_{n=0}^{\infty} \langle n| U(t) |0\rangle \rho_S(0) \langle 0| U^\dagger(t) |n\rangle \\
\rho_S(t) &= \sum_{n=0}^{\infty} K_n(t)\rho_S(0)K_n^\dagger(t)
\end{aligned} \tag{3.3}$$

where $K_n(t)$ are the Kraus operator given by,

$$K_n(t) = \langle n| U(t) |0\rangle, \tag{3.4}$$

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In the equations above, $|n\rangle$ and $|0\rangle$ are the states of n photons and the vacuum, respectively. The density operator of the system and environment at $t = 0$ can be written in the forms $\rho(0) = \rho_S(0) \otimes \rho_E(0)$ and $\rho_E(0) = |0\rangle\langle 0|$. One can prove that H_0 and H_i are commutative, $[H_0, H_i] = 0$ when the oscillation frequencies of atoms and photons are the same, $\omega_0 = \omega$. Then we have,

$$\exp[-iH_0 t]\, H_i\, \exp[iH_0 t] = H_i, \tag{3.5}$$

and hence

$$\begin{aligned} |\psi(t)\rangle &= \exp[-iH_i t]|0\rangle \\ &= U(t)|0\rangle \\ &= \exp[-iH_i t]|0\rangle = \exp[-i\lambda(\sigma_- a^\dagger + \sigma_+ a)t]|0\rangle. \end{aligned} \tag{3.6}$$

The Kraus operators are evaluated,

$$\begin{aligned} K_n(t) &= \langle n|\mathbb{1} + (-i\lambda t)(\sigma_- a^\dagger + \sigma_+ a) + \tfrac{1}{2!}(-i\lambda t)^2(\sigma_- a^\dagger + \sigma_+ a)^2 \\ &+ \tfrac{1}{3!}(-i\lambda t)^3(\sigma_- a^\dagger + \sigma_+ a)^3 + \tfrac{1}{4!}(-i\lambda t)^4(\sigma_- a^\dagger + \sigma_+ a)^4 \\ &+ \tfrac{1}{5!}(-i\lambda t)^5(\sigma_- a^\dagger + \sigma_+ a)^5 + \tfrac{1}{6!}(-i\lambda t)^6(\sigma_- a^\dagger + \sigma_+ a)^6 \\ &+ \tfrac{1}{7!}(-i\lambda t)^7(\sigma_- a^\dagger + \sigma_+ a)^7 + \ldots|0\rangle. \end{aligned} \tag{3.7}$$

For m=0,

$$\langle n|\mathbb{1}|0\rangle = \mathbb{1}\langle n|0\rangle = \mathbb{1}\delta_{n0}. \tag{3.8}$$

For m=1,

$$\begin{aligned} \langle n|(-i\lambda t)(\sigma_- a^\dagger + \sigma_+ a)|0\rangle &= (-i\lambda t)[\langle n|\sigma_- a^\dagger|0\rangle + \langle n|\sigma_+ a|0\rangle] \\ &= (-i\lambda t)[\langle n|\sigma_-|1\rangle] \end{aligned}$$

$$= (-i\lambda t)\sigma_-[\langle n|1\rangle]$$

$$= (-i\lambda t)\sigma_-\delta_{n1}. \tag{3.9}$$

For m=2,

$$\langle n|\frac{1}{2!}(-i\lambda t)^2(\sigma_-a^\dagger + \sigma_+a)^2|0\rangle = \frac{1}{2!}(-i\lambda t)^2[\langle n|(\sigma_-\sigma_-a^\dagger a^\dagger|0\rangle + \langle n|\sigma_-\sigma_+a^\dagger a|0\rangle$$

$$+ \langle n|\sigma_+\sigma_-aa^\dagger|0\rangle + \langle n|\sigma_+\sigma_+aa|0\rangle]$$

$$= \frac{1}{2!}(-i\lambda t)^2(\langle n|\sigma_+\sigma_-|0\rangle)$$

$$= \frac{1}{2!}(-i\lambda t)^2(\sigma_+\sigma_-)\langle n|0\rangle$$

$$= \frac{1}{2!}(-i\lambda t)^2(\sigma_+\sigma_-)\delta_{n0}. \tag{3.10}$$

For m=3,

$$\langle n|\frac{1}{3!}(-i\lambda t)^3(\sigma_-a^\dagger + \sigma_+a)^3|0\rangle = \frac{1}{3!}(-i\lambda t)^3[\langle n|\sigma_+\sigma_-\sigma_+a^\dagger aa^\dagger|0\rangle$$

$$= \frac{1}{3!}(-i\lambda t)^3[(\sigma_+\sigma_-\sigma_+)\langle n|a^\dagger aa^\dagger|0\rangle$$

$$= \frac{1}{3!}(-i\lambda t)^3[(\sigma_-)\langle n|1\rangle$$

$$= \frac{1}{3!}(-i\lambda t)^3(\sigma_-)\delta_{n1}. \tag{3.11}$$

For m=4,

$$\langle n|\frac{1}{4!}(-i\lambda t)^4(\sigma_-a^\dagger + \sigma_+a)^2|0\rangle = \frac{1}{4!}(-i\lambda t)^4[\langle n|\sigma_+\sigma_-\sigma_+\sigma_-aa^\dagger aa^\dagger|0\rangle$$

$$= \frac{1}{4!}(-i\lambda t)^4[(\sigma_+\sigma_-\sigma_+\sigma_-)\langle n|aa^\dagger aa^\dagger|0\rangle$$

$$= \frac{1}{4!}(-i\lambda t)^4[(\sigma_+\sigma_-\sigma_+\sigma_-)\langle n|0\rangle$$

$$= \frac{1}{4!}(-i\lambda t)^4(\sigma_+\sigma_-)\delta_{n0}. \tag{3.12}$$

After calculating for each term, we can see the pattern of the results as follows:

$$\begin{aligned}
m &= 0 \rightarrow \delta_{n0}\mathbb{1} \\
m &= 1 \rightarrow (-i\lambda t)\delta_{n1}\sigma_- \\
m &= 2 \rightarrow \frac{1}{2!}(-i\lambda t)^2\delta_{n0}(\sigma_+\sigma_-) \\
m &= 3 \rightarrow \frac{1}{3!}(-i\lambda t)^3\delta_{n1}\sigma_- \\
m &= 4 \rightarrow \frac{1}{4!}(-i\lambda t)^4\delta_{n0}(\sigma_+\sigma_-) \\
m &= 5 \rightarrow \frac{1}{5!}(-i\lambda t)^5\delta_{n1}\sigma_- \\
m &= 6 \rightarrow \frac{1}{6!}(-i\lambda t)^6\delta_{n0}(\sigma_+\sigma_-).
\end{aligned} \tag{3.13}$$

Adding all the terms together, we derive,

$$K_0(t) = \mathbb{1} - (\sigma_+\sigma_-) + [1 - \frac{1}{2!}(\lambda t)^2 + \frac{1}{4!}(\lambda t)^4$$

$$- \frac{1}{6!}(\lambda t)^6 + \frac{1}{8!}(\lambda t)^8 - \frac{1}{10!}(\lambda t)^{10} + ...](\sigma_+\sigma_-). \tag{3.14}$$

Comparing with the Taylor series of sine and cosine functions,

$$\cos(x) = 1 - \frac{1}{2!}(x)^2 + \frac{1}{4!}(x)^4 - \frac{1}{6!}(x)^6 + \frac{1}{8!}(x)^8 - \frac{1}{10!}(x)^{10} + ... \tag{3.15}$$

$$\sin(x) = x - \frac{1}{3!}(x)^3 + \frac{1}{5!}(x)^5 - \frac{1}{7!}(x)^7 + \frac{1}{9!}(x)^9 - \frac{1}{11!}(x)^{11} + ... \tag{3.16}$$

we can write the Kraus operator, $K_0(t)$ in the compact form,

$$K_0(t) = \mathbb{1} + [\cos(\lambda t) - 1](\sigma_+\sigma_-), \tag{3.17}$$

In the same way, we derive the Kraus operator, $K_1(t)$

$$\begin{aligned} K_1(t) &= -i[(\lambda t) - \frac{1}{3!}(\lambda t)^3 + \frac{1}{5!}(\lambda t)^5 \\ &\quad - \frac{1}{7!}(\lambda t)^7 + \frac{1}{9!}(\lambda t)^9 - \frac{1}{11!}(\lambda t)^{11} + ...](\sigma_-) \\ &= -i\sin(\lambda t)(\sigma_-), \end{aligned} \tag{3.18}$$

The Kraus operators may be written explicitly in the matrix form,

$$K_0(t) = \begin{pmatrix} \cos(\lambda t) & 0 \\ 0 & 1 \end{pmatrix}, \quad K_0^\dagger(t) = \begin{pmatrix} \cos(\lambda t) & 0 \\ 0 & 1 \end{pmatrix},$$

$$K_1(t) = \begin{pmatrix} 0 & 0 \\ -i\sin(\lambda t) & 0 \end{pmatrix}, \quad K_1^\dagger(t) = \begin{pmatrix} 0 & i\sin(\lambda t) \\ 0 & 0 \end{pmatrix}. \tag{3.19}$$

The relation $\sum_{n=0}^{1} K_n^\dagger(t)K_n(t) = \mathbb{1}$ is time independent and satisfied by Kraus operators, which is shown in Appendix A.

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3.2 Kraus representation for two qubits

The model of our system consists of two two-level atoms inside the same cavity. We assume that these atoms are identical and allowed to interact via photon exchange with the cavity. The total Hamiltonian is given by ($\hbar = 1$),

$$H_{total} = H_{atoms} + H_\gamma + H_i$$

$$= \frac{1}{2}\omega_0\Sigma_z + \sum_i^{\infty}\omega_i(a_i^{\dagger}a_i + \frac{1}{2}) + \sum_i^{\infty}(\lambda a_i\Sigma_+ + \lambda^* a_i^{\dagger}\Sigma_-) \qquad (3.20)$$

where$\Sigma_i = (\sigma_i + \tau_i)$, $\{+, -\}$ stand for the raising and lowering operations, $a_i(a_i^{\dagger})$ are the annihilation (creation) operators of photons, and λ and λ^* are the photon-atom coupling constants with the same strength. The spin operators of atoms A and B are represented by σ and τ, respectively. The dynamical equation (master equation) may be solved directly in the Kraus operator representation.

We consider in this work the total Hamiltonian of the two qubits system with single mode,

$$H_{total} = \frac{1}{2}\omega_0\Sigma_z + \omega_1(a^{\dagger}a + \frac{1}{2}) + (\lambda a\Sigma_+ + \lambda^* a^{\dagger}\Sigma_-) \qquad (3.21)$$

The Kraus operators are evaluated by using the unitary operator $U(t)|0\rangle = e^{-iH_i t}|0\rangle = e^{-i\lambda(\Sigma_- a^{\dagger} + \Sigma_+ a)t}|0\rangle$,

$$\begin{aligned}
K_n(t) &= \langle n|U(t)|0\rangle \\
&= \langle n|\sum_{m=0}^{\infty}\frac{1}{m!}(-i\lambda t)^m(\Sigma_- a^{\dagger} + \Sigma_+ a)^m|0\rangle \\
&= \langle n|\mathbb{1} + (-i\lambda t)(\Sigma_- a^{\dagger} + \Sigma_+ a) + \frac{1}{2!}(-i\lambda t)^2(\Sigma_- a^{\dagger} + \Sigma_+ a)^2 \\
&\quad + \frac{1}{3!}(-i\lambda t)^3(\Sigma_- a^{\dagger} + \Sigma_+ a)^3 + \frac{1}{4!}(-i\lambda t)^4(\Sigma_- a^{\dagger} + \Sigma_+ a)^4 \\
&\quad + \frac{1}{5!}(-i\lambda t)^5(\Sigma_- a^{\dagger} + \Sigma_+ a)^5 + \frac{1}{6!}(-i\lambda t)^6(\Sigma_- a^{\dagger} + \Sigma_+ a)^6 \\
&\quad + \frac{1}{7!}(-i\lambda t)^7(\Sigma_- a^{\dagger} + \Sigma_+ a)^7 + ...|0\rangle, \qquad (3.22)
\end{aligned}$$

Note that we have expanded the unitary operator in Taylor series. Starting from $m = 0$, we calculate each term of the above equation.

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For $m = 0$,

$$\langle n|\mathbb{1}|0\rangle = \mathbb{1}\langle n|0\rangle = \mathbb{1}\delta_{n0}. \tag{3.23}$$

For m=1,

$$\begin{aligned}
\langle n|(-i\lambda t)(\Sigma_- a^\dagger + \Sigma_+ a)|0\rangle &= (-i\lambda t)[\langle n|\Sigma_- a^\dagger|0\rangle + \langle n|\Sigma_+ a|0\rangle] \\
&= (-i\lambda t)[\langle n|\Sigma_-|1\rangle] \\
&= (-i\lambda t)\Sigma_-[\langle n|1\rangle] \\
&= (-i\lambda t)\Sigma_-\delta_{n1}.
\end{aligned} \tag{3.24}$$

For m=2,

$$\begin{aligned}
\langle n|\frac{1}{2!}(-i\lambda t)^2(\Sigma_- a^\dagger + \Sigma_+ a)^2|0\rangle &= \frac{1}{2!}(-i\lambda t)^2[\langle n|(\Sigma_-\Sigma_- a^\dagger a^\dagger|0\rangle + \langle n|\Sigma_-\Sigma_+ a^\dagger a|0\rangle \\
&+ \langle n|\Sigma_+\Sigma_- a a^\dagger|0\rangle + \langle n|\Sigma_+\Sigma_+ a a|0\rangle] \\
&= \frac{1}{2!}(-i\lambda t)^2[\sqrt{2}\langle n|\Sigma_-\Sigma_-|2\rangle + \langle n|\Sigma_+\Sigma_-|0\rangle] \\
&= \frac{1}{2!}(-i\lambda t)^2[\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle + (\Sigma_+\Sigma_-)\langle n|0\rangle] \\
&= \frac{1}{2!}(-i\lambda t)^2[\sqrt{2}(\Sigma_-\Sigma_-)\delta_{n2} + (\Sigma_+\Sigma_-)\delta_{n0}].
\end{aligned} \tag{3.25}$$

For m=3,

$$\begin{aligned}
\langle n|\frac{1}{3!}(-i\lambda t)^3(\Sigma_- a^\dagger + \Sigma_+ a)^3|0\rangle &= \frac{1}{3!}(-i\lambda t)^3[\langle n|\Sigma_-\Sigma_+\Sigma_- a^\dagger a a^\dagger|0\rangle + \langle n|\Sigma_+\Sigma_-\Sigma_- a a^\dagger a^\dagger|0\rangle \\
&= \frac{1}{3!}(-i\lambda t)^3[\langle n|\Sigma_-\Sigma_+\Sigma_-|1\rangle + 2\langle n|\Sigma_+\Sigma_-\Sigma_-|1\rangle] \\
&= \frac{1}{3!}(-i\lambda t)^3[2(\Sigma_-)\langle n|1\rangle + 2(\Sigma_+\Sigma_-\Sigma_-)\langle n|1\rangle]
\end{aligned}$$

$$= \frac{1}{3!}(-i\lambda t)^3[2(\Sigma_-) + 2(\ \Sigma_+\Sigma_-\Sigma_-)]\delta_{n1}. \qquad (3.26)$$

For m=4,

$$\langle n|\frac{1}{4!}(-i\lambda t)^4(\Sigma_- a^\dagger + \Sigma_+ a)^4|0\rangle = \frac{1}{4!}(-i\lambda t)^4[\langle n|\ \Sigma_-\Sigma_-\Sigma_+\Sigma_- a^\dagger a^\dagger a a^\dagger\ |0\rangle$$

$$+ \langle n|\ \Sigma_-\Sigma_+\Sigma_-\Sigma_- a^\dagger a a^\dagger a^\dagger\ |0\rangle + \langle n|\ \Sigma_+\Sigma_-\Sigma_+\Sigma_- a a^\dagger a a^\dagger\ |0\rangle$$

$$+ \langle n|\ \Sigma_+\Sigma_+\Sigma_-\Sigma_- a a a^\dagger a^\dagger\ |0\rangle]$$

$$= \frac{1}{4!}(-i\lambda t)^4[2\sqrt{2}\langle n|\Sigma_-\Sigma_-|2\rangle + 4\sqrt{2}\langle n|\Sigma_-\Sigma_-|2\rangle$$

$$+ 2\langle n|\Sigma_+\Sigma_-|0\rangle + 2\langle n|\Sigma_+\Sigma_+\Sigma_-\Sigma_-|0\rangle]$$

$$= \frac{1}{4!}(-i\lambda t)^4[2\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle + 4\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle$$

$$+ 2(\ \Sigma_+\Sigma_-)\langle n|0\rangle + 2(\ \Sigma_+\Sigma_+\Sigma_-\Sigma_-)\langle n|0\rangle]$$

$$= \frac{1}{4!}(-i\lambda t)^4[2(\Sigma_+\Sigma_-)\delta_{n0}$$

$$+ 2(\ \Sigma_+\Sigma_+\Sigma_-\Sigma_-)\delta_{n0} + 6\sqrt{2}(\Sigma_-\Sigma_-)\delta_{n2}]. \qquad (3.27)$$

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For m=5,

$$\langle n|\frac{1}{5!}(-i\lambda t)^5(\Sigma_- a^\dagger + \Sigma_+ a)^5|0\rangle = \frac{1}{5!}(-i\lambda t)^5[\langle n|\ \Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_- a^\dagger a a^\dagger a a^\dagger\ |0\rangle$$

$$+ \langle n|\ \Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_- a^\dagger a a a^\dagger a^\dagger\ |0\rangle$$

$$+ \langle n|\ \Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_- a a^\dagger a^\dagger a a^\dagger\ |0\rangle$$

$$+ \langle n|\ \Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_- a a^\dagger a a^\dagger a^\dagger\ |0\rangle]$$

$$= \frac{1}{5!}(-i\lambda t)^5[4\langle n|\Sigma_-|1\rangle + 4\langle n|\Sigma_+\Sigma_-\Sigma_-|1\rangle$$

$$+ 4\langle n|\Sigma_+\Sigma_-\Sigma_-|1\rangle + 8\langle n|\Sigma_+\Sigma_-\Sigma_-|1\rangle]$$

$$= \frac{1}{5!}(-i\lambda t)^5[4(\Sigma_-)\langle n|1\rangle + 4(\Sigma_+\Sigma_-\Sigma_-)\langle n|1\rangle$$

$$+ 4(\Sigma_+\Sigma_-\Sigma_-)\langle n|1\rangle + 8(\Sigma_+\Sigma_-\Sigma_-)\langle n|1\rangle]$$

$$= \frac{1}{5!}(-i\lambda t)^5[4(\Sigma_-) + 16(\Sigma_+\Sigma_-\Sigma_-)]\delta_{n1}. \quad (3.28)$$

For m=6,

$$\langle n|\frac{1}{6!}(-i\lambda t)^6(\Sigma_- a^\dagger + \Sigma_+ a)^6|0\rangle = \frac{1}{6!}(-i\lambda t)^6[\langle n|\,\Sigma_-\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_- a^\dagger a^\dagger a a^\dagger a a^\dagger\,|0\rangle$$

$$+ \langle n|\,\Sigma_-\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_- a^\dagger a^\dagger a a a^\dagger a^\dagger\,|0\rangle$$

$$+ \langle n|\,\Sigma_-\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_- a^\dagger a a^\dagger a^\dagger a a^\dagger\,|0\rangle$$

$$+ \langle n|\,\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_- a^\dagger a a^\dagger a a^\dagger a^\dagger\,|0\rangle$$

$$+ \langle n|\,\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_- a a^\dagger a a^\dagger a a^\dagger\,|0\rangle$$

$$+ \langle n|\,\Sigma_+\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_- a a^\dagger a a a^\dagger a^\dagger\,|0\rangle$$

$$+ \langle n|\,\Sigma_+\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_- a a a^\dagger a^\dagger a a^\dagger\,|0\rangle$$

$$+ \langle n|\,\Sigma_+\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_- a a a^\dagger a a^\dagger a^\dagger\,|0\rangle]$$

$$= \frac{1}{6!}(-i\lambda t)^6[4\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle + 8\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle$$

$$+ 8\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle + 16\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle$$

$$+ 4(\Sigma_+\Sigma_-)\langle n|0\rangle + 4(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)\langle n|0\rangle$$

$$+ 4(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)\langle n|0\rangle + 8(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)\langle n|0\rangle]$$

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$$= \frac{1}{6!}(-i\lambda t)^6[4(\Sigma_+\Sigma_-)\delta_{n0} + 16(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)\delta_{n0}$$

$$+ 36\sqrt{2}(\Sigma_-\Sigma_-)\delta_{n2}]. \tag{3.29}$$

For m=7,

$$\langle n|\frac{1}{7!}(-i\lambda t)^7(\Sigma_-a^\dagger + \Sigma_+a)^7|0\rangle = \frac{1}{7!}(-i\lambda t)^7[\langle n|\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-a^\dagger aa^\dagger aa^\dagger aa^\dagger|0\rangle$$

$$+ \langle n|\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_-a^\dagger aa^\dagger aaa^\dagger a^\dagger|0\rangle$$

$$+ \langle n|\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_-a^\dagger aaa^\dagger a^\dagger aa^\dagger|0\rangle$$

$$+ \langle n|\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_-a^\dagger aaa^\dagger aa^\dagger a^\dagger|0\rangle$$

$$+ \langle n|\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-aa^\dagger a^\dagger aa^\dagger aa^\dagger|0\rangle$$

$$+ \langle n|\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_-aa^\dagger a^\dagger aaa^\dagger a^\dagger|0\rangle$$

$$+ \langle n|\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_-aa^\dagger aa^\dagger a^\dagger aa^\dagger|0\rangle$$

$$+ \langle n|\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_-aa^\dagger aa^\dagger aa^\dagger a^\dagger|0\rangle$$

$$+ \langle n|\Sigma_+\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_-\Sigma_-aaa^\dagger a^\dagger aa^\dagger a^\dagger|0\rangle]$$

$$= \frac{1}{7!}(-i\lambda t)^7[8\langle n|\Sigma_-|1\rangle + 8\langle n|\Sigma_+\Sigma_-\Sigma_-|1\rangle$$

$$+ 8\langle n|\Sigma_+\Sigma_-\Sigma_-|1\rangle + 16\langle n|\Sigma_+\Sigma_-\Sigma_-|1\rangle]$$

$$+ 8\langle n|\Sigma_+\Sigma_-\Sigma_-|1\rangle + 16\langle n|\Sigma_+\Sigma_-\Sigma_-|1\rangle]$$

$$+ 16\langle n|\Sigma_+\Sigma_-\Sigma_-|1\rangle + 32\langle n|\Sigma_+\Sigma_-\Sigma_-|1\rangle]$$

$$= \frac{1}{7!}(-i\lambda t)^7[8(\Sigma_-)\langle n|1\rangle + 8(\Sigma_+\Sigma_-\Sigma_-)\langle n|1\rangle$$

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$$+ 8(\Sigma_+\Sigma_-\Sigma_-)\langle n|1\rangle + 16(\Sigma_+\Sigma_-\Sigma_-)\langle n|1\rangle]$$

$$+ 8(\Sigma_+\Sigma_-\Sigma_-)\langle n|1\rangle + 16(\Sigma_+\Sigma_-\Sigma_-)\langle n|1\rangle]$$

$$+ 16(\Sigma_+\Sigma_-\Sigma_-)\langle n|1\rangle + 32(\Sigma_+\Sigma_-\Sigma_-)\langle n|1\rangle]$$

$$= \frac{1}{7!}(-i\lambda t)^7[8(\Sigma_-) + 104(\Sigma_+\Sigma_-\Sigma_-)]\delta_{n1}. \qquad (3.30)$$

For m=8,

$$\langle n|\frac{1}{8!}(-i\lambda t)^8(\Sigma_-a^\dagger + \Sigma_+a)^8|0\rangle = \frac{1}{8!}(-i\lambda t)^8[\langle n|\Sigma_-\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-a^\dagger a^\dagger aa^\dagger aa^\dagger aa^\dagger|0\rangle$$

$$+ \langle n|\Sigma_-\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_-a^\dagger a^\dagger aaa^\dagger a^\dagger aa^\dagger|0\rangle$$

$$+ \langle n|\Sigma_-\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-a^\dagger aa^\dagger a^\dagger aa^\dagger aa^\dagger|0\rangle$$

$$+ \langle n|\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-aa^\dagger aa^\dagger aa^\dagger aa^\dagger|0\rangle$$

$$+ \langle n|\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_-aa^\dagger aa^\dagger aaa^\dagger a^\dagger|0\rangle$$

$$+ \langle n|\Sigma_+\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_-aa^\dagger aaa^\dagger aa^\dagger a^\dagger|0\rangle$$

$$+ \langle n|\Sigma_+\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-aaa^\dagger a^\dagger aa^\dagger aa^\dagger|0\rangle$$

$$+ \langle n|\Sigma_+\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_-aaa^\dagger a^\dagger aaa^\dagger a^\dagger|0\rangle$$

$$+ \langle n|\Sigma_+\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_-aaa^\dagger aa^\dagger a^\dagger aa^\dagger|0\rangle$$

$$+ \langle n|\Sigma_+\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_-aaa^\dagger aa^\dagger aa^\dagger a^\dagger|0\rangle$$

$$+ \langle n|\Sigma_-\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_-a^\dagger a^\dagger aa^\dagger aaa^\dagger a^\dagger|0\rangle$$

$$+ \langle n|\Sigma_-\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_-a^\dagger a^\dagger aaa^\dagger aa^\dagger a^\dagger|0\rangle$$

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$$+\langle n|\Sigma_-\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_-a^\dagger aa^\dagger a^\dagger aaa^\dagger a^\dagger|0\rangle$$

$$+\langle n|\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_-\Sigma_+\Sigma_-a^\dagger aa^\dagger aa^\dagger a^\dagger aa^\dagger|0\rangle$$

$$+\langle n|\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_-a^\dagger aa^\dagger aa^\dagger aa^\dagger a^\dagger|0\rangle$$

$$+\langle n|\Sigma_+\Sigma_-\Sigma_+\Sigma_-\Sigma_+\Sigma_+\Sigma_-\Sigma_-aa^\dagger aa^\dagger aaa^\dagger a^\dagger|0\rangle]$$

$$=\frac{1}{8!}(-i\lambda t)^8[8\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle$$

$$+16\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle+16\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle$$

$$+32\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle+16\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle$$

$$+32\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle+32\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle$$

$$+64\sqrt{2}(\Sigma_-\Sigma_-)\langle n|2\rangle+8(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)\langle n|0\rangle$$

$$+8(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)\langle n|0\rangle+16(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)\langle n|0\rangle]$$

$$+8(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)\langle n|0\rangle+16(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)\langle n|0\rangle$$

$$+16(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)\langle n|0\rangle]+32(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)\langle n|0\rangle$$

$$+8(\Sigma_+\Sigma_-)\langle n|0\rangle]$$

$$=\frac{1}{8!}(-i\lambda t)^8[8(\Sigma_+\Sigma_-)\delta_{n0}+104(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)\delta_{n0}$$

$$+216\sqrt{2}(\Sigma_-\Sigma_-)\delta_{n2}]. \tag{3.31}$$

After calculating for each term, we see the pattern of the results as follows:

$$m=0\rightarrow\delta_{n0}\mathbb{1}$$

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$$m = 1 \rightarrow \delta_{n1}\Sigma_-$$

$$m = 2 \rightarrow \frac{1}{2!}(-i\lambda t)^2[\delta_{n0}(\Sigma_+\Sigma_-) + \delta_{n2}\sqrt{2}(\Sigma_-\Sigma_-)]$$

$$m = 3 \rightarrow \frac{1}{3!}(-i\lambda t)^3(\delta_{n1}[2(\Sigma_-) + 2(\Sigma_+\Sigma_-\Sigma_-)])$$

$$m = 4 \rightarrow \frac{1}{4!}(-i\lambda t)^4[\delta_{n0}2(\Sigma_+\Sigma_-) + \delta_{n0}2(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)$$

$$+ \delta_{n2}6\sqrt{2}(\Sigma_-\Sigma_-)]$$

$$m = 5 \rightarrow \frac{1}{5!}(-i\lambda t)^5(\delta_{n1}[4(\Sigma_-) + 16(\Sigma_+\Sigma_-\Sigma_-)])$$

$$m = 6 \rightarrow \frac{1}{6!}(-i\lambda t)^6[\delta_{n0}4(\Sigma_+\Sigma_-) + \delta_{n0}16(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)$$

$$+ \delta_{n2}36\sqrt{2}(\Sigma_-\Sigma_-)]$$

$$m = 7 \rightarrow \frac{1}{7!}(-i\lambda t)^7(\delta_{n1}[8(\Sigma_-) + 104(\Sigma_+\Sigma_-\Sigma_-)])$$

$$m = 8 \rightarrow \frac{1}{8!}(-i\lambda t)^8[\delta_{n0}8(\Sigma_+\Sigma_-) + \delta_{n0}104(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)$$

$$+ \delta_{n2}216\sqrt{2}(\Sigma_-\Sigma_-)]$$

$$m = 9 \rightarrow \frac{1}{9!}(-i\lambda t)^9(\delta_{n1}[16(\Sigma_-) + 640(\Sigma_+\Sigma_-\Sigma_-)])$$

$$m = 10 \rightarrow \frac{1}{10!}(-i\lambda t)^{10}[\delta_{n0}16(\Sigma_+\Sigma_-) + \delta_{n0}640(\Sigma_+\Sigma_+\Sigma_-\Sigma_-)$$

$$+ \delta_{n2}1296\sqrt{2}(\Sigma_-\Sigma_-)]. \tag{3.32}$$

Next, we collect all the calculations to find the term Kraus operators. We have

$$K_0(t) = \mathbb{1} - (\Sigma_+\Sigma_-) - (\Sigma_+\Sigma_+\Sigma_-\Sigma_-) + [1 - \frac{1}{2!}(\lambda t)^2(1) + \frac{1}{4!}(\lambda t)^4(2)$$

$$
\begin{aligned}
&- \frac{1}{6!}(\lambda t)^6(4) + \frac{1}{8!}(\lambda t)^8(8) - \frac{1}{10!}(\lambda t)^{10}(16) + ...](\Sigma_+\Sigma_-) \\
&+ [1 - \frac{1}{2!}(\lambda t)^2(0) + \frac{1}{4!}(\lambda t)^4(2) - \frac{1}{6!}(\lambda t)^6(16) + \frac{1}{8!}(\lambda t)^8(104) \\
&- \frac{1}{10!}(\lambda t)^{10}(640) + ...](\Sigma_+\Sigma_+\Sigma_-\Sigma_-),
\end{aligned} \tag{3.33}
$$

$$
\begin{aligned}
K_1(t) &= [(\lambda t) - \frac{1}{3!}(\lambda t)^3(2) + \frac{1}{5!}(\lambda t)^5(4) - \frac{1}{7!}(\lambda t)^7(8) \\
&+ \frac{1}{9!}(\lambda t)^9(16) - ...](\Sigma_-) + [(\lambda t)(0) - \frac{1}{3!}(\lambda t)^3(2) \\
&+ \frac{1}{5!}(\lambda t)^5(16) - \frac{1}{7!}(\lambda t)^7(104) + \frac{1}{9!}(\lambda t)^9(640) - ...](\Sigma_+\Sigma_-\Sigma_-) \\
K_1(t) &= \frac{-i}{\sqrt{2}}\sin(\sqrt{2}\lambda t)(\Sigma_-) \\
&+ \frac{i}{6\sqrt{2}}[3\sin(\sqrt{2}\lambda t) + \sqrt{3}\sin(\sqrt{6}\lambda t)](\Sigma_+\Sigma_-\Sigma_-).
\end{aligned} \tag{3.34}
$$

$$
\begin{aligned}
K_2(t) &= [1 - \frac{1}{2!}(\lambda t)^2(1) + \frac{1}{4!}(\lambda t)^4(6) - \frac{1}{6!}(\lambda t)^6(36) \\
&+ \frac{1}{8!}(\lambda t)^8(216) - \frac{1}{10!}(\lambda t)^{10}(1296) + ...]\sqrt{2}(\Sigma_-\Sigma_-),
\end{aligned} \tag{3.35}
$$

Applying the power series,

$$
1, 2, 4, 8, 16, 32, ... = 2^{n-1} \tag{3.36}
$$

$$
0, 2, 16, 104, 640, ... = 2^{n-2}(3^{n-1} - 1) \tag{3.37}
$$

we get

$$
K_0(t) = \mathbb{1} + [\cos^2(\frac{\lambda t}{\sqrt{2}}) - 1](\Sigma_+\Sigma_-)
$$

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$$+[\frac{1}{12}(14-3\cos(\sqrt{2}\lambda t)+\cos(\sqrt{6}\lambda t))-1](\Sigma_+\Sigma_+\Sigma_-\Sigma_-). \tag{3.38}$$

$$K_1(t)=\frac{-i}{\sqrt{2}}\sin(\sqrt{2}\lambda t)(\Sigma_-)$$
$$+\frac{i}{6\sqrt{2}}[3\sin(\sqrt{2}\lambda t)+\sqrt{3}\sin(\sqrt{6}\lambda t)](\Sigma_+\Sigma_-\Sigma_-). \tag{3.39}$$

Applying the power series,

$$1,6,36,216,1296,...=6^{n-1} \tag{3.40}$$

we get

$$K_2(t)=[\frac{1}{3\sqrt{2}}(\cos(\sqrt{6}\lambda t)-1)](\Sigma_-\Sigma_-). \tag{3.41}$$

Now we have the Kraus operators,

$$K_n(t)=\delta_{n0}[\mathbb{1}+[\cos^2(\frac{\lambda t}{\sqrt{2}})-1](\Sigma_+\Sigma_-)$$
$$+[\frac{1}{12}(14-3\cos(\sqrt{2}\lambda t)+\cos(\sqrt{6}\lambda t))-1](\Sigma_+^2\Sigma_-^2)]$$
$$+\delta_{n1}[\frac{-i}{\sqrt{2}}\sin(\sqrt{2}\lambda t)(\Sigma_-)+\frac{i}{6\sqrt{2}}[3\sin(\sqrt{2}\lambda t)+\sqrt{3}\sin(\sqrt{6}\lambda t)](\Sigma_+\Sigma_-^2)]$$
$$+\delta_{n2}[\frac{1}{3\sqrt{2}}(\cos(\sqrt{6}\lambda t)-1)](\Sigma_-^2)]. \tag{3.42}$$

The relation $\sum_{n=0}^{2}K_n^\dagger(t)K_n(t)=\mathbb{1}$ is time independent and satisfied by Kraus operators, as shown in Appendix B.

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CHAPTER IV

SUMMARY

In this work, we have derived the Kraus operators of the entangled system consist of two two-level atoms interacting with the same cavity (photon bath),

$$K_0(t) = \begin{pmatrix} \frac{1}{3}(2+\cos(\sqrt{6}\lambda)) & 0 & 0 & 0 \\ 0 & \cos^2(\frac{\lambda}{\sqrt{2}}) & -\sin^2(\frac{\lambda}{\sqrt{2}}) & 0 \\ 0 & -\sin^2(\frac{\lambda}{\sqrt{2}}) & \cos^2(\frac{\lambda}{\sqrt{2}}) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$K_1(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{i}{\sqrt{6}}\sin(\sqrt{6}\lambda) & 0 & 0 & 0 \\ \frac{i}{\sqrt{6}}\sin(\sqrt{6}\lambda) & 0 & 0 & 0 \\ 0 & -\frac{i}{\sqrt{2}}\sin(\sqrt{2}\lambda) & -\frac{i}{\sqrt{2}}\sin(\sqrt{2}\lambda) & 0 \end{pmatrix},$$

$$K_2(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{3}(\cos(\sqrt{6}\lambda)-1) & 0 & 0 & 0 \end{pmatrix}. \tag{4.1}$$

where the oscillation frequencies of atoms and photons are the same. The relation $\sum\limits_{\eta=0}^{2} K_\eta^\dagger(t)K_\eta(t) = \mathbb{1}$ is time independent and satisfied. The results of the Kraus operators indicate that the system may have only three states, that is, no photon ($K_0(t)$), one photon ($K_1(t)$) and two photons ($K_2(t)$). Two photons in this system can be entangled between two atoms.

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The application of the Kraus operators to analyze the time evolution of the density matrix is underway. We expect to get the results in the near future.

The work here may be largely extended by doing the followings:

- Evaluate the Kraus operators for the entangled system consist of two two-level atoms interacting with the same cavity, where the oscillation frequencies of atoms and photons are different.
- Evaluate the Kraus operators for the entangled system consist of two two-level atoms interacting with the same cavity, where the interaction between the two atoms, for example, the spin-spin coupling, is included. The calculations may be carried out for both the equal and unequal oscillation frequencies of atoms and photons.
- Analyze the time evolution of the density matrix by applying the Kraus operators derived in all the cases mentioned above.

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REFERENCES

REFERENCES

A. Einstein, B. P. and Rosen, N. (1935). Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? **Phys. Rev.** 47(10): 777-780: 213.

Barnett, S. M. (2010). Quantum Information. Oxford Master Series in Physics. Oxford University Press, Oxford.

Breuer, H. P. and Petruccione, F. (2002). The Theory of Open Systems. Oxford University Press, New York.

Dodd, P. J. and Halliwell, J. J. (2004). Disentanglement and decoherence by open system dynamics. **Phys. Rev. A** 69: 052105.

Kraus, K. (1983). States, Effects, and Operations-Fundamental Notions of Quantum Theory. Springer, Berlin.

Liu, R. F. and Chen, C. C. (2006). Role of the Bell singlet state in the suppression of disentanglement. **Phys. Rev. A** 74: 024102.

M. ArsenijeviÄĞ, J. J.-D. and DugiÄĞ, M. (2017). Generalized Kraus Operators for the One-Qubit Depolarizing Quantum Channel. **Brazilian Journal of Physics** 47: 339-349.

Marinescu, D. C. and Marinescug, G. M. (2011). Classical and Quantum Information. Academic Press, Amsterdam.

Nielsen, M. A. and Chuang, I. L. (2010). Computation and Quantum Information. 10th Anniversary. Cambridge University Press, Cambridge.

Peres, A. (1993). Quantum Theory: Concepts and Methods. Dordrecht; Boston: Kluwer Academic, Boston.

Preskill, J. (2004). Lecture Notes on Quantum Information and Quantum Computation. Lecture Notes. California Institute of Technology, California.

Rivas, A. and Huelga, S. F. (2012). Open Quantum System: An Introduction. Springer, London.

Sych, D. and Leuchs, G. (2009). A complete basis of generalized Bell states. **New Journal of Physics.** 11: 9.

Tanas, R. and Ficek, Z. (2004). Entangling two atoms via spontaneous emission. **Journal of Optics B-quantum And Semiclassical Optics.** 63: S90-S97.

Yu, T. and Eberly, J. H. (2004). Finite-Time Disentanglement Via Spontaneous Emission. **Phys. Rev. Lett.** 93: 140404.

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APPENDICES

APPENDIX A

KRAUS REPRESENTATION FOR ONE-QUBIT

We could evaluate the results by using the relation $\sum_{n=0}^{\infty} K_n^\dagger(t) K_n(t) = \mathbb{1}$ which is time independent and satisfied by Kraus operators. Note that we have followed the condition of the two-qubit system in our calculation where the parameter $\lambda t \neq 0$).

Recall the Kraus matrice here,

$$K_0(t) = \begin{pmatrix} \cos(\lambda t) & 0 \\ 0 & 1 \end{pmatrix}, \quad K_0^\dagger(t) = \begin{pmatrix} \cos(\lambda t) & 0 \\ 0 & 1 \end{pmatrix},$$

$$K_1(t) = \begin{pmatrix} 0 & 0 \\ -i\sin(\lambda t) & 0 \end{pmatrix}, \quad K_1^\dagger(t) = \begin{pmatrix} 0 & i\sin(\lambda t) \\ 0 & 0 \end{pmatrix}. \tag{A.1}$$

We could check the results of Kraus operators as:

$$\sum_{n=0}^{1} K_n^\dagger K_n = K_0^\dagger(t) K_0(t) + K_1^\dagger(t) K_1(t), \tag{A.2}$$

$$\sum_{n=0}^{1} K_n^\dagger(t) K_n(t) = \begin{pmatrix} \cos(\lambda t) & 0 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \cos(\lambda t) & 0 \\ 0 & 1 \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & i\sin(\lambda t) \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 0 \\ -i\sin(\lambda t) & 0 \end{pmatrix}$$

$$\sum_{n=0}^{1} K_n^\dagger(t)K_n(t) = \begin{pmatrix} \cos^2(\lambda t) + \sin^2(\lambda t) & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathbb{1}, \tag{A.3}$$

where $\cos^2(x) + \sin^2(x) = 1$. Thus, the Kraus operators for one-qubit system is satisfied.

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APPENDIX B

KRAUS REPRESENTATION FOR TWO-QUBITS

We could evaluate the results by using the relation $\sum_{n=0}^{\infty} K_n^{\dagger}(t)K_n(t) = \mathbb{1}$ which is time independent and satisfied by Kraus operators. Note that we have followed the condition of the two-qubit system in our calculation where the parameter $\lambda t \neq 0$).

$$
K_0(t) = \begin{pmatrix} \frac{1}{3}(2+\cos(\sqrt{6}\lambda t)) & 0 & 0 & 0 \\ 0 & \cos^2(\frac{\lambda t}{\sqrt{2}}) & -\sin^2(\frac{\lambda t}{\sqrt{2}}) & 0 \\ 0 & -\sin^2(\frac{\lambda t}{\sqrt{2}}) & \cos^2(\frac{\lambda t}{\sqrt{2}}) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},
$$

$$
K_0^{\dagger}(t) = \begin{pmatrix} \frac{1}{3}(2+\cos(\sqrt{6}\lambda t)) & 0 & 0 & 0 \\ 0 & \cos^2(\frac{\lambda t}{\sqrt{2}}) & -\sin^2(\frac{\lambda t}{\sqrt{2}}) & 0 \\ 0 & -\sin^2(\frac{\lambda t}{\sqrt{2}}) & \cos^2(\frac{\lambda t}{\sqrt{2}}) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad \text{(B.1)}
$$

$$
K_1(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{i}{\sqrt{6}}\sin(\sqrt{6}\lambda t) & 0 & 0 & 0 \\ \frac{i}{\sqrt{6}}\sin(\sqrt{6}\lambda t) & 0 & 0 & 0 \\ 0 & -\frac{i}{\sqrt{2}}\sin(\sqrt{2}\lambda t) & -\frac{i}{\sqrt{2}}\sin(\sqrt{2}\lambda t) & 0 \end{pmatrix},
$$

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$$K_1^\dagger(t) = \begin{pmatrix} 0 & \frac{-i}{\sqrt{6}}\sin(\sqrt{6}\lambda t) & \frac{-i}{\sqrt{6}}\sin(\sqrt{6}\lambda t) & 0 \\ 0 & 0 & 0 & \frac{i}{\sqrt{2}}\sin(\sqrt{2}\lambda t) \\ 0 & 0 & 0 & \frac{i}{\sqrt{2}}\sin(\sqrt{2}\lambda t) \\ 0 & & & 0 \end{pmatrix}, \tag{B.2}$$

$$K_2(t) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\sqrt{2}}{3}(\cos(\sqrt{6}\lambda t) - 1) & 0 & 0 & 0 \end{pmatrix},$$

$$K_2^\dagger(t) = \begin{pmatrix} 0 & 0 & 0 & \frac{\sqrt{2}}{3}(\cos(\sqrt{6}\lambda t) - 1) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{B.3}$$

We could check the results of Kraus operators as:

$$\sum_{n=0}^{2} K_n^\dagger(t)K_n(t) = K_0^\dagger(t)K_0(t) + K_1^\dagger(t)K_1(t) + K_2^\dagger(t)K_2(t), \tag{B.4}$$

$$\sum_{n=0}^{2} K_n^\dagger(t)K_n(t) = \begin{pmatrix} \frac{1}{9}(2+\cos(\sqrt{6}\lambda t))^2 & 0 & 0 & 0 \\ 0 & \frac{1}{4}(3+\cos(2\sqrt{2}\lambda t)) & -\frac{1}{2}\sin^2(\sqrt{2}\lambda t) & 0 \\ 0 & -\frac{1}{2}\sin^2(\sqrt{2}\lambda t) & \frac{1}{4}(3+\cos(2\sqrt{2}\lambda t)) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$+ \begin{pmatrix} \frac{1}{3}\sin^2(\sqrt{6}\lambda t) & 0 & 0 & 0 \\ 0 & \frac{1}{2}\sin^2(\sqrt{2}\lambda t) & \frac{1}{2}\sin^2(\sqrt{2}\lambda t) & 0 \\ 0 & \frac{1}{2}\sin^2(\sqrt{2}\lambda t) & \frac{1}{2}\sin^2(\sqrt{2}\lambda t) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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$$+\begin{pmatrix} \frac{8}{9}\sin^4(\sqrt{\frac{3}{2}}\lambda t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\sum_{n=0}^{2} K_n^\dagger(t)K_n(t) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \mathbb{1}. \quad \text{(B.5)}$$

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