ENERGY EFFICIENT DESIGN OF 5G MASSIVE MIMO

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Suranaree University of Technology has approved this thesis submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy.

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อาราฟัต อาเหม็ด ข่าน : การออกแบบพลังงานอย่างมีประสิทธิภาพของระบบแมสซีพไมโมสำหรับการสื่อสารยุคที่ห้า (ENERGY EFFICIENT DESIGN OF 5G MASSIVE MIMO) อาจารย์ที่ปรึกษา : รองศาสตราจารย์ ดร.พีระพงษ์ อุฑารสกุล, 132 หน้า.

เทคโนโลยีไมโมขนาดใหญ่คือการใช้สายอากาศที่มีจำนวนมากร่วมกันส่งสัญญาณทำให้มีการใช้สเปคตรัมที่มีประสิทธิภาพสูงแต่ก็เกิดปัญหาในการใช้กำลังงานสูงด้วยเพื่อให้ลดการใช้พลังงานลงและให้มีความง่ายมากขึ้นในการประมวลผลสัญญาณวิทยานิพนธ์ฉบับนี้ได้ทำการออกแบบเทคโนโลยีไมโมขนานใหญ่ให้มีประสิทธิภาพการใช้กำลังงานโดยการพิจารณาจากวิธีการเชื่อมสัมพันธ์กันระหว่างสายอากาศปลายทางตามแนวกับการได้รับผลกระทบของตัวขยายแบบไม่เป็นเชิงเส้นในแต่ละลูกโซ่ของสายอากาศส่งโดยมีแนวคิดในการออกแบบระบบด้วยการคำนวณจำนวนสมดุลของตัวส่งและตัวรับกับการสมดุลค่าพลังงานที่ถูกส่งการวิเคราะห์จะให้ความสำคัญในส่วนของประสิทธิภาพกำลังงานที่คาดหวังของเทคโนโลยีไมโมขนานใหญ่ภายใต้เงื่อนไขช่องสัญญาณที่สมบูรณ์แบบและไม่สมบูรณ์แบบ ณ การใช้พลังงานที่แตกต่างกันและรวมถึงพื้นที่ครอบคลุมของสัญญาณวิทยานิพนธ์ฉบับนี้ได้เสนออัลกอริทึมทางเลือกใหม่และอัลกอริทึมแบบแบ่งโดเมนสำหรับการเพิ่มประสิทธิภาพสูงสุดของประสิทธิภาพการใช้กำลังงานและการคำนวณหาค่าที่ดีที่สุดของตัวแปรระบบที่แตกต่างกันโดยมีข้อดีคือความซับซ้อนในการคำนวณของอัลกอริทึมทางเลือกที่นำเสนอและอัลกอริทึมแบบแบ่งโดเมนมีค่าต่ำซึ่งไม่อิสระกับจำนวนของสายอากาศส่งและสายอากาศรับแบบลูกโซ่โดยได้เปรียบเทียบระหว่างวิธีการที่นำเสนอกับอัลกอริทึมที่อ้างอิงในลักษณะความซับซ้อนในการคำนวณซึ่งผลที่ได้พบว่าประสิทธิภาพอัลกอริทึมที่นำเสนอดีกว่า

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ARFAT AHMAD KHAN : ENERGY EFFICIENT DESIGN OF 5G MASSIVE MIMO. THESIS ADVISOR : ASSOC. PROF. PEERAPONG UTHANSAKUL, Ph.D., 132 PP.

MASSIVE MIMO/TIME DIVISION DUPLEX/ENERGY EFFICIENCY/POWER AMPLIFIERS/MUTUAL COUPLING

Massive Multiple-Input Multiple-Output (MIMO) alludes to the theory of having a large number of transmitter chains at the base station, which in turn provides the higher spectral and energy efficiency with reduced radiated power and greater simplicity in the signal processing. In this thesis, we have designed the energy efficient Massive MIMO by considering the effects of mutual coupling between the antenna terminal along with taking the effects of nonlinear amplifiers in each transmitter chain. We have designed the system by calculating the optimal number of transmitters and receivers with the optimal transmitted power and their corresponding spectral efficiency in terms of energy efficient prospective of Massive MIMO under both the perfect and imperfect channel conditions at different power consumption and area of coverage. We propose the alternative algorithm and the domain splitter algorithm for the optimization of energy efficiency and the computation of different optimal system parameters in this thesis. The computational complexity of the proposed alternative algorithm and domain splitter algorithm is not dependent on the number of transceiver chains, and the detailed comparison is presented between the proposed and the reference algorithms on the basis of the computational complexity, where the computatioinal complexities of the proposed algorithms are not dependent on the

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number of transceiver chains, which shows the effectiveness of the proposed algorithms.

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School of Telecommunication Engineering Student's Signature ______

Academic Year 2018 Advisor's Signature ______

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SYMBOLS AND ABBREVIATIONS

EE_1	=	Energy Efficiency under perfect CSI.
EE_2	=	Energy Efficiency under imperfect CSI.
$(\cdot)^{-1}$	=	Inverse operator.
$Ci(\cdot)$	=	Cosine integral.
$(\cdot)^T$	=	Transpose operator.
$Si(\cdot)$	=	Sine integral.
$(\cdot)^*$	=	Conjugate operator.
$(\cdot)^H$	=	Hermitian operator.
$\mathrm{E}[\cdot]$	=	Expectation operation.
$\lVert\cdot\rVert^2$	=	Norm operation.
Z_+	=	Set of positive integers.
$log_2(x)$	=	Logarithm of x with respect to base 2.
$ln(x)$	=	Logarithm of x with respect to base e.
$(\cdot)'$	=	First order derivative.
$(\cdot)''$	=	Second order derivative.
$\sim$	=	Approximation notation.
$var(\cdot)$	=	Variance notation.

SYMBOLS AND ABBREVIATIONS (Continued)

$\mathcal{CN}(x,y)$	=	Complex normal distribution with mean x and variance y .
$trace(\cdot)$	=	Matrix trace operation.
4G	=	Fourth Generation.
5G	=	Fifth Generation.
Arg Max	=	Arguments of the Maxima.
AWGN	=	Additive White Gaussian Noise.
BS	=	Base Station.
CSI	=	Channel State Information.
EE	=	Energy Efficiency.
FDD	=	Frequency Division Duplex.
LTE	=	Long Term Evaluation.
MC	=	Mutual Coupling.
MRC	=	Maximum Ratio Combining.
MIMO	=	Multiple Input Multiple Output.
MMSE	=	Minimum Mean Square Error.
PA	=	Power Amplifier.
PAPR	=	Peak to Average Power Ratio.
SNR	=	Signal to Noise Ratio.
TDD	=	Time Division Duplex.
ZF	=	Zero Forcing.

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CHAPTER I

INTRODUCTION

1.1 Background and Motivation

Wireless data traffic and the demand for bringing a higher data rate to a growing number of users has been increasing with every passing year and, in order to provide seamless connectivity, future generation networks will have to rely on denser deployment of infrastructure, reducing the inter and intra cell interference, simple signal processing, and reduction in the transmitted power along with improved energy and spectral efficiency (Marzetta et al, 2016; Ramezani et al, 2018). The global wireless data traffic is approximately 15.8 Exabyte in the 2018 and it is expected that the future 5G networks will have to deal with the 1000 fold increase in the data traffic. Furthermore, with the ever-increasing trend of advances in the wireless technologies, a large number of users are using these technologies by using the laptops, mobiles, tablets and the total number of the mobile communicating devices are approximately 10.2 billion in the 2018 around the world and it is expected that half of the population of the world will be using the smart phones in the near future as can be seen in the Figure 1.1 (Cisco 2014; Cisco 2017). In the conventional techniques, communication between the base station and users has happened in separate time-frequency resources by orthogonalizing the channel, but it results in interference when the number of users increases, because, in order to make sure the higher data rates, several users have to operate in the same time and frequency resources (Caire et al, 2010; Jose et al, 2011) and we have to use complex signal processing techniques like Dirty paper coding and maximum likely-hood multiuser detection (Verdu et al, 1998) in order to mitigate the interference (Viswanath et al, 2003; Weingarten et al, 2006). The deployment of multiple transmitters and receivers at the base station can significantly enhance the throughput and the performance of the system because the transmitters can send the multiple number of data streams spatially on the propagation channel by utilizing the same frequency and time resources, which in turn shifted the focus of the researchers towards the Multiple Transmitters and Multiple Receiver technologies (MIMO). The Multiple Transmitters and Multiple Receiver technologies are the source of attraction for the researchers over the last few decades and they are currently being deployed in the LTE and LTE advanced networks. The initial focus of the researchers was on Multiple-Input Multiple-Output (MIMO) technologies because they provide a substantial gain in area and spectral efficiency (Li et al, 2010; Hanzo et al; 2010). It has been seen that the deployment of a large antenna array at the base station (BS) results in substantial reduction in the intra cell interferences along with simple signal processing (Marzetta et al, 2010), which in turn have shifted the focus of researchers towards Massive MIMO.

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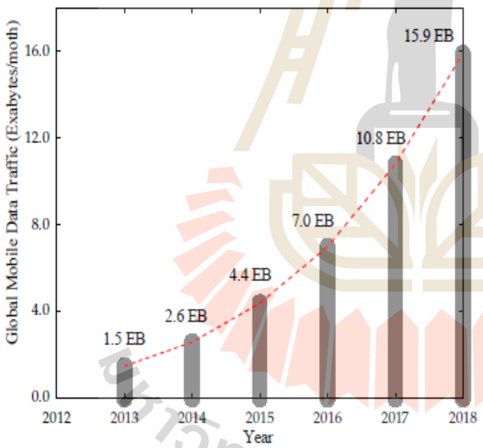
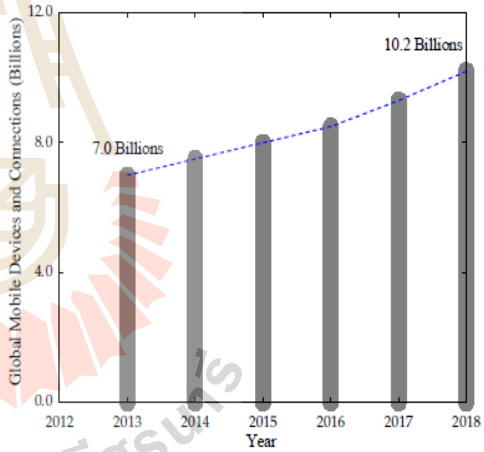
1.2 Requirements and the need of energy efficient communication devices

Information and communication technology (ICT) is one of fastest growing areas is wireless communications. The future 5G networks are expected to meet this growing trend of throughput and spectral efficiency, i.e. support latencies ranging from 1 millisecond (ms) to a few seconds, peak data rates up to 20 Giga bits per second

(Gbps), average data rates up to 100 Mega bits per second (Mbps), seamless connectivity for millions of IoT devices per square kilometre, and signaling loads ranging from 1% to almost 100% (Huawei, 2014) as shown in the Figure 1.2. Along with this growing demand of throughput and spectral efficiency, it is of vital important that our communication devices should have to be energy efficiency because the energy consumption of the communication devices is becoming vital economical and societal concern due to the emission of the carbon. The mobile phone devices are the main contributors in the overall carbon emissions and they are expected to emit around 300 million tons of greenhouse gases by the year 2020 (Ericsson, 2014).



(a) Global mobile data traffic.



(b) Global mobile devices and connections growth.

Figure 1.1 Internet data traffic along with the number of mobile devices (Cisco, 2003)

Furthermore, the industry of mobile Information and communication technology (ICT) is the fifth largest industry in terms of power consumption, and the mobile phone devices are the main contributors in the overall emissions of GHG as can be seen in the Figure 1.3, and the wireless communication networks consume significant amount of energy to overcome the fading, distortion, degradation (path loss), obstacles and interferences. Therefore, for a sustainable evolution into future 5G networks, it becomes critically important for future wireless technologies to not only address the multifold increase in service expectations, but also to operate at reduced power consumption levels. The energy efficiency (EE) of a wireless communication system can be increased by using methods which maximize the system throughput or minimize power consumption, or both. The Massive MIMO is the future 5G technology, therefore, we have contributed towards the energy efficient designing of the Massive MIMO in this thesis. Assume that the network of Massive MIMO needs to be designed from scratch to uniformly cover a given area with maximal energy efficiency. What are the optimal number of antennas, active users, and transmit power? The aim of this thesis is to answer this fundamental question.

.

1.3 Main Contributions

The contributions and novelties of this thesis are summarized as follows:

1. We have formulated and derived the general expressions of the spectral and the energy efficiency in the chapter III. Furthermore, we have formulated the power consumption of Massive MIMO in the chapter III and then optimized the energy efficiency of Massive MIMO along with the calculation of optimal system parameters by calculating the solution of the optimization problem.

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2. In the chapter IV, the energy efficient design of Massive MIMO along with the effects of nonlinear amplifiers under the perfect and imperfect channel conditions, and by using the realistic power consumption model, is first proposed and formulated. The Mathematical expressions of the spectral efficiency and energy efficiency are derived by considering the effects of nonlinear amplifiers in each transmitter branch under the perfect and imperfect channel conditions. The alternative algorithm is proposed to optimize the energy efficiency and calculation of optimal parameters. Simulation results are provided to support the mathematical modelling and investigate the relevant trend.

3. In the chapter V, the energy efficient design of Massive MIMO by considering the effects of Mutual Coupling between the antenna terminals under the perfect and imperfect channel conditions, and by using the realistic power consumption model, is first proposed and formulated. The Mathematical expressions of the spectral efficiency and energy efficiency are derived by considering the effect of mutual coupling between the antenna terminals under the perfect and imperfect channel conditions. The domain splitter algorithm is proposed to optimize the energy efficiency and calculation of optimal parameters. The computational complexity of the proposed domain splitter algorithm is not dependent on the number of transceiver chains, and the detailed comparison is presented between the proposed and the reference algorithms on the basis of the computational complexity, which shows the effectiveness of the proposed domain splitter algorithm. Simulation results are provided to support the mathematical modelling and investigate the relevant trend.

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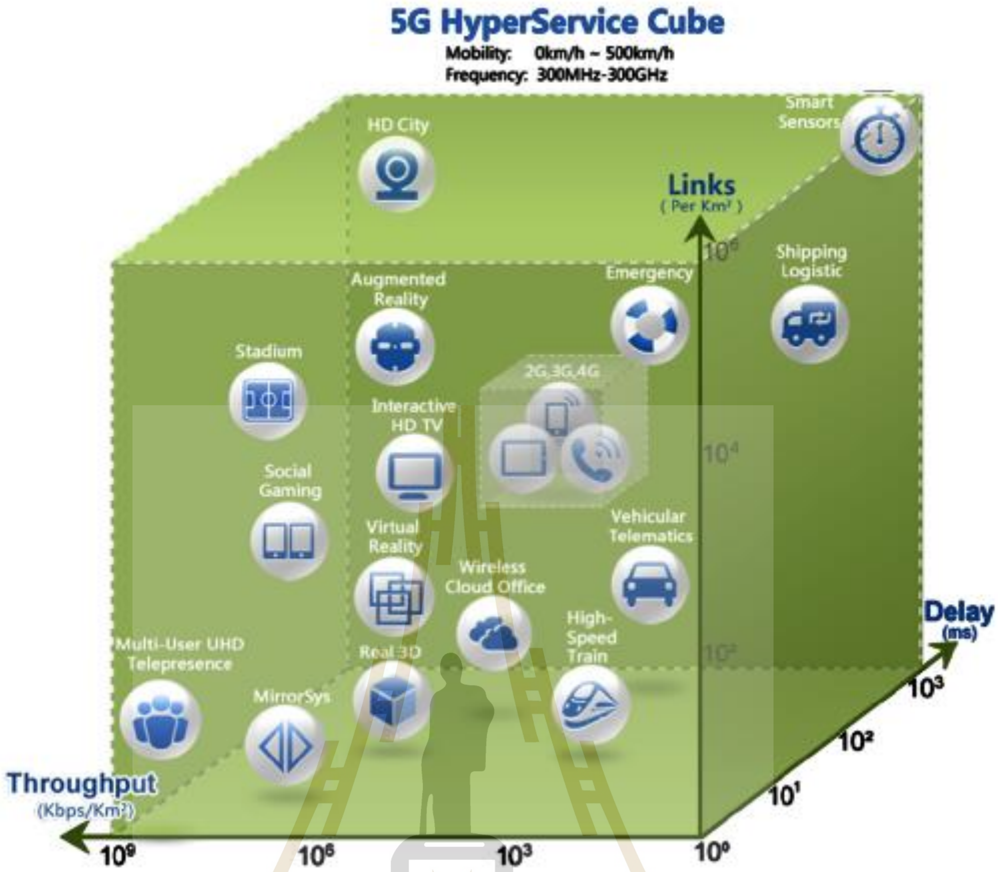

Figure 1.2 Overview of the Expected Throughput in the 5G Network

(Huawei, 2014)

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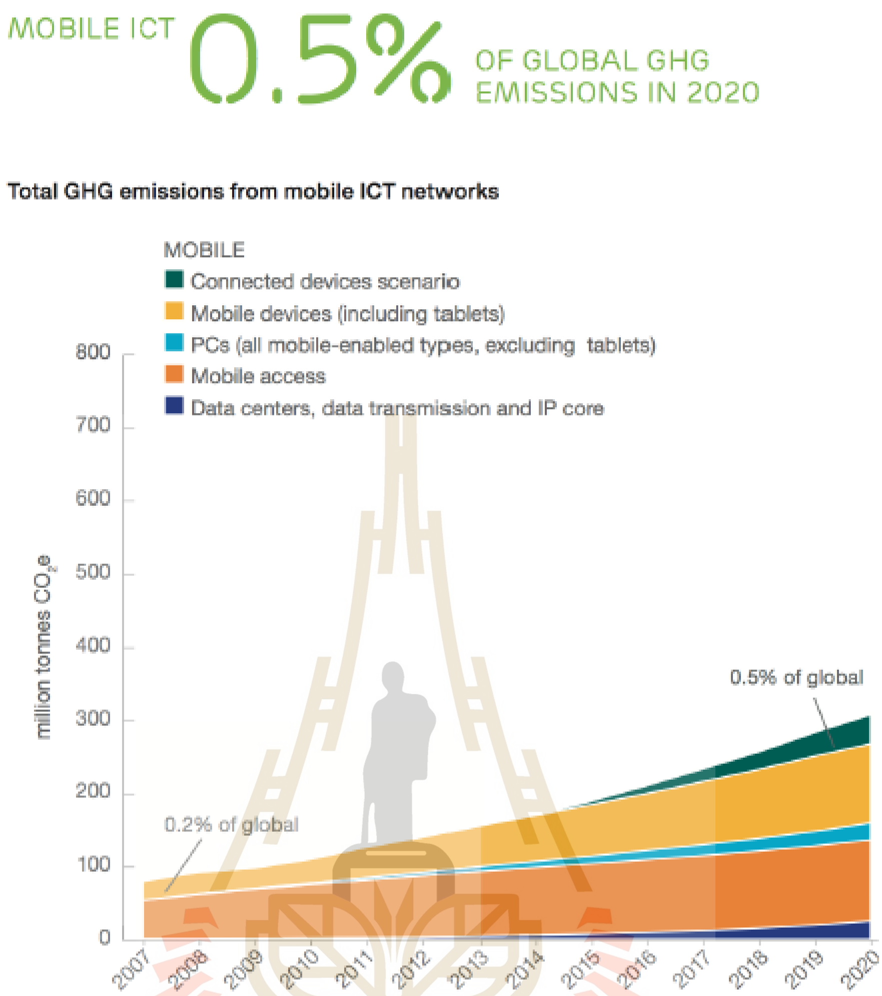

Figure 1.3 Expected figures of the CO_2 emission out of the communication devices (Huawei, 2014).

1.4 Thesis Layout

The remainder of the thesis is organized as follows. In the chapter II, we have discussed the discussed the literature review and the various linear precoding and the decoding scheme. In the chapter III, we have discussed the general designing of the energy efficient Massive MIMO under the perfect channel conditions. In the chapter IV, we have discussed the energy efficient designing of the Massive MIMO by considering the effects of nonlinear amplifiers in each transmitter chain under the perfect and imperfect channel conditions. In the chapter V, we have discussed the energy efficient designing of the Massive MIMO by considering the effects of mutual coupling between the antenna terminals under the perfect and imperfect channel conditions. The chapter VI belongs to the future work where we have discussed the future trend of the research on the basis of the research conducted in this thesis.

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CHAPTER II

Massive MIMO

2.1 Literature Review

Wireless communication is the most vital and key technology in this modern era and with the ever-increasing trend of advances in the wireless technologies, a large number of users are using these technologies, which in turn require having a network with higher energy and spectral efficiency together with the simple signal processing (Vu et al, 2018; Ramezani et al, 2017; Cisco, 2017). Multiple Input Multiple Output (MIMO) systems have gained a lot of attraction, both in industry and academia, due to their ability to significantly improve the spectral efficiency, but the acquisition of higher spectral efficiency comes at the cost of complex computation and signal processing in the conventional MIMO systems (Ma et al, 2011). Massive MIMO can significantly increase the performance of the system by just adopting the linear precoding and decoding schemes for the downlink transmission and the uplink reception due to the asymptotic orthogonality of the propagation channel, as compared to the conventional MIMO systems (Marzetta et al, 2010; Rusek et al, 2013; Kamga et al, 2017; Andrews et al, 2014).

In Massive MIMO, hundreds of antennas have to be stationed on the top of a building or tower, serving a comparatively less number of users (Björnson et al, 2016; Lu et al, 2014). In the urban environment, the electromagnetic interaction between the antenna elements is inevitable due to the space limitations for the deployment of a large number of antenna elements at the base station, thereby, leads to a phenomena named Mutual Coupling (Wang et al, 2017). In (Wallace et al, 2004; Clerckx et al, 2007), the authors have investigated the mutual coupling effect on the performance of the conventional MIMO systems by calculating the closed-form expression of the channel capacity, based on closely spaced antenna elements. In (Boccardi et al, 2014), the authors have examined and investigated the new challenges and issues regarding the deployment of hundreds of antenna elements at the Base Station (BS). In (Artiga et al, 2012; Ge et al, 2016), the authors have modeled the channel and derived the closed-form expression of the channel by considering the effect of mutual coupling among the antenna terminals of Massive MIMO. The mutual coupling effect can be minimized by using the hardware-circuit calibration method and the signal-space calibration method (Nishimori et al, 2014; Nishimori et al, 2014; Wei et al, 2016). The theory and benefits of the Massive MIMO are also applicable for the scenario of the wireless sensor networks where each sensor node can be taken as the user and the fusion center is equipped with large number of antennas, leading into the favorable propagation between the sensor nodes and the fusion center (Shirazinia et al, 2016; Ciuonzo et al, 2014). The initial focus of the researchers was on the spectral efficiency and the channel capacity of the communication devices, but the energy and power related pollution and the inevitable battery depletion of the wireless devices shifted the focus of the researchers to have more and more research on the green cellular networks in order to conserve as much energy as possible (Tombaz et al, 2011).

The energy efficiency is significantly dependent on the number of transceiver chains, transmitted and the consumed power (Zhang et al, 2016; Xiao et al, 2015). The power consumption of the conventional MIMO systems is always taken as a fixed quantity due to the limited number of transmitter chains (Tsinos et al, 2017). However, this assumption is not valid for the Massive MIMO, because the circuit power consumption is significantly dependent on the number of transmitter chains. The number of transceiver chains required for the system can be reduced by using the hybrid analogue to digital transceivers. In (Garcia et al,2016), the authors evaluate the performance of hybrid analogue to digital precoders by modelling the power consumptions of the RF chains. Motivated by the above research (Garcia et al,2016), the authors in (Tsinos et al,2017; Ngo et al, 2013) work on the energy efficient designing of the hybrid analogue to digital transceivers for the single carrier in (Tsinos et al,2017) and for the single and multi-carrier large antenna arrays in (Ngo et al, 2013) for the perfect channel conditions. In (Bjornson et al, 2014), the authors have investigated the energy and spectral efficiency of Massive MIMO at different channel situations and showed that the large number of antenna elements at the base station can significantly enhance the energy efficiency with orders of magnitude compared to the conventional MIMO systems, but, they fixed the total power consumption of the circuit, which in turn let them have an unbounded energy efficiency. In (Ha et al, 2013), the authors have investigated the energy efficiency and calculated the capacity limits of Massive MIMO, but the authors have not modelled the power consumption of the circuit in a correct way. They have not taken the power consumption during the decoding and coding, channel estimation and the linear processing, which in turn let them have the substantial enhancement in the energy efficiency. In (Yang et al, 2013; Bjornson et al, 2015; Xu et al, 2016) , the authors have investigated the tendency of the energy efficiency with respect to the increase in the number of transmitting antennas, users and the transmitted power, and showed that the energy efficiency depicts the response of quasi concave function in terms of the number of transmitting antennas in (Yang et al, 2013), and in terms of the number of users and the transmitted power in (Bjornson et al, 2015) and (Xu et al, 2016) respectively, by using the refined and appropriate model of the total circuit power consumption. Motivated by the above researches (Yang et al, 2013; Bjornson et al, 2015; Xu et al, 2016), the authors in (Khan et al, 2017) extended the results under different channel conditions, by using the refined and correct model of the total circuit power consumption. In (Guerreiro et al, 2016), Effects of nonlinear amplifiers on the spectral characterization of transmitted signals have been studied in case of Massive MIMO. Effects of nonlinear amplifiers can be reduced by designing the precoders for low Peak to Average Power Ratio (Studer et al, 2013; Mohammad et al, 2013). In (Emil et al, 2014), authors have calculated the energy efficiency of massive MIMO by considering the effects of nonlinear amplifiers and other hardware imperfections under the perfect channel situations but they have taken the circuit power consumption as a fixed quantity which is not correct because of the dependence of circuit power consumption on the number of transceiver chains and coherent participation of all BS antennas (Hien et al; 2013; Jiang et al, 2015).

2.2 Working of the Massive MIMO.

In this section, we have discussed the working of Massive MIMO during the uplink transmission and downlink reception of the transmitted signals. The largest of transmitter chains at the BS results into more number of possible signal paths and the performance of the system gets improved in terms of data rate and the link reliability, but, the improvement comes at the cost of a large number of RF amplifiers and energy consumption of the system at both ends.

2.2.1 Uplink Transmission:

In the uplink scenario, each user has to send signals to the BS. Let $s=[s_1,s_2,.....,s_k,....s_K]$ be the vector of the transmitted signals from the K number of users. The signals received from the kth user terminal at the BS can be written as:

$$y_k=\sqrt{p}g_k s_k+\sqrt{p}\sum_{k=1,k\neq k}^{K}\sqrt{p}g_k s_k+n \tag{2.1}$$

Where the first term in the above equation is the desired/useful signal from the kth user terminal and the second terms is representing the interferences among the transmitted signals. $\sqrt{p}$ is the average transmitted SNR and g is the channel matrix . n is the additive white Gaussian noise with zero mean and unity variance.

2.2.2 Downlink Transmission:

In the downlink scenario, base station has to send the transmitted signals to the K number of users. . Let $x=[x_1,x_2,.....,x_k,....x_K]$ be the vector of the transmitted signals from the BS to the K number of users. The signals received at the kth user terminal can be written as:

$$y_k=\sqrt{p}g_k x_k+\sqrt{p}\sum_{k=1,k\neq k}^{K}\sqrt{p}g_k x_k+n \tag{2.2}$$

Similarly, the first term in the above equation is the desired/useful signal from the kth user terminal and the second terms is representing the interferences among the transmitted signals. $\sqrt{p}$ is the average transmitted SNR and g is the channel matrix .

n is the additive white Gaussian noise with zero mean and unity variance, and independent of g

2.3 Linear processing

Massive MIMO offers the communication reliability by allowing the use of linear precoders and decoders in the uplink and downlink. It has been seen that the performance of the system approaches to the Shannon limits by employing the linear precoders and decoders in the uplink and downlink provided that the base station is equipped with the large number of antennas. Suppose A be a linear detection matrix with the dimension of $M \times K$, then the estimated signal $\hat{y} = \left[\hat{y}_1, \hat{y}_2, \hat{y}_3,,............, \hat{y}_k,......., \hat{y}_K \right]$ from the kth user terminal can be written as:

$$\hat{y} = \sqrt{p} A^H G x + A^H n \tag{2.3}$$

Similarly, the signal received from the kth user terminal can be written as:

$$\hat{y}_k = \sqrt{p} a_k^H g_k x_k + \sqrt{p} \sum_{i=1, i \neq k}^{K} a_i^H g_i x_i + a_k^H n$$

Where a_k^H is the kth column of the detection matrix and the corresponding Signal to Noise ratio (SNR) for the kth user terminal can be written as:

$$SNR_k = \frac{p \left| a_k^H h_k \right|^2}{p \sum_{i=1, i \neq k}^{K} \left| a_k^H h_i \right|^2 + \left| a_k \right|^2} \tag{2.4}$$

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The linear detection matrix A can be written as:

$$A = \begin{cases} G^{*} & \text{MRT} \\ G\left(G^{H}G\right)^{-1} & \text{ZF} \\ G\left(G^{H}G + \dfrac{K}{p}\right)^{-1} & \text{MMSE} \end{cases}$$

2.3.1 Maximum Ratio Combining Scheme.

In the MRC scheme, the BS ignores the interference between the transmitted signal and enhances the signal to noise ratio. Thus, the detection matrix for the kth user terminal when the maximum ratio combination scheme is implemented at the BS can be written as:

$$A = \text{ arg max } \frac{\text{signal power}}{\text{noise power}}$$

Thus, the detection matrix A for the kth user terminal can be written as:

$$a_k = \text{arg max } \frac{p\left|a_k^H g_k\right|^2}{\left|a_k^H\right|^2}$$

$$= \frac{p\left|a_k^H\right|^2 \left|g_k\right|^2}{\left|a_k^H\right|^2}$$

$$a_k = p\left|g_k\right|^2 \tag{2.5}$$

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Put the value of a_k from the equation 2.5 into equation 2.4:

$$SNR_k = \frac{p\left|h_k\right|^4}{p\sum_{i=1,i\neq k}^{K}\left|h_k^H h_i\right|^2 + \left|h_k\right|^2} \tag{2.6}$$

When the transmitted power p approaches to infinity then the signal to noise ratio for the kth user terminal can be written as:

$$SNR_k = \frac{\left|h_k\right|^4}{\sum_{i=1,i\neq k}^{K}\left|h_k^H h_i\right|^2} \tag{2.7}$$

It is easy to implement the maximum ratio combining detection scheme because of simple signal processing without much complexity, but this scheme does not perform well during the scenario of imperfect channel state information.

2.3.2 Zero Forcing.

Zero forcing is another linear detection scheme that works by cancelling the inter user interferences by multiplying the received signal with the pseudo inverse of the channel matrix:

$$\hat{y} = G\left(G^H G\right)^{-1} y \tag{2.8}$$

Put the value of y from the equation 2.3 into equation 2.8.

$$\hat{y} = G\left(G^H G\right)^{-1}\left(\sqrt{p}Gx + n\right)$$

Similarly, the estimated signal for the kth user terminal $\hat{y}_k$ can be written as:

$$\hat{y}_k = \sqrt{p}x_k + \left[G\left(G^H G\right)^{-1}\right]_k n$$

The correspond signal to noise ratio for the kth user terminal can be written as:

$$SNR_k = \frac{p}{\left[\left(G^H G\right)^{-1}\right]_k} \tag{2.9}$$

The implementation of the zero forcing detection scheme is also simply like the maximum ratio combining detection scheme and the zero forcing performs well by cancelling the inter user interferences by multiplying the received signal with the pseudo inverse of the propagation channel.

2.3.3 Minimum Mean Square Error Detection Scheme (MMSE):

Minimum Mean Square Error Detection works by the computing the minimum mean square error between the estimated and the received signal:

$$A = \arg\min \mathrm{E}\left[\left|A^H \hat{y} - x\right|^2\right]$$

The detection matrix A for the kth user terminal can be written as:

$$a_k = \arg\min \mathrm{E}\left[\left|a_k^H \hat{y}_k - x_k\right|^2\right]$$

$$a_k = \sqrt{p}\left(pGG^H + I_M\right)^{-1} \tag{2.10}$$

When the SNR is higher, the ZF approaches the MMSE detector and MRC approaches to the MMSE at the low SNR.

2.4 Channel Estimation in the Massive MIMO.

Two commonly used multiplexing schemes for the Massive MIMO are the Time Division Duplexing (T.D.D) and the Frequency Division Duplexing (F.D.D). Time Division Duplex is the most desirable scheme in the case of Massive MIMO, because of the reciprocity between the uplink and downlink channels during each coherence block, as compared to Frequency Division Duplex (F.D.D), where pilot overhead in each coherence block is dependent on the number of transmitting antennas at the BS. In FDD, we have to use different frequencies in the uplink and downlink and the pilot overhead of the channel estimation is proportional to the number of transmitting antennas, leading to a very challenging situation at the user terminals to acquire the downlink channel state information and then feedback the acquired channel state information to the base station. The Massive MIMO was basically imagined for TDD protocol, but can be applied also in FDD by using the various techniques.

2.4.1 Time Division Duplexing.

Figure 2.1 illustrates the frame structure of Massive MIMO in case of TDD protocol. An uplink and Downlink channel are reciprocal to each other in TDD operation and uses the same frequency spectrum during the uplink and downlink communications at different time slots. During the uplink operation, each user needs to send training signals or orthogonal pilots to the base station in order to estimate the CSI at the base station for T_p^{ul} channel uses followed by the transmission of data from all K users to BS in the same time-frequency resources for T_d^{ul} channel uses as shown in

Figure 2.1. BS uses the linear precoding to retrieve the signals transmitted from all K users together with channel estimation. In the downlink, BS uses the estimated channel in order to transmit the required signals to the intended users for T_d^{dl} channel uses. Number of transmitters M and users K are required to be same during the uplink and downlink operation in case of TDD protocol.

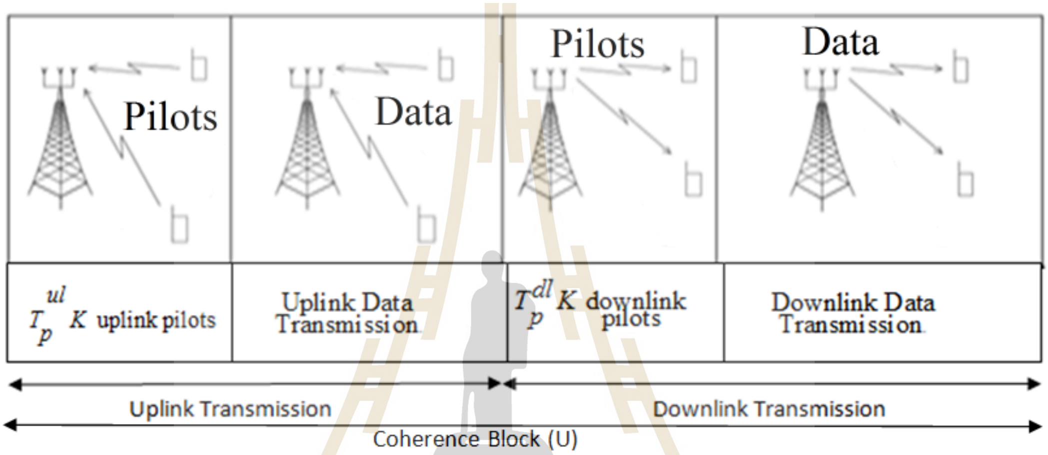

Figure 2.1 Frame Structure for the TDD scheme.

2.4.2 Frequency Division Duplexing.

The frequency Division Duplexing requires to have the different frequency bands for the uplink and the downlink transmission unlike the time division duplexing. The uplink transmission is same in the frequency division duplexing as the time division duplexing, but, we cannot use the estimated channel at the uplink transmission during the downlink transmission due to the different frequencies and the corresponding transmission channels.

CHAPTER III

Energy Efficient Designing of Massive MIMO

Demand for having a higher throughput and spectral efficiency has been getting increased exponentially with every passing year. Future generation networks will have to deal with large number of users, offering a higher spectral efficiency, less power consumption and with increased energy efficiency. Massive Multiple-Input Multiple-Output (MIMO) has proved to be an auspicious candidate in that context. It purveys higher spectral and energy efficiency by adopting a large number of transmitting antennas which in turn requires a large number of transceiver chains. In this chapter, we have assumed that the transmitting antennas are closely placed, resulting into the phenomena of mutual coupling because in practical situations, the transmitting antennas have to deal with some sort of mutual coupling. We have estimated the performance of Massive MIMO by calculating the achievable rate at different number of transmitting antennas and users by considering the overhead factor and mutual coupling. Moreover we have maximized the energy efficiency of Massive MIMO and calculated the optimal number of transmitters and receivers by incorporating with overhead factor and mutual coupling at different distances.

3.1 Channel Modelling of Massive MIMO

Consider a Figure 3.1 where M numbers of antennas are equipped with BS, serving K single antenna users. Single antenna users are uniformly distributed. The signal vector received at the BS is expressed as

$$y = \sqrt{p}Gx + n \tag{3.1}$$

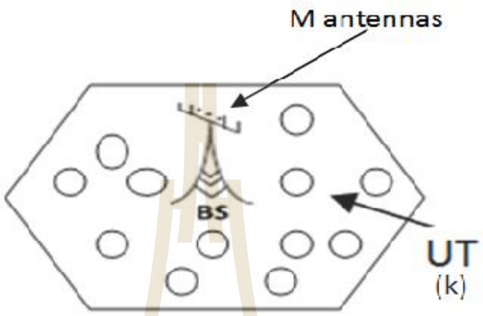

Figure 3.1 Illustration of massive MIMO (Base station with M number of antenna's serving K number of receivers)

Where y is the $M \times 1$ received signal vector, n is the additive white Gaussian noise (AWGN) with zero mean and having a dimension of $M \times 1$ and p is the average transmitted power of the signal. G having a dimension of $M \times K$ is the channel matrix. As explained above that Massive MIMO provides communication reliability because we can use linear precoders and detectors in the uplink and downlink and they can be described as follows:

$$V = \begin{cases} G^{*} \text{ for MRT} \\ G(G^{H}G)^{-1} \text{ for ZF} \\ G(G^{H}G + \dfrac{K}{p_d})^{-1} \text{ for MMSE} \end{cases}$$

When the antenna elements are closely placed then the field generated by one element will interact with the field of the other antenna. This electromagnetic interaction between two antenna elements give rise to a phenomena named as Mutual Coupling (MC). The mutual coupling matrix C is expressed as (Clerckx et al. 2007)

$$C = (Z_o + Z_L)(Z_L I_M + Z_C)^{-1} \tag{3.2}$$

Where Z_o is the self impedance, Z_L is the load impedance and Z_C is the mutual impedance matrix. The diagonal entries in Z_C represent the self impedances Z_o. When there will be no mutual coupling between two antennas then the non-diagonal entries of the Z_C will be zero. The mutual impedance between two antennas can be expressed as (Balanis, 2012)

$$Z_{mn} = \begin{cases} 30\begin{bmatrix} 0.577 + ln(2\pi) - Ci(2\pi) + \\ jSi(2\pi) \end{bmatrix} & \mathrm{m = n} \\ 30\begin{bmatrix} \begin{bmatrix} 2Ci(\beta d) - Ci(\beta\mu_1) \\ -Ci(\beta\mu_2) \end{bmatrix} \\ -30j\begin{bmatrix} 2Si(\beta d) - Si(\beta\mu_1) \\ -Si(\beta\mu_2) \end{bmatrix} \end{bmatrix} & \mathrm{m \neq n} \end{cases} \tag{3.3}$$

$$\beta = \frac{2\pi}{\lambda}$$
$$\mu_1 = \sqrt{d^2 + L^2} + L$$
$$\mu_2 = \sqrt{d^2 + L^2} - L$$

L is the length of the antenna and d is the inter-element distance of antenna array, and Ci(x), Si(x) are the cosine and sine integrals (Balanis, 2012)

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$$Ci(x) = \int_{-\infty}^{x} \frac{cos(x)}{x} dx$$

$$Si(x) = \int_{-\infty}^{x} \frac{sin(x)}{x} dx$$

Considering the effects of mutual coupling between antennas, the channel matrix can be expressed as

$$G = CHD^{\frac{1}{2}} \tag{3.4}$$

Where H is a $M \times \mathrm{K}$ Rayleigh small-scale fading matrix, C is a $M \times \mathrm{M}$ mutual coupling matrix defined as above and D is the large scale fading matrix. The large scale fading is a diagonal matrix of dimension $\mathrm{K} \times \mathrm{K}$ and can be expressed as:

$$D = diag\left(\beta_1, ..., \beta_k, ...\beta_K\right) \tag{3.5}$$

In this way we can model the wireless channel and the achievable rates using this channel model are discussed in the next section.

3.2 Achievable Rates:

According to Shannon theorem the achievable rates are defined as:

$$R = log_2\left(1 + SNR\right) \text{ (bits/s/Hz)} \tag{3.6}$$

Suppose that the zero forcing linear detectors are implemented at BS and detection matrix in case of linear detection is defined as

$$V = G\left(G^H G\right)^{-1} \tag{3.7}$$

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Thus, the received signal at the BS is expressed as

$$r = V^H y$$

Put the value of the received signal vector ‘ y ’ from the equation 3.1:

$$r = V^H \left(\sqrt{p} G x + n \right)$$

$$r = \sqrt{p} V^H G x + V^H n$$

$$r = \sqrt{p} x + V^H n$$

The signal received from the kth UT is defined as

$$r_k = \sqrt{p} x_k + V_k^H n$$

Therefore, corresponding SNR can be expressed as:

$$SNR_k = \frac{SNR}{\left\| V_{kk}^H \right\|^2} = \frac{p}{\left[\left(G^H G \right)^{-1} \right]_{kk}}$$

So, the corresponding achievable rate by having a zero forcing at the receiver end can be expressed as:

$$R_k = \mathrm{E}\left[log_2 \left(1 + SNR_k \right) \right]$$

$$R_k = \mathrm{E}\left[log_2 \left(1 + \frac{p}{\left[\left(G^H G \right)^{-1} \right]_{kk}} \right) \right]$$

The achievable rate by having an overhead factor can be expressed as

$$R_k = \left(1 - \frac{T_{sum}}{U}\right) \mathrm{E}\left[log_2 \left(1 + \frac{p}{\left[\left(G^H G\right)^{-1}\right]_{kk}} \right) \right] \tag{3.8}$$

Note that the pre-log term reflects the over-head of pilot sequence in each coherence block U and T_{sum} is the total relative pilot length.

3.3 Energy Efficiency

The energy efficiency of a system is defined as the sum-rate (the spectral efficiency) divided by the transmit power and all the power consumed or utilized.For a single cell Massive MIMO system, the spectral efficiency with perfect CSI available is defined as

$$R_p = \sum_{k=1}^{K} R_k \tag{3.9}$$

Where R_k is the achievable rate for kth user and R_p is the overall spectral efficiency in bits/s/Hz.

According to the definition of energy efficiency it can be written as

$$EE = \frac{R_p}{P_t + P_{uc}} \tag{3.10}$$

Where P_{uc} is the utilized power and P_t is the average power consumed by the power amplifiers and can be written as (Björnson et al, 2015)

$$P_t = \frac{B\alpha^2 p\delta_x K}{\eta} \tag{3.11}$$

Where η is the power amplifier efficiency, $B\alpha^2$ is the total noise power and δ_x is the path loss factor depending upon the uniform user distribution with in a cell of range [d_{min} d_{max}].

Whereas, the total power utilized P_{uc} by the Massive MIMO is equal to the power consumed starting from transmitter to receiver end i.e. transmitter and receiver chain power, power required by filter and mixers of transmitter as well as receiver, oscillator power, channel estimation power, coding and decoding power, processing power, power loses due to mutual coupling at the base station.

Power consumption of transmitter and receiver chain is defined as:

$$P_{TR} = MP_{CT} + KP_{CR} + P_{OSC} \tag{3.12}$$

Where P_{CT} is the total power consumption at the transmitter chains and P_{CR} is the total power consumption at the receiver chains.

In Massive MIMO, BS and UT have channel state information available by using the pilot sequence i.e. pilot sequences known to both transmitter and receiver are transmitted during the uplink and downlink to have an estimation of the channel. Power utilization during channel estimation of the uplink and downlink operation is given as (Björnson et al, 2015):

$$P_{CE} = \left(\frac{2BMK^2}{\Upsilon_{bs} U} + \frac{4BK^2}{\gamma_u U} \right) T_{sum} \tag{3.13}$$

Where Υ_{bs} and γ_u are the computational efficiency at the transmitter and the receiver end and B is the transmission bandwidth.

In the uplink and downlink BS and UT have to do coding, decoding, modulation and demodulation. Power utilization in this process can be written as

$$P_{C/D} = \sum_{k=1}^{K} (R_k)(P_{cod} + P_{dec}) \tag{3.14}$$

Where P_{cod} and P_{dec} are the coding and decoding powers and the total power utilization can be written as:

$$P_{uc} = MP_{CT} + KP_{CR} + P_{C/D} + P_{OSC} + P_{fix} \tag{3.15}$$

Where P_{fix} is the fixed power and put all the values of consumed power and achievable rates from the equations 3.11, 3.15, 3.9 in to EE equation 3.10 then energy efficiency can be re-written as

$$EE = \frac{K\sum_{k=1}^{K} R_k}{\left[P_t + P_{fix} + P_{CE} + MP_{CT} + KP_{CR} + P_{OSC} + K(P_{cod} + P_{dec})\sum_{k=1}^{K} R_k \right]} \tag{3.16}$$

We need to maximize the energy efficiency and in order to maximize the energy efficiency consider the following substitutions:

Let $$b = \frac{B\alpha^2 \delta_x K}{\eta}$$

$$z = \left[\left(G^H G \right)^{-1} \right]_{kk}$$

$$a = P_{fix} + P_{OSC} + MP_{CT} + KP_{CR} + P_{CE}$$

$$y = K\left(1 - \frac{T_{sum}}{U}\right)$$

$$s = y\left(P_{cod} + P_{dec}\right)$$

So, the energy efficiency (equation 3.16) can be written as:

$$EE(p) = \frac{y\,log(1+pz)}{a + bp + s\,log(1+pz)} \tag{3.17}$$

The problem of energy efficiency maximization can be expressed as the following mathematical optimization problem:

p=arg max{EE}

The response of the energy efficiency objective function has been plotted and it can be seen easily from Figure 3.2 that this objective function is a quasi concave function so its second order derivative should be less than zero and there exist a point such that $EE'(p^*) = 0$ where $p^* > \frac{-1}{z}$ will be a global maxima of the objective function. So

$$\frac{d}{dp}\left(EE(p)\right) = \frac{d}{dp}\left(\frac{y\,log(1+pz)}{a + bp + s\,log(1+pz)}\right)$$

$$\frac{d}{dp}\left(EE(p)\right) = \frac{\left[\begin{array}{l}\left(a + bp + s\,log(1+pz)\right)\frac{d}{dp}\left(y\,log(1+pz)\right) - \\ \left[\left(y\,log(1+pz)\right)\frac{d}{dp}\left(a + bp + s\,log(1+pz)\right)\right]\end{array}\right]}{\left[a + bp + s\,log(1+pz)\right]^2}$$

$$\frac{d}{dp}\left(EE(p)\right)=\frac{\left[\begin{array}{l}\left(a+bp+s\,log\left(1+pz\right)\right)\left(\dfrac{yz}{1+pz}\right)\\ -\left[\left(y\,log\left(1+pz\right)\right)\left(b+\dfrac{sz}{1+pz}\right)\right]\end{array}\right]}{\left[a+bp+s\,log\left(1+pz\right)\right]^{2}}$$

$$\frac{d}{dp}\left(EE(p)\right)=\frac{\left[\begin{array}{l}\left(a+bp+s\,log\left(1+pz\right)\right)\left(\dfrac{yz}{1+pz}\right)\\ -\left[\left(y\,log\left(1+pz\right)\right)\left(\dfrac{b\left(1+pz\right)+sz}{1+pz}\right)\right]\end{array}\right]}{\left[a+bp+s\,log\left(1+pz\right)\right]^{2}}$$

$$\frac{d}{dp}\left(EE(p)\right)=\frac{\dfrac{y}{1+pz}\left[\begin{array}{l}\left(a+bp+s\,log\left(1+pz\right)\right)z\\ -\left[\left(log\left(1+pz\right)\right)\left(b+bpz+sz\right)\right]\end{array}\right]}{\left[a+bp+s\,log\left(1+pz\right)\right]^{2}}$$

$$\frac{d}{dp}\left(EE(p)\right)=\frac{\left[\dfrac{y}{1+pz}\left[\begin{array}{l}az+bzp+sz\,log\left(1+pz\right)-\\ b\,log\left(1+pz\right)-bpz\,log\left(1+pz\right)-\\ sz\,log\left(1+pz\right)\end{array}\right]\right]}{\left[a+bp+s\,log\left(1+pz\right)\right]^{2}}$$

$$\frac{d}{dp}\left(EE(p)\right)=\frac{\dfrac{y}{1+pz}\left[az+bzp+log\left(1+pz\right)\left[\begin{array}{l}sz-b\\ -bpz-sz\end{array}\right]\right]}{\left[a+bp+s\,log\left(1+pz\right)\right]^{2}}$$

$$\frac{d}{dp}\left(EE(p)\right)=\frac{\dfrac{y}{1+pz}\left[az+bzp+log\left(1+pz\right)\left[-b-bpz\right]\right]}{\left[a+bp+s\,log\left(1+pz\right)\right]^{2}}$$

$$\frac{d}{dp}\left(EE(p)\right)=\frac{\dfrac{y}{1+pz}\left[az+bzp-log\left(1+pz\right)\left[b+bpz\right]\right]}{\left[a+bp+s\,log\left(1+pz\right)\right]^{2}}$$

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Equate this first order derivative to zero in order to find the global maxima.

$$\frac{\frac{y}{1+pz}\left[az+bzp-log\left(1+pz\right)\left[b+bpz\right]\right]}{\left[a+bp+s\,log\left(1+pz\right)\right]^{2}}=0$$

$$\frac{y}{1+pz}\left[az+bzp-log\left(1+pz\right)\left[b+bpz\right]\right]=0$$

$$\left[\frac{az+bzp}{1+pz}-\left(b\,log\left(1+pz\right)\right)\right]=0$$

Adding and subtracting b:

$$\left[\frac{az+bzp}{1+pz}-b+b-\left(b\,log\left(1+pz\right)\right)\right]=0$$

$$\left[\frac{az+bzp-b\left(1+pz\right)}{1+pz}+b-\left(b\,log\left(1+pz\right)\right)\right]=0$$

$$\frac{az+bzp-b-bzp}{1+pz}+b-\left(b\,log\left(1+pz\right)\right)=0$$

$$\frac{az-b}{1+pz}+b\left(1-log\left(1+pz\right)\right)=0$$

$$\frac{az-b}{1+pz}-b\left(log\left(1+pz\right)-1\right)=0 \qquad (3.18)$$

Let $u=log\left(1+pz\right)-1$ (3.19)

Taking exponential on both sides

$$e^{u} = e^{(log(1+pz)-1)}$$

$$e^{e}e = (1+pz) \tag{3.20}$$

So, the equation 3.18 can be written as after putting values from the equations 3.19 and 3.20.

$$\frac{az-b}{ee^{u}} = bu$$

$$ue^{u} = \frac{az-b}{be}$$

By using the omega (W) function, last equation can be written as:

$$u = W\left(\frac{az-b}{be}\right)$$ (Put this value in the equation 3.19)

$$W\left(\frac{az-b}{be}\right) = log(1+pz)-1$$

Taking exponential on both sides

$$e^{W\left(\frac{az-b}{be}\right)} = e^{log(1+pz)-1}$$

$$e^{W\left(\frac{az-b}{be}\right)} = (1+pz)e^{-1}$$

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$$1 + pz = e^{W\left(\frac{az-b}{be}\right)+1}$$

$$p = \frac{e^{W\left(\frac{az-b}{be}\right)+1} - 1}{z} \tag{3.21}$$

Energy efficiency can be maximized using this global maxima p and the optimal number of transmitters and receivers required for the system can be computed by using the following numerical approach.

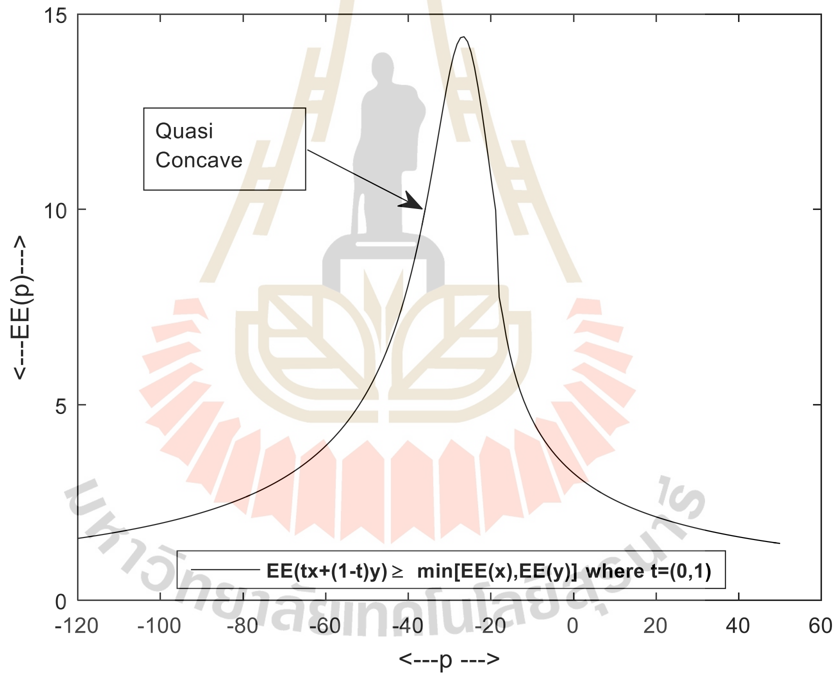

Figure 3.2 Response of EE Function

3.4 Numerical Algorithm

Assume that the given set of transmitters and receivers are given. Calculate the value of p for all combination of transmitters and receivers by using the equation 3.21

after setting all the required parameters and variables. Also calculate the corresponding transmitted power as well as achievable rates using this value of p from the equations 3.11 and 3.8. Furthermore, calculate the energy efficiency by using the equation 3.16 and then search exclusively for the maximum achievable energy efficiency with the optimal number of antennas and receivers. Simulation steps and the above mentioned discussions are summarized in the following flow chart (Figure 3.3).

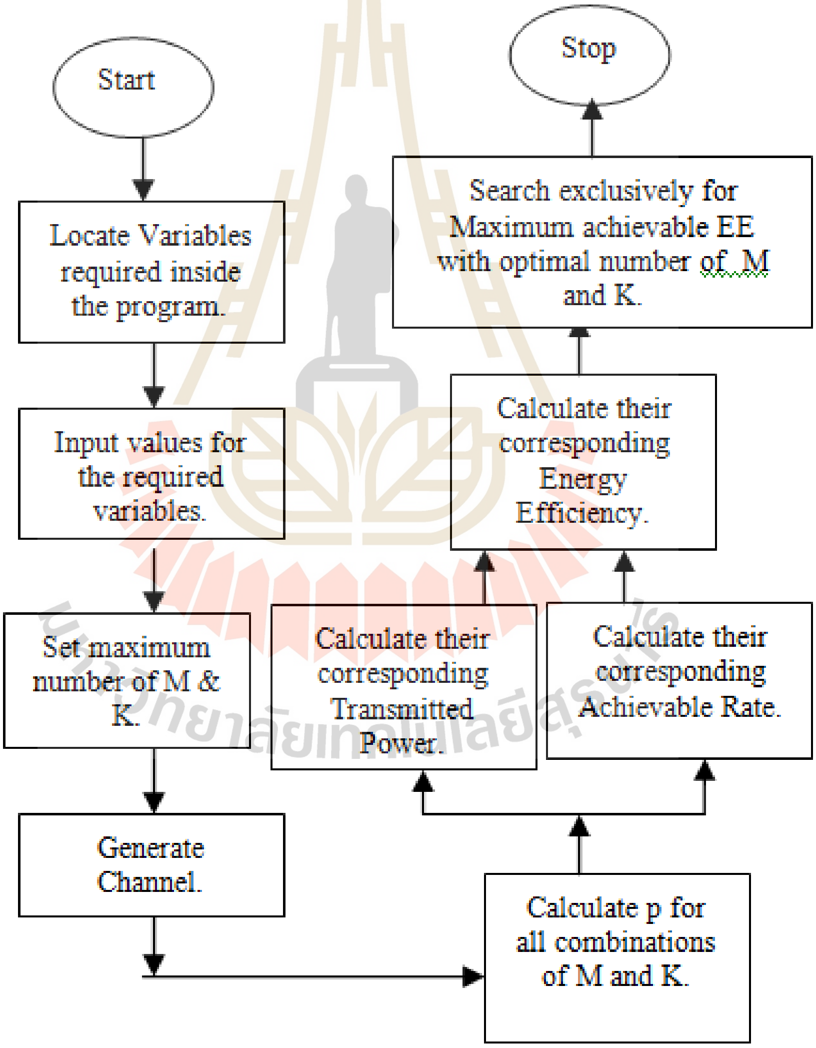

Figure 3.3 EE maximization with optimal number of transmitters and receivers

3.5 Simulations and Results

In this section we have performed simulations to prove the effectiveness of our mathematical models and discussions as in the previous sections. In the starting part, we have evaluated the performance of Massive MIMO by setting different number of transmitters and receivers and by adopting ZF processing under perfect CSI conditions and checked the effects on achievable rates when there are large numbers of transmitting antennas at the BS. We need to assign value to every variable shown in the achievable rate formula and we have assigned practical values to every variable based on different research journals. We have assumed the same type of antennas at the BS, the load impedance and self impedance of each antenna are assumed to be 50 Ohms. The large scale fading factor β_k is defined as $\beta_k = \frac{z}{(l_k / l_{resist})^v}$ where z is the random variable of log-normal distribution and v is the path loss coefficient and l_k is a uniformly distributed random variable ranging from 10 m to 140 m and v is assumed to be 3.8.

Next step is to maximize the energy efficiency after calculating the achievable rates of Massive MIMO with different number of transmitters and receivers over a given range of SNR, we need to define all the parameters required to optimize the energy of Massive MIMO. Realistic power consumption values have been assumed i.e. P_{cod} =0.7W, P_{dec} =0.2W, $\Upsilon_{bs} = \gamma_u$ =12.8Gflop/W,B=20MHz, $B\alpha^2$ =-0.97dB, P_{CT} =1W, P_{CR} =0.5w, P_{OSC} =2w, η=0.35. And coherence block U is assumed to be 1800 in simulations.

After setting all the required parameters, energy efficiency is computed by setting the minimum and maximum distance to be 30m and 220m using the optimization algorithm explained in section 3.4. 3-D representation of energy efficiency with optimal number of transmitters and receivers is shown in Figure 3.4. Star mark shows the maximum energy efficiency with optimal number of transmitters and receivers. X-axis shows the optimal number of users and Y-axis shows the optimal number of transmitters and where-as Z-axis shows the optimal energy efficiency. As it can be seen from Figure 3.4, the energy efficiency comes out to be approximately 29 (Mbits/joule) and the optimal number of antennas and receivers required for the system are approximately 182 and 111.

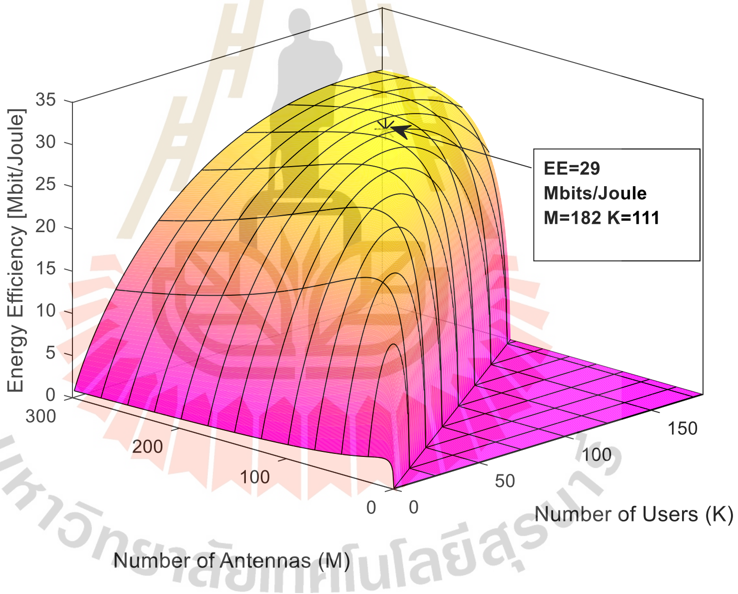

Figure 3.4 Energy efficiency by adopting ZF with area of coverage ranges from 30m to 220m

The energy efficiency of Massive MIMO has further investigated by setting different area of coverage as it was in the last scenario and this time the maximum distance is set to 300m. As it can be seen from Figure 3.5 the energy efficiency comes out to be approximately 21 (Mbits/joule) and the optimal number of antennas and receivers required for the system are approximately 222 and 118.

Furthermore, Energy efficiency of Massive MIMO is investigated further by setting the maximum distance to be 400 m. As shown in Figure 3.6, the energy efficiency has comes out to be approximately 16 (Mbits/joule) and the optimal number of antennas and receivers required for the system are approximately 277 and 128.
Figure 3.7 shows the maximum achievable energy efficiency with optimal number of antennas. X-axis shows the number of transmitting antennas and Y-axis shows the energy efficiency in Mbit/Joule. The circle mark on the Figure 3.7 shows the maximum achievable energy efficiency with the optimal number of transmitting antennas at different area of coverage. As it can be seen from Figure 3.7 that when the maximum distance is set to 220m then energy efficiency comes out to be 29 with optimal number of antennas 182. Similarly when the maximum distance is set to 300m then energy efficiency comes out to be 21 (Mbits/joule) with optimal number of antennas 222. Where-as when the maximum distance is 400m then energy efficiency comes out to be 16 (Mbits/joule) with optimal number of antennas 277

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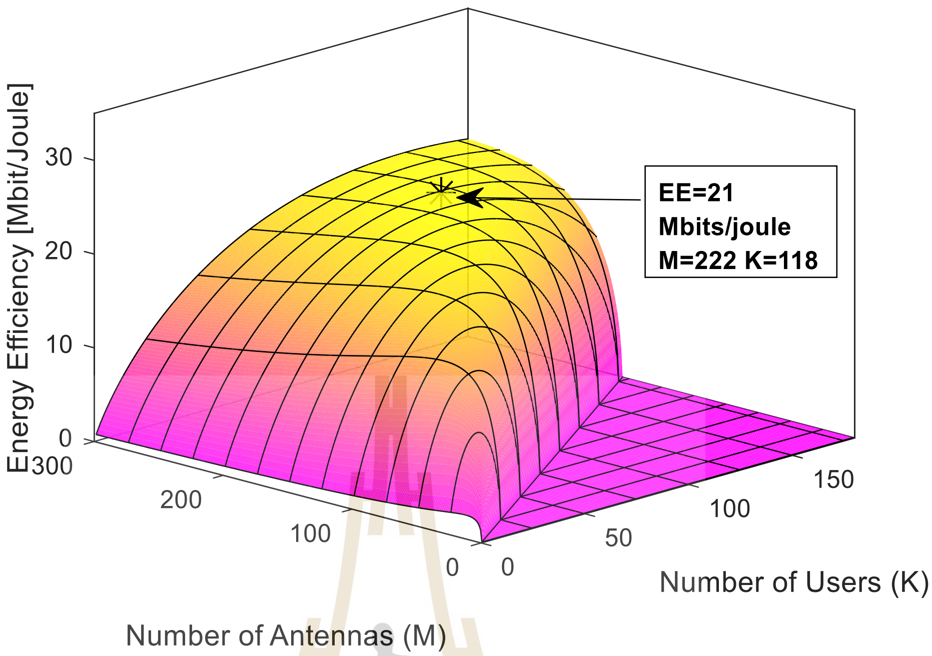

Figure 3.5 Energy efficiency by adopting the ZF with the maximum distance to be 300m

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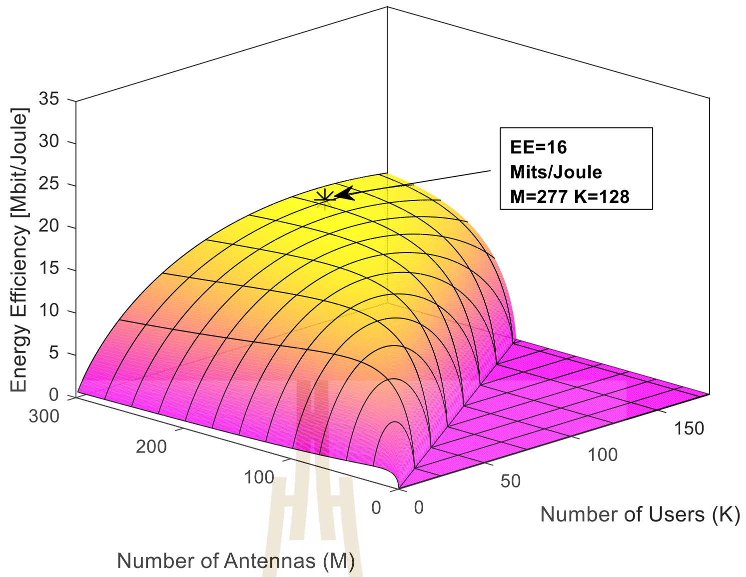

Figure 3.6 Energy efficiency by adopting the ZF with the maximum distance to be 400

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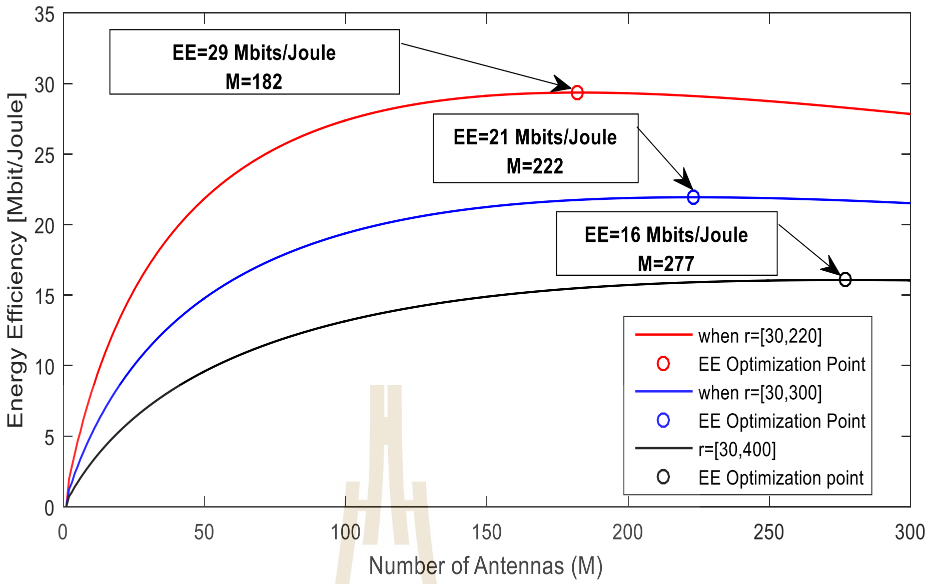

Figure 3.7 Comparison of EE with optimal number of transmitters at different area of coverage

3.6 Summary

This chapter mainly focused on improving the energy efficiency of Massive MIMO. We have improved the energy efficiency because we want our systems to be energy efficient with less power consumption. As energy efficiency is dependent on transmitted power and the power consumption of the system so we have modeled all the utilized and consumed power of Massive MIMO by starting from transmitter end and to finish at the receiver end as described in section 3.3 that all the power consumed by the Massive MIMO is the algebraic sum of transmitted power, mixer and filter power of the transmitter and receiver, channel estimation power, oscillator power, coding and decoding power and power loses due to mutual coupling. Energy efficiency is improved using the optimization algorithm, explained in the section 3.4.

Simulation results show the improved and the maximum achievable energy efficiency with optimal number of transmitters and receivers for the system over different area of coverage. We have performed simulations by using Matlab version of 2015 and assumed zero-forcing detection scheme. However, these results can easily be extended for other linear detection schemes.

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CHAPTER IV

Energy Efficient Designing of Massive MIMO by Considering the Effects of Nonlinear Amplifiers

In this chapter, we have maximized the energy efficiency of massive MIMO and calculated the optimal number of antennas and users along with optimal transmitted power and their corresponding achievable spectral efficiency under both the perfect and imperfect channel situations. Different from the existing studies, we have taken the overhead signaling factor into account and included the effects of nonlinear amplifiers in each transmitter branch under both the perfect and imperfect channel conditions and with proper modelling of circuit power consumptions. To the best of our knowledge, not much research has been done on the energy efficient designing of Massive MIMO by considering the effects of nonlinear amplifiers under the imperfect channel conditions and with proper modelling of circuit power consumptions. Moreover, we have calculated the optimal number of antennas and users along with optimal transmitted power and their corresponding achievable spectral efficiency under both the perfect and imperfect channel situations. Effects of nonlinear amplifiers on the energy efficiency of Massive MIMO are investigated by calculating the energy efficiency at different nonlinear power amplifier efficiencies and distortion loses under both the perfect and imperfect channel conditions. We have proposed an alternative optimization method that works for both perfect and imperfect channel conditions without much complexity and provides the optimal parameters by converging quickly.

4.1 Achievable Rates of Massive MIMO under Perfect CSI

Consider the data symbols $x=[x_1,x_2,.....x_k,...x_K]$ transmitted by the base station antennas intended for the K number of users as shown in Figure 4.1 then the transmitted vector S can be written as:

$$S = Ax \tag{4.1}$$

Where A is a linear precoding matrix and can be expressed as:

$$A = VP^{\frac{1}{2}} \tag{4.2}$$

Where V is a $M \times K$ beam forming vector and can be described as:

$$V = \begin{cases} G^* & \text{for MRT} \\ \mathrm{G}\left(G^H G\right)^{-1} & \text{for ZF} \\ \mathrm{G}\left(G^H G + \dfrac{K}{p_r}\right)^{-1} & \text{for MMSE} \end{cases}$$

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And $P = diag(p)$ where $P = [p_1, p_2, p_3, \dots, p_K]^T$ denotes the power allocation for all users as shown in Figure 4.1.

According to Bussgang's theorem, we can decompose the output of an amplifier as a sum of two uncorrelated components (input signal and the distortion). Let d_k be the distortion caused by the nonlinear amplifier as shown in Figure 4.1 then the signal received at the k^{th} user can be expressed as:

$$y_k = h_k^T G_k p_k^{1/2} x_k + \sum_{l=1, l\neq k}^{K} h_k^T G_l p_l^{1/2} x_l + d_k + n_k \tag{4.3}$$

The second term in the above equation is due to interference among data symbols and n_k is the Additive White Gaussian Noise (A.W.G.N) having zero mean and unity variance.

Let $i_k = \dfrac{\mathrm{E}\left[\left(\sum_{l=1, l\neq k}^{K} h_k^T G_l p_l x_l\right)^* d_k\right]}{p_k}$ is the correlation of d_k on the interference term and c_k are the power loses due to nonlinear amplifier, then equation 4.3 can be written as:

$$y_k = h_k^T G_k \left| p_k + c_k \right|^{1/2} x_k + i_k \sum_{l=1, l\neq k}^{K} h_k^T G_l p_l^{1/2} x_l + d_k + n_k \tag{4.4}$$

Whereas, c_k can also be seen as the affect of a nonlinear amplifier to the amplitude of the intended signal which can be termed as 'clipping' and in practical situations this contribution is negative i.e.

$$\left| p_k + c_k \right| < \left| p_k \right|$$

The corresponding clipping power p_c at the k^{th} user terminal can be written as:

$$p_c = \frac{p_k + c_k}{p_k} \tag{4.5}$$

The variance of the distortion at the k^{th} user terminal due to the nonlinear amplifier can be written as:

$$\sigma_k^2 = D_k = \frac{\mathrm{E}\left[\left|d_k\right|^2\right]}{p_k} \tag{4.6}$$

In order to have the equal rate for all the users, power allocation needs to be done in a clever way and by employing a technique from (Björnson et al, 2015) , it can be written as:

$$p_k^{(ZF)} = p\left(M - K\right) \tag{4.7}$$

Where p is the received signal to noise ratio and it is considered as an optimization parameter because optimizing the p is equal to the optimization of p_k. As we know that ZF suppresses the interference so the interference term will be zero:

$$i_k \sum_{l=1, l\neq k}^{K} h_k^T G_l \, p_l^{1/2} x_l = 0 \tag{4.8}$$

By using the equations 4.7 and 4.8, the equation (4.4) can be written as:

$$y_k = h_k^T G_k \left|p\left(M - K\right) + c_k\right|^{1/2} x_k + d_k + n_k \tag{4.9}$$

And the corresponding signal to noise ratio for the k^{th} user (SNR_k) can be computed as:

$$SNR_k = \left(\frac{p\left(M - K\right) + c_k}{D_k + 1}\right) \tag{4.10}$$

The corresponding achievable rates for the k^{th} user can be defined as:

$$R_k = \left[log_2 \left(1 + SNR_k\right)\right] \tag{4.11}$$

$$R_k = \mathrm{E}\left[log_2 \left(1 + \left(\frac{p\left(M-K\right)+c_k}{D_k+1}\right)\right)\right] \tag{4.12}$$

By considering the over-head factor, achievable rate for the k^{th} user can be expressed as:

$$R_k = \left(1 - \frac{T_{sum}K}{U}\right)\mathrm{E}\left[log_2 \left(1 + \left(\frac{p\left(M-K\right)+c_k}{D_k+1}\right)\right)\right] \tag{4.13}$$

Whereas, the factor $\left(1 - \frac{T_{sum}K}{U}\right)$ accounts for the pilot over-head in each coherence block U and T_{sum} is the total relative pilot length.

4.2 Achievable Rates of Massive MIMO by considering the affects of non-Linear Amplifiers under the imperfect channel condition.

In this subsection, we have calculated the achievable rate of Massive MIMO under imperfect channel conditions. Perfect channel conditions mean that the BS knows all the frequency components of the channel which results in improvement in the performance of the system. In practical situations, due to infinite precision of the electronic instruments and instantaneous nature of the transmission, achieving a perfect CSI is almost impossible. Imperfect CSI causes the inevitable interference among the

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users which in turn affects the performance of the system. We have assumed that the average attenuation (β_k) between the users and base station antennas is inversely proportional to transmission power of each user and for the k^{th} user it will be ($\frac{p\alpha^2}{\beta_k}$). As explained earlier, the transmission is divided into two phases, i.e., pilot transmission followed by data transmission.

During the pilot transmission phase, variance of the estimated channel by using MMSE estimator can be written as (D. Ciuonzo et al, 2015):

$$\sigma^2_{\hat{h}_k} = \frac{\beta_k}{1+\frac{1}{pKT_p}}$$

So, the variance of the channel estimation error ε can be written as:

$$var(\varepsilon) = var(G) - var\left(\hat{G}\right)$$

During the data transmission phase, achievable rates for the k^{th} user by assuming the *ZF* and treating the estimated channel as true channel, considering the effects of a nonlinear amplifier under imperfect channel conditions, can be written as:

$$R_{k,im} = \mathrm{E}\left[log_2\left(1+\left(\frac{p(M-K)+c_k \Big/ \left(1+\frac{1}{KpT_p}\right)}{\left[\left[var(\varepsilon)\right]\times Kp+1\right]+D_k}\right)\right)\right]$$

$$R_{k,im} = \mathrm{E}\left[log_2 \left(1 + \left(\frac{ p(M-K) + c_k \Big/ \left(1 + \frac{1}{KpT_p} \right) }{ \left[\left[1 - \frac{1}{1 + \frac{1}{Kp\mathrm{T}_p^{ul}}} \right] \times Kp + 1 \right] + D_k } \right) \right) \right].$$

$$R_{k,im} = \mathrm{E}\left[log_2 \left(1 + \left(\frac{ p(M-K) + c_k }{ D_k + 1 + \frac{1}{T_p} + \frac{1}{pKT_p} } \right) \right) \right],$$

where T_p is the same as that of T_p^{ul} and, similarly, achievable rates for the k^{th} user by considering the pilot overhead can be expressed as:

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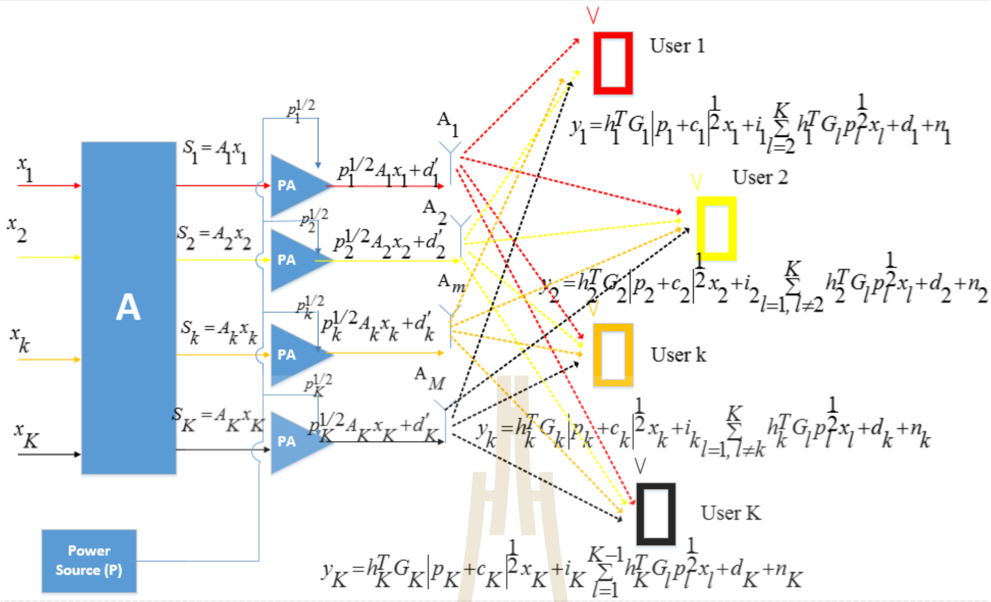

Figure 4.1 Block diagram of Massive MIMO with nonlinear amplifiers.

$$R_{k,im} = \left(1 - \frac{T_{sum}K}{U}\right) \mathrm{E}\left[log_2 \left(1 + \left(\frac{p(M-K)+c_k}{D_k + 1 + \frac{1}{T_p} + \frac{1}{pKT_p}} \right) \right) \right] \tag{4.2}$$

4.3 Modelling of Power Consumptions.

In this section we have modeled the power consumptions of Massive MIMO. The total power consumptions in the circuit of Massive MIMO can be composed into two parts:

$$P_{Tot} = P_{P.A} + P_{C.P} \tag{4.16}$$

Where $P_{P.A}$ is the total power consumed by the power amplifiers and can be illustrated as (Björnson et al, 2015):

$$P_{P.A} = \frac{\delta K p B \alpha^2}{\eta_{PA}} \tag{4.17}$$

Whereas δ is the path loss factor and when the required SNR will be fixed then this factor would be very important in order to calculate the total power consumption of the power amplifiers. Assume that the users are distributed uniformly in between the minimum d_{min} and the maximum distance d_{max}, then, δ the by using the uniform distribution can be expressed as:

$$\delta = \left(\frac{d_{max}^{\alpha+2} - d_{min}^{\alpha-2}}{\left(1+\frac{\alpha}{2}\right) d_{max}^2 - d_{min}^2} \right)$$

Where η_{PA} is the efficiency of the power amplifier and B is the bandwidth.

$P_{C.P}$ is the total circuit power consumptions of Massive MIMO i.e. power consumed in the transmitter and receiver chains, oscillator and filter power consumption, power required for the coding and decoding of the desired signals, power required for the channel estimation and linear processing. So, we need to model all the required or consumed power in the above mentioned processes.

Power consumed at the transmitter and receiver chain can be illustrated as:

$$P_{PTR} = M\left[P_{TC}\right] + K\left[P_{RC}\right] + P_{Os} \tag{4.18}$$

Where P_{PTR} is the total power consumption at the transmitter and receiver chains and P_{TC} is the power consumption at the transmitter chain and P_{RC} is the power consumption at the receiver chain i.e. power consumed at the filters, converters and mixers and P_{Os} is the oscillator power in order to synchronize the frequencies.

The power required for the coding and decoding of the desired signal can be demonstrated as:

$$P_{c/d} = R_K \left(P_c + P_d \right) \tag{4.19}$$

Where P_c and P_d denotes the corresponding power consumption during coding and decoding.

As explained in second section, Massive MIMO relies on CSI of the channel i.e. BS and Users have to send training or pilot signals during the uplink and downlink of the channel in order to get the frequency response of the channel during the coherence time. Power consumption during this process can be written as (Khan et al, 2017):

$$P_{ce} = \frac{2B}{U}\left[\frac{T^{ul} K^2 M}{\gamma_{bs}} + \frac{2T^{dl} K^2}{\gamma_{ue}} \right] \tag{4.20}$$

Where γ_{bs} and γ_{ue} are the computation efficiencies at the transmitter and receiver end. Consumption of power during linear processing by assuming ZF has been explained in (Björnson et al, 2015) and can be written as:

$$P_{ZF} = \frac{BK}{U\gamma_{bs}}\left(\frac{K^2}{3} + M\left(4K+1\right)\right) \tag{4.21}$$

So the total circuit power consumption of Massive MIMO by using the equations 4.18, 4.19, 4.20, and 4.21 can be expressed as:

$$P_{C.P} = P_{fix} + P_{PTR} + P_{ce} + P_{ZF}$$

$$P_{C.P} = P_{fix} + M\left[P_{TC}\right] + K\left[P_{RC}\right] + P_{Os} + \frac{2B}{U}\left(\frac{T^{ul}K^2M}{\gamma_{bs}} + \frac{2T^{dl}K^2}{\gamma_{ue}}\right) + \frac{BK}{U\gamma_{bs}}\left(\frac{K^2}{3} + M\left(4K+1\right)\right) \tag{4.22}$$

Where P_{fix} is the fixed power required for site cooling and the total power consumptions (equation 4.16) of Massive MIMO by using the equations 4.17 and 4.22 can be illustrated as:

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$$P_{Tot} = \frac{\delta KpB\alpha^2}{\eta_{PA}} + P_{fix} + M\left[P_{TC}\right] + K\left[P_{RC}\right] + P_{Os} + \frac{2BK^2}{U}\left(\frac{T^{ul}M}{\gamma_{bs}} + \frac{2T^{dl}}{\gamma_{ue}}\right) + \frac{BK}{U\gamma_{bs}}\left(\frac{K^2}{3} + M\left(4K+1\right)\right)$$

Total power consumptions can be written in more simplified and concentrated way:

$$P_{Tot} = \frac{\delta KpB\alpha^2}{\eta_{PA}} + \sum_{i=0}^{3} D_i K^i + M\sum_{i=0}^{2} E_i K^i \tag{4.23}$$

With the following substitutions:

$$D_0 = P_{fix} + P_{Os},\ D_1 = P_{RC},\ D_2 = \frac{4BT^{dl}}{U\gamma_{ue}},\ D_3 = \frac{B}{3U\gamma_{bs}}$$

$$E_0 = P_{TC},\ E_1 = \frac{B}{U\gamma_{bs}},\ E_2 = \frac{4B}{U\gamma_{bs}} + \frac{2B}{U\gamma_{bs}}$$

4.4 Energy Efficiency and Problem Formation

The energy efficiency of Massive MIMO can be defined as the spectral efficiency divided by the transmitted and the consumed power. As defined above, spectral efficiency of the system can be written as:

$$R_K = B\sum_{k=1}^{K} R_k .$$

where the total power can be written as the algebraic sum of transmitted and consumed power in the circuit of Massive MIMO as defined in the previous section on power modeling. So, Energy Efficiency (E.E) can be written as:

$$\text{E.E} = \frac{B\sum_{k=1}^{K} R_k}{P_{P.A} + P_{C.P}} .$$

In the following subsections, we have calculated and formulated the problem of energy efficiency maximization under perfect and imperfect CSI.

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4.4.1 Energy Efficiency under Perfect CSI

The spectral efficiency of Massive MIMO when the channel is perfectly known and by considering the effects of nonlinear power amplifiers can be written as:

$$R_K = \frac{K}{ln(2)}\left(1-\frac{T_{sum}K}{U}\right)B\left[ln\left(1+\left(\frac{p(M-K)+c_k}{D_k+1}\right)\right)\right].$$

The average total power as explained in the previous section can be written as:

$$P_{Tot} = \frac{\delta KpB\alpha^2}{\eta_{PA}} + \sum_{i=1}^{3} D_i K^i + M\sum_{i=0}^{2} E_i K^i .$$

So, the energy efficiency when the channel is perfectly known can be written as:

$$\text{E.E}_1 = \frac{\frac{K}{ln(2)}\left(1-\frac{T_{sum}K}{U}\right)B\left[ln\left(1+\left(\frac{p(M-K)+c_k}{D_k+1}\right)\right)\right]}{\frac{\delta KpB\alpha^2}{\eta_{PA}} + \sum_{i=1}^{3} D_i K^i + M\sum_{i=0}^{2} E_i K^i} .$$

We need to maximize the energy efficiency of Massive MIMO and in order to maximize the energy efficiency, consider the following mathematical optimization problem:

$$\begin{aligned} &\text{Maximize } \ \text{EE}_1(M,K,p) \\ &\text{Constraint to: } M \in Z_+,\ K \in Z_+ \\ &\qquad\qquad\qquad M > K\ ,\ p > 0 . \end{aligned}$$

As the number of BS antennas and users cannot be negative, they have been set positive in the first two constraints of optimization problem and the third constraint is the basic condition that holds for Massive MIMO (number of antennas are greater than number of users).

4.4.2 Energy Efficiency under Imperfect CSI

In this subsection, we have calculated the energy efficiency under imperfect channel conditions which results in inevitable interference among users. The spectral efficiency under the imperfect channel conditions can be written as:

$$R_{K,im} = \frac{K}{ln(2)}\left(1-\frac{T_{sum}K}{U}\right)B\left[ln\left(1+\left(\frac{p(M-K)+c_k}{D_k+1+\frac{1}{T_p}+\frac{1}{pKT_p}}\right)\right)\right]$$

.

So, the corresponding energy efficiency (E.E$_2$) under the imperfect channel conditions can be written as:

$$\text{E.E}_2 = \frac{\frac{K}{ln(2)}\left(1-\frac{T_{sum}K}{U}\right)B\left[ln\left(1+\left(\frac{p(M-K)+c_k}{D_k+1+\frac{1}{T_p}+\frac{1}{pKT_p}}\right)\right)\right]}{\frac{\delta KpB\alpha^2}{\eta_{PA}}+\sum_{i=1}^{3}D_iK^i+M\sum_{i=0}^{2}E_iK^i}.$$

The corresponding optimization problem of energy efficiency maximization under imperfect channel conditions can be illustrated as:

$$\begin{aligned}&\text{Maximize } \text{EE}_2(M,K,p)\\&\text{Constraint to: } M\in Z_+,\ K\in Z_+\\&\qquad\qquad M>K,\ p>0\end{aligned}.$$

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4.5 Problem Solution and Numerical Algorithm

In this section, we have designed an algorithm to solve the optimization problems EE_1 and EE_2. It is difficult to solve the optimization problem of EE_1 and EE_2 due to mixed nature of their corresponding objective functions with respect to *M*, *K* and *p*. Consider the following substitutions in order to simplify the objective functions of EE_1 and EE_2:

$$z_1 = K,\ z_2 = M / K,\ z_3 = Kp \ ,$$

where z_1 can be explicated as the number of active users, z_2 can be explicated as the number of active antennas per user and z_3 along with multiplication of some constant factor as described in Equation 4.17 can be explicated as the total power of power amplifiers. The simplified objective functions of EE_1 under perfect channel conditions can be written as:

$$\mathrm{E.E}_1 = \frac{\dfrac{z_1}{ln(2)}\left(1-\dfrac{T_{sum}z_1}{U}\right)B\left[ln\left(1+\left(\dfrac{z_3(z_2-1)+c_k}{D_k+1}\right)\right)\right]}{\dfrac{z_3\delta B\alpha^2}{\eta_{PA}}+\sum_{i=1}^{3}D_i z_1^i + z_2\sum_{i=0}^{2}E_i z_1^{i+1}}$$

with the following modified optimization problem:

$$\begin{aligned}
&\text{Maximize } \ \mathrm{EE}_1(z_1, z_2, z_3) \\
&\text{Constraint to: } z_1 > 0,\ z_2 > 1 \\
&\qquad\qquad\qquad\quad z_3 > 0 \ .
\end{aligned}$$

Similarly, objective function of energy efficiency under imperfect channel conditions following the above mentioned substitutions can be written as:

$$\mathrm{E.E}_2 = \frac{\dfrac{z_1}{ln(2)}\left(1-\dfrac{T_{sum}z_1}{U}\right)B\,ln\left(1+\left(\dfrac{z_3(z_2-1)+c_k}{D_k+1+\dfrac{1}{T_p}+\dfrac{1}{z_3T_p}}\right)\right)}{\dfrac{\delta z_3 B\alpha^2}{\eta_{PA}}+\sum_{i=1}^{3}D_i z_1^i+z_2\sum_{i=0}^{2}E_i z_1^{i+1}}, \tag{4.28}$$

With the following modified optimization problem:

$$\begin{aligned}&\text{Maximize } \mathrm{EE}_2(z_1,z_2,z_3)\\ &\text{Constraint to: } z_1>0,\ z_2>1\cdot\\ &\qquad\qquad\qquad\quad z_3>0\end{aligned}$$

Objective functions of optimization problems EE_1 and EE_2 follows a quasi-concave response because they are first increasing and then deceasing in each dimension while the other dimensions are fixed and their second order derivatives are less than zero. The proof of the quasi-concave nature of objective functions (EE_1 and EE_2) have been shown in the Appendixes A and B respectively.

According to Appendixes A and B, objective functions EE_1 and EE_2 undergo a peak point at the unique zero crossing of $\mathrm{EE}_1^{'}$ and $\mathrm{EE}_2^{'}$ in each dimension while the other dimensions are fixed. The following flow chart summarizes the above mentioned discussions and shows the simulation steps (Figure 4.2).

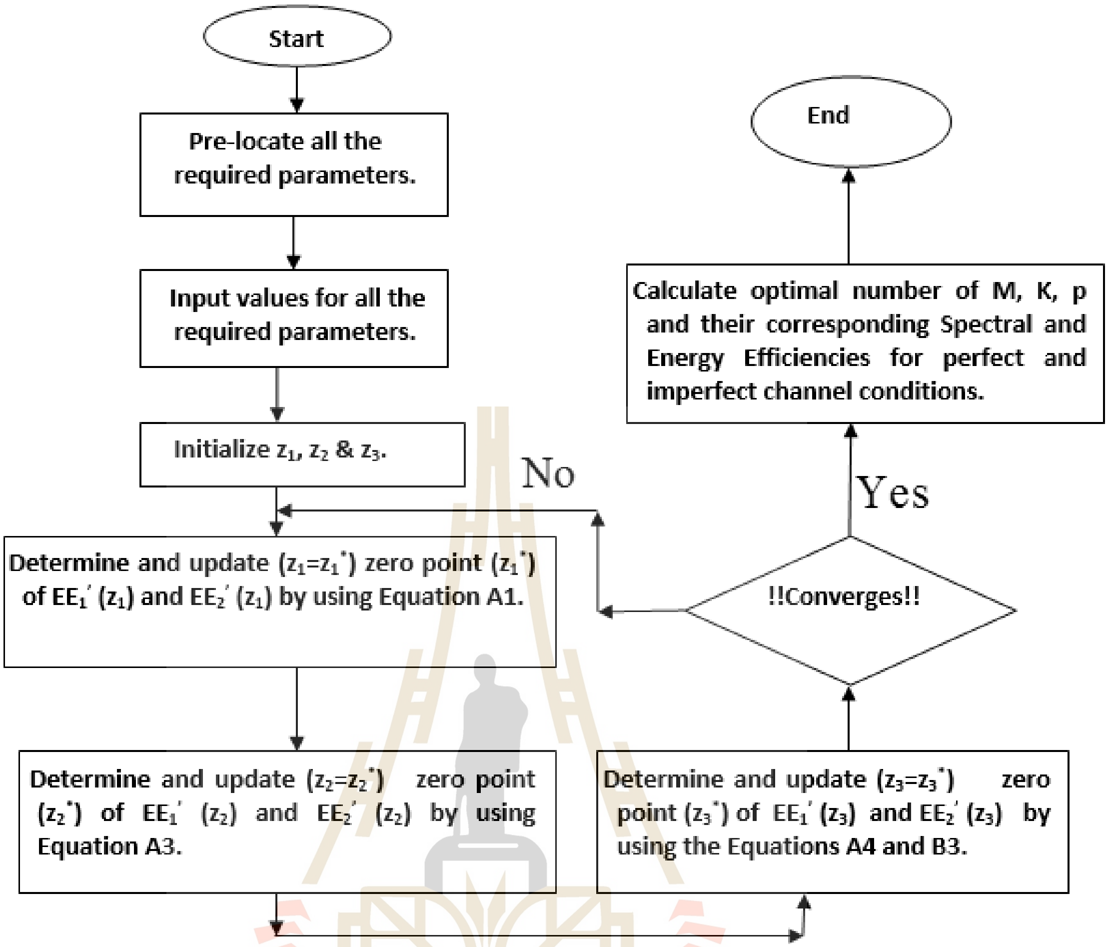

Figure 4.2 Alternative numerical algorithm for solving the optimization problem

4.6 Simulations and Numerical Results

In this section, we have performed simulations to test the mathematical and numerical algorithm discussed in the earlier sections. Realistic simulation parameters have been chosen for simulations as shown in the Table 4.1.

Table 4.1 Simulation parameters

Parameter	**Value**
Transmission Bandwidth (B)	20 MHz
Coherence Block (U)	1800
Computational efficiency at BSs (γ_{bs})	12.8 Gflops/W
Computational efficiency at Users (γ_{ue})	6 Gflops/W
Clipping power Loses (c_k)	–0.15 dB
Path loss exponent (α)	3.8
Distortion (D_k)	−25 dB
Total Noise Power ($B\alpha^2$)	−96 dBm
Pilot Lengths (T_p , T_{sum})	1 m, 2m
Power Amplifier Efficiency when fixed (η_{PA})	0.34

Figure 4.3 shows the optimal number of transmitters at different area of coverage ranges from 100 m to 500 m by setting different circuit power consumption levels under both the perfect and imperfect channel conditions As can be seen from Figure 4.3, when the coverage area increases, the optimal number of transmitters increases, respectively, in order to cover that area and when the channel condition is imperfect then more numbers of transmitters are required, whereas when the power consumptions of the circuit are less, optimal numbers of transmitters required for the system are less and vice versa.

Similarly, Figure 4.4 shows the optimal number of users at different area of coverage ranges from 100 m to 500 m at different circuit power consumption levels under both the perfect and imperfect channel situations. As can be seen from the Figure 4.4, more users can be accommodated at a higher area of coverage. Figure 4.5 shows the optimal transmitted or PA power at different area of coverage ranges from 100 m to 500 m by setting different circuit power consumption levels under both the perfect and imperfect channel conditions.

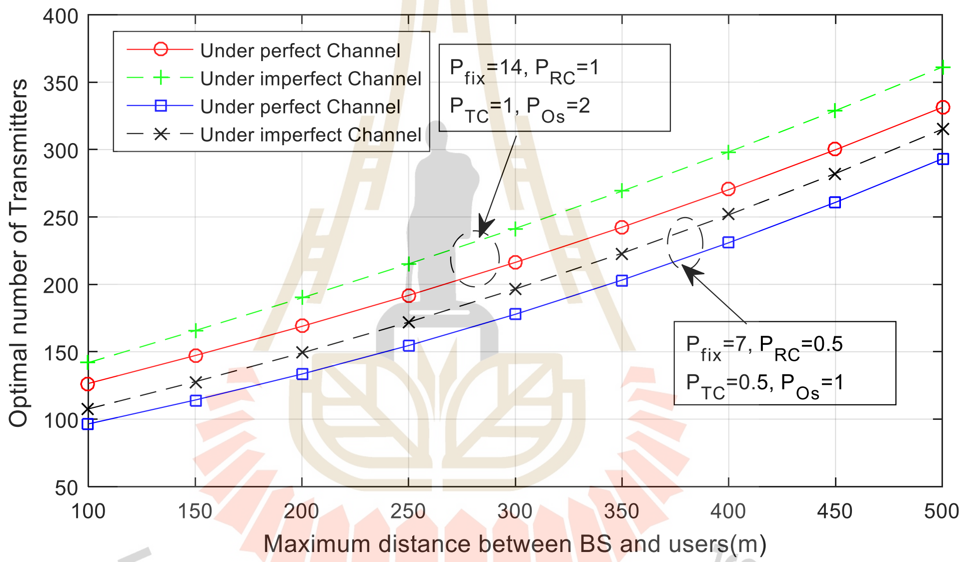

Figure 4.3 Optimal number of transmitters at different area of coverages

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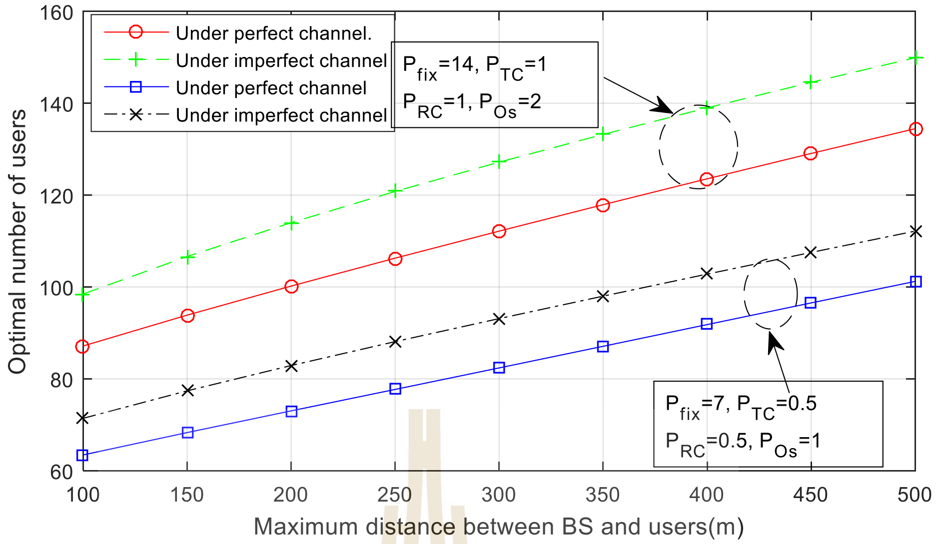

Figure 4.4 Optimal number of receivers at different area of coverages

As can be seen from Figure 4.5, more transmitted power is required in order to cover more distance and imperfect channel condition results in more transmitted power with the corresponding area throughput that maximizes the energy efficiency of Massive MIMO shown in Figure 4.6. Figure 4.7 shows the optimal energy efficiency and it can be seen from Figure 4.7 that less power consumptions of the circuit results in more achievable energy efficiency and under imperfect channel conditions energy efficiency is reduced because the system need to transmit more transmitted or PA power in order to mitigate the negative effects of imperfect channel conditions.

Figures 4.8 and 4.9 show the 3D representation of energy efficiency along with all the optimal parameters in which maximum distance is set to be 300 m and power consumption parameters are set to be $P_{fix}=14$, $P_{TC}=1$, $P_{RC}=1$ and $P_{Os}=2$ under both the perfect and imperfect channel conditions respectively. The optimal parameters come out to be $M=216$, $K=112$, $P_{P.A}=141.4$ W, EE = 19.5 Mbit/Joule whereas energy efficient area throughput to be 11.9 Gbits/Km^2 in the case of perfect channel conditions as shown in Figure 4.9 and when the channel conditions are not perfectly known then the optimal parameters comes out to be M = 241, K = 127, $P_{P.A}$ = 245 W, EE = 16.1 Mbit/Joule and area through put = 11.2 Gbits/Km^2 as shown in Figure 4.9.

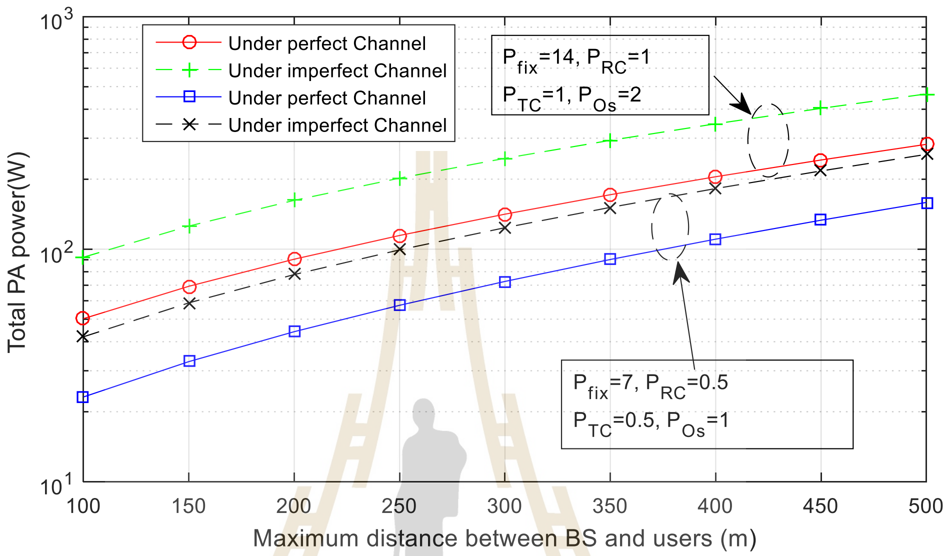

Figure 4.5 Optimal Power of non-linear amplifiers

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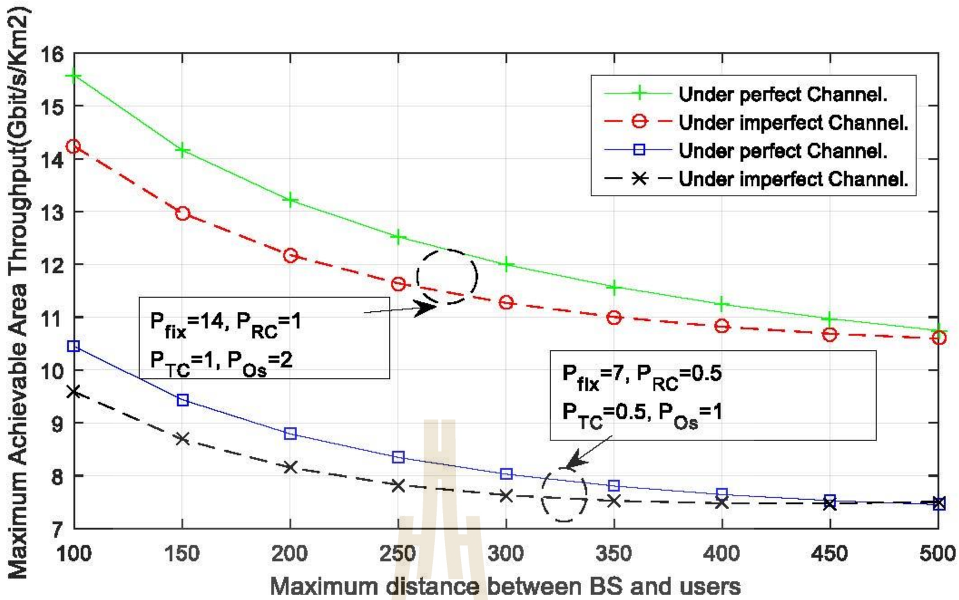

Figure 4.6. Area throughput.

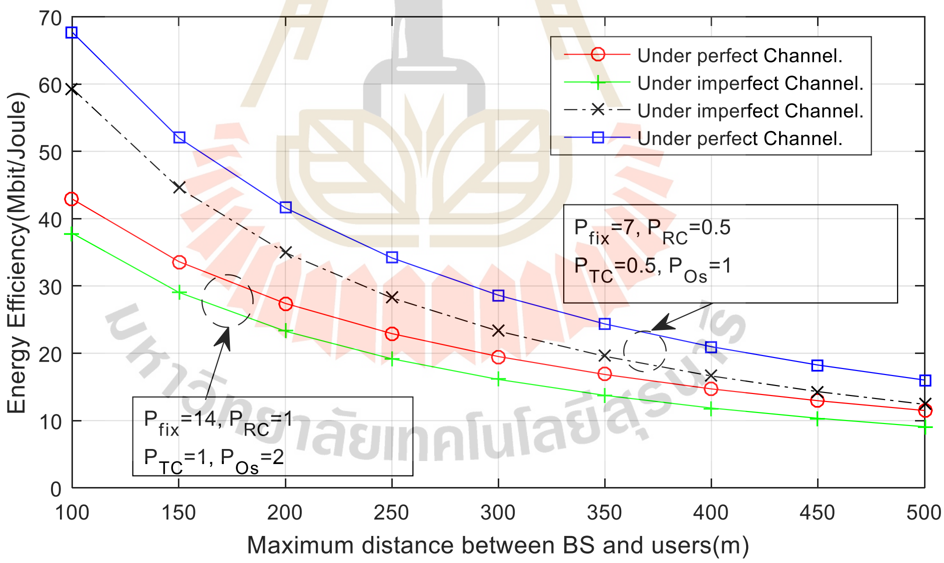

Figure 4.7 Energy efficiency (EE) at different area of coverages

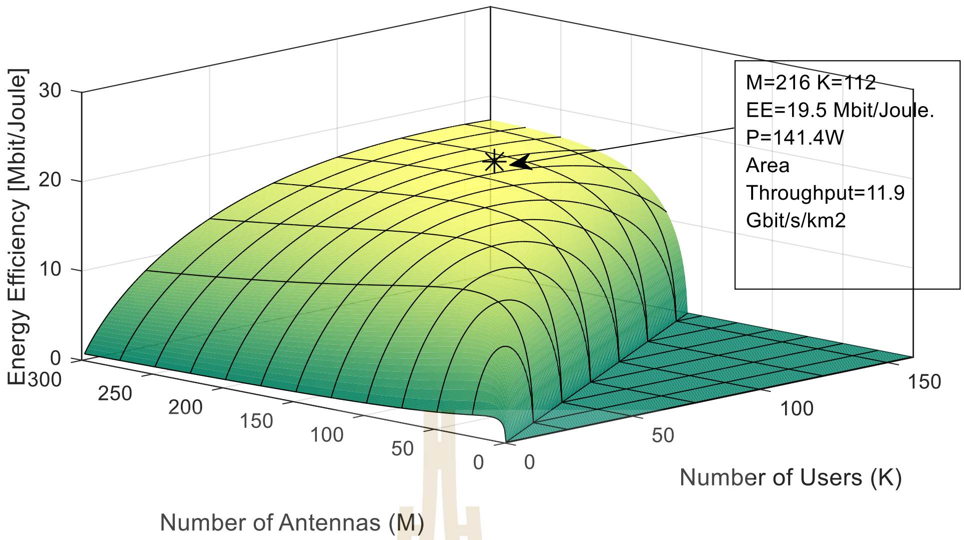

Figure 4.8 3-D representation of EE along with optimal parameters under perfect channel situation

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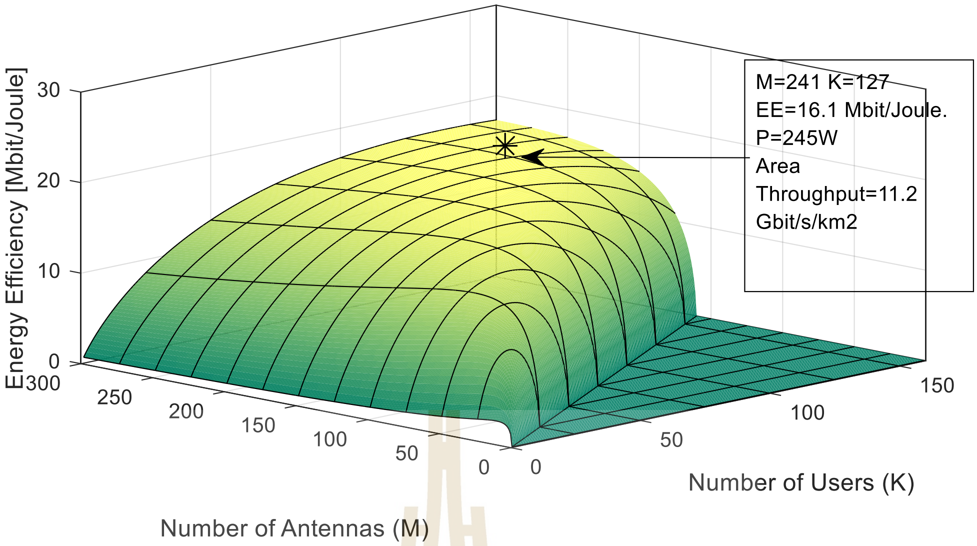

Figure 4.9 3-D representation of EE along with optimal parameters under Imperfect channel situation

Figure 4.10 shows the convergence of energy efficiency with respect to the number of iterations by using the numerical algorithm (discussed in Section 4.5) at various distances under the perfect and imperfect channel conditions. The computation complexity of the proposed algorithm at each iteration can be written as:

$$\text{Computation Complexity at each iteration} = \\ O\left(z_1^4\right)+O\left(z_2\, ln\left(1+z_2\right)\right)+O\left(z_3\, ln\left(z_3\right)\right),$$

where $O\left(z_1^4\right)$ represents the required computation complexity during the computation of z_1, and $O\left(z_2\, ln\left(1+z_2\right)\right)$ and $O\left(z_3\, ln\left(z_3\right)\right)$ represent the required computation complexity during the computation of z_2 and z_3 respectively at each iteration. As can be seen from the Figure 13, the energy efficiency converges completely at the third iteration, thus the overall computation complexity of the proposed algorithm can be written as $6\left[\mathrm{O}\left(n\,ln\left(n\right)\right)\right]$. The power consumptions parameters in Figure 13 are set to be $P_{fix}=7$, $P_{TC}=0.5$, $P_{RC}=0.5$ and $P_{Os}=1$.

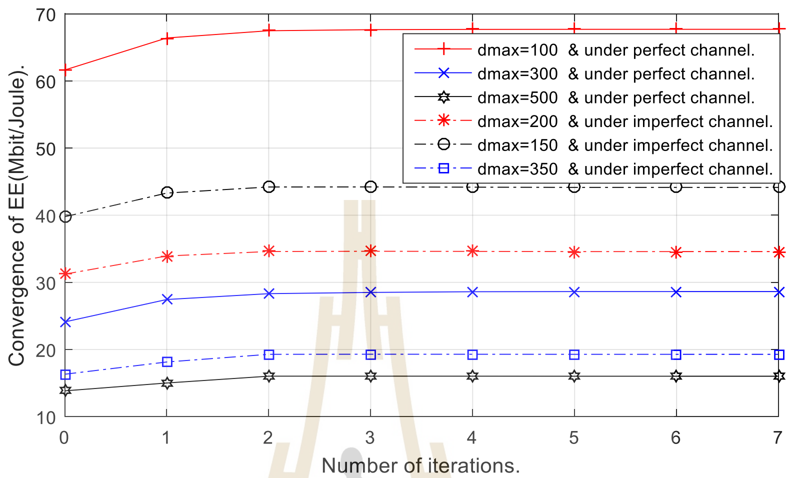

Figure 4.10 Convergence of energy efficiency

Figure 4.11 shows the impacts of power amplifier efficiencies on the energy efficiency of Massive MIMO and it can be seen easily that when the power amplifiers are operating at higher efficiency, energy efficiency is maximum and vice versa under both perfect and imperfect channel conditions. The power consumptions parameters are set to be $P_{fix}=7$, $P_{TC}=0.5$, $P_{RC}=0.5$ and $P_{Os}=1$ for simulations in Figure 4.11.

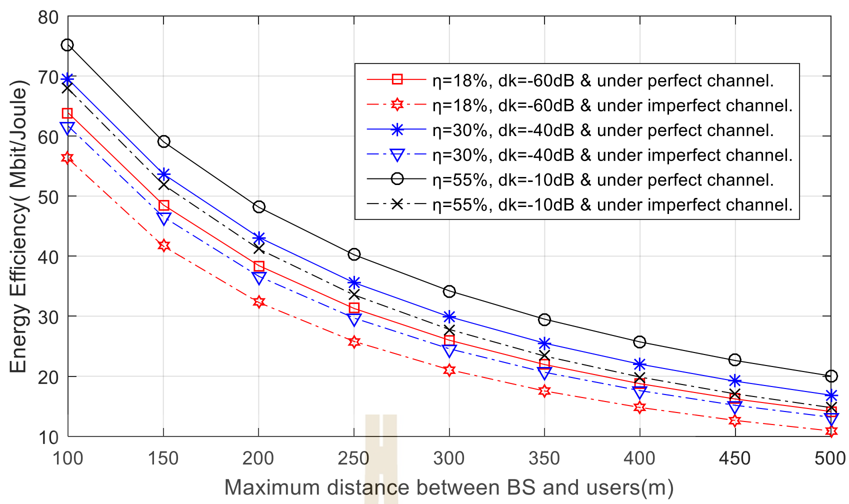

Figure 4.11 Energy efficiency at different distortion level and power amplifier efficiencies

4.7 Summary

This chapter mainly focused on the energy efficiency of Massive MIMO by considering the effects of nonlinear amplifiers. The impact of nonlinear amplifiers is investigated on the energy efficiency of massive MIMO along with calculation of optimal parameters by using the proposed alternative algorithm under both the perfect and imperfect channel conditions at different circuit power consumptions. Contrary to the existing work, we used a realistic circuit power consumption model that shows the dependence of circuit power consumption on the number of transmitters and users. We have seen that when the channel conditions are not perfectly known, then the system needs to transmit more power in order to overcome the negative effects of imperfect channel situations, and, owing to more transmitted PA power, the energy efficiency gets reduced as compared to the situation when the channel is perfectly known. Numerical results do not change much for a small change in the circuit power consumption but can otherwise change drastically. The alternative algorithm that we have used for joint calculation of optimal parameters works efficiently and converges quickly. Simulations result shows that when the power amplifiers are working at higher efficiency, then the energy efficiency of Massive MIMO also is increased, while it is better to have large cell coverage in the case of Massive MIMO along with less circuit power consumptions. In future, circuit power consumptions will be reduced, resulting in further improved energy efficiency with less transmitted or PA power, together with improved and simpler signal processing. The combination of energy efficient massive MIMO along with nonlinear amplifiers can be a fascinating option for low cost future wireless systems.

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CHAPTER V

Energy Efficient Designing of Massive MIMO Based on Closely Spaced Antennas: Mutual Coupling Effect

In this chapter, we have derived and formulated the mathematical expressions of the achievable spectral efficiency along with the energy efficiency by taking the effect of mutual coupling between the transmitting antennas at the base station with respect to the different channel conditions. We have designed the energy efficient Massive MIMO by using the correct model of the circuit power consumption and computed the number of optimal user terminals, transmitting antennas and the corresponding consumed as well as the transmitted power. Mutual coupling effect among the antenna elements have been thoroughly investigated on the energy efficiency and other optimal parameters, by varying the length of the transmitting antennas and the spacing between them. The optimization problem of energy efficiency has been formulated in order to calculate the optimal parameters with respect to the different channel conditions. We have proposed a domain splitter algorithm for the optimization of energy efficiency, and calculation of the optimal parameters. A detailed comparison is presented between the reference and the proposed algorithms in terms of the computation complexity. At the end, we have made simulations in order to support and show the effectiveness of the mathematical modelling, where the simulation result shows the optimally achieving energy efficiency and other optimal system parameters, with respect to different channel conditions and the spacing between the antenna elements.

5.1 Spectral Efficiency under the perfect channel condition:

Consider the M number of the antennas are equipped at the BS, having the length L and separated by a distance s, serving K number of uniformly distributed single antenna users as can be seen in Figure 5.1. Therefore, the received signal at the base station can be written as:

$$y = \sqrt{p}Gx + n \tag{5.1}$$

where $x = \left[x_1, x_2, ...x_K\right]$ is the matrix of the transmitted signals from the K number of uniformly distributed single antenna users and y is the received signal matrix with the dimension of $M \times 1$ at the base station. p can be deemed as the average transmitted power and n is the Additive White Guassian Noise with the dimension of $M \times 1$ and has the zero mean and unity variance, whereas G is the channel matrix with the dimension of $M \times K$.

As explained earlier, the transmitting antennas at the base station are closely spaced, leading into the effect of mutual coupling. Thus, the channel matrix by considering the mutual coupling effect among the base station antennas can be expressed as:

$$G = CHD^{1/2}$$

where C is the mutual coupling matrix with the dimension of $M \times M$ and H is the Rayleigh small scale fading matrix. The expressions of the mutual coupling matrix when the M number of antennas are equipped at the BS and the mutual impedance between two particular transmitting antennas i and j are explained in the equations 3.2 and 3.3.

Assume that, the zero forcing linear detection scheme is implemented at the base station, then, the signal received at the base station can be expressed as:

$$y = V^H\left(\sqrt{p}Gx + n\right)$$

Similarly, the signal received from the kth user terminal can be written as:

$$y_k = V_k{}^H\left(\sqrt{p}Gx_k + n\right)$$

$$y_k = \sqrt{p}x_k + v_k^H n$$

where v_k represent the kth column of the detection matrix V and the equivalent noise $v_k^H n$ can be expressed as: $v_k^H n \sim \mathrm{CN}\left(0, \left\|v_k^H\right\|^2\right)$. The Signal to Noise Ratio (S.N.R) at the corresponding kth link can be expressed as:

$$SNR_k = \frac{p}{\left\|V_k^H\right\|^2} \tag{5.2}$$

As we know $\|V\|^2 = VV^H$. Thus, the above equation can be written as:

$$SNR_k = \frac{p}{\left(G_k^H G_k\right)^{-1}} \tag{5.3}$$

The corresponding achievable rate can be illustrated as:

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$$R_k = \mathrm{E}\left[log_2\left(1 + \frac{p}{\left(G_k^H G_k \right)^{-1}} \right) \right] \tag{5.4}$$

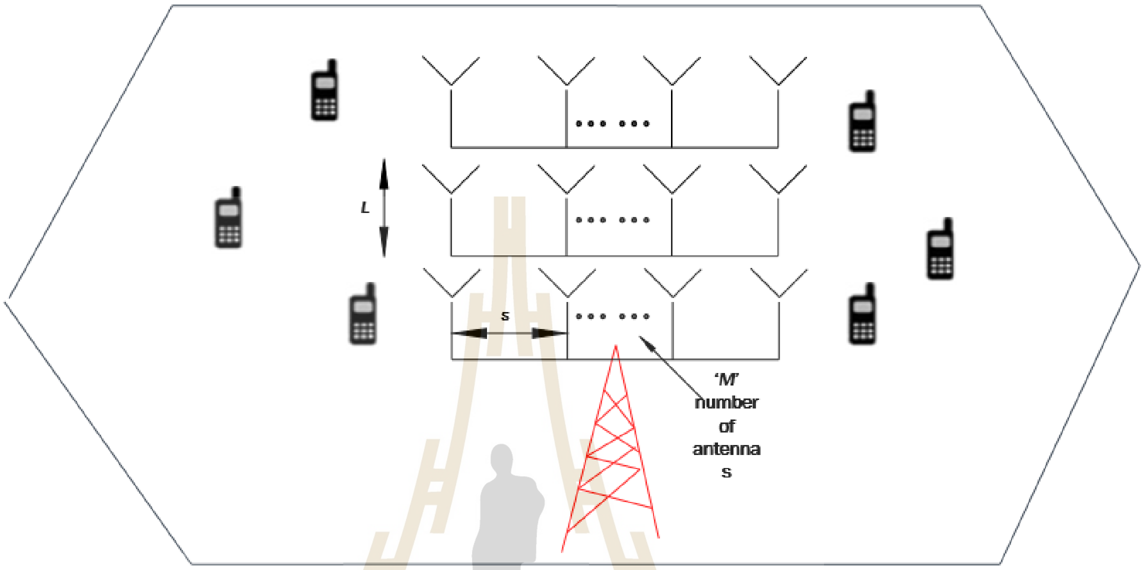

Figure 5.1 Massive MIMO with M antenna terminals

Theorem 1. In Rayleigh fading channel, when $M \geq K+1$ and ZF detector is implemented at the base station, then, the achievable rate for the kth user terminal can be written as:

.

$$R_k = \frac{1}{ln(2)} ln\left(1 + p\beta_k trace\left(C^H C \right) \times (M-K) \right) \tag{5.5}$$

Proof of the Theorem 1. See Appendix C.

The overall spectral efficiency (R_K) for the total K number of users can be written as:

$$R_K = B\sum_{k=1}^{K} R_k \tag{5.6}$$

Put the value of R_k from the equation 5.5 into equation 5.6:

$$R_K = \frac{B}{ln(2)}\sum_{k=1}^{K} ln\left(1+p\beta_k trace\left(C^H C\right)\times\left(M-K\right)\right)$$

$$R_K = K\frac{B}{ln(2)} ln\left(1+p\beta_k trace\left(C^H C\right)\times\left(M-K\right)\right) \tag{5.7}$$

The overall spectral efficiency by taking the overhead of the training signals into account can be written as:

$$R_K = K\left(1-\frac{\mathrm{T}K}{U}\right)\frac{B}{ln(2)} ln\left(1+\left(p\beta_k trace\left(C^H C\right)\times\left(M-K\right)\right)\right)$$

Whereas, T is the total length of the relative pilot sequence.

5.1.1 Computation of the Spectral Efficiency under the Scenario of Imperfect Channel Situation

The performance of the Massive MIMO is significantly dependent on the acquisition of the C.S.I. However, having a perfect (C.S.I) is almost unfeasible in terms of real-world scenario, which in turn leads to interference among the user terminals. During the uplink transmission, the C.S.I is computed by using the pilot signals, and, let the uplink power of the pilot signal for the kth user terminal be $\left(\frac{\sigma^2 p}{\beta_k}\right)$ and the total length of the orthogonal pilot signals be $\mathrm{T}_p^{ul}K$ as shown in the Figure 2.1. The estimated channel matrix $\hat{G}$ can be expressed as:

$$\hat{G} = G + \varepsilon$$

where ε is the channel estimation error during the acquisition of the frequency response of propagation channel. Therefore, the signal received at the BS can be written as:

$$y_{Im} = V^H\left(\sqrt{p}Gx + \varepsilon + n\right)$$

Theorem 2. In Rayleigh fading channel, when $M \geq K+1$ and ZF detector is implemented at the BS, then, the achievable rate for the kth user terminal by computing the estimated channel with the help of the MMSE estimator for the imperfect channel condition can be expressed as:

$$R_{k,Im} = log_2\left(1 + \frac{p\beta_k trace\left(C^H C\right)\times\left(M-K\right)}{1 + \frac{1}{\mathrm{T}_p^{ul}} + \frac{1}{pK\mathrm{T}_p^{ul}}}\right) \tag{5.8}$$

Proof of Theorem 2. See Appendix D.□

Similarly, the overall spectral efficiency by taking the overhead of the training signals into account for the imperfect channel conditions can be expressed as:

$$R_{K,Im} = K\left(1 - \frac{\mathrm{T}K}{U}\right)\frac{B}{ln(2)} ln\left(1 + \frac{p\beta_k trace\left(C^H C\right)\times\left(M-K\right)}{1 + \frac{1}{\mathrm{T}_p^{ul}} + \frac{1}{pK\mathrm{T}_p^{ul}}}\right) \tag{5.9}$$

5.2 Problem Definition

In this section, we have formulated the problem and derived the mathematical expressions of the energy efficiency for the perfect and imperfect channel situation. Energy efficiency can be expressed as the ratio of total spectral efficiency and the total power consumption i.e.

$$\mathrm{EE} = \frac{R_K}{P_{tot}}$$

Thus, the Energy efficiency of Massive MIMO $\mathrm{EE}_P(M,K,p)$ by using the equations 5.7 and 5.10 under the perfect channel condition can be expressed as:

$$\mathrm{EE}_P(M,K,p) = \frac{K\left(1-\frac{\mathrm{T}K}{U}\right)\frac{B}{ln(2)}\times ln\left(1+p\beta_k trace\left(C^H C\right)\times(M-K)\right)}{\frac{B\alpha^2 KpS_x}{\eta}+\sum_{i=0}^{3}C_i K^i + M\sum_{j=0}^{2}D_j K^j} \tag{5.11}$$

Energy efficiency needs to be optimized, so, the optimization problem of Energy efficiency can be formulated as:

$$\begin{aligned} &\text{Max} && \mathrm{EE}_P(M,K,p) \\ &\text{Subject to} && M \in \mathrm{Z}_+,\ \mathrm{K} \in \mathrm{Z}_+ \\ & && \mathrm{K}<\mathrm{M},\quad p>0 \end{aligned} \tag{5.12}$$

Similarly, the energy efficiency of Massive MIMO $\mathrm{EE}_{Im,P}(M,K,p)$ by using the equations 5.9 and 5.10 along with the optimization problem for the Imperfect channel condition can be expressed as:

$$\mathrm{EE}_{Im,P}\left(M,K,p\right)=\frac{K\left(1-\frac{\mathrm{T}K}{U}\right)\frac{B}{ln\left(2\right)}\times ln\left(1+\frac{p\beta_k trace\left(C^H C\right)\times\left(M-K\right)}{1+\frac{1}{\mathrm{T}_p^{ul}}+\frac{1}{pK\mathrm{T}_p^{ul}}}\right)}{\frac{B\alpha^2 KpS_x}{\eta}+\sum_{i=0}^{3}C_i K^i+M\sum_{j=0}^{2}D_j K^j} \tag{5.13}$$

$$\begin{aligned} &\text{Max} && \mathrm{EE}_{Im,P}\left(M,K,p\right) \\ &\text{Subject to} && M\in\mathrm{Z}_+,\ \mathrm{K}\in\mathrm{Z}_+ \\ & && \mathrm{K}<\mathrm{M},\quad p>0 \end{aligned} \tag{5.14}$$

We need to calculate the optimal number of BS antennas, users, and their corresponding transmitted power and energy efficiency. The M, K and p are closely associated and related to each other as it can be seen in the equations 5.11 and 5.13. Take the following substitutions and their corresponding interpretations (Table 5.1) into account for the simplification of $\mathrm{EE}_P\left(M,K,p\right)$ and $\mathrm{EE}_{Im,P}\left(M,K,p\right)$.

Table 5.1 Substitutions and physical interpretations

Substitution	**Interpretation**
$r_1=K$	Number of optimal user terminals.
$r_2=M/K$	Number of optimal active antennas per user terminal.
$r_3=pK$	Total transmitted power.

Following the above mentioned substitutions, the $\mathrm{EE}_P\left(M,K,p\right)$ and $\mathrm{EE}_{Im,P}\left(M,K,p\right)$ can be modified as:

$$\mathrm{EE}_P\left(r_1,r_2,r_3\right)=\frac{r_1\left(1-\frac{\mathrm{T}r_1}{U}\right)\frac{B}{ln(2)}\times ln\left(1+r_1\beta_k trace\left(C^H C\right)\times\left(r_2-1\right)\right)}{\frac{B\alpha^2 r_3 S_x}{\eta}+\sum_{i=0}^{3}C_i r_1^i++r_2\sum_{j=0}^{2}D_j r_1^{j+1}} \quad (5.15).$$

$$\mathrm{EE}_{Im,P}\left(r_1,r_2,r_3\right)=\frac{r_1\left(1-\frac{\mathrm{T}r_1}{U}\right)\frac{B}{ln(2)}\times ln\left(1+\frac{r_3\beta_k trace\left(C^H C\right)\times\left(r_2-1\right)}{1+\frac{1}{\mathrm{T}_p^{ul}}+\frac{1}{r_3\mathrm{T}_p^{ul}}}\right)}{\frac{B\alpha^2 r_3 S_x}{\eta}+\sum_{i=0}^{3}C_i r_1^i+r_2\sum_{j=0}^{2}D_j r_1^{j+1}} \quad (5.16)$$

Thus, the corresponding mathematical optimization problems for the perfect and imperfect channel situation can be simplified as:

$$\begin{aligned} &\text{Max} && \mathrm{EE}_P\left(r_1,r_2,r_3\right) \\ &\text{Subject to} && \mathrm{r}_1\in Z_+,\ \mathrm{r}_2\in Z_+ \\ &&& \mathrm{r}_3>0 \end{aligned} \quad (5.17)$$

And:

$$\begin{aligned} &\text{Max} && \mathrm{EE}_{Im,P}\left(r_1,r_2,r_3\right) \\ &\text{Subject to} && \mathrm{r}_1\in Z_+,\ \mathrm{r}_2\in Z_+ \\ &&& \mathrm{r}_3>0 \end{aligned} \quad (5.18)$$

5.4 Computation of the Optimal Parameters and the Complexity Comparison

In this section, we have proposed and discussed the domain splitter algorithm to calculate the optimal parameters and the comparison is presented between the proposed and the reference algorithms, in terms of the computation complexity. We have named the proposed algorithm as the domain splitter algorithm because the proposed algorithm splits the domains of $\mathrm{EE}_P(r_1,r_2,r_3)$ and $\mathrm{EE}_{Im,P}(r_1,r_2,r_3)$, and then find the solution in each respective domain as shown in Figure 5.2. The $\mathrm{EE}_P(r)$ and $\mathrm{EE}_{Im,P}(r)$ shows the response of the quasi concave and depicts the one and the only one zero crossing at the points $\mathrm{EE}'_P(r)=0$ and $\mathrm{EE}'_{Im,P}(r)=0$ in each respective domain $r=(r_1,r_2,r_3)$, as per the appendixes E and F. In the appendixes E and F, the solution of the energy efficiency is derived in each respective domain $r=(r_1,r_2,r_3)$ and the optimal values are computed at the points $\mathrm{EE}'_P(r)=0$ and $\mathrm{EE}'_{Im,P}(r)=0$ by using these solutions. The Figure 5.2 shows the domain splitter algorithm and the simulation methodology where the optimal values of $r=(r_1,r_2,r_3)$ are computed until the convergence. Finally, the acquired optimal values are used to compute the number of optimal user terminal, optimal transmitters along with the transmitted power, energy and the spectral efficiency.

Table 5.2 shows the computation complexity between the proposed and the reference schemes and $W(n)$ is the lambert omega operation. It can be seen from Table 5.2 that the computational complexity of the reference algorithms is dependent on the multiplication of the number of transmitting antennas at the BS with the number of user terminals $(M\times K)$. As Massive MIMO is dependent on the theory of large numbers of transmitting antennas at the BS, so, the computational complexity behind the reference algorithms is so high. The proposed algorithm (domain splitter algorithm) significantly improves the computation complexity where the energy efficiency gets saturated at the third iteration as shown in the Figure 5.11.

Table 5.2 Complexity Comparison

Algorithm	Computational Complexity
(Björnson et al. 2015) algorithm.	$M \times K\left(\mathrm{O}\left(2^{W(n)}\right)\right)$
(Khan et al. 2017) algorithm.	$M \times K\left(\mathrm{O}\left(n^2 2^{W(n)}\right)\right)$
(Biswas et al. 2016) Algorithm	$M \times K\left(\mathrm{O}\left(2^{W\left(n^2\right)}\right)\right)$
Proposed Algorithm	$3\left(\mathrm{O}\left(n^3\, ln\left(n^3\right)\right)\right)$

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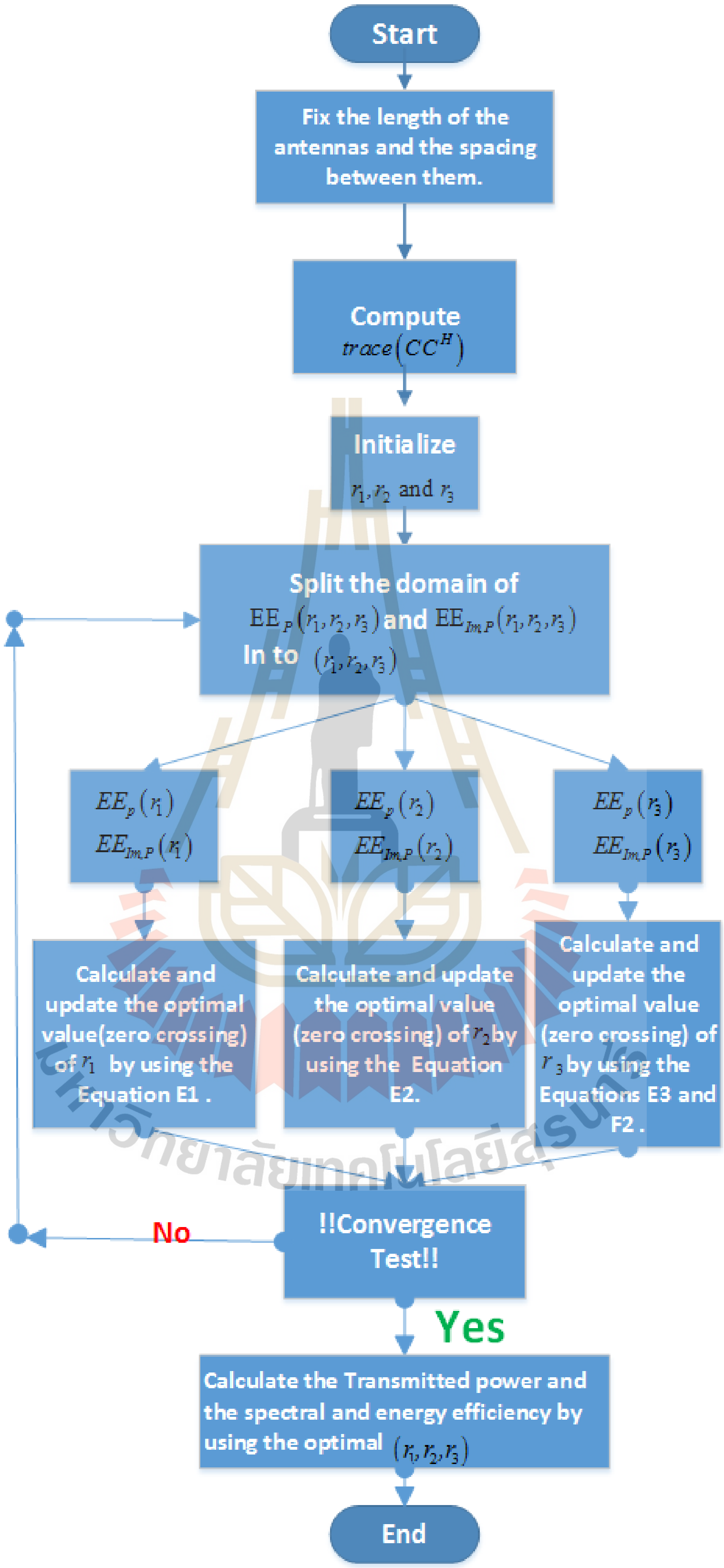

Figure 5.2 Domain Splitter Algorithm

5.5 Simulations

We have performed simulations in order to test and demonstrate the effectiveness of the domain splitter algorithm along with computation of different optimal system parameters. Table 5.3 unveils the basic simulation parameters used for the simulations. We have used the acronyms of PCSI and ICSI for the perfect and imperfect channel state information respectively in the simulation figures. Figure 5.3 unveils the input impedances by varying the length of the antenna, where the real part of the antenna impedance is showing the power that is either absorbed or radiated away and the imaginary part is showing the stored power. Input impedance depends significantly on the length of the antenna terminal, as it can be seen from Figure 5.3. Figure 5.4 unveils the mutual impedance between the antenna terminals with respect to the spacing among the antennas, calculated by using the equation 3.3, at different lengths of the antenna terminal.

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Table 5.3 Simulation Parameters.

Parameter	Value
Coherence Block Interval U	1800
Path loss exponent α	3.8 (Urban environment)
Computational efficiency at Base station (η_{Bs})	12 Gflops/W
Computational efficiency at the user end (η_{ue})	5.5 Gflops/W
Minimum distance d_{min}	30 m
Pilot lengths (T_p^{ul} , T)	1,2
PA efficiency (η)	35%

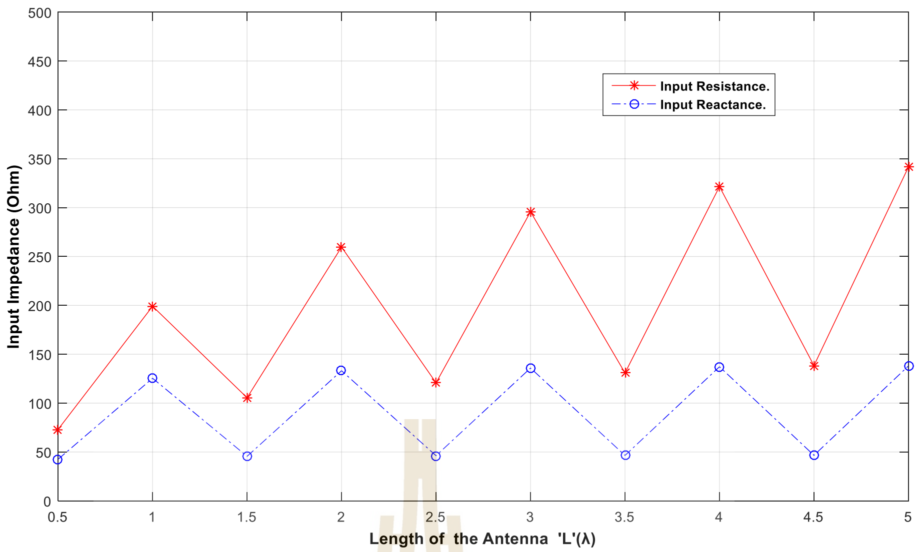

Figure 5.3 Input Impedance

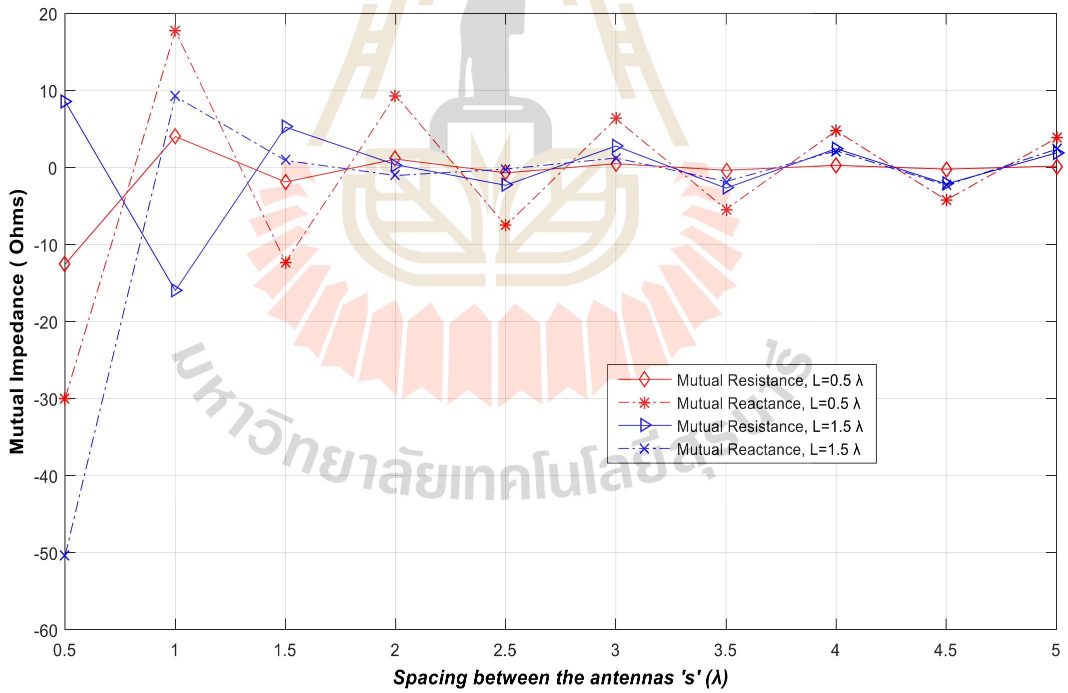

Figure 5.4 Mutual Impedances at different inter-antenna spacing

Figure 5.5 unveils the total number of optimal BS antennas with respect to the maximum distance between the base station and the users, at different circuit power consumption and at different spacing among the BS antennas. When the BS antennas are closely spaced, then it leads to mutual coupling and a higher number of the transmitting antennas are required. The effect of the mutual coupling and different antenna spacing on the optimal number of transmitters under the different channel situations can be seen in Figure 5.5.

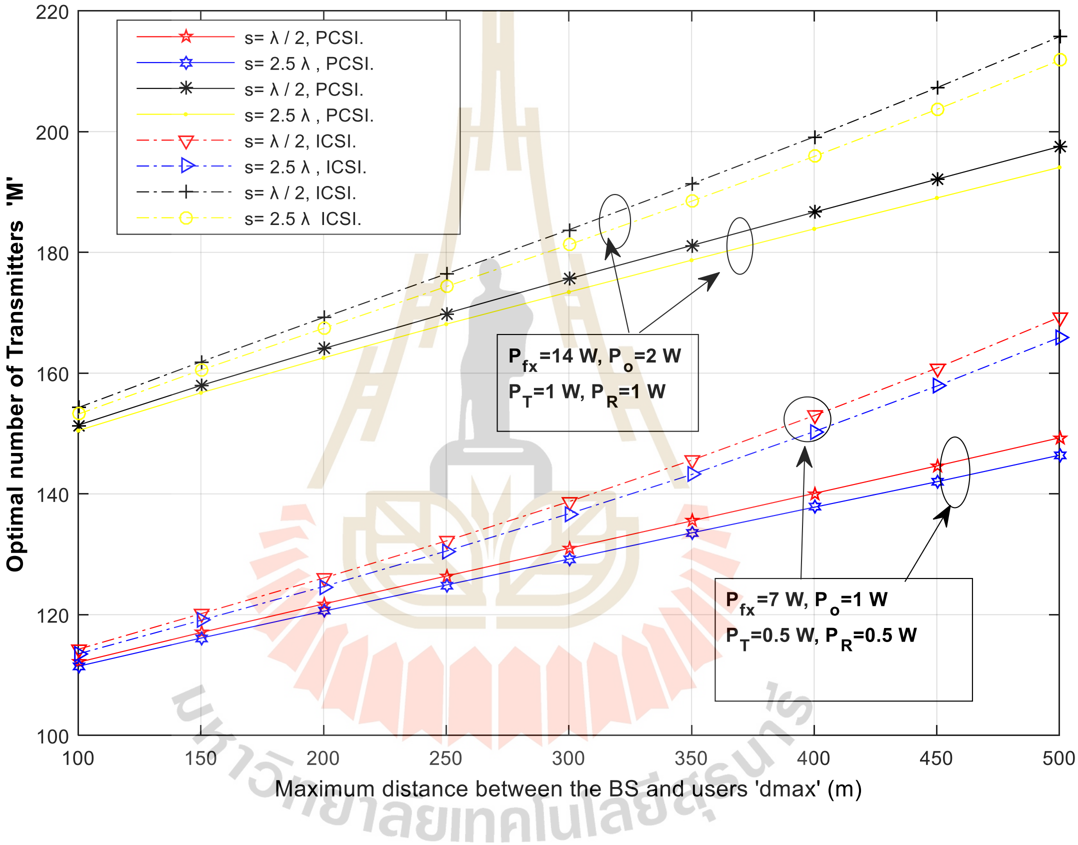

Figure 5.5 Number of optimal Transmitters

When the maximum distance between the base station and the end users gets increased, then, the number of optimal BS antennas required for the system will be increased, in order to cover more distance as shown in Figure 5.5. Similarly, Figure 5.6 shows the optimal number of user terminals with respect to the maximum distances (100–500 m) between the BS and the users, at different circuit power consumption and at different spacing among the BS antennas. More transmitted power is required when the BS antennas are closely spaced to overcome the mutual coupling effect along with the imperfect channel conditions as shown in Figure 5.7. Furthermore, high circuit power consumption leads to have more transmitted power to accommodate the higher number of BS antennas and the end users, and it is significantly dependent on the maximum distance between the BS and the users as shown in Figures 5.7 and 5.8. Figure 5.9 unveils the effect of mutual coupling on the achievable spectral efficiency of the system, and when the BS antennas are more closely spaced, then, the overall spectral efficiency gets reduced. As the system requires to have more transmitted power when the maximum distances between the BS and users gets increased, thereby, leads to have the reduction in the overall spectral efficiency of the system as shown in Figure 5.9.

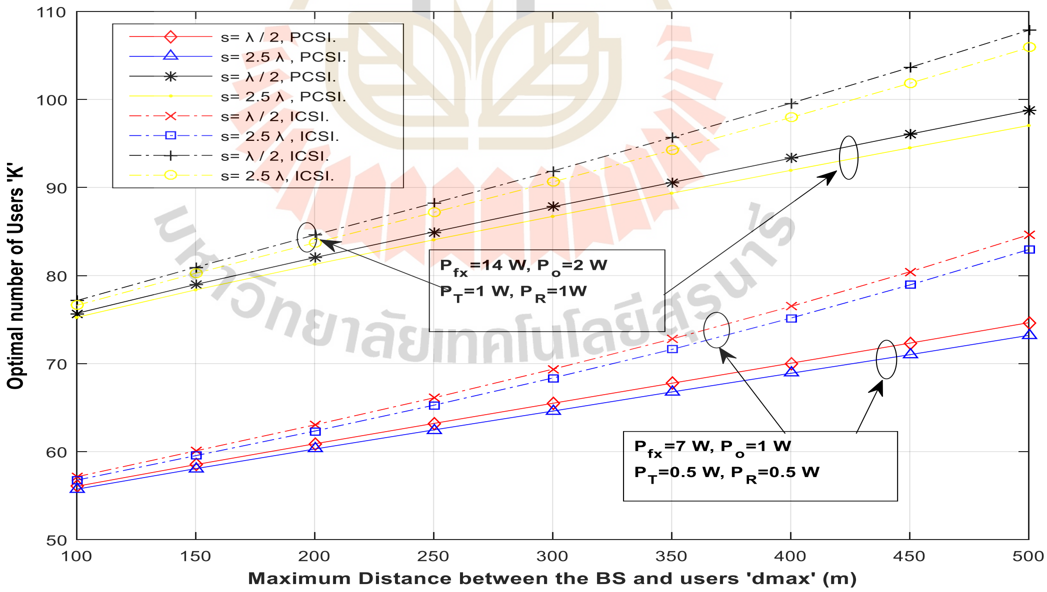

Figure 5.6 Number of optimal Users

Similarly, Figure 5.10 unveils the mutual coupling effect on the overall energy efficiency of the system with respect to the different maximum distances between the BS and the end users, and imperfect channel situation leads to have a reduction in the overall achievable energy efficiency, due to the higher consumed as well as the transmitted power. Figure 5.11 unveils the computational complexity and the convergence of the proposed domain splitter algorithm, where the convergence of the energy efficiency can be examined with respect to the number of iterations. The energy efficiency goes into the saturation at the 3rd iteration, so, the computational complexity of the domain splitter algorithm in terms of Landau's big O notation can be written as: $3\left(\mathrm{O}\left(n^3 ln\left(n^3\right)\right)\right)$. A detailed comparison between the proposed and the reference algorithms in terms of computation complexity has been presented and discussed in Table 5.3.

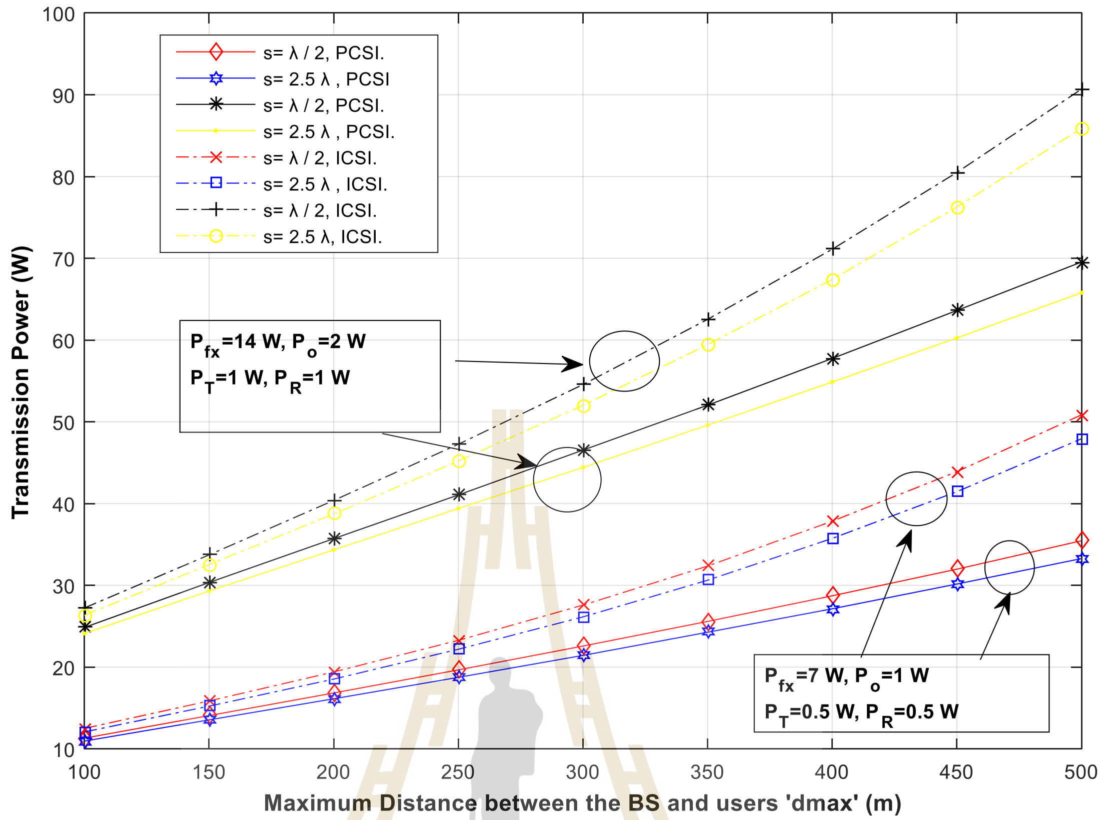

Figure 5.7 Transmission Power

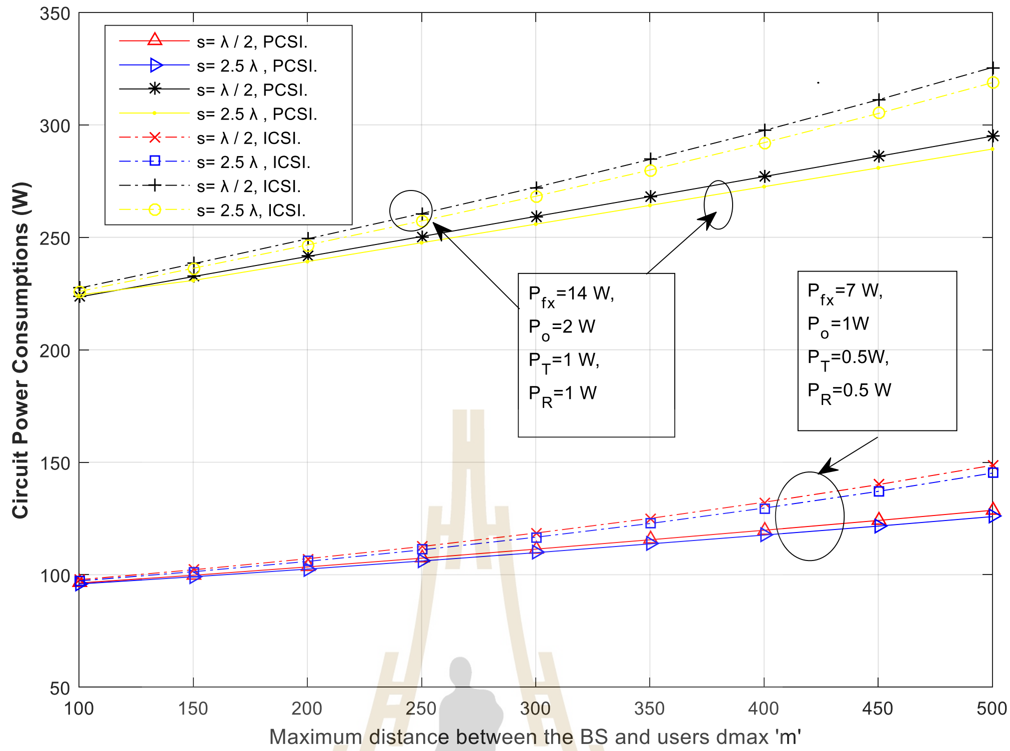

Figure 5.8 Circuit Power Consumption

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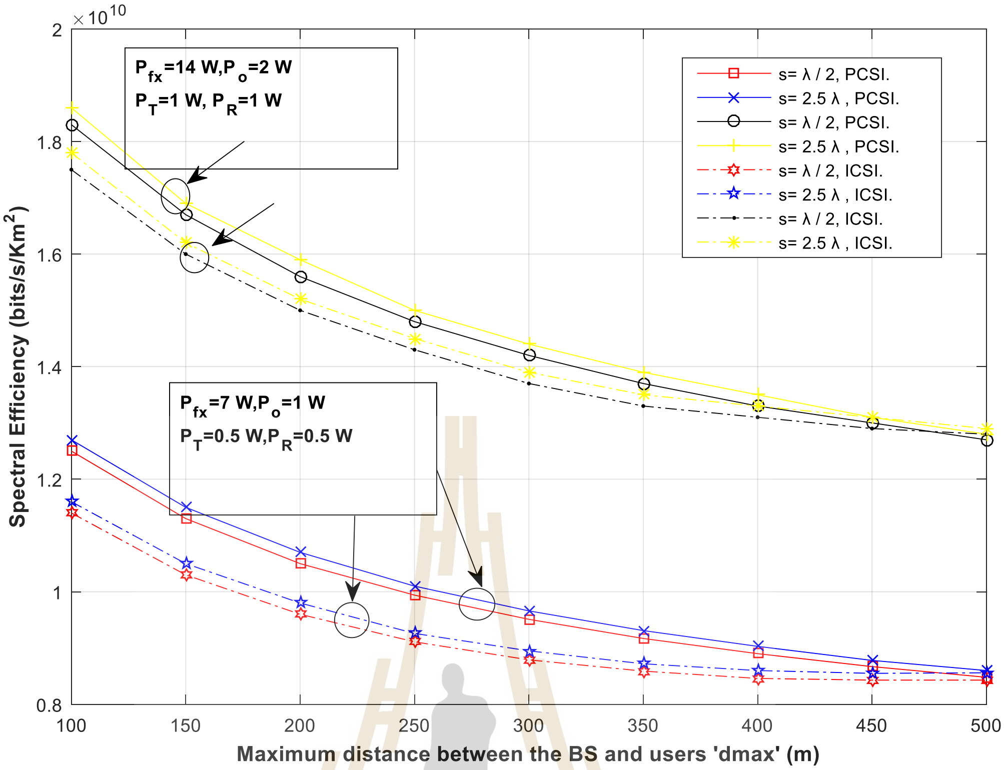

Figure 5.9 Spectral Efficiency

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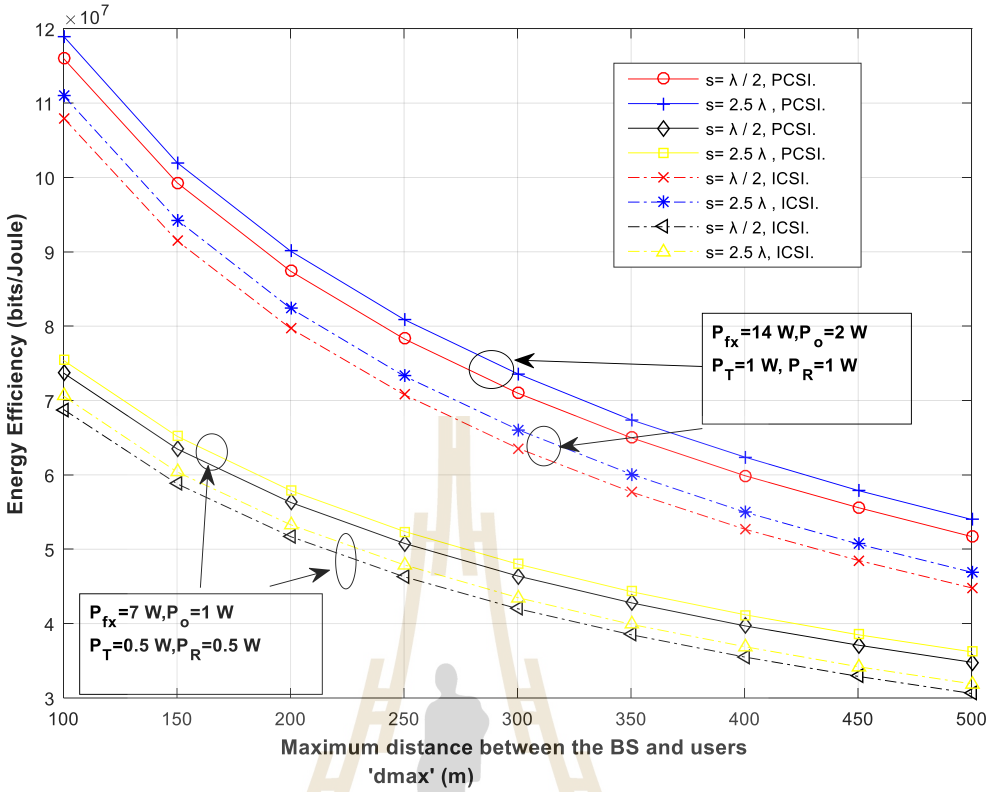

Figure 5.10 Energy Efficiency

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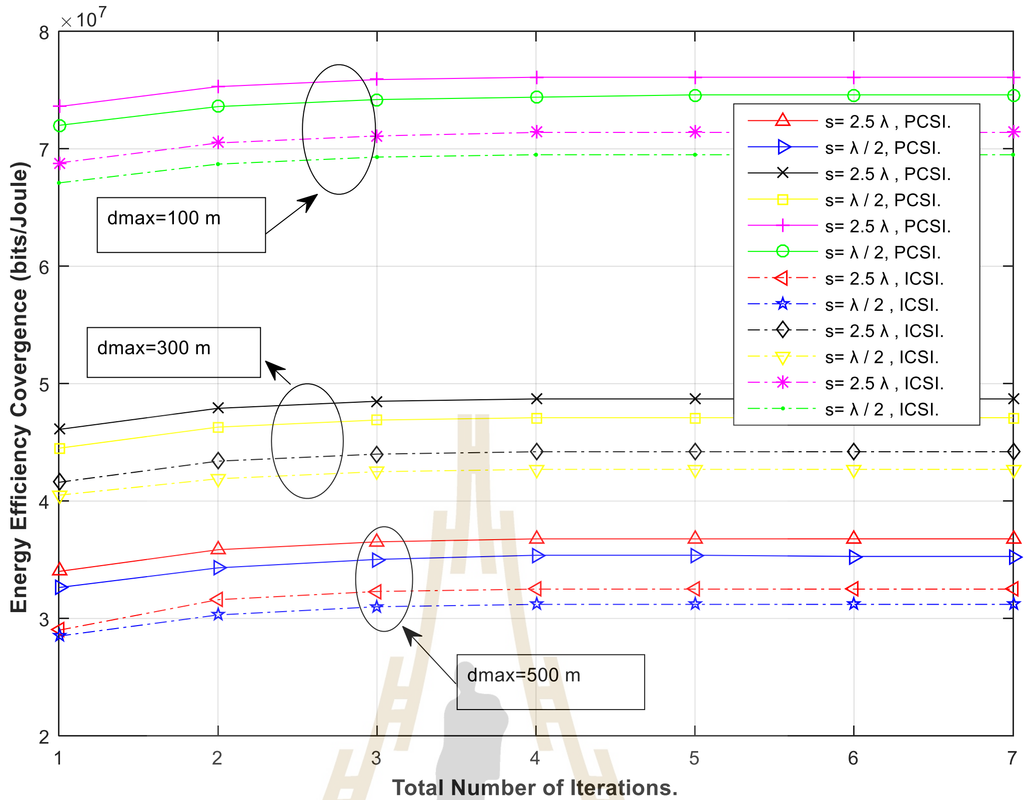

Figure 5.11 Convergence and the computation Complexity of the domain splitter Algorithm

5.6 Physical Placing of the antennas

In this subsection, we have provided the rough estimation regarding the placement of antennas in a physical space. Let's take the scenario when the maximum distance between the base station and users is 150m. As per the Fig.5, the optimal number of antennas required for the system, when the maximum distance is set to be 150m, the channel situation is considered to be imperfect, and the power consumption of the system is considered to be less, comes out to be 120 number of antennas. We have distributed the antennas into an array having the 4 rows and 40 columns. The carry

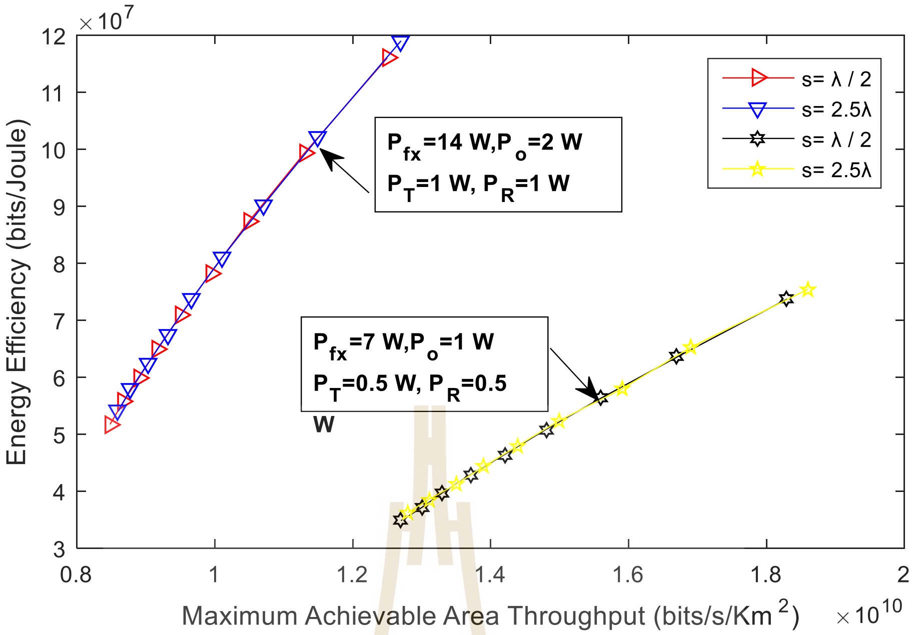

Figure 5.12 Energy Efficiency Vs Throughput

frequency used for the wireless communication is 2.5 GHz, the corresponding wavelength is lambda=0.12 m.

Length of each dipole antenna= lambda/2= 0.06m.

Radius of the dipole antenna wire= 10^-4.

Radius of the dipole antenna wire= 2* 10^-4.

Length of the antenna array (40 antennas are placed in a row separated by a spacing of Lambda/2) = 39(0.06)+40(2*10^-4)=2.348.

Width of the antenna array (40 antennas are placed in each of the four rows where the distance between the rows is assumed to be 5m) = 4*(Lamda/2)+3(5m)=15.24.

Total area= L * W= 35.7m^2

5.7 Summary

This chapter mainly concentrates on the energy efficient designing along with the selection of optimal parameters, with respect to different channel conditions and circuit power consumption levels, by considering the scenario of the urban environment for the Massive MIMO. The energy efficiency and the other optimal system parameters are calculated by using the realistic modelling of the circuit power consumption. The effect of mutual coupling has been deeply investigated on the energy efficiency and other optimal parameters of the system, by varying the length and the inter-distance spacing of the antenna terminals. There is always a tradeoff between the performance of the network (in terms of its throughput and the energy efficiency) and the spacing between the antennas. As we have noted, when the antennas are closely spaced (lambda/2) then the performance of the network gets reduced, and the system need to have more transmit power along with the more consumed power in the circuit of Massive MIMO. Owing to more transmit power, the achievable throughput and the energy efficiency of the system gets reduced as compared to the situation when the antennas are widely spaced. For best performance, we should aim to have the antenna spacing of 4 lambda, and for the most compact, the antenna spacing of 3/8 lambda may be still marginally useful. Closed form expressions of the energy and spectral efficiency are formulated and derived by taking the mutual coupling effect into account. The simulation result shows the mutual coupling effect on the overall optimum energy efficiency and other system parameters. According to the simulation results, we can conclude that it is useful to have the large cell coverage area together with the less power consumption of the circuit. As per the simulation results, the proposed algorithm (domain splitter algorithm) works efficiently without much computational effort. The detailed comparison (Table 5.2) is presented between the proposed and the reference algorithms. Contrary to the existing researchers, the computation complexity of the proposed domain splitter algorithm is not dependent on the number of the transceiver chains. The mathematical modelling and the simulation results can be so much more beneficial for the future trend of research in order to have the energy efficient communication technologies by controlling the power consumption of the communication devices.

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CHAPTER VI

Future Work and Recommendations

In this chapter, we have discussed and suggested the future trend of research of the basis of the research presented in this thesis. As, we have considered the situation of perfect and imperfect channel state information, the same research can be extended for the multi cellular situation. Furthermore, a lot of work has been done on the performance evaluation of the massive MIMO. The future trend of research will be on the hardware design of the Massive MIMO i.e. working on making the hardware of Massive MIMO to be less complex and cost efficient. The figure 6.1 unveils the general design of the transmitter. As, it can be seen that the basic components of the transmitter consist of the up sampling, up conversion and the low end amplifiers. In the up sampling, we have to use Analogue to Digital Convertors (A/D), so we need to have, the less complex and cost efficient design of the A/D's. Similarly the main components in the conversion process are the mixers and the oscillator, so working on the cost efficient and less complex design of the mixers and the oscillators can be a fascinating option for the future trend of research.

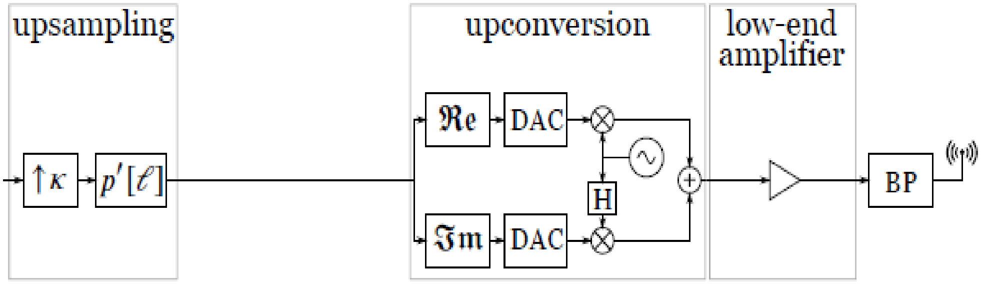

Figure 6.1 General Transmitter Design

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APPENDIX A

CALCULATION OF THE OPTIMAL PARAMETERS FOR THE ALTERNATIVE NUMERICAL ALGORITHM UNDER THE SCENARIO OF PERFECT CHANNEL CONDITIONS

- Check of quasi-concavity for $\mathrm{EE}_1(z_1)$ when the other parameters are fixed in the interval $[0,\mu]$

As we know that the energy efficiency under the perfect channel conditions (equation 4.25) can be written as:

$$\mathrm{EE}_1=\frac{\dfrac{z_1}{ln(2)}\left(1-\dfrac{T_{sum}z_1}{U}\right)\times B\times ln\left(1+\dfrac{z_3(z_2-1)+c_k}{D_k+1}\right)}{\dfrac{z_3\delta B\alpha^2}{\eta_{PA}}+\sum_{i=1}^{3}D_i z_1^i+z_2\sum_{i=0}^{2}E_i z_1^{i+1}}$$

$$\text{Let } a_1=\frac{z_3\delta B\alpha^2}{\eta_{PA}},\ a_2=D_1+z_2E_0,\ a_3=D_2+z_2E_1$$

$$a_4=D_3+z_2E_2,\ a_5=\frac{B}{ln(2)}\times ln\left(1+\frac{z_3(z_2-1)+c_k}{D_k+1}\right)$$

So, the equation 4.25 in terms of z_1 can be written as:

$$\mathrm{EE}_1(z_1)=\frac{z_1(\mu-z_1)\times a_5}{a_1+a_2z_1+a_3z_1^2+a_4z_1^3}$$

Differentiate with respect to z_1:

$$\frac{\mathrm{d}}{\mathrm{d}z_1}\left(\mathrm{EE}_1(z_1)\right)=\frac{a_5\left[\left[a_1+a_2z_1+a_3z_1^2+a_4z_1^3\right]\times\left[\mu-2z_1\right]-\left[\begin{array}{l}\left[z_1(\mu-z_1)\right]\times\\ \left[a_2+2a_3z_1+3a_4z_1^2\right]\end{array}\right]\right]}{\left[a_1+a_2z_1+a_3z_1^2+a_4z_1^3\right]^2}$$

Take out the numerator of $\frac{\mathrm{d}}{\mathrm{d}z_1}\left(\mathrm{EE}_1(z_1)\right)$ in order to find the optimal parameters and check the behavior.

$$\mathrm{Num}_1(z_1)=\left[a_1+a_2z_1+a_3z_1^2+a_4z_1^3\right]\left[\mu-2z_1\right]-\left[\left[z_1(\mu-z_1)\right]\left[a_2+2a_3z_1+3a_4z_1^2\right]\right] \qquad \text{(A 1)}$$

$$\mathrm{Num}_1(0)=\mu a_1>0\quad \&\quad \mathrm{Num}_1(\mu)=-\mu\left(a_1+a_2z_1+a_3z_1^2+a_4z_1^3\right)$$

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So, the given objective function $\mathrm{EE}_1(z_1)$ is first increasing and then decreasing with the peak value existed at $\mathrm{Num}_1(z_1)=0$ and the second order derivative should be less than zero:

$$\frac{\mathrm{d}\left(\mathrm{Num}_1(z_1)\right)}{\mathrm{dz}_1}=$$

$$\begin{bmatrix}\left[a_1+a_2z_1+a_3z_1^2+a_4z_1^3\right][-2]+\\ \left[a_2+2a_3z_1+3a_4z_1^2\right][\mu-2z_1]\end{bmatrix}-\begin{bmatrix}\left[\mu z_1-z_1^2\right][2a_3+6a_4z_1]+\\ [\mu-2z_1]\left[a_2+2a_3z_1+3a_4z_1^2\right]\end{bmatrix}$$

$$\frac{\mathrm{d}\left(\mathrm{Num}_1(z_1)\right)}{\mathrm{dz}_1}=$$

$$-\left[2a_1+2a_2z_1+2a_3z_1^2+2a_4z_1^3\right]-\left[\left[\mu z_1-z_1^2\right][2a_3+6a_4z_1]\right]<0$$

- Check of quasi-concavity for $\mathrm{EE}_1(z_2)$ when the other parameters are fixed in the interval $[1,\infty)$

Energy efficiency $\mathrm{EE}_1(z_2)$ in terms of z_2 can be written as:

$$\mathrm{EE}_1(z_2)=\frac{a_5\times ln\left(1+a_3\left[(z_2-1)+a_4\right]\right)}{a_1+z_2a_2} \tag{A2}$$

With the following substitutions:

$$a_1=\frac{z_3\delta B\alpha^2}{\eta_{PA}}+\sum_{i=1}^{3}D_iz_i^i,\ a_2=\sum_{i=0}^{2}E_iz_1^{i+1}$$

$$a_3=\frac{z_3}{D_k+1},\ a_4=\frac{c_k}{z_3},\ a_5=\frac{Bz_1}{ln(2)}\left(1-\frac{T_{sum}z_1}{U}\right)$$

Differentiate $\mathrm{EE}_1(z_2)$ with respect to z_2:

$$\frac{\mathrm{d}}{\mathrm{dz}_2}\left(\mathrm{EE}_1(z_2)\right)=\frac{a_5\left[(a_1+z_2a_2)\times\left(\dfrac{a_3}{1+a_3\left[(z_2-1)+a_4\right]}\right)-\left[a_2\times ln\left(1+a_3\left[(z_2-1)+a_4\right]\right)\right]\right]}{(a_1+z_2a_2)^2}$$

$$\frac{\mathrm{d}}{\mathrm{dz}_2}\left(\mathrm{EE}_1(z_2)\right)=\frac{a_5\left[a_3[a_1+z_2a_2]-\left[a_2\left(1+a_3\left((z_2-1)+a_4\right)\right)\times ln\left(1+a_3\left((z_2-1)+a_4\right)\right)\right]\right]}{(a_1+z_2a_2)^2}$$

Take out the numerator of $\frac{\mathrm{d}}{\mathrm{d}z_2}(\mathrm{EE}_1(z_2))$ in order to find the optimal parameters and check the behavior:

$$\mathrm{Num}_2(z_2)=$$

$$a_3[a_1+z_2a_2]-\left[a_2\left(1+a_3\left((z_2-1)+a_4\right)\right)\times ln\left(1+a_3\left((z_2-1)+a_4\right)\right)\right] \quad \text{(A3)}$$

$$\mathrm{Num}_2(1)=a_3(a_1+a_2)>0 \;\; \& \;\; \mathrm{Num}_{2_{z_2\to\infty}}(\infty)<0.$$

So, the given objective function $\mathrm{EE}_1(z_2)$ is first increasing and then decreasing with the peak value existed at $\mathrm{Num}_2(z_2)=0$ and the second order derivative should be less than zero:

$$\frac{\mathrm{d}\left(\mathrm{Num}_2(z_2)\right)}{\mathrm{d}z_2}=$$

$$a_3a_2-a_2\left[\left(1+a_3\left((z_2-1)+a_4\right)\right)\times\left(\frac{a_3}{\left(1+a_3\left((z_2-1)+a_4\right)\right)}\right)+a_3\times ln\left(1+a_3\left((z_2-1)+a_4\right)\right)\right]$$

$$\frac{\mathrm{d}\left(\mathrm{Num}_2(z_2)\right)}{\mathrm{d}z_2}=a_3a_2-a_3a_2-a_3a_2\times ln\left(1+a_3\left((z_2-1)+a_4\right)\right)$$

$$\frac{\mathrm{d}\left(\mathrm{Num}_2(z_2)\right)}{\mathrm{d}z_2}=-a_3a_2\times ln\left(1+a_3\left((z_2-1)+a_4\right)\right)<0$$

- Check of quasi-concavity for $\mathrm{EE}_1(z_3)$ when the other parameters are fixed in the interval $[1,\infty)$

Energy efficiency $\mathrm{EE}_1(z_3)$ in terms of z_3 can be written as:

$$\mathrm{EE}_1(z_3)=\frac{a_5\times ln\left(1+a_3(z_3+a_4)\right)}{a_1z_3+a_2}$$

With the following substitutions:

$$a_1 = \frac{\delta B \alpha^2}{\eta_{PA}},\ \mathrm{a}_2 = \sum_{i=1}^{3} D_i z_1^i + z_2 \sum_{i=0}^{2} E_i z_1^{i+1}$$
$$a_3 = \frac{z_2 - 1}{D_k + 1},\ a_4 = \frac{c_k}{z_2 - 1},\ a_5 = \frac{B z_1}{ln(2)}\left(1 - \frac{T_{sum} z_2}{U}\right)$$

Differentiate $\mathrm{EE}_1(z_3)$ with respect to z_3:

$$\frac{\mathrm{d}}{\mathrm{d}z_3}\left(\mathrm{EE}_1(z_3)\right) = \frac{a_5\left[(a_1 z_3 + a_2)\left(\frac{a_3}{1 + a_3(z_3 + a_4)}\right) - a_1 \times ln\left(1 + a_3(z_3 + a_4)\right)\right]}{(a_1 z_3 + a_2)^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}z_3}\left(\mathrm{EE}_1(z_3)\right) = \frac{a_5\left[a_3(a_1 z_3 + a_2) - a_1\left(1 + a_3(z_3 + a_4)\right) \times ln\left(1 + a_3(z_3 + a_4)\right)\right]}{(a_1 z_3 + a_2)^2}$$

Take out the numerator of $\frac{\mathrm{d}}{\mathrm{d}z_3}\left(\mathrm{EE}_1(z_3)\right)$ in order to find the optimal parameters and check the behavior:

$$\mathrm{Num}_3(z_3) = a_3(a_1 z_3 + a_2) - a_1\left(1 + a_3(z_3 + a_4)\right) \times ln\left(1 + a_3(z_3 + a_4)\right) \qquad \text{(A4)}$$

$$\mathrm{Num}_3(1) = a_3(a_1 + a_2) > 0 \ \ \& \ \ \mathrm{Num}_{3_{z_3 \to \infty}}(\infty) < 0.$$

So, the given objective function $\mathrm{EE}_2(z_3)$ is first increasing and then decreasing with the peak value existed at $\mathrm{Num}_3(z_3) = 0$ and the second order derivative should be less than zero:

$$\frac{\mathrm{d}\left(\mathrm{Num}_3(z_3)\right)}{\mathrm{d}z_3} = a_3 a_1 - a_1\left[\left(1 + a_3(z_3 + a_4)\right) \times \left(\frac{a_3}{1 + a_3(z_3 + a_4)}\right) + a_3 \times ln\left(1 + a_3(z_3 + a_4)\right)\right]$$

$$\frac{\mathrm{d}\left(\mathrm{Num}_3(z_3)\right)}{\mathrm{d}z_3} = a_3 a_1 - a_3 a_1 - a_1 a_3 \times ln\left(1 + a_3(z_3 + a_4)\right)$$

$$\frac{\mathrm{d}\left(\mathrm{Num}_3(z_3)\right)}{\mathrm{d}z_3} = -a_1 a_3 \times ln\left(1 + a_3(z_3 + a_4)\right) \ < 0$$

APPENDIX B

CALCULATION OF THE OPTIMAL PARAMETERS FOR THE ALTERNATIVE NUMERICAL ALGORITHM UNDER THE SCENARIO OF IMPERFECT CHANNEL CONDITIONS

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- Under imperfect channel conditions in case of $\mathrm{EE}_2(z_1)$, it comes out to be same (equation A1) when the other dimensions are fixed in the interval $[0,\mu]$. Similarly, for $\mathrm{EE}_2(z_2)$ it comes out to be same (equation A1) when the other dimensions are fixed in the interval $[1,\infty)$ but substitute $a_3 = \dfrac{z_3}{D_k + 1 + \frac{1}{T_p} + \frac{1}{z_3 T_p}}$ in the equations A2 and A3.

- Check of quasi concavity for $\mathrm{EE}_2(z_3)$ when the other parameters are fixed in the interval $[1,\infty)$

As we know that the energy efficiency under the imperfect channel conditions can be written as:

$$\mathrm{E.E}_2 = \frac{\dfrac{z_1}{ln(2)}\left(1-\dfrac{T_{sum}z_1}{U}\right) B\,ln\left(1+\left(\dfrac{z_3(z_2-1)+c_k}{D_k+1+\frac{1}{T_p}+\frac{1}{z_3T_p}}\right)\right)}{\dfrac{\delta z_3 B\alpha^2}{\eta_{PA}} + \sum_{i=1}^{3} D_i z_1^i + z_2 \sum_{i=0}^{2} E_i z_1^{i+1}} \tag{B1}$$

$$\text{Let } \mathrm{a}_1 = \frac{\delta B\alpha^2}{\eta_{PA}},\ \mathrm{a}_2 = \sum_{i=1}^{3} D_i z_1^i + z_2 \sum_{i=0}^{2} E_i z_1^{i+1}$$

$$\mathrm{a}_3 = c_k,\ \mathrm{a}_4 = D_k + 1 + T_p,\ \mathrm{a}_5 = \frac{Bz_1}{ln(2)}\left(1-\frac{T_{sum}z_1}{U}\right)$$

So, (B1) can be written as:

$$\mathrm{EE}_2(z_3) = \frac{a_5 \times ln\left(1+\dfrac{z_3a_6+a_3}{a_4+\frac{1}{z_3}}\right)}{a_1z_3+a_2}$$

Differentiate $\mathrm{EE}_2(z_3)$ with respect to z_3

$$\frac{d}{dz_3}\left(EE_2(z_3)\right)=\frac{a_5\left[(a_1z_3+a_2)\frac{d}{dz_3}\left(ln\left(1+\frac{z_3a_6+a_3}{a_4+\frac{1}{z_3}}\right)\right)-a_1\left(ln\left(1+\frac{z_3a_6+a_3}{a_4+\frac{1}{z_3}}\right)\right)\right]}{(a_1z_3+a_2)^2} \tag{B2}$$

Where:

$$\frac{d}{dz_3}\left(ln\left(1+\frac{z_3a_6+a_3}{a_4+\frac{1}{z_3}}\right)\right)=\frac{1}{\left(1+\frac{z_3a_6+a_3}{a_4+\frac{1}{z_3}}\right)}\times\frac{\left(a_4+\frac{1}{z_3}\right)(a_6)-(z_3a_6+a_3)\left(-z_3^{-2}\right)}{\left(a_4+\frac{1}{z_3}\right)^2}$$

$$\frac{d}{dz_3}\left(ln\left(1+\frac{z_3a_6+a_3}{a_4+\frac{1}{z_3}}\right)\right)=\frac{a_6\left(a_4+\frac{1}{z_3}\right)+(z_3a_6+a_3)\left(\frac{1}{z_3^2}\right)}{\left(\frac{a_4+\frac{1}{z_3}+z_3a_6+a_3}{a_4+\frac{1}{z_3}}\right)\left(a_4+\frac{1}{z_3}\right)^2}$$

$$\frac{d}{dz_3}\left(ln\left(1+\frac{z_3a_6+a_3}{a_4+\frac{1}{z_3}}\right)\right)=\frac{\left(a_4+\frac{1}{z_3}\right)\left(a_6+\frac{1}{z_3^2}\left(\frac{z_3a_6+a_3}{a_4+\frac{1}{z_3}}\right)\right)}{\left(a_4+\frac{1}{z_3}+z_3a_6+a_3\right)\left(a_4+\frac{1}{z_3}\right)}$$

$$\frac{d}{dz_3}\left(ln\left(1+\frac{z_3a_6+a_3}{a_4+\frac{1}{z_3}}\right)\right)=\frac{a_6+\frac{z_3a_6+a_3}{a_4z_3^2+z_3}}{a_4+\frac{1}{z_3}+z_3a_6+a_3}$$

(Put in the equation B2)

$$\frac{d}{dz_3}\left(EE_2(z_3)\right)=\frac{a_5\left[(a_1z_3+a_2)\left(\frac{\left(a_6+\frac{z_3a_6+a_3}{a_4z_3^2+z_3}\right)}{a_4+\frac{1}{z_3}+z_3a_6+a_3}\right)-\left(a_1\times ln\left(1+\frac{z_3a_6+a_3}{a_4+\frac{1}{z_3}}\right)\right)\right]}{(a_1z_3+a_2)^2}$$

Take out the numerator of $\frac{\mathrm{d}}{\mathrm{d}z_3}\left(\mathrm{EE}_2\left(z_3\right)\right)$ in order to find the optimal parameters and check the behavior:

$$\mathrm{Num}_{3,Im}\left(z_3\right)=\frac{\left(a_1 z_3+a_2\right)\left(a_6+\dfrac{z_3 a_6+a_3}{a_4 z_3^2+z_3}\right)}{\left(a_4+\dfrac{1}{z_3}+z_3 a_6+a_3\right)}-a_1\times ln\left(1+\frac{z_3 a_6+a_3}{a_4+\dfrac{1}{z_3}}\right)$$

$$\mathrm{Num}_{3,Im}\left(z_3\right)=\left(a_1 z_3+a_2\right)\left(a_6+\frac{z_3 a_6+a_3}{a_4 z_3^2+z_3}\right)-a_1\left(a_4+\frac{1}{z_3}+z_3 a_6+a_3\right)\times ln\left(1+\frac{z_3 a_6+a_3}{a_4+\dfrac{1}{z_3}}\right) \tag{B3}$$

$$\mathrm{Num}_{3,Im}\left(1\right)=\left(a_1 z_3+a_2\right)\left(a_6+\frac{z_3 a_6+a_3}{a_4 z_3^2+z_3^2}\right)>0\ \ \&\ \mathrm{Num}_{3,Im}\left(z_3\to\infty\right)<0$$

So, the given objective function $\mathrm{EE}_2\left(z_3\right)$ is first increasing and then decreasing with the peak value existed at $\mathrm{Num}_{3,Im}\left(z_3\right)=0$ and the second order derivative should be less than zero:

$$\frac{\mathrm{d}\left(\mathrm{Num}_{3,Im}\left(z_3\right)\right)}{\mathrm{d}z_3}=\left(a_1 z_3+a_2\right)\left(\frac{a_6\left(a_4 z_3^2+z_3\right)-\left(z_3 a_6+a_3\right)\left(2a_4 z_3+1\right)}{\left(a_4 z_3^2+z_3\right)^2}\right)+a_1\left(a_6+\frac{z_3 a_6+a_3}{a_4 z_3^2+z_3}\right)-$$
$$a_1\left(\begin{gathered}\left(a_4+\frac{1}{z_3}+z_3 a_6+a_3\right)\times\left(\frac{a_6+\dfrac{z_3 a_6+a_3}{a_4 z_3^2+z_3}}{a_4+\dfrac{1}{z_3}+z_3 a_6+a_3}\right)+\\ ln\left(1+\frac{z_3 a_6+a_3}{a_4+\dfrac{1}{z_3}}\right)\times\left(\frac{a_3-z_3^2}{z_3^2}\right)\end{gathered}\right)$$

$$\frac{\mathrm{d}\left(\mathrm{Num}_{3,Im}\left(z_3\right)\right)}{\mathrm{d}z_3}=$$

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$$\frac{\left(a_1 z_3 + a_2\right)\begin{pmatrix} a_6 a_4 z_3^2 + a_6 z_3 - \\ 2a_4 a_6 z_3^2 - 2a_4 a_3 z_3 - z_3 a_6 - a_3 \end{pmatrix}}{\left(a_4 z_3^2 + z_3\right)^2} + a_1\left(a_6 + \frac{z_3 a_6 + a_3}{a_4 z_3^2 + z_3}\right) - a_1\left(a_6 + \frac{z_3 a_6 + a_3}{a_4 z_3^2 + z_3}\right)$$

$$-a_1 \times ln\left(1 + \frac{z_3 a_6 + a_3}{a_4 + \frac{1}{z_3}}\right)\left(\frac{a_3 - z_3^2}{z_3^2}\right)$$

$$\frac{\mathrm{d}\left(\mathrm{Num}_{3,Im}\left(z_3\right)\right)}{\mathrm{d}z_3} = \frac{\left(a_1 z_3 + a_2\right)\left(-a_6 a_4 z_3^2 - 2a_4 a_3 z_3 - a_3\right)}{\left(a_4 z_3^2 + z_3\right)^2} - a_1 \times ln\left(1 + \frac{z_3 a_6 + a_3}{a_4 + \frac{1}{z_3}}\right)\left(\frac{a_3 - z_3^2}{z_3^2}\right)$$

$$\frac{\mathrm{d}\left(\mathrm{Num}_{3,Im}\left(z_3\right)\right)}{\mathrm{d}z_3} = -\frac{\left(a_1 z_3 + a_2\right)\left(a_6 a_4 z_3^2 + 2a_4 a_3 z_3 + a_3\right)}{\left(a_4 z_3^2 + z_3\right)^2} - a_1 \times ln\left(1 + \frac{z_3 a_6 + a_3}{a_4 + \frac{1}{z_3}}\right)\left(\frac{a_3 - z_3^2}{z_3^2}\right)$$

$$\frac{\mathrm{d}\left(\mathrm{Num}_{3,Im}\left(z_3\right)\right)}{\mathrm{d}z_3} < 0$$

APPENDIX C

PROOF OF THE THEOREM 1

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Proof of the Theorem 1: Equation 5.4 can be written as:

$$R_k = \left[log_2 \left(1 + \frac{p}{\mathrm{E}\left(G_k^H G_k\right)^{-1}} \right) \right]$$

$$R_k = \left[log_2 \left(1 + \frac{p}{\dfrac{\mathrm{E}\left(trace\left(H^H H\right)^{-1}\right)}{K\beta_k trace\left(C^H C\right)}} \right) \right] \tag{C1}$$

For a $m \times m$ Wishart central complex matrix ($W \sim W_m\left(n, I_n\right)$) with n degrees of freedom and provided that $n > m$, we get:

$$\mathrm{E}\left[trace\left(W\right)^{-1}\right] = \frac{m}{n-m} \tag{C2}$$

By using the identity (C2), equation C1 can be simplified as:

$$R_k = \left[log_2 \left(1 + \frac{p}{\dfrac{K}{K\beta_k trace\left(C^H C\right)\left(M-K\right)}} \right) \right]$$

$$R_k = \left[log_2 \left(1 + p\beta_k trace\left(C^H C\right) \times \left(M-K\right) \right) \right]$$

APPENDIX D

PROOF OF THE THEOREM 2

Proof of the Theorem 2: The mean and variance of the estimated channel $\hat{G}_k$ for the kth user terminal, with the help of MMSE estimator can be expressed as (D. Ciuonzo et al, 2015):

$$\hat{G}_k \sim CN\left(0, \frac{\beta_k}{1+\frac{1}{Kp\mathrm{T}^{ul}}}\right)$$

So, the variance of the channel estimation error ε can be written as:

$$var(\varepsilon) = var(G) - var\left(\hat{G}\right)$$

Thus, the achievable rates of the kth user terminal can be written as:

$$R_k = log_2\left(1+\frac{p/\left(1+\frac{1}{Kp\mathrm{T}_p^{ul}}\right)}{\left(\mathrm{E}\left(G_k^H G_k\right)^{-1}\times\left[var(\varepsilon)\right]Kp\right)+\mathrm{E}\left(G_k^H G_k\right)^{-1}}\right)$$

$$R_k = log_2\left(1+\frac{p/\left(1+\frac{1}{Kp\mathrm{T}_p^{ul}}\right)}{\mathrm{E}\left(G_k^H G_k\right)^{-1}\left[\left(var(\varepsilon)\right)Kp+1\right]}\right) \tag{D1}$$

By using the results of (C1) and (C2), (D1) can be written as:

$$R_k = log_2\left(1+\frac{p/\left(1+\frac{1}{Kp\mathrm{T}_p^{ul}}\right)}{\frac{K}{K\beta_k trace\left(C^H C\right)\left(M-K\right)}\times\left[\left(1-\frac{1}{1+\frac{1}{Kp\mathrm{T}_p^{ul}}}\right)Kp+1\right]}\right)$$

$$R_k = log_2\left(1+\frac{p\beta_k trace\left(C^H C\right)\times\left(M-K\right)}{1+\frac{1}{\mathrm{T}_p^{ul}}+\frac{1}{pK\mathrm{T}_p^{ul}}}\right)$$

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APPENDIX E

CALCULATION OF THE OPTIMAL PARAMETERS FOR THE DOMAIN SPLITTER ALGORITHM UNDER THE SCENARIO OF PERFECT CHANNEL CONDITIONS

- Calculation of the optimal parameter (r_1) for $\mathrm{EE}_P(r_1)$ in terms of r_1 while the remaining dimensions (r_2, r_3) in the interval $[0, \nu]$ are constant and fixed. The energy efficiency ($\mathrm{EE}_P(r_1)$) in terms of r_1 can be written as:

$$\mathrm{EE}_P(r_1) = \frac{r_1(\nu - r_1)d_2}{d_3 + d_4 r_1 + d_5 r_1^2 + d_6 r_1^3}$$

With the following substitutions:

$$d_1 = \beta_k trace\left(C^H C\right),\ d_2 = \frac{B}{ln(2)}\left(1 + d_1 r_3 (r_2 - 1)\right)$$

$$d_3 = C_0 + \frac{r_3 \delta_x B \alpha^2}{\eta},\ d_4 = C_1 + r_2 D_0,\ d_5 = C_2 + r_2 D_1$$

$$d_6 = C_3 + r_2 D_2$$

Differentiate $\mathrm{EE}_P(r_1)$ with respect to r_1 in order to calculate the optimal parameter:

$$\frac{d}{dr_1}\left(\mathrm{EE}_P(r_1)\right) = \frac{d}{dr_1}\left(\frac{r_1(\nu - r_1)d_2}{d_3 + d_4 r_1 + d_5 r_1^2 + d_6 r_1^3}\right)$$

$$\mathrm{EE}_P'(r_1) = \frac{\left[d_3 + d_4 r_1 + d_5 r_1^2 + d_6 r_1^3\right] \times \frac{d}{dr_1}\left(r_1(\nu - r_1)d_2\right) - \left[\begin{array}{l} r_1(\nu - r_1)d_2 \\ \times \frac{d}{dr_1}\left(d_3 + d_4 r_1 + d_5 r_1^2 + d_6 r_1^3\right)\end{array}\right]}{\left[d_3 + d_4 r_1 + d_5 r_1^2 + d_6 r_1^3\right]^2}$$

$$\mathrm{EE}_P'(r_1) = \frac{\left[d_3 + d_4 r_1 + d_5 r_1^2 + d_6 r_1^3\right]\left[d_2(\nu - 2r_1)\right] - \left[r_1(\nu - r_1)d_2\left(d_4 + 2d_5 r_1 + 3d_5 r_1^2\right)\right]}{\left[d_3 + d_4 r_1 + d_5 r_1^2 + d_6 r_1^3\right]^2}$$

Equate the above equation to zero in order to get the optimal parameter:

$$\mathrm{EE}_P'(r_1)=0$$

$$\left[d_3+d_4r_1+d_5r_1^2+d_6r_1^3\right]\left[(\nu-2r_1)\right] \\ -\left[r_1(\nu-r_1)\left(d_4+2d_5r_1+3d_5r_1^2\right)\right]=0 \tag{E1}$$

At $r_1=0;$ $\mathrm{EE}_P'(0)=\nu d_3$

At $r_1=\nu;$ $\mathrm{EE}_P'(\nu)=-\nu\left[d_3+d_4r_1+d_5r_1^2+d_6r_1^3\right]$

Thus, $\mathrm{EE}_P'(r_1)$ follows the quasi concave response because $\mathrm{EE}_P'(r_1)$ is first positive and then negative with the optimal value must be existed at $\mathrm{EE}_P'(r_1)=0$, and the $\mathrm{EE}_P''(r_1)$ must be less than zero:

$$\mathrm{EE}_P''(r_1)=\frac{d}{dr_1}\begin{pmatrix}\left[d_3+d_4r_1+d_5r_1^2+d_6r_1^3\right]\times\left[(\nu-2r_1)\right] \\ -\left[r_1(\nu-r_1)\left(d_4+2d_5r_1+3d_5r_1^2\right)\right]\end{pmatrix}$$

$$\mathrm{EE}_P''(r_1)=(\upsilon-2r_1)\left(d_4+2d_5r_1+3d_6r_1^2\right)-2\left(d_3+d_4r_1+d_5r_1^2+d_6r_1^3\right) \\ -\left[(\upsilon-2r_1)\left(d_4+2d_5r_1+3d_6r_1^2\right)+r_1(\nu-r_1)(2d_5+6d_6r_1)\right]$$

$$\mathrm{EE}_P''(r_1)=-\left[2\left(d_3+d_4r_1+d_5r_1^2+d_6r_1^3\right)+r_1(\nu-r_1)(2d_5+6d_6r_1)\right]$$

$$\mathrm{EE}_P''(r_1)<0$$

- Calculation of the optimal parameter (r_2) for $\mathrm{EE}_P(r_2)$ in terms of r_2 while the remaining dimensions (r_1,r_3) in the interval $[1,\infty)$ are constant and fixed. Thus, the energy efficiency ($\mathrm{EE}_P(r_2)$) in terms of r_2 can be written as:

$$\mathrm{EE}_P(r_2)=\frac{d_2\,ln(1+d_3r_2-d_3)}{d_4+d_5r_2}$$

With the following substitutions:

$$d_1 = \beta_k trace\left(C^H C\right),\ d_2 = r_1\left(1 - \frac{Tr_1}{U}\right)\frac{B}{ln(2)},$$

$$d_3 = d_1 r_3, d_4 = \frac{r_3 \delta_x B \alpha^2}{\eta} + \sum_{i=0}^{3} C_i r_1^i,\ d_5 = \sum_{j=0}^{2} D_j r_1^{j+1}$$

Differentiate $\mathrm{EE}_P\left(r_2\right)$ with respect to r_2 to calculate the optimal value:

$$\frac{d}{dr_1}\left(\mathrm{EE}_P\left(r_2\right)\right) = \frac{d}{dr_2}\left(\frac{d_2\, ln\left(1 + d_3 r_2 - d_3\right)}{d_4 + d_5 r_2}\right)$$

$$\mathrm{EE}_P^{'}\left(r_2\right) = \frac{d_2\left(d_4 + d_5 r_2\right)\frac{d}{dr_2}\left(ln\left(1 + d_3 r_2 - d_3\right)\right) - \left[d_2\, ln\left(1 + d_3 r_2 - d_3\right)\frac{d}{dr_2}\left(d_4 + d_5 r_2\right)\right]}{\left[d_4 + d_5 r_2\right]^2}$$

$$\mathrm{EE}_P^{'}\left(r_2\right) = \frac{d_2\left(d_4 + d_5 r_2\right)\left[\frac{d_3}{1 + d_3 r_2 - d_3}\right] - \left[d_2\, ln\left(1 + d_3 r_2 - d_3\right) d_5\right]}{\left[d_4 + d_5 r_2\right]^2}$$

Equate the above equation to zero to get the optimal value:

$$\mathrm{EE}_P^{'}\left(r_2\right) = 0$$

$$\left(d_4 + d_5 r_2\right)\left[\frac{d_3}{1 + d_3 r_2 - d_3}\right] - \left[ln\left(1 + d_3 r_2 - d_3\right) d_5\right] = 0$$

$$d_3\left(d_4 + d_5 r_2\right) - d_5\left(1 + d_3 r_2 - d_3\right) ln\left(1 + d_3 r_2 - d_3\right) = 0 \qquad \text{(E2)}$$

$$\text{At} \quad r_2 = 1 \quad \mathrm{EE}_P^{'}\left(1\right) = d_3\left[d_4 + d_5\right]$$

$$\text{At} \quad r_2 = \infty; \quad \mathrm{EE}_P^{'}\left(\infty\right) < 0$$

Thus, $\mathrm{EE}_P^{'}\left(r_2\right)$ follows the quasi concave response because $\mathrm{EE}_P^{'}\left(r_2\right)$ is first positive and then negative with the optimal value must be existed at $\mathrm{EE}_P^{'}\left(r_2\right) = 0$, and the $\mathrm{EE}_P^{''}\left(r_2\right)$ must be less than zero:

$$\mathrm{EE}_P''(r_2)=\frac{d}{dr_2}\left((d_4+d_5r_2)\left[\frac{d_3}{1+d_3r_2-d_3}\right]-\left[ln(1+d_3r_2-d_3)d_5\right]\right)$$

$$\mathrm{EE}_P''(r_2)=d_3d_5-d_5\left[\left(ln(1+d_3r_2-d_3)\times d_3\right)+\frac{(1+d_3r_2-d_3)d_3}{1+d_3r_2-d_3}\right]$$

$$\mathrm{EE}_P''(r_2)=\frac{d_3d_5(1+d_3r_2-d_3)-d_3d_5(1+d_3r_2-d_3)ln(1+d_3r_2-d_3)-d_3d_5(1+d_3r_2-d_3)}{(1+d_3r_2-d_3)}$$

$$\mathrm{EE}_P''(r_2)=-d_3d_5\,ln(1+d_3r_2-d_3)$$

$$\mathrm{EE}_P''(r_2)<0$$

- Calculation of the optimal parameter (r_3) for $\mathrm{EE}_P(r_3)$ in terms of r_3 while the remaining dimensions (r_1,r_2) in the interval $[1,\infty)$ are constant and fixed. Thus, the energy efficiency ($\mathrm{EE}_P(r_3)$) in terms of r_3 can be written as:

$$\mathrm{EE}_P(r_6)=\frac{d_2\,ln(1+d_3r_3)}{d_5r_3+d_4}$$

With the following substitutions:

$$d_1=\beta_k trace\left(C^HC\right),\ d_2=r_1\left(1-\frac{Tr_1}{U}\right)\frac{B}{ln(2)},$$
$$d_3=d_1(r_2-1),d_4=\sum_{i=0}^{3}C_ir_1^i+r_2\sum_{j=0}^{2}D_jr_1^{j+1},\ d_5=\frac{\delta_x B\alpha^2}{\eta}$$

Differentiate $\mathrm{EE}_P(r_3)$ with respect to r_3 to calculate the optimal value:

$$\frac{d}{dr_3}\left(\mathrm{EE}_P(r_3)\right)=\frac{d}{dr_3}\left(\frac{d_2\,ln(1+d_3r_3)}{d_5r_3+d_4}\right)$$

$$\mathrm{EE}_P'(r_3)=\frac{d_2(d_5r_3+d_4)\frac{d}{dr_2}\left(ln(1+d_3r_3)\right)-\left[d_2\,ln(1+d_3r_3)\frac{d}{dr_2}(d_5r_3+d_4)\right]}{\left[d_5r_3+d_4\right]^2}$$

$$\mathrm{EE}'_P\left(r_3\right)=\frac{d_2\left(d_5r_3+d_4\right)\left[\dfrac{d_3}{1+d_3r_3}\right]-\left[d_2\,ln\left(1+d_3r_3\right)d_5\right]}{\left[d_5r_3+d_4\right]^2}$$

Equate the above equation to zero in order to get the optimal value:

$$\mathrm{EE}'_P\left(r_3\right)=0$$

$$\frac{d_2\left(d_5r_3+d_4\right)\left[\dfrac{d_3}{1+d_3r_3}\right]-\left[d_2\,ln\left(1+d_3r_3\right)d_5\right]}{\left[d_5r_3+d_4\right]^2}=0$$

$$d_3\left(d_5r_3+d_4\right)-d_5\left(1+d_3r_3\right)ln\left(1+d_3r_3\right)=0 \tag{E3}$$

At $r_3=1;\quad \mathrm{EE}'_P\left(1\right)=d_3\left[d_5+d_4\right]$

At $r_3=\infty;\quad \mathrm{EE}'_P\left(\infty\right)<0$

Thus, $\mathrm{EE}'_P\left(r_3\right)$ follows the quasi concave response because $\mathrm{EE}'_P\left(r_3\right)$ is first positive and then negative with the optimal value must be existed at $\mathrm{EE}'_P\left(r_3\right)=0$, and the $\mathrm{EE}''_P\left(r_3\right)$ must be less than zero:

$$\mathrm{EE}''_P\left(r_3\right)=\frac{d}{dr_3}\begin{pmatrix}d_3\left(d_5r_3+d_4\right)\\-d_5\left(1+d_3r_3\right)ln\left(1+d_3r_3\right)\end{pmatrix}$$

$$\mathrm{EE}''_P\left(r_3\right)=d_3d_5-d_5\left[\left(1+d_3r_3\right)\left(\frac{d_3}{1+d_3r_3}\right)+d_3\,ln\left(1+d_3r_3\right)\right]$$

$$\mathrm{EE}''_P\left(r_3\right)=\frac{d_3d_5\left(1+d_3r_3\right)-d_3d_5\left(1+d_3r_3\right)-d_3d_5\left(1+d_3r_3\right)ln\left(1+d_3r_3\right)}{1+d_3r_3}$$

$$\mathrm{EE}''_P\left(r_3\right)=-d_3d_5\,ln\left(1+d_3r_3\right)$$

$$\mathrm{EE}''_P\left(r_3\right)<0$$

APPENDIX F

CALCULATION OF THE OPTIMAL PARAMETERS FOR THE DOMAIN SPLITTER ALGORITHM UNDER THE SCENARIO OF IMPERFECT CHANNEL CONDITIONS

- The optimal parameter (r_1) for $\mathrm{EE}_{Im,P}\left(r_1\right)$ in terms of r_1 when the other dimensions $\left(r_2, r_3\right)$ are fixed can be calculated by using the equation E1. Similarly, the optimal parameter (r_2) for $\mathrm{EE}_{Im,P}\left(r_2\right)$ in terms of r_2 when the other dimensions $\left(r_1, r_3\right)$ are fixed can be calculated by using the equation E2 but substitute $d_3 = \dfrac{\beta_k trace\left(C^H C\right)}{1+\dfrac{1}{\mathrm{T}_p^{ul}}+\dfrac{1}{r_3 \mathrm{T}_p^{ul}}} \times r_3$ in the equation E2.

- Calculation of the optimal parameter (r_3) for $\mathrm{EE}_{Im,P}\left(r_3\right)$ in terms of r_3 while the remaining dimensions $\left(r_1, r_2\right)$ in the interval $\left[1, \infty\right)$ are constant and fixed. Thus, the energy efficiency ($\mathrm{EE}_{Im,P}\left(r_3\right)$) in terms of r_3 can be written as:

$$\mathrm{EE}_{Im,P}\left(r_3\right) = \frac{d_1 \, ln\left(1+\dfrac{d_2 r_3}{d_6+\dfrac{d_3}{r_3}}\right)}{d_4 r_3 + d_5}$$

With the following substitutions:

$$d_1 = r_1\left(1-\frac{Tr_1}{U}\right)\frac{B}{ln(2)},\ d_2 = \left(r_2-1\right)\beta_k trace\left(C^H C\right),\ d_3 = \frac{1}{\mathrm{T}_p^{ul}},$$

$$d_4 = \frac{\delta_x B \alpha^2}{\eta},\ d_5 = \sum_{i=0}^{3} C_i r_1^i + r_2 \sum_{j=0}^{2} D_j r_1^{j+1},\ d_6 = 1 + d_3$$

Differentiate $\mathrm{EE}_{Im,P}\left(r_3\right)$ with respect to r_3 in order to calculate the optimal parameter:

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$$\frac{d}{dr_3}\left(\mathrm{EE}_{Im,P}\left(r_3\right)\right)=\frac{d}{dr_3}\left(\frac{d_1\,ln\left(1+\frac{d_2r_3}{d_6+\frac{d_3}{r_3}}\right)}{d_4r_3+d_5}\right)$$

$$\mathrm{EE}^{'}_{Im,P}\left(r_3\right)=\frac{d_1\left(d_4r_3+d_5\right)\frac{d}{dr_3}\left(ln\left(1+\frac{d_2r_3}{d_6+\frac{d_3}{r_3}}\right)\right)-d_1\,ln\left(1+\frac{d_2r_3}{d_6+\frac{d_3}{r_3}}\right)\frac{d}{dr_3}\left(\left(d_4r_3+d_5\right)\right)}{\left(d_4r_3+d_5\right)^2}$$

$$\mathrm{EE}^{'}_{Im,P}\left(r_3\right)=\frac{d_1\left(d_4r_3+d_5\right)\frac{d}{dr_3}\left(ln\left(1+\frac{d_2r_3}{d_6+\frac{d_3}{r_3}}\right)\right)-d_1d_4\,ln\left(1+\frac{d_2r_3}{d_6+\frac{d_3}{r_3}}\right)}{\left(d_4r_3+d_5\right)^2} \tag{F1}$$

Where

$$\frac{d}{dr_3}\left(ln\left(1+\frac{d_2r_3}{d_6+\frac{d_3}{r_3}}\right)\right)=\frac{1}{1+\frac{d_2r_3}{d_6+\frac{d_3}{r_3}}}\times\frac{\left(d_6+\frac{d_3}{r_3}\right)d_2-d_2r_3\left(-r_3x_3^{-2}\right)}{\left(d_6+\frac{d_3}{r_3}\right)^2}$$

$$\frac{d}{dr_3}\left(ln\left(1+\frac{d_2r_3}{d_6+\frac{d_3}{r_3}}\right)\right)=\frac{1}{\frac{d_6+d_3r_3^{-1}+d_2r_3}{d_6+d_3r_3^{-1}}}\times\frac{\left(d_6+\frac{d_3}{r_3}\right)d_2-d_2r_3\left(-r_3x_3^{-2}\right)}{\left(d_6+\frac{d_3}{r_3}\right)^2}$$

$$\frac{d}{dr_3}\left(ln\left(1+\frac{d_2 r_3}{d_6+\frac{d_3}{r_3}} \right) \right)=\frac{d_2\left[1+\frac{d_3}{d_6 r_3+d_3} \right]}{d_6+d_2 r_3+d_3 r_3^{-1}}$$

So, the equation F1 can be written as:

$$\mathrm{EE}'_{Im,P}\left(r_3\right)=\frac{d_1\left(d_4 r_3+d_5\right)\frac{d_2\left[1+\frac{d_3}{d_6 r_3+d_3} \right]}{d_6+d_2 r_3+d_3 r_3^{-1}}-d_1 d_4\, ln\left(1+\frac{d_2 r_3}{d_6+\frac{d_3}{r_3}} \right)}{\left(d_4 r_3+d_5\right)^2}$$

Equate the above equation to zero in order to get the optimal parameter:

$$\mathrm{EE}'_{Im,P}\left(r_3\right)=0$$

$$\left(d_4 r_3+d_5\right)\frac{d_2\left[1+\frac{d_3}{d_6 r_3+d_3} \right]}{d_6+d_2 r_3+d_3 r_3^{-1}}-d_4\, ln\left(1+\frac{d_2 r_3}{d_6+\frac{d_3}{r_3}} \right)=0$$

$$d_2\left(d_4 r_3+d_5\right)\left[1+\frac{d_3}{d_6 r_3+d_3} \right]-d_4\left(d_6+d_2 r_3+d_3 r_3^{-1}\right) ln\left(1+\frac{d_2 r_3}{d_6+\frac{d_3}{r_3}} \right)=0 \qquad \text{(F2)}$$

$$\text{At } r_3=1;\ \mathrm{EE}'_{Im,P}\left(1\right)=d_2\left(d_4 r_3+d_5\right)\left[1+\frac{d_3}{d_6 r_3+d_3} \right]$$

$$\text{At } r_3=\infty;\ \mathrm{EE}'_{Im,P}\left(\infty\right)<0$$

$$\mathrm{EE}'_{Im,P}\left(1\right)=d_2\left(d_4 r_3+d_5\right)\left[1+\frac{d_3}{d_6 r_3+d_3} \right]$$

$$\mathrm{EE}'_{Im,P}\left(\infty\right)<0$$

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Thus, $\mathrm{EE}'_{Im,P}(r_3)$ follows the quasi concave response because $\mathrm{EE}'_{Im,P}(r_3)$ is first positive and then negative with the optimal value must be existed at $\mathrm{EE}'_{Im,P}(r_3)=0$, and the $\mathrm{EE}''_{Im,P}(r_3)$ must be less than zero:

$$\mathrm{EE}''_{Im,P}(r_3)=\frac{d}{dr_3}\left(d_2\left(d_4r_3+d_5\right)\left[1+\frac{d_3}{d_6r_3+d_3}\right]-d_4\left(d_6+d_2r_3+d_3r_3^{-1}\right)\times ln\left(1+\frac{d_2r_3}{d_6+\frac{d_3}{r_3}}\right)\right)$$

$$\mathrm{EE}''_{Im,P}(r_3)=d_2\left(d_4r_3+d_5\right)\left[\frac{-d_3d_6}{\left(d_3+d_6r_3\right)^2}\right]+d_4d_2\left[1+\frac{d_3}{d_3+d_6r_3}\right]$$
$$-d_4\left[d_2\left(1+\frac{d_3}{d_6r_3+r_3}\right)+\left(d_2-\frac{d_3}{r_3^2}\right)\right]$$

$$\mathrm{EE}''_{Im,P}(r_3)=-\left[\frac{d_2d_3d_6\left(d_4r_3+d_5\right)}{\left(d_3+d_6r_3\right)^2}+\left(\frac{d_4d_2r_3^2-d_3d_4}{r_3^2}\right)\times ln\left(1+\frac{d_2r_3}{d_6+\frac{d_3}{r_3}}\right)\right]$$

$$\mathrm{EE}''_{Im,P}(r_3)<0$$

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APPENDIX G

PUBLICATIONS

List of Publications

Khan, A.A., Uthansakul, P., Duangmanee, P., Uthansakul, M. (2018). Energy Efficient Design of Massive MIMO by Considering the Effects of Nonlinear Amplifiers. **Energies.** 11:1045.

Uthansakul, P., Khan, A.A., P., Uthansakul, M. Duangmanee, P. (2018). Energy Efficient Design of Massive MIMO Based on Closely Spaced Antennas: Mutual Coupling Effect. **Energies.** 11:2029.

Khan, A.A., Uthansakul, P., Uthansakul, M. (2017). Energy Efficient Design of Massive MIMO by Incorporating with Mutual Coupling. **International Journal on the Communication Antenna and Propagation.** 7: 198-207.

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BIOGRAPGY

Mr. Arfat Ahmad Khan was born on December 13, 1991, in Lahore, Pakistan. He received the B.Eng degree (2013) in Electrical Engineering from The University of Lahore, Pakistan and M.Eng (2016) in Electrical Engineering from the Government College University Lahore, Pakistan. From 2008 to 2013, he worked as a mathematics and physics tutor. From 2013 to 2015, he was employed and worked as a site engineer in the G3 Engineering Consultants Pvt Ltd. From 2015 to 2016, he was employed and worked as an RF engineer in the leading telecommunication company “Etisalat” of United Arab Emirates. He has been the recipient of various awards and professional certificates throughout his career. He has been currently pursuing his PhD degree in Telecommunication and Computer Engineering at Suranaree University of Technology, Thailand. His research interests include Optimization and stochastic processes, Channel and the mathematical modelling, Wireless Sensor Networks, Zigbee, Green Communications, Massive MIMO, OFDM, Wireless technologies, Signal Processing and the advance wireless communications.