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FORECASTING THAI MORTALITY BY USING A MODIFIED LEE-CARTER MODEL

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FORECASTING THAI MORTALITY BY USING

A MODIFIED LEE-CARTER MODEL

Suranaree University of Technology has approved this thesis submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy.

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วิทยานิพนธ์ฉบับนี้มีวัตถุประสงค์เพื่อพยากรณ์อัตรามรณะของประชากรไทยและอัตรามรณะของประชากรในจังหวัดนครราชสีมาโดยประยุกต์ใช้ตัวแบบลี-คาร์เตอร์และตัวแบบที่ดัดแปลงมาจากตัวแบบลี-คาร์เตอร์ โดยใช้ตัวแบบที่เรียกว่า age-period cohort model

ในการศึกษาครั้งนี้วิธีการที่ใช้ในการประมาณค่าพารามิเตอร์ของตัวแบบโดยใช้วิธีการแบบดั้งเดิมและวิธีการที่พิจารณาภายใต้ข้อสมมติการแจกแจงความน่าจะเป็นของจำนวนการตายของประชากรแบบการแจกแจงปัวส์ซอง และการแจกแจงทวินามลบ ภายใต้สมมติฐานพารามิเตอร์โอเวอร์ดิซเพอชันขึ้นอยู่กับอายุ และพารามิเตอร์โอเวอร์ดิซเพอชันไม่ขึ้นอยู่กับอายุ นอกจากนี้ได้พยากรณ์อัตรามรณะของประชากรในจังหวัดนครราชสีมา โดยการประยุกต์ใช้ความน่าจะเป็นแบบเบย์ และความสัมพันธ์ของอัตรามรณะของประชากรกับอัตรามรณะของพอร์ตโฟลิโอ

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LEE-CARTER MODEL, AGE-PERIOD COHORT MODEL, MORTALITY RATE

The objectives of this thesis are to forecast the mortality rate and the mortality rate of a portfolio Nakorn Ratchasima province. We fit and forecast the mortality rate by using the Lee-Carter model and a modified Lee-Carter model which is an age-period-cohort model.

In this study, the method of parameter estimation of the model consists of the classical method, as well as the the method which assumes that the probability distribution of death counts be known. The distribution of the death counts is considered as a Poisson and negative binomial distribution. The age-dependent dispersion parameter and the age-independent dispersion parameter are proposed in a negative binomial distribution assumption. Furthermore, we also forecast the mortality rate for Nakorn Ratchasima province by applying the Bayesian probability theory and the relationship of the population mortality to the portfolio mortality.

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CHAPTER I

INTRODUCTION

1.1 Introduction

Throughout the recent century, technological advances in medicine and public health have caused a continuous decline in the mortality rate. Forecasting the mortality rate is important in studying the trends of the population because the death rate can be as important as the birth rate. In the insurance business, actuaries need to be able to create an accurate mortality table to determine insurance premiums and premium reserves. The mortality rate is also useful in other disciplines, such as medicine and public health, which use life tables to find the average lifespan and longevity of the population.

In life insurance, companies deal with two fundamental types of risk when issuing contracts: financial risk and demographic risk. Uncertainty about future mortality is a problem for financial businesses involved in selling annuities and all other life-contingent products. In demography and actuarial science, there have been many attempts to find an appropriate model to represent mortality.

Life expectancy at birth in Thailand increased in the period between 2003 and 2012 as shown in Figure 1.1. Hence, it is important for the government and insurance companies to be aware of this trend. This situation is called longevity risk, i.e. the risk that a pension scheme or an insurer's annuity portfolio will need to pay out more than expected due to increasing life expectancy.

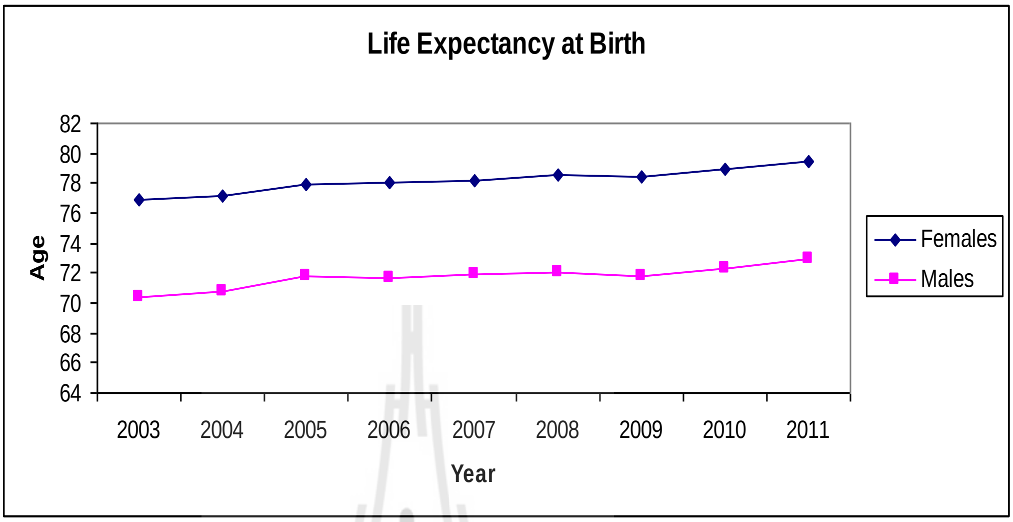


Figure 1.1 Life expectancy at birth in Thailand for the period between 2003 and 2012.

There are many ways to project the future mortality rate. A benchmark model was developed by R.D. Lee and L. Carter (Lee and Carter, 1992) who proposed a remarkably simple model for describing the secular change in mortality as a function of a single time index, specifying a log-bilinear form for the force of mortality. Lee and Carter (1992) applied Singular Value Decomposition (SVD) to estimate the parameters of the model. The Lee-Carter model has become popular and has been modified and extended in different ways of parameter estimation of the model by other researchers, such as Wilmonth (1993), who proposed the Weighted Least Square (WLS) acquiring an estimation of the Lee-Carter model parameters. Brouhns et al. (2002) implemented Wilmonth's recommendations for improving the Lee-Carter approach to forecasting the demographic component and applied Maximum

Likelihood Estimation (MLE) to estimate the parameters of the Lee-Carter equation under the assumption that the death rate has a Poisson distribution. The Lee-Carter model has proved to give good results for mortality in diverse countries. For example, in Japan (Wilmonth, 1993), Chile (Lee and Rofman, 1994), the seven most economically developed nations (G7) (Tulijapurkar et al., 2000), Austria (Carter and Prskawetz, 2001), Australia (Booth et al., 2002) Belgium (Brouhns et al., 2002), Canada (Lee and Nault, 2003), Japan and Taiwan (Yue, Huang and Yang, 2008), and China (Zhao, 2011). The model has many variants and extensions. For example, Lee and Miller (2001) proposed the adjustment of k_t to fit life expectancy at birth in year t rather than according to probable death rates which were used in the basic Lee-Carter Model, while Plat (2009) and O' Hare and Li (2012) proposed a multi-factor model. However, Plat's model does not give good results for a wider age range. Renshaw and Haberman (2006) proposed the age-period-cohort model version adding the cohort effect to the Lee-Carter model. So, in this thesis, we still prefer to use the single factor Lee-Carter model and the age-period-cohort model which are suitable for the aims of our investigation.

The objective of this thesis is to forecast the Thai mortality rate. We use the Lee-Carter model and the age-period-cohort model to fit and forecast the Thai mortality rate. We focus on the different methods of parameter estimation. The estimation methods are composed of the classical parameter estimation methods (Singular Value Decomposition, Weighted Least Square, and Maximum Likelihood Estimation). Using these methods, we consider the assumptions of the death count of the population. We focus on the Poisson setting and the negative binomial setting. Dispersion parameters of the negative binomial type refer to age-dependent and age-independent dispersion parameters. Furthermore, we also calculate a portfolio mortality rate (portfolio of Nakorn Ratchasima) by applying the Bayesian inference technique.

1.2 Outline of the thesis

This thesis consists of four parts. The first part; Chapter II describes some statistical and mathematical definitions. In the second part which is Chapter III we fit and forecast the Thai mortality rate using the Lee-Carter model by the classical parameter estimation methods; then we compare the results the best-suited method and calculate the life expectancy at birth. In the third part, in chapter IV, we apply the Lee-Carter model and the age-period-cohort model with the Poisson setting and the negative binomial setting. We introduce the negative binomial setting which is an extension of the age-period-cohort model underlying the assumption of dispersion parameter. We then apply two mortality models and all assumptions to the Thai population mortality data. We use the Thai mortality data to fit the mortality models and, to obtain suitable mortality model to forecast the Thai mortality rate and life expectancy. Also, we compute the life insurance premium using the forecasted mortality rate and the Thai mortality table (TMO2008).

In the final part, we apply the relationship between the past mortality rates of the general population and of a portfolio population to forecast the portfolio mortality rate. We use Nakorn Ratchasima population data as a representative of the portfolio mortality data and forecast its mortality rate using the Bayesian properties as presented in Chapter V. Finally, Chapter VI gives the conclusion to this thesis.

CHAPTER II

PRELIMINARIES

In this chapter, we introduce some statistical and mathematical definitions which related to this thesis.

The following definitions we formulate basic probability concepts by following Bowers et al. (1997), Cunningham et al. (2012) and Pitacco et al. (2009).

2.1 The survival function

Definition 2.1 Age at failure random variable X

Let X be the newborn's age at failure random variable, the domain of the random variable X is $X \geq 0$. We denote the distribution function of X by

$$F_0(x) = \Pr(X \leq x), \tag{2.1}$$

where $x \geq 0$.

The distribution function must satisfy requirements:

- $0 \leq F_0(x) \leq 1$ for all x.
- $F_0(x)$ is non decreasing.
- $F_0(x)$ is right-continuous.
- $\lim_{x\to-\infty} F_0(x) = 0$ and $\lim_{x\to\infty} F_0(x) = 1$.

In actuarial notation, the probability distribution is denoted by

$$ {}_{x}q_0 = F_0(x) = \Pr(X \leq x). $$

Definition 2.2 The survival distribution function for the survival random variable X.

We define the survival function by

$$ S_0(x) = 1 - F_0(x) = \Pr(X > x). \tag{2.2} $$

As a result:

- $0 \leq S_0(x) \leq 1$ for all x.
- $S_0(x)$ is non decreasing.
- $S_0(x)$ is right-continuous.
- $\lim_{x\to-\infty} S_0(x) = 1$ and $\lim_{x\to\infty} S_0(x) = 0$.

This probability distribution is denoted by ${}_{x}p_0 = S_0(x) = \Pr(X > x)$.

The probability that a newborn dies between ages x and z $(x < z)$ is

$$ \Pr(x < X \leq z) = F_0(z) - F_0(x) = S_0(x) - S_0(z). $$

Definition 2.3 The Probability Density Function of X.

The probability density function (PDF) for a continuous random variable x are defined as the derivative of CDF

$$ f_0(x) = \frac{d}{dx}F_0(x) = -\frac{d}{dt}S_0(x). \tag{2.3} $$

Then

$$ F_0(x) = \int_0^x f_0(y)\,dy \ . \tag{2.4} $$

and

$$S_0(x) = \int_x^{\infty} f_0(y)\, dy. \tag{2.5}$$

Also,

$$\int_0^{\infty} f_0(y)\, dy = 1 \tag{2.6}$$

for $y > 0$.

Definition 2.4 Time-until-Death for a person age X.

The conditional probability that a newborn will die between the ages x and z, given survival to age x (Bowers et al., 1997) is

$$\Pr(x < X \leq z | X > x) = \frac{F_0(z) - F_0(x)}{1 - F_0(x)} = \frac{S_0(x) - S_0(z)}{S_0(x)}. \tag{2.7}$$

Definition 2.5 The force of mortality and hazard rate.

Recall the definition of the force of mortality and hazard rate from Cunningham, Herzog, and London (2012) and Bowers et al. (1997).

The force of mortality is defined the following:

$$\mu(x) = \lim_{\Delta x \to 0} \frac{\Pr(x < X \leq x + \Delta x | X > x)}{\Delta x}. \tag{2.8}$$

The probability of death between age x and $x + \Delta x$, by equation (2.7) is

$$\Pr(x < X \leq x + \Delta x | X > x) = \frac{F_0(x + \Delta x) - F_0(x)}{1 - F_0(x)}$$

$$= \frac{F_0(x + \Delta x) - F_0(x)}{S_0(x)}. \tag{2.9}$$

The following function gives the conditional density of failure at age x, given survival to age x.It is called the *hazard rate* or *force of mortality* at age x, denoted by $\mu(x)$, and defined by

$$\mu(x) = \lim_{\Delta x \to 0} \frac{\Pr(x < X \leq x + \Delta x | X > x)}{\Delta x}.$$

So that by (2.9),

$$\mu(x) = \lim_{\Delta x \to 0} \frac{F_0(x+\Delta x) - F_0(x)}{\Delta x S_0(x)} = \frac{f_0(x)}{S_0(x)}.$$

$$= \frac{-\frac{d}{dt} S_0(x)}{S_0(x)} = -\frac{d}{dx} \ln S_0(x). \quad (2.10)$$

By integrating, we have

$$\int_0^x \mu(y) dy = -lnS_0(x), \quad (2.11)$$

or,

$$S_0(x) = \exp\left[-\int_0^x \mu(y) dy\right]. \quad (2.12)$$

The *cumulative hazard function* (CHF) can be defined as

$$\Lambda_0(x) = \int_0^x \mu(y) dy = -\ln(S_0(x)). \quad (2.13)$$

Definition 2.6 The remaining lifetime at age x random variable T_x.

The probability distribution function of T_x can be defined relating to X.

The cumulative distribution function of the age at death in year t, $F_t(x)$, is

$$F_t(x) = \Pr(T_x < t). \quad (2.14)$$

The probability of dying between age x and $x + t$, is

$$\Pr(x < X \leq x + t) = F_t(x + t) - F_t(x) = S_t(x) - S_t(x + t). \quad (2.15)$$

The conditional probability of dying between age x and $x+t$, given survival to age x, is

$$\Pr(x < X \le x+t | X > x) = \frac{F_t(x+t)-F_t(x)}{1-F_t(x)} = \frac{F_t(x+t)-F_t(x)}{S_t(x)}. \qquad (2.16)$$

Recall the hazard rate function of X, we can express the force of mortality by

$$\mu(x,t) = \lim_{t\to 0} \frac{\Pr(x < X \le x+t | X > x)}{t}$$

$$= \lim_{t\to 0} \frac{F_t(x+t)-F_t(x)}{t(1-F_t(x))} = -\frac{d}{dx}\ln S_t(x). \qquad (2.17)$$

The hazard rate is constant for the interval age $(x, x+1)$ integrating over age $(x, x+1)$,

$$\int_x^{x+1} \mu(y,t)dy = -\int_x^{x+1} \frac{d}{dy}\ln S_t(y)\, dy$$

$$= -\int_x^{x+1} d\ln S_t(y). \qquad (2.18)$$

Consequently,

$$\mu(x,t) = lnS_t(x) - lnS_t(x+1), \qquad (2.19)$$

or

$$-\mu(x,t) = ln\left[\frac{S_t(x+1)}{S_t(x)}\right]. \qquad (2.20)$$

So

$$\text{EXP}\big(-\mu(x,t)\big) = \frac{S_t(x+1)}{S_t(x)}. \qquad (2.21)$$

Definition 2.7 Survival function.

Let l_0 be a number of age-specific survival based on individual newborn. l_x is the number of age-specific of life aged x. For age interval $(x, x+t)$ where $x, t > 0$, the number dying between age $(x, x+t)$ is

$$d_x = l_x - l_{x+t}, \tag{2.22}$$

So that

$$d_x = l_x - l_{x+1}, \tag{2.23}$$

is the number of deaths over age interval $(x, x+1)$.

We treat $S(x)$ as survival function, that is

$$S(x) = \frac{l_x}{l_0}. \tag{2.24}$$

Then,

$$\begin{aligned} d_x &= l_x - l_{x+1} \\ &= S(x)l_0 - S(x+1)l_0 \\ &= l_0\big(S(x) - S(x+1)\big). \end{aligned} \tag{2.25}$$

To construct life table, the central death rate m_x between age x and $x+1$ is

$$m_x = \frac{d_x}{L_x}, \tag{2.26}$$

where L_x is the number of individuals living between age x and $x+1$.

The function T_x of total number of years lived after age x for l_0 initial individuals is denoted by

$$T_x = L_x + L_{x+1} + L_{x+2} + L_{x+3} + \cdots. \tag{2.27}$$

The life expectancy or the future lifetime expectancy, denoted by ${}^0e_x$, can be computed as

$${}^0e_x = \frac{T_x}{l_x}. \tag{2.28}$$

2.2 Mortality model

In our thesis, we use the Lee-Carter model and the age-period-cohort model to fit and forecast mortality rate.

2.2.1 The Lee-Carter model

In 1992, Lee and Carter proposed what is now called the Lee-Carter model. This model is describes as the logarithm of the mortality rate by the following equation:

$$\ln(m_{x,t}) = \alpha_x + \beta_x k_t + \varepsilon_{x,t}, \tag{2.29}$$

where $x = x_1, \ldots, x_p$ represent ages, and

$t = t_1, \ldots., t_n$ represent the calendar year.

Here α_x describes the age profile averaged overtime,

β_x is age-specific describe the change of mortality at each age relating with changing of k_t,

k_t represents the time series for the general level of mortality and measures the trend in mortality over time, and

$\varepsilon_{x,t}$ is the error term with mean zero and variance σ_ε^2. In order to make the model identifiable, one needs to impose the following constraints on the parameters,

$$\sum_{t=t_1}^{t_n} k_t = 0, \text{ and } \sum_{x=x_1}^{x_p} \beta_x = 1. \tag{2.30}$$

2.2.2 The age-period-cohort model

Renshaw and Haberman (2006) proposed the age-period-cohort model which considers the birth cohort, *cohort* = *period-age*, as follows

$$\ln(m_{x,t}) = \alpha_x + \beta_x^{(0)} k_t + \beta_x^{(1)} l_{t-x} + \varepsilon_{x,t}, \tag{2.31}$$

where $\beta_x^{(0)}$, $\beta_x^{(1)}$ measure the corresponding interaction with age,

l_{t-x} is a random cohort effect that is a function of the year of birth, $t-x$,

k_t is a period effect.

We use the following restriction to be able to estimate the parameters,

$$\sum_{x=x_1}^{x_p}\beta_x^{(0)}=1, \sum_{x=x_1}^{x_p}\beta_x^{(1)}=1, \sum_{t=t_1}^{t_n}k_t=0 \text{ and } \sum_{t=t_1}^{t_n}l_{t-x}=0. \quad (2.32)$$

2.3 Present value random variable

We follow this section from Cunningham, Herzog, and London (2012). The interval $(k-1,k]$ as failure with payment at time k, and let $K_x=k-1$. The present value random variable Z_x can be defined as

$$Z_x=\begin{cases} v^{K_x+1}, & K_x=0,1,2,\dots \\ v^{K_x^*}, & K_x^*=1,2,3,\dots \end{cases} \quad (2.33)$$

where v^k is a constant of compound interest rate a time k (the discount factor v).

The expected value of the random variable Z_x is denoted by A_x,

$$A_x=E[Z_x]=\sum_{k=1}^{\infty}v^k\,P(K_x^*=k). \quad (2.34)$$

The second moment of Z_x is

$$^2A_x=E[Z_x^2]=\sum_{k=1}^{\infty}(v^k)^2\,P(K_x^*=k). \quad (2.35)$$

The variance of the present value random variable Z_x is given by

$$Var(Z_x)={}^2A_x-A_x^2. \quad (2.36)$$

We consider the payment at precise time n if and only if failure occurs after the first n time intervals (only if $K_x^*\le n$) . The present value random variable is denoted by

$$Z_{x:\overline{n}|}^{\;\;1} = \begin{cases} v^n, K_x^* \le n \\ 0,\ K_x^* > n \end{cases}. \tag{2.37}$$

The first moment of $Z^1_{x:\overline{n}|}$ is given by

$$A_{x:\overline{n}|}^{\;\;1} = E\left[Z_{x:\overline{n}|}^{\;\;1}\right] = v^n P(K_x^* > n). \tag{2.38}$$

Since $P(K_x^* > n) = {}_t p_x$, then $A_{x:\overline{n}|}^{\;\;1} = v^t\ {}_t p_x$. It is the expected value of the n -year pure endowment.

An *n*-year endowment insurance

An *n*-year endowment insurance is combining the n-year term insurance model with the *n*-year pure endowment model. Letting $Z_{x:\overline{n}|}$ denoted the present value random variable for this model, we have

$$Z_{x:\overline{n}|} = \begin{cases} v^{K_x^*}, K_x^* \le n \\ v^n, K_x^* > n \end{cases}. \tag{2.39}$$

It is clear that

$$Z_{x:\overline{n}|} = Z^1_{x:\overline{t}|} + Z_{x:\overline{n}|}^{\;\;1} \tag{2.40}$$

where $Z^1_{x:\overline{t}|}$ is the random variable for an *n*-year temporary insurance and $Z_{x:\overline{n}|}^{\;\;1}$ is the random variable for an *n*-year pure endowment, since the benefit is paid to the insured if survival to age $x + n$ occurs.

The expected value for present value random variable is given by

$$A_{x:\overline{n}|} = A^1_{x:\overline{t}|} + A_{x:\overline{n}|}^{\;\;1}, \tag{2.41}$$

where $A^{1}_{x:\overline{t}|}$ is the expected value of random variable for an *n*-year temporary insurance and $A_{x:\overline{n}|}{}^{1}$ is the expected value of random variable for an *n*-year pure endowment.

Whole life annuity models

Suppose we have a status with identifying characteristic (x) as of time (t), such as a person alive at time (t), at age x, and a sequence of unit payment scheduled to be made at the end of the year as long as the status continues to survive. This model is called a whole life-contingent immediate annuity model. Let Y_x denote the present value random variable,

$$Y_x = \sum_{t=1}^{\infty} Z_{x:\overline{n}|}{}^{1}. \tag{2.42}$$

The expected value of the present value random variable Y_x is denoted a_x,

$$a_x = E[Y_x] = \sum_{t=1}^{\infty} E[\sum_{t=1}^{\infty} Z_{x:\overline{t}|}{}^{1}] = \sum_{t=1}^{\infty} A_{x:\overline{t}|}{}^{1} = \sum_{t=1}^{\infty} v^t \; {}_tp_x \,. \tag{2.43}$$

Temporary annuity models

The immediate *n*-year temporary annuity, payable to a status with identity characteristic (x) at time 0, will make payment at the end of the year for n years at most, provided the status continue to survive. Let $Y_{x:\overline{n}|}$ be the present value random variable for this model,

$$Y_{x:\overline{n}|} = \sum_{t=1}^{\infty} Z_{x:\overline{t}|}{}^{1}. \tag{2.44}$$

The expected value of this present value random variable is denoted $a_{x:\overline{n}|}$, and is given by

$$a_{x:\overline{n}|} = E[Y_{x:\overline{n}|}] = \sum_{t=1}^{n} E[Z_{x:\overline{t}|}{}^{1}] = \sum_{t=1}^{n} A_{x:\overline{t}|}{}^{1}$$

$$= \sum_{t=1}^{n} v^t \ {}_tp_x \ . \tag{2.45}$$

We refer to the expected value of the contingent annuity present random variable as the expected present value (EPV) or the actuarial present value (APV) or the net single premium (NSP).

CHAPTER III

CLASSICAL ESTIMATION METHOD FOR THE LEE-CARTER MODEL

3.1 Introduction

The objective of this chapter is to use the Lee-Carter model to estimate and forecast the mortality rate in Thailand. We use the three classical methods, i.e., Singular Value Decomposition (SVD), Weighted Least Square (WLS), and Maximum Likelihood Estimation (MLE) based on the Poisson statistical framework to estimate the parameter of the Lee-Carter model. With these methods, we investigate the goodness of fit for the mortality rate spanning the period 2003 to 2012. The fitted models are compared. The autoregressive moving average (ARIMA) is used to forecast the general index and mortality rate the time period from 2013 to 2022. The plan for this chapter is as follows: firstly, we describe the source of data. In Section 2, we briefly describe the three methodologies for deriving the parameters of the model. In section 3, the fitted model derived is presented. Section 4 is devoted to studying the forecasted mortality rate and life expectancy at birth. Finally, section 5 gives a summary of the results.

3.2 Source of data

The Thai Ministry of the Interior is the main source of data on the number of age-specific people in the population. The number of deaths was obtained from the Bureau of Policy and Strategy, Ministry of Public Health. We assume that the remaining lifetimes of the individuals aged on January 1 of year *t* are independent and identically distributed. The $\widehat{m}_{x,t}$ is the nonparametric estimation of $m_{x,t}$. This is given by the ratio of the observed number of deaths $D_{x,t}$ for age x and year t corresponding exposure to risk $ETR_{x,t}$.

$$\widehat{m}_{x,t} = \frac{D_{x,t}}{ETR_{x,t}}. \tag{3.1}$$

where $x = x_1, x_2,..., x_m$, $t = t_1, t_2,..., t_n$.

3.3 Estimation parameter approaches

In this section we use the classical method to estimate the parameters of the Lee-Carter model.

3.3.1 Singular Value Decomposition

In its original version, the Lee-Carter model (Lee and Carter, 1992) can not fit into ordinary regression methods because there are no given regressors on the right side of the equation. We have only parameters to be estimated and the unknown index k_t. The estimators of α_x, β_x, and $k_t$will be denoted by $\hat{\alpha}_x$, $\hat{\beta}_x$, and $\hat{k}_t$ respectively. Firstly, we estimate $\hat{\alpha}_x$ as the averaging $\ln(\widehat{m}_{x,t})$ over time t:

$$\hat{\alpha}_x = \frac{1}{t_n - t_1 + 1} \sum_{t=t_1}^{t_n} \ln(\widehat{m}_{x,t}). \tag{3.2}$$

The parameters β_x and k_t are computed by applying singular value decomposition (SVD) to the matrix

Z where

$$Z = \ln(\hat{m}_{x,t}) - \hat{\alpha}_x,$$

i.e.,

$$Z = \begin{bmatrix} \ln(\hat{m}_{x_1,t_1}) - \hat{\alpha}_{x_1} & \ln(\hat{m}_{x_1,t_2}) - \hat{\alpha}_{x_1} \;\dots\dots & \ln(\hat{m}_{x_1,t_n}) - \hat{\alpha}_{x_1} \\ \ln(\hat{m}_{x_2,t_1}) - \hat{\alpha}_{x_2} & \ln(\hat{m}_{x_2,t_2}) - \hat{\alpha}_{x_2} & \ln(\hat{m}_{x_2,t_n}) - \hat{\alpha}_{x_2} \\ \vdots & \ddots & \vdots \\ \vdots & \ddots & \vdots \\ \ln(\hat{m}_{x_{m-1},t_1}) - \hat{\alpha}_{x_{m-1}} & \ln(\hat{m}_{x_{m-1},t_2}) - \hat{\alpha}_{x_{m-1}} \;\dots\dots & \ln(\hat{m}_{x_{m,}-1,t_n}) - \hat{\alpha}_{x_{m-1}} \\ \ln(\hat{m}_{x_m,t_1}) - \hat{\alpha}_{x_m} & \ln(\hat{m}_{x_{m,},t_2}) - \hat{\alpha}_{x_m} \;\dots\dots & \ln(\hat{m}_{x_{m,},t_n}) - \hat{\alpha}_{x_m} \end{bmatrix}. \quad (3.3)$$

By applying SVD, one obtains the decomposition

$$Z = \sum_{i=1}^{r} \rho_i U_{x,i} V_{t,i}, \quad (3.4)$$

where r =rank[Z], ρ_i (i=1,2,....,r) are the increasingly ordered singular values with $U_{x,i}$ and $V_{t,i}$, as the corresponding left and right singular vectors. Then, the approximations β_x and k_t can be obtained as: $\hat{\beta}_x = U_{x,1}$ and $\hat{k}_t = \rho_1 V_{t,1}$.

Adjustment of the $\hat{k}_t$ by re-estimating to the total observed deaths.

In the first estimation, the estimation of k_t do not provide an adequate fit to the observed data. Thus, Lee and Carter (1992) re-estimate k_t to get the observed number of deaths equal to the fitted number of deaths, i.e.

$$D_t = \sum_x ETR_{x,t} exp(\alpha_x + \beta_x k_t), \quad (3.5)$$

where D_t is the total number of deaths in year *t* and $ETR_{x,t}$ is the population of age *x* in year *t*. There is need for a second-stage estimation for $\hat{k}_t$ of SVD method.

3.3.2 The Weighted Least Square

Wilmonth (1993) proposed the Weighted Least Square (WLS) approach to obtain an estimation of the Lee-Carter model parameters. Fitting the Lee-

Carter model into the Weighted Least Square (WLS) can be done by minimizing the equation

$$\sum_{x=x_1}^{x_p}\sum_{t=t_1}^{t_n}[D_{x,t}(\ln(m_{x,t}) - \alpha_x - \beta_x k_t)]^2. \tag{3.6}$$

To minimize (3.5), it is obtained by equation to 0 the first derivatives with respect to α_x, β_x and k_t. The parameters will be obtained numerically according to the following algorithm :

$$\hat{\alpha}_x = \frac{\sum_t D_{x,t}(\ln(m_{x,t}) - \beta_x k_t)}{\sum_t D_{xt}},$$

$$\hat{\beta}_x = \frac{\sum_t D_{x,t} k_t(\ln(m_{x,t}) - \alpha_x)}{\sum_t D_{xt}\,\hat{k}_t^2},$$

$$\hat{k}_t = \frac{\sum_t D_{x,t}\beta_x(\ln(m_{x,t}) - \alpha_x)}{\sum_t D_{xt}\,\hat{\beta}_x^2}.$$

This process continues until successive computation yields little or no change in parameter value (Wilmonth,1993).

3.3.3 Using Maximum Likelihood Estimation

Brouhns et al. (2002) implemented Wilmonth's recommendation for improving the Lee-Carter approach to forecast and proposed the maximum likelihood estimation (MLE) to give an optimal solution to the Lee-Carter equation under a Poisson model.

Distribution of the number of deaths

The number of deaths is a counting random variable and can be assumed to be Poisson distributed:

$$D_{x,t} \sim Poisson(\lambda_{x,t}), \tag{3.7}$$

where $\lambda_{x,t}$ is the parameter of the Poisson distribution which now equals:

$$\lambda_{x,t} = ETR_{x,t}e^{(\alpha_x+\beta_x k_t)}. \tag{3.8}$$

The mean and variance of $D_{x,t}$ can be obtained by $E\left[D_{x,t}\right] = Var\left[D_{x,t}\right] = \lambda_{x,t}$.

We wish to estimate the parameters α_x, β_x and k_t. Instead of using the SVD, we detemine these parameters by maximizing the log-likelihood of model (3.7) which is given by

$$L(\alpha,\beta,k) = \ln\left\{\prod_{t=t_1}^{t_n}\prod_{x=x_1}^{x=x_p}\left(\frac{\lambda_{x,t}^{D_{x,t}}exp(-\lambda_{x,t})}{\left(D_{x,t}\right)!}\right)\right\}$$

$$= \sum_{t=t_1}^{t_n}\sum_{x=x_1}^{x_p}\left\{D_{x,t}(\alpha_x+\beta_x k_t) - ETR_{x,t}\,exp(\alpha_x+\beta_x k_t)\right\} + constant. \tag{3.9}$$

Goodman (1979) proposed the iterative method for estimating the log-linear model for the maximum likelihood estimation.

$$\hat{\theta}^{(\nu+1)} = \hat{\theta}^{(\nu)} - \frac{\frac{\partial L^{(\nu)}}{\partial\theta}}{\frac{\partial^2 L^{(\nu)}}{\partial\theta^2}}, \tag{3.10}$$

where $L^{(\nu)} = L^{(\nu)}\left(\hat{\theta}^{(\nu)}\right)$.

The three sets of parameters are estimated, i.e. $\{\alpha_x\}$, $\{\beta_x\}$, and $\{k_t\}$. The updating scheme is as follows, starting with $\hat{\alpha}_x^{(0)} = 0$, $\hat{\beta}_x^{(0)} = 1$, and $\hat{k}_t^{(0)} = 0$ (random values can be used),

$$\hat{\alpha}_x^{(\nu+1)} = \hat{\alpha}_x^{(\nu)} - \frac{\sum_t(D_{x,t}-\hat{D}_{x,t}^{(\nu)})}{-\sum_t\hat{D}_{x,t}^{(\nu)}}, \hat{\beta}_x^{(\nu+1)} = \hat{\beta}_x^{(\nu)},\ \hat{k}_t^{(\nu+1)} = \hat{k}_t^{(\nu)},$$

$$\hat{k}_t^{(\nu+2)} = \hat{k}_t^{(\nu+1)} - \frac{\sum_t(D_{x,t}-\hat{D}_{x,t}^{(\nu)})\hat{\beta}_x^{(\nu+1)}}{-\sum_x\hat{D}_{x,t}^{(\nu)}\left(\hat{\beta}_x^{(\nu+1)}\right)^2},\ \ \hat{\alpha}_x^{(\nu+2)} = \hat{\alpha}_x^{(\nu+1)}, \hat{\beta}_x^{(\nu+2)} = \hat{\beta}_x^{(\nu+1)}, \tag{3.11}$$

$$\hat{\beta}_x^{(\nu+3)} = \hat{\beta}_x^{(\nu+2)} - \frac{\sum_t(D_{x,t}-\hat{D}_{x,t}^{(\nu+2)})\hat{k}_t^{(\nu+2)}}{-\sum_t\hat{D}_{x,t}^{(\nu+2)}\left(\hat{k}_t^{(\nu+2)}\right)^2},\ \hat{\alpha}_x^{(\nu+3)} = \hat{\alpha}_x^{(\nu+2)}, \hat{k}_t^{(\nu+3)} = \hat{k}_t^{(\nu+2)},$$

where $\hat{D}_{x,t}^{(\nu)} = E_{x,t}exp(\hat{\alpha}_x^{(\nu)} + \hat{\beta}_x^{(\nu)}\hat{k}_t^{(\nu)})$, and $\nu = 0,1,2,3,\ldots$.

We can stop the iteration when the value of the log-likelihood function is only little increasing.

3.4 Fitting the model

The Lee-Carter model $\ln(m_{x,t}) = \alpha_x + \beta_x k_t + \varepsilon_{xt}$,

where $x =$ <1,1,2,...,100,>100 , $t =$ 2003,...,2012. To fit the logarithm of mortality, we use the following equation

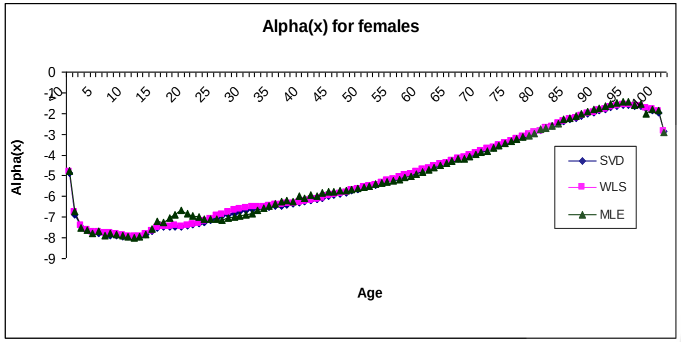
$$ln(\widehat{m}_{x,t}) = \hat{\alpha}_x + \hat{\beta}_x \hat{k}_t, \quad (3.12)$$

where the mortality rate $\widehat{m}_{x,t}$ is given by

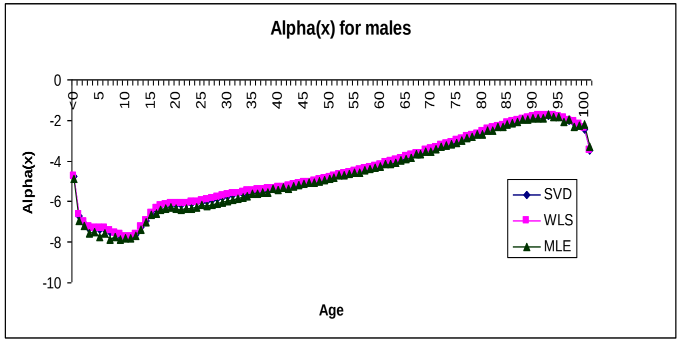
$$\widehat{m}_{x,t} = exp(\hat{\alpha}_x + \hat{\beta}_x \hat{k}_t). \quad (3.13)$$

Figure 3.1 shows a comparison of $\hat{\alpha}_x$, $\hat{\beta}_x$ and $\hat{k}_t$ with SVD, WLS, and MLE. Figure 3.1(a) shows graph of the estimations of the shape parameter α_x for females. The graph is high at the beginning of life, i.e. when x <1, and it declines in interval of age 1-14. Ti then rises gradually until it reaches the 97 age level. The graph of the estimations of the shape parameter α_x show the same trend for males as shown in Figure 3.1(b). Figure 3.1(c) shows the graph of the estimations of the parameter β_x for females. The graphs of SVD and WLS exhibit a similar trend but MLE is different from the others in the intervals of age 16-26 and over to 90 age level. Figure 3.1(d) shows the estimations of the parameter β_x for males with these three methods and all of them show the same trend. Figure 3.1(e) shows the graph of k_t for females and Figure 3.1(f) shows graph of k_t for males.

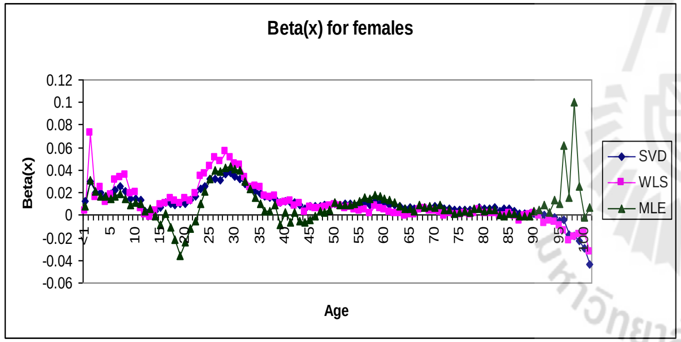
Figure 3.2 compares the mortality rate for the three methods in 2007 and 2011. Figure 3.2(a) and 3.2(b) show the mortality rate for females in 2007 and 2011 respectively. The graphs of all methods are close to the observed mortality rate, except that the MLE method is different from the observed data when the age is high. Figures 3.2(c) and 3.2(d) show the mortality rate for males in 2007 and 2011 respectively. The graphs indicate that the mortality rate of all methods is close to the graph of observed data.



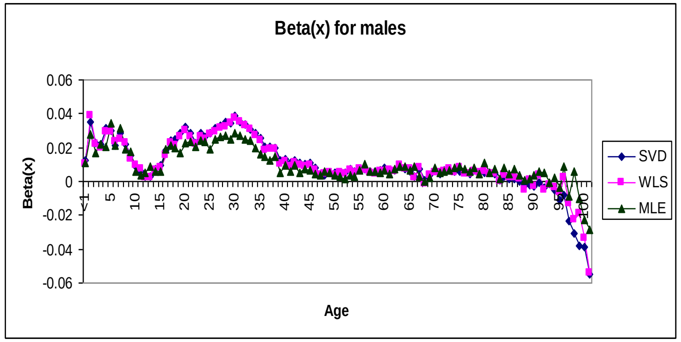
(a) Estimations of α_x, females.



(b) Estimations of α_x, males.

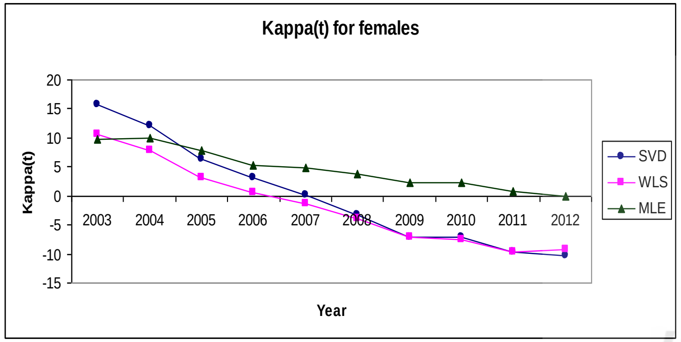


(c) Estimations of β_x, females.

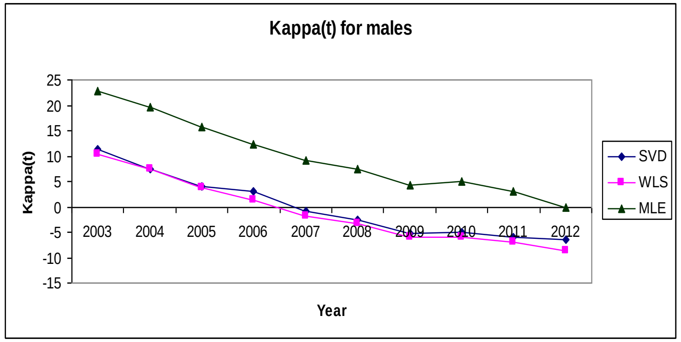


(d) Estimations of β_x, males.

Figure 3.1 Lee-Carter parameter estimations of α_x, β_x and k_t by the three methods (SVD, WLS, and MLE).

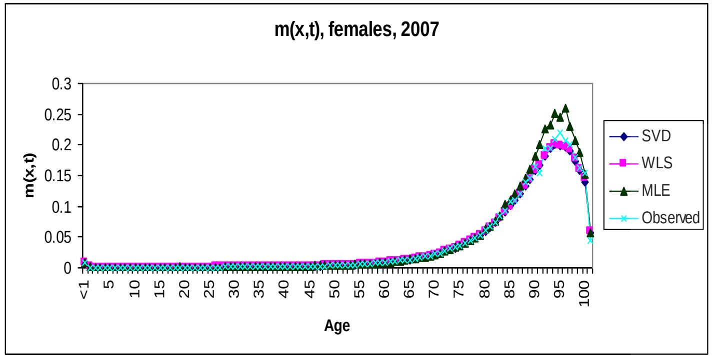


(e) Estimations of k_t, females.

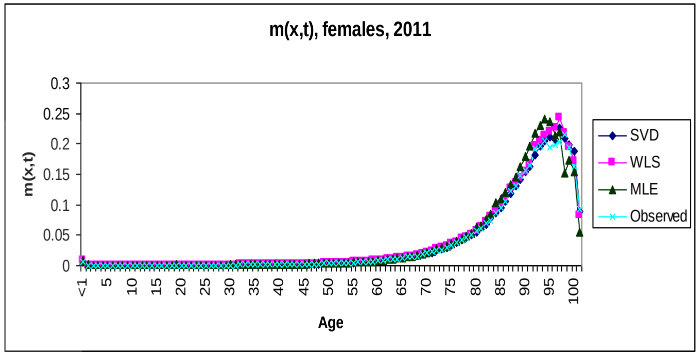


(f) Estimations of k_t, males.

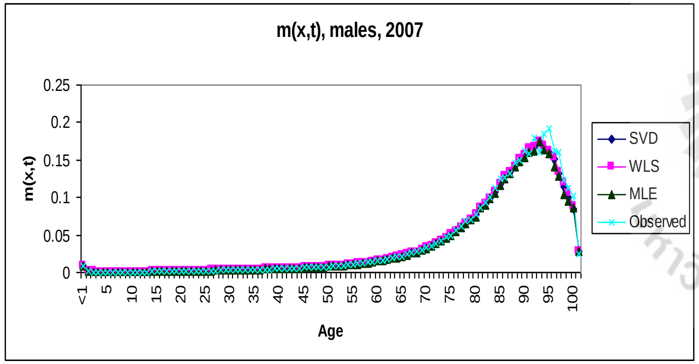
Figure 3.1 Lee-Carter parameter estimations of α_x, β_x and k_t by the three methods (SVD, WLS, and MLE) (Continued).



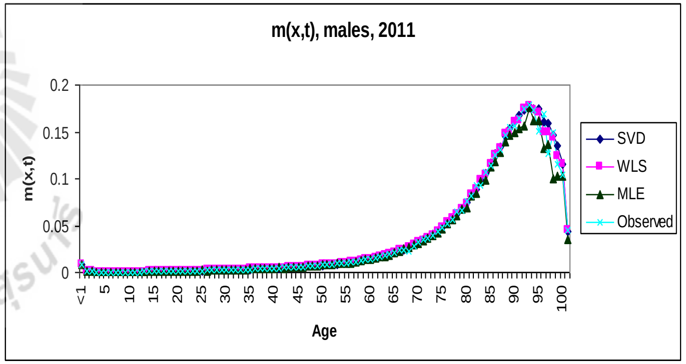
(a) Fitted $m_{x,t}$, females, 2007.



(b) Fitted $m_{x,t}$, females, 2011.



(c) Fitted $m_{x,t}$, males, 2007.



(d) Fitted $m_{x,t}$, males, 2011.

Figure 3.2 The fit of the mortality rate by three estimation methods as compare with the observed data in the year 2007 and 2011.

Table 3.1 Mean Square Error of the fitted mortality rate $\widehat{m}_{x,t}$. The bold entries show the smallest value of MSE for each gender.

MSE	SVD	WLS	MLE
Females	**0.007204**	0.078309	0.021715
Males	0.005707	**0.005332**	0.007073

We consider the fitted mortality rate. The data is a span from 2003 to 2012. The results appear in Table 3.1. It can be seen that, for females, the MSE of the SVD has the smallest value. For males, the MSE of the WLS gives the lowest value. This indicates that the SVD and the WLS are suitable methods for forecasting the mortality rate for females and males respectively.

3.5 Forecasting mortality rate and life expectancy at birth

3.5.1 ARIMA model for estimation k_t

We model the forecast index by using the time series model. Box-Jenkins methodology is used to estimate and forecast k_t with the appropriate ARIMA time series model. From the values of AIC and BIC in Table 3.2 and Table 3.3, we conclude ARIMA(0,1,0) with drift to be the most appropriate model based on the lower value of AIC and BIC. Other models have a higher AIC-value and BIC-value. The estimated model ARIMA(0,1,0) with drift is described by

$$\hat{k}_t - \hat{k}_{t-1} = \theta + \varepsilon_t, \text{ where } \varepsilon_t \sim N(0, \sigma_\varepsilon^2). \quad (3.14)$$

Table 3.2 ARIMA-models fitted to k_t for females. The bold entries show the smallest value of AIC and BIC.

		ARIMA					
		(2,1,2) with drift	(0,1,0) with drift	(1,1,0) with drift	(0,1,1) with drift	(1,1,1) with drift	(0,1,0)
SVD	AIC	49.19	**44.35**	46.34	46.32	47.21	57.19
	BIC	51.68	**45.15**	47.54	47.51	49.65	57.59
WLS	AIC	33.97	**28.31**	29.89	30.07	31.59	43.13
	BIC	35.16	**28.71**	30.48	30.67	32.38	43.33
MLE	AIC	36.13	**30.38**	32.23	32.23	34.22	46.24
	BIC	36.38	**30.77**	32.82	32.84	35.00	46.43

Table 3.3 ARIMA-models fitted to k_t for males. The bold entries show the smallest value of AIC and BIC.

		ARIMA					
		(2,1,2) with drift	(0,1,0) with drift	(1,1,0) with drift	(0,1,1) with drift	(1,1,1) with drift	(0,1,0)
	AIC	38.78	**32.55**	34.51	34.53	36.29	43.6
SVD	BIC	39.97	**32.95**	35.1	35.12	37.08	43.79
	AIC	37.12	**32.11**	34.07	34.09	35.93	44.82
WLS	BIC	38.67	**32.51**	34.66	34.68	36.72	45.02
	AIC	29.31	**23.64**	25.14	25.16	27.57	33.10
MLE	BIC	28.74	**24.04**	25.73	25.81	26.71	33.30

The estimation of ARIMA(0,1,0) with drift parameters are showed in Table 3.4:

Table 3.4 The parameters of ARIMA(0,1,0) with drift which were obtained by the three estimation methods (SVD, WLS, and MLE).

	Females		Males	
	$\hat{\theta}$	$\hat{\sigma}_\varepsilon$	$\hat{\theta}$	$\hat{\sigma}_\varepsilon$
SVD	-1.9685	0.5097	-2.3512	0.5754
WLS	-2.2038	0.4958	-2.1045	0.3909
MLE	-1.0831	0.292	-2.5297	0.4448

The forecasted $\hat{k}_{2012+s}$, s=1,2,...,10 are inserted into the formulas giving the force of mortality and provide

$$ln\ (\hat{m}_{x,2012+s}) = \hat{\alpha}_x + \hat{\beta}_x \hat{k}_{2012+s}. \tag{3.15}$$

We note that the life expectancy at x, e_x, is defined as follows: $e_x = \frac{T_x}{l_x}$ where T_x is the cumulative number of years lived by the cohort population in the age interval and all subsequent age intervals. The l_x value represents the number of living people at the beginning of the x age interval from a population of l_0 newborn babies. This is usually defined as $l_0 = 100{,}000$.

Firstly, we consider the problem of fitted life expectancy at birth. The data is a span from 2003 to 2012. The results appear in Table 3.5. It can be seen that, for females, the MSE of the SVD gives the smallest value. For males, the MSE of the WLS has the lowest value. This indicates that the SVD and the WLS are suitable methods for forecasting life expectancy for females and males respectively.

Table 3.5 Mean Square Error of the fitted life expectancy at birth. The bolded entries show the smallest value of MSE for each gender.

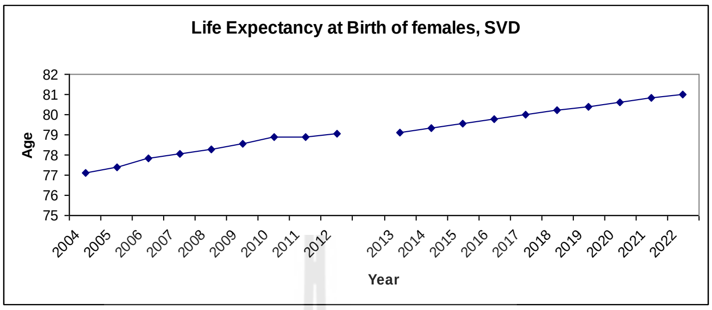
MSE	SVD	WLS	MLE
Females	**0.620004**	1.068034	1.549285
Males	0.815406	**0.771206**	1.264762

Next, we investigate the problem of forecasting life expectancy at birth. Our plan is to forecast from 2013 to 2022. As the results of our analysis (see Table 3.5), we shall use SVD and WLS to forecast the life expectancy at birth for females and males respectively. The forecasted results can be found in Table 3.6. We note further that, for females, the forecasted values of the life expectancy at birth by SVD increase from 79.12 years in 2013 to 81.01 year in 2022. For males, the forecasted values of the life expectancy at birth by WLS increase from 72.66 years in 2013 to 74.53 year in 2022.

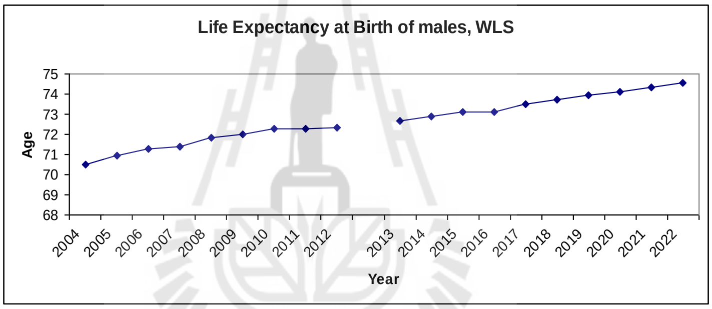
Table 3.6 The forecasted values of life expectancy at birth, for females by SVD and males by WLS respectively, 2013-2022.

Year	Females SVD	Males WLS	Year	Females SVD	Males WLS
2013	79.12	72.66	2018	80.19	73.73
2014	79.34	72.88	2019	80.4	73.93
2015	79.56	73.1	2020	80.61	74.14
2016	79.77	73.1	2021	80.81	74.33
2017	79.98	73.52	2022	81.01	74.53

Figure 3.3 shows the fitted and forecasted life expectancy at birth for females by SVD and for males by WLS. It indicates that the life expectancy at birth tends to increase from the past in the future.



(a)



(b)

Figure 3.3 Fit (2004-2012) and forecast (2013-2022) of life expectancy at birth for females by SVD and males by WLS respectively. (a) Life expectancy at birth of females, SVD. (b) Life expectancy at birth of males, WLS.

3.6 Conclusion

This chapter compares the estimation method of the parameters of the Lee-Carter model for mortality data in Thailand. The parameters of the model were estimated by using the Singular Value Decomposition (SVD) methods, the Weighted Least Square (WLS), and the Maximum Likelihood Estimate (MLE) methods.

Estimation of the parameters α_x, β_x, and k_t using each of these three methods show the same trend for males. For females, the estimation of β_x and k_t by MLE is different from the others. Moreover, the general mortality index k_t is a time series which shows a decreasing trend. There are some variations in the estimates of the time-dependent mortality index k_t.

For the comparison of fitted mortality rate, the results of the MSE show that the SVD is the best fit of parameter estimation for females and WLS is the best fit for males. Hence, we can conclude from our data for Thailand that the SVD method is appropriate for the estimation of the parameters of the Lee-Carter model for females. For males, WLS is the appropriate method for the estimation of the parameters of the Lee-Carter model.

By using SVD method for forecasting, it can be seen from Table 3.6 that life expectancy at birth for females increases by 2.38 percent from 2013 to 2022. Analogously, for males, WLS method gives a 2.57 percent increase from 2013 to 2022.

CHAPTER IV

A MODIFIED LEE-CARTER MORTALITY MODEL FOR FORECASTING

In this chapter, we discuss our modified mortality model. The assumption of the death count is based on the Poisson distribution and the negative binomial distribution. The main difference between the assumption of the Poisson distribution and negative binomial distribution is a mean-variance restriction of the death count. Brounhn et al. (2003) proposed that the Poisson model should be applied to the Lee-Carter model. Delwarde et al. (2007) and Li et al. (2009) proposed the negative binomial model to account for the death count which is called overdispersion for the Lee-Carter model. Under the assumption that the death count variable has a negative binomial distribution, the age-independent dispersion parameter and age-dependent dispersion parameter are constructed to be applied to the estimation of the parameters in the mortality model. In this chapter, we wish to apply the dispersion parameter in the negative binomial distribution with the age-period-cohort model.

This chapter consists of 9 parts. In the first part, we discuss the Poisson setting. Secondly, we introduce the negative binomial setting. Parameter estimation of the Lee-Carter and the age-period-cohort model are proposed in the third part. In

Part which follows, we apply the mortality model and the estimation method to Thai population data. In the fifth part, the fitted mortality result of the negative binomial setting is shown. In the sixth part, we forecast Thai mortality rate and life expectancy. We also simulate the death count of Thai data and find the 95% confidence interval of mortality rate forecasting in section 7. Furthermore, we calculate the insurance premium in section 8. Finally, the conclusion is discussed.

4.1 Poisson setting

4.1.1 Model fitting

We assume the death count variable has a Poisson distribution. Brouhns et al. (2002) proposed the Lee-Carter model by using a Poisson setting for the death count. Renshaw and Harberman (2003) proposed Poisson error structure by modeling the death count variable as an independent Poisson response variable

$$D_{x,t} \sim Poisson(ETR_{x,t} m_{x,t}). \tag{4.1}$$

We consider error structure and model fitting (Pitacco et al., 2009), which follows the generalized linear model. Since we model the death count as a Poisson error and then the first and second moment of the response $Y_{x,t}$, where $Y_{x,t} = D_{x,t}$,

$$E(Y_{x,t}) = E(D_{x,t}) = ETR_{x,t} m_{x,t}, \text{ and} \tag{4.2}$$

$$Var(Y_{x,t}) = Var(D_{x,t}) = \phi \frac{V(E(D_{x,t}))}{\omega_{x,t}} = \phi E(D_{x,t}) \tag{4.3}$$

with a scale parameter ϕ=1, $V\left(E(D_{x,t})\right) = E(D_{x,t})$, and $\omega_{x,t} = 1$ weight to indicate omitted data cells and $\omega_{x,t} = 0$ weight to indicate empty data cells, we assume to each mortality model by minimizing the Poisson deviance (Renshaw and Harberman

(2003a, 2003b, 2006)) with a standard generalized modeling. The iterative fitting is optimized by the Poisson maximum likelihood by considering the associated deviance (Renshaw and Harberman, 2003a) :

$$D(d_{x,t}, \hat{d}_{x,t}) = \sum_{x,t} dev(d_{x,t}, \hat{d}_{x,t})$$

$$= \sum_{x,t} 2\omega_{x,t} \left\{ d_{x,t} ln\left(\frac{d_{x,t}}{\hat{d}_{x,t}}\right) - (d_{x,t} - \hat{d}_{x,t}) \right\} \quad (4.4)$$

with weight $\omega_{x,t} = \begin{cases} 1, ETR_{x,t} > 0 \\ 0, ETR_{x,t} = 0 \end{cases}$, where $\hat{d}_{x,t} = ETR_{x,t}\hat{m}_{x,t}$.

4.1.2 Parameter estimation

To update the parameter, we used the iterative fitting procedure proposed by Goodman (1979) (Brouhns et al., 2002). In the iterative procedure, we can stop the iteration when the value of the deviance function is only little increasing.

4.1.2.1 The Lee-Carter model

The parameter updating is shown in the flowing steps, when i=1,2,3,… :

$$\text{Update } (\alpha_x): \begin{cases} \hat{\alpha}_x^{(i)} + \frac{\sum_t \omega_{x,t}(d_{x,t} - \hat{d}_{x,t}^{(i)})}{\sum_t \omega_{x,t}\hat{d}_{x,t}^{(i)}}, \\ \hat{\beta}_x^{(i+1)} = \hat{\beta}_x^{(i)}, \\ \hat{k}_t^{(i+1)} = \hat{k}_t^{(i)}. \end{cases}$$

$$\text{Update } (\beta_x): \begin{cases} \hat{\beta}_x^{(i+2)} = \hat{\beta}_x^{(i+1)} + \frac{\sum_t \omega_{x,t}(d_{x,t} - \hat{d}_{x,t}^{(i+1)})\hat{k}_t^{(i+1)}}{\sum_t \omega_{x,t}\hat{d}_{x,t}^{(i+1)}(\hat{k}_t^{(i+1)})^2}, \\ \hat{\alpha}_x^{(i+2)} = \hat{\alpha}_x^{(i+1)}, \\ \hat{k}_t^{(i+2)} = \hat{k}_t^{(i+1)}. \end{cases} \quad (4.5)$$

$$\text{Update}(k_t): \begin{cases} \hat{k}_t^{(i+3)} = \hat{k}_t^{(i+2)} + \frac{\sum_x \omega_{x,t}(d_{x,t} - \hat{d}_{x,t}^{(i+2)})\hat{\beta}_x^{(i+2)}}{\sum_x \omega_{x,t}\hat{d}_{x,t}^{(i+2)}\hat{\beta}_t^2}, \\ \hat{\alpha}_x^{(i+3)} = \hat{\alpha}_x^{(i+2)}, \\ \hat{\beta}_x^{(i+3)} = \hat{\beta}_x^{(i+2)}. \end{cases}$$

with the identical constraint $\sum_t k_t = 0$ and $\sum_x \beta_x = 1$.

4.1.2.2 The age-period-cohort model

The updating of the parameters at step i=1,2,3,… can be shown as follows:

$$\text{Update }(\alpha_x):\begin{cases}\hat{\alpha}_x^{(i+1)} = \hat{\alpha}_x^{(i)} + \frac{\sum_t \omega_{x,t}(d_{x,t}-\hat{d}_{x,t}^{(i)})}{\sum_t \omega_{x,t}\hat{d}_{x,t}},\\ (\hat{\beta}_x^{(0)})^{(i+1)} = (\hat{\beta}_x^{(0)})^{(i)},\\ \hat{k}_t^{(i+1)} = \hat{k}_t^{(i)},\\ (\hat{\beta}_x^{(1)})^{(i+1)} = (\hat{\beta}_x^{(1)})^{(i)},\\ \hat{\iota}_{t-x}^{(i+1)} = \hat{\iota}_{t-x}^{(i)}.\end{cases}$$

$$\text{Update}\left(\beta_x^{(0)}\right):\begin{cases}(\hat{\beta}_x^{(0)})^{(i+2)} = (\hat{\beta}_x^{(0)})^{(i+1)} + \frac{\sum_t \omega_{x,t}(d_{x,t}-\hat{d}_{x,t}^{(i+1)})\hat{k}_t^{(i+1)}}{\sum_t \omega_{x,t}\hat{d}_{x,t}^{(i+1)}(\hat{k}_t^{(i+1)})^2},\\ \hat{\alpha}_x^{(i+2)} = \hat{\alpha}_x^{(i+1)},\\ \hat{k}_t^{(i+2)} = \hat{k}_t^{(i+1)},\\ (\hat{\beta}_x^{(1)})^{(i+2)} = (\hat{\beta}_x^{(1)})^{(i+1)},\\ \hat{\iota}_{t-x}^{(i+2)} = \hat{\iota}_{t-x}^{(i+1)}.\end{cases}$$

$$\text{Update }(k_t):\begin{cases}\hat{k}_t^{(i+3)} = \hat{k}_t^{(i+2)} + \frac{\sum_x \omega_{x,t}(d_{x,t}-\hat{d}_{x,t}^{(i+2)})(\hat{\beta}_x^{(0)})^{(i+2)}}{\sum_x \omega_{x,t}\hat{d}_{x,t}^{(i+2)}((\hat{\beta}_x^{(0)})^{(i+2)})^2}\\ \hat{\alpha}_x^{(i+3)} = \hat{\alpha}_x^{(i+2)},\\ (\hat{\beta}_x^{(0)})^{(i+3)} = (\hat{\beta}_x^{(0)})^{(i+2)},\\ (\hat{\beta}_x^{(1)})^{(i+3)} = (\hat{\beta}_x^{(1)})^{(i+2)},\\ \hat{\iota}_{t-x}^{(i+3)} = \hat{\iota}_{t-x}^{(i+2)}.\end{cases} \tag{4.6}$$

$$\text{Update}\left(\beta_x^{(1)}\right):\begin{cases}(\hat{\beta}_x^{(1)})^{(i+4)} = (\hat{\beta}_x^{(1)})^{(i+3)} + \frac{\sum_t \omega_{x,t}(d_{x,t}-\hat{d}_{x,t}^{(i+3)})\hat{\iota}_{t-x}^{(i+3)}}{\sum_t \omega_{x,t}\hat{d}_{x,t}^{(i+3)}(\hat{\iota}_{t-x}^{(i+3)})^2},\\ \hat{\alpha}_x^{(i+4)} = \hat{\alpha}_x^{(i+3)},\\ (\hat{\beta}_x^{(0)})^{(i+4)} = (\hat{\beta}_x^{(0)})^{(i+3)},\\ \hat{k}_t^{(i+4)} = \hat{k}_t^{(i+3)},\\ \hat{\iota}_{t-x}^{(i+4)} = \hat{\iota}_{t-x}^{(i+3)}.\end{cases}$$

$$\text{Update}(\iota_{t-x}):\begin{cases}\hat{\iota}_{t-x}^{(i+5)}=\hat{\iota}_{t-x}^{(i+4)}+\frac{\sum_{x,t}\omega_{x,t}(d_{x,t}-\hat{d}_{x,t}^{(i+4)})(\hat{\beta}_x^{(1)})^{(i+4)}}{\sum_{x,t}\omega_{x,t}\hat{d}_{x,t}^{(i+4)}((\hat{\beta}_x^{(1)})^{(i+4)})^2},\\ \hat{\alpha}_x^{(i+5)}=\hat{\alpha}_x^{(i+4)},\\ (\hat{\beta}_x^{(0)})^{(i+5)}=(\hat{\beta}_x^{(0)})^{(i+4)},\\ \hat{k}_t^{(i+5)}=\hat{k}_t^{(i+4)},\\ (\hat{\beta}_x^{(1)})^{(i+5)}=(\hat{\beta}_x^{(1)})^{(i+4)}.\end{cases}$$

with the identical constraint $\sum_x \beta_x^{(0)} = \sum_x \beta_x^{(1)} = 1$, $\sum_{x,t} \iota_{t-x}$ and $\sum_t k_t = 0$ (Cairns et al., 2009).

4.2 Introduction of the negative binomial setting

4.2.1 Age-dependent dispersion parameter

The death count Poisson setting assumes homogeneity in the model of each age x at time t. Each age-period cell (age x at time t) may have different backgrounds for the death count, for example, health care, occupation, education, culture, environment, family status etc. We follow Kan (2012) to consider heterogeneity problem in mortality model. Li et al. (2009) consider heterogeneity into the model. Each age-period cell is segregated into N_x cluster which implies that each i^{th} cluster. It will have exposure-to-risk $\frac{ETR_{x,t}}{N_x}$ and death count $D_{x,t}^{(i)}$, where $i = 1,2,\dots,N_x$ and N_x is assumed that non-random. For clusters $\neq j$, the number of death $D_{x,t}^{(i)}$ and $D_{x,t}^{(j)}$ are assumed to be independent and the total number of deaths is $D_{x,t}$, which $D_{x,t} = \sum_{i=1}^{N_x} D_{x,t}$. With the i^{th} cluster $D_{x,t}^{(i)}$, the death count is assumed to be a Poisson-gamma model,

$$[D_{x,t}^{(i)}|z_x^{(i)}]\sim Poisson\left(ETR_{x,t}^{(i)}m_{x,t}z_x^{(i)}\right), \tag{4.7}$$

where $z_{x,t}^{(i)}$ is a gamma random variable accounting for the heterogeneity and an experience factor, then assigning $m_{x,t} z_x^{(i)}$ to cluster i.

The $z_x^{(i)}$ are assumed as having a gamma distribution,

$$z_x^{(i)} \sim Gamma(\gamma_x^{-1}, \gamma_x^{-1}) \tag{4.8}$$

where $\gamma_x > 0$. $z_x^{(i)}$ acts as the age-dependent experience factor.

We follow the assumption of the Gamma distribution with $E\left[z_x^{(i)}\right] = 1$ and $Var\left[z_x^{(i)}\right] = \gamma_x$.

The moment generating function for $z_x^{(i)}$ can be expressed as

$$M_{z_x(i)}(t) = \left(\frac{\gamma_x^{-1}}{\gamma_x^{-1} - t}\right)^{\gamma_x^{-1}}. \tag{4.9}$$

It can be shown that $E\left[D_{x,t}^{(i)}\right] = z_x^{(i)} \frac{ETR_{x,t}}{N_x} m_{x,t}$.

Consider $D_{x,t}^{(i)} | z_x^{(i)} \sim Poisson(z_x^{(i)} \frac{ETR_{x,t}}{N_x} m_{x,t})$

$$= Poisson(z_x^{(i)} e_{x,t}^{(i)} m_{x,t}) \text{ where } e_{x,t}^{(i)} = \frac{ETR_{x,t}}{N_x}.$$

To summarize the death count over the cluster, the independent Poisson distribution is considered. Then, the number of deaths has the following distribution:

$$D_{x,t} | z_x^{(1)}, z_x^{(2)}, \dots, z_x^{(N_x)} = \sum_{i=1}^{N_x} D_{x,t}^{(i)} | z_x^{(i)} \sim Poisson\left(\sum_{i=1}^{N_x} z_x^{(i)} e_{x,t}^{(i)} m_{x,t}\right)$$

$$= Poisson(\sum_{i=1}^{N_x} z_x^{(i)} \frac{ETR_{x,t}}{N_x} m_{x,t})$$

$$= Poisson(ETR_{x,t} m_{x,t} \sum_{i=1}^{N_x} \frac{z_x^{(i)}}{N_x})$$

$$= Poisson\left(\bar{z}_x ETR_{x,t} m_{x,t}\right). \tag{4.10}$$

Since the distribution of $z_x^{(i)}$ is

$$z_x^{(i)} \sim Gamma(\gamma_x^{-1}, \gamma_x^{-1}),$$

the moment generating function of the average of $z_x^{(i)}$, $\bar{z}_x = \sum_{i=1}^{N_x} \frac{z_x^{(i)}}{N_x}$, can be expressed as

$$M_{\bar{z}_x(i)}(t) = \left(\frac{N_x\gamma_x^{-1}}{N_x\gamma_x^{-1}-t}\right)^{N_x\gamma_x^{-1}} \quad (4.11)$$

Then $$\bar{z}_x \sim Gamma(N_x\gamma_x^{-1}, N_x\gamma_x^{-1})$$

$$= Gamma(\bar{\gamma}_x^{-1}, \bar{\gamma}_x^{-1}). \quad (4.12)$$

Consider the distribution of $\bar{z}_x ETR_{x,t} m_{x,t}$, the moment generating function is

$$M_{\bar{z}_x ETR_{x,t} m_{x,t}}(t) = \left(\frac{(ETR_{x,t} m_{x,t}\bar{\gamma}_x)^{-1}}{(ETR_{x,t} m_{x,t}\bar{\gamma}_x)^{-1}-t}\right)^{\bar{\gamma}_x^{-1}}. \quad (4.13)$$

Thus,

$$\bar{z}_x ETR_{x,t} m_{x,t} \sim Gamma\left(\bar{\gamma}_x^{-1}, \left(ETR_{x,t} m_{x,t}\bar{\gamma}_x\right)^{-1}\right). \quad (4.14)$$

Thus, a mortality Poisson-gamma model can be considered to have conditional distribution of $D_{x,t}$ in the following

$$D_{x,t}|\bar{z}_x \sim Poisson(\bar{z}_x ETR_{x,t} m_{x,t}) \quad (4.15)$$

and

$$\bar{z}_x ETR_{x,t} m_{x,t} \sim Gamma\left(\bar{\gamma}_x^{-1}, \left(ETR_{x,t} m_{x,t}\bar{\gamma}_x\right)^{-1}\right). \quad (4.16)$$

Therefore, we can conclude that the unconditional distribution of $D_{x,t}$ is the negative binomial underlying the age-dependent dispersion parameter, namely

$$D_{x,t} \sim NBin\left(\bar{\gamma}_x^{-1}, \frac{\left(ETR_{x,t} m_{x,t}\bar{\gamma}_x\right)^{-1}}{1+\left(ETR_{x,t} m_{x,t}\bar{\gamma}_x\right)^{-1}}\right)$$

$$= NBin\left(\bar{\gamma}_x^{-1}, \frac{1}{ETR_{x,t} m_{x,t}\bar{\gamma}_x+1}\right). \quad (4.17)$$

The unconditional mean and variance are

$$E\left[D_{x,t}\right] = ETR_{x,t} m_{x,t},$$

$$Var[D_{x,t}] = ETR_{x,t}m_{x,t} + \bar{\gamma}_x(ETR_{x,t}m_{x,t})^2. \quad (4.18)$$

4.2.2 Age-independent dispersion parameter

The previous sections showed that the number of deaths have the negative binomial distribution, after introducing the Poisson-gamma model and the age-dependent dispersion parameter by Li et al. (2009). In this section, we consider the age-independent dispersion parameter proposed by Delwarde et al. (2007) applied to the Lee-Carter model. We then wish to apply this dispersion parameter to the age-period cohort model.

We model the number of deaths according to the Poisson-gamma model

$$[D_{x,t}|\bar{z}] \sim Poisson(\bar{z}ETR_{x,t}m_{x,t}), \quad (4.19)$$

where the gamma distributed random variable $\bar{z}$ acts as the age-independent experience factor :

$$\bar{z} \sim Gamma(\bar{\gamma}^{-1}, \bar{\gamma}^{-1}). \quad (4.20)$$

Therefore, we can obtain the unconditional distribution of $D_{x,t}$,

$$D_{x,t} \sim NBin(\bar{\gamma}^{-1}, \frac{(ETR_{x,t}m_{x,t}\bar{\gamma})^{-1}}{1 + (ETR_{x,t}m_{x,t}\bar{\gamma})^{-1}})$$

$$= NBin\left(\bar{\gamma}^{-1}, \frac{1}{ETR_{x,t}m_{x,t}\bar{\gamma}+1}\right). \quad (4.21)$$

The unconditional mean can be shown as

$$E[D_{x,t}] = ETR_{x,t}m_{x,t},$$

and the variance

$$Var[D_{x,t}] = ETR_{x,t}m_{x,t} + \bar{\gamma}(ETR_{x,t}m_{x,t})^2. \quad (4.22)$$

4.2.3 Model fitting

The age-dependent dispersion parameter

We assume that the death count variable $D_{x,t}$ has a negative binomial distribution:

$$D_{x,t} \sim NBin\left(\bar{\gamma}_x^{-1}, \frac{1}{ETR_{x,t} m_{x,t} \bar{\gamma}_x + 1}\right). \tag{4.23}$$

The age-independent dispersion parameter

The death counts variable $D_{x,t}$ is assumed to be a negative binomial distribution:

$$D_{x,t} \sim NBin\left(\bar{\gamma}^{-1}, \frac{1}{ETR_{x,t} m_{x,t} \bar{\gamma} + 1}\right). \tag{4.24}$$

Equations (4.2) and (4.3) can be replaced by

$$E(D_{x,t}) = ETR_{x,t} m_{x,t}, \text{ and}$$

$$Var(D_{x,t}) = \phi \frac{V(E(D_{x,t}))}{\omega_{x,t}}, \tag{4.25}$$

with scale parameter and a variance function $V(D_{x,t}) = E[D_{x,t}] + \bar{\gamma}_x E[D_{x,t}]^2$ and $V(D_{x,t}) = E[D_{x,t}] + \bar{\gamma} E[D_{x,t}]^2$ for the age-dependent dispersion parameter and the age-independent dispersion parameter assumption respectively. In standard generalized linear modeling (GLM), fitting the parameters by maximizing the likelihood corresponds to minimizing to the deviance (Madsen and Thyegod, 2010). Thus, we fit the model by minimizing the negative binomial deviance:

4.2.3.1 Fitting mortality model for age-dependent dispersion parameter model

We fit the mortality model by minimizing the negative binomial deviance:

$$D\left(d_{x,t},\hat{d}_{x,t}\right)=\sum_{x,t} dev\left(d_{x,t},\hat{d}_{x,t}\right) \tag{4.26}$$

$$=\sum_{x,t} 2\omega_{x,t}\left\{d_{x,t}ln\left(\frac{d_{x,t}}{\hat{d}_{x,t}}\right)-(d_{x,t}+\frac{1}{\overline{\gamma}_x})\ln(\frac{1+\overline{\gamma}_x d_{x,t}}{1+\overline{\gamma}_x \hat{d}_{x,t}})\right\}$$

with weight $\omega_{x,t}=\begin{cases}1, ETR_{x,t}>0\\0, ETR_{x,t}=0\end{cases}$, where $\hat{d}_{x,t}=ETR_{x,t}\widehat{m}_{x,t}$.

4.2.3.2 Fitting mortality model for age-independent dispersion parameter model

We fit the mortality model by minimizing the negative binomial deviance:

$$D\left(d_{x,t},\hat{d}_{x,t}\right)=\sum_{x,t} dev\left(d_{x,t},\hat{d}_{x,t}\right) \tag{4.27}$$

$$=\sum_{x,t} 2\omega_{x,t}\left\{d_{x,t}ln\left(\frac{d_{x,t}}{\hat{d}_{x,t}}\right)-(d_{x,t}+\frac{1}{\overline{\gamma}})\ln(\frac{1+\overline{\gamma} d_{x,t}}{1+\overline{\gamma} \hat{d}_{x,t}})\right\},$$

with weight $\omega_{x,t}=\begin{cases}1, ETR_{x,t}>0\\0, ETR_{x,t}=0\end{cases}$, where $\hat{d}_{x,t}=ETR_{x,t}\widehat{m}_{x,t}$.

4.3 Parameter estimation

4.3.1 The parameter estimation for the model underlying the age-dependent dispersion parameter

4.3.1.1 The Lee-Carter model

When updating of the parameter, we used the iterative fitting procedure proposed by the Goodman algorithm (Brounhns et al., 2002). The parameter updating is shown in the following:

$$\text{Update}(\alpha_x):\begin{cases}\hat{\alpha}_x^{(i+1)} = \hat{\alpha}_x^{(i)} - \dfrac{\sum_t \left\{ \left(d_{x,t} + \frac{1}{\hat{\gamma}_x^{(i)}} \right) \left(\frac{\hat{d}_{x,t}^{(i)}}{\hat{d}_{x,t}^{(i)} + \frac{1}{\hat{\gamma}_x^{(i)}}} \right) - d_{x,t} \right\}}{\sum_t \left\{ \left(d_{x,t} + \frac{1}{\hat{\gamma}_x^{(i)}} \right) \frac{\frac{1}{\hat{\gamma}_x^{(i)}} \hat{d}_{x,t}^{(i)}}{\left(\hat{d}_{x,t}^{(i)} + \frac{1}{\hat{\gamma}_x^{(i)}} \right)^2} \right\}}, \\ \hat{\beta}_x^{(i+1)} = \hat{\beta}_x^{(i)}, \\ \hat{k}_t^{(i+1)} = \hat{k}_t^{(i)}, \\ \hat{\gamma}_x^{(i+1)} = \hat{\gamma}_x^{(i)}. \end{cases}$$

$$\text{Update}(\beta_x):\begin{cases}\hat{\beta}_x^{(i+2)} = \hat{\beta}_x^{(i+1)} - \dfrac{\sum_t \left\{ \left(d_{x,t} + \frac{1}{\hat{\gamma}_x^{(i+1)}} \right) \left(\frac{\hat{k}_t^{(i+1)} \hat{d}_{x,t}^{(i+1)}}{\hat{d}_{x,t}^{(i+1)} + \frac{1}{\hat{\gamma}_x^{(i+1)}}} \right) - \hat{k}_t^{(i+1)} d_{x,t} \right\}}{\sum_t \left\{ \left(d_{x,t} + \frac{1}{\hat{\gamma}_x^{(i+1)}} \right) \frac{(\hat{k}_t^{(i+1)})^2 \frac{1}{\hat{\gamma}_x^{(i+1)}} \hat{d}_{x,t}^{(i+1)}}{\left(\hat{d}_{x,t}^{(i+1)} + \frac{1}{\hat{\gamma}_x^{(i+1)}} \right)^2} \right\}}, \\ \hat{\alpha}_x^{(i+2)} = \hat{\alpha}_x^{(i+1)}, \\ \hat{k}_t^{(i+2)} = \hat{k}_t^{(i+1)}, \\ \hat{\gamma}_x^{(i+2)} = \hat{\gamma}_x^{(i+1)}. \end{cases} \quad (4.28)$$

$$\text{Update}(k_t):\begin{cases}\hat{k}_t^{(i+3)} = \hat{k}_t^{(i+2)} - \dfrac{\sum_x \left\{ \left(d_{x,t} + \frac{1}{\hat{\gamma}_x^{(i+2)}} \right) \left(\frac{\hat{\beta}_x^{(i+2)} \hat{d}_{x,t}^{(i+2)}}{\hat{d}_{x,t}^{(i+2)} + \frac{1}{\hat{\gamma}_x^{(i+2)}}} \right) - \hat{\beta}_x^{(i+2)} d_{x,t} \right\}}{\sum_x \left\{ \left(d_{x,t} + \frac{1}{\hat{\gamma}_x^{(i+2)}} \right) \frac{(\hat{\beta}_x^{(i+2)})^2 \frac{1}{\hat{\gamma}_x^{(i+2)}} \hat{d}_{x,t}^{(i+2)}}{\left(\hat{d}_{x,t}^{(i+2)} + \frac{1}{\hat{\gamma}_x^{(i+2)}} \right)^2} \right\}}, \\ \hat{\alpha}_x^{(i+3)} = \hat{\alpha}_x^{(i+2)}, \\ \hat{\beta}_x^{(i+3)} = \hat{\beta}_x^{(i+2)}, \\ \hat{\gamma}_x^{(i+3)} = \hat{\gamma}_x^{(i+2)}, \end{cases}$$

$$
\text{Update}(\bar{\gamma}_x): \begin{cases} \hat{\bar{\gamma}}_x^{(i+4)} = \hat{\bar{\gamma}}_x^{(i+3)} - \dfrac{\sum_t \left\{ \dfrac{\left(d_{x,t} + \frac{1}{\hat{\bar{\gamma}}_x^{(i+3)}}\right)}{(\hat{\bar{\gamma}}_x^{(i+3)})^2 \left(\hat{d}_{x,t}^{(i+3)} + \frac{1}{\hat{\bar{\gamma}}_x^{(i+3)}}\right)} \right\}}{\sum_t \left\{ \dfrac{\left(\hat{d}_{x,t}^{(i+3)} + \frac{1}{\hat{\bar{\gamma}}_x^{(i+3)}}\right) - \hat{\bar{\gamma}}_x^{(i+2)} \left(d_{x,t} + \frac{1}{\hat{\bar{\gamma}}_x^{(i+3)}}\right)\left(2\hat{d}_{x,t}^{(i+3)} + \frac{1}{\hat{\bar{\gamma}}_x^{(i+3)}}\right)}{(\hat{\bar{\gamma}}_x^{(i+3)})^2 \left(\hat{d}_{x,t}^{(i+3)} + \frac{1}{\hat{\bar{\gamma}}_x^{(i+3)}}\right)^2} \right\}}, \\ \hat{\alpha}_x^{(i+4)} = \hat{\alpha}_x^{(i+3)}, \\ \hat{\beta}_x^{(i+4)} = \hat{\beta}_x^{(i+3)}, \\ \hat{k}_t^{(i+4)} = \hat{k}_t^{(i+3)}. \end{cases}
$$

4.3.1.2 The age-period cohort model

The required parameters $\hat{\alpha}_x$, $\hat{\beta}_x^{(0)}$, $\hat{k}_t$, $\hat{\beta}_x^{(1)}$, $\hat{\iota}_{t-x}$ and $\hat{\bar{\gamma}}_x$ will be obtained numerically according to the following algorithm:

$$
\text{Update}(\alpha_x): \begin{cases} \hat{\alpha}_x^{(i+1)} = \hat{\alpha}_x^{(i)} - \dfrac{\sum_t \left\{ \left(d_{x,t} + \frac{1}{\hat{\bar{\gamma}}_x^{(i)}}\right)\left(\dfrac{\hat{d}_{x,t}^{(i)}}{\hat{d}_{x,t} + \frac{1}{\hat{\bar{\gamma}}_x^{(i)}}}\right) - \hat{d}_{x,t}^{(i)} \right\}}{\sum_t \left\{ \left(d_{x,t} + \frac{1}{\hat{\bar{\gamma}}_x^{(i)}}\right) \dfrac{\frac{1}{\hat{\bar{\gamma}}_x^{(i)}} \hat{d}_{x,t}^{(i)}}{\left(\hat{d}_{x,t}^{(i)} + \frac{1}{\hat{\bar{\gamma}}_x^{(i)}}\right)^2} \right\}}, \\ (\hat{\beta}_x^{(0)})^{(i+1)} = (\hat{\beta}_x^{(0)})^{(i)}, \\ \hat{k}_t^{(i+1)} = \hat{k}_t^{(i)}, \\ (\hat{\beta}_x^{(1)})^{(i+1)} = (\hat{\beta}_x^{(1)})^{(i)}, \\ \hat{\iota}_{t-x}^{(i+1)} = \hat{\iota}_{t-x}^{(i)}, \\ \hat{\bar{\gamma}}_x^{(i+1)} = \hat{\bar{\gamma}}_x^{(i)}. \end{cases}
$$

$$
\text{Update}\left(\beta_x^{(0)}\right):\begin{cases}
(\hat{\beta}_x^{(0)})^{(i+2)} = (\hat{\beta}_x^{(0)})^{(i+1)} - \dfrac{\sum_t\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+1)}}\right)\left(\frac{\hat{k}_t^{(i+1)}\hat{d}_{x,t}^{(i+1)}}{\hat{d}_{x,t}^{(i+1)}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+1)}}}\right)-\hat{k}_t^{(i+1)}d_{x,t}\right\}}{\sum_t\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+1)}}\right)\frac{(\hat{k}_t^{(i+1)})^2\frac{1}{\hat{\gamma}_x}\hat{d}_{x,t}^{(i+1)}}{\left(\hat{d}_{x,t}^{(i+1)}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+1)}}\right)^2}\right\}}, \\
\hat{\alpha}_x^{(i+2)} = \hat{\alpha}_x^{(i+1)}, \\
\hat{k}_t^{(i+2)} = \hat{k}_t^{(i+1)}, \\
(\hat{\beta}_x^{(1)})^{(i+2)} = (\hat{\beta}_x^{(1)})^{(i+1)}, \\
\hat{\iota}_{t-x}^{(i+2)} = \hat{\iota}_{t-x}^{(i+1)}, \\
\hat{\hat{\gamma}}_x^{(i+2)} = \hat{\hat{\gamma}}_x^{(i+1)}.
\end{cases}
$$

$$
\text{Update}(k_t):\begin{cases}
\left\{\hat{k}_t^{(i+3)} = \hat{k}_t^{(i+2)} - \dfrac{\sum_x\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+2)}}\right)\left(\frac{(\hat{\beta}_x^{(0)})^{(i+2)}\hat{d}_{x,t}^{(i+2)}}{\hat{d}_{x,t}^{(i+2)}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+2)}}}\right)-(\hat{\beta}_x^{(0)})^{(i+2)}d_{x,t}\right\}}{\sum_x\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+2)}}\right)\frac{((\hat{\beta}_x^{(0)})^{(i+2)})^2\frac{1}{\hat{\hat{\gamma}}_x^{(i+2)}}\hat{d}_{x,t}^{(i+2)}}{\left(\hat{d}_{x,t}^{(i+2)}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+2)}}\right)^2}\right\}}\right., \\
\hat{\alpha}_x^{(i+3)} = \hat{\alpha}_x^{(i+2)}, \\
(\hat{\beta}_x^{(0)})^{(i+3)} = (\hat{\beta}_x^{(0)})^{(i+2)}, \\
(\hat{\beta}_x^{(1)})^{(i+3)} = (\hat{\beta}_x^{(1)})^{(i+2)}, \\
\hat{\iota}_{t-x}^{(i+3)} = \hat{\iota}_{t-x}^{(i+2)}, \\
\hat{\hat{\gamma}}_x^{(i+3)} = \hat{\hat{\gamma}}_x^{(i+2)}.
\end{cases} \qquad (4.29)
$$

$$
\text{Update}\left(\beta_x^{(1)}\right):\begin{cases}
(\hat{\beta}_x^{(1)})^{(i+4)} = (\hat{\beta}_x^{(1)})^{(i+3)} - \dfrac{\sum_t\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+3)}}\right)\left(\frac{\hat{\iota}_{t-x}^{(i+3)}\hat{d}_{x,t}^{(i+3)}}{\hat{d}_{x,t}^{(i+3)}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+3)}}}\right)-\hat{\iota}_{t-x}^{(i+3)}d_{x,t}\right\}}{\sum_t\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+3)}}\right)\frac{(\hat{\iota}_{t-x}^{(i+3)})^2\frac{1}{\hat{\hat{\gamma}}_x^{(i+3)}}\hat{d}_{x,t}^{(i+3)}}{\left(\hat{d}_{x,t}^{(i+3)}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+3)}}\right)^2}\right\}}, \\
\hat{\alpha}_x^{(i+4)} = \hat{\alpha}_x^{(i+3)}, \\
(\hat{\beta}_x^{(0)})^{(i+4)} = (\hat{\beta}_x^{(0)})^{(i+3)}, \\
\hat{k}_t^{(i+4)} = \hat{k}_t^{(i+3)}, \\
\hat{\iota}_{t-x}^{(i+4)} = \hat{\iota}_{t-x}^{(i+3)}, \\
\hat{\hat{\gamma}}_x^{(i+4)} = \hat{\hat{\gamma}}_x^{(i+3)}.
\end{cases}
$$

$$
\text{Update}(\iota_{t-x}):\begin{cases}
\hat{\iota}_{t-x}^{(i+5)} = \hat{\iota}_{t-x}^{(i+4)} - \dfrac{\sum_{x,t}\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+4)}}\right)\left(\frac{(\hat{\beta}_x^{(1)})^{(i+4)}\hat{d}_{x,t}^{(i+4)}}{\hat{d}_{x,t}^{(i+4)}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+4)}}}\right)-(\hat{\beta}_x^{(1)})^{(i+4)}d_{x,t}\right\}}{\sum_{x,t}\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+4)}}\right)\frac{((\hat{\beta}_x^{(1)})^{(i+4)})^2\frac{1}{\hat{\hat{\gamma}}_x^{(i+4)}}\hat{d}_{x,t}^{(i+4)}}{\left(\hat{d}_{x,t}^{(i+4)}+\frac{1}{\hat{\hat{\gamma}}_x^{(i+4)}}\right)^2}\right\}}, \\
\hat{\alpha}_x^{(i+5)} = \hat{\alpha}_x^{(i+4)}, \\
(\hat{\beta}_x^{(0)})^{(i+5)} = (\hat{\beta}_x^{(0)})^{(i+4)}, \\
\hat{k}_t^{(i+5)} = \hat{k}_t^{(i+4)}, \\
(\hat{\beta}_x^{(1)})^{(i+5)} = (\hat{\beta}_x^{(1)})^{(i+4)}, \\
\hat{\hat{\gamma}}_x^{(i+5)} = \hat{\hat{\gamma}}_x^{(i+4)}.
\end{cases}
$$

$$
\text{Update}(\bar{\gamma}_x):\begin{cases}
\hat{\hat{\gamma}}_x^{(i+6)} = \hat{\hat{\gamma}}_x^{(i+5)} - \dfrac{\sum_t \left\{ \dfrac{\left(d_{x,t} + \frac{1}{\hat{\hat{\gamma}}_x^{(i+5)}} \right)}{(\hat{\hat{\gamma}}_x^{(i+5)})^2 \left(\hat{d}_{x,t}^{(i+5)} + \frac{1}{\hat{\hat{\gamma}}_x^{(i+5)}} \right)} \right\}}{\sum_t \left\{ \dfrac{\left(\hat{d}_{x,t}^{(i+5)} + \frac{1}{\hat{\hat{\gamma}}_x^{(i+5)}} \right) - \hat{\hat{\gamma}}_x^{(i+5)} \left(d_{x,t} + \frac{1}{\hat{\hat{\gamma}}_x^{(i+5)}} \right) \left(2\hat{d}_{x,t}^{(i+5)} + \frac{1}{\hat{\hat{\gamma}}_x^{(i+5)}} \right)}{(\hat{\hat{\gamma}}_x^{(i+5)})^2 \left(\hat{d}_{x,t}^{(i+5)} + \frac{1}{\hat{\hat{\gamma}}_x^{(i+5)}} \right)^2} \right\}} \\
\hat{\alpha}_x^{(i+6)} = \hat{\alpha}_x^{(i+5)}, \\
(\hat{\beta}_x^{(0)})^{(i+6)} = (\hat{\beta}_x^{(0)})^{(i+5)}, \\
\hat{k}_t^{(i+6)} = \hat{k}_t^{(i+5)}, \\
(\hat{\beta}_x^{(1)})^{(i+6)} = (\hat{\beta}_x^{(1)})^{(i+5)}, \\
\hat{\iota}_{t-x}^{(i+6)} = \hat{\iota}_{t-x}^{(i+5)}.
\end{cases}
$$

4.3.2 The parameter estimation for the model underlying the age-independent dispersion parameter

4.3.2.1 The Lee-Carter model

The parameter updating is as follows:

$$
\text{Update}(\alpha_x):\begin{cases}
\hat{\alpha}_x^{(i)} - \dfrac{\sum_t \left\{ \left(d_{x,t} + \frac{1}{\hat{\hat{\gamma}}^{(i)}} \right) \left(\dfrac{\hat{d}_{x,t}^{(i)}}{\hat{d}_{x,t}^{(i)} + \frac{1}{\hat{\hat{\gamma}}^{(i)}}} \right) - d_{x,t} \right\}}{\sum_t \left\{ \left(d_{x,t} + \frac{1}{\hat{\hat{\gamma}}^{(i)}} \right) \dfrac{\frac{1}{\hat{\hat{\gamma}}^{(i)}} \hat{d}_{x,t}^{(i)}}{\left(\hat{d}_{x,t}^{(i)} + \frac{1}{\hat{\hat{\gamma}}^{(i)}} \right)^2} \right\}}, \\
\hat{\beta}_x^{(i+1)} = \hat{\beta}_x^{(i)}, \\
\hat{k}_t^{(i+1)} = \hat{k}_t^{(i)}, \\
\hat{\hat{\gamma}}^{(i+1)} = \hat{\hat{\gamma}}^{(i)}.
\end{cases}
$$

$$\text{Update}(\beta_x):\begin{cases} \hat{\beta}_x^{(i+1)} - \dfrac{\sum_t\left\{\left(d_{x,t}+\frac{1}{\hat{\bar{\gamma}}^{(i+1)}}\right)\left(\frac{\hat{k}_t^{(i+1)}\hat{d}_{x,t}^{(i+1)}}{\hat{d}_{x,t}^{(i+1)}+\frac{1}{\hat{\bar{\gamma}}^{(i+1)}}}\right)-\hat{k}_t^{(i+1)}d_{x,t}\right\}}{\sum_t\left\{\left(d_{x,t}+\frac{1}{\hat{\bar{\gamma}}^{(i+1)}}\right)\frac{(\hat{k}_t^{(i+1)})^2\frac{1}{\hat{\bar{\gamma}}^{(i+1)}}\hat{d}_{x,t}^{(i+1)}}{\left(\hat{d}_{x,t}^{(i+1)}+\frac{1}{\hat{\bar{\gamma}}^{(i+1)}}\right)^2}\right\}}, \\ \hat{\alpha}_x^{(i+2)} = \hat{\alpha}_x^{(i+1)}, \\ \hat{k}_t^{(i+2)} = \hat{k}_t^{(i+1)}, \\ \hat{\bar{\gamma}}^{(i+2)} = \hat{\bar{\gamma}}^{(i+1)}. \end{cases} \tag{4.30}$$

$$\text{Update}(k_t):\begin{cases} \hat{k}_t^{(i+2)} - \dfrac{\sum_x\left\{\left(d_{x,t}+\frac{1}{\hat{\bar{\gamma}}^{(i+2)}}\right)\left(\frac{\hat{\beta}_x^{(i+2)}\hat{d}_{x,t}^{(i+2)}}{\hat{d}_{x,t}^{(i+2)}+\frac{1}{\hat{\bar{\gamma}}^{(i+2)}}}\right)-\hat{\beta}_x^{(i+2)}d_{x,t}\right\}}{\sum_x\left\{\left(d_{x,t}+\frac{1}{\hat{\bar{\gamma}}^{(i+2)}}\right)\frac{(\hat{\beta}_x^{(i+2)})^2\frac{1}{\hat{\bar{\gamma}}^{(i+2)}}\hat{d}_{x,t}^{(i+2)}}{\left(\hat{d}_{x,t}^{(i+2)}+\frac{1}{\hat{\bar{\gamma}}^{(i+2)}}\right)^2}\right\}}, \\ \hat{\alpha}_x^{(i+3)} = \hat{\alpha}_x^{(i+2)}, \\ \hat{\beta}_x^{(i+3)} = \hat{\beta}_x^{(i+2)}, \\ \hat{\bar{\gamma}}^{(i+3)} = \hat{\bar{\gamma}}^{(i+2)}. \end{cases}$$

$$\text{Update}(\bar{\gamma}):\begin{cases} \hat{\bar{\gamma}}^{(i+3)} - \dfrac{\sum_t\left\{\frac{\left(d_{x,t}+\frac{1}{\hat{\bar{\gamma}}^{(i+3)}}\right)}{(\hat{\bar{\gamma}}^{(i+3)})^2\left(\hat{d}_{x,t}^{(i+3)}+\frac{1}{\hat{\bar{\gamma}}^{(i+3)}}\right)}\right\}}{\sum_{x,t}\left\{\frac{\left(\hat{d}_{x,t}^{(i+3)}+\frac{1}{\hat{\bar{\gamma}}^{(i+3)}}\right)-\hat{\bar{\gamma}}_x^{(i+2)}\left(d_{x,t}+\frac{1}{\hat{\bar{\gamma}}^{(i+3)}}\right)\left(2\hat{d}_{x,t}^{(i+3)}+\frac{1}{\hat{\bar{\gamma}}^{(i+3)}}\right)}{(\hat{\bar{\gamma}}^{(i+3)})^2\left(\hat{d}_{x,t}^{(i+3)}+\frac{1}{\hat{\bar{\gamma}}^{(i+3)}}\right)^2}\right\}}, \\ \hat{\alpha}_x^{(i+4)} = \hat{\alpha}_x^{(i+3)}, \\ \hat{\beta}_x^{(i+4)} = \hat{\beta}_x^{(i+3)}, \\ \hat{k}_t^{(i+4)} = \hat{k}_t^{(i+3)}. \end{cases}$$

4.3.2.2 The age-period-cohort model

The parameter updating shows the following:

$$
\text{Update}(\alpha_x):\begin{cases}
\hat{\alpha}_x^{(i+1)} = \hat{\alpha}_x^{(i)} - \dfrac{\sum_t\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}^{(i)}}\right)\left(\frac{\hat{d}_{x,t}^{(i)}}{\hat{d}_{x,t}+\frac{1}{\hat{\hat{\gamma}}^{(i)}}}\right)-\hat{d}_{x,t}^{(i)}\right\}}{\sum_t\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}^{(i)}}\right)\frac{\frac{1}{\hat{\hat{\gamma}}^{(i)}}\hat{d}_{x,t}^{(i)}}{\left(\hat{d}_{x,t}^{(i)}+\frac{1}{\hat{\hat{\gamma}}^{(i)}}\right)^2}\right\}}, \\
(\hat{\beta}_x^{(0)})^{(i+1)} = (\hat{\beta}_x^{(0)})^{(i)}, \\
\hat{k}_t^{(i+1)} = \hat{k}_t^{(i)}, \\
(\hat{\beta}_x^{(1)})^{(i+1)} = (\hat{\beta}_x^{(1)})^{(i)}, \\
\hat{\iota}_{t-x}^{(i+1)} = \hat{\iota}_{t-x}^{(i)}, \\
\hat{\hat{\gamma}}^{(i+1)} = \hat{\hat{\gamma}}^{(i)}.
\end{cases}
$$

$$
\text{Update}\left(\beta_x^{(0)}\right):\begin{cases}
(\hat{\beta}_x^{(0)})^{(i+2)} = (\hat{\beta}_x^{(0)})^{(i+1)} - \dfrac{\sum_t\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}^{(i+1)}}\right)\left(\frac{\hat{k}_t^{(i+1)}\hat{d}_{x,t}^{(i+1)}}{\hat{d}_{x,t}^{(i+1)}+\frac{1}{\hat{\hat{\gamma}}^{(i+1)}}}\right)-\hat{k}_t^{(i+1)}d_{x,t}\right\}}{\sum_t\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}^{(i+1)}}\right)\frac{(\hat{k}_t^{(i+1)})^2\frac{1}{\hat{\hat{\gamma}}^{(i+1)}}\hat{d}_{x,t}^{(i+1)}}{\left(\hat{d}_{x,t}^{(i+1)}+\frac{1}{\hat{\hat{\gamma}}^{(i+1)}}\right)^2}\right\}}, \\
\hat{\alpha}_x^{(i+2)} = \hat{\alpha}_x^{(i+1)}, \\
\hat{k}_t^{(i+2)} = \hat{k}_t^{(i+1)}, \\
(\hat{\beta}_x^{(1)})^{(i+2)} = (\hat{\beta}_x^{(1)})^{(i+1)}, \\
\hat{\iota}_{t-x}^{(i+2)} = \hat{\iota}_{t-x}^{(i+1)}, \\
\hat{\hat{\gamma}}^{(i+2)} = \hat{\hat{\gamma}}^{(i+1)}..
\end{cases}
$$

$$
\text{Update}(k_t):\begin{cases}
\hat{k}_t^{(i+3)} = \hat{k}_t^{(i+2)} - \dfrac{\sum_x\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}^{(i+2)}}\right)\left(\frac{(\hat{\beta}_x^{(0)})^{(i+2)}\hat{d}_{x,t}^{(i+2)}}{\hat{d}_{x,t}^{(i+2)}+\frac{1}{\hat{\hat{\gamma}}^{(i+2)}}}\right)-(\hat{\beta}_x^{(0)})^{(i+2)}d_{x,t}\right\}}{\sum_x\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}^{(i+2)}}\right)\frac{((\hat{\beta}_x^{(0)})^{(i+2)})^2\frac{1}{\hat{\hat{\gamma}}^{(i+2)}}\hat{d}_{x,t}^{(i+2)}}{\left(\hat{d}_{x,t}^{(i+2)}+\frac{1}{\hat{\hat{\gamma}}^{(i+2)}}\right)^2}\right\}}, \\
\hat{\alpha}_x^{(i+3)} = \hat{\alpha}_x^{(i+2)}, \\
(\hat{\beta}_x^{(0)})^{(i+3)} = (\hat{\beta}_x^{(0)})^{(i+2)}, \\
(\hat{\beta}_x^{(1)})^{(i+3)} = (\hat{\beta}_x^{(1)})^{(i+2)}, \\
\hat{\iota}_{t-x}^{(i+3)} = \hat{\iota}_{t-x}^{(i+2)}, \\
\hat{\hat{\gamma}}^{(i+3)} = \hat{\hat{\gamma}}^{(i+2)}.
\end{cases} \qquad (4.31)
$$

$$
\text{Update}\left(\beta_x^{(1)}\right):\begin{cases}
(\hat{\beta}_x^{(1)})^{(i+4)} = (\hat{\beta}_x^{(1)})^{(i+3)} - \dfrac{\sum_t\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}^{(i+3)}}\right)\left(\frac{\hat{\iota}_{t-x}^{(i+3)}\hat{d}_{x,t}^{(i+3)}}{\hat{d}_{x,t}^{(i+3)}+\frac{1}{\hat{\hat{\gamma}}^{(i+3)}}}\right)-\hat{\iota}_{t-x}^{(i+3)}d_{x,t}\right\}}{\sum_t\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}^{(i+3)}}\right)\frac{(\hat{\iota}_{t-x}^{(i+3)})^2\frac{1}{\hat{\hat{\gamma}}^{(i+3)}}\hat{d}_{x,t}^{(i+3)}}{\left(\hat{d}_{x,t}^{(i+3)}+\frac{1}{\hat{\hat{\gamma}}^{(i+3)}}\right)^2}\right\}}, \\
\hat{\alpha}_x^{(i+4)} = \hat{\alpha}_x^{(i+3)}, \\
(\hat{\beta}_x^{(0)})^{(i+4)} = (\hat{\beta}_x^{(0)})^{(i+3)}, \\
\hat{k}_t^{(i+4)} = \hat{k}_t^{(i+3)}, \\
\hat{\iota}_{t-x}^{(i+4)} = \hat{\iota}_{t-x}^{(i+3)}, \\
\hat{\hat{\gamma}}^{(i+4)} = \hat{\hat{\gamma}}^{(i+3)}.
\end{cases}
$$

$$
\text{Update}(\iota_{t-x}):\begin{cases}
\hat{\iota}_{t-x}^{(i+5)} = \hat{\iota}_{t-x}^{(i+4)} - \dfrac{\sum_{x,t}\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}^{(i+4)}}\right)\left(\frac{(\hat{\beta}_x^{(1)})^{(i+4)}\hat{d}_{x,t}^{(i+4)}}{\hat{d}_{x,t}^{(i+4)}+\frac{1}{\hat{\hat{\gamma}}^{(i+4)}}}\right)-(\hat{\beta}_x^{(1)})^{(i+4)}d_{x,t}\right\}}{\sum_{x,t}\left\{\left(d_{x,t}+\frac{1}{\hat{\hat{\gamma}}^{(i+4)}}\right)\frac{((\hat{\beta}_x^{(1)})^{(i+4)})^2\frac{1}{\hat{\hat{\gamma}}^{(i+4)}}\hat{d}_{x,t}^{(i+4)}}{\left(\hat{d}_{x,t}^{(i+4)}+\frac{1}{\hat{\hat{\gamma}}^{(i+4)}}\right)^2}\right\}}, \\
\hat{\alpha}_x^{(i+5)} = \hat{\alpha}_x^{(i+4)}, \\
(\hat{\beta}_x^{(0)})^{(i+5)} = (\hat{\beta}_x^{(0)})^{(i+4)}, \\
\hat{k}_t^{(i+5)} = \hat{k}_t^{(i+4)}, \\
(\hat{\beta}_x^{(1)})^{(i+5)} = (\hat{\beta}_x^{(1)})^{(i+4)}, \\
\hat{\hat{\gamma}}^{(i+5)} = \hat{\hat{\gamma}}^{(i+4)}.
\end{cases}
$$

$$
\text{Update}(\bar{\gamma}):\begin{cases}
\hat{\bar{\gamma}}^{(i+6)} = \hat{\bar{\gamma}}^{(i+5)} - \dfrac{\sum_{x,t}\left\{\dfrac{\left(d_{x,t}+\frac{1}{\hat{\bar{\gamma}}^{(i+5)}}\right)}{(\hat{\bar{\gamma}}^{(i+5)})^2\left(\hat{d}_{x,t}^{(i+5)}+\hat{\bar{\gamma}}^{(i+5)}\right)}\right\}}{\sum_{x,t}\left\{\dfrac{\left(\hat{d}_{x,t}^{(i+5)}+\frac{1}{\hat{\bar{\gamma}}^{(i+5)}}\right)-\hat{\bar{\gamma}}_x^{(i+5)}\left(d_{x,t}+\frac{1}{\hat{\bar{\gamma}}^{(i+5)}}\right)\left(2\hat{d}_{x,t}^{(i+5)}+\frac{1}{\hat{\bar{\gamma}}^{(i+5)}}\right)}{(\hat{\bar{\gamma}}^{(i+5)})^2\left(\hat{d}_{x,t}^{(i+5)}+\frac{1}{\hat{\bar{\gamma}}^{(i+5)}}\right)^2}\right\}} \\
\hat{\alpha}_x^{(i+6)} = \hat{\alpha}_x^{(i+5)}, \\
(\hat{\beta}_x^{(0)})^{(i+6)} = (\hat{\beta}_x^{(0)})^{(i+5)}, \\
\hat{k}_t^{(i+6)} = \hat{k}_t^{(i+5)}, \\
(\hat{\beta}_x^{(1)})^{(i+6)} = (\hat{\beta}_x^{(1)})^{(i+5)}, \\
\hat{\iota}_{t-x}^{(i+6)} = \hat{\iota}_{t-x}^{(i+5)}.
\end{cases}
$$

4.4 Application to the population data for Thailand

4.4.1 Source of data

We investigate the goodness of fit for the mortality rate spanning the period 1998 to 2012.The Ministry of the Interior is the main source for data for the number of age-specific people in the population. The number of deaths was obtained from the Bureau of Policy and Strategy, Ministry of Public Health. We assume that the remaining lifetimes of the individuals aged on January 1 of year t are independent and identically distributed. The $\widehat{m}_{x,t}$ is the nonparametric estimation of $m_{x,t}$. This is given by the ratio of the observed number of deaths $D_{x,t}$ for age x and year t to $ETR_{x,t}$: $\widehat{m}_{x,t} = \frac{D_{x,t}}{ETR_{x,t}}$.

We use the two mortality models to fit and forecast the Thai mortality rate by the following methods:

a) by the Poisson setting

b) by the negative binomial setting

- age-independent dispersion parameter (denoted by NB(Case1))

- age-dependent dispersion parameter (denoted by NB(Case2))

4.4.2 Fitting the mortality model

4.4.2.1 Fitting of the Lee-Carter model

The Lee-Carter model is $m_{x,t} = exp(\alpha_x + \beta_x k_t + \varepsilon_{xt})$, where x = <1,1,2,...,100,>100 , t = 1998,...,2012. To fit the logarithm of mortality, we use the following equation

$$ln(\hat{m}_{x,t}) = \hat{\alpha}_x + \hat{\beta}_x \hat{k}_t,$$

where the mortality rate $\hat{m}_{x,t}$ is given by

$$\hat{m}_{x,t} = exp(\hat{\alpha}_x + \hat{\beta}_x \hat{k}_t). \tag{4.32}$$

4.4.2.2 Fitting of the age-period-cohort model

The age-period cohort model is

$$ln(m_{x,t}) = \alpha_x + \beta_x^{(0)} k_t + \beta_x^{(1)} \gamma_{t-x} + \varepsilon_{x,t}.$$

To fit the logarithm of mortality, we use the following equation

$$ln(\hat{m}_{x,t}) = \hat{\alpha}_x + \hat{\beta}_x^{(0)} \hat{k}_t + \hat{\beta}_x^{(1)} \hat{\gamma}_{t-x},$$

where the mortality rate $\hat{m}_{x,t}$ is given by

$$\hat{m}_{x,t} = exp(\hat{\alpha}_x + \hat{\beta}_x^{(0)} \hat{k}_t + \hat{\beta}_x^{(1)} \hat{\gamma}_{t-x}). \tag{4.33}$$

Our plan is to estimate the mortality rate from 1998 to 2012 and to forecast the mortality rate from 2013 to 2022. Thus, we fit the mortality model for Thai population data. As a result, the estimated parameter of all mortality model are shown in Appendix A. Figures 4.1 and 4.2 show graphs of the estimated parameters. Figure 4.1 shows a comparison of $\hat{\alpha}_x$, $\hat{\beta}_x$ and $\hat{k}_t$ with the Lee-Carter (LC) model by applying the Poisson setting and the negative binomial setting (NB(Case1) and NB(Case2)) respectively.

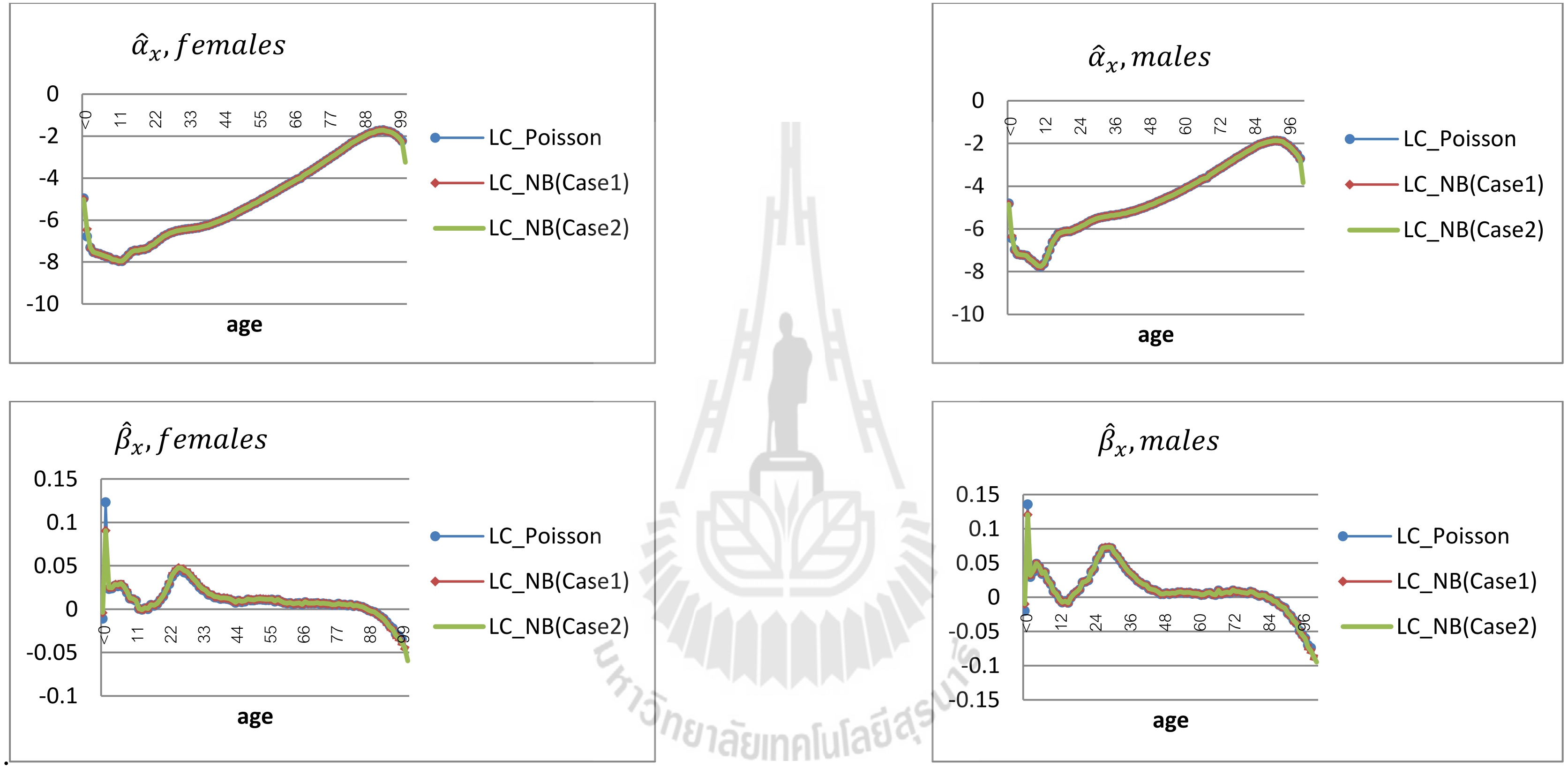


Figure 4.1 Lee-Carter parameter estimations of α_x, β_x and k_t by Poisson setting and negative binomial setting.

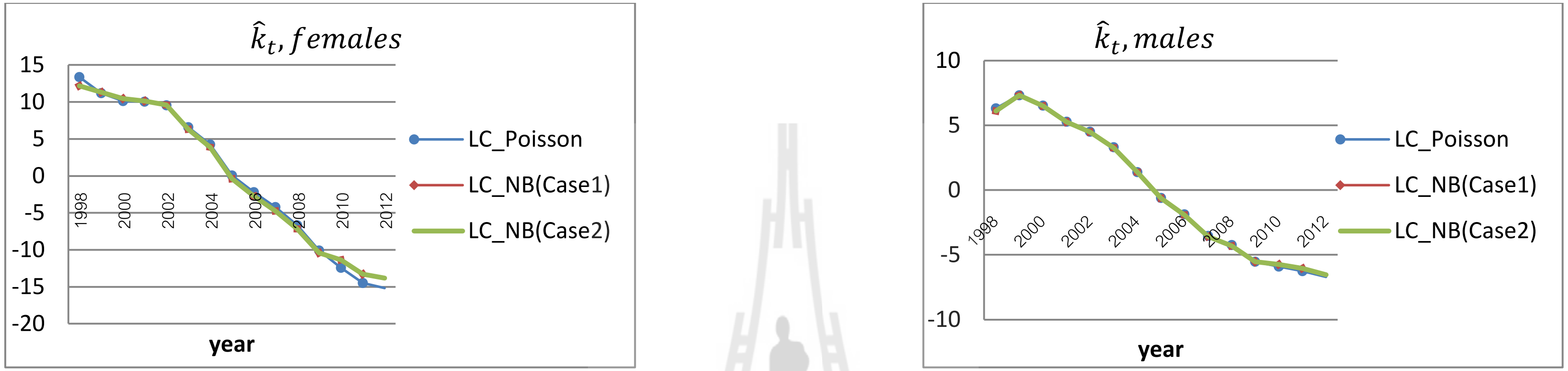


Figure 4.1 Lee-Carter parameter estimations of α_x, β_x and k_t by Poisson setting and negative binomial setting (Continued).

Next, the estimated parameters for the age-period-cohort model are showed in the following figures. Figure 4.2 shows a comparison of $\hat{\alpha}_x$, $\hat{\beta}_x^{(0)}$, $\hat{k}_t$, $\hat{\beta}_x^{(1)}$and $\hat{\gamma}_{t-x}$ with the age-period-cohort (APC) model by using the Poisson setting and the negative binomial setting (NB(Case1) and NB(Case2))**.**

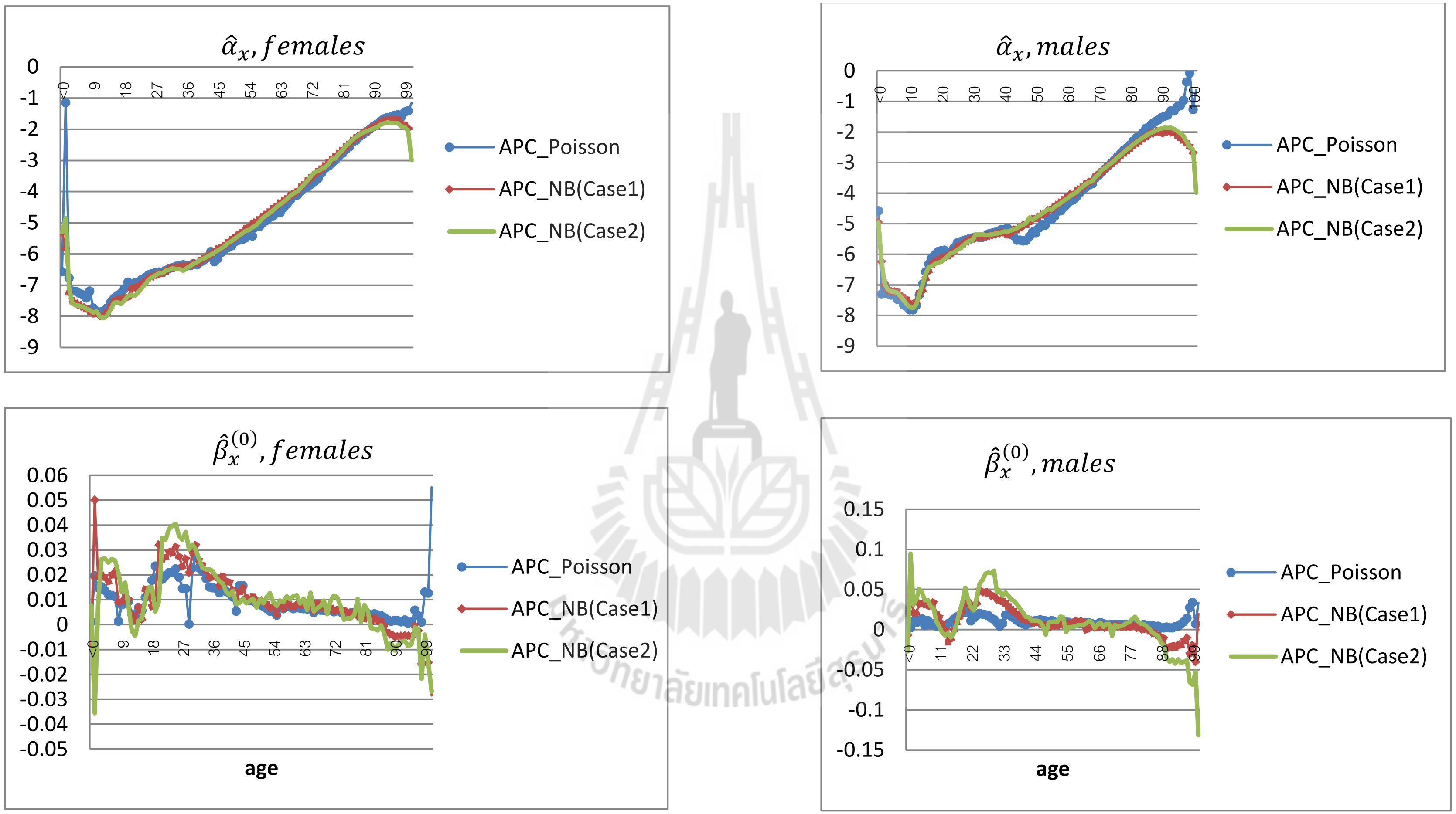


Figure 4.2 The age-period-cohort parameter is estimated by applying the Poisson setting and the negative binomial setting.

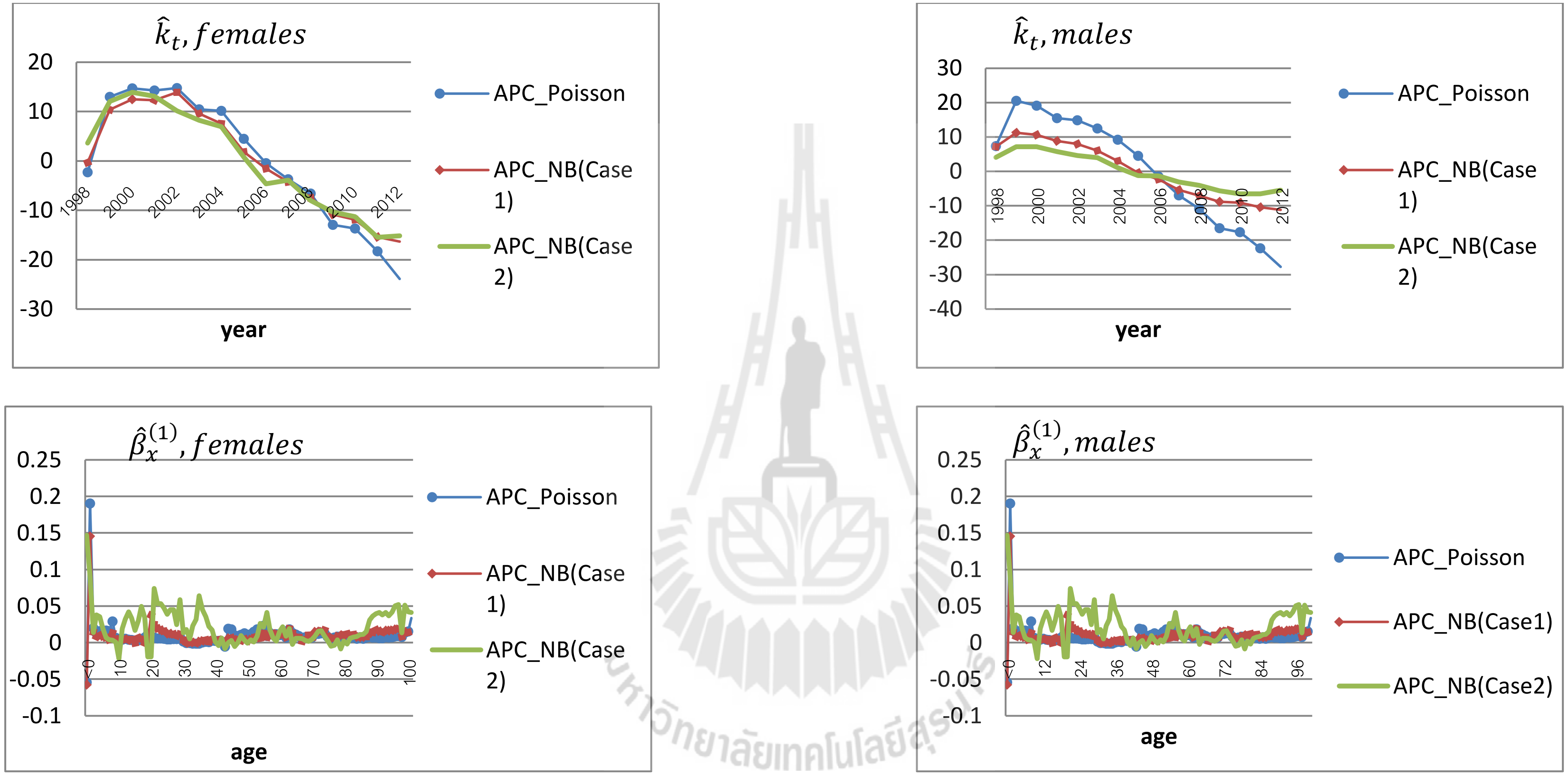


Figure 4.2 The age-period-cohort parameter is estimated by applying the Poisson setting and the negative binomial setting (Continued).

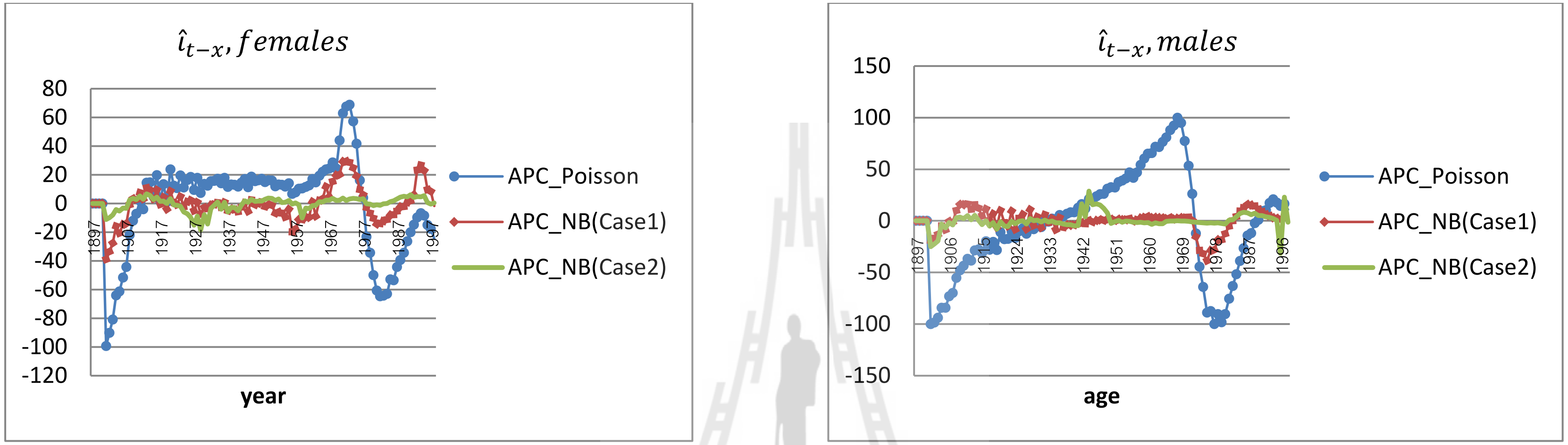


Figure 4.2 The age-period-cohort parameter is estimated by applying the Poisson setting and the negative binomial setting (Continued).

The estimated parameters of the Lee-Carter model and the age-period-cohort model are shown in the Appendix A.

4.4.2.3 Fitted mortality results of the Poisson setting

In this section, we calculate the error values of the fitted mortality rate from the Lee-Carter model and the age-period-cohort model under the Poisson setting for the death count. The mortality rate is fitted spanning in period 1998-2012.

Table 4.1 The error values of the fitted mortality rate $\widehat{m}_{x,t}$.

Error	Male		Female	
	Lee-Carter model	APC Model	Lee-Carter model	APC Model
MAD	**0.002303028**	0.006534078	**0.002390818**	0.014034036
MSE	**0.000030589**	0.005755655	**0.0000403588**	0.069088004
RMSE	**0.005530742**	0.075866036	**0.006352863**	0.26284597
MAPE	**5.763018238**	29.9754896	**6.701458816**	1361.395068

The entries in bold show the smallest error values of the fitted mortality rate. The results in Table 4.1 indicate that the Lee-Carter model is a suitable model for forecasting the mortality rate with the underlying the Poisson setting for both genders. Next, we wish to consider a mean-variance restriction of the death count, thus we will use an overdispersion test to detect overdispersion.

4.4.2.4 Overdispersion test

In this section, we use the Score Statistics proposed by Dean and Lawless (1989) and Dean (1992). These statistics are used to detect overdispersion. We consider the variance function of the negative binomial distribution of the mortality model as follows:

$$Var(d_{x,t}) = E[d_{x,t}] + \tau[E[d_{x,t}]]^2. \quad (4.34)$$

For testing the negative binomial distribution data, we wish to use the hypothesis $H_0: \tau = 0$ against $H_1: \tau > 0$.

The Score Statistic Q can be shown as follows,

$$Q = \frac{\sum_{x,t}((d_{x,t} - \hat{d}_{x,t})^2 - d_{x,t})}{\sqrt{2\sum_{x,t} \hat{d}_{x,t}^2}} \sim N(0,1). \quad (4.35)$$

Table 4.2 The values of the Score Statistics.

Gender	Lee-Carter model	Age-period-cohort model
Male	182.9386	5218.523
Female	209.9583	87839.65

In Table 4.2, all p-values are less than 0.00001, so we can reject the null hypothesis. This shows that the mortality model allows for overdispersion. Thus, this result leads to an alternative approach, namely that the mortality model should be based on the negative binomial distribution.

4.5 Fitted mortality result of the negative binomial setting

4.5.1 Evaluation of the efficiency of the forecasting by fitting the forecast to the mortality rate

In this section, we need to investigate the effectiveness of forecasting by our mortality model. The period for fitting mortality span the year 1998-2007 and 1998-2009, to forecast the periods 2008-2012 and 2010-2012 respectively.

Table 4.3 The error values measure the fitted period based on 1998-2007, males.

Error	Fitting period : 1998-2007 Forecast : 2008-2012 (Males)			
	Lee-Carter model		Age-period cohort model	
	Age-dependent	Age-independent	Age-dependent	Age-independent
MSE	0.000317	0.0448	0.000287	**0.000102343**
RMSE	0.017808	0.211659	0.016931	**0.010116496**
MAD	0.006014	0.06098	0.00645	**0.004204521**
MAPE	0.129237	0.693126	0.14487	**0.122434885**

Table 4.4 The error values measure the fitted period based on 1998-2007, females.

Error	Fitting period : 1998-2007 Forecast : 2008-2012 (Females)			
	Lee-Carter model		Age-period cohort model	
	Age-dependent	Age-independent	Age-dependent	Age-independent
MSE	0.000326	0.000326	0.000101	**9.76E-05**
RMSE	0.018062	0.018062	0.010035	**0.009877336**
MAD	0.00591	0.00591	**0.00384**	0.00391212
MAPE	0.145211	0.145212	0.150894	**0.109621392**

Table 4.5 The error values measure the fitted period based on 1998-2009, males.

Error	Fitting period : 1998-2009 Forecast : 2010-2012 (Males)			
	Lee-Carter model		Age-period cohort model	
	Age-dependent	Age-independent	Age-dependent	Age-independent
MSE	0.000232	0.000232	0.000118	**0.000106**
RMSE	0.015226	0.015225	0.010885	**0.010304**
MAD	0.005397	0.005397	0.004046	**0.00383**
MAPE	0.146526	0.146524	0.126314	**0.112204**

Table 4.6 The error values measure the fitted period based on 1998-2009, females.

Error	Fitting period : 1998-2009 Forecast : 2010-2012 (Females)			
	Lee-Carter model		Age-period cohort model	
	Age-dependent	Age-independent	Age-dependent	Age-independent
MSE	0.000308	0.000308	0.000279	**0.000127**
RMSE	0.017553	0.017556	0.016715	**0.011255**
MAD	0.005509	0.00551	0.005194	**0.003777**
MAPE	0.15357	0.15357	0.183032	**0.125659**

Next, we compute the error values of the fitted mortality spanning the period 1998-2012.

Table 4.7 The error values of the fitted mortality rate, $\hat{m}_{x,t}$, compared with the observed mortality rate based on the period 1998-2012 for males.

Error	Males			
	Lee-Carter model		APC Model	
	Age-dependent	Age-independent	Age-dependent	Age-independent
MSE	3.01E-05	3.01E-05	2.16E-05	**1.60E-05**
RMSE	0.005489	0.005489	0.004648	**0.004000062**
MAD	0.002287	0.002287	0.001636	**0.001579823**
MAPE	5.675554	5.677291	4.444869	**4.302597344**

Table 4.8 The error values of the fitted mortality rate, $\hat{m}_{x,t}$, compared with the observed mortality rate based on the period 1998-2012 for females.

Error	Females			
	Lee-Carter model		APC Model	
	Age-dependent	Age-independent	Age-dependent	Age-independent
MSE	3.67E-05	3.67E-05	**2.02E-05**	3.17E-05
RMSE	0.006058	0.006058	**0.004496**	0.005634379
MAD	0.002269	0.002269	0.001543	**0.001499114**
MAPE	6.329586	6.329738	4.344024	**4.219911831**

We consider the fitted mortality rate from the results which appear in Tables 4.3 to 4.8. As a result, we shall use the age-period-cohort model underlying the age-independent parameter to forecast the Thai mortality rate for males and females.

4.6 Forecasting Thai mortality rate and life expectancy

We model the forecast time mortality index by using a time series model. The Box-Jenkins method is used to forecast the mortality index k_t and ι_{t-x} with the appropriate ARIMA time series model.

- The Lee-Carter model

  The $\dot{k}_{t_n+s}$, $s = 1,2,3,\dots,10$ are the forecast mortality indices. The forecast of the mortality rate can be computed by the following equation:

$$\dot{m}_{x,t_n+s} = exp(\hat{\alpha}_x + \hat{\beta}_x \hat{k}_{t_n+s}). \qquad (4.36)$$

- The age-period-cohort model

  The $\dot{k}_{t_n+s}$, $s = 1,2,3,\dots,10$ are the forecast mortality indices. The estimated cohort effect, $\hat{\iota}_z$, $z\epsilon[t_1 - x_k, t_n - x_1]$ can be forecast by the following equation(Renshaw and Harberman , 2006):

$$\tilde{\iota}_{t_n-x+s} = \begin{cases} \hat{\iota}_{t_n-x+s}, & 0<s\leq x-x_1 \\ \dot{\iota}_{t_n-x+s}, & s>x-x_1 \end{cases}.$$

Then, the forecasted mortality rate is

$$\dot{m}_{x,t_n+s} = \widehat{m}_{x,t} ex\,p\left\{\hat{\beta}_x^{(0)}\left(\dot{k}_{t_n+s} - \hat{k}_{t_n}\right) + \hat{\beta}_x^{(1)}\left(\tilde{\iota}_{t_n-x+s} - \hat{\iota}_{t_n-x}\right)\right\}. \qquad (4.37)$$

4.6.1 Forecasted mortality rate

ARIMA model for estimation k_t

We model the forecast index by using the time series model. We use the Box-Jenkins methodology to estimate and forecast the k_t and the ι_{t-x} with the appropriate ARIMA time series model. We consider several ARIMA model by checking the diagnostic, the Ljung-Box Q-test, and then we choose the appropriate model by comparing the AIC and BIC.

For the Lee-Carter model, the estimation model ARIMA(0,1,0) with drift is the most suitable model for k_t, which is described by

$$\hat{k}_t = \theta + \hat{k}_{t-1} + \varepsilon_t, \text{ where } \varepsilon_t \sim N(0, \sigma_\varepsilon^2). \tag{4.38}$$

For the age-period-cohort model, the estimation model ARIMA(1,1,0) with drift is the most appropriate model for k_t, and ARIMA(1,0,2) for ι_{t-x} which are described by

$$\Delta\hat{k}_t = a_1 \Delta\hat{k}_{t-1} + \varepsilon_t, \text{ where } \Delta\hat{k}_t = \hat{k}_t - \hat{k}_{t-1} \text{ and } \varepsilon_t \sim N(0, \sigma_\varepsilon^2), \text{ and}$$

$$\hat{\iota}_{t-x} = a_1 \hat{\iota}_{t-x-1} + b_1 \varepsilon_{t-x-1} + b_2 \varepsilon_{t-x-2} \text{ and } \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right). \tag{4.39}$$

The following figure shows the observed mortality rate and the forecasted mortality rate from 2013 to 2022 using the age-period-cohort model underlying the age-independent dispersion parameter.

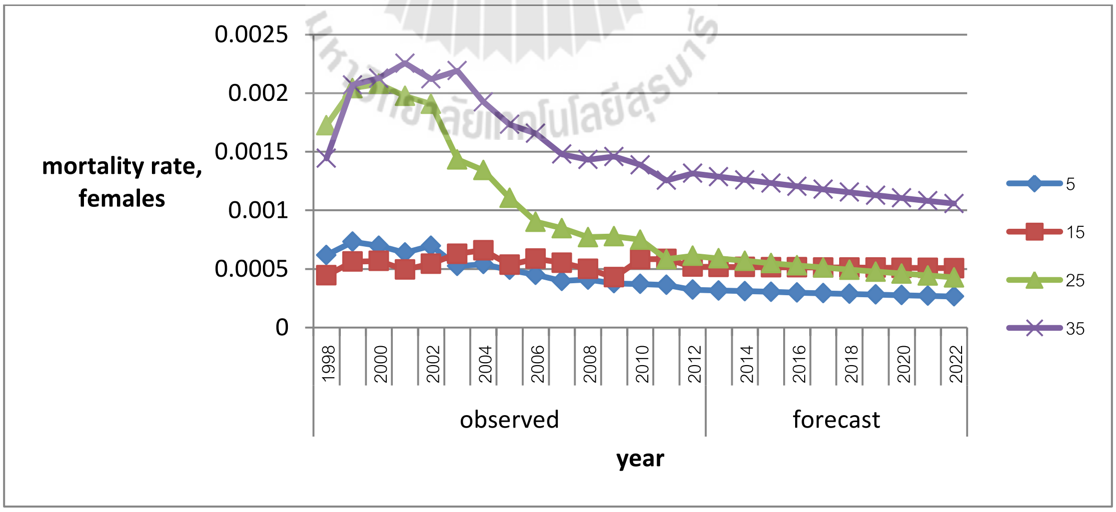


Figure 4.3 The observed mortality rate (1998-2012) and forecasted mortality rate (2013-2022), at age 5, 15, 25 and 35 years for females.

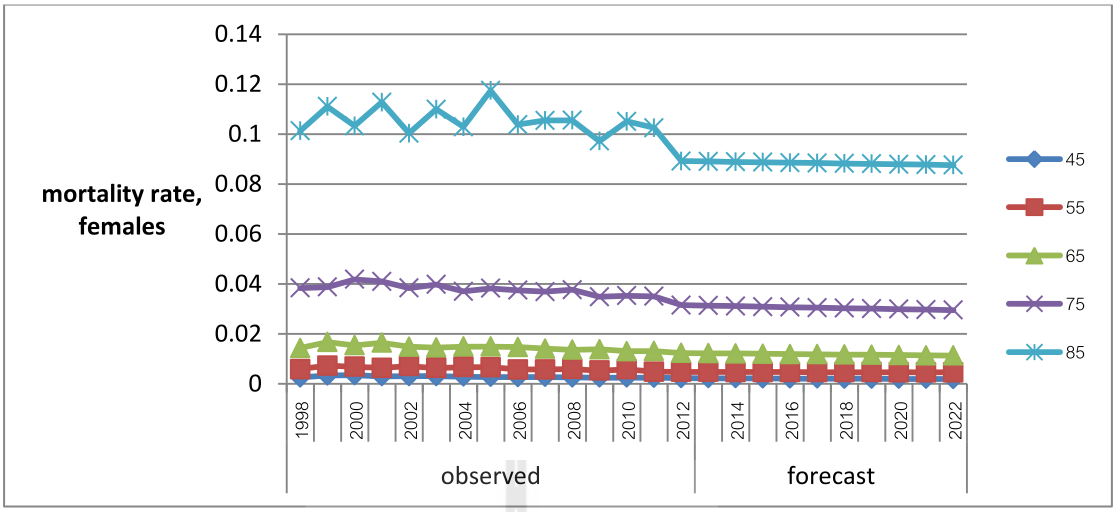


Figure 4.4 The observed mortality rate (1998-2012) and forecasted mortality rate (2013-2022), at age 45, 55, 65, 75 and 85 years for females.

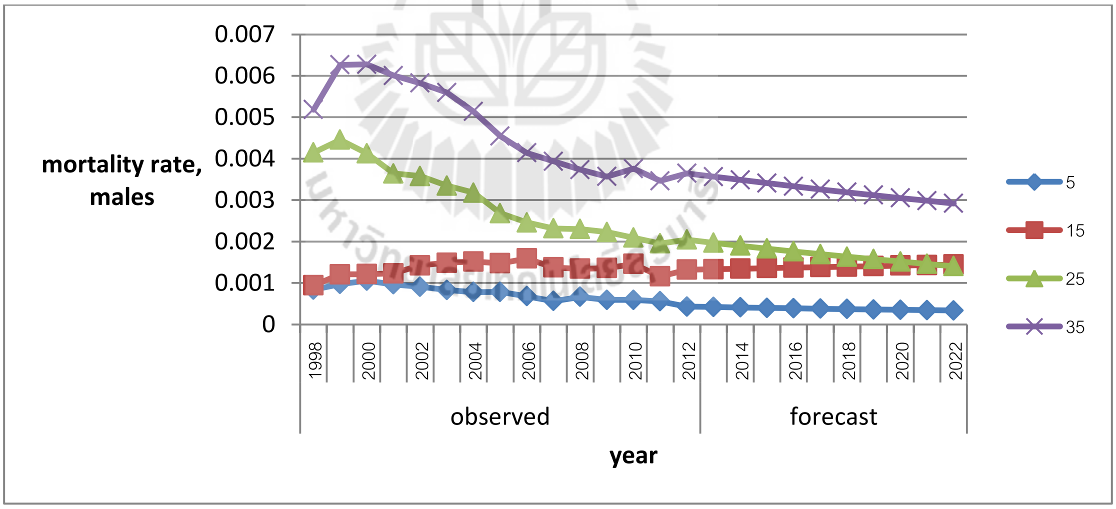


Figure 4.5 The observed mortality rate (1998-2012) and forecasted mortality rate (2013-2022), at age 5, 15, 25 and 35 years for males.

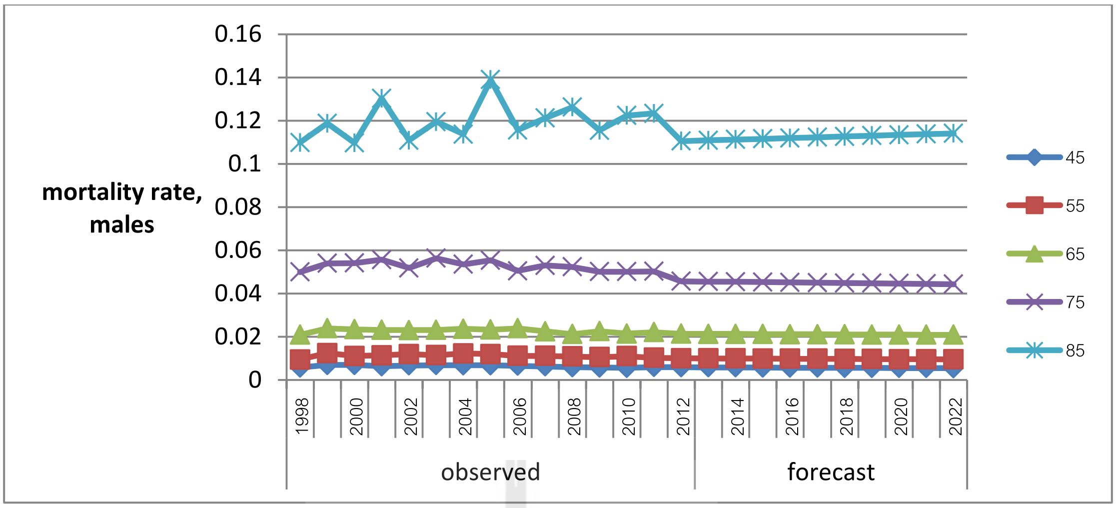


Figure 4.6 The observed mortality rate (1998-2012) and forecasted mortality rate (2013-2022), at age 45, 55, 65, 75 and 85 years for males.

In Figures 4.3 to 4.6, the graphs show the observed and forecasted mortality rate of the Thai data. The graphs show that the mortality rates are likely to decline from 1998 to 2022 for both genders. Further forecasted mortality rates can be seen in the Appendix B.

4.6.2 The forecast of life expectancy

Next, we investigate the problem of forecasting life expectancy. Our plan is to forecast for the period 2013 to 2022. We shall use the age-period-cohort model underlying the age-independent dispersion parameter to forecast the life expectancy of both males and females. The forecasted life expectancy at birth, 25, 50 60, 65 and 70 years can be found in Table 4.9 and 4.10. More forecasted life expectancies are in the Appendix C.

We note further that, for males, the forecasted values of life expectancy at birth increased from 72.07 years in 2013 to 72.82 year in 2022 which means an increase of 1.03%. For females, the forecasted values of life expectancy at birth increased from 78.90 years in 2013 to 79.59 years in 2022 which signifies an increase of 0.86%.

Table 4.9 The forecasted life expectancy, 2013-2022 using the age-period-cohort model underlying the age-independent dispersion parameter, males.

Year	At birth	25	50	60	65	70
2013	72.07	49.01	27.60	20.02	16.55	13.36
2014	72.16	49.08	27.62	20.03	16.55	13.36
2015	72.25	49.16	27.64	20.04	16.55	13.35
2016	72.34	49.23	27.66	20.05	16.55	13.35
2017	72.42	49.30	27.68	20.06	16.56	13.35
2018	72.50	49.37	27.70	20.06	16.56	13.35
2019	72.58	49.43	27.72	20.07	16.56	13.34
2020	72.66	49.50	27.74	20.08	16.56	13.34
2021	72.74	49.57	27.76	20.09	16.57	13.34
2022	72.82	49.63	27.78	20.10	16.57	13.34

Table 4.10 The forecasted life expectancy, 2013-2022 using the age-period-cohort model underlying the age-independent dispersion parameter, females.

Year	At birth	25	50	60	65	70
2013	78.90	55.12	31.66	22.99	18.98	15.23
2014	78.98	55.19	31.71	23.03	19.01	15.25
2015	79.06	55.27	31.75	23.06	19.04	15.27
2016	79.14	55.34	31.80	23.10	19.06	15.29
2017	79.21	55.41	31.84	23.13	19.09	15.31
2018	79.29	55.48	31.88	23.17	19.12	15.33
2019	79.37	55.55	31.93	23.20	19.15	15.35
2020	79.44	55.61	31.97	23.24	19.17	15.37
2021	79.51	55.68	32.01	23.27	19.20	15.38
2022	79.59	55.75	32.06	23.30	19.23	15.40

4.7 Simulation study and confidence interval

In this section, we generate a bootstrap sample for the number of deaths from the negative binomial distribution given by equation (4.21). Then, we forecast the mortality rate by using the age-period-cohort model underlying the age-independent dispersion parameter model both genders. We compute the 95% confidence interval of the forecasting mortality rate. Also, we calculate the confidence interval of life expectancy. The following table shows the 95% confidence interval of life expectancy at birth, 25, 50, 65, 70 years.

Table 4.11 The 95% confidence interval of life expectancy (at birth, 25, 50 years), males.

	95% CI of life expectancy, males					
	At birth		25		50	
Year	L	U	L	U	L	U
2013	71.9	72.39	48.87	49.26	27.47	27.77
2014	72.02	72.56	48.98	49.4	27.49	27.81
2015	72.13	72.74	49.07	49.54	27.5	27.86
2016	72.23	72.91	49.16	49.68	27.51	27.91
2017	72.32	73.08	49.25	49.82	27.52	27.97
2018	72.4	73.25	49.32	49.96	27.52	28.02
2019	72.48	73.41	49.4	50.09	27.53	28.07
2020	72.56	73.57	49.47	50.23	27.53	28.13
2021	72.63	73.73	49.54	50.36	27.53	28.18
2022	72.69	73.88	49.6	50.48	27.54	28.24

Table 4.12 The 95% confidence interval of life expectancy (at 60, 65, 70 years), males.

	95% CI of life expectancy, males					
	60		65		70	
Year	U	L	U	L	U	L
2013	19.91	20.15	16.44	16.66	13.26	13.45
2014	19.91	20.17	16.43	16.66	13.25	13.45
2015	19.9	20.19	16.42	16.67	13.23	13.45
2016	19.9	20.21	16.41	16.68	13.22	13.45
2017	19.89	20.23	16.39	16.69	13.21	13.45
2018	19.88	20.25	16.38	16.7	13.19	13.45
2019	19.88	20.28	16.37	16.72	13.18	13.45
2020	19.87	20.3	16.35	16.73	13.16	13.45
2021	19.86	20.33	16.34	16.74	13.15	13.45
2022	19.85	20.35	16.32	16.75	13.13	13.45

Table 4.13 The 95% confidence interval of life expectancy (at birth, 25, 50 years), females.

	95% CI of life expectancy, females					
	At birth		25		50	
Year	U	L	U	L	U	L
2013	78.66	79.11	54.95	55.29	31.53	31.79
2014	78.74	79.2	55.01	55.37	31.57	31.84
2015	78.82	79.28	55.08	55.44	31.61	31.88
2016	78.89	79.37	55.15	55.52	31.66	31.93
2017	78.96	79.45	55.21	55.59	31.7	31.98
2018	79.03	79.53	55.28	55.66	31.74	32.03
2019	79.1	79.62	55.34	55.74	31.77	32.07
2020	79.16	79.7	55.4	55.81	31.81	32.12
2021	79.23	79.78	55.46	55.88	31.85	32.17
2022	79.29	79.86	55.52	55.95	31.89	32.22

Table 4.14 The 95% confidence interval of life expectancy (at 60, 65, 70 years), females.

	95% CI of life expectancy, females					
	60		65		70	
Year	L	U	L	U	L	U
2013	22.88	23.1	18.88	19.08	15.13	15.32
2014	22.91	23.14	18.9	19.11	15.15	15.34
2015	22.94	23.17	18.93	19.14	15.17	15.36
2016	22.97	23.21	18.95	19.17	15.19	15.38
2017	23.01	23.25	18.98	19.2	15.2	15.4
2018	23.04	23.29	19	19.23	15.22	15.42
2019	23.07	23.32	19.03	19.26	15.24	15.45
2020	23.1	23.36	19.05	19.29	15.25	15.47
2021	23.13	23.4	19.07	19.32	15.27	15.49
2022	23.16	23.44	19.1	19.35	15.28	15.51

4.8 Insurance premium calculation

In this section, we apply the forecasted mortality rate to calculate the life insurance premiums. We use the average value of the forecasted mortality in 2013-2022 for this purpose. Moreover, we also compute the 95% confidence interval of the life insurance premium from the simulated mortality rate. We have taken an interest rate of $i = 5\%$ and compute the following net single premium paying 1000 :

- An n-year temporary insurance premium
- The whole life insurance premium
- An n-year endowment insurance premium

We compare the premium using our proposed, our forecasted data, with TMO2008 (The Thai mortality table 2008).

4.8.1 The premium calculation of the forecasted mortality rate

In this section, we apply the average of the mortality rate from 2013 to 2022 to compute the life insurance premium.

4.8.1.1 The n-year temporary insurance

We calculate the n-year temporary insurance premium by the following cases:

- 10-year temporary insurance premium
- 15-year temporary insurance premium
- 20-year temporary insurance premium

The premium of this case shows in Table 4.15 and 4.16.

Table 4.15 The premium of the10, 15 and 20-year temporary life insurance, males.

Age	10 years temporary		15 year temporary		20 year temporary	
	TMO 2008	Our Proposed	TMO 2008	Our Proposed	TMO 2008	Our Proposed
<1	5.43	10.73	7.36	12.04	10.43	14.92
1	5.02	3.63	7.28	5.42	10.73	8.38
2	4.75	3.27	7.4	5.56	11.2	8.55
3	4.62	3.05	7.72	5.91	11.82	8.92
4	4.66	3.08	8.23	6.48	12.58	9.46
5	4.88	3.33	8.91	7.13	13.46	10.06
6	5.28	3.95	9.81	7.82	14.46	10.8
7	5.87	4.54	10.85	8.46	15.55	11.5
8	6.63	5.37	12.01	9.3	16.74	12.37
9	7.55	6.24	13.26	10.14	18	13.28
10	8.6	7.24	14.57	11.08	19.31	14.36
11	9.82	8.15	15.92	12.05	20.67	15.47
12	11.1	9.07	17.27	13.04	22.03	16.71
13	12.38	10.05	18.58	14.06	23.37	18
14	13.61	10.93	19.83	15.05	24.68	19.27

Table 4.15 The premium of the10, 15 and 20-year temporary life insurance, males (Continued).

Age	10 years temporary		15 year temporary		20 year temporary	
	TMO 2008	Our Proposed	TMO 2008	Our Proposed	TMO 2008	Our Proposed
15	14.76	11.55	20.98	15.85	25.91	20.34
16	15.79	11.78	22.03	16.26	27.06	21.17
17	16.68	11.97	22.94	16.78	28.11	22.07
18	17.43	12.05	23.74	17.22	29.07	22.92
19	18.03	12.14	24.42	17.7	29.96	23.77
20	18.52	12.29	25.01	18.21	30.79	24.78
21	18.78	12.66	25.42	19.12	31.48	26.03
22	18.96	13.21	25.79	20.19	32.15	27.55
23	19.11	13.76	26.17	21.27	32.87	29
24	19.26	14.46	26.59	22.46	33.67	30.71
25	19.44	15.25	27.09	23.92	34.58	32.55
26	19.67	16.3	27.69	25.4	35.64	34.55
27	19.97	17.54	28.4	27.24	36.78	36.83
28	20.37	18.87	29.24	29.05	38.1	39.23
29	20.87	20.21	30.25	31.08	39.61	41.79
30	21.5	21.7	31.42	33.08	41.34	44.38
31	22.25	23.23	32.77	35.31	43.29	47.29
32	23.11	24.94	34.21	37.59	45.5	50.26
33	24.11	26.51	35.84	39.96	47.97	53.48
34	25.26	28.31	37.66	42.48	50.73	56.86
35	26.56	30.18	39.7	45.12	53.8	60.37
36	28.01	31.88	41.96	47.76	57.21	63.64
37	29.5	33.76	44.46	50.56	60.98	67.03
38	31.13	35.71	47.22	53.65	65.15	70.94
39	32.93	37.92	50.27	57.03	69.75	74.97
40	34.9	39.92	53.62	60.22	74.84	79.13
41	37.06	42.44	57.33	63.59	80.45	83.79
42	39.47	44.78	61.44	66.75	86.68	88.51
43	42.12	47.79	65.99	70.87	93.55	94.04
44	45.04	50.72	71.01	74.71	101.11	99.53
45	48.27	53.78	76.57	79.09	109.42	105.42

Table 4.15 The premium of the10, 15 and 20-year temporary life insurance, males (Continued).

Age	10 years temporary		15 year temporary		20 year temporary	
	TMO 2008	Our Proposed	TMO 2008	Our Proposed	TMO 2008	Our Proposed
46	51.84	56.69	82.74	83.76	118.51	111.99
47	56.02	59.58	89.78	88.78	128.61	119.03
48	60.67	63.25	97.59	94.39	139.57	126.32
49	65.85	66.72	106.23	100.15	151.38	134.03
50	71.62	70.81	115.76	106.34	164.07	142.35
51	78.05	74.94	126.21	113.11	177.62	151.31
52	85.21	79.54	137.61	120.52	192.07	161.04
53	93.19	84.49	149.97	127.86	207.41	170.75
54	102.05	89.56	163.3	135.71	223.66	181.42
55	111.84	95.18	177.58	144.36	240.83	192.91

Table 4.16 The premium of the 10, 15 and 20-year temporary life insurance, females.

Females	10 year temporary		15 year temporary		20 year temporary	
Age	TMO 2008	Our Proposed	TMO 2008	Our Proposed	TMO 2008	Our Proposed
<1	5.17	8.91	6.43	9.73	7.68	10.68
1	4.71	2.62	6.04	3.56	7.34	4.49
2	4.34	2.26	5.74	3.29	7.11	4.24
3	4.07	2.23	5.55	3.36	6.97	4.28
4	3.9	2.23	5.45	3.43	6.92	4.32
5	3.81	2.29	5.44	3.54	6.96	4.42
6	3.8	2.43	5.51	3.65	7.06	4.55
7	3.86	2.53	5.65	3.78	7.22	4.71
8	3.97	2.69	5.83	3.9	7.44	4.9
9	4.13	2.85	6.05	4.03	7.69	5.09
10	4.31	3.05	6.3	4.2	7.96	5.38
11	4.53	3.2	6.56	4.37	8.26	5.63
12	4.75	3.4	6.82	4.62	8.56	5.98
13	4.98	3.53	7.08	4.84	8.88	6.26
14	5.2	3.58	7.34	4.98	9.2	6.51
15	5.41	3.65	7.58	5.19	9.53	6.79

Table 4.16 The premium of 10, 15 and 20-year temporary life insurance for females (Continued).

Females	10 year temporary		15 year temporary		20 year temporary	
Age	TMO 2008	Our Proposed	TMO 2008	Our Proposed	TMO 2008	Our Proposed
16	5.6	3.63	7.82	5.29	9.86	7.05
17	5.78	3.73	8.06	5.51	10.2	7.38
18	5.95	3.78	8.31	5.64	10.55	7.65
19	6.11	3.84	8.56	5.85	10.91	8.02
20	6.27	4	8.83	6.1	11.29	8.43
21	6.41	4.18	9.08	6.5	11.67	8.95
22	6.55	4.43	9.35	6.88	12.05	9.53
23	6.71	4.69	9.65	7.32	12.46	10.14
24	6.89	5.03	9.97	7.88	12.91	10.92
25	7.09	5.4	10.33	8.45	13.41	11.76
26	7.33	5.88	10.73	9.09	13.97	12.6
27	7.61	6.27	11.15	9.74	14.59	13.54
28	7.92	6.64	11.61	10.34	15.28	14.51
29	8.27	7.18	12.12	11.16	16.06	15.63
30	8.65	7.61	12.69	11.94	16.94	16.73
31	9.06	8.19	13.31	12.8	17.93	18.03
32	9.47	8.75	13.99	13.74	19.05	19.36
33	9.92	9.38	14.74	14.84	20.32	20.87
34	10.39	10.12	15.56	15.99	21.76	22.53
35	10.89	10.93	16.48	17.22	23.4	24.34
36	11.44	11.58	17.52	18.45	25.27	26.07
37	12.04	12.52	18.7	19.91	27.42	28.04
38	12.71	13.57	20.05	21.48	29.86	30.33
39	13.47	14.59	21.62	23.18	32.66	32.59
40	14.34	15.74	23.43	25.11	35.85	35.29
41	15.34	16.99	25.55	27.02	39.47	38.17
42	16.57	18.34	28.05	29.05	43.63	41.21
43	18.01	19.88	30.94	31.53	48.33	44.8
44	19.7	21.49	34.25	33.89	53.62	48.58
45	21.68	23.26	38.04	36.68	59.57	52.57
46	24	25.15	42.36	39.87	66.25	57.11

Table 4.16 The premium of 10, 15 and 20-year temporary life insurance for females (Continued).

Females Age	10 year temporary TMO 2008	10 year temporary Our Proposed	15 year temporary TMO 2008	15 year temporary Our Proposed	20 year temporary TMO 2008	20 year temporary Our Proposed
47	26.69	26.97	47.25	43.03	73.74	61.96
48	29.81	29.18	52.77	46.71	82.1	67.1
49	33.4	31.38	58.98	50.82	91.39	72.82
50	37.48	34.14	65.95	55.18	101.66	78.93
51	42.11	37.05	73.73	59.91	112.94	85.87
52	47.3	40.07	82.4	65.2	125.24	93.17
53	53.1	43.76	92.01	70.85	138.53	101.06
54	59.56	47.68	102.63	76.95	152.77	109.86
55	66.72	51.73	114.3	83.39	167.91	119.46

Next, Figure 4.7 shows graphs of the 10, 15 and 20-year temporary insurance premium using the TMO2008 compared to our proposed premium.

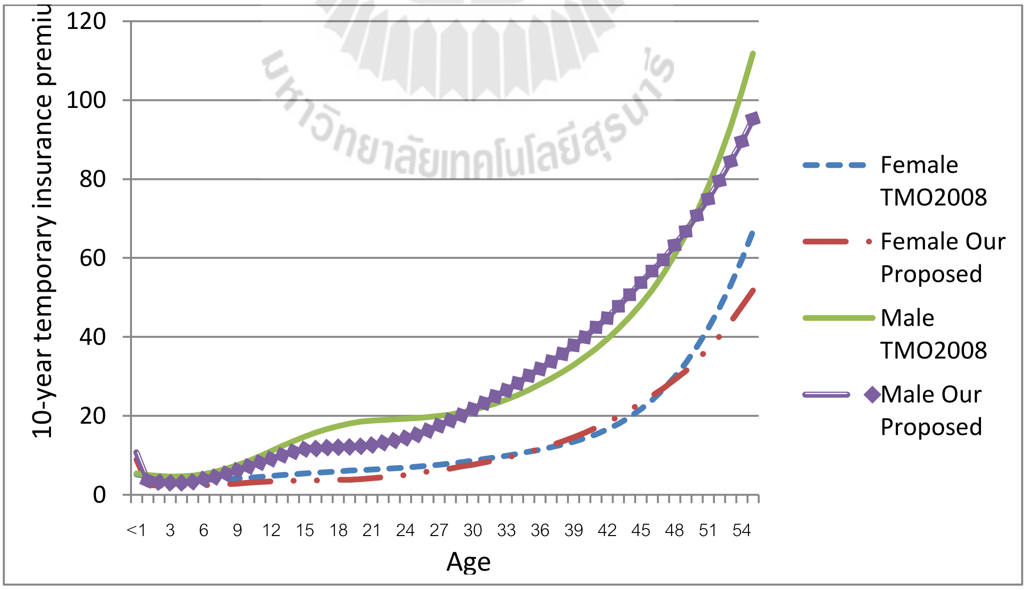


Figure 4.7 The premium of 10-year temporary insurance using the TMO2008 compared to our proposed mortality rate.

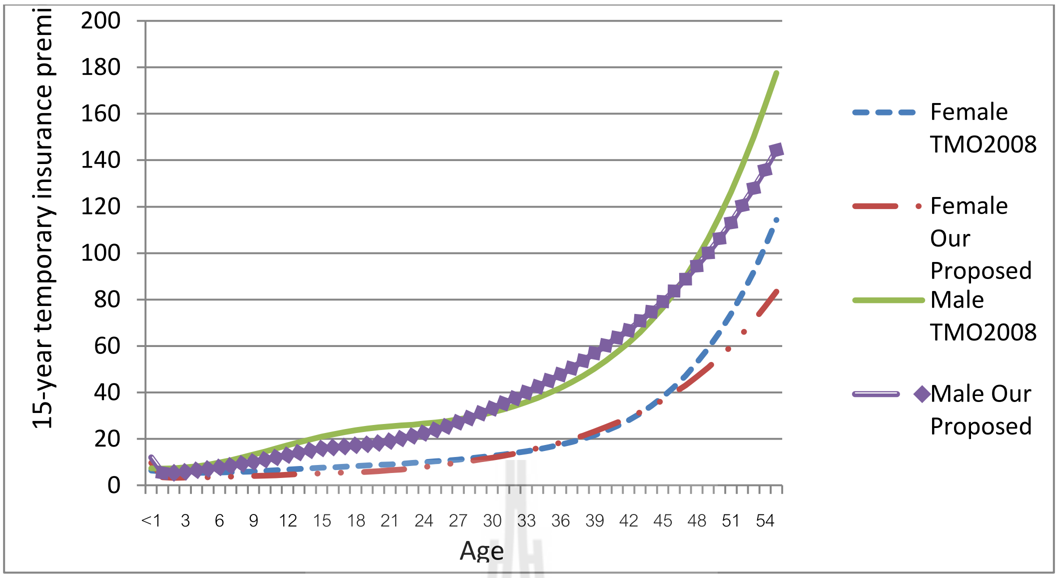


Figure 4.8 The premium of 15-year temporary insurance using the TMO2008 compared to our proposed mortality rate.

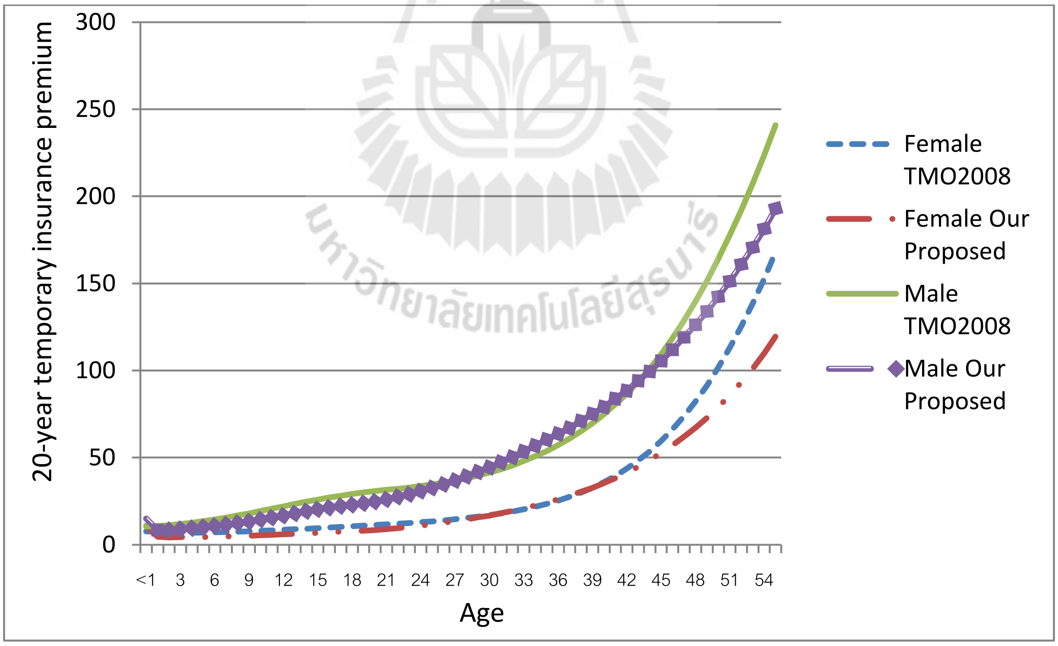


Figure 4.9 The premium of 15-year temporary insurance using the TMO2008 compared to our proposed mortality rate.

4.8.1.2 The whole life insurance premium

The whole life insurance premium is show in the Table 4.17.

Table 4.17 The premium of whole life insurance.

	Males		Females	
Age	TMO2008	Our Proposed	TMO2008	Our Proposed
<1	53	53.58	37.09	36.45
1	54.67	48.71	37.96	31.52
2	56.52	50.4	38.98	32.46
3	58.56	52.3	40.16	33.75
4	60.79	54.45	41.48	35.1
5	63.21	56.73	42.95	36.56
6	65.8	59.22	44.55	38.11
7	68.58	61.77	46.28	39.74
8	71.52	64.54	48.13	41.44
9	74.61	67.41	50.1	43.24
10	77.85	70.47	52.17	45.16
11	81.21	73.65	54.33	47.14
12	84.68	76.98	56.59	49.27
13	88.22	80.42	58.94	51.41
14	91.81	83.89	61.39	53.6
15	95.43	87.24	63.93	55.89
16	99.07	90.34	66.56	58.2
17	102.71	93.59	69.3	60.7
18	106.36	96.85	72.16	63.23
19	110.04	100.22	75.13	65.92
20	113.76	103.69	78.23	68.74
21	117.43	107.53	81.43	71.75
22	117.71	107.68	81.49	71.78
23	125.11	115.76	88.28	78.22
24	129.18	120.15	91.95	81.72
25	133.45	124.74	95.8	85.37
26	137.92	129.53	99.82	89.18
27	142.64	134.62	104.04	93.13
28	147.59	139.89	108.46	97.17
29	152.81	145.34	113.09	101.47
30	158.29	150.87	117.93	105.86

Table 4.17 The premium of whole life insurance (Continued).

	Males		Females	
Age	TMO2008	Our Proposed	TMO2008	Our Proposed
31	164.04	156.67	123	110.51
32	170.07	162.6	128.29	115.3
33	176.38	168.7	133.81	120.34
34	182.97	174.97	139.57	125.56
35	189.84	181.44	145.58	130.99
36	197	187.9	151.85	136.53
37	204.45	194.6	158.38	142.42
38	212.18	201.49	165.2	148.53
39	220.21	208.69	172.32	154.8
40	228.53	215.85	179.75	161.38
41	237.14	223.42	187.5	168.18
42	246.08	231	195.64	175.27
43	255.31	239.08	204.12	182.67
44	264.86	247.19	212.97	190.21
45	274.72	255.57	222.2	198.04
46	284.89	264.2	231.8	206.29
47	295.54	273.06	241.8	214.7
48	306.53	282.18	252.19	223.38
49	317.85	291.53	262.99	232.39
50	329.51	301.16	274.17	241.75
51	341.51	310.88	285.75	251.38
52	353.83	320.93	297.71	261.37
53	366.46	331.02	310.03	271.68
54	379.4	341.38	322.7	282.19
55	392.63	352.1	335.69	293.02
56	406.13	363.53	349	304.42
57	419.89	375.47	362.6	316.28
58	433.86	387.07	376.47	328.16
59	448.03	399.39	390.59	340.66
60	462.34	411.63	404.95	353.19
61	476.77	424.16	419.53	366.09
62	491.25	436.88	434.3	379.54
63	505.75	449.53	449.25	392.92
64	520.21	462.69	464.33	406.53
65	534.6	476.09	479.5	420.68

Table 4.17 The premium of whole life insurance (Continued).

	Males		Females	
Age	TMO2008	Our Proposed	TMO2008	Our Proposed
66	548.89	489.23	494.71	435.1
67	563.09	502.46	509.89	449.7
68	577.18	516.28	524.99	464.71
69	591.17	530.34	539.96	479.88
70	605.08	544.47	554.74	495.56
71	618.9	558.25	569.31	510.92
72	632.6	571.96	583.66	526.96
73	646.16	586.42	597.82	543.3
74	659.5	600.48	611.82	559.19
75	672.56	614.58	625.74	575.15
76	685.24	629.01	639.64	591.67
77	697.45	642.32	653.61	607.09
78	709.12	656.18	667.7	623.45
79	720.24	669.52	681.94	638.56

Next, Figure 4.10 shows a comparison of TMO2008 and our proposed the whole life insurance premium for both genders.

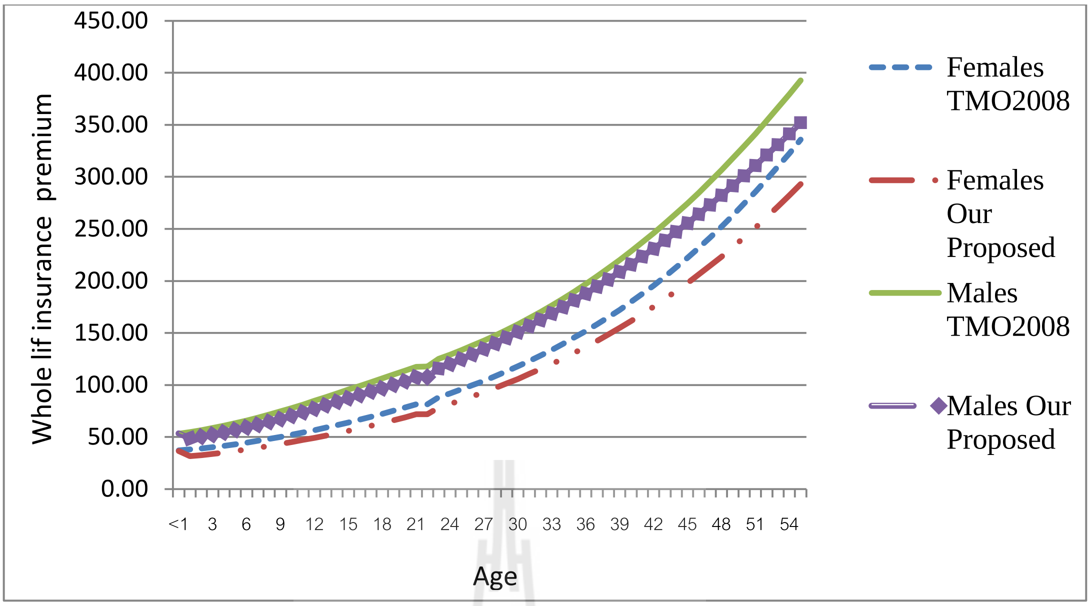


Figure 4.10 The graphs of the whole life insurance premium using the TMO2008 and our proposed mortality rate.

4.8.1.3 The n-year endowment insurance premium

We consider n-year endowment insurance premium by divided as 3 cases:

- 10- year endowment insurance
- 15- year endowment insurance
- 20- year endowment insurance

Table 4.18 The premium of the 10, 15 and 20-year endowment insurance, males.

Males	10- year endowment		15- year endowment		20- year endowment	
Age	TMO 2008	Our proposed	TMO 2008	Our proposed	TMO 2008	Our proposed
<1	662.99	660.23	529.31	527.9	423.48	422.66
1	664.96	660.01	530.96	526.38	424.79	420.1
2	667.03	661.95	532.65	527.78	426.14	421.52
3	669.18	664.02	534.39	529.32	427.61	423.11
4	671.4	666.2	536.19	530.96	429.2	424.85
5	673.67	668.4	538.04	532.68	430.94	426.72
6	675.99	670.55	539.97	534.53	432.83	428.74
7	678.33	672.51	541.92	536.57	434.87	430.84
8	680.7	674.64	543.98	538.8	437.07	433.14
9	683.1	676.78	546.16	541.09	439.42	435.53
10	685.54	679.05	548.47	543.56	441.92	438.05
11	688.03	681.41	550.91	546.14	444.56	440.61
12	690.5	684.05	553.48	548.87	447.32	443.32
13	693.06	686.87	556.18	551.76	450.18	446.09
14	695.7	689.73	559	554.72	453.14	448.89
15	698.46	692.68	561.93	557.65	456.16	451.62
16	701.32	695.64	564.96	560.44	459.24	454.21
17	704.31	698.73	568.08	563.38	462.36	456.8
18	707.43	701.97	571.28	566.31	465.52	459.41
19	710.68	705.3	574.57	569.3	468.72	462.08
20	714.07	708.72	577.96	572.35	471.98	464.87
21	717.57	712.23	581.39	575.59	475.22	467.73
22	717.74	712.03	581.43	574.92	475.03	466.92
23	725.04	719.7	588.57	582.18	481.92	473.9
24	729.02	723.57	592.34	585.65	485.42	477.22
25	733.16	727.56	596.24	589.31	489.02	480.56
26	737.48	731.67	600.28	592.92	492.75	484.04
27	741.98	735.79	604.46	596.83	496.59	487.7
28	746.66	740.06	608.81	600.73	500.64	491.45
29	751.51	744.46	613.3	604.9	504.83	495.31
30	756.54	749.04	617.94	609.02	509.14	499.2

Table 4.18 The premium of 10, 15 and 20-year endowment insurance, males (Continued).

Males	10- year endowment		15- year endowment		20- year endowment	
Age	TMO 2008	Our proposed	TMO 2008	Our proposed	TMO 2008	Our proposed
31	761.75	753.57	622.74	613.32	513.59	503.26
32	767.14	758.38	627.68	617.71	518.15	507.29
33	772.72	763.15	632.87	622.19	522.81	511.43
34	778.48	768.27	638.2	626.79	527.58	515.55
35	784.42	773.36	643.68	631.48	532.43	519.76
36	790.53	778.55	649.29	636.19	537.34	523.98
37	796.8	783.89	655.02	640.91	542.29	528.54
38	803.39	789.35	660.86	645.77	547.27	533.28
39	810.14	794.97	666.78	650.62	552.24	537.79
40	817.05	800.63	672.78	655.46	557.18	542.46
41	824.11	806.48	678.82	660.54	562.05	547.08
42	831.32	812.26	684.9	665.91	566.83	551.63
43	838.65	818.27	690.97	671.62	571.47	556.32
44	846.09	824.17	697.01	676.86	575.94	560.73
45	853.6	830.2	702.97	682.49	580.2	565.33
46	861.18	836.43	708.82	687.9	584.24	569.93
47	868.81	843.14	714.56	693.35	588.13	574.21
48	876.43	850.17	720.11	698.77	591.8	578.41
49	884.01	856.66	725.4	703.93	595.27	582.79
50	891.5	863.66	730.39	709.3	598.54	587.16
51	898.83	870.32	735.05	714.56	601.64	591.27
52	905.96	877.04	739.36	719.4	604.59	595
53	912.8	883.67	743.31	724	607.38	598.38
54	919.3	889.96	746.9	728.82	610.01	602.05
55	925.38	896.5	750.15	733.58	612.48	605.3

Table 4.19 The premium of the 10, 15 and 20-year endowment insurance, females.

Females	10 endowment		15- year endowment		20- year endowment	
Age	TMO 2008	Our proposed	TMO 2008	Our proposed	TMO 2008	Our proposed
<1	647.25	644.37	513.99	512.04	409.58	408.19
1	648.46	643.55	515.08	510.26	410.57	405.68
2	649.76	644.72	516.25	511.27	411.64	406.7
3	651.13	646.07	517.51	512.51	412.82	407.94
4	652.58	647.42	518.86	513.77	414.11	409.24
5	654.11	648.83	520.3	515.13	415.51	410.65
6	655.71	650.3	521.83	516.58	417.01	412.13
7	657.39	651.79	523.45	518.12	418.62	413.68
8	659.16	653.41	525.15	519.75	420.34	415.28
9	661	655.05	526.96	521.45	422.15	416.94
10	662.93	656.81	528.85	523.28	424.05	418.72
11	664.94	658.67	530.84	525.17	426.05	420.52
12	667.03	660.66	532.93	527.18	428.14	422.47
13	669.21	662.72	535.11	529.21	430.31	424.42
14	671.49	664.87	537.39	531.27	432.57	426.46
15	673.87	667.14	539.78	533.46	434.92	428.56
16	676.37	669.47	542.26	535.65	437.36	430.7
17	678.98	671.93	544.85	538.01	439.89	432.93
18	681.7	674.45	547.55	540.4	442.52	435.25
19	684.56	677.03	550.36	542.94	445.26	437.68
20	687.55	679.79	553.28	545.57	448.11	440.19
21	690.66	682.61	556.31	548.33	451.07	442.86
22	690.63	682.48	556.18	548.03	450.86	442.5
23	697.31	688.7	562.76	554.17	457.38	448.51
24	700.85	691.96	566.19	557.31	460.75	451.56
25	704.54	695.35	569.78	560.54	464.26	454.66
26	708.39	698.87	573.52	563.93	467.93	457.88
27	712.41	702.45	577.43	567.43	471.74	461.27
28	716.6	706.24	581.53	571.03	475.71	464.67
29	720.97	710.2	585.81	574.84	479.83	468.23
30	725.54	714.22	590.28	578.66	484.11	471.86

Table 4.19 The premium of 10, 15 and 20-year endowment insurance, females (Continued).

Females	10 endowment		15- year endowment		20- year endowment	
Age	TMO 2008	Our proposed	TMO 2008	Our proposed	TMO 2008	Our proposed
31	730.3	718.47	594.92	582.66	488.53	475.68
32	735.26	722.84	599.75	586.86	493.09	479.58
33	740.48	727.4	604.77	591.14	497.78	483.65
34	745.9	732.16	609.97	595.54	502.58	487.82
35	751.56	737	615.35	600.11	507.48	492.05
36	757.43	741.97	620.91	604.8	512.47	496.34
37	763.54	747.26	626.63	609.66	517.53	500.94
38	769.89	752.62	632.52	614.69	522.65	505.7
39	776.47	758.12	638.54	619.83	527.81	510.38
40	783.28	763.83	644.69	625.06	533.01	515.34
41	790.32	769.76	650.95	630.42	538.23	520.21
42	797.58	775.83	657.32	636.09	543.47	525.21
43	805.05	782.13	663.77	641.98	548.71	530.46
44	812.7	788.58	670.26	647.74	553.91	535.47
45	820.51	795.1	676.79	653.81	559.05	540.51
46	828.46	801.82	683.33	659.83	564.1	545.81
47	836.52	808.89	689.86	665.93	569.02	551.07
48	844.66	816.21	696.35	672.27	573.77	556.25
49	852.86	823.42	702.77	678.39	578.3	561.54
50	861.08	831.03	709.08	684.54	582.59	566.76
51	869.3	838.48	715.21	690.89	586.6	572.09
52	877.48	846.1	721.12	697.25	590.33	577
53	885.59	854.05	726.74	703.53	593.79	582.15
54	893.57	861.67	732	709.85	597	587.16
55	901.38	869.28	736.85	716	600.02	591.58

Next, Figures 4.11 and 4.13 show graphs of the 10, 15 and 20-year endowment insurance premium using the TMO2008 and our proposed premiums.

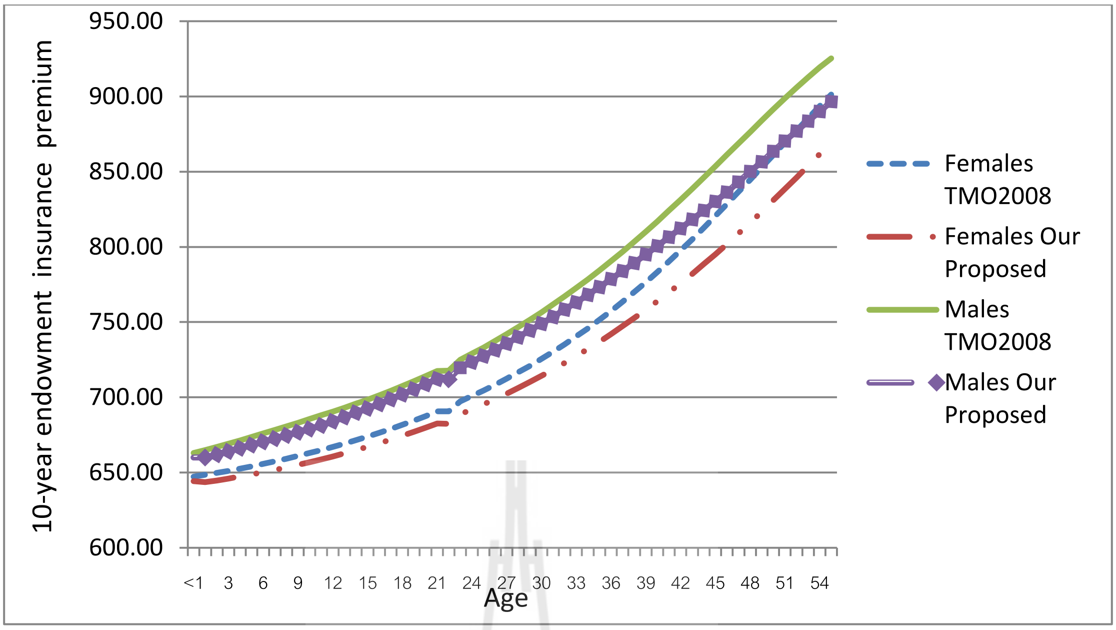


Figure 4.11 The 10-year endowment insurance premium using the TMO2008 and our proposed mortality rate for both genders.

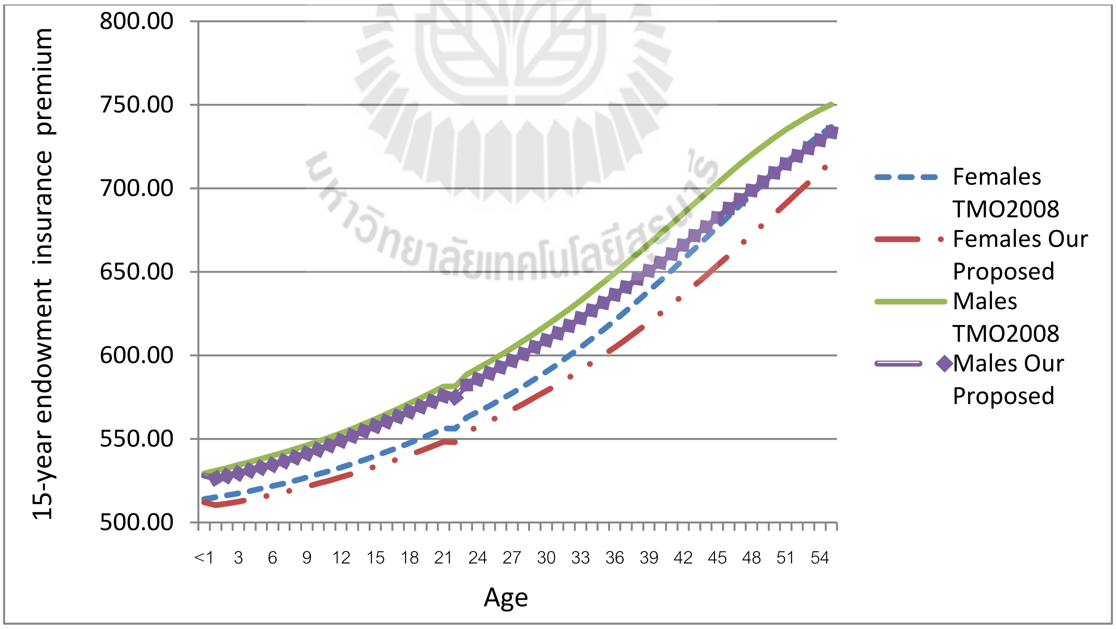


Figure 4.12 The 15-year endowment insurance premium using the TMO2008 and our proposed mortality rate for both genders.

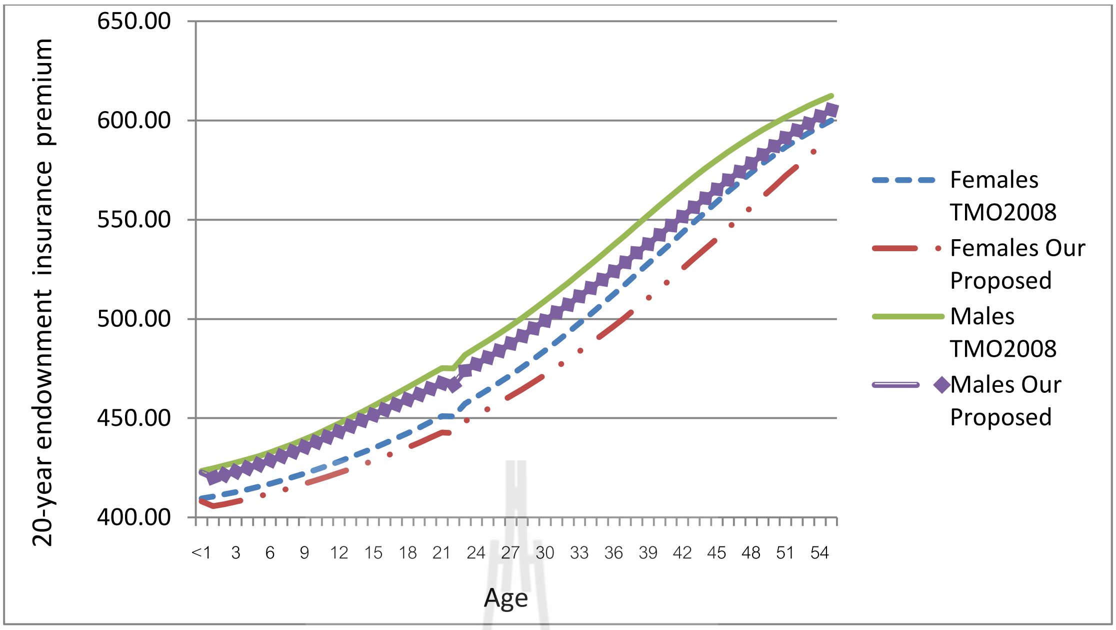


Figure 4.13 The 20-year endowment insurance premium using the TMO2008 and our proposed mortality rate for both genders.

The results of the insurance premium calculation show that the premiums which are computed by our method are lower than those computed using TMO2008. Moreover, all premiums of females are lower than the values of premiums for males.

In the next section, we calculate the 95% confidence interval of the premium.

4.8.2 The 95% confidence interval of the premium

4.8.2.1 The n-temporary insurance premium

Table 4.20 and Table 4.21 show the 95% confidence interval of the premium of 10, 15 and 20-year temporary insurance for males and females respectively.

Table 4.20 The 95% confidence interval of the 10, 15 and 20-year temporary insurance premium, males.

	n-year temporary insurance					
Males	n=10		n=15		n=20	
Age	L	U	L	U	L	U
<1	10.12	11.44	11.33	12.90	13.97	15.81
1	3.04	3.62	4.73	5.59	7.38	8.51
2	2.82	3.28	4.97	5.78	7.61	8.68
3	2.61	3.08	5.27	6.12	7.91	9.02
4	2.66	3.19	5.80	6.69	8.39	9.56
5	2.91	3.51	6.39	7.36	8.93	10.16
6	3.52	4.16	6.99	8.00	9.58	10.86
7	4.10	4.84	7.55	8.64	10.20	11.53
8	4.87	5.68	8.32	9.48	10.99	12.34
9	5.70	6.56	9.09	10.31	11.81	13.21
10	6.65	7.57	9.97	11.24	12.80	14.25
11	7.43	8.40	10.82	12.15	13.79	15.27
12	8.21	9.26	11.68	13.05	14.88	16.39
13	9.07	10.19	12.56	13.95	16.00	17.58
14	9.83	10.93	13.38	14.72	17.08	18.64
15	10.33	11.41	14.05	15.35	17.99	19.52
16	10.40	11.51	14.29	15.60	18.65	20.19
17	10.49	11.52	14.70	15.92	19.42	20.90
18	10.52	11.48	15.04	16.26	20.17	21.63
19	10.52	11.47	15.39	16.64	20.92	22.42
20	10.62	11.53	15.81	17.03	21.85	23.34
21	10.98	11.87	16.71	17.91	23.07	24.57
22	11.53	12.34	17.75	18.90	24.55	26.09
23	11.99	12.80	18.74	19.87	25.92	27.48
24	12.55	13.38	19.83	21.00	27.55	29.13
25	13.27	14.07	21.21	22.37	29.29	30.96
26	14.30	15.05	22.66	23.82	31.22	32.97
27	15.48	16.27	24.43	25.75	33.47	35.35
28	16.73	17.61	26.18	27.63	35.82	37.88
29	18.01	18.99	28.16	29.70	38.34	40.55
30	19.46	20.48	30.10	31.78	40.90	43.23
31	20.96	22.05	32.25	34.11	43.78	46.26
32	22.59	23.89	34.51	36.56	46.69	49.37
33	24.20	25.52	36.92	39.05	49.85	52.63

Table 4.20 The 95% confidence interval of the 10, 15 and 20-year temporary insurance premium, males (Continued).

	n-year temporary insurance					
Males	n=10		n=15		n=20	
Age	L	U	L	U	L	U
34	26.06	27.40	39.49	41.72	53.20	56.18
35	27.88	29.41	42.15	44.54	56.65	59.83
36	29.49	31.26	44.74	47.34	59.73	63.33
37	31.40	33.34	47.53	50.31	63.06	66.89
38	33.39	35.47	50.53	53.49	66.85	70.97
39	35.61	37.80	53.79	57.00	70.72	75.13
40	37.64	39.98	56.90	60.32	74.76	79.52
41	40.12	42.65	60.06	63.93	79.32	84.40
42	42.49	44.99	63.17	67.10	84.04	89.28
43	45.33	47.98	67.07	71.31	89.41	94.87
44	48.07	51.08	70.66	75.32	94.78	100.45
45	51.08	54.13	74.94	79.82	100.54	106.42
46	53.81	57.22	79.59	84.66	107.00	113.03
47	56.49	60.04	84.47	89.82	113.84	120.09
48	59.87	63.72	89.85	95.39	120.84	127.66
49	63.03	67.23	95.46	101.09	128.39	135.42
50	66.82	71.42	101.31	107.31	136.56	144.10
51	70.82	75.71	107.83	114.09	145.40	153.19
52	75.34	80.59	115.08	121.63	155.13	163.32
53	80.25	85.67	122.26	129.52	164.85	173.28
54	85.37	90.63	130.13	137.40	175.67	184.12
55	90.80	96.41	138.82	146.66	187.09	195.78

Table 4.21 The 95% confidence interval of the 10, 15 and 20-year temporary insurance premium, females.

	n-year temporary insurance					
Females	n=10		n=15		n=20	
Age	L	U	L	U	L	U
<1	8.59	9.30	9.38	10.21	10.23	11.19
1	2.43	2.86	3.31	3.86	4.17	4.86
2	2.07	2.45	3.05	3.57	3.92	4.59
3	2.07	2.43	3.12	3.64	3.98	4.65
4	2.06	2.41	3.15	3.68	4.01	4.67
5	2.13	2.49	3.24	3.79	4.08	4.76
6	2.24	2.62	3.36	3.93	4.21	4.90
7	2.35	2.72	3.49	4.06	4.37	5.05
8	2.50	2.87	3.62	4.19	4.54	5.24
9	2.64	3.04	3.75	4.34	4.74	5.45
10	2.83	3.26	3.92	4.53	5.01	5.77
11	2.97	3.44	4.08	4.71	5.24	6.03
12	3.17	3.67	4.31	4.96	5.58	6.40
13	3.27	3.80	4.48	5.17	5.82	6.66
14	3.33	3.86	4.62	5.32	6.06	6.92
15	3.35	3.89	4.78	5.51	6.30	7.19
16	3.36	3.91	4.89	5.65	6.56	7.49
17	3.46	3.99	5.11	5.87	6.87	7.81
18	3.51	4.07	5.25	6.02	7.17	8.13
19	3.60	4.13	5.48	6.22	7.56	8.50
20	3.75	4.31	5.75	6.50	7.99	8.93
21	3.90	4.46	6.09	6.87	8.47	9.43
22	4.13	4.69	6.44	7.24	9.02	10.01
23	4.36	4.91	6.87	7.68	9.62	10.62
24	4.68	5.26	7.40	8.24	10.36	11.40
25	5.06	5.66	7.99	8.85	11.22	12.28
26	5.49	6.14	8.60	9.49	12.03	13.13
27	5.86	6.58	9.24	10.20	12.96	14.11
28	6.28	6.97	9.89	10.83	13.99	15.11
29	6.80	7.51	10.69	11.65	15.09	16.26
30	7.27	7.95	11.50	12.45	16.21	17.38
31	7.84	8.54	12.34	13.31	17.49	18.67
32	8.40	9.15	13.28	14.27	18.79	20.03
33	9.03	9.83	14.42	15.44	20.29	21.58
34	9.77	10.58	15.55	16.62	21.92	23.26
35	10.59	11.39	16.78	17.86	23.73	25.10

Table 4.21 The 95% confidence interval of the 10, 15 and 20-year temporary insurance premium, females (Continued).

Females	n-year temporary insurance					
	n=10		n=15		n=20	
Age	L	U	L	U	L	U
36	11.26	12.04	18.02	19.08	25.45	26.85
37	12.24	12.99	19.47	20.55	27.42	28.84
38	13.28	14.03	20.99	22.11	29.65	31.14
39	14.29	15.1	22.67	23.84	31.87	33.45
40	15.42	16.28	24.56	25.8	34.53	36.19
41	16.66	17.51	26.43	27.73	37.36	39.1
42	17.96	18.83	28.42	29.74	40.34	42.15
43	19.46	20.36	30.86	32.25	43.88	45.78
44	21.04	21.98	33.16	34.65	47.67	49.63
45	22.76	23.78	35.9	37.49	51.55	53.66
46	24.61	25.71	39.03	40.72	56.04	58.28
47	26.38	27.56	42.11	43.95	60.82	63.24
48	28.48	29.79	45.68	47.68	65.85	68.41
49	30.62	32	49.82	51.82	71.48	74.1
50	33.33	34.78	54.05	56.2	77.52	80.23
51	36.19	37.78	58.74	61.06	84.38	87.4
52	39.19	40.89	64.01	66.49	91.55	94.76
53	42.85	44.66	69.63	72.21	99.31	102.68
54	46.84	48.69	75.65	78.34	108.04	111.59
55	50.76	52.76	82.02	84.79	117.47	121.27

4.8.2.2 The whole life insurance premium

Table 4.22 shows the 95% confidence interval of the premium of whole life insurance.

Table 4.22 The 95% confidence interval of the whole life insurance premium.

	Male		Female	
Age	L	U	L	U
<1	51.06	54.11	35.66	37.32
1	46.13	48.53	30.85	32.26
2	47.87	50.22	31.78	33.18
3	49.69	52.08	33.07	34.47
4	51.77	54.24	34.39	35.81
5	53.97	56.53	35.84	37.29
6	56.37	58.99	37.37	38.85
7	58.83	61.54	39.00	40.49
8	61.52	64.31	40.69	42.20
9	64.31	67.17	42.48	44.02
10	67.28	70.23	44.39	45.98
11	70.35	73.37	46.34	47.98
12	73.54	76.66	48.45	50.13
13	76.84	80.04	50.55	52.27
14	80.17	83.38	52.73	54.48
15	83.38	86.61	54.98	56.76
16	86.33	89.63	57.30	59.12
17	89.48	92.77	59.78	61.63
18	92.66	95.99	62.32	64.19
19	95.96	99.34	65.02	66.89
20	99.37	102.80	67.85	69.75
21	103.20	106.69	70.81	72.74
22	103.32	106.84	70.84	72.77
23	111.36	114.92	77.22	79.20
24	115.69	119.31	80.69	82.72
25	120.22	123.91	84.34	86.42
26	125.00	128.75	88.11	90.24
27	130.06	133.93	92.03	94.23
28	135.28	139.31	96.11	98.30
29	140.71	144.88	100.39	102.63
30	146.21	150.51	104.78	107.04
31	152.00	156.44	109.40	111.71
32	157.91	162.51	114.17	116.53
33	164.02	168.72	119.20	121.61
34	170.37	175.15	124.40	126.86
35	176.84	181.80	129.81	132.31
36	183.27	188.43	135.36	137.88
37	189.99	195.29	141.24	143.79
38	196.88	202.35	147.30	149.89

Table 4.22 The 95% confidence interval of the whole life insurance premium, both genders (Continued).

	Male		Female	
Age	L	U	L	U
39	204.1	209.66	153.56	156.21
40	211.24	217	160.1	162.82
41	218.81	224.73	166.88	169.65
42	226.42	232.34	173.92	176.72
43	234.44	240.51	181.29	184.13
44	242.54	248.78	188.81	191.72
45	250.94	257.24	196.58	199.57
46	259.63	265.96	204.8	207.86
47	268.42	274.82	213.17	216.32
48	277.46	283.98	221.75	225
49	286.79	293.38	230.75	234.03
50	296.31	303.08	240.05	243.4
51	306.01	312.87	249.66	253.11
52	316.03	322.99	259.63	263.14
53	326.2	333.27	269.92	273.48
54	336.71	343.68	280.43	284.06
55	347.47	354.59	291.21	294.91
56	359.13	365.96	302.61	306.35
57	371.15	378.01	314.43	318.24
58	382.9	389.69	326.29	330.13
59	395.48	402.09	338.83	342.67
60	407.83	414.36	351.29	355.2
61	420.41	426.86	364.19	368.14
62	433.13	439.44	377.63	381.59
63	445.86	452.27	390.97	394.96
64	459.12	465.68	404.49	408.59
65	472.7	479.24	418.67	422.78
66	486.07	492.59	433.06	437.2
67	499.4	505.88	447.61	451.72
68	513.48	519.4	462.59	466.78
69	527.71	533.62	477.8	482.07
70	541.78	547.39	493.43	497.82

Table 4.22 The 95% confidence interval of the whole life insurance premium, both genders (Continued).

	Male		Female	
Age	L	U	L	U
71	555.64	561.2	508.79	513.03
72	569.22	574.52	524.95	529.14
73	583.74	589.07	541.37	545.59
74	597.78	603.05	557.32	561.55
75	612	617.37	573.35	577.49
76	626.53	632	589.85	594.09
77	640.05	645.46	605.32	609.59
78	654.25	659.68	621.68	625.98
79	667.86	673.45	636.71	640.99

4.8.2.3 The n-year endowment insurance premium

Table 4.23 and Table 4.24 show the 95% confidence interval of the 10, 15 and 20-year endowment insurance premium for males and females respectively.

Table 4.23 The 95% confidence interval of the 10, 15 and 20-year endowment insurance premium, males.

	n-year endowment insurance					
Males	n=10		n=15		n-20	
Age	L	U	L	U	L	U
<1	657.25	661.2	524.91	528.88	419.7	423.73
1	657.45	660.26	523.65	526.64	417.38	420.5
2	659.43	662.11	525.08	527.97	418.89	421.94
3	661.4	664.14	526.51	529.51	420.44	423.57
4	663.48	666.3	528.07	531.19	422.11	425.35
5	665.51	668.49	529.73	532.97	423.91	427.28
6	667.52	670.62	531.51	534.85	425.88	429.33
7	669.36	672.59	533.51	536.97	427.92	431.5
8	671.35	674.73	535.68	539.28	430.2	433.86

Table 4.23 The 95% confidence interval of the 10, 15 and 20-year endowment insurance premium, males (Continued).

	n-year endowment insurance					
Males	n=10		n=15		n-20	
Age	L	U	L	U	L	U
9	673.40	676.92	537.90	541.62	432.56	436.29
10	675.59	679.24	540.30	544.14	435.05	438.87
11	677.89	681.63	542.82	546.75	437.57	441.46
12	680.48	684.35	545.48	549.53	440.22	444.22
13	683.22	687.24	548.32	552.46	442.93	447.01
14	686.03	690.06	551.27	555.37	445.68	449.77
15	688.93	692.97	554.19	558.28	448.37	452.51
16	691.82	695.93	556.93	561.08	450.91	455.10
17	694.92	698.99	559.88	563.98	453.49	457.63
18	698.17	702.24	562.82	566.92	456.09	460.26
19	701.52	705.59	565.79	569.96	458.73	462.97
20	704.95	709.04	568.85	573.10	461.47	465.79
21	708.49	712.61	572.12	576.41	464.34	468.73
22	708.32	712.42	571.48	575.75	463.53	467.93
23	716.04	720.14	578.77	583.07	470.43	474.97
24	719.90	724.09	582.23	586.60	473.68	478.32
25	723.90	728.21	585.83	590.33	476.97	481.71
26	728.06	732.39	589.45	594.01	480.39	485.27
27	732.17	736.61	593.32	598.04	483.96	489.08
28	736.39	741.07	597.06	602.12	487.55	492.95
29	740.76	745.61	601.15	606.40	491.30	496.92
30	745.27	750.35	605.21	610.66	495.08	500.95
31	749.79	755.02	609.42	615.12	499.05	505.10
32	754.54	759.99	613.69	619.72	502.96	509.29
33	759.21	764.90	618.06	624.28	507.05	513.56
34	764.30	770.09	622.61	628.96	511.21	517.87
35	769.32	775.38	627.17	633.85	515.30	522.37
36	774.38	780.79	631.75	638.70	519.47	526.82
37	779.63	786.35	636.38	643.60	523.87	531.69
38	784.91	791.94	641.14	648.65	528.54	536.65
39	790.45	797.62	646.02	653.69	533.00	541.38
40	795.96	803.49	650.71	658.86	537.55	546.39
41	801.70	809.44	655.76	664.16	542.06	551.24
42	807.51	815.25	661.05	669.74	546.55	555.85
43	813.41	821.43	666.66	675.66	551.00	560.66
44	819.30	827.60	671.80	681.19	555.38	565.28
45	825.28	833.88	677.39	687.11	560.04	570.02

Table 4.23 The 95% confidence interval of the 10, 15 and 20-year endowment insurance premium, males (Continued).

	n-year endowment insurance					
Males	n=10		n=15		n-20	
Age	L	U	L	U	L	U
46	831.61	840.22	682.77	692.65	564.69	574.78
47	838.16	847.24	688.06	698.19	568.93	579.22
48	845.1	854.46	693.2	703.69	572.97	583.52
49	851.55	861.18	698.38	708.98	577.03	588.1
50	858.39	868.59	703.72	714.56	581.27	592.64
51	864.94	875.43	708.96	720.03	585.02	596.73
52	871.48	882.29	713.8	725.12	588.65	600.55
53	877.81	889.02	718.29	729.9	591.7	603.94
54	884.33	895.31	722.87	734.86	595.52	607.59
55	890.95	902.1	727.6	739.87	598.63	610.95

Table 4.24 The 95% confidence interval of the 10, 15 and 20-year endowment insurance premium, females.

	n-year endowment insurance					
Females	n=10		n=15		n-20	
Age	L	U	L	U	L	U
<1	643.31	645.47	510.98	513.14	407.14	409.32
1	642.71	644.44	509.37	511.14	404.80	406.64
2	643.89	645.57	510.41	512.14	405.81	407.62
3	645.25	646.92	511.64	513.39	407.04	408.86
4	646.59	648.26	512.90	514.66	408.32	410.16
5	647.97	649.68	514.24	516.06	409.71	411.58
6	649.41	651.16	515.67	517.54	411.17	413.09
7	650.92	652.67	517.19	519.08	412.72	414.64
8	652.52	654.30	518.79	520.70	414.32	416.25
9	654.16	655.98	520.47	522.43	415.96	417.95
10	655.89	657.80	522.28	524.29	417.72	419.77
11	657.71	659.69	524.15	526.22	419.48	421.61
12	659.65	661.70	526.12	528.24	421.40	423.59
13	661.67	663.77	528.13	530.29	423.32	425.56
14	663.79	665.93	530.18	532.39	425.35	427.61

Table 4.24 The 95% confidence interval of the 10, 15 and 20-year endowment insurance premium, females (Continued).

	n-year endowment insurance					
Females	n=10		n=15		n-20	
Age	L	U	L	U	L	U
15	666.03	668.21	532.33	534.60	427.41	429.73
16	668.36	670.59	534.50	536.84	429.54	431.91
17	670.81	673.05	536.85	539.21	431.75	434.16
18	673.33	675.58	539.24	541.63	434.06	436.52
19	675.93	678.19	541.80	544.17	436.49	438.95
20	678.67	680.97	544.42	546.83	439.00	441.49
21	681.47	683.81	547.15	549.59	441.63	444.14
22	681.36	683.70	546.85	549.33	441.27	443.80
23	687.52	689.92	552.93	555.47	447.20	449.81
24	690.77	693.23	556.02	558.64	450.20	452.89
25	694.14	696.65	559.24	561.91	453.29	456.03
26	697.61	700.20	562.59	565.32	456.47	459.27
27	701.14	703.86	566.03	568.87	459.81	462.71
28	704.95	707.65	569.63	572.46	463.24	466.12
29	708.85	711.62	573.39	576.29	466.75	469.68
30	712.89	715.66	577.22	580.14	470.37	473.35
31	717.11	719.92	581.19	584.15	474.15	477.19
32	721.44	724.33	585.36	588.39	478.03	481.13
33	725.93	728.92	589.61	592.72	482.07	485.27
34	730.65	733.71	593.97	597.12	486.21	489.49
35	735.48	738.57	598.50	601.72	490.43	493.77
36	740.45	743.54	603.18	606.42	494.70	498.07
37	745.74	748.85	608.05	611.31	499.28	502.72
38	751.08	754.22	613.04	616.39	504.00	507.52
39	756.52	759.72	618.15	621.59	508.61	512.24
40	762.16	765.50	623.34	626.90	513.51	517.30
41	768.04	771.44	628.65	632.28	518.33	522.20
42	774.13	777.55	634.27	637.99	523.28	527.24
43	780.40	783.89	640.13	643.92	528.46	532.53
44	786.81	790.41	645.81	649.74	533.39	537.61
45	793.29	797.02	651.81	655.92	538.34	542.67
46	799.93	803.78	657.75	661.99	543.59	548.08
47	806.93	810.95	663.77	668.19	548.74	553.42
48	814.16	818.35	669.98	674.60	553.75	558.65
49	821.31	825.57	676.04	680.75	559.01	563.98

Table 4.24 The 95% confidence interval of the 10, 15 and 20-year endowment insurance premium, females (Continued).

	n-year endowment insurance					
Females	n=10		n=15		n-20	
Age	L	U	L	U	L	U
50	828.87	833.31	682.1	686.9	564.21	569.3
51	836.21	840.81	688.37	693.41	569.48	574.72
52	843.78	848.52	694.66	699.84	574.21	579.73
53	851.64	856.53	700.79	706.16	579.32	585.04
54	859.16	864.21	707.06	712.56	584.33	590.22
55	866.7	871.82	713.22	718.84	588.68	594.74

4.9 Discussion

In this section, we have calculated the Thai mortality rate using the Lee-Carter model and the age-period-cohort model. We have considered the distribution of the number of deaths and propose the negative binomial to the age-period-cohort model. The results show that the suitable mortality model for our data is the age-period-cohort model underlying the age-independent dispersion parameter. The forecasted mortality rate shows a decreasing trend. With this result, the Thai populations will have long lifespans in the future. As for the life insurance premium calculation, the results indicate that the premiums obtained by using our analysis (by the age-period-cohort model underlying the age-independent dispersion parameter) are lower than those obtained by using the Thai mortality table 2008 (TMO2008). Therefore, we can conclude that the mortality rate which tends to decrease effects the life insurance premium, which should make the life insurance premiums tend to decline in the future. Lastly, we have also derived the 95% confidence interval of the life insurance premium.

CHAPTER V

FORECASTING PORTFOLIO MORTALITY: PORTFOLIO OF NAKORN RATCHASIMA PROVINCE

In this chapter, we wish to consider the problem of forecasting portfolio mortality. In the insurance business, the insurer is interested in life tables dealing with life annuities since these provide information about the longevity risk. The longevity risks need to be quantified by using actuarial tools, for example, the binomial model and the Poisson model. We consider the negative binomial distribution to assess the longevity risk, by using the portfolio data from Nakorn Ratchasima province. This portfolio is composed of the observed mortality data (the number of deaths and the number of living people) for the years 1998 to 2012, and the portfolio is divided into males and females for each year and age. The portfolio mortality can be forecast based on the forecasting of the population mortality by using the relation of the portfolio mortality and the population. Finally, the portfolio mortality and life expectancy are forecast from the data observed. We also compute a 95% confidence interval for the mortality rate of the portfolio.

5.1 Introduction to Bayesian statistics theory

Bayesian statistics describe and study the uncertainty in decision making and reasoning. The parameters in Bayesian statistics are treated as random variables which have a probability distribution but unknown value. For classical statistics, the parameters are fixed. So the major difference between classical and Bayesian statistics is that Bayesian statistics takes into account the probability of the parameter in a statistical model.

In Bayesian statistics (Carlin and Louis, 2009), let the observed data $Y = y_1, y_2, \dots, y_n$ be given an unknown parameter vector. A probability distribution is given by

$$p(Y|\theta). \tag{5.1}$$

The vector θ of unknown parameters has a prior distribution $\pi(\theta|\eta)$ with η as a vector of hyper parameters. By Bayes' Theorem, the probability distribution of θ which is called the posterior distribution is given by

$$\begin{aligned} p(\theta|y,\eta) &= \frac{p(y,\theta|\eta)}{p(y|\eta)} \\ &= \frac{p(y,\theta|\eta)}{\int p(y,u|\eta)du} \\ &= \frac{p(y|\theta)\pi(\theta|\eta)}{\int p(y|u)\pi(u|\eta)du}. \end{aligned} \tag{5.2}$$

The posterior distribution can be written as follows:

$$\begin{aligned} p(\theta|y) &= \frac{p(y,\theta)}{p(y)} \\ &= \frac{p(y,\theta)}{\int p(y,u)du} \\ &= \frac{p(y|\theta)\pi(\theta)}{\int p(y|u)\pi(u)du}. \end{aligned} \tag{5.3}$$

This clearly shows that the posterior distribution is proportional to the likelihood times the prior distribution,

$$p(\theta|y) \propto f(y|\theta)\pi(\theta). \quad (5.4)$$

The Poisson distribution: Conjugate prior

Let $y_i, i = 1,2, \dots, n,$ be observed random variables from an exponential family distribution. For an exponential family, we can write the density as follows:

$$p(y_i|\theta) = c(y_i)e^{\eta(\theta)^T t(x_i) - A(\theta)}, \quad (5.5)$$

where $\eta(\theta)$ is the natural parameter,

$t(x_i)$ is the sufficient statistic,

$c(x_i)$ is the underlying measure, and

$A(\theta)$ is the log normalizer.

The general likelihood function for the exponential family density is the following:

$$f(y|\theta) \propto A(\theta)^n e^{\eta(\theta)^T \sum_i t(x_i)}. \quad (5.6)$$

The conjugate prior density is:

$$\pi(\theta) \propto A(\theta)^a e^{\eta(\theta)^T b}. \quad (5.7)$$

So the posterior distribution can be written as

$$p(\theta|y) \propto f(y|\theta)\pi(\theta)$$

$$= A(\theta)^{n+a} e^{\eta(\theta)^T (b + \sum_i t(x_i))}. \quad (5.8)$$

Conjugate prior to the Poisson distribution

Let $Y = y$ be the observed data from the Poisson distribution, the likelihood is given by

$$f(y|\lambda) = e^{-\lambda}\frac{\lambda^y}{y!}. \tag{5.9}$$

A conjugate prior $\pi(\lambda)$ with Poisson likelihood is a Gamma distribution given by

$$\pi(\lambda) = \frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda}, \tag{5.10}$$

so then we can compute the posterior distribution as follows

$$p(\lambda|y) \propto f(y|\lambda)\pi(\lambda)$$

$$= e^{-\lambda}\frac{\lambda^y}{y!}\frac{\beta^\alpha}{\Gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta\lambda}$$

$$= e^{-(\beta+1)\lambda}\frac{\lambda^{\alpha+y-1}}{y!}\frac{\beta^\alpha}{\Gamma(\alpha)}$$

$$\propto \lambda^{(\alpha+y)-1}e^{-(\beta+1)\lambda}. \tag{5.11}$$

Thus, the posterior distribution is the Gamma distribution $Gamma(\alpha + y, \beta + 1)$.

In general, the insurer uses life tables dealing with life annuities, but one cannot access the data and the method of construction of the life table which is necessary for the insurer to calculate the mortality risk or longevity risk. Forecasting portfolio mortality is described by Olivieri and Pitacco (2009), Wijk (2012) and Kan (2012). The impact of this risk needs to be quantified. We consider the risk as random fluctuations and systematic deviation. Representation of systematic deviation can be the result of an increase in the mortality rate or death count, lifestyle, living conditions, epidemic diseases, or certain weather conditions (Olivieri and Pitacco, 2009). These situations are changeable. Under the condition of the deviation, we apply a simple model proposed by Workgroup PLT (2010). Then, we construct a portfolio random mortality rate $m_{x,t}^{PORT}$ that can be shown by the following

$$m_{x,t}^{PORT} = \bar{z}_x m_{x,t}^*, \tag{5.12}$$

where $m_{x,t}^{PORT}$ expresses the mortality rate of the insured, $m_{x,t}^*$ is the mortality rate of the population and $\bar{z}_x$ is the portfolio random-coefficient factor so that

$$0 \le m_{x,t}^{PORT} \le 1. \tag{5.13}$$

The mortality rate of the insured (portfolio) and the mortality rate of the population are stable over time. The portfolio mortality can be forecasted by using Equation (5.13) based on the mortality rate of the population.

This section focuses on the portfolio random-coefficient factor which is dependent on age, and $\bar{z}_x^{PORT}$ which is the conjugate property of the Poisson-gamma distribution. We follow Olivieri and Pitacco (2009), Wijk (2012) and Kan (2012) to consider the portfolio modeling.

Let $\sum_{t=1}^{T-1} D_{x,t} | \bar{z}_x^{PORT}$ be the Poisson random variable with parameters $\bar{z}_x^{PORT} e_x^{PORT} m_x^*$ where $e_x^{PORT} m_x^* = \sum_{t=1}^{T-1} e_{x,t}^{PORT} m_{x,t}^*$, and $\bar{z}_x^{PORT}$ be the Gamma random variable, $Gamma(\bar{\alpha}, \bar{\beta})$,

$$[\textstyle\sum_{t=1}^{T-1} D_{x,t} | \bar{z}_x^{PORT}] \sim Poisson(\bar{z}_x^{PORT} e_x^{PORT} m_x^*), \tag{5.14}$$

$$[\bar{z}_x^{PORT}] \sim Gamma(\bar{\alpha}, \bar{\beta}). \tag{5.15}$$

We will determine the posterior distribution of $\bar{z}_x^{PORT}$ by using the conjugate property of the Poisson distribution. The prior distribution of $\bar{z}_x^{PORT} e_x^{PORT} m_x^*$ can be shown as

$$[\bar{z}_x^{PORT} e_x^{PORT} m_x^*] \sim Gamma(\bar{\alpha}, \frac{\bar{\beta}}{\sum_{t=1}^{T-1} e_{x,t}^{PORT} m_{x,t}^*}). \tag{5.16}$$

The posterior distribution of $\bar{z}_x^{port} e_x^{port} m_x^*$ after we observe $D_{x,1}, D, \ldots, D_{x,T-1}$,

$$\left[\bar{z}_x^{PORT} e_x^{PORT} m_x^* | \sum_{t=1}^{T-1} D_{x,t}\right] \sim Gamma\left(\bar{\alpha}, \frac{\bar{\beta}}{\sum_{t=1}^{T-1} e_{x,t}^{PORT} m_{x,t}^*}\right). \tag{5.17}$$

We also have the posterior of $\bar{z}_x^{PORT}$,

$$\left[\bar{z}_x^{PORT} | \sum_{t=1}^{T-1} D_{x,t}\right] \sim Gamma\left(\bar{\alpha} + \sum_{t=1}^{T-1} d_{x,t}, \bar{\beta} + \sum_{t=1}^{T-1} e_{x,t}^{PORT} m_{x,t}^*\right). \tag{5.18}$$

Now, we will consider the next year,

$$\left[m_{x,T}^* \bar{z}_x^{PORT} | \sum_{t=1}^{T-1} D_{x,t}\right] \sim Gamma\left(\bar{\alpha} + \sum_{t=1}^{T-1} d_{x,t}, \frac{\bar{\beta} + \sum_{t=1}^{T-1} e_{x,t}^{PORT} m_{x,t}^*}{m_{x,T}^*}\right). \tag{5.19}$$

and

$$\left[e_{x,T}^{PORT} m_{x,T}^* \bar{z}_x^{PORT} | \sum_{t=1}^{T-1} D_{x,t}\right] \sim Gamma\left(\bar{\alpha} + \sum_{t=1}^{T-1} d_{x,t}, \frac{\bar{\beta} + \sum_{t=1}^{T-1} e_{x,t}^{PORT} m_{x,t}^*}{e_{x,t}^{PORT} m_{x,T}^*}\right). \tag{5.20}$$

Then at time T, the unconditional distribution of the number of deaths of the portfolio can be shown as

$$\left[D_{x,T}^{PORT}\right] \sim NBin\left(\bar{\alpha} + \sum_{t=1}^{T-1} d_{x,t}, \frac{\frac{\bar{\beta} + \sum_{t=1}^{T-1} e_{x,t}^{PORT} m_{x,t}^*}{m_{x,T}^*}}{\frac{\bar{\beta} + \sum_{t=1}^{T-1} e_{x,t}^{PORT} m_{x,t}^*}{m_{x,T}^*} + 1}\right). \tag{5.21}$$

5.2 Portfolio mortality forecasting

We forecast the portfolio mortality after we consider the posterior distribution of the portfolio random-coefficient factor. By equation (5.12), the forecasting portfolio mortality rate can be computed as

$$\widehat{m}_{x,t_n+T}^{port} = \bar{z}_x \widehat{m}_{x,t_{n+T}}^{*}, \tag{5.22}$$

where $\widehat{m}_{x,t_n+T}^{port}$ expresses the forecasted mortality rate of the insured (portfolio), $\widehat{m}_{x,t_{n+T}}^{*}$ is the forecasted mortality rate of the population and $\bar{z}_x$ is the portfolio random-coefficient.

5.2.1 Forecasting the mortality rate of the portfolio of Nakorn Ratchasima

In chapter IV we obtained the age-period-cohort model under the assumption of the age-independent dispersion parameter as a suitable model to fit and forecast to the mortality model for Thailand. So, we now use the Bayesian property in this chapter and the appropriate estimation method in chapter IV to predict the mortality rate of the portfolio, using a pension portfolio for Nakorn Ratchasima province. The historical data is for the period from 1998 to 2012. We wish to forecast the portfolio mortality for the period from 2013 to 2022.

The following steps show the forecast the portfolio mortality rate:

(1) We estimate the parameters of the model and forecast the mortality rate for Thailand by using the age-period-cohort model underlying the age-independent dispersion parameter.

(2) We generate the portfolio random-coefficient factor from the equation (5.18).

(3) We forecast the portfolio mortality by using the equation (5.22).

5.2.2 The forecasted portfolio mortality rate

The following tables show the portfolio random-coefficient factor.

Table 5.1 The portfolio random-coefficient factor, females.

age(x)	$\bar{z}_x$	age(x)	$\bar{z}_x$	age(x)	$\bar{z}_x$	age(x)	$\bar{z}_x$
<1	1.000005	26	1	52	0.999995	78	0.999997
1	1.000014	27	1.000003	53	1.000018	79	1.000003
2	0.999999	28	1.00002	54	0.999993	80	0.999974
3	1.000007	29	1.000004	55	0.999978	81	0.99998
4	0.999993	30	0.999983	56	1	82	0.999994
5	1.000011	31	0.999964	57	0.999998	83	0.999984
6	0.999987	32	1.000017	58	0.999995	84	1.000016
7	1.000007	33	1.000007	59	0.999984	85	0.999993
8	0.999999	34	1.000011	60	0.999987	86	1.000004
9	1.000004	35	0.999989	61	0.999998	87	0.999991
10	0.999996	36	0.999984	62	0.999986	88	1.00001
11	1.000011	37	1.000026	63	1.000005	89	0.999995
12	0.999991	38	0.99998	64	0.999987	90	1.000005
13	1	39	0.999998	65	0.999996	91	1.000011
14	0.999997	40	0.999991	66	0.999989	92	1.000003
15	1.000015	41	1.000009	67	0.999966	93	0.999991
16	1.000013	42	0.999985	68	1.000014	94	0.99999
17	1.000007	43	1.000019	69	0.999993	95	1.000007
18	1.000002	44	1.00001	70	1.000005	96	1.000023
19	0.999982	45	1.000004	71	1.000003	97	1.000014
20	0.999964	46	0.999981	72	1.000016	98	1.000032
21	1.000014	47	0.999997	73	0.999996	99	1.000025
22	1.000019	48	0.999995	74	1.000007	100	1.000011
23	0.99999	49	0.999995	75	0.999973	>100	1.000001
24	1.000018	50	1.000011	76	1.000012		
25	1.000019	51	1.000005	77	1.000004		

Table 5.2 The portfolio random-coefficient factor, males.

age(x)	$\bar{z}_x$	age(x)	$\bar{z}_x$	age(x)	$\bar{z}_x$	age(x)	$\bar{z}_x$
<1	0.999994	26	1.000016	52	1.000002	78	1.000008
1	0.999992	27	1.000021	53	0.999987	79	0.999995
2	1.000006	28	0.999987	54	0.999986	80	0.999991
3	1.000028	29	1.000023	55	1.000014	81	1.000006
4	1.000016	30	1.000012	56	0.999988	82	0.999995
5	0.999988	31	1.00001	57	1.00005	83	1.000015
6	0.999997	32	1.000013	58	1.000003	84	0.999952
7	0.999986	33	0.999994	59	0.999987	85	1.000002
8	1	34	1.000021	60	1.000004	86	1.00001
9	1.000004	35	0.999988	61	0.999992	87	0.999987
10	0.999993	36	1.000002	62	0.999985	88	1.000015
11	1.000033	37	1.000002	63	1.000006	89	1.000015
12	0.999985	38	0.99997	64	0.999993	90	0.999985
13	0.99999	39	1.000012	65	0.999981	91	1.000001
14	1.000002	40	0.99998	66	1.000005	92	0.999992
15	1.000013	41	1.000001	67	0.999996	93	0.999996
16	1.000001	42	1	68	0.999998	94	0.999978
17	0.999995	43	0.999995	69	0.999998	95	1
18	1	44	1.000018	70	1.000012	96	1.000017
19	0.999978	45	0.999985	71	0.999998	97	0.999996
20	1.000005	46	0.999997	72	1.000021	98	1.000016
21	1.000011	47	1.000019	73	0.999988	99	0.999989
22	0.999988	48	1	74	1.000027	100	0.999999
23	1.000003	49	1.000005	75	0.999995	>100	0.999959
24	1.000012	50	1.000013	76	1.000022		
25	0.999986	51	0.999991	77	1.000021		

5.2.3 Forecasted portfolio mortality and life expectancy

The generated portfolio random coefficients shown in Table 5.1 and 5.2 are close to 1. As a result, the forecasted portfolio mortality rates are very close to the mortality rate of the total population. This means that the portfolio of Nakorn Ratchasima mortality rates differ very little from the population mortality rate. The following table shows the forecasted life expectancy of the Nakorn Ratchasima portfolio at birth, 25, 50, 65 and 70 years. More forecasted life expectancies of the portfolio are in the Appendix C.

Table 5.3 The forecasted values of life expectancy of the portfolio, males.

year	At birth	25	50	60	65	70
2013	72.07	49.01	27.60	20.02	16.55	13.36
2014	72.16	49.08	27.62	20.03	16.55	13.36
2015	72.25	49.16	27.64	20.04	16.55	13.35
2016	72.34	49.23	27.66	20.05	16.55	13.35
2017	72.42	49.30	27.68	20.06	16.56	13.35
2018	72.50	49.37	27.70	20.06	16.56	13.35
2019	72.58	49.43	27.72	20.07	16.56	13.34
2020	72.66	49.50	27.74	20.08	16.56	13.34
2021	72.74	49.57	27.76	20.09	16.57	13.34
2022	72.82	49.63	27.78	20.10	16.57	13.34

Table 5.4 The forecasted values of life expectancy of the portfolio, females.

year	At birth	25	50	60	65	70
2013	78.90	55.12	31.66	22.99	18.98	15.23
2014	78.98	55.19	31.71	23.03	19.01	15.25
2015	79.06	55.27	31.75	23.06	19.04	15.27
2016	79.14	55.34	31.80	23.10	19.06	15.29
2017	79.21	55.41	31.84	23.13	19.09	15.31
2018	79.29	55.48	31.88	23.17	19.12	15.33
2019	79.37	55.55	31.93	23.20	19.15	15.35
2020	79.44	55.61	31.97	23.24	19.17	15.37
2021	79.51	55.68	32.01	23.27	19.20	15.38
2022	79.59	55.75	32.06	23.30	19.23	15.40

In Table 5.3 and 5.4, the portfolio life expectancy and the population life expectancy (in chapter IV) are almost the same values. The following figure shows the percent of the difference life expectancy values between the Thai data and Nakorn Ratchasima data. The graph indicates that the percentages of difference are less than 0.0005% of males and less than 0.0007% for females.

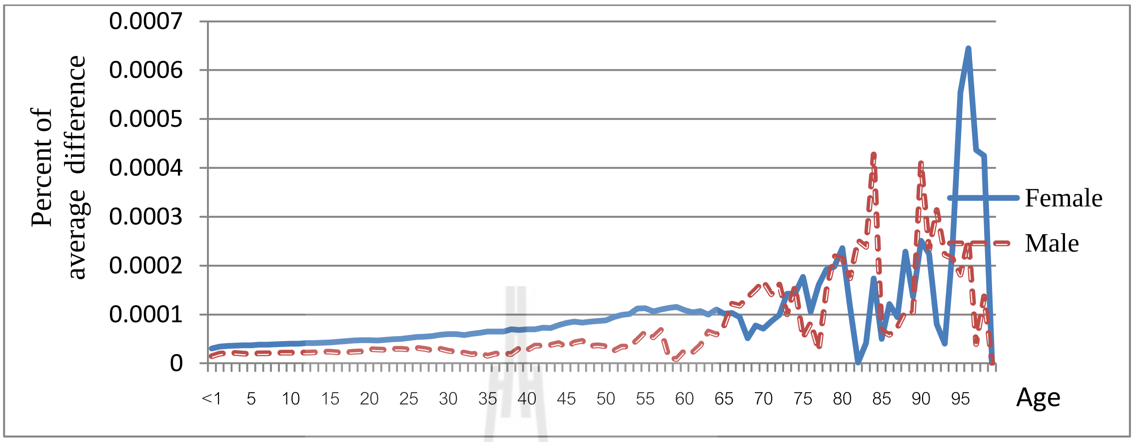


Figure 5.1 The percentage of the different life expectancy values between the Thai population data and Nakorn Ratchasima data.

5.3 Discussion

In this chapter, we have forecasted the portfolio mortality rate. We have used the data of Nakorn Ratchasima province which represents the portfolio in an insurance company. We have introduced the conjugate property in the Bayesian probability of Poisson-gamma distribution and applied it to forecast the portfolio mortality rate. The result shows that the values of portfolio random-coefficient factor are closed to 1. Consequently, the forecasted portfolio mortality rates are almost the same as the population mortality rate. Thus, the forecasted mortality rate and life expectancy of Nakorn Ratchasima portfolio are very close to the mortality rate in Thailand. We can conclude that the posterior of the portfolio data is not affected by the mortality rate of the historical portfolio.

CHAPTER VI

CONCLUSION

6.1 Discussion and conclusion

In this thesis, we have studied the mortality rate of the population of Thailand and Nakhon Ratchasima by using the Lee-Carter model and the age-period-cohort model. The first part of the thesis explains the application of the Lee-Carter model by using the classical estimation method.With these methods, we investigate the goodness of fit for the mortality rate spanning the period 2003 to 2012. The results show that the SVD is the best fit of parameter estimation for females and WLS is the best fit for males.

The second part of the thesis is shown in Chapter IV. We consider the use of the Lee-Carter model and the age-period-cohort model underlying the Poisson setting and the negative binomial setting. We then apply the negative binomial setting underlying the dispersion parameter i.e. the age-dependent dispersion parameter and the age-independent dispersion parameter. The results show that the age-period-cohort model underlying the age-independent dispersion parameter is a suitable method for our data. We forecast the Thai mortality rate and life expectancy for the period 2013 to 2022. The forecasted mortality rate shows a decreasing trend. The life expectancy at birth increases 0.87% from 2013 to 2022 for females. Similarly, the life expectancy at birth for males shows a 1.03% increase from 2013 to 2022. Thus the Thai populations are likely to experience longer lifespan in the future. Finally, we compute the 10, 15 and 20-years temporary insurance premium, whole life insurance premium, and 10, 15 and 20-years endowment insurance premium (n=10, 15 and 20 years). All premium types from our analysis give lower premiums than those obtained from the Thai mortality table in 2008 (TMO2008). We can infer that the decrease of the future Thai mortality rate effects a reduction in the premium. This evidence is very useful for an insurer.

In the third part of the thesis, we forecast the portfolio mortality rate using Nakorn Ratchasima data. We use a Bayesian approach to the Poisson-gamma model. The results show that the values of the forecasted portfolio mortality and life expectancy hardly differ from the values of the Thai population. Based on the Bayesian property, we can conclude that the posterior of the portfolio data is not affected by the mortality rate of the historical portfolio.

6.2 Recommendation for further research

In this thesis, the assumptions of the mortality model and the parameter estimation method have an impact on the forecasted mortality model. Thus, the forecasting of the mortality rate depends on the mortality model used and the method of estimation. There are many other mortality models which are very interesting to investigate, for example, the age-cohort model and the other extension of Lee-Carter model. In further research, other mortality models and methods for the estimation of parameters can be applied to mortality data for the population. These will enable us to forecast the mortality rate more accurately in the future.

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APPENDICES

APPENDIX A

ESTIMATED PARAMETERS OF THE MODEL

The estimated parameters of the mortality model are shown in the following Tables. We separate into three cases which are the results of Chapter IV.

Case1: Poisson setting

Case 2: The negative binomial setting under assumption of age-independent dispersion parameter

Case 3: The negative binomial setting under assumption of age-dependent dispersion parameter

The estimated parameters of the Lee-Carter model.

The following tables, from Table A1 to A4, show the estimated parameters of the Lee-Carter model of females.

Table A1 The estimated parameters, $\hat{\alpha}_x$, females.

Age	Case 1	Case 2	Case 3	age	Case 1	Case 2	Case 3
<1	-4.97005	-5.02024	-5.02024	26	-6.70181	-6.70357	-6.70357
1	-6.78428	-6.45236	-6.45241	27	-6.62432	-6.62609	-6.62609
2	-7.30426	-7.30819	-7.30819	28	-6.57498	-6.57681	-6.5768
3	-7.52243	-7.51962	-7.51962	29	-6.51932	-6.52025	-6.52024
4	-7.58444	-7.58567	-7.58567	30	-6.49875	-6.49951	-6.49951
5	-7.61601	-7.61711	-7.61711	31	-6.45865	-6.45952	-6.45952
6	-7.67921	-7.68069	-7.68069	32	-6.44329	-6.44366	-6.44366
7	-7.74261	-7.74353	-7.74353	33	-6.421	-6.42159	-6.42159
8	-7.78323	-7.78437	-7.78437	34	-6.40292	-6.40331	-6.40331
9	-7.87657	-7.87645	-7.87645	35	-6.37507	-6.37548	-6.37548
10	-7.89906	-7.8989	-7.8989	36	-6.36093	-6.361	-6.361
11	-7.95382	-7.95334	-7.95334	37	-6.32637	-6.32634	-6.32634
12	-7.95437	-7.95436	-7.95436	38	-6.28271	-6.28269	-6.28269
13	-7.84097	-7.841	-7.841	39	-6.25341	-6.25349	-6.25349
14	-7.67306	-7.67305	-7.67305	40	-6.19047	-6.19047	-6.19047
15	-7.51478	-7.51478	-7.51478	41	-6.13923	-6.13922	-6.13921
16	-7.4527	-7.45275	-7.45275	42	-6.08074	-6.08067	-6.08067
17	-7.45843	-7.45853	-7.45853	43	-6.01593	-6.01588	-6.01588
18	-7.41398	-7.41414	-7.41414	44	-5.95291	-5.95285	-5.95285
19	-7.39622	-7.39646	-7.39646	45	-5.88609	-5.88603	-5.88603
20	-7.33753	-7.33839	-7.33839	46	-5.82181	-5.82178	-5.82178
21	-7.22564	-7.22613	-7.22613	47	-5.74451	-5.74442	-5.74442
22	-7.15026	-7.15133	-7.15133	48	-5.65563	-5.65554	-5.65554
23	-7.02822	-7.02908	-7.02908	49	-5.58297	-5.58287	-5.58287
24	-6.90517	-6.90686	-6.90685	50	-5.49755	-5.49748	-5.49748
25	-6.77437	-6.77564	-6.77563				

Table A1 The estimated parameter $\hat{\alpha}_x$, females (Continued).

Age	Case 1	Case 2	Case 3	age	Case 1	Case 2	Case 3
51	-5.41856	-5.41845	-5.41845	77	-3.08346	-3.08347	-3.08347
52	-5.3484	-5.34827	-5.34827	78	-2.96883	-2.96882	-2.96882
53	-5.25243	-5.2523	-5.2523	79	-2.88304	-2.88299	-2.88299
54	-5.18712	-5.18712	-5.18712	80	-2.77401	-2.774	-2.774
55	-5.10387	-5.10379	-5.10379	81	-2.67525	-2.67531	-2.67531
56	-5.01144	-5.01144	-5.01144	82	-2.5629	-2.56307	-2.56307
57	-4.92961	-4.92964	-4.92964	83	-2.46684	-2.46715	-2.46715
58	-4.84061	-4.84063	-4.84063	84	-2.3392	-2.33938	-2.33938
59	-4.75695	-4.75714	-4.75714	85	-2.25987	-2.26005	-2.26005
60	-4.67474	-4.67498	-4.67498	86	-2.17359	-2.17354	-2.17354
61	-4.58885	-4.58909	-4.58909	87	-2.08707	-2.08715	-2.08715
62	-4.48579	-4.48602	-4.48602	88	-2.01065	-2.01072	-2.01072
63	-4.41003	-4.41021	-4.41021	89	-1.93539	-1.93549	-1.93549
64	-4.32395	-4.32406	-4.32406	90	-1.86387	-1.86391	-1.86391
65	-4.23858	-4.23862	-4.23862	91	-1.82682	-1.82699	-1.82699
66	-4.14132	-4.14128	-4.14128	92	-1.75857	-1.7588	-1.7588
67	-4.06204	-4.06206	-4.06206	93	-1.7347	-1.73517	-1.73518
68	-4.00459	-4.00456	-4.00456	94	-1.71871	-1.71885	-1.71885
69	-3.87789	-3.87781	-3.87781	95	-1.75909	-1.75928	-1.75928
70	-3.77526	-3.77521	-3.77521	96	-1.78749	-1.78761	-1.78761
71	-3.69106	-3.69106	-3.69106	97	-1.84839	-1.84837	-1.84837
72	-3.58954	-3.58963	-3.58963	98	-1.94079	-1.94957	-1.94957
73	-3.4836	-3.48373	-3.48373	99	-2.0709	-2.08896	-2.08896
74	-3.39032	-3.39029	-3.39029	100	-2.23414	-2.25426	-2.25426
75	-3.28549	-3.2855	-3.2855	>100	-3.22271	-3.24139	-3.24139
76	-3.18076	-3.18064	-3.18064				

Table A2 The estimated parameter, $\hat{\beta}_x$, females, Lee-Carter model.

Age	Case 1	Case 2	Case 3	age	Case 1	Case 2	Case 3
<1	-0.01081	-0.00387	-0.00387	26	0.04526	0.04717	0.04717
1	0.12356	0.09069	0.09070	27	0.04254	0.04437	0.04437
2	0.02331	0.02475	0.02476	28	0.04137	0.04314	0.04314
3	0.02408	0.02451	0.02451	29	0.03792	0.03940	0.03940
4	0.02731	0.02848	0.02848	30	0.03386	0.03517	0.03517
5	0.02668	0.02781	0.02781	31	0.03062	0.03189	0.03189
6	0.02833	0.02966	0.02966	32	0.02540	0.02640	0.02640
7	0.02514	0.02627	0.02627	33	0.02244	0.02348	0.02348
8	0.01932	0.02050	0.02050	34	0.02084	0.02175	0.02175
9	0.01237	0.01281	0.01281	35	0.01695	0.01780	0.01780
10	0.01155	0.01198	0.01198	36	0.01602	0.01668	0.01668
11	0.00980	0.00990	0.00990	37	0.01386	0.01441	0.01441
12	0.00053	0.00052	0.00052	38	0.01345	0.01403	0.01403
13	-0.00057	-0.00078	-0.00078	39	0.01211	0.01275	0.01275
14	0.00143	0.00146	0.00146	40	0.01227	0.01286	0.01286
15	0.00024	0.00047	0.00047	41	0.01192	0.01257	0.01257
16	0.00494	0.00517	0.00517	42	0.01141	0.01194	0.01194
17	0.00396	0.00422	0.00422	43	0.00992	0.01034	0.01034
18	0.00605	0.00636	0.00636	44	0.00733	0.00773	0.00773
19	0.01067	0.01108	0.01108	45	0.00978	0.01031	0.01031
20	0.01472	0.01557	0.01557	46	0.00834	0.00875	0.00875
21	0.02149	0.02236	0.02236	47	0.00903	0.00953	0.00953
22	0.02897	0.03025	0.03025	48	0.01119	0.01174	0.01174
23	0.03777	0.03923	0.03923	49	0.01035	0.01090	0.01090
24	0.04224	0.04402	0.04402	50	0.01009	0.01057	0.01057
25	0.04579	0.04757	0.04757				

Table A2 The estimated parameter, $\hat{\beta}_x$, females, Lee-Carter model (Continued).

Age	Case 1	Case 2	Case 3	age	Case 1	Case 2	Case 3
51	0.010957	0.011516	0.011516	76	0.004942	0.005182	0.005182
52	0.011377	0.011914	0.011914	77	0.005025	0.005193	0.005193
53	0.011016	0.011487	0.011487	78	0.006321	0.006586	0.006586
54	0.010773	0.011135	0.011135	79	0.005151	0.005385	0.005385
55	0.010706	0.011151	0.011151	80	0.005244	0.005447	0.005447
56	0.011301	0.011675	0.011675	81	0.004422	0.004563	0.004563
57	0.008889	0.009198	0.009198	82	0.00551	0.00567	0.00567
58	0.010791	0.011231	0.011231	83	0.003615	0.00364	0.00364
59	0.00888	0.009138	0.009138	84	0.004402	0.004528	0.004528
60	0.007459	0.007664	0.007664	85	0.003296	0.003353	0.003353
61	0.006688	0.0069	0.0069	86	0.002603	0.00276	0.00276
62	0.007111	0.007374	0.007374	87	0.001055	0.001046	0.001046
63	0.0058	0.005971	0.005971	88	-0.00102	-0.0011	-0.0011
64	0.00673	0.006971	0.006971	89	-0.00229	-0.00246	-0.00246
65	0.007039	0.007308	0.007308	90	-0.00314	-0.00331	-0.00331
66	0.005274	0.005551	0.005551	91	-0.00555	-0.00587	-0.00587
67	0.008306	0.008598	0.008598	92	-0.00826	-0.00876	-0.00876
68	0.006684	0.00697	0.00697	93	-0.0108	-0.0117	-0.0117
69	0.006686	0.006958	0.006957	94	-0.01478	-0.0158	-0.0158
70	0.006693	0.006891	0.006891	95	-0.01907	-0.02044	-0.02044
71	0.007204	0.007396	0.007396	96	-0.02195	-0.02348	-0.02348
72	0.006277	0.006382	0.006382	97	-0.02842	-0.0301	-0.0301
73	0.006667	0.006805	0.006805	98	-0.03046	-0.03318	-0.03318
74	0.006097	0.006328	0.006328	99	-0.03461	-0.03854	-0.03854
75	0.006088	0.006281	0.006281	100	-0.0403	-0.04461	-0.04461
				>100	-0.05554	-0.05949	-0.05949

Table A3 The estimated parameter, $\hat{k}_t$, females.

year	Case 1	Case 2	Case 3
1998	13.38557933	12.16938496	12.16974609
1999	11.19751517	11.34034780	11.34020021
2000	10.14452681	10.47468649	10.47447762
2001	10.03931392	10.12868216	10.12851914
2002	9.53632402	9.61187571	9.61172975
2003	6.61471371	6.30594334	6.30582475
2004	4.23806650	3.88750795	3.88757158
2005	0.07903826	-0.38408383	-0.38409828
2006	-2.19647985	-2.73379365	-2.73394117
2007	-4.19551597	-4.76274093	-4.76300203
2008	-6.69089370	-7.15378988	-7.15402588
2009	-10.08823824	-10.42133366	-10.42119944
2010	-12.41046753	-11.35302054	-11.35261975
2011	-14.48099098	-13.29942138	-13.29904818
2012	-15.17249145	-13.81024455	-13.81013441

Next, the following tables, Table A4-A6, show the estimated parameters of the Lee-Carter model of males.

Table A4 The estimated parameter, $\hat{k}_t$, males.

Year	Case 1	Case 2	Case 3
1998	6.30770873	6.10374406	6.10377107
1999	7.33419488	7.32127191	7.32126146
2000	6.51625306	6.50701981	6.50699552
2001	5.27522338	5.24902187	5.24901698
2002	4.51874944	4.49505859	4.49506311
2003	3.30511760	3.27027393	3.27027142
2004	1.39783879	1.37905520	1.37905722
2005	-0.61068162	-0.63850783	-0.63847589
2006	-1.86567800	-1.91863555	-1.91863632
2007	-3.52936263	-3.59089659	-3.59087232
2008	-4.24408407	-4.32207393	-4.32209258
2009	-5.53375075	-5.53997040	-5.53997298
2010	-5.90327395	-5.73655739	-5.73655061
2011	-6.26203036	-6.04633050	-6.04634776
2012	-6.70622450	-6.53247317	-6.53248832

Table A5 The estimated parameter, $\hat{\alpha}_x$, males.

Age	Case 1	Case 2	Case 3	age	Case 1	Case 2	Case 3
<1	-4.8068	-4.84755	-4.84755	26	-5.8012	-5.80093	-5.80093
1	-6.4389	-6.35471	-6.35472	27	-5.7154	-5.71519	-5.71519
2	-6.97209	-6.97975	-6.97975	28	-5.63268	-5.63262	-5.63262
3	-7.17588	-7.18283	-7.18283	29	-5.57435	-5.57425	-5.57425
4	-7.21257	-7.21251	-7.21251	30	-5.52557	-5.5255	-5.5255
5	-7.22305	-7.2231	-7.2231	31	-5.47511	-5.47516	-5.47516
6	-7.25458	-7.25476	-7.25476	32	-5.45094	-5.45092	-5.45092
7	-7.38736	-7.38733	-7.38733	33	-5.42374	-5.4238	-5.4238
8	-7.49607	-7.49603	-7.49603	34	-5.40518	-5.40522	-5.40522
9	-7.61533	-7.61536	-7.61536	35	-5.37353	-5.37361	-5.37361
10	-7.73011	-7.73009	-7.73009	36	-5.3621	-5.36213	-5.36213
11	-7.76088	-7.76087	-7.76087	37	-5.34509	-5.34513	-5.34513
12	-7.63832	-7.63832	-7.63832	38	-5.32263	-5.32265	-5.32265
13	-7.31713	-7.31714	-7.31714	39	-5.28915	-5.28918	-5.28918
14	-6.97977	-6.97977	-6.97977	40	-5.26723	-5.26725	-5.26725
15	-6.61782	-6.61783	-6.61783	41	-5.22837	-5.22838	-5.22838
16	-6.40481	-6.4048	-6.4048	42	-5.19282	-5.19282	-5.19282
17	-6.23312	-6.23311	-6.23311	43	-5.1533	-5.1533	-5.1533
18	-6.16538	-6.16538	-6.16538	44	-5.10941	-5.10941	-5.10941
19	-6.13222	-6.13222	-6.13222	45	-5.06066	-5.06066	-5.06066
20	-6.1141	-6.1141	-6.1141	46	-5.01309	-5.01308	-5.01308
21	-6.11392	-6.11395	-6.11395	47	-4.97577	-4.97576	-4.97576
22	-6.06289	-6.06287	-6.06287	48	-4.91046	-4.91045	-4.91045
23	-5.99295	-5.99292	-5.99292	49	-4.86197	-4.86195	-4.86195
24	-5.93971	-5.93966	-5.93966	50	-4.80072	-4.80066	-4.80066
25	-5.85767	-5.85748	-5.85748				

Table A5 The estimated parameter $\hat{\alpha}_x$, males (Continued).

Age	Case 1	Case 2	Case 3	age	Case 1	Case 2	Case 3
51	-4.73858	-4.73852	-4.73852	76	-2.86287	-2.86289	-2.86289
52	-4.6797	-4.67966	-4.67966	77	-2.78532	-2.78534	-2.78534
53	-4.61787	-4.61783	-4.61783	78	-2.68262	-2.68266	-2.68266
54	-4.54799	-4.54802	-4.54802	79	-2.61234	-2.61233	-2.61233
55	-4.4976	-4.4976	-4.4976	80	-2.53208	-2.53212	-2.53212
56	-4.43742	-4.43743	-4.43743	81	-2.44056	-2.44058	-2.44058
57	-4.36582	-4.36579	-4.36579	82	-2.36074	-2.36076	-2.36076
58	-4.30224	-4.30224	-4.30223	83	-2.28433	-2.28433	-2.28433
59	-4.22404	-4.22405	-4.22405	84	-2.20717	-2.20717	-2.20717
60	-4.15676	-4.15679	-4.15679	85	-2.12911	-2.12912	-2.12912
61	-4.08111	-4.08113	-4.08113	86	-2.05838	-2.05838	-2.05838
62	-4.01374	-4.01376	-4.01376	87	-2.01437	-2.01439	-2.01439
63	-3.94158	-3.9416	-3.9416	88	-1.97448	-1.97442	-1.97442
64	-3.85905	-3.85906	-3.85906	89	-1.92353	-1.92344	-1.92344
65	-3.78793	-3.78793	-3.78793	90	-1.89989	-1.89985	-1.89985
66	-3.69499	-3.69498	-3.69498	91	-1.8736	-1.87361	-1.87361
67	-3.64091	-3.64093	-3.64093	92	-1.87141	-1.87142	-1.87142
68	-3.61029	-3.61028	-3.61028	93	-1.88271	-1.88266	-1.88266
69	-3.47027	-3.47027	-3.47027	94	-1.91696	-1.91679	-1.91679
70	-3.3845	-3.3845	-3.3845	95	-2.01037	-2.01019	-2.01019
71	-3.30545	-3.30545	-3.30545	96	-2.08983	-2.0897	-2.0897
72	-3.21118	-3.21121	-3.21121	97	-2.21961	-2.21952	-2.21952
73	-3.12734	-3.12739	-3.12739	98	-2.35271	-2.35985	-2.35985
74	-3.04618	-3.04617	-3.04617	99	-2.50401	-2.51974	-2.51974
75	-2.95271	-2.95274	-2.95274	100	-2.71764	-2.73443	-2.73443
				>100	-3.84273	-3.83661	-3.83661

Table A6 The estimated parameter, $\hat{\beta}_x$, males.

Age	Case 1	Case 2	Case 3	age	Case 1	Case 2	Case 3
<1	-0.0198	-0.00976	-0.00976	26	0.062279	0.062882	0.062882
1	0.135976	0.120616	0.120617	27	0.070953	0.071676	0.071676
2	0.029886	0.032013	0.032013	28	0.072021	0.07281	0.07281
3	0.037037	0.039041	0.039041	29	0.072273	0.073052	0.073052
4	0.048945	0.049435	0.049435	30	0.071853	0.072629	0.072629
5	0.044559	0.045076	0.045075	31	0.064165	0.064912	0.064912
6	0.034016	0.034538	0.034538	32	0.060239	0.060905	0.060905
7	0.036681	0.037079	0.037079	33	0.054053	0.054716	0.054716
8	0.024756	0.025017	0.025017	34	0.0477	0.04828	0.04828
9	0.019196	0.019501	0.019501	35	0.041717	0.042274	0.042274
10	0.008497	0.008603	0.008603	36	0.036553	0.03702	0.03702
11	0.004676	0.004751	0.004751	37	0.032852	0.033306	0.033306
12	-0.00334	-0.00329	-0.00329	38	0.029411	0.029814	0.029814
13	-0.00769	-0.00777	-0.00777	39	0.023922	0.024285	0.024285
14	-0.00552	-0.0056	-0.0056	40	0.021288	0.021595	0.021595
15	-0.00813	-0.00826	-0.00826	41	0.018267	0.018584	0.018584
16	-0.00047	-0.00051	-0.00051	42	0.017751	0.018048	0.018048
17	0.00456	0.004564	0.004564	43	0.012545	0.012707	0.012707
18	0.007771	0.007828	0.007828	44	0.011091	0.011237	0.011237
19	0.011519	0.011622	0.011622	45	0.010289	0.010435	0.010435
20	0.021831	0.022055	0.022055	46	0.008128	0.008279	0.008279
21	0.022517	0.0228	0.0228	47	0.004744	0.004825	0.004825
22	0.024943	0.025219	0.025219	48	0.004639	0.004721	0.004721
23	0.037315	0.037747	0.037747	49	0.005841	0.005965	0.005965
24	0.041763	0.042251	0.042251	50	0.004918	0.005084	0.005084
25	0.055299	0.055855	0.055855				

Table A6 The estimated parameter, $\hat{\beta}_x$, males (Continued).

Age	Case 1	Case 2	Case 3	Age	Case 1	Case 2	Case 3
51	0.005918	0.006083	0.006083	76	0.006523	0.006594	0.006594
52	0.005659	0.005791	0.005791	77	0.005553	0.005606	0.005606
53	0.007151	0.007302	0.007302	78	0.008358	0.008446	0.008446
54	0.007562	0.007644	0.007644	79	0.007146	0.007265	0.007265
55	0.006562	0.00666	0.00666	80	0.004115	0.004138	0.004138
56	0.006644	0.006729	0.006729	81	0.002044	0.002052	0.002052
57	0.006719	0.006847	0.006847	82	0.002772	0.002793	0.002793
58	0.005388	0.005477	0.005477	83	0.002455	0.002499	0.002499
59	0.005398	0.00548	0.00548	84	-0.00111	-0.00113	-0.00113
60	0.004983	0.005033	0.005033	85	-0.00177	-0.00182	-0.00182
61	0.003517	0.003568	0.003568	86	-0.00562	-0.00571	-0.00571
62	0.003878	0.003935	0.003933	87	-0.00623	-0.00636	-0.00636
63	0.005942	0.006043	0.006044	88	-0.01117	-0.0113	-0.0113
64	0.006635	0.006727	0.006727	89	-0.01385	-0.01399	-0.01399
65	0.004267	0.004336	0.004336	90	-0.01558	-0.01578	-0.01578
66	0.003209	0.003311	0.003311	91	-0.02321	-0.02352	-0.02352
67	0.010034	0.01014	0.01014	92	-0.02698	-0.02737	-0.02737
68	0.004678	0.004763	0.004762	93	-0.03365	-0.03419	-0.03419
69	0.006223	0.006304	0.006304	94	-0.03776	-0.03834	-0.03834
70	0.007268	0.007345	0.007345	95	-0.04881	-0.04967	-0.04967
71	0.006075	0.006153	0.006153	96	-0.05441	-0.05538	-0.05538
72	0.010128	0.010227	0.010227	97	-0.05958	-0.06059	-0.06059
73	0.008235	0.008288	0.008288	98	-0.07007	-0.07275	-0.07275
74	0.008123	0.008237	0.008237	99	-0.07376	-0.07855	-0.07855
75	0.006923	0.006978	0.006978	100	-0.08268	-0.08807	-0.08807
				>100	-0.09612	-0.09479	-0.09479

The estimated parameters of the age-period-cohort model

The following tables, Table A7 to A11, show the estimated parameters of the age-period cohort model of females.

Table A7 The estimated parameter, $\hat{\alpha}_x$, females.

Age	Case 1	Case 2	Case 3	Age	Case 1	Case 2	Case 3
<1	-6.57481	-5.32271	-5.30086	26	-6.63689	-6.72928	-6.74272
1	-1.14891	-5.8187	-4.85532	27	-6.59715	-6.67924	-6.68069
2	-6.76897	-7.24804	-7.07726	28	-6.58692	-6.64496	-6.6141
3	-7.19486	-7.50694	-7.59017	29	-6.58045	-6.62804	-6.61596
4	-7.19863	-7.58866	-7.65052	30	-6.52783	-6.54442	-6.52692
5	-7.26422	-7.63693	-7.65548	31	-6.46607	-6.46508	-6.50353
6	-7.31465	-7.70135	-7.69215	32	-6.43656	-6.45968	-6.46141
7	-7.41421	-7.76558	-7.75901	33	-6.39494	-6.4342	-6.48034
8	-7.19245	-7.85206	-7.79913	34	-6.36553	-6.40508	-6.48538
9	-7.74011	-7.90477	-7.88069	35	-6.34604	-6.40439	-6.53478
10	-7.84061	-7.91726	-7.83929	36	-6.38142	-6.40582	-6.46101
11	-7.86411	-7.98369	-8.01417	37	-6.38086	-6.38082	-6.38956
12	-7.8253	-7.98195	-8.05463	38	-6.3157	-6.30076	-6.31239
13	-7.73548	-7.85771	-7.97491	39	-6.35371	-6.30126	-6.26302
14	-7.55999	-7.69336	-7.77729	40	-6.25916	-6.21515	-6.19238
15	-7.42601	-7.5134	-7.56546	41	-6.19926	-6.15346	-6.14145
16	-7.33176	-7.46675	-7.5422	42	-6.11635	-6.08208	-6.07912
17	-7.26601	-7.46114	-7.60088	43	-5.92685	-6.01655	-6.02098
18	-7.12025	-7.41558	-7.49729	44	-6.25417	-5.92817	-5.95352
19	-6.90168	-7.38842	-7.35572	45	-6.15566	-5.8495	-5.88479
20	-6.96253	-7.11961	-7.30302	46	-5.96034	-5.79701	-5.82631
21	-6.93029	-7.07986	-7.34966	47	-5.86671	-5.72437	-5.74529
22	-6.91492	-7.04733	-7.22728	48	-5.7968	-5.63199	-5.65164
23	-6.82025	-6.95418	-7.09983	49	-5.74588	-5.54207	-5.58411
24	-6.74817	-6.86766	-6.96764	50	-5.60342	-5.48039	-5.49869
25	-6.67422	-6.77253	-6.82027				

Table A7 The estimated parameter, $\hat{\alpha}_x$, females (Continued).

Age	Case 1	Case 2	Case 3	Age	Case 1	Case 2	Case 3
51	-5.56484	-5.38465	-5.42126	76	-3.26655	-3.17499	-3.21817
52	-5.54285	-5.30482	-5.35022	77	-3.18204	-3.07733	-3.11394
53	-5.49387	-5.19731	-5.2592	78	-3.09475	-2.96664	-2.9486
54	-5.35941	-5.15478	-5.2328	79	-2.98811	-2.87765	-2.93512
55	-5.42469	-5.03998	-5.14632	80	-2.87867	-2.77128	-2.76728
56	-5.14465	-4.98225	-5.09274	81	-2.77762	-2.67184	-2.68162
57	-5.12673	-4.90132	-4.9641	82	-2.65333	-2.56875	-2.53004
58	-5.0058	-4.80477	-4.84912	83	-2.56879	-2.47802	-2.43883
59	-4.92806	-4.72972	-4.75855	84	-2.41092	-2.3562	-2.32325
60	-4.8256	-4.64977	-4.6887	85	-2.3562	-2.28034	-2.24664
61	-4.79137	-4.55355	-4.59711	86	-2.23767	-2.20022	-2.17116
62	-4.59492	-4.47233	-4.48897	87	-2.16344	-2.11742	-2.10208
63	-4.68246	-4.3746	-4.40671	88	-2.0842	-2.05931	-2.06534
64	-4.52483	-4.30033	-4.32543	89	-1.99067	-1.99598	-2.01871
65	-4.40592	-4.22644	-4.23626	90	-1.90106	-1.93167	-1.97233
66	-4.27745	-4.12756	-4.13543	91	-1.83633	-1.87609	-1.9334
67	-4.13176	-4.05946	-4.05297	92	-1.74115	-1.78891	-1.85159
68	-4.11703	-3.99608	-3.99702	93	-1.67271	-1.73674	-1.81094
69	-3.99111	-3.84968	-3.87112	94	-1.62831	-1.70518	-1.77688
70	-3.87727	-3.7611	-3.75821	95	-1.60991	-1.70854	-1.7975
71	-3.85259	-3.6516	-3.6428	96	-1.56149	-1.69414	-1.79558
72	-3.7611	-3.55251	-3.49389	97	-1.54058	-1.69259	-1.8025
73	-3.67252	-3.4535	-3.37843	98	-1.6165	-1.83575	-1.90345
74	-3.57479	-3.3598	-3.33249	99	-1.44713	-1.85615	-1.92718
75	-3.40231	-3.2744	-3.27468	100	-1.40846	-1.97121	-2.05766
				>100	-1.16246	-2.92386	-2.98468

Table A8 The estimated parameter, $\beta_x^{(0)}$, females.

Age	Case 1	Case 2	Case 3	Age	Case 1	Case 2	Case 3
<1	0.001198	-0.00758	0.00782	26	0.019097	0.027435	0.035641
1	0.019666	0.050212	-0.03564	27	0.014587	0.023406	0.034163
2	0.015006	0.019183	0.007885	28	0.014464	0.026471	0.037226
3	0.015332	0.019914	0.0263	29	0.000248	0.020865	0.030223
4	0.013841	0.019027	0.026602	30	0.022768	0.029024	0.032248
5	0.012123	0.017257	0.024886	31	0.027041	0.03205	0.030142
6	0.011801	0.019834	0.026423	32	0.02285	0.025672	0.026405
7	0.011202	0.021292	0.025758	33	0.021615	0.023951	0.022774
8	0.00138	0.009076	0.020764	34	0.018587	0.02172	0.022088
9	0.008046	0.009374	0.013142	35	0.015227	0.019068	0.022288
10	0.009921	0.011371	0.016831	36	0.01487	0.019278	0.02142
11	0.009359	0.010273	0.007421	37	0.014623	0.019027	0.019159
12	0.004354	0.003831	-0.00256	38	0.012775	0.015524	0.01747
13	0.003179	0.001991	-0.00453	39	0.015328	0.019438	0.0165
14	0.006893	0.007117	0.00166	40	0.013377	0.017162	0.013151
15	0.005727	0.002223	0.00237	41	0.012553	0.017098	0.011858
16	0.011071	0.014272	0.010087	42	0.011147	0.014005	0.013569
17	0.013003	0.011746	0.015001	43	0.005371	0.010466	0.008254
18	0.017881	0.007367	0.015117	44	0.015712	0.013137	0.008747
19	0.02356	0.010447	0.005369	45	0.015706	0.014859	0.010522
20	0.019487	0.032266	0.009266	46	0.010295	0.010847	0.008615
21	0.018337	0.026406	0.034913	47	0.009721	0.010336	0.009619
22	0.019747	0.027211	0.03416	48	0.010262	0.011375	0.01046
23	0.020991	0.029267	0.038431	49	0.008557	0.009784	0.006889
24	0.02107	0.028838	0.039665	50	0.008015	0.009754	0.010364
25	0.022459	0.031311	0.040564				

Table A8 The estimated parameter, $\beta_x^{(0)}$, females (Continued).

Age	Case 1	Case 2	Case 3	Age	Case 1	Case 2	Case 3
51	0.007479	0.009195	0.008605	76	0.004517	0.00482	0.002493
52	0.006602	0.008344	0.011183	77	0.004759	0.004846	0.002492
53	0.005412	0.006752	0.012696	78	0.004817	0.004745	0.003814
54	0.00568	0.006907	0.009039	79	0.004361	0.003473	0.010347
55	0.00386	0.004143	0.007602	80	0.004463	0.003483	0.001002
56	0.007794	0.007993	0.009715	81	0.004184	0.002406	0.007858
57	0.006384	0.006503	0.009044	82	0.004566	0.00327	0.005676
58	0.00867	0.00867	0.011402	83	0.003716	0.001261	-0.00146
59	0.007089	0.007722	0.010132	84	0.004395	0.002762	-0.00167
60	0.006415	0.007224	0.011498	85	0.004005	0.001612	-0.00234
61	0.007055	0.008194	0.011847	86	0.003577	0.001524	-0.00029
62	0.006685	0.008059	0.008416	87	0.002894	-0.00039	-0.00374
63	0.006472	0.008199	0.01047	88	0.001907	-0.00324	-0.0101
64	0.006327	0.00787	0.005893	89	0.001398	-0.00395	-0.00734
65	0.006499	0.007586	0.012852	90	0.001772	-0.00476	-0.01043
66	0.004836	0.005568	0.005327	91	0.001567	-0.00462	-0.00823
67	0.007036	0.008471	0.006605	92	0.001123	-0.0044	-0.00694
68	0.005918	0.007044	0.007494	93	0.00196	-0.00432	-0.00642
69	0.005815	0.006517	0.009669	94	0.000489	-0.00432	-0.00873
70	0.005702	0.006547	0.004235	95	0.001151	-0.00458	-0.0078
71	0.006224	0.006689	0.009537	96	0.005921	-0.00046	-0.00103
72	0.005248	0.005551	0.011726	97	0.003684	-0.00242	-0.0019
73	0.005653	0.006091	0.010927	98	0.001106	-0.01603	-0.02175
74	0.005056	0.005442	0.007304	99	0.013184	-0.00934	-0.004
75	0.005242	0.005684	0.001954	100	0.012743	-0.0152	-0.01519
				>100	0.055158	-0.02841	-0.027

Table A9 The estimated parameter, $\beta_x^{(1)}$, females.

Age	Case 1	Case 2	Case 3	Age	Case 1	Case 2	Case 3
<1	-0.05416	-0.05737	0.146887	26	0.004719	0.012388	0.045052
1	0.190571	0.145791	-0.90036	27	0.004931	0.01167	0.044917
2	0.019989	0.014543	-0.13878	28	0.004948	0.009444	0.024691
3	0.012913	0.008227	0.037799	29	0.008141	0.011523	0.058583
4	0.016019	0.010116	0.035249	30	0.001747	0.00371	0.013363
5	0.015485	0.008932	0.019631	31	-0.00015	-0.00028	0.018075
6	0.016787	0.007011	0.006533	32	-0.00055	0.000723	0.004936
7	0.015877	0.004399	0.003935	33	-0.00121	0.000383	0.022995
8	0.029217	0.013398	0.003241	34	-0.00134	-0.00022	0.031793
9	0.006769	0.005131	0.000442	35	-0.00103	0.001594	0.064021
10	0.002963	0.002753	-0.02202	36	0.000397	0.002747	0.045033
11	0.004214	0.004919	0.019335	37	0.001376	0.003801	0.035249
12	0.00547	0.005069	0.032035	38	0.00089	0.001362	0.022189
13	0.004086	0.003494	0.042001	39	0.003165	0.005083	0.016996
14	0.003993	0.004679	0.033306	40	0.002468	0.003582	0.001322
15	0.002825	-0.00042	0.016783	41	0.002554	0.004015	-0.00414
16	0.003517	0.006819	0.029959	42	0.001775	0.00188	0.006416
17	0.005133	0.005092	0.049509	43	-0.00549	0.000473	-0.00827
18	0.007304	-0.00022	0.034244	44	0.019014	0.006339	-4.8E-05
19	0.011732	0.001436	-0.01958	45	0.018239	0.006767	0.003002
20	0.008765	0.037479	-0.01954	46	0.009868	0.004069	-0.0053
21	0.007093	0.02501	0.0741	47	0.009212	0.002806	-0.00027
22	0.006009	0.019541	0.053133	48	0.011003	0.003174	0.009427
23	0.005907	0.017715	0.053451	49	0.012947	0.00519	0.003493
24	0.005475	0.015768	0.047446	50	0.008339	0.002455	-0.00086
25	0.004626	0.012392	0.038831				

Table A9 The estimated parameter, $\beta_x^{(1)}$, females (Continued).

Age	Case 1	Case 2	Case 3	Age	Case 1	Case 2	Case 3
51	0.011588	0.0046	0.003872	76	0.005852	0.005852	-0.00512
52	0.015133	0.006437	0.000706	77	0.006952	0.005705	-0.00379
53	0.018437	0.008739	0.008601	78	0.008443	0.00895	0.003906
54	0.012581	0.006157	0.026112	79	0.0075	0.010587	-0.00867
55	0.023312	0.012786	0.021286	80	0.007634	0.009855	0.002232
56	0.009264	0.007635	0.041241	81	0.00801	0.012126	-0.00187
57	0.013295	0.010389	0.017656	82	0.006818	0.009424	0.006
58	0.011241	0.012631	0.005739	83	0.008047	0.011325	0.00751
59	0.011642	0.01068	0.002086	84	0.00546	0.007811	0.008508
60	0.010275	0.010519	0.015347	85	0.007561	0.007587	0.010752
61	0.01392	0.015035	0.021601	86	0.005328	0.008095	0.011481
62	0.007303	0.007455	0.009001	87	0.007004	0.008362	0.015066
63	0.018303	0.019165	0.019365	88	0.007796	0.012877	0.030643
64	0.013431	0.013416	-0.00229	89	0.006957	0.014356	0.036304
65	0.01112	0.00756	0.005944	90	0.007136	0.017578	0.039367
66	0.009103	0.007746	0.006025	91	0.006439	0.015894	0.041147
67	0.004627	0.002025	0.005312	92	0.006254	0.014752	0.037817
68	0.007623	0.004519	0.003165	93	0.007388	0.016624	0.041084
69	0.008043	0.010653	0.001702	94	0.006974	0.016506	0.036717
70	0.007072	0.006363	0.004351	95	0.007748	0.017199	0.041451
71	0.011557	0.015063	0.008705	96	0.008963	0.01853	0.050561
72	0.012159	0.0154	0.015266	97	0.009266	0.018817	0.052069
73	0.013125	0.015248	0.016467	98	0.008156	0.008312	0.013172
74	0.012843	0.016928	0.008824	99	0.01372	0.014919	0.05114
75	0.007978	0.008449	0.002745	100	0.015302	0.014183	0.042221
				>100	0.033782	0.011812	0.04123

Table A10 The estimated parameter, $\hat{\gamma}_{t-x}$, females.

Year	Case 1	Case 2	Case 3	Year	Case 1	Case 2	Case 3
1897	-0.0003	-0.0450	0.0533	1935	13.9980	-2.4950	-4.0059
1898	-0.0003	-0.0450	0.0533	1936	17.9435	0.8256	0.8622
1899	-0.0003	-0.0450	0.0533	1937	11.5117	-5.0641	-5.8356
1900	-0.0003	-0.0450	0.0533	1938	13.3095	-3.7244	-2.8112
1901	-99.3659	-38.9328	-11.1004	1939	12.9277	-4.2394	-2.8398
1902	-90.1173	-33.4494	-9.9279	1940	11.6822	-5.6740	-5.0429
1903	-80.8153	-27.6529	-8.0354	1941	14.4095	-2.6003	-1.1484
1904	-64.0278	-15.7255	-4.2210	1942	17.0631	0.4067	1.8166
1905	-61.2706	-20.6221	-5.2603	1943	11.3407	-5.4472	0.8898
1906	-51.6209	-14.4821	-3.1668	1944	18.6521	2.5718	2.9746
1907	-44.2934	-13.9074	-3.3830	1945	15.6525	-0.8281	1.5083
1908	-21.9074	-0.8487	2.5782	1946	16.1524	-0.0785	1.6732
1909	-11.9291	3.4366	3.5698	1947	17.1446	0.9795	1.7627
1910	-7.2528	2.7148	2.6313	1948	14.8748	-2.0734	3.0202
1911	1.8377	7.5754	5.2075	1949	16.0924	-0.8075	3.7512
1912	-4.0188	3.1831	2.9142	1950	15.9577	-1.3207	3.4515
1913	14.5243	10.9565	6.6313	1951	12.0680	-6.7534	2.0170
1914	14.6381	9.4656	5.6162	1952	13.1587	-5.8320	3.1662
1915	4.8101	3.0832	2.6774	1953	13.0614	-6.7297	0.6479
1916	19.6835	9.3626	4.3236	1954	11.7376	-9.5287	0.8343
1917	1.4422	-0.7274	1.3519	1955	14.0405	-4.1075	2.0242
1918	13.4023	4.3530	1.7233	1956	6.6524	-19.7992	-0.5360
1919	0.9703	-3.9746	0.0138	1957	7.7495	-17.6327	0.3074
1920	23.7752	8.6189	3.4453	1958	10.3061	-11.0929	-0.9023
1921	12.5044	0.9174	-1.1093	1959	10.5608	-11.2119	-10.1738
1922	10.8034	-0.8107	-1.2991	1960	11.5341	-9.5587	-3.2507
1923	19.4627	4.7783	-1.9295	1961	12.6113	-9.8065	-3.5320
1924	11.0372	-2.2456	-5.9162	1962	16.9546	1.2090	-1.4250
1925	16.3379	1.3222	-7.2882	1963	14.6902	-8.7937	-1.3386
1926	18.4721	2.1984	-9.6223	1964	19.3650	2.1129	1.1870
1927	9.4244	-5.1821	-15.2721	1965	22.0017	3.3432	1.1109
1928	17.7722	0.9198	-8.9296	1966	23.8188	11.7187	2.2478
1929	7.3229	-7.4086	-18.6395	1967	24.3321	4.6717	2.3374
1930	13.7198	-2.7626	-7.9198	1968	28.6913	15.3194	3.5470
1931	12.4894	-3.5398	-14.4310	1969	24.4773	19.7507	2.1218
1932	15.5503	-1.1330	-3.4803	1970	44.0372	20.1211	2.3187
1933	15.7675	-0.6584	-1.3200	1971	62.9834	29.2151	3.6638
1934	17.3092	0.5454	1.1250	1972	67.5454	29.4730	2.1551

Table A10 The estimated parameter, $\hat{\gamma}_{t-x}$, females (Continued).

Year	Case 1	Case 2	Case 3	Year	Case 1	Case 2	Case 3
1973	68.7775	28.5790	3.0397	1993	-9.6197	23.4494	4.8451
1974	57.3521	24.7416	3.6563	1994	-6.8241	26.6279	4.6532
1975	41.7788	19.0960	3.3406	1995	-8.3789	23.0360	5.4821
1976	16.2525	9.6309	2.6461	1996	-15.0195	9.7465	0.6241
1977	3.9016	6.0576	3.3984	1997	-17.8166	8.4488	-0.4095
1978	-23.2151	-2.7102	-0.1951	1998	-24.5408	-1.6961	0.4234
1979	-34.4310	-6.8270	-0.4509	1999	-27.6912	-6.4945	0.5773
1980	-49.8652	-10.2949	-1.4757	2000	-26.1589	-4.5712	0.6085
1981	-60.6389	-14.2597	-0.8763	2001	-27.3169	-6.3400	1.0755
1982	-64.6963	-13.9889	-1.2060	2002	-27.5091	-5.6794	1.1264
1983	-64.2463	-12.2925	-0.2198	2003	-29.3525	-7.3643	1.4109
1984	-63.0942	-11.2376	-0.0036	2004	-30.6860	-8.0629	2.3231
1985	-52.9266	-7.9348	1.2484	2005	-30.4748	-7.2418	2.4762
1986	-53.6034	-7.8689	1.8890	2006	-29.9166	-6.2584	2.5447
1987	-44.2899	-5.2260	3.0890	2007	-30.2137	-6.2849	2.8801
1988	-39.3620	-2.2811	4.3035	2008	-29.7970	-4.6920	2.6229
1989	-34.4250	-2.0396	4.9416	2009	-0.0003	-0.0450	0.0533
1990	-26.3796	0.8349	4.7032	2010	-0.0003	-0.0450	0.0533
1991	-19.9328	1.8384	6.0844	2011	-0.0003	-0.0450	0.0533
1992	-14.8068	6.1251	4.2284	2012	-0.0003	-0.0450	0.0533

Table A11 The estimated parameter, $\hat{k}_t$, females.

Year	Case 1	Case 2	Case 3	Year	Case 1	Case 2	Case 3
1998	-2.3182	-0.3470	3.6451	2006	-0.4494	-1.5740	-4.6566
1999	12.9870	10.4221	12.0523	2007	-3.7283	-4.2284	-3.9080
2000	14.7255	12.4624	13.9028	2008	-6.6193	-7.4551	-8.0113
2001	14.2888	12.3011	13.0742	2009	-12.9521	-10.8348	-10.3316
2002	14.7647	13.9204	10.1476	2010	-13.6704	-11.9210	-11.2522
2003	10.4830	9.5864	8.2176	2011	-18.2743	-15.4300	-15.4820
2004	10.1431	7.5784	6.9660	2012	-23.8835	-16.3231	-15.1777
2005	4.5033	1.8425	0.8141				

The following tables, Table A12 to A16, show the estimated parameter of the age-period cohort model of females.

Table A12 The estimated parameter, $\hat{\alpha}_x$, males.

Age	Case 1	Case 2	Case 3	Age	Case 1	Case 2	Case 3
<1	-4.5776	-4.9641	-4.9644	26	-5.6118	-5.7248	-5.7969
1	-7.3064	-6.2390	-6.2516	27	-5.5550	-5.6469	-5.7078
2	-6.9970	-6.9803	-6.9668	28	-5.5195	-5.5794	-5.6209
3	-7.3031	-7.1730	-7.1676	29	-5.4777	-5.5164	-5.5508
4	-7.3363	-7.2131	-7.2164	30	-5.4592	-5.4839	-5.5363
5	-7.2854	-7.2162	-7.2271	31	-5.4369	-5.4480	-5.3353
6	-7.4703	-7.2206	-7.2640	32	-5.4405	-5.4410	-5.3636
7	-7.4696	-7.3620	-7.3795	33	-5.4430	-5.4291	-5.3341
8	-7.6534	-7.4260	-7.5022	34	-5.3970	-5.4160	-5.3774
9	-7.7203	-7.4749	-7.6258	35	-5.3314	-5.3852	-5.3605
10	-7.8033	-7.6283	-7.7306	36	-5.3075	-5.3688	-5.3488
11	-7.8154	-7.6604	-7.7631	37	-5.2774	-5.3498	-5.3252
12	-7.6687	-7.5518	-7.6403	38	-5.2617	-5.3239	-5.3136
13	-7.3369	-7.2799	-7.3189	39	-5.1885	-5.2840	-5.2909
14	-6.9704	-7.1739	-6.9834	40	-5.2008	-5.2970	-5.2559
15	-6.5717	-6.7927	-6.6363	41	-5.1461	-5.3717	-5.2592
16	-6.3212	-6.5383	-6.4215	42	-5.3410	-5.2912	-5.2265
17	-6.1126	-6.3391	-6.3869	43	-5.3917	-5.2017	-5.1862
18	-6.0058	-6.2337	-6.3068	44	-5.5333	-5.1844	-5.1401
19	-5.9089	-6.1776	-6.2661	45	-5.5342	-5.1298	-5.0778
20	-5.8855	-6.1306	-6.2449	46	-5.5592	-5.0498	-5.0361
21	-5.8636	-6.1035	-6.1557	47	-5.5421	-5.0345	-4.9769
22	-6.0189	-6.0417	-6.0671	48	-5.4197	-4.9243	-4.7960
23	-5.9390	-5.9765	-5.9620	49	-5.2694	-4.8518	-4.8738
24	-5.7943	-5.9130	-5.9355	50	-5.3094	-4.8334	-4.8017
25	-5.6280	-5.7666	-5.8569				

Table A12 The Estimated parameter, $\hat{\alpha}_x$, males (Continued).

Age	Case 1	Case 2	Case 3	Age	Case 1	Case 2	Case 3
51	-5.1451	-4.7610	-4.7323	76	-2.8031	-2.8643	-2.8367
52	-5.0187	-4.7066	-4.6702	77	-2.7044	-2.7844	-2.7626
53	-5.0485	-4.6703	-4.5921	78	-2.5924	-2.6874	-2.6624
54	-4.7834	-4.5555	-4.5194	79	-2.5112	-2.6163	-2.5689
55	-4.8726	-4.5311	-4.5367	80	-2.4227	-2.5371	-2.5027
56	-4.7605	-4.4532	-4.4504	81	-2.3005	-2.4446	-2.4057
57	-4.5975	-4.3713	-4.3902	82	-2.1855	-2.3739	-2.3294
58	-4.5768	-4.2965	-4.3219	83	-2.1538	-2.2923	-2.2633
59	-4.4552	-4.2091	-4.2433	84	-2.0300	-2.2212	-2.1841
60	-4.3557	-4.1312	-4.1740	85	-1.8736	-2.1486	-2.1191
61	-4.2620	-4.0420	-4.0949	86	-1.8343	-2.0727	-2.0516
62	-4.2230	-4.0292	-4.0303	87	-1.7303	-2.0419	-2.0098
63	-4.1092	-3.9177	-3.9618	88	-1.6680	-2.0075	-1.9636
64	-3.9816	-3.8641	-3.8651	89	-1.6057	-1.9490	-1.9164
65	-3.8959	-3.7894	-3.7867	90	-1.5221	-2.0407	-1.8830
66	-3.8128	-3.6916	-3.7045	91	-1.4849	-1.9994	-1.8587
67	-3.7303	-3.6295	-3.6378	92	-1.4474	-1.9822	-1.8633
68	-3.6893	-3.6026	-3.6093	93	-1.3005	-2.0050	-1.8617
69	-3.5106	-3.4603	-3.4701	94	-1.3135	-2.0117	-1.9050
70	-3.4128	-3.3837	-3.3864	95	-1.1780	-2.1147	-1.9696
71	-3.3211	-3.2963	-3.2361	96	-1.1355	-2.1769	-2.0413
72	-3.2124	-3.2076	-3.1850	97	-0.9609	-2.2721	-2.1458
73	-3.1165	-3.1245	-3.1030	98	-0.3629	-2.3601	-2.3231
74	-3.0207	-3.0417	-3.0193	99	-0.0823	-2.4801	-2.4762
75	-2.9166	-2.9529	-2.9370	100	-1.2645	-2.6795	-2.5786
				>100	-0.6260	-3.5218	-3.9799

Table A13 The Estimated parameter, $\beta_x^{(0)}$, males.

Age	Case 1	Case 2	Case 3	Age	Case 1	Case 2	Case 3
<1	0.003648	-0.00695	-0.02709	26	0.01904	0.04665	0.06199
1	0.002474	0.043493	0.094494	27	0.01868	0.0471	0.06968
2	0.010954	0.021101	0.031005	28	0.01746	0.04492	0.07102
3	0.008473	0.020626	0.036488	29	0.01429	0.04262	0.0702
4	0.012062	0.03254	0.050936	30	0.01269	0.04227	0.07348
5	0.013347	0.032852	0.045845	31	0.00894	0.03769	0.04672
6	0.004976	0.031868	0.03665	32	0.00412	0.03686	0.04884
7	0.011365	0.030838	0.037111	33	0.00765	0.03529	0.04166
8	0.006356	0.029335	0.028814	34	0.01822	0.03234	0.04639
9	0.007405	0.033881	0.026667	35	0.01951	0.02889	0.04204
10	0.00438	0.019171	0.00984	36	0.01657	0.02477	0.0372
11	0.004346	0.015289	0.003485	37	0.01425	0.02234	0.03546
12	0.002381	0.00756	-0.00375	38	0.01158	0.01959	0.03172
13	0.004299	-0.00018	-0.00751	39	0.009	0.01628	0.02623
14	0.007529	-0.01556	-0.00449	40	0.00721	0.01314	0.02174
15	0.008843	-0.0117	-0.00963	41	0.00564	0.0114	0.01606
16	0.012906	-0.00072	-0.00431	42	0.00815	0.01223	0.01523
17	0.015849	0.00878	0.007794	43	0.00812	0.00899	0.01118
18	0.01784	0.015809	0.021221	44	0.01102	0.00901	0.01115
19	0.021121	0.022621	0.033321	45	0.01156	0.00902	0.01083
20	0.022309	0.03268	0.052213	46	0.01216	0.00699	0.00983
21	0.021389	0.034405	0.036551	47	0.01167	0.00642	0.00473
22	0.010888	0.027435	0.027775	48	0.01078	0.00389	-0.00611
23	0.014649	0.030144	0.024068	49	0.0094	0.00328	0.00927
24	0.017429	0.032693	0.040171	50	0.01121	0.005	0.00934
25	0.020275	0.04866	0.056864				

Table A13 The estimated parameter, $\beta_x^{(0)}$, males (Continued).

Age	Case 1	Case 2	Case 3	Age	Case 1	Case 2	Case 3
51	0.009564	0.004963	0.010287	76	0.0066	0.00371	0.00845
52	0.008154	0.004878	0.009694	77	0.00645	0.0032	0.00928
53	0.010218	0.006846	0.015949	78	0.00713	0.00508	0.01314
54	0.006889	0.005126	0.015636	79	0.00654	0.00457	0.01598
55	0.009572	0.007039	-0.00351	80	0.0057	0.00233	0.01043
56	0.00917	0.007715	0.003358	81	0.00541	0.00061	0.00712
57	0.00769	0.007423	0.003193	82	0.00637	0.00096	0.00691
58	0.008757	0.010522	0.003896	83	0.00443	0.00078	0.00433
59	0.008176	0.009725	0.005269	84	0.00464	-0.0017	0.00064
60	0.007678	0.010347	0.005623	85	0.00524	-0.0041	-0.00383
61	0.007123	0.009987	0.005738	86	0.00378	-0.0048	-0.00579
62	0.008715	-0.00035	0.007678	87	0.00441	-0.0073	-0.00799
63	0.008619	0.006974	0.010999	88	0.00299	-0.0108	-0.01618
64	0.007427	0.003841	0.008662	89	0.003	-0.0105	-0.01932
65	0.0067	0.002873	0.001667	90	0.00336	-0.0235	-0.03502
66	0.006912	0.002004	0.006208	91	0.00218	-0.0224	-0.04019
67	0.00871	0.006105	0.008704	92	0.0025	-0.0201	-0.03728
68	0.007213	0.002597	0.003315	93	0.00404	-0.0223	-0.04248
69	0.006018	0.003396	0.007019	94	0.00461	-0.0186	-0.03704
70	0.005982	0.004661	0.007125	95	0.00719	-0.0196	-0.04166
71	0.00618	0.002575	-0.00796	96	0.00988	-0.0146	-0.04039
72	0.006446	0.005443	0.006012	97	0.01448	-0.0103	-0.03804
73	0.006437	0.003736	0.002906	98	0.02772	-0.0304	-0.06561
74	0.006315	0.003928	0.004555	99	0.03386	-0.0195	-0.06874
75	0.00593	0.003869	0.005138	100	0.00701	-0.04	-0.05256
				>100	0.03347	0.01944	-0.1317

Table A14 The estimated parameter, $\beta_x^{(1)}$, males.

Age	Case 1	Case 2	Case 3	Age	Case 1	Case 2	Case 3
<1	-0.05031	-0.16308	-0.02703	26	0.00312	0.0057	0.00131
1	0.147915	0.304627	0.043297	27	0.00289	0.00482	0.00162
2	0.00177	-0.00962	0.000801	28	0.00238	0.00365	0.00589
3	0.015613	0.021309	0.000731	29	0.00254	0.00393	0.01401
4	0.013717	-0.00544	-0.0005	30	0.00245	0.00293	-0.00995
5	0.006072	-0.01817	-0.00087	31	0.00251	0.00195	0.10034
6	0.020422	-0.03787	-0.00208	32	0.0031	0.00078	0.07375
7	0.007088	-0.01832	0.003234	33	0.00212	-0.0004	0.08378
8	0.014123	-0.03071	-0.00147	34	-0.00048	-0.001	0.02555
9	0.009362	-0.04254	-0.00521	35	-0.0014	-0.0013	0.01165
10	0.007403	-0.02284	0.000357	36	-0.00134	-0.001	0.01173
11	0.006445	-0.01788	0.003071	37	-0.00133	-0.0013	0.01708
12	0.005129	-0.01312	0.001262	38	-0.00103	-0.0007	0.00806
13	0.007152	-0.00506	0.000778	39	-0.00151	-0.0036	-0.00054
14	0.007393	0.02544	0.001162	40	-0.00094	0.01058	0.00905
15	0.007232	0.022831	0.005071	41	-0.00113	0.05078	-0.02345
16	0.006734	0.018739	0.007803	42	0.00202	0.0382	-0.0246
17	0.006266	0.016902	0.038443	43	0.00336	0.01936	-0.02372
18	0.005924	0.013973	0.037105	44	0.0063	0.03158	-0.02191
19	0.006492	0.013081	0.03779	45	0.0075	0.03041	-0.01163
20	0.00544	0.011819	0.041065	46	0.00919	0.01642	-0.01607
21	0.005129	0.010426	0.014719	47	0.01017	0.0284	-0.00063
22	0.00082	0.005126	0.00145	48	0.00972	0.00687	0.07352
23	0.000921	0.002479	-0.01829	49	0.00825	-0.0052	-0.01248
24	0.002319	0.002664	-0.00354	50	0.01095	0.01788	-0.01119
25	0.00362	0.007304	0.002999				

Table A14 The estimated parameter, $\beta_x^{(1)}$, males (Continued).

Age	Case 1	Case 2	Case 3	Age	Case 1	Case 2	Case 3
51	0.009375	0.01338	-0.00605	76	0.0113	0.00582	0.02819
52	0.00835	0.017793	-0.00404	77	0.01173	0.00604	0.01939
53	0.011456	0.044593	-0.00732	78	0.01169	0.00457	0.01757
54	0.006757	0.007859	-0.00518	79	0.01061	0.00694	0.02775
55	0.011701	0.0541	0.006169	80	0.00988	0.0044	0.01767
56	0.010829	0.038686	0.001654	81	0.01072	0.00527	0.01757
57	0.008302	0.021458	0.003418	82	0.01256	0.00655	0.01652
58	0.010848	0.042639	0.002945	83	0.00822	0.00399	0.00966
59	0.010036	0.03001	0.003053	84	0.01028	0.00439	0.01042
60	0.009526	0.030232	0.00279	85	0.01345	0.00573	0.00551
61	0.009669	0.028908	0.002396	86	0.0108	0.00308	0.00348
62	0.012327	-0.01278	0.002856	87	0.0125	0.00514	0.00338
63	0.011257	0.01308	0.003302	88	0.01231	0.0054	0.00881
64	0.009483	-0.0027	0.001067	89	0.01176	0.00333	0.00865
65	0.009691	-0.00078	-0.00038	90	0.01262	0.01772	0.02801
66	0.012343	0.001994	0.002536	91	0.0116	0.01523	0.03028
67	0.011656	0.005587	-0.00127	92	0.01139	0.01338	0.02614
68	0.012765	0.004304	-0.00086	93	0.01386	0.01725	0.03221
69	0.009456	0.004823	0.000349	94	0.01313	0.01387	0.02297
70	0.008642	0.000607	-0.0037	95	0.0164	0.01861	0.0267
71	0.010583	0.006112	0.041246	96	0.01719	0.01937	0.01761
72	0.008387	0.003647	0.018518	97	0.02068	0.02138	0.01655
73	0.010209	0.005318	0.022733	98	0.03083	0.00935	0.00407
74	0.009629	0.006757	0.026826	99	0.03458	0.01566	0.00212
75	0.009198	0.002496	0.019808	100	0.01871	0.0061	0.01433
				>100	0.03911	0.04162	-0.01174

Table A15 The estimated parameter, $\hat{\gamma}_{t-x}$, males.

Year	Case 1	Case 2	Case 3	Year	Case 1	Case 2	Case 3
1897	-0.00512	0.013266	-0.04934	1935	0.5091	-8.82547	-1.93513
1898	-0.00512	0.013266	-0.04934	1936	5.609845	3.038739	-1.90047
1899	-0.00512	0.013266	-0.04934	1937	4.415139	-6.21935	-3.21008
1900	-0.00512	0.013266	-0.04934	1938	6.749749	-4.12107	-3.76477
1901	-100.05	-19.1622	-25.1718	1939	8.32075	-4.21866	-3.67687
1902	-98.6995	-16.7737	-22.5892	1940	8.245077	-3.94955	-4.98516
1903	-93.891	-13.0645	-19.4495	1941	12.66225	-2.28953	-4.46011
1904	-84.3429	-4.77714	-3.00403	1942	16.17966	-1.80461	16.63785
1905	-84.1369	-8.65093	-6.84924	1943	11.86368	-2.78902	3.784734
1906	-73.1172	-0.51842	-1.56733	1944	21.24186	0.423284	28.74731
1907	-69.8133	-1.48535	-4.1123	1945	19.21225	-1.16536	15.57084
1908	-55.2478	10.11841	2.941257	1946	23.85019	0.424286	16.44711
1909	-47.9781	15.93042	3.618213	1947	25.85983	0.855182	15.30898
1910	-43.415	15.62447	2.180972	1948	26.20185	0.518591	12.22196
1911	-36.9024	16.15744	5.130231	1949	31.22857	1.82868	7.833528
1912	-38.4619	11.27523	0.884553	1950	32.58411	1.660822	-2.60442
1913	-28.7152	15.59882	5.339964	1951	32.24713	0.469124	-0.68711
1914	-27.8069	12.38497	0.675477	1952	37.48629	1.587812	0.430897
1915	-31.2295	4.825792	-3.74866	1953	39.00986	1.118901	-1.24807
1916	-19.7817	11.60433	0.030447	1954	41.21497	0.90353	-0.28201
1917	-28.0038	0.673444	-5.19978	1955	46.85689	2.056309	-0.53388
1918	-20.2815	6.690515	-1.74145	1956	41.82019	-0.32036	-1.33226
1919	-28.4275	-6.31762	-8.13819	1957	46.984	0.87575	-1.60097
1920	-11.1772	10.03029	-0.7733	1958	54.40074	2.586073	-1.48513
1921	-17.6446	-1.77403	-5.07005	1959	60.30641	3.345919	-1.89646
1922	-17.549	-5.27091	-5.47745	1960	65.0676	4.268168	-1.64818
1923	-10.0142	9.481731	-0.62889	1961	65.76479	3.152248	-3.30849
1924	-16.4657	-8.35805	-2.96753	1962	71.9124	3.232256	-1.84351
1925	-10.0624	2.436685	-0.45509	1963	71.58698	2.98607	-2.77632
1926	-7.12477	6.12112	0.223674	1964	76.44081	2.694601	-2.89616
1927	-12.4284	-8.08774	-3.14932	1965	80.89113	2.159943	-0.60453
1928	-1.94148	10.86864	2.181776	1966	87.81964	2.924829	-0.08706
1929	-8.00983	-4.45073	-0.02309	1967	92.19841	1.747827	-0.18964
1930	-1.93983	3.866115	0.779265	1968	100.0394	2.481668	0.053693
1931	-6.06416	-6.62892	-1.07488	1969	94.84912	2.896259	-0.13245
1932	1.802358	5.833259	0.698197	1970	77.53342	2.788523	-0.34208
1933	-0.72795	-1.67529	-1.18715	1971	53.44219	3.671532	-0.48724
1934	2.011925	2.382729	-0.3366	1972	26.0153	0.44682	-0.94833

Table A15 The estimated parameter, $\hat{\gamma}_{t-x}$, males (Continued).

Year	Case 1	Case 2	Case 3	Year	Case 1	Case 2	Case 3
1973	-12.1912	-15.4641	-1.29387	1993	17.46159	4.766616	3.266599
1974	-44.557	-28.63	-1.71641	1994	20.78868	2.911652	2.008186
1975	-64.0458	-31.9403	-1.89328	1995	17.55696	2.568971	0.307494
1976	-88.9219	-38.6333	-2.30038	1996	14.38061	1.671264	-31.6546
1977	-87.5519	-28.5249	-2.01956	1997	16.35065	3.483736	22.98837
1978	-100.05	-27.9271	-2.26004	1998	10.93701	0.293593	-1.39196
1979	-90.3421	-18.3956	-1.88645	1999	8.266675	-0.75763	-11.2785
1980	-98.4024	-18.4745	-2.4193	2000	8.924416	-0.37974	-5.65772
1981	-90.4535	-12.567	-3.13658	2001	7.461602	-0.83013	-7.76855
1982	-75.3952	-5.1373	0.632517	2002	7.205344	-0.73426	-7.29964
1983	-63.1275	0.202875	0.916148	2003	5.827527	-1.0438	-6.45269
1984	-51.588	4.072876	3.952279	2004	3.497015	-1.68189	-8.88649
1985	-38.7605	9.441294	7.047868	2005	3.811011	-1.24032	-7.69866
1986	-26.5933	12.58534	8.383062	2006	3.56708	-1.1184	-6.38384
1987	-14.288	15.77757	7.792379	2007	3.366941	-0.90234	-4.59345
1988	-12.0419	15.39056	5.892603	2008	3.383556	-0.83445	-2.19627
1989	-2.21001	14.45426	7.470382	2009	-0.00512	0.013266	-0.04934
1990	1.068985	11.32698	5.137039	2010	-0.00512	0.013266	-0.04934
1991	9.369029	9.014101	5.175464	2011	-0.00512	0.013266	-0.04934
1992	10.42376	6.272926	3.896559	2012	-0.00512	0.013266	-0.04934

Table A16 The estimated parameter, $\hat{k}_t$, males.

Year	Case 1	Case 2	Case 3	Year	Case 1	Case 2	Case 3
1998	7.3398	7.1413	4.1014984	2006	-1.524891	-2.2884	-1.299453
1999	20.514928	11.24	7.1638393	2007	-6.930566	-5.4131	-3.067702
2000	19.121853	10.578	7.1393894	2008	-10.8651	-7.1017	-4.049322
2001	15.501523	8.8208	5.7387271	2009	-16.47547	-8.8594	-5.673021
2002	14.837917	7.9474	4.5794377	2010	-17.6561	-9.0929	-6.499328
2003	12.452202	6.0486	4.0000646	2011	-22.32603	-10.445	-6.487198
2004	9.2237796	3.0373	1.1083741	2012	-27.74197	-11.183	-5.498671
2005	4.5281261	-0.43	-1.256635				

APPENDIX B

FORECASTED MORTALITY RATE

The forecasted the Thai mortality rate using the age-period-cohort model underlying age-independent dispersion parameter is shown in this appendix.

The following tables show the forecasted mortality rate for males.

Table B1 The forecasted mortality rate, females.

Age	2013	2014	2015	2016	2017
<1	0.00674	0.006798	0.006858	0.006917	0.006977
1	0.000836	0.000789	0.000745	0.000704	0.000665
2	0.00039	0.000382	0.000374	0.000366	0.000358
3	0.000385	0.000376	0.000368	0.00036	0.000352
4	0.000336	0.000328	0.000321	0.000314	0.000308
5	0.000316	0.00031	0.000304	0.000298	0.000292
6	0.000312	0.000305	0.000298	0.000291	0.000285
7	0.000334	0.000326	0.000318	0.00031	0.000303
8	0.000298	0.000295	0.000292	0.000289	0.000286
9	0.000268	0.000265	0.000263	0.00026	0.000257
10	0.000307	0.000303	0.000299	0.000296	0.000292
11	0.000256	0.000253	0.00025	0.000247	0.000244
12	0.000347	0.000345	0.000344	0.000342	0.000341
13	0.000402	0.000401	0.0004	0.000399	0.000398
14	0.000428	0.000424	0.000421	0.000417	0.000414
15	0.000518	0.000516	0.000515	0.000514	0.000513
16	0.000468	0.000461	0.000453	0.000446	0.000439
17	0.000574	0.000567	0.000559	0.000552	0.000544
18	0.000534	0.000529	0.000525	0.00052	0.000516
19	0.000529	0.000523	0.000516	0.00051	0.000504
20	0.000545	0.000525	0.000506	0.000488	0.00047
21	0.000526	0.000511	0.000495	0.000481	0.000466
22	0.000546	0.000529	0.000513	0.000497	0.000482
23	0.000518	0.000501	0.000485	0.000469	0.000454
24	0.000545	0.000528	0.000511	0.000494	0.000478

Table B1 The forecasted mortality rate, females (Continued).

Age	2013	2014	2015	2016	2017
25	0.00059	0.000569	0.000549	0.00053	0.000511
26	0.000648	0.000628	0.000609	0.00059	0.000572
27	0.000758	0.000738	0.000719	0.0007	0.000681
28	0.000709	0.000688	0.000667	0.000647	0.000628
29	0.000854	0.000834	0.000814	0.000795	0.000776
30	0.00084	0.000812	0.000786	0.00076	0.000735
31	0.000971	0.000936	0.000902	0.00087	0.000839
32	0.000942	0.000914	0.000888	0.000862	0.000838
33	0.001029	0.001001	0.000974	0.000948	0.000922
34	0.001085	0.001059	0.001033	0.001008	0.000983
35	0.001287	0.001259	0.001232	0.001205	0.001179
36	0.001211	0.001184	0.001159	0.001133	0.001109
37	0.001308	0.00128	0.001252	0.001225	0.001199
38	0.001477	0.001451	0.001425	0.0014	0.001376
39	0.001527	0.001494	0.001461	0.001429	0.001398
40	0.001658	0.001626	0.001594	0.001563	0.001533
41	0.001752	0.001718	0.001685	0.001652	0.00162
42	0.001792	0.001764	0.001736	0.001708	0.001681
43	0.002068	0.002044	0.002019	0.001995	0.001972
44	0.002252	0.002218	0.002185	0.002153	0.002121
45	0.002247	0.002209	0.002172	0.002135	0.002099
46	0.002555	0.002523	0.002492	0.002462	0.002431
47	0.002796	0.002763	0.002731	0.002699	0.002667
48	0.002986	0.002947	0.002909	0.002872	0.002835
49	0.003138	0.003104	0.003069	0.003035	0.003001
50	0.003453	0.003415	0.003377	0.00334	0.003303
51	0.00367	0.003631	0.003594	0.003556	0.003519
52	0.003954	0.003917	0.003879	0.003843	0.003806
53	0.004446	0.004412	0.004378	0.004345	0.004311
54	0.004815	0.004777	0.00474	0.004702	0.004665
55	0.004782	0.00476	0.004737	0.004715	0.004693
56	0.00514	0.005093	0.005047	0.005001	0.004956
57	0.006061	0.006016	0.005972	0.005927	0.005884
58	0.006213	0.006152	0.006091	0.006031	0.005972
59	0.007278	0.007214	0.007151	0.007088	0.007026
60	0.00782	0.007756	0.007692	0.007629	0.007567
61	0.008185	0.008109	0.008034	0.007959	0.007885
62	0.009654	0.009566	0.009478	0.009392	0.009306
63	0.010647	0.010548	0.01045	0.010352	0.010256
64	0.011156	0.011056	0.010957	0.010859	0.010762
65	0.012245	0.01214	0.012035	0.011931	0.011828

Table B1 The forecasted mortality rate, females (Continued).

Age	2013	2014	2015	2016	2017
66	0.013475	0.013389	0.013304	0.01322	0.013136
67	0.014662	0.014521	0.014382	0.014243	0.014106
68	0.016214	0.016084	0.015955	0.015827	0.015701
69	0.017179	0.017051	0.016925	0.016799	0.016675
70	0.020111	0.019962	0.019813	0.019666	0.019519
71	0.020992	0.020833	0.020674	0.020517	0.020361
72	0.022784	0.02264	0.022497	0.022355	0.022214
73	0.026729	0.026544	0.02636	0.026177	0.025996
74	0.029475	0.029293	0.029111	0.028931	0.028752
75	0.031305	0.031103	0.030902	0.030702	0.030504
76	0.03763	0.037424	0.037218	0.037014	0.036811
77	0.038825	0.038611	0.038398	0.038186	0.037975
78	0.046555	0.046303	0.046053	0.045805	0.045557
79	0.049889	0.049692	0.049495	0.0493	0.049105
80	0.054536	0.054319	0.054104	0.053889	0.053675
81	0.06083	0.060663	0.060497	0.060331	0.060166
82	0.069569	0.06931	0.069052	0.068795	0.068539
83	0.073976	0.073869	0.073763	0.073657	0.073551
84	0.087086	0.086812	0.086539	0.086266	0.085995
85	0.089035	0.088871	0.088708	0.088545	0.088382
86	0.104499	0.104317	0.104136	0.103955	0.103774
87	0.113403	0.113454	0.113505	0.113556	0.113607
88	0.124423	0.124885	0.125348	0.125813	0.12628
89	0.147399	0.148066	0.148736	0.149408	0.150084
90	0.150237	0.151055	0.151878	0.152705	0.153537
91	0.156681	0.157509	0.158341	0.159177	0.160018
92	0.187882	0.188828	0.189779	0.190734	0.191694
93	0.164191	0.165004	0.16582	0.166641	0.167465
94	0.202948	0.203951	0.204959	0.205973	0.206991
95	0.187469	0.18845	0.189437	0.190429	0.191426
96	0.198146	0.19825	0.198353	0.198457	0.19856
97	0.195211	0.19575	0.196291	0.196833	0.197377
98	0.192566	0.196121	0.199742	0.20343	0.207186
99	0.201873	0.204035	0.20622	0.208428	0.210661
100	0.170217	0.173195	0.176226	0.17931	0.182447
>100	0.084239	0.087015	0.089882	0.092844	0.095904

Table B1 The forecasted mortality rate, females (Continued).

Age	2018	2019	2020	2021	2022
<1	0.007038	0.007099	0.007161	0.007223	0.007285
1	0.000628	0.000593	0.00056	0.000528	0.000499
2	0.00035	0.000342	0.000335	0.000328	0.000321
3	0.000344	0.000336	0.000328	0.000321	0.000314
4	0.000301	0.000295	0.000288	0.000282	0.000276
5	0.000286	0.000281	0.000275	0.00027	0.000265
6	0.000278	0.000272	0.000266	0.00026	0.000254
7	0.000296	0.000289	0.000282	0.000275	0.000268
8	0.000283	0.00028	0.000277	0.000275	0.000272
9	0.000254	0.000252	0.000249	0.000246	0.000244
10	0.000288	0.000284	0.000281	0.000277	0.000273
11	0.000241	0.000239	0.000236	0.000233	0.00023
12	0.000339	0.000338	0.000336	0.000335	0.000333
13	0.000398	0.000397	0.000396	0.000395	0.000394
14	0.000411	0.000407	0.000404	0.000401	0.000398
15	0.000511	0.00051	0.000509	0.000507	0.000506
16	0.000432	0.000425	0.000418	0.000411	0.000404
17	0.000537	0.00053	0.000523	0.000516	0.000509
18	0.000512	0.000507	0.000503	0.000499	0.000495
19	0.000498	0.000492	0.000486	0.000481	0.000475
20	0.000453	0.000437	0.000421	0.000406	0.000391
21	0.000453	0.000439	0.000426	0.000413	0.000401
22	0.000467	0.000453	0.000439	0.000426	0.000413
23	0.000439	0.000424	0.00041	0.000397	0.000384
24	0.000463	0.000448	0.000433	0.000419	0.000405
25	0.000493	0.000476	0.000459	0.000443	0.000428
26	0.000554	0.000537	0.000521	0.000505	0.000489
27	0.000664	0.000646	0.000629	0.000612	0.000596
28	0.000609	0.000591	0.000574	0.000557	0.00054
29	0.000758	0.00074	0.000723	0.000706	0.000689
30	0.000711	0.000688	0.000666	0.000644	0.000623
31	0.000809	0.00078	0.000752	0.000725	0.000699
32	0.000813	0.00079	0.000767	0.000745	0.000723
33	0.000897	0.000873	0.000849	0.000827	0.000804
34	0.000959	0.000935	0.000913	0.00089	0.000868
35	0.001154	0.001129	0.001105	0.001081	0.001058
36	0.001085	0.001061	0.001038	0.001015	0.000993
37	0.001173	0.001148	0.001123	0.001099	0.001076
38	0.001351	0.001328	0.001304	0.001282	0.001259
39	0.001367	0.001337	0.001308	0.001279	0.001251
40	0.001503	0.001474	0.001445	0.001417	0.00139

Table B1 The forecasted mortality rate, females (Continued).

Age	2018	2019	2020	2021	2022
41	0.001589	0.001558	0.001528	0.001498	0.001469
42	0.001655	0.001628	0.001602	0.001577	0.001552
43	0.001948	0.001925	0.001902	0.00188	0.001857
44	0.002089	0.002058	0.002027	0.001997	0.001968
45	0.002064	0.002029	0.001995	0.001962	0.001929
46	0.002401	0.002372	0.002343	0.002314	0.002285
47	0.002636	0.002605	0.002574	0.002544	0.002514
48	0.002798	0.002762	0.002727	0.002691	0.002657
49	0.002968	0.002935	0.002903	0.00287	0.002838
50	0.003266	0.00323	0.003194	0.003159	0.003124
51	0.003482	0.003446	0.00341	0.003374	0.003339
52	0.00377	0.003734	0.003699	0.003664	0.003629
53	0.004278	0.004246	0.004213	0.004181	0.004148
54	0.004629	0.004592	0.004556	0.004521	0.004485
55	0.004671	0.004649	0.004627	0.004605	0.004583
56	0.004911	0.004866	0.004822	0.004778	0.004735
57	0.00584	0.005797	0.005754	0.005711	0.005669
58	0.005913	0.005855	0.005797	0.00574	0.005684
59	0.006964	0.006903	0.006842	0.006782	0.006723
60	0.007504	0.007443	0.007382	0.007321	0.007261
61	0.007811	0.007739	0.007667	0.007595	0.007525
62	0.00922	0.009136	0.009052	0.008969	0.008887
63	0.01016	0.010066	0.009972	0.009879	0.009787
64	0.010666	0.010571	0.010476	0.010383	0.01029
65	0.011726	0.011625	0.011525	0.011426	0.011327
66	0.013053	0.012971	0.012888	0.012807	0.012726
67	0.013971	0.013836	0.013703	0.013571	0.013441
68	0.015575	0.01545	0.015327	0.015204	0.015082
69	0.016551	0.016429	0.016307	0.016186	0.016066
70	0.019374	0.01923	0.019087	0.018945	0.018804
71	0.020206	0.020053	0.0199	0.019749	0.019599
72	0.022073	0.021934	0.021795	0.021658	0.021521
73	0.025816	0.025637	0.025459	0.025283	0.025108
74	0.028574	0.028397	0.028221	0.028047	0.027873
75	0.030306	0.030111	0.029916	0.029722	0.02953
76	0.036609	0.036408	0.036209	0.03601	0.035813
77	0.037766	0.037558	0.03735	0.037144	0.03694
78	0.045311	0.045066	0.044823	0.044581	0.04434
79	0.04891	0.048717	0.048524	0.048332	0.048141
80	0.053462	0.05325	0.053039	0.052829	0.052619

Table B1 The forecasted mortality rate, females (Continued).

Age	2018	2019	2020	2021	2022
81	0.060001	0.059836	0.059672	0.059508	0.059345
82	0.068283	0.068029	0.067776	0.067523	0.067272
83	0.073445	0.07334	0.073234	0.073129	0.073024
84	0.085724	0.085455	0.085186	0.084918	0.08465
85	0.088219	0.088057	0.087896	0.087734	0.087573
86	0.103594	0.103414	0.103234	0.103055	0.102876
87	0.113658	0.113709	0.11376	0.113811	0.113862
88	0.126748	0.127218	0.12769	0.128164	0.128639
89	0.150763	0.151445	0.15213	0.152818	0.153509
90	0.154373	0.155214	0.156059	0.156909	0.157764
91	0.160864	0.161714	0.162568	0.163427	0.16429
92	0.192659	0.193629	0.194604	0.195584	0.196568
93	0.168294	0.169126	0.169963	0.170804	0.171649
94	0.208014	0.209042	0.210076	0.211114	0.212158
95	0.192428	0.193436	0.194449	0.195467	0.196491
96	0.198664	0.198768	0.198871	0.198975	0.199079
97	0.197923	0.19847	0.199018	0.199568	0.20012
98	0.211011	0.214907	0.218875	0.222916	0.227032
99	0.212917	0.215197	0.217502	0.219831	0.222185
100	0.18564	0.188888	0.192193	0.195556	0.198978
>100	0.099064	0.102328	0.1057	0.109184	0.112781

The following tables show the forecasted mortality rate for males.

Table B2 The forecasted mortality rate, males.

Age	2013	2014	2015	2016	2017
<1	0.007807	0.007849	0.007891	0.007933	0.007976
1	0.000914	0.000884	0.000855	0.000827	0.0008
2	0.000706	0.000694	0.000683	0.000672	0.000661
3	0.000526	0.000518	0.00051	0.000502	0.000494
4	0.000518	0.000505	0.000493	0.000481	0.000469
5	0.000421	0.00041	0.0004	0.00039	0.00038
6	0.000492	0.00048	0.000468	0.000457	0.000446
7	0.000375	0.000367	0.000358	0.00035	0.000342
8	0.000422	0.000412	0.000403	0.000394	0.000385
9	0.000375	0.000365	0.000356	0.000346	0.000338
10	0.000401	0.000395	0.00039	0.000384	0.000378

Table B2 The forecasted mortality rate, males (Continued).

Age	2013	2014	2015	2016	2017
11	0.000398	0.000394	0.000389	0.000384	0.00038
12	0.000462	0.000459	0.000457	0.000454	0.000451
13	0.000593	0.000593	0.000593	0.000593	0.000593
14	0.000881	0.000892	0.000903	0.000913	0.000924
15	0.001335	0.001347	0.001359	0.001371	0.001383
16	0.001389	0.00139	0.001391	0.001392	0.001392
17	0.001626	0.001615	0.001604	0.001593	0.001582
18	0.001731	0.00171	0.00169	0.001669	0.001649
19	0.001852	0.00182	0.001788	0.001758	0.001727
20	0.0017	0.001657	0.001616	0.001576	0.001537
21	0.001654	0.001611	0.001569	0.001528	0.001488
22	0.001767	0.00173	0.001694	0.001659	0.001624
23	0.001764	0.001723	0.001684	0.001645	0.001608
24	0.001818	0.001773	0.001729	0.001686	0.001644
25	0.001972	0.001899	0.00183	0.001762	0.001698
26	0.001879	0.001812	0.001749	0.001687	0.001628
27	0.002004	0.001933	0.001864	0.001798	0.001734
28	0.002114	0.002042	0.001972	0.001906	0.001841
29	0.002374	0.002298	0.002224	0.002152	0.002083
30	0.002389	0.002313	0.002239	0.002167	0.002098
31	0.002589	0.002515	0.002443	0.002374	0.002306
32	0.002787	0.002709	0.002634	0.00256	0.002488
33	0.002955	0.002876	0.002799	0.002724	0.002651
34	0.00312	0.003043	0.002968	0.002896	0.002824
35	0.003565	0.003486	0.00341	0.003335	0.003262
36	0.003651	0.003582	0.003514	0.003448	0.003383
37	0.003846	0.003781	0.003716	0.003653	0.003591
38	0.003893	0.003835	0.003778	0.003721	0.003666
39	0.004436	0.004381	0.004326	0.004272	0.004219
40	0.004347	0.004304	0.00426	0.004218	0.004175
41	0.004866	0.004823	0.004781	0.004739	0.004698
42	0.004778	0.004734	0.004689	0.004645	0.004602
43	0.005279	0.005243	0.005206	0.005171	0.005135
44	0.005534	0.005496	0.005458	0.00542	0.005383
45	0.005832	0.005791	0.005751	0.005711	0.005672
46	0.006144	0.006111	0.006079	0.006046	0.006014
47	0.006471	0.006439	0.006407	0.006375	0.006344
48	0.006836	0.006815	0.006795	0.006775	0.006754
49	0.007187	0.007169	0.007151	0.007133	0.007115
50	0.007911	0.00788	0.00785	0.00782	0.00779

Table B2 The forecasted mortality rate, males (Continued).

Age	2013	2014	2015	2016	2017
51	0.008267	0.008235	0.008204	0.008172	0.008141
52	0.0091	0.009066	0.009032	0.008998	0.008964
53	0.009669	0.009618	0.009568	0.009517	0.009467
54	0.010016	0.009976	0.009937	0.009898	0.009859
55	0.010003	0.009949	0.009896	0.009842	0.009789
56	0.010293	0.010232	0.010171	0.010111	0.010051
57	0.012085	0.012016	0.011948	0.01188	0.011812
58	0.012209	0.012111	0.012013	0.011916	0.01182
59	0.013681	0.013579	0.013478	0.013378	0.013278
60	0.014607	0.014491	0.014376	0.014262	0.014149
61	0.015716	0.015596	0.015477	0.015358	0.015241
62	0.016823	0.016828	0.016832	0.016837	0.016841
63	0.017913	0.017817	0.017722	0.017627	0.017533
64	0.019001	0.018945	0.018889	0.018833	0.018778
65	0.02132	0.021273	0.021226	0.021179	0.021132
66	0.022982	0.022946	0.022911	0.022876	0.022841
67	0.024153	0.02404	0.023927	0.023815	0.023704
68	0.025564	0.025513	0.025463	0.025412	0.025361
69	0.027897	0.027824	0.027752	0.027679	0.027607
70	0.03142	0.031307	0.031195	0.031084	0.030972
71	0.034023	0.033955	0.033888	0.033821	0.033754
72	0.035441	0.035292	0.035145	0.034998	0.034852
73	0.039459	0.039346	0.039233	0.039121	0.039009
74	0.042754	0.042625	0.042496	0.042368	0.04224
75	0.045539	0.045404	0.045269	0.045135	0.045001
76	0.052716	0.052566	0.052416	0.052267	0.052118
77	0.055137	0.055002	0.054866	0.054731	0.054597
78	0.061769	0.061528	0.061288	0.061049	0.060811
79	0.065868	0.065637	0.065407	0.065178	0.064949
80	0.073014	0.072883	0.072752	0.072622	0.072492
81	0.079846	0.079808	0.079771	0.079733	0.079695
82	0.088524	0.088458	0.088393	0.088327	0.088262
83	0.093961	0.093905	0.093848	0.093792	0.093736
84	0.107819	0.107958	0.108098	0.108237	0.108377
85	0.11087	0.111224	0.111579	0.111936	0.112294
86	0.125421	0.125889	0.126358	0.12683	0.127303
87	0.133966	0.134719	0.135476	0.136237	0.137002
88	0.137715	0.138867	0.140029	0.1412	0.142381
89	0.15879	0.160081	0.161383	0.162695	0.164018
90	0.14899	0.151709	0.154478	0.157297	0.160168

Table B2 The forecasted mortality rate, males (Continued).

Age	2013	2014	2015	2016	2017
91	0.162788	0.165619	0.168498	0.171428	0.174409
92	0.181567	0.184391	0.187258	0.190171	0.193128
93	0.162639	0.165452	0.168314	0.171225	0.174187
94	0.174374	0.176885	0.179432	0.182015	0.184636
95	0.154766	0.157112	0.159494	0.161911	0.164365
96	0.156785	0.158553	0.160342	0.16215	0.163979
97	0.126382	0.127388	0.128402	0.129425	0.130455
98	0.122422	0.125317	0.12828	0.131314	0.13442
99	0.113197	0.114909	0.116646	0.11841	0.120201
100	0.101359	0.104526	0.107792	0.11116	0.114633
>100	0.039506	0.03892	0.038342	0.037773	0.037212

APPENDIX C

THE LIFE EXPECTANCY

The following tables show the forecasted life expectancy of Thai population.

Table C1 The forecasted life expectancies, females.

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
<1	78.9	78.98	79.06	79.14	79.21	79.29	79.37	79.44	79.51	79.59
1	78.43	78.51	78.6	78.68	78.77	78.85	78.93	79.01	79.09	79.17
2	77.49	77.58	77.66	77.74	77.82	77.9	77.98	78.05	78.13	78.21
3	76.52	76.61	76.69	76.77	76.85	76.93	77	77.08	77.16	77.23
4	75.55	75.63	75.72	75.8	75.87	75.95	76.03	76.1	76.18	76.25
5	74.58	74.66	74.74	74.82	74.9	74.97	75.05	75.13	75.2	75.28
6	73.6	73.68	73.76	73.84	73.92	74	74.07	74.15	74.22	74.29
7	72.62	72.7	72.78	72.86	72.94	73.02	73.09	73.17	73.24	73.31
8	71.65	71.73	71.81	71.88	71.96	72.04	72.11	72.19	72.26	72.33
9	70.67	70.75	70.83	70.91	70.98	71.06	71.13	71.21	71.28	71.35
10	69.69	69.77	69.85	69.92	70	70.08	70.15	70.22	70.3	70.37
11	68.71	68.79	68.87	68.94	69.02	69.1	69.17	69.24	69.32	69.39
12	67.73	67.81	67.88	67.96	68.04	68.11	68.19	68.26	68.33	68.4
13	66.75	66.83	66.91	66.98	67.06	67.14	67.21	67.28	67.36	67.43
14	65.78	65.86	65.93	66.01	66.09	66.16	66.24	66.31	66.38	66.45
15	64.8	64.88	64.96	65.04	65.11	65.19	65.26	65.34	65.41	65.48
16	63.84	63.92	63.99	64.07	64.15	64.22	64.3	64.37	64.44	64.51
17	62.87	62.95	63.02	63.1	63.18	63.25	63.32	63.4	63.47	63.54
18	61.9	61.98	62.06	62.13	62.21	62.28	62.36	62.43	62.5	62.57
19	60.94	61.01	61.09	61.17	61.24	61.32	61.39	61.46	61.53	61.6
20	59.97	60.05	60.12	60.2	60.27	60.35	60.42	60.49	60.56	60.63
21	59	59.08	59.15	59.23	59.3	59.37	59.44	59.52	59.59	59.65
22	58.03	58.11	58.18	58.25	58.33	58.4	58.47	58.54	58.61	58.68
23	57.06	57.14	57.21	57.28	57.36	57.43	57.5	57.57	57.63	57.7
24	56.09	56.17	56.24	56.31	56.38	56.45	56.52	56.59	56.66	56.72
25	55.12	55.19	55.27	55.34	55.41	55.48	55.55	55.61	55.68	55.75

Table C1 The forecasted life expectancies, females (Continued).

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
26	54.15	54.23	54.30	54.37	54.44	54.50	54.57	54.64	54.71	54.77
27	53.19	53.26	53.33	53.40	53.47	53.53	53.60	53.67	53.73	53.80
28	52.23	52.30	52.37	52.44	52.50	52.57	52.64	52.70	52.77	52.83
29	51.27	51.33	51.40	51.47	51.54	51.60	51.67	51.73	51.79	51.86
30	50.31	50.38	50.44	50.51	50.58	50.64	50.70	50.77	50.83	50.89
31	49.35	49.42	49.48	49.55	49.61	49.68	49.74	49.80	49.86	49.92
32	48.40	48.46	48.53	48.59	48.65	48.72	48.78	48.84	48.90	48.96
33	47.44	47.51	47.57	47.63	47.69	47.75	47.82	47.88	47.93	47.99
34	46.49	46.55	46.62	46.68	46.74	46.80	46.86	46.92	46.97	47.03
35	45.54	45.60	45.66	45.72	45.78	45.84	45.90	45.96	46.02	46.07
36	44.60	44.66	44.72	44.78	44.84	44.89	44.95	45.01	45.06	45.12
37	43.65	43.71	43.77	43.83	43.89	43.94	44.00	44.05	44.11	44.16
38	42.71	42.77	42.82	42.88	42.94	42.99	43.05	43.10	43.16	43.21
39	41.77	41.83	41.88	41.94	42.00	42.05	42.10	42.16	42.21	42.27
40	40.84	40.89	40.95	41.00	41.05	41.11	41.16	41.21	41.27	41.32
41	39.90	39.96	40.01	40.06	40.12	40.17	40.22	40.27	40.32	40.37
42	38.97	39.02	39.08	39.13	39.18	39.23	39.28	39.33	39.38	39.43
43	38.04	38.09	38.14	38.19	38.25	38.30	38.35	38.40	38.44	38.49
44	37.12	37.17	37.22	37.27	37.32	37.37	37.42	37.47	37.52	37.56
45	36.20	36.25	36.30	36.35	36.40	36.45	36.49	36.54	36.59	36.64
46	35.28	35.33	35.38	35.43	35.47	35.52	35.57	35.61	35.66	35.71
47	34.37	34.42	34.46	34.51	34.56	34.60	34.65	34.70	34.74	34.79
48	33.46	33.51	33.56	33.60	33.65	33.69	33.74	33.78	33.83	33.87
49	32.56	32.61	32.65	32.70	32.74	32.79	32.83	32.88	32.92	32.96
50	31.66	31.71	31.75	31.80	31.84	31.88	31.93	31.97	32.01	32.06
51	30.77	30.82	30.86	30.90	30.94	30.99	31.03	31.07	31.11	31.15
52	29.88	29.93	29.97	30.01	30.05	30.09	30.13	30.18	30.22	30.26
53	29.00	29.04	29.08	29.12	29.16	29.20	29.24	29.29	29.32	29.36
54	28.13	28.17	28.21	28.25	28.29	28.33	28.37	28.41	28.45	28.48
55	27.26	27.30	27.34	27.38	27.42	27.46	27.50	27.53	27.57	27.61
56	26.39	26.43	26.47	26.51	26.54	26.58	26.62	26.66	26.70	26.73
57	25.52	25.56	25.60	25.64	25.67	25.71	25.75	25.79	25.82	25.86
58	24.67	24.71	24.75	24.79	24.82	24.86	24.90	24.93	24.97	25.00
59	23.82	23.86	23.90	23.93	23.97	24.00	24.04	24.07	24.11	24.14
60	22.99	23.03	23.06	23.10	23.13	23.17	23.20	23.24	23.27	23.30
61	22.17	22.21	22.24	22.27	22.31	22.34	22.37	22.40	22.44	22.47
62	21.35	21.38	21.41	21.45	21.48	21.51	21.54	21.57	21.60	21.63

Table C1 The forecasted life expectancies, females (Continued).

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
63	20.55	20.58	20.61	20.64	20.67	20.7	20.73	20.76	20.79	20.82
64	19.77	19.8	19.82	19.85	19.88	19.9	19.94	19.97	20	20.02
65	18.98	19.01	19.04	19.06	19.09	19.1	19.15	19.17	19.2	19.23
66	18.21	18.24	18.26	18.29	18.31	18.3	18.36	18.39	18.41	18.44
67	17.45	17.48	17.5	17.52	17.55	17.6	17.6	17.62	17.64	17.67
68	16.7	16.72	16.75	16.77	16.79	16.8	16.84	16.86	16.88	16.9
69	15.97	15.99	16.01	16.03	16.05	16.1	16.09	16.11	16.13	16.15
70	15.23	15.25	15.27	15.29	15.31	15.3	15.35	15.37	15.38	15.4
71	14.53	14.55	14.57	14.58	14.6	14.6	14.64	14.65	14.67	14.69
72	13.83	13.85	13.86	13.88	13.89	13.9	13.92	13.94	13.95	13.97
73	13.14	13.15	13.17	13.18	13.19	13.2	13.22	13.23	13.25	13.26
74	12.48	12.49	12.5	12.52	12.53	12.5	12.55	12.56	12.57	12.58
75	11.84	11.85	11.86	11.87	11.88	11.9	11.9	11.91	11.92	11.93
76	11.2	11.21	11.21	11.22	11.23	11.2	11.25	11.25	11.26	11.27
77	10.61	10.62	10.62	10.63	10.63	10.6	10.64	10.65	10.66	10.66
78	10.01	10.01	10.02	10.02	10.03	10	10.03	10.04	10.04	10.04
79	9.46	9.46	9.47	9.47	9.47	9.47	9.47	9.47	9.48	9.48
80	8.92	8.92	8.92	8.92	8.92	8.92	8.92	8.92	8.92	8.92
81	8.39	8.39	8.39	8.39	8.39	8.38	8.38	8.38	8.38	8.37
82	7.89	7.89	7.88	7.88	7.87	7.87	7.87	7.86	7.86	7.86
83	7.42	7.42	7.41	7.4	7.4	7.39	7.39	7.38	7.37	7.37
84	6.95	6.95	6.94	6.93	6.92	6.92	6.91	6.9	6.9	6.89
85	6.54	6.53	6.52	6.51	6.5	6.49	6.48	6.47	6.46	6.45
86	6.1	6.09	6.08	6.07	6.06	6.05	6.03	6.02	6.01	6
87	5.72	5.71	5.69	5.68	5.66	5.65	5.64	5.62	5.61	5.59
88	5.35	5.33	5.32	5.3	5.29	5.27	5.26	5.24	5.23	5.21
89	4.99	4.98	4.96	4.95	4.93	4.92	4.9	4.89	4.87	4.86
90	4.71	4.69	4.68	4.67	4.65	4.64	4.62	4.61	4.6	4.58
91	4.39	4.38	4.37	4.35	4.34	4.33	4.32	4.31	4.29	4.28
92	4.05	4.04	4.03	4.02	4.01	4	3.99	3.98	3.97	3.96
93	3.79	3.78	3.77	3.76	3.75	3.75	3.74	3.73	3.72	3.71
94	3.38	3.37	3.36	3.36	3.35	3.34	3.34	3.33	3.32	3.31
95	3.03	3.02	3.02	3.01	3.01	3	3	2.99	2.99	2.98
96	2.55	2.55	2.54	2.54	2.54	2.53	2.53	2.53	2.53	2.52
97	2	2	1.99	1.99	1.99	1.98	1.98	1.98	1.97	1.97
98	1.32	1.32	1.32	1.32	1.31	1.31	1.31	1.3	1.3	1.3
99	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table C2 The forecasted life expectancies, males.

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
<1	72.07	72.16	72.25	72.34	72.42	72.50	72.58	72.66	72.74	72.82
1	71.64	71.73	71.82	71.91	72.00	72.08	72.17	72.25	72.33	72.41
2	70.70	70.79	70.88	70.97	71.05	71.14	71.22	71.30	71.38	71.46
3	69.75	69.84	69.93	70.02	70.10	70.19	70.27	70.35	70.43	70.51
4	68.79	68.88	68.96	69.05	69.14	69.22	69.30	69.38	69.46	69.54
5	67.82	67.91	68.00	68.08	68.17	68.25	68.33	68.41	68.49	68.57
6	66.85	66.94	67.03	67.11	67.19	67.28	67.36	67.43	67.51	67.59
7	65.88	65.97	66.06	66.14	66.22	66.30	66.38	66.46	66.54	66.62
8	64.91	64.99	65.08	65.16	65.25	65.33	65.41	65.48	65.56	65.64
9	63.94	64.02	64.11	64.19	64.27	64.35	64.43	64.51	64.58	64.66
10	62.96	63.04	63.13	63.21	63.29	63.37	63.45	63.53	63.60	63.68
11	61.98	62.07	62.15	62.24	62.32	62.40	62.47	62.55	62.62	62.70
12	61.01	61.09	61.18	61.26	61.34	61.42	61.50	61.57	61.65	61.72
13	60.04	60.12	60.20	60.29	60.37	60.45	60.52	60.60	60.67	60.75
14	59.07	59.16	59.24	59.32	59.40	59.48	59.56	59.64	59.71	59.78
15	58.12	58.21	58.29	58.38	58.46	58.54	58.62	58.69	58.77	58.84
16	57.20	57.29	57.37	57.46	57.54	57.62	57.70	57.77	57.85	57.93
17	56.28	56.37	56.45	56.53	56.62	56.70	56.78	56.85	56.93	57.01
18	55.37	55.46	55.54	55.62	55.71	55.79	55.86	55.94	56.02	56.09
19	54.46	54.55	54.63	54.72	54.80	54.88	54.95	55.03	55.11	55.18
20	53.56	53.65	53.73	53.81	53.89	53.97	54.04	54.12	54.19	54.27
21	52.66	52.74	52.82	52.90	52.97	53.05	53.12	53.20	53.27	53.34
22	51.74	51.82	51.90	51.98	52.05	52.12	52.20	52.27	52.34	52.41
23	50.83	50.91	50.99	51.06	51.13	51.21	51.28	51.35	51.42	51.48
24	49.92	50.00	50.07	50.14	50.22	50.29	50.36	50.42	50.49	50.56
25	49.01	49.08	49.16	49.23	49.30	49.37	49.43	49.50	49.57	49.63
26	48.11	48.18	48.25	48.31	48.38	48.45	48.51	48.57	48.64	48.70
27	47.20	47.26	47.33	47.39	47.46	47.52	47.58	47.65	47.71	47.76
28	46.29	46.35	46.42	46.48	46.54	46.60	46.66	46.72	46.78	46.83
29	45.39	45.45	45.51	45.57	45.63	45.68	45.74	45.80	45.85	45.90
30	44.49	44.55	44.61	44.66	44.72	44.77	44.83	44.88	44.93	44.98
31	43.60	43.65	43.71	43.76	43.81	43.86	43.91	43.97	44.02	44.06
32	42.71	42.76	42.81	42.86	42.91	42.96	43.01	43.06	43.10	43.15
33	41.83	41.88	41.92	41.97	42.02	42.06	42.11	42.15	42.20	42.24
34	40.95	41.00	41.04	41.08	41.13	41.17	41.21	41.26	41.30	41.34
35	40.08	40.12	40.16	40.20	40.24	40.28	40.32	40.36	40.40	40.44
36	39.22	39.26	39.30	39.33	39.37	39.41	39.45	39.49	39.52	39.56
37	38.36	38.40	38.43	38.47	38.50	38.54	38.58	38.61	38.64	38.68
38	37.51	37.54	37.57	37.61	37.64	37.67	37.71	37.74	37.77	37.81
39	36.65	36.68	36.71	36.75	36.78	36.81	36.84	36.87	36.90	36.93
40	35.81	35.84	35.87	35.90	35.93	35.96	35.99	36.02	36.05	36.08

Table C2 The forecasted life expectancies, males (Continued).

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
41	34.96	34.99	35.02	35.05	35.08	35.11	35.14	35.16	35.19	35.22
42	34.13	34.16	34.19	34.21	34.24	34.27	34.3	34.32	34.35	34.38
43	33.29	33.32	33.35	33.37	33.4	33.42	33.45	33.47	33.5	33.52
44	32.47	32.49	32.52	32.54	32.57	32.59	32.62	32.64	32.66	32.69
45	31.64	31.67	31.69	31.72	31.74	31.76	31.79	31.81	31.83	31.86
46	30.83	30.85	30.87	30.9	30.92	30.94	30.96	30.98	31.01	31.03
47	30.01	30.04	30.06	30.08	30.1	30.12	30.14	30.17	30.19	30.21
48	29.21	29.23	29.25	29.27	29.29	29.31	29.33	29.35	29.37	29.39
49	28.4	28.42	28.44	28.46	28.48	28.51	28.53	28.55	28.57	28.58
50	27.6	27.62	27.64	27.66	27.68	27.7	27.72	27.74	27.76	27.78
51	26.82	26.84	26.86	26.88	26.9	26.92	26.94	26.95	26.97	26.99
52	26.04	26.06	26.08	26.09	26.11	26.13	26.15	26.17	26.19	26.2
53	25.27	25.29	25.31	25.33	25.34	25.36	25.38	25.4	25.41	25.43
54	24.51	24.53	24.55	24.56	24.58	24.6	24.61	24.63	24.65	24.66
55	23.75	23.77	23.79	23.8	23.82	23.83	23.85	23.87	23.88	23.9
56	22.99	23	23.02	23.03	23.05	23.06	23.08	23.09	23.11	23.12
57	22.22	22.23	22.25	22.26	22.28	22.29	22.3	22.32	22.33	22.34
58	21.48	21.5	21.51	21.52	21.53	21.55	21.56	21.57	21.58	21.6
59	20.74	20.75	20.76	20.77	20.78	20.8	20.81	20.82	20.83	20.84
60	20.02	20.03	20.04	20.05	20.06	20.06	20.07	20.08	20.09	20.1
61	19.31	19.31	19.32	19.33	19.33	19.34	19.35	19.35	19.36	19.37
62	18.61	18.61	18.61	18.62	18.62	18.63	18.63	18.64	18.64	18.65
63	17.91	17.92	17.92	17.93	17.93	17.94	17.94	17.94	17.95	17.95
64	17.23	17.23	17.23	17.24	17.24	17.24	17.25	17.25	17.25	17.25
65	16.55	16.55	16.55	16.55	16.56	16.56	16.56	16.56	16.57	16.57
66	15.89	15.9	15.9	15.9	15.9	15.9	15.9	15.9	15.91	15.91
67	15.25	15.25	15.25	15.25	15.26	15.26	15.26	15.26	15.26	15.26
68	14.61	14.61	14.61	14.61	14.61	14.61	14.61	14.61	14.61	14.61
69	13.98	13.98	13.98	13.97	13.97	13.97	13.97	13.97	13.97	13.96
70	13.36	13.36	13.35	13.35	13.35	13.35	13.34	13.34	13.34	13.34
71	12.77	12.77	12.76	12.76	12.75	12.75	12.75	12.74	12.74	12.73
72	12.19	12.19	12.18	12.18	12.17	12.17	12.16	12.16	12.15	12.15
73	11.62	11.61	11.6	11.6	11.59	11.58	11.57	11.57	11.56	11.55
74	11.06	11.06	11.05	11.04	11.03	11.02	11.01	11	10.99	10.99
75	10.53	10.52	10.5	10.49	10.48	10.47	10.46	10.45	10.44	10.43
76	9.99	9.98	9.97	9.96	9.94	9.93	9.92	9.91	9.89	9.88
77	9.51	9.49	9.48	9.46	9.45	9.43	9.42	9.41	9.39	9.38

Table C2 The forecasted life expectancies, males (Continued).

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
78	9.02	9	8.98	8.97	8.95	8.93	8.92	8.9	8.89	8.87
79	8.56	8.54	8.52	8.5	8.48	8.46	8.44	8.42	8.4	8.38
80	8.11	8.09	8.06	8.04	8.02	7.99	7.97	7.95	7.93	7.9
81	7.69	7.66	7.63	7.61	7.58	7.56	7.53	7.51	7.48	7.45
82	7.28	7.26	7.23	7.2	7.17	7.14	7.11	7.09	7.06	7.03
83	6.91	6.88	6.85	6.82	6.79	6.75	6.72	6.69	6.66	6.63
84	6.54	6.51	6.47	6.44	6.4	6.37	6.34	6.3	6.27	6.23
85	6.23	6.19	6.16	6.12	6.08	6.04	6.01	5.97	5.93	5.89
86	5.91	5.87	5.83	5.78	5.74	5.7	5.66	5.63	5.59	5.55
87	5.63	5.59	5.54	5.5	5.46	5.42	5.37	5.33	5.29	5.25
88	5.37	5.32	5.28	5.23	5.19	5.14	5.1	5.05	5.01	4.97
89	5.08	5.04	5	4.95	4.91	4.86	4.82	4.77	4.73	4.68
90	4.88	4.83	4.78	4.74	4.69	4.65	4.6	4.55	4.51	4.46
91	4.58	4.54	4.5	4.46	4.42	4.38	4.34	4.3	4.26	4.22
92	4.3	4.27	4.24	4.21	4.17	4.14	4.11	4.07	4.04	4.01
93	4.06	4.04	4.01	3.98	3.96	3.93	3.9	3.88	3.85	3.82
94	3.69	3.67	3.66	3.64	3.62	3.6	3.58	3.56	3.54	3.52
95	3.3	3.29	3.28	3.26	3.25	3.24	3.22	3.21	3.2	3.18
96	2.77	2.77	2.76	2.75	2.74	2.74	2.73	2.72	2.71	2.7
97	2.16	2.16	2.15	2.15	2.14	2.14	2.14	2.13	2.13	2.12
98	1.38	1.38	1.38	1.38	1.37	1.37	1.37	1.37	1.36	1.36
99	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

The following tables show the forecasted life expectancy of Nakorn Ratchasima.

Table C3 The forecasted life expectancies of Nakorn Ratchasima, females.

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
<1	78.9	78.98	79.06	79.14	79.21	79.29	79.37	79.44	79.51	79.59
1	78.43	78.51	78.6	78.68	78.77	78.85	78.93	79.01	79.09	79.17
2	77.49	77.58	77.66	77.74	77.82	77.9	77.98	78.05	78.13	78.21
3	76.52	76.61	76.69	76.77	76.85	76.93	77	77.08	77.16	77.23
4	75.55	75.63	75.72	75.8	75.87	75.95	76.03	76.1	76.18	76.25
5	74.58	74.66	74.74	74.82	74.9	74.97	75.05	75.13	75.2	75.28
6	73.6	73.68	73.76	73.84	73.92	74	74.07	74.15	74.22	74.29

Table C3 The forecasted life expectancies of Nakorn Ratchasima, females (Continued).

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
7	72.62	72.7	72.78	72.86	72.94	73.02	73.09	73.17	73.24	73.31
8	71.65	71.73	71.81	71.88	71.96	72.04	72.11	72.19	72.26	72.33
9	70.67	70.75	70.83	70.91	70.98	71.06	71.13	71.21	71.28	71.35
10	69.69	69.77	69.85	69.92	70	70.08	70.15	70.22	70.3	70.37
11	68.71	68.79	68.87	68.94	69.02	69.1	69.17	69.24	69.32	69.39
12	67.73	67.81	67.88	67.96	68.04	68.11	68.19	68.26	68.33	68.41
13	66.75	66.83	66.91	66.98	67.06	67.14	67.21	67.28	67.36	67.43
14	65.78	65.86	65.93	66.01	66.09	66.16	66.24	66.31	66.38	66.45
15	64.8	64.88	64.96	65.04	65.11	65.19	65.26	65.34	65.41	65.48
16	63.84	63.92	63.99	64.07	64.15	64.22	64.3	64.37	64.44	64.51
17	62.87	62.95	63.02	63.1	63.18	63.25	63.32	63.4	63.47	63.54
18	61.9	61.98	62.06	62.13	62.21	62.28	62.36	62.43	62.5	62.57
19	60.94	61.01	61.09	61.17	61.24	61.32	61.39	61.46	61.53	61.6
20	59.97	60.05	60.12	60.2	60.27	60.35	60.42	60.49	60.56	60.63
21	59	59.08	59.15	59.23	59.3	59.37	59.44	59.52	59.59	59.65
22	58.03	58.11	58.18	58.25	58.33	58.4	58.47	58.54	58.61	58.68
23	57.06	57.14	57.21	57.28	57.36	57.43	57.5	57.57	57.63	57.7
24	56.09	56.17	56.24	56.31	56.38	56.45	56.52	56.59	56.66	56.72
25	55.12	55.19	55.27	55.34	55.41	55.48	55.55	55.61	55.68	55.75
26	54.15	54.23	54.3	54.37	54.44	54.5	54.57	54.64	54.71	54.77
27	53.19	53.26	53.33	53.4	53.47	53.53	53.6	53.67	53.73	53.8
28	52.23	52.3	52.37	52.44	52.5	52.57	52.64	52.7	52.77	52.83
29	51.27	51.33	51.4	51.47	51.54	51.6	51.67	51.73	51.79	51.86
30	50.31	50.38	50.44	50.51	50.58	50.64	50.7	50.77	50.83	50.89
31	49.35	49.42	49.48	49.55	49.61	49.68	49.74	49.8	49.86	49.92
32	48.4	48.46	48.53	48.59	48.65	48.72	48.78	48.84	48.9	48.96
33	47.44	47.51	47.57	47.63	47.69	47.75	47.82	47.88	47.93	47.99
34	46.49	46.55	46.62	46.68	46.74	46.8	46.86	46.92	46.97	47.03
35	45.54	45.6	45.66	45.72	45.78	45.84	45.9	45.96	46.02	46.07
36	44.6	44.66	44.72	44.78	44.84	44.89	44.95	45.01	45.06	45.12
37	43.65	43.71	43.77	43.83	43.89	43.94	44	44.05	44.11	44.16
38	42.71	42.77	42.82	42.88	42.94	42.99	43.05	43.1	43.16	43.21
39	41.77	41.83	41.88	41.94	42	42.05	42.1	42.16	42.21	42.27
40	40.84	40.89	40.95	41	41.05	41.11	41.16	41.21	41.27	41.32

Table C3 The forecasted life expectancies of Nakorn Ratchasima, females (Continued).

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
41	39.9	39.96	40.01	40.06	40.12	40.17	40.22	40.27	40.32	40.37
42	38.97	39.02	39.08	39.13	39.18	39.23	39.28	39.33	39.38	39.43
43	38.04	38.09	38.14	38.19	38.25	38.3	38.35	38.4	38.44	38.49
44	37.12	37.17	37.22	37.27	37.32	37.37	37.42	37.47	37.52	37.56
45	36.2	36.25	36.3	36.35	36.4	36.45	36.49	36.54	36.59	36.64
46	35.28	35.33	35.38	35.43	35.47	35.52	35.57	35.61	35.66	35.71
47	34.37	34.42	34.46	34.51	34.56	34.6	34.65	34.7	34.74	34.79
48	33.46	33.51	33.56	33.6	33.65	33.69	33.74	33.79	33.83	33.87
49	32.56	32.61	32.65	32.7	32.74	32.79	32.83	32.88	32.92	32.96
50	31.66	31.71	31.75	31.8	31.84	31.88	31.93	31.97	32.01	32.06
51	30.77	30.82	30.86	30.9	30.94	30.99	31.03	31.07	31.11	31.15
52	29.88	29.93	29.97	30.01	30.05	30.09	30.13	30.18	30.22	30.26
53	29	29.04	29.08	29.12	29.16	29.2	29.24	29.29	29.32	29.36
54	28.13	28.17	28.21	28.25	28.29	28.33	28.37	28.41	28.45	28.48
55	27.26	27.3	27.34	27.38	27.42	27.46	27.5	27.53	27.57	27.61
56	26.39	26.43	26.47	26.51	26.54	26.58	26.62	26.66	26.7	26.73
57	25.52	25.56	25.6	25.64	25.67	25.71	25.75	25.79	25.82	25.86
58	24.67	24.71	24.75	24.79	24.82	24.86	24.9	24.93	24.97	25
59	23.82	23.86	23.9	23.93	23.97	24	24.04	24.07	24.11	24.14
60	22.99	23.03	23.06	23.1	23.13	23.17	23.2	23.24	23.27	23.3
61	22.17	22.21	22.24	22.27	22.31	22.34	22.37	22.4	22.44	22.47
62	21.35	21.38	21.41	21.45	21.48	21.51	21.54	21.57	21.6	21.63
63	20.55	20.58	20.61	20.64	20.67	20.7	20.73	20.76	20.79	20.82
64	19.77	19.8	19.82	19.85	19.88	19.91	19.94	19.97	20	20.02
65	18.98	19.01	19.04	19.06	19.09	19.12	19.15	19.17	19.2	19.23
66	18.21	18.24	18.26	18.29	18.31	18.34	18.36	18.39	18.41	18.44
67	17.45	17.48	17.5	17.52	17.55	17.57	17.6	17.62	17.64	17.67
68	16.7	16.72	16.75	16.77	16.79	16.81	16.84	16.86	16.88	16.9
69	15.97	15.99	16.01	16.03	16.05	16.07	16.09	16.11	16.13	16.15
70	15.23	15.25	15.27	15.29	15.31	15.33	15.35	15.37	15.38	15.4
71	14.53	14.55	14.57	14.58	14.6	14.62	14.64	14.65	14.67	14.69
72	13.83	13.85	13.86	13.88	13.89	13.91	13.92	13.94	13.95	13.97
73	13.14	13.15	13.17	13.18	13.19	13.21	13.22	13.23	13.25	13.26
74	12.48	12.49	12.5	12.52	12.53	12.54	12.55	12.56	12.57	12.58
75	11.84	11.85	11.86	11.87	11.88	11.89	11.9	11.91	11.92	11.93

Table C3 The forecasted life expectancies of Nakorn Ratchasima, females (Continued).

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
76	11.2	11.21	11.21	11.22	11.23	11.24	11.25	11.25	11.26	11.27
77	10.61	10.62	10.62	10.63	10.63	10.64	10.64	10.65	10.66	10.66
78	10.01	10.01	10.02	10.02	10.03	10.03	10.03	10.04	10.04	10.04
79	9.46	9.46	9.47	9.47	9.47	9.47	9.47	9.47	9.48	9.48
80	8.92	8.92	8.92	8.92	8.92	8.92	8.92	8.92	8.92	8.92
81	8.39	8.39	8.39	8.39	8.39	8.38	8.38	8.38	8.38	8.37
82	7.89	7.89	7.88	7.88	7.87	7.87	7.87	7.86	7.86	7.86
83	7.42	7.42	7.41	7.4	7.4	7.39	7.39	7.38	7.37	7.37
84	6.95	6.95	6.94	6.93	6.92	6.92	6.91	6.9	6.9	6.89
85	6.54	6.53	6.52	6.51	6.5	6.49	6.48	6.47	6.46	6.45
86	6.1	6.09	6.08	6.07	6.06	6.05	6.03	6.02	6.01	6
87	5.72	5.71	5.69	5.68	5.66	5.65	5.64	5.62	5.61	5.59
88	5.35	5.33	5.32	5.3	5.29	5.27	5.26	5.24	5.23	5.21
89	4.99	4.98	4.96	4.95	4.93	4.92	4.9	4.89	4.87	4.86
90	4.71	4.69	4.68	4.67	4.65	4.64	4.62	4.61	4.6	4.58
91	4.39	4.38	4.37	4.35	4.34	4.33	4.32	4.31	4.29	4.28
92	4.05	4.04	4.03	4.02	4.01	4	3.99	3.98	3.97	3.96
93	3.79	3.78	3.77	3.76	3.75	3.75	3.74	3.73	3.72	3.71
94	3.38	3.37	3.36	3.36	3.35	3.34	3.34	3.33	3.32	3.31
95	3.03	3.02	3.02	3.01	3.01	3	3	2.99	2.99	2.98
96	2.55	2.55	2.54	2.54	2.54	2.53	2.53	2.53	2.53	2.52
97	2	2	1.99	1.99	1.99	1.98	1.98	1.98	1.97	1.97
98	1.32	1.32	1.32	1.32	1.31	1.31	1.31	1.3	1.3	1.3
99	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

Table C4 The forecasted life expectancies of Nakorn Ratchasima, males.

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
<1	72.07	72.16	72.25	72.34	72.42	72.5	72.58	72.66	72.74	72.82
1	71.64	71.73	71.82	71.91	72	72.08	72.17	72.25	72.33	72.41
2	70.7	70.79	70.88	70.97	71.05	71.14	71.22	71.3	71.38	71.46
3	69.75	69.84	69.93	70.02	70.1	70.19	70.27	70.35	70.43	70.51
4	68.79	68.88	68.96	69.05	69.14	69.22	69.3	69.38	69.46	69.54
5	67.82	67.91	68	68.08	68.17	68.25	68.33	68.41	68.49	68.57

Table C4 The forecasted life expectancies of Nakorn Ratchasima, males (Continued).

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
6	66.85	66.94	67.03	67.11	67.19	67.28	67.36	67.43	67.51	67.59
7	65.88	65.97	66.06	66.14	66.22	66.3	66.38	66.46	66.54	66.62
8	64.91	64.99	65.08	65.16	65.25	65.33	65.41	65.48	65.56	65.64
9	63.94	64.02	64.11	64.19	64.27	64.35	64.43	64.51	64.58	64.66
10	62.96	63.04	63.13	63.21	63.29	63.37	63.45	63.53	63.6	63.68
11	61.98	62.07	62.15	62.24	62.32	62.4	62.47	62.55	62.62	62.7
12	61.01	61.09	61.18	61.26	61.34	61.42	61.5	61.57	61.65	61.72
13	60.04	60.12	60.2	60.29	60.37	60.45	60.52	60.6	60.67	60.75
14	59.07	59.16	59.24	59.32	59.4	59.48	59.56	59.64	59.71	59.78
15	58.12	58.21	58.29	58.38	58.46	58.54	58.62	58.69	58.77	58.84
16	57.2	57.29	57.37	57.46	57.54	57.62	57.7	57.77	57.85	57.93
17	56.28	56.37	56.45	56.53	56.62	56.7	56.78	56.85	56.93	57.01
18	55.37	55.46	55.54	55.62	55.71	55.79	55.86	55.94	56.02	56.09
19	54.46	54.55	54.63	54.72	54.8	54.88	54.95	55.03	55.11	55.18
20	53.56	53.65	53.73	53.81	53.89	53.97	54.04	54.12	54.19	54.27
21	52.66	52.74	52.82	52.9	52.97	53.05	53.12	53.2	53.27	53.34
22	51.74	51.82	51.9	51.98	52.05	52.12	52.2	52.27	52.34	52.41
23	50.83	50.91	50.99	51.06	51.13	51.21	51.28	51.35	51.42	51.48
24	49.92	50	50.07	50.14	50.22	50.29	50.36	50.42	50.49	50.56
25	49.01	49.08	49.16	49.23	49.3	49.37	49.43	49.5	49.57	49.63
26	48.11	48.18	48.25	48.31	48.38	48.45	48.51	48.57	48.64	48.7
27	47.2	47.26	47.33	47.39	47.46	47.52	47.58	47.65	47.71	47.76
28	46.29	46.35	46.42	46.48	46.54	46.6	46.66	46.72	46.78	46.83
29	45.39	45.45	45.51	45.57	45.63	45.68	45.74	45.8	45.85	45.9
30	44.49	44.55	44.61	44.66	44.72	44.77	44.83	44.88	44.93	44.98
31	43.6	43.65	43.71	43.76	43.81	43.86	43.91	43.97	44.02	44.06
32	42.71	42.76	42.81	42.86	42.91	42.96	43.01	43.06	43.1	43.15
33	41.83	41.88	41.92	41.97	42.02	42.06	42.11	42.15	42.2	42.24
34	40.95	41	41.04	41.08	41.13	41.17	41.21	41.26	41.3	41.34
35	40.08	40.12	40.16	40.2	40.24	40.28	40.32	40.36	40.4	40.44
36	39.22	39.26	39.3	39.33	39.37	39.41	39.45	39.49	39.52	39.56
37	38.36	38.4	38.43	38.47	38.5	38.54	38.58	38.61	38.64	38.68
38	37.51	37.54	37.57	37.61	37.64	37.67	37.71	37.74	37.77	37.81
39	36.65	36.68	36.71	36.75	36.78	36.81	36.84	36.87	36.9	36.93
40	35.81	35.84	35.87	35.9	35.93	35.96	35.99	36.02	36.05	36.08
41	34.96	34.99	35.02	35.05	35.08	35.11	35.14	35.16	35.19	35.22
42	34.13	34.16	34.19	34.21	34.24	34.27	34.3	34.32	34.35	34.38
43	33.29	33.32	33.35	33.37	33.4	33.42	33.45	33.47	33.5	33.52
44	32.47	32.49	32.52	32.54	32.57	32.59	32.62	32.64	32.66	32.69
45	31.64	31.67	31.69	31.72	31.74	31.76	31.79	31.81	31.83	31.86
46	30.83	30.85	30.87	30.9	30.92	30.94	30.96	30.98	31.01	31.03
47	30.01	30.04	30.06	30.08	30.1	30.12	30.14	30.17	30.19	30.21

Table C4 The forecasted life expectancies of Nakorn Ratchasima, males (Continued).

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
48	29.21	29.23	29.25	29.27	29.29	29.31	29.33	29.35	29.37	29.39
49	28.4	28.42	28.44	28.46	28.48	28.51	28.53	28.55	28.57	28.58
50	27.6	27.62	27.64	27.66	27.68	27.7	27.72	27.74	27.76	27.78
51	26.82	26.84	26.86	26.88	26.9	26.92	26.94	26.95	26.97	26.99
52	26.04	26.06	26.08	26.09	26.11	26.13	26.15	26.17	26.19	26.2
53	25.27	25.29	25.31	25.33	25.34	25.36	25.38	25.4	25.41	25.43
54	24.51	24.53	24.55	24.56	24.58	24.6	24.61	24.63	24.65	24.66
55	23.75	23.77	23.79	23.8	23.82	23.83	23.85	23.87	23.88	23.9
56	22.99	23	23.02	23.03	23.05	23.06	23.08	23.09	23.11	23.12
57	22.22	22.23	22.25	22.26	22.28	22.29	22.3	22.32	22.33	22.34
58	21.48	21.5	21.51	21.52	21.53	21.55	21.56	21.57	21.58	21.6
59	20.74	20.75	20.76	20.77	20.78	20.8	20.81	20.82	20.83	20.84
60	20.02	20.03	20.04	20.05	20.06	20.06	20.07	20.08	20.09	20.1
61	19.31	19.31	19.32	19.33	19.33	19.34	19.35	19.35	19.36	19.37
62	18.61	18.61	18.61	18.62	18.62	18.63	18.63	18.64	18.64	18.65
63	17.91	17.92	17.92	17.93	17.93	17.94	17.94	17.94	17.95	17.95
64	17.23	17.23	17.23	17.24	17.24	17.24	17.25	17.25	17.25	17.25
65	16.55	16.55	16.55	16.55	16.56	16.56	16.56	16.56	16.57	16.57
66	15.89	15.9	15.9	15.9	15.9	15.9	15.9	15.9	15.91	15.91
67	15.25	15.25	15.25	15.25	15.26	15.26	15.26	15.26	15.26	15.26
68	14.61	14.61	14.61	14.61	14.61	14.61	14.61	14.61	14.61	14.61
69	13.98	13.98	13.98	13.97	13.97	13.97	13.97	13.97	13.97	13.96
70	13.36	13.36	13.35	13.35	13.35	13.35	13.34	13.34	13.34	13.34
71	12.77	12.77	12.76	12.76	12.75	12.75	12.75	12.74	12.74	12.73
72	12.19	12.19	12.18	12.18	12.17	12.17	12.16	12.16	12.15	12.15
73	11.62	11.61	11.6	11.6	11.59	11.58	11.57	11.57	11.56	11.55
74	11.06	11.06	11.05	11.04	11.03	11.02	11.01	11	10.99	10.99
75	10.53	10.52	10.5	10.49	10.48	10.47	10.46	10.45	10.44	10.43
76	9.99	9.98	9.97	9.96	9.94	9.93	9.92	9.91	9.89	9.88
77	9.51	9.49	9.48	9.46	9.45	9.43	9.42	9.41	9.39	9.38
78	9.02	9	8.98	8.97	8.95	8.93	8.92	8.9	8.89	8.87
79	8.56	8.54	8.52	8.5	8.48	8.46	8.44	8.42	8.4	8.38
80	8.11	8.09	8.06	8.04	8.02	7.99	7.97	7.95	7.93	7.9
81	7.69	7.66	7.63	7.61	7.58	7.56	7.53	7.51	7.48	7.45
82	7.28	7.26	7.23	7.2	7.17	7.14	7.11	7.09	7.06	7.03
83	6.91	6.88	6.85	6.82	6.79	6.75	6.72	6.69	6.66	6.63
84	6.54	6.51	6.47	6.44	6.4	6.37	6.34	6.3	6.27	6.23
85	6.23	6.19	6.16	6.12	6.08	6.04	6.01	5.97	5.93	5.89
86	5.91	5.87	5.83	5.78	5.74	5.7	5.66	5.63	5.59	5.55
87	5.63	5.59	5.54	5.5	5.46	5.42	5.37	5.33	5.29	5.25

Table C4 The forecasted life expectancies of Nakorn Ratchasima, males (Continued).

Age	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
88	5.37	5.32	5.28	5.23	5.19	5.14	5.1	5.05	5.01	4.97
89	5.08	5.04	5	4.95	4.91	4.86	4.82	4.77	4.73	4.68
90	4.88	4.83	4.78	4.74	4.69	4.65	4.6	4.55	4.51	4.46
91	4.58	4.54	4.5	4.46	4.42	4.38	4.34	4.3	4.26	4.22
92	4.3	4.27	4.24	4.21	4.17	4.14	4.11	4.07	4.04	4.01
93	4.06	4.04	4.01	3.98	3.96	3.93	3.9	3.88	3.85	3.82
94	3.69	3.67	3.66	3.64	3.62	3.6	3.58	3.56	3.54	3.52
95	3.3	3.29	3.28	3.26	3.25	3.24	3.22	3.21	3.2	3.18
96	2.77	2.77	2.76	2.75	2.74	2.74	2.73	2.72	2.71	2.7
97	2.16	2.16	2.15	2.15	2.14	2.14	2.14	2.13	2.13	2.12
98	1.38	1.38	1.38	1.38	1.37	1.37	1.37	1.37	1.36	1.36
99	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5

APPENDIX D

ARIMA MODEL

Definition Autoregressive integrated moving average (ARIMA) model (Cowpertwait and Metcalfe, 2009)

A time series $\{X_t\}$ follows an ARIMA(*p*,*d*,*q*) process if the *d*th difference of the $\{X_t\}$ series is an ARMA(*p*,*q*) process.

If we introduce $$y_t = (1-B)^d X_t,$$

then $$\theta_p(B) y_t = \phi_q(B) w_t.$$

We can now substitute for y_t to obtain the more succinct form for an ARIMA(p,d,q) process as

$$\theta_p(B)(1-B)^d X_t = \phi_q w_t$$

where θ_p and ϕ_q are polynomials of order *p* and *q*, respectively.

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