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Action Principle and Modification of the Faddeev–Popov Factor in Gauge Theories

K. Limboonsong[1,2] and E. B. Manoukian[1]

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The quantum action (dynamical) principle is exploited to investigate the nature and origin of the Faddeev–Popov (FP) factor in gauge theories without recourse to path integrals. Gauge invariant as well as gauge non-invariant interactions are considered to show that the FP factor needs to be modified in more general cases and expressions for these modifications are derived. In particular we show that a gauge invariant theory does not necessarily imply the familiar FP factor for proper quantization.

KEY WORDS: action principle; gauge theories; Faddeev–Popov factor; quantization rules.

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1. INTRODUCTION

In earlier communications (Manoukian, 1986, 1987; Manoukian and Siranan, 2005), we have seen that the quantum action (dynamical) principle (Schwinger, 1951a,b, 1953a,b, 1954, 1972, 1973; Lam, 1965; Manoukian, 1985) may be used to quantize gauge theories in constructing the vacuum-to-vacuum transition amplitude and the Faddeev–Popov (FP) factor (Faddeev and Popov, 1967), encountered in non-abelian gauge theories (e.g., (Abers and Lee, 1973; Rivers, 1987; 't Hooft, 2000; Veltman, 2000; Gross, 2005; Politzer, 2005; Wilczek, 2005)), may be obtained *directly* from the action principle without much effort. No appeal was made to path integrals, and there was not even the need to go into the well-known complicated structure of the Hamiltonian (Fradkin and Tyutin, 1970) in non-abelian gauge theories. For extensive references on the gauge problem in gauge theories

[1] School of Physics, Institute of Science, Suranaree University of Technology, Nakhon Ratchasima, 30000, Thailand.
[2] To whom correspondence should addressed be at School of Physics, Institute of Science, Suranaree University of Technology, Nakhon Ratchasima, 30000, Thailand; e-mail: manoukian_eb@hotmail.com.

see Manoukian and Siranan (2005). The latter reference traces its historical development from early papers to most recent ones.

In the present investigation, we consider the generic non-abelian gauge theory Lagrangian density

$$\mathscr{L}_{\mathrm{T}} = \mathscr{L} + \mathscr{L}_{\mathrm{S}} \tag{1}$$

and modifications thereof, where

$$\mathscr{L} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2i}[(\partial_\mu \overline{\psi})\gamma^\mu \psi - \overline{\psi}\gamma^\mu \partial_\mu \psi] - m_0 \overline{\psi}\psi + g_0 \overline{\psi}\gamma_\mu A^\mu \psi \tag{2}$$

$$\mathscr{L}_{\mathrm{S}} = \overline{\eta}\psi + \overline{\psi}\eta + J^\mu_a A^a_\mu \tag{3}$$

$$A_\mu = A^a_\mu t_a, \qquad G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i g_0 [A_\mu A_\nu] \tag{4}$$

$$G_{\mu\nu} = G^a_{\mu\nu} t_a \tag{5}$$

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_0 f^{abc} A^b_\mu A^c_\nu. \tag{6}$$

The t^a are generators of the underlying algebra, and the f^{abc}, totally antisymmetric, are the structure constants satisfying the Jacobi identity, $[t^a, t^b] = i f^{abc} t^c$. Note that A_μ is a matrix. $\mathscr{L}_{\mathrm{S}}$ is the source term with the J^a_μ classical functions, while η, $\overline{\eta}$ are so-called anti-commuting Grassmann variables.

The Lagrangian density $\mathscr{L}$ in (2) is invariant under simultaneous local gauge transformations:

$$\psi \longrightarrow U\psi, \qquad \overline{\psi} \longrightarrow \overline{\psi} U^{-1}, \tag{7}$$

$$A_\mu \longrightarrow U A_\mu U^{-1} + \frac{i}{g_0} U \partial_\mu U^{-1} \tag{8}$$

$$G_{\mu\nu} \longrightarrow U G_{\mu\nu} U^{-1} \tag{9}$$

where $U = U(\theta) = \exp[i g_0 \theta^a t^a]$, $\theta = \theta^a t^a$, $\theta = \theta(x)$.

Upon setting

$$\nabla_\mu = \partial_\mu - i g_0 A_\mu \tag{10}$$

with

$$\nabla^{ab}_\mu = \delta^{ab} \partial_\mu + g_0 f^{acb} A^c_\mu \tag{11}$$

we have the basic commutator

$$[\nabla_\mu, \nabla_\nu] = -i g_0 G_{\mu\nu} \tag{12}$$

and the identity

$$\nabla_\mu^{ab}\nabla_\nu^{bc}G_c^{\mu\nu} = 0. \tag{13}$$

[The latter generalizes the elementary identity $\partial_\mu\partial_\nu F^{\mu\nu} = 0$, in abelian gauge theory, to non-abelian ones, where $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$.]

We consider gauge invariant (Section 3.) as well as gauge non-invariant (Section 4.) modifications of the Lagrangian density and show by a systematic use of the quantum action principle that the familiar FP factor needs to be modified in more general cases and explicit expressions for these modifications are derived. In particular, we show that a gauge invariant theory does *not* necessarily imply the familiar FP factor for proper quantization, as may be perhaps expected (cf. Rivers (1987, p. 204), and modifications thereof may be necessary. Before doing so, however, we use the action principle to derive, in Section 2., the FP factor and investigate its origin for the classic Lagrangian density $\mathscr{L}$, without recourse to path integrals, as an anticipation of what to expect in more general cases. Throughout, we work in the celebrated Coulomb gauge $\partial_k A_a^k = 0$, $k = 1, 2, 3$.

2. ACTION PRINCIPLE AND THE ORIGIN OF THE FP FACTOR

To obtain the expression for the vacuum-to-vacuum transition amplitude $\langle 0_+ \,|\, 0_- \rangle$, in the presence of external sources J_μ^a, η^a, $\overline{\eta}^a$, as the generator of all the Green functions of the theory, *no* restrictions may be set, in particular, on the external current J_μ^a, coupled to the gauge fields A_a^μ, such as $\partial^\mu J_\mu^a = 0$, so that *variations of the components of* J_μ^a *may be carried out independently*, until the entire analysis is completed, and all functional differentiations are carried out to generate Green functions. This point cannot be overemphasized. As we will see, the *generality* condition that must be adopted on the external current J_μ^a together with the presence of *dependent* gauge field components in (A_a^μ), as a result of the structure of the Lagrangian density $\mathscr{L}$ in (2) and the gauge constraint, are responsible for the *origin* and the presence of the FP factor in the theory for a proper quantization in the realm of the quantum action principle.

We define the Green operator $D^{ab}(x, x')$ satisfying the differential equation

$$\left[\delta^{ac}\vec{\partial}^2 + g_0 f^{abc} A_k^b \partial_k\right] D^{cd}(x, x') = \delta^4(x, x')\delta^{ad}. \tag{14}$$

Since the differential operator on the left-hand side of $D^{cd}(x, x')$ is independent of the time derivative, $D^{cd}(x, x')$ involves a $\delta(x^0 - x'^0)$ factor. Using the gauge constraint, one may, for example, eliminate A_a^3 in favor of A_a^1, A_a^2. That is, we may treat the A_a^3 as dependent fields.

The field equations are given by

$$\nabla_\mu^{ab} G_b^{\mu\nu} = -\left(\delta^\nu{}_\sigma\,\delta^{ac} - g^{\nu k}\partial_k D^{ab}\nabla_\sigma^{bc}\right)\left[J_c^\sigma + g_0\overline{\psi}\gamma^\sigma t_c\psi\right] \tag{15}$$

with $\mu, \nu = 0, 1, 2, 3$, $k = 1, 2, 3$, and

$$\left[\gamma^{\mu}\frac{\nabla_{\mu}}{i} + m_0\right]\psi \;\; = \eta \tag{16}$$

$$\overline{\psi}\left[\gamma^{\mu}\frac{\overleftarrow{\nabla}^{*}_{\mu}}{i} - m_0\right] = -\overline{\eta} \tag{17}$$

where ∇_{μ} is defined in (10).

The canonical conjugate variables to A^1_a, A^2_a, are given by

$$\pi^{i}_{a} = G^{i0}_{a} - \partial_3^{-1}\partial^{i} G^{30}_{a}, \qquad i = 1, 2. \tag{18}$$

With $\pi^0_a = 0$, $\pi^3_a = 0$, we may rewrite (18) as

$$\pi^{\mu}_{a} = G^{\mu 0}_{a} - \partial_3^{-1} g^{\nu k}\partial_k G^{30}_{a} \tag{19}$$

$k = 1, 2, 3$. One may then readily express $G^{\mu 0}_a$ as follows:

$$G^{\mu 0}_{a} = \pi^{\mu}_{a} - g^{\mu k}\partial_k D_{ab}\left[J^0_b + g_0\overline{\psi}\gamma^0 t_b\psi + \nabla^{bc}_{\nu}\pi^{\nu}_{c}\right]. \tag{20}$$

We note that the right-hand side of (20) is expressed in terms of the independent fields A^1_a, A^2_a, their canonical conjugate momenta and involves no time derivatives. Here we recall that A^3_a is expressed in terms of A^1_a, A^2_a with no time derivative. Accordingly, with the (independent) fields and their canonical conjugate momenta kept *fixed*, we obtain the following functional derivative

$$\frac{\delta}{\delta J^{\nu}_{b}(x')} G^{\mu 0}_{a}(x) = -g^{\mu k}\delta^{0}{}_{\nu}\,\partial_k D_{ab}(x, x') \tag{21}$$

$\mu, \nu = 0, 1, 2, 3,\; k = 1, 2, 3$. On the other hand, $G^{kl}_a = \partial^k A^l_a - \partial^l A^k_a,\; k, l = 1, 2, 3$, may be expressed in terms of the independent fields A^1_a, A^2_a and involves no time derivatives. Accordingly with A^1_a, A^2_a and their canonical conjugate variables kept fixed, we also have

$$\frac{\delta}{\delta J^{\nu}_{b}(x')} G^{kl}_{a}(x) = 0. \tag{22}$$

Similarly, with ψ and $\overline{\psi}$ kept fixed, we have the obvious functional derivative expression

$$\frac{\delta}{\delta J^{\nu}_{b}(x')}[\overline{\psi}(x)\gamma^{\mu}t^{a}\psi(x)] = 0. \tag{23}$$

The action principle gives

$$\frac{\partial}{\partial g_0}\langle 0_+ \,|\, 0_-\rangle = i\left\langle 0_+ \left| \int (dx)\,\hat{\mathscr{L}}_{\rm I} \right| 0_- \right\rangle \tag{24}$$

where

$$\hat{\mathscr{L}}_{\rm I} = \frac{\partial}{\partial g_0}\mathscr{L} = -\frac{1}{2} f^{abc} A^b_\mu A^c_\nu G^{\mu\nu}_a + \overline{\psi}\gamma^\mu A_\mu \psi. \tag{25}$$

We may also write

$$f^{abc} A^b_\mu A^c_\nu G^{\mu\nu}_a = 2 f^{abc} A^b_k A^c_0 G^{k0}_a + f^{abc} A^b_k A^c_l G^{kl}_a \tag{26}$$

and set $(-i)\delta/\delta J^\mu_a = A'^a_\mu$, $(-i)\delta/\delta\overline{\eta} = \psi'$, $(-i)\delta/\delta\eta = \overline{\psi}'$. [Here we note that $G'^a_{\mu\nu}$ on the right-hand side of (5.30) of Manoukian (1986) should be replaced by $F'^a_{\mu\nu} = \partial_\mu A'^a_\nu - \partial_\nu A'^a_\mu$.]

Now we use the rule of functional differentiations (cf. Manoukian (2006), Ch. 11) that for an operator $\mathcal{O}(x)$

$$\begin{aligned}(-i)\frac{\delta}{\delta J^\mu_a(x')}\langle 0_+ | \mathcal{O}(x) | 0_-\rangle \;&= \left\langle 0_+ \left| \left(A^a_\mu(x')\mathcal{O}(x)\right)_+ \right| 0_- \right\rangle \\ &\quad - i\left\langle 0_+ \left| \frac{\delta}{\delta J^\mu_a(x')}\mathcal{O}(x) \right| 0_- \right\rangle \end{aligned}\tag{27}$$

where $(\ldots)_+$ denotes the time-ordered product, and the functional derivative of $\mathcal{O}(x)$ in the second term on the right-hand of (27) is taken as in (21)–(23) with the (independent) fields and their canonical conjugate momenta kept fixed. Here we recall that A^3_a may be expressed in terms of A^1_a, A^2_a and involves no time derivatives.

From (24)–(27), together with (21)–(23), we obtain

$$\frac{\partial}{\partial g_0}\langle 0_+ | 0_-\rangle = \int (dx)\left[i\hat{\mathscr{L}}'_{\rm I}(x) - f^{bca} A'^b_k \partial^k D'^{ac}(x,x)\right]\langle 0_+ | 0_-\rangle\,. \tag{28}$$

Using a matrix notation

$$D^{ab}(x,x') = \left[\left\langle x \left| \left(\frac{1}{\vec{\partial}^2 - i g_0 A_k \partial_k}\right) \right| x' \right\rangle\right]^{ab}, \tag{29}$$

the notation

$$Tr[f] = \int (dx)\, f^{aa}(x,x), \tag{30}$$

and the fact that $f^{bca} A^b_k = i(A_k)^{ca}$, we may rewrite the second factor within the square brackets in (28) as

$$Tr\left\{ -i A'_k\, \partial^k \frac{1}{[\vec{\partial}^2 - i g_0 A'_l\, \partial^l]}\right\}. \tag{31}$$

An elementary integration over g_0 from 0 to some g_0 value then gives the familiar FP factor for $\langle 0_+ \,|\, 0_- \rangle$ in (28)

$$\det\left[1 - i g_0 \frac{1}{\vec{\partial}^2} A'_k\, \partial^k\right]. \tag{32}$$

3. GAUGE INVARIANCE AND MODIFICATION OF THE FP FACTOR

Now consider the modification of the Lagrangian density $\mathscr{L}$ in (2):

$$\mathscr{L} \longrightarrow \mathscr{L} + \lambda \overline{\psi} \psi G^a_{\mu\nu} G^{\mu\nu}_a \equiv \mathscr{L}_1 \tag{33}$$

which is obviously gauge invariant under the simultaneous local gauge transformations in (7)–(9).

The field equations corresponding to the Lagrangian density $\mathscr{L}_{1\mathrm{T}} = \mathscr{L}_1 + \mathscr{L}_{\mathrm{S}}$, where $\mathscr{L}_{\mathrm{S}}$ is defined in (3), are given by

$$\nabla^{ab}_\mu \left(\left[1 - 4\lambda \overline{\psi} \psi\right] G^{\mu\nu}_b \right) = -\left(\delta^\nu{}_\sigma\, \delta^{ac} - g^{\nu k} \partial_k D^{ab} \nabla^{bc}_\sigma \right) \times \left[J^\sigma_c + g_0 \overline{\psi} \gamma^\sigma t_c \psi \right] \tag{34}$$

$$\left[\gamma^\mu \frac{\nabla_\mu}{i} - \lambda G^a_{\mu\nu} G^{\mu\nu}_a + m_0 \right] \psi = \eta \tag{35}$$

$$\overline{\psi} \left[\gamma^\mu \frac{\overleftarrow{\nabla}^*_\mu}{i} + \lambda G^a_{\mu\nu} G^{\mu\nu}_a - m_0 \right] = -\overline{\eta}. \tag{36}$$

The canonical conjugate momenta to A^1_a, A^2_a are given by

$$\pi^i_a = [1 - 4\lambda \overline{\psi} \psi] G^{i0}_a - \partial_3^{-1} \partial^i [1 - 4\lambda \overline{\psi} \psi] G^{30}_a \tag{37}$$

$i = 1, 2$. One may then express G^{k0}_a as follows:

$$\left[1 - 4\lambda \overline{\psi}(x) \psi(x)\right] G^{k0}_a(x) = \pi^k_a(x) - \partial_k \int (dx')\, D_{ab}(x, x') \left[J^0_b(x') + g_0 \overline{\psi}(x') \gamma^0 t_b \psi(x') + \nabla'^{bc}_j \pi^j_c(x') \right] \tag{38}$$

$k = 1, 2, 3$, with π^3_a set equal to zero.

With the (independent) fields and their canonical conjugate momenta kept fixed, we then have

$$\left[1 - 4\lambda \overline{\psi}(x) \psi(x)\right] \frac{\delta}{\delta J^\nu_b(x')} G^{k0}_a(x) = -\partial_k D_{ab}(x, x')\, \delta^0{}_\nu. \tag{39}$$

The equal time commutation relations of the independent fields $A_a^1(x)$, $A_a^2(x)$ are given by

$$\delta(x^0 - x'^0)\big[A_a^i(x), \pi_b^j(x')\big] = i\delta_{ab}\delta^{ij}\delta^4(x - x') \tag{40}$$

with $i, j = 1, 2$. From the gauge constraint, we may then write

$$\delta(x^0 - x'^0)\left[A_a^k(x), \pi_b^l(x')\right] = i\delta_{ab}\left[\delta^{kl} - \delta^{k3}\partial_3^{-1}\partial^l\right]\delta^4(x - x') \tag{41}$$

with now $k, l = 1, 2, 3$.

From (38), (41), we then obtain the commutation relation

$$\begin{aligned}
&\left[1 - 4\lambda\overline{\psi}(x)\psi(x)\right]\left[A_{ka}(x'), G_a^{k0}(x)\right]\delta(x^0 - x'^0)\\
&= 2i\delta_{aa}\delta^4(x - x') - \partial_k\int(dx'')\,D_{ab}(x, x'')\nabla_j''^{bc}\\
&\quad\times\left[A_{ka}(x'), \pi_c^j(x'')\right]\delta(x^0 - x'^0),
\end{aligned} \tag{42}$$

where we recall that $D_{ab}(x, x'')$ involves the factor $\delta(x^0 - x''^0)$. The latter then implies that the last term in (42) is given by

$$-i\partial_k\int(dx'')\,D_{ab}(x, x'')\nabla_j''^{ba}[\delta^{kj} - \delta^{k3}\partial_3'^{-1}\partial'^j]\delta^3(\mathbf{x}' - \mathbf{x}'')\,\delta(x^0 - x'^0). \tag{43}$$

Now we take the limit $\mathbf{x}' \to \mathbf{x}$ in the latter and integrate over $d^3\mathbf{x}$ to obtain

$$-i\int(dx'')\int d^3\mathbf{x}\left[\partial_j - \partial_j\right]D_{ab}(x, x'')\nabla_j''^{ba}\delta^3(\mathbf{x} - \mathbf{x}'')\,\delta(x^0 - x'^0) = 0. \tag{44}$$

This result will be used later in deriving the modification of the FP factor.

The action principle gives

$$\frac{\partial}{\partial\lambda}\langle 0_+\,|\,0_-\rangle = i\int(dx)\left\langle 0_+\left|\overline{\psi}(x)\psi(x)G_{\mu\nu}^a(x)G_a^{\mu\nu}(x)\right|0_-\right\rangle. \tag{45}$$

Consider the matrix element

$$\begin{aligned}
\left\langle 0_+\left|\left(G_{\mu\nu}^a(x)G_a^{\mu\nu}(x')\right)_+\right|0_-\right\rangle &= 2\left\langle 0_+\left|\left(G_{k0}^a(x)G_a^{k0}(x')\right)_+\right|0_-\right\rangle\\
&\quad+\left\langle 0_+\left|\left(G_{kl}^a(x)G_a^{kl}(x')\right)_+\right|0_-\right\rangle.
\end{aligned} \tag{46}$$

The second term is simply equal to

$$G_{kl}'^a(x)G_a'^{kl}(x')\,\langle 0_+\,|\,0_-\rangle \tag{47}$$

expressed in terms of functional derivatives using our notation below Eq. (26). While to determine the first term, we rewrite

$$G_{k0}^a(x) = \int(dz)\,\delta^4(x - z)\nabla_k^{ac}(z)A_0^c(z) - \int(dz)\,\delta^4(x - z)\,\partial_0^z A_k^a(z). \tag{48}$$

We then have

$$\left\langle 0_+ \left| \left(G^a_{k0}(x) G^{k0}_a(x') \right)_+ \right| 0_- \right\rangle = G'^a_{k0}(x) G'^{k0}_a(x') \, \langle 0_+ \,|\, 0_- \rangle$$
$$+ \int (dz) \, \delta^4(x-z) \, \delta(z^0 - x'^0) \left\langle 0_+ \left| \left[A^a_k(z) \, , \, G^{k0}_a(x') \right] \right| 0_- \right\rangle$$
$$-i \int (dz) \, \delta^4(x-z) \nabla'^{ac}_k(z) \left\langle 0_+ \left| \frac{\delta}{\delta J^0_c(z)} G^{k0}_a(x') \right| 0_- \right\rangle \tag{49}$$

where the second term comes from the non-commutativity of the time derivative and the time ordering operation as resulting from the last term in (48), and the third term follows from the rule of functional differentiation in (27) as resulting from the first integral in (48).

From (38), (42), (44), the right-hand side of (49) simplifies for $x' \to x$ to

$$\left[G'^a_{k0}(x) G'^{k0}_a(x) + \Delta'(x) \right] \langle 0_+ | 0_- \rangle \tag{50}$$

where

$$\Delta'(x) = 2 \int (dz) \, \frac{\delta^4(z-x)}{[1 - 4\lambda \overline{\psi}'(x) \psi'(x)]} K'(x, z) \tag{51}$$

$$K'(x, z) = i \left[\delta_{aa} \delta^4(0) + \frac{1}{2} \partial^x_k \nabla'^{ac}_k(z) D'_{ac}(x, z) \right] \tag{52}$$

involving a familiar $\delta^4(0)$ term.

All told, the expression (45) becomes

$$\frac{\partial}{\partial \lambda} \, \langle 0_+ \,|\, 0_- \rangle = i \int (dx) \, \overline{\psi}'(x) \psi'(x) G'^a_{\mu\nu}(x) G'^{\mu\nu}_a(x) \, \langle 0_+ \,|\, 0_- \rangle$$
$$+ \, 2i \int (dx) \, \overline{\psi}'(x) \psi'(x) \Delta'(x) \, \langle 0_+ \,|\, 0_- \rangle \tag{53}$$

which upon an elementary integration over λ leads to

$$\langle 0_+ \,|\, 0_- \rangle = e^{iM'} \exp\left[i\lambda \int (dx) \, \overline{\psi}'(x) \psi'(x) G'^a_{\mu\nu}(x) G'^{\mu\nu}_a(x) \right] \langle 0_+ \,|\, 0_- \rangle_{\lambda=0} \tag{54}$$

where

$$M' = - \int (dx)(dz) \, \delta^4(x-z) \, \ln[1 - 4\lambda \overline{\psi}'(x) \psi'(x)] K'(x, z) \tag{55}$$

and $\langle 0_+ \,|\, 0_- \rangle_{\lambda=0}$ is the vacuum-to-vacuum amplitude corresponding to the Lagrangian density $\mathscr{L}_{\mathrm{T}}$ in (1) involving the FP factor in (32). That is, the familiar FP factor gets modified by a multiplicative factor $\exp[iM']$ for the gauge invariant Lagrangian density $\mathscr{L}_1$ in (33).

4. GAUGE BREAKING INTERACTIONS

In the present section we consider the addition of a gauge breaking term to the Lagrangian density $\mathscr{L}$ in (2). It is well known that even the addition of the simple source term $\mathscr{L}_S$ in (3) to $\mathscr{L}$ causes difficulties (cf. Rivers (1987), p. 204) in the quantization problem in the path integral formalism as the action $\int(dx)\mathscr{L}_T(x)$, with $\mathscr{L}_T(x)$ defined in (1), is not gauge invariant. We will see how easy it is to handle the addition of a gauge breaking term to $\mathscr{L}_T$.

Consider the Lagrangian density

$$\mathscr{L}_{2T} = \mathscr{L}_T + \frac{\lambda}{2} A^a_\mu A^\mu_a \overline{\psi}\psi. \tag{56}$$

Then an analysis similar to the one in Section 3. shows that

$$G^{k0}_a = \pi^k_a - \partial_k D_{ab}\left[J^0_b + \lambda A^0_b \overline{\psi}\psi + g_0 \overline{\psi}\gamma^0 t_b \psi + \nabla^{bc}_\nu \pi^\nu_c\right]. \tag{57}$$

Using the fact that

$$\partial_k G^{k0}_a = \nabla^{ab}_k \partial_k A^0_b \tag{58}$$

we obtain upon multiplying (57) by

$$\nabla^{ca}_l \partial^l \frac{1}{\vec{\partial}^2} \partial_k$$

and using (14), we obtain

$$\left(\nabla^{ca}_l \partial^l \frac{1}{\vec{\partial}^2} \nabla^{ab}_k \partial_k\right) A^0_b = -J^0_c - \lambda A^0_c \overline{\psi}\psi + \ldots \tag{59}$$

where the dots correspond to terms *independent* of J^0_b and A^0_b. We introduce the Green operator $N^{be}(x, x')$ satisfying

$$\left[\nabla^{ca}_l \partial^l \frac{1}{\vec{\partial}^2} \nabla^{ab}_k \partial_k + \lambda \delta^{cb} \overline{\psi}(x)\psi(x)\right] N^{be}(x, x') = \delta^{ce}\delta^4(x - x') \tag{60}$$

to obtain from (59)

$$\frac{\delta}{\delta J^0_b(x)} A^0_b(x) = -N^{bb}(x, x). \tag{61}$$

Hence the action principle and (61) give

$$\begin{aligned}\frac{\partial}{\partial\lambda}\langle 0_+ \,|\, 0_-\rangle &= \tfrac{i}{2}\int(dx)\, A'^a_\mu(x) A'^\mu_a(x) \overline{\psi}'(x)\psi'(x)\,\langle 0_+ \,|\, 0_-\rangle \\ &\quad - \tfrac{1}{2}\int(dx)\, \overline{\psi}'(x)\psi'(x) N'^{bb}(x, x)\,\langle 0_+ \,|\, 0_-\rangle\,. \end{aligned} \tag{62}$$

Upon integrating the latter over λ, by using in the process (60), we obtain

$$\langle 0_+ \,|\, 0_- \rangle = \exp\left[-\frac{1}{2} Tr \ln\left(1 + \frac{\lambda}{\nabla_l' \partial^l (\vec{\partial}^2)^{-1} \nabla_k' \partial_k} \overline{\psi}' \psi' \right) \right]$$

$$\times \exp\left[i \frac{\lambda}{2} \int (dx)\, A_\mu'^a(x) A_a'^\mu(x) \overline{\psi}'(x) \psi'(x) \right] \langle 0_+ \,|\, 0_- \rangle_{\lambda=0} \quad (63)$$

showing an obvious modification of the FP factor with the latter occurring in $\langle 0_+ \,|\, 0_- \rangle_{\lambda=0}$.

5. CONCLUSION

The quantum action (dynamical) principle leads systematically to the FP of non-abelian gauge theories with no much effort. It is emphasized, in the process of the analysis, that no restrictions may be set on the external current J_μ^a, coupled to the gauge field A_a^μ (such as $\partial^\mu J_\mu^a = 0$), until all functional differentiations with respect to it are taken so that all of its components may be varied independently. We have considered gauge invariant as well as gauge non-invariant interactions and have shown that the FP factor needs to be modified in more general cases and expressions for these modifications were derived. [It is well known that even the simple gauge breaking source term $\mathscr{L}_S$ in (3) causes complications in the path integral formalism. The path integral may, of course, be readily derived from the action principle.] The presence of the source term $\mathscr{L}_S$ in the Lagrangian density is essential in order to generate the Green functions of the theory from the vacuum-to-vacuum transition amplitude, as a generating functional, by functional differentiations. We have also shown, in particular, that a gauge invariant theory does not necessarily imply the familiar FP factor for proper quantization. Finally we note that even for abelian gauge theories, as obtained from the bulk of the paper by taking the limit of f^{abc} to zero and replacing t^a by the identity, may lead to modifications, as multiplicative factors in $\langle 0_+ \,|\, 0_- \rangle$, as clearly seen from the expressions in (55) and (63).

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