Eur. Phys. J. D **31**, 137–143 (2004)
DOI: 10.1140/epjd/e2004-00131-2

THE EUROPEAN
PHYSICAL JOURNAL D

Speed dependent polarization correlations in QED and entanglement*

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Received 5 January 2004 / Received in final form 25 June 2004
Published online 5 October 2004 –

Abstract. Exact computations of polarizations correlations probabilities are carried out in QED, to the leading order, for initially *polarized* as well as *unpolarized* particles. Quite generally they are found to be *speed dependent* and are in clear violation of Bell's inequality of Local Hidden Variables (LHV) theories. This dynamical analysis shows how speed dependent entangled states are generated. These computations, based on QED are expected to lead to new experiments on polarization correlations monitoring speed in the light of Bell's theorem. The paper provides a full QED treatment of the dynamics of entanglement

PACS. 12.20.Ds Specific calculations – 12.20.Fv Experimental tests – 03.65.Ud Entanglement and quantum nonlocality (e.g. EPR paradox, Bell's inequalities, GHZ states, etc.)

1 Introduction

We carry out exact computations of joint probabilities of particle polarizations correlations in QED, to the leading order, for initially *polarized* and *unpolarized* particles. The interesting lesson we have learn from such studies is that the mere fact that particles emerging from a process have non-zero speeds to reach detectors implies, in general, that their polarizations correlations probabilities *depend* on speed [1]. The present extended, and needless to say, dynamical analysis shows that this is true, in general. This is unlike formal arguments based simply on combining angular momenta. As a byproduct of this work, we obtain clear violations with Bell's inequality (cf. [2–4]) of LHV theories. We will also see how QED generates speed dependent entangled states.

Several experiments have been performed in recent years (cf. [4–8]) on particles' polarizations correlations. And, it is expected that the novel properties recorded here by explicit calculations following directly from field theory, which is based on the principle of relativity and quantum theory, will lead to new experiments on polarization correlations monitoring speed in the light of Bell's Theorem. We hope that theses computations will be also useful in such areas of physics as quantum teleportation and quantum information in general.

The relevant quantity of interest here in testing Bell's inequality of LHV [2] theories is, in a standard notation,

$$S = \frac{p_{12}(a_1,a_2)}{p_{12}(\infty,\infty)} - \frac{p_{12}(a_1,a_2')}{p_{12}(\infty,\infty)} + \frac{p_{12}(a_1',a_2)}{p_{12}(\infty,\infty)} + \frac{p_{12}(a_1',a_2')}{p_{12}(\infty,\infty)} - \frac{p_{12}(a_1',\infty)}{p_{12}(\infty,\infty)} - \frac{p_{12}(\infty,a_2)}{p_{12}(\infty,\infty)} \qquad (1.1)$$

as is *computed from* QED. Here a_1, a_2 (a_1', a_2') specify directions along which the polarizations of two particles are measured, with $p_{12}(a_1,a_2)/p_{12}(\infty,\infty)$ denoting the joint probability, and $p_{12}(a_1,\infty)/p_{12}(\infty,\infty)$, $p_{12}(\infty,a_2)/p_{12}(\infty,\infty)$ denoting the probabilities when the polarization of only one of the particles is measured. [$p_{12}(\infty,\infty)$ is normalization factor.] The corresponding probabilities as computed from QED will be denoted by $P[\chi_1,\chi_2]$, $P[\chi_1,-]$, $P[-,\chi_2]$ with χ_1, χ_2 denoting angles the polarization vectors make with certain axes spelled out in the bulk of the paper. To show that QED is in violation with Bell's inequality of LHV, it is sufficient to find one set of angles χ_1, χ_2, χ_1', χ_2' and speed β, such that S, as computed in QED, leads to a value of S with $S > 0$ or $S < -1$. In this work, it is implicitly assumed that the polarization parameters in the particle states are directly observable and may be used for Bell-type measurements as discussed.

The need of a relativistic treatment based on explicit quantum field *dynamical* calculations in testing Bell-like inequalities is critically important. An intriguing and very recent reference [9], which appeared after the submission of our paper for publication, discusses the role of relativity in quantum information, in general, and traces the

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historical development of its role, and most importantly, in the light of our present investigations, emphasizes the need of quantum field theory as necessary for a consistent description of interactions. Most earlier analyses dealing with relativistic aspects, relevant to information theory and Bell-like tests are kinematical of nature or deal with basic general properties of local operators associated with bounded regions of spacetime setting limits on measurements and localizability of quantum systems. These probabilities are well documented in some of the recent monographs [10–12] on the subject. Notable important other recent references on such general aspects which are, however, non-dynamical of nature are [13–17], and a paper by Czachor [18] indicating how a possible decrease in violation of Bell's inequalities may occur. In the present work, we are interested in dynamical aspects and related uniquely determined probabilities (intensities) of correlations based on QED, as a fully relativistic quantum field theory (i.e., encompassing quantum theory and relativity) that meet the verdict of experiments. QED is a non-speculative theory and as Feynman [19] puts it, it is the most precise theory we have in fundamental physics. The closest investigation to our own is that of reference [20], a reference we encountered after the submission of our paper for publication, which considers spin-spin interactions, in a QED setting, for non-relativistic electrons and, unfortunately, does not compute their polarizations correlations which are much relevant experimentally. In the present paper, exact fully relativistic QED, computations, to the leading order, of polarizations *correlations* are explicitly carried out for initially polarized and unpolarized particles. The importance of also considering unpolarized spin stems from the fact that we discover the existence of non-trivial correlations, in the outcome of the processes, even for such mixed states (since one averages over spin) and not only for pure states arising from polarized spins, leading, in particular, in both cases to speed dependent probabilities. The main results of our paper are given in (2.20), (2.22), (2.23), (2.41–2.43), (3.10), (3.12–3.19). All of these probabilities lead to a violation of Bell's inequality of LHV theories. As the computations are based on the fully relativistic QED, it is of some urgency that relevant experiments are carried out by monitoring speed.

2 Polarizations correlations: initially polarized particles

We consider the process $e^-e^- \to e^-e^-$, in the c.m., with initially polarized electrons with one spin up, along the z-axis, and one spin down. With $\mathbf{p}_1 = \gamma m\beta(0,1,0) = -\mathbf{p}_2$ denoting the momenta of the initial electrons, $\gamma = 1/\sqrt{1-\beta^2}$, we consider momenta of the emerging electrons with

$$\mathbf{p}_1' = \gamma m\beta(\sin\theta, 0, \cos\theta) = -\mathbf{p}_2' \tag{2.1}$$

where θ is measured from the z-axis.

For the four-spinors of the initial electrons, we have $(p^0 = \gamma m)$

$$u(p_1) = \left(\frac{p^0+m}{2m}\right)^{1/2} \begin{pmatrix} \begin{pmatrix}1\\0\end{pmatrix} \\ \mathrm{i}\rho\begin{pmatrix}0\\1\end{pmatrix}\end{pmatrix} \tag{2.2}$$

$$u(p_2) = \left(\frac{p^0+m}{2m}\right)^{1/2} \begin{pmatrix} \begin{pmatrix}0\\1\end{pmatrix} \\ \mathrm{i}\rho\begin{pmatrix}1\\0\end{pmatrix}\end{pmatrix} \tag{2.3}$$

$$\rho = \frac{\gamma\beta}{\gamma+1} = \frac{\beta}{1+\sqrt{1-\beta^2}} \tag{2.4}$$

and for the final ones

$$u(p_1') = \left(\frac{p^0+m}{2m}\right)^{1/2} \begin{pmatrix} \xi_1 \\ \frac{\sigma\cdot\mathbf{p}_1'}{p^0+m}\xi_1\end{pmatrix} \tag{2.5}$$

$$u(p_2') = \left(\frac{p^0+m}{2m}\right)^{1/2} \begin{pmatrix} \xi_2 \\ -\frac{\sigma\cdot\mathbf{p}_1'}{p^0+m}\xi_2\end{pmatrix} \tag{2.6}$$

where the two-spinors ξ_1, ξ_2 will be specified later.

The expression for the amplitude of the process is well-known (cf. [21])

$$A \propto \left[\frac{\overline{u}(p_1')\gamma^\mu u(p_1)\overline{u}(p_2')\gamma_\mu u(p_2)}{(p_1'-p_1)^2} - \frac{\overline{u}(p_2')\gamma^\mu u(p_1')\overline{u}(p_1')\gamma_\mu u(p_2)}{(p_2'-p_1)^2}\right]. \tag{2.7}$$

The following matrix elements are needed and are readily calculated

$$\overline{u}(p_1')\gamma^0 u(p_1) = \frac{p^0+m}{2m}\xi_1^\dagger \begin{pmatrix}1+\mathrm{i}\rho^2\sin\theta\\ -\mathrm{i}\rho^2\cos\theta\end{pmatrix} \tag{2.8}$$

$$\overline{u}(p_2')\gamma^0 u(p_2) = \frac{p^0+m}{2m}\xi_2^\dagger \begin{pmatrix}-\mathrm{i}\rho^2\cos\theta\\ 1-\mathrm{i}\rho^2\sin\theta\end{pmatrix} \tag{2.9}$$

$$\overline{u}(p_1')\gamma^0 u(p_2) = \frac{p^0+m}{2m}\xi_1^\dagger \begin{pmatrix}\mathrm{i}\rho^2\cos\theta\\ 1+\mathrm{i}\rho^2\sin\theta\end{pmatrix} \tag{2.10}$$

$$\overline{u}(p_2')\gamma^0 u(p_1) = \frac{p^0+m}{2m}\xi_2^\dagger \begin{pmatrix}1-\mathrm{i}\rho^2\sin\theta\\ \mathrm{i}\rho^2\cos\theta\end{pmatrix} \tag{2.11}$$

$$\overline{u}(p_1')\gamma^j u(p_1) = \frac{p^0+m}{2m}\rho\xi_1^\dagger \left[\begin{pmatrix}\mathrm{i}+\sin\theta\\ -\cos\theta\end{pmatrix}\delta^{j1} + \mathrm{i}\begin{pmatrix}-\mathrm{i}+\sin\theta\\ -\cos\theta\end{pmatrix}\delta^{j2} + \begin{pmatrix}-\cos\theta\\ -\mathrm{i}+\sin\theta\end{pmatrix}\delta^{j3}\right] \tag{2.12}$$

$$\overline{u}(p_2')\gamma^j u(p_2) = \frac{p^0+m}{2m}\rho\xi_2^\dagger\Bigg[\begin{pmatrix}-\cos\theta\\ \mathrm{i}-\sin\theta\end{pmatrix}\delta^{j1} + \mathrm{i}\begin{pmatrix}\cos\theta\\ \mathrm{i}+\sin\theta\end{pmatrix}\delta^{j2} + \begin{pmatrix}\mathrm{i}+\sin\theta\\ -\cos\theta\end{pmatrix}\delta^{j3}\Bigg] \tag{2.13}$$

$$\overline{u}(p_1')\gamma^j u(p_2) = \frac{p^0+m}{2m}\rho\xi_1^\dagger\Bigg[\begin{pmatrix}\cos\theta\\ \mathrm{i}+\sin\theta\end{pmatrix}\delta^{j1} + \mathrm{i}\begin{pmatrix}-\cos\theta\\ \mathrm{i}-\sin\theta\end{pmatrix}\delta^{j2} + \begin{pmatrix}\mathrm{i}-\sin\theta\\ \cos\theta\end{pmatrix}\delta^{j3}\Bigg] \tag{2.14}$$

$$\overline{u}(p_2')\gamma^j u(p_1) = \frac{p^0+m}{2m}\rho\xi_2^\dagger\Bigg[\begin{pmatrix}\mathrm{i}-\sin\theta\\ \cos\theta\end{pmatrix}\delta^{j1} - \mathrm{i}\begin{pmatrix}\mathrm{i}+\sin\theta\\ -\cos\theta\end{pmatrix}\delta^{j2} - \begin{pmatrix}\cos\theta\\ \mathrm{i}+\sin\theta\end{pmatrix}\delta^{j3}\Bigg]. \tag{2.15}$$

For $\theta = 0$, (see Fig. 1), we obtain from (2.7–2.15)

$$A \propto \xi_1^\dagger\xi_2^\dagger\Bigg\{(1+6\rho^2+\rho^4)\left[\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 - \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2\right] + 4\mathrm{i}\rho^2\left[\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 + \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right]\Bigg\} \tag{2.16}$$

generating the speed dependent (normalized) entangled state of the emerging electrons

$$|\psi\rangle = \frac{1}{N}\Bigg\{\frac{(1+6\rho^2+\rho^4)}{\sqrt{2}}\left[\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 - \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2\right] + \frac{4\mathrm{i}\rho^2}{\sqrt{2}}\left[\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2 + \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2\right]\Bigg\} \tag{2.17}$$

where

$$N = \left[(1+6\rho^2+\rho^4)^2 + 16\rho^4\right]^{1/2} \tag{2.18}$$

$$\xi_j = \frac{1}{\sqrt{2}}\begin{pmatrix}\mathrm{e}^{-\mathrm{i}\chi_j/2}\\ \mathrm{e}^{\mathrm{i}\chi_j/2}\end{pmatrix}, \quad j = 1,2 \tag{2.19}$$

ρ is defined in (2.4), and the angles are measured relative to the x-axis (see Fig. 1).

The joint probability of the electrons polarizations correlations is then given by

$$P[\chi_1,\chi_2] = \left\|\xi_1^\dagger\xi_2^\dagger\,|\psi\rangle\right\|^2 = \frac{\left[(1+6\rho^2+\rho^4)\sin\left(\frac{\chi_1-\chi_2}{2}\right) - 4\rho^2\cos\left(\frac{\chi_1+\chi_2}{2}\right)\right]^2}{2\left[(1+6\rho^2+\rho^4)^2+16\rho^4\right]}. \tag{2.20}$$

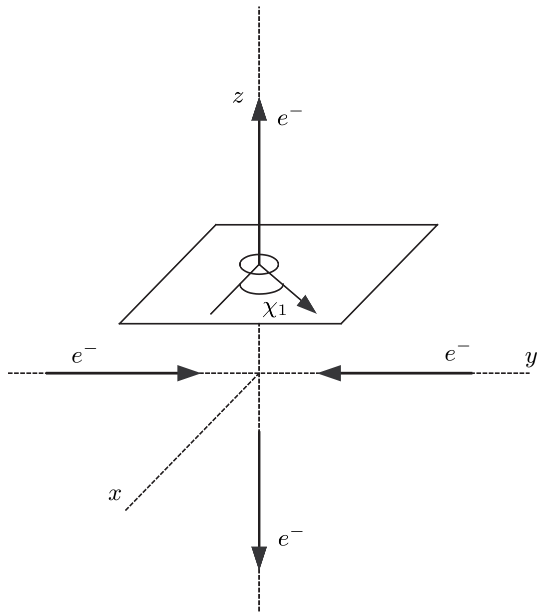

Fig. 1. The figure depicts e^-e^- scattering, with the electrons initially moving along the y-axis, while the emerging electrons moving along the z-axis. The angle χ_1, measured relative to the x-axis, denotes the orientation of spin of one of the emerging electrons may make.

[For $\beta \to 0$, one obtains a rather familiar expression $P[\chi_1,\chi_2] = \sin^2\left[(\chi_1-\chi_2)/2\right]/2$.]

If only one of the spins is measured, say, corresponding to χ_1, we then have to form the state

$$\xi_1^\dagger\,|\psi\rangle = \frac{[1+6\rho^2+\rho^4]}{2N}\begin{pmatrix}\mathrm{e}^{-\mathrm{i}\chi_1/2}\\ -\mathrm{e}^{\mathrm{i}\chi_1/2}\end{pmatrix}_2 + \frac{4\mathrm{i}\rho^2}{2N}\begin{pmatrix}\mathrm{e}^{\mathrm{i}\chi_1/2}\\ \mathrm{e}^{-\mathrm{i}\chi_1/2}\end{pmatrix}_2 \tag{2.21}$$

from which we obtain the corresponding probability

$$P[\chi_1,-] = \left\|\xi_1^\dagger\,|\psi\rangle\right\|^2 = \frac{1}{2} - \frac{4\rho^2(1+6\rho^2+\rho^4)}{(1+6\rho^2+\rho^4)^2+16\rho^4}\sin\chi_1 \tag{2.22}$$

and similarly

$$P[-,\chi_2] = \left\|\xi_2^\dagger\,|\psi\rangle\right\|^2 = \frac{1}{2} + \frac{4\rho^2(1+6\rho^2+\rho^4)}{(1+6\rho^2+\rho^4)^2+16\rho^4}\sin\chi_2. \tag{2.23}$$

The probability $P[\chi_1,-]$ may be *equivalently* obtained by summing $P[\chi_1,\chi_2]$ over the two angles

$$\chi_2, \quad \chi_2+\pi \tag{2.24}$$

for any arbitrarily chosen fixed χ_2, i.e.,

$$P[\chi_1,\chi_2] + P[\chi_1,\chi_2+\pi] = P[\chi_1,-] \tag{2.25}$$

as is easily checked, and similarly for $P[-,\chi_2]$.

For all $0 \leqslant \beta \leqslant 1$, angles χ_1, χ_2, χ'_1, χ'_2 are readily found leading to a violation of Bell's inequality of LHV theories. For example, for $\beta = 0.3$, $\chi_1 = 0°$, $\chi_2 = 137°$, $\chi'_1 = 12°$, $\chi'_2 = 45°$, $S = -1.79$ violating the inequality from below.

The speed dependence of $P[\chi_1, \chi_2]$ generally holds true for other angles as well. For $\theta = \pi/2$, however, it is readily verified that (2.7) leads to the entangled state

$$|\psi\rangle_0 = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}0\\1\end{pmatrix}_1\begin{pmatrix}1\\0\end{pmatrix}_2 - \begin{pmatrix}1\\0\end{pmatrix}_1\begin{pmatrix}0\\1\end{pmatrix}_2\right] \tag{2.26}$$

for all $0 \leqslant \beta \leqslant 1$, leading to a rather familiar expression $P[\chi_1, \chi_2] = \sin^2\left[(\chi_1 - \chi_2)/2\right]/2$.

Now we consider the process $e^+e^- \to 2\gamma$, in the c.m. of e^-, e^+ with spins up, along the z-axis, and down, respectively. With $\mathbf{p}_1 = \mathbf{p}(e^-) = \gamma m\beta(0,1,0) = -\mathbf{p}(e^+) = -\mathbf{p}_2$, we have for e^-, e^+ the spinors given by

$$u = \left(\frac{p^0+m}{2m}\right)^{1/2}\begin{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\\ \mathrm{i}\rho\begin{pmatrix}0\\1\end{pmatrix}\end{pmatrix} \tag{2.27}$$

$$v = \left(\frac{p^0+m}{2m}\right)^{1/2}\begin{pmatrix}\mathrm{i}\rho\begin{pmatrix}0\\1\end{pmatrix}\\ \begin{pmatrix}1\\0\end{pmatrix}\end{pmatrix} \tag{2.28}$$

with ρ defined in (2.4), and we consider momenta of the photons

$$\mathbf{k}_1 = \gamma m(\sin\theta, 0, \cos\theta) = -\mathbf{k}_2 \tag{2.29}$$

where we have used the facts that

$$|\mathbf{k}_1| = |\mathbf{k}_2| = k_1^0 = k_2^0 = p^0(e^\pm) \equiv p^0 = \gamma m. \tag{2.30}$$

The amplitude for the process is given by (cf. [21])

$$A \propto \overline{v}\left[\frac{\gamma^\mu\gamma k_1\gamma^\nu}{2p_1k_1} + \frac{\gamma^\nu\gamma k_2\gamma^\mu}{2p_1k_2} + \frac{\gamma^\mu p_1^\nu}{p_1k_1} + \frac{\gamma^\nu p_1^\mu}{p_1k_2}\right] u\, e_1^\nu e_2^\mu \tag{2.31}$$

where $e_1^\mu = (0, \mathbf{e}_1)$, $e_2^\mu = (0, \mathbf{e}_2)$ are the polarizations of the photons with $(j = 1, 2)$

$$\mathbf{e}_j = (-\cos\theta\cos\chi_j, \sin\chi_j, \sin\theta\cos\chi_j) \equiv \left(e_j^{(1)}, e_j^{(2)}, e_j^{(3)}\right). \tag{2.32}$$

The following matrix elements are readily derived

$$\overline{v}\left(\gamma^i\gamma^0\gamma^j\right)u = \frac{p^0+m}{2m}2\mathrm{i}\varepsilon_{ij2}\rho \tag{2.33}$$

$$\overline{v}\gamma^i u = \frac{p^0+m}{2m}(1-\rho^2)\delta^{i3} \tag{2.34}$$

$$\overline{v}\left(\gamma^i\gamma^m\gamma^j\right)u = \frac{p^0+m}{2m}\left(-\delta^{mj}\delta^{i3} - \delta^{mi}\delta^{j3} + \delta^{ij}\delta^{m3}\right) \times (1-\rho^2) - \mathrm{i}\frac{p^0+m}{2m}(1+\rho^2)\varepsilon_{imj}. \tag{2.35}$$

Upon setting,

$$\frac{\mathbf{k}_1}{|\mathbf{k}_1|} = \mathbf{n} \tag{2.36}$$

the amplitude A is then given by

$$A \propto -\mathrm{i}(1+\rho^2)\mathbf{n}\cdot(\mathbf{e}_1\times\mathbf{e}_2) + \beta(1-\rho^2)\left(e_1^{(2)}e_2^{(3)} + e_1^{(3)}e_2^{(2)}\right). \tag{2.37}$$

For $\theta = \pi/2$, this gives

$$A \propto -\left(0, \sin\chi_1, \sin\chi_1\right)_1\left(0, \sin\chi_2, \sin\chi_2\right)_2 \times\Bigg\{\mathrm{i}(1+\rho^2)\left[\begin{pmatrix}0\\1\\0\end{pmatrix}_1\begin{pmatrix}0\\0\\1\end{pmatrix}_2 - \begin{pmatrix}0\\0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\\0\end{pmatrix}_2\right] -\beta(1-\rho^2)\left[\begin{pmatrix}0\\1\\0\end{pmatrix}_1\begin{pmatrix}0\\0\\1\end{pmatrix}_2 + \begin{pmatrix}0\\0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\\0\end{pmatrix}_2\right]\Bigg\} \tag{2.38}$$

(see Fig. 2), generating a speed dependent (normalized) entangled state for the photons given by

$$|\phi\rangle = \frac{1}{N}\Bigg\{\frac{\mathrm{i}(1+\rho^2)}{\sqrt{2}}\left[\begin{pmatrix}0\\1\\0\end{pmatrix}_1\begin{pmatrix}0\\0\\1\end{pmatrix}_2 - \begin{pmatrix}0\\0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\\0\end{pmatrix}_2\right] -\beta\frac{(1-\rho^2)}{\sqrt{2}}\left[\begin{pmatrix}0\\1\\0\end{pmatrix}_1\begin{pmatrix}0\\0\\1\end{pmatrix}_2 + \begin{pmatrix}0\\0\\1\end{pmatrix}_1\begin{pmatrix}0\\1\\0\end{pmatrix}_2\right]\Bigg\} \tag{2.39}$$

with

$$N = \left[(1+\rho^2)^2 + \beta^2(1-\rho^2)^2\right]^{1/2}. \tag{2.40}$$

Therefore the joint probability of photons polarizations correlations is given by

$$P[\chi_1, \chi_2] = \|(0, \sin\chi_1, \cos\chi_1)_1(0, \sin\chi_2, \cos\chi_2)_2\,|\phi\rangle\|^2 = \frac{(1+\rho^2)^2\sin^2(\chi_1-\chi_2) + \beta^2(1-\rho^2)^2\cos^2(\chi_1+\chi_2)}{2[(1+\rho^2)^2 + \beta^2(1-\rho^2)^2]} \tag{2.41}$$

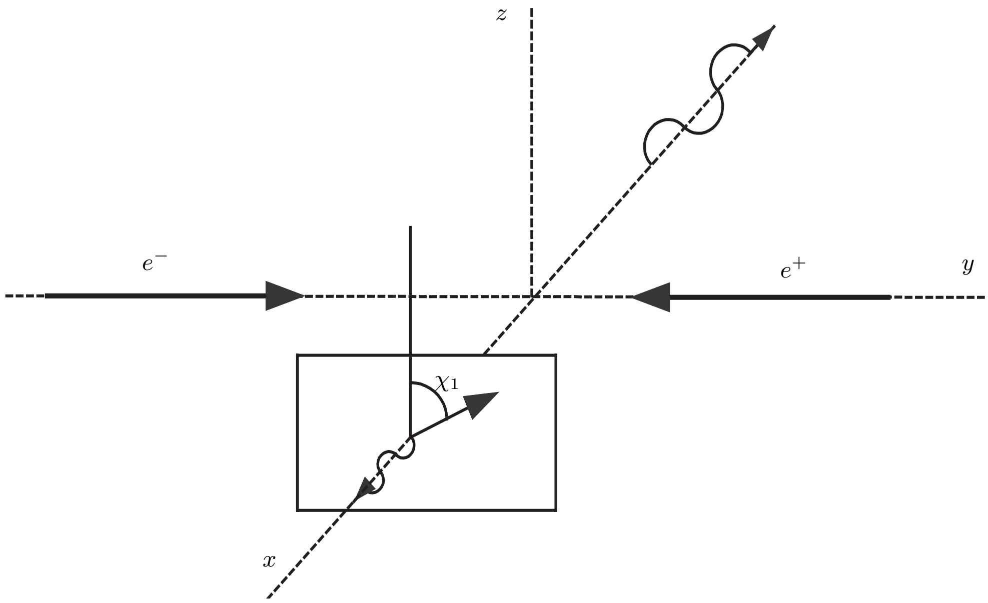

Fig. 2. The figure depicts e^+e^- annihilation into 2γ, with e^+, e^- moving along the y-axis, and the emerging photons moving along the x-axis. χ_1 denotes the angle the polarization vector of one of the photons may make with the z-axis.

and

$$P[\chi_1, -] = \|(0, \sin\chi_1, \cos\chi_1)_1 \, |\phi\rangle\|^2 = \frac{1}{2} \tag{2.42}$$

$$P[-, \chi_2] = \|(0, \sin\chi_2, \cos\chi_2)_2 \, |\phi\rangle\|^2 = \frac{1}{2} \tag{2.43}$$

$P[\chi_1, -]$ is also *equivalently* obtained by summing $P[\chi_1, \chi_2]$ over

$$\chi_2, \quad \chi_2 + \frac{\pi}{2} \tag{2.44}$$

for any arbitrarily chosen χ_2, i.e.,

$$P[\chi_1, \chi_2] + P\left[\chi_1, \chi_2 + \frac{\pi}{2}\right] = P[\chi_1, -] \tag{2.45}$$

and similarly for $P[-, \chi_2]$.

For all $0 \leqslant \beta \leqslant 1$, angles χ_1, χ_2, χ_1', χ_2' are readily found leading to a violation of Bell's inequality of LHV theories. For example, for $\beta = 0.2$, $\chi_1 = 0°$, $\chi_2 = 23°$, $\chi_1' = 45°$, $\chi_2' = 67°$, $S = -1.187$ violating the inequality from below.

Again the speed dependence of $P[\chi_1, \chi_2]$ generally holds true for other angles as well. For $\theta = 0$, however, it is readily checked that (2.37) leads to the entangled state

$$|\phi_0\rangle = \frac{1}{\sqrt{2}}\left[\begin{pmatrix}0\\1\\0\end{pmatrix}_1 \begin{pmatrix}0\\0\\1\end{pmatrix}_2 - \begin{pmatrix}0\\0\\1\end{pmatrix}_1 \begin{pmatrix}0\\1\\0\end{pmatrix}_2\right] \tag{2.46}$$

for all $0 \leqslant \beta \leqslant 1$ giving a rather familiar expression $P[\chi_1, \chi_2] = \left[\sin^2(\chi_1 - \chi_2)\right]/2$.

3 Polarizations correlations: initially unpolarized particles

For the process $e^-e^- \to e^-e^-$, in the c.m., with initially unpolarized spins, with momenta $\mathbf{p}_1 = \gamma m\beta(0, 1, 0) = -\mathbf{p}_2$, we take for the final electrons

$$\mathbf{p'}_1 = \gamma m\beta(1, 0, 0) = -\mathbf{p}_2' \tag{3.1}$$

and for the four-spinors

$$u(p_1') = \left(\frac{p^0 + m}{2m}\right)^{1/2} \begin{pmatrix}\xi_1 \\ \frac{\sigma\cdot\mathbf{p}_1'}{p^0+m}\xi_1\end{pmatrix}, \quad \xi_1 = \begin{pmatrix}-\mathrm{i}\cos\chi_1/2 \\ \sin\chi_1/2\end{pmatrix} \tag{3.2}$$

$$u(p_2') = \left(\frac{p^0 + m}{2m}\right)^{1/2} \begin{pmatrix}\xi_2 \\ -\frac{\sigma\cdot\mathbf{p}_1'}{p^0+m}\xi_2\end{pmatrix}, \; \xi_2 = \begin{pmatrix}-\mathrm{i}\cos\chi_2/2 \\ \sin\chi_2/2\end{pmatrix}. \tag{3.3}$$

A straightforward but tedious computation of the corresponding probability of occurrence with initially unpolarized electrons, (2.7) leads to

$$\begin{aligned}
\text{Prob} \propto\ & [\overline{u}(p_1')\gamma^\mu(-\gamma p_1 + m)\gamma^\sigma u(p_1')] \\
& \times [\overline{u}(p_2')\gamma_\mu(-\gamma p_2 + m)\gamma_\sigma u(p_2')] \\
& - [\overline{u}(p_1')\gamma^\mu(-\gamma p_1 + m)\gamma^\sigma u(p_2')] \\
& \times [\overline{u}(p_2')\gamma_\mu(-\gamma p_2 + m)\gamma_\sigma u(p_1')] \\
& - [\overline{u}(p_2')\gamma^\mu(-\gamma p_1 + m)\gamma^\sigma u(p_1')] \\
& \times [\overline{u}(p_1')\gamma_\mu(-\gamma p_2 + m)\gamma_\sigma u(p_2')] \\
& + [\overline{u}(p_2')\gamma^\mu(-\gamma p_1 + m)\gamma^\sigma u(p_2')] \\
& \times [\overline{u}(p_1')\gamma_\mu(-\gamma p_2 + m)\gamma_\sigma u(p_1')]
\end{aligned} \tag{3.4}$$

which after simplification and of collecting terms reduces to

$$\begin{aligned}\text{Prob} \propto (1-\beta^2)(1+3\beta^2)\sin^2\left(\frac{\chi_1-\chi_2}{2}\right)\\ +\beta^4\cos^2\left(\frac{\chi_1+\chi_2}{2}\right)+4\beta^4\\ \equiv F[\chi_1,\chi_2] \end{aligned} \quad (3.5)$$

where we have used the expressions for the spinors in (3.2), (3.3).

Given that the process has occurred, the conditional probability that the spins of the emerging electrons make angles χ_1, χ_2 with the z-axis, is directly obtained from (3.5) to be

$$P[\chi_1,\chi_2] = \frac{F[\chi_1,\chi_2]}{C}. \quad (3.6)$$

The normalization constant C is obtained by summing over the polarizations of the emerging electrons. This is equivalent to summing of $F[\chi_1,\chi_2]$ over the pairs of angles

$$(\chi_1,\chi_2),\ (\chi_1+\pi,\chi_2),\ (\chi_1,\chi_2+\pi)\ (\chi_1+\pi,\chi_2+\pi) \quad (3.7)$$

for any arbitrarily chosen fixed χ_1, χ_2, corresponding to the orthonormal spinors

$$\begin{pmatrix}-\mathrm{i}\cos\chi_j/2\\ \sin\chi_j/2\end{pmatrix},\quad \begin{pmatrix}-\mathrm{i}\cos(\chi_j+\pi)/2\\ \sin(\chi_j+\pi)/2\end{pmatrix} = \begin{pmatrix}\mathrm{i}\sin\chi_j/2\\ \cos\chi_j/2\end{pmatrix} \quad (3.8)$$

providing a complete set, for each $j=1,2$, in reference to (3.2), (3.3). This is,

$$\begin{aligned}C &= F[\chi_1,\chi_2]+F[\chi_1+\pi,\chi_2]\\ &\quad + F[\chi_1,\chi_2+\pi]+F[\chi_1+\pi,\chi_2+\pi]\\ &= 2(1+2\beta^2+6\beta^4)\end{aligned} \quad (3.9)$$

which as expected is independent of χ_1, χ_2, giving

$$P[\chi_1,\chi_2] = \frac{(1-\beta^2)(1+3\beta^2)\sin^2\left(\frac{\chi_1-\chi_2}{2}\right)+\beta^4\cos^2\left(\frac{\chi_1+\chi_2}{2}\right)+4\beta^4}{2(1+2\beta^2+6\beta^4)}. \quad (3.10)$$

By summing over

$$\chi_2,\quad \chi_2+\pi \quad (3.11)$$

for any arbitrarily fixed χ_2, we obtain

$$P[\chi_1,-] = \frac{1}{2} \quad (3.12)$$

and similarly,

$$P[-,\chi_2] = \frac{1}{2} \quad (3.13)$$

for the probabilities when only one of the photons polarizations is measured.

A clear violation of Bell's inequality of LHV theories was obtained for all $0 \leqslant \beta \leqslant 0.45$. For example, for $\beta = 0.3$, with $\chi_1 = 0°$, $\chi_2 = 45°$, $\chi_1' = 90°$, $\chi_2' = 135°$ give $S = -1.165$ violating the inequality from below. For larger β values, alone, one cannot discriminate between LHV theories and quantum theory for this process. A violation of Bell's inequality for at least some β values, as seen, however, automatically violates LHV theories.

The probability of photon polarizations correlations in $e^+e^- \to 2\gamma$ with initially unpolarized e^+, e^-, has been given in [1] to be

$$P[\chi_1,\chi_2] = \frac{1-\left[\cos(\chi_1-\chi_2)-2\beta^2\cos\chi_1\cos\chi_2\right]^2}{2[1+2\beta^2(1-\beta^2)]} \quad (3.14)$$

$$P[\chi_1,-] = \frac{1+4\beta^2(1-\beta^2)\cos^2\chi_1}{2[1+2\beta^2(1-\beta^2)]} \quad (3.15)$$

$$P[-,\chi_2] = \frac{1+4\beta^2(1-\beta^2)\cos^2\chi_2}{2[1+2\beta^2(1-\beta^2)]} \quad (3.16)$$

and a clear violation of Bell's inequality of LHV theories was obtained for all $0 \leqslant \beta \leqslant 0.2$. Again, for larger values of β, alone, one cannot discriminate between LHV theories and quantum theory for this process. A violation of Bell's inequality for at least some β values, as seen, however, automatically occurs violating LHV theories.

For completeness, we mention that for the annihilation of the spin 0 pair into 2γ the following probabilities are similarly worked out:

$$P[\chi_1,\chi_2] = \frac{\left(\cos(\chi_1-\chi_2)-2\beta^2\cos\chi_1\cos\chi_2\right)^2}{2[1-2\beta^2(1-\beta^2)]} \quad (3.17)$$

$$P[\chi_1,-] = \frac{1-4\beta^2(1-\beta^2)\cos^2\chi_1}{2[1-2\beta^2(1-\beta^2)]} \quad (3.18)$$

$$P[-,\chi_2] = \frac{1-4\beta^2(1-\beta^2)\cos^2\chi_2}{2[1-2\beta^2(1-\beta^2)]} \quad (3.19)$$

and violates Bell's inequality of LHV theories for all $0 \leqslant \beta \leqslant 1$.

4 Conclusion

We have seen by explicit dynamical computations based on QED, that the polarizations correlations probabilities of particles emerging in processes *depend* on speed, for initially *polarized* as well as *unpolarized* particles, in general. We have also seen how QED leads directly to speed dependent entangled states. For processes with initially polarized particles (as well as for spin 0 pairs annihilation into 2γ), a clear violation of Bell's inequality of LHV theories was obtained for all speeds. This clear violation was also true for several speeds for processes with initially unpolarized particles, but the tests are more sensitive on the speed for such processes. The main results of the paper are given in (2.20), (2.22), (2.23), (2.41–2.43), (3.10), (3.12–3.19). We feel that it is a matter of some urgency that the relevant experiments are carried out by monitoring speed.

The authors would like to acknowledge with thanks for being granted a “Royal Golden Jubilee Award” by the TRF No. PHD/0022/2545 for especially carrying out this project. The authors would like also to thank the referees for for pointing out some pertinent references for this work and for valuable suggestions.

References

1. N. Yongram, E.B. Manoukian, Int. J. Theor. Phys. **42**, 1755 (2003)
2. J.F. Clauser, M.A. Horne, Phys. Rev. D **10**, 526 (1974); J.F. Clauser, A. Shimoney, Rep. Prog. Phys. **41**, 1881 (1978)
3. F. Selleri, in: *Quantum Mechanics Versus Local Realism*, edited by F. Selleri (Plenum, New York, 1988)
4. A. Aspect, J. Dalibard, G. Roger, Phys. Rev. Lett. **49**, 1804 (1982)
5. E.S. Fry, Quant. Opt. **7** 227 (1995)
6. L.R. Kaday, J.D. Ulman, C.S. Wu, Nuovo Cim. B **25**, 633 (1975)
7. S. Osuch, M. Popskiewicz, Z. Szeflinski, Z. Wilhelmi, Acta Phys. Polon. B **27**, 567 (1996)
8. J.D. Irby, Phys. Rev. A **67**, 034102 (2003)
9. A. Peres, D.R. Terno, Rev. Mod. Phys. **76**, 93 (2004)
10. R. Haag, *Local Quantum Fields, Particles, Algebras* (Springer, Berlin, 1996)
11. H. Araki, *Mathematical Theory of Quantum Fields* (Oxford, Oxford, 1999)
12. O. Bratelli, D.W. Robinson, *Operator Algebras and Quantum Statistical Mechanics*, 2nd edn. (Springer, New York, 1987)
13. S.J. Summers, R. Werner, Phys. Rev. Lett. A **110**, 257(1985); J. Math. Phys. **28**, 2440 (1987)
14. L.J. Landau, Phys. Rev. Lett. A **120**, 54 (1987)
15. P.M. Alsing, G.J. Milburn, preprint `arXiv:quant-ph/0203051`; R.M. Gingrich, C. Adami, preprint `arXiv:quant-ph/0205179`; S.D. Bartlett, D.R. Terno, preprint `arXiv:quant-ph/0403014`
16. A.J. Bergou, R.M. Gingrich, C. Adami, Phys. Rev. A **68**, 042102 (2003)
17. D.R. Terno, Phys. Rev. A **67**, 014102 (2003)
18. R.P. Czachor, Phys. Rev. A **55**, 72 (1997)
19. R.P. Feynman, *QED: The Strange Theory of Light and Matter* (Princeton, New Jersey, 1985)
20. J. Pachos, E. Solono, preprint `arXiv:quant-ph/0203065`
21. C. Itzykson, J.-B. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980)