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Cherenkov radiation near a dielectric medium at finite temperatures

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Abstract

A field theoretical derivation is given for the average number of Cherenkov photon emission by a charged particle, in a dielectric medium of permitivity ε_1, moving parallel to the plane surface of a different dielectric medium of permitivity $\varepsilon_2 > \varepsilon_1$ at finite temperatures. Near threshold for the speed of the charged particle, it is shown that an enhancement of about 31% of this number is possible in the presence of the second medium, by choosing specific windows, obtained from a general formula, centered about points of the spectrum at any temperature and arbitrary values of the permitivities $\varepsilon_1, \varepsilon_2 > \varepsilon_1$. The conditions for this 31% enhancement are explicitly worked out for blue and red light.

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1. Introduction

There has been much interest in the literature (cf. [1–5]) for years in the so-called Cherenkov radiation emitted by a charged particle in a medium, moving with a speed greater than the speed of light *in* the medium, since its discovery [6] and early theoretical explanation [7]. In the present paper, we investigate Cherenkov photon-emission in a dielectric medium of permitivity ε_1, by charged particle moving parallel to the plane surface of a different dielectric medium of permitivity $\varepsilon_2 > \varepsilon_1$ at finite temperatures, and hence the derivation includes Planck's constant h. More precisely, we derive the expression for the average number of Cherenkov photons emitted within the frequency ranges at finite T, by the charged particle the time that it moves a given distance. It is shown that as we move away from the threshold value $c/\sqrt{\varepsilon_1}$ of the speed v of the particle and very near to it, an explicit enhancement of about 31% of the Cherenkov photon number is possible, over the corresponding case with no second medium present, for photons emitted through "selective windows", obtained from a general formula (Eq. (49)), centered about points of the spectrum for any temperature and arbitrary values of the permitivities $\varepsilon_2 > \varepsilon_1$. This 31% enhancement provides a criterion for the indirect detectability of the presence of the second medium. Applications are then carried out for blue and red light, and the conditions for this enhancement are worked out. The treatment of the recoil of

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the charged particle is beyond the scope of the present work. The analysis is given from a quantum field theory view point (cf. [1]) via the vacuum-to-vacuum transition amplitude (cf. [1, 5, 8]).

2. Cherenkov photon emission

In the temporal gauge for the vector potential $A^0 = 0$, the photon propagator $D^{ij}(x', x) = i\langle 0_+ | (A^i(x')A^j(x))_+ | 0_-\rangle / \langle 0_+ | 0_- \rangle$ satisfies a well-known differential equation (cf. [1, 9]):

$$[(-\partial'^z + \varepsilon(z')\partial'^{02})\partial^{ik} + \partial'^i\partial'^k] D^{kj}(x', x) = \delta^{ij}\,\delta(x', x), \tag{1}$$

where $x = (x^0, \mathbf{x})$, $x^0 = ct$, and

$$\varepsilon(z') = \begin{cases} \varepsilon_1, & z' > 0, \\ \varepsilon_2, & z' < 0 \end{cases} \tag{2}$$

and we consider $\varepsilon_2 > \varepsilon_1$. The charged particle moves in the $z' > 0$ region parallel to the x–y plane specifying the surface of the second dielectric medium. The charged particle will be described by a current $J^1(x)$. The expectation values of the electric and magnetic field components E^i, B^i, due to the charged particle, are given by [1]

$$\langle E^i(x') \rangle = \frac{1}{c} \int (\mathrm{d}x)\, \partial'^0 D^{i1}(x', x) J^1(x), \tag{3}$$

$$\langle B^i(x') \rangle = \frac{\varepsilon^{ijk}}{c} \int (\mathrm{d}x)\, \partial'^j D^{k1}(x', x) J^1(x), \tag{4}$$

The boundary conditions are: $\langle E^a(x') \rangle$, $a = 1, 2$, $\langle B^i(x') \rangle$, $i = 1, 2, 3$, and $\varepsilon(z')\langle E^3(x') \rangle$ are continuous across the boundary surface $z' = 0$. By applying these boundary conditions, the photon Green's function $D^{ij}(x', x)$ has been worked out in detail in Ref. [10], and here we need only the $D^{11}(x', x)$ component for $z' > 0$, $z > 0$. The latter is given [10] by

$$D^{11}(x', x) = i \int \frac{\mathrm{d}^2 \mathbf{K}}{(Z\pi)^2} \int \frac{\mathrm{d}q}{2\pi} \frac{\mathrm{e}^{i\mathbf{K}\cdot(\mathbf{x}'_{11} - \mathbf{x}_{11})}\, \mathrm{e}^{-i|\mathbf{k}|/\varepsilon_1 |x'^0 - x^0|}}{2\sqrt{\varepsilon_1}\,|k|} \times \left(\mathrm{e}^{iq(z'-z)} \left[1 - \left(\frac{K^1}{|\mathbf{k}|} \right)^2 \right] + \mathrm{e}^{-q(z'+z)} \times G_+^{11}(\mathbf{K}, q) \right), \tag{5}$$

where $\mathbf{k} = (\mathbf{K}, q)$, $\mathbf{x}_{11} = (x, y)$, and

$$G_+^{11}(\mathbf{K}, q) = \left(A_+ - \left(\frac{K^1}{|\mathbf{k}|} \right)^2 B_+ \right), \tag{6}$$

$$A_+ = -1 + \frac{2q}{q + Q}, \tag{7}$$

$$B_+ = -1 + \frac{2\varepsilon_1 q}{\varepsilon_1 Q + \varepsilon_2 q}, \tag{8}$$

$$Q = \sqrt{\left(\frac{\varepsilon_2}{\varepsilon_1} - 1 \right) \mathbf{k}^2 + q^2}\; \mathrm{sgn}\, q. \tag{9}$$

The vacuum-to-vacuum transition amplitude [1, 5, 8, 11], first at $T = 0$, may be then written as

$$\langle 0_+ | 0_- \rangle = \exp\left[\frac{\mathrm{i}}{2\hbar c^3} \int (\mathrm{d}x')(\mathrm{d}x)\, J^1(x') D^{11}(x', x) J^1(x) \right]. \tag{10}$$

We will specialize in currents of the form

$$J^1(x) = f(x^1, t)\, \delta(x^2)\, \delta(x^3 - a), \tag{11}$$

where a denotes the distance at which a charged particle moves from the surface of the second medium. The function $f(x^1, t)$ is quite general for the time being and a the particular functional form for it, for the problem at hand, will be chosen later. We consider a causal arrangement of currents $f_+(x^1, t)$ and $f_-(x^1, t)$ defined through

$$f(x^1, t) = f_+(x^1, t) + f_-(x^1, t), \tag{12}$$

where the current $f_+(x^1, t)$ is switched on after that the current $f_-(x^1, t)$ is switched off. The components $f_-(x^1, t)$ and $f_+(x^1, t)$ are considered as the emission and detection sources of photons, respectively. The contribution to the exponent in (10) coming from such a causal arrangement of emitter and detector of photons is then given from (5) to be

$$\frac{\mathrm{i}}{\hbar c^3} \int \mathrm{d}x'^1\, \mathrm{d}t'\, \mathrm{d}x'\, \mathrm{d}t\, f_+(x^1, t') D(x'^1 t', x^1 t) f_-(x^1, t), \tag{13}$$

where

$$D(x'^1 t', x^1 t) = \mathrm{i} \int \frac{\mathrm{d}^2 \boldsymbol{K}}{(2\pi)^2} \int \frac{\mathrm{d}q}{2\pi} \frac{e^{\mathrm{i}\boldsymbol{K}\cdot(x'^1 - x^1)} e^{-\mathrm{i}|\boldsymbol{k}|/\varepsilon_1 |x'^0 - x^0|}}{2\sqrt{\varepsilon_1}|k|} \times \left\{ \left[1 - \left(\frac{K^1}{|\boldsymbol{k}|} \right)^2 \right] + \mathrm{e}^{-2\mathrm{i}qa} G_+^{11}(\boldsymbol{K}, q) \right\}, \tag{14}$$

and $G_+^{11}(K, q)$ is defined in (6–9). To simplify the integral in (14), we first insert the unit operator in the integrand

$$1 = \int_0^\infty \mathrm{d}\omega\, \delta\left(\omega - \frac{|\boldsymbol{k}|c}{\sqrt{\varepsilon_1}} \right), \tag{15}$$

where ω will be identified with the angular frequency of a photon. Secondly, we integrate over the K^2, q integrals in polar coordinates $K^2 = R\cos\theta$, $q = \mathrm{R}\sin\theta$, $\mathrm{d}K^2\,\mathrm{d}q = R\mathrm{d}R\,\mathrm{d}\theta$, $0 \leqslant R < \infty$, $0 \leqslant \theta \leqslant 2\pi$. We note, in particular, that due to

$$R^2 = \boldsymbol{k}^2 - (K^1)^2 = \frac{\varepsilon_1 \omega^2}{c^2} - (K^1)^2. \tag{16}$$

the delta function in (15), in turn, allows us to integrate over R in the expression

$$D(x'^1, t', x^1 t) = \mathrm{i} \int_{-\infty}^{\infty} \frac{\mathrm{d}K^1}{2\pi} \int_0^\infty \mathrm{d}\omega \frac{\mathrm{e}^{-(\omega/c)(x'^0 - x^0)}}{2\varepsilon_1(\omega/c)} \times \int_0^\infty R\,\mathrm{d}R\, \delta\left(\omega - \sqrt{(K^1)^2 + R^2} \frac{c}{\sqrt{\varepsilon_1}} \right) F_R(K^1, \omega), \tag{17}$$

where

$$F_R(K^1, \omega) = \int_0^{2\pi} \frac{\mathrm{d}\theta}{2\pi} \left\{ \left(1 - \frac{(K^1)^2 c^2}{\omega^2 \varepsilon_1} \right) [1 - \cos(2Ra\cos\theta)] + 2\cos(2Ra\cos\theta)\cos\theta \left[\frac{1}{\sqrt{A^2 + \cos^2\theta}\,\mathrm{sgn}\cos\theta + \cos\theta} - \frac{(K^1)^2 c^2/(\omega^2\varepsilon_1)}{\sqrt{A^2 + \cos^2\theta}\,\mathrm{sgn}\cos\theta + (\varepsilon_2\cos\theta/\varepsilon_1)} \right] \right\}, \tag{18}$$

$$A^2 = (\varepsilon_2 - \varepsilon_1)\left(\varepsilon_1 - \frac{(K^1)^2 c^2}{\omega^2} \right)^{-1} \tag{19}$$

by writing

$$\delta\left(\omega - \sqrt{(K^1)^2 + R^2} \frac{c}{\sqrt{\varepsilon_1}} \right) = \frac{\delta\left(R - \sqrt{\frac{\omega^2}{c^2}\varepsilon_1 - (K^1)^2}\right)}{Rc^2} \omega\varepsilon_1 \tag{20}$$

with the obvious necessary condition that

$$\frac{\omega^2}{c^2}\varepsilon_1 - (K^1)^2 \geqslant 0, \tag{21}$$

which immediately follows since $[1 - (K^1/(|\boldsymbol{K}|)^2] \geqslant 0$. All told, (13) simplifies to

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}K^1}{2\pi} \int_0^\infty \mathrm{d}\omega\, \mathrm{i}f_+^*\left(K^1, \frac{\omega}{c} \right) \times P(K^1, \omega)\, \mathrm{i}f_-\left(K^1, \frac{\omega}{c} \right), \tag{22}$$

where

$$P(K^1, \omega) = \frac{1}{8\pi^2 c}\left[1 - \frac{(K^1)^2 c^2}{\omega^2 \varepsilon_1} \right] I(K^1, \omega), \tag{23}$$

$$I(K^1, \omega) = \int_0^{2\pi} \mathrm{d}\theta \left\{ 1 - \cos\left(\frac{2\omega a}{c}\sqrt{D}\cos\theta \right) + \frac{2\varepsilon_1}{D}\cos(2\omega a\sqrt{D}\cos\theta)\cos\theta \times \left[\frac{1}{\sqrt{A^2 + \cos^2\theta}\,\mathrm{sgn}\cos\theta + \cos\theta} - \frac{(K^1)^2 c^2/(\omega^2\varepsilon_1)}{\sqrt{A^2 + \cos^2\theta}\,\mathrm{sgn}\,\theta + \frac{\varepsilon_2}{\varepsilon_1}\cos\theta} \right] \right\}, \tag{24}$$

$$D = \left(\varepsilon_1 - \frac{(K^1)^2 c^2}{\omega^2} \right). \tag{25}$$

In the Appendix, it is shown that $I(K^1, \omega)$, and hence also $P(K^1, \omega)$, are real and positive definite. That is, in the expression (22), one may define the effective sources

$$S_\pm\left(K^1, \frac{\omega}{c} \right) = f_\pm\left(K^1, \frac{\omega}{c} \right)\sqrt{P(K^1, \omega)}, \tag{26}$$

and the problem at hand is, in particular, reduced to a two-dimensional problem. With effective sources as just defined, Eq. (22) may be rewritten as

$$\int_{-\infty}^{\infty}\frac{dK^1}{2\pi}\int_0^{\infty}\mathrm{d}\omega\,\mathrm{i}\,S_+^*\left(K^1,\frac{\omega}{c}\right)i\,S_-\left(K^1,\frac{\omega}{c}\right). \quad (27)$$

The inclusion of temperature may be done in the usual manner [11]. We first replace the amplitude $\langle 0_+|0_-\rangle$ by an amplitude $\langle N; N_1, N_2, \ldots N; N_1, N_2, \ldots\rangle$ involving an arbitrary number N of photons, N_1 of which have each an energy $\hbar\omega_1$, with their first component of the wave-vector $\boldsymbol{K}$ being equal to K_1^1 N_2 of which have each energy $\hbar\omega_2$, with their first component of the wave-vector being equal to K_2^1, and so on, in a convenient discrete (cf. [12]) notation for ω and K^1. Then an average of the latter amplitude is made with the statistical factor $\Pi_i \exp(-\hbar\omega_i/kT)$, where $1/kT$ is the Boltzmann factor, and a sum over $N_1, N_2, \ldots$, and N as well is made. This gives the thermal average ground-state persistent amplitude $\langle G_+|G_-\rangle$ containing an arbitrary number of photons of all possible energies. The procedure is carried out systematically in Ref. [11] and gives

$$\langle G_+|G_-\rangle$$

$$= \langle 0_+|0_-\rangle \exp -\frac{1}{\hbar c^3}\int_0^{\infty}\mathrm{d}\omega\int_{-\infty}^{\infty}\frac{\mathrm{d}K^1}{(2\pi)}\left|f\left(K^1,\frac{\omega}{c}\right)\right|^2 \times P(K^1,\omega)\left(\exp\left(\left(\frac{\hbar\omega}{kT}\right)-1\right)^{-1}\right). \quad (28)$$

The average number of photons emitted by a current described in (11) is [1, 5, 11] then

$$\langle N\rangle_T = \frac{1}{\hbar c^3}\int_0^{\infty}\mathrm{d}\omega\int_{-\infty}^{\infty}\frac{\mathrm{d}K^1}{(2\pi)}\left|f\left(K^1,\frac{\omega}{c}\right)\right|^2 \times P(K^1,\omega)\coth\left(\frac{\hbar\omega}{2kT}\right). \quad (29)$$

Since $P(K^1,\omega)$ is positive definite, the average number density of photons with angular frequency ω is then

$$\langle N(\omega)\rangle_T = \frac{1}{\hbar c^3}\int_{-\infty}^{\infty}\frac{\mathrm{d}K^1}{(2\pi)}\left|f\left(K^1,\frac{\omega}{c}\right)\right|^2 \times P(K^1,\omega)\coth\left(\frac{\hbar\omega}{2kT}\right). \quad (30)$$

Note the presence of the $\hbar$ factors in the expression, and that

$$\langle N\rangle_T = \int_0^{\infty}\mathrm{d}\omega\,\langle N(\omega)\rangle_T, \quad (31)$$

We now specialize the problem at hand by choosing $f(x^1, t)$ in (11) to be

$$f(x^1,t) = ev\,\delta(x^1 - vt), \quad (32)$$

where v is the speed of the charged particle. Then

$$f\left(K^1,\frac{\omega}{c}\right) = evc\int_{-\infty}^{\infty}\mathrm{d}t\,\mathrm{e}^{-\mathrm{i}K^1x^1}\mathrm{e}^{\mathrm{i}\omega t}\delta(x^1-vt) = 2\pi ec\,\delta\left(K^1-\frac{\omega}{v}\right). \quad (33)$$

Therefore, for a charged particle moving a distance L, we may formally write

$$\left|f\left(K^1,\frac{\omega}{c}\right)\right|^2 = 2\pi e^2c^2\,\delta\left(K^1-\frac{\omega}{v}\right)\int_0^L \mathrm{d}x^1. \quad (34)$$

That is, the average number density of photons of frequency ω emitted by the charged particle during the time it traverses a distance L is from (23), (24), (30) at finite T is

$$\langle N(\omega,L)\rangle_T = \frac{\alpha L}{c\varepsilon_1}\left(\varepsilon_1 - \frac{1}{\beta^2}\right)\times\left[1-F(x)\right]\coth\left(\frac{\hbar\omega}{2kT}\right), \quad (35)$$

where $\alpha = e^2/4\pi\hbar c$,

$$F(x) = \frac{2}{\pi}\int_0^{\pi/2}\mathrm{d}\theta\cos(x\cos\theta)\times\left\{1-\frac{2\varepsilon_1}{D}\left[\frac{\cos\theta}{\sqrt{A^2+\cos^2\theta}+\cos\theta}-\frac{1}{\varepsilon_1\beta^2}\times\frac{\cos\theta}{\sqrt{A^2+\cos^2\theta}+\frac{\varepsilon_2}{\varepsilon_1}\cos\theta}\right]\right\}, \quad (36)$$

$$x = \frac{2\omega a}{c}\sqrt{\varepsilon_1 - \frac{1}{\beta^2}} = \frac{4\pi \nu a}{c}\sqrt{\varepsilon_1 - \frac{1}{\beta^2}}, \quad \beta = v/c, \tag{37}$$

$$A^2 = (\varepsilon_2 - \varepsilon_1)/D, \tag{38}$$

$$D = \left(\varepsilon_1 - \frac{1}{\beta^2}\right) \tag{39}$$

and the condition (21) for the validity of (35) translates to the threshold condition

$$\beta^2 > \frac{1}{\varepsilon_1}. \tag{40}$$

We will eventually apply our formula (35) to the visible spectrum. Near the threshold condition for v: $\sqrt{\varepsilon_1 - (1/\beta^2)} \ll 1$, the function $F(x)$ may be simplified to

$$F(x) \cong -\frac{2}{\pi}\int_0^{\pi/2} \mathrm{d}\theta \cos(2\theta)\cos(x\cos\theta). \tag{41}$$

We introduce the measure

$$\rho(x) = \frac{2kT}{\hbar c \varepsilon_1}\left(\varepsilon_1 - \frac{1}{\beta^2}\right)\ln(\mathrm{e}^{bx} - \mathrm{e}^{-bx}),$$
$$b \equiv \frac{\hbar c}{4kTa\sqrt{\varepsilon_1 - \frac{1}{\beta^2}}} \tag{42}$$

That is,

$$\mathrm{d}\rho(x) = \frac{1}{2\varepsilon_1 a}\sqrt{\varepsilon_1 - \frac{1}{\beta^2}}\coth(bx)\,\mathrm{d}x$$
$$= \frac{1}{\varepsilon_1 c}\left(\varepsilon_1 - \frac{1}{\beta^2}\right)\coth(bx)\,\mathrm{d}\omega. \tag{43}$$

Therefore, the average number of photons emitted, with angular frequencies within a range (ω_1, ω_2), during the time the charged particle traverses a distance L at temperature T is

$$\int_{\omega_1}^{\omega_2} \langle N(\omega, L)\rangle_T \mathrm{d}\omega = \alpha L[\rho(x_2) - \rho(x_1)]$$
$$\times\left[1 - \frac{\int_{x_1}^{x_2} F(x)\,\mathrm{d}\rho(x)}{\rho(x_2) - \rho(x_1)}\right], \tag{44}$$

where

$$x_1 = \frac{4\pi \nu_1}{c} a\sqrt{\varepsilon_1 - \frac{1}{\beta^2}}, \quad x_2 = \frac{4\pi \nu_2}{c} a\sqrt{\varepsilon_1 - \frac{1}{\beta^2}}. \tag{45}$$

The multiplicative factor

$$\left[1 - \frac{\int_{x_1}^{x_2} F(x)\,\mathrm{d}\rho(x)}{\rho(x_2) - \rho(x_1)}\right], \tag{46}$$

gives the correction, due to the presence of the second dielectric medium, to the average number of photons emitted in the range (ω_1, ω_2) during the time the charged particle moves a distance L. From the mean value theorem, we may write

$$F(x^*) = \frac{\int_{x_1}^{x_2} F(x)\,\mathrm{d}\rho(x)}{\rho(x_2) - \rho(x_1)}, \quad x_1 < x^* < x_2. \tag{47}$$

Clearly (47), in particular, holds when $F(x)$ is almost a constant in the range (x_1, x_2): $F(x_1) \cong F(x^*) \cong F(x_2)$. Near the threshold, the minimum value of (47) is obtained, numerically, for $x_1 \cong 6.65$, $x_2 \cong 6.75$, $x^* \cong 6.7$ with the minimum value

$$F(x^*) = \frac{\int_{x_1}^{x_2} F(x)\,\mathrm{d}\rho(x)}{\rho(x_2) - \rho(x_1)} \cong -0.310 \tag{48}$$

independently of T, ε_1 and $\varepsilon_2 > \varepsilon_1$. Eq. (48) says that as long as one chooses a window centered about a given point ν_0, such that

$$\frac{4\pi \nu_1}{c} a\sqrt{\varepsilon_1 - \frac{1}{\beta^2}} \cong 6.65, \quad \frac{4\pi \nu_2}{c} a\sqrt{\varepsilon_1 - \frac{1}{\beta^2}} \cong 6.75, \tag{49}$$

there will be about a 31% enhancement of the number of photons emitted in the presence of a second dielectric medium, compared to with corresponding no second dielectric medium for any T, ε_1 and $\varepsilon_2 > \varepsilon_1$, as we gradually "increase" v near its threshold value. Eqs. (49) lead to

$$\nu_1 = \frac{13.3}{13.4}\nu_0, \qquad \nu_2 = \frac{13.5}{13.4}\nu_0 \tag{50}$$

and

$$a\sqrt{\varepsilon_1 - \frac{1}{\beta^2}} = \frac{6.7c}{4\pi\nu_0}. \tag{51}$$

Eq. (51) will allow us to quantify the near threshold condition below.

Practically, such an enhancement of 31%, with a window (ν_1, ν_2) defined in (50) about a point ν_0, will be meaningful if in turn

$$\alpha L[\rho(x_2 - \rho(x_1) = \frac{3.9c\alpha L}{8\pi\varepsilon_1 a^2 \nu_0} \times\left[(x_2 - x_1) + \frac{1}{b}\ln\left(\frac{1-e^{-2bx_2}}{1-e^{-2bx_1}}\right)\right] \gg 1, \tag{52}$$

as the coefficient of the corrective factor (46) in (44), to ensure the detactibility of at least some photons in the absence of a second dielectric medium.

For blue light with $\nu_0 \cong 6.172\times10^{14}$ Hz,

$$\nu_1 \cong 6.126\times10^{14}\ \text{Hz}, \quad \nu_2 \cong 6.218\times10^{14}\ \text{Hz} \tag{53}$$

and

$$a\sqrt{\varepsilon_1 - \frac{1}{\beta^2}} \cong 0.26\times10^{-6}\ \text{m}. \tag{54}$$

The definition of b in (42) then gives

$$b = \frac{2214.5}{[T/\text{K}]}. \tag{55}$$

We note that for $0 \leqslant T < 5000$ K, with $x_2 = 6.75$, $x_1 = 6.65$,

$$(x_2 - x_1) + \frac{1}{b}\ln\left(\frac{1-e^{-2bx_2}}{1-e^{-2bx_1}}\right) \cong (x_2 - x_1) = 0.10. \tag{56}$$

Hence the criterion in (52) leads to

$$\left[\frac{a}{m}\right] \ll 0.74\times10^{-5}\frac{\sqrt{L/\text{m}}}{\sqrt{\varepsilon_1}}, \tag{57}$$

where m is the unit of meter. For a $\cong 10^{-6}\sqrt{(L/\text{m})}/\sqrt{\varepsilon_1}$, Eq. (54) gives $\sqrt{\varepsilon_1 - (1/\beta^2)} \cong 0.26\sqrt{\varepsilon_1}/\sqrt{L/\text{m}}$. For the charged particle moving, e.g., in water $\sqrt{\varepsilon_1} \simeq \frac{4}{3}$ over a 100 m, the latter gives

$$\sqrt{\varepsilon_1 - \frac{1}{\beta^2}} \simeq 0.035 \ll 1$$

which is near the threshold.

For red light with $\nu_0 \cong 4.283\times10^{14}$ Hz,

$$\nu_1 \cong 4.25\times10^{14}\ \text{Hz}, \qquad \nu_2 \cong 4.315\times10^{14}\ \text{Hz} \tag{58}$$

and

$$a\sqrt{\varepsilon_1 - \frac{1}{\beta^2}} \cong 0.37\times10^{-6}\ \text{m}. \tag{59}$$

The criterion in (52) leads to

$$\left[\frac{a}{m}\right] \ll 0.85\times10^{-5}\frac{\sqrt{L/\text{m}}}{\sqrt{\varepsilon_1}}. \tag{60}$$

Again for $a \cong 10^{-6}\sqrt{(L/\text{m})}/\sqrt{\varepsilon_1}$ Eq. (59) leads to $\sqrt{\varepsilon_1 - (1/\beta^2)} \cong 0.37\sqrt{\varepsilon_1}/\sqrt{L/\text{m}} \cong 0.05$ for $\sqrt{\varepsilon_1} \cong \frac{4}{3}$ for the charged particle moving 100 m, which is near the threshold.

The curve $G(x) = -F(x)$, with the latter defined in (36), and its asymptotic threshold curve $G_0(x) = -F(x)$, with the latter defined in (41), are plotted in the Fig. 1 about the optimal value $x^* = 6.7$ in (48). The solid line denotes the curve $G_0(x)$ and the dotted and dashed lines represent the exact curves for $\varepsilon_2/\varepsilon_1 = 10$ and 2, respectively, with both curves approaching the threshold solid curve in the threshold limit. In particular, for blue light threshold limit is defined in (54) and for red light it is defined in (59). The windows for maximum enhancement of photon emission for blue light and red light are worked in (53): (6.126×10^{14} Hz, 6.218×10^{14} Hz) and (4.251×10^{14}, 4.315×10^{14} Hz) in (58) in the frequency variable, respectively. The universality character of the 31% enhancement at the threshold limit is emphasized by working with the variable x, defined in (45), with the window defined by ($x_1 = 6.65, x_2 = 6.75$) as given in (49), applicable, in particular, for both blue and red

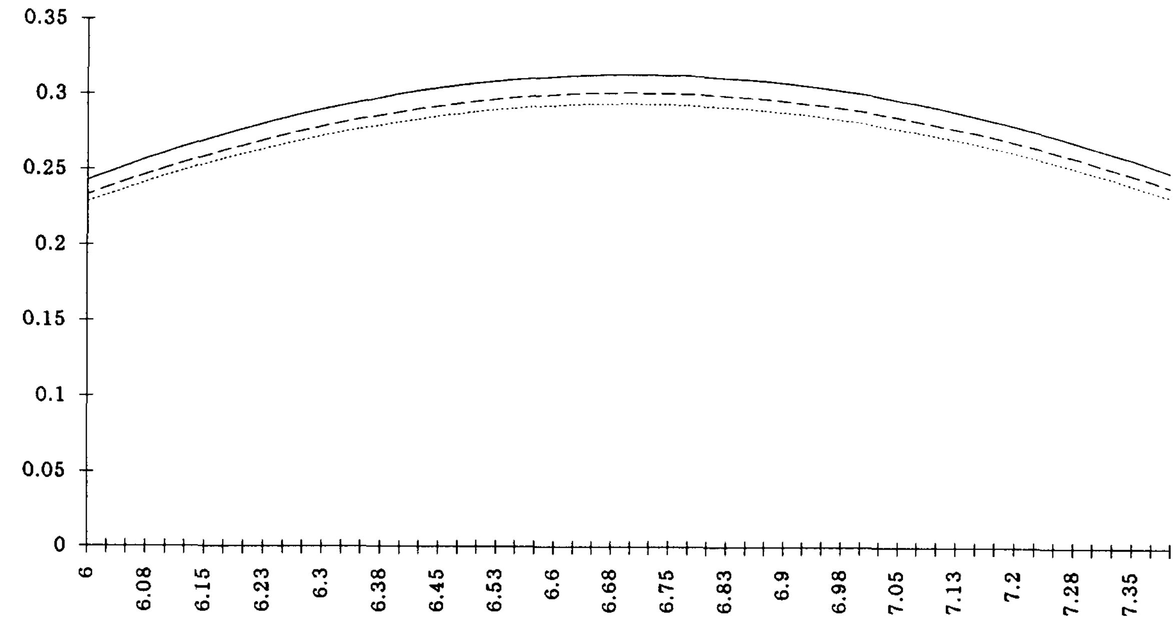

Fig. 1. Plot of $G(x) = -F(x)$ and $G_0(x)$ versus as given, respectively, in (36) and (41) with the latter denoting the asymptotic threshold behavior and is represented by the solid curve. The dashed and the dotted curves correspond to $\varepsilon_2/\varepsilon_1 = 2, 10$, respectively, with both approaching the threshold curve in the threshold limit with estimates, in particular, for blue and red light given in (54) and (59). The universality of maximum enhancement of number of photon emission is emphasized by working with the x variable defining a window (6.65, 6.75) about the optimal point $x^* = 6.7$. A wide range for x was chosen to show the gradual decrease of the curves about the point 6.70. The precise x value for blue (6.6999) and red light (6.7001) frequencies are too close to be identified on the x-axis.

lights. This window is about a frequency $\nu_0 \cong 6.172 \times 10^{14}$ Hz for blue light corresponding to an x value of 6.6999 and a frequency $\nu_0 \cong 4.283 \times 10^{14}$ Hz for red light which corresponds to an x value of 6.7001 as determined from (50) and the $x_1 = 6.65$.

Applications to other points of the spectrum, such as in the microwave region, cannot be readily extended since we have effectively set the permeabilities to be equal. The generalization to include the permeabilities is beyond the scope of the present work and will be attempted in a future work.

Appendix

A direct proof of the important positive definiteness of the obviously real integral $I(K^1, \omega)$ in (24) follows. The integrand of the latter may rewritten as

$$1 - \cos(x\cos\theta) \times \left\{1 - \frac{2}{1-a}\left[\frac{\cos\theta}{\sqrt{A^2 + \cos^2\theta}\,\mathrm{sgn}\cos\theta + \cos\theta} - \frac{a\cos\theta}{\sqrt{A^2 + \cos^2\theta}\,\mathrm{sgn}\cos\theta + b\cos\theta}\right]\right\}, \quad \text{(A.1)}$$

where

$$a = (K^1)^2 c^2/(\omega^2 \varepsilon_1) \leqslant 1, \quad \text{(A.2)}$$

$$b = \varepsilon_2/\varepsilon_1 > 1,$$

$$x = \frac{2\omega a}{c}\varepsilon_1(1-a), \quad \text{(A.3)}$$

$$A^2 = (b-1)/(1-a). \quad \text{(A.4)}$$

The positive definiteness of $I(K^1, \omega)$ will follow if the expression in (A.1) is positive. The expression multiplying $\cos(x\cos\theta)$) in (A.1) may be rewritten as

$$\frac{(b-1)-(1+a)(b-1)\cos^2\theta+(1-a)(b-1)\sqrt{A^2+\cos^2\theta}|\cos\theta|}{(b-1)+(1-a)(b+1)\cos^2\theta+(1-a)(b+1)\sqrt{A^2+\cos^2\theta}|\cos\theta|}\equiv E. \tag{A.5}$$

The denominator is *positive*, and by a direct comparison of the numerator and the denominator in (A.5), we note that

$$E \leqslant 1. \tag{A.6}$$

Now we show that $E \geqslant -1$. Suppose that the contrary is true, that is $E < 1$, and hence

$$\begin{aligned}&(b-1)-(1+a)(b-1)\cos^2\theta\\ &\quad+(1-a)(b-1)\sqrt{A^2+\cos^2\theta}|\cos\theta|\\ &<-(b-1)-(1-a)(b+1)\cos^2\theta\\ &\quad-(1-a)(b+1)\sqrt{A^2+\cos^2\theta}\,|\cos\theta|\end{aligned} \tag{A.7}$$

or

$$\begin{aligned}&(b-1)+(1-a)b\sqrt{A^2+\cos^2\theta}|\cos\theta|\\ &\quad<(ba-1)\cos^2\theta.\end{aligned} \tag{A.8}$$

Further, using a lower bound to the left-hand side of this inequality, we may write

$$(b-1)+(1-a)b\cos^2\theta<(ba-1)\cos^2\theta \tag{A.9}$$

or

$$(b-1)<(2ab-1-b)\cos^2\theta. \tag{A.10}$$

If $(2ba-1-b)\leqslant 0$, we run into a contradiction that $b<1$ since $\cos^2\theta \geqslant 0$. On the other hand, if $(2ba-1-b)>0$, we may devide (A.10) by this expression to obtain

$$\frac{(b-1)}{(2ab-1-b)}<\cos^2\theta\leqslant 1, \tag{A.11}$$

which leads to the inequality: $(b-1)<2ab-1-b$, or to the contradiction that $1<a$. Therefore, $E \geqslant -1$. That is from (A.6), we have $|E| \leqslant 1$. Eq. (A.1), may be rewritten as $[1-\cos(x\cos\theta)E]$, which is obviously bounded below by 0, since from what we have just established for E, $|\cos(x\cos\theta))E| \leqslant 1$.

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