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Technical note

Quantum correction to the photon number emission in synchrotron radiation

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Received 25 May 2006; accepted 29 August 2006

Abstract

The quantum correction to the mean *number* $\langle N \rangle$ of photons emitted per revolution, to the order $\hbar$, is derived in closed form in synchrotron radiation which supplements our explicit expression obtained earlier for $\langle N \rangle$ which was based on the classical analysis.

1. Introduction

The historical development of the theory of synchrotron radiation and the fascinating story behind it are well documented in the literature (see, e.g., Pollock, 1983) and relatively recent theoretical progress in the field as well as extensive references may be found in Bordovitsyn (1999). Although many features of synchrotron radiation have been well known for a long time, there is room for further developments and certainly for improvements. For example, in a recent investigation (Manoukian and Jearnkulprasert, 2000), an explicit expression for the mean *number* $\langle N \rangle_C$ of photons emitted per revolution was derived, based on the classical analysis, involving a remarkably simple one-dimensional integral, with C, here, standing for classical. The latter is given by

$$\langle N \rangle_C = 2\alpha\beta^2 \int_0^\infty \frac{dx}{x^2} \left[\frac{(\sin x/x)^2 - \cos(2x)}{1 - \beta^2 (\sin x/x)^2} \right], \tag{1.1}$$

where $\beta = v/c$, v is the speed of the charged particle, c is the speed of light and α is the fine-structure constant. For high energetic charged particles, Eq. (1.1) gives (Manoukian and Jearnkulprasert, 2000)

$$\langle N \rangle_C \simeq \frac{5\pi\alpha}{\sqrt{3(1-\beta^2)}} + a_0\alpha + \mathcal{O}\left(\sqrt{1-\beta^2}\right), \tag{1.2}$$

where the constant a_0 is overwhelmingly large in magnitude and is given by

$$a_0 = 2 \int_0^\infty \frac{dx}{x^2} \left[\frac{6(\sin x/x)^2 - \cos(2x) - 5}{1 - (\sin x/x)^2} \right] = -9.55797 \tag{1.3}$$

and the second term on the right hand of (1.2) gives an important contribution for high energetic particles and was unfortunately missing in the earlier investigations (see, e.g., Particle Data Group, 2004).

For example, the relative errors in (1.2) are quite satisfactory with 4.11%, 1.34%, 0.063% for $\beta = 0.8$, 0.9, 0.99, respectively, to be compared with the relative errors of 160%, 82%, 17% of the well known expression tabulated earlier (Particle Data Group, 2004) involving only the first term on the right-hand side of (1.2). A systematic asymptotic analysis for high energetic

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doi:10.1016/j.radphyschem.2006.08.012

relativistic particles has been also carried out more recently by Manoukian et al. (2004), based on (1.1), providing additional corrections to the ones on the right-hand side of (1.2) as functions of $\sqrt{1-\beta^2}$.

The purpose of this communication is to derive the quantum correction $\langle N\rangle_{\mathrm{Q}}$ in closed form to the mean *number* $\langle N\rangle$ of photons emitted per revolution, to the order $\hbar$, to supplement our explicit expression in (1.1), which was based on the classical analysis, giving the final result $\langle N\rangle = \langle N\rangle_{\mathrm{C}} + \langle N\rangle_{\mathrm{Q}}$, where $\langle N\rangle_{\mathrm{Q}}$ is given in (2.16).

2. Quantum correction

The explicit integral expression for the quantum correction, to the order $\hbar$, to the mean *number* $\langle N\rangle$ may be obtained from that of the formula of the power of (Schwinger, 1949, Eqs. (III), (6), (7); 1954, Eq. (24)) and Schwinger and Tsai (1978, Eq. (C.11)) given by

$$\langle N\rangle_{\mathrm{Q}} = \alpha\hbar\left(\frac{\delta}{\delta\hbar}F(\hbar)\right)_{\hbar\to 0}, \tag{2.1}$$

where

$$F(\hbar) = \int_0^\infty \mathrm{d}\omega \int_{-\infty}^{\infty} \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}\omega(1+\hbar\omega/E)\tau}\frac{(\beta^2\cos\omega_0\tau - 1)}{\beta\sin(\omega_0\tau/2)} \times \sin\left(2\beta\frac{\omega}{\omega_0}\left(1+\frac{\hbar\omega}{E}\right)\sin\frac{\omega_0\tau}{2}\right) \tag{2.2}$$

and, in our notation,

$$\frac{E}{mc^2} = \frac{1}{\sqrt{1-\beta^2}}, \quad \omega_0 = \frac{\beta c}{R} = \frac{cqB\sqrt{1-\beta^2}}{mc^2}, \tag{2.3}$$

with q, R and B denoting, respectively, the magnitude of the charge of the particle, the radius of the classical circular motion, and the magnetic field in question. Eq. (2.2) gives

$$\left.\frac{\delta}{\delta\hbar}F(\hbar)\right|_{\hbar\to 0} = \int_0^\infty \mathrm{d}\omega \int_{-\infty}^{\infty} \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}\omega\tau}\frac{(\beta^2\cos\omega_0\tau - 1)}{\beta\sin(\omega_0\tau/2)} \times\left[-\mathrm{i}\frac{\omega^2}{E}\tau\sin\left(2\beta\frac{\omega}{\omega_0}\sin\frac{\omega_0\tau}{2}\right) + \cos\left(2\beta\frac{\omega}{\omega_0}\sin\frac{\omega_0\tau}{2}\right) \times\left(2\beta\frac{\omega}{\omega_0}\frac{\omega}{E}\sin\frac{\omega_0\tau}{2}\right)\right]. \tag{2.4}$$

Let

$$\frac{\omega}{\omega_0} = z, \quad \omega_0\tau = x, \quad 2\beta\sin\frac{\omega_0\tau}{2} = a(x) \tag{2.5}$$

in (2.4) to rewrite the latter in the more convenient form:

$$\frac{\delta}{\delta\hbar}F(\hbar)|_{\hbar\to 0} = \left(\frac{\omega_0}{E}\right)\int_{-\infty}^{\infty}\mathrm{d}x(\beta^2\cos x - 1)\int_0^\infty \mathrm{d}z\, z^2 \times\left[\mathrm{e}^{-\mathrm{i}z(x+a(x))}\frac{x+a(x)}{a(x)} + \mathrm{e}^{-\mathrm{i}z(x-a(x))}\frac{x-a(x)}{-a(x)}\right]. \tag{2.6}$$

This seemingly complicated double integral may be computed in closed form. To this end, let

$$x + a(x) = \xi \tag{2.7}$$

and note that

$$\frac{\mathrm{d}x}{\mathrm{d}\xi} = \frac{1}{(1+\beta\cos(x/2))}. \tag{2.8}$$

Also for $x\to\pm\infty$, $\xi\to\pm\infty$, and for $\xi = 0$, $x = 0$.

Accordingly, $\langle N\rangle_{\mathrm{Q}}$ in (2.1) may be obtained by taking the real part of (2.6) giving

$$\langle N\rangle_{\mathrm{Q}} = \alpha\pi\left(\frac{\hbar\omega_0}{E}\right)\int_{-\infty}^{\infty}\mathrm{d}\xi[I(\xi,\beta) + I(\xi,-\beta)], \tag{2.9}$$

where

$$I(\xi,\beta) = \left(-\frac{\mathrm{d}^2}{\mathrm{d}\xi^2}\delta(\xi)\right)\left(\frac{\beta^2\cos x - 1}{2\beta\sin(x/2)}\right)\left(\frac{x+2\beta\sin(x/2)}{1+\beta\cos(x/2)}\right) \tag{2.10}$$

and we have used the integral

$$\mathrm{Re}\int_0^\infty \mathrm{d}z\,\mathrm{e}^{-\mathrm{i}z\xi} = \pi\delta(\xi). \tag{2.11}$$

To evaluate the integral in (2.9), we use the relations

$$\frac{\mathrm{d}^2}{\mathrm{d}\xi^2} = \frac{1}{(1+\beta\cos(x/2))^2}\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{(\beta/2)\sin(x/2)}{1+\beta\cos(x/2)}\frac{\mathrm{d}}{\mathrm{d}x}\right], \tag{2.12}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left.\left(\frac{x}{\sin(x/2)}\right)\right|_{x=0} = 0, \quad \frac{\mathrm{d}^2}{\mathrm{d}x^2}\left.\left(\frac{x}{\sin(x/2)}\right)\right|_{x=0} = \frac{1}{6}, \tag{2.13}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left.\left(\frac{\beta^2\cos x - 1}{1+\beta\cos(x/2)}\right)\right|_{x=0} = 0, \quad \frac{\mathrm{d}^2}{\mathrm{d}x^2}\left.\left(\frac{\beta^2\cos x - 1}{1+\beta\cos(x/2)}\right)\right|_{x=0} = \frac{-\beta(1+3\beta)}{4(1+\beta)}, \tag{2.14}$$

from which we obtain

$$\int_{-\infty}^{\infty}\mathrm{d}\xi I(\xi,\beta) = \frac{1}{(1+\beta)^2}\left[\frac{(1-\beta)}{12\beta} + \frac{1+3\beta}{4}\right]. \tag{2.15}$$

Please cite this article as: E.B. Manoukian, P. Viriyasrisuwattana, Quantum correction to the photon number emission in synchrotron radiation, Radiation Physics and Chemistry (2006), doi:10.1016/j.radphyschem.2006.08.012

The quantum correction $\langle N\rangle_{\mathrm{Q}}$ then emerges from (2.9) and (2.15) to be

$$\langle N\rangle_{\mathrm{Q}} = \frac{8\pi}{3}\alpha\left(\frac{\hbar\omega_0}{mc^2}\right)\left[\left(\frac{E}{mc^2}\right) - \left(\frac{E}{mc^2}\right)^3\right]. \tag{2.16}$$

For $\beta \to 1$, this gives the truly asymptotic formula

$$\langle N\rangle_{\mathrm{Q}} \to -\frac{8\pi}{3}\alpha\left(\frac{\hbar\omega_0}{mc^2}\right)\left(\frac{E}{mc^2}\right)^3. \tag{2.17}$$

The expression in (2.16) supplements our explicit form $\langle N\rangle_{\mathrm{C}}$ in (1.1) in a quantum mechanical setting.

For the synchrotron in our institution, $R = 2.78\,\mathrm{m}$ and $E = 1.2\,\mathrm{GeV}$. This gives the estimates $\langle N\rangle_{\mathrm{C}} \sim \alpha 2.14 \times 10^4$ and $\langle N\rangle_{\mathrm{Q}} \sim \alpha 1.51 \times 10^{-2}$, in magnitudes, as based on (1.1)/(1.2) and (2.16)/(2.17), respectively. The latter is indeed relatively small but may be, however, significant for several revolutions in the magnetic field. This small quantum correction is not necessarily to be dismissed on practical grounds and may be reminiscent of small radiative corrections such as the Lamb shift contribution to the spectrum of the hydrogen atom which has been measured with very high accuracy and has led to much new physics. The quantum correction given in this work may be equally challenging to detect experimentally. It is interesting to note that a singularity in β for $\beta \to 1$ in $\langle N\rangle_{\mathrm{Q}}$ arises as in the classical treatment. Our quantum correction is based on a leading $\hbar$-contribution. At present it is not clear what would be the expression for $\langle N\rangle_{\mathrm{Q}}$ in an exact $\hbar$-treatment. Would such an expression compete with its classical counterpart and would it be practically relevant? Would it be singular in β for $\beta \to 1$? The exact $\hbar$-treatment of $\langle N\rangle_{\mathrm{Q}}$ as well as its experimental detection remain formidable problems and will be hopefully confronted with in the near future.

Acknowledgments

One of the authors (E.B.M.) would like to thank Asst. Prof. Dr. Prayoon Songsiriritthigul for the information provided on our synchrotron. The authors would also like to thank the reviewers for valuable comments and suggestions.

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Please cite this article as: E.B. Manoukian, P. Viriyasrisuwattana, Quantum correction to the photon number emission in synchrotron radiation, Radiation Physics and Chemistry (2006), doi:10.1016/j.radphyschem.2006.08.012