

# NEURO-TABU-FUZZY CONTROLLER TO STABILIZE AN INVERTED PENDULUM SYSTEM

S. Khwan-on, T. Kulworawanichpong, A. Srikaew, and S. Sujitjorn

School of Electrical Engineering, Suranaree University of Technology  
111 University Avenue, Muang District, Nakhon Ratchasima, 30000, THAILAND  
Phone: +66-4422-4400 Fax: +66-4422-4220 E-mail: [sarawut@ccs.sut.ac.th](mailto:sarawut@ccs.sut.ac.th)

## ABSTRACT

*This paper proposes a new control structure with its application in stabilizing an inverted pendulum system. The structure is neuro-fuzzy control of which initial parameters of the neural network are obtained from the adaptive tabu search, hence the name "neuro-tabu-fuzzy controller". This proposed controller consists of the Single Input Rule Modules (SIRMs) and the dynamic importance degrees (DIDs). The learning of the neural network results in the DIDs. The simulation results indicate that the proposed neuro-tabu-fuzzy controller has an ability to stabilize a wide range of an inverted pendulum system.*

## 1. INTRODUCTION

Nowadays, modern industrial plants increase their complexity and demand flexibility that makes the control design difficult. It is known that the conventional control system design requires an explicit mathematical model of the plant. The practical characteristics of the plant such as nonlinearity, complexity, uncertainty, etc, are restrictions of the convention control because the plant model cannot be easily obtained. Intelligent control can be a practical alternative since control design can be based on the knowledge and experience of human. This method does not require any explicit mathematical model of the plant. The intelligent control such as fuzzy logic, neural network, combination of neural network and fuzzy logic was developed to apply for many complex control problems. An inverted pendulum system is a typically unstable nonlinear system often used as a benchmark for verifying the performance and effectiveness of control methods. One can find a great number of research articles concerning stabilization of an inverted pendulum on cart. Some recent works related to our work are reviewed as follows. Xu [1] constructed a fuzzy logic controller using simplified table-lookup method to build a set of 625 fuzzy rules for this purpose. Alata [2] used adaptive neuro-fuzzy inference to construct the rules for the fuzzy gain schedule to control an inverted pendulum system. Sakai [3] applied a nonlinear optimization method to

learn fuzzy control rules for an inverted pendulum system by using the vector simplex method. Omatu [4] used a neural network controller to tune the gains of a PID controller. Yi [5] applied a fuzzy logic controller based on the single input rule modules (SIRM) to stabilize an inverted pendulum system.

In this article, we propose a new control structure, which is an advanced form of the SIRM, to stabilize an inverted pendulum system. The structure is based on neural network, fuzzy logic, and adaptive tabu search, so called neuro-tabu-fuzzy controllers. The learning of the neural network uses a backpropagation algorithm [6]. For fast convergence and avoiding local minimum entrapment, adaptive tabu search [7] is adopted to find suitable initial values of the connection weights and the thresholds. This proposed controller has a simple structure that decreases the number of fuzzy rules.

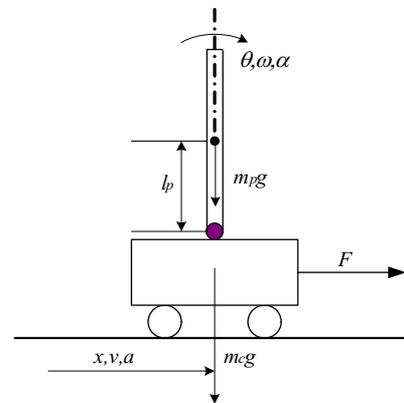


Fig. 1 Structure of the inverted pendulum system

## 2. INVERTED PENDULUM SYSTEM

Fig.1 depicts an inverted pendulum system in which  $F$  is the driving force. Under stable mode, the inverted pendulum is standing on a cart being controlled to move on horizontally on a frictionless rail. The dynamic equations of the inverted pendulum system can be expressed as [5]

$$\alpha = \frac{(m_c + m_p)g \sin\theta - \{F + m_p l_p \omega^2 \sin\theta\} \cos\theta}{\left\{ \frac{4}{3}(m_c + m_p) - m_p (\cos\theta)^2 \right\} l_p} \quad (1)$$

$$a = \frac{\frac{4}{3}\{F + m_p l_p \omega^2 \sin\theta\} - m_p g \sin\theta \cos\theta}{\left\{\frac{4}{3}(m_c + m_p) - m_p (\cos\theta)^2\right\}} \quad (2)$$

where,  $m_c$  and  $m_p$  are the mass of the cart and the pendulum, respectively.  $g=9.8 \text{ m/s}^2$  is the gravitation. The parameter  $l_p$  is the half length of the pendulum. The variables  $\theta, \omega, \alpha$  represent the angular displacement, the angular velocity, and the acceleration of the pendulum, respectively. The variables  $x, v, a$  denote the position, the velocity, and the acceleration of the cart, respectively.

### 3. NEURO-TABU-FUZZY CONTROLLER

The conventional fuzzy inference model sets all the input variables into the antecedent part of each fuzzy rule. This approach tends to increase the total number of fuzzy rules exponentially with the number of the input variables [5] and has a difficulty in setting up suitable fuzzy rules. The Single Input Rule Modules (SIRMs) [5] are adopted to handle these problems by using only one input variable in the antecedent part of the fuzzy rules. The SIRMs can be described as

$$\text{SIRM-}i : \left\{ R_i^j : \text{if } x_i = A_i^j \text{ then } f_i = C_i^j \right\}_{j=1}^{m_i} \quad (3)$$

where, SIRM- $i$  denotes the SIRM of the  $i$ th input variable and  $R_i^j$  is the  $j$ th in the SIRM- $i$ .  $A_i^j$  and  $C_i^j$  are the membership functions of the  $x_i$  and  $f_i$  in the  $j$ th rule of the SIRM- $i$ .

The inference result  $f_i^0$  of the consequent variable  $f_i$  of the fuzzy rules can be determined by using the simplified fuzzy reasoning method [8] defined as

$$f_i^0 = \frac{\sum_{j=1}^{m_i} A_i^j(x_i) C_i^j}{\sum_{j=1}^{m_i} A_i^j(x_i)} \quad (4)$$

Because each input variable plays a different role on system performance, the dynamic importance degree (DID) is set up for each input variable. The learning of neural network results in the DIDs. Then, the output  $f$  can be expressed as

$$f = \sum_{i=1}^n \text{DID}_i * f_i^0 \quad (5)$$

The structure of the proposed neuro-tabu-fuzzy controller is shown in Fig.2. In order to adjust the DID for each input variable, the learning of the neural network based on the backpropagation algorithm [6] is adopted. Fig.3 shows the basic structure of a three

layer feedforward neural network applied in proposed control structure. The learning of the backpropagation algorithm can be described by the following four steps.

*Step 1.* Find the initial values [-1, 1] of the weights and the thresholds by using adaptive tabu search.

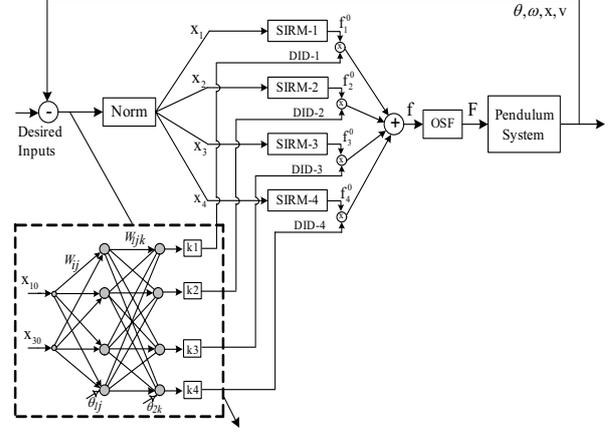


Fig. 2 Neuro-tabu-fuzzy control structure

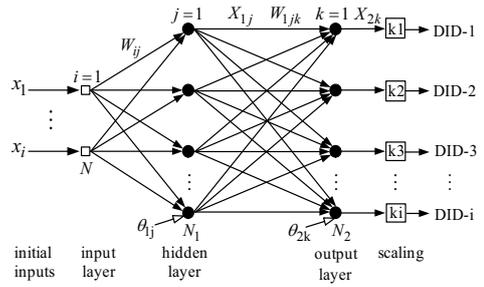


Fig. 3 Multilayer feedforward neural network

*Step 2.* Compute the output of each layer

- hidden layer;  $X_{1j}$

$$X_{1j}(t) = \frac{1}{1 + \exp(-O_{1j} - \theta_{1j})}$$

where  $O_{1j} = \sum_{i=1}^N w_{ij} X_i$  ;  $j=1,2,\dots,N_1$

- output layer;  $X_{2k}$

$$X_{2k}(t) = \frac{1}{1 + \exp(-O_{2k} - \theta_{2k})}$$

where  $O_{2k} = \sum_{j=1}^{N_1} w_{1jk} X_{1j}(t)$  ;  $k=1,2,\dots,N_2$

*Step 3.* Update the weights and the thresholds

- update the weights from the hidden to the output layer;  $w_{1jk}$

$$w_{1jk}(t + \Delta t) = w_{1jk}(t) + \Delta w_{1jk}$$

where  $\Delta w_{1jk} = \eta_1 \delta_{1k} X_{1j}(t)$

and  $\delta_{1k} = (X_{2kd}(t) - X_{2k}(t)) X_{2k}(t) (1 - X_{2k}(t))$  (6)

- update the weights from the input to the hidden layer;  $W_{ij}$

$$W_{ij}(t + \Delta t) = W_{ij}(t) + \Delta W_{ij}$$

where  $\Delta W_{ij} = \eta \delta_j X_i$

and 
$$\delta_j = \left[ \sum_{k=1}^{N_s} \delta_{1k} W_{1jk}(t + \Delta t) \right] X_{1j}(t) (1 - X_{1j}(t))$$

- update the thresholds;  $\theta_{2k}$ ,  $\theta_{1j}$

$$\theta_{2k}(t + \Delta t) = \theta_{2k}(t) + \eta_{1\theta} \delta_{1k}$$

$$\theta_{1j}(t + \Delta t) = \theta_{1j}(t) + \eta_{\theta} \delta_j$$

where  $\eta$ ,  $\eta_1$ ,  $\eta_{\theta}$  and  $\eta_{1\theta}$  are the learning rate.

*Step 4.* Go back to Step 2 and repeat the process until the selected error criterion or the max-count iteration is met.

Because the network output,  $X_{2kd}$ , is not obtainable, this problem can be overcome by taking the system output error to adjust the weights and the threshold [6]. So, eq. (6) is replaced by

$$\delta_{1k} = (y_{kd}(t) - y_k(t)) \cdot D \cdot X_{2k}(t) (1 - X_{2k}(t)) \quad (7)$$

where  $D = \text{sign} \left( \frac{\partial y_k(t)}{\partial X_{2k}(t)} \right)$

The adaptive tabu search (ATS) is the mechanism to search for the suitable initial values of the weights and the thresholds. It provides fast convergence and its procedures are detailed in [7].

#### 4. SIMULATION RESULTS

The parameters of the inverted pendulum system are  $m_p = 0.1$  kg,  $m_c = 1$  kg and  $l_p = 0.5$  m. The moving range of the cart are limited to  $[-2.4, +2.4]$  m. The variables  $\theta$ ,  $\omega$ ,  $x$ ,  $v$  of the inverted pendulum are input variables of the SIRMs. The membership functions of NB (Negative Big), ZO (Zero), and PB (Positive Big) of the antecedent part are defined in Fig. 4 and the fuzzy rules for each SIRM can be summarized in Table 1. The learning of the neural network requires the initial angle of the pendulum and the initial position of the cart to adjust the DIDs.

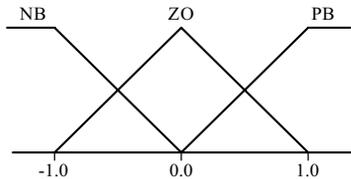


Fig. 4 Membership function for each SIRM

Antecedent variable $x_i$ ( $i = 1, 2, 3, 4$ )	Consequent variable $f_i$ ( $i = 1, 2, 3, 4$ )
NB	-1.0
ZO	0.0
PB	+1.0

Table 1. SIRM for each variable

The simulation results are used to verify the performance and effectiveness of the proposed neuro-tabu-fuzzy controller.

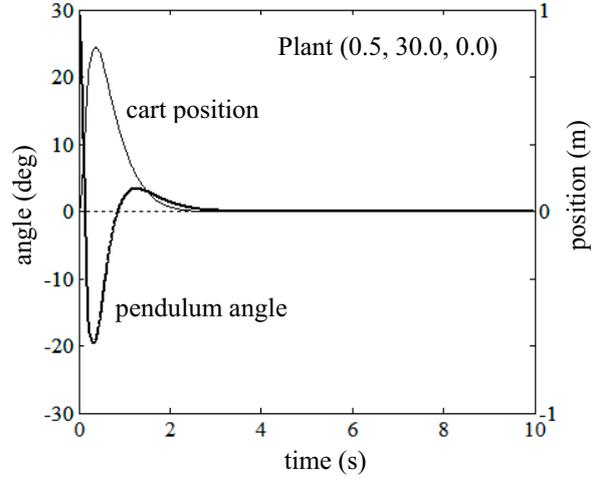


Fig. 5 Control result when  $l_p = 0.5$  m

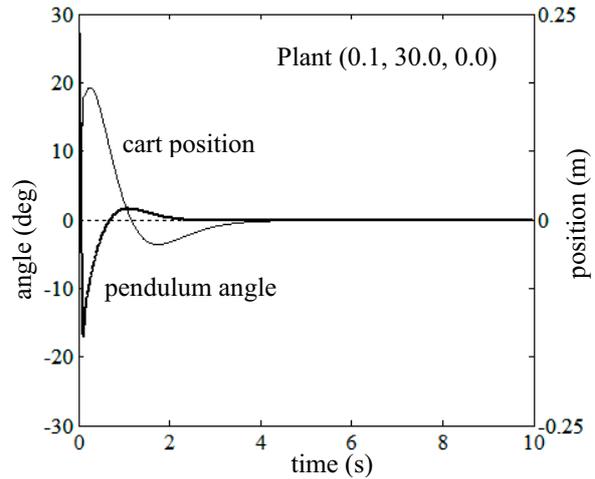


Fig. 6 Control result when  $l_p = 0.1$  m

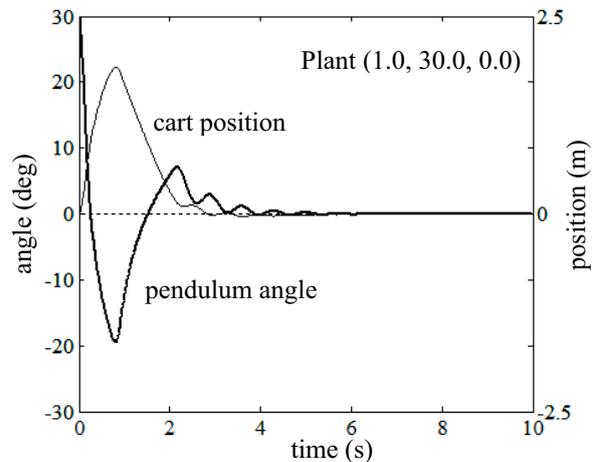


Fig. 7 Control result when  $l_p = 1.0$  m

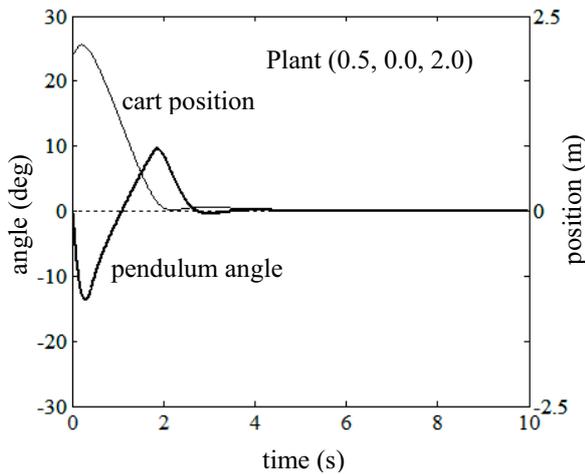


Fig. 8 Control result when the initial position of the cart is 2.0 m

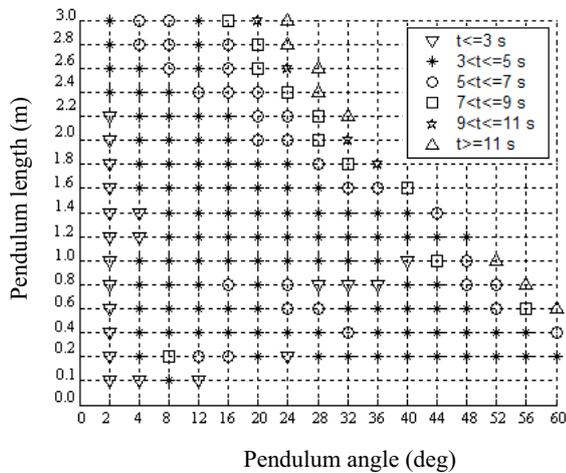


Fig. 9 Relation of the pendulum length with the initial angle of the pendulum

Fig. 5 shows the result, where the initial angle of the pendulum is 30 degrees and the other initial values are all zero. The sampling period is 0.01 s, following [5]. The numbers in Plant (0.5, 30.0, 0.0) represent the half length of the pendulum, the initial angle, and the initial position, respectively. In this case, the stabilization time is 3.48 s. To check the ability of the proposed controller, the length of the pendulum is changed. Fig. 6 shows the result when the length of the pendulum is 0.2 m, the stabilization time in this case is 3.57 s. After increasing the length of pendulum to 2.0 m, stabilization time is about 6.17 s as indicated by Fig. 7. Fig. 8 depicts the result when the initial position of the cart is 2.0 m. As a result, the inverted pendulum system can be stabilized in 5.14 s. Fig. 9 shows the relation of the pendulum length with the initial angle of the pendulum. The symbols are located to indicate the complete stabilization time. The simulation results show that the proposed controller has ability to rapidly stabilize an inverted pendulum system of wide ranges.

## 5. CONCLUSION

In this article, we propose the neuro-tabu-fuzzy control structure to stabilize an inverted pendulum system. The proposed controller follows the basic structure of the SIRMs and the DIDs that can decrease the number of the fuzzy rules. The learning of neural network can adjust the DIDs. The adaptive tabu search is adopted to find the suitable initial parameters of neural network. The simulation results indicate that the neuro-tabu-fuzzy controller can rapidly stabilize an inverted pendulum system of various dimensions.

## 6. ACKNOWLEDGMENTS

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