Stochastic Dynamic Business Game

Overview

A Stochastic Dynamic Business Game (SDBG) is a mathematical model designed to simulate the real business situation faced by business firm. Each business firm is regarded as a player and at the same time a competitor to the others in the market (or industry) where they compete in. Basically, the industry is formed and shaped dynamically and stochastically (randomly) by both uncontrollable factors (i.e., business environment) and controllable factors (firm's strategy). Furthermore, a crucial aspect of competition known as strategic behavior (i.e., strategic game) also influences firm's strategy and the overall market as well.

The SDBG works systematically by a mathematical model representing the nature that governs the industry. The nature aspects of dynamic game and stochastic dynamic are captured by the mathematical modeling approach. Once the inputs, the information of both controllable and uncontrollable factors up to a current time, are plugged in the SDBG will continuously gather new information and update its model (parameters) then compute business results of both market level and firm level. Each player's business results will be based partly on her strategy on key business functions namely, marketing, operations, and finance, and partly on current industry condition. The player's job is to achieve business objective: to maximize shareholders' wealth in the long run (a predetermined time period), by managing her business strategy properly.

Objective

To provide a learning tool for business student to assimilate and practice business strategy decision making.

The System

The mechanism of the SDBG can be explained diagrammatically as shown in figure 1. Business environment data such as economic variables: economic growth, interest rates, commodity prices, firms' stock prices, etc., and firms' strategies are the input to the model. Economic variables change continuously with some degree of uncertainty (mathematically speaking, their values follow stochastic processes) and also each player can freely adjust and input her business strategy at any time which means all players need not to move simultaneously as required by standard business simulation game. Rather, the players can either move simultaneously or alternately. This resembles a typical characteristic of business competition. The model is designed to calculate, update itself, and produce results continuously. Hence, the player can monitor her business results as closely as she needs as a part of strategic control.

Shareholders' wealth of the firm at a pre-specified time is the only key criterion in judging the game winner.

The output consists of two classes of data: information and intelligence. Market information includes firm's internal data (e.g., sales report, cost structure) and publicly accessible data (e.g., other firms' marketing budgets, product market prices) while market intelligence, not disclosed to the firm, is comprised of other firms' internal data and other key aspects of the market such as consumer behavior, market structure, retail audit. Nevertheless, market intelligence can be partially acquired through buying market research whose price varies with types of data sought. It must be noted that there is no guarantee for the accuracy of the research. For the rest of market intelligence the player needs to form her own expectations regarding, for example, other firms' strategies, government policy, several economic variables, etc.

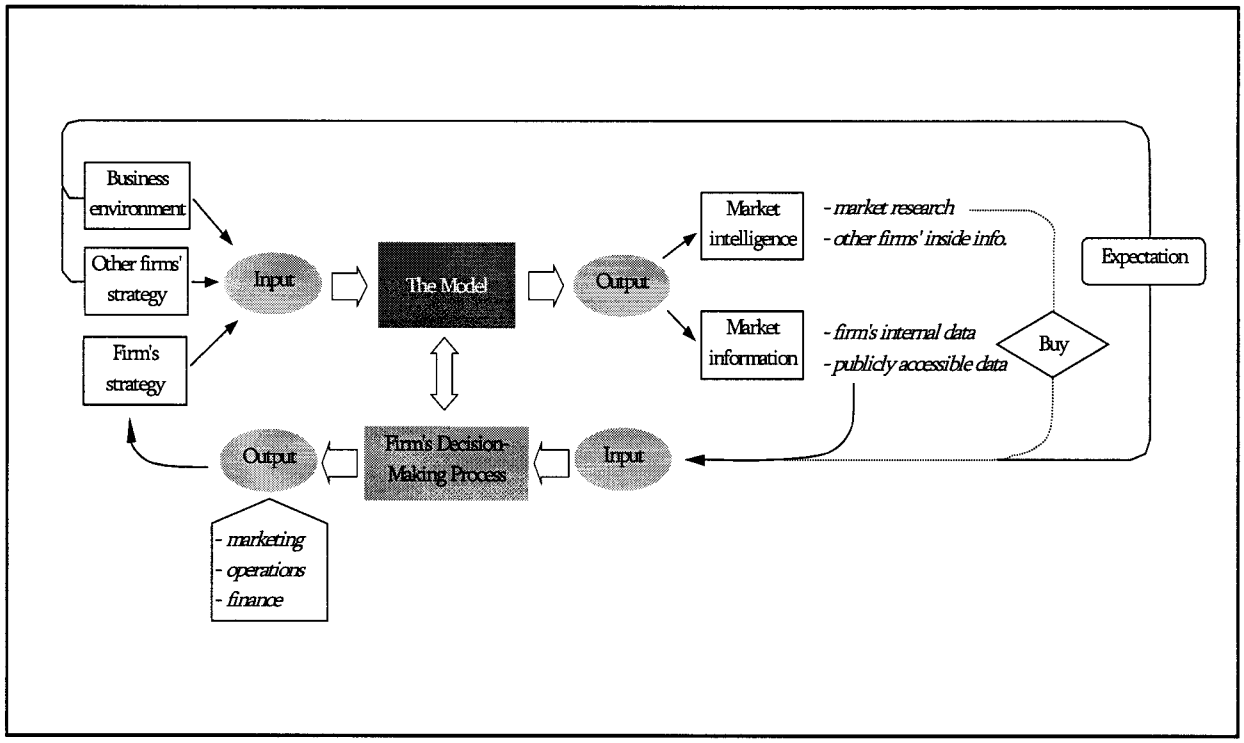

Figure 1. The System of SDBG

The Model

Mathematical model of SDBG consists of two parts: the market response model and the asset pricing model.

1) THE MARKET RESPONSE MODEL

The model produces firm's business performance in accordance with its competitiveness in the market and environmental conditions. The model translates marketing inputs (marketing mix), competitive actions (other firms' marketing mix) and environmental variables (e.g., GDP, substitutes, stochastic disturbances) into market outputs within the framework of a marketing decision model.

Demand function

1. *Market demand* ($Q_{m,t}$) is comprised of three parts as follows
   - Autonomous or core demand ($Q_{a,t}$): demand that occurs regardless of market condition.
   - Responding demand ($Q_{r,t}$): demand that is stimulated by marketing activities.
   - Demand shock ($\varepsilon_{m,t}$): demand which is resulted from uncontrollable factor. Its magnitude and occurrence are random.

All three components are assumed independent hence, they add up to be market demand (linear form). For the responding demand, to capture both the effect of individual marketing mix element and that of interaction between marketing-mix elements the demand is modeled in multiplicative and additive form as given by

$$Q_{m,t} = Q_{a,t} + Q_{r,t} + \varepsilon_t \quad [1]$$

$$Q_{a,t} = \alpha_{yt} Y_t^{e_y} \quad [2]$$

$$Q_{r,t} = \sum_{i=1}^{n} f^i(X_{i,t}) + \sum_{i=1}^{n} \sum_{j=1_{i\neq j}}^{n} f^i(X_{i,t}) f^j(X_{j,t}) \quad [3]$$

$$\varepsilon_{m,t} \overset{iid}{\sim} \mathrm{N}(\mu_m, \sigma_m) \quad [4]$$

where

Y_t = nominal GDP at time t
e_y = income elasticity of market demand
μ_m, σ_m are mean and standard deviation of demand shock
$f^i(X_{i,t})$ = response function of individual marketing-mix element
n = the number of marketing-mix elements

2. *Firm's demand* ($Q_{i,t}$) is straightforward from market demand so the system of firm's demand is similar to equation [1]-[4] with subscription i instead of m and slight modification as follows

$$q_{i,t} = q_{a,t} + q_{r,t} + \varepsilon_t \quad [1.1]$$

$$q_{a,t} = q_0 \quad [2.1]$$

$$q_{r,t} = \sum_{j=1}^{n} f^i(X_{j,t}) + \sum_{j=1}^{n} \sum_{k=1_{j\neq k}}^{n} f^i(X_{j,t}) f^j(X_{k,t}) \quad [3.1]$$

$$\varepsilon_{i,t} \overset{iid}{\sim} \mathrm{N}(\mu_i, \sigma_i) \quad [4.1]$$

Sales response function

Equation [2] consists of several response functions whose functional forms are different according to the influence of each individual marketing-mix element. However, only six elements are considered in this model including

- Average product quality (R)
- Average price (P)
- Overall distribution (D)
- Total advertising spending (A)
- Total consumer promotion spending (C)
- Total personal selling expenditure (S)

Response function of each individual element is the followings

1. *Product quality response function*

   We assume a concave functional form to reflect effectiveness and diminishing return (response). The functional form is chosen to be of Cobb-Douglas form.

$$Q_{R,t} = a_R(\alpha_{R,t} R_t)^{e_R} \quad [6]$$

where

R_t = market average product quality index at time t

α_R = marketing effectiveness of a dollar spent by company (with $\alpha = 1.00$ for average effectiveness)

e_R = elasticity of product quality, $0 < e < 1$ (constant)

a_R = a scale factor

2. *Price response function*

   The sales-price relationship is by law of demand negative sloping and by product life cycle theory its elasticity is increasing through time as competition become more intense.

$$Q_t = a_P P_t^{-b_P \sqrt{t}} \quad [7]$$

where

P_t = market average product price at time t
a_P = a constant
$-b_P\sqrt{t}$ = price elasticity

3. *Distribution response function*

Distribution effort renders diminishing return until reaches and maintains at zero marginal return at some high distribution coverage level where products are 100% distributed and the influence of distribution effort is saturated. The response function is assumed to be of (modified) exponential form.

$$Q_t = a_D(1 - e^{-bD}) + c_D \qquad [8]$$

where

D_t = total trade promotion expenses at time t
a_D, b_D, and c_D are constant
e = exponential constant

4. *Advertising response function*

The effect of advertising follows an S-shaped function to reflect a sluggish response over some low spending and diminishing return. Furthermore a key feature of carry-over effect or momentum influence of advertising is also captured by including lagged terms. The carry-over effect however declines geometrically. By incorporating previous sales into a response function, so called autoregressive process therefore, current sales is influenced to some extent by all previous advertising since Q_t depends on Q_{t-1} which in turn depends on S_{t-1} and Q_{t-1} also depends on Q_{t-2} and so on. The functional form is assumed to be in the class of the ADBUDG model.

$$Q_t = b_A + (a_A - b_A)\frac{A_t^{c_A}}{d_A + A_t^{c_A}} + \lambda Q_{t-1} \qquad [9]$$

where

A_t = industry advertising spending at time t
a_A, b_A, c_A, and d_A are constant and $c_A > 1$
λ is constant < 1

Solving equation [9] n-period backward yields

$$Q_t = \left(\frac{1}{1-\lambda}\right)b_A + (a_A - b_A)\sum_{i=0}^{n-1}\lambda^i \frac{A_{t-i}^{c_A}}{d_A + A_{t-i}^{c_A}} \qquad [10]$$

Equation [10] suggests that the n^{th}-period sales depends on all previous advertising spendings up to the present whereas recent spending has more impact on sales than older ones.

5. *Consumer promotion response function*
A key distinction between consumer promotion and advertising is the carry-over effect. Consumer promotion impact is immediate but rather short-lived (within the period the campaign is introduced). The response function is of an S-shaped form like that of advertising with no lagged effect.

$$Q_t = b_C + (a_C - b_C)\frac{C_t^{c_C}}{d_C + C_t^{c_C}} \qquad [11]$$

where

C_t = total consumer promotion spending at time t
a_C, b_C, c_C, and d_C are constant

6. *Personal selling response function*
The effect of personal selling is somewhat straightforward and displays no diminishing return as long as there is still room in the market. Hence the relationship is modeled in linear form.

$$Q_t = a_S + b_S S_t \qquad [12]$$

where

S_t = salaries, commission, and all other expenses related to sales department at time t

Replacing equation [6]-[12] into equation [3] completes the system of market demand [1]-[4].

2. THE ASSET PRICING MODEL

Financial planning as well as operations planning are subject to asset price risk. The SDBG takes into account prices of three basic assets namely interest rate, common stock, and commodity (raw material) that influence business costs. Each asset price is described by a stochastic differential equation (SDE) as the followings

1. *Interest rate*

Debt financing involves both short term (r) and long term (l) interest rates. Their processes are assumed to follow the mean-reverting *Ornstein-Uhlenbeck* processes.

$$dr_t = \kappa_r(\mu_r - r_t)dt + \sigma_r dW_t^1 \quad [13]$$

$$dl_t = \kappa_l(\mu_l - l_t)dt + \sigma_l dW_t^2 \quad [14]$$

where

r_t and l_t are short and long rate at time t, respectively

κ_r and κ_l are constant rate of adjustment coefficients of short and long rate respectively

μ_r and μ_l are short term and long term interest rate means, respectively

σ_r and σ_l are short term and long term interest rate volatilities, espectively

W^1 and W^2 are two independent Wiener processes with zero mean and variance = t

The solution of mean-reverting processes as [13] and [14] is given by

$$r_t = e^{\kappa_r t}\left[r_0 + \int e^{\kappa_s t}(\sigma_u dW_u + \kappa_u \mu_u du)\right] \quad [15]$$

The exact discretization of [15] are of the form

$$r_{t+\Delta t} = r_t + (\mu_r - r_t)(1 - e^{-\kappa_r \Delta t}) + \int_t^{t+\Delta t} \sigma_r e^{(\kappa_r u - \kappa_r (t+\Delta t))} dW_u \quad [16]$$

$$l_{t+\Delta t} = l_t + (\mu_l - l_t)(1 - e^{-\kappa_l \Delta t}) + \int_t^{t+\Delta t} \sigma_l e^{(\kappa_l u - \kappa_l (t+\Delta t))} dW_u \quad [17]$$

2. *Stock price*

Stock price (s) process is assumed to follow the *Ornstein-Uhlenbeck* process or geometric Brownian process.

$$\frac{ds_t}{s_t} = \mu_s dt + \sigma_s dW_t^3 \quad [18]$$

where

s_t is stock price at time t

μ_s and σ_s are mean and volatility of stock price

W^3 is Wiener process with zero mean and variance = t

The solution to equation [15] is given by

$$s_t = s_0 \exp[(\mu_s - \tfrac{1}{2}\sigma_s^2)t + \sigma_s W_t] \qquad [19]$$

Discretization of [16] gets and estimation as

$$s_{t+\Delta t} = s_t \exp[(\mu_s - \tfrac{1}{2}\sigma_s^2)\Delta t + \sigma_s \Delta W_{t+\Delta t}] \qquad [20]$$

3. *Commodity price*

Commodity price (m) process is assumed to follow a geometric Brownian process.

$$\frac{dm_t}{m_t} = \mu_m dt + \sigma_m dW_t^4 \qquad [21]$$

where

m_t is commodity price at time t
μ_m and σ_m are mean and volatility of commodity price
W^4 is Wiener process with zero mean and variance = t

The solution is as given earlier, i.e.

$$m_{t+\Delta t} = m_t \exp[(\mu_m - \tfrac{1}{2}\sigma_m^2)\Delta t + \sigma_m \Delta W_{t+\Delta t}] \qquad [22]$$

Input Data

Players are required to submit there strategies as input data. Business strategy decision-making involves three key areas as follows

1. Marketing plan

In practice, key marketing-mix decision-making has to be concluded in marketing budget that represents firm's marketing effort. According to the market response model constructed earlier the marketing budget each player has to input into the system, must include

1.1 *Sales forecast* both in units and value
1.2 *Marketing expenditures* of the followings
1.2.1 *Marketing promotion*
-Advertising outlay (to stimulate sales)

-Consumer promotion budget (to improve sales)

-Trade promotion budget (for product distribution purpose)

-Personal selling expenditure (salaries and commissions)

1.2.2 *R&D in marketing*

-Human Resource Development (HRD) expenses for salespersons (to improve productivity of salespersons)

-Product Development (PD) expense (to improve product quality)

-Marketing research expense (to monitor firm's performance and current market situation)

2. Operations plan

Key tasks in this function involves two areas

2.1 Production

2.1.1 *Capacity planning*: from a given initial production capacity firm has to decide in advance whether to expand or even reduce its capacity. Too much capacity causes too much cost and expenses while too little capacity results in lost sales. Expanding capacity requires additional capital investment and time while reducing capacity or divestment is subject to liquidation cost.

2.1.2 *Raw material requirement*: must be ordered in advance as well. Price of raw material is subject to commodity market price thus firm is exposed to price risk where no hedging is available. If firm orders too many raw materials (and then produces too much) then firm has to carry too much inventory cost. On the other hand, too few raw materials will cause lost sales and wasting marketing expenses.

2.1.3 *R&D in production technology*

-HRD spending on production workers can improve labor productivity so lower the production cost. This expense is included in production labor cost.

-Technology R&D spending can improve capital productivity as well as overall production productivity.

2.2 *Administration* HRD spending on supporting staff can reduce % administration expenses to sales.

3. Financial plan

Financial plan is the final expected business results and must be prepared in accordance with the other plans. Player must conclude his/her business result in the following forms

3.1 *Pro forma financial statements* (balance sheet and income statement) are the conclusion of business plan

3.2 *Financing plan:* either firm seeks additional funds from internal (retained earnings) or external sources (debt and equity) of funds firm must abide by the bank conditions. Different financial structure of the firm will result in different cost of funds. However both interest rates and stock price are subject to change therefore firm is inevitably exposed to financial risk.

Output Data

The model renders three classes of data as follows

Firm's internal data: firm's own information includes

-Sales report

-Actual financial statements

Public information: some other market information and business environment condition including

-Product quality index of all firms

-Product prices of other players

-Total advertising spending

-Current interest rates (which are used in producing actual interest expense)

-Current stock price (use in deciding whether to issue new stock)

-Current commodity price (to estimate raw material cost)

-Key events occurred in the previous period that effected the market.

Market intelligence: is provided by the system at some accuracy level. This source of data can be acquired by player at the cost of marketing research expense. The information includes

-Market structure (size, market share analysis)

-Marketing activities (other firms' marketing spendings)

-Market outlook and forecast demand

-Other firms' financial statements

-Analysis of interest rates, stock price and commodity trend

Game Onset

At the beginning of the game (t_0) each player will be assigned with the same size of initial investment and financial structure (i.e., initial financial statements) and basic information at the start-up stage of the firm as follows:

Marketing

1. Each player will be supplied by the same market studies to be used in marketing planning.
2. All new product launched will have initial product quality index of zero (from 10 quality scale).
3. Product quality can be improved by investing on *Product Development (PD).*

Operations

1. Firm's initial capacity is given according to initial investment.
2. Firm's production function is assumed to follow Cobb-Douglas function as follows

$$Q_{it} = A\alpha_{L_{it}} L_{it}^{e_{L_{it}}} \alpha_{K_{it}} K_{it}^{e_{K_{it}}}$$

where

Q_{it} is production units
L_{it} is the number of labor of firm i at time t
K_{it} is the value of operating assets (capital) of firm i at time t
α_{Lit} and α_{Kit} are labor and capital effectiveness of a Baht spent by firm i at time t
e_{Lit} and e_{Kit} are elasticities of labor and capital of firm i at time t
A is a scale factor

3. Labor productivity or effectiveness improves with *Human Resource Development (HRD) investment* but at decreasing rate.
4. Capital productivity increases at decreasing rate with *Technology Research and Development (TRD) investment.*
5. Raw materials must be ordered one period in advance.
6. If firm is to expand its capacity (additional investment) it will take two periods to complete the project.
7. Depreciation of fixed assets is 10% a year.

Finance

1. Firm's initial financial structure is 10% short-term debt and 90% equity.
2. To borrow additional funds bank has requirements of 2.5 minimum current ratio and 70% maximum debt ratio. All debts are charged with floating interest rates.

3. To raise fund by issuing new common stock there is 1% *floatation cost* of total proceeds.
4. Divestment can take effect immediately with liquidation cost 30%.
5. Accumulated loss is tax deductible.
6. Dividend payout ratio is fixed at 10% of net income.

References

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